# DIGITAL COMPUTER SIMULATION OF THE MECHANICAL AND THERMAL RESPONSE OF COMPLEX HYDRAULIC SYSTEMS

By

LYNN ROYCE EBBESEN

Bachelor of Science
South Dakota School of Mines and Technology
Rapid City, South Dakota
1970

Master of Science
Oklahoma State University
Stillwater, Oklahoma
1972

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of the Oklahoma State University
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Thesis Approved:

Henry R Lebesta
Thesis Adviser

K. M. Reid

Jerald D Parker

Ronald D Rhota

J. E. Bose

Norman N Durham
Dean of the Graduate College

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NOMENCLATURE

$A$ - Wall area normal to the Z direction ($in.^2$)

$A_i$ - Area at the inside surface of a component wall ($in.^2$)

$a$ - Acoustic velocity of the hydraulic fluid (in./sec.)

$a_1, a_2$ - Coefficients in the assumed wall temperature function

$c$ - Specific heat of the component wall ($Btu/(lb_m °R)$)

$c_p$ - Specific heat of the hydraulic fluid at constant pressure ($Btu/(lb_m °R)$)

$c_v$ - Specific heat of the hydraulic fluid at constant volume ($Btu/(lb_m °R)$)

$c_1, c_2, c_3$ - Coefficients defined in the solution for the wall temperature

$D$ - Denominator of the fluid temperature transfer function

$E$ - Error in satisfying the conduction equation after substituting the assumed wall temperature function

$E_F$ - Flow energy of a fluid stream (Btu)

$E_I$ - Internal energy of a fluid stream (Btu)

$E_M$ - Mechanical energy into or out of a component (Btu)

$E_S$ - Internal energy of the fluid in a component (Btu)

$E_W$ - Energy transferred from the fluid in a component to the component wall (Btu)

$e$ - Base of the natural logarithms

$f$ - Darcy-Weisbach friction factor

$g_c$ - Gravitational constant ($386.04\ in.\ lb_m/(sec.^2\ lb_f)$)

$h_i$ - Convection coefficient at the inside component wall surface (Btu/(sec. in.$^2$°R))

$h_o$ - Convection coefficient at the outside component wall surface (Btu/(sec. in.$^2$°R))

$I_0, I_1$ - Integrals appearing in the approximate solution of the wall conduction equation

J - Conversion constant (9336 in. $lb_f$/Btu)

k - Thermal conductivity of the component wall (Btu/(sec. in.°R))

$k_f$ - Thermal conductivity of the hydraulic fluid (Btu/(sec. in.°R))

$\ell$ - Transmission line length (in.)

$\dot{m}$ - Mass flow rate ($lb_m$/sec.)

$m_f$ - Fluid mass within a component ($lb_m$)

N - Numerator of the fluid temperature transfer function

$N_P$ - Prandtl number

$N_R$ - Reynold's number

$O_N$ - Order of the numerator of an approximate fluid temperature transfer function

$O_D$ - Order of the denominator of an approximate fluid temperature transfer function

P - Pressure ($lb_f$/in.$^2$)

$P_D$ - A dependent variable at a component port (exclusive of temperature variables)

$P_D^T$ - Dependent temperature at a component port (°R)

$P_I$ - An independent variable at a component port (exclusive of temperature variables)

$P_I^T$ - Independent temperature at a component port (°R)

Q - Volumetric fluid flow rate (in.$^3$/sec.)

$q_w$ - Rate of heat transfer from the hydraulic fluid to a component wall (Btu/sec.)

$r$ - Radial dimension of a transmission line (in.)

$r_i$ - Inside radius of a transmission line (in.)

$r_o$ - Outside radius of a transmission line (in.)

$S$ - A discrete state vector

$s$ - The Laplace variable

$T$ - Temperature (°R)

$T_e$ - Environmental temperature (°R)

$T_f$ - Hydraulic fluid temperature (°R)

$T_i$ - Inside component wall surface temperature (°R)

$T_o$ - Outside component wall surface temperature (°R)

$T_{inlet}$ - Hydraulic fluid temperature at the inlet of a transmission line (°R)

$T_p$ - Temperature of the fluid at a port (°R)

$\Delta T$ - Temperature difference (°R)

$t$ - The independent variable, time (sec.)

$\Delta t$ - The integration time step (sec.)

$U$ - Overall heat transfer coefficient (Btu/(sec. $in.^2$ °R)

$u$ - Hydraulic fluid internal energy as a function of pressure and temperature (Btu/$lb_m$)

$V$ - Fluid velocity (in./sec.)

$v$ - Volume of wall element ($in.^3$)

$w$ - Unknown function of time in the assumed wall temperature function (°R)

$\dot{w}$ - Derivative of w with respect to t

$X$ - Continuous state vector

$x$ - Axial position along a transmission line (in.)

$Y$ - An algebraic vector of unknowns or Young's modulus ($lb_f/in.^2$)

$Z$ - Component wall coordinate normal to the outside wall surface (in.)

$\Delta Z$ - Component wall thickness (in.)

$\alpha$ - Coefficient of thermal expansion ($°R^{-1}$)

$\beta$ - Hydraulic fluid compressibility ($lb_f/in.^2$)

$\Gamma$ - Normalized heat transfer coefficient

$\gamma$ - Component wall thermal diffusivity ($in.^2/sec.$)

$\eta_i$ - Mechanical input efficiency

$\eta_o$ - Mechanical output efficiency

$\rho$ - Density ($lb_m/in.^3$)

$T$ - Normalized temperature

$T_{inlet}$ - Normalized line inlet temperature

$\tau$ - Normalized time

$\tau_{rx}$ - Shear stress normal to the r direction acting in the x direction ($lb_f/in.^2$)

$\tau_w$ - Wall shear stress ($lb_f/in.^2$)

$\tau_{xx}$ - Normal stress in the x direction ($lb_f/in.^2$)

$\chi$ - Normalized axial coordinate in a transmission line

$\omega$ - Radian frequency ($sec.^{-1}$)

# CHAPTER I

## INTRODUCTION

Digital simulation of continuous dynamic systems has developed in conjunction with the modern digital computer. Such simulation requires a digital computer of sufficient size and computation speed, a mathematical description of the dynamic system, and appropriate algorithms to allow the solution of the system response to be calculated. The size and computation speed of the computer influence the practicality of simulating the response of any particular dynamic system. The physical characteristics of the system to be simulated indirectly determine whether available models and algorithms are appropriate. If current models and algorithms are not applicable, the investigator must either develop the missing portions or not perform the simulation.

This study is concerned with the simulation of the transient mechanical and thermal response of hydraulic systems. The term mechanical response is used to denote the pressure, flow, velocity, and position responses of the system components. Thermal response includes the temperature of both the hydraulic fluid and the system components as well as heat transfer effects. The study considers hydraulic systems to be constructed of interconnected components which are each modeled individually. This approach allows component models to be developed without any assumptions which restrict the configuration of hydraulic systems which can be simulated.

The results of this study are directly applicable to the simulation of hydraulic power systems. This includes systems such as those used on aircraft, mobile equipment, and agricultural tractors. The capability developed in this study makes it possible to investigate hydraulic system responses during warm-up, in the presence of extremely high temperatures, or with part or all of the system operating in a cold environment. The above mentioned systems and operating conditions are only examples which indicate the variety of conditions to which this study applies. It is the intention that any hydraulic power system should be able to be analyzed using the model forms and algorithms developed herein.

Systems other than hydraulic power systems can also be considered. As an example, the fuel delivery and injection system on a diesel or gasoline engine could be studied with the techniques that have been developed. Also, simulations of the transient pressure, temperature, and flow response of pipe networks can be performed. Networks which can be simulated include, for example, residential and commercial hot water delivery systems and long-distance liquid transmission lines such as those used to transport petroleum products. In general, the results are applicable to non-power systems which are characterized by transient liquid flow through transmission lines.

## Objective and Scope of Study

The objective of this study was the development of mathematical models and numerical algorithms for the digital simulation of the thermal response of hydraulic systems which do not operate at constant temperature. It was assumed that electrical, mechanical, and pressure/flow models were available and thus only thermal response models needed to be

developed for the hydraulic system components. (Multiport component models which could be inter-connected to form a system model were sought.) The equation forms which were allowed were sets of first-order, ordinary differential equations, sets of first-order difference equations, and sets of algebraic equations. (Algorithms were developed to allow the component models to be used in a system simulation program which had the capability to predict combined mechanical, electrical, thermal, and pressure/flow transient responses.)

The study also included the development of a prototype digital program used to demonstrate the models and to verify the algorithms. The program was designed in a manner that allowed the computer storage requirements to change with the number of components being modeled. A sparse matrix approach was investigated for use in solving the algebraic equations in a system model. The program structure used included the capability to accept any new component models which could be expressed in the above mentioned forms.

The prototype routine was the second portion of a conceptual program which utilizes a preprocessor to interpret user-supplied input data. The investigation included the design and specifications for the preprocessor but did not include the programming of the actual routine.

## Major Results

The most significant result of this work is the development of the capability to predict the combined thermal and mechanical response of hydraulic systems. (The developments in this study which make the prediction possible include contributions in both system modeling and simulation algorithms.) The models and solutions which are attributable to

this study are a transmission line model with both an operational mathematics and method of characteristics solution, a one-dimensional thermal wall model with a solution utilizing the heat balance integral, and a thermal response model for general hydraulic components.

There are three algorithmic developments in this study which are most significant. The first is the development of an algorithm for defining temperature at a component port such that the flow direction is not defined a priori. The second development makes it possible to utilize a sparse matrix solution for algebraic equations in a system model. An important result of the sparse matrix approach is the ability to use a very simple syntax within individual component model routines. The third contribution is the specification of a preprocessor and simulator approach for the simulation of hydraulic systems. The approach makes it possible to describe systems as coupled components and to develop new component models which can be implemented with very little effort. A distinguishing feature of the preprocessor and simulator approach developed herein is that it is component oriented as contrasted to being equation oriented.

## Plan of Presentation

A summary of the literature reviewed for this study is presented in Chapter II. Thermal response models and solutions appear in Chapter III with additional details in Appendices A and B. The inclusion of thermal response in a system simulation is considered in Chapter IV where an approach is developed to allow fluid temperature at a component port to be considered in a general manner. The response of the thermal models is demonstrated by two examples in Chapter V. One example consists of a

step input to a transmission line and the second is a small hydraulic system. The conclusions and recommendations appear in the final chapter. Appendices C and D contain programming details with the first considering the preprocessor and simulator approach to simulation and the second dealing with the use of a sparse matrix equation solver within a simulation program.

# CHAPTER II

## LITERATURE SURVEY

Digital simulation of hydraulic systems requires an understanding of both the physical system and the algorithms used by the computer program. The literature survey presented in this chapter contains two sections dealing with the phenomena of mechanical and thermal response and one section concerned only with algebraic equation algorithms. Both the modeling and the solution of the physical responses are considered in the review.

In brief, the following literature survey reveals that:

1) there is no existing hydraulic system simulation program designed for or well-suited for the inclusion of dynamic thermal response models;
2) transient heat transfer and temperature response investigations have considered components similar to those found in hydraulic systems (for example, transmission lines have been studied extensively) but no method for investigating entire systems is available; and
3) sparse matrix methods are used in structural analyses but have not been extensively applied in other types of dynamic system simulation.

## Mechanical Response

The determination of the response of a hydraulic circuit is a problem in continuous system simulation. Several programs have been developed which could be used to model or simulate a hydraulic system if one wishes to work with the individual component equations. A review of the various programs has been presented by Smith (1). These programs have been developed independent of any particular type of dynamic system, and therefore contain no specialization for hydraulic systems. The most significant programs are MIMIC, DSL/90, and CSMP/360 (2,3,4). Several electrical circuit analysis programs which have been developed are reviewed by Murali (5). The better known programs are CIRCUS, ECAP, NET, and SCEPTRE. The programs are specialized for electrical circuit analysis and are not well suited for adaption to hydraulic circuits.

A major study of available hydraulic system simulation programs was conducted by Boeing Commercial Aircraft (6) for the Air Force in 1972-1973. The investigation identified five transient mechanical response programs including HYTRAN (Boeing), HYTRAN (McDonnell), LQST1 (NASA-University of Georgia) (7), TS7 (General Dynamics), and HYDSIM (Oklahoma State University (1). Of the five, the only program with the capability to handle algebraic sets crossing component boundaries is HYDSIM. An improved version of the McDonnell HYTRAN is currently being developed under contract for the Air Force (8). This program is very strongly dependent on the method of characteristics line model (9,10,11,12), and uses the properties of the model to avoid algebraic equation sets.

The general programs MIMIC, DSL/90, and CSMP/360 utilize a preprocessor and simulator execution structure. Each program accepts input in a specific format and creates computer code to be used by the

simulator. The MIMIC processor reads user input and creates a machine language subprogram which is used during the actual simulation portion of the execution. The CSMP/360 preprocessor translates the user input into a Fortran subroutine which is then compiled and used by the CSMP/360 simulator. The approach used in DSL/90 is similar to CSMP/360 except that the program also provides a portion of the job control statements required for its own execution.

## Thermal Response

The Boeing investigation identified only one program, HEATEV (13,14), designed for hydraulic system thermal analysis. The program performs steady-state analysis using lumped parameter models. The same study also identified several "fluid system analyzers" and "thermal analyzers", none of which were concluded as being suited to hydraulic system thermal response studies. The "fluid system analyzers" are designed for use in steady-state environmental control system analysis, and the "thermal analyzers" are intended for problems involving the temperature distributions in structures. McDonnell is currently under contract to develop a hydraulic system thermal analysis program under the same contract as the HYTRAN extensions (8). No results have been published, and it is expected that the program will have the same restrictions as HYTRAN and will perform only quasi-transient analysis.

Heat transfer to fluid flowing in a pipe has been studied by many investigators. Numerous papers deal with determining the Nusselt number for steady laminar or turbulent pipe flow. Work in this area has been reviewed by Fand (15), Hughmark (16), Kalinin (17), and Petukhov (18). The most basic result is the determination of the Nusselt number as a

function of the Reynolds number, Prandtl number, and fluid temperature. Steady-state pipe flow and heat transfer has also been investigated analytically. Problems of this type are termed a Graetz problem due to the fact that Graetz presented the first analytical solution for the laminar case in 1885 (19). The works of Chung (20), Dunduchenko (21), Gaertner (22), Siegel (23), Thomas (24), and Tseng (25) are representative. The works generally consider specified inside wall temperature or heat flux, no axial conduction, and either steady solutions or approximate transient solutions following step inputs.

Analytical investigations involving heat transfer and pipe flow which consider the wall conductivity have been reported by Forghieri (26), Hayasi (27), and Mori (28). All of these studies considered a thin wall, a steady parabolic or uniform velocity profile, and specified outside wall conditions. Approximate analytical solutions are found for the steady temperature profile (28) and for step inputs (26,27).

The earliest work applicable to transient pressure, flow, and temperature was presented by Dussinberre (29). The approach consisted of a difference equation model for lumped fluid elements in a line. A method for calculating transient pressure, flow, and temperature was developed by Benson (30). A method of characteristics solution was used and applied to the flow of gas through a super-charged two-cycle engine. Equivalent methods have been used by Wright (31), Kot (32), Issa (33), Jonsson (34), and Kawahashi (35). All of the solutions consider a compressible fluid and use the equation of state for a gas. No equivalent investigation for liquids has been located.

Heat transfer during pulsating or periodic flow has been considered by several authors. Siegel (36,37) has reviewed the early work in the

area and presented an analytical study. The experimental work is often in conflict with investigators reporting both an increase and decrease in the Nusselt number for pulsating flow as compared to steady flow for the same average velocities. The results by Siegel indicate that for pulsations not involving flow reversals, there is only a slight increase in the average heat transfer due to pulsations. Jenkins (38) extended the work of Siegel to the circular tube with the same general results. Experimental studies by Galitseyskiy (39) and Keil (40) show conflicting results typical of those reviewed by Siegel. Thomas has considered the problem from a turbulent flow point of view, and the results indicate an increase in heat transfer due to pulsations (41). Flow reversals have been considered by Niida with even larger predicted increases in total heat transfer (42). The experimental studies have shown the pulsatile Nusselt number to differ from the steady value by a factor of 0.7 to 2.0. Theoretical predictions range from 1.0 to 10.0 depending on pipe location and pulsation amplitude and frequency.

Heat transfer involving irregular geometries must be considered in the analysis of such components as valves, pumps, orifices, etc. Several authors have presented methods for determining equivalent heat transfer coefficients (43,44,45,46,47). Recent conduction heat transfer research is largely oriented toward integral methods which are directly related to the finite element method popular in structural analysis (48). A review of the basic approaches has been presented by Goodman (49) and Weiss (50). The three approaches used are the heat balance integral by Goodman (50,51), a variational method by Biot (52,49), and the more general method of weighted residuals (53).

## Algebraic Equation Sets

The construction of the algebraic equation sets related to a particular fluid power circuit has been accomplished by Smith (1). Smith's HYDSIM program uses either a Newton-Raphson or a closed-form differential solution to solve the algebraic equations. An alternate applicable method has been presented by Powell (54). All of the methods require the repeated solution of a set of linear equations which will often be sparse, i.e., the coefficient matrix contains a relatively large number of zero values. Mondkar (55) and Wilson (56) have presented methods for solving sparse equations when the coefficient matrix is banded, symmetric, and positive-definite. The more general method of Key (57) is more applicable to fluid power circuit analysis. Coefficient matrix storage algorithms are used by all of the mentioned methods. DeVilliers has presented a discussion and evaluation of several hashing methods for coefficient storage (58). The paper concludes that there is no significant reason why the non-zero terms in the coefficient matrix must be stored in any sequential pattern but rather they need only be stored in a deterministic manner.

# CHAPTER III

## DEVELOPMENT OF THERMAL RESPONSE MODELS

The total response of a hydraulic system component includes both the mechanical and thermal response. Mechanical response implies the position, velocity, force, torque, pressure, or flow response of the component. Thermal response, as used in this study, describes the change in temperature of the hyraulic fluid and the component mass which occurs simultaneously with the mechanical response. An alternate definition of thermal response could be stated in terms of the change in internal energy which occurs in the hyraulic fluid and in the material of the component walls. If one assumes that internal energy depends only on temperature, then either internal energy or temperature can be used in describing thermal response. This assumption is made in this study, and, in general, thermal response is expressed in terms of the more convenient temperature rather than internal energy.

This chapter defines the type of block-oriented thermal response models sought in the study. Requirements of the block-orientation are considered to ensure the applicability of the models to a system simulation. Thermal models are then developed for the fluid mass both in general hydraulic system components and for fluid transmission lines. The mathematical model for the component mass is developed in the last section of the chapter.

## Multiport Component Model Considerations

The thermal response models developed in this study are intended for implementation in a block-oriented simulation program. This requires that components be modeled in terms of the physical variables which are defined at the component ports (1,59,60,61). (These variables are called port variables where "port" refers to the points at which components are interconnected.)

A multiport representation of a system from the thesis by Smith (1) is shown in Figure 1. The figure contains three components with two ports each, and one component, the valve, which contains three ports. The port variables for classical multiport component models are chosen in one of two ways. One method defines port variables such that the product of the two variables at a port equals the rate at which energy passes through the port. This convention is followed for each of the six pairs of port variables in Figure 1 except the force/stroke port for the valve. The product of these two variables is the net energy which has crossed the port and is an example of the second method for selecting port variables.

The arrows in Figure 1 define the causality of the port variables. At each port there is one arrow pointing toward the component with an associated independent port variable. (When the component model is developed, the independent port variables are treated as inputs which will be defined "independent" of the component being modeled. The arrow pointing away from each port has an associated dependent port variable which must be defined by the component model. These variables are the outputs of the component model and can be thought of as being "dependent" on the model.)

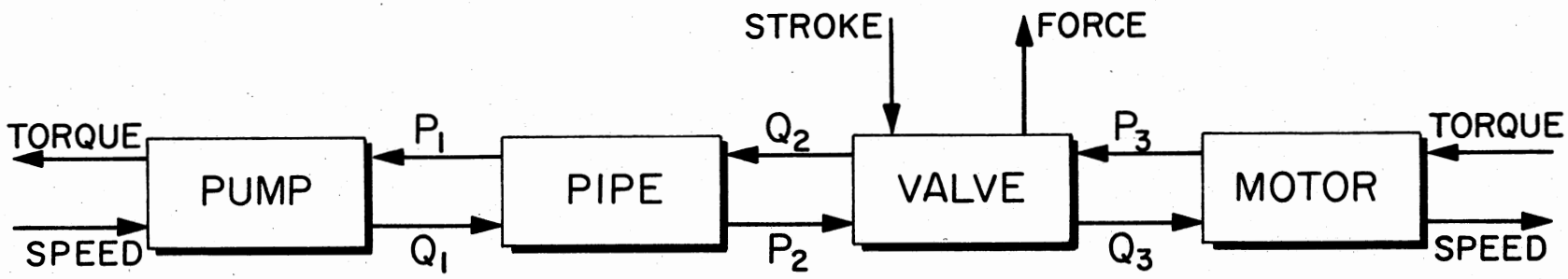


Figure 1. Multiport System Representation

The coupling of component models to form a system model causes the independent and dependent variables at connected ports to be interrelated. In Figure 1, $Q_1$ is dependent with respect to the pump and independent with respect to the pipe. Similarly, $P_1$ is dependent and independent contingent on the reference component. The variables are, in general, functionally related due to port coupling. For example, if the pipe model in Figure 1 defines $P_1 = f(Q_1)$ and the pump model defines $Q_1 = g(P_1)$, then $(P_1, Q_1)$ must be determined simultaneously. This example demonstrates that a port variable can be independent with respect to one component, dependent with respect to the connected component, and unknown with respect to the system model. The algorithms and equation forms necessary to allow a correct simultaneous prediction of the system model response for nonlinear models were defined by Smith (1,59). Rosenberg has defined the requirements for linear models described in terms of basic elements and sources (60,61).

Hydromechanical systems can be modeled in terms of five port variable pairs which are (pressure, flow), (force, position), (force, velocity), (torque, angular position), and (torque, angular velocity). (The addition of electrical components requires defining (voltage, current) ports.) If thermal effects are considered, two additional port types must be defined to include the effect of heat transfer and to include the internal energy in a fluid stream and a wall mass.

From standpoint of dynamic simulation only P-F, F,V T,a-v.

## Heat Transfer Ports

A heat transfer port can be described with entropy rate and temperature as the port variables (62). The product of the two variables will then be the heat transfer rate which is the rate at which thermal energy

passes through the port. Entropy rate is analogous to flow rate, force, and current, and temperature can be compared to pressure, velocity, and voltage.

Even though the variables constitute a port description which is conceptually acceptable, entropy rate and temperature are not attractive for use in component modeling. It is desirable to select a pair of variables which are in common engineering usage and which are readily useful. A port assigned the variables heat transfer rate and temperature would be convenient from an applications viewpoint. Each port variable would represent a quantity which is of interest in the determination of the thermal response of a hydraulic system. In practice these variables describe the heat rejected (or gained) by the component and the port temperature.

A conceptual difficulty with using temperature and heat transfer rate is that the product of the two is not an energy or power term. However, this does not impair the usage of such ports in block-oriented simulation programs unless the program requires the product be energy or power. The prototype program discussed in Chapter IV as well as those developed by Smith (59) and McDonnell Aircraft (8) have no such requirement. Therefore, heat transfer ports appearing in the models developed herein will use heat transfer rate and temperature as the port variables even though these variables are not analogous to other port designations.[1]

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[1]This exception for energy due to heat transfer also exists in system models as presented by Shearer, et al. (62).

Internal Energy Ports

The internal energy of fluid flowing through the hyraulic fluid ports of a component must be included in thermal response models. An acceptable internal energy port can be defined by selecting the mass flow rate and the internal energy of the fluid per unit mass as the port variables. The product of these two quantities yields the energy rate, and the port is analagous to mechanical or electrical ports.

Fluid temperature and mass flow rate would be more practical for use as port variables. The fluid temperature is of primary interest in analysis of the thermal response of the fluid and is equivalent to internal energy given the specific heat. Therefore, all internal energy ports appearing in this study will use temperature and mass flow rate as the port variables.

Internal energy is a property of the fluid entering and leaving a hydraulic component, and an internal energy port exists if and only if there is a corresponding pressure and flow port. These two facts imply that internal energy ports cannot be modeled independent of a pressure and flow port. It is also implied, although not obvious, that the selection of independent and dependent port variables cannot be done arbitrarily. The following example indicates the requirements of internal energy port modeling.

Consider three components as shown in Figure 2. Component 2 is a rigid, adiabatic line with no thermal capacitance in the line wall. Components 1 and 3 can be thought of as any components having a pressure and flow port with the flow specified as mass rate. The variables $P_1$ and $P_2$ have been specified as being independent for the line model requiring that $\dot{m}_1^P$ and $\dot{m}_2^P$ be dependent at the pressure and flow ports. This

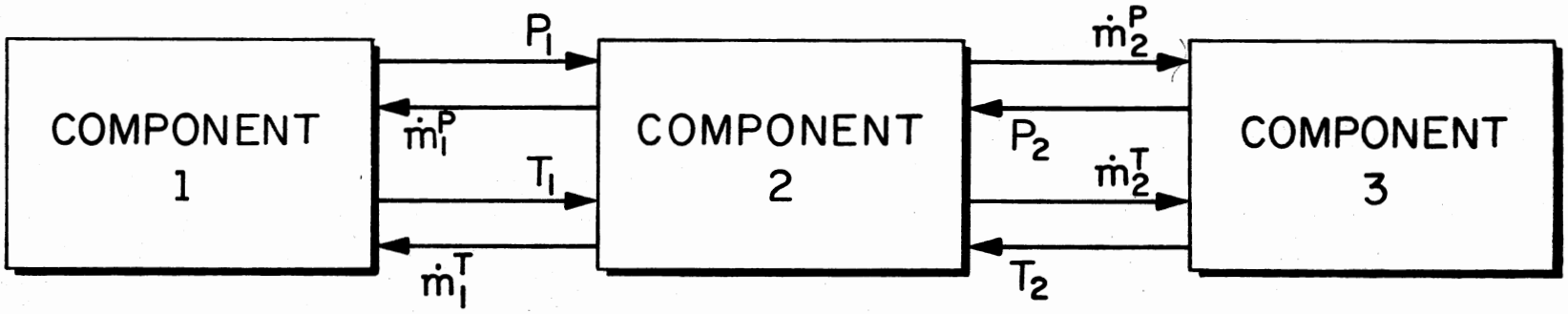



Figure 2. Pressure-Flow and Temperature-Flow Multiport Representation

selection is arbitrary, and either P or $\dot{m}$ could be dependent at either port.

Mass flow rates used to model the pressure and flow response of a line are the same as the mass flow rates used to determine the internal energy at the ports. That is, $\dot{m}_1^P = \dot{m}_1^T$ and $\dot{m}_2^P = \dot{m}_2^T$ in Figure 2. Also, it is implied that the causality of mass flow rate at an internal energy port is determined by the causality of pressure. If one then considers that each port will have a dependent and independent variable, it would seem that the causality of temperature should be the same as that of pressure. Thus Figure 2 might appear to be a properly represented diagram, in which the mass rates could simply be represented as $\dot{m}_1$ and $\dot{m}_2$ with no superscripts to indicate the associated pressure or temperature.

The causality specified in Figure 2 does not indicate either the sign convention for mass flow rate at a port or the instantaneous direction of mass flow rate. As a first case, one can assume that mass is flowing from component 1 to component 3. The line receives the flow $\dot{m}_1$ at temperature $T_1$ and discharges fluid at $T_2$. The line exit temperature $T_2$ depends on the line inlet temperature $T_1$ plus an increase due to frictional losses in the adiabatic line. Component 3 receives fluid at temperature $T_2$ and cannot influence this temperature except indirectly through $P_2$ and the associated frictional losses in the line. It must be realized that a component receives fluid mass and its associated internal energy by virtue of the direction of flow. The fluid property internal energy cannot be specified by the component physically receiving the flow. It follows that a component model can only specify the temperature of mass leaving the component; and thus, temperature can only be dependent at a port if fluid is leaving the port.

The dependence of $T_2$ as shown in Figure 2 contradicts the requirement that $T_2$ be dependent on the line model if flow is into component 3. The dependence would be correct as shown if fluid was entering both ends of the line, which is possible for a compressible fluid or elastic line. It is also quite possible for flow to be leaving both line ports simultaneously or for flow to be from component 3 to component 1. Since all of these conditions may occur within a single simulation, it is not possible, in general, to specify the causality of port emperatures a priori. It must rather be realized that port temperature causality changes with the direction of flow and models and programs must be developed in such a way that the dependence of port temperature can change accordingly. The models and program developed in this study satisfy this requirement.

Pressure and flow ports which have an associated temperature can be compactly represented as in Figure 3. The suggested notation indicates the causality of pressure and flow in the conventional manner. Temperature causality is shown with the double arrow, indicating both possible dependencies. The presence of only one mass flow is consistent with the physical phenomena being represented. The notation in Figure 3 is used in the remainder of this study.

## Hydraulic Fluid Model for General Components

Thermal response models developed in this work are based on an unsteady energy balance. An energy balance is performed on the fluid mass within a component and on the mass of the component itself, with the allowance for heat transfer between the two masses. The physical quantity being modeled is the internal energy of the masses which is represented in the models by "bulk" temperature. This subsection develops

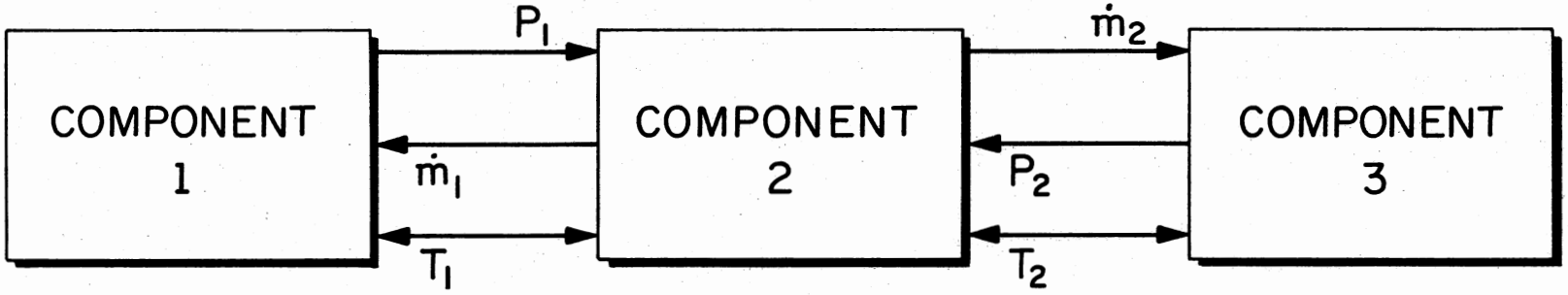



Figure 3. Condensed Pressure, Flow, and Temperature Multiport Representation

the fluid model for hydraulic system components other than transmission lines. Lines are considered separately due to the axial variation in temperature which may exist.

A hydraulic fluid model which includes the known effects should consider internal energy as a function of three spatial variables and one temporal variable and should also contain a detailed description of all geometries and associated boundary conditions. Such a model would not be suitable for use in a hydraulic system simulation for two major reasons. The first is that an accurate, detailed description of fluid cavities in a component is not a practical model requirement. Secondly, the task of solving the model equations would be too complex and time consuming to be included within a system simulation. An alternate approach must be used to develop a practical model for the thermal response of the fluid in a hydraulic component.

