Undergraduate Honors Thesis: Analyzing Student Understanding of Linear Function Concepts After Completing Basic Algebra Coursework

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Abstract

The purpose of this project is to investigate student understanding of the concept of linear functions after completing basic algebra coursework. Students that were asked to participate have taken basic algebra coursework, defined as either College Algebra or Mathematical Functions and Their Uses. A survey was distributed to students in a SMED 3153: Teaching Mathematics at the Primary Level course. The survey included a questionnaire on their background experiences in mathematics courses followed by questions over the concept of linear functions in different representations: tabular, contextual, and graphical. Specifically, the goal was to analyze the assessments to learn how participants' prior background and experiences affect their understanding of linear functions. There were 14 total participants. Out of the 14 participants, there were seven students who completed both College Algebra and Mathematical Functions and Their Uses, four students who completed College Algebra, and three students who completed Mathematical Functions and Their Uses. The results indicated that there is little distinction between participants' prior background experiences and their understanding of linear functions in different representations. This is due to the responses from participants being relatively consistent among the subgroups, whether the results indicated a deep understanding or a weak understanding of the concept for a subgroup.

Introduction

At Oklahoma State University, there are two introductory level mathematics courses. These include MATH 1483: Mathematical Functions and Their Uses, and MATH 1513: College Algebra. In addition to gathering information from literature, I observed several classes of MATH 1483 and MATH 1513 at OSU. Through my observations I found that Math 1513 introduced functions primarily through their algebraic equations in a manner that was independent of context, whereas in Math 1483, functions were primarily introduced through context. With that, in analyzing degree sheets at OSU, most degrees require at least MATH 1513, even if that is the only math course the degree requires. To enroll in MATH 1513 at OSU, one must score at least a 45 out of 75 on the Math Placement Exam administered by OSU for all incoming freshman. If one does not achieve this score, they will enroll in MATH 1483. A passing score of "C" or higher in this course allows one to enroll in MATH 1513. Some degree sheets even allow a choice between MATH 1483 and MATH 1513. However, at least one of these courses is required unless a student has already achieved college credit for a mathematics course equivalent or higher to either one of these courses.

The first questions we asked ourselves is when MATH 1483 was introduced at OSU and why was it created. In the 1996 - 1997 OSU Course Catalog, we are able to see MATH 1483 for the first time. The course was originally introduced as a choice for students who did not intend to take Calculus but needed to take College Algebra. The course description includes, "Analysis of functions from their graphs. Linear, exponential, logarithmic, periodic functions and rates of change. Special emphasis on applications to the natural sciences, agriculture, business and the social sciences" (OSU Course Catalog, 1996 - 1997, p. 261). Also within the 1996 - 1997 Course

Catalog, we can see that MATH 1483 was utilized as a prerequisite for MATH 3403 and MATH 3603, courses for Elementary Education majors. MATH 1483 was not utilized as a prerequisite for MATH 1513 at this time, again because it was not originally intended to be a gateway to Calculus courses. The history of MATH 1513 has not really changed since the 1996 - 1997 Course Catalog. MATH 1513 has consistently been a prerequisite for several other courses at OSU including Business Calculus, discrete mathematics courses, and Calculus I, II, and III. The course description has also remained consistent.

A change seen within MATH 1483 since its introduction in 1996 can be found in the 2013 - 2014 Course Catalog. The courses description reads, “Analysis of functions and their graphs from the viewpoint of rates of change. Linear, exponential, logarithmic, and other functions. Applications to the natural sciences, agriculture, business and the social sciences.” (Course Descriptions, 2013 - 2014, p. 293) This description, compared to the first course description of this course in 1996, reads as if the course is more advanced. For instance, analysis of graphs from the viewpoint of rates of change will certainly prepare a student more for Calculus courses they could potentially complete after completing MATH 1483. In addition to this change in the course description, in 2013 MATH 1483 was also allowed as a prerequisite for Business Calculus. This description and adding this course to a prerequisite for Business Calculus indicates that MATH 1483 can provide more of a connection between an introductory level mathematics course to advanced mathematics courses such as Calculus. The shift in the ideas of this course from 1996 to 2013 developed around the same time that the teaching of mathematics was also beginning to shift.

Beginning in the 1980s, research began to develop on how Calculus courses were taught and what methods were starting to become ineffective. David Tall found that “teacher[s] know conceptual questions are rarely answered correctly, [and therefore] the vicious circle of procedural questions is set in motion” (Tall, 1992, p. 4). Other research shows “it is possible that procedural, technique-oriented secondary school courses in Calculus may predispose students to attend more to the procedural aspects of the college course” (Ferrini-Mundy & Gaudard, 1992, p. 68). This opens up the possibility for students to continue in mathematics courses due to enjoying the procedural aspects the courses follow without having a true understanding of the symbols in mathematics problems, and therefore struggle with applying their knowledge to contextual based problems. It was posed that “individuals who know how to perform algebraic manipulations, but do not consider the possible relevance of symbols to reveal the structure of a problem that has aroused their curiosity, have not fully developed their symbol sense” (Arcavi, 1994, p. 25). Thus, it became clear that students were moving to upper level mathematics courses with a strong ability to solve procedural based problems but little ability to solve contextual based problems. As more research on this idea developed, “the calculus reform movement was born in several conferences sponsored by mathematic societies and private foundations” (Culotta, 1992, p. 1060).

