Revisiting the plastic number theory from the perspective of perceptual psychology

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Architect Hans van der Laan spent his lifetime studying human perception of proportion. His work led to the development of the plastic number system. In this system every step represents a group of fractions that describe objects of similar sizes. Indeed, each step is defined as the 'representative size' of a group of sizes that are perceptually indiscriminate from one another. Over decades of research, Van der Laan conducted a series of experiments to test his system. The clarity with which he approached the perception of proportion provides a suitable ground for testing the plastic number by scientific means. To take the first step in that direction, it is useful to consider Van der Laan's approach within the context of sensory psychophysics and Gestalt psychology. Highlighting these connections suggests specific directions for future research of the plastic number and opens a new chapter in studies on proportion in architecture.

Keywords: proportion, margin of size, just noticeable difference, type of size, perceptual grouping

Introduction[1]

Dutch architect and Benedictine monk Hans van der Laan (1904-1991) initiated and completed his lifelong research during a half century of monastic seclusion, studying human perceptions of proportion in a search to understand how we detect and discern objects' size and proportion. This work led him to develop the plastic number proportional system, meant to address the limitations of human perception through architecture. In doing so, he addressed some hypotheses and questions already considered by psychophysics and perceptual organization.

The plastic number is a proportional system with high additive properties resulting by the mathematical equation $x^3 = x + 1$, in which the sum of two consecutive measures is equal to the next measure, skipping over one (Aarts et all 2001; Kruijtzer

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1998; Navon 2011; Stewart 1996). However, the mathematics behind the system is only "an aid to gaining an understanding of the plastic reality of these proportions" (Van der Laan 1977, 99), which starts from an investigation into the human ability to perceive, discern, and discriminate size.

Despite being recognized as the inventor of the plastic number system, Van der Laan viewed it as a discovery of a set of related proportions that resonate with human perception. He was not knowledgeable about sensory psychophysics and Gestalt psychology. He was, however, fully aware that the theory he was bringing to an architectural audience was a broader one that derived its basic principles from an abiding interest in the mechanisms that inform human perception of the external world. Interested in the perceptive and cognitive value of proportion, Van der Laan built a theory in between perceptual science and architecture. In so doing, as with psychophysics with psychology, he categorized the experience of architecture as "quantifiable" and therefore subject to scientific experimentation.

Coming from a family of architects, Van der Laan began his education at the School of Architecture in Delft (1923-1927). Before completing his coursework, he left the School and joined the St. Paul Abbey in Oosterhout as a novice, where he studied theology and established the foundations of his more mature thinking (1929-1934). In 1968, Van der Laan moved to the St. Benedict Abbey in Vaals, the expansion of which became his architectural masterpiece and built manifesto of the plastic number theory (1956-1986) (Remery 2010). From the earliest days of his education, Van der Laan searched for an approach to architecture originating in the observation of things and developed a painstaking and systematic analysis of the world as presented to us from our senses. He continually tested his hypotheses and theories against direct observation of the world, rather than a priori reasoning. Although Van der Laan never had the chance to subject his theory to scientific tests, it is, in fact, particularly suitable for psychophysical experimentation.

The present analysis centers on two aspects of Van der Laan's theory, defined by the master as "margin of size" and "type of size." The investigation involves comparing these two concepts, respectively, with psychophysical laws describing just noticeable differences on the one hand, and Gestalt laws of perceptual organization on the other. The first section introduces the core of Van der Laan's theory using his concepts of size discrimination and typification. The second section illustrates the notions of just noticeable difference and perceptual organization, using selected experiments from perceptual psychology and experimental phenomenology to suggest how the experiments and their findings may inform plastic number theory. The analysis concludes by proposing that different aspects of the plastic number system can be tested using scientific methods and introduces a psychophysical study inspired by the plastic number theory (Proietti and Gepshtein 2020). This new study focuses on the perceptibility of proportion and is the beginning of a research program that will consider different roles that proportional systems could play in the experience of architecture.

The plastic number theory

Margin of size and type of size

Van der Laan's various writings recount a childhood experience that inspired his development of the plastic number (Graatsma and Naalden 1982, 7-9; Ferlenga and Verde 2000, 195). Having spent time on the construction sites of buildings designed by his father Leo van der Laan, the young boy became fascinated by different construction processes. The process of sifting gravel, in particular, instilled in Van der Laan a curiosity that would occupy his mind for long time: If the pebbles bigger than the sieve's holes stay on the top and the smaller pass through it, how to explain the lack of pebbles of the same size of the sieve's hole? Where are they? The fundamental answer, already in the mind of the young aspiring architect, established the basis of Van der Laan's experiments and the formulation of the concept of margin of size. The pebbles of the same size as the sieve's hole do not exist, because the sieve is a human product, while gravel belongs to nature.