A general component is represented by Figure 4 which depicts the types of energy which may exist within a fluid power component model. Figure 4 is appropriate for use in defining a control volume for an unsteady energy balance. The assumptions to be made for the energy balance are:

1) the fluid within the component can be described by a bulk fluid temperature, $T_f$;
2) heat transfer between the fluid and the component wall is due to convection;
3) fluid propreties are dependent on temperature; the rate of change of these properties is negligible;
4) the inside and outside wall temperatures, $T_i$ and $T_o$, can be evaluated by applying the models developed in the last section

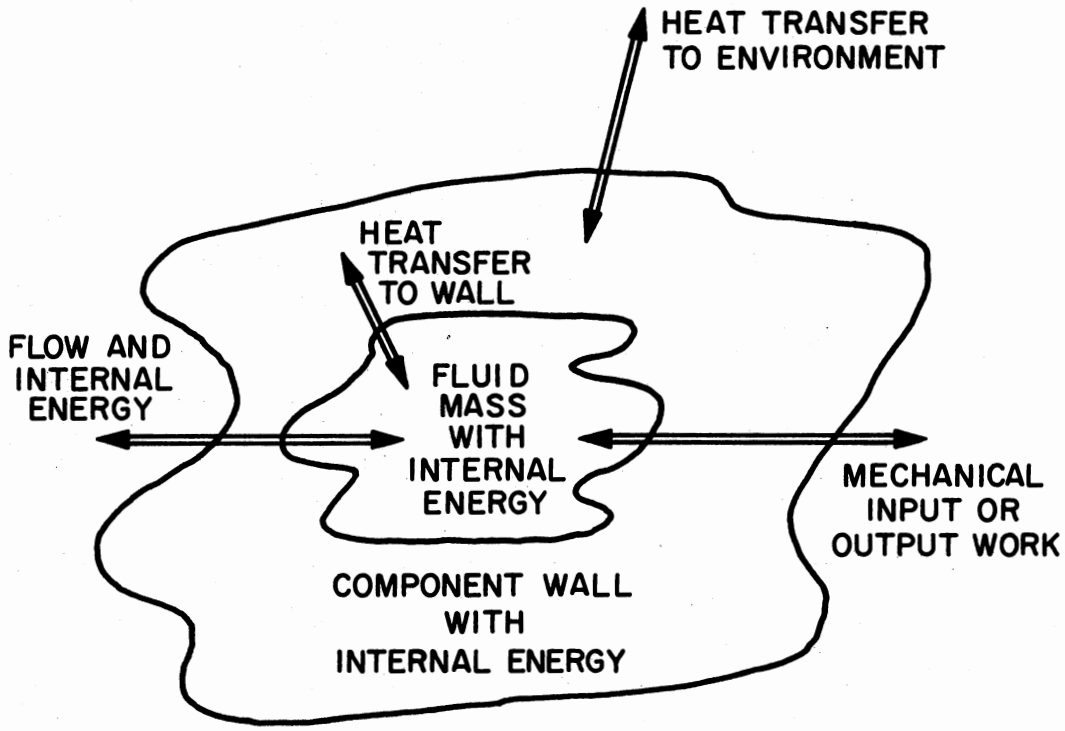



Figure 4. Control Volume for a Hydraulic Component

of this chapter;

5) internal energy is a function of temperature and is independent of pressure;

6) changes in kinetic and potential energy are negligible; and

7) energy dissipated due to mechanical inefficiency is transferred directly to the fluid to increase the fluid internal energy.

The first two assumptions above can be justified for components such as a reservoir, a small linear actuator, or fittings where thorough mixing of the fluid can be expected. For components having very irregular internal fluid cavities such as pumps, motors, and valves, the assumption allows a tractable description of the fluid mass without the detail required by a distributed model. A practical model probably could not be developed for components of irregular geometry without such an assumption.

Assumption 3 is concerned mainly with the dependence of specific heat on temperature. Specific heat is not a strong function of temperature for practical hydraulic fluids, thus allowing one to assume that the rate of change is negligible. Assumptions 5 and 6 are justified due to the relatively small amounts of energy which are associated with the compression, velocity, and position of hydraulic oils.

The last assumption must be evaluated based on experimental results. There are no known results in the literature to support or disprove the assumption. If the assumption is disproved by future studies, the models which follow will need to be modified to account for the direct transfer of a portion of the dissipated energy to the component wall. The dissipated energy may have to be distributed between the fluid mass and the wall mass and an energy flux term may need to be added to the component

wall model.

An energy balance for the fluid in a component such as that shown in Figure 4 requires the following differential terms:

1) internal energy

$$dE_I = \dot{m}\, c_p\, T_p\, dt \quad ;$$

2) flow energy

$$dE_F = \frac{\dot{m}}{J\rho} P\, dt \quad ;$$

3) mechanical energy (in terms of efficiency)

$$dE_m = \frac{\dot{m}}{J\rho} \frac{\Delta P}{\eta_i} dt \text{ (work done on the component}$$

$$= \frac{-\dot{m}\eta_o}{J\rho} \Delta P\, dt \text{ (work done by the component)} \quad ;$$

4) energy transfer to the wall

$$dE_w = h_i\, A_i (T_f - T_i)\, dt \quad ;$$

5) stored internal energy

$$dE_s = m_f\, c_p\, dT_f \quad .$$

An energy balance including on the above terms yields

$$dE_s = dE_w + dE_M + dE_F + dE_I \quad .$$

Hydraulic system components may possess more than one pressure and flow port. This requires that terms 1 and 2 above simply be applied for each port. If fluid is leaving a port, then $T_p$ for that port equals the fluid temperature $T_f$. The temperature of fluid entering a port is

assumed to be defined by the component connected to that port.

The unsteady energy equation for the bulk fluid temperature can be expressed as

$$m_f c_p \frac{dT_f}{dt} = h_i A_i (T_i - T_f) + \frac{\dot{m}_k \Delta P}{J\rho} \left(\frac{1}{\eta_i} - \eta_o\right) + \sum_{j=1}^{n} \dot{m}_j \left(\frac{P_j}{J\rho} + c_p T_{pj}\right) \tag{3.1}$$

where

$\dot{m}_k$ is the mass flow at the high pressure port of a pump or motor;

$\Delta P$ is inlet pressure minus exit pressure;

$n$ is the number of pressure and flow ports including the $\dot{m}_k$ port;

$T_{pj}$ is equal to $T_f$ if $m_j$ is negative (flow out of port j) and is defined by the connected component if $m_j$ is positive (flow into port j).

Equation 3.1 contains more terms than are applicable to all hydraulic system components. The mechanical energy terms will be present if the component is a pump or an actuator. For pumps, the $\eta_i$ term will be included, and the $-\eta_o$ term is applicable to motors or actuators. If significant mechanical work is present and is not expressible as a function of efficiency and pressure difference, the product of force and velocity or torque and angular velocity at a port may be more appropriate and would implicitly include the inefficiency effect.

Valves and other throttling device models will generally have no mechanical terms, and one may also ignore heat transfer if a small surface area is involved. Components such as lossless manifolds or tees may contain only the internal energy term if the heat transfer area is again small. If a component contains a small fluid mass, $m_f$, the assumption that the response of $T_f$ is instantaneous can be used to reduce Equation

3.1 to a steady energy equation. This assumption would be appropriate, for example, in the modeling of a tee which would contain negligible fluid mass compared to the fluid mass in the total system.

A reservoir model requires all of the terms in Equation 3.1 except the mechanical energy terms. If $m_f$ cannot be considered constant, then the previous assumption that the rate of change of $c_p$ is negligible allows one to write

$$\frac{dE_S}{dt} = m_f c_p \frac{dT_f}{dt} + c_p T_f \frac{dm_f}{dt} \quad . \tag{3.2}$$

The equivalent to Equation 3.1 is then

$$m_f c_p \frac{dT_f}{dt} = - c_p T_f \frac{dm_f}{dt} + h_i A_i (T_i - T_f) + \frac{\dot{m}_k \Delta P}{J\rho} \left(\frac{1}{\eta_i} - \eta_o\right) + \sum_{j=1}^{n} \dot{m}_j \left(\frac{P_j}{J\rho} + c_p T_{pj}\right) \tag{3.3}$$

where certain terms may be eliminated for individual components. If Equation 3.3 is required, conservation of mass also requires that

$$\frac{dm_f}{dt} = \sum_{j=1}^{n} \dot{m}_j \tag{3.4}$$

be satisfied simultaneously.

Equation 3.1 or Equations 3.3 and 3.4 constitute a lumped-parameter thermal model for the fluid within a component. The model is applicable when the assumptions on page 22 are valid for the component being

analyzed. The inside wall temperature, $T_i$, is defined by the wall temperature model derived in the last section of this chapter. The one type of component which should not be described by a lumped-parameter model is a "long" transmission line. Distributed line models are developed in the following section.

## Hydraulic Fluid Model for Transmission Lines

The transient pressure and flow response of fluid power transmission lines has been studied extensively in recent years as indicated by the available literature (10,11,12,63). The analytical models for liquid transmission lines are based on a one-dimensional distributed parameter line model with the assumption that temperature is held constant. The resulting set of partial differential equations has been solved by the various investigators with two distinctly separate approaches. One approach involves using operational mathematics to develop a transfer matrix which relates the pressure and flow at opposite ends of a line. Models of this type have been discussed by Goodson (63). The second approach to determining transient line response employs a numerical method of characteristics to determine pressure and flow distributed along a line (10,11,12). This latter approach has the advantages of being able to include frequency dependent friction (acceleration) effects in the predicted response.

This section utilizes the energy equation from Appendix A as a model for the transient thermal response of the fluid within a transmission line. The equation is solved by two methods which parallel the pressure and flow solutions discussed above. The two solutions are consistent with the available mechanical response solutions and can be combined with

them to predict the combined pressure, flow, and fluid temperature response in a transmission line.

## The Model for Fluid Within a Transmission Line

A fluid transmission line can be modeled by three partial differential equations which are the momentum equation, the continuity equation, and the thermal energy equation. The development of the momentum and continuity equations has been presented in detail by Krane (64) and Streeter (9) in a form equivalent to that used in typical transmission line studies such as those reviewed by Goodson (63) or in characteristic solutions (10,11,12). The momentum equation is

$$\frac{\partial P}{\partial x} + \frac{\rho}{g_c}\left(\frac{\partial V}{\partial t} + V\frac{\partial V}{\partial x}\right) = -\frac{2\tau_w}{r_i} \qquad (3.5)$$

and the continuity equation is

$$\frac{\partial P}{\partial t} + V\frac{\partial P}{\partial x} + \frac{a^2\rho}{g_c}\frac{\partial V}{\partial x} = 0 \qquad (3.6)$$

The assumptions for the above equations are that the flow is one-dimensional with fluid properties varying in the axial direction only. The fluid pressure is above vapor pressure at all times, and the fluid fills the line. The frictional losses are modeled by a shear stress $\tau_w$ concentrated at the wall. For method of characteristics solutions this stress can depend on fluid acceleration and can also be a nonlinear function of velocity (10,11,12). It is assumed that the wall elasticity can be modeled by modifying the velocity of sound in the fluid as shown in detail by Krane (64). The line is assumed to be circular, and no energy dissipation occurs due to expansion or compression in the line wall or in the fluid.

The thermal energy equation in cylindrical coordinates appears commonly in texts on heat and mass transfer (65). For axial flow with only axial and radial temperature dependence the thermal energy equation for a fluid of constant conductivity can be stated as

$$\rho c_v \left(\frac{\partial T_f}{\partial t} + V \frac{\partial T_f}{\partial x}\right) = - \frac{T}{J} \left(\frac{\partial P}{\partial T}\right)_\rho \frac{\partial V}{\partial x}$$

$$+ k_f \left[ \frac{\partial^2 T_f}{\partial x^2} + \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T_f}{\partial r}\right) \right] + \frac{1}{J} \tau_{xx} \frac{\partial V}{\partial x} + \frac{1}{J} \tau_{rx} \frac{\partial V}{\partial r} \quad . \qquad (3.7)$$

If one assumes that that temperature is uniform across the diameter of a fluid line and that friction exists only at the wall, the equation can be written as

$$\rho c_v \left(\frac{\partial T_f}{\partial t} + V \frac{\partial T_f}{\partial x}\right) = - \frac{\alpha T_f}{J} \left(\frac{\partial P}{\partial t} + V \frac{\partial P}{\partial x}\right)$$

$$+ k_f \frac{\partial^2 T_f}{\partial x^2} + \frac{2}{r_i} h_i (T_i - T_f) + \frac{2}{J r_i} |\tau_w V| \quad . \qquad (3.8)$$

The development of the above equation and the required assumptions are shown in Appendix A. The terms in Equations 3.7 and 3.8 have been written in a corresponding order. The basic differences are attributable to the expression of the radial heat transfer in terms of a convection term, the determination of the partial derivative of pressure with respect to temperature at constant density in terms of the fluid properties, and the assumption that friction (represented by $\tau_w$) exists only at the wall.

Appendix A contains an order of magnitude analysis of the terms in Equation 3.8 with the conclusion that the equation can be written as

$$\rho c_v \left(\frac{\partial T_f}{\partial t} + V \frac{\partial T_f}{\partial x}\right) = \frac{2}{r_i} h_i (T_i - T_f) \quad . \qquad (3.9)$$

The assumptions required for this simplification basically depend on the properties of common hydraulic fluids and do not restrict the use of Equation 3.9 for many applications. Appendix A discusses the assumptions which allow the simplification. The most significant assumption is that losses due to friction are included in the momentum equation but are not significant in the energy equation unless the resulting pressure drop is relatively large.

Operational Solution

The simplified fluid thermal energy Equation 3.9 is a nonlinear hyperbolic partial differential which together with Equations 3.5 and 3.6 form a complete model for the fluid in a line. The thermal energy equation is coupled to the momentum and continuity equations by the velocity, V, and could also be coupled through the convection coefficient if one utilizes a model of the form $h_i = h_i(V,\tau_w)$. The coupling of the continuity and momentum equations to the thermal energy equation is through the dependence of the fluid properties, i.e., viscosity and density, on temperature. If $h_i$ depends only on V and V is known, Equation 3.9 can be solved independent of Equations 3.5 and 3.6.

The operational solutions available in the literature for the momentum and continuity equations generally assume that a nominal fluid velocity, $V_0$, and density, $\rho_0$, exist in the transmission line (62,66, 67). A steady pressure drop exists due to the nominal through flow, and the solution is used to predict a transient perturbation from the nominal. If the terms which appear as coefficients in Equations 3.5, 3.6 and 3.9

are replaced by nominal values, $V_0$, $\rho_0$, $c_{v0}$, then the following set of linear hyperbolic partial differential equations results:

$$\frac{\partial P}{\partial x} + \frac{\rho_0}{g_c}\left(\frac{\partial V}{\partial t} + V_0 \frac{\partial V}{\partial x}\right) = -\frac{2\tau_w}{r_i} \quad ; \qquad (3.10)$$

$$\frac{\partial P}{\partial t} + V_0 \frac{\partial P}{\partial x} + \frac{a^2 \rho_0}{g_c} \frac{\partial V}{\partial x} = 0 \quad ; \qquad (3.11)$$

$$\rho_0 c_{v0}\left(\frac{\partial T_f}{\partial t} + V_0 \frac{\partial T_f}{\partial x}\right) = \frac{2}{r_i} h_i (T_i - T_f) \quad . \qquad (3.12)$$

It is now possible to solve Equation 3.12 as discussed below.

The methods employed in the literature for solving Equations 3.10 and 3.11 can be summarized in four basic steps. These consists of transforming the equations from the time and space domain to the Laplace and space domain. The transformed equations are then solved to determine pressure and velocity at the ends of a line in terms of the Laplace variable, s. Since the original equations are partial and not ordinary differential equations, the solutions in s contain hyperbolic sine and cosine functions and cannot conveniently be inverse transformed. If the hyperbolic functions are expressed in terms of truncated product expansions, one can develop rational polynomial transfer functions which relate the pressure and velocity at the two ends of a transmission line. These transfer functions can be transformed to the time domain to complete the solution which consists of a set of ordinary differential equations. This approach will be used to solve Equation 3.12.

The solution to be derived for Equation 3.12 is intended for application to hydraulic lines. If one can assume that the thermal capacitance of the line wall is negligible, the equation becomes

$$\frac{\partial T_f}{\partial t} + V \frac{\partial T_f}{\partial x} = \frac{2}{\rho_0 c_{v0} r_i} U(T_e - T_f) \qquad (3.13)$$

where $T_e$ is the environmental temperature and

$$U = \frac{1}{\frac{1}{h_i} + \frac{r_o - r_i}{k} + \frac{1}{h_o}} \quad .$$

A non-dimensional temperature can be defined as

$$T = \frac{T_f - T_e}{T_e}$$

and a parameter can be defined as

$$\Gamma = \frac{2U\ell}{\rho_0 c_{v0} V_0 r_i} \quad .$$

The spatial variable can be scaled by

$$\chi = \frac{x}{\ell}$$

and the temporal variable can be non-dimensionalized by the use of

$$\tau = \frac{V_0 t}{\ell} \quad .$$

The above four definitions allow Equation 3.13 to be written in a non-dimensional form as

$$\frac{\partial T}{\partial \tau} + \frac{\partial T}{\partial \chi} + \Gamma T = 0 \qquad (3.14)$$

which is linear and homogeneous. The boundary conditions for Equation 3.14 are:

$$T = T(\chi, \tau) \quad ;$$

$$T(0,\tau) = T_{inlet}(\tau) \quad ;$$

$$T(\chi,0) = 0 \quad .$$

The initial condition is selected for convenience and can be changed in the final solution. Thus, the Laplace transform of Equation 3.14 with respect to $\tau$ is:

$$\mathcal{L}\left(\frac{\partial T}{\partial \tau} + \frac{\partial T}{\partial \chi} + \Gamma T\right) = \mathcal{L}(0) \quad ; \qquad (3.15)$$

$$sT(\chi,s) + \frac{d}{d\chi} T(\chi,s) + \Gamma T(\chi,s) = 0 \quad ; \qquad (3.16)$$

$$\frac{d}{d\chi} T(\chi,s) + (s + \Gamma)T(\chi,s) = 0 \quad . \qquad (3.17)$$

The solution of Equation 3.17 subject to $T(0,s) = T_{inlet}(s)$ is:

$$T(\chi,s) = T_{inlet}(s)e^{-(s+\Gamma)\chi} \qquad (3.18)$$

$$= T_{inlet}(s)e^{-s\chi}e^{-\Gamma\chi} \quad . \qquad (3.19)$$

Equation 3.19 can be inverse transformed to

$$T(\chi,\tau) = T_{inlet}(\tau - \chi)e^{-\Gamma\chi} \qquad (3.20)$$

and unnormalized for $\chi = 1$ to

$$T_f(t) = T_e + (T_{inlet}(t - \frac{\ell}{V_0}) - T_e)e^{-\frac{2U\ell}{\rho_0 c_{v0} V_0 r_i}} \quad , \quad t \geq \frac{\ell}{V_0}$$

which is a solution for Equation 3.14.

This solution consists of a time delay of $\ell/V_0$ plus the exponential term due to heat transfer. Since Equation 3.9 does not contain the friction, conduction, or pressure effects, one would expect the solution to depend only on heat transfer and the history of the input temperature.

A transfer function for the response of $T_f$ to $T_{inlet}$ can be derived by expanding Equation 3.18 and truncating the expansion at the desired order. This type of solution has been presented for the momentum and continuity equations by Goodson (66) and Oldenberger (67), for example, and is a type of solution most useful for digital simulation.

The desired transfer function for $\chi$ equal to one is of the form

$$\frac{T(s)}{T_{inlet}(s)} = e^{-(s+\Gamma)} \approx \frac{N(s)}{D(s)} \tag{3.21}$$

where N(s) and D(s) are rational polynomials and the order of N(s) is no greater than the order of D(s). The expansion of the exponential function into a ratio of polynomials is thus required to develop the desired transfer function.

The exponential function need not be expanded as in infinite product as done by Goodson (66) and Oldenberger (67) for hyperbolic functions. The result one seeks in expanding the exponential function is a transfer function with constant magnitude independent of frequency and with an acceptable accuracy in phase. This can be accomplished by expanding the exponential using ratios of rational polynomials as described by Pade (68). The expansions for $e^{-s}$ are shown in Table I.

It is sufficient to expand the exponential if Equation 3.21 is re-written as

$$\frac{T(s)}{e^{-\Gamma\chi}T_{inlet}(s)} = e^{-s} \tag{3.22}$$

since $e^{-\Gamma\chi}$ is a constant. Figure 5 shows the phase response for

$$\frac{N'(s)}{D'(s)} \approx e^{-s} \tag{3.23}$$

TABLE I

SELECTED PADE EXPANSIONS FOR THE EXPONENTIAL FUNCTION $e^{-s}$

| $O_N = O_D - 2$ | $O_N = O_D - 1$ | $O_N = O_D$ |
|---|---|---|
| $\frac{2}{s^2+2s+2}$ | $\frac{-2s+6}{s^2+4s+6}$ | $\frac{s^2-6s+12}{s^2+6s+12}$ |
| $\frac{-6s+24}{s^3+6s^2+18s+24}$ | $\frac{3s^2-24s+60}{s^3+9s^2+36s+60}$ | $\frac{-s^3+12s^2-60s+120}{s^3+12s^2+60s+120}$ |
| $\frac{12s^2-120s+360}{s^4+12s^3+72s^2+240s+360}$ | $\frac{-4s^3+60s^2-360s+840}{s^4+16s^3+120s^2+480s+840}$ | $\frac{s^4-20s^3+180s^2-840s+1680}{s^4+20s^3+180s^2+840s+1680}$ |

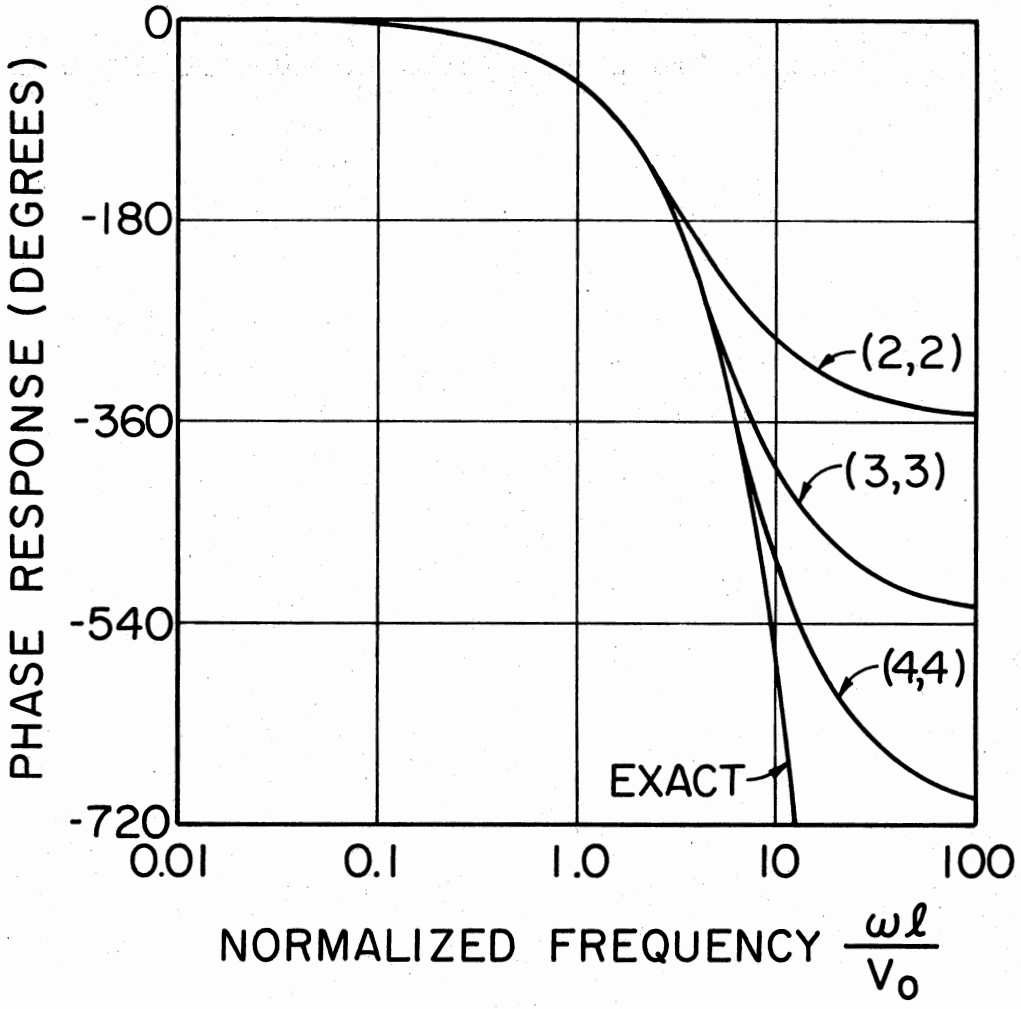



Figure 5. Phase Response of the Approximate Temperature Transfer Functions With $O_N = O_D$

for the expansions in the last column of Table I as well as for the exponential. As can be seen each additional term adds only a small amount of phase bandwidth, and below the normalized frequency of 1, the approximations are essentially equivalent in phase. It should be noted that the magnitude response is exact for all the Pade expansions in the last column of Table I. The notation $(O_N,O_D)$ used in these figures indicates the order of the numerator and denominator for each response.

The amplitude responses for the remaining functions from Table I appear in Figure 6 with the corresponding phase responses in Figure 7. Since the order of the numerator is less than that of the denominator, the approximations have a finite magnitude response bandwidth as shown in Figure 6. This implies a limit on the normalized frequency for which the approximations are useful, but it does not necessarily mean that one should use the approximations with $O_N = O_D$. An example step response in Chapter V shows that the time response achieved for $O_N < O_D$ is more representative of the actual phenomena than the response for $O_N = O_D$. Additionally, the phase responses in Figures 5 and 7 do not improve greatly with an increase in model order which implies that the low order approximations may be sufficient for many investigations.

The solution developed in the preceding discussion eliminated spatial dependence and involved a transfer function in terms of the end conditions for a transmission line. The applicability of the transfer function presented in Equation 3.22 is discussed in Chapter V. An alternate approach which retains spatial dependence and requires fewer assumptions follows.

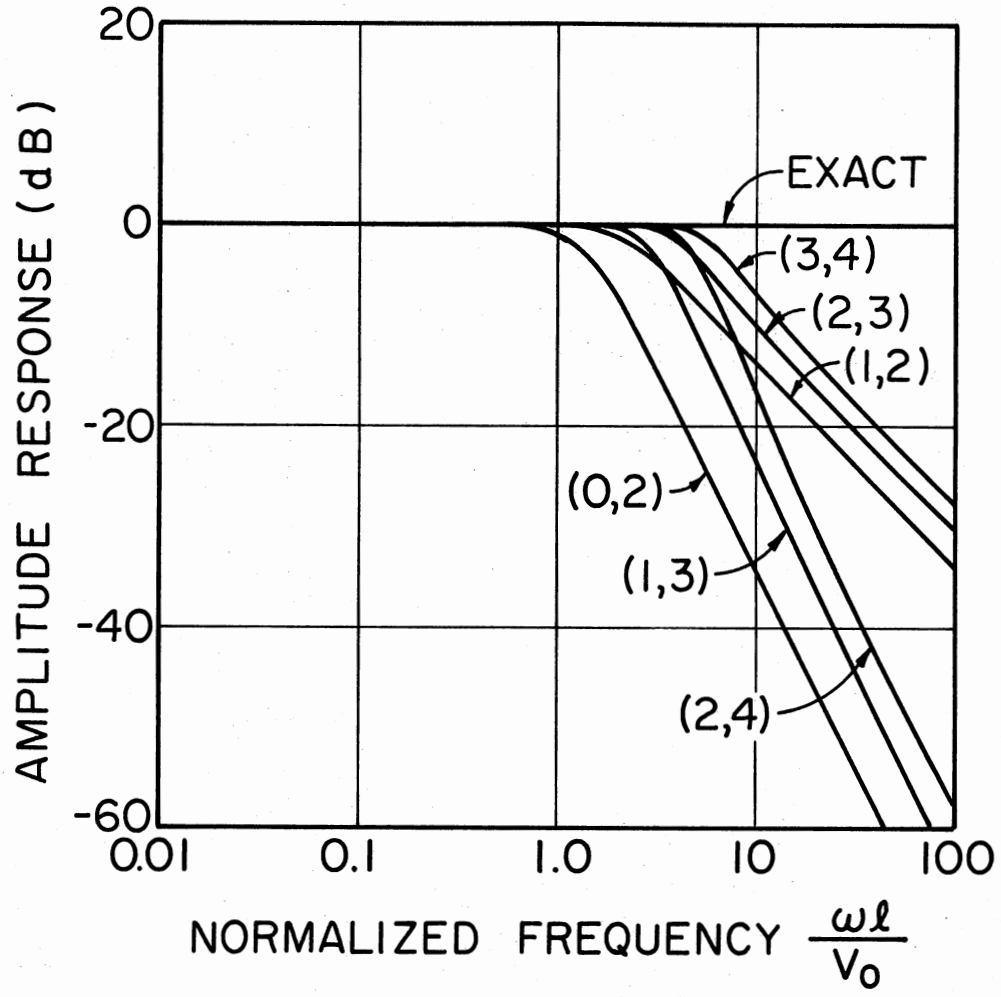



Figure 6. Magnitude Response of the Approximate Temperature Transfer Functions With $O_N < O_D$

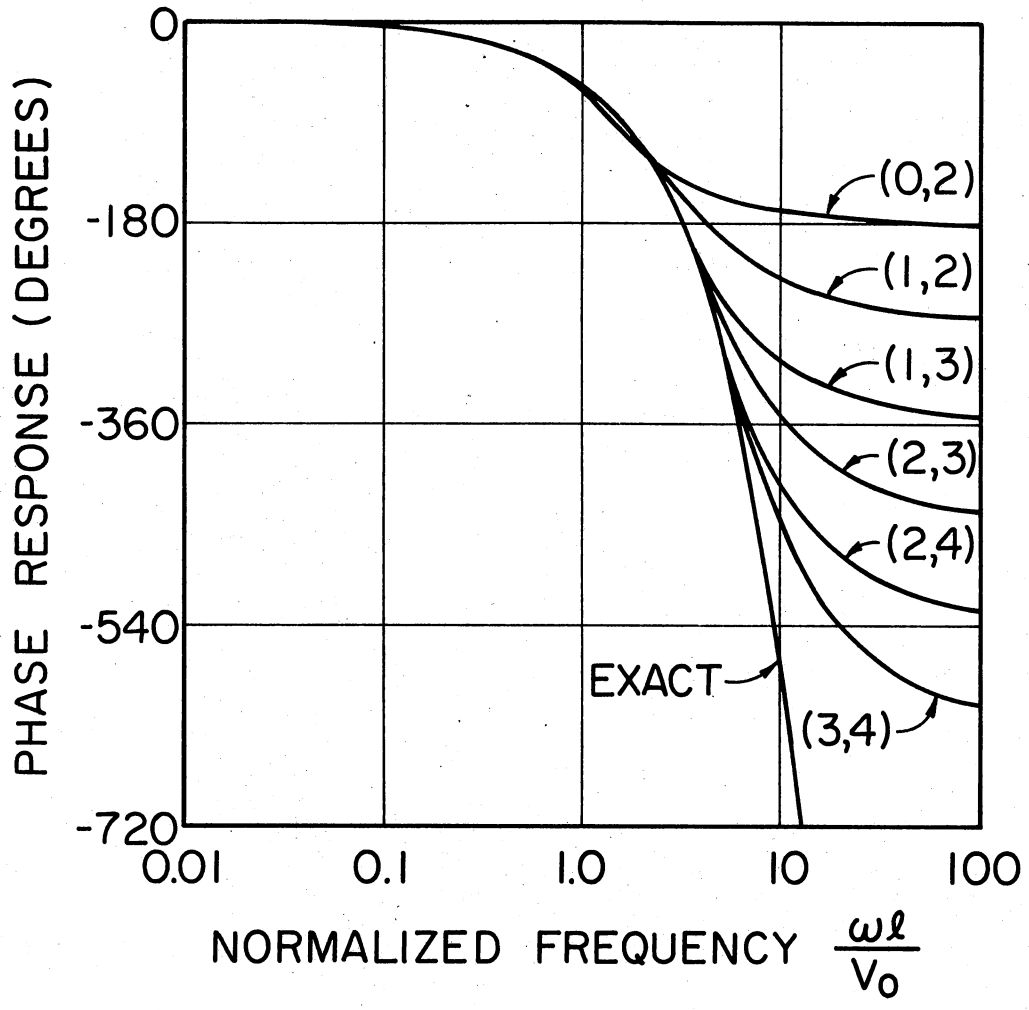



Figure 7. Phase Response of the Approximate Temperature Transfer Functions With $O_N < O_D$

Method of Characteristics Solution

The line model equations presented earlier as Equations 3.5, 3.6, and 3.8 are a set of simultaneous hyperbolic partial differential equations. As such, they could be solved by the method of characteristics if the equations were linear. Even though the equations are not linear, acceptable solutions can be determined by the method of characteristics if the fluid velocity, V, in Equations 3.5 and 3.6, is small compared to the acoustic velocity. The solution of Equation 3.9 by the characteristic method also requires that terms other than the heat transfer to the wall be relatively small. This requirement is met as discussed in Appendix A and allows the small terms to be treated as additional forcing functions functions in a manner analogous to that presented by Zielke (10).