The reform movement for teaching Calculus developed drastically due to Deb Hughes-Hallett and William McCallum. In 1992, the two decided the teaching of Calculus should fall on the

Rule of Three: teaching Calculus "algebraically, graphically, and numerically" (Culotta, 1992, p. 1060). In fact, Hughes-Hallett and McCallum eventually adopted the "'Rule of Four,' ideas [that] are presented graphically, numerically, symbolically, and verbally" (Hughes-Hallet et al., 2009, p. v). Hughes-Hallett presented her ideas on why Calculus, and mathematics in general, should be taught with the Rule of Four at a talk at Harvard University. She indicated that "mathematics needs to be taught showing its connections to other fields" as well as that "problems are needed that probe student conceptual understanding, otherwise some students only memorize" (Hughes-Hallett, 'The Teaching of Mathematics'). As these ideas became increasingly popular, "universities have experimented with new syllabi, [and] technology, [and] some have changed their courses significantly" (Hughes-Hallett, 'The Teaching of Mathematics'). Within new courses that developed due to reformers such as Hughes-Hallett and McCallum, "there's less memorizing and more thinking about real-world problems," and the courses include "writing, stress mathematical experimentation, foster teamwork, and in many cases rely on computers and graphing calculators" (Culotta, 1992, p. 1060). Hughes-Hallett and McCallum developed a textbook for Calculus that focused on multiple representations of functions through tabular data, graphical representations, and word problems. The two indicate that because so many ideas taught in Calculus courses require several procedural steps that "therein lies the danger in teaching calculus: it is possible to teach the subject as nothing but the rules and procedures—thereby losing sight of both the mathematics and of its practical value" (Hughes-Hallet et al., 2009, p. v). Thus, within this textbook the authors explain in the preface that the "book emphasizes at every stage the meaning (in practical, graphical or numerical terms) of the symbols [the students] are using. There is much less emphasis on 'plug-and-chug' and using formulas, and much more emphasis on the interpretation of the formulas that [the students] may expect" (Hughes-Hallett et al., 2009, p. xi). Hughes-Hallett and McCallum indicate the very importance of incorporating contextual problems regularly in their text. The two believe "Calculus has been so successful because of its extraordinary power to reduce complicated problems to simple rules and procedures" (Hughes-Hallet et al., 2009, p. v).

This idea inspired others to incorporate more contextual texts for Calculus courses, which also translated to creating mathematics courses that focus on contextual based problems in algebraic, graphical, and tabular forms. Thus, MATH 1483 was developed shortly after Deb Hughes-Hallett and William McCallum inspired a new teaching of mathematics. In particular, since MATH 1483 was first introduced at OSU in the 1996 - 1997 Course Catalog, around the time researched developed that showed students had little understanding of contextual problems but strong understanding of procedural problems, its possible the inspiration of the course was to provide students with a strong foundational understanding of conceptual problems in mathematics before they arrive in Calculus courses and other higher level mathematics courses.

This seems to be the case due to the nature of the course textbook for MATH 1483, *Functions and Modeling*. The textbook's preface indicates "the dual goals of this text are to show students the importance of mathematics in everything from business to science to politics and to show them that they can not only succeed but also excel at sophisticated mathematics" (Crauder, Evans, & Noell, 2014, p. xi). Thus, the textbook and the course developed around the textbook, are intended to provide students with an understanding of applications of fundamental ideas in mathematics to other subjects, as well as potentially reshape any negative attitudes they may already have towards mathematics. The preface then goes on to explain that "linear functions are defined not using a formula, but as those functions having a constant rate of change. Similarly, exponential functions are defined as those functions that show a constant percentage change" (Crauder, Evans, & Noell, 2014, p. xi). Introducing fundamental ideas, such as linear and exponential functions, through contextual situations and definitions rather than formulas certainly aligns with the goal of providing students with a more conceptual understanding of these ideas.

Another possible inspiration for the creation of MATH 1483 is due to enrollment numbers in Calculus courses. It is clear from the course description for MATH 1513 in the OSU Course Catalog that MATH 1513 is intended to prepare students for Calculus courses. For example, in the 2018 - 2019 Course Catalog, the course description reads, "Quadratic equations, functions and graphs, inequalities, systems of equations, exponential and logarithmic functions, theory of equations, sequences, permutations and combinations" (Course Catalog, 2018 - 2019, p. 569). Each of these ideas are fundamental ideas in higher level mathematics courses. Thus, the proper and early introduction of them in MATH 1513 will prepare students for Calculus courses. This idea is also clear due to the nature of the preface for the textbook of the course, *College Algebra*. The preface specifically indicates that "for many students, college algebra serves as a gateway course to the higher levels of mathematics needed for a variety of careers" (Gerken & Miller, 2014, p. iii). However, research about enrollment numbers from the University of Nebraska at Lincoln indicate that "only a minority of college algebra students (43% [of 1,458 students]) ever go on to take a Calculus course,…about 32% went on to take Business Calculus,…only 11% were enrolled in Calculus I after [three weeks],…[and] only 4% stayed in Calculus II [after three weeks]" (Herriott & Dunbar, 2009, p. 76). This data clearly indicates that "the College Algebra course is not primarily a feeder for Calculus" (Herriott & Dunbar, 2009, p. 76). Dr. Evans from the OSU Department of Mathematics gave a presentation discussing the College Algebra course at OSU and indicated that "the curriculum committees of national mathematics organizations have uniformly called for replacing the current College Algebra course with one in which students address problems presented as real world situations by creating and interpreting mathematical models" (Evans, 'Renewal of College Algebra'). We also found that "national organizations and policy makers have made repeated calls for precalculus level curricula to place greater emphasis on functions…many mathematics education research have found that student difficulty in understanding key ideas of calculus are rooted in their weak understanding of the function concept and their inability to use functions to reason about and represent the relationships between quantities and how they change together" (Carlson, Madison, & West, 2010, p. 2). Given that the MATH 1483 course is built around different representations of functional relationships, it seems this course could potentially close the gap in understanding functional relationships.

