DEVELOPMENT OF AN IN-SITU SYSTEM AND ANALYSIS PROCEDURE FOR MEASURING GROUND THERMAL PROPERTIES[1].

Warren A. Austin, III, Associate Member ASHRAE
Cenk Yavuzturk, Student Member ASHRAE
Jeffrey D. Spitler, Ph.D., PE, Member ASHRAE

Abstract

Determination of the ground's thermal conductivity is a significant challenge facing designers of Ground Source Heat Pump (GSHP) systems applied in commercial buildings. The number of boreholes and the depth and cost of each borehole are highly dependent on the ground thermal properties. Hence, depending on the geographic location and the local drilling costs, the ground thermal properties strongly influence the initial cost to install a GSHP system. In order to be able to predict ground thermal properties, an experimental apparatus has been built capable of imposing a heat flux on a test borehole, and measuring its temperature response. Parameter estimation techniques in conjunction with a two-dimensional numerical model are used to determine the thermal conductivity of the surrounding ground. Independent measurements of the soil conductivity test results are reported for several test boreholes and a laboratory experiment. An uncertainty analysis of the thermal conductivity prediction is presented.

KEYWORDS: conduction, geothermal energy, ground coupled, heatpump, heat exchanger, simulation, thermal response, thermal storage.

AUTHORS' NOTE: ***W. A. Austin, III*** is staff mechanical engineer with EMC Engineers in Denver and associate member of ASHRAE. ***C. Yavuzturk*** is assistant professor at the University of Wyoming and student member of ASHRAE. ***J. D. Spitler*** is professor at the Oklahoma State University and member of ASHRAE.

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Introduction

Although originating in the residential building sector, ground source heat pump systems have become increasingly popular for use in commercial and institutional buildings. Many of these buildings require comparatively large and expensive ground loop heat exchangers. Although "ground source heat pump systems" may include a range of system configurations, closed-loop systems, where the ground loop heat exchanger consists of a series of vertical boreholes, are usually preferred in commercial applications due to minimal required surface area and ease of maintenance. Particularly for large systems, an extensive effort is made to design the ground loop heat exchangers so that they are not too large (resulting in too high of a first cost) or too small (resulting in entering water temperatures to the heat pumps being too high or too low).

There are a number of design tools used to size ground loop heat exchangers (Ingersoll 1954, Kavanaugh 1984, Eskilson 1987, Deerman 1991, Cane 1991, IGSHPA 1991 and Spitler et al. 1996). All of the design tools rely on some estimate of the ground thermal conductivity and volumetric specific heat. This estimate is critical to the design, yet it is very difficult to make. The required borehole depth or length is highly dependent on the thermal properties of the ground. This in turn strongly influences the cost of the system and its competitiveness with conventional systems.

The traditional approach to estimating the ground thermal properties has been to first ascertain the type (or types) of soil or rock that surrounds the borehole. Once the type of soil or rock is determined, its thermal conductivity can be estimated from tabulated data, such as that contained in the Soil and Rock Classification for the Design of Ground-Coupled Heat Pump Systems Field Manual (EPRI, 1989). For each rock type, a horizontal band is drawn to indicate the range of thermal conductivity expected. Considering one rock type, "Quartzose sandstone, wet", the thermal conductivity varies from about 1.8 Btu/h-ft-°F (3.1 W/m-K) to about 4.5 Btu/h-ft-°F (7.8 W/m-K). This is a significant variation, and the prudent designer will probably choose the lower value of about 1.8 Btu/h-ft-°F (3.1 W/m-K), even though the extra borehole depth required may not allow the ground loop system to be competitive on either a first cost basis or a life cycle cost basis.

A method for more accurately estimating the ground thermal conductivity is therefore highly desirable. This paper focuses on a method for experimentally measuring the ground thermal conductivity using a test borehole. The portable experimental apparatus for data collection and a parameter-estimation-based data analysis procedure are described. The parameter estimation method used utilizes the downhill simplex minimization algorithm of Nelder and Mead (1965) to estimate the ground thermal conductivity. A transient, two-dimensional numerical finite volume model for the vertical borehole (Yavuzturk et al. 1999) is used to evaluate the performance of a ground loop heat exchanger for parameter estimation.

Background

The ground thermal conductivity can not be directly measured – its value must be inferred from temperature and heat flux measurements. The method presented in this paper relies on an experimental measurement of the ground thermal response to a heat flux imposed on a test borehole. Mogensen (1983) described the concept of using such a measurement to estimate the ground thermal conductivity. Subsequently, development of an experimental apparatus began in 1995 at Oklahoma State University and was described by Austin (1998). Simultaneously and independently, a similar approach was taken by Eklof and Gehlin (1996) who present and discuss a mobile testing facility that is used to determine the thermal capacity of underground thermal energy storage systems based on thermal response tests of underground storage volumes. Gehlin and Nordell (1998) report on results from in-situ thermal response tests conducted using the mobile testing facility at various locations in Sweden to predict ground thermal conductivities.

In order to determine the ground thermal conductivity from the temperature and heat flux measurements, some model of the heat transfer in the ground must be utilized. A number of different ground heat transfer models, typically used for estimating the performance of vertical ground loop heat exchangers, are available. They are of interest here for possible inverse use—estimating the ground thermal properties from the performance rather than the performance from the ground thermal properties. Specifically, we are interested in imposing a heat pulse of "short" duration (1-7 days) and determining the ground thermal properties by analysis of the temperature response of the ground.

One of the models currently used for inverse application is the line source model. Ingersoll and Plass (1948) applied the model to ground loop heat exchangers. Mogensen (1983) applied the model to estimate the ground thermal conductivity from an experimental test. The second model that is currently used is the cylinder source model. Carslaw and Jaeger (1947) developed analytical solutions with varying boundary conditions for regions bounded by cylinder geometry. Ingersoll et al. (1948, 1954) investigated the applicability of the line source approach to buried pipes as used in applications of ground source heat pump systems. Kavanaugh and Rafferty (1997) describe the use of the cylinder source model in designing ground loop heat exchangers.

Although the line source and the cylinder source approaches may be used inversely to estimate the ground's thermal conductivity, they require several simplifying assumptions, the effects of which cannot easily be quantified. In fact, the authors know of no published analysis of the uncertainties resulting from using either simplified approach. A detailed numerical model reduces the uncertainties associated with these simplifying assumptions by providing a detailed representation of the borehole geometry and thermal properties of the fluid, pipe, grout, and ground. It may therefore be expected to provide a more accurate estimate of the ground thermal conductivity.

Methodology

Parameter Estimation using the Nelder-Mead Simplex Method.

Parameter estimation involves minimizing the differences between experimentally obtained results and results predicted through an analytical or numerical model by adjusting inputs to the model. In this case, the results from a transient, two-dimensional numerical finite volume model of the borehole and surrounding ground are compared to the experimental results. Some inputs to the model, such as time-varying power and borehole geometry are fixed, and other inputs, such as the thermal conductivity of the ground and the thermal conductivity of the grout are allowed to vary. By systematically varying the thermal conductivity of the ground and the thermal conductivity of the grout so that the minimum difference between the experimental results and the numerical model is attained, a best estimate of the thermal conductivities may be found.

The objective function algorithm uses the following as inputs:

* power input in 2.5 minute intervals (obtained from experimental measurement)
* average borehole temperatures in 2.5 minute intervals as a response to the power input (obtained from experimental measurement, determined by averaging the inlet and outlet temperatures of the loop)
* undisturbed ground temperature (measured at beginning of the experiment.)

* geometric information: (pipe size, pipe wall thickness, borehole diameter, pipe spacing, borehole depth)
* ground thermal properties (conductivity and volumetric specific heat)
* grout thermal properties (conductivity and volumetric specific heat)
* pipe thermal properties (conductivity and volumetric specific heat)
* fluid properties (conductivity, volumetric specific heat, flow rate and viscosity)

Most of the inputs will be determined based on knowledge of the borehole installation. A few, however, will be treated as independent variables in an optimization. The objective function for the optimization is the sum of the squares of the errors (SSE) between the numerical model solution and the experimental results, specifically:

$$\text{SSE} = \sum_{n=1}^{N} (T_{\text{exp}} - T_{\text{num}})^2 \qquad (1)$$

Where,

N = The total number of data points over the duration of the experiment.

T_{exp} = Average of the calibrated input and output temperature at the n^{th} data point.

T_{num} = Average fluid temperature at n^{th} data point as predicted by the numerical model.

SSE = Sum of the squares of the errors.

The optimization is performed with a non-linear “downhill simplex” optimization technique of Nelder and Mead (1965) although other methods might be used. The independent variables for the optimization may be almost any of the inputs, although the obvious choices include the ground thermal properties, the grout thermal properties and the pipe spacing. The summary information flow diagram for the parameter estimation algorithm is provided in Figure 1.

