THE GENERALIZED SAGNAC EFFECT WITH THE RINGLASER AND OTHER OPTRADICHES IN THE PPN GRAVITATIONAL THEORY

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Submitted to the Faculty of the Graduate College
of the Oklahoma State University
in partial fulfillment of the requirements
for the Degree of
DOCTOR OF PHILOSOPHY
July, 1976

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PREFACE

The object of the study is an experiment to measure the "dragging of inertial frames" effect using two closed-loop beams traveling in opposite directions. This experiment has been analyzed in the literature before, with General Relativity used as the gravitational theory. However, the Parametrized Post-Newtonian Formalism is used here. One reason is that it provides a weak-field approximation scheme which can be easily applied to Solar System cases; another is that it is widely used by experimentalists to analyze their experiments (W3) (W4) (W5) [These numbers within parentheses are citations of items in the bibliography; another example: (M3, Section 40.7)]. A theoretical description of the experiment is developed from a review of the literature. Two simple cases are analyzed to yield numerical results for Earth-based loops and Sun-, Jupiter-, and Earth-orbiting loops.

The viewpoint of the thesis in general is that of the experimentalist who wants to begin to learn how to judge the scientific worth and feasibility of the experiment, and, in case he decides that such experiments should be implemented, in planning, design and constructing it. Thus, equations are written using the SI system of units (with the symbols for the units) rather than these theoretical systems in which the speed of light is unity, and so on.

Footnotes in a chapter are placed at the end of that chapter; and some notations, conventions, etc. are detailed in Section I.C.

Quite possibly, if it were not for my parents, Drs. Elvin Odell Campbell and Lois Evelyn Franklin Campbell, and Drs. William L. Hughes, Hans R. Bilger, and W. Kent Stowell, I may never be writing a preface to a Ph.D. thesis. To the above, a lot of thanks!

I wish to express my appreciation to my thesis adviser, Dr. Hans R. Bilger, for suggesting the thesis topic, guiding and assisting the research and thesis preparation, making numerous suggestions, and prodding me--he was very helpful. Appreciation is also expressed to the other committee members, Drs. William L. Hughes, Eugene K. McLachlan, and Nyayapathi V. V. J. Swamy, for serving so well on my committee, and also for being so helpful.

Heinz Dehnen, an expert in the thesis field, has written a review of the thesis that was helpful, and has made some suggestions (see especially Section III.C). These are appreciated. Danke!

Charlene Fries was invaluable in typing the thesis. She's a pro! Thank you, Charley.

The staff of the Oklahoma State University Library should also be singled out and thanked. If there are any others which I should have mentioned and thanked here but inadvertently did not, please accept my apologies and know that your help is also appreciated. To the others too numerous to mention, a megagram of thanks!

I want to acknowledge and show my appreciation for the financial support provided by my parents and by Dr. W. L. Hughes, Head of the Electrical Engineering Department at Oklahoma State University.

This thesis is dedicated to the memory of my father,

Doctor Elvin Odell Campbell.

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CHAPTER I

INTRODUCTION AND PRELIMINARIES

I.A General Introduction

Although General Relativity, among other gravitational theories, appears to fit current experimental data best, and although there are now numerous experiments already done or seriously considered to test it and other gravitational theories, it is at least desirable to seek further ways to test the currently viable and future gravitational theories. The type of experiment considered here has already been proposed and analyzed several times before (see Chapter II), but has not been seriously considered otherwise. Heretofore only General Relativity has been used to analyze it, but it seems desirable here to use the Parametrized Post Newtonian Formalism (see Section I.D). A reason is that the latter formalism is a weak-field approximation scheme that can easily be applied, e.g., to the case of a gyroscope in orbit about the sun (some special cases in the Solar System are considered in Chapter III).

A theoretical description--the Sagnac Effect is generalized--of the experiment is developed from a study of the literature (Chapter II).

The type of instrument on which the experiment is based is introduced in the next section. An example is the ringlaser (Section I.E), whose sensitivity could be developed to the point where the experiment becomes practical.