In the preface for the course textbook for MATH 1513, the authors indicate that "the textbook is filled with robust applications and numerous opportunities for mathematical modeling for those instructors looking to incorporate these features into the course" (Gerken & Miller, 2014, p. vii). However, if there were an instructor not looking to incorporate those features, they might never be included as they are just an extended portion of the text and not the entire goal of the course or textbook itself. Since the content MATH 1483 introduces relies heavily on contextual based problems in algebraic, graphical, and tabular form, this course fits the type of course wanted to help further prepare students for higher level mathematics courses. It's possible that in an effort to increase enrollment numbers in Calculus as well as provide students with a more foundational understanding of basic mathematics concepts was an inspiration to introduce MATH 1483. We also believe this is a possibility given that "studies have consistently revealed that student success in learning calculus requires that students view a function, in each of its representational forms (words, formula, graph, table), as a means of associating input values from a function's domain to output values in the function's range for the purpose of representing how values of one quantity change with the values of another quantity" (Carlson, Madison, & West, 2010, p. 3). Since MATH 1483 is built around each of these factors, one can imagine a hope for the course is to eventually increase the enrollment numbers in higher level mathematics courses through providing students with a deeper conceptual understanding of functions.

As Deb Hughes-Hallett and William McCallum originally indicated mathematics courses should teach through the Rule of Three, this study focuses on student understanding of linear functions in graphical, tabular, and contextual representations. This study compares the depth of student understanding of linear functions in each of these forms by focusing on students who have taken both or either one of MATH 1483 and MATH 1513. The inspiration of this study is to indicate if there is a connection between the depth of student understanding of linear functions in each of the three forms and a student's history of basic algebra coursework.

Research Questions

1. Is there a difference in the way students understand graphical representations of linear functions based on their prior experience with either College Algebra or Mathematical Functions and Their Uses?
2. Is there a difference in the way students understand tabular representations of linear functions based on their prior experience with either College Algebra or Mathematical Functions and Their Uses?
3. Is there a difference in the way students understand contextual representations of linear functions based on their prior experience with either College Algebra or Mathematical Functions and Their Uses?
4. Is there a difference in students sense of self-efficacy of linear functions based on their prior experience with either College Algebra or Mathematical Functions and Their Uses?

Methods

At Oklahoma State University in the Fall 2018 semester, SMED 3153: Teaching Mathematics at the Primary Level, was offered for one section that met for two 110 minutes sessions a week starting at 12:30pm and ending at 2:20pm. While enrollment was unrestricted, this course is typically one that Elementary Education majors complete. On the Elementary Education degree sheet for 2015-2016, 2016-2017, 2017-2018, and 2018-2019, students are required to complete at least two courses from the following list of Analytical and Quantitative Thought courses: MATH 1483: Mathematical Functions and Their Uses, MATH 1493: Applications of Modern Mathematics, MATH 1513: College Algebra, MATH 1613: Trigonometry, MATH 2103: Business Calculus, MATH 2144: Calculus I, and STAT 2013 Elementary Statistics. Each course in that list has either MATH 1483: Mathematical Functions and Their Uses or MATH 1513: College Algebra as an option for prerequisites, with other options being an ALEKS Placement Exam score of a specific level or an AP Calculus Exam AB or BC score of a 3 or higher. We decided that students in SMED 3153 would have already taken either MATH 1483 or MATH 1513, taken both courses, or they would be currently taking one of the courses. It was decided to administer a survey to this group of students.

A survey was created that included two questions asking the participants of their background experience in mathematics courses, four questions about their attitudes towards mathematics, and five content questions over the concept of linear functions. From an advising report, SMED 3153 had 15 students enrolled in the course. The survey was administered on a day where there were 14 students present, and each student voluntarily chose to participate. The survey questions are listed below:

I. Background and Experience

1. Did you complete a College Algebra course? (Please check one.)

Yes _________ *No* _________ *If so, what year?* ____________

2. Did you complete a Mathematical Functions and Their Uses course? (Please check one.)

Yes _________ *No* _________ *If so, what year?* ___________

The attitude questions below stemmed from *The Mathematics Attitudes and Perceptions Survey* by Code, Merchant, Maciejewski, Thomas, & Lo. Within their work, they present a survey with 32 statements in which students either agree or disagree. Their survey was administered to students in order to "deepen our understanding of undergraduate mathematics education" (Code, Merchant, Maciejewski, Thomas, & Lo, 2016, p. 1). Also within their work, they sorted each of the 32 statements in their survey into the following categories: Growth mindset, Real world, Confidence, Interest, Persistence, Sense making, Answers, No category but scored for expertise, Filter statement, Expertise (expert consensus). The statements included in our survey stemmed from the authors Real world category. MATH 1483 is typically a contextual course and MATH 1513 typically emphasizes equation manipulation. We decided the statements from the Real world category were appropriate to possibly indicate a difference in the attitudes of students taking either MATH 1483 or MATH 1513, as each course exposes students to different representations of mathematics problems providing the students with a variety of experiences.

Please circle one number between 1 and 5 to indicate the level in which you agree to the statements below.