In order to group pebbles of the right size for the foundation of the building, workers used two sieves. The first sieve had larger holes and the second had smaller holes. By sifting gravel, they grouped pebbles of similar size in between two pre-defined limits. The gravel in between the two sieves constituted a group that Van der Laan named "type of size" (Van der Laan 1977, 54). Sizes belonging to the same group differ by a small increment. This increment Van der Laan named "margin of size" (Van der Laan 1977, 54) and it is the just perceptible difference from one size to another.

Starting with the concepts of margin of size and type of size, Van der Laan searched for an alternative way to measure objects. Indeed, according to Van der Laan, whenever we measure objects, we delude ourselves into thinking we have access to measures that perfectly correspond to real sizes. As a matter of fact, by using pre-defined units of measure (meter, kilogram, foot, etc.), we only "count," or rather "repeat," units in order to cover a certain range of dimensions. However, units of measure are artificial and do not correspond to the continuous extension of the real object we want to measure, just like the hole of the sieve did not perfectly correspond to any of the pebbles' sizes.

Van der Laan's investigations aimed to bridge the gap between these two contrasting aspects of measuring objects: the "discrete quantity" – a product of human beings expressed by artificial units of measures – and the "continuous quantity" – a product of nature expressed by measures stretching to the infinite (Van der Laan 1977, 46). The first is the representation of the act of counting (how many), while the second is the representation of the act of measuring (how much).

By grouping sizes according to differences in margins of size, the continuous quantity is translated into a number of types of size within limits or thresholds. Each type of size is represented by a size defined by Van der Laan the "inward visualization of size" (Van der Laan 1977, 48). The number of types of size sifting the continuous quantity through certain limits (discrete quantity) are related one to another and constitute an order of measures that Van der Laan defines "order of size" (Van der Laan 1977, 54). Thus, he concludes, "margin, type and order of size are the three stages in which continuous quantity reveals itself to our intellectual insight. They form the basis for a kind of number special to continuous quantity" (Van der Laan 1977, 55).

Van der Laan's experiments

The first experiment developed by Van der Laan to demonstrate the concept of margin of size consists of the division of a piece of paper without the assistance of measurement tools. Subjects were given a 50 cm long sheet of paper and asked to trace a line in order to divide it in two equal parts. He found that one part of the divided paper consistently turned out to be slightly larger than the other: "The estimated middle proves to be on average half cm out of true. The two pieces, estimated by eye to be equal in size, are thus in the proportion 25 ½ : 24 ½ , so they differ from each other by 1/25" (Van der Laan 1977, 48-49).

Thanks to this experiment, Van der Laan realized that the margin of size, or discrimination threshold, is not constant, but, in the case of the paper experiment, proportional to the length of the sheet. Therefore, if the paper were longer than 50 cm, the central line would have been placed so that the small, unperceivable difference from one side to the other would have been bigger. According to Van der Laan, a margin of size of 1/25 is just discernible by the eye; those below 1/50 are not (Van der Laan 1960, 24).

The paper experiment brought Van der Laan to the following conclusion: "This proves that in order to test experimentally how we sift the continuous sequence of sizes that confronts us, it will be enough to use a series of objects whose sizes increase by increments of 1/25" (Van der Laan 1977, 49). Following on observation, Van der Laan experimented with 36 squares made out of cardboard. He demonstrated that the concept of margin of size also applied to pebbles by using pebbles ranging in size differentials of about 4% (1/25) one from another (Padovan 1994, 99-100) (Figure 1).

In the 36 squares experiment, each square's length grew by 1/25 of the previous one, from 40 to 165 mm (Figure 2). The pieces were presented as two-dimensional objects by keeping the height below the same ratio (1/25), or about 2 mm. Subjects were asked to group the objects according to "a like size" (Van der Laan 1977, 49-53). Objects' measurements for the experiment were chosen so that the smallest square precisely equalled the difference between the two largest measures of the last two groups (Van der Laan 1977, 54). Subjects were asked to group all pieces starting from the smallest object (Figure 3). All these pre-imposed conditions brought Van der Laan to the definition of six groups, or types of size (later expanded to eight), each "numbering an average of seven sizes" (Van der Laan 1977, 52) (Figure 4).

