EXACT AND ESTIMATED EXACT TESTS USING THE RANK TRANSFORM IN DESIGNED EXPERIMENTS

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CHAPTER ONE

INTRODUCTION

In experiments to determine if one or more factors have an effect on a response, the researcher typically can choose between one of two classes of analyses: parametric procedures which require that certain assumptions be made about the form of the sampled population; or nonparametric techniques which do not.

R.A. Fisher (1935) proposed a nonparametric test in which the sampling distribution of the test statistic is found by finding the value of the statistic for all possible permutations of the observed data. He considered this the most logical and efficient way to determine exact significance. Although most agreed with his assessment, the computational complexity of finding all possible permutations made this permutation test too impractical to use for all except the smallest sample sizes. In addition, the test requires a new sampling distribution be derived for each new set of observed data.

Dwass (1957) modified the permutation test by using a random sample of all possible permutations to approximate the sampling distribution, which alleviated the problem of finding all possible permutations. It did not, however, solve the problem of having to derive a new sampling distribution for each set of data, and a large number of permutations were still needed to obtain a close approximation.

Another modification to Fisher's test was to replace the data by their respective ranks. Thus, for a given sample size, only one sampling distribution need be constructed to determine significance, allowing tables of critical values to be constructed. But these tables have only been constructed for small sample sizes, and the methods have generally relied on asymptotic distributions for larger samples. More importantly, neither class has been widely applicable to complex experimental designs involving interactions, such as factorial and split-plot designs. Procedures that have been proposed are generally theoretically rigorous but difficult to use in applied situations. A method proposed by Conover and Iman (1976) using rank transformed data in standard parametric procedures appeared promising early, but has since been determined to not be suitable as a test for interactions in complex designs (as well as in other situations). A modification of the rank method, in which the observations are "aligned" before ranking, was proposed by Hodges and Lehmann (1962). This method is theoretically rigorous, but has not been widely investigated in applied situations, although some studies have suggested that it is an improvement over the traditional rank transform method, especially when testing for interaction.

This research develops an exact testing procedure for testing main effects and interaction in complex designs that is easy to use in applied situations. First, a common parametric test statistic will be computed using the ranks of the data, as well as using the aligned ranks. However, significance is determined using either the permutation distribution of the statistic (for sample sizes as large as computing power will allow), or an estimate of the permutation distribution based on a random sample of all possible permutations. Tables of critical values are derived for certain designs, and comparisons of these tests are made to the parametric F-ratio tests. These comparisons are made for different distributional assumptions and using different magnitudes of treatment effects to compare power and nominal type I error rates.

CHAPTER TWO

LITERATURE REVIEW

Nonparametric tests have long been considered as alternatives to normal theory based tests due to the fact that fewer (or no) assumptions must be made regarding the form of the sampled population in order for the test to be valid. Addressing the almost blind application of normal theory tests by researchers, R.C. Geary wrote in 1947: "Amends might be made in the interest of the new generation of students by printing in leaded type in all new textbooks: Normality is a myth; there never was, and never will be, a normal distribution. This is an overstatement from the practical point of view, but it represents a safer initial mental attitude than in fashion during the past two decades." During this time, R. A. Fisher had been developing tests based on the assumption of normality, which were and still are being widely accepted and used. Ironically, it was also Fisher who is generally credited with promoting interest in nonparametric techniques.

Fisher's idea was to determine significance of a test statistic by referring to a permutation distribution of the observations; i.e., the distribution of test statistic values for all possible permutations of the observed data. When discussing parametric tests in relation to this permutation test, he stated: "conclusions have no justification beyond the fact that they agree with those which could have been arrived at by this elementary method (the permutation test)" (Fisher, 1936).

Kempthorne (1955), on parametric tests, cautioned: "The making of assumptions of normality and applying the statistical tests is not a satisfactory basis for experimental inference, because the extent to which the reliability of an inference depends on the assumptions made in the analysis is usually unknown. Even though the application of the general linear hypotheses theory appears in many cases to lead to inferences which are essentially correct for Fisher's criterion, the validity of such normal theory inferences in the case of some designs . . . is highly questionable." It is generally agreed in statistical literature that to make an externally valid interpretation of the analysis of variance (ANOVA) the observations must be independent. The ANOVA is also very sensitive to the assumption of homogeneity of variance. Thus, it would appear that Fisher's permutation test would be preferred over any parametric test. However, obtaining the sampling distribution of the test statistic is difficult, due to the problem of calculating all possible permutations to obtain critical values, and thus the method is impractical to use, except for very small sample sizes.

Dwass (1957) proposed "the almost obvious procedure of examining a 'random sample' of permutations of the observations and making the decision to accept or reject H_0 on the basis of those permutations only." Dwass applied this method to the two sample case, and asserted that "the power of the modified test will be 'close' to that of the most powerful nonparametric test (Fisher's permutation test)." This "closeness" was quantified by a bound on the ratio of the power of the original procedure to the modified one.

More recently, Edgington (1995) and Manly (1991) have promoted these "randomization tests" applied to randomized block designs and completely randomized factorial experiments, among others, and Manly provides workable programs for obtaining critical values. However, there seems to have been no attempt to apply this technique to more complex designs. One drawback of this method (and also of the permutation test) is that a new sampling distribution must be derived for each new set of data to which the test is applied. This makes the procedure unattractive to many practitioners, since programming expertise is required to implement the tests. In addition, the results of the randomization test may vary depending on which permutations are sampled.

Still and White (1981) proposed a test for interaction in which the main effects are subtracted and a randomization test applied to the residuals. This test suffers from the same drawbacks as the ordinary randomization test and is difficult to apply. Bradley (1979) proposed a test for interactions in which the data are entered into a matrix, then collapsed and reduced over the main effects, similar to Still and White's method. Then a nonparametric test, such as the Kruskal-Wallis test, is performed on the residuals. This procedure is restricted to the balanced case, and the value of the test statistic is dependent upon how the data are entered into the matrix. Bhapkar and Gore (1974) proposed a similar test. None of these methods, however, have been thoroughly investigated to determine how well they perform, and neither have they gained any degree of popularity in applied situations.

The problem of needing to derive a new sampling distribution for each new set of observed data can be eliminated by transforming the data into their respective ranks before doing the analysis. Although the idea of nonparametric tests pre-dates Fisher's proposals (As far back as 1710, John Arbuthnott used the Sign Test in an attempt to prove the wisdom of divine providence (Bradley, 1968)), most of the work in this area began after 1935. Some of the more famous tests for two sample situations were proposed by Fisher (1935), Wilcoxon (1945) and Mann and Whitney (1947). Kruskal and Wallis (1952) developed a test for the multi-sample case, and Pitman (1938), Friedman (1937) and Quade (1972,1979) devised tests for randomized block designs. For many of these tests, tables of exact critical values of the test statistics are available, but only for very small sample sizes. For larger samples, the tests are based on known theoretical distributions, using the asymptotic properties of the test statistics.

However, methods for more complex designs involving interactions were not as forthcoming. Bradley (1968) noted that distribution-free tests for high-order interaction "tend to be complicated, awkward, and limited in application. Furthermore, many of them are inexact, their derivations being based upon the limiting case of infinite sample sizes and involving 'asymptotic' formulas for the test statistic. Thus, they lack many of the virtues possessed by distribution-free tests for 'main-effects' or first order interactions." As recently as 1990, Sawilowsky stated that historically, there have been no satisfactory nonparametric tests for interaction in the analysis of variance.

Hodges and Lehmann (1962), Puri and Sen (1969), Koch (1969) and Hettmansperger (1984) (among others) discussed tests for interactions in complex designs based on a ranking after alignment procedure. This procedure involves "aligning" each observation by subtracting from it an estimate of location of each main effect, and then ranking these "aligned" observations. A nonparametric procedure is then performed on the "aligned" ranks. These tests are mathematically rigorous, and the asymptotic properties of the statistics have been investigated. Since the ranking after alignment procedure produces transformed variables which are usually dependent, most of these tests are only conditionally distribution-free, since certain regularity conditions have to be assumed in order for the test statistic to be distribution-free. Puri and Sen (1985) developed a test based on a large sample approximation which does not rely on the "aligned" ranks. None of these techniques seem to have been widely used, and there are no known software packages which have adopted them. Thus, they are generally not easy to implement for practitioners. In addition, little is known about the small sample behavior of these tests. Harwell and Serlin (1989) did investigate the test of Puri and Sen (1985) and found the test to lack power for small sample sizes ($n < 40$). Conover and Iman (1976) compared the common parametric F-test to both the aligned rank procedure and the traditional rank transform procedure for a model with lognormally distributed errors, and found that the rank tests tended to be more powerful than their parametric counterparts for testing both main effects and interaction. Fawcett and Salter (1984) and Groggel (1987) found the aligned rank technique to be a viable competitor to the F-test for testing treatment effects in a randomized block design, especially when the classical assumptions are violated.

Higgins and Tashtoush (1994) investigated the aligned rank technique for testing interaction in a two-way factorial in completely randomized and split-plot designs and found it to be an improvement over the traditional rank transform technique. However, they used the traditional rank transform technique applied to the aligned data, and thus used the F-distribution as the sampling distribution for the test statistics, and not the exact sampling distribution. In addition, they did not examine the aligned rank technique for testing for main effects.

A slight modification to the rank transform was proposed by Fisher and Yates (1949) as well as Bell and Doksum (1965). They suggested a random normal scores transform, where the observed data are replaced by randomly drawn normal random variates. Hoeffding (1952) and Terry (1952) suggested yet another modification: using expected normal scores. Although these tests were shown in some cases to be more robust and more powerful than using the ordinary rank transform, they did not compare favorably to the parametric ANOVA, especially for small samples, and thus were never serious competitors to the ANOVA.

Scheirer, et al. (1976) proposed a modified extension of the Kruskal-Wallis test for analysis of ranked data arising from completely randomized factorial designs. They showed that the well known Kruskal-Wallis H-statistic was equivalent to the ratio of the sum of squares for treatment divided by the "mean square" for the total variability, where both quantities are computed using the ranks of the data. Their extension to the KW test was based upon this statistic. Toothaker and Chang (1980) studied the Scheirer et al. method, however, and concluded, based on Monte Carlo studies, that "under no circumstances could the tests be recommended for use," due to lack of power and inability to control nominal type I error rates. They suggested that researchers consider aligned rank methods instead.

A twist on the rank transform idea that did gain widespread acceptance was proposed by Conover and Iman (1976). Their idea was to transform the data to their respective ranks, and then run the usual parametric analysis, where the theoretical distribution of the test statistic, based on the parametric assumptions, is used to obtain critical values. This method, which became known simply as the "rank transform method", held much appeal to practitioners since this allowed a nonparametric analysis to be performed for any type of experimental design, and almost all statistical computer packages could run such an analysis. Hora and Conover (1984) showed that the limiting null distribution of the usual F-statistic for main effects in the two-way layout has the same limiting distribution when applied to ranks as when applied to normal data. Iman (1974) showed that the rank transform had greater power than the F-test for certain nonnormal distributions. Other studies also supported the procedure for different situations: Hora and Iman (1988), Iman, et al. (1984), Kepner and Robinson (1988) and Thompson and Ammann (1989). The procedure was hailed as a "bridge between parametric and nonparametric statistics" (Conover and Iman, 1981). Even SAS, in its discussion of nonparametric analysis of variance procedures, stated: "The NPAR1WAY procedure is available to perform a nonparametric one-way analysis of variance. *Other nonparametric tests can be performed by taking ranks of the data and using a regular parametric procedure to perform the analysis.*" (SAS User's Guide: Statistics, 1985; SAS/STAT User's Guide, 1990). The honeymoon soon came to an end, however, beginning with Fligner (1981) who cautioned that until each new application of the rank transform was investigated it should not be used. Blair and Higgins (1985) found that a loss of power occurred in related samples tests if samples were correlated. Blair et al. (1987) found that nominal type I error rates for testing interaction became seriously inflated for certain models. Thompson and Ammann (1990) found that the test for interaction broke down in the presence of main effects. Subsequent studies have shown that the rank transform is neither a robust nor powerful alternative to the factorial ANOVA, especially as a test for interaction when both main effects are present. Sawilowsky (1990), discussing tests of interaction, stated that the rank transform should not be used, based on poor Monte Carlo results.

CHAPTER THREE

FINDING EXACT DISTRIBUTIONS

3.1 All Possible Permutations

As was mentioned in Chapter One, the goal of this research is to develop an exact test using ranks that is easy to apply to for testing main effects and interaction in multi-factor experiments. To make the method easy to apply, for any given test, the usual F-ratio calculated in a parametric ANOVA computed on the ranks of the data is used as a test statistic. Then the permutation distribution of the statistic is found, and tables of critical values are constructed to use to determine significance. Much work has been invested in an attempt to use modern computing power to obtain exact critical values by finding the value of the statistic for all possible permutations of the data. Program 1 in the Appendix was used to derive tables of exact critical values for tests for main effects and interaction in a two factor experiment with two observations per treatment combination (See tables 3.1-3.2). This procedure for obtaining the exact distributions became impractical for larger sample sizes, due to a prohibitive amount of computer time. For example, the program to derive the exact sampling distribution for a design with twelve observations was eventually terminated after four days of execution without completing its task.

Table 3.1.

Design: two factors, each with two levels, two observations per treatment combination. Exact upper tail permutation distribution for test of main effect: F = MSTRT/MSE, calculated on the ranks of the data.

F_{calc}	$P(F \leq F_{calc})$
3.78947353	0.898412645
4.54545403	0.904761851
4.79999924	0.911111057
5.14285660	0.923809469
5.76470566	0.926984072
6.53333282	0.933333278
6.54545403	0.939682484
7.53846073	0.942857087
8.90909004	0.949206293
10.0000000	0.952380896
10.8888884	0.958730102
12.0000000	0.965079308
12.7999992	0.968253911
14.2222214	0.974603117
16.0000000	0.980952322
19.5999908	0.984126925
21.3333282	0.990476131
25.5999908	0.996825337
64.0000000	1.00000000

Table 3.2

Design: two factors, each with two levels, two observations per treatment combination. Exact upper tail permutation distribution for test of interaction: F = MSAB/MSE, calculated on the ranks of the data.

F_{calc}	$P(F\leq F_{calc})$
3.78947353	0.898412645
4.54545403	0.904761851
4.79999924	0.911111057
5.14285660	0.923809469
5.76470566	0.926984072
6.53333282	0.933333278
6.54545403	0.939682484
7.53846073	0.942857087
8.90909004	0.949206293
10.0000000	0.952380896
10.8888884	0.958730102
12.0000000	0.965079308
12.7999992	0.968253911
14.2222214	0.974603117
16.0000000	0.980952322
19.5999908	0.984126925
21.3333282	0.990476131
25.5999908	0.996825337
64.0000000	1.00000000

3.2 An Alternative to Finding All Possible Permutations

Alternatives to having to find all possible permutations in order to obtain the exact distribution of a test statistic have also been sought. One such alternative which appeared promising was proposed by Pagano and Tritchler (1981). They suggested a two-step method of finding the exact distribution of a linear rank statistic by first finding the characteristic function of the statistic, and then inverting it to obtain the distribution. Suppose we have two samples $x_1, \ldots, x_m$ and $y_1, \ldots, y_n$ ($m \leq n$) that, when combined, may be written $z_1, \ldots, z_N$ ($N = m+n$), and when ranked, yield the ranks $R_1, \ldots, R_m$ and $R_{m+1}, \ldots, R_{m+n}$. Consider the class of statistics S that may be written

$$S = \sum_{j=1}^{N} a(R_j) I_j$$

for some function $a(\bullet)$, where I_j is one for $j \leq m$ and zero otherwise. To find the characteristic function, ϕ, first define

$$\psi(m,N,\theta) = {}_N C_m \, \phi(\theta)\,,$$

where ${}_N C_m$ is the number of different samples of size m that can be selected from N elements, and then define

$$\psi(m, N, \theta) = \sum_{j} \prod_{k=1}^{m} \exp(i\theta a(R_{jk})) \qquad (1)$$

where the summation is over all ${}_NC_m$ samples of size m, ($j_1, \ldots, j_m$), from the first N natural numbers, and R_{jk} denotes the rank of the value in the k^{th} position of the j^{th} combination. Using the above equations would still require obtaining all ${}_NC_m$ combinations of the ranks, which would not be worth the added complexity of involving the characteristic function. However, using the following theorem, enumerating all possible combinations of the ranks is not necessary, and the value of the characteristic function can be obtained in approximately 2mN (complex) multiplications and additions.

Theorem 3.1 (Pagano and Tritchler, 1981). Define $\psi(j,k,\theta) = 0$ for $j > k$ and $=1$ for $j = k = 0$. Then

$$\psi(j,k,\theta) = \exp(i\theta\ a(R_k))\ \psi(j-1,k-1,\theta) + \psi(j,k-1,\theta), \quad \text{for } 1 \le j \le k = 1,2,\ldots, \qquad (2)$$

where R_k is the rank of the value in the k^{th} position.

Proof: Consider all the samples of size j formed from the first k observations. These can be split into two groups, those that contain the kth observation and those that do not. The ones that do contain the kth observation can be obtained by adjoining the kth observation to each sample of size $(j-1)$ from the first $(k-1)$ observations (the first term in (2)). The ones that do not contain the kth observation are the samples of size j obtained from the first $(k-1)$ observations (the second term in (2)). □

To see the advantage of the recursive relation, consider the case where N=5 and m=2. Using (1), with a(R) = R,

$$\psi(2,5,\theta) = \sum_{j=1}^{{}_5C_2} \prod_{k=1}^{2} \exp(i\theta R_{jk})$$

$$= \exp[i\theta(R_{11}+R_{12})] +$$

$$\exp[i\theta(R_{21}+R_{22})] +$$

$$\bullet$$

$$\bullet$$

$$\bullet$$

$$\exp[i\theta(R_{10,1}+R_{10,2})]$$

which requires obtaining all 10 combinations of the ranks. However, using (2), it is not necessary to enumerate all possible combinations, and the problem reduces to one of just taking a series of complex exponentials of the ranks 1 through 5:

$$\psi(2,5,\theta) = \exp[i\theta(R_5+R_1)] + \exp[i\theta(R_4+R_1)] + \exp[i\theta(R_3+R_1)] +$$
$$\exp[i\theta(R_2+R_1)]$$

In general, using the recursive expression (2), $\psi(m,N,\theta)$ will be the sum of ${}_{N-1}C_{m-1}$ (the number of samples which contain the N^{th} rank) exponential expressions. If m is chosen to be the smaller sample, then the maximum number of exponential expressions needed will be one half of the ${}_NC_m$ total possible samples, and this will occur when sample sizes are equal. When sample sizes are different, the number of terms in the expression can be reduced greatly. For example, if N = 20, when sample sizes are equal, m = 10 and using the recursive formula will result in an expression for $\psi(10,20,\theta)$ with ${}_{19}C_9 = 92{,}378$ exponential terms instead of ${}_{20}C_{10} = 184{,}756$ needed for complete enumeration of all combinations. If instead m = 8, now only ${}_{19}C_7 = 50{,}388$ exponential terms are required to find the exact distribution, which is only 40% of the ${}_{20}C_8 = 125{,}970$ total combinations.

Then, let X be a discrete random variable with distribution $P(X = j) = p_j$, $j = 0, 1, \ldots, U$, where U is the maximum value of X, and characteristic function

$$\phi(\theta) = \sum_{j=0}^{U} p_j \exp(ij\theta), \quad \theta \in [0, 2\pi). \qquad (3)$$

Since X is defined on a finite integer lattice, we may use the following basic theorem found in most sources on Fourier series to find the p_j :

Theorem 3.2. For any integer Q > U and j = 0, . . ., U,

$$p_j = \frac{1}{Q}\sum_{k=0}^{Q-1}\phi\left(\frac{2\pi k}{Q}\right)\exp\left(-\left(\frac{2\pi jk}{Q}\right)\right)$$

That is, knowing the characteristic function at these Q equispaced points on $[0, 2\pi)$ is equivalent to knowing it everywhere. And, if it is known at these points, one may use a fast Fourier transform (FFT) to invert it and obtain the p_j. Thus, equation (2) must be evaluated at Q equispaced points on $[0, 2\pi)$, and this set of values represents Q values of the Fourier series given by (3). By theorem 2, the probabilities p_j can be obtained, as well as the exact distribution of S, using the Fourier transform. A FORTRAN program, which used IMSL™ subroutines for performing the FFT, was written to test the method for the two sample case, and the method did indeed determine the exact distribution of S easily, using very little computer time.

An extension to the multi-sample problem was also proposed. For the three sample case, the following recursive relation holds:

Lemma 3.1 (Pagano & Tritchler, 1981): For $j, k = 1, 2, \ldots$ such that $j + k \leq l = 1, 2, \ldots,$

$$\begin{aligned}\psi(j, k, l, \theta_1, \theta_2) = {} & \exp(i\theta_1 u_l)\, \psi(j\text{-}1, k, l\text{-}1, \theta_1, \theta_2) + \\ & \exp(i\theta_2 u_l)\, \psi(j, k\text{-}1, l\text{-}1, \theta_1, \theta_2) + \\ & \psi(j, k, l\text{-}1, \theta_1, \theta_2)\end{aligned}$$

However, the characteristic function now becomes a function of two parameters, and the characteristic function must now be evaluated at $U_1 \bullet U_2$ pairs (θ_1, θ_2), where U_1 and U_2 are the maximum values taken by S_1 and S_2, the sums of the ranks of the first two samples, respectively. Thus the computational complexity of calculating and inverting the characteristic function increases exponentially in the number of samples. Even for the three sample case, the additional complexity renders this method impractical to use. In addition, the method is restricted to test statistics which are linear functions of sums of the ranks, so that common F-ratio test statistics used for analysis of factorial experiments could not be used. It was for these reasons that this method was eventually abandoned as a means of determining exact distributions of test statistics for analyzing factorial designs.

3.3 Estimating Exact Distributions

Thus, for more complex designs, and for situations with larger sample sizes, the exact distribution of the test statistic will be estimated based on a random sample of all possible permutations of the data. This method was first proposed by Dwass (1957), and tests based on this method of determining significance have become known as "Randomization Tests" (Manly, 1991 ; Edgington, 1995). This technique, when used on the actual observations, has the somewhat undesirable property that a possibly unique sampling distribution must be constructed for each set of data. In addition, two researchers performing a randomization test independently on the same set of data would likely obtain slightly different p-values. For a large (at least 10,000) random sample of permutations, however, it is unlikely that two independent tests would arrive at different conclusions regarding significance. For example, for estimating the cumulative probability associated with the 95^{th} percentile of a sampling distribution based on a random sample of 10,000 permutations, the expected error of estimation, with 99% confidence, would be about .0056, or .56%.

Applied to rank transformed data, however, a unique sampling distribution would need to be derived only for each possible sample size. Thus, it is possible to create tables of estimated critical values, given a particular sample size. Programs were written to generate such tables, Tables 3.3 and 3.4 present the values which are used in the simulations of Chapters Five and Six.

Table 3.3.

Estimated percentiles of the sampling distributions of F-ratios computed using the ranks of the data. All ratios are computed as MS (effect)/MS (error), for a two-way layout with four levels of factor A and three levels of factor B, in a completely randomized design, where n is the number of observations per treatment combination, and are based on a random sample of 20,000 permutations.

n	Effect	Percentile point		
		.90	.95	.99
2	A	2.669	3.560	6.000
	B	2.820	3.914	7.098
	AB	2.356	3.056	4.814
5	A	2.175	2.816	4.320
	B	2.396	3.207	5.296
	AB	1.920	2.322	3.282
10	A	2.118	2.680	4.003
	B	2.345	3.125	5.088
	AB	1.822	2.183	2.986
20	A	2.136	2.644	3.902
	B	2.325	3.038	4.785
	AB	1.802	2.146	2.866

Table 3.4

Estimated percentiles of the sampling distributions of F-ratios computed using the ranks of the data. All ratios are computed as MS (effect)/MS (error), for a two-way factorial in a split-plot experiment with four levels of the main unit treatment in a randomized block design with three blocks and three levels of the sub-unit treatment, and are based on a random sample of 20,000 permutations.

Effect	Percentile point		
	.90	.95	.99
MU Trt	3.363	4.830	10.200
SU Trt	2.712	3.666	6.569
Interaction	2.218	2.792	4.352

CHAPTER FOUR

APPLICATIONS TO COMPLETELY RANDOMIZED TWO-FACTOR FACTORIAL EXPERIMENTS

4.1 Problems with the Rank Transform in Factorial Experiments

Conover and Iman (1976, 1981) suggested that most parametric procedures may be performed using rank transformed data instead of the raw data, especially when the parametric assumptions may be violated. Although this technique works well in some situations, it has been widely publicized in recent years that many situations exist where this procedure does not perform well. The most notable of these involves the test for interaction in two factor experiments. Several studies have found that the rank transform test can be affected by nuisance parameters, or effects present which are not being tested. Blair, et al. (1987) suggested that the rank transformation can, in some situations, introduce interaction effects in the ranked data that are not present in the original data. This is due to the fact that the expected value of the rank of any particular cell depends nonlinearly on the means of all other cells. Addressing this, Blair et al. (1987) stated the following:

Theorem 4.1: Let X_i be an observation from population i and Y_j an observation from population j, j=1,2, . . ., k. Then the expected rank of X_i is given by

$$E[R(X_i)] = \frac{n+1}{2} + \sum_{j \neq i} nP(Y_j < X_i),$$

and thus, if the k populations have normal distributions with means $\mu_1, \mu_2, \ldots, \mu_k$, respectively, and standard deviation σ, then

$$E[R(X_i)] = \frac{n+1}{2} + \sum_{j \neq i} n\Phi\left(\frac{\mu_i - \mu_j}{\sqrt{2\sigma^2}}\right).$$

Since Blair, et al. (1987) did not include a proof of this result, one is provided here.

Proof: First assume that n observations have been selected at random from each of k continuous populations. Then if X_i is an observation from population i and Y_j is an observation from population j, j = 1, 2, . . ., k, the rank of X_i can be expressed as

$$R(X_i) = 1 + \text{the number of observations in population i less than } X_i$$
$$+ \sum_{j \neq i} \text{the number of observations in population j less than } X_i.$$

Let Z equal the number of observations in population i less than X_i. Since the observations within each sample have been randomly selected, each possible permutation of n ranks is equally likely to occur. So, Z is a random variable with $P(Z=i) = \frac{1}{n}$, $i = 0, 1, \ldots, n-1$. Thus,

$$E(Z) = \frac{1}{n}\sum_{i=1}^{n-1} i = \frac{1}{n}\left[\frac{n(n-1)}{2}\right] = \frac{n-1}{2}.$$

Next, if W_j equals the number of observations in population j less than X_i, $i \neq j$, then W_j is a binomial random variable with mean $nP(Y_j < X_i) = E(W_j)$. Therefore,

$$E[R(X_i)] = 1 + E(Z_i) + \sum_{j \neq i} E(W_j)$$

$$= 1 + \frac{n-1}{2} + \sum_{j \neq i} nP(Y_j < X_i)$$

$$= \frac{n+1}{2} + \sum_{j \neq i} nP(Y_j < X_i) .$$

Further, if populations are normally distributed with means $\mu_1, \mu_2, \ldots, \mu_k$, respectively, and common variance σ^2, then

$$P(Y_j < X_i) = P(Y_j - X_i < 0)$$

$$= P\left[\frac{(Y_j - X_i) - (\mu_j - \mu_i)}{\sqrt{2\sigma^2}} < \frac{0 - (\mu_j - \mu_i)}{\sqrt{2\sigma^2}}\right]$$

$$= P\left[Z < \frac{\mu_i - \mu_j}{\sqrt{2\sigma^2}}\right]$$

$$= \Phi\left[\frac{\mu_i - \mu_j}{\sqrt{2\sigma^2}}\right],$$

and thus,

$$E[R(X_i)] = \frac{n+1}{2} + \sum_{j \neq i} n\Phi\left(\frac{\mu_i - \mu_j}{\sqrt{2\sigma^2}}\right). \quad \square$$

Clearly the expected rank of any observation depends (non-linearly) on the means of all other populations. For a two-way layout with "a" fixed levels of factor A and "b" fixed levels of factor B, the population means can be expressed as $\mu_{11}, \mu_{12}, \ldots, \mu_{1b}, \mu_{21}, \mu_{22}, \ldots, \mu_{2b}, \ldots, \mu_{ab}$. Then $\mu_{ij} = A_i + B_j + (AB)_{ij}$. It is not surprising that increasing the magnitude of effects A and/or B would have an effect on the expected rank of an observation. Even when no interaction is modeled, nominal type-I error rates for testing interaction can become quite inflated if the magnitudes of effects are large (or if samples sizes are large). This can result in a test which in certain cases can be expected to detect interaction in rank transformed data where none existed in the original data. Figure 4.1, based on simulation results in Chapter Five, illustrates an example of this behavior It was found that this problem was most serious when both main effects were present in the model.

The rank transform method has also been shown to have a serious power disparity compared to the F-test when testing for interaction in the presence of both main effects and interaction, although the disparity is much less evident whenever the assumptions of normality and equality of variances are violated. Figure 4.2 shows an example, using simulation results from Chapter Five.

Figure 4.1 (Note: Effect magnitude is in standard deviation units).