	Strongly Disagree	**Disagree**	**Neutral**	**Agree**	**Strongly Agree**
3. Learning math changes my ideas about how the world works.	1	2	3	4	5
4. Reasoning skills used to understand math can be helpful to me in my everyday life.	1	2	3	4	5
5. School mathematics has little to do with what I experience in the real world.	1	2	3	4	5
6. When learning something new in math, I relate it to what I already know rather than just memorizing it the way it is presented.	1	2	3	4	5

(Code, Merchant, Maciejewski, Thomas, & Lo, 2016, p. 937)

For the content questions below, we decided to focus on the concept of linear functions. As this is a fundamental concept in mathematics, it has several direct and useful applications to the real world and is seen in higher level mathematics. In creating and deciding the questions to include in the survey, we wanted to include questions that would allow us to analyze student understanding of a contextual representation of linear functions verses student understanding of an algebraic representation of linear functions. We wanted to analyze this distinction because having a contextual understanding and an algebraic understanding of linear functions implies a deeper understanding of the concept as a whole. Having solely an understanding of algebraic representations of linear functions but an inability to apply this concept to contextual situations implies a lower level of understanding of this concept. Because the two courses consistently emphasize different forms of representations of linear functions, MATH 1483 including more contextual problems and MATH 1513 including more algebraic problems, we wanted to analyze if there is a distinction between the level of understanding of the concept of linear functions once a student has completed either one, or both, of these courses. Further, we decided to include questions for three different representations of functions: tabular form, written form, and graphical form. When considering questions to include, we referenced the textbooks used at Oklahoma State University for MATH 1483 and MATH 1513. The MATH 1483 textbook utilized was *Functions and Change: A Modeling Approach to College Algebra* by Crauder, Evans, and Noell, and the MATH 1513 textbook utilized was *College Algebra* by Miller.

Throughout the MATH 1483 textbook, questions of the tabular form appear consistently in a contextual sense. While tabular questions also appeared in the MATH 1513 textbook, they were far more apparent in the MATH 1483 textbook. Thus, we decided to formulate question 1a and 1b based on the table below. We decided to create a question of this form that would allow us to assess whether students understand how to move from a contextual representation of a linear function, to a mathematical representation, and then accurately evaluate their solution in the context of the question.

II. Assessment
Please answer the following questions that use different ways of describing linear functions.

1. *The cost for postage on a first class letter is given in the table.*

    a. *Determine the rate of change of the postage cost per ounce of increase in the weight of the letter.*

Weight	Postage
1 oz	\$0.50
4 oz	\$1.13
6 oz	\$1.55
10 oz	\$2.39

    b. *Determine the postage cost for a letter that weighs 3 oz.*

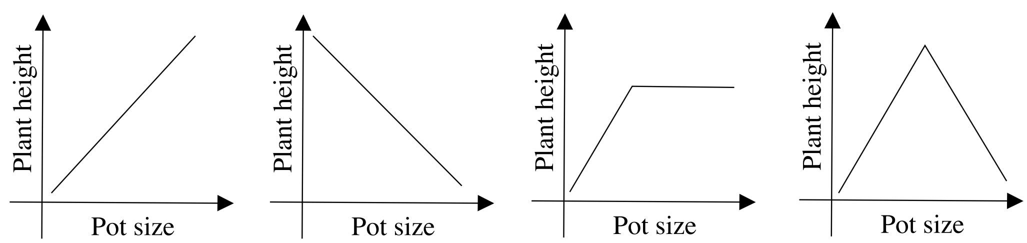
Throughout both the MATH 1483 and MATH 1513 textbooks, questions were often formulated in the form of our second question listed below, tasking students with writing an expression that represents a contextual situation. In particular, we assessed exams from MATH 1483 and MATH 1513. On a MATH 1513 Exam 1 from the Spring 2016 semester at Oklahoma State University, a question of the form below was included. We decided to create a question of this form that would allow us to assess whether students understand how to represent a contextual situation in the form of a mathematical expression.

2. *A taxicab charges $3 a ride, plus $1.50 per mile. Write a model for the cost C of a taxicab ride, in dollars, based on the length x of a ride, in miles.*

Throughout both the MATH 1483 and MATH 1513 textbooks, questions in a graphical form were included consistently. Graphs I, II, and II as well as statements A and B stemmed from a dissertation by Zhixia You, *Preservice Teachers' Knowledge of Linear Functions Within Multiple Representation Modes*. Zhixia You noted that "although there is no exact equation listed in the question, information about an important component of linear function, slope, [is] given in detail" (p. 100). We decided tasking students with evaluating a graph in a contextual sense would allow us to assess students' understanding of linear functions in a graphical form. From Zhixia You's dissertation, we decided to change Graph IV, as the original graph represented a quadratic equation. As the original quadratic equation was negative, we decided to include a positive and negative piecewise linear function for Graph IV to represent a similar idea within the question.

3. *Which graph is best described by each of the following statements below?*

I. II. III. IV.



a. *As the pot size increases, the plant height decreases.*

b. *As the pot size increases, the plant height increases up to a certain pot size. With large pots, plant height remains the same.*

(You, 2006, p. 100)

Analysis of Results

When analyzing the survey responses, we recorded the number of responses for each of the attitude questions, calculated the average response for each of the attitude questions, recorded the number of correct responses for each question, and calculated the percentage of participants who responded correctly for each question. We then divided the participants into three subcategories: participants who took both MATH 1483 and MATH 1513, participants who took MATH 1483, and participants who took MATH 1513. Within each of the subcategories, we performed the same calculations as we did for the overall data: recorded the number of responses for each of the attitude questions, calculated the average response for each of the attitude questions, recorded the number of correct responses for each question, and calculated the percentage of participants who responded correctly for each question.

