Illustrative Applications of Optimal Control Theory Techniques to Problems in Agricultural Economics

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Illustrative Applications of Optimal Control Theory Techniques to Problems in Agricultural Economics

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Optimal control theory is a mathematical technique for analyzing systems under alternative sets of controls. Specifically optimal control theory is a technique to determine the optimal values for particular control variables in a system. The technique has been used primarily by engineers and mathematicians in dealing with control problems in physical systems.

Many industrial processes that make use of automated control devices were designed and calibrated using optimal control techniques. Such processes make use of some type of sensing device that measures chemical reactions, solution mixture, pressure, etc. Signals from the devices are analyzed and converted into mechanical commands. For example, an autopilot in an airplane consists of a system of gyrocompasses, hydrolic gauges and pumps, electrical circuits, etc., which sense the position, speed, wind resistance, etc., of the plane and in turn adjust wing flaps, throttle settings, etc. The mechanical systems used to conduct automatic control procedures such as this have been designed and calibrated using optimal control techniques.

Optimal control theory can be readily applied to many agricultural economics problems. Agricultural economists, like engineers, are dealing with complex systems that emit reactions and signals which require management responses. Optimal control analysis can assist in designing information systems and managerial decision procedures that will create desired economic results.

Traditionally, optimal control theory has been viewed as applicable only to continuous time systems described with differential equations. In practice most agricultural economic models do not fall in this category, rather, in most cases, they are discrete time models. This discrepancy, plus the fact that optimal control theory is typically described with complicated mathematical expositions has caused many agricultural economist to be hesitant in learning to apply optimal control theory. Optimal control techniques can, however, be applied to discrete time models and the basic concepts of such applications can be understood and applied without the use of advanced mathematics.

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For discrete time models, or continuous models for which discrete numerical approximations can be found, the optimal control problem can be viewed as the problem of choosing variables to maximize an objective function. From this perspective, optimal control becomes the process of maximizing a generalized non-linear, perhaps constrained, objective function. The maximization process may be either static or dynamic, depending on the nature of the model, but is generally thought of in control theory as being dynamic. Recent advances in the technique of numerical optimization of non-linear functions make this perspective of optimal control a useful tool for analysis of many applied problems in the area of agricultural economics.

The objective of this bulletin is to demonstrate the use of optimal control to solve applied problems in the area of agricultural economics. This bulletin presents the principles of optimal control theory in a nonmathematical form to allow researchers to focus on the application of optimal control techniques. The first section briefly reviews the origin of optimal control theory and its use in economics. The second section describes a particular numerical optimization procedure which can be utilized in solving optimal control problems. The last section presents three examples of how optimal control theory techniques can be used in applied economic research.

Origin of Control Theory

The first application of control theory was on a single variable optimization problem in the field of engineering, specifically it was a study by Maxwell [1868] concerning the use of governors for speed control. This work led to other applications in the engineering area and during the second World War control theory was used extensively for studying military systems. Following the war, control theory was expanded to handle multi-variable optimization problems and later become widely used in aerospace and industrial development problems [Jacobs, 1975]. It was during this later stage that applied mathematicians contributed to the technique by developing numerous application oriented algorithms [Box, 1965; Goldfeld, et al., 1966; Kendrick and Taylor, 1970; Swann, 1974; Fair, 1974; and Chow, 1976]. Recent control theory contributions have included the introduction of stochastic and adaptive controls [Kirk, 1970; Schweppe, 1973; Cooper and Fischer, 1974; and, Rausser and Freebairn, 1975].

Economists have only recently been actively working with control theory. General economists including Intriligator [1971], Pindyck [1973], Chow [1975, 1976], Arrow [1968], Theil [1965], Dorfman [1969], Livesey [1971], Kendrick and Taylor [1970], Pindyck and Roberts [1974], Cooper and Fischer [1974], Arzac and Wilkinson [1977] and numerous others have made use of optimal control techniques to solve economic problems. Relatively few agricultural economists have applied the technique. Tinter [1969], Raulerson and Langham [1970], Rausser and Freebairn [1974a, 1974b], Rausser and Howitt [1975], Trapp [1977], Taylor and Talpaz [1977], Frohberg and Taylor [1977], Richardson [1978] and others have demonstrated the use of optimal control techniques in analyzing problems in agricultural economics.

Principles of Control Theory

The objective of optimal control theory is to determine the values of control variables that cause a particular system to maximize (or minimize) a given performance measure subject to a set of boundary constraints [Jacobs, 1975; Kirk, 1970; and Sage, 1968]. Formulation of a control problem involves three steps: 1) development of a mathematical model of the system to be controlled; 2) a statement of the boundary constraints on the control, input and output, variables; and, 3) a statement of the performance measure for the system [Kirk, 1970]. As is true for other applications of mathematical models, the model should be a structurally accurate representation of the system and should include linkages between the various sectors. The structural coefficients of the model may be estimated econometrically, obtained from known constant physical relations, or in some cases, iteratively estimated as part of the optimal control problem.

In control theory literature, the endogenous variables in the model are referred to as the state variables are denoted as: $x_1(t)$, $x_2(t)$, ..., $x_n(t)$ for time period t (e.g., production mix, profits, net worth at micro level; supply and utilization components, prices, government costs, stock levels, etc. in a macro application). The subset of state variables used in the performance measure are referred to as the output variables, and are designated as: $y_1(t)$, $y_2(t)$, ... $y_k(t)$ (e.g., profit, net worth or stock levels and government cost). Uncontrollable exogenous variables, (e.g., weather, unemployment level, interest rates) are denoted as $z_1(t)$, $z_2(t)$, ..., $z_q(t)$. The exogenous variables that can be manipulated or controlled by the decision maker, such as fertilizer use by an individual farmer or the level of loan rates by government officials, are referred to as control inputs (controls). Controls for period t are represented by: $u_1(t)$, $u_2(t)$, ..., $u_m(t)$. Values for the control variables over the period analyzed (t_o to t_f) constitute the control path and values for the state variables over the period analyzed make up the state trajectory [Kirk, 1970].

The model equations that describe the state (or endogenous) variables can be a function of the controls, other state variables, time and the noncontrollable exogenous variables. In order for the system to be controlled, one or more of the equations describing the state variables must contain a control variable. In turn, controls are normally a function of one or more of the state variables and/or time and other variables. When controls are a function of state variables, dynamic feed back from the system can be used to throttle successive control values. This circular causal flow which relates control values to state values and then back to the controls is called a closed loop control problem. When controls are not a function of the state variables the system is an open-loop control problem.

Boundary constraints are usually imposed on the control variables, and can be imposed on the state variables. The constraints limit the states and controls within boundaries established by the user in light of physical, economic and political limits of the system. The constraints reduce the number of alternative control paths that must be investigated since the model is only solved for admissible controls and admissible trajectories. An admissible control is a control path that satisfies all constraints on the controls for each time period and an admissible trajectory is a state trajectory that satisfies all constraints on state variables for each time period. Realistic boundary constraints on the controls allows more accurate modeling of the system while reducing the number of feasible trajectories and the cost of solving for the optimal control path.

A single valued performance measure, the criterion for evaluating the admissible control paths, must be developed for the particular problem being investigated. The performance measure (F) is defined by a mathematical equation or set of equations that sums weighted functions of the output variables for each state trajectory generated by the system being controlled. Exogenous information such as priority rankings, target levels, etc., may be integrated with the output variables in the mathematical formulation which determines the single performance measure value associated with each state trajectory and its corresponding control path. In application, values for the controls are selected by a control procedure in an iterative process that ultimately leads to the set of controls (or control path) that cause the performance measure to be optimized.

A diagrammatic illustration of a dynamic control system is presented in Figure 1. The model is simulated to obtain values of the state variables, using as input the following variables: the controls (u_j), initial or lagged values of the states (x_j), and values for any uncontrollable exogenous variables (z_j). The equations in the model are used to estimate the values for the state variables (x_j). The estimated values for a subset of the state variables, which have been referred to as output variables (y_j), are used in conjunction with user provided weights (r_j) to compute the value of the performance measure (F). The control mechanism (or numerical optimization routine as used here) computes new values for the control variables (u_j) for each iteration, based upon previous values of the performance measure and controls until the objective function value is optimized.

Rausser and Freebairn [1974b] propose a three step procedure for specifying and estimating the performance measure in a control theory problem. The steps to the procedure are: 1) select the relevant state variables in the model as the output variables, 2) determine the appropriate mathematical form, and 3)

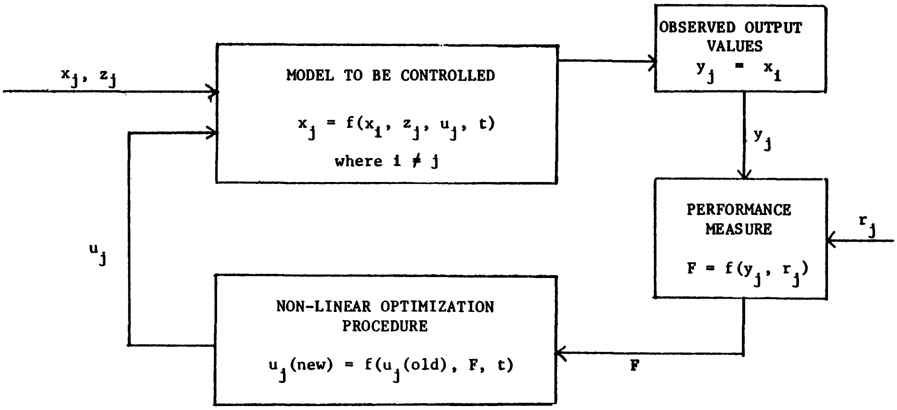

Figure 1. A Dynamic Control System

obtain estimates of the parameters or weights for the output variables. The guideline for selecting the output variables to include in the performance measure is quite obvious, select variables that are important to decision makers. Selection of the appropriate mathematical form and parameters or weights for output variables is slightly more difficult.

In general, the functional form of the performance measure should formalize assumptions regarding the rate of substitution among the output variables. In application, the functional form needs to be as simple as possible in its assignment of a unique real number to each set of output variables. The nature of the functional form for the performance measure depends upon the type of problem being analyzed. For example, a terminal control problem attempts to minimize the system's deviations from some desired level for the output variables in the final year (t_f) or:

$$\text{Minimize: } F = \sum_{i=1}^{n} r_i\,[y_i(t_f) - s_i\,(t_f)]^2$$

where t_f is the final year or stage of the system, s_i is the target value for output variable i, and r_i is the parameter weight assigned to the i^{th} output variable measure [Kirk, 1970]. Another type of performance measure is for tracking problems where the objective is to keep the output variable, $y_i(t)$, as close as possible to a series of target value, $s_i(t)$, over the interval t_o to t_f:

$$\text{Minimize: } F = \sum_{j=t_o}^{t_f} \left(\sum_{i=1}^{n} r_{ij}[y_i(t_j) - s_i(t_j)]^2 \right)$$

where r_{ij} is the weight assigned to the deviation for output variable i in time period j from the target value s_{ij} [Kirk, 1970; Theil, 1965; Ryan, 1974]. Theil referred to this functional form as a quadratic preference or performance function and used it for analyzing economic problems despite its obvious problems, that of using constant weights for under and over shooting the target level and the need to establish single valued target levels for each output variable for each period.

The performance measure developed by Richardson [1978] is a modified version of the tracking function or quadratic preference function; it allows the analysts to target output variables within acceptable ranges and provides a weighting procedure that differentiates between positive and negative deviations from the desired ranges. These improvements generalize the performance function suggested by Theil [1965] by allowing different penalties for over and under shooting target values. Also, the modified functional form does not force the analyst to provide single valued targets for each observation in the trajectory of output variables but only targets for the upper and lower boundaries for each output variable. The performance measure is expressed as:

If lower bound is violated -

$$JL_{ij} = H_{ij} \, |y_{ij} - LB_{ij}|$$

If upper bound is violated -

$$JU_{ij} = I_{ij} \, |y_{ij} - UB_{ij}|$$

$$\text{Minimize: } F = \sum_{j=1}^{4} \left(\sum_{i=1}^{n} (JL_{ij} + JU_{ij}) \right)$$

where H_{ij} is the weight for output variable y_i violating lower boundary limit LB_i in period j; I_{ij} is the weight for output variable y_i violating upper boundary limit UB_i in period j. The JL_{ij} or JU_{ij} is set to zero when the boundary level of an output variable is not violated, so the objective function is not penalized when the values of the output variables fall within their acceptable boundary limits. Values for the upper and lower boundary limits can be specified from observing prior decisions by decision makers and by questioning decision makers as to the acceptable ranges for the output variables. Theil's quadratic preference function is a special case of Richardson's performance measure, for if $LB_{ij} = UB_{ij}$, $I_{ij} = H_{ij}$ and the deviations from the targets are squared we obtain Theil's quadratic preference function.