I.B The Optradich

In this thesis, "loop" means a simple closed curve such that it is the boundary of at least one surface such that Stokes' theorem is applicable to the curve and the surface. Let there be two rays which travel on a loop, one in each direction, and a device or process which gives information about the difference in the times required by the rays to complete opposite circuits around the loop (the trip times). The loop may be defined by three or more mirrors (as in Figure 1), by fiber light guides (V1), by media with variable indices of refraction (C1), etc. In this thesis, such a system is generically called an optradich: the word is an acronym of OPpositely Traveling RAy DIfferential CHronometer (the origin of the latter two terms: S8, page 401). Examples, besides the ringlaser (Figure 1), are the interferometric optradich (Figure 2), and a system using three satellites which is described briefly in Chapter IV. As these examples show, there is a variety of ways in which the optradich can be excited and a variety of kinds of information that the optradich can give on the difference in the trip times. Strictly speaking, the "rays" in the optradich are pencils of rays or beams, but the beams in an optradich are usually so monochromatic and narrow that they can be approximated by "rays"; henceforth, "ray," "beam," and "signal" are used interchangeably.

According to the well-known reciprocity theorem in electromagnetics, the trip time difference should be zero for an optradich at rest in an inertial frame of reference, but there are some phenomena exhibiting "nonreciprocity": e.g., Fresnel drag in moving media (B2) (S7), or the Faraday Effect (K1, page 53). The Sagnac Effect, to be described soon, and some gravitational effects considered in this thesis may also produce