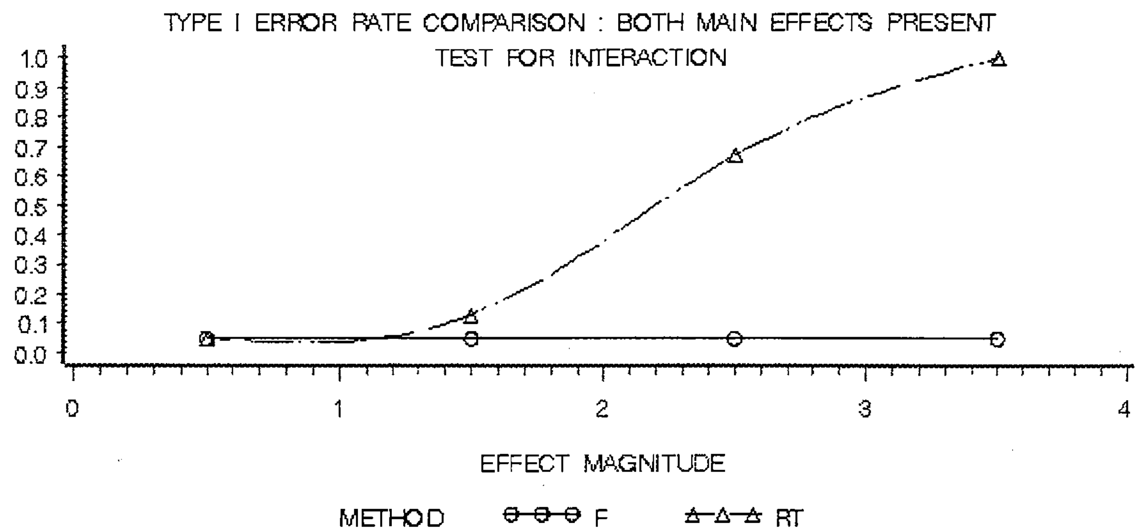
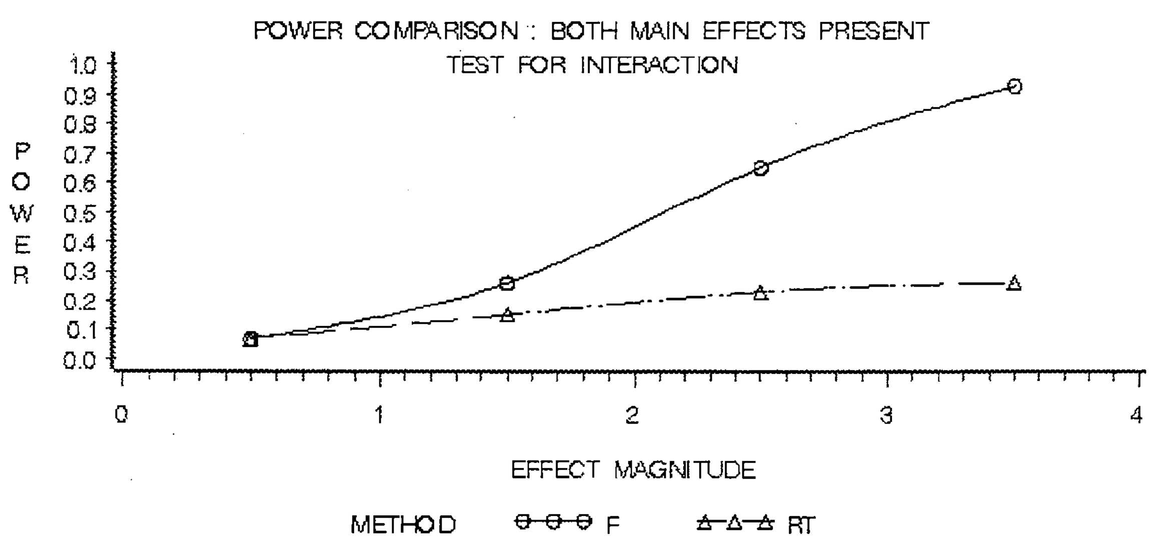


Figure 4.2 (Note: Effect magnitude is in standard deviation units).



4.2 Ranking After Alignment

The idea of somehow isolating the effect that is to be tested seems to have been first proposed by Hodges and Lehmann (1962). Observations are aligned by subtracting estimates of the unwanted effects from each observation. The remaining residual is expected to contain (on average) only the effect of interest, and thus no other "nuisance" effects would be expected (on average) to influence the outcome of the test. This can easily be demonstrated for testing the effect of interaction by an argument similar to the previous section. Once again, consider a two-way layout with "a" fixed levels of factor A and "b" fixed levels of factor B, where the mean of each population is given by $\mu_{ij} = A_i + B_j + (AB)_{ij}$, $i=1,2,\ldots,a$, $j=1,2,\ldots,b$.

Corollary 4.1: Let $(AX)_{ij} = X_{ij} - \hat{A}_i - \hat{B}_j$, $(AY)_{kl} = Y_{kl} - \hat{A}_k - \hat{B}_l$ be aligned observations, where $\hat{A}$ and $\hat{B}$ are unbiased estimators of A and B, respectively. Then if all populations are normally distributed, the expected rank of an aligned observation is independent of effects A and B.

Proof: If we wish to test for the effect of interaction (AB), each observation is "aligned" by subtracting estimates of factors A and B. Since $E(X_{ij}) = \mu_{ij} = A_i + B_j + (AB)_{ij}$, it follows that

$$E[(AX)_{ij}] = E(X_{ij}) - A_i - B_j = (AB)_{ij}, \text{ and } E[(AY)_{kl}] = E(Y_{kl}) - A_k - B_l = (AB)_{kl}$$

Also, if X and Y are normally distributed,

$$(AX)_{ij} \sim N[(AB)_{ij},\ \sigma_A^2) \text{ and } (AY)_{kl} \sim N[(AB)_{kl},\ \sigma_A^2),$$

where $\sigma_A^2 = \mathrm{Var}(X_{ij} - \hat{A}_i - \hat{B}_j)$ for all i, j. This implies that

$$P[(AY)_{kl} < (AX)_{ij}] = \Phi\left(\frac{(AB)_{ij} - (AB)_{kl}}{\sqrt{2\sigma_A^2}}\right).$$

This shows that the expected rank of an aligned observation depends only on the effect of interaction for each cell. Further, if $(AB)_{ij} = 0$ for all i, j, then

$$P[(AY)_{kl} < (AX)_{ij}] = \Phi(0) = \frac{1}{2} \text{ for all i, j,}$$

and then

$$E[R((AX)_{ij})] = \frac{n+1}{2} + \sum_{j \neq i} \frac{1}{2} n = \frac{1}{2}(1 + nab) .$$

So, if the original data contains no interaction, neither will the ranks of the aligned observations. □

This procedure has been found to perform favorably compared to the F-test in some limited applications, both for testing for interaction and for testing for main effects when interaction is not present. It has been noted by some that a shortcoming of this method is the inability to remove an interaction effect in order to test for main effects, but it is doubtful this scenario would be considered in practice. For example, in analyzing data in a two-way layout, the test for interaction would be performed first. If significant interaction was detected, there would be little use in testing for main effects. On the other hand, if the effect of interaction was determined to be not significant, it is likely that the interaction effect would not interfere with the tests for main effects. In this case, there would be no need to "remove" the interaction effect. However, the fact that the procedure allows main effects to be removed makes it an excellent candidate to be an improvement over the rank transform procedure.

CHAPTER FIVE

SIMULATION STUDY FOR A COMPLETELY RANDOMIZED TWO-FACTOR FACTORIAL EXPERIMENT

5.1 Simulation Procedure

Simulated data sets were generated to examine the performance of the three methods: the parametric F-test procedure (FT), the estimated exact rank transform test procedure (RT), and the estimated exact aligned rank transform test procedure (ART). For both rank tests, the estimated exact sampling distribution of the test statistics was used to obtain critical values. The following model was used to generate the observations:

$$Y_{ijk} = \mu + A_i + B_j + (AB)_{ij} + e_{ijk},$$

where A_i is the effect of the i^{th} level of treatment A, B_j is the effect of the j^{th} level of treatment B, $(AB)_{ij}$ is the effect of the interaction between the i^{th} level of factor A and the j^{th} level of factor B, and e_{ijk} is the random error effect, and where i=1,2 3,4, j=1,2,3, and k=1,2, . . .,n. Standard normal (both with homogeneous and heterogeneous variances), uniform [-3,3], and exponential (μ=3) distributions were used to model the error distributions. In addition, different degrees of heterogeneity were considered. It was desired to observe both "moderately large" and "very large" degrees of heterogeneity. To get some idea of these degrees, Hartley's F-max test was used to determine the approximate ratio between largest and smallest variances that would be considered moderately large and very large. Thus, for all models, ratios between the largest and the smallest variances of 10:1 (moderately large) and 30:1 (very large) were studied (in addition, some models with very, very large degrees of heterogeneity were observed). Effect sizes (denoted by "c" in the tabled results) are in standard deviation units, and range in magnitude from 0.5 (very small) to 3.5 (very large). Effects were chosen so that many different modelings of main effects and interaction could be investigated. The model containing only both main effects and the model containing all effects were the same as those for which Blair, et al. (1987) found that the rank transform procedure performed poorly. The values a_i, b_j, and ab_{ij} referred to in the tables that follow represent the values assigned to A_i, B_j, and $(AB)_{ij}$, respectively, for each model. All effects not referred to were set to zero. Critical values for both rank tests were estimated by calculating the value of the test statistic for a random sample of twenty thousand permutations of the ranks of the data. Ten thousand samples were generated, and the proportion of test statistic values greater than or equal to the critical values for the respective sampling distributions was calculated. For the simulations in this chapter, as well as those in chapter six, when estimating a nominal type I error rate of 0.05, the simulated values can be expected to be within 0.0056 of the true proportion, with 99% confidence (in the following tables, nominal levels in bold indicate values which are significantly different from 0.05). For power estimation, the simulated values have a *maximum* error of estimation of 0.014, with 99% confidence. All simulations were programmed in

FORTRAN using Microsoft® Fortran PowerStation (*Professional Edition)* 4.0™ for Windows 95™, using IMSL™ MATH/LIBRARY® and STAT/LIBRARY® subroutines.

5.2 Simulation Results

5.2.1 Normal errors, equal variances (see Tables 5.1-5.7).

The ART consistently showed power almost equal to that of the F-test. The RT tended to compare favorably in most cases, but showed poor power when both main effects and interaction were present in the model, especially for testing interaction (see Table 5.3). In addition, for all models the RT had nominal type I error rates that inflated as the magnitude of the effects increased. This occurred not only for tests for interaction in the presence of only both main effects, as reported by Blair, et al. (1987), but also for the test for the main effect not modeled when only one main effect was present. As can be seen in Table 5.2, these error rates approached 1.0 for the test for interaction for large sample sizes. The ART often had slightly inflated nominal type I error rates, but the inflation was never severe (usually only .01-.02 above the nominal level), and did not appear to be affected by the magnitude of the modeled effects.

Table 5.1.

Proportion of rejections at α=0.05, normally distributed errors with equal variance, based on 10,000 samples. A main effect present (a_1=c, a_3=-c).

n = 2

Test for:	Method	c 0.5	1.5	2.5	3.5
Factor A	FT	.215	.969	1.00	1.00
	RT	.206	.956	1.00	1.00
	ART	.208	.959	1.00	1.00
Factor B	FT	.052	.052	.052	.052
	RT	.053	.055	**.057**	**.060**
	ART	.055	.055	.055	.055
Interaction	FT	.050	.050	.050	.050
	RT	.054	.053	**.060**	**.069**
	ART	.056	.056	.056	.056

n = 10

Test for:	Method	c 0.5	1.5	2.5	3.5
Factor A	FT	.901	1.00	1.00	1.00
	RT	.888	1.00	1.00	1.00
	ART	.886	1.00	1.00	1.00
Factor B	FT	.052	.052	.052	.052
	RT	.050	.049	.049	.050
	ART	.051	.051	.051	.051
Interaction	FT	.049	.049	.049	.049
	RT	.051	.054	**.060**	**.066**
	ART	.050	.050	.050	.050

Table 5.2.

Proportion of rejections at $\alpha = .05$, normally distributed errors with equal variance, based on 10,000 samples. A and B main effects present ($a_2=b_1=c$, $a_3=b_2= -c$).

n = 2

Test for:	Method	c 0.5	1.5	2.5	3.5
Factor A	FT	.210	.968	1.00	1.00
	RT	.199	.942	1.00	1.00
	ART	.199	.959	1.00	1.00
Factor B	FT	.329	.999	1.00	1.00
	RT	.317	.996	1.00	1.00
	ART	.319	.998	1.00	1.00
Interaction	FT	.050	.050	.050	.050
	RT	.054	.054	.054	**.068**
	ART	.056	.056	.056	.056

n = 10

Test for:	Method	c 0.5	1.5	2.5	3.5
Factor A	FT	.904	1.00	1.00	1.00
	RT	.887	1.00	1.00	1.00
	ART	.889	1.00	1.00	1.00
Factor B	FT	.984	1.00	1.00	1.00
	RT	.978	1.00	1.00	1.00
	ART	.979	1.00	1.00	1.00
Interaction	FT	.049	.049	.049	.049
	RT	.051	**.134**	**.671**	**.997**
	ART	.050	.050	.050	.050

Table 5.3.

Proportion of rejections at $\alpha = .05$, normally distributed errors with equal variance, based on 10,000 samples. A, B and interaction effects present ($ab_{11}=c$, $b_1=ab_{41}= -c$).

n = 2

Test for:	Method	c 0.5	1.5	2.5	3.5
Factor A	FT	.066	.213	.527	.830
	RT	.066	.132	.193	.218
	ART	.065	.153	.252	.290
Factor B	FT	.139	.780	.997	1.00
	RT	.134	.652	.940	.994
	ART	.140	.732	.989	1.00
Interaction	FT	.069	.260	.655	.931
	RT	.066	.153	.230	.264
	ART	.075	.251	.617	.909

n = 10

Test for:	Method	c 0.5	1.5	2.5	3.5
Factor A	FT	.156	.907	1.00	1.00
	RT	.145	.691	.896	.939
	ART	.151	.829	.993	.999
Factor B	FT	.622	1.00	1.00	1.00
	RT	.582	1.00	1.00	1.00
	ART	.589	1.00	1.00	1.00
Interaction	FT	.214	.991	1.00	1.00
	RT	.195	.908	.994	.999
	ART	.210	.988	1.00	1.00

Table 5.4.

Proportion of rejections at $\alpha = .05$, normally distributed errors with equal variance, based on 10,000 samples. A main effect (large) and interaction (small) effects present ($ab_{11}=ab_{12}=c$, $ab_{31}=ab_{32}=-c$, $a_2=2c$).

n = 2

Test for:	Method	c 0.5	1.5	2.5	3.5
Factor A	FT	.383	1.00	1.00	1.00
	RT	.369	1.00	1.00	1.00
	ART	.374	1.00	1.00	1.00
Factor B	FT	.052	.052	.052	.052
	RT	.053	.049	**.042**	**.043**
	ART	.053	.053	.053	.053
Interaction	FT	.069	.259	.659	.940
	RT	.071	.272	.591	.760
	ART	.074	.250	.621	.912

n = 10

Test for:	Method	c 0.5	1.5	2.5	3.5
Factor A	FT	.997	1.00	1.00	1.00
	RT	.996	1.00	1.00	1.00
	ART	.995	1.00	1.00	1.00
Factor B	FT	.052	.052	.052	.052
	RT	.050	.050	**.039**	**.035**
	ART	.050	.048	.048	.046
Interaction	FT	.216	.991	1.00	1.00
	RT	.216	.991	1.00	1.00
	ART	.207	.985	1.00	1.00

Table 5.5.

Proportion of rejections at $\alpha = .05$, normally distributed errors with equal variance, based on 10,000 samples. A main effect (small) and interaction effect (large) present ($ab_{11}=ab_{12}=ab_{33}=c$, $ab_{13}=ab_{31}=ab_{32}= -c$).

n = 2

Test for:	Method	c 0.5	1.5	2.5	3.5
Factor A	FT	.068	.215	.530	.831
	RT	.064	.186	.461	.680
	ART	.067	.181	.445	.637
Factor B	FT	.052	.052	.052	.052
	RT	.054	.055	**.058**	**.062**
	ART	.055	.054	**.058**	**.057**
Interaction	FT	.130	.834	1.00	1.00
	RT	.128	.811	.999	1.00
	ART	.133	.799	.998	1.00

n = 10

Test for:	Method	c 0.5	1.5	2.5	3.5
Factor A	FT	.162	.901	1.00	1.00
	RT	.158	.934	1.00	1.00
	ART	.158	.933	1.00	1.00
Factor B	FT	.050	.050	.050	.050
	RT	.050	.048	.046	.045
	ART	.049	.046	.045	.044
Interaction	FT	.760	1.00	1.00	1.00
	RT	.740	1.00	1.00	1.00
	ART	.734	1.00	1.00	1.00

Table 5.6.

Proportion of rejections at $\alpha = .05$, normally distributed errors with equal variance, based on 10,000 samples. A main effect (large) and interaction effect (large) present ($ab_{12}=ab_{23}=ab_{41}=c$, $ab_{22}=ab_{31}=ab_{33}= -c$).

n = 2

Test for:	Method	c 0.5	1.5	2.5	3.5
Factor A	FT	.100	.568	.957	.999
	RT	.097	.541	.942	.998
	ART	.097	.546	.942	.998
Factor B	FT	.052	.052	.052	.052
	RT	.049	.052	**.057**	**.062**
	ART	.053	.051	.050	.053
Interaction	FT	.111	.699	.992	1.00
	RT	.105	.670	.987	1.00
	ART	.114	.699	.991	1.00

n = 10

Test for:	Method	c 0.5	1.5	2.5	3.5
Factor A	FT	.431	1.00	1.00	1.00
	RT	.414	1.00	1.00	1.00
	ART	.415	1.00	1.00	1.00
Factor B	FT	.052	.052	.052	.052
	RT	.049	.050	.052	.054
	ART	.052	.055	**.064**	**.074**
Interaction	FT	.612	1.00	1.00	1.00
	RT	.589	1.00	1.00	1.00
	ART	.589	1.00	1.00	1.00

Table 5.7.

Proportion of rejections at $\alpha = .05$, normally distributed errors with equal variance, based on 10,000 samples. Interaction effect present ($ab_{11}=ab_{33}=c$, $ab_{13}=ab_{31}=-c$).

n = 2

Test for:	Method	c 0.5	1.5	2.5	3.5
Factor A	FT	.050	.050	.050	.050
	RT	.050	.049	.050	.055
	ART	.051	.047	.048	.048
Factor B	FT	.052	.052	.052	.052
	RT	.055	.055	.054	.056
	ART	.054	.049	.050	.048
Interaction	FT	.109	.701	.995	1.00
	RT	.108	.626	.975	1.00
	ART	.114	.652	.983	1.00

n = 10

Test for:	Method	c 0.5	1.5	2.5	3.5
Factor A	FT	.050	.050	.050	.050
	RT	.051	.047	.044	.042
	ART	.050	.046	.041	.039
Factor B	FT	.052	.052	.052	.052
	RT	.049	.045	**.039**	**.039**
	ART	.050	.044	**.035**	**.031**
Interaction	FT	.061	1.00	1.00	1.00
	RT	.058	1.00	1.00	1.00
	ART	.058	1.00	1.00	1.00

5.2.2. Non-normal errors (see Tables 5.8-5.21). When the errors were uniformly distributed (Tables 5.8-5.14), all three methods had considerably less power than when errors were normally distributed. Relatively, however, the results were almost identical to the case for normally distributed errors, with the F-test having the most power, followed closely by the ART and then the RT. The ART again often had slightly inflated nominal type I error rates for testing interaction (see Tables 5.8-5.9).

When the errors were exponentially distributed (see Tables 5.15-5.21), both rank tests had superior power to the F-test (although the power of all tests was lower than either the uniform or normal error case). A notable exception was the model which had both main effects and interaction present, where again the RT had less power for testing interaction than in other models (see Table 5.17). Even though for most models the power of the RT was about the same as the FT (except when effect magnitudes became very large, where the FT usually had more power), it was still outperformed by the ART. When only one main effect was present, along with interaction, the RT usually had slightly higher power for testing interaction than the ART, except when effect sizes were small (see Table 5.15).

Interestingly, for small sample sizes (n=2 and n=5 observations per cell), when the error distributions were non-normal, the nominal type I error rates for the RT did not show a tendency to inflate as the magnitudes of the effects increased (see Tables 5.9 and 5.16). The inflation was evident for larger sample sizes ($n \geq 10$ observations per cell), but was much less severe than in the case of normally distributed errors.

The reader should exercise caution, however, when interpreting power disparities between different error distributions. In these simulations, all methods had less power when the error distributions were non-normal. It should be noted, however, that parameters for the two non-normal distributions could have been chosen so that all methods would have had more power for non-normally distributed errors than for normally distributed errors. However, the parameters in this study were chosen to facilitate the comparison of powers between the different methods. Thus, while the relative performance of the methods for each of the distributions can be generalized, the same is not true for the performance of any given method across the different distributions.

Table 5.8.

Proportion of rejections at $\alpha=0.05$, uniformly distributed errors with equal variance, based on 10,000 samples. A main effect present ($a_1=c$, $a_3=-c$).

n = 2

Test for:	Method	c 0.5	1.5	2.5	3.5
Factor A	FT	.094	.541	.968	1.00
	RT	.091	.477	.948	1.00
	ART	.090	.487	.952	1.00
Factor B	FT	.052	.052	.052	.052
	RT	.051	.051	.054	.054
	ART	.055	.055	.055	.055
Interaction	FT	.054	.054	.054	.054
	RT	.052	.051	.055	**.057**
	ART	**.058**	**.058**	**.058**	**.058**

n = 10

Test for:	Method	c 0.5	1.5	2.5	3.5
Factor A	FT	.422	1.00	1.00	1.00
	RT	.395	1.00	1.00	1.00
	ART	.389	1.00	1.00	1.00
Factor B	FT	.051	.051	.051	.051
	RT	.049	.049	.048	.050
	ART	.048	.048	.048	.048
Interaction	FT	.050	.050	.050	.050
	RT	.051	.053	.056	**.058**
	ART	.050	.050	.050	.050

Table 5.9.

Proportion of rejections at $\alpha = .05$, uniformly distributed errors with equal variance, based on 10,000 samples. A and B main effects present ($a_2=b_1=c$, $a_3=b_2=-c$).

n = 2

Test for:	Method	c = 0.5	1.5	2.5	3.5
Factor A	FT	.097	.540	.965	1.00
	RT	.089	.465	.926	.998
	ART	.093	.489	.948	1.00
Factor B	FT	.131	.776	.999	1.00
	RT	.124	.716	.997	1.00
	ART	.130	.745	.999	1.00
Interaction	FT	.054	.054	.054	.054
	RT	.051	.050	.052	.049
	ART	**.058**	**.058**	**.058**	**.058**

n = 10

Test for:	Method	c = 0.5	1.5	2.5	3.5
Factor A	FT	.422	1.00	1.00	1.00
	RT	.382	1.00	1.00	1.00
	ART	.392	1.00	1.00	1.00
Factor B	FT	.617	1.00	1.00	1.00
	RT	.556	1.00	1.00	1.00
	ART	.562	1.00	1.00	1.00
Interaction	FT	.050	.050	.050	.050
	RT	.051	**.058**	**.108**	**.273**
	ART	.050	.050	.050	.050

Table 5.10.

Proportion of rejections at $\alpha = .05$, uniformly distributed errors with equal variance, based on 10,000 samples. A, B and interaction effects present (ab_{11}=c, b_1=ab_{41}=-c).

n = 2

Test for:	Method	c 0.5	1.5	2.5	3.5
Factor A	FT	.055	.094	.188	.340
	RT	.054	.081	.119	.161
	ART	.056	.085	.139	.202
Factor B	FT	.079	.325	.751	.972
	RT	.077	.272	.590	.831
	ART	.080	.300	.678	.933
Interaction	FT	.061	.110	.231	.437
	RT	.057	.088	.145	.195
	ART	.061	.111	.223	.404

n = 10

Test for:	Method	c 0.5	1.5	2.5	3.5
Factor A	FT	.086	.431	.884	1.00
	RT	.083	.322	.656	.817
	ART	.083	.358	.752	.952
Factor B	FT	.234	.984	1.00	1.00
	RT	.212	.952	1.00	1.00
	ART	.211	.958	1.00	1.00
Interaction	FT	.094	.603	.987	1.00
	RT	.090	.457	.879	.975
	ART	.092	.537	.969	1.00

Table 5.11.

Proportion of rejections at $\alpha = .05$, uniformly distributed errors with equal variance, based on 10,000 samples. A main effect (large) and interaction (small) effects present ($ab_{11}=ab_{12}=c$, $ab_{31}=ab_{32}=-c$, $a_2=2c$).

n = 2

Test for:	Method	c 0.5	1.5	2.5	3.5
Factor A	FT	.143	.861	1.00	1.00
	RT	.133	.811	1.00	1.00
	ART	.134	.828	1.00	1.00
Factor B	FT	.052	.052	.052	.052
	RT	.051	.051	.052	**.043**
	ART	.055	.054	.051	.050
Interaction	FT	.059	.111	.233	.434
	RT	.054	.107	.243	.412
	ART	.063	.108	.220	.403

Table 5.12.

Proportion of rejections at $\alpha = .05$, uniformly distributed errors with equal variance, based on 10,000 samples. A main effect (small) and interaction effect (large) present ($ab_{11}=ab_{12}=ab_{33}=c$, $ab_{13}=ab_{31}=ab_{32}= -c$).

n = 2

Test for:	Method	c 0.5	1.5	2.5	3.5
Factor A	FT	.057	.094	.187	.337
	RT	.054	.086	.150	.274
	ART	.053	.082	.150	.280
Factor B	FT	.052	.052	.052	.052
	RT	.051	.051	.054	.055
	ART	.054	.050	.052	.053
Interaction	FT	.075	.327	.791	.992
	RT	.074	.290	.734	.972
	ART	.079	.301	.738	.975

Table 5.13.

Proportion of rejections at $\alpha = .05$, uniformly distributed errors with equal variance, based on 10,000 samples. A main effect (large) and interaction effect (large) present ($ab_{12}=ab_{23}=ab_{41}=c$, $ab_{22}=ab_{31}=ab_{33}= -c$).

n = 2

Test for:	Method	c 0.5	1.5	2.5	3.5
Factor A	FT	.066	.202	.503	.836
	RT	.062	.182	.456	.772
	ART	.063	.188	.465	.787
Factor B	FT	.052	.052	.052	.052
	RT	.050	.050	.055	.053
	ART	.054	.055	.051	.050
Interaction	FT	.071	.245	.644	.948
	RT	.068	.219	.577	.904
	ART	.074	.241	.623	.934

Table 5.14.

Proportion of rejections at $\alpha = .05$, uniformly distributed errors with equal variance, based on 10,000 samples. Interaction effect present ($ab_{11}=ab_{33}=c$, $ab_{13}=ab_{31}=-c$).

n = 2

Test for:	Method	c 0.5	1.5	2.5	3.5
Factor A	FT	.050	.050	.050	.050
	RT	.050	.050	.050	.051
	ART	.049	.051	.049	.046
Factor B	FT	.052	.052	.052	.052
	RT	.052	.053	.054	.055
	ART	.055	.053	.053	.052
Interaction	FT	.068	.249	.643	.946
	RT	.068	.214	.531	.856
	ART	.072	.235	.583	.898

Table 5.15.

Proportion of rejections at $\alpha=0.05$, identically exponentially distributed errors, based on 10,000 samples. A main effect present ($a_1=c$, $a_3=-c$).

n = 2

Test for:	Method	c 0.5	1.5	2.5	3.5
Factor A	FT	.066	.242	.576	.831
	RT	.090	.357	.687	.888
	ART	.083	.329	.662	.875
Factor B	FT	.047	.047	.047	.047
	RT	.053	.053	.054	.052
	ART	**.059**	**.059**	**.059**	**.059**
Interaction	FT	.055	.055	.055	.055
	RT	.055	**.058**	**.059**	**.057**
	ART	**.074**	**.074**	**.074**	**.074**

n = 10

Test for:	Method	c 0.5	1.5	2.5	3.5
Factor A	FT	.170	.902	1.00	1.00
	RT	.359	.994	1.00	1.00
	ART	.334	.994	1.00	1.00
Factor B	FT	.047	.047	.047	.047
	RT	.048	.048	.046	.048
	ART	.048	.048	.048	.048
Interaction	FT	.048	.048	.048	.048
	RT	.052	**.057**	**.057**	**.057**
	ART	**.061**	**.061**	**.061**	**.061**

Table 5.16.

Proportion of rejections at $\alpha = .05$, identically exponentially distributed errors, based on 10,000 samples. A and B main effects present ($a_2=b_1=c$, $a_3=b_2=-c$).

n = 2

Test for:	Method	c 0.5	1.5	2.5	3.5
Factor A	FT	.066	.246	.574	.828
	RT	.083	.314	.621	.834
	ART	.086	.335	.665	.877
Factor B	FT	.084	.386	.762	.943
	RT	.119	.497	.825	.956
	ART	.113	.485	.839	.966
Interaction	FT	.055	.055	.055	.055
	RT	**.058**	**.059**	**.059**	**.057**
	ART	**.074**	**.074**	**.074**	**.074**

n = 10

Test for:	Method	c 0.5	1.5	2.5	3.5
Factor A	FT	.172	.898	1.00	1.00
	RT	.329	.985	1.00	1.00
	ART	.332	.993	1.00	1.00
Factor B	FT	.251	.977	1.00	1.00
	RT	.477	.999	1.00	1.00
	ART	.463	1.00	1.00	1.00
Interaction	FT	.048	.048	.048	.048
	RT	.053	**.060**	**.078**	**.121**
	ART	**.061**	**.061**	**.061**	**.061**

Table 5.17.

Proportion of rejections at $\alpha = .05$, identically exponentially distributed errors, based on 10,000 samples. A, B and interaction effects present (ab_{11}=c, b_1=ab_{41}=-c).

n = 2

Test for:	Method	c 0.5	1.5	2.5	3.5
Factor A	FT	.049	.063	.097	.154
	RT	.054	.073	.094	.121
	ART	.057	.080	.113	.151
Factor B	FT	.057	.155	.362	.610
	RT	.073	.224	.405	.576
	ART	.072	.208	.420	.634
Interaction	FT	.058	.075	.113	.186
	RT	.059	.082	.109	.142
	ART	.076	.100	.153	.234

n = 10

Test for:	Method	c 0.5	1.5	2.5	3.5
Factor A	FT	.059	.167	.412	.707
	RT	.077	.238	.443	.616
	ART	.075	.268	.549	.774
Factor B	FT	.113	.638	.961	1.00
	RT	.200	.832	.986	1.00
	ART	.185	.841	.992	1.00
Interaction	FT	.065	.227	.592	.891
	RT	.089	.335	.634	.836
	ART	.091	.412	.846	.984

Table 5.18.

Proportion of rejections at $\alpha = .05$, identically exponentially distributed errors, based on 10,000 samples. A main effect (large) and interaction (small) effects present ($ab_{11}=ab_{12}=c$, $ab_{31}=ab_{32}=-c$, $a_2=2c$).

n = 2

Test for:	Method	c 0.5	1.5	2.5	3.5
Factor A	FT	.082	.436	.831	.968
	RT	.123	.569	.896	.981
	ART	.114	.536	.885	.980
Factor B	FT	.047	.047	.047	.047
	RT	.053	.054	.051	.049
	ART	**.058**	**.058**	.056	.056
Interaction	FT	.056	.074	.114	.191
	RT	.062	.098	.163	.243
	ART	.077	.102	.156	.239

Table 5.19.