There were a total of 14 participants. Of those participants, seven completed both MATH 1483 and MATH 1513, three completed MATH 1483, and four completed MATH 1513. When dividing the participants into the subcategories, there was one participant who indicated they took neither MATH 1483 or MATH 1513 because they tested out. We decided to include this participant in the category of participants who took MATH 1513. We made this decision because, after analyzing the prerequisites for each course, testing out of the courses could mean achieving a specific score on the ALEKS Placement Exam or on an AP Calculus Exam. The ability to score well on these courses requires the understanding of mathematical topics represented in MATH 1513. While the participant has been exposed to contextual situations such as the context in MATH 1483, we decided they most likely did not engage in the amount and level of contextual problems represented in MATH 1483. Thus, this participant fits best in the subcategory of participants who took MATH 1513.

Results

Overall Results

Overall Responses

	1	2	3	4	5
Question 3 Learning math changes my ideas about how the world works.	0	2	5	7	0
Question 4 Reasoning skills used to understand math can be helpful to me in my everyday life.	1	0	2	9	2
Question 5 School mathematics has little to do with what I experience in the real world.	1	7	3	3	0
Question 6 When learning something new in math, I relate it to what I already know rather than just memorizing it the way it is presented.	0	2	3	6	3

Overall Average Responses

Question	Average Response
Question 3 Learning math changes my ideas about how the world works.	3.357
Question 4 Reasoning skills used to understand math can be helpful to me in my everyday life.	3.786
Question 5 School mathematics has little to do with what I experience in the real world.	2.571
Question 6 When learning something new in math, I relate it to what I already know rather than just memorizing it the way it is presented.	3.714

These results indicate that the participants have a general appreciation towards mathematics, and they are either neutral or they see a slight value in learning mathematics. These results also indicate that in general, the participants do not have a negative view of mathematics, and do not completely view mathematics as pointless.

Accuracy Of Overall Responses

Question	Correct Responses	Percent of Participants Who Responded Correctly
1a	4	28.571%
1b	5	35.714%
2	13	92.857%
3a	13	92.857%
3b	13	92.857%

These results indicate that the participants in general have a good understanding of questions 2, 3a, and 3b, but a weak understanding of questions 1a and 1b.

I am now going to analyze the results in the following subcategories: participants who completed both MATH 1483 and MATH 1513, participants who completed MATH 1483, and participants who completed MATH 1513. Within each of these subcategories, I will analyze the correct number of responses for each question, the average response for each question, and the accuracy of the participants' responses for each question. For this portion of the analysis, I will be focusing on questions three through six on the survey since those are the content related questions. It must be noted again that these subcategories may be inaccurate due to possible errors of memory in the participants self-identifications on the survey.

Subcategory I

Responses From Participants Who Completed Both MATH 1483 and MATH 1513

	1	**2**	**3**	**4**	**5**
Question 3	0	2	1	4	0
Question 4	1	0	1	4	1
Question 5	0	5	2	0	0
Question 6	0	1	1	4	1

Average Responses From Participants Who Completed Both MATH 1483 and MATH 1513

Question	**Average Response**
Question 3	3.286
Question 4	3.571
Question 5	2.286
Question 6	3.714

These results indicate that in general, these participants see a slight benefit to learning mathematics. However, the average response is so similar to each question, it is difficult to conclude a significant amount of detail about the participants in this category.

Accuracy Of Participants Who Completed Both MATH 1483 and MATH 1513

Question	Correct Responses	Percent of Participants Who Responded Correctly
1a	1	14.286%
1b	1	14.286%
2	7	100%
3a	7	100%
3b	7	100%

These results indicate participants in this category each have a deep understanding of questions 2, 3a, and 3b, and a weak understanding of questions 1a and 1b. It is very interesting out of seven participants who completed two mathematics courses that discussed the concept of linear functions in different representations several times, only one participant was able to correctly apply their knowledge. While all of these questions are over the same topic, there is a distinct difference in the participants' ability to understand linear functions in a tabular form and participants' ability to understand linear functions in a written or graphical form.

Subcategory II

Responses From Participants Who Completed MATH 1483

	1	2	3	4	5
Question 3	0	0	2	1	0
Question 4	0	0	0	3	0
Question 5	0	2	0	1	0
Question 6	0	1	0	1	1

Average Responses From Participants Who Completed MATH 1483

Question	Average Response
Question 3	3.333
Question 4	4
Question 5	2.667
Question 6	3.667

These results indicate that participants of this category slightly have a positive attitude towards mathematics and learning mathematics. They also indicate that the participants in this category do not have a negative attitude towards learning mathematics.

Accuracy Of Participants Who Completed MATH 1483

Question	Correct Responses	Percent of Participants Who Responded Correctly
1a	2	66.667%
1b	2	66.667%
2	3	100%
3a	3	100%
3b	2	66.667%

These results indicate that the participants have a strong understanding of questions 1a and 1b, a deep understanding of questions 2 and 3a, and a good understanding of question 3b. While the percentage of participants who responded correctly to questions 1a and 1b is lower, it must be noted that there are only three students in this subcategory. Thus, two out of the three students correctly responding to questions 1a and 1b indicate a strong understanding of question 1 for this subcategory. It is also interesting to note that more participants in this category than the category of participants who completed MATH 1513 were able to write correct mathematical sentences about their work. Participants in this category had a higher tendency to show their work and explain what they are doing when they are solving the problems.

Subcategory III

Responses From Participants Who Completed MATH 1513

	1	2	3	4	5
Question 3	0	0	2	2	0
Question 4	0	0	1	2	1
Question 5	1	0	1	2	0
Question 6	0	0	2	1	1

Average Responses From Participants Who Completed MATH 1513

Question	Average Response
Question 3	3.5
Question 4	4
Question 5	3
Question 6	3.75

These results indicate that participants of this category have a relatively positive view of learning mathematics. These participants also, for the most part, do not have a negative view of learning mathematics.