A method of characteristics solution requires determining characteristic curves in the (x,t) plane along which the partial differential equations reduce to ordinary differential equations. If this can be accomplished, the ordinary differential equations can be integrated numerically, and a solution can be found which propagates through space and time. It is desirable to determine the solution at a set of fixed grid points as shown in Figure 8. It is assumed that initial conditions are available and that it is desired to propagate the solution one increment, $\Delta t$, in time.

The characteristic of Equation 3.8 can be determined by applying the identities

$$\frac{dT_f}{dt} = \frac{\partial T_f}{\partial t} + \frac{\partial T_f}{\partial x}\frac{dx}{dt} \quad ,$$

$$\frac{dP}{dt} = \frac{\partial P}{\partial t} + \frac{\partial P}{\partial x}\frac{dx}{dt} \quad ,$$

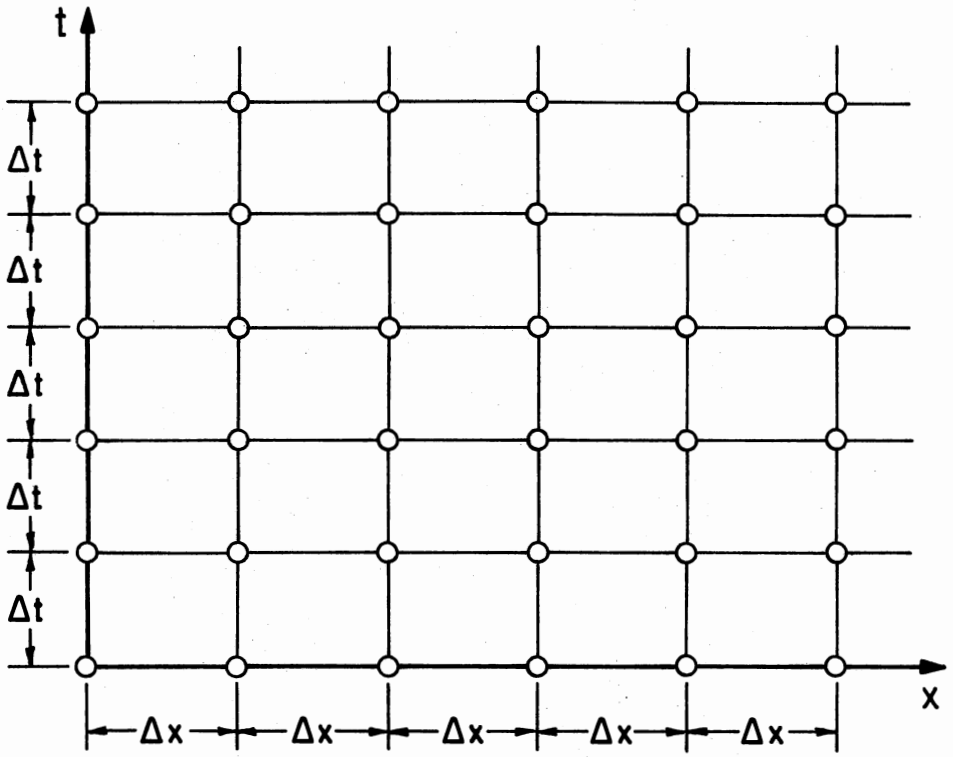



Figure 8. Fixed Grid for Method of Characteristics Solution

and

$$\frac{dx}{dt} = V \quad .$$

With the above identities, the equation can be written as

$$\rho c_v \frac{dT_f}{dt} = - \frac{\alpha T_f}{J} \frac{dP}{dt} + k_f \frac{\partial^2 T_f}{\partial x^2} + \frac{2}{r_i}(h_i (T_i - T_f) + \frac{|\tau_w V|}{J}) \qquad (3.24)$$

which is applicable along the curves defined by

$$\frac{dt}{dx} = \frac{1}{V} \quad .$$

A characteristic along which Equation 3.24 is applicable is shown in Figure 9. Velocity, pressure, and temperature are available at points A, B, and C, and the same quantities are to be determined at point D. Equation 3.24 is applicable along the line from point H to point D, thus implying that the temperature and pressure must be known at H. If the quantities at H are determined by interpolation, Equation 3.24 can be used to calculate temperature at point D in a manner consistent with existing pressure and flow solutions (8,9,10,11,12,65).

A first-order approximation of the integral of Equation 3.24 from H to D is

$$\rho c_v (T_D - T_H) = - \frac{\alpha}{J} T_H (P_D - P_H) + k_f (\frac{\partial^2 T}{\partial x^2})_H \Delta t + \frac{2}{r_i}(h_i (T_i - T_f) + \frac{|\tau_w V|}{J})_H \Delta t \quad . \qquad (3.25)$$

Solving for the unknown temperature yields

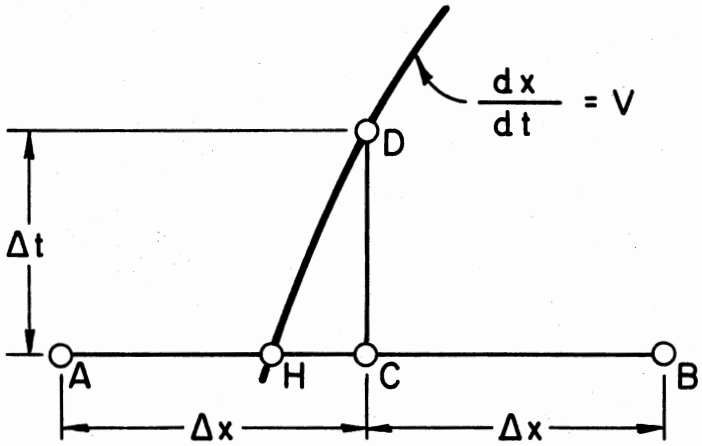



Figure 9. Illustration of the Thermal Energy Characteristics

$$T_D = T_H - \frac{\alpha}{J\rho c_v} T_H (P_D - P_H) + \frac{k_f}{\rho c_v} \left(\frac{\partial^2 T_f}{\partial x^2}\right)_H \Delta t$$

$$+ \frac{2}{\rho c_v r_i} \left(h_i (T_i - T_f) + \frac{|\tau_w V|}{J}\right)_H \Delta t \quad . \qquad (3.26)$$

The quantities $P_H$, $T_H$, and the second derivative of temperature are obtained from the known values at points A, B, and C. The pressure and temperature can be determined by linear or quadratic interpolation, and the second derivative of temperature can be determined by quadratic interpolation using $T_A$, $T_B$, and $T_C$.

A higher order of integration could be used to integrate Equation 3.24 if the gradient of P was known. The gradient of P can be considered to be known along the curve from H to D if one assumes that P is a linear function of time. The error introduced by this assumption should be tolerable since it has been shown in Appendix A that the pressure gradient is part of a relatively small term. The pressure and flow equations which must be solved along with Equation 3.24 to predict $P_D$, and thus the gradient can be replaced by

$$\frac{dP}{dt} = \frac{P_D - P_H}{\Delta t}$$

in Equation 3.24. In this form Equation 3.24 can be numerically integrated with a higher-order numerical algorithm. However, the examples and discussion in Chapter V demonstrate that the effect of grid size is dominant in the prediction of thermal response and imply that high-order integration would not greatly improve the predicted response.

Equation 3.26 and Equations B.9 and B.10 can be used to propagate the pressure, velocity, and temperature in a grid such as that shown in Figure 8. The inclusion of the temperature propagation is due to this

study, whereas the pressure and flow propagation has only been modified within this investigation. The modifications allow fluid properties to vary with temperature, thus coupling pressure and flow response to thermal response. Considerations in programming the combined responses are discussed in Chapter IV, and examples appear in Chapter V. The wall temperature $T_i$ is determined by the model developed in the following section.

## Wall Model for Hydraulic Components

The models developed in the preceding sections describe the thermal response of the fluid within a component. Each of the models has included an inside wall temperature, $T_i$, which appears in the convective heat transfer terms. This section develops the component wall model which is required to predict the wall temperature. The model to be developed is based on a simplified conduction equation in order to avoid the complexity of a complete distributed temperature prediction. The simplification is done in part to obtain a model which is practical to implement within a hydraulic system simulation.

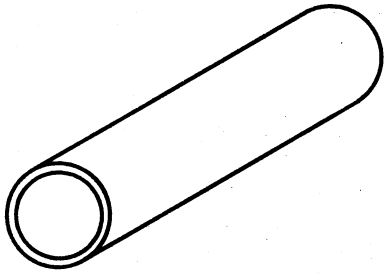
Preliminary to developing a component wall model the purpose of the model should be considered. A component wall performs two functions which are the most pertinent to system simulation. It stores energy due to thermal capacitance and it transmits energy between the hydraulic fluid and the environment. A wall model should not be included to allow one to perform a detailed study of the temperature distribution within a single component. The wall model should be included in a manner which depicts the effect which the walls have on the total system response. From the system response viewpoint, component walls allow heat transfer

between the environment and the hydraulic fluid. The energy transferred effects the fluid temperature which in turn influences system response. Thus the purpose of a wall model should be to provide a means for determining the amount of energy exchanged between the wall and the fluid due to heat transfer.

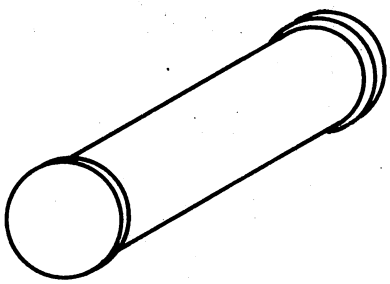
The geometries of hydraulic components must also be considered before a wall model can be developed. A hydraulic line is a most ideal geometry to model, and can be described with a relatively simple model. Components with irregular wall geometry present a much more difficult problem both in defining the component geometry and in determining the temperature response. It would seem difficult to justify a detailed wall model for a component such as a valve if one contrasts the complexity of the geometry with the effect that the component has on system response due to heat transfer. The wall model developed below assumes that components can be described in an idealized manner which does not include geometrical details. The above discussion is the basis for the simplified modeling.

Figure 10 contains three geometries which will be considered in the development of a wall model. It is assumed that hydraulic components can be represented by one of the selected shapes. Some of the more direct selections would be to model a line with shape (a), a rectangular reservoir with (c), or an accumulator as (b). Greater simpliciation would allow describing a pump with (c), a control valve as (b) or (c), or a relief valve as (c).

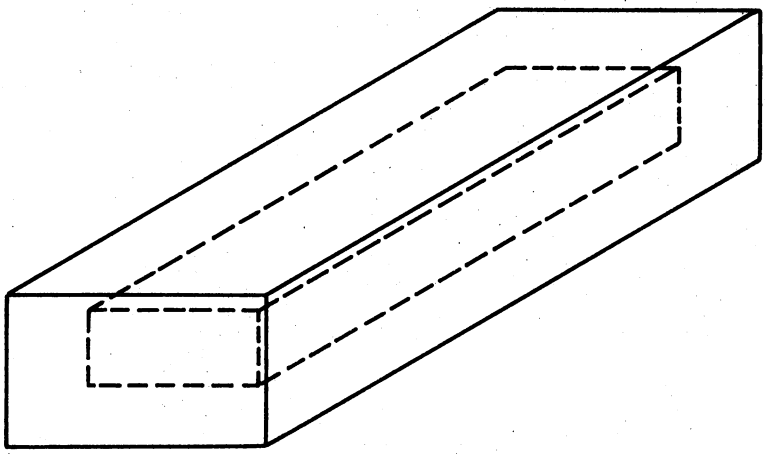
Each of the selected wall geometries are essentially a structure which entraps fluid in an internal cavity. The fluid in the cavity is at temperature $T_f$, and the environment is at $T_e$. Any effect of the fluid



(a) CYLINDER



(b) CAPPED CYLINDER



(c) HOLLOW RECTANGULAR BODY

Figure 10. Selected Wall Model Geometries

flowing through wall passages to enter or leave the internal cavity is ignored in the model development. It is assumed that the internal cavity is the same shape as the outside wall and that all the thermal capacitance of a component exists in the wall. Heat transfer to the wall is by convection from the fluid and the environment, and it will be assumed that no energy is transferred between the walls of connected components.

Additional assumptions will be made concerning conduction through component walls. The first assumption is that temperature depends on only one spatial dimension. It is assumed that energy is transferred by conduction only between the two parallel faces of any wall shown in Figure 10. For shape (c) this implies that the body is made up of six walls and that energy can be transferred through any wall by conduction, but that no energy is transferred between the walls. For shape (a) the implication is that there is radial conduction but no axial conduction exists. In composite shape (b) energy can be transferred through each wall, but each cap is essentially insulated from the cylinder. It is also assumed that the temperature of each inside and outside wall face is uniform over the face. These assumptions allow the development of the actual wall model.

A wall model can be developed by considering a differential element as shown in Figure 11. At side 1 the face area is $A_1$, and at side 2, the area $A_2$ is

$$A_2 = A_1 + \left.\frac{\partial A}{\partial Z}\right|_1 dZ \qquad . \qquad (3.27)$$

The temperature at side 1 is $T_1$ and at side 2 is

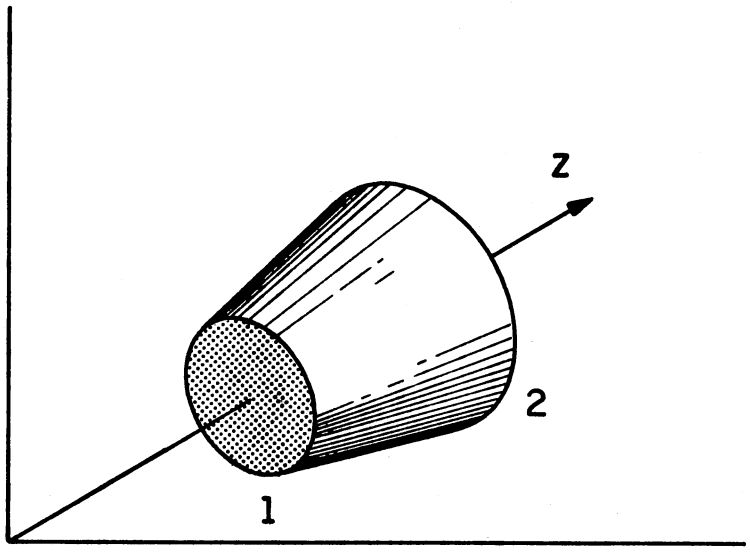



Figure 11. Example of Tapered, Parallel-Face Wall Element

$$T_2 = T_1 + \left.\frac{\partial T}{\partial Z}\right|_1 dZ \qquad (3.28)$$

An energy balance on the element yields

$$\rho cv \frac{\partial T}{\partial t} = A_1 q_1 - A_2 q_2 \qquad (3.29)$$

where $q_1$ and $q_2$ are the energy rate at the two faces. The energy rates are

$$q_1 = - k \left.\frac{\partial T}{\partial Z}\right|_1 \qquad (3.30)$$

and

$$q_2 = - k \left.\frac{\partial T}{\partial Z}\right|_2$$

$$= - k \frac{\partial}{\partial Z} \left(T_1 + \left.\frac{\partial T}{\partial Z}\right|_1 dZ\right) \qquad (3.31)$$

The storage rate can be expressed as

$$\rho cv \frac{\partial T}{\partial t} = \rho c\left(\left(\frac{A_1 + A_2}{2}\right) dZ\right) \frac{\partial T}{\partial t}$$

$$= \frac{\rho c}{2} \left(A_1 + A_1 + \left.\frac{\partial A}{\partial Z}\right|_1 dZ\right) dZ \frac{\partial T}{\partial t} .$$

The substitution of Equations 3.27 through 3.31 into the above equation yields

$$\frac{\partial T}{\partial t} = \gamma\left(\frac{1}{A(Z)} \frac{\partial A}{\partial Z} \frac{\partial T}{\partial Z} + \frac{\partial^2 T}{\partial Z^2}\right) \qquad (3.32)$$

where

$$\gamma = \frac{k}{\rho c}$$

and all terms multiplied by $dZ^2$ have been eliminated.

Equation 3.32 described the transient temperature distribution in a body under the assumption of one-dimensional temperature variation. If the cross-section is constant then the equation reduces to

$$\frac{\partial T}{\partial t} = \gamma \frac{\partial^2 T}{\partial Z^2}$$

which is the Fourier conduction equation in one dimension. For a cylin der (where Z becomes the radial direction), the area function is

$$A(r) = 2\pi r \ell \quad ;$$

$$\frac{\partial A}{\partial r} = 2\pi \ell \quad ;$$

$$\frac{\partial T}{\partial t} = \gamma\left(\frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial r^2}\right) \quad ,$$

which is the Fourier equation in cylindrical coordinates with angular and axial effects neglected. Thus Equation 3.32 is actually a general one-dimension conduction equation for heat transfer in the direction normal to two parallel faces of a slab. The equation describes each wall present in Figure 10, and will be used to develop a solution for wall temperature.

Assumptions stated earlier require that the wall temperature distribution must satisfy boundary conditions which are

$$k \left.\frac{\partial T}{\partial Z}\right|_i = h_i(T - T_f) \tag{3.33}$$

and

$$-k \left.\frac{\partial T}{\partial Z}\right|_o = h_o(T - T_e) \quad . \tag{3.34}$$

An approximate solution to Equation 3.32 which satisfies the above conditions can be developed using the heat balance integral approach (51).

The approach consists of assuming a solution which matches the boundary conditions and satisfies Equation 3.32 in an average sense. Such a solution can be developed by assuming

$$T(Z,t) = w(t)(1 + a_1 Z + a_2 Z^2) \tag{3.35}$$

where $w(t)$ is unknown and $(a_1, a_2)$ must be selected to satisfy the conditions of Equations 3.33 and 3.34. If the inside wall face is defined to be $Z = 0$ and the wall thickness is $\Delta Z$, the determination of $a_1$ and $a_2$ yields

$$a_1 = \frac{w(t)h_i - h_i T_f}{kw(t)} \tag{3.36}$$

and

$$a_2 = \frac{h_i(T_f - w(t))(k + h_o\Delta Z) + kh_o(T_e - w(t))}{kw(t)(2k\Delta Z + h_o\Delta Z^2)} . \tag{3.37}$$

The substitution of the above expressions into the assumed temperature expression results in

$$T(Z,t) = w(t) + \frac{h_i}{k}(w(t) - T_f)Z + \left(\frac{h_i(T_f - w(t))(k + h_o\Delta Z) + kh_o(T_e - w(t))}{k(2k\Delta Z + h_o\Delta Z^2)}\right)Z^2 . \tag{3.38}$$

The definition of

$$c_1 = -\frac{h_i}{k} ,$$

$$c_2 = \frac{h_i(k + h_o\Delta Z)}{k(2k\Delta Z + h_o\Delta Z^2)} ,$$

and

$$c_3 = \frac{h_o}{2k\Delta Z + h_o \Delta Z^2}$$

simplifies this to

$$T(Z,t) = w(t) + (c_1 Z + c_2 Z^2)(T_f - w(t)) + c_3 Z^2 (T_e - w(t)) \quad . (3.39)$$

Equation 3.39 can be substituted into the conduction Equation 3.32, but the equality will not be retained. If the resulting error is defined as

$$\begin{aligned} E(Z,t) &= \frac{\partial T}{\partial t} - \gamma\left(\frac{1}{A(Z)} \frac{\partial A}{\partial Z} \frac{\partial T}{\partial Z} + \frac{\partial^2 T}{\partial Z^2}\right) \\ &= \dot{w}(t) - (c_1 Z + c_2 Z^2 + c_3 Z^2)\dot{w}(t) \\ &\quad - \gamma\left(\frac{1}{A(Z)} \frac{\partial A}{\partial Z} ((c_1 + 2c_2 Z)(T_f - w(t)) \right. \\ &\quad \left. + 2c_3 Z(T_e - w(t))\right) \\ &\quad + 2c_2(T_f - w(t)) + 2c_3(T_e - w(t))) \quad , \end{aligned} \quad (3.40)$$

then the heat balance integral can be satisfied by requiring

$$0 = \int_0^{\Delta Z} E(Z,t)dZ \quad .$$

The result of the above integration is

$$\begin{aligned} \dot{w}(t)\left(1 - \frac{1}{2} c_1 \Delta Z^2 - \frac{1}{3}(c_2 + c_3)\Delta Z^3\right) &= 2\gamma \Delta Z(c_2(T_f - w(t)) + c_3(T_e - w(t))) \\ - \gamma \int_0^{\Delta Z} &\left(\frac{1}{A(Z)} \frac{dA}{dZ} ((c_1 + 2c_2 Z)(T_f - w(t)) + 2c_3 Z(T_e - w(t)))\right)dZ \quad . \end{aligned} \quad (3.41)$$

If one defines

$$\begin{aligned} I_0 &= \int_0^{\Delta Z} \frac{1}{A(Z)} \frac{dA}{dZ} dZ \\ &= \ell n(A(Z))\Big|_0^{\Delta Z} \end{aligned}$$

and

$$I_1 = \int_0^{\Delta Z} \frac{Z}{A(Z)} \frac{dA}{dZ} dZ \quad ,$$

the above equation can be written as

$$\dot{w}(t)(1 - \frac{1}{2} c_1 \Delta Z^2 - \frac{1}{3} (c_2 + c_3)\Delta Z^3) = 2\gamma\Delta Z(c_2(T_f - w(t)) + c_3(T_e - w(t)))$$

$$- \gamma c_1(T_f - w(t))I_0 - 2\gamma(c_2(T_f - w(t)) + c_3(T_e - w(t)))I_1 \quad . \tag{3.42}$$

This is a first-order ordinary differential equation which together with Equation 3.39 describes the thermal response of a hydraulic component wall. In the above form the equation is applicable to a single wall of a component. As such it is applicable as a hydraulic line wall model or as a wall model for a component assumed to be a cube with a cubic internal cavity. If a component consists of several different wall sections, the error in Equation 3.32 can be averaged over all the walls to derive

$$\dot{w}(t) \sum_{i=1}^{n} (1 - \frac{1}{2} c_1 \Delta Z_i^2 - \frac{1}{3} (c_2 + c_3)\Delta Z_i^3)$$

$$= \sum_{i=1}^{n} (2\gamma\Delta Z_i(c_2(T_f - w(t)) + c_3(T_e - w(t)))$$

$$- \gamma c_1(T_f - w(t))$$

$$- 2\gamma(c_2(T_f - w(t)) + c_3(T_e - w(t)))) \tag{3.43}$$

where n is the number of wall sections and $\Delta Z_i$ is the thickness of each section. In this form the model describes the thermal response of a total component which is the desired result.

The initial condition for w(t) follows directly from Equation 3.39. If at the initial time the component is at the environmental temperature then $T = T_e = T_f$, and it follows that $w(0) = T_e$. If the initial temperature is not the environmental temperature, then an initial condition can be derived by approximately satisfying an initial known temperature T(0) by requiring

$$0 = \sum_{i=1}^{n} \{\int_{0}^{\Delta Z_i} (T(0) - T(Z,0))dZ\}$$

which results in a w(0) with zero average error. An initial steady state condition could also be used. This follows from Equation 3.43 with $\dot{w}(t) = 0$. The selection of a particular initial condition must be made when a particular component is being modeled, but the above three possibilities should include the realistic situations.

The integrals $I_0$ and $I_1$ depend on the geometry of a single wall. If a wall is of constant cross-section such as the flat end cap in Figure 8(b), then the derivative of the area is zero and $I_0 = I_1 = 0$. For a cylinder wall of inside radius $r_i$ and outside radius $r_o$, the functions are:

$$A(r) = 2\pi \ell r \quad , \qquad r_i \leq r \leq r_o \quad ,$$

$$\frac{\partial A}{\partial r} = 2\pi \ell \quad ,$$

$$I_0 = \ell n(\frac{r_o}{r_i}) \quad ,$$

$$I_1 = r_o - r_i - r_i \ \ell n(\frac{r_o}{r_i}) \quad .$$

The rectangular shape in Figure 10(c) is composed of six tapered walls. If each wall is considered as shown in Figure 12, then the

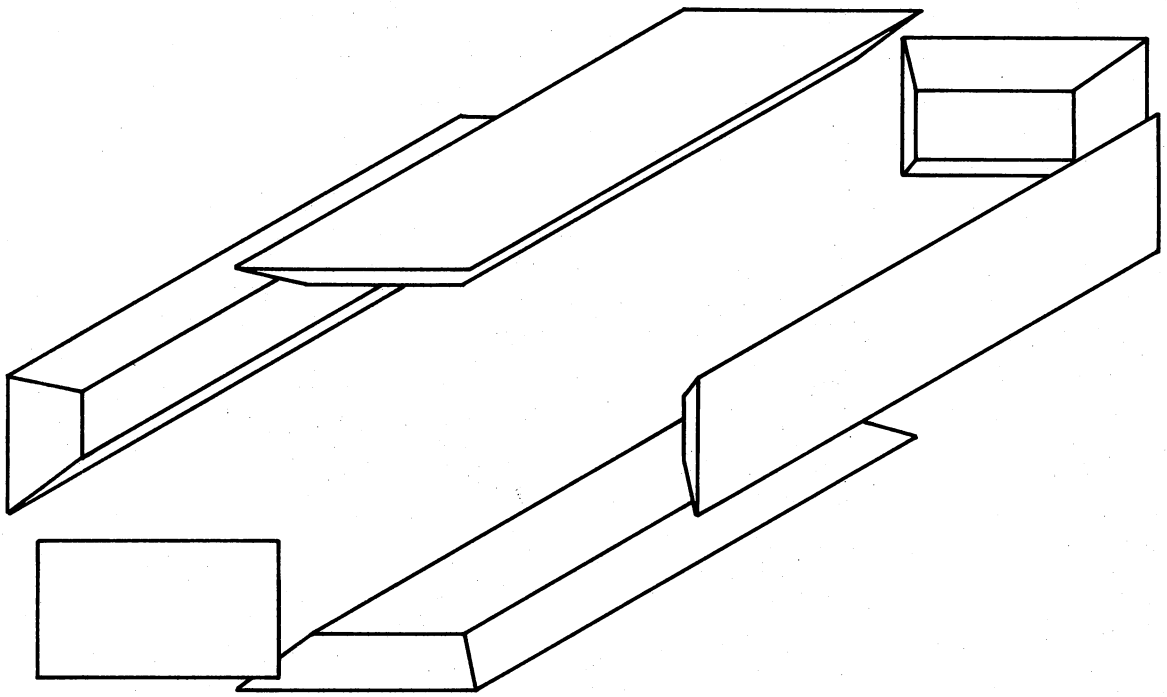


Figure 12. Representation of a Hollow Rectangular Body as a Composite of Six Tapered Walls

functions are:

$$A(Z) = a + bZ + cZ^2 \quad ,$$

$$a = Y_i X_i \quad ,$$

$$b = \frac{Y_i(X_o - X_i) + X_i(Y_o - Y_i)}{\Delta Z} \quad ,$$

$$c = \frac{(Y_o - Y_i)(X_o - X_i)}{\Delta Z^2} \quad ,$$

$$I_0 = \ell n\left(\frac{a + b\Delta Z + c\Delta Z^2}{a}\right) \quad ,$$

$$= \ell n\left(\frac{Y_o X_o}{Y_i X_i}\right) \quad ,$$

$$I_1 = 2\Delta Z - \frac{b}{2c}\,\ell n\left(\frac{a + b\Delta Z + c\Delta Z^2}{a}\right) + \frac{\sqrt{b^2 - 4ac}}{2c}\,\ell n\left(\frac{2a + (b + \sqrt{b^2 - 4ac})\Delta Z}{2a + (b - \sqrt{b^2 - 4ac})\Delta Z}\right),$$

where $(X_i, Y_i)$ are the dimensions of an inside face and $(X_o, Y_o)$ are the dimensions of an outside face.

The above integrals complete the thermal response model. The application of Equations 3.39 and 3.42 with the appropriate initial condition and integrals $(I_0, I_1)$ will result in a model for each of the geometries in Figure 10. The model satisfies the boundary conditions of Equations 3.33 and 3.34, and satisfies the heat balance integral which requires zero average error in Equation 3.32. The model involves the environmental temperature $T_e$ and the internal fluid temperature $T_f$. The wall model combined with one of the fluid models developed earlier in the chapter results in a complete hydraulic component thermal response model. The total model predicts the temperature response of both a component and

the entrapped fluid.

The thermal models which interact with mechanical models to describe the total response of hydraulic components have been developed in this chapter. The total model for a system can be developed by coupling component models in an appropriate manner. The next chapter considers the coupling of components to form such systems. The algorithms required to solve the resulting system of equations in a general form are developed.

# CHAPTER IV

## ALGORITHMS FOR INCLUDING THERMAL RESPONSE IN HYDRAULIC SYSTEM SIMULATION

The simulation of hydraulic systems can be considered as a type of network analysis. One characteristic which distinguishes hydraulic system simulation from other network analyses is the inherent nonlinear response exhibited by many hydraulic components. A second distinction is the need to represent hydraulic components as multiport models in which the port variables can seldom be decoupled. Both of these characteristics contrast electrical network simulation in which components are often characterized as being linear and in which components with high input impedance isolate segments of a circuit.