The third step in Rausser and Freebairn's preformance measure specification, estimating parameters for the performance measure, is the most difficult step in applying control theory to problems in economics. The problem of specifying the appropriate parameters for the performance measure (r_{ij}'s) has been of little importance in the past, since the functions used by engineers in optimal control applications require only that the weights cause the model to follow a prescribed trajectory or achieve a final targeted value. Such weights can be found through experimentation or by studying the physical relationships in the system. The performance measures developed for economic applications of control theory are not generally of the tracking function form so meaningful values for the weights must be developed [Bray, 1974; Rausser and Freebairn, 1974a, 1974b, 1975].

Bray [1974] suggests that the parameter weights may be determined through interviews with decision makers and government planners. Rausser and Freebairn [1974b] include Bray's suggestion in their direct approach and add to this two other approaches. The indirect approach involves studying past political decisions and the arbitrary approach involves the analyst assigning arbitrary values for the parameter weights.

Numerical Solution of Optimal Control Problems

Theoretical descriptions of optimal control theory problems generally utilize calculus of variation and assume the systems is represented in the form of a set of first order differential equations which is referred to as the state form. Direct-solution techniques are available for solving control problems in the state form by maximizing the implicit Lagrangian functions [Chow, 1975; Kirk, 1970]. However, as Swann [1974] points out, direct-solution techniques may not be practical due to the lengthy and complicated calculations involved in solving the derivatives. The problem often can be overcome with finite-difference approximations but this tends to introduce truncation and cancellation errors which can cause problems in obtaining the final solution.

An alternative to using direct-solution techniques is to use direct-search or numerical techniques. Numerical techniques do not require the model be in the state form and obtain the final (optimal) solution without solving derivatives. Kirk [1970] and Swann [1974] describe several direct-search methods available for solving constrained optimization problems. In general, the direct-search techniques are hill climbing procedures that utilize alternative methods of searching the surface of the performance measure for its global maximum (or minimum). In application, the control mechanism selects values for the control variables, determines their impacts on the system's output variables and evaluates the performance measure based on the values of the relevant output variables. This process is repeated in an iterative fashion until any change in the control variables results in a reduction in the value of the performance measure.

The direct-search technique described in this report is Box's Complex Procedure. The Complex Procedure, developed by Box [1965], is capable of solving for the optimal set of controls in a multi-variable model, that is in the form of a closed-loop feedback problem. Swann [1974] indicates that the Complex Procedure has been used quite extensively and successfully to solve a wide range of constrained optimization problems. The procedure has the flexibility of handling non-linear inequality constraints on the control variables and has been shown to be reliable when compared to more sophisticated mathematical techniques [Box, 1965; Goldfeld, et al., 1966]. Since Complex is a direct-search technique, the procedure can be applied to an existing model without reprogramming the model to the state form. This was a major consideration in selecting the technique, since most models in the field of agricultural economics are not stated in terms of the state form. (A computer algorithm for Complex is available in Kuester and Mize [1973] and a listing of the revised computer algorithm used for this bulletin is in Appendix A.)

Box's Complex Procedure

The objective of Box's Complex Procedure is to maximize a performance measure (F) subject to the boundary constraints on the control variables or:

Maximize: $F(y_1, y_2, ..., y_n, r_1, r_2, ..., r_n)$
Subject to: $G_j \leq u_j \leq H_j, j = 1, 2, 3, ..., m$

where $y_1, ..., y_n$ are output variables, $r_1, ..., r_n$ are user provided parameter weights, and G_j and H_j are lower and upper boundary constraints for control variable j, respectively. Values for the admissible control paths (u_j's) are used as input in a model of the system to be controlled, to obtain predicted or simulated values for the system's state variables (x_j's), i.e., the state trajectory. The output variables (y_j's) are used in the performance measure (F) to obtain a unique real number to be associated with the control path being evaluated. This process continues iteratively. With each iteration a new control path is computed by systematically changing the values of the control variables. The new control path is then evaluated by using it in the model to simulate values for the state trajectory and using the predicted values of the output variables in the performance measure. The final solution is reached when no improvement in the value of the performance measure can be made.

The computer program for the Complex Procedure is written in Fortran. The program consists of the following subroutines: COMPLX (acts as the MAIN), CONSX, CHECK, CENTR, CONSTT, and OBJT (see the computer listing in Appendix A). The researcher can link the Complex Procedure to the system to be controlled in one of two ways: code the control model directly into subroutine OBJT and use COMPLX as the MAIN or call subroutine COMPLX from another computer program and code the control model in subroutine OBJT. The performance measure must be provided by the researcher in subroutine OBJT. Also, the researcher must provide the upper and lower boundary constraints for the control variables in the CONSTT subroutine. Whether COMPLX is used as the MAIN or as a separate subroutine it has two functions, they are to read the data cards and to print an output table of the optimal values for the control variables.

The Complex Procedure begins each control problem by generating or reading sets of initial values for the m control variables. The number of sets of initial values required to identify the performance surface is the number of controls plus 1. Thus, if there are two controls, the domain of the surface is 3 and can be visualized as a three dimensional graph with the performance measure on the vertical axis and control variables on the two horizontal axes. The number of sets of initial values for the control would be three in this case. In general, a control problem with m controls has m+1 or k control paths to mathematically identify the performance surface. Each of the k control paths has values for each of the m control variables. Each path or set of values is considered to be a coordinate for one point on the surface of the performance measure. The control paths are stored in a k by m matrix (X), with the rows containing the k different control paths and the columns containing the values for the m different control variables.[1] The initial control paths can be user supplied or they can be random values, uniformly distributed between the respective lower and upper boundary constraints. The source of the initial control paths is determined by the user, depending upon the data input option specified on the I-0 Card (see Appendix B).

Once the X matrix is initialized with starting values for the control variables, each control path is checked to be sure it is admissible (subroutine CHECK). The value of each control variable is compared to its respective lower and upper boundary constraints, provided by the user in subroutine CONSTT, to be sure the control is admissible. If a value is inadmissible, the value is moved inside the violated boundary constraint by a small amount DELTA, say 0.001.

After determining that the initial control paths are admissible, the performance measure is evaluated for each of the k control paths. The OBJT subroutine contains the performance measure and the user supplied model of the system to be controlled so it is called each time a control path is evaluated. To evaluate the initial control paths, subroutine OBJT is called k times, each time a different control path is used as input in the researcher's model. Simulated values of the output variables are used in the performance measure to obtain a unique real number for evaluating the particular control path. The values of the performance measure are stored in the F array which is a kxl array.

After evaluating the k^{th} initial control path, Complex begins the iterative procedure that leads to the optimal control path for the given performance measure. The first step in each iteration is to identify the control path (or row of the X matrix) associated with the minimum value of the performance measure, say row i. The control mechanism then replaces the rejected row, i, with a control path that is associated with a higher point on the surface of the performance measure.

[1]This definition of the X matrix in the Complex procedure should not be confused with the x vectors discussed earlier which denote the values of a state trajectory.

New values for control path i are calculated by the following formula:

$$X_{ij}(\text{new}) = \overline{X}_{jc} + \alpha(X_{jc} - X_{ij}(\text{old})); \; j = 1, 2, ..., m$$

where X_{ij} (new) is the new value of control variable j in coordinate or control path i, α is the reflection factor (Box [1965] recommends using 1.3), and $\overline{X}_{jc}$ is the centroid for control variable j. The new centroid for the jth control variable, $\overline{X}_{jc}$, for each iteration is the average of the control variable excluding the one that is rejected. The centroid for each of the m control variables is calculated in subroutine CENTR. The reflection factor, α, is greater than one to insure that the control mechanism searches both sides of the centroid in its approach to the optimal control values.

The new values for the control variables (X_{ij} (new)) are then checked against the lower and upper boundary constraints to assure that the control path is admissible. The value of the performance measure for the ith control path is obtained by using control path i as input in the user supplied model and simulating values for the endogenous or state variables in the model. If the ith control path is no longer associated with the minimum point on the performance measure the first iteration is complete. However, if the ith control path repeats as the lowest point, new control values are selected, checked and evaluated until the ith path is no longer associated with the minimum point on the performance measure. In the next iteration this procedure is followed for the row which now has the lowest performance measure value and so on.

By rejecting the control path associated with the minimum value for the performance measure and replacing it with a control path that has a higher value, the procedure will ultimately find the maximum value of the performance measure. Each of the k sets or control paths will eventually coverge to the optimal control path. The control path associated with the maximum point on the performance measure surface is considered to be optimal for the given performance measure.

At the end of each iteration the convergence criteria is checked to see if the performance measure is at a maximum (subroutine CONSX). A maximum is declared if for Y iterations the highest and lowest values of the performance measure remain within ß units of each other. (Values for Y and ß are provided by the user on the data cards, see Appendix A.)

To insure that the final solution is at the global maximum for the performance measure, the problem should be run several times. Each time a different set of initial control paths should be used so the procedure searches a different set of values for the control variables. If the procedure returns the same answer several times, the analyst can feel fairly certain of having found the global maximum.

The boundary constraints for the control variables are critical to the use of the Complex Procedure. The user must provide values for the boundary constraints in the user provided constraint subroutine CONSTT. The lower boundary constraints (G_i's) are programmed in the G array and the upper boundary constraints (H_i's) are programmed in the H array. The order of the variables in the G and H arrays must correspond exactly to the order in the X matrix, since G and H are mxl arrays and X is the kxm matrix of control variables.

Application of Optimal Control Techniques

Box's Complex Procedure can be used to solve many different problems in the general area of agricultural economics. To demonstrate the flexibility of the procedure, and to provide examples of typical optimal control problems encountered in agricultural economics, three widely different problem examples will be discussed. The first application is a constrained profit maximization problem for a firm producing three outputs with four inputs. This problem represents a static control problem. However, the main purpose of its use here is display the capabilities and nature of the complex procedure.

The second application demonstrates how the procedure can be used to estimate characteristics of the aggregate population of cattle being placed on feed and in turn use this information to aid in forecasting beef supply. This example is the classical dynamic control problem described in the preceding control theory discussion. In this case the time path of control variables sought are characteristics and numbers of cattle placed on feed which will generate accurate tracking of cattle on feed slaughter.

The third application of the procedure is in the area of agricultural policy analysis. The complex procedure is used in conjunction with a National Agricultural Policy Simulator model to compute "optimal" values for policy instruments given expected conditions and performance criteria. This example can be viewed as an application of optimal control theory to assist in system design. In this case the agricultural program, consisting of support prices, target prices, etc., is viewed as a part of the total agricultural system structure. By altering the nature of the agricultural program a different set of consequences can be generated from a set of expected future conditions (i.e., scenarioed or forecasted model inputs). Optimal control is used to select the program features which, given model inputs, lead to the desired results as described by the simulated output.

Application of Control Techniques to Profit Maximization

One of the most common problems faced in the area of agricultural economics is the problem of determining the profit maximizing level of production and input use when the quantity of inputs available is constrained in some way. Such problems are usually solved by setting up a constrained profit function and maximizing it by simultaneously solving a system of first derivatives for the profit function [Henderson and Quandt, 1958]. In general this method is simple if the problem is limited to one or two products that are a function of a small number of inputs and the prices for both the outputs and the inputs are fixed. If the products are produced in less than perfect competition, i.e., face a downward sloping demand curve and the inputs are associated with a positively sloped marginal factor cost function, the problem of finding the profit maximizing level of production becomes more difficult.

The Problem. To demonstrate how a constrained profit maximization problem can be solved using Box's Complex Procedure, consider a firm with three outputs (y_1, y_2, and y_3), four inputs (x_1, x_2, x_3, and x_4) and constraints on the maximum amount of each input that can be used. The problem can be stated as:

Maximize: profits for outputs y_1, y_2, and y_3

Subject to: production functions -

$$y_1 = x_{11}^{.33} \; x_{12}^{.17} \; x_{13}^{.20} \; x_{14}^{.30}$$

$$y_2 = x_{21}^{.10} \; x_{22}^{.08} \; x_{23}^{.25} \; x_{24}^{.40}$$

$$y_3 = x_{31}^{.09} \; x_{32}^{.19} \; x_{33}^{.15} \; x_{34}^{.20}$$

Output demand functions (prices) -

$Py_1 = 1050.0 - 0.5y_1$

$Py_2 = 1000.0 - 0.25(y_2)^2$

$Py_3 = 100.0 - 0.15(y_3)^2$

Input constraints -

$2000.0 \geqslant x_{11} + x_{21} + x_{31} = \text{sum } x_1$

$3000.0 \geqslant x_{12} + x_{22} + x_{23} = \text{sum } x_2$

$2100.0 \geqslant x_{13} + x_{23} + x_{33} = \text{sum } x_3$

$1000.0 \geqslant x_{14} + x_{24} + x_{34} = \text{sum } x_4$

Input marginal costs -

$Px_1 = 3.0 + 0.0009 \text{ sum } x_1$

$Px_2 = 6.0 + 0.00011 \text{ sum } x_2$

$Px_3 = 9.0 + 0.0003 \text{ sum } x_3$

$Px_4 = 7.0 + 0.000199 \text{ sum } x_4$

And: $x_{ij} > 0.0$ for $i = 1, 2, 3$ and $j = 1, 2, 3, 4$.