A proportional series of threshold measures

In the square grouping experiment, Van der Laan related the concept of proportion to the human ability to typify sizes. He referred to proportion as a set of measures representative of groups of measures whose difference in size is just perceptible. This difference is the “margin of size.” Each group constitutes a “type of size.” The types of size all together constitute the “order of size.” Since each measure of each type of size grows by an equal fraction (1/25), and all groups were defined by Van der Laan’s experiments as comprising the same number of measures, a constant proportion links not only each single measure, but also all of the transition measures (Figure 5). The latter are the measures that are judged to be, at the same time, the smallest and the largest measures of two consecutive groups.

These measures, according to Van der Laan, “answer to an identical image of their size that we have formed in our mind” (Van der Laan 1977, 53). Therefore, they are “representative” of each type of size: the image we retain in our mind of the type they belong to, or the “inward visualization of size.” Van der Laan calls these measures ‘authentic measures’ (1/1; 3/4; 4/7; 3/7; 1/3; 1/4; 1/5; 1/7). By expanding the system from six to eight measures, the plastic number increased its additive properties. Indeed, the expanded system includes eight other measures that Van der Laan named “derived measures” (6/7; 2/3; 1/2; 2/5; 2/7; 2/9; 1/8). These measures are the harmonic mean between two consecutive authentic measures (Figure 6) (Proietti 2015, Voet 2016).

Authentic and derived measures are related by mutual proportions, within limits, or thresholds deriving from humans’ ability to discriminate size. Van der Laan summarized the thresholds of the plastic number system as follows:

> (a) Within the limits of a type of size, we call all concrete measures identical: there is as yet no question of proportion.
> (b) Within the limits of an order of size, the types of size can be compared with each other; here it is a question of proportion.
> (c) Beyond the limits of an order of size, no relation is any more possible between types of size; there can no longer be any question of proportion (Van der Laan 1960, 28).

According to Van der Laan, these limits are precisely definable. The first are the limits within which sizes grow by 1/25 and appear as equal (a). Within these limits, the question of proportion still does not emerge. The first threshold of the system is 1/25 and concerns size discrimination in linear extensions. The second regards the limits of the order of size. Within these limits, the transition measures of each type of size grow by 1/4 (b). This is the second threshold of the system, 1/4, and concerns aspect ratio discrimination. Here proportion is fully manifested. This threshold is significant in terms of three-dimensional objects and plays a bigger role in the experience of architecture. The last are the limits that expand out of the order of size. Within these limits, proportional relationships are no longer possible since measures are out of the system. Further proportional relationships are possible when successive orders of size grow by a constant ratio, which, according to Van der Laan, is 1/7, thereby expanding the system. The latter is the last threshold and concerns proportional relationships among parts at different scales in the built environment (Figure 6) (Padovan 2002).

As Padovan explains, "Psychologists estimate the smallest difference that can be distinguished by the eye when two sizes are compared directly as about 4% [1/25] of the sizes concerned. But the instantly recognizable difference that concerns Van der Laan is much larger, as we shall see: about 25% [1/4]" (Padovan 2015, 410). Indeed, 1/4 is the threshold that most interested Van der Laan. The following section of this article proposes drawing insight from perceptual psychology in order to study the perceptibility of proportion in architecture, starting from Van der Laan's 1/4 threshold.

Corresponding Psychophysical Concepts

Just noticeable difference

With the plastic number system, Van der Laan invited us to look at objects' proportion not in terms of fixed ratios, but in terms of types of size and their corresponding threshold measures. The concept of margin of size, or the just perceptible difference among sizes of a type of size, constitutes a core concept of Van der Laan's proportional system.

Van der Laan's margin of size has a correspondent definition in psychophysics known as just noticeable difference (JND). The JND refers to the amount of change that is needed in order to perceive a difference between one stimulus and another. If, for example, we imagine taking one sheet of paper and placing it into the hand of a blindfolded subject and then adding another sheet of paper, the subject will be able, in most instances, to detect the additional weight. If, however, we collect our sheets of paper in a book and then add a single sheet of paper, the subject will not be able to tell the difference. Even if the same increment was added, the magnitude of the original stimulus – or reference stimulus – determined the ability of the subject to detect the increment. In other words, if a physical stimulus of a certain intensity is presented to a subject, the stimulus must be changed by a certain increment relative to the original in order for the subject to perceive the variation. Therefore, “a greater stimulus increment is necessary for a stronger stimulus than for a weaker stimulus in order for the addition to be just noticeable as an increment, or to be equally noticeable” (Fechner 1860, 50).