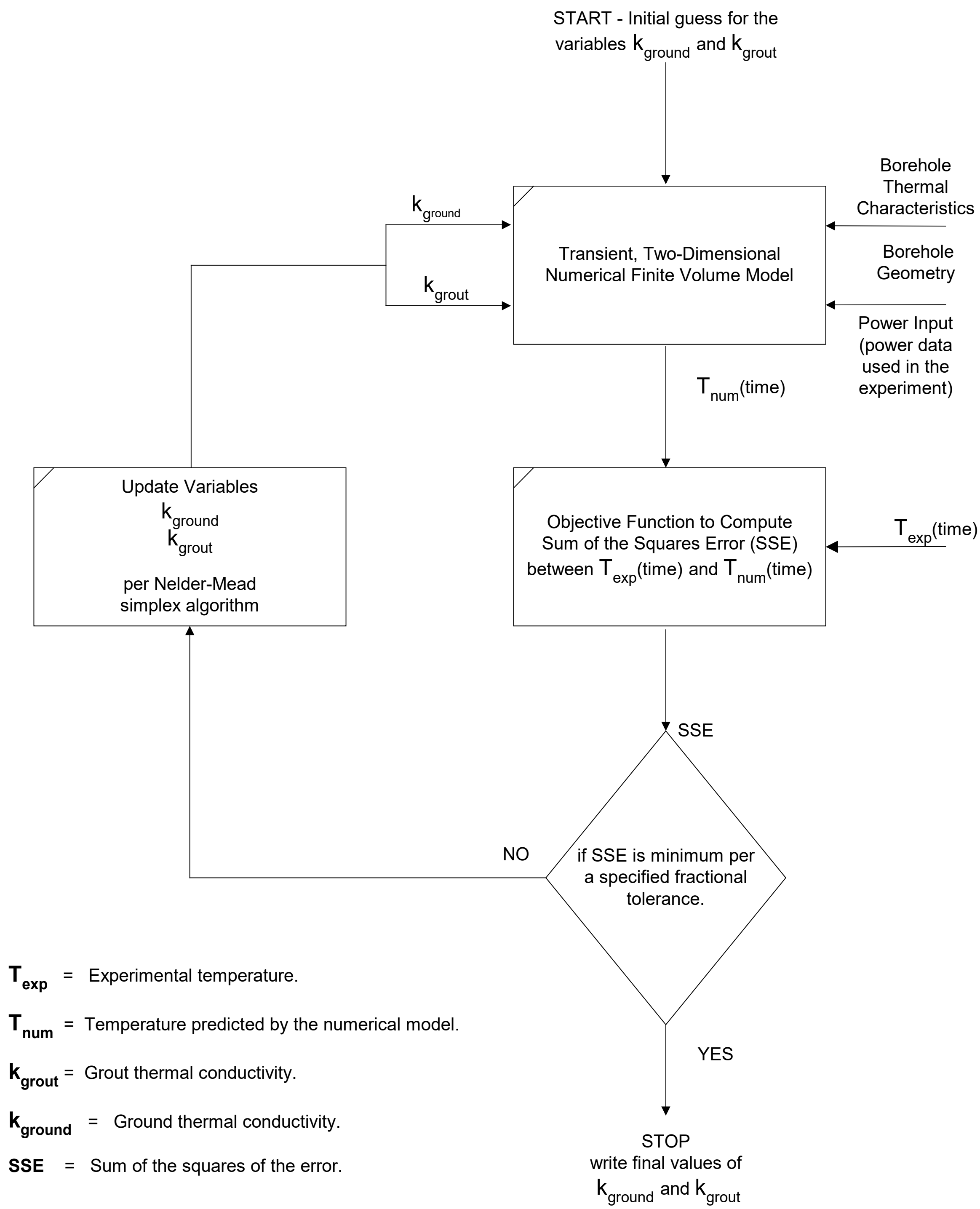

Figure 1 *Information flow diagram for the parameter estimation algorithm.*

Two-Dimensional, Numerical Finite Volume Model.

The line source approach assumes an infinitely long heat source in the ground and attempts to determine ground temperatures at certain radial distances from the line source by solving a one-dimensional, transient heat conduction problem with a source term. This model has no way of accounting for geometric characteristics of the borehole elements, such as the U-tube pipes. The line source model may be used for vertically buried pipes with significant errors, especially for temperatures near the borehole. The cylinder source model, on the other hand, may be used to represent the ground loop heat exchanger as a cylinder by introducing a so-called equivalent diameter to represent the two pipes of the U-tube heat exchanger as a single pipe.

A numerical approach can more accurately model the ground loop heat exchanger by representing each component of a ground loop heat exchanger (U-tube, grout-filled borehole, and the surrounding ground). This section will briefly summarize the steps taken to model the borehole using a numerical modeling technique based on a transient, two-dimensional finite volume model in polar coordinates described by Yavuzturk, et al. (1999).

A sketch of the numerical domain is provided in Figure 2. Since there is a symmetry axis through the borehole, only one half of the borehole is modeled. For a typical borehole, a grid resolution of about 100 finite volume cells in the angular direction and about 150 to 200 cells in the radial direction is utilized. The exact grid resolution is a function of the borehole and U-tube pipe geometry and is determined by an automated parametric grid generation algorithm. The radius of the numerical domain is 12.0 ft (3.6 m) to allow for reasonably long simulation times.

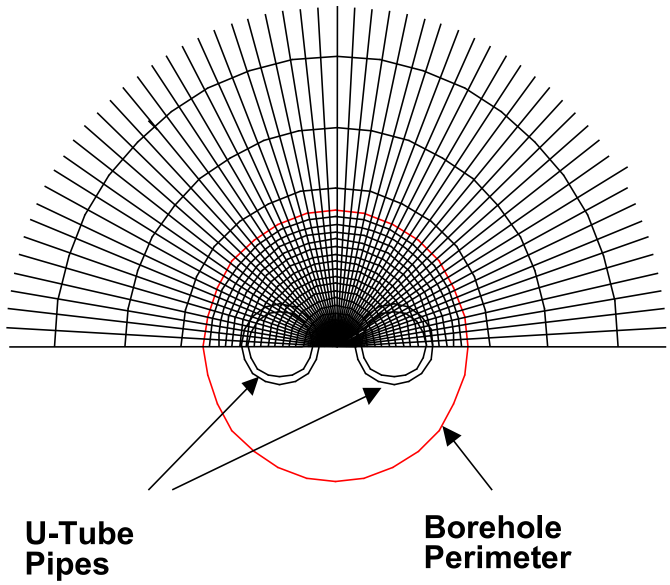

Figure 2 *Simplified representation of the borehole numerical model domain.*

The geometry of the circular U-tube pipes is approximated by "pie-sectors" over which a constant flux is assumed to be entering the numerical domain for each time step. The pie-sector approximation attempts to simulate the heat transfer conditions through a circular pipe by matching the inside perimeter of the circular pipe to the inside perimeter of the pie-sector and by establishing identical heat flux and resistance conditions near the pipe walls. The convection resistance due to the heat transfer fluid flow inside the U-tubes is accounted for using fluid properties through an adjustment on the conductivity of the pipe wall material.

The initial condition of the numerical model stipulates a constant, undisturbed domain temperature corresponding to the far field temperature. Due to the symmetry in the numerical domain a zero heat flux condition is implemented in the angular direction while the heat transfer from/to the U-tube pipes (the pie-sectors that model the U-tube pipes) are input as time-varying boundary flux conditions. Since the total amount of boundary heat flux over each U-tube pipe is not the same, a 60% vs. 40% heat transfer distribution over the pipes of the U-tube is assumed. Finally, the boundary condition in the radial axis is set to be the constant far field temperature. The simulation time step is 2.5 minutes.

The numerical model requires two input files, one of which gives borehole thermal and geometry parameters such as the fluid properties, borehole radius and depth, ground undisturbed far-field temperature, etc. The grid is automatically generated from this data. The other file gives the experimental input power at 2.5-minute intervals. The power input over the specified time interval is assumed to be the average between the measurement at the beginning of the time interval and the measurement at the end of the time interval.

Using the experimentally measured power as an input to the numerical model allows the estimation procedure to adapt to the typical power fluctuations introduced with the use of portable power generators or utility power supply lines.

Experimental Apparatus

Description of the Experimental Configuration

The experimental apparatus is housed in a trailer that can be towed to the site and contains everything needed to perform a test – the apparatus, two generators, and a purge tank containing 80 gallons (304 l) of water. A simplified schematic of the test system is shown in Figure 3. Once connected to a U-tube that has been inserted into a borehole, and after the system has been purged, a heat flux is imposed on the borehole using the three in-line water heaters, and the temperature response (average of inlet and outlet fluid temperatures, which changes with time) of the borehole is measured. A brief description of the experimental apparatus follows. A more detailed description is available in Austin (1998).

In addition to the components shown in Figure 3, a purge tank and two additional pumps are used to remove all air from the piping system before the heat pulse phase of the experiment begins. Also, when electricity is not otherwise available, two 7000 W capacity gasoline generators are used to power the experiment.

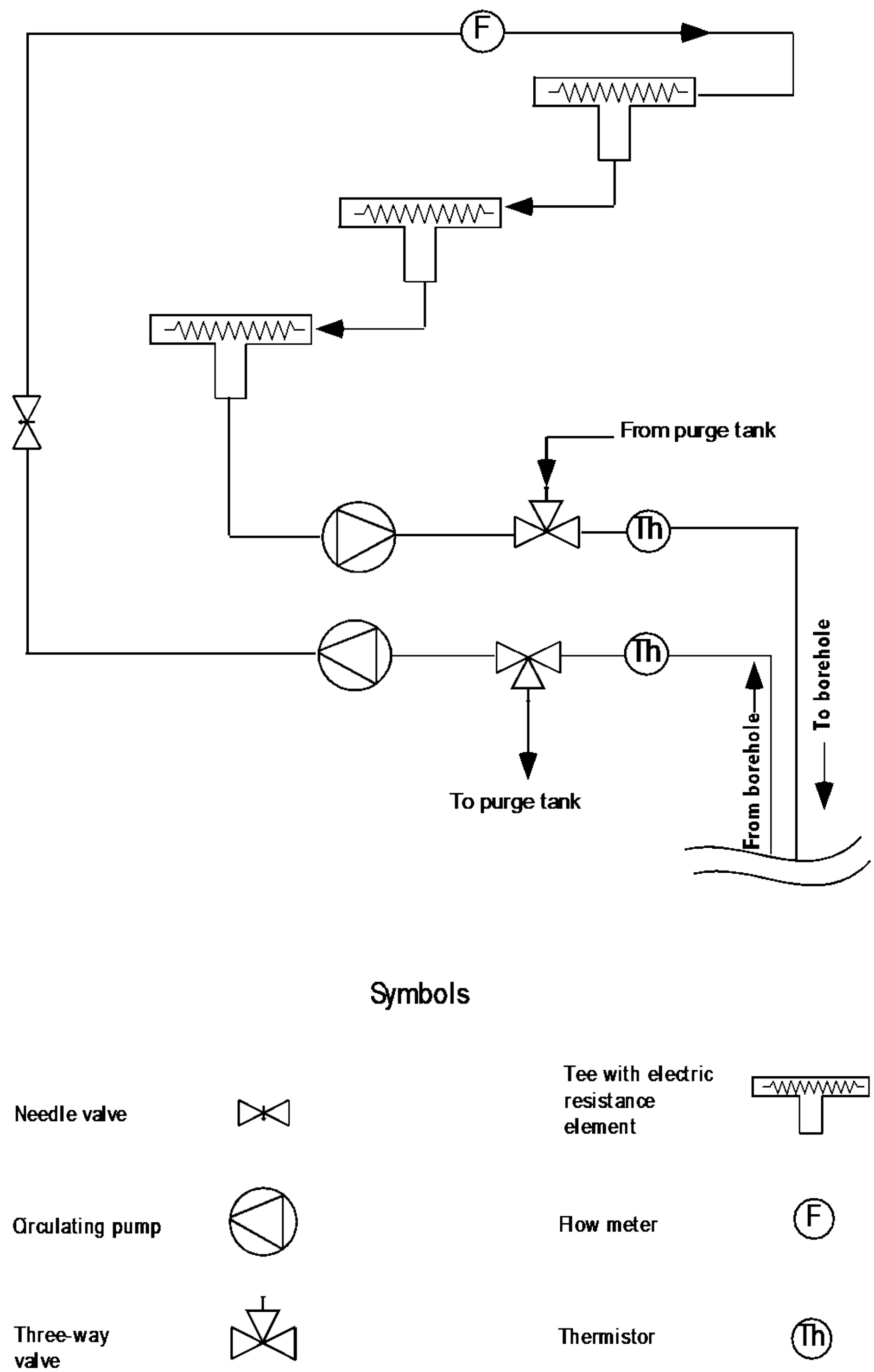

Figure 3 *In-situ thermal conductivity test system schematic.*

Once the system has been purged, the three-way valves shown in Figure 3 are turned to close the connection to the purge system. The following components are then used to impose a heat pulse on the borehole, and measure both the power and the temperature:

- The circulating water inside the closed loop system is heated with (up to) three in-line water heaters shown schematically in Figure 3. The water heaters are ordinary water heating elements (typically used in residential water heaters) mounted in piping tees. The heater elements are rated at 1, 1.5, and 2 kW. The 2 kW water heater element is connected to an electronic power controller, so that by switching individual elements on or off, and by adjusting the controller, the power can be adjusted continuously between 0 and 4.5 kW.
- Two circulating pumps are used to circulate heated water through the U-tube in the borehole.
- A needle valve is used to adjust the flow rate. Typically, a flow rate of approximately (2.50 gpm [0.16 l/s]) was used.
- All of the plumbing, inside and outside is insulated. The interior pipe insulation is fiberglass, about 1.5 inches (38.1 mm) thick. The tubing between the trailer and the borehole is insulated with three layers of insulation, a 0.5 inch (12.7 mm) thick foam rubber, and two layers of fiberglass duct insulation, 5 inches (127.0 mm) and 9 inches (228.6 mm) thick respectively.
- The power consumption of the heaters and the circulating pumps is measured using a watt transducer.
- Inlet and outlet temperatures are measured using two high accuracy thermistors, immersed in the circulating fluid.
- The flow rate is measured using an in-line flow meter.

Experimental measurements are made every 2.5 minutes using a data logger, and the power input, the entering/exiting fluid temperatures of the loop and the volumetric flow rate are downloaded to an on-board computer.

Model Validation

A completely independent estimate of the ground thermal conductivity is required for validation of the parameter estimation model predictions. To accomplish this, several tests have been conducted where the ground conductivity was established independently.

One test was performed on a borehole that was drilled with a coring bit. The core samples were carefully preserved in sealed PVC cases and stored in climate-controlled rooms to avoid changes in the moisture content of the sample. The conductivity of 19 representative samples was then measured in a guarded hot plate apparatus (Smith 1998; Smith, et al. 1999a) to obtain an independent estimate for its thermal conductivity. The guarded hot plate apparatus requires core samples 3.0 inches (76.2 mm.) in length and 3.0 inches (76.2 mm.) in diameter. A constant heat flux is imposed on one end of the sample, while the other end is cooled. The resulting temperature difference is used to determine the sample's thermal conductivity. The method has been validated on stainless steel samples, which have a thermal conductivity that is about 3 to 5 times higher than soil, with an error of about $\pm$ 1%.

Another test was performed using a medium-scale laboratory experiment (Smith 1998; Smith, et al. 1999b) where the geometry and thermal characteristics of a borehole are replicated under controlled conditions. The thermal conductivity of the soil material used in the experiment was determined independently with a calibrated soil conductivity probe.

Various other types of indirect confirmation have also been looked at to verify that the parameter estimation method works correctly. For example, measurements of thermal conductivity taken at nearby boreholes with different grout types and pipe types should give approximately the same value of thermal conductivity. Austin (1998) reports on extensive field experience obtained from a series of in-situ tests at various locations in Oklahoma. However, the results presented in this paper focus on the tests with independent measurements of thermal conductivity.

Cored Borehole (Oklahoma State University, Test Site A #6)

A series of test boreholes were drilled at an experimental field on the premises of the Oklahoma State University in Stillwater, Oklahoma (Test Site A). Core samples of the soil from one (Site A borehole #6) of the boreholes were obtained and analyzed using a modified guarded hot plate method as implemented by Smith (1998) to determine the effective thermal conductivity of the borehole core.

At present, 19 representative samples have been analyzed. (Analysis of additional samples is ongoing, and may eventually result in an improved estimate of the average ground thermal conductivity surrounding the borehole.) The samples were chosen so that they represent identifiable layers. Since the thermal conductivity of the formational layers of the core sample varies, a thickness-weighted average thermal conductivity value is calculated. The resulting thermal conductivity then represents the effective thermal conductivity for the Test Site A #6 borehole.

The results of the guarded hot plate tests are provided in Figure 4 (Smith 1998; Smith, et al. 1999a) where the measured ground conductivity for various layers of the borehole core is plotted against the depth of the borehole. The weighted average ground conductivity is calculated to be approximately 1.351 Btu/hr-ft-°F (2.337 W/m-K). The ground thermal conductivity varies between approximately 1.9 Btu/hr-ft-°F (3.3 W/m-K) and 0.9 Btu/hr-ft-°F (1.6 W/m-K). The strong variation in the thermal conductivity along the depth of a given borehole serves to reinforce the fact that the average thermal conductivity is really an "effective" thermal conductivity for the ground surrounding the borehole.

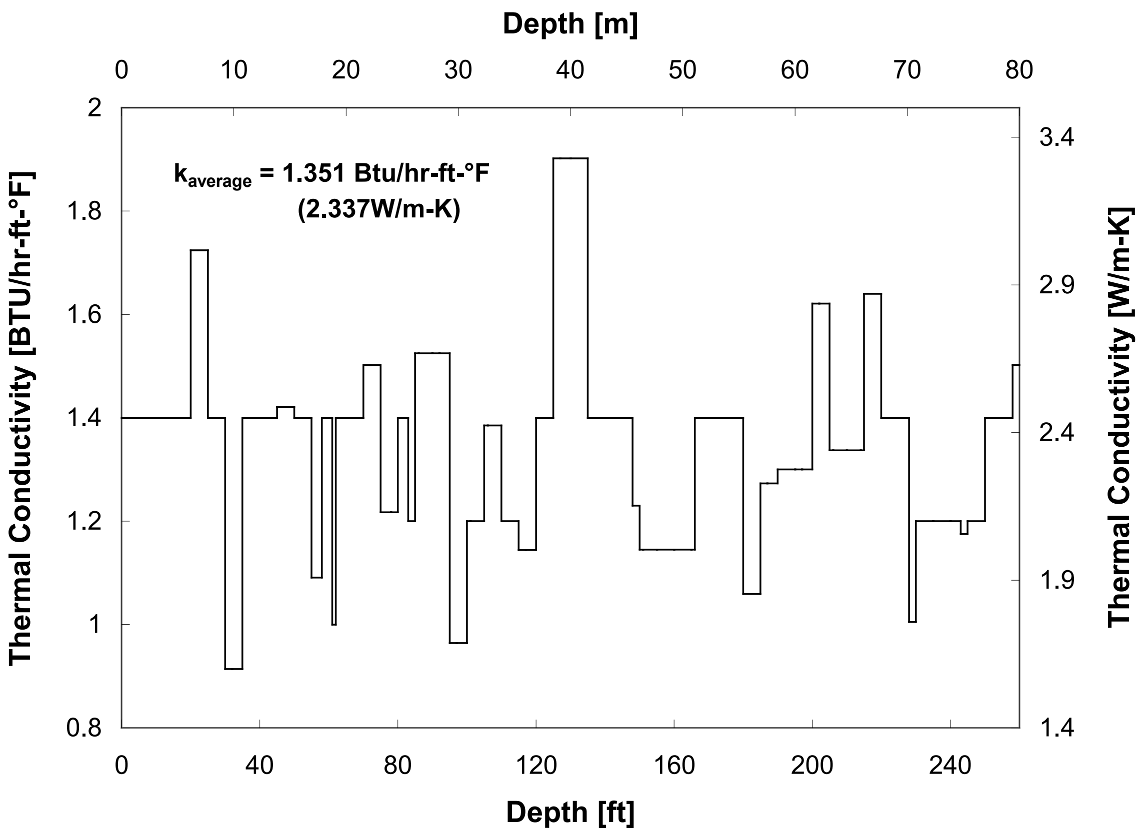

Figure 4 *Thermal conductivity vs. the cored borehole depth based on the guarded hot plate core experiments for Oklahoma State University site A #6 borehole.*

Medium-Scale Laboratory Experiment

A medium-scale laboratory test where a homogeneous soil surrounds a simulated borehole was conducted to provide a validation for the in situ measurement procedure (Smith 1998; Smith, et al. 1999b). The flexible configuration of the simulated borehole allows for a series of borehole parameters such as the shank spacing of the U-tube and the exact geometry of the borehole to be controlled, as it is also easily modified for various grout and soil types for testing. The test apparatus utilizes its own data acquisition system, rather than the in situ apparatus described above.

The dimensions of the wooden structure that contains a homogenous soil (either dry or saturated sand) are 48.0 ft (14.6 m.) in depth, 4.0 ft (1.2 m) in width and height. The simulated borehole is created by placing a U-tube and bentonite-based grout inside of a horizontal 5.25-inch (133 mm) diameter aluminum pipe. The U-tube position inside the borehole is controlled with spacers, and the aluminum pipe is centered within the wooden structure.

Saturated and dry sands were tested. The thermal conductivity of the sands was independently determined using a 6 inch (150 mm) probe at various locations in the test apparatus. The thermal conductivity of the dry sand was determined to be between 0.142 Btu/hr-ft-°F (0.246 W/m-K) and 0.155 Btu/hr-ft-°F (0.268 W/m-K) based on five different measurement locations with an average of 0.149 Btu/hr-ft-°F (0.258 W/m-K). Similarly, the thermal conductivity of the saturated sand was measured to be between 1.272 Btu/hr-ft-°F (2.201 W/m-K) and 1.565 Btu/hr-ft-°F (2.708 W/m-K) with an average of 1.353 Btu/hr-ft-°F (2.341 W/m-K). The dry and saturated sands were chosen for the medium scale laboratory tests since they represent a relatively wide range of ground thermal conductivities in addition to being relatively homogenous and readily available. The dry sand, however, is representative of extremely low ground conductivity.