Production functions in the Cobb-Douglas form are used since they are non-linear and the functional form is used in most mathematical economics textbooks [Henderson and Quandt, 1958]. Demand functions for y_1, y_2, and y_3 are used to demonstrate an additional dimension, that of less than perfect competition in the output market. The constraints on the inputs x_1 through x_4 are incorporated, to make the example a constrained profit maximization problem. The marginal input prices or costs for the four inputs are functions of the quantities used, instead of a fixed price or cost for unlimited use of the inputs. The linear marginal input prices allows for input markets that are not operating in perfect competition. The final constraint on the problem, that of non-zero levels of input use, is imposed to prevent the model from selecting a zero level of input which in a multiplicative production function causes the output level to be zero.

Setting Up the Problem. To incorporate the profit maximization problem into the Complex Procedure the first step is to identify the control variables and determine lower and upper boundary constraints for the individual controls. For the problem presented above, the controls are the levels of inputs used in each product; more specifically the control varaiables are:

Control variable 1 X(i, 1), G(1), H(1) input x_{11}
Control variable 2 X(i, 2), B(2), H(2) input x_{21}
Control variable 3 X(i, 3), G(3), H(3) input x_{31}
Control variable 4 X(i, 4), G(4), H(4) input x_{12}
Control variable 5 X(i, 5), G(5), H(5) input x_{22}
Control variable 6 X(i, 6), G(6), H(6) input x_{32}
Control variable 7 X(i, 7), G(7), H(7) input x_{13}
Control variable 8 X(i, 8), G(8), H(8) input x_{23}
Control variable 9 X(i, 9), G(9), H(9) input x_{33}
Control variable 10 X(i, 10), G(10), H(10) input x_{14}
Control variable 11 X(i, 11), G(11), H(11) input x_{24}
Control variable 12 X(i, 12), G(12), H(12) input x_{34}

where the X matrix is the location of values for the control variables selected by the control mechanism, G is the lower boundary constraint array and H is the upper boundary constraint array. The boundary constraints must be provided by the user in the CONSTT subroutine. The listing of the program in Appendix A includes the profit maximization problem presented here, to demonstrate how the user provides the boundary constraints and the performance measure. The lower boundary constraint for each of the 12 control variables is zero so G(i) = 0.0 for i = 1, 2, 3, ..., 12 as indicated in CONSTT. The upper boundary constraints for the control variables are given in the problem statement, for example, the maximum level of x_1 that could be used to produce one product (zero amounts of other products) is 2000.0 units.[2] Thus H(1), H(2), and H(3) equal 2000.0 in subroutine CONSTT. The upper boundary constraints for the remaining control variables are set in a similar manner.

The second step in setting up the profit maximization problem is to program the model or system to be optimized and the performance measure into the OBJT subroutine. The production functions for y_1, y_2 and y_3 are

[2]Strictly speaking, input use would only approach 2000.0 units since the X_{ij} are constrained to be non-zero.

programmed in Fortran using the appropriate locations in the X matrix as the input variables x_1, x_2, x_3 and x_4 (see subroutine OBJT in Appendix A). The control mechanism selects values for the control variables and puts them in the ith row of the X matrix, the order of the controls in the X matrix is the same as the order used for the G and H arrays presented above. The simulated output levels for y_1, y_2 and y_3 are used in the product demand equations to obtain prices for the outputs (Py_1, Py_2 and Py_3). The total quantity of each input used is calculated and used to compute the prices or costs for the four factors of production (Px_1, Px_2, Px_3 and Px_4). The performance measure to be maximized is a constrainted profit function (F):

$$F = \sum_{i=1}^{3} Py_i * y_i - \sum_{j=1}^{4} Px_j * \text{sum } x_j - \sum_{\ell=1}^{4} (UB_\ell - \text{sum } x_\ell)^2$$

where Py_i is the price of output y_i, Px_j is the cost of input x_j and UB is the maximum amount of x_ℓ available. Values for UB_ℓ are 2000, 3000, 2100 and 1000 for sum x_1, sum x_2, sum x_3, and sum x_4, respectively. If sum x_ℓ is less than UB_ℓ the last part of F is ignored thus only penalizing the performance measure if excessive quantities of x_ℓ are used.

The final step in solving the profit maximization problem with Box's Complex procedure is to code the data cards and run the program. Coding instructions for the data cards are presented in Appendix B. For the results presented in the next section, the random number seed provided on the I-0 Card is 999991.0 and the initial set of values for the control variables are selected at random. The values entered on the Parameter Card are the following: Alpha = 1.3, Beta = 0.30, Delta = 1.0, Gamma = 5, the number of control variables is 12, and the maximum number of iterations is 700. (Definitions of the parameters are presented in an earlier section of this bulletin.)

Results of the Example Problem. The optimal solution obtained from using the Complex Procedure to solve the profit maximization problem is presented in Table 1. The maximum value of the performance measure is \$545,090.4 and comes from producing 1,000.9 units of y_1, 35.9 units of y_2, and 10.1 units of y_3.

The sum of x_1 used in the production of y_1, y_2 and y_3 is 1998.0 units, approximately equal to the maximum amount of x_1 available (2,000 units). A similar situation exists for input x_4, in that the total quantity used is 998 units and the maximum available is 1000 units. The x_2 and x_3 inputs do not restrain on the profit maximization solution since the optimal levels for these inputs are substantially below their respective upper constraints. A change in the demand function for any of the three outputs or a change in the input cost function for any of the four inputs causes the solution to the problem to be altered. Also, changes in the form of the performance measure can alter the optimal solution reported in Table 1.

Table 1. Results of the Sample Problem - A Profit Maximization With Three Outputs and Four Inputs.

Inputs	Outputs y_1	y_2	y_3	Sum of Inputs (x_i)
x_1	1972.04	10.85	15.89	1998.78
x_2	1202.95	7.67	59.67	1270.29
x_3	396.24	198.40	11.50	606.14
x_4	793.34	103.94	101.10	998.38
Total Production	1000.9	35.9	10.1	
Profit				$545,090.4

The costs of using Box's Complex Procedure to solve the profit maximization problem are quite small in comparison to the alternatives, namely solving the problem by hand or using parametric programming. The time required to program the problem is less than one hour and the computer time on an IBM 370-75 to solve the problem is about 30 seconds; even though the program runs about 800 iterations to reach the final solution. This particular problem requires a large number of iterations because of the number of control variables (12) and the non-linearities in the production functions.

Many other problems of this type can be solved by using Box's Complex Procedure. For example the problem of how to allocate "given quantities" of x_1, x_2, x_3 and x_4 among the three outputs to maximize profits can be solved by simply changing the performance measure to:

$$F = \sum_{i=1}^{3} Py_i * y_i - \sum_{j=1}^{4} Px_j * \text{sum } x_j - \sum_{\ell=1}^{4} (DS_\ell{}^* - \text{sum } x_\ell)^2$$

where Py_i is the price of output y_i, Px_j is the input cost of all x_j used and DS is the desired level of use for input x . In this case a penalty is forthcoming if the level of input use is different from the desired level of use.

Application of Control Techniques to Beef Supply Models

Agricultural outlook economists have made extensive use of cattle on feed data to make short-run beef supply forecasts. Cattle on feed statistics report the number of animals on feed by sex and weight. Weight of cattle on feed is reported in 200 lb. increments, i.e., 500 lbs. and under, 500-700 lbs., 700-900 lbs., 900-1100 lbs., and 1100 lbs. and above. Cattle on feed data also report the number of animals placed on feed during the last reporting period. Placement data does not describe the sex or weight characteristics of the animals placed on feed.

The traditional method outlook economists have used in making short-run supply forecasts based upon cattle on feed data, has been to assume various percentages of the cattle, in different weight groups, will be marketed within 30 days, 60 days, etc. Statistical models have been developed to predict slaughter one to six months in advance from the cattle on feed data. These statistical models regress slaughter for the period in question upon current cattle on feed numbers by weight groupings. An example of such a model is specified below.

$$\text{Slg} = a + b\left(\begin{array}{c}\#\text{ of 700-900 lb.}\\ \text{cattle on feed}\end{array}\right) + c\left(\begin{array}{c}\#\text{ of 900-1100 lb.}\\ \text{cattle on feed}\end{array}\right)$$

Short-run beef slaughter forecasts made in this manner have proven to be quite reliable, especially when tempered with experience and judgment. Forecasts made by this method, however, ignore substantial amounts of information known about beef growth processes. This is because aggregate cattle on feed data do not describe the nature of the animals on feed in enough detail to make such information useful. For example, if the precise weight distribution of animals on feed were known as well as exact placement weights and rations, then growth models of beef animals could predict the future weight of animals for a given date quite accurately. Growth models capable of such predictions have been developed by Fox and Black [1977] and Gill [1975]. These models are heavily based upon Lofgreen and Garrett's [1968] net energy equations. It is the premise of the modeling and optimal control applications to be described here that more detailed knowledge of the weight of cattle on feed and the weight at which they are placed on feed coupled with beef growth models and/or common knowledge of typical beef growth rates will permit more accuracy to be developed in making short-term beef supply forecasts.

Optimal feedback control techniques have been applied to a cattle on feed growth simulator to estimate specific cattle on feed weight distributions and placement weights. In this procedure cattle placement weights sex of the animals placed and growth rates are treated as control parameters. Box's Complex Procedure is used to adjust the control parameters to optimize the tracking of historical cattle on feed data. The mathematical relations used to describe the inventory of cattle on feed and their growth process and eventual slaughter consists of a set of continuous differential equations[3]. Because the equations describing the growth process and placement weight distributions are continuous with respect to time and weight, inventories of cattle on feed can be computationally broken into single pound increments with respect to the current weight distribution of cattle on feed and their placement weights. Even though the model is continuous in nature, tracking must be done in a discrete sense because the data to be tracked are discrete. This is achieved by integrating the continuous functions over the desired time and/or weight ranges and comparing the results with the discrete data. Hence, the continuous flows of the model interpolate between discrete data points in such a way that accurate discrete tracking is obtained. The time path of control variables

[3]See Llewellyn [1966] or Manetsch and Park [1974] for presentations of differential equation modeling.

derived to generate accurate tracking of the discrete cattle on feed data provide information which can be used to assist in predicting future placement weights, sex of animals and aggregate growth rates.

The optimal feedback control framework used is depicted in Figure 1. Observed cattle on feed and cattle placed on feed coupled with base values of growth rates and placement weights are initially supplied to the cattle on feed simulation model. The simulation model is then operated to generate predictions of beef slaughter and ending cattle on feed by 200 lb. weight increments. These predicted values are compared to observed values and an error squared performance measure calculated. The performance measure is defined as follows:

$$\begin{aligned} OBJ = {} & (M - \hat{M}/M)^2 + (COF + \widehat{COF}/COF)^2 * 8 + (S5 - \widehat{S5}/S5)^2 + \\ & (S57 - \widehat{S57}/S57)^2 + (S79 - \widehat{S79}/S79)^2 + (S9 - \widehat{S9}/S9)^2 + (H5 - \\ & \widehat{H5}/H5)^2 + (H57 - \widehat{H57}/H57)^2 + (H79 - \widehat{H79}/H79)^2 + (H9 - \\ & \widehat{H9}/H9)^2 \end{aligned}$$

where the symbol ^ denotes model predictions and

OBJ = performance value to be minimized;
M = cattle on feed marketed, i.e., slaughtered;
COF = total cattle on feed;
S5 = steer under 500 lbs. on feed;
S57 = steer between 500-699 lbs. on feed;
S79 = steer between 700-899 lbs. on feed;
S9 = steer 900 lbs. and over on feed;
H5 = heifers under 500 lbs. on feed;
H57 = heifers 500-699 lbs. on feed;
H79 = heifers 700-899 lbs. on feed; and,
H9 = heifers 900 lbs. and over on feed.

A heavier weight or penalty is given to error in tracking total cattle on feed since it is the summation of individual categories of cattle on feed and is believed to be reported more accurately than individual categories of cattle on feed.

The numerical optimization routine receives performance "feedback" information from each setting of the control variables. This information is in terms of weighted percent of tracking error squared. By recording the marginal change in performance (improved tracking accuracy) associated with a given marginal change in the control variables (growth rates and placement weights by sex), the optimization routine iteratively adjusts the control variable settings in a systematic manner until the performance measure is minimized.

This procedure of estimating growth rates and placement weights constitutes a closed-loop feedback control procedure according to the traditional definition, i.e., the control variables are a function of state variables. In this case the state variables are estimates of cattle on feed and cattle on feed marketed.