Proportion of rejections at $\alpha = .05$, identically exponentially distributed errors, based on 10,000 samples. A main effect (small) and interaction effect (large) present ($ab_{11}=ab_{12}=ab_{33}=c$, $ab_{13}=ab_{31}=ab_{32}= -c$).

n = 2

Test for:	Method	c 0.5	1.5	2.5	3.5
Factor A	FT	.049	.067	.099	.157
	RT	.054	.079	.112	.168
	ART	.057	.078	.115	.166
Factor B	FT	.047	.047	.047	.047
	RT	.052	.052	.052	.053
	ART	**.058**	**.057**	.055	.055
Interaction	FT	.064	.146	.366	.634
	RT	.074	.230	.489	.712
	ART	.086	.196	.415	.665

Table 5.20.

Proportion of rejections at $\alpha = .05$, identically exponentially distributed errors, based on 10,000 samples. A main effect (large) and interaction effect (large) present ($ab_{12}=ab_{23}=ab_{41}=c$, $ab_{22}=ab_{31}=ab_{33}= -c$).

n = 2

Test for:	Method	c 0.5	1.5	2.5	3.5
Factor A	FT	.054	.102	.229	.418
	RT	.064	.153	.303	.477
	ART	.063	.145	.293	.467
Factor B	FT	.047	.047	.047	.047
	RT	.054	.053	.052	.051
	ART	**.057**	.055	.053	.055
Interaction	FT	.062	.121	.276	.515
	RT	.066	.179	.375	.591
	ART	.083	.163	.341	.572

Table 5.21.

Proportion of rejections at $\alpha = .05$, identically exponentially distributed errors, based on 10,000 samples. Interaction effect present ($ab_{11}=ab_{33}=c$, $ab_{13}=ab_{31}=-c$).

n = 2

Test for:	Method	c 0.5	1.5	2.5	3.5
Factor A	FT	.049	.049	.049	.049
	RT	.052	.052	.048	.049
	ART	.056	.055	.054	.054
Factor B	FT	.047	.047	.047	.047
	RT	.053	.054	.054	.053
	ART	**.059**	**.058**	**.057**	.056
Interaction	FT	.062	.121	.280	.515
	RT	.069	.180	.369	.567
	ART	.083	.162	.329	.548

5.2.3. Normal errors, unequal variances (see Tables 5.22-5.28). This situation was a much more serious problem than the lack of normality. As in the case of non-normally distributed errors, however, the power for all methods was less than in the equal variance case, and this decrease in power became more severe as the degree of heterogeneity between variances increased. In this case, however, since all errors were normally distributed with mean zero, the observed power disparity can be attributed to variance heterogeneity alone. Also as in the non-normal case, however, both rank tests consistently outperformed the FT in the power category, except for the RT in the previously discussed model (see Table 5.24). The FT did, however, often have slightly higher power for very small effect magnitudes. In addition, the ART usually had more power for testing interaction than the RT. These last two observations deserve some comment. Examination of nominal type I error rates for testing interaction when none was modeled revealed that these rates were inflated for all three methods, with more severe inflation occurring when the variances were more variable (see Tables 5.22, 5.23). This indicated that variance heterogeneity actually tends to introduce interaction into the data more often than would be expected. The ART seemed to be the most sensitive to this interaction, which is not surprising since the alignment procedure isolates the effect of interaction, followed by the FT and then the RT. Thus, it is not surprising that the ART showed more power when interaction was actually modeled. In addition, the RT, which was the least sensitive to interaction, usually "caught up" to the other two tests' type I error nominal levels as the magnitude of the effects became very large. This was the same behavior that was observed in the equal variance case.

The problem of nominal type I error rate inflation was not limited only to the test for interaction, however. When only one main effect was modeled along with an interaction effect, the nominal type I error rates for testing the unmodeled main effect were also inflated for all methods. Thus, it is apparent that variance heterogeneity can produce very erratic behavior in the data.

Although the results reported in this paper are all based an a nominal type I error rate of 0.05, simulations were also conducted using nominal type I error rates of 0.10 and 0.01. The results obtained were similar for all three levels.

Table 5.22.

Proportion of rejections at α=0.05, normally distributed errors with unequal variance, based on 10,000 samples. A main effect present (a_1=c, a_3=-c).

n = 2 (10:1 ratio)	Test for:	Method	c 0.5	1.5	2.5	3.5
	Factor A	FT	.106	.394	.829	.985
		RT	.105	.477	.904	.995
		ART	.107	.465	.898	.994
	Factor B	FT	**.069**	**.069**	**.069**	**.069**
		RT	**.060**	**.066**	**.070**	**.072**
		ART	**.063**	**.063**	**.063**	**.063**
	Interaction	FT	**.090**	**.090**	**.090**	**.090**
		RT	**.069**	**.076**	**.085**	**.090**
		ART	**.097**	**.097**	**.097**	**.097**

n = 2 (30:1 ratio)	Test for:	Method	c 0.5	1.5	2.5	3.5
	Factor A	FT	.108	.215	.476	.758
		RT	.095	.296	.661	.905
		ART	.098	.273	.626	.892
	Factor B	FT	**.083**	**.083**	**.083**	**.083**
		RT	**.065**	**.071**	**.076**	**.083**
		ART	**.067**	**.067**	**.067**	**.067**
	Interaction	FT	**.113**	**.113**	**.113**	**.113**
		RT	**.077**	**.085**	**.098**	**.109**
		ART	**.134**	**.134**	**.134**	**.134**

Table 5.22 continued.

n = 2 (60:1 ratio)	Test for:	Method	c 0.5	2.0	2.5	3.5
	Factor A	FT	.111	.167	.307	.510
		RT	.095	.226	.487	.763
		ART	.100	.206	.441	.721
	Factor B	FT	**.090**	**.090**	**.090**	**.090**
		RT	**.069**	**.074**	**.080**	**.087**
		ART	**.071**	**.071**	**.071**	**.071**
	Interaction	FT	**.127**	**.127**	**.127**	**.127**
		RT	**.082**	**.089**	**.102**	**.117**
		ART	**.159**	**.159**	**.159**	**.159**

n =10 (30:1 ratio)	Test for:	Method	c 0.5	1.5	2.5	3.5
	Factor A	FT	.141	.885	1.00	1.00
		RT	.234	.991	1.00	1.00
		ART	.232	.990	1.00	1.00
	Factor B	FT	**.057**	**.057**	**.057**	**.057**
		RT	.052	.052	.053	.055
		ART	.052	.052	.052	.052
	Interaction	FT	**.091**	**.091**	**.091**	**.091**
		RT	**.062**	**.067**	**.076**	**.082**
		ART	**.130**	**.130**	**.130**	**.130**

Table 5.23.

Proportion of rejections at $\alpha = .05$, normally distributed errors with unequal variance, based on 10,000 samples. A and B main effects present ($a_2=b_1=c$, $a_3=b_2=-c$).

n = 2			c			
(10:1 ratio)	Test for:	Method	0.5	1.5	2.5	3.5
	Factor A	FT	.108	.394	.812	.979
		RT	.101	.428	.816	.976
		ART	.103	.453	.870	.991
	Factor B	FT	.124	.573	.944	.999
		RT	.125	.607	.941	.998
		ART	.132	.631	.963	1.00
	Interaction	FT	**.090**	**.090**	**.090**	**.090**
		RT	**.071**	**.084**	**.086**	**.090**
		ART	**.097**	**.097**	**.097**	**.097**

n = 2			c			
(30:1 ratio)	Test for:	Method	0.5	1.5	2.5	3.5
	Factor A	FT	.108	.218	.475	.753
		RT	.096	.280	.562	.802
		ART	.097	.279	.613	.874
	Factor B	FT	.108	.313	.651	.887
		RT	.102	.380	.718	.914
		ART	.105	.406	.757	.945
	Interaction	FT	**.113**	**.113**	**.113**	**.113**
		RT	**.080**	**.099**	**.110**	**.111**
		ART	**.134**	**.134**	**.134**	**.134**

Table 5.23 continued.

n = 2 (60:1 ratio)	Test for:	Method	c 0.5	1.5	2.5	3.5
	Factor A	FT	.112	.171	.305	.510
		RT	.094	.220	.418	.626
		ART	.098	.211	.438	.703
	Factor B	FT	.105	.216	.433	.674
		RT	.093	.288	.550	.763
		ART	.097	.302	.582	.806
	Interaction	FT	**.127**	**.127**	**.127**	**.127**
		RT	**.085**	**.105**	**.121**	**.125**
		ART	**.159**	**.159**	**.159**	**.159**

n = 10 (30:1 ratio)	Test for:	Method	c 0.5	1.5	2.5	3.5
	Factor A	FT	.145	.865	1.00	1.00
		RT	.228	.967	1.00	1.00
		ART	.231	.977	1.00	1.00
	Factor B	FT	.201	.947	1.00	1.00
		RT	.306	.993	1.00	1.00
		ART	.309	.995	1.00	1.00
	Interaction	FT	**.091**	**.091**	**.091**	**.091**
		RT	**.076**	**.129**	**.149**	**.153**
		ART	**.130**	**.130**	**.130**	**.130**

Table 5.24.

Proportion of rejections at $\alpha = .05$, normally distributed errors with unequal variance, based on 10,000 samples. A, B and interaction effects present (ab_{11}=c, b_1=ab_{41}= -c).

n = 2 (10:1 ratio)	Test for:	Method	c 0.5	1.5	2.5	3.5
	Factor A	FT	.084	.111	.166	.252
		RT	.069	.094	.136	.176
		ART	.071	.098	.141	.188
	Factor B	FT	.088	.245	.538	.810
		RT	.076	.217	.473	.728
		ART	.081	.236	.528	.808
	Interaction	FT	.093	.128	.206	.320
		RT	.069	.091	.129	.178
		ART	.102	.146	.222	.334

n = 2 (30:1 ratio)	Test for:	Method	c 0.5	1.5	2.5	3.5
	Factor A	FT	.097	.110	.132	.167
		RT	.076	.090	.115	.146
		ART	.082	.093	.118	.144
	Factor B	FT	.090	.160	.291	.481
		RT	.075	.144	.275	.455
		ART	.075	.153	.302	.506
	Interaction	FT	.117	.132	.164	.211
		RT	.078	.086	.110	.140
		ART	.135	.157	.193	.248

Table 5.24 continued.

n = 2 (60:1 ratio)	Test for:	Method	c 0.5	1.5	2.5	3.5
	Factor A	FT	.106	.112	.125	.145
		RT	.081	.090	.104	.125
		ART	.088	.097	.111	.130
	Factor B	FT	.094	.132	.209	.316
		RT	.074	.116	.198	.315
		ART	.075	.124	.218	.350
	Interaction	FT	.130	.136	.154	.183
		RT	.083	.091	.102	.118
		ART	.160	.171	.193	.225

n = 10 (30:1 ratio)	Test for:	Method	c 0.5	1.5	2.5	3.5
	Factor A	FT	.089	.147	.287	.516
		RT	.079	.165	.363	.598
		ART	.080	.167	.351	.586
	Factor B	FT	.101	.517	.931	.999
		RT	.094	.534	.934	.999
		ART	.096	.541	.942	.999
	Interaction	FT	.101	.197	.429	.736
		RT	.068	.136	.326	.620
		ART	.157	.356	.718	.958

Table 5.25.

Proportion of rejections at $\alpha = .05$, normally distributed errors with unequal variance, based on 10,000 samples. A main effect (large) and interaction (small) effects present ($ab_{11}=ab_{12}=c$, $ab_{31}=ab_{32}=-c$, $a_2=2c$).

n = 2 (10:1 ratio)			c			
	Test for:	Method	0.5	1.5	2.5	3.5
	Factor A	FT	.138	.656	.977	1.00
		RT	.139	.729	.991	1.00
		ART	.142	.721	.990	1.00
	Factor B	FT	**.069**	**.069**	**.069**	**.069**
		RT	**.061**	**.066**	**.070**	**.065**
		ART	**.063**	**.063**	**.066**	**.067**
	Interaction	FT	.093	.123	.196	.326
		RT	.074	.128	.210	.329
		ART	.102	.143	.226	.356

n = 2 (30:1 ratio)			c			
	Test for:	Method	0.5	1.5	2.5	3.5
	Factor A	FT	.122	.353	.740	.952
		RT	.115	.471	.870	.988
		ART	.115	.449	.851	.986
	Factor B	FT	**.083**	**.083**	**.083**	**.083**
		RT	**.067**	**.073**	**.083**	**.085**
		ART	**.066**	**.069**	**.071**	**.074**
	Interaction	FT	.115	.127	.155	.201
		RT	.082	.120	.167	.219
		ART	.134	.157	.193	.252

Table 5.25 continued.

n = 2 (60:1 ratio)	Test for:	Method	c 0.5	1.5	2.5	3.5
	Factor A	FT	.119	.235	.496	.768
		RT	.107	.345	.708	.926
		ART	.110	.319	.671	.905
	Factor B	FT	**.090**	**.090**	**.090**	**.090**
		RT	**.069**	**.075**	**.087**	**.093**
		ART	**.070**	**.073**	**.075**	**.077**
	Interaction	FT	.128	.135	.150	.174
		RT	.086	.116	.156	.188
		ART	.160	.173	.194	.232

Table 5.26.

Proportion of rejections at $\alpha = .05$, normally distributed errors with unequal variance, based on 10,000 samples. A main effect (small) and interaction effect (large) present ($ab_{11}=ab_{12}=ab_{33}=c$, $ab_{13}=ab_{31}=ab_{32}= -c$).

n = 2 (10:1 ratio)	Test for:	Method	c 0.5	1.5	2.5	3.5
	Factor A	FT	.082	.106	.159	.258
		RT	.070	.105	.161	.245
		ART	.075	.108	.157	.232
	Factor B	FT	**.069**	**.069**	**.069**	**.069**
		RT	**.061**	**.066**	**.071**	**.076**
		ART	**.063**	**.067**	**.071**	**.077**
	Interaction	FT	.104	.252	.589	.880
		RT	.090	.291	.646	.904
		ART	.115	.286	.613	.883

Table 5.26 continued.

n = 2 (30:1 ratio)	Test for:	Method	c 0.5	1.5	2.5	3.5
	Factor A	FT	.098	.108	.129	.162
		RT	.078	.102	.139	.177
		ART	.082	.103	.133	.169
	Factor B	FT	**.083**	**.083**	**.083**	**.083**
		RT	**.065**	**.072**	**.081**	**.085**
		ART	**.066**	**.072**	**.078**	**.085**
	Interaction	FT	.121	.177	.316	.529
		RT	.093	.198	.407	.641
		ART	.143	.221	.384	.600

n = 2 (60:1 ratio)	Test for:	Method	c 0.5	1.5	2.5	3.5
	Factor A	FT	.106	.111	.122	.143
		RT	.081	.102	.126	.160
		ART	.090	.107	.126	.151
	Factor B	FT	**.090**	**.090**	**.090**	**.090**
		RT	**.069**	**.075**	**.084**	**.089**
		ART	**.070**	**.077**	**.082**	**.088**
	Interaction	FT	.130	.161	.227	.349
		RT	.093	.165	.300	.478
		ART	.166	.211	.302	.441

Table 5.27.

Proportion of rejections at $\alpha = .05$, normally distributed errors with unequal variance, based on 10,000 samples. A main effect (large) and interaction effect (large) present ($ab_{12}=ab_{23}=ab_{41}=c$, $ab_{22}=ab_{31}=ab_{33}= -c$).

n = 2 (10:1 ratio)	Test for:	Method	c 0.5	1.5	2.5	3.5
	Factor A	FT	.088	.171	.360	.608
		RT	.077	.178	.382	.614
		ART	.081	.177	.371	.608
	Factor B	FT	**.069**	**.069**	**.069**	**.069**
		RT	**.063**	**.065**	**.067**	**.072**
		ART	**.063**	**.065**	**.065**	**.066**
	Interaction	FT	.102	.211	.463	.752
		RT	.085	.226	.509	.782
		ART	.111	.244	.508	.783

n = 2 (30:1 ratio)	Test for:	Method	c 0.5	1.5	2.5	3.5
	Factor A	FT	.100	.132	.208	.333
		RT	.079	.133	.239	.378
		ART	.084	.134	.228	.359
	Factor B	FT	**.083**	**.083**	**.083**	**.083**
		RT	**.067**	**.073**	**.076**	**.081**
		ART	**.067**	**.069**	**.075**	**.076**
	Interaction	FT	.119	.161	.260	.420
		RT	.086	.165	.315	.505
		ART	.141	.204	.325	.500

Table 5.27 continued.

n = 2 (60:1 ratio)	Test for:	Method	c 0.5	1.5	2.5	3.5
	Factor A	FT	.107	.128	.164	.228
		RT	.083	.122	.189	.277
		ART	.088	.123	.177	.258
	Factor B	FT	**.090**	**.090**	**.090**	**.090**
		RT	**.068**	**.075**	**.082**	**.084**
		ART	**.068**	**.074**	**.080**	**.083**
	Interaction	FT	.130	.153	.201	.286
		RT	.088	.144	.244	.370
		ART	.162	.200	.269	.371

Table 5.28.

Proportion of rejections at $\alpha = .05$, normally distributed errors with unequal variance, based on 10,000 samples. Interaction effect present ($ab_{11}=ab_{33}=c$, $ab_{13}=ab_{31}=-c$).

n = 2 (10:1 ratio)	Test for:	Method	c 0.5	1.5	2.5	3.5
	Factor A	FT	**.080**	**.080**	**.080**	**.080**
		RT	**.068**	**.071**	**.074**	**.076**
		ART	**.070**	**.075**	**.075**	**.073**
	Factor B	FT	**.069**	**.069**	**.069**	**.069**
		RT	**.061**	**.067**	**.069**	**.069**
		ART	**.061**	**.065**	**.067**	**.065**
	Interaction	FT	.100	.209	.464	.767
		RT	.085	.225	.499	.773
		ART	.113	.236	.488	.760

Table 5.28 continued.

n = 2 (30:1 ratio)	Test for:	Method	c 0.5	1.5	2.5	3.5
	Factor A	FT	**.095**	**.095**	**.095**	**.095**
		RT	**.075**	**.082**	**.087**	**.088**
		ART	**.081**	**.086**	**.090**	**.089**
	Factor B	FT	**.083**	**.083**	**.083**	**.083**
		RT	**.066**	**.071**	**.076**	**.078**
		ART	**.068**	**.070**	**.075**	**.079**
	Interaction	FT	.119	.162	.259	.424
		RT	.088	.167	.320	.510
		ART	.140	.201	.319	.483

n = 2 (60:1 ratio)	Test for:	Method	c 0.5	1.5	2.5	3.5
	Factor A	FT	**.106**	**.106**	**.106**	**.106**
		RT	**.081**	**.086**	**.091**	**.094**
		ART	**.089**	**.092**	**.098**	**.099**
	Factor B	FT	**.090**	**.090**	**.090**	**.090**
		RT	**.068**	**.074**	**.080**	**.083**
		ART	**.070**	**.074**	**.080**	**.084**
	Interaction	FT	.130	.152	.201	.285
		RT	.091	.144	.247	.380
		ART	.163	.198	.269	.372

Table 5.28 continued.

n = 10 (30:1 ratio)	Test for:	Method	c 0.5	1.5	2.5	3.5
	Factor A	FT	**.084**	**.084**	**.084**	**.084**
		RT	**.071**	**.073**	**.076**	**.078**
		ART	**.070**	**.073**	**.077**	**.080**
	Factor B	FT	**.057**	**.057**	**.057**	**.057**
		RT	**.059**	**.075**	**.079**	**.073**
		ART	**.058**	**.073**	**.077**	**.072**
	Interaction	FT	.115	.493	.979	1.00
		RT	.140	.888	1.00	1.00
		ART	.221	.890	1.00	1.00

5.3 Conclusion for Analysis of Completely Randomized Factorial Experiments

The exact aligned rank procedure appears to be the overall best choice for performing tests in a general factorial experiment. When the error distribution was symmetric and error variances were homogeneous (situations in which the F-test is generally assumed to work well), the ART was nearly as powerful as the F-test, with an almost negligible difference in power between the two methods. For a skewed error distribution, the ART was clearly more powerful than the F-test. When the error variances were heterogeneous, both methods had problems maintaining nominal type I error levels for testing interaction, but the ART showed superior power for detecting main effects and interaction. Thus as a general purpose method, the ART appears to be superior to the F-test. It is possible that the ART procedure could benefit from an additional adjustment to stabilize variances. If, in addition to aligning the observations with regard to location, the observations could also be scaled to correct for possible problems with unequal variance, then the tendency for the ART to have inflated nominal type I error rates could be eliminated.

The problems with the rank transform method in two-factor experiments are not alleviated by using the exact permutation distribution of the test statistic computed on the ranks. Based upon the results of this and other studies, the rank transform procedure should not be used to analyze data in a factorial arrangement, due to the serious type I error rate inflations caused by the transformation of data to ranks, and also to the poor power exhibited for some models. This implies that the rank transform procedure should be avoided in any design that allows for interaction between factors.

CHAPTER SIX

SIMULATION STUDY FOR A SPLIT-UNIT EXPERIMENT

6.1 Simulation Procedure

Simulated data sets were generated to examine the performance of the three methods: the parametric F-test procedure (FT), the exact rank transform test procedure (RT), and the exact aligned rank transform test procedure (ART). A split-unit experiment with main units in a randomized complete block design was considered. The following model was used to generate the observations:

$$Y_{ijk} = B_i + M_j + (BM)_{ij} + S_k + (MS)_{jk} + E_{ijk},$$

where B_i is the random effect of the i^{th} block, M_j is the fixed effect of the j^{th} level of the main unit treatment, $(BM)_{ij}$ is the random effect of the interaction between the i^{th} block and the j^{th} level of the main unit treatment, S_k is the fixed effect of the k^{th} level of the sub-unit treatment, $(MS)_{jk}$ is the fixed effect of the interaction between the j^{th} level of the sub-unit treatment with the k^{th} level of the main unit treatment, and E_{ijk} is the random sub-unit error effect. The random effect $(BM)_{ij}$ was used as error to test for the effect of the main unit treatment, while the random effect E_{ijk} was used as error to test both the sub-unit treatment effect, S_k, and the interaction effect, $(MS)_{jk}$. Standard normal (both with homogeneous and heterogeneous variances), exponential (μ=3) and uniform [-3,3] distributions were used to model the error distributions. Using notation analogous to Chapter Five, The values m_i, s_j, and ms_{ij} referred to in the tables that follow represent the values assigned to M_i, S_j, and $(MS)_{ij}$, respectively, for each model. Ten thousand samples were generated, and the proportion of test statistic values greater than or equal to the critical values for the respective sampling distributions was calculated.

For the aligned rank procedure, three different methods of aligning were used, depending upon the effect being tested. For testing main unit treatment effect, the observations were aligned by subtracting estimates of both block and sub-unit treatment effects. For testing sub-unit treatment effect, estimates of both block and main unit treatment effects were subtracted from each observation. Finally, for testing interaction, the observations were aligned by subtracting block, main unit and sub-unit effect estimates.

6.2 Simulation Results

6.2.1. Normal errors, equal variances (see Tables 6.1-6.5). In this situation, all random effects were modeled as identically distributed standard normal distributions. The three methods performed almost identically to the previous study of the two-way layout in a completely randomized design. Both rank tests consistently had power almost equal to that of the F-test. As in the completely randomized case, the RT again showed poor power for testing interaction when both main and sub-unit main effects and interaction were present in the model (see Table 6.4). Also, Table 6.3 shows that when only main and sub-unit effects were in the model, the RT again had type I error rates that inflated as the magnitude of the effects increased. This behavior was not as evident for other models, however.

Table 6.1.

Proportion of rejections at α=0.05, normally distributed errors with equal variance, based on 10,000 samples. Sub-unit main effect present (s_1=-c, s_3=c).

Test for:	Method	c 0.5	1.5	2.5	3.5
MU Trt	FT	.048	.048	.048	.048
	RT	.054	.049	.050	.047
	ART	.053	.053	.053	.053
SU Trt	FT	.050	1.00	1.00	1.00
	RT	.046	1.00	1.00	1.00
	ART	.048	1.00	1.00	1.00
Interaction	FT	.055	.055	.055	.055
	RT	.044	.049	.048	.047
	ART	.049	.051	.051	.049

Table 6.2.

Proportion of rejections at α=0.05, normally distributed errors with equal variance, based on 10,000 samples. Main unit main effect present (m_1=c, m_3=-c).

Test for:	Method	c 0.5	1.5	2.5	3.5
MU Trt	FT	.087	.476	.900	.994
	RT	.092	.480	.894	.994
	ART	.096	.484	.898	.995
SU Trt	FT	.049	.049	.049	.049
	RT	.047	.046	.048	.050
	ART	.050	.050	.050	.050
Interaction	FT	.049	.049	.049	.049
	RT	.044	.044	.047	.053
	ART	.049	.049	.049	.049

Table 6.3.

Proportion of rejections at $\alpha = .05$, normally distributed errors with equal variance, based on 10,000 samples. MU and SU main effects present ($m_2=s_1=c$, $m_3=s_2= -c$).

Test for:	Method	c 0.5	1.5	2.5	3.5
MU Trt	FT	.088	.474	.900	.994
	RT	.091	.467	.889	.993
	ART	.096	.481	.897	.993
SU Trt	FT	.500	1.00	1.00	1.00
	RT	.449	1.00	1.00	1.00
	ART	.473	1.00	1.00	1.00
Interaction	FT	.049	.049	.049	.049
	RT	.046	.047	**.077**	**.148**
	ART	.049	.049	.049	.049

Table 6.4.

Proportion of rejections at $\alpha = .05$, normally distributed errors with equal variance, based on 10,000 samples. MU, SU main effects and interaction effect present ($ms_{11}=-c$, $s_1=ms_{41}=c$).

Test for:	Method	c 0.5	1.5	2.5	3.5
MU Trt	FT	.052	.087	.168	.298
	RT	.057	.078	.114	.146
	ART	.058	.087	.123	.155
SU Trt	FT	.187	.942	1.00	1.00
	RT	.168	.875	.998	1.00
	ART	.179	.911	1.00	1.00
Interaction	FT	.079	.416	.894	.997
	RT	.070	.269	.497	.642
	ART	.075	.383	.850	.991

Table 6.5.

Proportion of rejections at $\alpha = .05$, normally distributed errors with equal variance, based on 10,000 samples. Interaction effect present ($ms_{11}=ms_{33}=c$, $ms_{13}=ms_{31}=-c$).

Test for:	Method	c 0.5	1.5	2.5	3.5
MU Trt	FT	.048	.048	.048	.048
	RT	.053	.052	.048	.047
	ART	.053	.052	.049	.052
SU Trt	FT	.049	.049	.049	.049
	RT	.048	.045	.045	.044
	ART	.049	.048	**.040**	**.033**
Interaction	FT	.149	.919	1.00	1.00
	RT	.128	.852	.999	1.00
	ART	.140	.878	1.00	1.00

6.2.2. Non-normal errors (see Tables 6.6-6.29). Four different cases were considered. In the first three cases, one random effect was modeled as either exponentially or uniformly distributed, while the other two random effects were modeled as normally distributed. In addition, one case was investigated with both random error effects uniformly distributed.

When the block effect was exponentially distributed, the behavior of the tests did not deviate significantly from the case of all normally distributed random effects. When the main unit error was exponentially distributed, although all tests had less power than when all random effects were normally distributed, both rank tests usually had superior power to the F-test (see Tables 6.11-6.15). One exception was the model which had both main effects and interaction present, where again the RT had much less power for testing interaction than the other two methods, as can be seen in Table 6.14. Another exception was the model where only interaction was present (see Table 6.15). Here, the F-test was not outperformed, but had slightly more power than either of the two rank tests. Table 6.13 indicates that the RT also had inflated type I error rates in tests for interaction when the model included only both main and sub-unit main effects. When the sub-unit error effect was exponentially distributed, both rank tests had more power than the F-test for all models (see Tables 6.16-6.20). When all fixed effects were in the model, Table 6.19 shows that the power of the ART was clearly superior to the other two, although the drop-off in power for the RT was not as severe as had been observed in previous situations.

Uniformly distributed errors were examined for the models with both main effects present (both alone, and with interaction present), and with only interaction present (see Tables 6.21-6.29). When only one of the error distributions was uniform, the power for all tests was much less than in the normally distributed case, but the relative performance was essentially the same, with very similar power for all tests, except for the rank transform which had much less power when all effects were present. When both errors were uniformly distributed, the power of all methods for testing main and sub-unit treatment effects was diminished even more (compare Tables 6.21, 6.24 and 6.27). When only the main and sub-unit effects were present, the rank transform method again had nominal type I error rates for testing interaction that became inflated as the magnitude of the effects became larger.

Table 6.6.

Proportion of rejections at α=0.05, exponentially distributed block effect, normally distributed main and sub-unit errors, based on 10,000 samples. Sub-unit main effect present (s_1=-c, s_3=c).

		c			
Test for:	Method	0.5	1.5	2.5	3.5
MU Trt	FT	.049	.049	.049	.049
	RT	.046	.047	.045	.048
	ART	.051	.051	.051	.051
SU Trt	FT	.050	1.00	1.00	1.00
	RT	.044	1.00	1.00	1.00
	ART	.048	1.00	1.00	1.00
Interaction	FT	.049	.049	.049	.049
	RT	.044	**.039**	**.034**	**.030**
	ART	.049	.049	.049	.049

Table 6.7.

Proportion of rejections at α=0.05, exponentially distributed block effect, normally distributed main and sub-unit errors, based on 10,000 samples. Main unit main effect present (m_1=c, m_3=-c).

		c			
Test for:	Method	0.5	1.5	2.5	3.5
MU Trt	FT	.088	.471	.896	.996
	RT	.088	.451	.874	.992
	ART	.094	.477	.893	.995
SU Trt	FT	.052	.052	.052	.052
	RT	.049	.048	.047	.051
	ART	.051	.051	.051	.051
Interaction	FT	.049	.049	.049	.049
	RT	.045	.046	.047	.051
	ART	.049	.049	.049	.049

Table 6.8.

Proportion of rejections at $\alpha = .05$, exponentially distributed block effect, normally distributed main and sub-unit errors, based on 10,000 samples. MU and SU main effects present ($m_2=s_1=c$, $m_3=s_2= -c$).