The algorithms necessary to simulate hydraulic systems have been developed in a general manner by Smith (1). A less general formulation has been made by Zielke (69) and is being applied in HYTRAN in the current contract effort by McDonnell Aircraft (8). As stated earlier, the HYTRAN program is dependent on the method of characteristics solution for line models. The use of this method results in a system model in which the response of each component can be calculated almost independently. However, this requires that a line must be used to couple any two components to form a system and that only pressure and flow ports are allowable. The algorithms used do not allow the definition of any ports which cannot be coupled through a transmission line. Since the HYTRAN

approach can be included within a more general development, it will not be used as a basis for the inclusion of thermal effects.)

(The algorithms developed by Smith and implemented in HYDSIM (59) assume that a system can be represented as a set of coupled components. Each component is described by differential and algebraic equations, and port variables provide the inputs and outputs for each component. There are no restrictions concerning acceptable ports as long as two variables exist at each port with one being independent and the other dependent with respect to a reference component. Smith has not considered that difference equations may appear in a system model, and it follows that the HYDSIM program is not readily amenable to the inclusion of the method of characteristics line model. The two major requirements which must be met in order to include thermal effects involve a capability to implement difference equation models and a capability to include pressure, mass flow rate, and temperature ports as discussed in Chapter III. These requirements can be met by the following extensions of the work done by Smith.

A statement of the assumed form for each component model is required before the appropriate algorithms can be presented. It is assumed for this work that each component may be modeled by a coupled set of ordinary differential, algebraic, and difference equations of the form

$$\dot{X}(t) = f(X,Y,S,P_I,t) \quad , \qquad (4.1)$$

$$0 = g(X,Y,S,P_I,t) \quad , \qquad (4.2)$$

$$S(t + \Delta t) = L(X,Y,S,P_I,t) \quad , \qquad (4.3)$$

with initial conditions

$$X(t_0) = X_0$$

$$S(t_0) = S_0$$

known. The continuous state vector X is of dimension n, the algebraic vector Y is of dimension m, and the discrete state S is of dimension k. The functions (f,g,h) are of dimension (n,m,k), respectively. Each dependent port variable may be defined either as

$$P_{Di} = P_{Di}(X,S,t) \tag{4.4}$$

or

$$P_{Di} = Y_j \tag{4.5}$$

where $Y_j$ is any element in the algebraic variable vector Y. The range of i is one to the total number of port variables present. If thermal effects are modeled, each pressure and flow port has a dependent temperature and an independent temperature variable in addition to the port variables pressure and flow. Other ports will have one dependent variable. The independent vector $P_I$ is of the same dimension as $P_D$. The dimensioning of $P_I$ and $P_D$ is discussed further in the next section which considers the inclusion of temperature as a port variable.

## Algorithm for Including Temperature as a Port Variable

The representation of a pressure and flow port with temperature was discussed in Chapter III. It was stated that the dependence of port temperature could not, in general, be specified a priori, which results in a special algorithmic requirement for temperature at a fluid flow port.

Consider defining an algebraic variable $Y_i$ as the temperature of the fluid at port j of a component. Such a quantity can, from physical considerations, be determined in either of two ways which are:

1) the temperature is equal to that of the fluid being discharged by the adjacent component at the port; or
2) the temperature is dependent on the fluid temperature within the component itself.

The first case implies flow is into the port; $Y_i$ is thus specified by the adjacent component; and the port temperature is independent. In the second situation, flow is out of the port, and the temperature is dependent with respect to the component. Thus $Y_i$ must have two definitions which are

$$Y_i = P_{Ij}^T \tag{4.6}$$

for flow <u>into</u> a port where $P_{Ij}^T$ is the port temperature calculated by the coupled component and

$$Y_i = \hat{g}_i(X,Y,S,t) \tag{4.7}$$

for flow <u>out of</u> a port. The corresponding algebraic equations are

$$\begin{aligned} 0 &= g_i(Y_i,P_{Ij}^T) \\ &= Y_i - P_{Ij}^T \end{aligned} \tag{4.8}$$

for flow <u>into</u> a port and

$$\begin{aligned} 0 &= g_i(X,Y,S,P_I,t) \\ &= Y_i - \hat{g}_i(X,Y,S,P_I,t) \end{aligned} \tag{4.9}$$

for flow out of a port. At any time only one of the above equations is applicable depending on the direction of flow. Functionally, $g_i$ must be defined as depending on the arguments of $\hat{g}_i$ and $P^T_{Ij}$.

Equations 4.8 and 4.9 must be included for each flow port of a component model. The solution of the appropriate form for $g_i$ results in $Y_i$ being equal to the port temperature. If flow is out of a port, the connected component must use an equation similar to Equation 4.8 to calculate port temperature. This implies that port variable

$$P^T_{Dj} = Y_i \tag{4.10}$$

so that the independent temperature of fluid entering the connected component can be defined as

$$P^{Tc}_I = P^T_{Dj} \tag{4.11}$$

where the superscript c denotes the coupled component.

The fluid thermal response models developed in Chapter III include port temperature variables. In order to implement the models it is necessary to first define an algebraic temperature variable for every flow port and then to substitute the resulting variables into the model equations. This assures that the model equations are expressed in terms of specified quantities since each of the temperature variables are determined by Equations 4.8 and 4.9. It should be noted that in many cases the necessary definition will reduce to $Y_i = T_f$.

The requirements of including temperature at a flow port can be summarized as:

1) define an algebraic variable for each flow port to be the temperature at the port;

2) substitute the algebraic variables into the model Equations 4.1 through 4.3;
3) determine the form of Equation 4.9 for each port temperature algebraic variable and add Equations 4.8 and 4.9 to the algebraic equation set for the component. (One equation will be added for each flow port;) and
4) define the independent temperature at each port according to Equation 4.10.

The impact of including thermal response on the number of equations can be assessed by considering a component with N ports of which M are pressure and flow ports. The total component model, in terms of Equations 4.1 through 4.3, will then contain N + M independent and dependent variables and m + M algebraic variables and equations. Thus the inclusion of fluid temperature adds one dependent variable, one independent variable, and one algebraic variable per flow port in addition to the requirements of the models as developed in Chapter III.

## Including Difference Equations in a Component Model

The method of characteristics transmission line model defines the transient response of a line with a set of difference equations. These equations result from the integration of a set of simultaneous differential equations which are defined along the pressure wave and particle pathline characteristics. Since the difference equations are related to a set of ordinary differential equations, it should be determined whether Equation 4.3 should be included in a model or whether the differential equations should be added to the vector Equation 4.1. Another

possibility is to consider the difference equations to be a special set of algebraic equations which could be appended to the vector Equation 4.2.

The manner in which a method of characteristics line model should be implemented in a general program can be determined by considering the form of the model equations. Figure 9 in Chapter III illustrates that in a method of characteristics solution the propagation of all quantities along interior spatial grid lines depends only on the values along the current time grid line. Thus all interior point quantities can be propagated one time increment independent of the boundary conditions which are imposed by components coupled to each end of a line. This implies that the propagation of the solution at interior points can be performed independently within a component model. It is advantageous to include the method of characteristics solution in this manner because it allows the component model to interact with other components at the line ends only. There is no need to propagate the interior point solutions simultaneously with other differential equations or to include the difference equations as algebraic equations.

The grid points on the boundaries of a line cannot be propagated independent of the connected component. As shown in Appendix B, the pressure and flow at the line ends are indeterminate because one wave characteristic is not present. Thus the propagation of the solution for the line end grid points can only be achieved correctly by defining an algebraic equation in terms of the wave characteristic which is present plus the independent variable at the port. This algebraic equation is B.9 or B.10 depending on the line end and must be included in vector Equation 4.2.

Temperature must also be defined by an algebraic equation. If flow is into a port, then Equation 4.8 is appropriate; if the opposite is true, Equation 3.26 applies as the form for 4.9. This is consistent with the concept of defining algebraic port temperatures as discussed in the preceding section. Again, each of the ports adds one equation to the vector set 4.2.

# CHAPTER V

## APPLICATION EXAMPLES

This chapter contains three examples which demonstrate the model forms developed in Chapter III. The first example involves the step response of the operational line model followed by an example of the step response of the method of characteristics model. The final example consists of a hydraulic circuit simulated under two sets of initial and environmental conditions. The chapter also contains a discussion of results for each example.

### Transmission Line Step Response

#### Line Configuration

The first two examples consist of simulating the pressure, velocity, and temperature response of a transmission line following a step change in the inlet temperature. The system is shown in Figure 13 and consists of two constant pressure sources which produce a pressure differential of one hundred pounds per square inch (psi). The fluid properties are those given in reference (8) for MIL-H-5606.

The line friction model used is the Darcy-Weisbach equation available in standard texts and used in (8). The convection coefficient models used for the method of characteristics model are

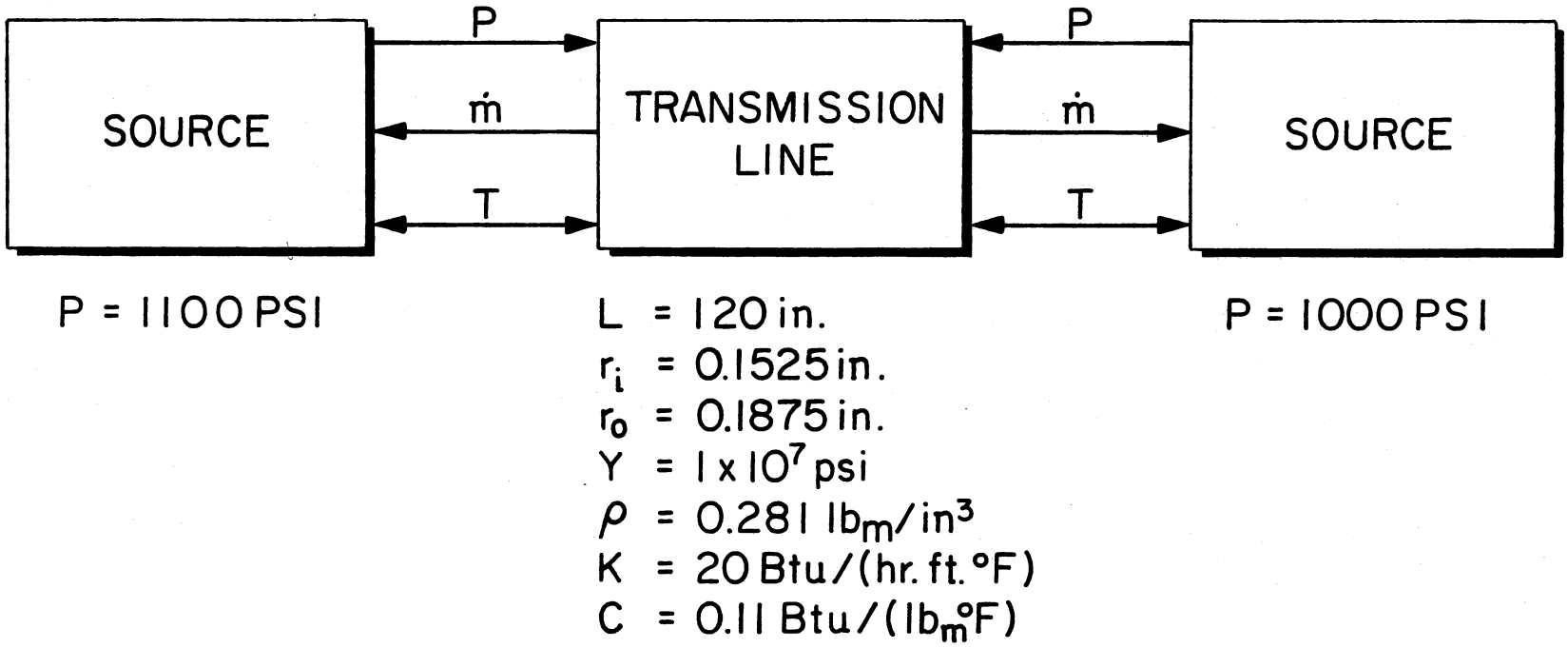



Figure 13. Transmission Line Step Response System

$$\frac{2r_o h_i}{k_f} = 3.66 + \frac{0.668\left(\frac{2r_o}{\ell}\right)N_R N_P}{1 + 0.04\left(\left(\frac{2r_o}{\ell}\right)N_R N_P\right)^{\frac{2}{3}}}$$

for $N_R \leq 1200$ and

$$\frac{2r_o h_i}{k_f} = \frac{0.0396\ N_R^{\frac{3}{4}}\ N_P}{1 + 2.44\ N_R^{-\frac{1}{8}}\ (N_P - 1)}$$

for $N_R > 1200$ with

$$h_o = 0.27\left(\frac{\Delta T}{2r_o}\right)^{\frac{1}{4}} ,$$

all in (ft - hr - Btu) set of units where

$$\Delta T = T_o - T_e$$

for the method of characteristics solution and

$$\Delta T = T_f - T_e$$

for the operational solution. For the operational solution the overall heat transfer coefficient model was chosen to be

$$U = h_o$$

with $h_o$ defined above since $h_o$ dominates in the calculation of the overall heat transfer coefficient U. Additional parameter values for the line wall are shown in Figure 13.

The simulation consisted of initializing the line model in steady state with all temperatures at zero degrees Fahrenheit; at time zero plus

the temperature at the 1100 psi source was increased to 100 degrees. The resulting responses are considered in the next two sections.

## Operational Model

The operational temperature response model developed in Chapter III was used together with the pressure and flow model from Oldenburger (67). The pressure and flow model used was the "n = 1" model for the response of the output flow to constant pressure inputs with time varying fluid properties. For this case the "n = 1" solution reduced to a third-order transfer function. Fluid properties were evaluated at the average of the inlet and outlet temperatures and pressures. Since the operational solution is only valid at the ends of the transmission lines, no prediction of pressure, temperature, or flow at interior points could be made.

The outlet temperature and velocity responses for six of the models from Table I are shown in Figures 14, 15, 16, and 17. As can be seen from Figures 14 and 16, the temperature response models with $O_N = O_D$ predict an instantaneous change in outlet temperature followed by an oscillatory response which undershoots the initial value. For $O_N = O_D - 1$ there is no instantaneous change, but the response again goes below the initial fluid temperature and thus contradicts what must physically occur. For $O_N = O_D - 2$ the undershoot is eliminated for the second-order model and decreased for the fourth-order model. The implication is that although the Pade approximations with high-order numerators have more desirable frequency responses, the time response is less desirable.

The integral of the velocity responses from Figures 15 and 17 were calculated as part of the step response simulation. For each of the curves the integral equals the line length of 120 inches at a time of

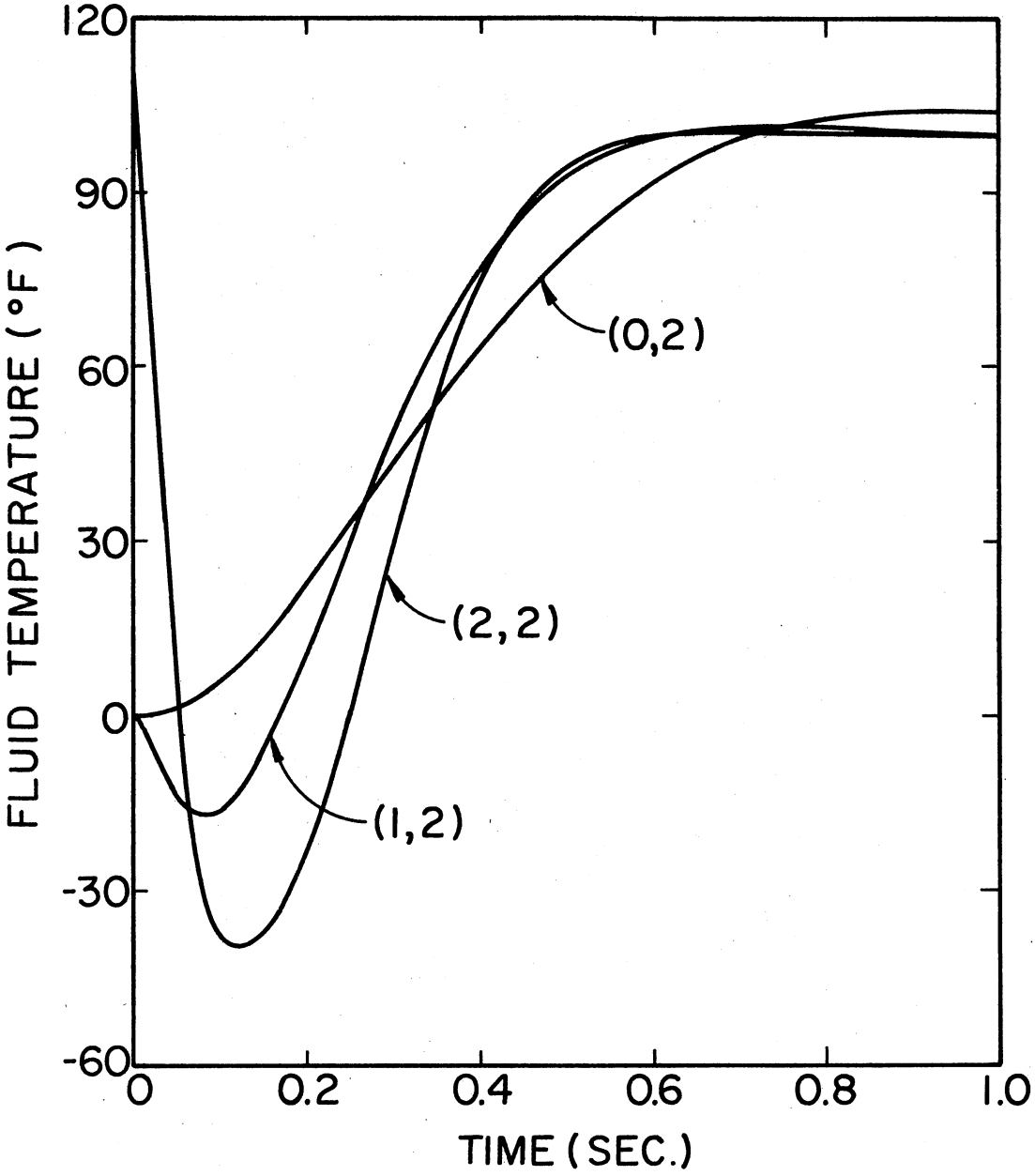



Figure 14. Outlet Temperature Response With Second-Order Operational Model

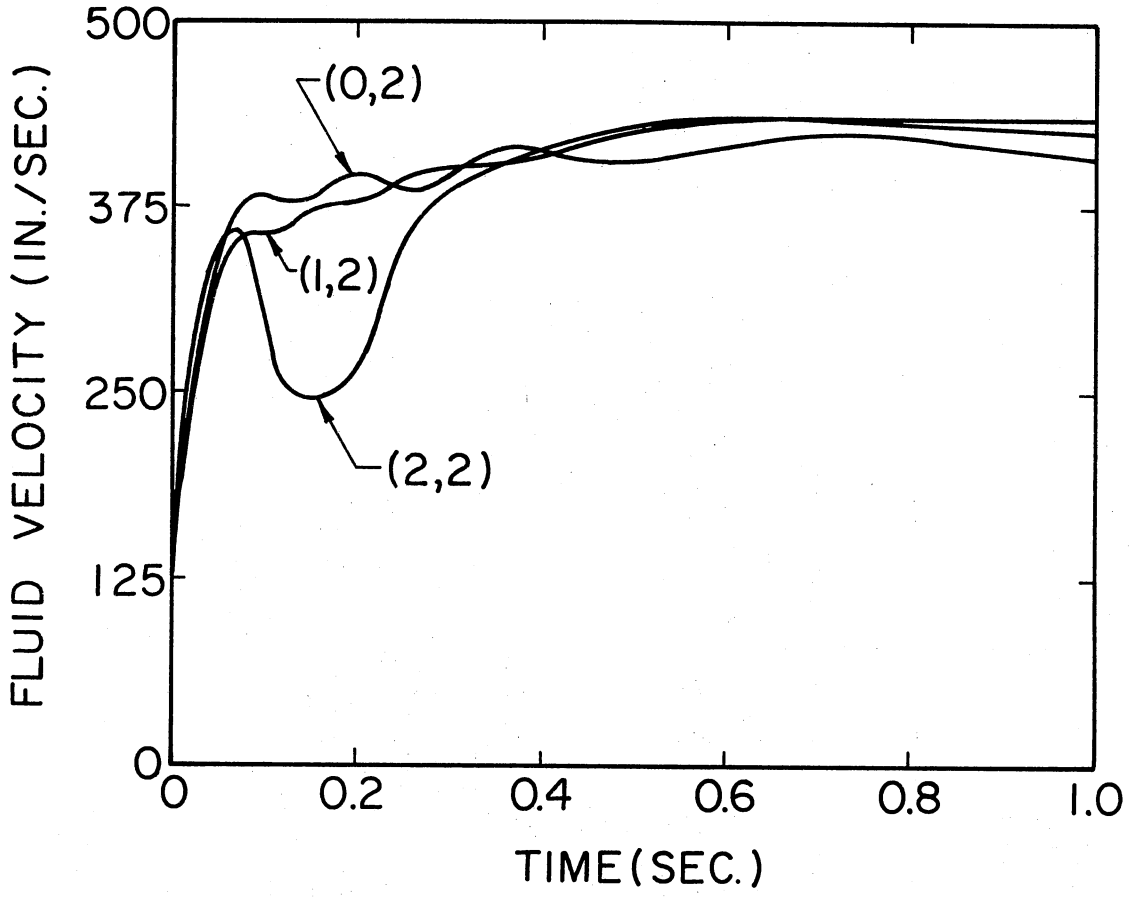



Figure 15. Velocity Response With Second-Order Operational Model

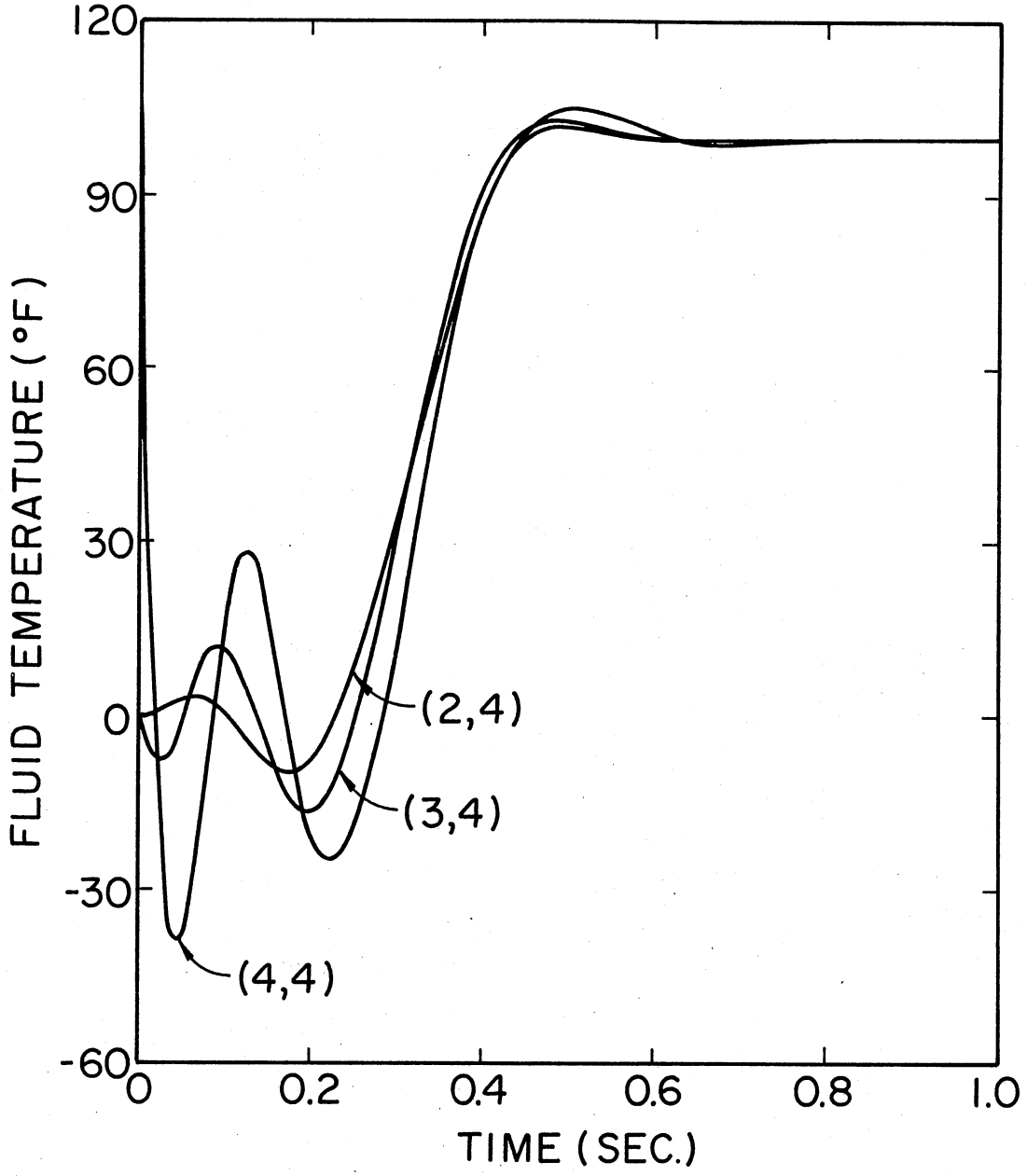



Figure 16. Outlet Temperature Response With Fourth-Order Operational Model

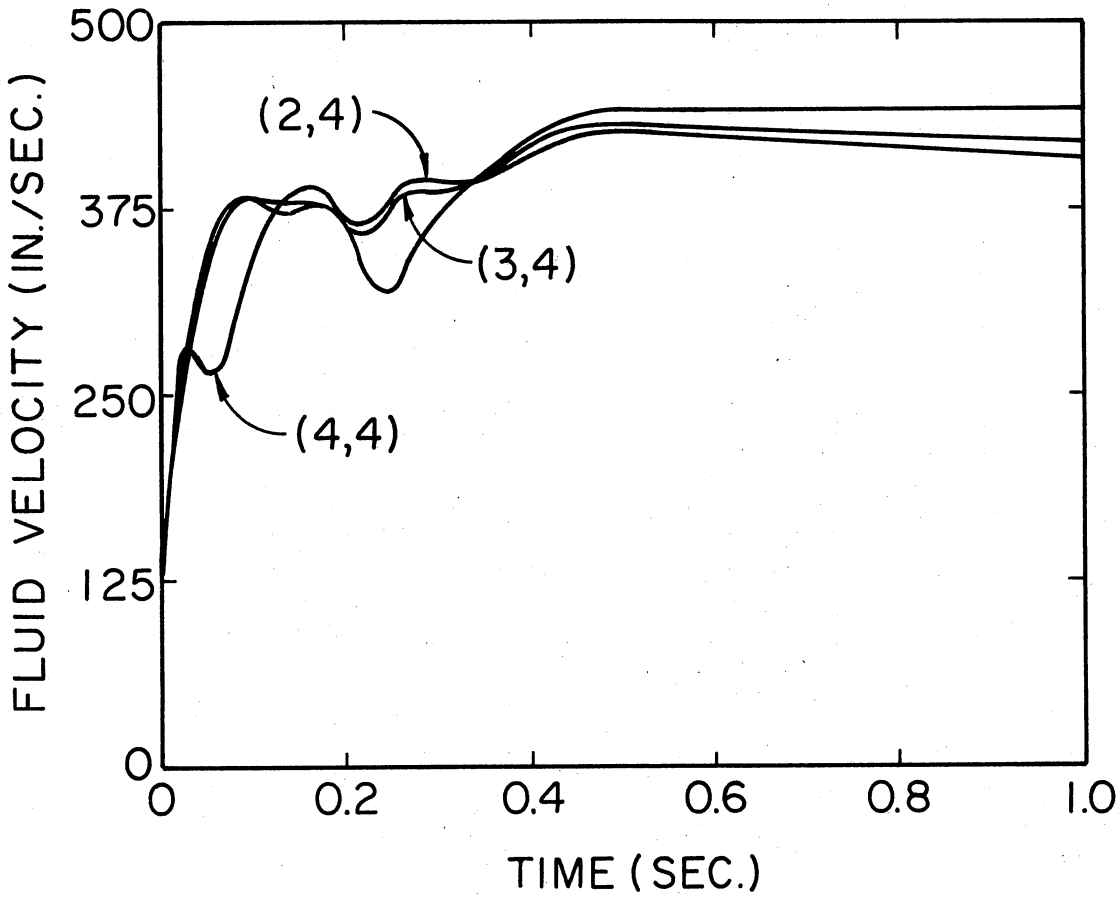



Figure 17. Velocity Response With Fourth-Order Operational Model

approximately 0.35 seconds. This corresponds to the approximate time at which the temperature responses in Figures 14 and 16 indicate an outlet temperature of 50 degrees. It can be seen that independent of the temperature model, the predicted outlet temperature is approximately midway between the initial and final value when the integral of the velocity predicts that the temperature front reaches the outlet.

Two distinct problems arise in attempting to utilize the operational solution for transmission line temperature predictions. One problem is the need to define fluid properties which depend on pressure and temperature and thus vary spatially as well as temporally. Since only line end conditions are available, it does not seem possible to accurately determine an effective bulk temperature and pressure. Averaging the end values as done in the above example is a first approximation to the bulk quantities, but it is obviously a poor approximation. Considering the above example, one can readily recognize that immediately after the step in inlet temperature the bulk temperature is not the average of the end conditions but is rather the bulk temperature prior to the step change. However, since spatial dependence was eliminated in the development of the solution, there does not appear to be an alternative to calculating bulk temperature and pressure based on the instantaneous end conditions.

The second problem in utilizing the operational model is implicit in the specification of an inlet and outlet for the transmission line. When the model is implemented, the inlet and outlet must be selected, and if the flow direction changes, the thermal response model is no longer valid. Ideally, one could switch between two models each of which are valid for one flow direction, but this results in having to reinitialize the alternate model at each flow reversal. The reinitialization cannot,

in general, be done so as to maintain a continuously correct solution making the whole concept of allowing flow reversals very undesirable. In contrast, flow reversals occur in physical transmission lines and should not be ignored if one is to have a realistic transmission line model.

One alternative to reduce the above problems is to model each line as a series of short lines. This does not eliminate either problem, but it does localize errors in determining bulk properties and temperature dependence due to flow direction. However, such segmenting actually is self-defeating since the effect is to reintroduce the spatial dependence which the operational solution eliminates. It seems that rather than segment lines, one should conclude that the operational solution derived in Chapter III is not a practical model for use in general hydraulic system simulation applications. A model which does appear practical is the method of characteristics model which is considered in the following example.

## Method of Characteristics Model

This section parallels the preceding section in presenting and discussing the results of simulated pressure, velocity, and temperature response following a step change in inlet temperature. The system is depicted in Figure 13 with all conditions as discussed earlier. The thermal response model used for this example is the method of characteristics model from Chapter III, and the pressure and velocity model is from Appendix B. Four sets of results are compared in this section with each set resulting from using a different number of grid points along the transmission line while keeping the total length constant.

As discussed in Appendix B, the points R and S in Figure 31 must lie

between A and B for the method of characteristics solution to be stable. This implies that for the maximum magnitude of (V + a), one must assure that

$$\Delta t \leq \frac{\Delta x}{|V + a|}$$

and thus the grid size $\Delta X$ implicitly determines the allowable step size, $\Delta t$, or conversely. For this example with a line 120 inches long the time steps used were 0.0001, 0.00025, 0.005, and 0.001 seconds which correspond to grid increments of 5.45, 13.33, 30.0, and 60.0 inches, respectively.