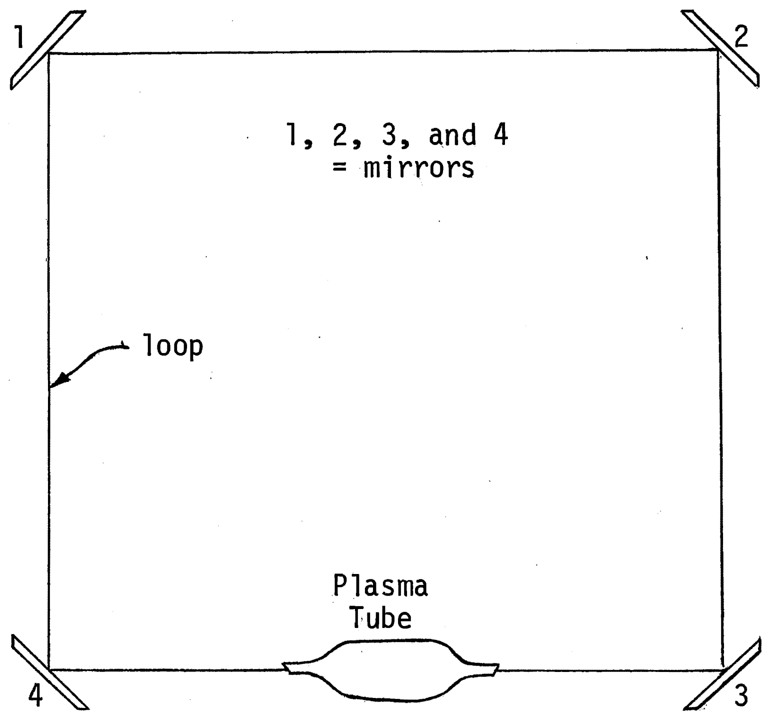

Figure 1. The Ringlaser

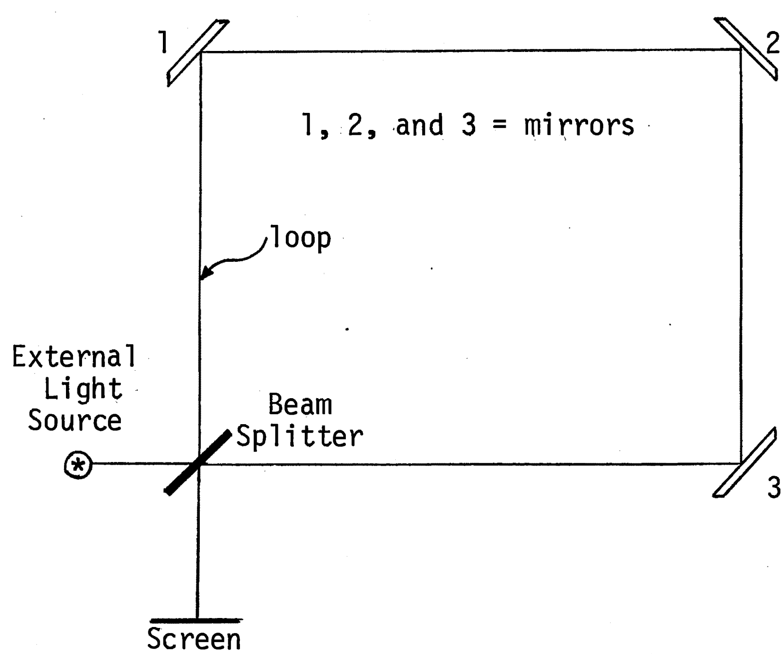

Figure 2. The Interferometric Optradich

time nonreciprocities, albeit the optradich is not then generally at rest in an inertial frame of reference.

A simple treatment of the Sagnac Effect is made by Post (P3, pages 479-80): an optradich of circular shape is subjected to a rotation, Ω, about its axis of symmetry relative to an inertial frame of reference. He shows how the trip time difference, ΔT, is related to Ω:

$$\Delta T = \frac{4}{c^2}\,\Omega A, \tag{I.B.1}$$

where A is the area of the plane surface enclosed by the circle, c is the speed of light in vacuum, and $\Omega R \ll c$, where R is the radius of the circle. This can be generalized to apply to a loop C not necessarily planar. If Γ is any surface having only C as its boundary and Γ is oriented in the usual mathematical sense, then

$$\Delta T = \frac{4}{c^2}\int_{\Gamma} \vec{\Omega}\cdot\vec{n}_u \; dA, \tag{I.B.2}$$

where $\vec{\Omega}$ is now a vector quantity, $\vec{n}_u$ is the unit vector normal to the surface pointing in the positive direction with respect to the orientation of Γ, and dA is the area of a surface element of Γ (vector notation is explained in the next section). In the next chapter, the Sagnac Effect is generalized to include gravitationally induced time nonreciprocities.

The next section details some important notations, conventions, definitions, and symbols used in this thesis.

I.C Some Notations, Conventions, Definitions, and Symbols

The standards adopted in this thesis are closer to those of Misner, Thorne, and Wheeler (M3) than to those of any other work, albeit the thesis does not go far in following their "modern notation."

Two mathematical relational symbols are used: $\equiv$, identity or definition; $\simeq$, "is approximated by" or "is approximate to."

Greek indices on tensors range over 0, 1, 2, and 3; Latin ones, over 1, 2, and 3, unless otherwise noted (exceptions: x, y, z, and u). Einstein's summation convention is used: whenever an index--Greek or Latin--appears twice in a mathematical term, summation over that index is implied. The line element in Riemann geometry is denoted by ds: $ds^2 = g_{\alpha\beta}dx^\alpha dx^\beta$, where $g_{\alpha\beta}$ is the metric tensor and x^α are the coordinates. The signature of $g_{\alpha\beta}$ is +2. To lower a contravariant index, $g_{\alpha\beta}$ is used: $v_\alpha \equiv g_{\alpha\beta}v^\beta$; $g^{\alpha\beta}$--where $g^{\alpha\mu}g_{\mu\beta} = 1$ if $\alpha = \beta$, or 0 if $\alpha \neq \beta$--is used to raise a covariant index: $v^\alpha \equiv g^{\alpha\beta}v_\beta$. Partial differentiation is indicated by a comma: $T^\alpha{}_{,\beta} \equiv \partial T^\alpha/\partial x^\beta$; covariant differentiation, by a semicolon: $T^\alpha{}_{;\beta} \equiv T^\alpha{}_{;\beta} + \Gamma^\alpha_{\gamma\beta}T^\gamma$, where $\Gamma^\alpha_{\beta\gamma}$ is the connection coefficient (M3, page 210). Since ds^2 is negative on a time-like trajectory in spacetime, $c\,d\tau = (-g_{\alpha\beta}dx^\alpha dx^\beta)^{1/2}$ is used, instead: τ, the <u>proper time</u> of the trajectory, is used to parametrize it.