The cattle on feed growth simulation model will not be described in detail here.[4] A basic understanding of the structure of the model can be obtained by studying Figure 2 which outlines the placement and cattle on feed categories described within the model. The model simulates the daily rate of gain of cattle on feed by considering the effect of placement weight, current weight, sex and season of the year upon growth rates. Briefly, the effect of each of these factors as modeled is the following: steers have been observed to grow faster than heifers and grade choice at heavier weights than heifers and, hence, are typically slaughtered at heavier weights; animals placed at heavier weights, once on full feed experience "compensitory growth" and gain weight at faster marginal daily rates of gain than other animals of the same current weight but placed at a lighter weight; as animals on feed become heavier, their marginal daily rate of gain slows because more energy from their ration is required for body maintenance leaving less for growth; seasonal changes in temperature, rainfall, etc., cause different growth rates.

Results of operating the model over the period 1962-1977 in a feedback control framework indicate that seasonal patterns exist for aggregate growth rates, placement weights and sex ratios of cattle placed on feed. Table 2 presents the results found for seasonal growth rates, sex ratios and average weight of cattle on feed. Growth rates were estimated to be most rapid in the first and fourth quarters and the slowest in the third quarter. These results would tend to indicate that heat stress in the third quarter hampers animal growth more than cold temperatures in the first and fourth quarter. A definite seasonal change in the steer to heifer ratio (sex ratio) of cattle placed on feed is estimated. The ratios indicate that proportionately fewer heifers are placed in first and fourth quarters. Lastly, the estimates of average weight of cattle on feed reported in Table 2, Column 3, indicates that the heaviest average weight

[4]See Trapp, James N., "A New Approach to Beef Supply Modeling Using Differential Equations and Optimal Control Techniques," forthcoming, Oklahoma State University Experiment Station Technical Bulletin.

Table 2. Selected Average Estimated Characteristics of Cattle on Feed and Placed on Feed by Quarter, 1962-1977.

Quarter	Growth Rate Index	Sex Ratio of Cattle Placed on Feed Steers/Heifers	Average Wt. of Cattle on Feed
1	104	3.20	815
2	100	2.15	834
3	89	1.92	821
4	105	2.24	768

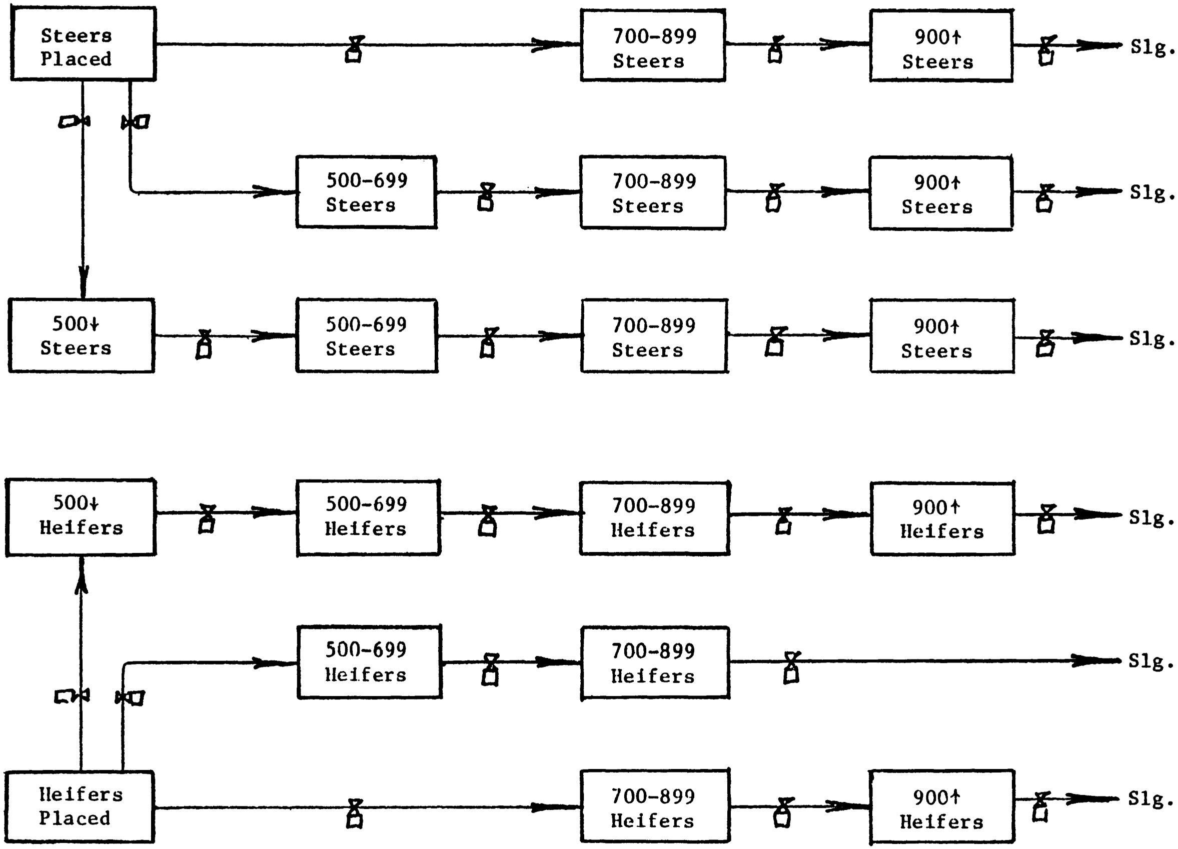

Figure 2. Cattle on Feed Population and Growth Model

Table 3. Estimated Seasonal Distribution and Average Weight of Cattle Placed on Feed, 1962-1977.

Quarter	Percent Placed Under 500 lbs.	500-700 lbs.	700-900 lbs.	Average Weight
1	53.2	40.3	6.6	508
2	26.7	66.3	7.6	557
3	26.5	43.7	29.8	581
4	64.4	26.5	9.1	490
Annual Average	42.2	44.5	13.2	534

of cattle on feed occurs in the second quarter, with the next heaviest weight occurring in the third quarter. This pattern of weights for cattle on feed appears to be due to the seasonal fluctuation of cattle placement weights and numbers of cattle placed.

The placement weight information generated by the model is perhaps the most interesting and valuable. Somewhat surprisingly the estimates indicate that a significant portion of cattle placed weigh less than 500 lbs., i.e., 42.2 percent (Table 3). This is not so surprising if one considers that the turnover rate of cattle on feed under 500 lbs. is the most rapid of any reported weight group of cattle on feed. Cattle typically gain only 50-75 lbs. while in this weight classification as compared to 200 lbs. in others. Hence, to maintain a given inventory of cattle on feed under 500 lbs. requires more placements than to maintain the same inventory in wider ranged weight classes where the turnover rate is three to four times slower.

The estimates reported in Table 3 indicate that the majority of the under 500 lb. placements occur during the first and fourth quarters. This factor contributes to causing the low average weight estimates for cattle on feed reported for these quarters in Table 3. The heaviest average placement weights occur in the second and third quarters. The largest percentage of cattle placed in the second quarter is in the 500-700 lb. weight range, i.e., 61 percent. This group of cattle likely consists of spring calves that have been wintered, grazed on spring pasture and sent to the feedlot. The third quarter average placement weight, unlike the other quarters, is derived from a relatively uniform distribution of placement weights.

The time series paths of the annual average values found for the control variables are presented in Figures 3A-3C. The sex ratio (Figure 3A) is correlated with the cattle cycle (Figure 3D) as measured in terms of the annual index of the rate of change in the size of the cow herd. The simple correlation coefficient is +.67. The sex ratio is hypothesized to rise during period of

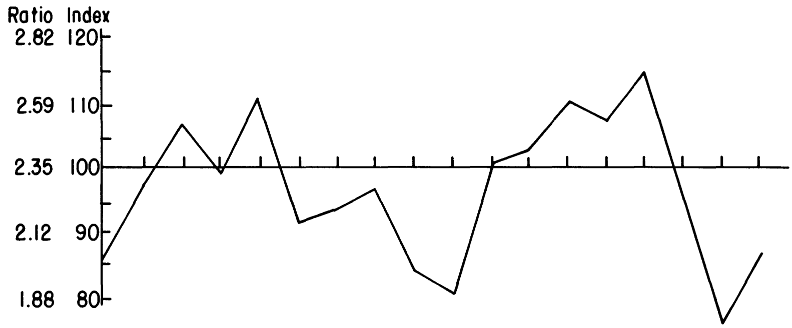

Figure 3A. Steer/Heifer Sex Ratio of Cattle Placed on Feed, 1960-77

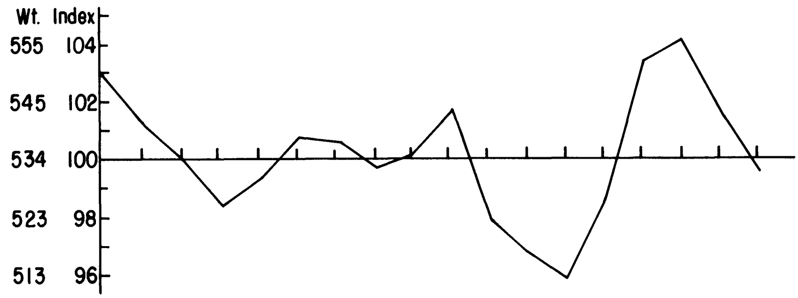

Figure 3B. Average Weight of Cattle Placed on Feed, 1960-77

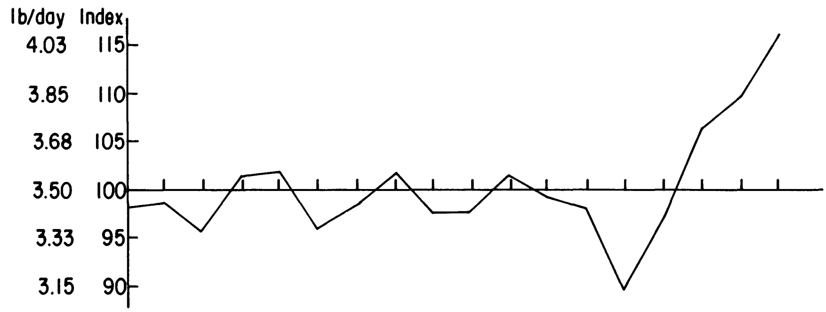

Figure 3C. Daily Growth Rates for 900 lb. Cattle on Feed, 1960-77

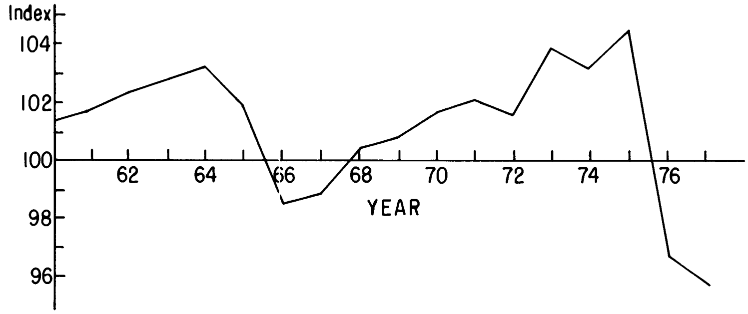

Figure 3D. Rate of Change in the Size of the Beef Cow Herd, 1960-77

expansion due to more heifers being held for replacements, thus causing the steer/heifer ratio to rise. The placement weight series is not strongly correlated to the cattle cycle as measured here but does appear to be cyclical. During 1974 and 1975 when feed prices were high relative to cattle prices and "grass fed" beef was common, placement weights were estimated to be the highest observed for the period 1960-1977.

The index of growth rates does not seem to follow the cycle either. When regressed against time it shows a significant positive trend (a t-value of 3.76 was found). This trend is primarily due to the unprecedented rise in growth rates occurring since 1973. The drop in growth rates estimated from 1970 to 1973 may be due to the legal actions taken against growth hormones and feed additives being used at that time.

Information generated from the estimated time paths of the control variables can likely be used to aid making cattle on feed forecasts via the traditional approaches. Their best use for assisting in making forecasts would appear to be in conjunction with continuous models of the beef growth process such as the one described and used in this study. By forecasting the control variables and using them as inputs to the continuous cattle on feed inventory and growth model, continuous (with respect to time and weight) projections of cattle on feed inventories and marketing can be made. Forecasts of the control variables (placement weights, sex ratios and growth rates) can be made either subjectively or via econometric methods. Research using econometric methods to estimate structural relations between the estimated values for the control variables and other observed economic variables is currently underway.

Figure 4 is presented as an example of the type of forecasts that can be made with the continuous cattle on feed inventory and growth model. It compares the 1977 optimal tracking path for cattle on feed marketed against a simulated 1978 projection. The daily slaughter rate simulated for January 1 of 1977 is used as a base value.

The 1978 projection depicted in Figure 4 was made immediately after the release of the 1977 fourth quarter cattle on feed report. The detailed (broken into one pound weight increments) December 31st ending cattle on feed estimates made by the model were used as an input into the projection. All other inputs were provided by making subjective assumptions of the expected changes from the reported or estimated (estimated in the cases where input data are not reported) 1977 values for the inputs. The input assumptions were as follows: a) placements would decline by five percent; b) slaughter weights would decline by 20 pounds; c) placements weights would decline by 30-40 pounds; d) growth rates would slow by five percent; and e) the steer/heifer ratio would increase by 15 percent.