The outlet temperature and velocity responses for the method of characteristics model with the above time steps are shown in Figures 18 and 19. As expected, the smaller grid sizes produce a sharper rise in outlet temperature, and one would expect the response to approach a step change with even smaller time steps and grid increments. For the largest step, which corresponds to only two grids for the entire line, the response reflects the reduction in the order of the resulting model, and one cannot expect a sharp temperature rise to occur. Unlike the operational solution, there is no oscillation in any of the responses which is a result that is consistent with the actual results one would expect to observe in the laboratory.

Figure 20 is the simulated pressure response at the midpoint of the transmission line. The results are consistent with Figures 18 and 19, and again, the larger grid sizes produce faster responses. The pressure rise which accompanies the movement of the temperature front through the line is an interesting phenomena which the operational model could not demonstrate. The interaction of pressure and temperature mainly through

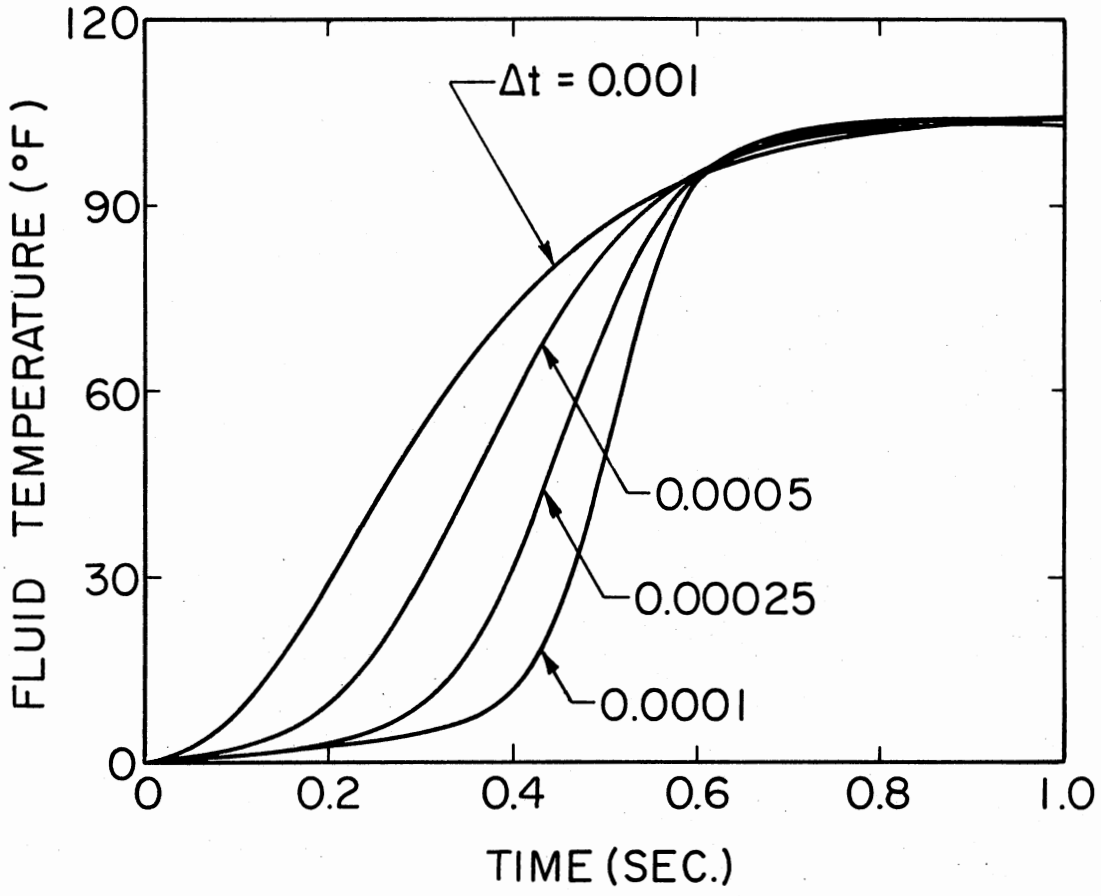



Figure 18. Outlet Temperature Response With Method of Characteristics Model

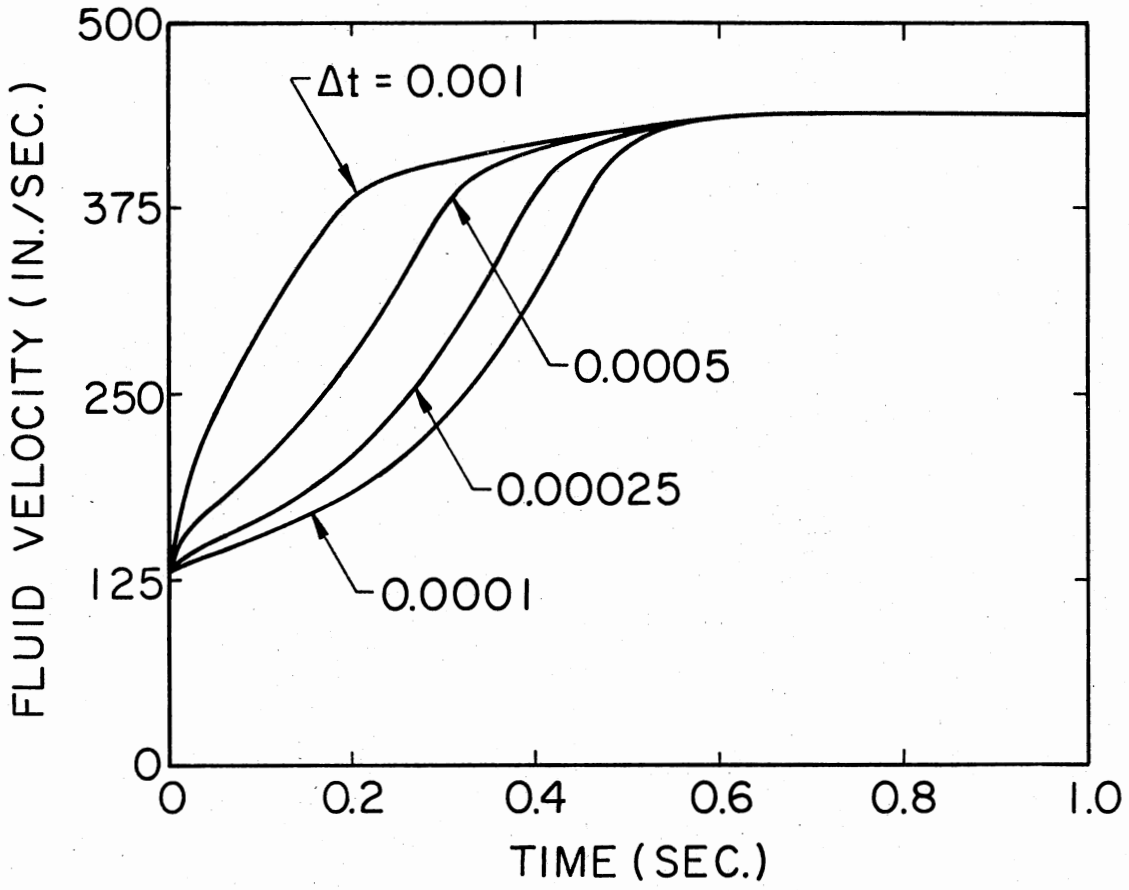



Figure 19. Velocity Response With Method of Characteristics Model

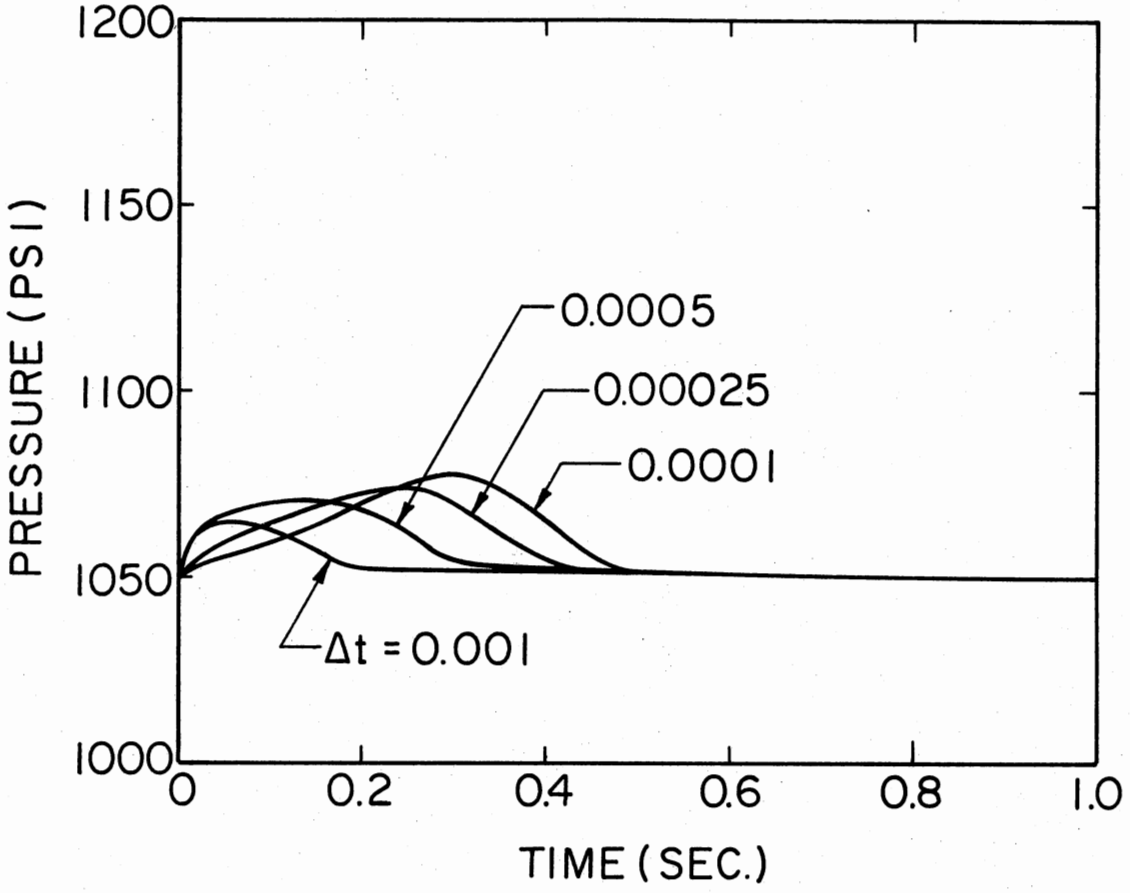



Figure 20. Pressure Response at the Line Center With Method of Characteristics Model

viscosity is clearly shown by Figure 20.

The one-dimensional wall model developed in Chapter III was used for the method of characteristics line simulation. Figure 21 shows the resulting responses for the inside and outside wall temperature at the line inlet, at the center, and at the outlet. The results shown are for a time step of 0.0001 seconds, and the responses for the other time steps are very similar. At the final time, the predicted wall temperatures are increasing at approximately 30 degrees Fahrenheit per second. Since the inside convection coefficient is much higher than the outside value, one would expect that if the simulation had continued, the wall temperature would approach that of the fluid in an additional four to five seconds.

The method of characteristics line model can accommodate flow reversals with no difficulty. The inlet and outlet are not specified a priori for the method of characteristics, and all of the instantaneous line end combinations of inlet-outlet, outlet-inlet, inlet-inlet, and outlet-outlet are valid. Since for this example the constant pressure sources were used, the terms inlet and outlet have been utilized for discussion purposes only.

The problem which exists with the operational model of determining fluid properties also exists in the method of characteristics. The difference in velocity and pressure responses due to grid size variation is shown by Figures 19 and 20. These differences are caused by the variation in the predicted temperature responses in Figure 18. As the model for the larger grid sizes predicts a faster temperature rise, there is a corresponding decrease in the most significant fluid property, viscosity. The decreased viscosity leads to higher accelerations and thus the increased velocity response as shown in Figure 19. The increased rate of

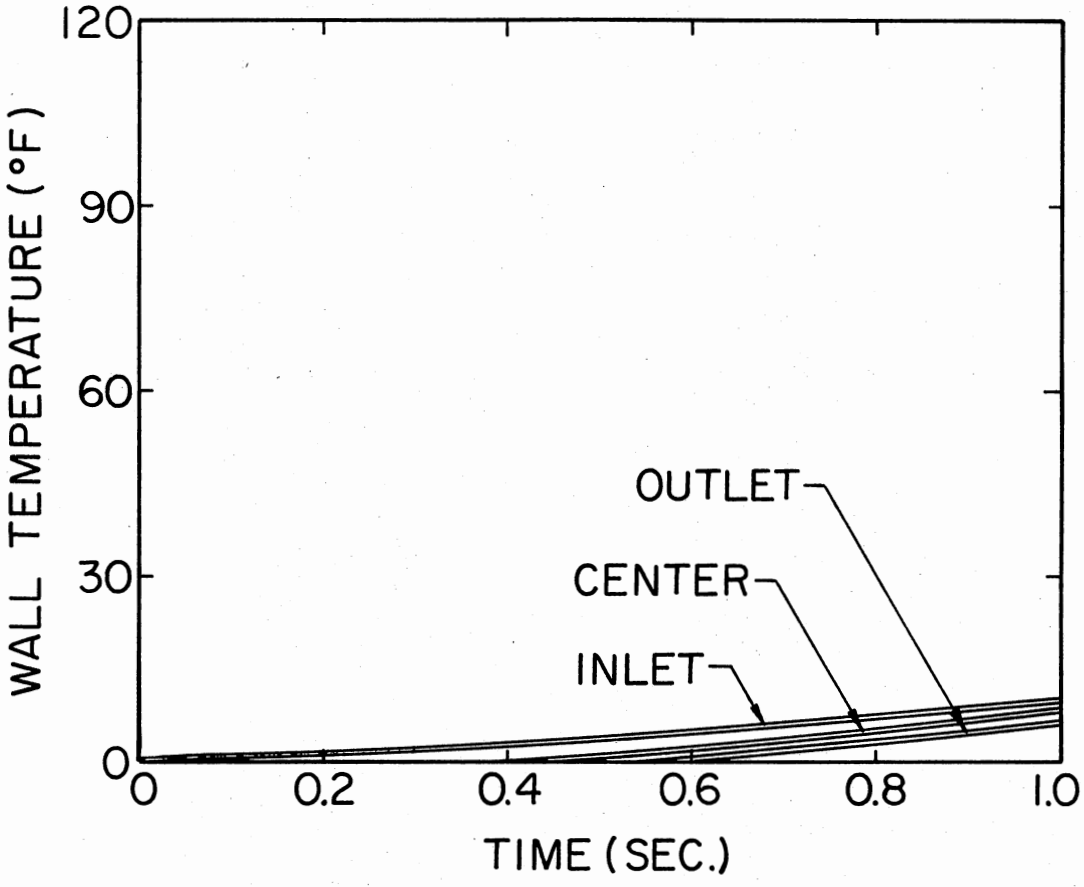



Figure 21. Wall Temperature Response With Method of Characteristic Model and One-Dimensional Wall Model

temperature response results from interpolating the state of the fluid between grid points.

Figure 22 depicts four grid points from a general grid as shown in Figure 8. In order to achieve an accurate pressure and velocity solution, $\Delta x$ and $\Delta t$ are chosen such that points R and S are near A and B, respectively. Since the magnitude of the acoustic velocity, a, is much greater than the fluid velocity, V, one can, for discussion purposes, consider R approaching A, S approaching B, and H relatively close to C. Calculating the pressure and velocity at D based on interpolated values at R and S (basically A and B) is a good approximation which is accepted to be of reasonable accuracy (8,9,10). However, point H must not be taken as C or the effect of fluid velocity on temperature response will be negated. Defining the temperature at point H by interpolating from A to C (or from C to B) gives a poor estimate when a temperature front exists as in the example. There appear to be two alternatives one can employ in simulations which will involve large temperature variations.

The most simple approach to achieve more accurate results is simply to use a small time step and grid size as demonstrated by Figures 18 and 19. The second alternative which appears to merit investigation consists of utilizing a smaller spatial grid for propagating temperature than that used for propagating pressure and velocity. The smaller grid with the corresponding temperatures could be used to predict fluid properties needed for the pressure and velocity calculations, and interpolated pressure and velocity values could be used for temperature propagations. This dual grid approach would require more calculation per time increment, implying a need to investigate both the accuracy and computational load as compared to using a small step size with only one grid size. The

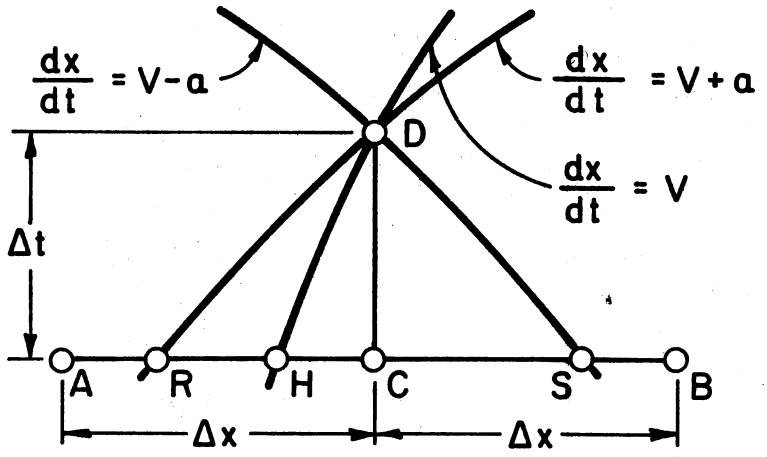



Figure 22. Illustration of the Characteristics Involved in Propagating Interior Grid Points

author has not investigated the dual grid approach.

Variable and moving grids have been used by investigators in flow field studies. The study which utilized a variable grid and most nearly parallels transmission line response was reported by Benson (30). While the variable grid method used by Benson could be implemented for the reported study involving one component, it appears that such a method would be extremely difficult to implement effectively in a general hydraulic system simulation program.

The first two example responses have demonstrated the response of the transmission line models subject to a severe input, i.e., a step change. The following example demonstrates the response of a small hydraulic system with parts of the system subjected to different temperature conditions.

## Simulation of a Position Control Circuit

This section discusses the simulation of the position control circuit shown in Figure 23. The system consists of a closed-center position system made up of the valve, actuator, linkage, and load with a pressure-compensated variable-displacement pump for the hydraulic power source. The line between the pump and valve is represented by a method of characteristics model with the same parameters used for the preceding examples.

The full details of each component model will not be presented here, but the most important points will be summarized. The pump is modeled as a nine piston pump with a maximum flow rate of approximately 33 gallons per minute at 3750 revolutions per minute. Initially the pump is assumed to be delivering maximum flow at the set pressure of 3000 pounds per square inch. The pump is modeled by a set of five first-order

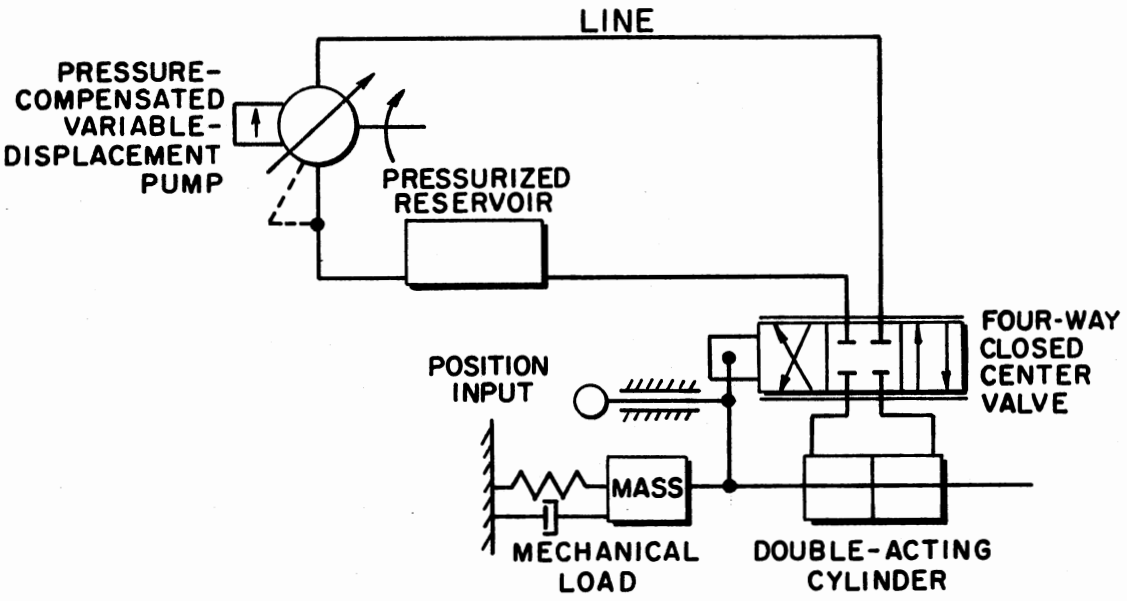



Figure 23. Schematic Representation of the Simulated Hydraulic Circuit

differential equations.

The valve is modeled as being closed-centered with an overlap of $10^{-10}$ inches. It is assumed that the discharge coefficient for the metering orifices is 0.65 with a spool diameter of 0.375 inches. The linkage which connects the valve, actuator, and external input is designed to have a position gain of 10.

The actuator load consists of a 1000 pound weight plus a spring with a spring rate of 2000 pounds per inch. The damping is assumed to be 105 pound seconds per inch at a temperature of 100 degrees Fahrenheit and 150 pound seconds per inch at 0 degrees. These damping values assume that 5 percent of the damping at 100 degrees is due to internal damping in the actuator and that 95 percent is external to the actuator and independent of actuator temperature. The increase in damping results from the increased viscosity of the fluid in the actuator as the temperature is decreased. The piston diameter for the actuator is 2.75 inches with a total stroke of 10 inches. The original null position is chosen with the actuator extended 5 inches.

The model for the pump, line, actuator, and reservoir includes thermal response for both the fluid and the component wall. In each case the wall model is from Chapter III with the reservoir, line, and actuator modeled as cylindrical walls with the pump modeled as a cube. The thermal model for the valve assumes the valve to be adiabatic with negligible fluid and wall mass.

The multiport representation for the system model is shown in Figure 24. Each pressure and flow port contains a temperature variable which is represented as discussed in Chapter III. The models for all pressure flow ports were derived according to Chapter IV and assume that flow can

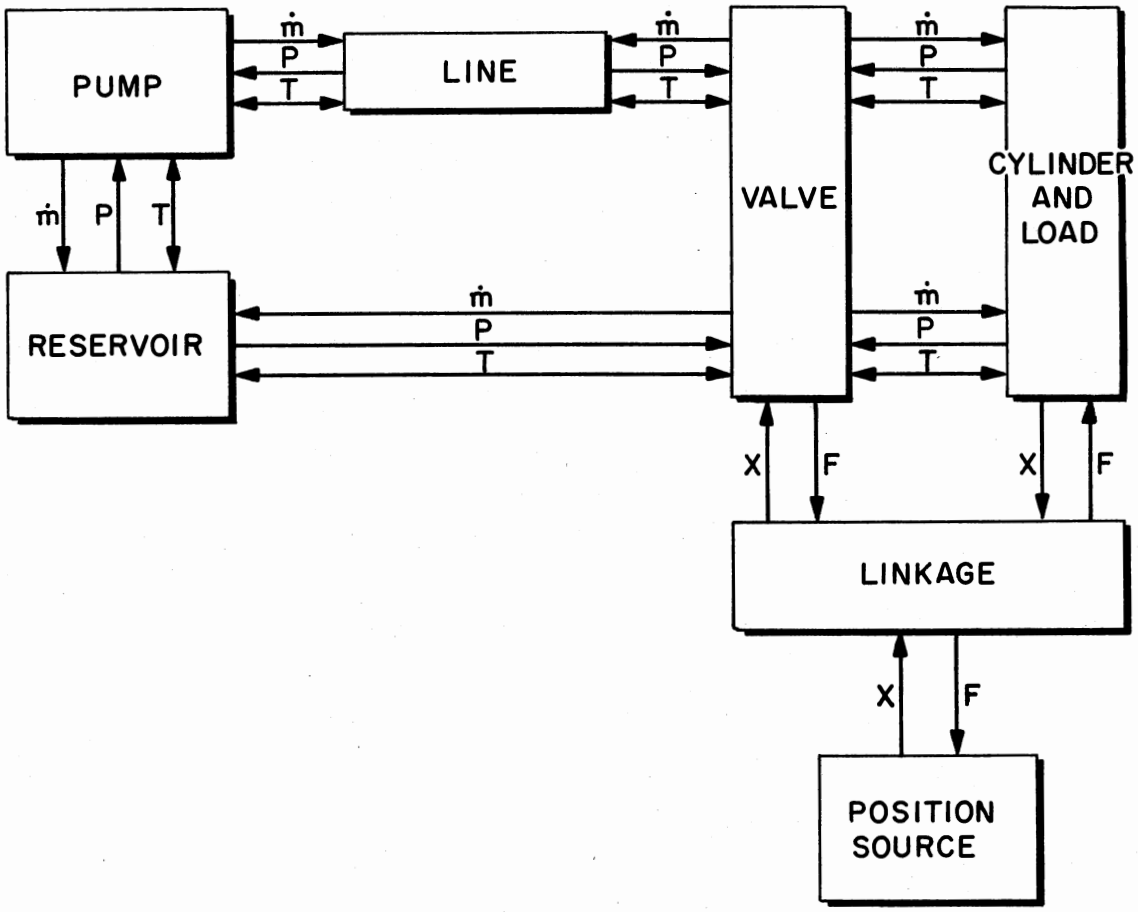



Figure 24. Multiport Representation of the Simulated Hydraulic Circuit

be in either direction at any port. The algebraic equations for the total system model were solved using a sparse matrix method as discussed in Appendix D with a prototype simulator as presented in Appendix C.

The system represented by Figures 23 and 24 was simulated using a truncated ramp input to the linkage. The ramp was chosen to produce an input of 0.1 inches in 5 milliseconds which implies an actuator position change of 1 inch. For one simulation all initial temperatures were assumed to be 100 degrees Fahrenheit and the environmental temperature was chosen to be a constant 100 degrees. For a second simulation the initial temperature of the line and the actuator were reduced to zero degrees with the environment also at zero. The following discussion considers the results of the two simulations.

The position response of the actuator to the ramp input is shown in Figure 25. The two responses show very little difference and may lead one to conclude either that temperature has no effect or that the model is not predicting any appreciable differences in response due to changes in temperature. However, the corresponding actuator velocity shown in Figure 26 shows that temperature is indeed having an effect on the predicted response. For the high temperature case, there is less damping in the actuator with a resulting resonant response as the actuator reaches the new null position. The lower temperature with the corresponding high damping is less oscillatory as the valve closes and the fluid entrapped in the actuator increases the effective spring rate for the actuator and load. Figure 26 also shows, as expected, that the initial velocity is higher for the warmer system.