Three-dimensional (spatial) vectors are denoted by superior arrows: $\vec{v}$. They are treated as Cartesian-like. Vector operations are denoted: the dot or scalar product by $\vec{v}\cdot\vec{w}$; the cross or vector product by $\vec{v} \times \vec{w}$; the gradient of a scalar, ψ, by $\nabla\psi$; the divergence of $\vec{v}$ by $\vec{\nabla}\cdot\vec{v}$; the curl of $\vec{w}$ by $\vec{\nabla} \times \vec{w}$; and the magnitude of $\vec{v}$ by $|\vec{v}|$. The subscript u on a vector--e.g., $\vec{n}_u$--means the vector is a unit vector: $\vec{n}_u\cdot\vec{n}_u = 1$; the symbol u as a subscript is never used as an index.

The components of a matrix (or tensor) are sometimes shown within double lines, as below:

$$||\eta_{\alpha\beta}|| \equiv \begin{Vmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{Vmatrix}, \qquad \text{(I.C.1)}$$

where α indicates the row, and β indicates the column.

In the rest of the thesis, let

$$g \equiv \text{determinant of } ||g_{\alpha\beta}||, \qquad \text{(I.C.2)}$$

$$\delta_{\alpha\beta} \equiv \delta^{\alpha}_{\beta} \equiv \delta^{\alpha\beta} \equiv \begin{cases} 1 \text{ if } \alpha = \beta \\ 0 \text{ if } \alpha \neq \beta, \end{cases} \qquad \text{(I.C.3)}$$

$$\varepsilon_{\alpha\beta\gamma\delta} \equiv (-g)^{1/2} [\alpha\beta\gamma\delta], \qquad \text{(I.C.4)}$$

where in general

$$[\alpha\beta\gamma \ . \ . \ .] \equiv \begin{cases} 1 & \text{if } \alpha\beta\gamma \ . \ . \ . \text{ is an even permutation of } 1\ 2\ 3\ . \ . \ . \ , \\ -1 & \text{if } \alpha\beta\gamma \ . \ . \ . \text{ is an odd permutation of } 1\ 2\ 3\ . \ . \ . \ , \\ 0 & \text{if otherwise.} \end{cases} \qquad \text{(I.C.5)}$$

The local inertial frames of reference with Cartesian coordinates, as used in Special Relativity, are denoted in this thesis as Lorentz frames. In such a frame, $g_{\alpha\beta} = \eta_{\alpha\beta}$, where $\eta_{\alpha\beta}$ is as in Equation (I.C.1). A local inertial frame in general is denoted an IFR; a noninertial frame, NIFR. The term, "acceleration," when used without qualification, always means acceleration relative to nearby IFRs.

The SI system of units, with the symbols for them, is used; however, Gaussian electrodynamic units are also used in one subsection for reasons given there. The symbol, c, denotes the speed of light in vacuum as measured in an IFR (2.9979 x 10^8 m s^{-1}), and G is the Cavendish (gravitational) constant (6.673 x 10^{-11} N m^2 kg^{-2}).

I.D The PPN Formalism

This section gives what is a little more than an abstract or summary from the references (M3, Chapter 39) (W3) (W4) (W5) (W6). It serves the purpose of giving some necessary background and establishing notation.