The length of the tests was limited to between 46 and 50 hours to avoid edge effects. The far-field temperature of the ground was estimated to be the average initial temperature of the sand at five different locations at different radial distances from the center of the borehole. The temperature at the outer domain boundary of the wooden structure was observed throughout the experiment and the numerical simulation to insure that the domain temperature was unchanged from the initial 'far-field' temperature.

Overview of the Parameter Estimation Results

There are a number of ways that the parameter estimation might be approached. Specifically, one, two, or more parameters might be estimated simultaneously. Although a number of approaches were tried, including estimating up to five parameters (soil conductivity, grout conductivity, soil volumetric specific heat, grout volumetric specific heat, and shank spacing) simultaneously, only the most promising approach is presented in this paper.

The approach used involves simultaneous estimation of both soil conductivity and grout conductivity. This has the advantage of allowing for an approximate accounting for several borehole-related parameters: grout conductivity, shank spacing and even borehole diameter. (The borehole will not necessarily be exactly the diameter of the drill bit.) The estimated grout conductivity might be considered as effective grout conductivity in this case.

Austin (1998) first attempted a single variable approach, involving only the estimation of the soil conductivity. This has the advantages of simplicity and computational speed, since only one parameter is varied for each function evaluation. The disadvantage of using only one variable is that all of the other inputs must be "correct": shank spacing, grout conductivity, and grout volumetric specific heat, etc. While the grout conductivity and grout volumetric specific heat might be independently determined, the actual location of the U-tube in the borehole and the effective shank spacing cannot be determined with typical installation techniques. Although it is possible to control some of the parameters such as the shank spacing and the U-tube spacing in the borehole, further investigation is needed to determine its practicality.

Nevertheless, parameter estimation of only one variable cannot adequately account for uncertainties in the tube placement, grout conductivity and the exact borehole geometry. Although the ground thermal conductivity will obviously still be one of the estimated variables, a second variable is needed to be estimated to account for these uncertainties in the borehole. In this respect, the grout conductivity as the second independent variable is a good surrogate for the other borehole parameters.

As discussed by Austin (1998), other approaches that involved estimation of additional parameters often gave very good fits to the experimental data. Unfortunately, some of the estimated parameters, especially the volumetric specific heats, were outside of what might be considered physically possible. Also, as more simultaneous parameters are estimated, more computational time is required.

Furthermore, simultaneous estimation of both soil conductivity and soil volumetric specific heat is problematic. In a transient conduction heat transfer problem, the governing equation is often written with only the thermal diffusivity, the ratio of the thermal conductivity to the volumetric specific heat. From this, one might conclude that it is impossible to estimate conductivity and volumetric specific heat simultaneously, as there are an infinite number of combinations that represent the same value of diffusivity. However, the boundary condition at the wall of the pipe is an imposed heat flux, and therefore $\left(k_{Grout}\frac{\partial T}{\partial n}\right)$ is fixed at any point in time. This does allow simultaneous estimation of thermal conductivity and volumetric specific heat, even if the results are not always satisfactory.

Consequently, the recommended procedure expects that the engineer analyzing the test will estimate the volumetric specific heat based on knowledge of the rock/soil formation and treat it as a known value. The effect of this assumption on the thermal conductivity prediction is discussed below.

Validation of the Parameter Estimation Procedure

A summary of the two-dimensional parameter estimations on the simulated borehole and the cored borehole configurations is provided in Table 1 along with the independently measured values of the thermal conductivities. The parameter estimations used between 46 and 50 hours of measured data, as discussed in the next section. A comparison shows a very reasonable agreement between the predicted values of thermal conductivities using the parameter estimation method based on the downhill simplex algorithm with the numerical model of the borehole and the known and/or measured values for the same. A maximum deviation of about 2.1% is observed (cored borehole Okla. State Univ. Site A6) while the simulated borehole with dry sand and the simulated borehole with saturated sand display a deviation of only about 2.0% and 1.3% respectively. As expected, the errors associated with the predictions of the thermal conductivity of the grout are greater since the second independent parameter is used as a surrogate to account for uncertainties in the borehole.

TABLE 1 Thermal conductivity estimations for the cored borehole and the simulated borehole configuration.

	Okla. State University SiteA6		Experiment - Dry Sand		Experiment - Saturated Sand	
	predicted	indep. measured	predicted	indep. measured	predicted	indep. measured
k_{ground} Btu/hr-ft-°F (W/m-K)	1.379 (2.386)	1.351 (2.337)	0.152 (0.263)	0.149 (0.258)	1.336 (2.311)	1.353 (2.341)
k_{grout} Btu/hr-ft-°F (W/m-K)	0.758 (1.311)	0.850 (1.471)	0.540 (0.934)	0.430 (0.744)	0.496 (0.858)	0.430 (0.744)
Avg. Error of the Fit °F (°C)	0.11 (0.06)	N/A	0.23 (0.13)	N/A	0.22 (0.12)	N/A
Iterations	47	N/A	72	N/A	83	N/A

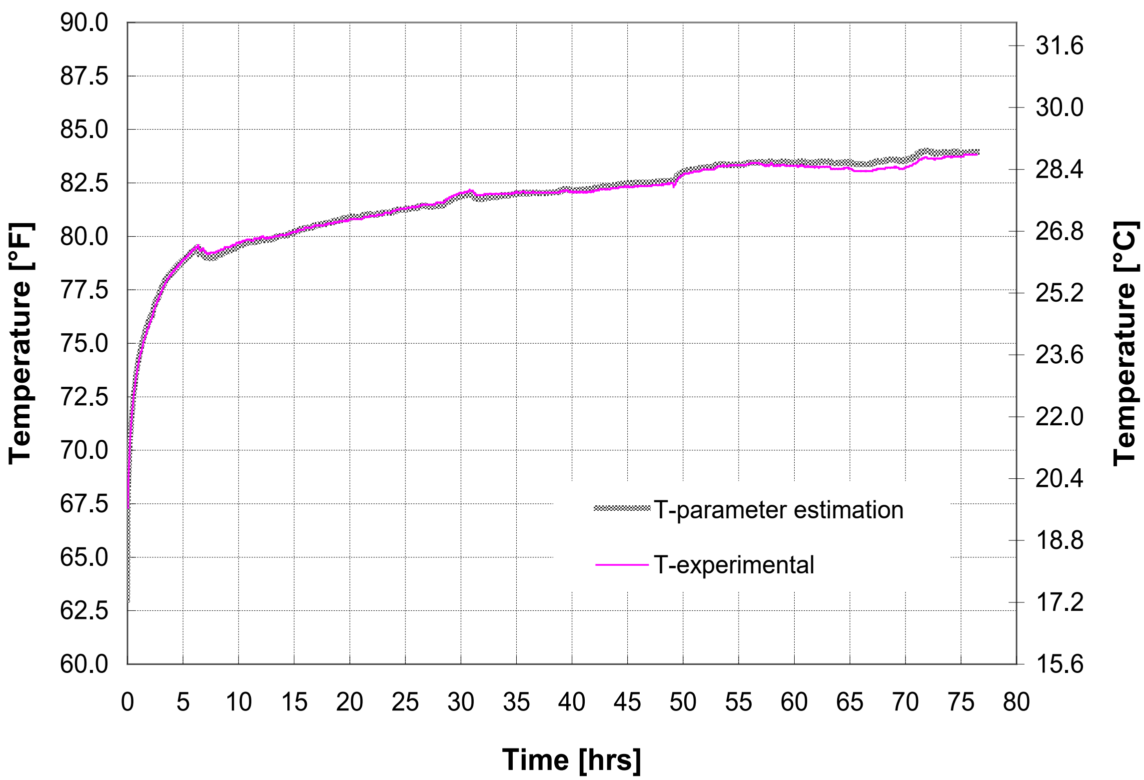

Figure 5 *Comparison of in-situ experimental temperatures to predicted temperatures using the numerical function evaluation model based on the estimated parameters (k_{soil} and k_{grout}) using the Nelder-Mead simplex minimization. Oklahoma State University site A #6 borehole.*

The absolute average error of the predicted temperatures using the estimated parameters ranges from about 0.11°F (0.06°C) to about 0.23°F (0.13°C). Figure 5 shows a typical comparison between the in-situ measured temperatures and the predicted temperatures with the numerical finite volume model using estimated ground and grout thermal conductivities. The temperature versus time plot in Figure 5 is provided for the cored borehole (OSU Site A #6). Although fluctuating power input was observed from the in-situ test, the parameter estimation method was capable of predicting the ground conductivity within about ±2% of the measured value.

Sensitivity Analyses

A series of sensitivity analyses have been performed to evaluate the influence of a number of input parameters that cannot be determined exactly, but estimated with some uncertainty. (The term "input parameters" refers here to parameters that are not estimated with the parameter estimation procedure, e.g. far-field temperature, volumetric specific heats, shank spacing, borehole radius) The uncertainty in the input parameters has a corresponding uncertainty in the estimated ground thermal conductivity. In addition, the duration of the test and experimental errors impact the results, so a sensitivity analysis is performed for both. The sensitivity analyses are described individually below. Because all of the other uncertainties depend on the length of test, sensitivity of the predictions to the length of test is described first.

Length of In-Situ Testing

One of the most commonly asked questions about in situ testing is "How long does the test need to be?" One of the best approaches available for answering this question may be to run long tests, and to observe the sensitivity of the ground thermal conductivity estimations to the length of the data used. As the duration of data used increases, there should be a point in time beyond which the estimated value of the ground thermal conductivity does not change very much.

Field experience suggests that the estimate of the ground thermal conductivity reaches convergence between 80 and 100 hours. To illustrate the point, Figure 6 shows the typical dependency of the ground thermal conductivity on the test duration observed for three test boreholes (Oklahoma State University site A#1 and A #2 boreholes and a test borehole located in Chickasha, Oklahoma). The total duration of the in-situ test on the site A#2 borehole was slightly longer than170 hours while the in-situ tests on site A#1 borehole and the Chickasha borehole were each about 100 hours long. For each data set, the ground thermal conductivity is estimated for various data lengths starting from the 20th hour. The data sets shorter than 170 hours have been logarithmically extrapolated up to the 170th hour for comparison. The estimated ground thermal conductivity values appear to converge after about 80 to 100 hours from which time on no significant changes are observed.