The differential pressure across the actuator is shown in Figure 27. The responses are consistent with Figure 26, and again the warmer system

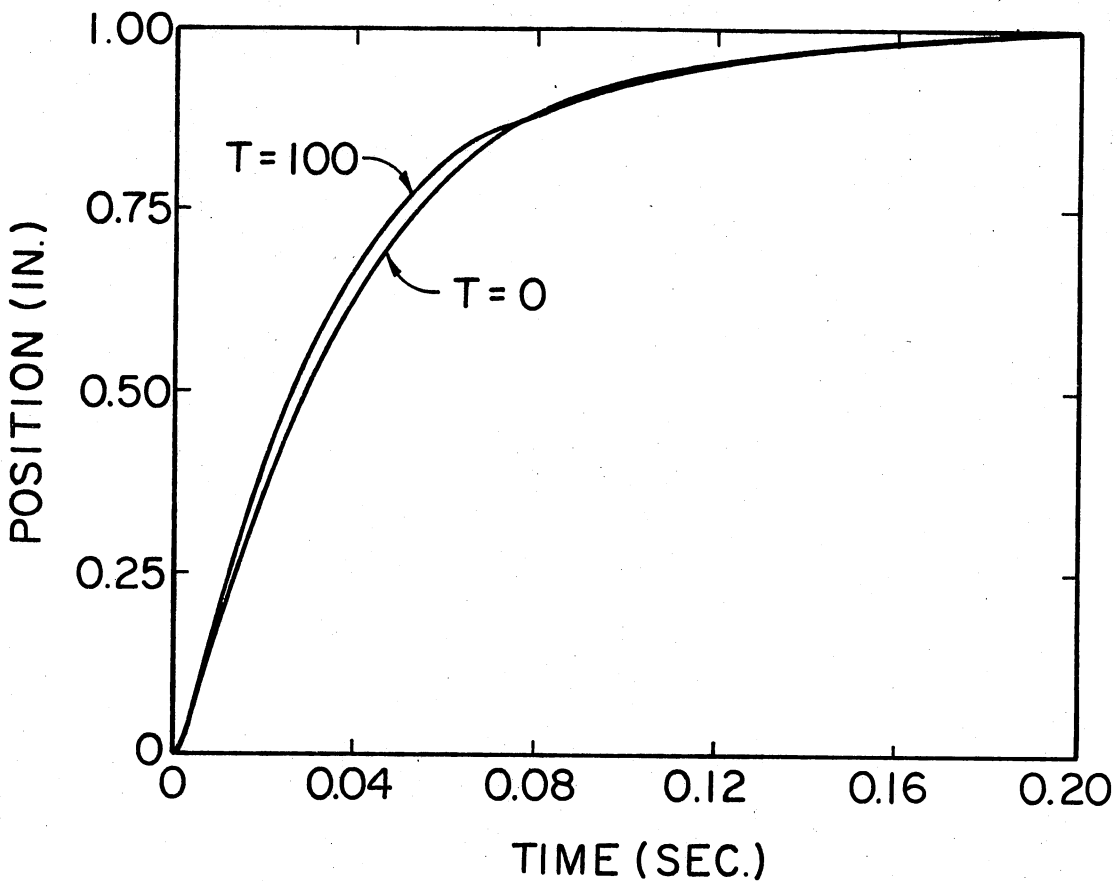



Figure 25. Position Response of the Actuator and Load Versus Time

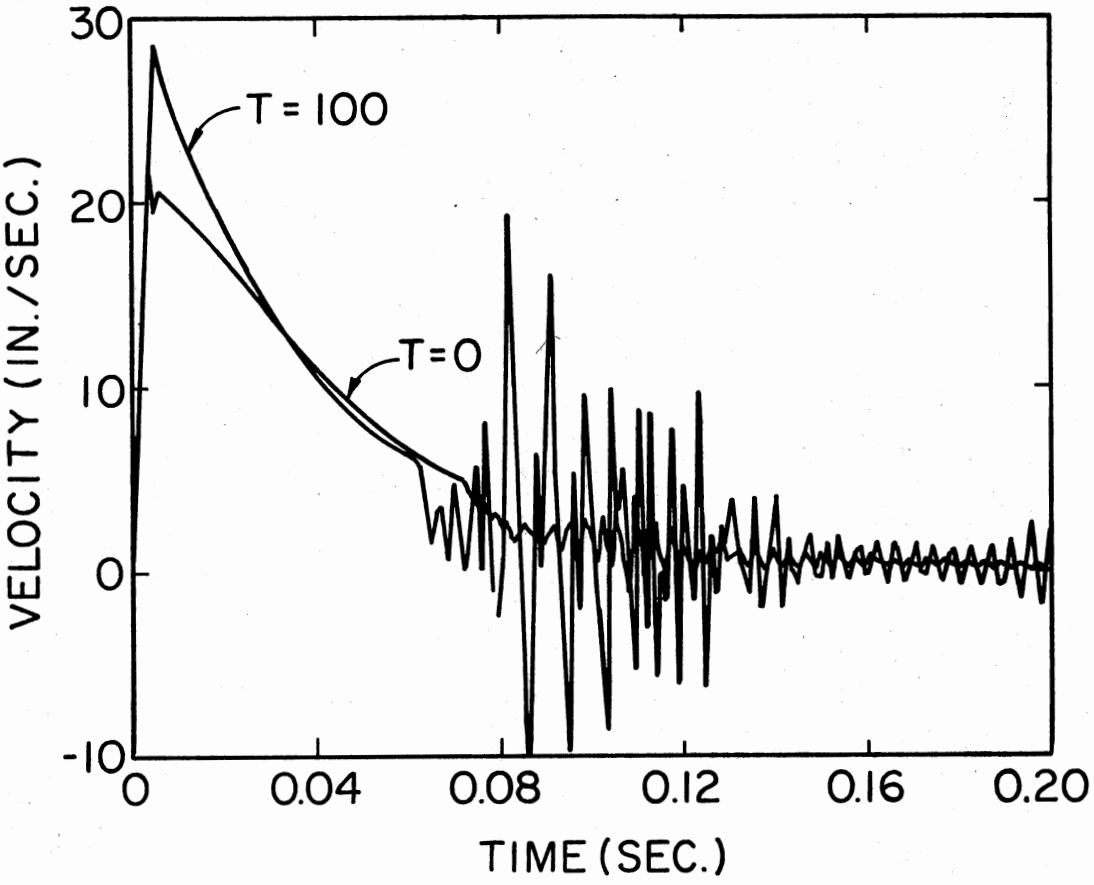



Figure 26. Velocity Response of the Actuator and Load Versus Time

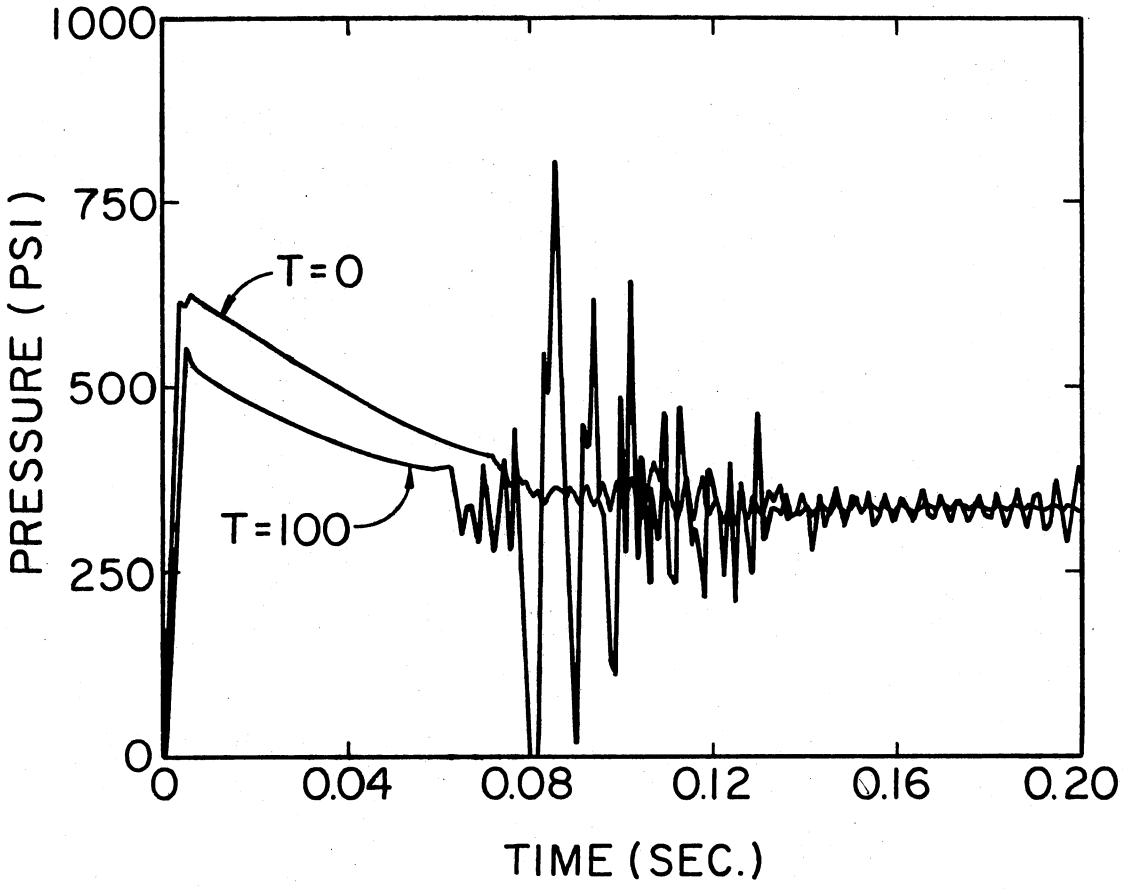



Figure 27. Pressure Differential Across the Actuator Versus Time

is more oscillatory as the control valve closes. The final differential pressure corresponds to that required to offset the 2000 pound spring force with the actuator area of 6 square inches. The initial pressure differential is zero due to the assumed initial steady-state conditions.

The pressure and temperature responses at the connections of the actuator and valve and the line and valve are shown in Figures 28 and 29. The initial conditions for the line were 3000 pounds per square inch with the fluid at zero velocity. Both the pressure and temperature responses show a relatively smooth response until the valve begins to close after which the effects of the mechanical oscillation are seen. The initial temperature drops shown in Figure 29 are not what the author expected, although a temperature drop could occur since fluid expansion is included in the line model. The temperature for the low temperature simulation does not appear to approach the pump outlet temperature of approximately 100 degrees Fahrenheit since the through flow is not sufficient for the temperature front to have reached the valve. A larger input should result in a temperature profile at the downstream end of the line which is similar to the response shown in Figure 18 with a time step of 0.0001 seconds.

The routines developed for the examples in this chapter were written to demonstrate the models and algorithms. As a result, the coding was not particularly clean and many direct but slow computational approaches were used. For example, the acoustic velocity was recalculated for every grid point in the method of characteristics line model based on the current interpolated fluid properties. Calculations such as this can be simplified or eliminated in a final program and thus make it possible to then evaluate the cost of a simulation in a realistic manner. Due to

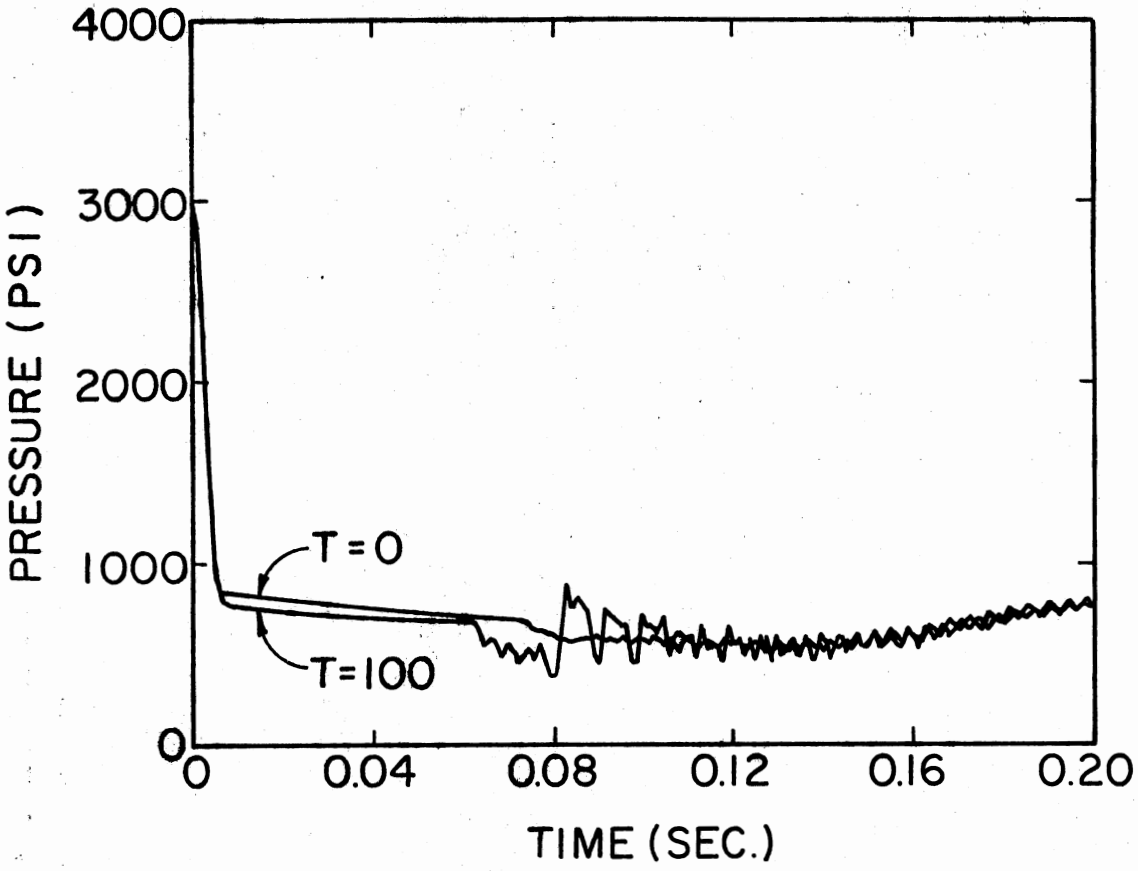



Figure 28. Pressure in the High-Pressure End of the Actuator Versus Time

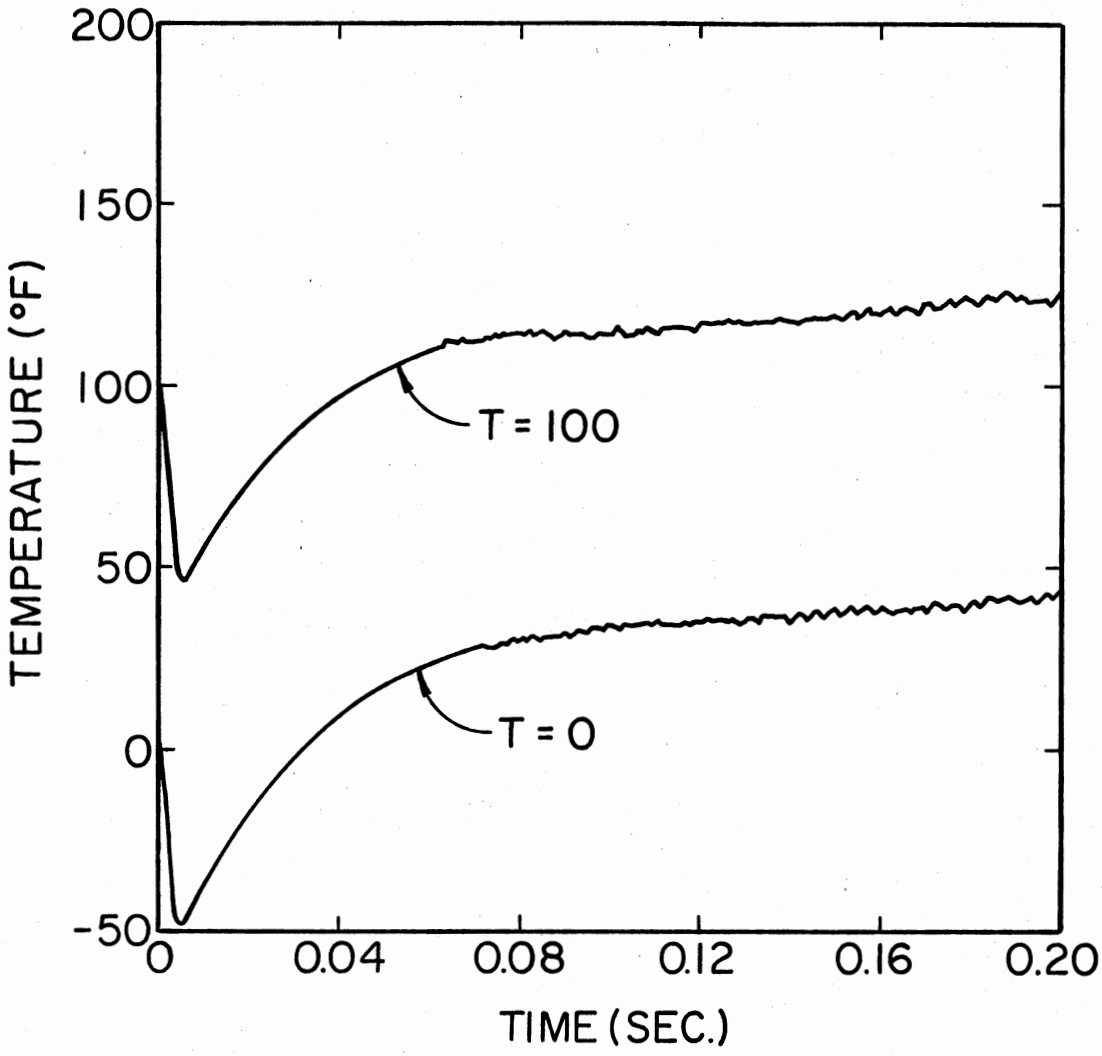



Figure 29. Temperature at the Connection of the Valve and Line Versus Time

reasons such as this an evaluation of the computation time for the example simulations would be of little value.

This example has demonstrated the application of models and algorithms developed in this work to a small system simulation. The programs used for the simulation were based on the algorithms and concepts from Chapter IV and Appendices C and D with the models from Chapter III and Appendices A and B. The work of the preprocessor from Appendix C was done manually to construct the system model for the prototype simulation program. Each component as shown in Figure 24 was modeled by a separate Fortran subroutine using the storage concepts from Appendix C. There did not appear to be any obstacles which would prevent the full implementation of a hydraulic system simulation program based on the preprocessor and simulator approach.

# CHAPTER VI

## CONCLUSIONS AND RECOMMENDATIONS

Thermal response can be predicted simultaneously with mechanical response for general hydraulic circuits. The models of Chapter III demonstrate that transient thermal effects can be included in a system model in a direct manner and that the equations for thermal response are of the same forms as for mechanical response. Except for the operational line model, thermal models can be implemented with no a priori flow direction assumptions, which enhances the consistency between thermal and mechanical models. The component wall thermal response model in Chapter III allows wall temperature to depend on one spatial dimension with the same order differential equation which is required for a lumped temperature model.

A Fortran program can be written based on the design in Appendix C and the sparse matrix implementation concepts from Appendix D. The component orientation throughout the program design assures that the program will be readily expandable to any system and that new component routines can be added easily. Component routine syntax can be very simple due to both the use of Fortran argument lists and the use of the sparse matrix solution technique.

## Recommendations for Further Study

The simulation of hydraulic systems is an area in which much work is still required. This study in thermal response must also be followed by other investigations. Some recommendations which apply in part to thermal response studies and in part to the general area of system simulation are the following.

1) Develop the preprocessor and simulator in Fortran for execution on at least IBM and CDC computers. Also develop a component library which can be expanded continually.
2) Design and perform experimental studies to allow the verification of the thermal models developed in this study and to identify appropriate heat transfer coefficients.
3) Develop alternate operational thermal line models which will allow flow reversals to occur.
4) Develop an algorithm for calculating the steady state response of a general dynamic system both for initial conditions and for steady state analysis.
5) Investigate the possibility of combining the algorithms by Smith (1), the integration algorithms by Gear (70) and Iyengar (71), and this work within a component-oriented simulation program.

The first recommendation should be able to be accomplished with little difficulty. The experimental work must be done in order to verify thermal models which can then be used with confidence in system simulations. The additional thermal line modeling work is important for the reasons discussed in Chapter V. The last two recommendations are applicable to simulation in general and are important studies independent of

the particular systems considered. It seems possible that 4 and 5 may be achieved if Gear's integration method can be implemented successfully in a component oriented program. No matter what approach is successful, the final result must be user-oriented since any of these efforts can expect to be applied only if they are available in a highly usable form.

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# APPENDIX A

## DEVELOPMENT OF THE THERMAL ENERGY EQUATION FOR A HYDRAULIC TRANSMISSION LINE

The momentum and continuity equations generally used to model fluid transmission lines constitute a one-dimensional line model with friction assumed to be concentrated at the line wall. A thermal energy model which is consistent with the pressure and flow model is developed in this appendix. This energy model is used in Chapter III to develop the solutions for the thermal response of the fluid in a transmission line.

A cylindrical fluid element is shown in Figure 30. An energy balance on the element can be expressed as

$$\left[\begin{array}{c}\text{rate of accumulation}\\ \text{of internal and}\\ \text{kinetic energy}\end{array}\right] = \left[\begin{array}{c}\text{rate of internal and}\\ \text{kinetic energy in}\\ \text{by convection}\end{array}\right]$$

$$- \left[\begin{array}{c}\text{rate of internal and}\\ \text{kinetic energy out}\\ \text{by convenction}\end{array}\right] + \left[\begin{array}{c}\text{net rate of energy}\\ \text{added by}\\ \text{heat transfer}\end{array}\right]$$

$$- \left[\begin{array}{c}\text{net rate of work}\\ \text{done by element}\\ \text{on surrounding}\end{array}\right] + \left[\begin{array}{c}\text{increase in internal}\\ \text{energy due to}\\ \text{viscous dissipation}\end{array}\right] .$$

This statement of the unsteady energy balance on the element is similar to that presented by Bird, et al. (64). If one assumes that gravitational effects are negligible and that the last term is exactly the frictional losses from the flow model, then each of the terms can be

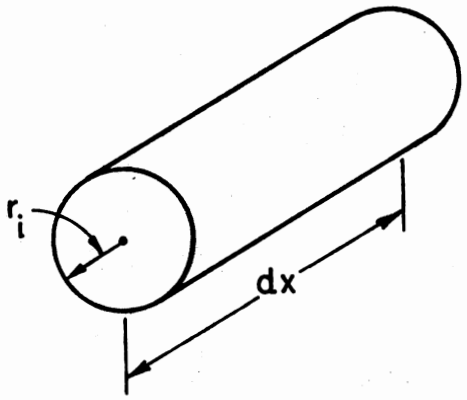


Figure 30. Representation of a Cylindrical Fluid Element

evaluated as follows:

$$\left[\begin{array}{c}\text{rate of accumulation}\\ \text{of internal and}\\ \text{kinetic energy}\end{array}\right] = \frac{\partial}{\partial t}\left(\pi r_i^2 \rho u dx + \frac{\pi r_i^2}{2Jg_c}\rho V^2 dx\right) \quad ,$$

$$\left[\begin{array}{c}\text{rate of internal and}\\ \text{kinetic energy in}\\ \text{by convection}\end{array}\right] = \pi r_i^2 \rho V\left(u + \frac{V^2}{2Jg_c}\right) \quad ,$$

$$\left[\begin{array}{c}\text{rate of internal and}\\ \text{kinetic energy out}\\ \text{by convection}\end{array}\right] = \pi r_i^2 \rho V\left(u + \frac{V^2}{2Jg_c}\right) + \frac{\partial}{\partial x}\left(\pi r_i^2 \rho V\left(u + \frac{V^2}{2Jg_c}\right)\right)dx \quad ,$$

$$\left[\begin{array}{c}\text{net rate of energy}\\ \text{added by}\\ \text{heat transfer}\end{array}\right] = \pi r_i^2 \frac{\partial}{\partial x}\left(k \frac{\partial T_f}{\partial x}\right)dx + 2\pi r_i q_w dx \quad ,$$

$$\left[\begin{array}{c}\text{net rate of work}\\ \text{done by element}\\ \text{on surrounding}\end{array}\right] = -\pi r_i^2\left(\frac{PV}{J} - \left(\frac{PV}{J} + \frac{1}{J}\frac{\partial}{\partial x}(PV)dx\right)\right) - \frac{2\pi r_i}{J}\tau_w V dx \quad ,$$

and

$$\left[\begin{array}{c}\text{increase in internal}\\ \text{energy due to}\\ \text{viscous dissipation}\end{array}\right] = \frac{2\pi r_i}{J}\left|\tau_w V\right| dx \quad .$$

The term $q_w$ is defined as the heat transfer from the wall to the fluid element and will later be expressed as convective heat transfer.

If the algebraic terms are substituted into the above equality with the terms of order dx retained, an energy equation can be written as

$$\frac{\partial}{\partial t} P\left(u + \frac{1}{2Jg_c} V^2\right) = - \frac{\partial}{\partial x} \left(\rho V\left(u + \frac{1}{2Jg_c} V^2\right)\right) + k \frac{\partial^2 T_f}{\partial x^2}$$

$$+ \frac{2}{r_i} q_w - \frac{1}{J} \frac{\partial}{\partial x} (PV) - \frac{2}{Jr_i} \tau_w V + \frac{2}{Jr_i} |\tau_w V| . \quad (A.1)$$

The corresponding momentum, continuity, and state equations used to model transmissions as presented in references (10), (11), and (62) are:

$$\frac{\partial P}{\partial x} + \frac{\rho}{g_c} \left(\frac{\partial V}{\partial t} + V \frac{\partial V}{\partial x}\right) = - \frac{2\tau_w}{r_i} \quad , \quad (A.2)$$

$$\frac{\partial P}{\partial t} + V \frac{\partial P}{\partial x} + \frac{a^2 \rho}{g_c} \frac{\partial V}{\partial x} = 0 \quad , \quad (A.3)$$

and

$$\left.\frac{\partial \rho}{\rho}\right|_{T_f} = \left.\frac{\partial \rho}{\beta}\right|_{T_f} \quad (A.4)$$

where

$$a = \sqrt{\frac{\beta}{\rho}}$$

and the concentrated wall shear stress $\tau_w$ creates a steady loss which can be expressed in terms of the widely used friction factor, f, as

$$\tau_w = \frac{f}{4} \frac{\rho}{g_c} \frac{V^2}{2} .$$

An unsteady, frequency-dependent shear effect may also be included as discussed by Zielke (10), Brown (11), and Trikha (12).

Equation A.1 is the total energy and can be simplified by subtracting the mechanical energy equation which is the velocity, V, times Equation A.2. If one multiplies V/J times A.2 and subtracts the result from A.1, the final result is

$$\frac{\partial}{\partial t}(\rho u) = -\frac{1}{2Jg_c} V^2 \left(\frac{\partial \rho}{\partial t} + V \frac{\partial \rho}{\partial x} + \rho \frac{\partial V}{\partial x}\right) - u \frac{\partial}{\partial x}(\rho V) - \rho V \frac{\partial u}{\partial x} + k \frac{\partial^2 T_f}{\partial x^2} + \frac{2}{r_i} q_w$$

$$- \frac{P}{J} \frac{\partial V}{\partial x} + \frac{2}{J r_i} |\tau_w V| \quad . \qquad (A.5)$$

The term

$$\frac{\partial \rho}{\partial t} + V \frac{\partial \rho}{\partial x} + \rho \frac{\partial V}{\partial x}$$

is the continuity equation in terms of density and velocity and is identically zero. This reduces Equation A.5, after rearranging, to

$$\rho \frac{\partial u}{\partial t} + u\left(\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho V)\right) = -\rho V \frac{\partial u}{\partial x} + k \frac{\partial^2 T_f}{\partial x^2} + \frac{2}{r_i} q_w - \frac{P}{J} \frac{\partial V}{\partial x} + \frac{2}{J r_i} |\tau_w V| \qquad (A.6)$$

where the term in parenthesis is again identically the continuity equation and is thus zero. The thermal energy equation in terms of internal energy can then be stated as

$$\rho\left(\frac{\partial u}{\partial t} + V \frac{\partial u}{\partial x}\right) = -\frac{P}{J} \frac{\partial V}{\partial x} + k \frac{\partial^2 T_f}{\partial x^2} + \frac{2}{r_i} q_w + \frac{2}{J r_i} |\tau_w V| \quad . \qquad (A.7)$$

This equation can be stated in terms of pressure and temperature if one assumes $u = u(P, T_f)$ as done by Bird, et al. (64). An identity then is

$$du = \left(\frac{\partial u}{\partial \rho}\right)_{T_f} d\rho + \left(\frac{\partial u}{\partial T_f}\right)_{\rho} dT_f$$

$$= -\left(-P + T_f\left(\frac{\partial P}{\partial T_f}\right)_{\rho}\right) \frac{d\rho}{J\rho^2} + c_v dT_f \qquad (A.8)$$

where $c_v$ is the specific heat at constant volume. If identity A.8 is substituted into A.7 and the continuity equation is substituted twice, the result is

$$\rho c_v \left(\frac{\partial T_f}{\partial t} + V \frac{\partial T_f}{\partial x}\right) = - \frac{T_f}{J} \left(\frac{\partial P}{\partial T_f}\right)_\rho \frac{\partial V}{\partial x} + k \frac{\partial^2 T_f}{\partial x^2} + \frac{2}{r_i} q_w + \frac{2}{J r_i} |\tau_w V| \quad . \quad (A.9)$$

The partial derivative of pressure with respect to temperature at constant density must be evaluated in order to apply Equation A.9. This can be accomplished as follows. In general, $\rho = \rho(P, T_f)$, and constant density implies $d\rho = 0$. Thus

$$d\rho = 0 = \left(\frac{\partial \rho}{\partial T_f}\right)_P dT_f + \left(\frac{\partial \rho}{\partial P}\right)_{T_f} dP$$

$$\left(\frac{dP}{dT_f}\right)_\rho = - \frac{\left(\frac{\partial \rho}{\partial T_f}\right)_P}{\left(\frac{\partial \rho}{\partial P}\right)_{T_f}} \quad . \quad (A.10)$$

The state Equation A.4 for constant temperature can be rearranged to

$$\left(\frac{\partial \rho}{\partial P}\right)_{T_f} = \frac{\rho}{\beta}$$

and a coefficient of thermal expansion can be defined as

$$\alpha = \frac{1}{\rho} \left(\frac{\partial \rho}{\partial T_f}\right)_P \quad .$$

The final thermal energy equation can be written as

$$\rho c_v \left(\frac{\partial T_f}{\partial t} + V \frac{\partial T_f}{\partial x}\right) = - \frac{\alpha T_f}{J} \left(\frac{\partial P}{\partial t} + V \frac{\partial P}{\partial x}\right) + k \frac{\partial^2 T_f}{\partial x^2} + \frac{2h_i}{r_i} (T_i - T_f) + \frac{2}{J r_i} |\tau_w V| \quad (A.11)$$

where $T_i$ is the inside line wall temperature and the convective heat transfer expression

$$q_w = h_i (T_i - T_f)$$

has been substituted. This is the form of the energy used for the method of characteristics solution in Chapter III. The operational solution, also in Chapter III, is based on a simplified form of Equation A.11.

Reference (8) contains fluid property data for three hydraulic fluids which are MIL-H-5606, MIL-H-83282, and Skydrol 500B. Based on the data for these fluids, the maximum value of $(\alpha T_f/J)$ can be calculated to be $3.6 \times 10^{-5}$. Thus, the term containing this coefficient is insignificant. The thermal conductivity of hydraulic fluids is of the order of $10^{-6}$ Btu/(sec-in-°F) which makes the thermal conductivity term negligible unless a large second derivative of temperature exists; however, the conductivity term can be neglected if the length to radius ratio for a line is large which implies that heat transfer to the wall will dominate.

The friction term can be eliminated by realizing that the pressure drop due to friction can be expressed in terms of $\tau_w$ and then performing a simple order of magnitude analysis. If $\tau_w$ and fluid properties are constant, one can express the steady-state temperature increase due to friction for a line of length $\ell$ as follows:

$$\rho c_v \Delta T_f = \frac{\Delta P}{J} \quad ,$$

$$\frac{2\tau_w}{r_i} = \frac{\Delta P}{\ell} \quad ,$$

$$\Delta T_f = \frac{2\tau_w \ell}{\rho c_v r_i J} \quad ,$$

$$= \frac{\Delta P}{\rho c_v J} \quad .$$

If one uses nominal values for $\rho$ and $c_v$ of 0.03 $lb_m/in^3$ and 0.5 Btu/($lb_m$°F) and uses the (in-lb-sec) units, then the result is

$$\Delta T_f = 7.1 \times 10^{-3} \Delta P$$

where $\Delta T_f$ is the total temperature increase in steady-state due to the pressure drop $\Delta P$. Obviously a large pressure drop must be considered before the pressure drop (frictional effect) will be significant.

If all of the terms on the right of Equation A.11 are eliminated except the heat transfer term, then a simplified equation can be written as

$$\rho c_v \left(\frac{\partial T_f}{\partial t} + V \frac{\partial T_f}{\partial x}\right) = \frac{2h_i}{r_i} (T_i - T_f) \quad . \qquad (A.12)$$

This is the equation used for the operational solution in Chapter III.

## APPENDIX B

## SOLUTION OF THE MOMENTUM AND CONTINUITY EQUATIONS BY THE METHOD OF CHARACTERISTICS

The method of characteristics solution for the thermal response of fluid in a line has been developed in Chapter III. This appendix presents a brief summary of the corresponding solution for pressure and flow which must be performed simultaneously. Details of the solution presented here can be found in references (8,9,10,11,12, and 65).