The JND is expressed by the Ernst Weber’s law with the formula $\Delta S = kS$, wherein S is the reference stimulus, k is a constant that depends on the type of stimuli analysed, and ΔS is the difference between the compared stimuli. The ΔS, or the JND, refers to the minimum change in stimuli’s intensity that can be perceived and is also defined as the discrimination threshold (Baird and Elliot 1978, 25-34; Kingdom and Prins 2010).

In order to better illustrate how the concept of just noticeable difference might inform Van der Laan’s theory, it is helpful to recall experimental psychologists’ studies with particular attention to proportion discrimination.

In 1992, psychologists David Regan and Stanley Hamstra investigated humans’ ability to discriminate between shapes of different aspect ratios, defined as the relationship between the height (a) and the width (b) of two-dimensional rectangular or elliptical objects (a/b). In the first experiment, rectangles were computer generated and spaced asymmetrically from a central point of fixation. A series of aspect ratios from a horizontal bar-like shape to a vertical one, with the middle ratio being a square, were created [6.0, 3.3, 1.4, 1.0, 1/1.4, 1/3.3, 1/6.0]. In each set of trials, one rectangle served as the standard stimulus and the others as comparison stimuli. Generally, in such experiments, the standard stimulus stays constant while the comparisons change.

Subjects were asked to report whether the second rectangle (comparison) was greater or smaller than the first rectangle (standard). Different viewing distances were used in order to test how increasing or decreasing the area of the rectangles affected the discrimination task. Two buttons were provided for the answer: one standing for greater and the other for smaller (also known as “two alternative forced choice”). Stimuli were presented in random order. In psychophysics, this method of experimentation is called “method of constant stimuli” (Kingdom and Prins 2010, 13-14).

Data were collected through psychometric functions in order to show the change of JND across aspect ratios. The psychometric function shows clearly the change of value in JND over trials. The y-axis represents the JND, while the x-axis represents the aspect ratios (Figure 7). As shown from the V-shaped curve with the vertex marking the lowest JND, subjects, listed in rows, performed better for aspect ratios closer to 1:1. Namely, the increment, or JND, that was necessary in order to discern the difference from one shape to another was less for the square and increasingly more for the bar-like aspect ratio. With this experiment, Regan and Hamstra demonstrated that the JND changes according to aspect ratios and increases for more elongated ratios. This finding supports other experiments in the field arguing for the specialness of a 1:1 aspect ratio in discrimination tasks (Appelle and Goodnow 1970; Appelle and Gravetter 1980; Morgan 2005; Regan and Hamstra 1991).

By taking advantage of Regan and Hamstra's scientific methodology and findings, similar experiments could be conducted by using the plastic number aspect ratios. In order to test the 1/4 threshold of Van der Laan's system, the current author suggests using pairs of three-dimensional objects. Different aspect ratios, from the square to the bar-like shape, are compared. Bars are presented in both their vertical and horizontal orientation, as in Regan and Hamstra's experiments. Testing the threshold of the plastic number system over different aspect ratios – from a square to a bar-like shape, vertically or horizontally oriented – offers material to understand better the degree of perceptibility of specific proportions.

Another interesting aspect of proportion discrimination threshold was investigated by Regan and Hamstra in the same set of experiments. In order to ascertain whether subjects were instinctively using the width or height change in dimension as cue for the rectangle's aspect ratio, the same experiment was repeated by deleting the contours of the shape. More specifically, one experiment of the same series used as stimuli only rectangles' diagonals, without contours, while another experiment used only dots placed at the center of all sides of the rectangles. Due to the absence of rectangles' contours, subjects were forced to focus on angles and consequently picture the rectangle with its aspect ratios. Results were in close agreement with JND values collected in the first experiment (Figure 8). This may suggest that subjects tend to use imaginary lines, such as diagonals, as cue to rectangles' aspect ratio.