Accuracy Of Participants Who Completed MATH 1513

Question	Correct Responses	Percent of Participants Who Responded Correctly
1a	1	25%
1b	2	50%
2	3	75%
3a	3	75%
3b	4	100%

These results indicate these participants have a weak understanding of questions 1a and 1b, a good understanding of questions 2 and 3a, and a deep understanding of questions 3b. It is interesting to note that students accuracy consistently improved as they completed the survey. It is also interesting to note that these results are similar to the results of the participants who completed MATH 1483, as both groups have a weak understanding of questions 1a and 1b.

Conclusions and Discussion

In regard to our first research question, there is no difference in the way students understand graphical representations of linear functions based on their prior experience with either College Algebra or Mathematical Functions and Their Uses. In each of the three subcategories, there was a high percentage of participants that were able to accurately answer to the questions regarding graphical representation of linear functions. Particularly, 100% of participants who completed both MATH 1483 and MATH 1513 correctly answered questions 3a and 3b, 100% of participants who completed MATH 1483 correctly answered question 3a and 66.667% correctly answered question 3b, and 75% of the participants who completed MATH 1513 correctly answered question 3a and 100% correctly answered question 3b. This indicates that participants' prior mathematical experience with either of these courses does not make a difference in their understanding of graphical representations of linear functions.

It is interesting to note that while question one is formulated in a contextual situation, participants could potentially write a mathematical expression that accurately represents the situation without understanding what they wrote, specifically without understanding what the rate of change means in equation form. For example, "slope-intercept form, written as $y = mx + b$, is the most frequently introduced symbolisms of linear function in the middle school mathematics curriculum" (You, 2006, p. 82). Because this form is so frequently covered in mathematics courses, it is possible students could memorize this expression and understand which values from a contextual situation go in the correct places in the expression without understanding what the slope value (m) really means in a contextual situation. Understanding how to write an expression and understanding what the expression means are two separate levels of understanding linear functions. It would have been interesting to include a question on the survey that asked participants to explain which value in their expression is the rate of change and why. We believe this would have given us more insight as to students' ability in connecting algebraic representations of linear functions to contextual representations of linear functions. Including a question of that sort could have also given us insight as to why roughly 92.857% of the participants could correctly represent an expression given a contextual situation, but only 28.571% of the participants could accurately apply the same reasoning to question 1.

In regard to our second research question, there is no difference in the way students understand tabular representations of linear functions based on their prior experience with either College Algebra or Mathematical Functions and Their Uses. In fact, in each of the three subcategories, there was a weak understanding of the tabular representation of linear functions. Particularly, in the overall analysis of the responses, only 28.571% of the total participants correctly answered question 1a (4 out of 14) and 35.714% of the total participants correctly answered question 1b (5 out of 14). It is interesting to note that question 1a asks the participants to find the rate of change within the table, which is a concept covered heavily in both courses. Because each of the subcategories were so small, there is no true distinction between the way participants from one category understood the tabular form of linear functions versus another category. In general, the participants as a whole had trouble applying their knowledge of linear functions in the form of a table from a real-world situation to solve a problem.

In regard to our third research question, there is no difference in the way students understand contextual representations of linear functions based on their prior experience with either College Algebra or Mathematical Functions and Their Uses. In particular, each of the subcategories indicated a high level of understanding this representation. 100% of the participants who completed both MATH 1483 and MATH 1513 correctly responded to question 2, 100% of the participants who completed MATH 1483 correctly responded to question 2, and 75% of participants who completed MATH 1513 correctly responded to question 2. While the participants who completed MATH 1513 make up the only subcategory that did not have a 100%

accuracy rate for this question, this is really due to only one participant within the category because there was a total of 4 students in the category. Therefore, the slight difference in accuracy is not enough to distinguish if students who completed MATH 1483 have a better understanding of this representation of linear functions. Again, it would have been interesting to include a question asking participants to explain what the rate of change within their expression is and why. Responses to a question of that sort could possibly have given us the ability to make a distinction between the subcategories, as well as possibly understand why participants overall had a weak understanding of question 1.

In regard to our fourth research question, there is no difference in students self-efficacy of linear functions based on their prior experience with either College Algebra or Mathematical Functions and Their Uses. Participants from each of the subcategories were either neutral or had slightly a positive attitude towards learning mathematics. Because the average answer in the categories was typically 3, and the number 3 represents 'Neutral' on the survey, it is difficult to truly conclude anything about the participants view of learning mathematics and mathematics' usefulness in the real-world.

We must also note that the survey was anonymous, and therefore the papers had no identifying characteristics on them. When we asked students to identify if they had taken MATH 1483 or MATH 1513 on the survey, there is no way to verify if their responses are accurate. It is possible the participants incorrectly indicated which course they completed due to errors of memory. Thus, the results within each of the subcategories may differ from results if the participants were to accurately distinguish their background experience or if we could verify their responses.