Test for:	Method	c 0.5	1.5	2.5	3.5
MU Trt	FT	.085	.471	.899	.995
	RT	.083	.445	.878	.992
	ART	.093	.480	.896	.994
SU Trt	FT	.500	1.00	1.00	1.00
	RT	.446	1.00	1.00	1.00
	ART	.480	1.00	1.00	1.00
Interaction	FT	.049	.049	.049	.049
	RT	.043	.036	.044	**.087**
	ART	.049	.049	.049	.049

Table 6.9.

Proportion of rejections at $\alpha = .05$, exponentially distributed block effect, normally distributed main and sub-unit errors, based on 10,000 samples. MU, SU main effects and interaction effect present ($ms_{11}=-c$, $s_1=ms_{41}=c$).

Test for:	Method	c 0.5	1.5	2.5	3.5
MU Trt	FT	.052	.088	.168	.302
	RT	.051	.079	.119	.159
	ART	.055	.087	.125	.157
SU Trt	FT	.195	.945	1.00	1.00
	RT	.171	.867	.998	1.00
	ART	.188	.913	1.00	1.00
Interaction	FT	.079	.417	.894	.996
	RT	.067	.258	.516	.675
	ART	.077	.382	.852	.990

Table 6.10.

Proportion of rejections at $\alpha = .05$, exponentially distributed block effect, normally distributed main and sub-unit errors, based on 10,000 samples. Interaction effect present ($ms_{11}=ms_{33}=c$, $ms_{13}=ms_{31}=-c$).

Test for:	Method	c 0.5	1.5	2.5	3.5
MU Trt	FT	.049	.049	.049	.049
	RT	.046	.048	.045	.044
	ART	.051	.049	.047	.049
SU Trt	FT	.052	.052	.052	.052
	RT	.048	**.043**	**.040**	**.037**
	ART	.052	.046	**.039**	**.032**
Interaction	FT	.152	.923	1.00	1.00
	RT	.124	.831	.999	1.00
	ART	.140	.883	.999	1.00

Table 6.11.

Proportion of rejections at $\alpha=0.05$, exponentially distributed main unit errors, normally distributed block effect and sub-unit errors, based on 10,000 samples. Sub-unit main effect present ($s_1=-c$, $s_3=c$).

Test for:	Method	c 0.5	1.5	2.5	3.5
MU Trt	FT	**.041**	**.041**	**.041**	**.041**
	RT	**.057**	.056	.053	.050
	ART	.052	.052	.052	.052
SU Trt	FT	.050	1.00	1.00	1.00
	RT	.042	1.00	1.00	1.00
	ART	.044	1.00	1.00	1.00
Interaction	FT	.049	.049	.049	.049
	RT	**.042**	.045	.048	.047
	ART	.050	.050	.050	.050

Table 6.12.

Proportion of rejections at α=0.05, exponentially distributed main unit errors, normally distributed block effect and sub-unit errors, based on 10,000 samples. Main unit main effect present (m_1=c, m_3=-c).

Test for:	Method	c 0.5	1.5	2.5	3.5
MU Trt	FT	.048	.116	.264	.458
	RT	.068	.159	.329	.520
	ART	.063	.145	.307	.490
SU Trt	FT	.049	.049	.049	.049
	RT	.049	.046	.048	.050
	ART	.050	.050	.050	.050
Interaction	FT	.049	.049	.049	.049
	RT	**.041**	.044	.046	.046
	ART	.050	.050	.050	.050

Table 6.13.

Proportion of rejections at α = .05, exponentially distributed main unit errors, normally distributed block effect and sub-unit errors, based on 10,000 samples. MU and SU main effects present (m_2=s_1=c, m_3=s_2= -c).

Test for:	Method	c 0.5	1.5	2.5	3.5
MU Trt	FT	.050	.113	.267	.458
	RT	.067	.149	.316	.502
	ART	.062	.148	.308	.492
SU Trt	FT	.050	1.00	1.00	1.00
	RT	.042	1.00	1.00	1.00
	ART	.044	1.00	1.00	1.00
Interaction	FT	.049	.049	.050	.049
	RT	.042	.050	**.070**	**.103**
	ART	.050	.050	.050	.050

Table 6.14.

Proportion of rejections at $\alpha = .05$, exponentially distributed main unit errors, normally distributed block effect and sub-unit errors, based on 10,000 samples. MU, SU main effects and interaction effect present (ms_{11}=-c, s_1=ms_{41}=c).

Test for:	Method	c 0.5	1.5	2.5	3.5
MU Trt	FT	.042	.088	.168	.232
	RT	.060	.079	.119	.140
	ART	.053	.087	.125	.145
SU Trt	FT	.195	.945	1.00	1.00
	RT	.167	.867	.998	1.00
	ART	.169	.913	1.00	1.00
Interaction	FT	.078	.417	.894	.976
	RT	.064	.258	.516	.610
	ART	.075	.382	.852	.957

Table 6.15.

Proportion of rejections at $\alpha = .05$, exponentially distributed main unit errors, normally distributed block effect and sub-unit errors, based on 10,000 samples. Interaction effect present (ms_{11}=ms_{33}=c, ms_{13}=ms_{31}=-c).

Test for:	Method	c 0.5	1.5	2.5	3.5
MU Trt	FT	**.041**	**.041**	**.041**	**.041**
	RT	.058	.056	.054	.052
	ART	.054	.052	.048	.047
SU Trt	FT	.049	.049	.049	.049
	RT	.048	.046	.047	.046
	ART	.049	.054	.049	**.042**
Interaction	FT	.146	.914	1.00	1.00
	RT	.116	.775	.990	1.00
	ART	.130	.815	.994	1.00

Table 6.16.

Proportion of rejections at $\alpha=0.05$, exponentially distributed sub-unit errors, normally distributed block effect and main unit errors, based on 10,000 samples. Sub-unit main effect present ($s_1=-c$, $s_3=c$).

Test for:	Method	c 0.5	1.5	2.5	3.5
MU Trt	FT	.051	.051	.051	.051
	RT	.052	.053	.053	.053
	ART	.054	.054	.054	.054
SU Trt	FT	.095	.547	.905	.990
	RT	.133	.689	.963	.998
	ART	.127	.653	.950	.997
Interaction	FT	.044	.044	.044	.044
	RT	.048	.049	.049	.049
	ART	**.058**	**.058**	**.058**	**.058**

Table 6.17.

Proportion of rejections at $\alpha=0.05$, exponentially distributed sub-unit errors, normally distributed block effect and main unit errors, based on 10,000 samples. Main unit main effect present ($m_1=c$, $m_3=-c$).

Test for:	Method	c 0.5	1.5	2.5	3.5
MU Trt	FT	.067	.199	.466	.755
	RT	.076	.250	.549	.816
	ART	.076	.241	.534	.803
SU Trt	FT	**.041**	**.041**	**.041**	**.041**
	RT	.049	.048	.048	.050
	ART	.051	.051	.051	.051
Interaction	FT	.044	.044	.044	.044
	RT	.048	.049	.051	.051
	ART	**.058**	**.058**	**.058**	**.058**

Table 6.18.

Proportion of rejections at $\alpha = .05$, exponentially distributed sub-unit errors, normally distributed block effect and main unit errors, based on 10,000 samples. MU and SU main effects present ($m_2=s_1=c$, $m_3=s_2= -c$).

Test for:	Method	c 0.5	1.5	2.5	3.5
MU Trt	FT	.066	.198	.470	.748
	RT	.074	.234	.513	.770
	ART	.074	.240	.542	.801
SU Trt	FT	.095	.543	.909	.989
	RT	.126	.657	.948	.996
	ART	.125	.655	.952	.997
Interaction	FT	.044	.044	.044	.044
	RT	.049	.049	.049	.055
	ART	**.058**	**.058**	**.058**	**.058**

Table 6.19.

Proportion of rejections at $\alpha = .05$, exponentially distributed sub-unit errors, normally distributed block effect and main unit errors, based on 10,000 samples. MU, SU main effects and interaction effect present ($ms_{11}=-c$, $s_1=ms_{41}=c$).

Test for:	Method	c 0.5	1.5	2.5	3.0
MU Trt	FT	.054	.068	.096	.138
	RT	.055	.070	.094	.120
	ART	.056	.074	.098	.132
SU Trt	FT	.061	.220	.518	.778
	RT	.076	.282	.574	.778
	ART	.076	.274	.582	.805
Interaction	FT	.050	.080	.160	.288
	RT	.055	.094	.155	.227
	ART	.064	.105	.198	.345

Table 6.20.

Proportion of rejections at $\alpha = .05$, exponentially distributed sub-unit errors, normally distributed block effect and main unit errors, based on 10,000 samples. Interaction effect present ($ms_{11}=ms_{33}=c$, $ms_{13}=ms_{31}=-c$).

Test for:	Method	c 0.5	1.5	2.5	3.5
MU Trt	FT	.044	.044	.044	.044
	RT	.054	.052	.051	.051
	ART	.052	.051	.050	.049
SU Trt	FT	.045	.045	.045	.045
	RT	.047	.047	.049	.048
	ART	.054	.054	.054	.054
Interaction	FT	.056	.164	.443	.730
	RT	.061	.194	.470	.793
	ART	.063	.189	.466	.766

Table 6.21.

Proportion of rejections at $\alpha = .05$, uniformly distributed main unit errors, normally distributed block effect and sub-unit errors, based on 10,000 samples. MU and SU main effects present ($m_2=s_1=c$, $m_3=s_2= -c$).

Test for:	Method	c 0.5	1.5	2.5	3.5
MU Trt	FT	.074	.201	.488	.816
	RT	.077	.201	.482	.816
	ART	.077	.203	.481	.800
SU Trt	FT	.487	1.00	1.00	1.00
	RT	.433	1.00	1.00	1.00
	ART	.444	1.00	1.00	1.00
Interaction	FT	.055	.055	.055	.055
	RT	.052	.050	**.074**	**.136**
	ART	.051	.051	.051	.051

Table 6.22.

Proportion of rejections at $\alpha = .05$, uniformly distributed main unit errors, normally distributed block effect and sub-unit errors, based on 10,000 samples. MU, SU main effects and interaction effect present (ms_{11}=-c, s_1=ms_{41}=c).

Test for:	Method	c 0.5	1.5	2.5	3.5
MU Trt	FT	.059	.071	.098	.144
	RT	.063	.072	.087	.106
	ART	.062	.074	.092	.112
SU Trt	FT	.187	.939	1.00	1.00
	RT	.163	.871	.998	1.00
	ART	.172	.895	1.00	1.00
Interaction	FT	.088	.425	.895	.997
	RT	.076	.291	.567	.731
	ART	.078	.372	.828	.984

Table 6.23.

Proportion of rejections at $\alpha = .05$, uniformly distributed main unit errors, normally distributed block effect and sub-unit errors, based on 10,000 samples. Interaction effect present (ms_{11}=ms_{33}=c, ms_{13}=ms_{31}=-c).

Test for:	Method	c 0.5	1.5	2.5	3.5
MU Trt	FT	**.057**	**.057**	**.057**	**.057**
	RT	**.060**	**.059**	.056	.055
	ART	**.061**	**.060**	.055	.053
SU Trt	FT	.055	.055	.055	.055
	RT	.053	.053	.052	.053
	ART	.055	.055	.044	**.032**
Interaction	FT	.160	.916	1.00	1.00
	RT	.134	.827	.997	1.00
	ART	.139	.857	.999	1.00

Table 6.24.

Proportion of rejections at $\alpha = .05$, uniformly distributed sub-unit errors, normally distributed block effect and sub-unit errors, based on 10,000 samples. MU and SU main effects present ($m_2=s_1=c$, $m_3=s_2=-c$).

Test for:	Method	c 0.5	1.5	2.5	3.5
MU Trt	FT	.077	.331	.748	.961
	RT	.076	.315	.714	.948
	ART	.079	.325	.741	.957
SU Trt	FT	.185	.945	1.00	1.00
	RT	.164	.900	1.00	1.00
	ART	.169	.918	1.00	1.00
Interaction	FT	.051	.051	.051	.051
	RT	.047	.048	.053	**.061**
	ART	.050	.050	.050	.050

Table 6.25.

Proportion of rejections at $\alpha = .05$, uniformly distributed sub-unit errors, normally distributed block effect and sub-unit errors, based on 10,000 samples. MU, SU main effects and interaction effect present ($ms_{11}=-c$, $s_1=ms_{41}=c$).

Test for:	Method	c 0.5	1.5	2.5	3.5
MU Trt	FT	.050	.074	.126	.214
	RT	.051	.070	.101	.132
	ART	.052	.072	.111	.155
SU Trt	FT	.096	.485	.926	.999
	RT	.087	.403	.829	.979
	ART	.091	.431	.881	.995
Interaction	FT	.060	.144	.382	.698
	RT	.053	.116	.230	.350
	ART	.059	.135	.333	.629

Table 6.26.

Proportion of rejections at $\alpha = .05$, uniformly distributed sub-unit errors, normally distributed block effect and sub-unit errors, based on 10,000 samples. Interaction effect present ($ms_{11}=ms_{33}=c$, $ms_{13}=ms_{31}=-c$).

		c			
Test for:	Method	0.5	1.5	2.5	3.5
MU Trt	FT	.049	.049	.049	.049
	RT	.047	.047	.047	.047
	ART	.050	.047	.047	.045
SU Trt	FT	.052	.052	.052	.052
	RT	.050	.051	.049	.051
	ART	.052	.052	.049	**.043**
Interaction	FT	.084	.412	.897	.999
	RT	.074	.342	.819	.992
	ART	.078	.361	.847	.996

Table 6.27.

Proportion of rejections at $\alpha = .05$, uniformly distributed main and sub-unit errors, normally distributed block effect, based on 10,000 samples. MU and SU main effects present ($m_2=s_1=c$, $m_3=s_2= -c$).

		c			
Test for:	Method	0.5	1.5	2.5	3.5
MU Trt	FT	.066	.178	.424	.730
	RT	.067	.176	.410	.715
	ART	.069	.185	.424	.729
SU Trt	FT	.185	.948	1.00	1.00
	RT	.166	.902	1.00	1.00
	ART	.169	.915	1.00	1.00
Interaction	FT	.053	.053	.053	.053
	RT	.048	.049	.054	**.062**
	ART	.053	.053	.053	.053

Table 6.28.

Proportion of rejections at $\alpha = .05$, uniformly distributed main and sub-unit errors, normally distributed block effect, based on 10,000 samples. MU, SU main effects and interaction effect present (ms_{11}=-c, s_1=ms_{41}=c).

		c			
Test for:	Method	0.5	1.5	2.5	3.5
MU Trt	FT	.056	.067	.087	.123
	RT	.056	.066	.080	.098
	ART	.059	.069	.086	.108
SU Trt	FT	.091	.488	.927	1.00
	RT	.085	.406	.838	.981
	ART	.086	.434	.878	.994
Interaction	FT	.064	.149	.376	.697
	RT	.057	.118	.244	.388
	ART	.060	.134	.327	.623

Table 6.29.

Proportion of rejections at $\alpha = .05$, uniformly distributed main and sub-unit errors, normally distributed block effect, based on 10,000 samples. Interaction effect present (ms_{11}=ms_{33}=c, ms_{13}=ms_{31}=-c).

		c			
Test for:	Method	0.5	1.5	2.5	3.5
MU Trt	FT	.054	.054	.054	.054
	RT	.055	.055	.054	.049
	ART	**.057**	**.057**	.052	.049
SU Trt	FT	.051	.051	.051	.051
	RT	.050	.050	.049	.049
	ART	.050	.051	.049	.045
Interaction	FT	.084	.406	.890	.999
	RT	.073	.325	.803	.986
	ART	.078	.355	.835	.992

6.2.3. Normal errors, unequal variances (see Tables 6.30-6.52). Two cases were considered. One of the errors was modeled as normally distributed with heterogeneous variances, while the other was modeled as normally distributed with homogeneous variances. In each case, the block effect was modeled as having a standard normal distribution. As in the completely randomized case, different degrees of heterogeneity were considered. For all models, ratios between the largest and the smallest variances of 10:1 (moderately large) and 30:1 (very large) were studied (in addition, some models with very, very large degrees of heterogeneity were observed). As in the completely randomized case, unequal error variances turned out to be a more serious problem than the lack of normality. However, while in the completely randomized case, the performance of the rank tests was generally better than that of the F-test, in the split-unit case the results were mixed.

The power of all tests was lower when the main units had heterogeneous variances, and the power became worse as the degree of the heterogeneity increased, as evidenced in Tables 6.30-6.39. The rank tests had more power for detecting the main unit treatment effect when it was the only effect present. When only the sub-unit effect was present, as in Tables 6.32 and 6.33, the FT actually had slightly more power than either rank test, while all methods had inflated nominal type I error rates for testing main unit treatment effect. When only main unit and sub-unit treatment effects were present (see Tables 6.34 and 6.35), the rank tests had better power for testing for main unit treatment effect, but slightly less power for testing for sub-unit treatment effect. In addition, the RT had nominal type I error rates that increased steadily with increasing effect magnitudes. Tables 6.36 and 6.37 indicate that when all effects were present, the FT had the best power, with the ART close behind and the RT a distant third. When only the interaction effect was present (see Tables 6.38 and 6.39), the results were similar to the equal variance case, where the FT had slightly higher power, except that nominal type I error rates were inflated for all tests when testing for the effect of the main unit treatment (this inflation became more severe as the degree of heterogeneity increased).

The rank tests performed consistently better than the FT when then sub-unit error effect had unequal variances (see Tables 6.40-6.49). When only the effect of the main unit treatment was present, as in Tables 6.40-6.41, the power of the rank tests was higher than that of the FT, although all tests showed a tendency to have inflated nominal type I error rates for testing for sub-unit treatment and interaction effects. When only the sub-unit effect was present, there was essentially no difference in power for the three tests when the maximum to minimum variance ratio was 10:1 (see Table 6.42). When the ratio increased to 30:1 (see Table 6.43), however, the rank tests had more power. For all methods, there was also a slight nominal type I error rate inflation for testing the interaction effect, which became more severe as the variance ratio increased. Surprisingly, the RT showed less inflation than the either the FT or the ART. When only both main and sub-unit effects were modeled, the rank tests were much more powerful, with some nominal type I error rate inflation for testing interaction evident for all methods (see Tables 6.44 and 6.45). However, while the FT and the ART nominal rates remained

Table 6.30.

Proportion of rejections at α=0.05, normally distributed errors, unequal main unit error variances, based on 10,000 samples. Ratio largest:smallest variance 10:1. Main unit main effect present (m_1=c, m_3=-c).

		c			
Test for:	Method	0.5	1.5	2.5	3.5
MU Trt	FT	.075	.180	.407	.677
	RT	.086	.203	.435	.695
	ART	.084	.199	.420	.681
SU Trt	FT	.050	.050	.050	.050
	RT	.049	.049	.049	.050
	ART	.047	.047	.047	.047
Interaction	FT	.052	.052	.052	.052
	RT	.051	.048	.048	.050
	ART	.053	.053	.053	.053

Table 6.31.

Proportion of rejections at α=0.05, normally distributed errors, unequal main unit error variances, based on 10,000 samples. Ratio largest:smallest variance 30:1. Main unit main effect present (m_1=c, m_3=-c).

		c			
Test for:	Method	0.5	1.5	2.5	3.5
MU Trt	FT	.087	.126	.225	.370
	RT	.096	.154	.271	.431
	ART	.091	.146	.257	.400
SU Trt	FT	.050	.050	.050	.050
	RT	.056	.049	.051	.052
	ART	.050	.050	.050	.050
Interaction	FT	.052	.052	.052	.052
	RT	.054	.051	.049	.048
	ART	.050	.050	.050	.050

Table 6.32.

Proportion of rejections at α=0.05, normally distributed errors, unequal main unit error variances, based on 10,000 samples. Ratio largest to smallest variance 10:1. Sub-unit main effect present (s_1=-c, s_3=c).

Test for:	Method	c 0.5	 1.5	 2.5	 3.5
MU Trt	FT	**.065**	**.065**	**.065**	**.065**
	RT	**.072**	**.071**	**.068**	**.067**
	ART	**.071**	**.071**	**.071**	**.071**
SU Trt	FT	.498	1.00	1.00	1.00
	RT	.429	1.00	1.00	1.00
	ART	.447	1.00	1.00	1.00
Interaction	FT	.052	.052	.052	.052
	RT	.049	**.062**	**.068**	**.065**
	ART	.053	.053	.053	.053

Table 6.33.

Proportion of rejections at α=0.05, normally distributed errors, unequal main unit error variances, based on 10,000 samples. Ratio largest to smallest variance 30:1. Sub-unit main effect present (s_1=-c, s_3=c).

Test for:	Method	c 0.5	 1.5	 2.5	 3.5
MU Trt	FT	**.083**	**.083**	**.083**	**.083**
	RT	**.090**	**.092**	**.092**	**.093**
	ART	**.084**	**.084**	**.084**	**.084**
SU Trt	FT	.500	1.00	1.00	1.00
	RT	.416	1.00	1.00	1.00
	ART	.434	1.00	1.00	1.00
Interaction	FT	.052	.052	.052	.052
	RT	.054	**.092**	**.128**	**.128**
	ART	.050	.050	.050	.050

Table 6.34.

Proportion of rejections at α=0.05, normally distributed errors, unequal main unit error variances, based on 10,000 samples. Ratio largest to smallest variance 10:1. MU and SU main effects present (m_2=s_1=c, m_3=s_2= -c).

		c			
Test for:	Method	0.5	1.5	2.5	3.5
MU Trt	FT	.076	.180	.398	.667
	RT	.083	.195	.415	.677
	ART	.084	.198	.420	.678
SU Trt	FT	.509	1.00	1.00	1.00
	RT	.435	1.00	1.00	1.00
	ART	.463	1.00	1.00	1.00
Interaction	FT	.052	.052	.052	.052
	RT	.052	.059	**.076**	**.123**
	ART	.053	.053	.053	.053

Table 6.35.

Proportion of rejections at α=0.05, normally distributed errors, unequal main unit error variances, based on 10,000 samples. Ratio largest to smallest variance 30:1. MU and SU main effects present (m_2=s_1=c, m_3=s_2= -c).

		c			
Test for:	Method	0.5	1.5	2.5	3.5
MU Trt	FT	.088	.130	.223	.366
	RT	.095	.151	.257	.405
	ART	.090	.142	.258	.407
SU Trt	FT	.509	1.00	1.00	1.00
	RT	.422	1.00	1.00	1.00
	ART	.440	1.00	1.00	1.00
Interaction	FT	.052	.052	.052	.052
	RT	.057	**.080**	**.107**	**.120**
	ART	.050	.050	.050	.050

Table 6.36.

Proportion of rejections at α=0.05, normally distributed errors, unequal main unit error variances, based on 10,000 samples. Ratio largest to smallest variance 10:1. MU, SU main effects and interaction effect present (ms_{11}=-c, s_1=ms_{41}=c).

		c			
Test for:	Method	0.5	1.5	2.5	3.5
MU Trt	FT	.067	.077	.098	.131
	RT	.073	.079	.088	.104
	ART	.071	.082	.096	.111
SU Trt	FT	.194	.936	1.00	1.00
	RT	.144	.773	.990	1.00
	ART	.159	.838	.997	1.00
Interaction	FT	.082	.421	.890	.996
	RT	.065	.192	.405	.580
	ART	.076	.340	.797	.974

Table 6.37.

Proportion of rejections at α=0.05, normally distributed errors, unequal main unit error variances, based on 10,000 samples. Ratio largest to smallest variance 30:1. MU, SU main effects and interaction effect present (ms_{11}=-c, s_1=ms_{41}=c).

		c			
Test for:	Method	0.5	1.5	2.5	3.5
MU Trt	FT	.084	.088	.097	.109
	RT	.091	.092	.094	.101
	ART	.085	.087	.094	.103
SU Trt	FT	.194	.936	1.00	1.00
	RT	.133	.691	.969	1.00
	ART	.144	.777	.991	1.00
Interaction	FT	.082	.421	.890	.996
	RT	.067	.152	.302	.458
	ART	.070	.307	.735	.947

Table 6.38.

Proportion of rejections at α=0.05, normally distributed errors, unequal main unit error variances, based on 10,000 samples. Ratio largest to smallest variance 10:1. Interaction effect present ($ms_{11}=ms_{33}=c$, $ms_{13}=ms_{31}=-c$).

		c			
Test for:	Method	0.5	1.5	2.5	3.5
MU Trt	FT	**.065**	**.065**	**.065**	**.065**
	RT	**.071**	**.073**	**.071**	**.071**
	ART	**.071**	**.075**	**.074**	**.071**
SU Trt	FT	.050	.050	.050	.050
	RT	.049	.053	.052	.052
	ART	.053	.051	.048	**.035**
Interaction	FT	.155	.921	1.00	1.00
	RT	.140	.853	.999	1.00
	ART	.146	.879	.999	1.00

Table 6.39.

Proportion of rejections at α=0.05, normally distributed errors, unequal main unit error variances, based on 10,000 samples. Ratio largest to smallest variance 30:1. Interaction effect present ($ms_{11}=ms_{33}=c$, $ms_{13}=ms_{31}=-c$).

		c			
Test for:	Method	0.5	1.5	2.5	3.5
MU Trt	FT	**.083**	**.083**	**.083**	**.083**
	RT	**.090**	**.094**	**.096**	**.099**
	ART	**.086**	**.092**	**.099**	**.099**
SU Trt	FT	.050	.050	.050	.050
	RT	.055	**.063**	**.066**	**.066**
	ART	.049	.055	.053	.049
Interaction	FT	.155	.921	1.00	1.00
	RT	.144	.843	.998	1.00
	ART	.147	.869	.999	1.00

Table 6.40.

Proportion of rejections at α=0.05, normally distributed errors, unequal sub-unit error variances, based on 10,000 samples. Ratio largest:smallest variance 10:1. Main unit main effect present (m_1=c, m_3=-c).

		c			
Test for:	Method	0.5	1.5	2.5	3.5
MU Trt	FT	.071	.262	.626	.901
	RT	.077	.285	.649	.910
	ART	.078	.286	.652	.912
SU Trt	FT	**.064**	**.064**	**.064**	**.064**
	RT	**.062**	**.065**	**.067**	**.065**
	ART	**.060**	**.060**	**.060**	**.060**
Interaction	FT	**.071**	**.071**	**.071**	**.071**
	RT	**.060**	**.063**	**.068**	**.069**
	ART	**.076**	**.076**	**.076**	**.076**

Table 6.41.

Proportion of rejections at α=0.05, normally distributed errors, unequal sub-unit error variances, based on 10,000 samples. Ratio largest:smallest variance 30:1. Main unit main effect present (m_1=c, m_3=-c).

		c			
Test for:	Method	0.5	1.5	2.5	3.5
MU Trt	FT	.062	.150	.352	.620
	RT	.069	.193	.438	.706
	ART	.066	.190	.438	.699
SU Trt	FT	**.074**	**.074**	**.074**	**.074**
	RT	**.074**	**.076**	**.079**	**.078**
	ART	**.068**	**.068**	**.068**	**.068**
Interaction	FT	**.083**	**.083**	**.083**	**.083**
	RT	**.063**	**.073**	**.078**	**.084**
	ART	**.105**	**.105**	**.105**	**.105**

Table 6.42.

Proportion of rejections at α=0.05, normally distributed errors, unequal sub-unit error variances, based on 10,000 samples. Ratio largest to smallest variance 10:1. Sub-unit main effect present (s_1=-c, s_3=c).

		c			
Test for:	Method	0.5	1.5	2.5	3.5
MU Trt	FT	.052	.052	.052	.052
	RT	.051	.054	.051	.051
	ART	.054	.054	.054	.054
SU Trt	FT	.150	.775	.996	1.00
	RT	.149	.777	.997	1.00
	ART	.148	.788	.997	1.00
Interaction	FT	**.071**	**.071**	**.071**	**.071**
	RT	**.058**	.055	.052	.050
	ART	**.076**	**.076**	**.076**	**.076**

Table 6.43.

Proportion of rejections at α=0.05, normally distributed errors, unequal sub-unit error variances, based on 10,000 samples. Ratio largest to smallest variance 30:1. Sub-unit main effect present (s_1=-c, s_3=c).

		c			
Test for:	Method	0.5	1.5	2.5	3.5
MU Trt	FT	.052	.052	.052	.052
	RT	.053	.053	.052	.046
	ART	.052	.052	.052	.052
SU Trt	FT	.107	.364	.789	.974
	RT	.109	.413	.854	.990
	ART	.102	.396	.839	.987
Interaction	FT	**.083**	**.083**	**.083**	**.083**
	RT	**.064**	**.061**	**.059**	**.054**
	ART	**.105**	**.105**	**.105**	**.105**

Table 6.44.

Proportion of rejections at $\alpha=0.05$, normally distributed errors, unequal sub-unit error variances, based on 10,000 samples. Ratio largest to smallest variance 10:1. MU and SU main effects present ($m_2=s_1=c$, $m_3=s_2=-c$).

Test for:	Method	c 0.5	1.5	2.5	3.5
MU Trt	FT	.711	.257	.625	.899
	RT	.767	.268	.609	.874
	ART	.761	.288	.657	.914
SU Trt	FT	.139	.866	1.00	1.00
	RT	.173	.913	1.00	1.00
	ART	.169	.929	1.00	1.00
Interaction	FT	**.071**	**.071**	**.071**	**.071**
	RT	**.060**	**.065**	**.068**	**.069**
	ART	**.076**	**.076**	**.076**	**.076**

Table 6.45.

Proportion of rejections at $\alpha=0.05$, normally distributed errors, unequal sub-unit error variances, based on 10,000 samples. Ratio largest to smallest variance 30:1. MU and SU main effects present ($m_2=s_1=c$, $m_3=s_2=-c$).

Test for:	Method	c 0.5	1.5	2.5	3.5
MU Trt	FT	.063	.155	.350	.619
	RT	.070	.184	.389	.625
	ART	.067	.191	.437	.701
SU Trt	FT	.095	.411	.911	.999
	RT	.131	.666	.985	1.00
	ART	.114	.636	.984	1.00
Interaction	FT	**.083**	**.083**	**.083**	**.083**
	RT	**.065**	**.074**	**.081**	**.083**
	ART	**.105**	**.105**	**.105**	**.105**

Table 6.46.