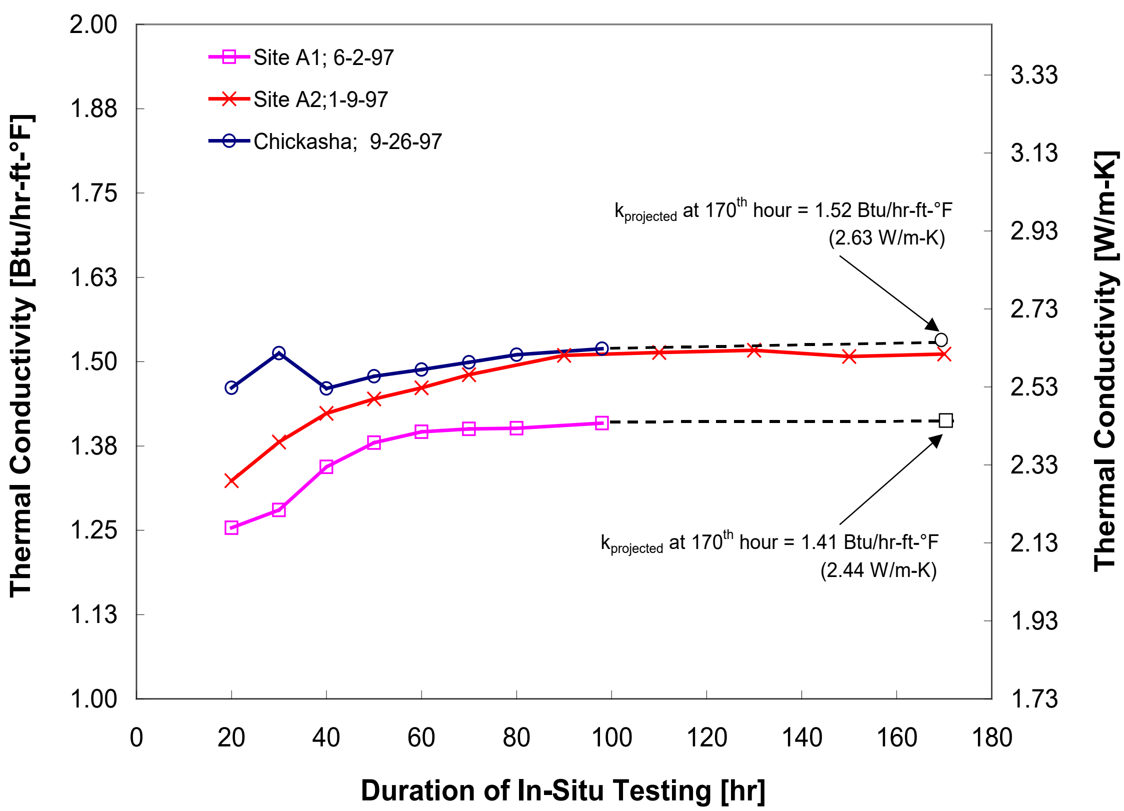

Figure 6 *Ground thermal conductivity estimation vs. in-situ test duration. Oklahoma State University site A #1 and #2, and Chickasha test boreholes. (Dotted lines indicate logarithmic extrapolations.)*

It is often not feasible to conduct a test of this length. Therefore, a significant effort has been made to find a suitable compromise between test length and test accuracy. Although the choice is somewhat subjective, the authors have settled on a test length of 50 hours based on analyses conducted on the current in-situ test data and field experience (Austin 1998). With in-situ tests shorter than 50 hours, the error in the ground thermal conductivity prediction can be significant. This error is quantified in Table 2 where the thermal conductivity estimations and associated errors from the converged value for the Oklahoma State University Site A#1, #2 and Chickasha test boreholes are provided. The deviation between the ground thermal conductivity estimation of the 20-hour test and the estimations of the 170-hour test for the site A#2 borehole is approximately 14.2%. The absolute error diminishes rapidly as the length of data is increased. It is about 4.6% by the 50th hour. A very similar trend is observed on the site A#1 and Chickasha test boreholes where the absolute errors at the 50th hour from the converged estimations are observed to be about 2.2% and 2.8% respectively.

TABLE 2 Thermal conductivity estimations and associated errors from the converged value for the Okla. State University Site A#1, #2 and Chickasha test boreholes.

Duration of In-Situ Testing [hr]	Okla. St. Uni.;Site A1; 6-2-97		Okla. St. Uni.;Site A2; 1-9-97		Chickasha; 9-26-97	
	k_{ground} Btu/hr-ft-°F(W/m-K)	Error [%]	k_{ground} Btu/hr-ft-°F(W/m-K)	Error [%]	k_{ground} Btu/hr-ft-°F(W/m-K)	Error [%]
20	1.254 (2.169)	12.48	1.323 (2.289)	14.20	1.461 (2.528)	4.04
30	1.280 (2.214)	10.15	1.381 (2.389)	9.46	1.513 (2.618)	0.49
40	1.344 (2.325)	4.93	1.423 (2.462)	6.18	1.460 (2.526)	4.10
50	1.380 (2.387)	2.20	1.445 (2.500)	4.60	1.478 (2.557)	2.81
60	1.396 (2.415)	1.00	1.461 (2.528)	3.43	1.488 (2.574)	2.15
70	1.400 (2.422)	0.70	1.480 (2.560)	2.08	1.499 (2.593)	1.40
80	1.401 (2.424)	0.64			1.510 (2.612)	0.65
90			1.509 (2.611)	0.15		
100	1.409 (2.438)	0.08			1.519 (2.628)	0.07
110			1.514 (2.619)	0.15		
130			1.517 (2.624)	0.36		
150			1.508 (2.609)	0.24		
170	1.410 (2.439)*	0.00	1.511 (2.614)	0.00	1.520 (2.630)*	0.00
	(*) Projected					

A series of in-situ tests on other nearby boreholes at the Oklahoma State University test site A were performed. Although one additional long-term test (longer than 100 hours) was conducted, the majority of the tests were about 70 hours. Analysis of the 100+ hour tests and the 70 hour tests indicated that the estimated ground thermal conductivity values, based on 50-hour test length, were typically within

±6.5% of the converged value, although about half of the long tests had values within ±2.5%. On the shorter (approximately 70 hours) tests, it was not always possible to determine the converged value. Therefore, it is possible that the uncertainty associated with the thermal conductivity estimates at the 50th hour might be somewhat greater, although current field experience appears to bound its range within ±6.5%.

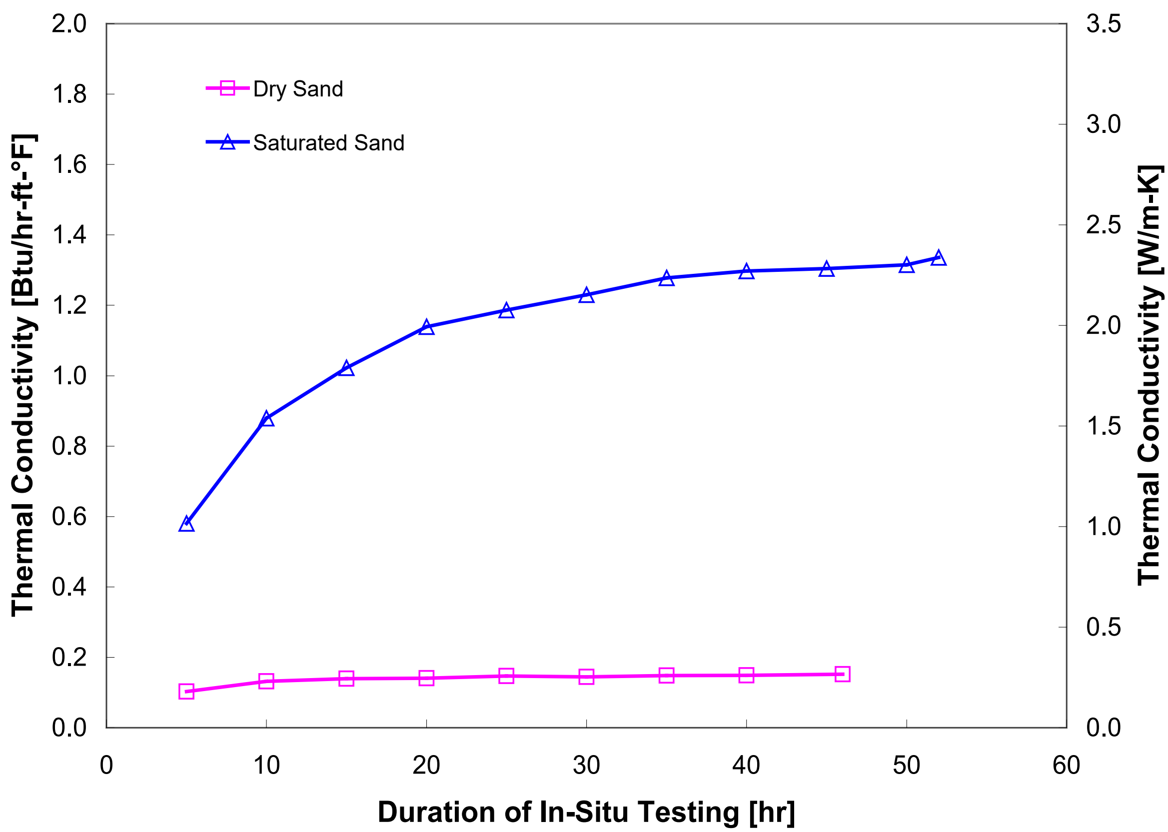

Figure 7 *Thermal conductivity estimation vs. duration of in-situ testing. Simulated borehole.*

In addition, there appears to be a correlation between the ground thermal conductivity and the required length of in-situ testing. Figure 7 illustrates that the dry sand with low conductivity and low diffusivity converges significantly faster than the saturated sand with higher conductivity, while in each test case identical grout of thermal conductivity of 0.43 Btu/hr-ft-°F (0.74 W/m-K) was used. As shown in Figure 7, the simulated borehole tests with dry sand estimates the converged conductivity within ± 8%

with only 15 hours of in-situ test data, while the simulated borehole test with saturated sand requires about 35 hours of in-situ test data to achieve the same accuracy.

Far-Field Temperature

Since the borehole temperature response to an imposed heat flux is sensitive to the undisturbed far field temperature of the ground, the value of the far field temperature has a significant impact on the estimated ground thermal conductivity. There are several maps (IGSHPA 1991) available that give a general idea of the undisturbed ground temperatures for the continental U.S. using well water isotherms. However, such maps cannot possibly yield locally accurate information. Although several experimental procedures have been tried for obtaining the undisturbed ground temperature, the best procedure seems to be lowering a thermocouple (or other calibrated temperature sensor) down the U-tube and measuring the temperature of the heat transfer fluid along the borehole depth before each test. The undisturbed far field temperature is then determined by averaging the measured temperatures along the depth of the borehole below 10ft (3m). Even then, there is some uncertainty in the measurement. Although the ground thermal conductivity predictions will be strongly affected by variations in the assumed far field temperature, the impact on the borehole design is mitigated as long as the design value and the value used for the parameter estimation are the same.