At the date of this writing the January and February seven state cattle on feed reports were available. They incidate that 1978 seven state cattle on feed marketings as a percent of 1977 seven state cattle on feed marketings were 109 and 106 percent respectively for January and February. The model is based upon 23 state quarterly marketings, hence comparisons of absolute values is not possible but ratio comparisons are valid. The model forecasts of 1978 marketings as a percent of simulated 1977 marketings for January and February were 111 percent and 108 percent respectively.

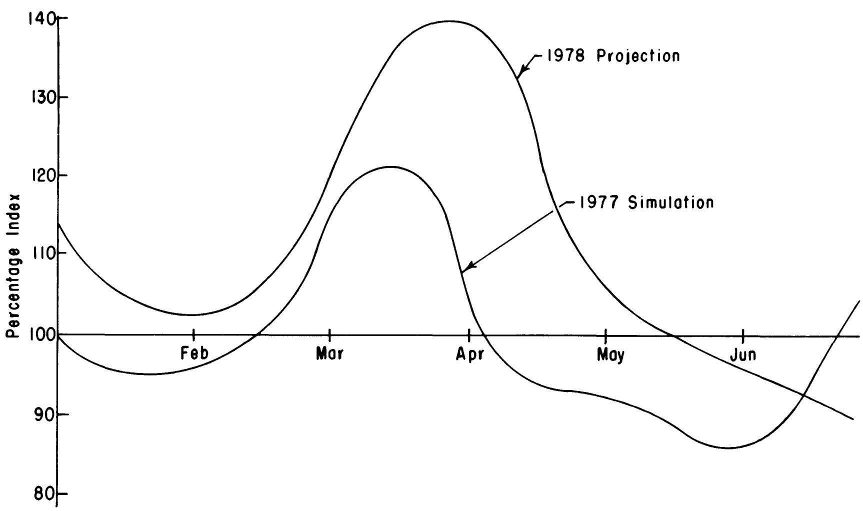

Figure 4. Simulated Indices of Daily Slaughter Rates for January Through June of 1977 and 1978

Application of Control Techniques to Agricultural Policy

The POLYSIM model is a disaggregated simulation model of the national agricultural economy developed by Ray and Richardson [1978] at Oklahoma State University. The model makes full use of forecasted data as a reference baseline. Included are the five-year baseline projections of commodity supplies, prices, and utilization made by ERS. Commodity specialists develop these projections using formal and informal forecasting models tempered with their own experienced judgments. The projections contain explicit assumptions concerning the rates of change in population, per capita incomes, consumer preferences, export demand, technology (including crop yields and livestock grains), and other supply and demand shifters. These projections also assume a specific set of Government farm programs. The user starts a simulation by changing one or more of the policy assumptions used in the base conditions, for example, by using a different series of loan rates. The simulation procedure traces through the effects on production, price, utilization, and farm income for each of the eleven commodity groups and on agriculture in the aggregate.

As indicated in Figure 5, the Complex Procedure is linked to the POLYSIM model by calling subroutine COMPLX from POLYSIM. Subroutine call statements for the execution subroutines in POLYSIM are included in subroutine OBJT so the simulation model is executed each time an admissible control path is selected by the optimization procedure.

A grain reserve farm program with acreage set-aside and loan rate provisions is analyzed to demonstrate how optimal control theory can be used to select loan rates and acreage set-aside levels for feed grains, wheat and cotton, that cause the Commodity Credit Corporation (CCC) to maintain a fixed reserve of grains. A grain reserve of 20 million tons of feed grains and 500 million bushels of wheat is assumed to be established in 1977 by the CCC. The farm program is analyzed for the four year period of 1978-1981.

The control variables, loan rates and set-aside levels for the three crops, are constrained to the upper and lower boundary constraints for these variables (Tables 4 and 5). The performance measure used for the analysis is reported in Table IV of Richardson [1978]. In general, the performance measure seeks to maximize net farm income subject to constraints on government expenditures, consumer food costs and maintenance of a critical level of stocks.

The 24 control variables for the farm program reported here are loan rates and acreage set-aside levels for feed grains, wheat and cotton in 1978, 1979, 1980 and 1981. The objective is to determine loan rates and acreage set-aside levels that maximize the performance measure. The CCC release rule used for the farm program is the following: release CCC held reserves if the average market price exceeds the loan rate by 50 percent and release only the amount

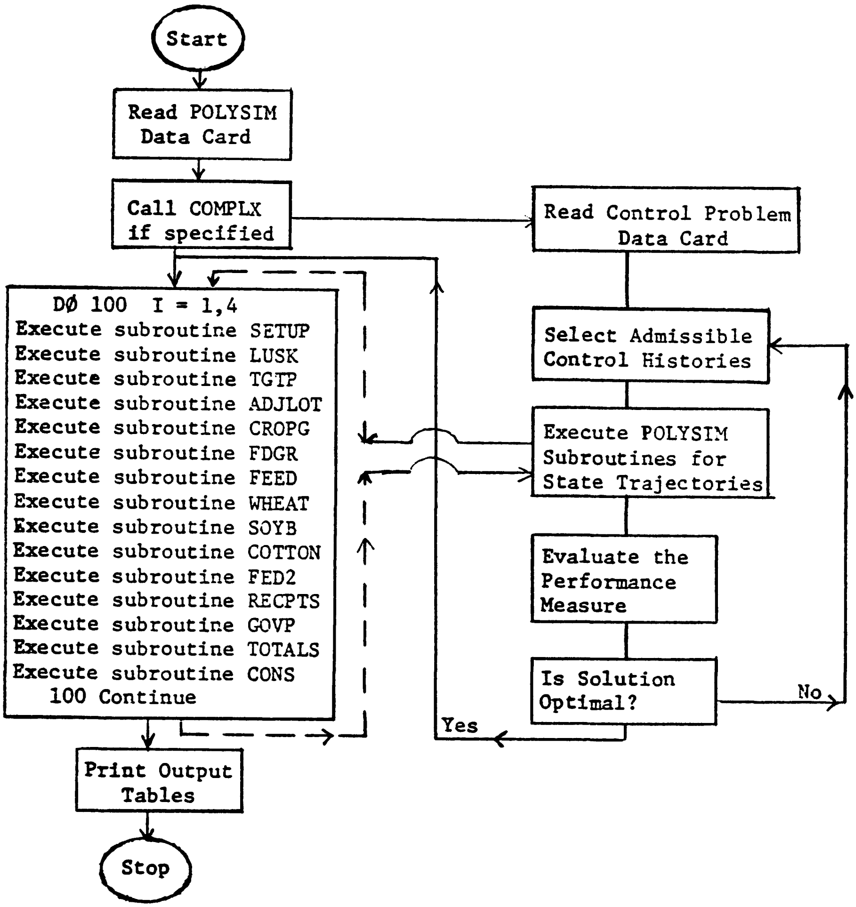

Figure 5. Flowchart of POLYSIM and its Modifications for the Control Theory Option

of stocks needed to lower the average market price to 150 percent of the loan rate.

For the control mechanism to maximize the performance measure it must select values for the control variables (loan rates and acreage set-aside levels for feed grains, wheat and cotton in 1978-1981) with respect to their estimated impacts on the state variables in POLYSIM and the output variables in the performance measure. Both immediate impacts (one year) and longer run impacts (two or more years) are considered by the control mechanism.

To select a value for the wheat loan rate in 1978 the control mechanism must consider the immediate impacts in 1978, as well as, the longer run impacts in 1979-1981, on the state variables in the model and particularly the

Table 4. Upper and Lower Boundary Constraints for Loan Rates for Wheat, Corn, and Cotton 1978-1981.

Year	Wheat Lower	Wheat Upper	Corn Lower	Corn Upper	Cotton Lower	Cotton Upper
	$/bu.		$/bu.		$/lb.	
1978	2.00	3.00	1.75	2.10	.37	.52
1979	2.00	3.10	1.75	2.21	.37	.55
1980	2.00	3.34	1.75	2.34	.37	.58
1981	2.00	3.52	1.75	2.47	.37	.61

Source: Lower boundaries for wheat and corn 1978-81 are minimum legal values established in the 1977 Act; the legal minimum for wheat and corn is about 88 percent of the 1977 loan rate, using this for cotton we get a minimum of about 0.37; upper boundaries for all crops 1978-81, are estimated target prices over the life of the 1977 Act.

Table 5. Upper and Lower Boundary Constraints for Acreage Set-Aside Levels for Wheat, Feed Grains, and Cotton 1978-1981.

Year	Wheat Lower	Wheat Upper	Feed Grains Lower	Feed Grains Upper	Cotton Lower	Cotton Upper
	m. ac.					
1978	0	24.7	0	37.7	0	3.2
1979	0	24.8	0	37.7	0	3.3
1980	0	24.8	0	37.6	0	3.1
1981	0	24.8	0	37.5	0	3.2

Source: The Agricultural Act of 1977 specifies that the maximum acreage set-aside for cotton is 28 percent of planted acreage in the previous year. For cotton, planted acreage is about equal to harvested acreage so the maximum set-aside for cotton is 28 percent of harvested acreage in the previous year. For feed grains and wheat, planted acreage is often much larger than harvested acreage so the maximum set-aside is 35 percent of harvested acreage in the previous year.

impacts on the output variables. The immediate impacts on the following state variables must be considered: the market price of wheat, the quantity of domestic and export demands for wheat, and wheat cash receipts, as well as their impacts on the output variables in the performance measure. The longer run impacts that must be considered are impacts on state variables such as: harvested acreage and supply of wheat, feed grains, cotton and soybeans, wheat yields, market prices of wheat, feed grains, cotton and soybeans, the quantity of domestic and export demands for the four model crops and cash receipts for all four model crops, because of their linkages to the output variables.

To select a value for the corn loan rate in 1978 the control mechanism must consider the immediate impacts on the following state variables: the market price for corn and the other feed grains, export and domestic demands for feed grains, feed grains cash receipts, and livestock feed costs, because of the linkages between these state variables and the output variables in the performance measure. Also, the control mechanism must consider the longer run impacts (1979-1981) on the following state variables: livestock production, prices and cash receipts, harvested acreage for feed grains, wheat, soybeans, and cotton, feed grain yields, supplies and prices of the four model crops, domestic and export demands for the model crops, total cash receipts for crops and livestock feed costs due to their linkages to farm income, government payments, CCC costs, food costs and ending year carryovers for the four model crops.

The above discussion assumes only the selection of the 1978 loan rates to illustrate the linkages in POLYSIM. Actually, the control mechanism simultaneously selects values for the loan rates of corn, wheat and cotton in 1978, 1979, 1980 and 1981, after considering the impacts of the loan rates on the output variables in the performance measure. The immediate and longer run interrelationships described above for 1978 thus become confused with the immediate and longer run impacts due to selecting loan rates in each of the remaining years.

In addition to selecting values for the loan rates, the control mechanism also selects the acreage set-aside levels for feed grains, wheat and cotton in 1978, 1979, 1980 and 1981. The immediate impacts that the control mechanism must consider are the same as those for changing the loan rates, as well as the impacts on: harvested acreage, production and supply for each of the three crops, of course, the longer run (1979-81) impacts on the output and other state variables must also be considered.

The optimal values for the control variables are presented in Table 6. Loan rates are not used by the control mechanism to support the market price in this particular farm program since the support action results in the CCC acquiring control of additional stocks. So acreage set-aside is the predominate control variable for the farm program. The optimal acreage set-aside levels for wheat and cotton are equal to the crop's respective upper boundary constraints in each of the four years simulated. Optimal acreage set-aside levels for feed grains range from 12 million acres to 32 million acres over the period simulated. So the feed grain acreage diversion levels are less than the boundary constraints (about 37 million acres).

The high levels of acreage set-aside for feed grains, wheat and cotton cause the average market prices for these crops to be greater than the respective market prices in the baseline for each of the years simulated. The corn loan rate is increased from year to year but is never greater than the market price and it is never less than the market price by more than 50 percent. So the CCC release and acquisition rule for corn is never activated. A similar situation exists for wheat.