In this regard, the 1/4 threshold can be tested by abstracting surfaces of the three-dimensional objects with the use of rectangles' diagonals, minus contours, dots, or other defining elements. This may confirm that such cues can also assist the discrimination of aspect ratio in three-dimensional objects. This point is particularly relevant in addressing one of the biggest historical issues of the perceptibility of proportion, namely the possibility to discern proportion in the built environment through geometrical constructions or tracing lines.

Michael Morgan (2005) similarly addressed the question of how subjects discern different aspect ratios. As was done in Regan and Hamstra's experiments, subjects were asked to "compare the areas of two shapes with randomly-differing widths and heights" (Morgan 2005, 2565) and decide whether the comparison shape was larger or smaller than the standard shape. The methods of constant stimuli and two alternative forced choice were also applied to these experiments. Shapes were ellipses and rectangles presented on a grey background. A central fixation point defined the location of the standard and comparison stimuli. Width, height, and aspect ratio thresholds were measured concurrently in order to see if thresholds for aspect ratio perception was based on the relationship between height and width, rather than on one single dimension (height or width separately). In other words, whether proportion can be perceived "holistically" or whether its perception depends on objects' linear extensions was tested.

Results show how the discrimination threshold changes for the four parameters (Figure 9). The graphs show that the JND was better for width and height, but worsened especially for area discrimination. The tasks involving area discrimination resulted in higher JND, suggesting that observers tend to attempt different strategies by combining height and width in order to deduce the area of a shape. When the conditions facilitate this process (width and height changing by the same value and in the same direction) the threshold tends to decrease. This implies that determination of areas strongly depends on width and height discrimination and strategies of calculation (Appelle and Goodnow 1970; Appelle and Gravetter 1980; Nachmias 2008). In other words, our ability to discern shapes' aspect ratio depends heavily on our perception of their linear extensions (height and width). Morgan suggests that "the encoding of 3-D shape attributes, such as volume, is derived directly from 1-D measurement using a variety of heuristics, without involving an explicit 2-D intermediate" (Morgan 2005, 2570).

In the squares experiment, Van der Laan focused exclusively on the discernment of linear extensions by testing the 1/25 threshold, which he considered to be the closest to the continuous quantity of nature. According to him, the difference of two objects growing by 1/25 is just perceptible. Even though one can discern that the two objects are not identical, however, their difference is so small such that it is not possible to assign to each one of them a specific name. On the contrary, according to Van der Laan, the 1/4 threshold of the plastic number system allows for ascribing a specific aspect ratio to each element. We are able to register smaller differences at the level of linear extension, but the act of linking these differences to an "inner representation of form" emerges only at a larger threshold. As Morgan's experiments suggest, our judgment of linear extensions is very refined, while the aspect ratio judgment is in some way dependent upon the former. As a consequence, in order to test whether the 1/4 threshold results in better judgments in the discrimination of aspect ratios, subjects should be asked to perform the task not using any measurement technique (Appelle and Gravetter 1980).

Perceptual grouping

While Van der Laan's concept of margin of size finds its scientific analogue in the just noticeable difference, the concept of type of size strongly relates to Gestalt laws of perceptual organization. These laws developed from Max Wertheimer's pivotal text "Laws of Organization in Perceptual Forms" ((1923) 1938), which proposes a number of factors affecting perceptual grouping, including proximity, similarity, common fate, objective set, direction, closure, good curve, and past experience or habit. These grouping factors, both individually and collectively, determine how we perceive and understand groups of shapes. Among these many factors, similarity, or "the tendency of like parts to bend together" (Wertheimer (1923) 1938, 75), most closely characterizes Van der Laan's experiments.

As demonstrated by Wertheimer ((1923) 1938), the grouping factors interact with and, according to specific conditions, influence one another. For example, if we imagine organizing a series of circular shapes of the same diameter on a squared grid and change the horizontal spacing so that the vertical axes gain more or less space relative to one another, we notice how the vertical axes start grouping accordingly. The circles are no longer perceived as independent elements, but rather as being grouped by proximity. Similarly, if we take the same squared grid with its circles and replace both the central vertical and the first horizontal axes with void circles, we notice that the two axes in a T-like shape start clustering according to similarity.

Grouping by similarity can occur under various parameters such as color, size, form, luminance, orientation, among other attributes. The grouping can occur easily and intuitively, or with more difficulty according to the location of the objects, their mutual distance, or brightness, to give some examples. Factors can compete against one another by resulting in ambiguous grouping or can find their balance by reinforcing the perception of groups (Brooks 2014).