Proportion of rejections at α=0.05, normally distributed errors, unequal sub-unit error variances, based on 10,000 samples. Ratio largest to smallest variance 10:1. MU, SU main effects and interaction effect present (ms_{11}=-c, s_1=ms_{41}=c).

		c			
Test for:	Method	0.5	1.5	2.5	3.5
MU Trt	FT	.054	.072	.109	.173
	RT	.057	.078	.108	.138
	ART	.058	.082	.119	.158
SU Trt	FT	.087	.351	.812	.988
	RT	.093	.402	.829	.980
	ART	.093	.412	.856	.990
Interaction	FT	.075	.124	.264	.509
	RT	.066	.130	.241	.347
	ART	.085	.144	.304	.560

Table 6.47.

Proportion of rejections at α=0.05, normally distributed errors, unequal sub-unit error variances, based on 10,000 samples. Ratio largest to smallest variance 30:1. MU, SU main effects and interaction effect present (ms_{11}=-c, s_1=ms_{41}=c).

		c			
Test for:	Method	0.5	1.5	2.5	3.5
MU Trt	FT	.053	.059	.079	.111
	RT	.057	.075	.095	.122
	ART	.054	.073	.101	.135
SU Trt	FT	.081	.159	.370	.682
	RT	.090	.240	.537	.816
	ART	.078	.210	.510	.814
Interaction	FT	.085	.102	.143	.219
	RT	.070	.107	.170	.242
	ART	.108	.135	.193	.294

Table 6.48.

Proportion of rejections at α=0.05, normally distributed errors, unequal sub-unit error variances, based on 10,000 samples. Ratio largest to smallest variance 10:1. Interaction effect present ($ms_{11}=ms_{33}=c$, $ms_{13}=ms_{31}=-c$).

		c			
Test for:	Method	0.5	1.5	2.5	3.5
MU Trt	FT	.052	.052	.052	.052
	RT	.054	.056	**.058**	.055
	ART	.055	**.057**	.056	.053
SU Trt	FT	**.064**	**.064**	**.064**	**.064**
	RT	**.065**	**.064**	**.063**	**.061**
	ART	**.059**	**.061**	**.060**	.056
Interaction	FT	.088	.287	.687	.938
	RT	.078	.280	.683	.924
	ART	.098	.310	.696	.932

Table 6.49.

Proportion of rejections at α=0.05, normally distributed errors, unequal sub-unit error variances, based on 10,000 samples. Ratio largest to smallest variance 30:1. Interaction effect present ($ms_{11}=ms_{33}=c$, $ms_{13}=ms_{31}=-c$).

		c			
Test for:	Method	0.5	1.5	2.5	3.5
MU Trt	FT	.052	.052	.052	.052
	RT	**.057**	**.057**	.061	.061
	ART	.052	.054	**.058**	.056
SU Trt	FT	**.074**	**.074**	**.074**	**.074**
	RT	**.073**	**.076**	**.076**	**.074**
	ART	**.066**	**.068**	**.069**	**.068**
Interaction	FT	.092	.159	.315	.547
	RT	.075	.166	.366	.612
	ART	.113	.197	.369	.600

constant as the magnitude of the effects increased, the RT showed its familiar inflation as an increasing function of effect magnitude. When all fixed effects were in the model, the ART had much more power than the other two methods for testing interaction. As can be seen in Tables 6.46 and 6.47, the power of the FT was slightly better than the RT when the variance ratio was 10:1, but fell behind when the ratio increased to 30:1. Finally, with only interaction present in the model (see Tables 6.48 and 6.49), the rank tests had better power for detecting interaction than the FT.

Investigation of the nominal type I error rates when the main or sub-unit variances were unequal revealed a problem of inflated nominal type I error rates similar to that of the completely randomized experiment (see Tables 6.50-6.51). When the main unit variances were heterogeneous, nominal type I error rates for testing the main unit treatment effect were often larger than expected. When the sub-unit variances were heterogeneous, nominal type I error rates for tests for sub-unit treatment and interaction effects were always inflated. However, heterogeneous main unit variances did not adversely affect the nominal levels of the sub-unit tests, and vice-versa. Once again, the inflation of the nominal rates for the RT was often a function of the magnitude of the modeled effects, while the inflation of the nominal rates for the FT and the ART seemed to be independent of the effect magnitude. Once more this indicates that when error variances are heterogeneous, test results may be misleading, especially when testing for interaction. Table 6.52 indicates that this was not a problem when one of the underlying populations was skewed (exponentially distributed).

Table 6.50.

Nominal type I error rate at $\alpha=0.05$, normally distributed errors, based on 10,000 samples. Unequal main unit variances.

Test for:	Method	$Var_{max}:Var_{min}$ 1:1	10:1	30:1	50:1
MU Trt	FT	.053	**.065**	**.083**	**.090**
	RT	.056	**.072**	**.090**	**.097**
	ART	**.060**	**.071**	**.084**	**.085**
SU Trt	FT	.050	.050	.050	.050
	RT	.048	.051	.056	.054
	ART	.052	.047	.050	.050
Interaction	FT	.052	.052	.052	.052
	RT	.047	.051	.051	.053
	ART	.053	.053	.050	.050

Table 6.51.

Nominal type I error rate at $\alpha=0.05$, normally distributed errors, based on 10,000 samples. Unequal sub-unit variances.

Test for:	Method	$Var_{max}:Var_{min}$ 1:1	10:1	30:1	50:1
MU Trt	FT	.053	.052	.052	.052
	RT	.056	.052	.055	.055
	ART	**.060**	.054	.052	.052
SU Trt	FT	.050	**.064**	**.074**	**.078**
	RT	.048	**.064**	**.073**	**.075**
	ART	.052	**.060**	**.068**	**.069**
Interaction	FT	.052	**.071**	**.083**	**.089**
	RT	.047	**.060**	**.065**	**.065**
	ART	.053	**.076**	**.105**	**.118**

Table 6.52.

Nominal type I error rate at α=0.05, one random exponentially distributed, the other two random effects normally distributed, based on 10,000 samples.

Test for:	Method	Exponentially distributed: Block effect	Main unit errors	Sub-unit errors
MU Trt	FT	.049	.041	.051
	RT	.049	.059	.052
	ART	.051	.052	.054
SU Trt	FT	.052	.049	.041
	RT	.050	.049	.049
	ART	.051	.050	.051
Interaction	FT	.049	.049	.044
	RT	.046	.040	.048
	ART	.049	.050	.058

6.3 Conclusion for Analysis of Split-unit Experiments

Although the results were not as consistent as for the completely randomized case, the aligned rank procedure appears to be viable alternative to the normal theory F-test for performing tests in a split-unit factorial design, and is certainly a better choice than the rank transform method. Once more, when the error distributions were normal and error variances were homogeneous (situations in which the F-test is known to work well), the ART was always nearly as powerful, with usually an almost negligible difference in power between the two methods. For exponential error distributions, the ART was clearly more powerful than the F-test. When the error variances were heterogeneous, both methods tended to have problems maintaining nominal type I error levels for interaction, although this problem was less severe in the split-unit case, while the ART usually had superior power for detecting main effects. Although the FT outperformed the ART in some cases, even when parametric assumptions were violated, the ART still appears in general to be superior to the F-test, especially when the assumptions of normality and homogeneity of variance are suspected to be violated. Even though the simulation results indicate that a nonexistent interaction effect can be introduced when error variances are unequal, this phenomenon occurs for both the FT and the ART. Since typically the analysis is performed without the benefit of definite knowledge of the nature of the error variances, and since the ART generally has more power than the FT when variances are unequal, the ART seems a logical choice over the FT. The results once again suggest that the ART procedure could possibly benefit from an additional adjustment to stabilize variances, perhaps by scaling to correct for unequal error variances. The unpredictable performance of the RT for the split-unit experiment adds to the growing body of evidence that the RT is not a good choice for multi-factor experiments.

CHAPTER SEVEN

EPILOGUE

7.1. Approximation of Exact Distributions of Rank Statistics Using the F-Distribution

The goal of this research was to develop an exact rank test applicable to a wide variety of factorial designs. Thus, for certain designs, the exact sampling distributions of certain F-ratio statistics computed on the ranks of the data were estimated, and these were used in the simulations in this paper. One somewhat surprising result was that the upper tails of these estimated exact sampling distributions were approximated well by the F-distribution (although the approximation becomes poorer beyond the 95th percentile). See table 7.1 as an example for the two-way layout. Similar results were obtained for the split-unit design, although the F-distribution consistently underestimated the exact values, which would result in a more liberal test if the F-approximation was used (see table 7.2). Although Hora and Conover (1984) showed that the F-distribution is the limiting distribution of the F-ratio statistic computed using the ranks for the two-way layout, it was suspected that for small sample sizes this would not necessarily be true. It appears, however, that the F-distribution gives a reasonable approximation for the sampling distributions of F-ratio statistics computed using the ranks of the data, even for small sample sizes.

Table 7.1.

Comparison of the percentiles of the sampling distributions of F-ratios computed using the ranks of the data. All ratios are computed as MS (effect)/MS (error), for a two-way layout with four levels of factor A and three levels of factor B, in a completely randomized design, where n is the number of observations per treatment combination. "Exact" values are actually estimates, based on a sample of 20,000 permutations of the ranks.

n	Effect	Percentile point .90		.95		.99	
		Exact	F	Exact	F	Exact	F
2	A	2.669	2.660	3.560	3.587	6.000	6.217
	B	2.820	2.860	3.914	3.982	7.098	7.206
	AB	2.356	2.389	3.056	3.095	4.814	5.069
5	A	2.175	2.202	2.816	2.798	4.320	4.218
	B	2.396	2.417	3.207	3.191	5.296	5.077
	AB	1.920	1.901	2.322	2.295	3.282	3.204
10	A	2.118	2.135	2.680	2.689	4.003	3.968
	B	2.345	2.352	3.125	3.080	5.088	4.807
	AB	1.822	1.829	2.183	2.184	2.986	2.973
20	A	2.136	2.108	2.644	2.644	3.902	3.869
	B	2.325	2.326	3.038	3.035	4.785	4.699
	AB	1.802	1.800	2.146	2.138	2.866	2.882

Table 7.2.

Comparison of the percentiles of the sampling distributions of F-ratios computed using the ranks of the data. All ratios are computed as MS (effect)/MS (error), for a two-way factorial in a split-plot design with four levels of the main unit treatment in a completely randomized block design with three blocks and three levels of the sub-unit treatment. "Exact" values are actually estimates, based on a sample of 20,000 permutations of the ranks.

Effect	Percentile point					
	.90		.95		.99	
	Exact	F	Exact	F	Exact	F
MU Trt	3.363	3.289	4.830	4.757	10.200	9.780
SU Trt	2.712	2.668	3.666	3.634	6.569	6.226
Interaction	2.218	2.178	2.792	2.741	4.352	4.202

7.2. Extending the Aligned Rank Technique to Experiments with More than Two Factors.

The aligned rank procedure discussed previously can be adapted to analyze experiments with more than two factors. Higgins and Tashtoush (1994) suggest a pattern for aligning observations in completely randomized designs for testing higher order interactions. For example, to test for three-way interaction in a three factor experiment, the alignment suggested is:

$$(AY)_{ijk} = Y_{ijkl} - (\text{sum of 2-way means involving i, j, and k})$$
$$+ (\text{sum of one-way means involving i, j, and k})$$
$$- \text{overall mean}$$

The pattern for more than three factors is apparent. After aligning the data, the data are ranked, and tests are carried out by applying the usual analysis of variance to the ranked data, ignoring all tests so obtained except for the test of interaction of interest (Higgins and Tashtoush, 1994).

7.3. Future Research

Since the ART generally has better power than the FT when variances are unequal, there is interest in trying to alleviate the problem of inflated nominal type I error rates for the ART. A possible improvement would be to scale the observations in some way to remove the "effect" of unequal variance. Another area to investigate is the application of the ART to situations where sample sizes are unequal, since this is also a situation where the FT often suffers a loss of power. In addition, since no known statistical software packages perform the aligned rank procedure, future work may include developing SAS programs for use in analyzing data in factorial arrangements using the ART.

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APPENDIX

Program 1.

PROGRAM TO FIND THE EXACT (TAIL) DISTRIBUTION OF THE F-RATIO STATISTIC COMPUTED USING THE RANKS OF THE DATA. TWO FACTORS WITH TWO LEVELS EACH AND TWO OBSERVATIONS PER TREATMENT IN A COMPLETELY RANDOMIZED DESIGN.

```
  INTEGER IG(100,3),IDF(8),IC1(20),IC2(8,8),NL(20),M,N,P,Q,OMIT
  REAL R(8),MS(8),SS(8),SUM1(20),SS1(20),SUM2(2,2),SS2(20,20),F
  DATA CM/0.0/, NL/20*0/, IC1/20*0/, NREP/0/
  DATA IG(1,1),IG(1,2),IG(2,1),IG(2,2),IG(3,1),IG(3,2),IG(4,1),
1 IG(4,2),IG(5,1),IG(5,2),IG(6,1),IG(6,2),IG(7,1),IG(7,2),IG(8,1),
2 IG(8,2)/1,1,1,1,2,1,2,1,1,2,1,2,2,2,2,2/, IC2/64*0/
  NC=8
  NREP=2
  NPERMS=0
  NF=2
  NP=0
  OMIT=0
  OPEN (UNIT=1,FILE='TWDATA',ACCESS='SEQUENTIAL',FORM='FORMATTED',
1 STATUS='NEW')

  DO 300 I=1,NC
  DO 295 J=1,NC
  IF (J.NE.I) THEN
  DO 290 K=1,NC
  IF (K.NE.I .AND. K.NE.J) THEN
  DO 285 L=1,NC
  IF (L.NE.I .AND. L.NE.J .AND. L.NE.K) THEN
  DO 280 M=1,NC
  IF (M.NE.L .AND. M.NE.K .AND. M.NE.J .AND. M.NE.I) THEN
  DO 275 N=1,NC
  IF (N.NE.M .AND. N.NE.L .AND. N.NE.K .AND. N.NE.J .AND. N.NE.I) THEN
  DO 270 P=1,NC
  IF (P.NE.N .AND. P.NE.M .AND. P.NE.L .AND. P.NE.K .AND. P.NE.J .AND. P.NE.I) THEN
  DO 265 Q=1,NC
  IF (Q.NE.P .AND. Q.NE.N .AND. Q.NE.M .AND. Q.NE.L.AND. Q.NE.K .AND. Q.NE.J .AND.
     Q.NE.I) THEN
  R(1)=I
  R(2)=J
  R(3)=K
  R(4)=L
  R(5)=M
  R(6)=N
  R(7)=P
```

```
      R(8)=Q
      NPERMS=NPERMS+1

C     TWO FACTOR ANALYSIS

C     CALCULATE SS FOR MAIN EFFECTS

      SUMX=0.0
      SST=0
      DO 166 I1=1,NC
      SUMX=SUMX+R(I1)
      SST=SST+(R(I1))**2
      IC1(IG(I1,1))=IC1(IG(I1,1))+1
      DO 30 K11=1,NF
      IF (IG(I1,K11).GT.NL(K11)) THEN
      NL(K11)=IG(I1,K11)
      END IF
   30 CONTINUE
      IC2(IG(I1,1),IG(I1,2))=IC2(IG(I1,1),IG(I1,2))+1
  166 CONTINUE
      CM=SUMX**2/NC
      SST=SST-CM
      IDF(1)=NL(1)-1
      IDF(2)=NL(2)-1
      IDF(3)=IDF(1)*IDF(2)
      IDF(4)=NL(1)*NL(2)*(NREP-1)
      NP=3
      IF (NREP.EQ.1) THEN
      NP=NP-1
      ENDIF
      DO 160 J1=1,2
      DO 160 K1=1,NL(J1)
  160 SUM2(K1,J1)=0.0
      DO 170 I11=1,NC
      DO 170 J2=1,2
  170 SUM2(IG(I11,J2),J2)=SUM2(IG(I11,J2),J2)+R(I11)
      DO 180 J3=1,2
      SS(J3)=0.0
      DO 190 K2=1,NL(J3)
  190 SS(J3)=SS(J3)+(SUM2(K2,J3))**2
      MM=NC/NL(J3)
  180 SS(J3)=SS(J3)/MM-CM

C     CALCULATE INTERACTION SS

      DO 200 I2=1,NL(1)
      DO 200 J4=1,NL(2)
      SUM2(I2,J4)=0.0
      SS2(I2,J4)=0.0
  200 CONTINUE
      DO 210 I3=1,NC
      SUM2(IG(I3,1),IG(I3,2))=SUM2(IG(I3,1),IG(I3,2))+R(I3)
  210 SS2(IG(I3,1),IG(I3,2))=SS2(IG(I3,1),IG(I3,2))+(R(I3))**2
      SS(3)=0.0
```

```
      DO 220 I4=1,NL(1)
      DO 220 J5=1,NL(2)
  220 SS(3)=SS(3)+(SUM2(I4,J5))**2
      SS(3)=SS(3)/NREP-CM-SS(1)-SS(2)

C     FIND ERROR SUM OF SQUARES AND MEAN SQUARES

      SS(4)=SST-SS(1)-SS(2)-SS(3)
      IF (NP.EQ.3) THEN
      MS(4)=SS(4)/IDF(4)
      END IF
      DO 230 I5=1,3
  230 MS(I5)=SS(I5)/IDF(I5)

      IF (MS(4).EQ.0.0) THEN
      F=9999.0
      ELSE
      F=MS(3)/MS(4)
      END IF
      IF (F.GT.1.31) THEN
      WRITE (1,*) F
      ELSE
      OMIT=OMIT+1
      END IF
      END IF
  265 CONTINUE
      END IF
  270 CONTINUE
      END IF
  275 CONTINUE
      END IF
  280 CONTINUE
      END IF
  285 CONTINUE
      END IF
  290 CONTINUE
      END IF
  295 CONTINUE
  300 CONTINUE

      CLOSE (UNIT=1)

      END
```

Program 2.

PROGRAM TO FIND THE ESTIMATED EXACT SAMPLING DISTRIBUTION OF F-RATIO STATISTIC COMPUTED ON THE RANKS OF THE DATA, FOR A 4 BY 3 FAT IN A COMPLETELY RANDOMIZED DESIGN.

```
      USE MSIMSL
      INTEGER IG(24,3),IDF(8),IC1(20),IC2(24,24),NL(20)
      INTEGER IPER(24), ISEED,SUMX,SST,NOUT,IPERM(20000),Z,A
      INTEGER J,FRQ(20000),C,HOLD(500)
      REAL MS(8),SS(8),SUM2(20,20),SS2(20,20),F
      REAL LIST(20000),TEMP(20000),CUM,PVAL
      DATA CM/0.0/, NL/20*0/, IC1/20*0/, NREP/0/,FRQ/20000*1/
      DATA IC2/576*-1.0/,LIST/20000*9999.0/

      NR=0
      NC=24
      NL(1)=4
      NL(2)=3
      NREP=2
      NPERMS=20000
      NF=2
      NP=0
      Z=1
      INCX=1

C     ROUTINE TO FILL IG VECTOR

      C=1
      DO 2 I=1,NL(1)
      DO 4 J=1,NL(2)
      DO 6 K=1,NREP
      HOLD(C)=J
      HOLD(C+1)=I
      C=C+2
 6    CONTINUE
 4    CONTINUE
 2    CONTINUE
      C=1
      DO 12 I=1,NC
      DO 14 J=NF,1,-1
      IG(I,J)=HOLD(C)
      C=C+1
14    CONTINUE
12    CONTINUE

      OPEN (UNIT=4,FILE='C:\MSDEV\DATA\TW432AR.TXT')
      CALL UMACH(2,NOUT)
      ISEED=62064
      CALL RNSET(ISEED)
      DO 1 A=1,NPERMS
      CALL RNPER(NC,IPER)
```

```
C     TWO FACTOR ANALYSIS

C     CALCULATE SS FOR MAIN EFFECTS
      SUMX=0
      SST=0
      DO 10 I=1,NC
      SUMX=SUMX+IPER(I)
      SST=SST+(IPER(I))**2
      IC1(IG(I,1))=IC1(IG(I,1))+1
      IC2(IG(I,1),IG(I,2))=IC2(IG(I,1),IG(I,2))+1
 10   CONTINUE
      CM=SUMX**2/NC
      SST=SST-CM
      IDF(1)=NL(1)-1
      IDF(2)=NL(2)-1
      IDF(3)=IDF(1)*IDF(2)
      IDF(4)=NL(1)*NL(2)*(NREP-1)
      NP=3
      IF (NREP.EQ.1) THEN
      NP=NP-1
      ENDIF
      DO 210 J=1,2
      DO 210 K=1,NL(J)
      SUM2(K,J)=0.0
210   CONTINUE
      DO 220 I=1,NC
      DO 220 J=1,2
      SUM2(IG(I,J),J)=SUM2(IG(I,J),J)+IPER(I)
220   CONTINUE
      DO 230 J=1,2
      SS(J)=0.0
      DO 240 K=1,NL(J)
      SS(J)=SS(J)+(SUM2(K,J))**2
240   CONTINUE
      MM=NC/NL(J)
      SS(J)=SS(J)/MM-CM
230   CONTINUE

C     CALCULATE INTERACTION SS

      DO 250 I=1,NL(1)
      DO 250 J=1,NL(2)
      SUM2(I,J)=0.0
      SS2(I,J)=0.0
250   CONTINUE
      DO 260 I=1,NC
      SUM2(IG(I,1),IG(I,2))=SUM2(IG(I,1),IG(I,2))+IPER(I)
      SS2(IG(I,1),IG(I,2))=SS2(IG(I,1),IG(I,2))+(IPER(I))**2
260   CONTINUE
      SS(3)=0.0
      DO 270 I=1,NL(1)
      DO 270 J=1,NL(2)
      SS(3)=SS(3)+(SUM2(I,J))**2
```

```
270 CONTINUE
    SS(3)=SS(3)/NREP-CM-SS(1)-SS(2)

C   FIND ERROR SUM OF SQUARES AND MEAN SQUARES

    SS(4)=SST-SS(1)-SS(2)-SS(3)
    IF (NP.EQ.3) THEN
    MS(4)=SS(4)/IDF(4)
    END IF
    DO 280 I=1,3
280 MS(I)=SS(I)/IDF(I)
    IF (NP.EQ.3) THEN
    IF (MS(4).EQ.0.0) THEN
    F=999.0
    ELSE
    F=MS(1)/MS(3)
    END IF
    END IF
    DO 300 I=1,NPERMS
    IPERM(I)=I
300 CONTINUE

C   ROUTINE TO CREATE TABLE OF CRITICAL VALUES AND PROPORTIONS

    CALL SRCH(NPERMS,F,LIST,INCX,INDEX)
    IF (INDEX .LT. 0) THEN
    LIST(Z)=F
    Z=Z+1
    ELSE
    FRQ(INDEX)=FRQ(INDEX)+1
    NR=NR+1
    END IF
    CALL SVRGP(NPERMS,LIST,LIST,IPERM)
    DO 310 I=1,NPERMS
    TEMP(I)=FRQ(IPERM(I))
310 CONTINUE
    DO 320 I=1,NPERMS
    FRQ(I)=TEMP(I)
320 CONTINUE

  1 CONTINUE

C   END OF MAIN LOOP

C   ROUTINE TO WRITE CRITCAL VALUES AND PROPORTIONS

    CUM=0.0
    DO 330 I=1,NPERMS
    IF (LIST(I) .LT. 9999.0) THEN
    CUM=CUM+FRQ(I)
    PVAL=1-(CUM-1)/REAL(NPERMS)
    IF (PVAL .LE. 0.101) THEN
    WRITE (4,*) LIST(I),PVAL
    END IF
```

```
      END IF
330   CONTINUE

      END
```
□

Program 3.

PROGRAM TO FIND ESTIMATED EXACT SAMPLING DISTRIBUTION FOR F-RATIO STATISTICS COMPUTED ON THE RANKS OF THE DATA IN A THREE FACTOR COMPLETELY RANDOMIZED DESIGN

```
      INTEGER IG(48,3),IDF(8),IC1(20),NL(20),NN
      INTEGER IPER(16), ISEED,K,SUMX,SST,NOUT,IC3(20,20,20),NPERMS
      INTEGER INCX,INDEX,FRQ(20000),TEMP(20000),IPERM(20000),Z,COUNT
      INTEGER LENGTH
      REAL MS(8),SS(8),SUM2(20,20),F,CUM
      REAL SUM3(20,20,20),SS3(20,20,20),M,LIST(20000)
      DATA CM/0.0/, NL/20*0/, IC1/20*0/, NREP/0/
      DATA IG(1,1),IG(1,2),IG(1,3),IG(2,1),IG(2,2),IG(2,3),IG(3,1),
     1 IG(3,2),IG(3,3),IG(4,1),IG(4,2),IG(4,3),IG(5,1),IG(5,2),IG(5,3),
     2 IG(6,1),IG(6,2),IG(6,3),IG(7,1),IG(7,2),IG(7,3),IG(8,1),IG(8,2),
     3 IG(8,3),IG(9,1),IG(9,2),IG(9,3),IG(10,1),IG(10,2),IG(10,3),
     4 IG(11,1),IG(11,2),IG(11,3),IG(12,1),IG(12,2),IG(12,3),IG(13,1),
     5 IG(13,2),IG(13,3),IG(14,1),IG(14,2),IG(14,3),IG(15,1),
     6 IG(15,2),IG(15,3),IG(16,1),IG(16,2),IG(16,3)/1,1,1,1,
     7 1,1,1,1,2,1,1,2,1,2,1,1,2,1,1,2,2,1,2,2,2,1,1,2,1,1,2,1,2,2,1,2,
     8 2,2,1,2,2,1,2,2,2,2,2,2/,IC3/8000*-1.0/,
     9 FRQ/20000*1/,LIST/20000*-999.0/
      NN=16
      NC=16
      NREP=2
      NPERMS=10000
      NF=3
      NP=0
      INCX=1
      Z=1
      NL(1)=2
      NL(2)=2
      NL(3)=2
      COUNT=0
      LENGTH=20000
      OPEN (UNIT=4,FILE='C:\MSDEV\DATA\OUT3W.TXT')
      CALL UMACH(2,NOUT)
```

```
      ISEED=40396
      CALL RNSET(ISEED)
      DO 1 A=1,NPERMS
      CALL RNPER(NN,IPER)

C     CALCULATE SS FOR MAIN EFFECTS

      SUMX=0
      SST=0
      DO 10 I=1,NC
      SUMX=SUMX+IPER(I)
      SST=SST+(IPER(I))**2
      IC1(IG(I,1))=IC1(IG(I,1))+1
      DO 20 K=1,NF
      IF (IG(I,K) .GT. NL(K)) THEN
      NL(K)=IG(I,K)
      END IF
 20   CONTINUE
      IC3(IG(I,1),IG(I,2),IG(I,3))=IC3(IG(I,1),IG(I,2),IG(I,3))+1
 10   CONTINUE
      CM=SUMX**2/NC
      SST=SST-CM

C     THREE FACTOR ANALYSIS

 300  IDF(1)=NL(1)-1
      IDF(2)=NL(2)-1
      IDF(3)=NL(3)-1
      IDF(4)=IDF(1)*IDF(2)
      IDF(5)=IDF(1)*IDF(3)
      IDF(6)=IDF(2)*IDF(3)
      IDF(7)=IDF(4)*IDF(3)
      IDF(8)=NL(1)*NL(2)*NL(3)*(NREP-1)
      NP=7
      IF (NREP .EQ. 1) NP=NP-1
      DO 305 I=1,3
      DO 305 J=1,NL(I)
      SUM2(J,I)=0.0
 305  CONTINUE

C     FIND SS FOR MAIN EFFECTS

      DO 310 I=1,NC
      DO 310 J=1,3
      SUM2(IG(I,J),J)=SUM2(IG(I,J),J)+IPER(I)
 310  CONTINUE
      DO 315 J=1,3
      SS(J)=0.0
      DO 320 K=1,NL(J)
      SS(J)=SS(J)+SUM2(K,J)**2
 320  CONTINUE
      M=REAL(NC)/REAL(NL(J))
      SS(J)=SS(J)/M-CM
 315  CONTINUE
```

```
C     FIND SS FOR TWO FACTOR INTERACTIONS

      NLMAX=MAX(NL(1),NL(2),NL(3))
      DO 325 I=1,NLMAX
      DO 325 J=1,NLMAX
      DO 325 K=1,3
      SUM3(I,J,K)=0.0
  325 CONTINUE
      DO 330 I=1,NC
      SUM3(IG(I,1),IG(I,2),1)=SUM3(IG(I,1),IG(I,2),1)+IPER(I)
      SUM3(IG(I,1),IG(I,3),2)=SUM3(IG(I,1),IG(I,3),2)+IPER(I)
      SUM3(IG(I,2),IG(I,3),3)=SUM3(IG(I,2),IG(I,3),3)+IPER(I)
  330 CONTINUE
      SS(4)=0.0
      DO 335 I=1,NL(1)
      DO 335 J=1,NL(2)
      SS(4)=SS(4)+SUM3(I,J,1)**2
  335 CONTINUE
      SS(4)=SS(4)/(NL(3)*NREP)-SS(1)-SS(2)-CM
      SS(5)=0.0
      DO 340 I=1,NL(1)
      DO 340 K=1,NL(3)
      SS(5)=SS(5)+SUM3(I,K,2)**2
  340 CONTINUE
      SS(5)=SS(5)/(NL(2)*NREP)-SS(1)-SS(3)-CM
      SS(6)=0.0
      DO 345 J=1,NL(2)
      DO 345 K=1,NL(3)
      SS(6)=SS(6)+SUM3(J,K,3)**2
  345 CONTINUE
      SS(6)=SS(6)/(NL(1)*NREP)-SS(2)-SS(3)-CM

C     FIND SS FOR THREE FACTOR INTERACTION AND ERROR

      IF (NREP .GT. 1) GOTO 350
      SS(7)=SST-SS(1)-SS(2)-SS(3)-SS(4)-SS(5)-SS(6)
      SS(8)=0.0
      GOTO 355
  350 DO 360 I=1,NL(1)
      DO 360 J=1,NL(2)
      DO 360 K=1,NL(3)
      SUM3(I,J,K)=0.0
      SS3(I,J,K)=0.0
  360 CONTINUE
      DO 365 I=1,NC
      SUM3(IG(I,1),IG(I,2),IG(I,3))=SUM3(IG(I,1),IG(I,2),IG(I,3))
     1 +IPER(I)
      SS3(IG(I,1),IG(I,2),IG(I,3))=SS3(IG(I,1),IG(I,2),IG(I,3))
     1 +IPER(I)**2
  365 CONTINUE
      SS(7)=0.0
      DO 370 I=1,NL(1)
      DO 370 J=1,NL(2)
```

```
      DO 370 K=1,NL(3)
      SS(7)=SS(7)+SUM3(I,J,K)**2
  370 CONTINUE
      SS(7)=SS(7)/NREP-SS(1)-SS(2)-SS(3)-SS(4)-SS(5)-SS(6)-CM
      SS(8)=SST-SS(1)-SS(2)-SS(3)-SS(4)-SS(5)-SS(6)-SS(7)

C     FIND MEAN SQUARES AND F-VALUES

      IF (NP .EQ. 7) MS(8)=SS(8)/IDF(8)
  355 DO 375 I=1,7
      MS(I)=SS(I)/IDF(I)
  375 CONTINUE
      IF (MS(8) .EQ. 0.0) THEN
      F=999.0
      ELSE
      F=MS(1)/MS(8)
      END IF
      DO 380 IS=1,LENGTH
      IPERM(IS)=IS
  380 CONTINUE

C     FILL DATA FILE WITH UNIQUE F-VALUES AND FREQUENCIES

      CALL SRCH(LENGTH,F,LIST,INCX,INDEX)
      IF (INDEX .LT. 0) THEN
      LIST(Z)=F
      Z=Z+1
      ELSE
      FRQ(INDEX)=FRQ(INDEX)+1
      COUNT=COUNT+1
      END IF
      CALL SVRGP(LENGTH,LIST,LIST,IPERM)
      DO 385 IP=1,LENGTH
      TEMP(IP)=FRQ(IPERM(IP))
  385 CONTINUE
      DO 390 IC=1,LENGTH
      FRQ(IC)=TEMP(IC)
  390 CONTINUE

    1 CONTINUE

C     WRITE DISTRIBUTION OF F TO FILE

      CUM=0.0
      DO 395 I=1,LENGTH
      IF (LIST(I) .GE. 0.0) THEN
      CUM=CUM+FRQ(I)
      WRITE (4,*) LIST(I),FRQ(I),CUM/REAL(NPERMS)
      END IF
  395 CONTINUE

      CLOSE (UNIT=4)

      END
```