The total government payments for miscellaneous farm programs and acreage set-aside is less than the $3.7 billion upper limit imposed on the performance measure, in each year simulated. The upper limit is almost

Table 6. Optimal Values of Control Variables and the Simulated Values of Selected State Variables for a Farm Program with Loan Rates and Acreage Set-Aside Provisions[1,2]

Item	Unit	Baseline Values				Simulated Values			
		1978	1979	1980	1981	1978	1979	1980	1981
CONTROL VARIABLES									
Price Support Levels									
Corn	$/bu.	2.00	2.00	2.00	2.00	1.80	1.94	2.10	2.18
Wheat	ds.	2.35	2.35	2.35	2.35	2.23	2.26	2.44	2.46
Cotton	$/lb.	.51	.51	.51	.51	.38	.38	.42	.46
Income Support Levels									
Corn	$/bu.	2.10	2.21	2.34	2.47	0.0	0.0	0.0	0.0
Wheat	ds.	3.00	3.16	3.34	3.52	0.0	0.0	0.0	0.0
Cotton	$/lb.	.52	.55	.58	.61	0.0	0.0	0.0	0.0
Set-Aside									
Feed grains	m. ac.	0.0	0.0	0.0	0.0	11.8	20.9	27.8	31.9
Wheat	ds.	0.0	0.0	0.0	0.0	24.6[U]	24.8[U]	24.8[U]	24.8[U]
Cotton	ds.	0.0	0.0	0.0	0.0	3.2[U]	3.3[U]	3.1[U]	3.2[U]
STATE VARIABLES									
Harvested Acreage									
Feed grains	m. ac.	107.7	107.7	107.4	107.2	100.6	96.3	92.9	90.3
Wheat	ds.	70.7	71.1	71.1	71.1	55.9	57.7	58.5	58.6
Cotton	ds.	11.6	11.4	11.7	11.2	9.7	9.9	10.7	10.0
Yield									
Feed grains	T./ac.	2.06	2.09	2.12	2.15	2.06	2.10	2.16	2.22
Wheat	bu./ac.	31.00	31.50	32.00	32.49	31.00	31.89	32.71	33.37
Cotton	lb./ac.	480.00	480.00	480.00	480.00	480.00	488.66	497.21	501.24

[1]Optimal control variables that equal their lower boundary constraints are denoted by superscript "L" and those that equal their upper boundary constraints are denoted by superscript "U".

[2]The performance measure for the optimal solution presented is the lower range performance measure in Table IV.

Table 6. Continued.

Item	Unit	Baseline Values				Simulated Values			
		1978	1979	1980	1981	1978	1979	1980	1981
Export Levels									
Feed grains	m. t.	50.4	52.2	53.7	55.4	49.1	46.9	45.9	46.0
Wheat	m. bu.	1025.0	1070.0	1110.0	1160.0	900.4	854.6	857.8	894.0
Cotton	m. bales	4.5	4.5	4.4	4.4	4.2	4.0	3.9	3.8
Total Utilization									
Feed grains	m. t.	206.2	213.3	223.0	228.6	204.2	201.0	201.2	204.3
Wheat	m. bu.	1925.0	1953.0	1991.0	2049.0	1770.6	1688.6	1681.6	1723.5
Cotton	m. bales	11.6	11.7	11.6	11.8	11.2	11.0	10.8	10.9
Ending Year Carryovers									
Feed Grains	m. t.	70.4	82.6	87.5	89.3	57.8	59.5	59.7	55.8
Wheat	m. bu.	1539.0	1827.0	2112.0	2374.0	1235.2	1385.9	1620.0	1852.6
Cotton	m. bales	4.3	4.2	4.5	4.1	2.7	2.0	2.5	2.1
CCC Inventory and Outstanding Loans									
Feed grains	m. t.	0.0	0.0	0.0	0.0	20.0	20.0	20.0	20.0
Wheat	m. bu.	776.0	1130.0	1497.0	1848.0	500.0	500.0	500.0	500.0
Cotton	m. bales	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Commodity Prices									
Corn	$/bu.	2.00	2.00	2.00	2.00	2.11	2.37	2.45	2.49
Wheat	ds.	2.35	2.35	2.35	2.35	2.92	3.11	3.11	3.09
Soybeans	ds.	5.60	5.60	5.70	5.80	4.32	4.88	6.16	6.49
Cotton	$/lb.	.54	.55	.52	.55	.60	.65	.61	.65
Cattle and Calves	ds.	.42	.45	.49	.50	.42	.46	.52	.53
Hogs	ds.	.35	.41	.40	.37	.35	.42	.45	.41

Table 6. Continued.

Item	Unit	Baseline Values 1978	Baseline Values 1979	Baseline Values 1980	Baseline Values 1981	Simulated Values 1978	Simulated Values 1979	Simulated Values 1980	Simulated Values 1981
Total Government Payments	B. $	2.019	3.549	4.712	5.850	2.544	3.144	3.359	3.696
Total CCC Storage and Interest Costs	B. $	0.150	0.310	0.452	0.599	0.150	0.253	0.253	0.253
Consumer's Food Expenditures	B. $	188.3	196.8	205.0	214.0	188.3	197.9	208.7	218.1
Livestock Producer's Net Income	B. $	17.312	18.844	19.967	21.289	17.169	18.425	21.549	23.232
Realized Net Farm Income	B. $	18.118	18.949	18.812	19.550	18.641	18.995	21.634	22.911
Performance Measure									123,162.0

passed in 1981 with total government payments of $3,696 billion. Additional acreage set-aside of feed grains is possible in 1981; however, higher levels of set-aside would increase total government payments over the upper limit and penalize the performance measure. Realized net farm income for the farm program is higher than the baseline values in each year simulated, and over the four years the simulated net farm income is nine percent greater than the baseline.

Summary and Conclusions

Optimal control theory is a mathematical technique to determine the values for the control variables that cause a particular system to satisfy a given set of constraints and optimize a given performance criteria. The objective of this report is to demonstrate the use of a non-linear optimization technique to solve applied problems in the area of agricultural economics.

The principles of optimal control theory are presented in a non-mathematical form, with an emphasis on application of the technique rather than the mathematics involved. Box's Complex Procedure, a direct-search algorithm for controlling a non-linear constrained system, is described. Also, three illustrative applications of the Complex Procedure for solving problems in the area of agricultural economics are presented. The examples are: 1) a constrained profit maximization for a firm producing three outputs with four inputs and facing less than perfectly elastic output demand and input supply functions; 2) a procedure for predicting beef supplies based on growth rates and SRS's cattle on feed reports; and 3) a farm policy problem where the control mechanism selects optimal values for the farm policy variables.

Optimal control techniques are important additions to the kit of tools available to agricultural economists for use in applied research. A large portion of the problems considered by agricultural economists can be cast into a maximization (or minimization) framework. The ability of optimal control techniques to handle nonlinearities, multiple objectives and non-normative behavioral response parameters makes it potentially a very powerful analyses tool.