Experimental phenomenologists have studied quantitatively Wertheimer's principles and expanded upon them. It is important to introduce some examples in order to assist this revision of Van der Laan's plastic number system.

In the 1960s, Jacob Beck studied the effectiveness of differences in shape and orientation in perceptual grouping by similarity (Beck 1966, 1967). In 1967, stimuli used for the first experiment included 2-line figures in the shape of a T, inverted T, upright T rotated of 45º, a +, a + rotated of 45º, an L, and an L rotated 45º to form a V. In the second experiment, 3-line figures were used: an upright F, an F on its side, an F rotated of 45º, and an upright H. Each stimulus was repeated into a pattern. Two patterns were used, one standard pattern and one comparison (Figure 10). The standard pattern was constant, while the comparison changed lines-figures and orientations.

Subjects were asked to report when they observed the two patterns grouped and when they appeared as two different groups. Answers were given on a scale from zero to six, where zero indicated that the two patterns cannot be seen as separate and six signified that they appeared to be clearly two separate groups.

It was found that the rotation of 45º facilitated the grouping, but that the same rotation is less effective in the case of + figure rotated into the shape of an x. The experiment also demonstrated that not all figures composed of vertical and horizontal lines were effective in the same way for the grouping. The backward L resulted in grouping better than the other figures with vertical and horizontal lines. This may suggest that the right angle facilitates grouping (Figure 11).

Beck demonstrated that line orientation is a significant variable in perceptual grouping. In particular, he determined that, “if the orientation of a figure’s line are changed to 45º and 135º, perceptual grouping relative to figures with vertical and horizontal lines will be improved” (Beck 1967, 494). Similar results were found by Richard Olson and Fred Attneave, who, like Beck, concluded that slope is an effective major grouping variable (1970).

In the squares experiment, Van der Laan did not establish rules for objects’ placement, distance, and orientation. Objects were randomly placed. Imagining a more refined and controlled experiment, objects could be showed in pairs. This would allow one to focus fully on similarity and consequent groups of size. The control of objects’ orientation is especially significant for the 1/4 threshold and three-dimensional objects. Indeed, different orientations of objects in space may strongly alter the capacity of judgment.

Interactions between similarity and proximity in grouping tasks is another significant aspect of perceptual organization that further advance the present analysis. Mercedes Ben-Av and Dov Sagi’s experiments (1994) have tested the effects of similarity in grouping tasks by combining similarity variables with proximity. Stimuli used for the experiments were elements in the shape of an X or L arranged on an array of columns and rows. The Xs and Ls were composed of perpendicular segments of equal length. The elements were arranged with different orientations. Spacing among elements and their luminance varied across trials. Parameters of proximity (spacing) and similarity (shape and luminance) were investigated in order to understand how they might interfere in the grouping task. Two stimuli were used for all experiments: the uniform stimulus (X-shaped elements on all rows and columns) and the combined stimulus (X shapes on odd columns or rows, and L shapes on even columns or rows).

In the first experiment, Ben-Av and Sagi varied the distance between rows and columns and kept the luminance fixed. Nine spacing ratios were used. In the second experiment, they kept the distance between elements fixed and changed the luminance across seven ratios. Luminance variations were displayed on alternating columns with the shape variations on alternating rows. In the third experiment both parameters varied at the same time. Subjects were asked to report whether the perceptual grouping was by horizontal rows or vertical columns and to press the zero button on a keyboard for horizontal grouping and the one button for vertical. Subjects were exposed to each stimulus for 60 msec and 160 msec. Stimuli were presented with or without a mask (Figure 12) used to transform the main pattern.

The graphs in Figure 13, for Experiment 1, show how the judgment of vertical grouping (y-axis), in both the uniform (marked with an x) and combined stimulus (marked with a void rhombus), changed according to distance ratios (x-axis). Listed in rows are showed timing and mask trials: 60 msec, 160 msec, and no application of mask. In the 60 msec trials, the two curves for uniform and combined stimulus are very close and almost touch, meaning that the effect of similarity is minimal and that proximity informs the judgment. In other words, the more that the vertical columns are spaced out, the more that the subjects perceive them as groups of elements vertically oriented, even when the pattern is composed by similar objects on alternate rows. With the 160 msec trials, the curves start slightly to detach and the grouping by similarity starts to affect the judgment. Indeed, the researchers' findings show that "perceptual organization is time dependent" and that "proximity grouping can be perceived much faster than similarity (shape or luminance) based grouping" (Ben-Av and Sagi 1994, 861). With increasing processing time, grouping by similarity was demonstrated to be the dominant parameter.