Program 4.

```
PROGRAM "3WAY " TO PERFORM RANDOMIZATION TEST FOR THREE FACTOR
ANALYSIS OF VARIANCE


 INTEGER IG(36,3),IDF(8),IC1(20),NL(20),NN,NRM,NRS,NRI
 INTEGER IPER(36), ISEED,K,SUMX,SST,NOUT,IC3(20,20,20),NPERMS
 INTEGER INCX,INDEX,FRQM(20000),TEMP(20000),IPERM(20000),COUNT
 INTEGER ZM,ZS,ZI,FRQS(20000),FRQI(20000)
 REAL LISTI(20000)
 REAL MS(8),SS(8),SUM2(20,20),FMAIN,FSUB,FINT,CUM,PVALM,PVALS,PVALI
 REAL SUM3(20,20,20),SS3(20,20,20),M,LISTM(20000),LISTS(20000)
 DATA CM/0.0/, NL/20*0/, IC1/20*0/, NREP/0/
 DATA IG(1,1),IG(1,2),IG(1,3),IG(2,1),IG(2,2),IG(2,3)/1,1,1,1,1,2/
 DATA IG(3,1),IG(3,2),IG(3,3),IG(4,1),IG(4,2),IG(4,3)/1,1,3,1,2,1/
 DATA IG(5,1),IG(5,2),IG(5,3),IG(6,1),IG(6,2),IG(6,3)/1,2,2,1,2,3/
 DATA IG(7,1),IG(7,2),IG(7,3),IG(8,1),IG(8,2),IG(8,3)/1,3,1,1,3,2/
 DATA IG(9,1),IG(9,2),IG(9,3),IG(10,1),IG(10,2)/1,3,3,1,4/
 DATA IG(10,3),IG(11,1),IG(11,2),IG(11,3),IG(12,1)/1,1,4,2,1/
 DATA IG(12,2),IG(12,3),IG(13,1),IG(13,2),IG(13,3)/4,3,2,1,1/
 DATA IG(14,1),IG(14,2),IG(14,3),IG(15,1),IG(15,2)/2,1,2,2,1/
 DATA IG(15,3),IG(16,1),IG(16,2),IG(16,3),IG(17,1)/3,2,2,1,2/
 DATA IG(17,2),IG(17,3),IG(18,1),IG(18,2),IG(18,3)/2,2,2,2,3/
 DATA IG(19,1),IG(19,2),IG(19,3),IG(20,1),IG(20,2)/2,3,1,2,3/
 DATA IG(20,3),IG(21,1),IG(21,2),IG(21,3),IG(22,1)/2,2,3,3,2/
 DATA IG(22,2),IG(22,3),IG(23,1),IG(23,2),IG(23,3)/4,1,2,4,2/
 DATA IG(24,1),IG(24,2),IG(24,3),IG(25,1),IG(25,2)/2,4,3,3,1/
 DATA IG(25,3),IG(26,1),IG(26,2),IG(26,3),IG(27,1)/1,3,1,2,3/
 DATA IG(27,2),IG(27,3),IG(28,1),IG(28,2),IG(28,3)/1,3,3,2,1/
 DATA IG(29,1),IG(29,2),IG(29,3),IG(30,1),IG(30,2)/3,2,2,3,2/
 DATA IG(30,3),IG(31,1),IG(31,2),IG(31,3),IG(32,1)/3,3,3,1,3/
 DATA IG(32,2),IG(32,3),IG(33,1),IG(33,2),IG(33,3)/3,2,3,3,3/
 DATA IG(34,1),IG(34,2),IG(34,3),IG(35,1),IG(35,2)/3,4,1,3,4/
 DATA IG(35,3),IG(36,1),IG(36,2),IG(36,3)/2,3,4,3/
 DATA IC3/8000*-1.0/, FRQM/20000*1/,LISTM/20000*-999.0/
 DATA FRQS/20000*1/,LISTS/20000*-999.0/
 DATA FRQI/20000*1/,LISTI/20000*-999.0/

 NN=36
 NC=36
 NREP=1
 NPERMS=20000
 NF=3
 NP=0
 INCX=1
 ZM=1
 ZS=1
 ZI=1
 NL(1)=3
 NL(2)=4
 NL(3)=3
```

```
      COUNT=0
      OPEN (UNIT=4,FILE='C:\WINDOWS\SCOTT\3WAYDAT2.TXT')
      CALL UMACH(2,NOUT)
      ISEED=40396
      CALL RNSET(ISEED)
      DO 1 A=1,NPERMS
      CALL RNPER(NN,IPER)

C     CALCULATE SS FOR MAIN EFFECTS

      SUMX=0
      SST=0
      DO 10 I=1,NC
      SUMX=SUMX+IPER(I)
      SST=SST+(IPER(I))**2
      IC1(IG(I,1))=IC1(IG(I,1))+1
      DO 20 K=1,NF
      IF (IG(I,K) .GT. NL(K)) THEN
      NL(K)=IG(I,K)
      END IF
   20 CONTINUE
      IC3(IG(I,1),IG(I,2),IG(I,3))=IC3(IG(I,1),IG(I,2),IG(I,3))+1
   10 CONTINUE
      CM=SUMX**2/NC
      SST=SST-CM

C     THREE FACTOR ANALYSIS

  300 IDF(1)=NL(1)-1
      IDF(2)=NL(2)-1
      IDF(3)=NL(3)-1
      IDF(4)=IDF(1)*IDF(2)
      IDF(5)=IDF(1)*IDF(3)
      IDF(6)=IDF(2)*IDF(3)
      IDF(7)=IDF(4)*IDF(3)
      IDF(8)=NL(1)*NL(2)*NL(3)*(NREP-1)
      NP=7
      IF (NREP .EQ. 1) NP=NP-1
      DO 305 I=1,3
      DO 305 J=1,NL(I)
      SUM2(J,I)=0.0
  305 CONTINUE

C     FIND SS FOR MAIN EFFECTS

      DO 310 I=1,NC
      DO 310 J=1,3
      SUM2(IG(I,J),J)=SUM2(IG(I,J),J)+IPER(I)
  310 CONTINUE
      DO 315 J=1,3
      SS(J)=0.0
      DO 320 K=1,NL(J)
      SS(J)=SS(J)+SUM2(K,J)**2
  320 CONTINUE
```

```
      M=REAL(NC)/REAL(NL(J))
      SS(J)=SS(J)/M-CM
  315 CONTINUE

C     FIND SS FOR TWO FACTOR INTERACTIONS

      NLMAX=MAX(NL(1),NL(2),NL(3))
      DO 325 I=1,NLMAX
      DO 325 J=1,NLMAX
      DO 325 K=1,3
      SUM3(I,J,K)=0.0
  325 CONTINUE
      DO 330 I=1,NC
      SUM3(IG(I,1),IG(I,2),1)=SUM3(IG(I,1),IG(I,2),1)+IPER(I)
      SUM3(IG(I,1),IG(I,3),2)=SUM3(IG(I,1),IG(I,3),2)+IPER(I)
      SUM3(IG(I,2),IG(I,3),3)=SUM3(IG(I,2),IG(I,3),3)+IPER(I)
  330 CONTINUE
      SS(4)=0.0
      DO 335 I=1,NL(1)
      DO 335 J=1,NL(2)
      SS(4)=SS(4)+SUM3(I,J,1)**2
  335 CONTINUE
      SS(4)=SS(4)/(NL(3)*NREP)-SS(1)-SS(2)-CM
      SS(5)=0.0
      DO 340 I=1,NL(1)
      DO 340 K=1,NL(3)
      SS(5)=SS(5)+SUM3(I,K,2)**2
  340 CONTINUE
      SS(5)=SS(5)/(NL(2)*NREP)-SS(1)-SS(3)-CM
      SS(6)=0.0
      DO 345 J=1,NL(2)
      DO 345 K=1,NL(3)
      SS(6)=SS(6)+SUM3(J,K,3)**2
  345 CONTINUE
      SS(6)=SS(6)/(NL(1)*NREP)-SS(2)-SS(3)-CM

C     FIND SS FOR THREE FACTOR INTERACTION AND ERROR

      IF (NREP .GT. 1) GOTO 350
      SS(7)=SST-SS(1)-SS(2)-SS(3)-SS(4)-SS(5)-SS(6)
      SS(8)=0.0
      GOTO 355
  350 DO 360 I=1,NL(1)
      DO 360 J=1,NL(2)
      DO 360 K=1,NL(3)
      SUM3(I,J,K)=0.0
      SS3(I,J,K)=0.0
  360 CONTINUE
      DO 365 I=1,NC
      SUM3(IG(I,1),IG(I,2),IG(I,3))=SUM3(IG(I,1),IG(I,2),IG(I,3))+IPER(I)
      SS3(IG(I,1),IG(I,2),IG(I,3))=SS3(IG(I,1),IG(I,2),IG(I,3))+IPER(I)**2
  365 CONTINUE
      SS(7)=0.0
      DO 370 I=1,NL(1)
```

```
      DO 370 J=1,NL(2)
      DO 370 K=1,NL(3)
      SS(7)=SS(7)+SUM3(I,J,K)**2
  370 CONTINUE
      SS(7)=SS(7)/NREP-SS(1)-SS(2)-SS(3)-SS(4)-SS(5)-SS(6)-CM
      SS(8)=SST-SS(1)-SS(2)-SS(3)-SS(4)-SS(5)-SS(6)-SS(7)

C     FIND MEAN SQUARES AND F VALUES

      IF (NP .EQ. 7) MS(8)=SS(8)/IDF(8)
  355 DO 375 I=1,7
      MS(I)=SS(I)/IDF(I)
  375 CONTINUE

      FMAIN=MS(2)/MS(4)
      FSUB=MS(3)/(MS(5)+MS(7))
      FINT=MS(6)/(MS(5)+MS(7))

      DO 380 I=1,NPERMS
      IPERM(I)=I
  380 CONTINUE
      CALL SRCH(NPERMS,FMAIN,LISTM,INCX,INDEX)
      IF (INDEX .LT. 0) THEN
      LISTM(ZM)=FMAIN
      ZM=ZM+1
      ELSE
      FRQM(INDEX)=FRQM(INDEX)+1
      NRM=NRM+1
      END IF
      CALL SVRGP(NPERMS,LISTM,LISTM,IPERM)
      DO 410 I=1,NPERMS
      TEMP(I)=FRQM(IPERM(I))
  410 CONTINUE
      DO 420 I=1,NPERMS
      FRQM(I)=TEMP(I)
  420 CONTINUE
      CALL SRCH(NPERMS,FSUB,LISTS,INCX,INDEX)
      IF (INDEX .LT. 0) THEN
      LISTS(ZS)=FSUB
      ZS=ZS+1
      ELSE
      FRQS(INDEX)=FRQS(INDEX)+1
      NRS=NRS+1
      END IF
      CALL SVRGP(NPERMS,LISTS,LISTS,IPERM)
      DO 510 I=1,NPERMS
      TEMP(I)=FRQS(IPERM(I))
  510 CONTINUE
      DO 520 I=1,NPERMS
      FRQS(I)=TEMP(I)
  520 CONTINUE
      CALL SRCH(NPERMS,FINT,LISTI,INCX,INDEX)
      IF (INDEX .LT. 0) THEN
      LISTI(ZI)=FINT
```

```
      ZI=ZI+1
      ELSE
      FRQI(INDEX)=FRQI(INDEX)+1
      NRI=NRI+1
      END IF
      CALL SVRGP(NPERMS,LISTI,LISTI,IPERM)
      DO 610 I=1,NPERMS
      TEMP(I)=FRQI(IPERM(I))
  610 CONTINUE
      DO 620 I=1,NPERMS
      FRQI(I)=TEMP(I)
  620 CONTINUE

    1 CONTINUE

C     ROUTINE TO WRITE CRITCAL VALUES AND P-VALUES
C
      CUM=0.0

      DO 630 I=1,NPERMS
      IF (LISTM(I) .LT. 9999.0) THEN
      CUM=CUM+FRQM(I)
      PVALM=1-CUM/REAL(NPERMS)
      IF (PVALM .LE. 0.101) THEN
      WRITE (4,*) 'FMAIN=',LISTM(I),PVALM
      END IF
      END IF
  630 CONTINUE

      CUM=0
      DO 640 I=1,NPERMS
      IF (LISTS(I) .LT. 9999.0) THEN
      CUM=CUM+FRQS(I)
      PVALS=1-CUM/REAL(NPERMS)
      IF (PVALS .LE. 0.101) THEN
      WRITE (4,*) 'FSUB=',LISTS(I),PVALS
      END IF
      END IF
  640 CONTINUE

      CUM=0
      DO 650 I=1,NPERMS
      IF (LISTI(I) .LT. 9999.0) THEN
      CUM=CUM+FRQI(I)
      PVALI=1-CUM/REAL(NPERMS)
      IF (PVALI .LE. 0.101) THEN
      WRITE (4,*) 'FINT=',LISTI(I),PVALI
      END IF
      END IF
  650 CONTINUE

      CLOSE (UNIT=4)

      END
```

Program 5.