The sensitivity of the parameter estimation model to the uncertainties in the measurement of the undisturbed far-field ground temperature can be seen in Figures 8 and 9 for the cored borehole and the simulated borehole in the medium-scale laboratory tests. For one particular experimental data set, five different far-field temperatures were used as input parameters. The analyses demonstrate the ground thermal conductivity prediction sensitivity based on a $\pm$ 1.0 °F ($\pm$ 0.6 °C) error range. For each far-field temperature point, all other input parameters were kept constant.

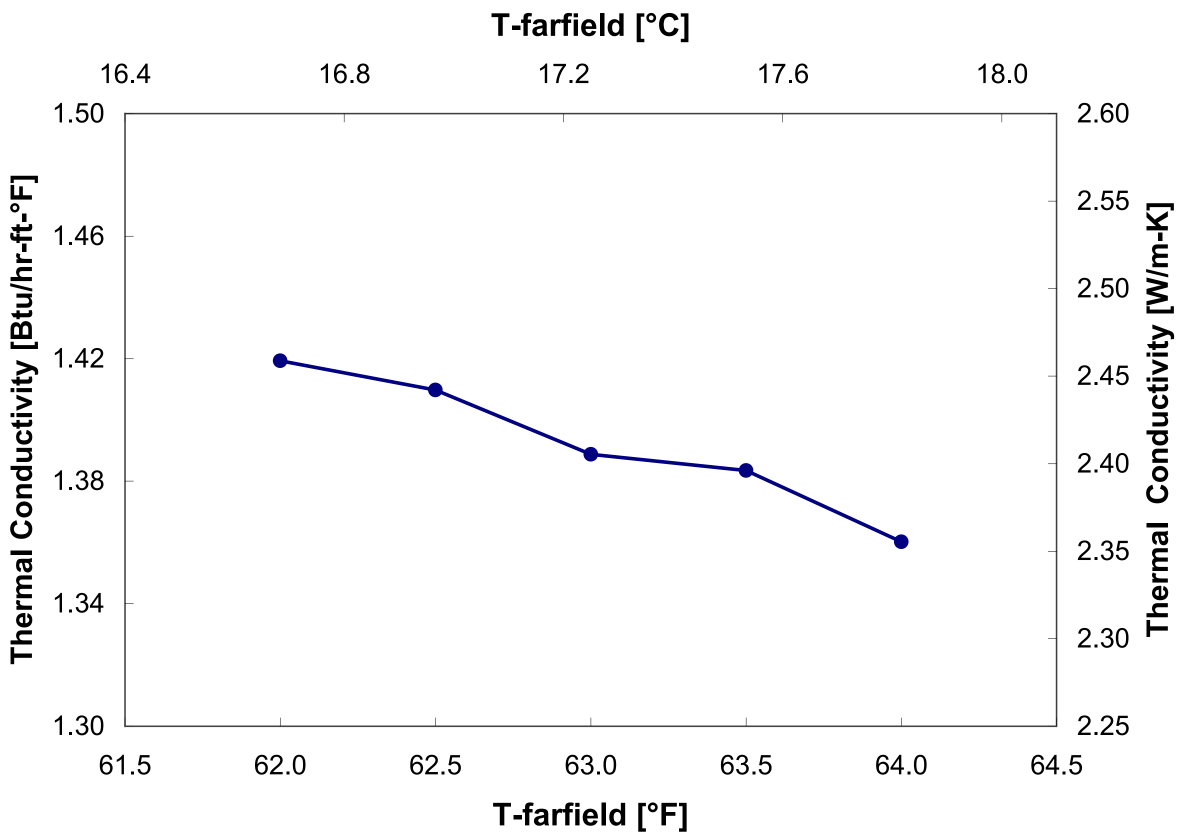

Figure 8 *Ground thermal conductivity estimation vs. the undisturbed far-field ground temperature. Oklahoma State University site A #6 borehole.*

Figures 8 and 9 show that the parameter estimation model is very sensitive to the estimate of the ground far-field temperature. It is also observed that this sensitivity is stronger for high thermal conductivity soils than for low thermal conductivity soils. As expected, the predicted ground thermal conductivity decreases with increasing far-field temperature, since, for unchanged series of heat transfer rates the temperature differences between the average borehole temperatures and the far-field temperature becomes larger. The analyses based on the simulated and cored boreholes show that if the ground far-field temperature can be determined within ± 1.0 °F (± 0.6 °C) the associated error in the thermal conductivity estimation will be limited to about ± 4.9%.

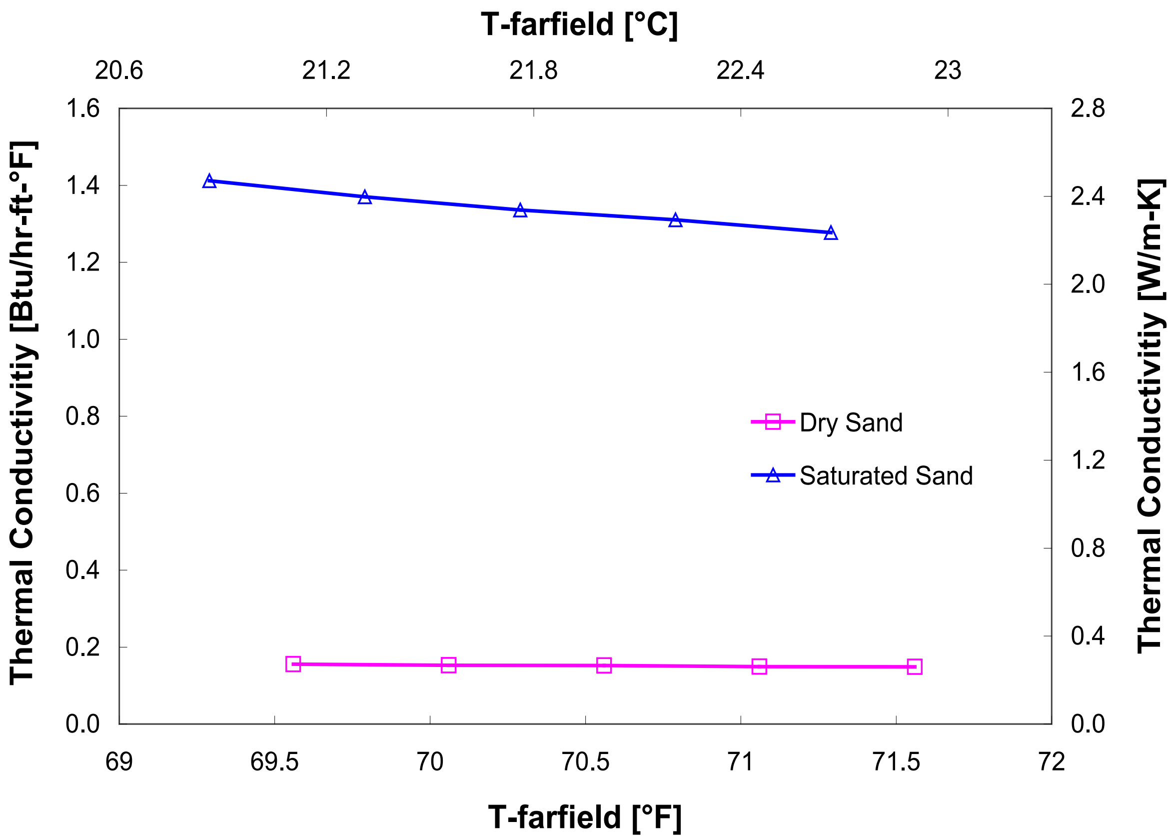

Figure 9 *Thermal conductivity estimations vs. the undisturbed far-field temperature. Simulated borehole.*

Shank Spacing

The sensitivity of the ground thermal conductivity estimations to uncertainties in the shank spacing (the distance between the two pipes from pipe outer wall to pipe outer wall of a U-tube) is presented in this section. Since it is difficult in practice to control the shank spacing, this parameter was varied to examine its sensitivity to the ground thermal conductivity estimations.

Figure 10 shows the results obtained from the cored borehole and the simulated borehole tests. In each of these cases, five different shank spacing values that would not violate the borehole geometry were used. Since the inclusion of the second independent variable (k_{grout}) in the parameter estimation is expected to act as a surrogate for the uncertainties in the shank spacing, the sensitivity analyses have shown that even significant uncertainties (errors in the initial estimate) in the U-tube shank spacing only yield small changes in the ground conductivity predictions. A ±40% change in the 'effective' shank spacing only causes a ±1.6% change in the ground thermal conductivity estimation.

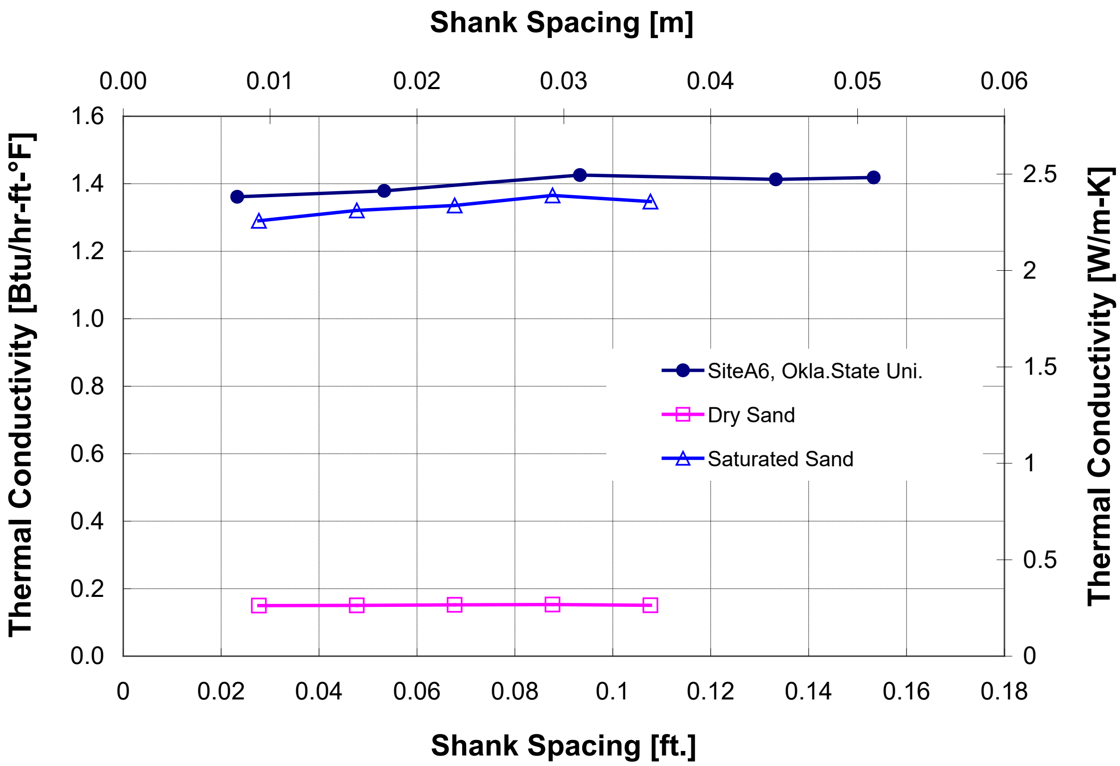

Figure 10 *Thermal conductivity vs. the shank spacing of the U-tube. Oklahoma State University site A #6 borehole, and the simulated borehole in the medium-scale test unit with dry and saturated sand.*

Although there is a strong correlation between the grout thermal conductivity estimates and the shank spacing values, the ground thermal conductivity is affected only slightly. The simulated borehole tests with dry and saturated sands suggest that very low thermal conductivity sand is significantly less sensitive to uncertainties in the shank spacing than saturated sand with higher thermal conductivity.