In his experiments, Van der Laan had asked participants to perform the required tasks as quickly as possible. He was aware that timing affects experiments' results. Nonetheless he did not define the time of exposure to stimuli. As demonstrated by Ben-Av and Sagi, giving a specific time frame guarantees that all subjects are exposed to the stimuli for the same period, resulting in more reliable data. In addition, as demonstrated by Ben-Av and Sagi, proximity can have an influence on similarity according to timing. In order to address this point, as mentioned above, the use of pairs of objects can allow subjects to focus on the similarity among aspect ratios and reduce the interaction with the factor of proximity.

The perceptibility of proportion: a first psychophysical experiment

With the concept of margin of size and type of size, Van der Laan combined the investigation into discrimination threshold with perceptual grouping. In his grouping experiment, he questioned not only the similarity among squares, but also the degree of similarity. In other words, he investigated how sizes group into families by similarity and determined to what degree members of the groups are similar one to another. Heretofore, scientists had typically separated the two issues.

Having compared the plastic number theory with basic scientific concepts of psychophysics and Gestalt theory, one can see how the latter fields of experimentation may inform Van der Laan's proportional theory. This view suggests the need for a broader investigation into the role of proportion in architecture, one for which Van der Laan's approach has provided inspiration and laid a solid foundation.

In light of the discussed scientific findings, an interdisciplinary team comprising an architect (the author of the present article) and a scientist have launched a psychophysical study aiming to measure perceptibility of proportions of three-dimensional objects, starting with the plastic number system (Proietti and Gepshtein 2020). The objects used in this work are sampled from Van der Laan's proportional tool called the "morphotheek." The morphotheek consists of 120 solid objects making up a pyramid extruded in three dimensions following the plastic number authentic measures. As described, each measure grows by 1/4. The objects are divided into 60 "blocks," 20 "bars," 20 "slabs," and 20 "white forms" (Van der Laan 1977: 126) (Figure 14).

In a typical experiment as part of this study, pairs of objects are presented to participants on a square grid covering a horizontal surface. At each location on the grid, the objects are positioned at one of several orientations. The different locations and orientations of objects produce the desired range of perspectival distortion, allowing researchers to test how proportion is perceived under a wide range of conditions.

In a two-alternative forced choice procedure, participants are asked to compare aspect ratios of morphotheek objects in monocular and binocular viewing conditions, while a mechanical shutter controls the duration of viewing. This way, the threshold of discrimination between proportions is determined under perspective distortion while the compared objects belong to the same of different types of size (as defined by Van der Laan's system). Subjects are asked not to use any measurement technique to assist their judgment.

This experiment aims to study the perceptibility of proportion in three-dimensional volumes under the conditions of perspective distortion by starting from the

1/4 threshold of the plastic number. It is asked whether this threshold gives the necessary margin of difference from one aspect ratio to another to be discriminated. Solid objects are used to simulate the conditions of perception of proportion in space. Different orientations result in different perspective distortions in order to study how they may affect the judgment of similarity.

The data collected in this study will be used as the basis of broader research into alternative roles of proportion. After understanding which proportions are discriminable and under which spatial conditions, future research will investigate how proportion can affect movement in space and, therefore, walking pace, awareness of location in space, among other experiential components. Subsequent experiments will involve moving objects and moving subjects in both virtual and physical environments. Results could, for example, reveal that proportional systems may facilitate the perception of the structure of the built environment and thus help to mediate architectural affordances. When applied correctly and intentionally, specific proportions may facilitate wayfinding, help visitors gain the sense of the rhythm of the built environment, and thus adjust the pace and speed of their movement, along with improving other aspects of visitors' well-being.

In summary, the aim of this new study is to analyse proportion from the perspective of spatial awareness and human behaviour, beginning with the human aptitude to discriminate aspect ratios. Beyond aesthetic and symbolic interpretations of proportion, the analysis and the example of a psychophysical experiment presented in this paper suggest numerous benefits of employing the scientific method in order to reinvigorate the debate on the role of proportion in architecture by bringing attention to how we perceive proportion and, consequently, how it can inform our movement and behaviour in the built environment.

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