If one multiplies the momentum Equation A.2 by $g_c/\rho$ and the continuity Equation A.3 by $g_c/(\rho a)$, the respective results are

$$\frac{g_c}{\rho a}\frac{\partial P}{\partial x} + \left(\frac{\partial V}{\partial t} + V\frac{\partial V}{\partial x}\right) + \frac{2g_c\tau_w}{\rho r_i} = 0 \qquad (B.1)$$

and

$$\frac{g_c}{\rho a}\frac{\partial P}{\partial t} + \frac{g_c}{\rho a}V\frac{\partial P}{\partial x} + a\frac{\partial V}{\partial x} = 0 \qquad (B.2)$$

The sum and difference of Equation B.1 and B.2 are

$$\frac{g_c}{\rho a}\left[\frac{\partial P}{\partial t} + (a+V)\frac{\partial P}{\partial x}\right] + \left[\frac{\partial V}{\partial t} + (a+V)\frac{\partial P}{\partial x}\right] + \frac{2g_c\tau_w}{\rho r_i} = 0 \qquad (B.3)$$

and

$$-\frac{g_c}{\rho a}\left[\frac{\partial P}{\partial t} + (-a+V)\frac{\partial P}{\partial x}\right] + \left[\frac{\partial V}{\partial t} + (-a+V)\frac{\partial P}{\partial x}\right] + \frac{2g_c\tau_w}{\rho r_i} = 0 \qquad (B.4)$$

Each of the bracketed terms define a total time derivative according to

the identity

$$\frac{d}{dt} = \frac{\partial}{\partial t} + \frac{\partial}{\partial x}\frac{dx}{dt}$$

along the curve

$$\frac{dx}{dt} = V + a \tag{B.5}$$

or

$$\frac{dx}{dt} = V - a \tag{B.6}$$

Equations B.5 and B.6 define the pressure wave characteristics in the (x,t) plane as shown in Figure 31. With the appropriate substitutions, Equations B.3 and B.4 can be written as

$$\frac{g_c}{\rho a}\frac{dP}{dt} + \frac{dV}{dt} = -\frac{2g_c\tau_w}{\rho r_i} \tag{B.7}$$

along the curves

$$\frac{dx}{dt} = V + a$$

and

$$-\frac{g_c}{\rho a}\frac{dP}{dt} + \frac{dV}{dt} = -\frac{2g_c\tau_w}{\rho r_i} \tag{B.8}$$

along the curves

$$\frac{dx}{dt} = V - a$$

The pressure, velocity, and fluid properties at the points R and S in Figure 31 can be evaluated by interpolation since all of the quantities are known at points A, B, and C. If one assumes that the wave characteristics are straight lines from R to D and S to D, a first-order

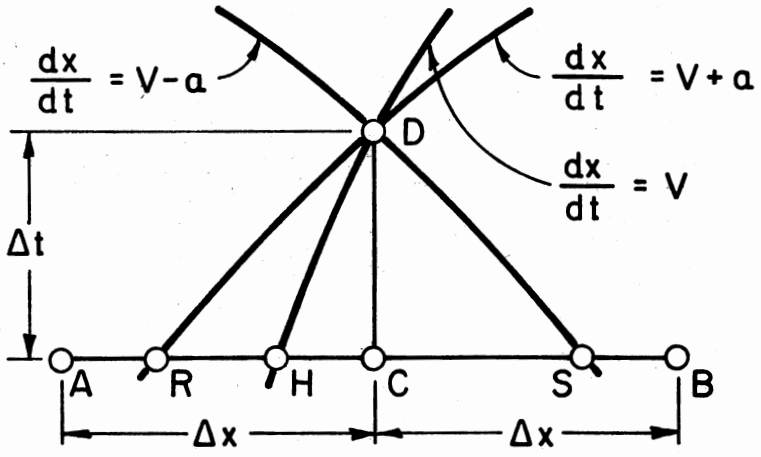


Figure 31. Illustration of the Pressure Wave and Thermal Energy Characteristics

integration of Equations B.7 and B.8 and simultaneous solution for the pressure and velocity at point D results in

$$P_D = -\frac{1}{2}(C_R + C_L) \quad , \tag{B.9}$$

and

$$V_D = \frac{1}{2Z_c}(C_R - C_L) \tag{B.10}$$

where

$$C_R = -P_S + Z_c V_S - \left.\frac{2a\tau_w}{r_i}\right|_S \Delta t \quad ,$$

$$C_L = -P_R + Z_c V_R + \left.\frac{2a\tau_w}{r_i}\right|_R \Delta t \quad ,$$

and

$$Z_c = \frac{a\rho}{g_c} \quad .$$

It is shown in reference (31) that the solution is stable if points R and S lie between A and B. This implies that the thermal energy solution developed in Chapter III is stable whenever the solution given by Equations B.9 and B.10 is stable. This can be verified by observing the pathline or thermal characteristic shown passing through point H in Figure 31. For all values of acoustic velocity, a, point H must lie between R and S, and since, in general, $|V| << a$, $\Delta x$ and $\Delta t$ can be chosen such that R is very near A, S is very near B, and H is relatively near C. This is considered further in Chapter V.

# APPENDIX C

## PREPROCESSOR AND SIMULATOR APPROACH TO HYDRAULIC SYSTEM SIMULATION

This appendix considers the conceptual design of a hydraulic system simulation package which is based on a preprocessor and simulator approach. The concepts presented are valid whether or not thermal effects are to be simulated, and in fact, any type of system which can be represented as a set of coupled components could be considered. The contents of this appendix are a result of the author's analysis of existing hydraulic system simulation programs and particular general programs for continuous systems. The purpose of the appendix is to document a programming approach which can be used to develop a more versatile hydraulic system simulation program than is presently available.

The two most significant hydraulic system simulation programs which are currently available are the McDonnell Aircraft HYTRAN (8) and HYDSIM by C. K. Smith (59,1). The HYDSIM program structure is more general and has the capability to accept more types of component models than the HYTRAN program. HYTRAN is designed to use the method of characteristics line model only, which is one model type which cannot be easily implemented in HYDSIM. Both of the routines have two characteristics which are undesirable from the viewpoint of a user. These are as follow.

1) The programs depend on a component model routine library. The addition of new components to library is, at best, very

difficult. In HYDSIM, the difficulties arise from a very complex model routine structure, and in HYTRAN, the lack of port variables other than pressure and flow causes the difficulty.

2) The amount of computer core storage is nearly independent of the complexity of the system being simulated. This independence derives from the use of Fortran common blocks for component variable storage and from the concept of loading the entire component library for all simulations. This makes small system simulations unnecessarily costly, makes the expansion to very large systems difficult, and implicitly penalizes having both simple and complex component models in the component library.[1]

It is the author's opinion that desirable characteristics for a simulation program include the following.

1) Component orientation must allow a system to be described in terms of coupled components. Permissible port variables at the component interfaces should include all those variables of interest which occur in the physical systems to be simulated.

2) User-supplied data to specify a system configuration and the parameters for a simulation should be in a free format which is convenient for the user.

3) The inclusion of new models should be possible both temporarily and permanently with as little effort as possible. Model routines should have a simple syntax; the specification of new models to the simulation program should be straightforward; and

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[1]It is understood by the author that a HYTRAN version not yet released will not use Fortran common blocks for component routine variable storage.

the interfacing of component ports should be generalized.

4) The computer memory storage requirements of a simulation program should reflect the complexity of the system being simulated.

5) The program should be designed and coded in an efficient manner but not in such a way that user-orientation is reduced.

It is not possible to achieve all of the above objectives with a simulation routine written as one Fortran program. A single Fortran program cannot expand as required by 4) above, and item 3) requires coding changes if one is to permanently add routines to a program such as HYDSIM or HYTRAN. Also, if item 2) is to be achieved the result will be a rather large input processor which will be used during the initialization of a simulation and will not be needed while the simulated response is being calculated. Characteristics 1) and 5) are matters of program design and can be achieved if the overall simulation package is correctly structured.

## Preprocessor and Simulator Concept

Several of the general purpose simulation programs such as CSMP/360 (4), DSL/90 (3), and MIMIC (2) utilize a preprocessor and simulator approach to system simulation. The preprocessor portion of each program reads the user data and produces output to be used by the simulator. As examples, the model description which a user supplies for CSMP/360 is sorted and processed into a Fortran subroutine. This routine is compiled by the regular Fortran compiler and is used by the simulator for the total description of the system model. Similarly, MIMIC translates the user input data directly to a machine language routine which is used by the simulator. In each case the user input is read by a preprocessor

whose function is complete once the input data has been translated to a second form. The preprocessor is then replaced by a simulator that calculates and reports the system response. A similar approach can be used to develop a Fortran based hydraulic system simulation which will have the characteristics listed above.

The conceptual operation of a block-orientated preprocessor and simulator can be represented as in Figure 32. In this figure the input to be supplied by the user is illustrated by the card images. The primary user input to the preprocessor is used in setting up the simulation to be done by the simulator. The preprocessor should produce three Fortran routines which are specific for the hydarulic system defined by the user. These are a main program to allocate array storage via Fortran dimension statements and call the simulator, a routine to direct the calling of model routines, and a routine to direct the solution of algebraic equation sets within the system model. The advantages of this approach are as follow.

1) The preprocessor can be large since it is not part of the simulator.
2) Array storage can be allocated to match the size of the system being simulated since a small main program is always written and compiled.
3) Only the component routines which are actually needed from the component library need be loaded thus reducing the simulation program size.
4) The optional new model routines can be permanently added to the simulation package by adding them to the data file which describes model routines and to the component library. No

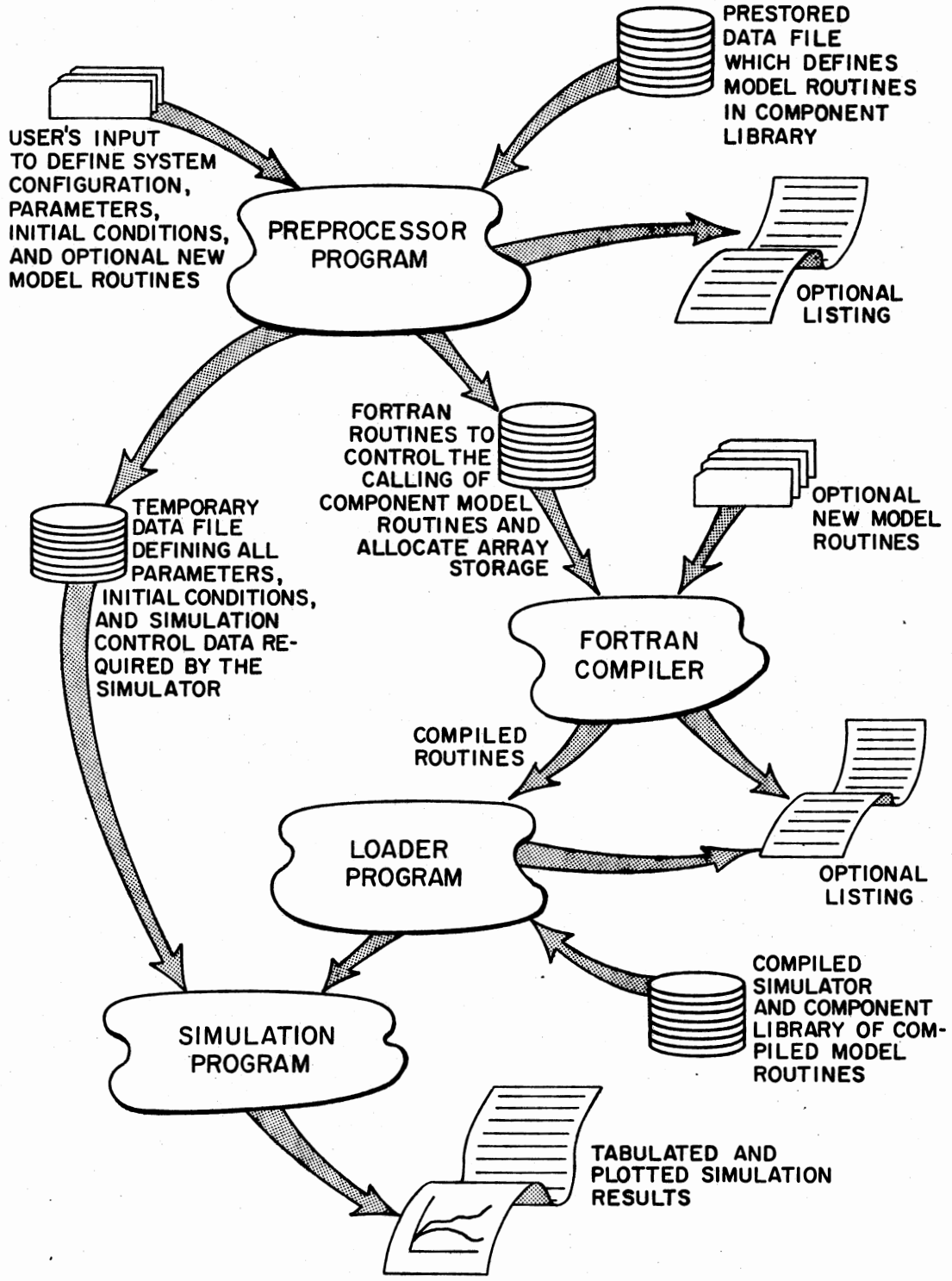


Figure 32. Information Flow in a Preprocessor and Simulator Program

modifications of the preprocessor or simulator should be required.

The preprocessor approach to simulation can be contrasted to that shown in Figure 33. The programs HYDSIM and HYTRAN both follow Figure 33 in which the primary user input is not used until the simulation package is all loaded and is executing. This results in a fixed program size and in little possibility for adding new models without changing the simulation program coding. The optional new model routines shown in Figure 33 are temporary additions which at most replace dummy routines in the component library. The only apparent advantage to the approach in Figure 33 is that it parallels the regular execution of a Fortran program thus requiring no special job control requirements.

The details needed to complete Figure 32 can be outlined to complete the conceptual design of a preprocessor and simulator for block-oriented hydraulic system simulation. The following section contains this part of the design.

## Program Design Requirements

### User Input Data

The user input data for a hydraulic system simulation must specify four types of information which are a description of new model routines, a description of the system topology, the parameters for component models, and simulation control variables which set quantities such as the simulation time, variables to be tabulated and plotted, integration step size, etc. The topology and other data can be specified in a manner similar to that used in HYDSIM. Component port connections can be specified on data cards as

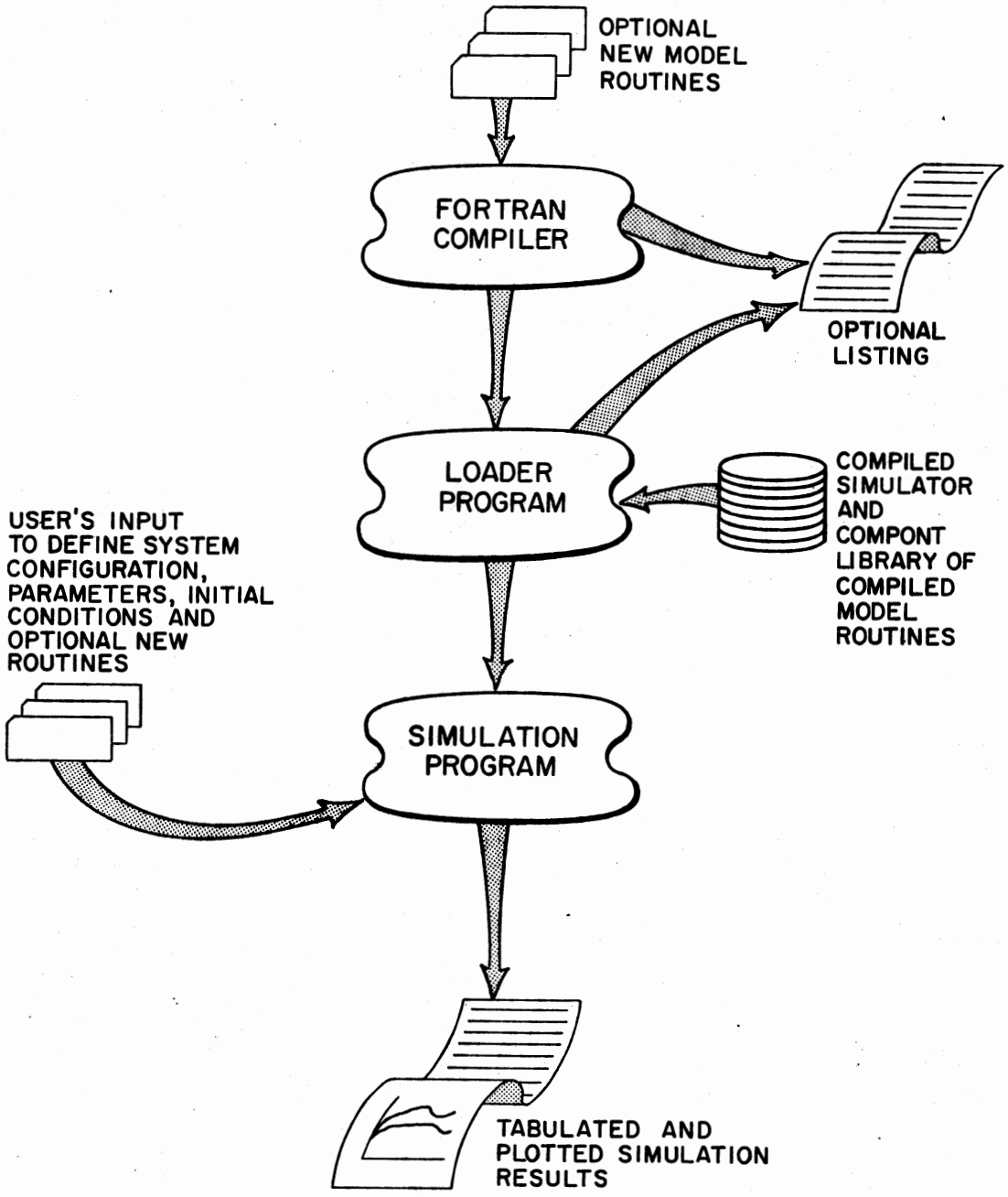


Figure 33. Information Flow in a Typical Fortran Simulation Program

```
3(1) = 4(6)PG
```

to indicate that port 1 of component 3 is connected to port 6 of component 4. The "P" and "G" are optional and indicate that the port variables at this connection are to be printed and graphed. The parameters for a component can be specified as

```
4/2.1, 1.0E4, -6, 5.1/
```

to indicate that the five parameters for component 4 are the values between the slashes. The type of component can be given by an expression such as

```
PUMP/3/
```

to indicate that component 3 is the model routine PUMP from the component library. The simulation control variables can simply be specified such as

```
DELT = 0.001
```

for each of the control variables.

This completes a model format for each type of information except the definition of a new model routine. It is intended that each of the above types of data can be punched anywhere on a card and that one card may contain several data items in any order. In general, one should not expect the user to adhere to a set of strict input rules which can be avoided by allowing the preprocessor to accept data liberally.

The data needed to define a new component model consists of three parts which are the specification of the number of variables used by the model, a specification of each dependent port variable, and a

specification of the algebraic equation sets within the model. Assume that a variable notation such as that shown in Table II is used and that a new routine by the name of SAMPL is to be described. If the characteristics of the model are as shown in Table III and one requires that all model routines are subroutines with the argument list (X, DX, S, Y, P, A, G, PVIND, PVDEP), then all of the user data required to define SAMPL can be presented as shown in Table IV.

Table IV is a set of data cards which specify all of the information in Table III. These cards can be read by the preprocessor along with the data described earlier and again no specific placement of the data is assumed. The information is sufficient to define a new model routine and is, in fact, the same type of data which the component data file in Figure 32 must contain for each component model routine.

For some model routines the array storage requirements depend on parameters. For example, a method of characteristics line model storage requirements depends on the line length, fluid properties, and the integration step size. Such models can be implemented by having the user write a Fortran subprogram which is passed the component routine name or a code number, the parameters for the component, and the integration step. The routine must calculate the amount of X and S storage required and return the same. Actually, two such storage definition routines are required with one to define new model requirements and one to define storage requirements for model routines in the component library.

## Library Component Data File

A component data file is necessary to define the characteristics of the model routines in the component library. The data file must contain

TABLE II

VARIABLE NOTATION FOR COMPONENT ROUTINE MODELS

| Variable | Definition |
|---|---|
| A(J, I) | The derivative of the Ith algebraic equation with respect to the Jth variable in the equation. |
| DELT | The integration time step. |
| DX(I) | The Ith state derivative. |
| G(I) | The value of algebraic equation I. |
| NP | The number of parameters, P(i), used in a component model. NP $\geq$ 0. |
| NPORTS | The number of ports defined for a component. |
| NS | The number of storage variables, S(i), used in a component model. NS $\geq$ 0. |
| NX | The number of state variables, X(i), used in a component model. NX $\geq$ 0. |
| NY | The number of algebraic variables, Y(i), used in a component. NY $\geq$ 0. |
| P(I) | The parameters needed to describe a component. |
| PVDEP(1, I) | The dependent variable at port I. |
| PVDEP(2, I) | The dependent temperature at port I. |
| PVIND(1, I) | The independent variable at port I. |
| PVIND(2, I) | The independent temperature at port I. |
| S(I) | A storage array which can be used within a component model routine for storing model information, solving internal algebraic equations, or propagating difference equations. |
| X(I) | The Ith state variable. |
| Y(I) | The Ith algebraic variable. |

TABLE III

SPECIFICATIONS FOR EXAMPLE MODEL ROUTINE SAMPL

| | |
|---|---|
| Continuous State Variables | $X_1$, $X_2$ |
| Discrete State Variables or Internal Storage | $S_1$, $S_2$, $S_3$, $S_4$ |
| Algebraic Equations | Set 1<br>$g_1(Y_1, Y_3, PVIND_{1,1})$<br>$g_2(Y_1, Y_3)$<br>Set 2<br>$g_1(Y_2, Y_4, PVIND_{1,2})$<br>$g_2(Y_2, Y_4, PVIND_{2,3})$ |
| Parameters Required | $P_1$, $P_2$, ..., $P_6$ |
| Ports | Ports 2 and 3 are pressure and flow ports with pressure independent.<br>Port 1 is a force and velocity port with force independent. |
| Port Variables Defined by Algebraic Variables | $PVDEP_{1,1} = Y_1$<br>$PVDEP_{1,3} = Y_2$<br>$PVDEP_{2,2} = Y_3$<br>$PVDEP_{2,3} = Y_4$<br>All other independent port variables are defined in terms of X, S, and time. |

TABLE IV

SAMPLE DATA TO DEFINE NEW MODEL ROUTINE SAMPL

```
SUBROUTINE SAMPL (X(2), DX(2), S(4), Y(4), P(6),
                  PVIND(3), PVDEP (3))


SET = 1
     G1(Y(1), Y(3), PVIND(1,1))
     G2(Y(1), Y(3))


SET = 2
     G1(Y(2), Y(4), PVIND(1,2))
     G2(Y(2), Y(4), PVIND(2,3))


PVIND(1,1) = PRES, PVIND(1,3) = PRES,
PVIND(1,2) = FORCE
PVDEP(1,1) = Y(1), PVDEP(1,3) = Y(2)
PVDEP(2,2) = Y(4), PVDEP(2,3) = Y(4)
```

one set of information for each component to allow the processor to include that component in a simulation. The information which must be stored in the file is identical to that shown for the "new model" in Table IV. The data can be stored more efficiently than actually storing card images in the file at the discretion of the programmer. It is assumed that a special program must be written to create the data file and that the input for the creation program will be similar to that in Table IV. This program would be rerun each time a model routine was to be permanently added to the component library.

Component Model Routine Structure

Component model routines should be structured in a manner which encourages the user to add new models as opposed to being overwhelmingly complex. This can be achieved if the routines all follow one skeleton form and use a simple variable set such as shown in Table II. All array variables should be passed to a model routine via an argument list which implies that all routines should begin in a manner similar to

```
SUBROUTINE SAMPL (X, DX, S, Y, P, A, G, PVIND, PVDEP)
DIMENSION X(1), DX(1), S(1), Y(1), P(1), A(NG,1), G(1),
          PVIND(2,1), PVDEP(2,1)
COMMON . . . scalar variables . . .
GO TO (100, 200, 300, . . .), MODE
```

where NG could be specified through common. The variable MODE would specify whether the routine is to perform initialization, evaluate state derivatives, evaluate port variables, etc. MODE would be defined by the simulation routine, and a user would only have to code the operations

required by each MODE value.

Each array used by each routine would begin at the first element such as X(1), Y(1), A(1,1), etc., and would proceed to the maximum element to be used in the routine. The preprocessor must allocate individual routine storage space in a large array, but the user need not be concerned with this except to the extent required by Table IV. The variables from the list in Table II which must be defined by a routine include DX, S, A, G, and PVDEP. Each must be defined as discussed in Chapter IV or as done in the HYDSIM (59) program, although HYDSIM uses a much more complex variable syntax.

Preprocessor Program

The preprocessor program must interpret the user input data and construct a system model which corresponds to the user's hydraulic circuit. It should write a main program which dimensions an array large enough for all the storage required by the component routines that are to be used. The main program must call the simulator and pass the storage array as an argument. The simulator must control the calculation of the simulated response and must have available a model description routine also written by the preprocessor.

The simultor needs to call only one routine and pass the storage array as an argument. The routine must be written by the preprocessor and must call the component model routines for the user's system. When the calls to model routines are made, the arguments must be elements of the storage array. For example, assume the main program contains DIMENSION B(500) and that B is passed to the major simulation routine which in turn passes it to the model routine. If the routine SAMPL from

the earlier discussion is allocated the storage from B(9) to B(37), then a call to SAMPL could appear as

```
CALL SAMPL (B(9), B(11), B(13), B(16), B(20), A,F
            B(26), B(32)).
```

Thus, X(1) and X(2) in SAMPL would correspond to B(9) and B(10), DX(1) and DX(2) would correspond to B(11) and B(12), etc. The bookkeeping associated with allocating and using space in such a manner can be done within the preprocessor and the simulator without being of concern to the user. The user only need be concerned with developing and presenting models as discussed earlier.

A second subroutine which must be written by the simulator must control the calling of component model routines during the solution of algebraic equation sets. This routine must supply the Jacobian matrix and equation values to a solution method such as the block-oriented Newton-Raphson algorithm defined by Smith (1) and can do so by appropriately calling component routines. The component routines in turn evaluate the Jacobian elements for the equations within the component. When all of the component routines related to an equation have been called, the full Jacobian will be available as will the equation values. If a sparse matrix approach is used to eliminate the zeroes in the Jacobian, the algebraic equation control routine written by the preprocessor must also define the sparse solution information discussed in Appendix D.

Figure 32 also indicates that the preprocessor must write a temporary data file to be read by the simulator. This file must contain all of the parameters from the user input data plus other control vectors

needed to do the simulation. The control vectors will include quantities such as the subscripts of variables to be printed and plotted, pointers to locate state variables in the large storage array, integration control data, and the information required to solve the algebraic equation sets. If a sparse matrix method is used, the algebraic equation set information would be similar to that discussed in Appendix D. The data from the file must be read by the simulator during initialization. After the data is read, the simulator should immediately be ready to begin the simulation since no error checking need be done except to assure that a preprocessor data file was read. It is conceivable that the data file could contain information for multiple runs, and that at the end of each run the simulator would attempt to read another set of data until an end of file occurred.

A flowchart for the preprocessor is shown in Figure 34. The flowchart is intended to identify the major operations which must be performed in the preprocessing. The handling of user input data is further detailed in Figure 35. An assumption in Figure 35 is that one data card may only contain one entry of the types discussed earlier. This assumption simplifies the understanding of the steps in processing the input data, but is not a requirement for actual program implementation. The details of determining storage requirements are shown in Figure 36. The main feature of the storage specification algorithm is the option of calculating the requirements based on the parameters for a particular component. The equation sorting required in Figure 34 is detailed in the reference cited and can be implemented directly.