Volumetric Specific Heat

Since the transient conduction heat transfer problem depends strongly, but not solely, on the thermal diffusivity, it is inevitable that the estimated thermal conductivity will be dependent on the assumed value of the volumetric specific heat of the ground.

In order to determine the sensitivity, the effect of volumetric specific heat values ranging from 20 Btu/ft^3-°F (1340 kJ/m^3-K) to 50 Btu/ft^3-°F (3350 kJ/m^3-K) have been investigated. This range of volumetric specific heat, as reported by EPRI (1989) represents almost the entire practical range for commonly occurring soil types. In order to accommodate the medium-scale laboratory test cases involving dry sands with very low diffusivity, a relatively low volumetric specific heat value of 14 Btu/ft^3-°F (938 kJ/m^3-K) is also investigated.

Figure 11 shows the results of the sensitivity analyses for the simulated borehole tests. A relatively strong correlation is observed between ground thermal conductivity and ground volumetric specific heat. The ground thermal conductivity estimations decrease as the volumetric specific heat of the ground increases, although this trend is not as strong in the case of the saturated sand as it is for soils with very low thermal conductivities. The low thermal conductivity soils appear to be more sensitive to uncertainties in the soil's volumetric specific than higher conductivity soils that are typical for soil types encountered in practice.

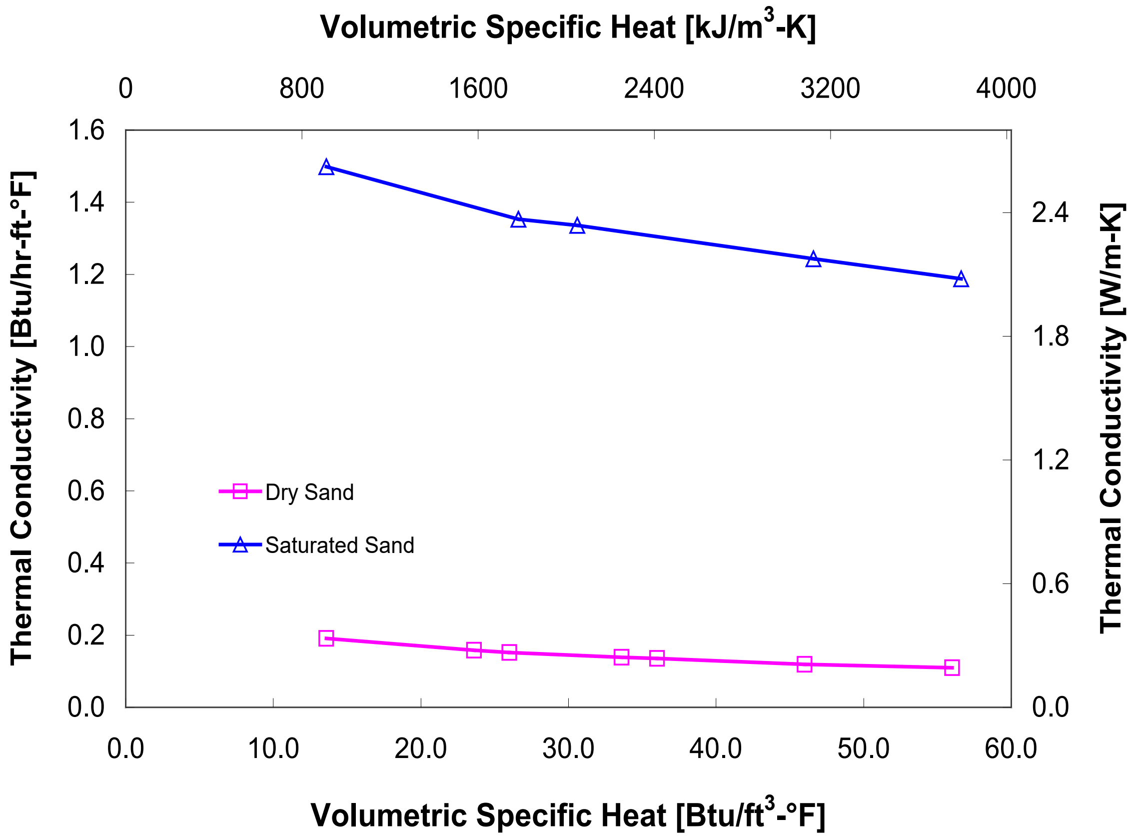

Figure 11 *Thermal conductivity estimations vs. the volumetric specific heat. Simulated borehole.*

However, the analyses show that, if the volumetric specific heat of the ground can be estimated within ± 5 Btu/ft^3-°F (± 335 kJ/m^3-K), which represents about 10% to 25% of the practical range for commonly occurring soil types, the ground thermal conductivity estimations vary by about ± 2.6% for the cored borehole and the simulated borehole with saturated sand while it varies by about ± 6.3% for very dry sand.

Power Input and Temperature Calibration

Errors due to improperly calibrated instrumentation can affect the in-situ test data. This may manifest itself in the form of incorrect power-input (errors on the watt transducer) and/or entering and exiting loop temperature readings (errors in temperature sensor calibration). Therefore, an error estimate for the experimentally collected data is required to investigate the sensitivity of the ground thermal conductivity predictions to uncertainties in power and/or temperature measurements

In order to accomplish this, artificial errors were introduced to the power and temperature sensor calibrations. For the temperature data, the slope of the sensor calibration curve was increased by 2% for both the borehole entering and exiting fluid temperatures. The experimental average borehole temperature was then 're-computed' based on the artificially adjusted loop temperatures. The ground thermal conductivity estimations were then obtained based on the actual power-input data and the modified temperature response data. The results are reported in Table 3.

The sensitivity analysis of the ground thermal conductivity to uncertainties in the power-input measurements is implemented by an artificial modification of the power-input values. The power-input values for each time step were increased by 5% while the corresponding temperature responses to the changes in power were unchanged. The results of the power sensitivity analyses are reported in Table 3.

The analyses for the specific cases investigated show an almost linearly proportional relationship between an increase of the calibration curve slope of the temperature sensors used or the increase in the power input, and the predicted thermal conductivity values for the cored and the simulated borehole cases.

TABLE 3 Change in ground thermal conductivity Btu/hr-ft-°F (W/m-K) estimations based on changes in power input and temperature measurement

	Base	Power up 5%	Change [%]	Base	Temp. Calib. Coeff. up 2%	Change [%]
$k_{ground-OSU\ SiteA6}$	1.379 (2.386)	1.445 (2.500)	***4.79***	1.379 (2.386)	1.400 (2.422)	***1.52***
$k_{ground\ -\ Dry\ Sand}$	0.152 (0.263)	0.160 (0.277)	***5.26***	0.152 (0.263)	0.154 (0.266)	***1.32***
$k_{ground\ -\ Sat.\ Sand}$	1.336 (2.311)	1.428 (2.470)	***6.89***	1.336 (2.311)	1.364 (2.360)	***2.10***

In summary, a $\pm$2% change in the slope of the thermistor calibration curve causes an estimated uncertainty of about $\pm$2%. However, based on a simple statistical analysis of the sensor calibration, the uncertainty in the slope is expected to be less than ±0.12%. This will cause a negligible uncertainty in the ground thermal conductivity estimate.

The watt transducer used in the experimental apparatus has an accuracy of approximately $\pm$1.5% for the conditions encountered during in-situ tests. Based on this, the resulting uncertainty in the thermal conductivity estimations is projected to be about $\pm$1.5%.

Borehole Geometry

The drilling of boreholes under field conditions introduces uncertainties due to drilling processes used and the ground conditions at the field. The actual borehole diameter may be both larger than the drill bit in some places, and smaller than the drill bit in other places. Since it is not feasible that these occurrences be controlled (and are not controlled in typical practice) a series of sensitivity analyses are required to assess the impact of uncertainties introduced through inaccurate borehole depth and radius.

These uncertainties are analyzed only for the cored borehole. The borehole radius was varied between 0.149 ft (0.045 m) and 0.229 ft (0.070 m), a range that is within $\pm$20% of the nominal borehole radius. Again, for each estimation, all other input parameters were kept constant.

Figure 12 illustrates the dependency of the ground thermal conductivity estimations on the uncertainty of borehole radius for the cored borehole. As the radius of the borehole becomes larger the estimated ground conductivity increases due to increased borehole resistance. This is expected, since, as the borehole resistance increases through the larger borehole diameter, the estimates for the ground conductivity have to increase to adjust for the unchanged average borehole temperatures. However, analyses suggest that, if the borehole radius can be determined within $\pm$0.04 ft ($\pm$0.012 m), the uncertainty in estimating the ground thermal conductivity is reduced to about $\pm$3.6%.