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Appendix A

Listing of the Source Program for COMPLX

```
C/////   THIS VERSION OF BOX CONTAINS COMMON STATEMENTS AND RANG AS A
C/////   RANDOM NO. GENERATOR FOR THE RANDOM POINTS ON THE SURFACE.
C/////   ALSO, THE VERSION HAS THE PROVISION FOR THE USER TO PROVIDE
C/////   THE POINTS FOR ALL OF THE INITIAL POINTS ON THE SURFACE.
C/////   REVIZED BY JWR 6/77.
      INTEGER GAMMA
      COMMON  /BMAIN1/  ITMAX ,IQ, R(60,60) , NO, ALPHA, BETA, GAMMA
      INTEGER   BEG,END, BEG2
      COMMON  /BMAIN2/ IBASE,    DELTA, KODE, IPRINT ,IC ,BEG,END,BEG2
      COMMON  /BMAIN3/  X(60,99) ,N,M,K,IEV1,IEV2,  K1,F(60),G(99),H(99)
     1,XC(60),NDEBUG  ,KNB,NB2,NB3,NB1
      COMMON  /BMAIN4/ NBFILE(99) ,I,J
      DEFINE FILE  16(100,60,U,JCOM)
    1 FORMAT(I1,9X,F10.0,3I4)
    2 FORMAT(I1,9X,3F10.0,9I4)
    4 FORMAT (8F10.4)
    5 FORMAT (' ',I4,10(10F10.5,/))
    6 FORMAT (' ',T5,I4,T12,20X,T40,3(F10.4,5X))
    7 FORMAT('0',T8,'J',T17,'                    ',T42,'X(1,J)',T59,'G(J)',
     1 T74,'H(J)')
    8 FORMAT(' ',//, ' THE USER PROVIDED VALUES FOR POINTS 1-K')
   10 FORMAT (1H1,//,18X,24HCOMPLEX PROCEDURE OF BOX)
   11 FORMAT(' ',/,T3,'PARAMETERS',/
     1,T5,'NO. OF EXPLICIT CONTROL VAR(N) =',I4,/
     1,T5,'NO. OF IMPLICIT CONTROL VAR(IC =',I4,/
     3,T5,'NO. OF TOTAL    CONTROL VAR(M) =',I4,//
     3,T5,'NO. OF POINTS ON SURFACE(K)    =',I4,/
     3,T5,'NO. OF MAXIMUM ITERATION(ITMAX)=',I4,/
     3,T5,'NO. OF REPEAT ITERATIONS(GAMMA)=',I4,//
     3,T5,'REFLECTION FACTOR     (ALPHA)    =', F6.2,/
     3,T5,'DEGREE OF ACCURACY    (BETA)     =', F6.2    ,/
     3,T5,'WITHIN BOUNDS ADJUST (DELTA)     =', F8.4,/
   12 FORMAT (//,2X,14HRANDOM NUMBERS)
   13 FORMAT (/,3(2X,2HR(,I2,1H,,I2,4H) = ,F6.4,2X))
   14 FORMAT (///,2X,30HFINAL VALUE OF THE FUNCTION = ,E20.8)
   15 FORMAT (//,2X,14HFINAL X VALUES)
   16 FORMAT (/,2X,2HX(,I2,4H) = ,4X,20X,F30.10,10X,I4)
   17 FORMAT (///,2X,38HTHE NUMBER OF ITERATIONS HAS EXCEEDED ,I4,10X,
     118HPROGRAM TERMINATED)
   18 FORMAT(' ', ' RANDOM NO. SEED IS = ',2X,F12.0  ,/)
   19 FORMAT('1',' JOB TERMINATED BECAUSE CARDS FOR COMPLX ARE OUT OF OR
     1DER')
      NI = 5
      NO = 6
C     READ THE I-O CARD
      READ(5,1) IKO, ANAR,       IPRINT, NDEBUG, IBASE
      IF(IKO.NE.7) GO TO 29
C     READ THE PARAMETER CARD.
      READ(5,2) IKO, ALPHA, BETA, DELTA, GAMMA, ITMAX ,END
      IF(IKO.NE.8) GO TO 29
      GO TO 32
   29 WRITE(6,19)
      STOP
   32 NAR=ANAR
      BEG=1
C     N IS NO. OF EXPLICIT IND. VARIABLES.                           60
      N=END
C     M IS NO. OF IMPLICIT & EXPLICIT CONTROL VARIABLES
      M=END
      BEG2=END+1
C     IC IS NO. OF IMPLICIT CONTROL VARIABLES IC=M-N
      IC=M-N
C     K IS NO. OF POINTS ON THE COMPLEX.                             30 MAX
      K=END+ 1
C     PRINT THE PARAMETER SUMMARY
      WRITE (NO,010)
```

```
      WRITE(6,11) N,IC,M,K,ITMAX,GAMMA,ALPHA,BETA,DELTA
C      ZERO OUT THE X MATRIX
      DO 41 II=1,K
      DO 31 J=BEG,M
   31 X(II,J) = 0.0
   41 CONTINUE
C     WHEN THE USER PROVIDES ONLY THE INITAL VALUES FOR THE FIRST
C     SET OF CONTROL VARIABLES, READ THE  STARTING VALUE CARD.
      IF(IBASE.NE.0) GO TO 40
      READ(5,4) (X(1,J),J=BEG,END)
      WRITE(6,3)(X(1,J),J= BEG, END)
      GO TO 450
C         READ THE USER SUPPLIED VALUES FOR X FOR POINTS 1 THROUGH K,
C         THE STARTING VALUE CARDS.
   40 IF(IBASE.NE.1) GO TO 450
      WRITE(6,8)
      DO 425 L=1,K
      READ(5,4) (X(L,J),J=BEG, END)
  425 WRITE(6,3) L, (X(L,J),J=BEG,END)
      GO TO 210
  450 CONTINUE
      I=1
      CALL CONSTT
      WRITE(6,7)
      DO 250 J=BEG,M
      L=J+2
  250 WRITE(6,6) J,                   X(1,J),G(J),H(J)
      IF(IBASE.EQ.3) GO TO 210
      DO 100 II=1,K
      DO 100 JJ=BEG,END
      R(II,JJ) = RANG(NAR)
  100 CONTINUE
      WRITE (NO,012)
      WRITE(6,18)ANAR
      DO 200  J=1,K
      WRITE (NO,013) (J, L, R(J,L), L= BEG,END)
  200 CONTINUE
  210 CONTINUE
C     CALL SUBROUTINE CONSX TO BEGIN OPTIMTZATION.
      CALL CONSX
C     RETURN EITHER WITH OPTIMAL SOLUTION OR AFTER GOING TO THE MAX ITER
      IF (IQ-ITMAX) 20,20,30
   20 WRITE (NO,014) F(IEV2)
      WRITE (NO,015)
C     WRITE OPTIMAL VALUES OF THE CONTROL VARIABLES.
      DO 300 J=BEG,M
      L=J+2
      WRITE (NO,016) J,                  X(IEV2,J)
  300 CONTINUE
      GO TO 999
C     MAX NO. OF ITERATIONS EXCEEDED SO PRINT THE VALUES OF THE CONTROLS
   30 WRITE (NO,017) ITMAX
      DO 850 I=1,K
      DO 900 J=BEG,M
      L=J+2
      WRITE (NO,016) J,                  X(I,J) ,I
  900 CONTINUE
  850 CONTINUE
C     STORE THE POINTS ON DISK FOR COLD START .'0003' IN CC 28-32 I-O CD
      DO 875 J=1,M
  875 WRITE(16' J) (X(I,J),I=1,K)
  999 CONTINUE
      RETURN
      END
C**********************************************************************
      SUBROUTINE CONSX
C**********************************************************************
      INTEGER GAMMA
      COMMON  /BMAIN1/  ITMAX ,IQ, R(60,60) , NO, ALPHA, BETA, GAMMA
      INTEGER   BEG,END, BEG2
      COMMON  /BMAIN2/ IBASE,    DELTA, KODE, IPRINT ,IC ,BEG,END,BEG2
```

```
      COMMON  /BMAIN3/  X(60,99) ,N,M,K,IEV1,IEV2,  K1,F(60),G(99),H(99)
     1,XC(60),NDEBUG  ,KNB ,NB2,NB3,NB1
      COMMON  /BMAIN4/ NBFILE(99) ,I,J
    1 FORMAT(' ',' GOING TO 170 FOR TIME NO. ',I4,3E15.5)
   16 FORMAT(' ','  STORED K POINTS ON DISK FOR ITERATION NO. ',I4)
   17 FORMAT(' ','  DATA FOR K POINTS READ FROM UNIT 16')
  018 FORMAT (//,2X,30HCOORDINATES OF INITIAL COMPLEX)
  019 FORMAT (/,5(1X,2HX(, I2,1H,,I2,4H) = ,  E13.6))
  021 FORMAT (/,2X,22HVALUES OF THE FUNCTION )
   22 FORMAT (   /,5(1X,2HF(,I2,4H) = ,  E13.6))
  023 FORMAT (//,2X,17HITERATION NUMBER ,I5)
  024 FORMAT (/,2X,30HCOORDINATES OF CORRECTED POINT)
  025 FORMAT (/,2X,27HCOORDINATES OF THE CENTROID)
  026 FORMAT (/,5(1X,2HX(, I2,6H,C) = ,  E13.6))
 1234 FORMAT(' ',2X, 'SUBROUTINE CONSX')
      IF(NDEBUG.NE.0) WRITE(6,1234)
C     IQ     = ITERATION INDEX
C     IEV1   = INDEX OF POINT WITH MINIMUM FUNCTION VALUE.
C     IEV2   = INDEX OF POINT WITH MAXIMUM FUNCTION VALUE.
C     I      = POINT INDEX.
C     KODE   = CONTROL KEY USED TO DETERMINE IF IMPLICIT CONSTRAINTS
C              ARE PROVIDED.
C     K1     = DO LOOP LIMIT
      IQ = 1
      KODE = 0
      IF (M-N) 20,20,10
   10 KODE = 1
   20 CONTINUE
C     CALCULATE COMPLEX POINTS AT RANDOM FROM UNIFORMLY DISTRIBUTED
C     NOS. & THE BOUNDARY CONSTRAINTS.
      IF(IBASE.EQ.1 .OR. IBASE.EQ.3) GO TO 61
      IROW1 = 2
      IF(IBASE.EQ.2) IROW1 = 1
      DO 65 II=IROW1,K
      DO 50 J=BEG,END
      I = II
      CALL CONSTT
      X(II,J) = G(J) + R(II,J)*(H(J)-G(J))
   50 CONTINUE
C     CHECK THE VALUES OF EXPLICIT VARIABLES
      DO 350 J=BEG,END
      IF(X(I,J)- G(J)) 320,320,330
  320 X(I,J) = G(J) + DELTA
      GO TO 350
  330 IF( H(J)-X(I,J)) 340,340,350
  340 X(I,J) = H(J)- DELTA
  350 CONTINUE
      CALL CONSTT
      K1 = II
      CALL CHECK
      IF (II-2) 51, 51, 55
   51 IF (IPRINT) 52, 65, 52
   52 WRITE (NO,018)
      IO = 1
      WRITE (NO,019) (IO, J, X(IO,J), J= BEG,END)
   55 IF (IPRINT) 56, 65, 56
   56 WRITE (NO,019) (II, J, X(II,J), J= BEG,END)
   65 CONTINUE
      GO TO 69
C        ENTER HERE IF THE USER HAS PROVIDED X VALUES FOR 1 THROUGH K
C        CALL CONST TO CALCULATE OTHER X VALUES & GET READY TO CALL FUNC
   61 CONTINUE
      IF(IBASE.EQ.1) GO TO 63
C     READ THE K POINTS FROM DISK, UNIT 16.
      DO 62 L=1,M
   62 READ(16' L) (X(IKK,L),IKK=1,K)
      WRITE(6,17)
   63 CONTINUE
      WRITE (NO,018)
      DO  64  I=1,K
      CALL CONSTT
```

```
      K1=I
      CALL CHECK
      WRITE (NO,019) (I , J, X(I ,J), J= BEG,END)
   64 CONTINUE
   69 K1 = K
      DO 70 I=1,K
      CALL OBJT
   70 CONTINUE
      KOUNT = 1
      IA = 0
      IF (IPRINT) 72, 80, 72
   72 WRITE (NO,021)
      WRITE (NO,022) (J, F(J), J=1,K)
C
C        THE PROGRAM WORKS BETWEEN HERE AND '240 RETURN'
C        UNTIL AN OPTIMUM IS REACHED.
C
   80 IEV1 = 1
C     FIND THE INDEX FOR THE MINIMUM OF F(I) ,I=1,K
      DO 100 ICM=2,K
      IF (F(IEV1)-F(ICM)) 100,100,90
   90 IEV1 = ICM
  100 CONTINUE
C     FIND POINT WITH HIGHEST FUNCTION VALUE
      IEV2 = 1
      DO 120 ICM=2,K
      IF (F(IEV2)-F(ICM)) 110,110,120
  110 IEV2 = ICM
  120 CONTINUE
C
C     CHECK CONVERGENCE CRITERIA
C
      IF (F(IEV2)-(F(IEV1)+BETA)) 140,130,130
  130 KOUNT = 1
      GO TO 150
  140 KOUNT = KOUNT + 1
      IF (KOUNT-GAMMA) 150,240,240
C
C     REPLACE POINT WITH LOWEST FUNCTION VALUE
  150 CONTINUE
      CALL CENTR
      DO 160  JJ=BEG,END
  160 X(IEV1,JJ) = (1.0+ALPHA)*(XC(JJ))-ALPHA*(X(IEV1,JJ))
      I = IEV1
      CALL CHECK
      CALL OBJT
      IEV3 = IEV2
      ICOUNT=0
  170 CONTINUE
C
C     REPLACE NEW POINT IF IT REPEATS AS LOWEST FUNCTION VALUE
C
C     FIND THE INDEX FOR THE F() WITH THE MINIMUM VALUE.
      ICOUNT=1+ ICOUNT
      IEV2 = 1
      DO 190 ICM=2,K
      IF (F(IEV2)-F(ICM)) 190,190,180
  180 IEV2 = ICM
  190 CONTINUE
      IF (IEV2-IEV1) 220,200,220
  200 DO 210 JJ=BEG,END
      L=K/4
      IF(K.GT.2 .AND. ICOUNT.GE.L) XC(JJ)=X(IEV3,JJ)
      X(IEV1,JJ)=(X(IEV1,JJ) + XC(JJ))/2.0
  210 CONTINUE
      I = IEV1
      CALL CHECK
      CALL OBJT
      IF(IPRINT) 480,485,480
  480 WRITE(6,1) ICOUNT,F(IEV1),F(IEV2) ,F(IEV3)
      WRITE (NO,022) (I, F(I), I=BEG,K)
```

```
  485 CONTINUE
C       IF(ICOUNT.EQ. K) GO TO 220
        GO TO 170
  220 CONTINUE
      IF (IPRINT) 230, 228, 230
  230 WRITE (NO,023) IQ
      WRITE (NO,024)
      WRITE (NO,019) (IEV1, JC, X(IEV1,JC), JC= BEG,END)
      WRITE (NO,021)
      WRITE (NO,022) (I, F(I), I=BEG,K)
      WRITE (NO,025)
      WRITE (NO,026) (JC, XC(JC), JC=BEG,END)
  228 IQ = IQ + 1
C          STORE THE X MATRIX ON DISK AT THE END OF EVERY TENTH ITERATION
C         FOR A COLD START, '0003' IN CC 28-32 OF I-O CARD.
      IF(MOD(IQ,10).NE.0) GO TO 239
      DO 238 L=1,M
  238 WRITE(16' L) (X(IKK,L),IKK=1,K)
      WRITE(6,16) IQ
      IF(IPRINT) 242,241,242
  241 CONTINUE
      WRITE (NO,024)
      WRITE (NO,019) (IEV1, JC, X(IEV1,JC), JC= BEG,END)
      WRITE (NO,021)
      WRITE (NO,022) (I, F(I), I=BEG,K)
  242 CONTINUE
  239 IF (IQ-ITMAX) 80,80,240
  240 RETURN
      END
C***********************************************************************
      SUBROUTINE CHECK
C***********************************************************************
      INTEGER GAMMA
      COMMON /BMAIN1/  ITMAX ,IQ, R(60,60) , NO, ALPHA, BETA, GAMMA
      INTEGER   BEG,END, BEG2
      COMMON /BMAIN2/ IBASE,    DELTA, KODE, IPRINT ,IC ,BEG,END,BEG2
      COMMON /BMAIN3/  X(60,99) ,N,M,K,IEV1,IEV2,  K1,F(60),G(99),H(99)
     1,XC(60),NDEBUG   ,KNB,NB2,NB3,NB1
      COMMON /BMAIN4/ NBFILE(99) ,I,J
    1 FORMAT( ' ',2I4,3F15.4)
 1234 FORMAT(' ',2X, 'SUBROUTINE CHECK')
      IF(NDEBUG.NE.0) WRITE(6,1234)
      ICOUNT=0
   10 KT = 0
      ICOUNT=1+ ICOUNT
      CALL CONSTT
C     CHECK AGAINST EXPLICIT CONSTRAINTS
      DO 50 J=BEG,END
      IF (X(I,J)-G(J)) 20,20,30
   20 X(I,J) = G(J) + DELTA
      GO TO 50
   30 IF (H(J)-X(I,J)) 40,40,50
   40 X(I,J) = H(J) - DELTA
   50 CONTINUE
      IF (KODE) 110,110,60
C     CHECK AGAINST THE IMPLICIT CONSTRAINTS
   60 NN = END + 1
      DO 100 J=NN,M
      CALL CONSTT
      IF(NDEBUG.NE.0) WRITE(6,1) J,I,X(I,J),G(J), H(J)
      IF (X(I,J)-G(J)) 80,70,70
   70 IF (H(J)-X(I,J)) 80,100,100
   80 IEV1 = I
      KT = 1
      CALL CENTR
      DO 90 JJ=BEG,END
      X(I,JJ) = (X(I,JJ) + XC(JJ))/2.0
   90 CONTINUE
  100 CONTINUE
      IF (KT) 110, 110, 10
  110 RETURN
      END
```

```
C***********************************************************************
      SUBROUTINE CENTR
C***********************************************************************
      INTEGER   BEG,END, BEG2
      COMMON  /BMAIN2/ IBASE,    DELTA, KODE, IPRINT ,IC ,BEG,END,BEG2
      COMMON  /BMAIN3/  X(60,99) ,N,M,K,IEV1,IEV2,  K1,F(60),G(99),H(99)
     1,XC(60),NDEBUG   ,KNB,NB2,NB3,NB1
      COMMON  /BMAIN4/ NBFILE(99) ,I,J
 1234 FORMAT(' ',2X, 'SUBROUTINE CENTR')
      IF(NDEBUG.NE.0) WRITE(6,1234)
      DO 20  J=BEG,END
      XC(J) = 0.0
      DO 10  IL=1,K1
   10 XC(J) = XC(J) + X(IL,J)
      RK = K1
   20 XC(J) = (XC(J)-X(IEV1,J))/(RK-1.0)
      RETURN
      END
C***********************************************************************
      FUNCTION RANG(NARG)
C***********************************************************************
C  GENERATES PSEUDO-RANDOM NUMBERS, UNIFORMLY DISTRIBUTED ON (0,1).
C  THIS VERSION IS FOR THE IBM 360.
      EQUIVALENCE (RAN,JRAN)
      DIMENSION N(128)
      DATA NFIRST/7/,K/7654321/,L/3141593/,M/271828183/
      DATA MK/231525/,ML/282629/,MM/253125/
      IF(NARG)20,10,20
   10 IF(NFIRST)30,60,30
   20 KLM=IABS(2*NARG+1)
      K=KLM
      L=KLM
      M=KLM
      NARG=0
   30 NFIRST=0
      NDIV=16777216
      RDIV=32768.*65536.
      DO 50 J=1,128
      K=K*MK
   50 N(J)=K
   60 L=L*ML
      J=1+IABS(L)/NDIV
      M=M*MM
      NR=IABS(N(J)+L+M)
      RAN=FLOAT(NR)/RDIV
      IF(J.GT.64  .AND.  RAN.LT.1.)  JRAN=JRAN+1
      RANG=RAN
      K=K*MK
      N(J)=K
      RETURN
      END
C***********************************************************************
      SUBROUTINE OBJT
C***********************************************************************
C     SUBROUTINE OBJT IS PROVIDED FOR THE USER TO ENTER THE MODLE TO
C     BE OPTIMIZED AND THE OBJECTIVE FUNCTION TO BE MAXIMIZED.
C     ENTER THE MODLE PRIOR TO THE FUNCTION , USE THE VALUES OF THE
C     CONTROLS IN 'X(I,1-N)' AS THE EXOGENOUS DATA IN THE MODLE
C     AND THE FUNCTION IS 'F(I)'.
      INTEGER   BEG,END, BEG2
      COMMON  /BMAIN2/ IBASE,    DELTA, KODE, IPRINT ,IC ,BEG,END,BEG2
      COMMON  /BMAIN3/  X(60,99) ,N,M,K,IEV1,IEV2,  K1,F(60),G(99),H(99)
     1,XC(60),NDEBUG   ,KNB,NB2,NB3,NB1
      COMMON  /BMAIN4/ NBFILE(99) ,I,J
    2 FORMAT(' ',' THE VALUE OF THE PEFORMANCE MEASURE = ',I2,F20.1,3F10
     1.2)
 1234 FORMAT(' ',2X, 'SUBROUTINE OBJT')
      IF(NDEBUG.NE.0) WRITE(6,1234)
      IB=I
C     EXAMPLE PROFIT MAXIMIZATION PROBLEM WITH 3 OUTPUTS,Y1, Y2, Y3.
      Y1=(X(I,1)**0.33)*(X(I,4)**0.17) * (X(I,7)**0.20)* (X(I,10)**0.3)
```

```
      Y2=(X(I,2)**0.10)*(X(I,5)**0.08) * (X(I,8)**0.25)* (X(I,11)**0.4)
      Y3=(X(I,3)**0.09)*(X(I,6)**0.19) * (X(I,9)**0.15)* (X(I,12)**0.2)
C     THE FOUR INPUTS X1, X2, X3, AND X4.
      SUMX1= X(I,1) + X(I,02) + X(I,03)
      SUMX2= X(I,04)+ X(I,05) + X(I,06)
      SUMX3= X(I,07)+ X(I,08) + X(I,09)
      SUMX4= X(I,10)+ X(I,11) + X(I,12)
C     PRICES OF THE THREE OUTPUTS
      PY1 = 1050.0 -0.5 * Y1
      PY2 = 1000.0 -0.25*(Y2** 2)
      PY3 = 100.00 -0.15*(Y3** 2)
C     PRICES OF THE FOUR INPUTS.
      PX1= 3.0 + 0.0009 * SUMX1
      PX2=6.0 + 0.000111 * SUMX2
      PX3=9.0 + 0.0003 * SUMX3
      PX4 = 7.0 + 0.000199 * SUMX4
C     SET UP THE CONSTRAINTS ON THE MAXIMUM AMOUNT OF RESOURCES AVAILABE
      PENAL1 = 0.0
      PENAL2 = 0.0
      PENAL3 = 0.0
      PENAL4 = 0.0
      IF(SUMX1.GT.2000.0) PENAL1 =((2000.0 - SUMX1)** 2 ) * 200000.
      IF(SUMX2.GT.3000.0) PENAL2 =((3000.0 - SUMX2)** 2 ) * 200000.
      IF(SUMX3.GT.2100.0) PENAL3 =((2100.0 - SUMX3)** 2 ) * 200000.
      IF(SUMX4.GT.1000.0) PENAL4 =((1000.0 - SUMX4)** 2 ) * 200000.
C     THE OBJECTIVE FUNCTION - A CONSTRAINED PROFIT MAXIMIZATION.
      F(IB) = PY1 * Y1 + PY2 * Y2 + PY3*Y3
     1-(PX1*SUMX1+ PX2 * SUMX2+ PX3*SUMX3 + PX4 * SUMX4)
     2 -(PENAL1 + PENAL2 + PENAL3 + PENAL4)
      WRITE(6,2) IB,F(IB),Y1,Y2,Y3
      I=IB
      RETURN
      END
C*********************************************************************
      SUBROUTINE CONSTT
C*********************************************************************
C     SUBROUTINE CONSTT IS PROVIDED FOR THE USER TO ENTER THE
C     LOWER & UPPER BOUNDARY CONSTRAINTS FOR THE CONTROL VARIABLES.
C     THE LOWER BOUNDARY CONSTRAINTS ARE ENTERED IN THE 'G()' ARRAY,
C     AND THE UPPER BOUNDAYR CONSTRAINTS ARE ENTERED IN THE 'H()' ARRAY.
      COMMON /BMAIN3/  X(60,99) ,N,M,K,IEV1,IEV2,  K1,F(60),G(99),H(99)
     1,XC(60),NDEBUG  ,KNB,NB2,NB3,NB1
      COMMON /BMAIN4/ NBFILE(99) ,I,J
 1234 FORMAT(' ',2X, 'SUBROUTINE CONSTT')
      IF(NDEBUG.NE.0) WRITE(6,1234)
C     LOWER BOUNDARY CONSTRAINTS.
      DO 1 L=1,12
    1 G(L) = 0.0
C     UPPER BOUNDARY CONSTRAINTS.
      H(01) = 2000.0
      H(02) = 2000.0
      H(03) = 2000.0
      H(04) = 3000.0
      H(05) = 3000.0
      H(06) = 3000.0
      H(07) = 2100.0
      H(08) = 2100.0
      H(09) = 2100.0
      H(10) = 1000.0
      H(11) = 1000.0
      H(12) = 1000.0
      RETURN
      END
```