For further research on this topic, it would be helpful to gather results from a larger group of participants. For example, there were three participants within the subcategory that took MATH 1483 and did not take MATH 1513. It can be hard to draw conclusions from these results given there is a small variety in the answers the participants can provide. Ensuring a large number of participants for each subcategory would potentially reveal results that are unable to be seen within this study. In addition to gathering a larger number of participants, it would also be helpful to design an IRB approved study that could collect identifying information on students to confirm their history in basic mathematics courses such as MATH 1483 and MATH 1513. This could avoid the potential issue of human error in self-identifying on the survey we distributed regarding the participants' math history. It might also be beneficial to assess student understanding of linear functions of students who are currently enrolled in MATH 1483 and MATH 1513, and create a new subcategory for these participants. The results these participants could provide might reveal new information regarding the depth of student understanding of linear functions in terms of the length of time they were introduced to the concept. Finally, given that Hughes-Hallett and McCallum eventually moved from the Rule of Three to the Rule of Four, it would be helpful to adopt a survey that followed the Rule of Four. Our survey incorporated function relationships in the form of a table, graph, and contextual problem. It's possible new results could be revealed if we were to design a survey that also incorporated a purely formula based question as well.

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Appendix A

Survey Administered to Participants

Questionnaire

I. Background and Experience
Please answer the following questions about your math background and experience.

1. Did you complete a College Algebra course? (Please check one.)

   Yes _________ No _________ If so, what year? ____________

2. Did you complete a Mathematical Functions and Their Uses course? (Please check one.)

   Yes _________ No _________ If so, what year? ____________

Please circle one number between 1 and 5 to indicate the level in which you agree to the statements below.

	Strongly Disagree	Disagree	Neutral	Agree	Strongly Agree
3. Learning math changes my ideas about how the world works.	1	2	3	4	5
4. Reasoning skills used to understand math can be helpful to me in my everyday life.	1	2	3	4	5
5. School mathematics has little to do with what I experience in the real world.	1	2	3	4	5
6. When learning something new in math, I relate it to what I already know rather than just memorizing it the way it is presented.	1	2	3	4	5

II. Assessment
Please answer the following questions that use different ways of describing linear functions.

1. The cost for postage on a first class letter is given in the table.

   a. Determine the rate of change of the postage cost per ounce of increase in the weight of the letter.

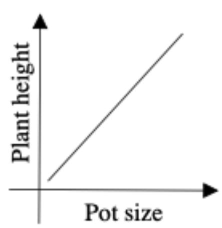
Weight	Postage
1 oz	$0.50
4 oz	$1.13
6 oz	$1.55
10 oz	$2.39

b. Determine the postage cost for a letter that weighs 3 oz.

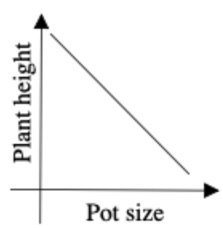
2. A taxicab charges \$3 a ride, plus \$1.50 per mile. Write a model for the cost C of a taxicab ride, in dollars, based on the length x of the ride, in miles.

3. Which graph is best described by each of the following statements below?

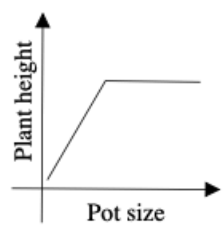
I.



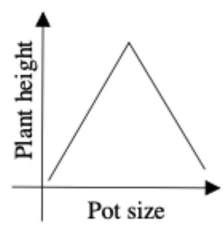
II.



III.



IV.



a. As the pot size increases, the plant height decreases.

b. As the pot size increases, the plant height increases up to a certain pot size. With larger pots, plant height remains the same.

Appendix B

Answer Key to Survey Administered to Participants

Questionnaire

I. Background and Experience
Please answer the following questions about your math background and experience.

1. Did you complete a College Algebra course? (Please check one.)

   Yes _________ No _________ If so, what year? ____________

2. Did you complete a Mathematical Functions and Their Uses course? (Please check one.)

   Yes _________ No _________ If so, what year? ____________

Please circle one number between 1 and 5 to indicate the level in which you agree to the statements below.

	Strongly Disagree	Disagree	Neutral	Agree	Strongly Agree
3. Learning math changes my ideas about how the world works.	1	2	3	4	5
4. Reasoning skills used to understand math can be helpful to me in my everyday life.	1	2	3	4	5
5. School mathematics has little to do with what I experience in the real world.	1	2	3	4	5
6. When learning something new in math, I relate it to what I already know rather than just memorizing it the way it is presented.	1	2	3	4	5

II. Assessment
Please answer the following questions that use different ways of describing linear functions.

1. The cost for postage on a first class letter is given in the table.

   a. Determine the rate of change of the postage cost per ounce of increase in the weight of the letter.

Weight	Postage
1 oz	$0.50
4 oz	$1.13
6 oz	$1.55
10 oz	$2.39

$0.21/ounce

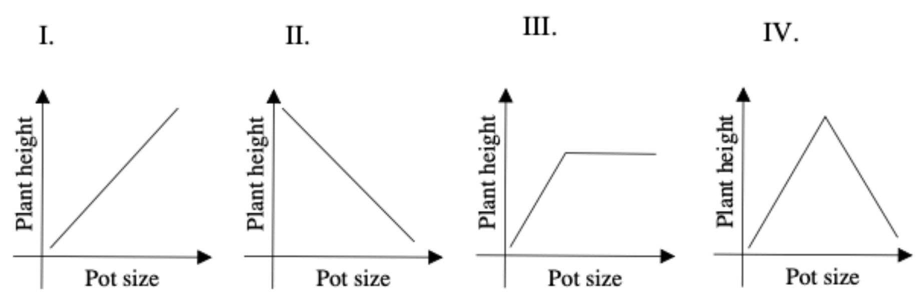
b. Determine the postage cost for a letter that weighs 3 oz.

$0.92

2. A taxicab charges $3 a ride, plus $1.50 per mile. Write a model for the cost *C* of a taxicab ride, in dollars, based on the length *x* of the ride, in miles.