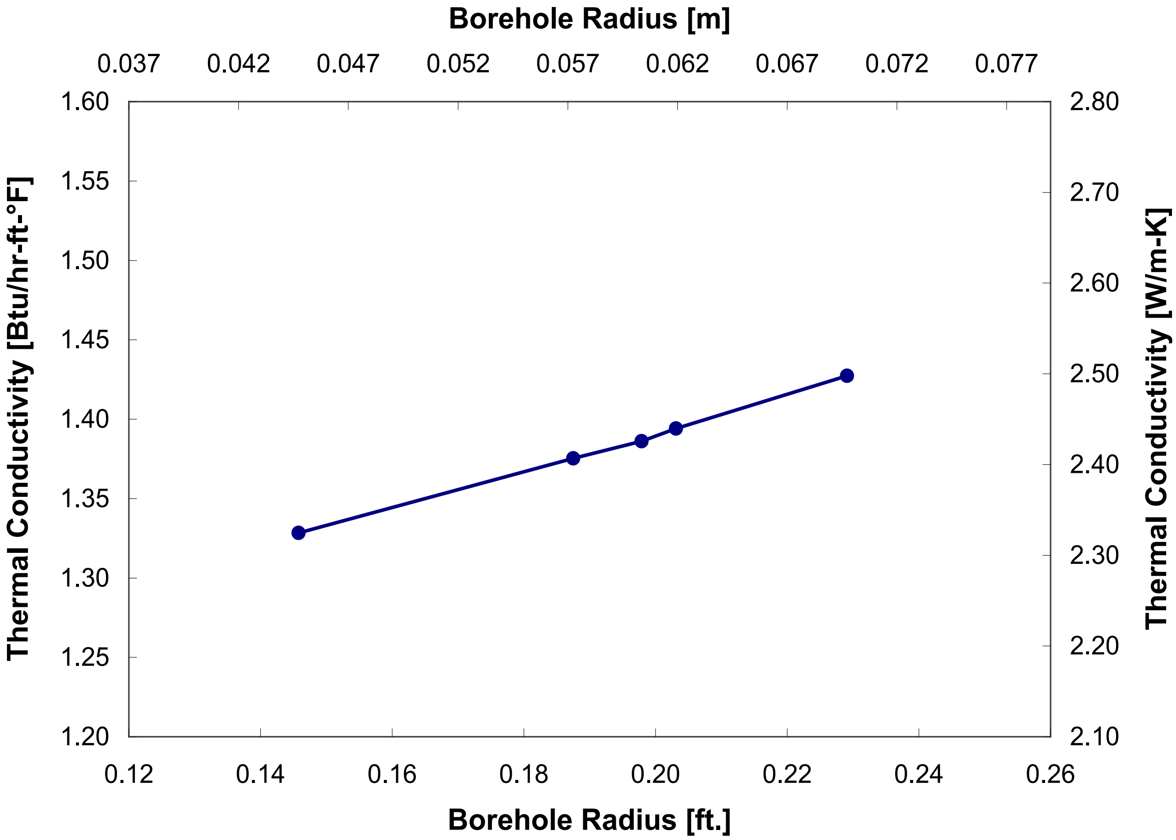

Figure 12 *Ground thermal conductivity estimation vs. the borehole radius. Cored borehole Oklahoma State University site A#6.*

The sensitivity of the estimated ground thermal conductivity to uncertainties in the depth of the borehole was also investigated. The estimated conductivity decreases with increasing depth, since, in the analyses, the total amount of heat transferred over the borehole is unchanged. Consequently, for shorter borehole depths, the amount of heat transferred per unit borehole depth increases while the average borehole temperatures and other input parameters are kept unmodified, resulting in higher ground thermal conductivity estimations. Similarly, lower ground thermal conductivities are estimated for increased borehole depths. The analyses indicate that the uncertainty in the ground thermal conductivity due to a $\pm$ 0.5 ft (0.15 m) uncertainty in the borehole depth is $\pm$ 0.15% for a 250 ft (76.2 m) deep borehole. This uncertainty is negligible when added in quadrature with the other uncertainties.

In addition to the uncertainties discussed above, the numerical finite volume model of the borehole also represents a source of uncertainty in the estimation of the ground's thermal conductivity. Yavuzturk et al. (1999) provides detailed discussion on the numerical model and its validation against an analytical solution using six different test cases that simulate a typical range of heat flux, model geometry and thermal properties. However, the analytical solution does not correspond exactly to the borehole geometry with differing ground and grout conductivities. Therefore, it is difficult to determine the exact impact of the uncertainties in the numerical model on the estimate of the ground conductivity. It appears, but probably cannot be proven, that the inaccuracies in the numerical model are reflected in the estimate of the grout conductivity. Accordingly, a heuristic estimate of the impact of the uncertainty in the numerical model on the ground conductivity is made. The $\pm$ 1.2% uncertainty corresponds to the error in the numerical model results at 12 hours.

Summary of Uncertainties on the Ground Thermal Conductivity Estimations.

A summary of the sources of uncertainties and their effect on the ground thermal conductivity estimation is given in Table 4. Since the uncertainties described in Table 4 pertain to parameters that are all independent or nearly independent from each other they may be added in quadrature. Thus, the total estimated uncertainty of the ground thermal conductivity estimations falls within a range of about 9.6% - 11.2% depending on the level of the estimated thermal conductivity, since very low conductivity sands appear to be more sensitive to the estimate of the volumetric specific heat. The overall uncertainties compare very well with the range of values that was obtained from other tests in nearby locations (Austin 1998). (Uncertainties smaller than 0.2 % have been ignored, as their contribution to the overall uncertainty is negligible.)

TABLE 4 Summary of primary sources of uncertainties in the estimation of thermal conductivity of the ground.

Source	**Estimated uncertainty in predicted k_{ground}**
Length of Test – approx. 50 hours	**± 6.5%**
Power Measurement. **(± 1.5% uncertainty.)**	**± 1.5%**
Estimate of the volumetric specific heat of the ground. **(± 5 Btu/ft^3-°F [± 335 kJ/m^3-K])**	**± 2.6% (average soils)** **or** **± 6.3% (extremely dry soils)**
Estimate of the borehole radius. **(± 0.5 inches [12.7 mm])**	**± 3.6%**
Estimate of the shank spacing. **(± 40%)**	**± 1.6%**
The numerical model.	**± 1.2%**
Estimate of the far-field temperature. **(± 1 °F [± 0.6 °C])**	**± 4.9%**
Total Estimated Uncertainty	**± 9.6% - 11.2%**

It is obvious that the estimated uncertainty is somewhat higher than the errors found when the parameter estimation procedure was applied to the validation test cases. It should be noted that, for these cases, a number of the input parameters, e.g. far field temperature, volumetric specific heat, borehole radius, and shank spacing were determined more accurately than what might be feasible under typical field conditions.

Conclusions and Recommendations

An experimental apparatus has been described that is capable of imposing a heat pulse on a test borehole, and measuring its temperature response. The ground thermal conductivity is estimated using a parameter estimation technique in conjunction with a two-dimensional numerical model. Independent measurements of soil conductivity test results are reported for a cored borehole and a simulated borehole with different types of sands in a medium scale laboratory experiment to validate the parameter estimation method. The sensitivity of the estimated ground thermal conductivity is investigated to assess the uncertainties associated with determining the values of volumetric specific heat of the ground, the undisturbed far-field temperature, the borehole geometry and instrumentation.

Specific conclusions and recommendations regarding the design of the in-situ test apparatus and experimental procedure are discussed in detail by Austin (1998). Additional conclusions and recommendations related to the length of in-situ test and the parameter estimation procedure, and overall accuracy of the estimates are as follows:

- Using the in-situ procedure described, the length of test should be no less than 50 hours to obtain a value of ground conductivity that would be within about $\pm$ 6.5% of that obtained with a much longer tests. Preliminary analyses suggest that the ground thermal conductivity to be estimated may have a significant influence on the length of in-situ testing. It appears that low thermal conductivity soils require less time to converge than higher thermal conductivity soils.
- An error analysis suggests that with data measured by the experimental apparatus, the two-variable parameter estimation procedure can be expected to predict the ground thermal conductivity within a range of about $\pm$ 9.6% and $\pm$ 11.2%.
- Validation test cases using saturated and dry sands under laboratory conditions and the cored borehole show that the two-variable parameter estimation model estimates the ground thermal conductivity within a maximum range of $\pm$ 2.1%. As noted, the errors here are smaller than the general error estimate because several of the input parameters were estimated more accurately than what might be feasible under typical field conditions.

It is obviously desirable that the required time for the in situ test and parameter estimation be reduced. To that end, the following recommendations for further investigation are offered:

- In order to quantify the relationship between the required length of in-situ tests and the ground's thermal conductivity, further research is suggested utilizing test data from an even wider range of ground thermal conductivities.
- Since the duration of the test depends on the desired accuracy, any improvement in accuracy of the method may allow for a shorter test. Accordingly, methods for reducing the uncertainty of the input

parameters should be investigated. In particular, methods for more accurately estimating the far-field temperature, the average borehole radius (perhaps by measuring the total grout volume), and the ground volumetric specific heat should be pursued.

- The current recommended duration of the in situ test is 50 hours. In practice, it is highly desirable to be able to do the test in a significantly shorter amount of time. One possible approach for this is to improve the model's accuracy in the first few hours. This might be done by extending the numerical model to 3 dimensions and/or more closely matching the actual geometry by using a boundary-fitted coordinate grid. Presumably, any improvements made in the first few hours will help allow for a shorter test. At the same time, it will probably be useful to physically control the position of the U-tube in the borehole. Whether the reduced test time will be worth the increased computational time for the parameter estimation remains to be seen.
- The parameter estimation algorithm is a computationally intensive procedure. For acceptable estimation accuracy, about 50-80 objective function evaluations are typically required, with each one requiring a simulation using the detailed numerical model of the borehole. In order to reduce the computational time, a better initial guess for the conductivities may be made by using a simple analytical model in conjunction with the parameter estimation procedure. This estimate can be made very quickly, and used to reduce the number of objective function evaluations made with the detailed numerical model
- In order to reduce the time from the start of the experiment to final parameter estimation results, the parameter estimation may be performed simultaneously (on-line) instead of subsequently (off-line). The suitability of on-line parameter estimation methods, such as recursive and/or adaptive techniques should be investigated. This could also have the advantage of being able to tell the operator when the experiment is "done", rather than running a predetermined number of hours.

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Nomenclature

k = conductivity (Btu/hr-ft-°F [W/m-K]).

Q = Heat transfer rate (Btu/hr-ft [W/m]).

r = radius (ft [m]).

T = temperature (°F [°C]).

t = time (hr).

SSE = sum of the squares of the error.

Array variables and Subscripts

N = number of data points.

exp = experimental.

num = numerical.

ff = far field.

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