PROGRAM "SM4B3AR" TO COMPUTE SIGNIFICANCE LEVELS FOR F-RATIO, EXACT RANK TRANSFORM AND EXACT ALIGNED RANK TRANSFORM TESTS FOR A 2 FACTOR CRD WITH N LEVELS PER TRT AND NREP OBS PER TRT COMB

```
USE MSIMSL
PARAMETER (NC=120,NLA=4,NLB=3,NREP=10)
INTEGER IG(NC,3),IDF(8),IC1(20),IC2(NC,NC),NL(20),N,Z
INTEGER FYAREJ10,FYBREJ10,FYABREJ10,FRAREJ10,FRBREJ10,FRABREJ10
INTEGER FARAREJ10,FARBREJ10,FARABREJ10
INTEGER HOLD(250),W,P,Q
INTEGER FYAREJ05,FYBREJ05,FYABREJ05,FRAREJ05,FRBREJ05,FRABREJ05
INTEGER FARAREJ05,FARBREJ05,FARABREJ05
INTEGER FYAREJ01,FYBREJ01,FYABREJ01,FRAREJ01,FRBREJ01,FRABREJ01
INTEGER FARAREJ01,FARBREJ01,FARABREJ01
REAL R(NC),Y(NC),MS(8),SS(8),SUM2(20,20),SS2(20,20)
REAL RA(NC),RB(NC),RAB(NC),CONS,SIG
REAL FYA,FRA,FYA10PV,FRA10PV,FARA10PV
REAL FYB,FRB,FYB10PV,FRB10PV,FARB10PV
REAL FYAB,FRAB,FYAB10PV,FRAB10PV,FARAB10PV
REAL FYA05PV,FRA05PV,FARA05PV
REAL FYB05PV,FRB05PV,FARB05PV
REAL FYAB05PV,FRAB05PV,FARAB05PV
REAL FYA01PV,FRA01PV,FARA01PV
REAL FYB01PV,FRB01PV,FARB01PV
REAL FYAB01PV,FRAB01PV,FARAB01PV
REAL A(NLA),B(NLB),AB(NLA,NLB),E(NC),ER(NC),YFIX(NC),P01,P05,P10
REAL DFNA,DFNB,DFNAB,DFD
REAL M(NLA,NLB,NREP),AMAB(NLA,NLB,NREP),AYAB(NC),AYB(NC),UE(1)
REAL MA(NLA),MB(NLB),SUM,RY(NC),AMA(NLA,NLB,NREP)
REAL AMB(NLA,NLB,NREP)
REAL AYA(NC)
REAL CRITFA10,CRITFB10,CRITFAB10,CRITRA10,CRITRB10,CRITRAB10
REAL CRITFA05,CRITFB05,CRITFAB05,CRITRA05,CRITRB05,CRITRAB05
REAL CRITFA01,CRITFB01,CRITFAB01,CRITRA01,CRITRB01,CRITRAB01
REAL SUMFYA,SUMFYB,SUMFYAB,SUMFRA,SUMFRB,SUMFRAB

DATA CM/0.0/, NL/20*0/, IC1/20*0/,IC2/14400*0/
DATA A(1),A(2),A(3),A(4)/.0,.0,.0,.0/
DATA B(1),B(2),B(3)/.0,-.0,-.0/
DATA AB(1,1),AB(1,2),AB(1,3)/.0,3.50,-.0/
DATA AB(2,1),AB(2,2),AB(2,3)/0,-3.50,3.50/
DATA AB(3,1),AB(3,2),AB(3,3)/-3.50,-.0,-3.50/
DATA AB(4,1),AB(4,2),AB(4,3)/3.50,0,0/
OPEN (UNIT=4,FILE='C:\MSDEV\DATA\SIM4310.TXT')

N=10000
NL(1)=NLA
NL(2)=NLB
CONS=1.0
```

```
      NPERMS=0
      NF=2
      FYREJ=0
      FRREJ=0
      FRTREJ=0

C     CRITICAL VALUES

      P10=.90
      P05=.95
      P01=.99
      DFNA=3
      DFNB=2
      DFNAB=6
      DFD=108
      CRITFA10=FIN(P10,DFNA,DFD)
      CRITFB10=FIN(P10,DFNB,DFD)
      CRITFAB10=FIN(P10,DFNAB,DFD)
      CRITRA10=2.11847
      CRITRB10=2.344881
      CRITRAB10=1.821623
      CRITFA05=FIN(P05,DFNA,DFD)
      CRITFB05=FIN(P05,DFNB,DFD)
      CRITFAB05=FIN(P05,DFNAB,DFD)
      CRITRA05=2.680210
      CRITRB05=3.124526
      CRITRAB05=2.182787
      CRITFA01=FIN(P01,DFNA,DFD)
      CRITFB01=FIN(P01,DFNB,DFD)
      CRITFAB01=FIN(P01,DFNAB,DFD)
      CRITRA01=4.003309
      CRITRB01=5.087671
      CRITRAB01=2.985842

      NP=0
      Z=1

C     FILL IG VECTOR

      C=1
      DO 2 I=1,NL(1)
      DO 4 J=1,NL(2)
      DO 6 K=1,NREP
      HOLD(C)=J
      HOLD(C+1)=I
      C=C+2
  6   CONTINUE
  4   CONTINUE
  2   CONTINUE
      C=1
      DO 12 I=1,NC
      DO 14 J=NF,1,-1
      IG(I,J)=HOLD(C)
      C=C+1
```

```
 14  CONTINUE
 12  CONTINUE

     CALL RNSET(62064)
     DO 10 S=1,N

C    GENERATE OBSERVATIONS

     W=1
     SIG=1
     DO 1 I=1,NL(1)
     DO 3 J=1,NL(2)
     DO 5 K=1,NREP
     CALL RNNOA(1,UE)
     CALL SSCAL(1,SIG,UE,1)
C     CALL RNEXP(1,UE)
C     CALL SSCAL(1,3.0,UE,1)
C     CALL RNUN(1,UE)
C     CALL SSCAL(1,6.0,UE,1)
C     CALL SADD(1,-3.0,UE,1)

     Y(W)=A(I)+B(J)+AB(I,J)+UE(1)
     W=W+1
  5  CONTINUE
C    SIG=CONS*SIG
  3  CONTINUE
     SIG=CONS*SIG
C    SIG=1
  1  CONTINUE

C    ALIGN OBSERVATIONS

C    FILL MATRIX WITH OBSERVATIONS

     P=1
     DO 51 I=1,NL(1)
     DO 52 J=1,NL(2)
     DO 53 K=1,NREP
     M(I,J,K)=Y(P)
     P=P+1
 53  CONTINUE
 52  CONTINUE
 51  CONTINUE

C    COMPUTE FACTOR A MEANS

     SUM=0
     DO 61 I=1,NL(1)
     DO 62 J=1,NL(2)
     DO 63 K=1,NREP
     SUM=SUM+M(I,J,K)
 63  CONTINUE
 62  CONTINUE
     MA(I)=SUM/(NL(2)*NREP)
```

```
      SUM=0
   61 CONTINUE

C     COMPUTE FACTOR B MEANS

      SUM=0
      DO 71 J=1,NL(2)
      DO 72 I=1,NL(1)
      DO 73 K=1,NREP
      SUM=SUM+M(I,J,K)
   73 CONTINUE
   72 CONTINUE
      MB(J)=SUM/(NL(1)*NREP)
      SUM=0
   71 CONTINUE

C     COMPUTE OVERALL MEAN

      SUM=0
      DO 76 I=1,NL(2)
      SUM=SUM+MB(I)
   76 CONTINUE
      MAB=SUM/NL(2)

C     COMPUTE ALIGNED OBSERVATIONS

      DO 81 I=1,NL(1)
      DO 82 J=1,NL(2)
      DO 83 K=1,NREP
      AMAB(I,J,K)=M(I,J,K)-(MA(I)+MB(J))
      AMA(I,J,K)=M(I,J,K)-MB(J)
      AMB(I,J,K)=M(I,J,K)-MA(I)
   83 CONTINUE
   82 CONTINUE
   81 CONTINUE

C     RETURN ALIGNED MATRIX ELEMENTS TO SINGLE ARRAY

      Q=1
      DO 91 I=1,NL(1)
      DO 92 J=1,NL(2)
      DO 93 K=1,NREP
      AYAB(Q)=AMAB(I,J,K)
      AYA(Q)=AMA(I,J,K)
      AYB(Q)=AMB(I,J,K)
      Q=Q+1
   93 CONTINUE
   92 CONTINUE
   91 CONTINUE

C     FIND THE RANKS OF THE ALIGNED DATA

      CALL RANKS(NC,AYAB,.000000001,0,0,RAB)
      CALL RANKS(NC,AYA,.000000001,0,0,RA)
```

```
      CALL RANKS(NC,AYB,.000000001,0,0,RB)
      CALL RANKS(NC,Y,.000000001,0,0,R)

C     TWO FACTOR ANALYSIS : F-TEST ON RAW DATA

C     CALCULATE SS FOR MAIN EFFECTS

      SUMX=0.0
      SST=0
      DO 166 I=1,NC
      SUMX=SUMX+Y(I)
      SST=SST+(Y(I))**2
      IC1(IG(I,1))=IC1(IG(I,1))+1
      DO 30 K=1,NF
      IF (IG(I,K).GT.NL(K)) THEN
      NL(K)=IG(I,K)
      END IF
   30 CONTINUE
      IC2(IG(I,1),IG(I,2))=IC2(IG(I,1),IG(I,2))+1
  166 CONTINUE
      CM=SUMX**2/NC
      SST=SST-CM
      IDF(1)=NL(1)-1
      IDF(2)=NL(2)-1
      IDF(3)=IDF(1)*IDF(2)
      IDF(4)=NL(1)*NL(2)*(NREP-1)
      NP=3
      IF (NREP.EQ.1) THEN
      NP=NP-1
      ENDIF
      DO 160 J=1,2
      DO 160 K=1,NL(J)
      SUM2(K,J)=0.0
  160 CONTINUE
      DO 170 I=1,NC
      DO 170 J=1,2
      SUM2(IG(I,J),J)=SUM2(IG(I,J),J)+Y(I)
  170 CONTINUE
      DO 180 J=1,2
      SS(J)=0.0
      DO 190 K=1,NL(J)
  190 SS(J)=SS(J)+(SUM2(K,J))**2
      MM=NC/NL(J)
  180 SS(J)=SS(J)/MM-CM

C     CALCULATE INTERACTION SS

      DO 200 I=1,NL(1)
      DO 200 J=1,NL(2)
      SUM2(I,J)=0.0
      SS2(I,J)=0.0
  200 CONTINUE
      DO 210 I=1,NC
      SUM2(IG(I,1),IG(I,2))=SUM2(IG(I,1),IG(I,2))+Y(I)
```

```
210 SS2(IG(I,1),IG(I,2))=SS2(IG(I,1),IG(I,2))+(Y(I))**2
    SS(3)=0.0
    DO 220 I=1,NL(1)
    DO 220 J=1,NL(2)
    SS(3)=SS(3)+(SUM2(I,J))**2
220 CONTINUE
    SS(3)=SS(3)/NREP-CM-SS(1)-SS(2)

C   FIND ERROR SUM OF SQUARES AND MEAN SQUARES

    SS(4)=SST-SS(1)-SS(2)-SS(3)
    IF (NP.EQ.3) THEN
    MS(4)=SS(4)/IDF(4)
    END IF
    DO 230 I=1,3
230 MS(I)=SS(I)/IDF(I)
    IF (NP.EQ.3) THEN
    IF (MS(4).EQ.0.0) THEN
    FY=9999.0
    ELSE
    FYA=MS(1)/MS(4)
    FYB=MS(2)/MS(4)
    FYAB=MS(3)/MS(4)
    SUMFYA=SUMFYA+FYA
    SUMFYB=SUMFYB+FYB
    SUMFYAB=SUMFYAB+FYAB
    END IF
    ELSE
    IF (MS(3).EQ.0.0) THEN
    FY=9999.0
    ELSE
    FY=MS(1)/MS(3)
    END IF
    END IF
    IF (FYA .GE. CRITFA10) THEN
    FYAREJ10=FYAREJ10+1
    END IF
    IF (FYB .GE. CRITFB10) THEN
    FYBREJ10=FYBREJ10+1
    END IF
    IF (FYAB .GE. CRITFAB10) THEN
    FYABREJ10=FYABREJ10+1
    END IF
    IF (FYA .GE. CRITFA05) THEN
    FYAREJ05=FYAREJ05+1
    END IF
    IF (FYB .GE. CRITFB05) THEN
    FYBREJ05=FYBREJ05+1
    END IF
    IF (FYAB .GE. CRITFAB05) THEN
    FYABREJ05=FYABREJ05+1
    END IF
    IF (FYA .GE. CRITFA01) THEN
    FYAREJ01=FYAREJ01+1
```

```
      END IF
      IF (FYB .GE. CRITFB01) THEN
      FYBREJ01=FYBREJ01+1
      END IF
      IF (FYAB .GE. CRITFAB01) THEN
      FYABREJ01=FYABREJ01+1
      END IF

C     TWO FACTOR ANALYSIS : F-TEST ON RAW RANKS

C     CALCULATE SS FOR MAIN EFFECTS

      SUMX=0.0
      SST=0
      DO 4166 I=1,NC
      SUMX=SUMX+R(I)
      SST=SST+(R(I))**2
      IC1(IG(I,1))=IC1(IG(I,1))+1
      DO 430 K=1,NF
      IF (IG(I,K).GT.NL(K)) THEN
      NL(K)=IG(I,K)
      END IF
  430 CONTINUE
      IC2(IG(I,1),IG(I,2))=IC2(IG(I,1),IG(I,2))+1
 4166 CONTINUE
      CM=SUMX**2/NC
      SST=SST-CM
      IDF(1)=NL(1)-1
      IDF(2)=NL(2)-1
      IDF(3)=IDF(1)*IDF(2)
      IDF(4)=NL(1)*NL(2)*(NREP-1)
      NP=3
      IF (NREP.EQ.1) THEN
      NP=NP-1
      ENDIF
      DO 4160 J=1,2
      DO 4160 K=1,NL(J)
      SUM2(K,J)=0.0
 4160 CONTINUE
      DO 4170 I=1,NC
      DO 4170 J=1,2
      SUM2(IG(I,J),J)=SUM2(IG(I,J),J)+R(I)
 4170 CONTINUE
      DO 4180 J=1,2
      SS(J)=0.0
      DO 4190 K=1,NL(J)
 4190 SS(J)=SS(J)+(SUM2(K,J))**2
      MM=NC/NL(J)
 4180 SS(J)=SS(J)/MM-CM

C     CALCULATE INTERACTION SS

      DO 4200 I=1,NL(1)
      DO 4200 J=1,NL(2)
```

```
      SUM2(I,J)=0.0
      SS2(I,J)=0.0
 4200 CONTINUE
      DO 4210 I=1,NC
      SUM2(IG(I,1),IG(I,2))=SUM2(IG(I,1),IG(I,2))+R(I)
 4210 SS2(IG(I,1),IG(I,2))=SS2(IG(I,1),IG(I,2))+(R(I))**2
      SS(3)=0.0
      DO 4220 I=1,NL(1)
      DO 4220 J=1,NL(2)
      SS(3)=SS(3)+(SUM2(I,J))**2
 4220 CONTINUE
      SS(3)=SS(3)/NREP-CM-SS(1)-SS(2)

C     FIND ERROR SUM OF SQUARES AND MEAN SQUARES

      SS(4)=SST-SS(1)-SS(2)-SS(3)
      IF (NP.EQ.3) THEN
      MS(4)=SS(4)/IDF(4)
      END IF
      DO 4230 I=1,3
 4230 MS(I)=SS(I)/IDF(I)
      IF (NP.EQ.3) THEN
      IF (MS(4).EQ.0.0) THEN
      FY=9999.0
      ELSE
      FRA=MS(1)/MS(4)
      FRB=MS(2)/MS(4)
      FRAB=MS(3)/MS(4)
      SUMFRA=SUMFRA+FRA
      SUMFRB=SUMFRB+FRB
      SUMFRAB=SUMFRAB+FRAB
      END IF
      ELSE
      IF (MS(3).EQ.0.0) THEN
      FY=9999.0
      ELSE
      FY=MS(1)/MS(3)
      END IF
      END IF
      IF (FRA .GE. CRITRA10) THEN
      FRAREJ10=FRAREJ10+1
      END IF
      IF (FRB .GE. CRITRB10) THEN
      FRBREJ10=FRBREJ10+1
      END IF
      IF (FRAB .GE. CRITRAB10) THEN
      FRABREJ10=FRABREJ10+1
      END IF
      IF (FRA .GE. CRITRA05) THEN
      FRAREJ05=FRAREJ05+1
      END IF
      IF (FRB .GE. CRITRB05) THEN
      FRBREJ05=FRBREJ05+1
      END IF
```

```
      IF (FRAB .GE. CRITRAB05) THEN
      FRABREJ05=FRABREJ05+1
      END IF
      IF (FRA .GE. CRITRA01) THEN
      FRAREJ01=FRAREJ01+1
      END IF
      IF (FRB .GE. CRITRB01) THEN
      FRBREJ01=FRBREJ01+1
      END IF
      IF (FRAB .GE. CRITRAB01) THEN
      FRABREJ01=FRABREJ01+1
      END IF

C     TWO FACTOR ANALYSIS : F-TEST ON ALIGNED RANKS , TEST FOR INTERACTION

C     CALCULATE SS FOR MAIN EFFECTS

      SUMX=0.0
      SST=0
      DO 1166 I=1,NC
      SUMX=SUMX+RAB(I)
      SST=SST+(RAB(I))**2
      IC1(IG(I,1))=IC1(IG(I,1))+1
      DO 130 K=1,NF
      IF (IG(I,K).GT.NL(K)) THEN
      NL(K)=IG(I,K)
      END IF
  130 CONTINUE
      IC2(IG(I,1),IG(I,2))=IC2(IG(I,1),IG(I,2))+1
 1166 CONTINUE
      CM=SUMX**2/NC
      SST=SST-CM
      IDF(1)=NL(1)-1
      IDF(2)=NL(2)-1
      IDF(3)=IDF(1)*IDF(2)
      IDF(4)=NL(1)*NL(2)*(NREP-1)
      NP=3
      IF (NREP.EQ.1) THEN
      NP=NP-1
      ENDIF
      DO 1160 J=1,2
      DO 1161 K=1,NL(J)
      SUM2(K,J)=0.0
 1161 CONTINUE
 1160 CONTINUE
      DO 1170 I=1,NC
      DO 1171 J=1,2
      SUM2(IG(I,J),J)=SUM2(IG(I,J),J)+RAB(I)
 1171 CONTINUE
 1170 CONTINUE
      DO 1180 J=1,2
      SS(J)=0.0
      DO 1190 K=1,NL(J)
 1190 SS(J)=SS(J)+(SUM2(K,J))**2
```

```
      MM=NC/NL(J)
1180  SS(J)=SS(J)/MM-CM

C     CALCULATE INTERACTION SS

      DO 1200 I=1,NL(1)
      DO 1201 J=1,NL(2)
      SUM2(I,J)=0.0
      SS2(I,J)=0.0
1201  CONTINUE
1200  CONTINUE
      DO 1210 I=1,NC
      SUM2(IG(I,1),IG(I,2))=SUM2(IG(I,1),IG(I,2))+RAB(I)
1210  SS2(IG(I,1),IG(I,2))=SS2(IG(I,1),IG(I,2))+(RAB(I))**2
      SS(3)=0.0
      DO 1220 I=1,NL(1)
      DO 1221 J=1,NL(2)
      SS(3)=SS(3)+(SUM2(I,J))**2
1221  CONTINUE
1220  CONTINUE
      SS(3)=SS(3)/NREP-CM-SS(1)-SS(2)

C     FIND ERROR SUM OF SQUARES AND MEAN SQUARES

      SS(4)=SST-SS(1)-SS(2)-SS(3)
      IF (NP.EQ.3) THEN
      MS(4)=SS(4)/IDF(4)
      END IF
      DO 1230 I=1,3
1230  MS(I)=SS(I)/IDF(I)

      IF (NP.EQ.3) THEN
      IF (MS(4).EQ.0.0) THEN
      FR=9999.0
      ELSE
      FARAB=MS(3)/MS(4)
      SUMFARAB=SUMFARAB+FARAB
      END IF
      ELSE
      IF (MS(3).EQ.0.0) THEN
      FARAB=9999.0
      ELSE
      FARAB=9999.0
      END IF
      END IF
      IF (FARAB .GE. CRITRAB10) THEN
      FARABREJ10=FARABREJ10+1
      END IF
      IF (FARAB .GE. CRITRAB05) THEN
      FARABREJ05=FARABREJ05+1
      END IF
      IF (FARAB .GE. CRITRAB01) THEN
      FARABREJ01=FARABREJ01+1
      END IF
```

```
C     TWO FACTOR ANALYSIS : F-TEST ON ALIGNED RANKS,  TEST FOR FACTOR A

C     CALCULATE SS FOR MAIN EFFECTS

      SUMX=0.0
      SST=0
      DO 2266 I=1,NC
      SUMX=SUMX+RA(I)
      SST=SST+(RA(I))**2
      IC1(IG(I,1))=IC1(IG(I,1))+1
      DO 2130 K=1,NF
      IF (IG(I,K).GT.NL(K)) THEN
      NL(K)=IG(I,K)
      END IF
2130 CONTINUE
      IC2(IG(I,1),IG(I,2))=IC2(IG(I,1),IG(I,2))+1
2266 CONTINUE
      CM=SUMX**2/NC
      SST=SST-CM
      IDF(1)=NL(1)-1
      IDF(2)=NL(2)-1
      IDF(3)=IDF(1)*IDF(2)
      IDF(4)=NL(1)*NL(2)*(NREP-1)
      NP=3
      IF (NREP.EQ.1) THEN
      NP=NP-1
      ENDIF
      DO 2160 J=1,2
      DO 2161 K=1,NL(J)
      SUM2(K,J)=0.0
2161 CONTINUE
2160 CONTINUE
      DO 2170 I=1,NC
      DO 2171 J=1,2
      SUM2(IG(I,J),J)=SUM2(IG(I,J),J)+RA(I)
2171 CONTINUE
2170 CONTINUE
      DO 2180 J=1,2
      SS(J)=0.0
      DO 2190 K=1,NL(J)
2190 SS(J)=SS(J)+(SUM2(K,J))**2
      MM=NC/NL(J)
2180 SS(J)=SS(J)/MM-CM

C     CALCULATE INTERACTION SS

      DO 2200 I=1,NL(1)
      DO 2201 J=1,NL(2)
      SUM2(I,J)=0.0
      SS2(I,J)=0.0
2201 CONTINUE
2200 CONTINUE
      DO 2210 I=1,NC
```

```
      SUM2(IG(I,1),IG(I,2))=SUM2(IG(I,1),IG(I,2))+RA(I)
2210 SS2(IG(I,1),IG(I,2))=SS2(IG(I,1),IG(I,2))+(RA(I))**2
      SS(3)=0.0
      DO 2220 I=1,NL(1)
      DO 2221 J=1,NL(2)
      SS(3)=SS(3)+(SUM2(I,J))**2
2221 CONTINUE
2220 CONTINUE
      SS(3)=SS(3)/NREP-CM-SS(1)-SS(2)

C     FIND ERROR SUM OF SQUARES AND MEAN SQUARES

      SS(4)=SST-SS(1)-SS(2)-SS(3)
      IF (NP.EQ.3) THEN
      MS(4)=SS(4)/IDF(4)
      END IF
      DO 2230 I=1,3
2230 MS(I)=SS(I)/IDF(I)
      IF (NP.EQ.3) THEN
      IF (MS(4).EQ.0.0) THEN
      FR=9999.0
      ELSE
      FARA=MS(1)/MS(4)
      SUMFARA=SUMFARA+FARA
      END IF
      ELSE
      IF (MS(3).EQ.0.0) THEN
      FR=9999.0
      ELSE
      FR=MS(1)/MS(3)
      END IF
      END IF
      IF (FARA .GE. CRITRA10) THEN
      FARAREJ10=FARAREJ10+1
      END IF
      IF (FARA .GE. CRITRA05) THEN
      FARAREJ05=FARAREJ05+1
      END IF
      IF (FARA .GE. CRITRA01) THEN
      FARAREJ01=FARAREJ01+1
      END IF

C     TWO FACTOR ANALYSIS : F-TEST ON ALIGNED RANKS, TEST FOR FACTOR B

C     CALCULATE SS FOR MAIN EFFECTS

      SUMX=0.0
      SST=0
      DO 3266 I=1,NC
      SUMX=SUMX+RB(I)
      SST=SST+(RB(I))**2
      IC1(IG(I,1))=IC1(IG(I,1))+1
      DO 3230 K=1,NF
      IF (IG(I,K).GT.NL(K)) THEN
```

```
      NL(K)=IG(I,K)
      END IF
 3230 CONTINUE
      IC2(IG(I,1),IG(I,2))=IC2(IG(I,1),IG(I,2))+1
 3266 CONTINUE
      CM=SUMX**2/NC
      SST=SST-CM
      IDF(1)=NL(1)-1
      IDF(2)=NL(2)-1
      IDF(3)=IDF(1)*IDF(2)
      IDF(4)=NL(1)*NL(2)*(NREP-1)
      NP=3
      IF (NREP.EQ.1) THEN
      NP=NP-1
      ENDIF
      DO 3260 J=1,2
      DO 3261 K=1,NL(J)
      SUM2(K,J)=0.0
 3261 CONTINUE
 3260 CONTINUE
      DO 3270 I=1,NC
      DO 3271 J=1,2
      SUM2(IG(I,J),J)=SUM2(IG(I,J),J)+RB(I)
 3271 CONTINUE
 3270 CONTINUE
      DO 3280 J=1,2
      SS(J)=0.0
      DO 3290 K=1,NL(J)
 3290 SS(J)=SS(J)+(SUM2(K,J))**2
      MM=NC/NL(J)
 3280 SS(J)=SS(J)/MM-CM

C     CALCULATE INTERACTION SS

      DO 3300 I=1,NL(1)
      DO 3301 J=1,NL(2)
      SUM2(I,J)=0.0
      SS2(I,J)=0.0
 3301 CONTINUE
 3300 CONTINUE
      DO 3310 I=1,NC
      SUM2(IG(I,1),IG(I,2))=SUM2(IG(I,1),IG(I,2))+RB(I)
 3310 SS2(IG(I,1),IG(I,2))=SS2(IG(I,1),IG(I,2))+(RB(I))**2
      SS(3)=0.0
      DO 3320 I=1,NL(1)
      DO 3321 J=1,NL(2)
      SS(3)=SS(3)+(SUM2(I,J))**2
 3321 CONTINUE
 3320 CONTINUE
      SS(3)=SS(3)/NREP-CM-SS(1)-SS(2)

C     FIND ERROR SUM OF SQUARES AND MEAN SQUARES

      SS(4)=SST-SS(1)-SS(2)-SS(3)
```

```
      IF (NP.EQ.3) THEN
      MS(4)=SS(4)/IDF(4)
      END IF
      DO 3330 I=1,3
3330  MS(I)=SS(I)/IDF(I)

      IF (NP.EQ.3) THEN
      IF (MS(4).EQ.0.0) THEN
      FR=9999.0
      ELSE
      FARB=MS(2)/MS(4)
      SUMFARB=SUMFARB+FARB
      END IF
      ELSE
      IF (MS(3).EQ.0.0) THEN
      FR=9999.0
      ELSE
      FARB=MS(2)/MS(3)
      END IF
      END IF
      IF (FARB .GE. CRITRB10) THEN
      FARBREJ10=FARBREJ10+1
      END IF
      IF (FARB .GE. CRITRB05) THEN
      FARBREJ05=FARBREJ05+1
      END IF
      IF (FARB .GE. CRITRB01) THEN
      FARBREJ01=FARBREJ01+1
      END IF

10    CONTINUE

      FYA10PV=REAL(FYAREJ10)/REAL(N)
      FRA10PV=REAL(FRAREJ10)/REAL(N)
      FARA10PV=REAL(FARAREJ10)/REAL(N)
      FRTA10PV=REAL(FRTAREJ10)/REAL(N)
      FYB10PV=REAL(FYBREJ10)/REAL(N)
      FRB10PV=REAL(FRBREJ10)/REAL(N)
      FARB10PV=REAL(FARBREJ10)/REAL(N)
      FYAB10PV=REAL(FYABREJ10)/REAL(N)
      FRAB10PV=REAL(FRABREJ10)/REAL(N)
      FARAB10PV=REAL(FARABREJ10)/REAL(N)
      FYA05PV=REAL(FYAREJ05)/REAL(N)
      FRA05PV=REAL(FRAREJ05)/REAL(N)
      FARA05PV=REAL(FARAREJ05)/REAL(N)
      FYB05PV=REAL(FYBREJ05)/REAL(N)
      FRB05PV=REAL(FRBREJ05)/REAL(N)
      FARB05PV=REAL(FARBREJ05)/REAL(N)
      FYAB05PV=REAL(FYABREJ05)/REAL(N)
      FRAB05PV=REAL(FRABREJ05)/REAL(N)
      FARAB05PV=REAL(FARABREJ05)/REAL(N)
      FYA01PV=REAL(FYAREJ01)/REAL(N)
      FRA01PV=REAL(FRAREJ01)/REAL(N)
      FARA01PV=REAL(FARAREJ01)/REAL(N)
```

```
FYB01PV=REAL(FYBREJ01)/REAL(N)
FRB01PV=REAL(FRBREJ01)/REAL(N)
FARB01PV=REAL(FARBREJ01)/REAL(N)
FYAB01PV=REAL(FYABREJ01)/REAL(N)
FRAB01PV=REAL(FRABREJ01)/REAL(N)
FARAB01PV=REAL(FARABREJ01)/REAL(N)

WRITE (4,*) 'ALPHA = 0.10'
WRITE (4,*) 'FYAPVAL=  ',FYA10PV
WRITE (4,*) 'FRAPVAL=  ',FRA10PV
WRITE (4,*) 'FARAPVAL= ',FARA10PV
WRITE (4,*) 'FYBPVAL=  ',FYB10PV
WRITE (4,*) 'FRBPVAL=  ',FRB10PV
WRITE (4,*) 'FARBPVAL= ',FARB10PV
WRITE (4,*) 'FYABPVAL= ',FYAB10PV
WRITE (4,*) 'FRABPVAL= ',FRAB10PV
WRITE (4,*) 'FARABPVAL=',FARAB10PV

WRITE (4,*) 'ALPHA = 0.05'
WRITE (4,*) 'FYAPVAL=  ',FYA05PV
WRITE (4,*) 'FRAPVAL=  ',FRA05PV
WRITE (4,*) 'FARAPVAL= ',FARA05PV
WRITE (4,*) 'FYBPVAL=  ',FYB05PV
WRITE (4,*) 'FRBPVAL=  ',FRB05PV
WRITE (4,*) 'FARBPVAL= ',FARB05PV
WRITE (4,*) 'FYABPVAL= ',FYAB05PV
WRITE (4,*) 'FRABPVAL= ',FRAB05PV
WRITE (4,*) 'FARABPVAL=',FARAB05PV

WRITE (4,*) 'ALPHA = 0.01'
WRITE (4,*) 'FYAPVAL=  ',FYA01PV
WRITE (4,*) 'FRAPVAL=  ',FRA01PV
WRITE (4,*) 'FARAPVAL= ',FARA01PV
WRITE (4,*) 'FYBPVAL=  ',FYB01PV
WRITE (4,*) 'FRBPVAL=  ',FRB01PV
WRITE (4,*) 'FARBPVAL= ',FARB01PV
WRITE (4,*) 'FYABPVAL= ',FYAB01PV
WRITE (4,*) 'FRABPVAL= ',FRAB01PV
WRITE (4,*) 'FARABPVAL=',FARAB01PV

CLOSE (UNIT=4)

END
```

Program 6.