C = $1.50*x* + 3

3. Which graph is best described by each of the following statements below?



a. As the pot size increases, the plant height decreases.

II.

b. As the pot size increases, the plant height increases up to a certain pot size. With larger pots, plant height remains the same.

III.

Appendix C

Course Descriptions at Oklahoma State University (2018 - 2019)

- MATH 1483: Mathematical Functions and Their Uses

  “Analysis of functions and their graphs from the viewpoint of rates of change. Linear, exponential, logarithmic and other functions. Applications to the natural sciences, agriculture, business and the social sciences. Prerequisite(s): An acceptable placement score - see placement.okstate.edu."

- MATH 1513: College Algebra

  “Quadratic equations, functions and graphs, inequalities, systems of equations, exponential and logarithmic functions, theory of equations, sequences, permutations and combinations. Combined credit toward a degree for MATH 1513, MATH 1613 and MATH 1715 limited to six hours. Prerequisite(s): An acceptable placement score (see placement.okstate.edu). Two years of high school algebra recommended.”

Retrieved from the registration page on Student Portal in the Banner System for OSU students.

Appendix D

Elementary Education Degree Sheet 2018 - 2019

ELEMENTARY EDUCATION, BS

Requirements for Students Matriculating in or before Academic Year 2018-2019. Learn more about University Academic Regulation 3.1 (http://catalog.okstate.edu/university-academic-regulations/#matriculation).

Minimum Overall Grade Point Average: 2.75
Total Hours: 124

Code	Title	Hours
General Education Requirements		
*Minimum grade of "C" or "P" in each course		
*Minimum GPA 2.75 required in combination with Major Requirements		
*Certification requirements that meet GE requirements		
*English Composition courses**		
See Academic Regulation 3.5 (http://catalog.okstate.edu/university-academic-regulations/#english-composition)		
ENGL 1113	Composition I	3
or ENGL 1313	Critical Analysis and Writing I	
Select one of the following courses*:		3
ENGL 1213	Composition II	
ENGL 1413	Critical Analysis and Writing II	
ENGL 3323	Technical Writing	
American History & Government		
Select one of the following courses*:		3
HIST 1103	Survey of American History	
HIST 1483	American History to 1865	
HIST 1493	American History Since 1865	
POLS 1113	American Government	3
Analytical & Quantitative Thought (A)		
Select 6 hours of the following courses*:		6
MATH 1483	Mathematical Functions and Their Uses (A)	
MATH 1493	Applications of Modern Mathematics (A)	
MATH 1513	College Algebra (A)	
MATH 1613	Trigonometry (A)	
MATH 2103	Business Calculus (A)	
MATH 2144	Calculus I (A)	
STAT 2013	Elementary Statistics (A)	
Humanities (H)		
Select one of the following courses*:		3
ENGL 2413	Conversations in Literature (DH)	
ENGL 1923	Great Works of Literature (H)	
ENGL 2883	Survey of American Literature II (DH)	
Select one of the following courses*:		3
ART 1503	Art History Survey I (H)	
ART 1513	Art History Survey II (H)	
MUSI 2573	Introduction to Music (H)	
Natural Sciences (N)		
Must include one Laboratory Science (L) course*		
BIOL 1114	Introductory Biology (LN)	4
CHEM 1014	Chemistry In Civilization (LN)	4
or PHYS 1014	Descriptive Physics (N)	
Select one of the following courses*:		4
GEOG 1114	Physical Geography (LN)	
GEOL 1014	Geology and Human Affairs (LN)	
GEOL 1114	Physical Geology (LN)	
*Social & Behavioral Sciences (S) courses**		
GEOG 1113	Introduction to Cultural Geography (IS)	3
or GEOG 1713	World Regional Geography (IS)	
or GLST 1713	World Regional Geography (IS)	
PSYC 1113	Introductory Psychology (S)	3
or SOC 1113	Introductory Sociology (S)	
Additional General Education		
Select one of the following courses*:		3
ENGL 2243	Language, Text and Culture (HI)	
ENGL 2513	Introduction to Creative Writing (H)	
SPCH 2713	Introduction to Speech Communication (S)	
Hours Subtotal		45
Diversity (D) & International Dimension (I)		
May be completed in any part of the degree plan		
Select at least one Diversity (D) course		
Select at least one International Dimension (I) course		
College/Departmental Requirements		
EDUC 1111	First Year Seminar	1
Select 10 hours of electives (3 hours may need to be foreign language)		10
Hours Subtotal		11
Major Requirements		
Minimum GPA 2.75 with a minimum grade of "C" or "P" in each course		
CIED 3133	Children's Literature Across the Curriculum	3
CIED 3253	Teaching Language Arts in the Elementary and Middle School	3
CIED 3293	Teaching Reading in the Elementary and Middle School	3
CIED 4213	Introduction to Visual Arts in the Curriculum	3
CIED 4233	Literacy Assessment and Instruction (Students must pass the Oklahoma Reading Test to receive a passing grade)	3
HLTH 2603	Total Wellness (S)	3
MATH 3403	Geometric Structures for Early Childhood and Elementary Teachers	3
MATH 3603	Mathematical Structures for Early Childhood and Elementary Teachers	3
SMED 3153	Teaching Mathematics at the Primary Level	3
Taken as a block semester prior to student teaching:		
CIED 3430	Early Lab and Clinical Experience in Elementary Education II [1]	1
CIED 4323	Social Studies in the Elementary School Curriculum [1]	3
CIED 4362	Design and Management of the Elementary School Classroom [1]	2
CIED 4073	Elementary School Curriculum Design and Development [1]	3