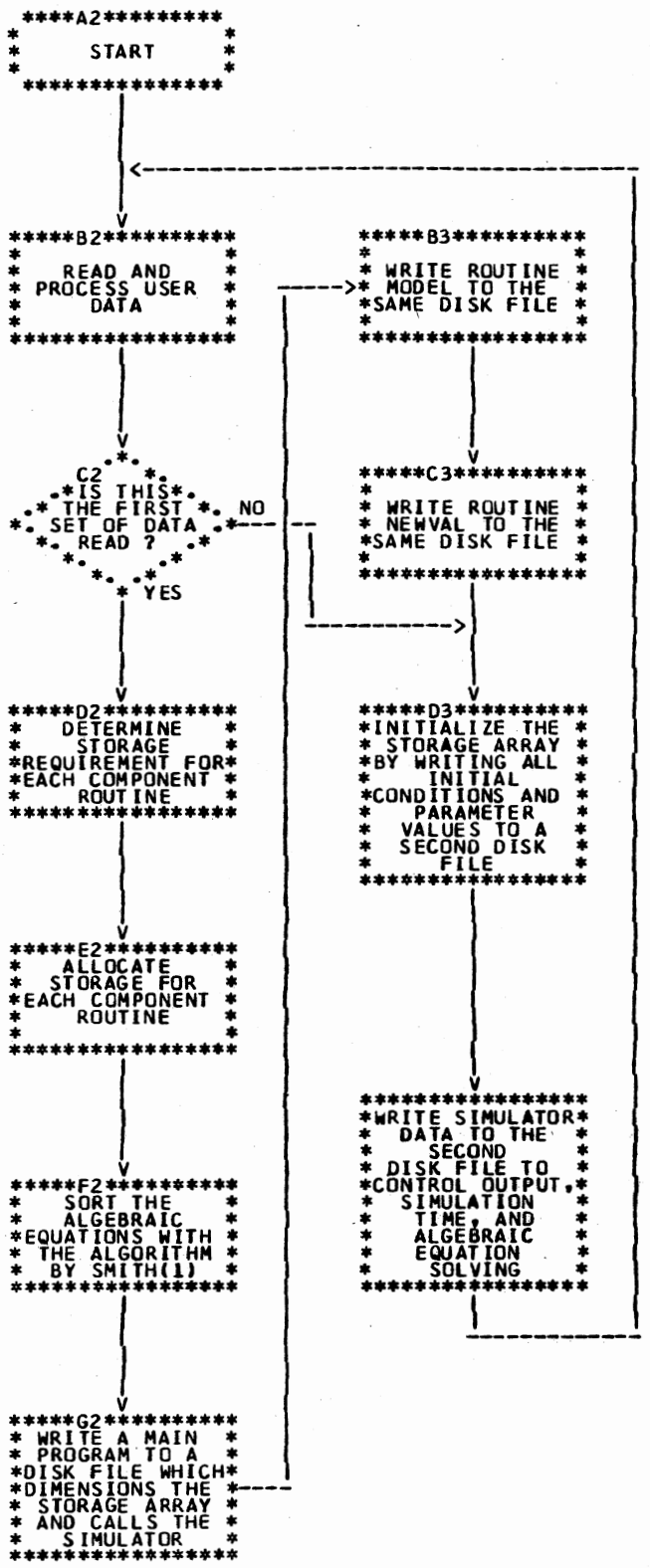


Figure 34. Flow Diagram for the Preprocessor Program

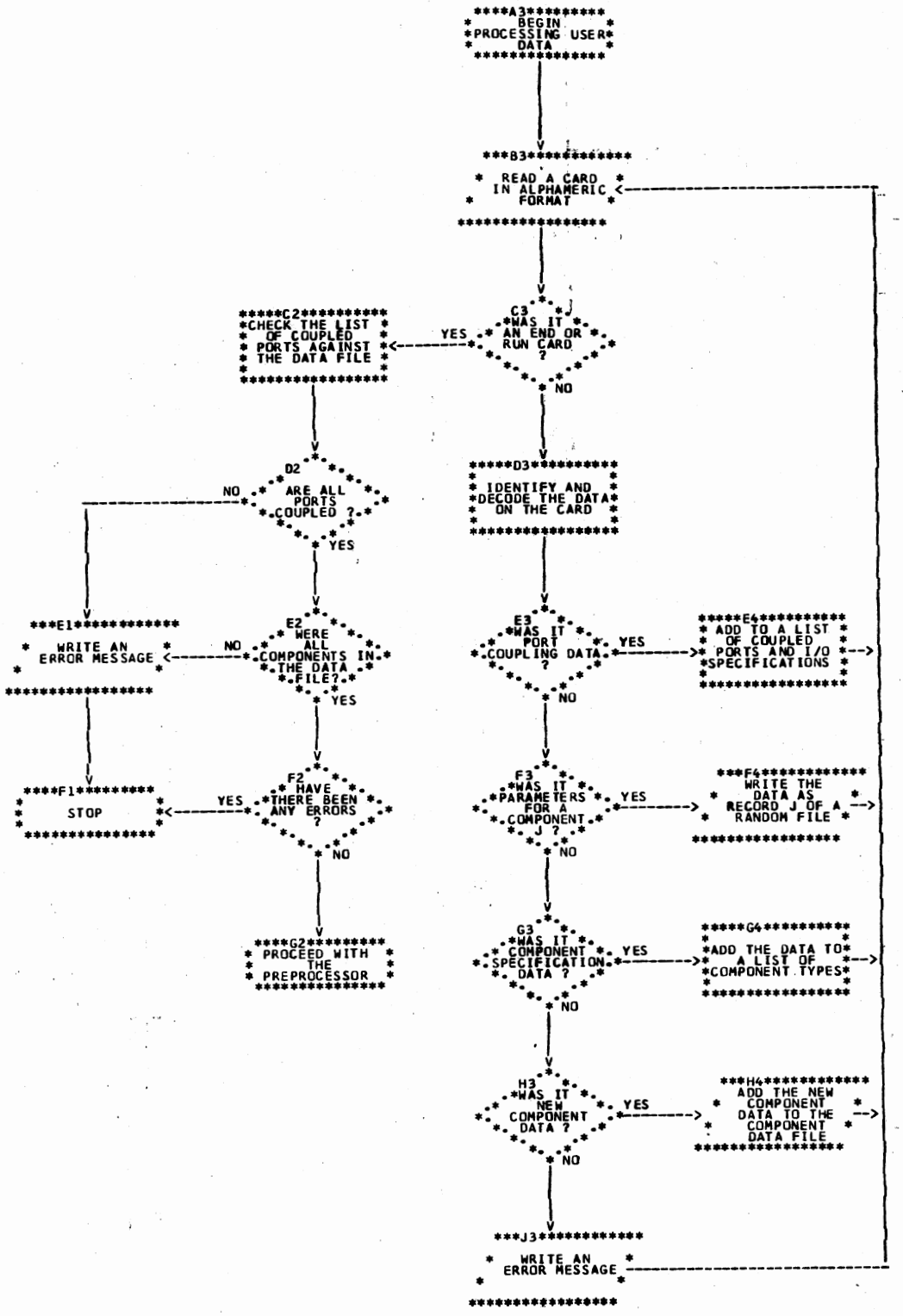


Figure 35. Flow Diagram for User Input Data Processing

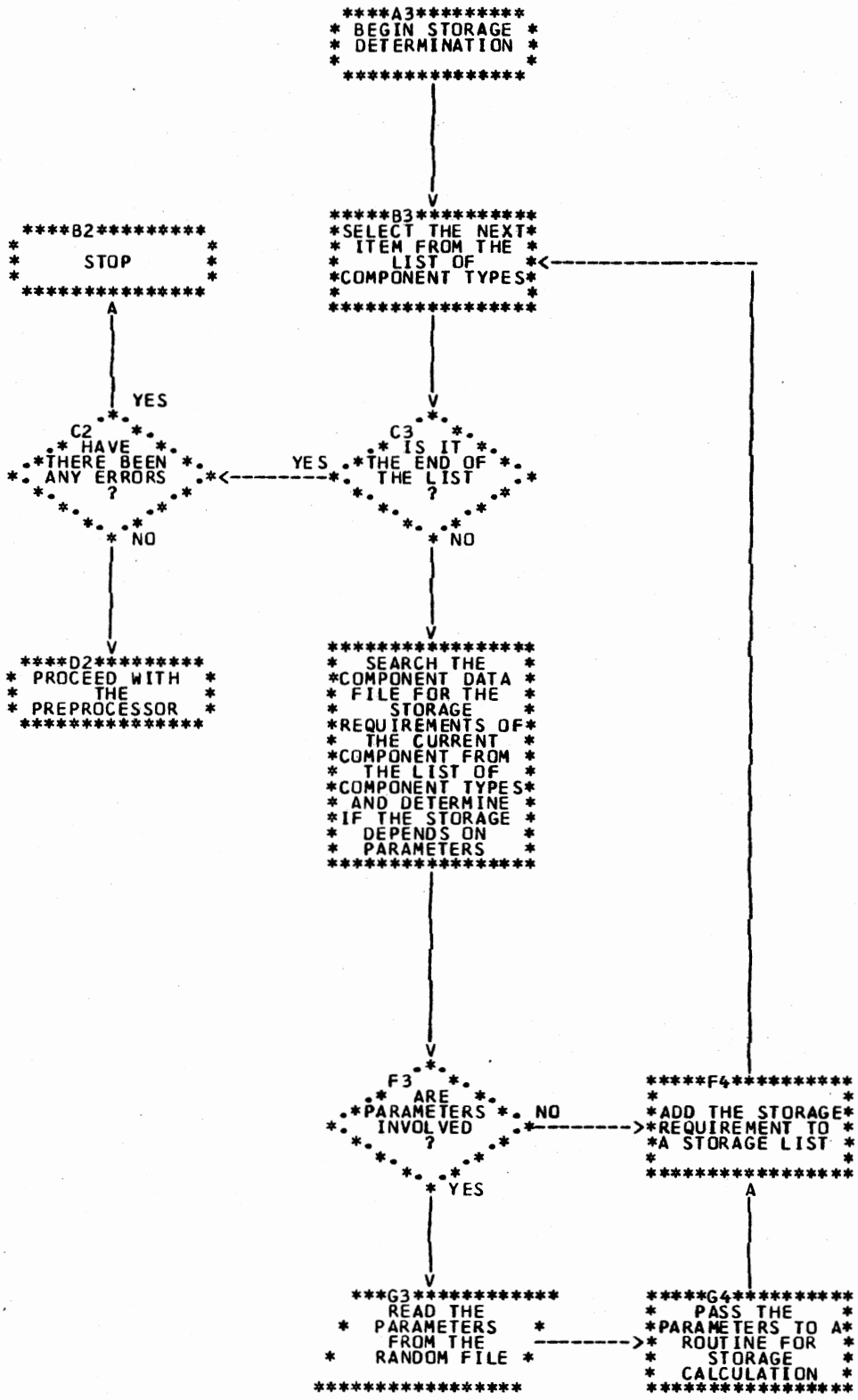


Figure 36. Flow Diagram for Storage Requirements Determination

## Simulator Program

The simulator program shown in Figure 32 controls the calculation of the simulated response. The major functions which the simulator must perform are integration, printing tabular results, and plotting results. The integration depends on the routine to call model routines discussed earlier and also on the solution of the algebraic equations which is controlled external to the major simulator routine.

A convenient way to achieve control of algebraic equation solving is to have the routine which the preprocessor produces call an equation solver. The equation solver can in turn call the algebraic equation evaluation routine from the preprocessor to obtain the Jacobian and equation values. If this approach is used the routine which controls the simulation need only call one routine and all derivative values can be made available on return. If no algebraic equation sets exist in a system model, the call to the solver can be omitted with no change in the flow of the central control routine.

It would be useful for the central integrator to define a variable which indicates the current status of the integration algorithm. For example, if an integration algorithm requires four derivative evaluations per time step, a variable could be set to 1, 2, 3, or 4, to indicate the evaluation being done. Routines such as a method of characteristics line model could then propagate internal difference equations based on the integrator status. This would simplify the propagation of discrete states along with continuous state variables.

A flowchart for the simulator is shown in Figure 37. The figure indicates the basic operations which must be performed for a simulation. The basic functions of routine MODEL are shown in Figure 38. The routine

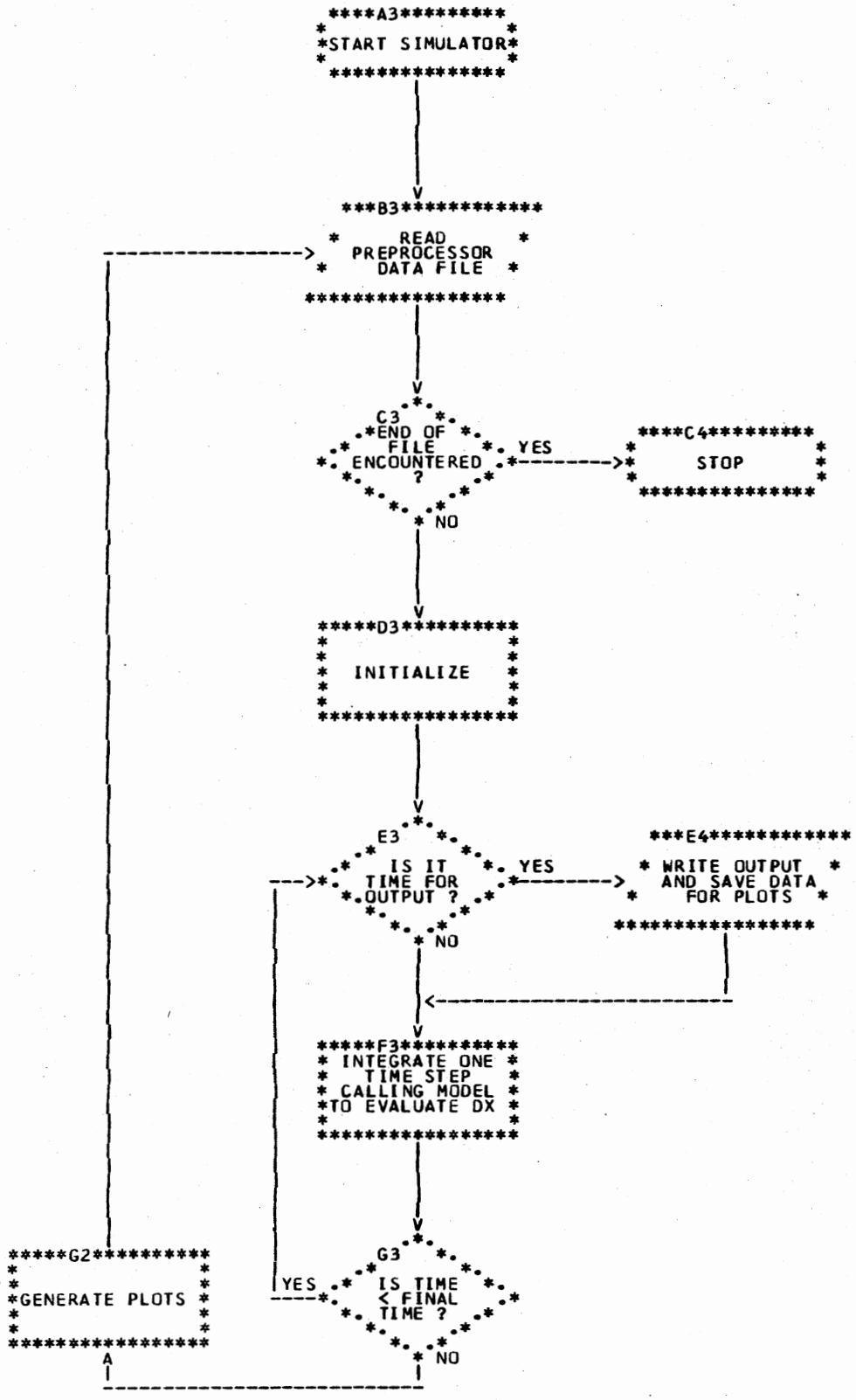


Figure 37. Flow Diagram for the Simulator

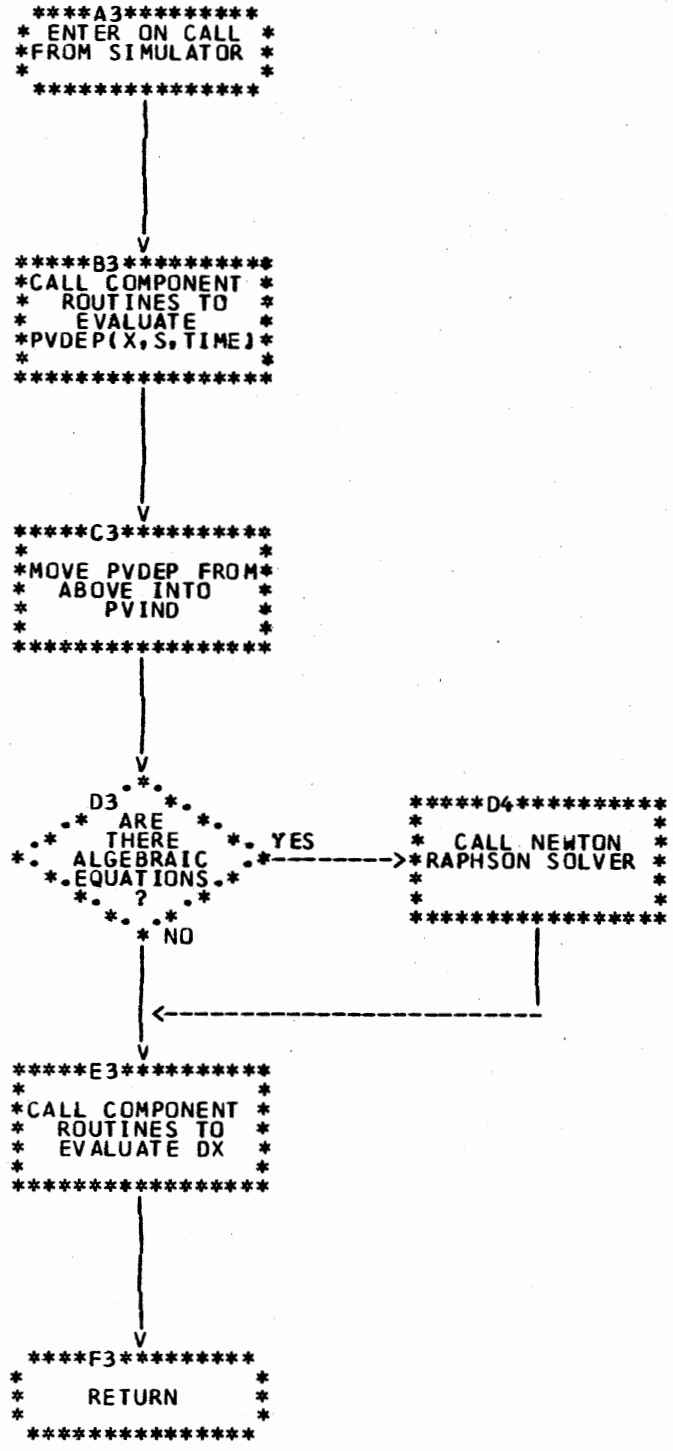


Figure 38. Flow Diagram for Subroutine Model

essentially consists of calls to component routines along with a call to an equation solver if algebraic equation sets are present. The solution of algebraic equation sets requires a routine as depicted in Figure 39 to evaluate the Jacobian and equations for each set. The individual steps in evaluating the Jacobian for one equation set when multiple components are involved are shown in Figure 39.

This appendix has briefly considered design concepts which can be implemented to form a versatile, block-oriented simulation program. The example simulations presented in Chapter V were performed with a program based on the concepts presented here. The preprocessor portion of the simulation was done manually for expediency and to aid in verifying the concepts for this appendix. Aside from programming details there seems to be no obstacles to prevent implementing a computer code which follows Figure 32 and possesses the desirable characteristics stated earlier.

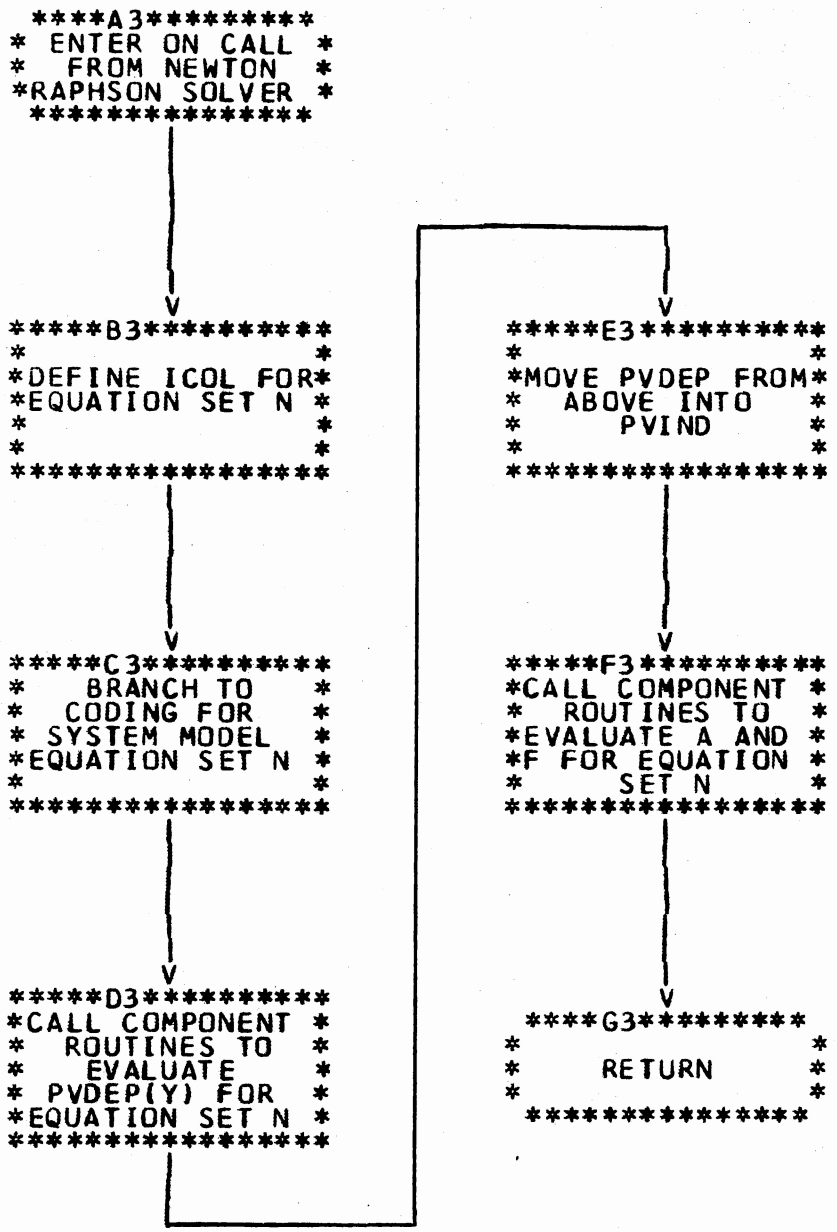


Figure 39. Flow Diagram for Jacobian Evaluation

# APPENDIX D

## USE OF A SPARSE MATRIX SOLUTION IN BLOCK-ORIENTED SIMULATION

The discussion in Chapter IV and Appendix C assumes that algebraic equations can be used to describe a component model. Smith has detailed an algorithm for solving the sets of coupled algebraic equations which result when components are coupled into systems as well as an algorithm for identifying independent equation sets (1). His effort was directed toward constructing the component models in such a way that equation sets could cross component boundaries and could still be solved in a general manner.

Implicit in the Newton-Raphson solution used by Smith is the need to repeatedly solve a linear equation set of the form

$$\left.\frac{\partial g}{\partial Y}\right|_{Y_i} (Y_{i+1} - Y_i) = g\Big|_{Y_i} \qquad (D.1)$$

where g is a vector of n functions, Y is a vector of n unknowns, and the partial derivative is the Jacobian of g. The Jacobians which appear in hydraulic system models often contain many zero terms thus suggesting that a sparse solution method may be applicable. This appendix develops an approach which allows a general sparse matrix solution method to be implemented within a block-oriented system simulation program.

The literature contains numerous discussions which consider the solution of linear equation sets of the form

$$AY = G$$

where A is n x n, Y is n x 1, and G is n x 1. However, the majority of the discussions are specialized for cases such as A being symmetric, A being banded or partitioned, or A being too large to store within computer memory. Since none of these conditions exist, in general, in hydraulic system models, it appears that the only applicable method is that presented by Key for general A matrices (57). The study presented by Key shows the method to be both time and storage effective if the A matrix is at least fifty percent sparse. The method can be readily adapted for simulation applications with small changes.

The solution algorithm by Key requires the representation of the coefficient matrix in a compressed form. The compressed form consists of a matrix A' and a pointer matrix ICOL defined such that A'(I,J) is the coefficient of the ICOL(I,J) unknown in the J'th equation for J ranging from one to the number of non-zero terms of row I in A. For example, the equation set

$$\begin{bmatrix} 1 & 4 & 0 \\ 0 & 6 & 0 \\ 0 & 2 & 5 \end{bmatrix} \begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 7 \\ 9 \end{bmatrix}$$

would be represented in compressed form as

$$A' = \begin{bmatrix} 1 & 4 \\ 6 & - \\ 2 & 5 \end{bmatrix}, \text{ and } ICOL = \begin{bmatrix} 1 & 2 \\ 2 & 0 \\ 2 & 3 \end{bmatrix}.$$

Although Key required that the A matrix be compressed by simply moving

row elements to the left without changing order, the above matrices could also be represented as

$$A' = \begin{bmatrix} 4 & 1 \\ 6 & - \\ 2 & 5 \end{bmatrix}, \text{ and } ICOL = \begin{bmatrix} 2 & 1 \\ 2 & 0 \\ 2 & 3 \end{bmatrix}$$

where the first row has been reordered. This change required modification in Key's implementation of the algorithm but in no way changes the basic procedures. It should be noted that the end of a row or the presence of a zero in ICOL indicates the end of the coefficients for one equation in A'.

It is useful to consider how the A' matrix, which is in fact a Jacobian, must be evaluated and stored in a Fortran, block-oriented simulation program. Each row in A' is associated with one equation defined by one component model routine. If a component model contains several coupled equations, then successive rows can be associated with the equations for one component. However, successive row elements in a Fortran matrix are not contiguous storage locations since Fortran matrices are allocated by column. If one evaluates A' transpose then successive terms defined by one equation will appear as column entries and will be stored in successive locations in a Fortran matrix. The result is that a component routine can be passed the beginning location for a column in $A'^{T}$, and one can proceed to define one column for each equation in the component model.

As an example consider that two component routines together form a set of three equations in three unknowns. Suppose that a component i defines a function $g_1^i$ which is the first equation in the set, and that a

component j defines $g_1^j$ and $g_2^j$ which are the second and third equations in the set. $A'^T$ will be defined as shown below.

$$A'^T = \begin{bmatrix} \vdots & \vdots & \vdots \\ g_1^i & g_1^j & g_2^j \\ \vdots & \vdots & \vdots \end{bmatrix}$$

If $A'^T$ is a 3 x 3 Fortran matrix, then the actual storage locations relative to the (1,1) position will be

$$\begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix} .$$

Thus $g_1^i$ will define elements 1, 2, and 3; $g_1^j$ will define 4, 5, and 6; and $g_2^j$ will define elements 7, 8, and 9. If the $A'^T$ matrix is sparse and no equation contains more than two terms, then only six elements will be required and can be represented as

$$\begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{bmatrix} .$$

In either case one column is associated with one equation and consists of consecutive storage which can be addressed relative to the first entry in the column.

Each component model routine can contain the statement DIMENSION A(NG,1) where NG is defined through a common block. The routine defining $g_1^i$ can be called as CALL $name_i$(..., A(1,1), G(1), ...) to evaluate the first column in A. The second routine can be entered with

CALL name$_j$(. . ., A(1,2), G(2), . . .) which will make A(1,1) within the routine be the (1,2) element in the Jacobian transpose. Similarly A(1,2) within the routine will be A(1,3) in the Jacobian transpose. Within a component routine the elements in A must be defined as A(K,L) as the derivative of the L'th equation in a set of coupled equations appearing in the component with respect to the K'th variable in the equation. Thus within the coding for the routine SAMPL from Appendix C one would include:

```
      SUBROUTINE SAMPL (. . ., A, G, . . .)
      DIMENSION A(NG, 1), G(1)
      COMMON . . ., NG, . . .
                .       .       .
                .       .       .
C     FOR SET 1
      G(1) = g1(Y(1), Y(3), PVIND(1,1))
      G(2) = g2(Y(1), Y(3))
      A(1,1) = ∂g1/∂Y(1)
      A(2,1) = ∂g1/∂Y(3)
      A(3,1) = ∂g1/∂PVIND(1,1)
      A(1,2) = ∂g2/∂Y(1)
      A(2,2) = ∂g2/∂Y(3)
                .       .       .
                .       .       .
```

```
C     FOR SET 2
      G(1) = g1(Y(2), Y(4), PVIND(1,2))
      G(2) = g2(Y(2), Y(4), PVIND(2,3))
      A(1,1) = ∂g1/∂Y(2)
      A(2,1) = ∂g1/∂Y(4)
      A(3,1) = ∂g1/∂PVIND(1,2)
      A(1,2) = ∂g2/∂Y(2)
      A(2,2) = ∂g2/∂Y(4)
      A(3,2) = ∂g2/∂PVIND(2,3)
               .       .       .
               .       .       .       .
```

It should be noted that the above skeleton coding uses only the elements of A and G which one would use if the equations for model SAMPL were a complete set. The placing of the columns of A defined within SAMPL is done by the call to the routine as discussed above. Also, the only derivatives which are required are those which can be non-zero. Thus for set one above A(2,1) corresponds to a derivative with respect to Y(3), but A(2,1) would be the derivative with respect to PVIND(1,1) if G(2) did not involve Y(3).

Only non-zero derivatives need be included in the Fortran matrix A since it is actually a compressed Jacobian transpose to be used in solving D.1 for $(Y_{i+1} - Y_i)$ by a sparse matrix method. However, it is necessary to specify exactly which variables appear in each equation in order to construct the ICOL matrix which identifies the elements in the compressed A matrix. This can be realized through the following example.

Routine SAMPL from Appendix C is component j and is coupled to component i at port G of each component. In terms of the local variables

and equations, the equation to be solved is:

for component i (assumed):

$$G(1) = g_1^i(Y^i(1), PVIND^i(1,1))$$

$$PVDEP^j(1) = Y^j(1) \quad ,$$

for component j (from SAMPL, set 1):

$$G(1) = g_1^j(Y^j(1), Y^j(3), PVIND^j(1,1)$$

$$G(2) = g_2^j(Y^j(1), Y^j(3))$$

$$PVDEP^j(1,1) = Y^j(1) \quad ,$$

and

due to port coupling:

$$PVIND^j(1,1) = PVDEP^i(1,1)$$

$$PVIND^i(1,1) = PVDEP^j(1,1) \quad .$$

The complete equation set is then

$$g_1^i(Y^i(1), PVIND^i(1,1) = 0$$

$$g_1^j(Y^j(1), Y^j(3), PVIND^j(1,1) = 0$$

$$g_2^j(Y^j(1), Y^j(3)) = 0$$

with unknowns $Y^i(1)$, $Y^j(1)$, $Y^j(3)$. Assume as before that Y(1) and Y(3) for component j, SAMPL, are allocated storage B(16) and B(18) and further that $Y^i(1)$ is in B(83). The unknowns in terms of storage locations are then B(16), B(18), and B(83) which are the actual quantities which must be determined.

A Newton-Raphson solution as described by Smith (1) requires the derivatives of each equation with respect to each unknown. The non-zero terms of the compressed Jacobian transpose for the entire set are

$$A(1,1) = \frac{\partial g_1^i}{\partial Y^i(1)} \quad ,$$

$$A(2,1) = \frac{\partial g_1^i}{\partial Y^j(1)} = \frac{\partial g_1^i}{\partial PVIND^i(1,1)} \quad ,$$

$$A(1,2) = \frac{\partial g_1^j}{\partial Y^j(1)} \quad ,$$

$$A(2,2) = \frac{\partial g_1^j}{\partial Y^j(3)} \quad ,$$

$$A(3,2) = \frac{\partial g_1^j}{\partial Y^i(1)} = \frac{\partial g_1^j}{\partial PVIND^j(1,1)} \quad ,$$

$$A(1,3) = \frac{\partial g_2^j}{\partial Y^j(1)} \quad ,$$

$$A(2,3) = \frac{\partial g_2^j}{\partial Y^j(3)} \quad .$$

The A(2,1) and A(3,2) terms are obtained by the chain rule. The A(2,1) term is actually

$$A(2,1) = \frac{\partial g_1^i}{\partial Y^j(1)}$$

$$= \frac{\partial g_1^i}{\partial PVIND^i(1,1)} \frac{\partial PVIND^i(1,1)}{\partial PVDEP^j(1,1)} \frac{\partial PVDEP^j(1,1)}{\partial Y^j(1)}$$

$$= \frac{\partial g_1^i}{\partial PVIND^i(1,1)} (1) (1)$$

$$= \frac{\partial g_1^i}{\partial PVIND^i(1,1)} \quad .$$

An ICOL matrix to accompany the above equation set would be

$$ICOL = \begin{bmatrix} 83 & 16 & 16 \\ 16 & 18 & 18 \\ 0 & 83 & 0 \end{bmatrix} .$$

The preceding discussion and example can be summarized as follows.

1) Component model routines can be developed to define sets of equations G(1), G(2), ..., G(N) as functions of algebraic variables Y(1), Y(2), ..., and independent port variables. It is necessary to define A(I,J) as the derivative of G(J) with respect to the I'th variable in G(J).

2) An algebraic equation and port variable description can be as shown in Table IV. This information along with the port coupling information for a given system is sufficient to identify sets of algebraic equations which cross component boundaries as described by Smith (1).

3) An ICOL matrix must be constructed to identify the actual storage locations for unknowns in V terms of a global array such as G used in the examples. This matrix can best be defined by a preprocessor such as that discussed in Appendix C.

4) Actual elements of the compressed Jacobian transpose can be evaluated by calls to component routines to overlay the A(1,1) element in a component routine onto the A(1,J) element in the A matrix for an entire equation set. The row dimension of A must be made available to component routines through common. Component routines also evaluate each function G.

5) Given the ICOL from 3) above, which can be constructed by a preprocessor, and the total A from 4) above, whose columns are defined by component routines, plus G from 4) above, Equation D.1

can be solved for $Y_{i+1} - Y_i$ by a modified Key method (57). The required modifications are:

a) coding the algorithm to accept A as the transpose of the coefficient matrix; and

b) coding the algorithm to accept ICOL as a transposed matrix with elements of A appearing in any order.

6) Implicit in the above steps is the definition of dependent port variables which are not functions of only state or time as PVDEP(•,•) = Y(•). Also it is assumed that models are developed in such a way that the port coupling equations are of the form $PVDEP^j(\bullet,\bullet) = PVIND^i(\bullet,\bullet)$.

The method outlined in this appendix was used in the programming of the examples in Chapter V. The advantages of the approach are that zero terms are eliminated and, more importantly, that the Jacobian can be formed in a very unrestrictive manner. This allows component model routines to be developed using the simple variable names A, G, Y, etc., without any complex subscripting as done in HYDSIM (1,59). If a system model results in large sets of algebraic equations, the approach is also attractive due to savings in both execution time and storage requirements.

VITA 2

Lynn Royce Ebbesen

Candidate for the Degree of

Doctor of Philosophy

Thesis: DIGITAL COMPUTER SIMULATION OF THE MECHANICAL AND THERMAL RESPONSE OF COMPLEX HYDRAULIC SYSTEMS

Major Field: Mechanical Engineering

Biographical:

Personal Data: Born in Rapid City, South Dakota, January 27, 1948, the son of Mr. and Mrs. Neaph N. Ebbesen.

Education: Graduated from Hurley High School, Hurley, South Dakota in May, 1966; received the Bachelor of Science degree, with a major in Mechanical Engineering, from the South Dakota School of Mines and Technology, Rapid City, in May, 1970; received the Master of Science degree, with a major in Mechanical Engineering, from Oklahoma State University, Stillwater, in May, 1972; completed the requirements for the Degree of Doctor of Philosophy in December, 1976.

Professional Experience: Plant Engineer, Caterpillar Tractor Company, Summer, 1970; Project Assistant on research projects at Oklahoma State University, 1970 to 1974; Instructor in Mechanical Engineering at Oklahoma State University, 1974 to date.

Professional Organizations: Member of American Society of Mechanical Engineers; Pi Tau Sigma; Phi Kappa Phi; Pi Mu Epsilon.