PROGRAM TO SIMULATE SPLIT PLOT EXPERIMENT

```
PARAMETER (NC=36,NOBSA=6,NOBSB=8,NLA=3,NLB=4,NLC=3,NREP=1)

INTEGER IG(NC,3),IDF(8),IC1(20),NL(20),N,Z
INTEGER HOLD(75),W,P,Q,NOBSA,NOBSB,NMISS
INTEGER FYMREJ,FYSREJ,FYIREJ,FRMREJ,FRSREJ,FRIREJ
INTEGER FARMREJ,FARSREJ,FARIREJ
INTEGER FRTMREJ,FRTSREJ,FRTIREJ
INTEGER ISEED,K,NOUT,IC3(20,20,20),NPERMS
INTEGER INCX,INDEX,COUNT,NN
REAL R(NC),Y(NC),MS(8),SS(8),SUM2(20,20),SS2(20,20)
REAL RC(NC),RB(NC),RBC(NC),CONS,SIG,QPROP(1),MDA(1),MDB(1)
REAL FYM,FRM,MEDA(NLA),MEDB(NLB)
REAL FARM,FARS,FARI,SUMX,SST
REAL FYS,FRS,FYI,FRI,M,SSE,MSE
REAL FYMPV,FRMPV,FARMPV,FRTMPV,ARRAYA(NOBSA)
REAL FYSPV,FRSPV,FARSPV,FRTSPV,ARRAYB(NOBSB)
REAL FYIPV,FRIPV,FARIPV,FRTIPV
REAL A(NLA),MT(NLB),MST(NLB,NLC),P05,AMC(NLA,NLB,NLC)
REAL DFNA,DFNB,DFNAB,DFD,MC(NLC),AMBC(NLA,NLB,NLC)
REAL MX(NLA,NLB,NLC),AYB(NC),AYC(NC),UE(1),ST(NLC)
REAL MA(NLA),MB(NLB),SUM,AMB(NLA,NLB,NLC),AYBC(NC)
REAL CRITFM,CRITFS,CRITFI,CRITRM,CRITRS,CRITRI
REAL SUM3(20,20,20),SS3(20,20,20)
REAL BEV(1),MEV(1),SEV(1)
REAL BE(NLA),ME(NLA*NLB),SE(NLA*NLB*NLC)
REAL SIGB,SIGM,SIGS,SUMARM(NC),SUMARS(NC),SUMARI(NC)
DATA CM/0.0/, NL/20*0/, IC1/20*0/
DATA IG(1,1),IG(1,2),IG(1,3),IG(2,1),IG(2,2),IG(2,3)/1,1,1,1,1,2/
DATA IG(3,1),IG(3,2),IG(3,3),IG(4,1),IG(4,2),IG(4,3)/1,1,3,1,2,1/
DATA IG(5,1),IG(5,2),IG(5,3),IG(6,1),IG(6,2),IG(6,3)/1,2,2,1,2,3/
DATA IG(7,1),IG(7,2),IG(7,3),IG(8,1),IG(8,2),IG(8,3)/1,3,1,1,3,2/
DATA IG(9,1),IG(9,2),IG(9,3),IG(10,1),IG(10,2)/1,3,3,1,4/
DATA IG(10,3),IG(11,1),IG(11,2),IG(11,3),IG(12,1)/1,1,4,2,1/
DATA IG(12,2),IG(12,3),IG(13,1),IG(13,2),IG(13,3)/4,3,2,1,1/
DATA IG(14,1),IG(14,2),IG(14,3),IG(15,1),IG(15,2)/2,1,2,2,1/
DATA IG(15,3),IG(16,1),IG(16,2),IG(16,3),IG(17,1)/3,2,2,1,2/
DATA IG(17,2),IG(17,3),IG(18,1),IG(18,2),IG(18,3)/2,2,2,2,3/
DATA IG(19,1),IG(19,2),IG(19,3),IG(20,1),IG(20,2)/2,3,1,2,3/
DATA IG(20,3),IG(21,1),IG(21,2),IG(21,3),IG(22,1)/2,2,3,3,2/
DATA IG(22,2),IG(22,3),IG(23,1),IG(23,2),IG(23,3)/4,1,2,4,2/
DATA IG(24,1),IG(24,2),IG(24,3),IG(25,1),IG(25,2)/2,4,3,3,1/
DATA IG(25,3),IG(26,1),IG(26,2),IG(26,3),IG(27,1)/1,3,1,2,3/
DATA IG(27,2),IG(27,3),IG(28,1),IG(28,2),IG(28,3)/1,3,3,2,1/
DATA IG(29,1),IG(29,2),IG(29,3),IG(30,1),IG(30,2)/3,2,2,3,2/
DATA IG(30,3),IG(31,1),IG(31,2),IG(31,3),IG(32,1)/3,3,3,1,3/
DATA IG(32,2),IG(32,3),IG(33,1),IG(33,2),IG(33,3)/3,2,3,3,3/
DATA IG(34,1),IG(34,2),IG(34,3),IG(35,1),IG(35,2)/3,4,1,3,4/
DATA IG(35,3),IG(36,1),IG(36,2),IG(36,3)/2,3,4,3/
```

```
      DATA IC3/8000*-1.0/
      DATA MT(1),MT(2),MT(3),MT(4)/.0,.0,-.0,.0/
      DATA ST(1),ST(2),ST(3)/3.50,-.0,.0/
      DATA MST(1,1),MST(1,2),MST(1,3)/-3.50,.0,-.0/
      DATA MST(2,1),MST(2,2),MST(2,3)/0,-.0,.0/
      DATA MST(3,1),MST(3,2),MST(3,3)/-.0,.0,.0/
      DATA MST(4,1),MST(4,2),MST(4,3)/3.50,0,0/

      OPEN (UNIT=4,FILE='C:\MSDEV\DATA\SIMSPLIT.TXT')

      WRITE (4,*)
      WRITE (4,*)
      WRITE (4,*) '3*4*3 SPLIT PLOT, ALL TESTS USING'
      WRITE (4,*) 'USING POOLED ERROR; NORMAL ERRORS EQUAL VARIANCE'
C      WRITE (4,*) 'ST1=-3.5;ST3=3.5'
C      WRITE (4,*) 'MT2=ST1=3.50, MT3=ST2=-3.50'
      WRITE (4,*) 'ST1=MST41=3.5, MST11=-3.5'
C      WRITE (4,*) 'MST11=MST33=3.5;MST13=MST31=-3.5'
C       WRITE (4,*) 'SUB UNIT EFFECT PRESENT'
      WRITE (4,*) 'ALL EFFECTS PRESENT'
C       WRITE (4,*) 'MAIN AND SUB UNIT EFFECTS PRESENT'
C       WRITE (4,*) 'INTERACTION EFFECT PRESENT'

C       CONS=1.77
      NN=36
      NPERMS=10000
      N=10000
      NF=3
      NP=0
      INCX=1
      ZM=1
      ZS=1
      ZI=1
      NL(1)=3
      NL(2)=4
      NL(3)=3
      COUNT=0
      QPROP=.5

C       CRITICAL VALUES

      P05=.95
      DFNM=3
      DFDM=6
      DFNS=2
      DFDS=16
      DFNI=6
      DFDI=16
      CRITFM=FIN(P05,DFNM,DFDM)
      CRITFS=FIN(P05,DFNS,DFDS)
      CRITFI=FIN(P05,DFNI,DFDI)
      CRITRM=4.829662
      CRITRS=3.666049
      CRITRI=2.792083
```

```
      Z=1

      CALL RNSET(62064)
      DO 10 S=1,N

C     GENERATE OBSERVATIONS

      W=1
      W1=1
      W2=1
      SIGB=1.0
      SIGM=1.0
      SIGS=1.0

      CALL RNNOA(NC,BE)
C     CALL SSCAL(NC,SIGB,BE,1)
C     CALL RNEXP(NC,BE)
C     CALL SSCAL(NC,3.0,BE,1)

      CALL RNNOA(NC,ME)
C     CALL SSCAL(NC,SIGM,ME,1)
C     CALL RNEXP(NC,ME)
C     CALL SSCAL(NC,3.0,ME,1)

      CALL RNNOA(NC,SE)
C     CALL SSCAL(NC,SIGS,SE,1)
C     CALL RNEXP(NC,SE)
C     CALL SSCAL(NC,3.0,SE,1)

      DO 1 I=1,NL(1)
C     CALL RNNOA(1,BEV)
C     CALL SSCAL(1,SIGB,BEV,1)

      DO 3 J=1,NL(2)

C     CALL RNNOA(1,MEV)
C     CALL SSCAL(1,SIGM,MEV,1)
C     CALL RNUN(1,MEV)
C     CALL SSCAL(1,6.0,MEV,1)
C     CALL SADD(1,-3.0,MEV,1)

      DO 5 K=1,NL(3)

C      CALL RNNOA(1,SEV)
C      CALL SSCAL(1,SIGS,SEV,1)

C      CALL RNEXP(1,SEV)
C      CALL SSCAL(1,3.0,SEV,1)
C      CALL RNUN(1,SEV)
C      CALL SSCAL(1,6.0,SEV,1)
C      CALL SADD(1,-3.0,SEV,1)

      Y(W)=MT(J)+ST(K)+MST(J,K)+BE(I)+ME(W1)+SE(W)
C      Y(W)=MT(J)+ST(K)+MST(J,K)+BEV(1)+MEV(1)+SEV(1)
```

```
      W=W+1
  5   CONTINUE
      W1=W1+1
C      SIGM=CONS*SIGM
  3   CONTINUE
      W2=W2+1
C      SIGB=CONS*SIGB
C      SIGM=1
  1   CONTINUE

C      ALIGN OBSERVATIONS

C      FILL MATRIX WITH OBSERVATIONS

      P=1
      DO 51 I=1,NL(1)
      DO 52 J=1,NL(2)
      DO 53 K=1,NL(3)
      MX(I,J,K)=Y(P)
      P=P+1
 53   CONTINUE
 52   CONTINUE
 51   CONTINUE

C      COMPUTE FACTOR A MEANS AND MEDIANS
      SUM=0
      DO 61 I=1,NL(1)
C      Q=1
      DO 62 J=1,NL(2)
      DO 63 K=1,NL(3)
      SUM=SUM+MX(I,J,K)
C     ARRAYA(Q)=M(I,J,K)
C     Q=Q+1
 63   CONTINUE
 62   CONTINUE
C      CALL EQTIL(NOBSA,ARRAYA,1,QPROP,MDA,XLO,XHI,NMISS)
C      MEDA(I)=MDA(1)
      MA(I)=SUM/(NL(2)*NL(3))
      SUM=0
 61   CONTINUE

C    COMPUTE FACTOR B MEANS AND MEDIANS
      SUM=0
      DO 71 J=1,NL(2)
C          Q=1
      DO 72 I=1,NL(1)
      DO 73 K=1,NL(3)
      SUM=SUM+MX(I,J,K)
C                  ARRAYB(Q)=M(I,J,K)
C                  Q=Q+1
 73   CONTINUE
 72   CONTINUE
C       CALL EQTIL(NOBSB,ARRAYB,1,QPROP,MDB,XLO,XHI,NMISS)
```

```
C        MEDB(J)=MDB(1)
       MB(J)=SUM/(NL(1)*NL(3))
       SUM=0
  71 CONTINUE

C    COMPUTE FACTOR C MEANS AND MEDIANS

       SUM=0
       DO 710 K=1,NL(3)
C           Q=1
       DO 720 I=1,NL(1)
       DO 730 J=1,NL(2)
       SUM=SUM+MX(I,J,K)
C                  ARRAYB(Q)=M(I,J,K)
C                  Q=Q+1
 730 CONTINUE
 720 CONTINUE
C        CALL EQTIL(NOBSB,ARRAYB,1,QPROP,MDB,XLO,XHI,NMISS)
C        MEDB(J)=MDB(1)
       MC(K)=SUM/(NL(1)*NL(2))
       SUM=0
 710 CONTINUE

C    COMPUTE OVERALL MEAN
       SUM=0
       DO 760 I=1,NL(2)
       SUM=SUM+MB(I)
 760 CONTINUE
       MAB=SUM/NL(2)

C    COMPUTE ALIGNED OBSERVATIONS
       DO 81 I=1,NL(1)
       DO 82 J=1,NL(2)
       DO 83 K=1,NL(3)
       AMBC(I,J,K)=MX(I,J,K)-MA(I)-MB(J)-MC(K)
       AMB(I,J,K)=MX(I,J,K)-MA(I)-MC(K)
       AMC(I,J,K)=MX(I,J,K)-MA(I)-MB(J)
  83 CONTINUE
  82 CONTINUE
  81 CONTINUE

C    RETURN ALIGNED MATRIX ELEMENTS TO SINGLE ARRAY
       Q=1
       DO 91 I=1,NL(1)
       DO 92 J=1,NL(2)
       DO 93 K=1,NL(3)
       AYBC(Q)=AMBC(I,J,K)
       AYB(Q)=AMB(I,J,K)
       AYC(Q)=AMC(I,J,K)
       Q=Q+1
  93 CONTINUE
  92 CONTINUE
  91 CONTINUE
```

```
C    FIND THE RANKS OF THE ALIGNED AND RAW DATA

      CALL RANKS(NC,AYBC,.000000001,0,0,RBC)
      CALL RANKS(NC,AYB,.000000001,0,0,RB)
      CALL RANKS(NC,AYC,.000000001,0,0,RC)
      CALL RANKS(NC,Y,.000000001,0,0,R)

      DO 1000 I=1,NC
      SUMARM(I)=SUMARM(I)+RB(I)
      SUMARS(I)=SUMARS(I)+RC(I)
      SUMARI(I)=SUMARI(I)+RBC(I)
1000 CONTINUE

C    PERFORM ANALYSIS ON RAW DATA

C    CALCULATE SS FOR MAIN EFFECTS

      SUMX=0
      SST=0
      DO 101 I=1,NC
      SUMX=SUMX+Y(I)
      SST=SST+(Y(I))**2
      IC1(IG(I,1))=IC1(IG(I,1))+1
      DO 101 K=1,NF
      IF (IG(I,K) .GT. NL(K)) THEN
      NL(K)=IG(I,K)
      END IF
 102 CONTINUE
      IC3(IG(I,1),IG(I,2),IG(I,3))=IC3(IG(I,1),IG(I,2),IG(I,3))+1
 101 CONTINUE
      CM=SUMX**2/NC
      SST=SST-CM
      IDF(1)=NL(1)-1
      IDF(2)=NL(2)-1
      IDF(3)=NL(3)-1
      IDF(4)=IDF(1)*IDF(2)
      IDF(5)=IDF(1)*IDF(3)
      IDF(6)=IDF(2)*IDF(3)
      IDF(7)=IDF(4)*IDF(3)
      IDF(8)=NL(1)*NL(2)*NL(3)*(NREP-1)
      NP=7
      IF (NREP .EQ. 1) NP=NP-1
      DO 105 I=1,3
      DO 105 J=1,NL(I)
      SUM2(J,I)=0.0
 105 CONTINUE

C    FIND SS FOR MAIN EFFECTS

      DO 110 I=1,NC
      DO 110 J=1,3
      SUM2(IG(I,J),J)=SUM2(IG(I,J),J)+Y(I)
 110 CONTINUE
      DO 115 J=1,3
```

```
      SS(J)=0.0
      DO 120 K=1,NL(J)
      SS(J)=SS(J)+SUM2(K,J)**2
  120 CONTINUE
      M=REAL(NC)/REAL(NL(J))
      SS(J)=SS(J)/M-CM
  115 CONTINUE

C     FIND SS FOR TWO FACTOR INTERACTIONS

      NLMAX=MAX(NL(1),NL(2),NL(3))
      DO 125 I=1,NLMAX
      DO 125 J=1,NLMAX
      DO 125 K=1,3
      SUM3(I,J,K)=0.0
  125 CONTINUE
      DO 130 I=1,NC
      SUM3(IG(I,1),IG(I,2),1)=SUM3(IG(I,1),IG(I,2),1)+Y(I)
      SUM3(IG(I,1),IG(I,3),2)=SUM3(IG(I,1),IG(I,3),2)+Y(I)
      SUM3(IG(I,2),IG(I,3),3)=SUM3(IG(I,2),IG(I,3),3)+Y(I)
  130 CONTINUE
      SS(4)=0.0
      DO 135 I=1,NL(1)
      DO 135 J=1,NL(2)
      SS(4)=SS(4)+SUM3(I,J,1)**2
  135 CONTINUE
      SS(4)=SS(4)/(NL(3)*NREP)-SS(1)-SS(2)-CM
      SS(5)=0.0
      DO 140 I=1,NL(1)
      DO 140 K=1,NL(3)
      SS(5)=SS(5)+SUM3(I,K,2)**2
  140 CONTINUE
      SS(5)=SS(5)/(NL(2)*NREP)-SS(1)-SS(3)-CM
      SS(6)=0.0
      DO 145 J=1,NL(2)
      DO 145 K=1,NL(3)
      SS(6)=SS(6)+SUM3(J,K,3)**2
  145 CONTINUE
      SS(6)=SS(6)/(NL(1)*NREP)-SS(2)-SS(3)-CM

C     FIND SS FOR THREE FACTOR INTERACTION AND ERROR

      IF (NREP .GT. 1) GOTO 150
      SS(7)=SST-SS(1)-SS(2)-SS(3)-SS(4)-SS(5)-SS(6)
      SS(8)=0.0
      GOTO 155
  150 DO 160 I=1,NL(1)
      DO 160 J=1,NL(2)
      DO 160 K=1,NL(3)
      SUM3(I,J,K)=0.0
      SS3(I,J,K)=0.0
  160 CONTINUE
      DO 165 I=1,NC
      SUM3(IG(I,1),IG(I,2),IG(I,3))=SUM3(IG(I,1),IG(I,2),IG(I,3))+Y(I)
```

```
      SS3(IG(I,1),IG(I,2),IG(I,3))=SS3(IG(I,1),IG(I,2),IG(I,3))+Y(I)**2
  165 CONTINUE
      SS(7)=0.0
      DO 170 I=1,NL(1)
      DO 170 J=1,NL(2)
      DO 170 K=1,NL(3)
      SS(7)=SS(7)+SUM3(I,J,K)**2
  170 CONTINUE
      SS(7)=SS(7)/NREP-SS(1)-SS(2)-SS(3)-SS(4)-SS(5)-SS(6)-CM
      SS(8)=SST-SS(1)-SS(2)-SS(3)-SS(4)-SS(5)-SS(6)-SS(7)

C     FIND MEAN SQUARES AND F-VALUES

      IF (NP .EQ. 7) MS(8)=SS(8)/IDF(8)
  155 DO 175 I=1,7
      MS(I)=SS(I)/IDF(I)
  175 CONTINUE
      SSE=SS(5)+SS(7)
      MSE=SSE/(IDF(5)+IDF(7))
      IF (MS(4) .EQ. 0.0) THEN
      FYM=999.0
      ELSE
      FYM=MS(2)/MS(4)
      END IF
      IF (MSE .EQ. 0.0 ) THEN
      FYS=999.0
      FYI=999.0
      ELSE
      FYS=MS(3)/MSE
      FYI=MS(6)/MSE
      END IF
      IF (FYM .GE. CRITFM) THEN
      FYMREJ=FYMREJ+1
      END IF
      IF (FYS .GE. CRITFS) THEN
      FYSREJ=FYSREJ+1
      END IF
      IF (FYI .GE. CRITFI) THEN
      FYIREJ=FYIREJ+1
      END IF

C     PERFORM ANALYSIS ON RANKS

C     CALCULATE SS FOR MAIN EFFECTS

      SUMX=0
      SST=0
      DO 201 I=1,NC
      SUMX=SUMX+R(I)
      SST=SST+(R(I))**2
      IC1(IG(I,1))=IC1(IG(I,1))+1
      DO 202 K=1,NF
      IF (IG(I,K) .GT. NL(K)) THEN
      NL(K)=IG(I,K)
```

```
      END IF
  202 CONTINUE
      IC3(IG(I,1),IG(I,2),IG(I,3))=IC3(IG(I,1),IG(I,2),IG(I,3))+1
  201 CONTINUE
      CM=SUMX**2/NC
      SST=SST-CM
      IDF(1)=NL(1)-1
      IDF(2)=NL(2)-1
      IDF(3)=NL(3)-1
      IDF(4)=IDF(1)*IDF(2)
      IDF(5)=IDF(1)*IDF(3)
      IDF(6)=IDF(2)*IDF(3)
      IDF(7)=IDF(4)*IDF(3)
      IDF(8)=NL(1)*NL(2)*NL(3)*(NREP-1)
      NP=7
      IF (NREP .EQ. 1) NP=NP-1
      DO 205 I=1,3
      DO 205 J=1,NL(I)
      SUM2(J,I)=0.0
  205 CONTINUE

C     FIND SS FOR MAIN EFFECTS

      DO 210 I=1,NC
      DO 210 J=1,3
      SUM2(IG(I,J),J)=SUM2(IG(I,J),J)+R(I)
  210 CONTINUE
      DO 215 J=1,3
      SS(J)=0.0
      DO 220 K=1,NL(J)
      SS(J)=SS(J)+SUM2(K,J)**2
  220 CONTINUE
      M=REAL(NC)/REAL(NL(J))
      SS(J)=SS(J)/M-CM
  215 CONTINUE

C     FIND SS FOR TWO FACTOR INTERACTIONS

      NLMAX=MAX(NL(1),NL(2),NL(3))
      DO 225 I=1,NLMAX
      DO 225 J=1,NLMAX
      DO 225 K=1,3
      SUM3(I,J,K)=0.0
  225 CONTINUE
      DO 230 I=1,NC
      SUM3(IG(I,1),IG(I,2),1)=SUM3(IG(I,1),IG(I,2),1)+R(I)
      SUM3(IG(I,1),IG(I,3),2)=SUM3(IG(I,1),IG(I,3),2)+R(I)
      SUM3(IG(I,2),IG(I,3),3)=SUM3(IG(I,2),IG(I,3),3)+R(I)
  230 CONTINUE
      SS(4)=0.0
      DO 235 I=1,NL(1)
      DO 235 J=1,NL(2)
      SS(4)=SS(4)+SUM3(I,J,1)**2
  235 CONTINUE
```

```
      SS(4)=SS(4)/(NL(3)*NREP)-SS(1)-SS(2)-CM
      SS(5)=0.0
      DO 240 I=1,NL(1)
      DO 240 K=1,NL(3)
      SS(5)=SS(5)+SUM3(I,K,2)**2
  240 CONTINUE
      SS(5)=SS(5)/(NL(2)*NREP)-SS(1)-SS(3)-CM
      SS(6)=0.0
      DO 245 J=1,NL(2)
      DO 245 K=1,NL(3)
      SS(6)=SS(6)+SUM3(J,K,3)**2
  245 CONTINUE
      SS(6)=SS(6)/(NL(1)*NREP)-SS(2)-SS(3)-CM

C     FIND SS FOR THREE FACTOR INTERACTION AND ERROR

      IF (NREP .GT. 1) GOTO 250
      SS(7)=SST-SS(1)-SS(2)-SS(3)-SS(4)-SS(5)-SS(6)
      SS(8)=0.0
      GOTO 255
  250 DO 260 I=1,NL(1)
      DO 260 J=1,NL(2)
      DO 260 K=1,NL(3)
      SUM3(I,J,K)=0.0
      SS3(I,J,K)=0.0
  260 CONTINUE
      DO 265 I=1,NC
      SUM3(IG(I,1),IG(I,2),IG(I,3))=SUM3(IG(I,1),IG(I,2),IG(I,3))+R(I)
      SS3(IG(I,1),IG(I,2),IG(I,3))=SS3(IG(I,1),IG(I,2),IG(I,3))+R(I)**2
  265 CONTINUE
      SS(7)=0.0
      DO 270 I=1,NL(1)
      DO 270 J=1,NL(2)
      DO 270 K=1,NL(3)
      SS(7)=SS(7)+SUM3(I,J,K)**2
  270 CONTINUE
      SS(7)=SS(7)/NREP-SS(1)-SS(2)-SS(3)-SS(4)-SS(5)-SS(6)-CM
      SS(8)=SST-SS(1)-SS(2)-SS(3)-SS(4)-SS(5)-SS(6)-SS(7)

C     FIND MEAN SQUARES AND F-VALUES

      IF (NP .EQ. 7) MS(8)=SS(8)/IDF(8)
  255 DO 275 I=1,7
      MS(I)=SS(I)/IDF(I)
  275 CONTINUE
      SSE=SS(5)+SS(7)
      MSE=SSE/(IDF(5)+IDF(7))
      IF (MS(4) .EQ. 0.0) THEN
      FRM=999.0
      ELSE
      FRM=MS(2)/MS(4)
      END IF
      IF (MSE .EQ. 0.0) THEN
      FRS=999.0
```

```
      FRI=999.0
      ELSE
      FRS=MS(3)/MSE
      FRI=MS(6)/MSE
C     FRTS=MS(3)/MSE
C     FRTI=MS(6)/MSE
      END IF
      IF (FRM .GE. CRITRM) THEN
      FRMREJ=FRMREJ+1
      END IF
      IF (FRS .GE. CRITRS) THEN
      FRSREJ=FRSREJ+1
      END IF
      IF (FRI .GE. CRITRI) THEN
      FRIREJ=FRIREJ+1
      END IF
      IF (FRM .GE. CRITFM) THEN
      FRTMREJ=FRTMREJ+1
      END IF
      IF (FRS .GE. CRITFS) THEN
      FRTSREJ=FRTSREJ+1
      END IF
      IF (FRI .GE. CRITFI) THEN
      FRTIREJ=FRTIREJ+1
      END IF

C     PERFORM ANALYSIS ON ALIGNED RANKS: TEST FOR MAIN UNIT TRT

C     CALCULATE SS FOR MAIN EFFECTS

      SUMX=0
      SST=0
      DO 401 I=1,NC
      SUMX=SUMX+RB(I)
      SST=SST+(RB(I))**2
      IC1(IG(I,1))=IC1(IG(I,1))+1
      DO 402 K=1,NF
      IF (IG(I,K) .GT. NL(K)) THEN
      NL(K)=IG(I,K)
      END IF
  402 CONTINUE
      IC3(IG(I,1),IG(I,2),IG(I,3))=IC3(IG(I,1),IG(I,2),IG(I,3))+1
  401 CONTINUE
      CM=SUMX**2/NC
      SST=SST-CM
      IDF(1)=NL(1)-1
      IDF(2)=NL(2)-1
      IDF(3)=NL(3)-1
      IDF(4)=IDF(1)*IDF(2)
      IDF(5)=IDF(1)*IDF(3)
      IDF(6)=IDF(2)*IDF(3)
      IDF(7)=IDF(4)*IDF(3)
      IDF(8)=NL(1)*NL(2)*NL(3)*(NREP-1)
      NP=7
```

```
      IF (NREP .EQ. 1) NP=NP-1
      DO 405 I=1,3
      DO 405 J=1,NL(I)
      SUM2(J,I)=0.0
  405 CONTINUE

C     FIND SS FOR MAIN EFFECTS

      DO 410 I=1,NC
      DO 410 J=1,3
      SUM2(IG(I,J),J)=SUM2(IG(I,J),J)+RB(I)
  410 CONTINUE
      DO 415 J=1,3
      SS(J)=0.0
      DO 420 K=1,NL(J)
      SS(J)=SS(J)+SUM2(K,J)**2
  420 CONTINUE
      M=REAL(NC)/REAL(NL(J))
      SS(J)=SS(J)/M-CM
  415 CONTINUE

C     FIND SS FOR TWO FACTOR INTERACTIONS

      NLMAX=MAX(NL(1),NL(2),NL(3))
      DO 425 I=1,NLMAX
      DO 425 J=1,NLMAX
      DO 425 K=1,3
      SUM3(I,J,K)=0.0
  425 CONTINUE
      DO 430 I=1,NC
      SUM3(IG(I,1),IG(I,2),1)=SUM3(IG(I,1),IG(I,2),1)+RB(I)
      SUM3(IG(I,1),IG(I,3),2)=SUM3(IG(I,1),IG(I,3),2)+RB(I)
      SUM3(IG(I,2),IG(I,3),3)=SUM3(IG(I,2),IG(I,3),3)+RB(I)
  430 CONTINUE
      SS(4)=0.0
      DO 435 I=1,NL(1)
      DO 435 J=1,NL(2)
      SS(4)=SS(4)+SUM3(I,J,1)**2
  435 CONTINUE
      SS(4)=SS(4)/(NL(3)*NREP)-SS(1)-SS(2)-CM
      SS(5)=0.0
      DO 440 I=1,NL(1)
      DO 440 K=1,NL(3)
      SS(5)=SS(5)+SUM3(I,K,2)**2
  440 CONTINUE
      SS(5)=SS(5)/(NL(2)*NREP)-SS(1)-SS(3)-CM
      SS(6)=0.0
      DO 445 J=1,NL(2)
      DO 445 K=1,NL(3)
      SS(6)=SS(6)+SUM3(J,K,3)**2
  445 CONTINUE
      SS(6)=SS(6)/(NL(1)*NREP)-SS(2)-SS(3)-CM

C     FIND SS FOR THREE FACTOR INTERACTION AND ERROR
```

```
      IF (NREP .GT. 1) GOTO 450
      SS(7)=SST-SS(1)-SS(2)-SS(3)-SS(4)-SS(5)-SS(6)
      SS(8)=0.0
      GOTO 455
  450 DO 460 I=1,NL(1)
      DO 460 J=1,NL(2)
      DO 460 K=1,NL(3)
      SUM3(I,J,K)=0.0
      SS3(I,J,K)=0.0
  460 CONTINUE
      DO 465 I=1,NC
      SUM3(IG(I,1),IG(I,2),IG(I,3))=SUM3(IG(I,1),IG(I,2),IG(I,3))+RB(I)
      SS3(IG(I,1),IG(I,2),IG(I,3))=SS3(IG(I,1),IG(I,2),IG(I,3))+RB(I)**2
  465 CONTINUE
      SS(7)=0.0
      DO 470 I=1,NL(1)
      DO 470 J=1,NL(2)
      DO 470 K=1,NL(3)
      SS(7)=SS(7)+SUM3(I,J,K)**2
  470 CONTINUE
      SS(7)=SS(7)/NREP-SS(1)-SS(2)-SS(3)-SS(4)-SS(5)-SS(6)-CM
      SS(8)=SST-SS(1)-SS(2)-SS(3)-SS(4)-SS(5)-SS(6)-SS(7)

C     FIND MEAN SQUARES AND F-VALUES

      IF (NP .EQ. 7) MS(8)=SS(8)/IDF(8)
  455 DO 475 I=1,7
      MS(I)=SS(I)/IDF(I)
  475 CONTINUE
      IF (MS(4) .EQ. 0.0) THEN
      FARM=999.0
      ELSE
      FARM=MS(2)/MS(4)
      END IF
      IF (FARM .GE. CRITRM) THEN
      FARMREJ=FARMREJ+1
      END IF

C     PERFORM ANALYSIS ON ALIGNED RANKS: TEST FOR SUB UNIT TRT

C     CALCULATE SS FOR MAIN EFFECTS

      SUMX=0
      SST=0
      DO 501 I=1,NC
      SUMX=SUMX+RC(I)
      SST=SST+(RC(I))**2
      IC1(IG(I,1))=IC1(IG(I,1))+1
      DO 502 K=1,NF
      IF (IG(I,K) .GT. NL(K)) THEN
      NL(K)=IG(I,K)
      END IF
  502 CONTINUE
```

```
      IC3(IG(I,1),IG(I,2),IG(I,3))=IC3(IG(I,1),IG(I,2),IG(I,3))+1
  501 CONTINUE
      CM=SUMX**2/NC
      SST=SST-CM
  500 IDF(1)=NL(1)-1
      IDF(2)=NL(2)-1
      IDF(3)=NL(3)-1
      IDF(4)=IDF(1)*IDF(2)
      IDF(5)=IDF(1)*IDF(3)
      IDF(6)=IDF(2)*IDF(3)
      IDF(7)=IDF(4)*IDF(3)
      IDF(8)=NL(1)*NL(2)*NL(3)*(NREP-1)
      NP=7
      IF (NREP .EQ. 1) NP=NP-1
      DO 505 I=1,3
      DO 505 J=1,NL(I)
      SUM2(J,I)=0.0
  505 CONTINUE

C     FIND SS FOR MAIN EFFECTS

      DO 510 I=1,NC
      DO 510 J=1,3
      SUM2(IG(I,J),J)=SUM2(IG(I,J),J)+RC(I)
  510 CONTINUE
      DO 515 J=1,3
      SS(J)=0.0
      DO 520 K=1,NL(J)
      SS(J)=SS(J)+SUM2(K,J)**2
  520 CONTINUE
      M=REAL(NC)/REAL(NL(J))
      SS(J)=SS(J)/M-CM
  515 CONTINUE

C     FIND SS FOR TWO FACTOR INTERACTIONS

      NLMAX=MAX(NL(1),NL(2),NL(3))
      DO 525 I=1,NLMAX
      DO 525 J=1,NLMAX
      DO 525 K=1,3
      SUM3(I,J,K)=0.0
  525 CONTINUE
      DO 530 I=1,NC
      SUM3(IG(I,1),IG(I,2),1)=SUM3(IG(I,1),IG(I,2),1)+RC(I)
      SUM3(IG(I,1),IG(I,3),2)=SUM3(IG(I,1),IG(I,3),2)+RC(I)
      SUM3(IG(I,2),IG(I,3),3)=SUM3(IG(I,2),IG(I,3),3)+RC(I)
  530 CONTINUE
      SS(4)=0.0
      DO 535 I=1,NL(1)
      DO 535 J=1,NL(2)
      SS(4)=SS(4)+SUM3(I,J,1)**2
  535 CONTINUE
      SS(4)=SS(4)/(NL(3)*NREP)-SS(1)-SS(2)-CM
      SS(5)=0.0
```

```
      DO 540 I=1,NL(1)
      DO 540 K=1,NL(3)
      SS(5)=SS(5)+SUM3(I,K,2)**2
  540 CONTINUE
      SS(5)=SS(5)/(NL(2)*NREP)-SS(1)-SS(3)-CM
      SS(6)=0.0
      DO 545 J=1,NL(2)
      DO 545 K=1,NL(3)
      SS(6)=SS(6)+SUM3(J,K,3)**2
  545 CONTINUE
      SS(6)=SS(6)/(NL(1)*NREP)-SS(2)-SS(3)-CM

C     FIND SS FOR THREE FACTOR INTERACTION AND ERROR

      IF (NREP .GT. 1) GOTO 550
      SS(7)=SST-SS(1)-SS(2)-SS(3)-SS(4)-SS(5)-SS(6)
      SS(8)=0.0
      GOTO 555
  550 DO 560 I=1,NL(1)
      DO 560 J=1,NL(2)
      DO 560 K=1,NL(3)
      SUM3(I,J,K)=0.0
      SS3(I,J,K)=0.0
  560 CONTINUE
      DO 565 I=1,NC
      SUM3(IG(I,1),IG(I,2),IG(I,3))=SUM3(IG(I,1),IG(I,2),IG(I,3))+RC(I)
      SS3(IG(I,1),IG(I,2),IG(I,3))=SS3(IG(I,1),IG(I,2),IG(I,3))+RC(I)**2
  565 CONTINUE
      SS(7)=0.0
      DO 570 I=1,NL(1)
      DO 570 J=1,NL(2)
      DO 570 K=1,NL(3)
      SS(7)=SS(7)+SUM3(I,J,K)**2
  570 CONTINUE
      SS(7)=SS(7)/NREP-SS(1)-SS(2)-SS(3)-SS(4)-SS(5)-SS(6)-CM
      SS(8)=SST-SS(1)-SS(2)-SS(3)-SS(4)-SS(5)-SS(6)-SS(7)

C     FIND MEAN SQUARES AND F-VALUES

      IF (NP .EQ. 7) MS(8)=SS(8)/IDF(8)
  555 DO 575 I=1,7
      MS(I)=SS(I)/IDF(I)
  575 CONTINUE
      SSE=SS(5)+SS(7)
      MSE=SSE/(IDF(5)+IDF(7))
      IF (MSE .EQ. 0.0) THEN
      FARS=999.0
      ELSE
      FARS=MS(3)/MSE
      END IF
      IF (FARS .GE. CRITRS) THEN
      FARSREJ=FARSREJ+1
      END IF
```

```
C     PERFORM ANALYSIS ON ALIGNED RANKS: TEST FOR INTERACTION

C     CALCULATE SS FOR MAIN EFFECTS

      SUMX=0
      SST=0
      DO 601 I=1,NC
      SUMX=SUMX+RBC(I)
      SST=SST+(RBC(I))**2
      IC1(IG(I,1))=IC1(IG(I,1))+1
      DO 602 K=1,NF
      IF (IG(I,K) .GT. NL(K)) THEN
      NL(K)=IG(I,K)
      END IF
  602 CONTINUE
      IC3(IG(I,1),IG(I,2),IG(I,3))=IC3(IG(I,1),IG(I,2),IG(I,3))+1
  601 CONTINUE
      CM=SUMX**2/NC
      SST=SST-CM
      IDF(1)=NL(1)-1
      IDF(2)=NL(2)-1
      IDF(3)=NL(3)-1
      IDF(4)=IDF(1)*IDF(2)
      IDF(5)=IDF(1)*IDF(3)
      IDF(6)=IDF(2)*IDF(3)
      IDF(7)=IDF(4)*IDF(3)
      IDF(8)=NL(1)*NL(2)*NL(3)*(NREP-1)
      NP=7
      IF (NREP .EQ. 1) NP=NP-1
      DO 605 I=1,3
      DO 605 J=1,NL(I)
      SUM2(J,I)=0.0
  605 CONTINUE

C     FIND SS FOR MAIN EFFECTS

      DO 610 I=1,NC
      DO 610 J=1,3
      SUM2(IG(I,J),J)=SUM2(IG(I,J),J)+RBC(I)
  610 CONTINUE
      DO 615 J=1,3
      SS(J)=0.0
      DO 620 K=1,NL(J)
      SS(J)=SS(J)+SUM2(K,J)**2
  620 CONTINUE
      M=REAL(NC)/REAL(NL(J))
      SS(J)=SS(J)/M-CM
  615 CONTINUE

C     FIND SS FOR TWO FACTOR INTERACTIONS

      NLMAX=MAX(NL(1),NL(2),NL(3))
      DO 625 I=1,NLMAX
      DO 625 J=1,NLMAX
```

```
      DO 625 K=1,3
      SUM3(I,J,K)=0.0
  625 CONTINUE
      DO 630 I=1,NC
      SUM3(IG(I,1),IG(I,2),1)=SUM3(IG(I,1),IG(I,2),1)+RBC(I)
      SUM3(IG(I,1),IG(I,3),2)=SUM3(IG(I,1),IG(I,3),2)+RBC(I)
      SUM3(IG(I,2),IG(I,3),3)=SUM3(IG(I,2),IG(I,3),3)+RBC(I)
  630 CONTINUE
      SS(4)=0.0
      DO 635 I=1,NL(1)
      DO 635 J=1,NL(2)
      SS(4)=SS(4)+SUM3(I,J,1)**2
  635 CONTINUE
      SS(4)=SS(4)/(NL(3)*NREP)-SS(1)-SS(2)-CM
      SS(5)=0.0
      DO 640 I=1,NL(1)
      DO 640 K=1,NL(3)
      SS(5)=SS(5)+SUM3(I,K,2)**2
  640 CONTINUE
      SS(5)=SS(5)/(NL(2)*NREP)-SS(1)-SS(3)-CM
      SS(6)=0.0
      DO 645 J=1,NL(2)
      DO 645 K=1,NL(3)
      SS(6)=SS(6)+SUM3(J,K,3)**2
  645 CONTINUE
      SS(6)=SS(6)/(NL(1)*NREP)-SS(2)-SS(3)-CM

C     FIND SS FOR THREE FACTOR INTERACTION AND ERROR

      IF (NREP .GT. 1) GOTO 650
      SS(7)=SST-SS(1)-SS(2)-SS(3)-SS(4)-SS(5)-SS(6)
      SS(8)=0.0
      GOTO 655
  650 DO 660 I=1,NL(1)
      DO 660 J=1,NL(2)
      DO 660 K=1,NL(3)
      SUM3(I,J,K)=0.0
      SS3(I,J,K)=0.0
  660 CONTINUE
      DO 665 I=1,NC
      SUM3(IG(I,1),IG(I,2),IG(I,3))=SUM3(IG(I,1),IG(I,2),IG(I,3))+RBC(I)
      SS3(IG(I,1),IG(I,2),IG(I,3))=SS3(IG(I,1),IG(I,2),IG(I,3))+RBC(I)**2
  665 CONTINUE
      SS(7)=0.0
      DO 670 I=1,NL(1)
      DO 670 J=1,NL(2)
      DO 670 K=1,NL(3)
      SS(7)=SS(7)+SUM3(I,J,K)**2
  670 CONTINUE
      SS(7)=SS(7)/NREP-SS(1)-SS(2)-SS(3)-SS(4)-SS(5)-SS(6)-CM
      SS(8)=SST-SS(1)-SS(2)-SS(3)-SS(4)-SS(5)-SS(6)-SS(7)

C     FIND MEAN SQUARES AND F-VALUES
```

```
      IF (NP .EQ. 7) MS(8)=SS(8)/IDF(8)
  655 DO 675 I=1,7
      MS(I)=SS(I)/IDF(I)
  675 CONTINUE
      SSE=SS(5)+SS(7)
      MSE=SSE/(IDF(5)+IDF(7))
      IF (MSE .EQ. 0.0) THEN
      FARI=999.0
      ELSE
      FARI=MS(6)/MSE
      END IF
      IF (FARI .GE. CRITRI) FARIREJ=FARIREJ+1

10    CONTINUE


      FYMPV=REAL(FYMREJ)/REAL(N)
      FRMPV=REAL(FRMREJ)/REAL(N)
      FRTMPV=REAL(FRTMREJ)/REAL(N)
      FARMPV=REAL(FARMREJ)/REAL(N)
      FYSPV=REAL(FYSREJ)/REAL(N)
      FRSPV=REAL(FRSREJ)/REAL(N)
      FRTSPV=REAL(FRTSREJ)/REAL(N)
      FARSPV=REAL(FARSREJ)/REAL(N)
      FYIPV=REAL(FYIREJ)/REAL(N)
      FRIPV=REAL(FRIREJ)/REAL(N)
      FRTIPV=REAL(FRTIREJ)/REAL(N)
      FARIPV=REAL(FARIREJ)/REAL(N)

      WRITE (4,*) 'ALPHA = 0.05'
      WRITE (4,*)
      WRITE (4,*) 'FYMPVAL=  ',FYMPV
      WRITE (4,*) 'FRMPVAL=  ',FRMPV
      WRITE (4,*) 'FRTMPVAL= ',FRTMPV
      WRITE (4,*) 'FARMPVAL= ',FARMPV
      WRITE (4,*)
      WRITE (4,*) 'FYSPVAL=  ',FYSPV
      WRITE (4,*) 'FRSPVAL=  ',FRSPV
      WRITE (4,*) 'FRTSPVAL= ',FRTSPV
      WRITE (4,*) 'FARSPVAL= ',FARSPV
      WRITE (4,*)
      WRITE (4,*) 'FYIPVAL= ',FYIPV
      WRITE (4,*) 'FRIPVAL= ',FRIPV
      WRITE (4,*) 'FRTIPVAL=',FRTIPV
      WRITE (4,*) 'FARIPVAL=',FARIPV

      CLOSE (UNIT=4)

      END
```
□

VITA

Scott James Richter

Candidate for the Degree of

Doctor of Philosophy

Thesis: EXACT AND ESTIMATED EXACT TESTS USING THE RANK TRANSFORM IN DESIGNED EXPERIMENTS

Major Field: Statistics

Biographical:

Education: Graduated from Terry Parker High School, Jacksonville, Florida in June 1982; received a Bachelor of Science degree in Mathematics from Jacksonville University, Jacksonville, Florida in December 1986; received a Master of Arts degree in Mathematical Science from the University of North Florida in May 1991. Completed the requirements for the Doctor of Philosophy degree with a major in Statistics at Oklahoma State University in July 1997.

Experience: Employed as a mathematics teacher at Paxon Senior High School, Jacksonville, Florida from 1987 to 1989. Employed as a graduate teaching assistant in the Department of Mathematics and Statistics, University of North Florida, from 1989 to 1991. Employed as a Professor of Mathematics at Florida Community College at Jacksonville from 1991 to 1993. Employed as a graduate teaching associate in the Department of Statistics at Oklahoma State University from 1993 to present.

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