Appendix B

Data Cards for Box's Complex Procedure

Data cards necessary for Box's Complex Procedure are presented in this Appendix. The first data card for the Complex Procedure is an I-0 Card. The I-0 Card provides a means for the user to indicate the type of input to be provided and the type of printed output desired from the model. The second data card, the Parameter Card is provided for the user to input parameters necessary for the optimization routine. The Parameter Card is the last data card unless the user selects the option of providing the initial values for each of the m control variables. In such a case, m+1 (or k) Starting Values Cards follow the Parameter Card.

I-0 Card

The I-0 Card is the first data card for the program; it provides three options for entering the initial values for the control variables (policy variables) and three options for printing the output. Code the I-0 Card as follows:

Card Column

1	Punch a '7'.
2-10	Punch 'I-0 CARD'.
11-20	An odd, six-digit number, to be used as a random number generator seed (punched with decimal point) as '999991.0'.
21-24	Code '00000' if a minimum amount of output is desired. Code '0001' to print the value of the control variables, and the performance measure on each iteration.
25-28	Code '0001' to use the de-bug option for locating logic errors within the program.
29-32	The option to indicate the source of the k initial values for the m control variables. A '0001' indicates the user will provide data cards for the initial values of all control variables. A '0002' indicates that the initial values for all K points are to be selected at random. A '0003' indicates that initial values for all k points were stored on a direct access disk (unit 16) in a previous run and are to be used for this run.
33-80	Blank.

Parameter Card

The computer program used in this study to execute Box's Complex Procedure requires values for five parameters. These parameters are provided by the user on the Parameter Card. (Integers must be right justified.)

Card Column

1	Punch an '8'.
2-10	Punch 'PARAMETER'.
11-20	The reflection factor ALPHA, Box [1965] suggests using '1.3'.
21-30	The convergence parameter BETA, as '0.50'.
31-40	The within bounds accuracy for the constraints, DELTA, as '0.001'.

41-44	The number of interations to continue searching after finding an optimal, GAMMA, as '0005'.
45-48	The maximum number of iterations the search program can go through in trying to locate the maximum of the performance measure, as '0400'.
49-52	Enter the number of control variables in the problem to be run, as '0008'.
53-80	Blank.

When a control path (a coordinate on the surface of the performance measure) is rejected the new values for the control variables are moved ALPHA units closer to the centroid. By using a value greater than one, say 1.3, we are assured of searching both sides of the centroid for the optimal control path. The Complex Procedure assumes convergence when the value of the performance measure for the k points on the surface is within BETA units for GAMMA iterations. When control values are selected that lie outside the boundary constraints, the control value is moved inside the violated boundary by DELTA units.

Starting Value Cards

The Starting Value Cards are used when the user chooses to provide the m+1 (or k) initial values (or control paths) for the m control variables (option '0001' in card columns 28-32 of the I-0 Card). The starting values are stored in a k by m matrix, X. Each Starting Value Card provides values for m control variables, so k different Starting Value Cards must be provided. The order of the control variables on the Starting Value Cards depends upon the order of the control variables in the X matrix, established by the user in subroutine CONSTT and OBJT. Row i of arrays G and H is the same variable as column i of the X matrix.

The initial values for the first set of eight control variables are coded as:

Card Column

1-10	Value for X(1,1), punched with decimal point.
11-20	Value for X(1,2), punched with decimal point.
21-30	Value for X(1,3), punched with decimal point.
31-40	Value for X(1,4), punched with decimal point.
41-50	Value for X(1,5), punched with decimal point.
51-60	Value for X(1,6), punched with decimal point.
61-70	Value for X(1,7), punched with decimal point.
71-80	Value for X(1,9), punched with decimal point.

This same format is used for the next set of eight control variables if necessary for the problem being simulated or X(1,9) through X(1,16). (If more than 16 control variables are being used, continue on a third and fourth card, until reaching control variable X(1,m).) Repeat the process for the second set of initial control values or X(2,i), i=1, 2, ..., m. The process is complete after coding k sets of the Starting Value Cards.

As new values for the control variables are calculated, during the solution of the Complex Procedure, they are stored in the X matrix. The X matrix is stored on a direct access disk (unit 16) every tenth iteration, so if the programs stops prematurely the calculations can be resumed at the last solution set stored on disk. Calculations can be resumed by re-submitting the program with option '0003' specified in card column 28-32 of the I-0 Card. The process of re-submitting the program can be repeated as many times as necessary to get the program to a maximum value for the performance measure.

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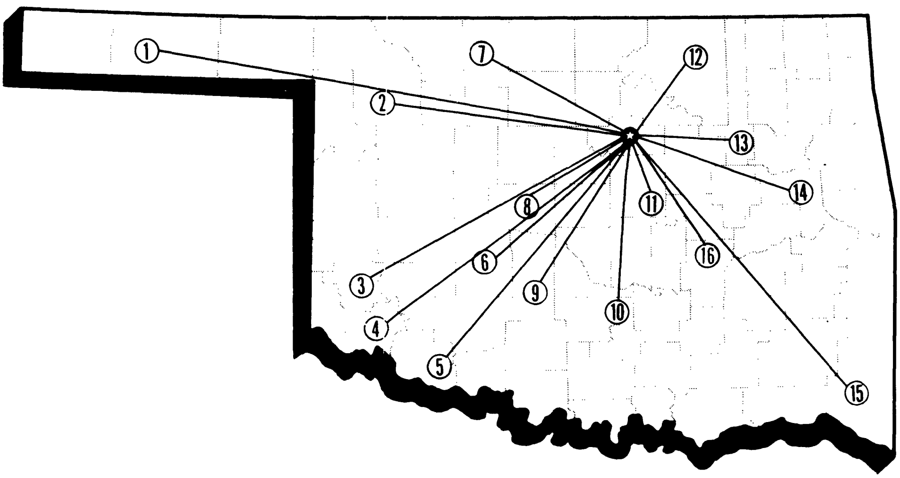

Main Station — *Stillwater, Perkins and Lake Carl Blackwell*

1. Panhandle Research Station — *Goodwell*
2. Southern Great Plains Field Station — *Woodward*
3. Sandyland Research Station — *Mangum*
4. Irrigation Research Station — *Altus*
5. Southwest Agronomy Research Station — *Tipton*
6. Caddo Research Station — *Ft. Cobb*
7. North Central Research Station — *Lahoma*
8. Southwestern Livestock and Forage Research Station — *El Reno*
9. South Central Research Station — *Chickasha*
10. Agronomy Research Station — *Stratford*
11. Pecan Research Station — *Sparks*
12. Veterinary Research Station — *Pawhuska*
13. Vegetable Research Station — *Bixby*
14. Eastern Research Station — *Haskell*
15. Kiamichi Field Station — *Idabel*
16. Sarkeys Research and Demonstration Project — *Lamar*