Will and others have discovered that in the important case of weak gravitational fields and low matter velocities, such as is given by the Solar System, many important theories including General Relativity can be fitted by a common, general form: the Parametrized Post Newtonian (PPN) Formalism. In simple, brief terms, these theories, which have been called "metric theories," all agree on how test mass-particles of negligible mass and extent behave in spacetime, which has an assigned Riemann geometry; but they differ on how matter-energy shapes the geometry. The PPN Formalism provides a general metric having parameters (the PPN parameters) whose numerical values depend on how a particular "metric theory" describes the dynamic influence of matter-energy on the geometry.

Most non-Newtonian gravitational experiments can be analyzed in this formalism (exceptions: the gravitational radiation experiments and a few others). After any one of these experiments is performed and the resulting data is analyzed, estimates of the pertinent PPN parameters' values can then be made. If these estimates are sufficiently tight, they can be used to eliminate one "metric theory" or more as nonviable.

In this formalism, for a gravitationally bound system like the Solar System, the components of the metric tensor are expanded in orders of a small dimensionless parameter, ε, where

$\varepsilon^2 \equiv$ maximum value of U anywhere in the system, and

$\varepsilon^2 \gtrsim$ other dimensionless physical quantitites which are functions of velocity, pressure, mass, density, etc., that characterize the state of matter-energy in the system, (I.D.1)

where

$$U \equiv -\frac{\Phi}{c^2}, \qquad \text{(I.D.2)}$$

a dimensionless, positive "gravitational potential," and where Φ is Newton's gravitational potential.

Various physical and mathematical arguments lead to the "Newtonian" or first approximation to the metric tensor in the formalism:

$$g_{oo} = -1 + 2U + O(\varepsilon^4),$$
$$g_{oj} = O(\varepsilon^3),$$
$$g_{ij} = \delta_{ij} + O(\varepsilon^2), \qquad \text{(I.D.3)}$$

and then to the post-Newtonian approximation, which is parametrized by the PPN parameters Δ_1, Δ_2, α_1, α_2, and γ:

$$g_{oo} = -1 + 2U + O(\varepsilon^4),$$
$$g_{oj} = -\frac{7}{2}\Delta_1 V_j - \frac{1}{2}\Delta_2 W_j + (\alpha_2 - \frac{1}{2}\alpha_1)\frac{w_j}{c}U$$
$$- \alpha_2 \frac{w_k}{c} U_{kj} + O(\varepsilon^5),$$
$$g_{ij} = \delta_{ij}(1 + 2\gamma U) + O(\varepsilon^4), \qquad \text{(I.D.4)}$$

where V_j, W_j, and U_{kj} are other dimensionless "gravitational potentials," and w_j are the components of the velocity of the PPN frame of reference relative to the "preferred universal rest frame" that certain metric theories single out. General Relativity is not such a one. There are more terms in g_{oo} than presented here but they are negligible in this thesis.

Table I shows the PPN parameters used in this thesis (first column), their heuristic significances (second column), their values in General

TABLE I

HEURISTIC DESCRIPTION OF SOME PPN PARAMETERS[1]

PPN Parameter	What it Measures, Relative to General Relativity	Value in General Relativity	Value in Brans-Dicke-Jordan Theory*	Experimental Bounds
γ	How much curvature is produced by unit rest mass?	1	$\frac{1+\tilde{\omega}}{2+\tilde{\omega}}$	1.03 ± 0.022
α_1	To what extent and in what way does the theory single out a preferred Universal rest frame?†	0	0	0 ± 0.2
α_2		0	0	0 ± 0.03
Δ_1	How much dragging of inertial frames is produced by unit momentum?	1	$\frac{10+7\tilde{\omega}}{14+7\tilde{\omega}}$	1.03 ± 0.02
Δ_2	How much easier is it for momentum to drag inertial frames radially (towards the observer) than in a transverse direction?	1	1	1 ± 0.03

*$\tilde{\omega}$ is the Dicke coupling constant, a free parameter whose value is set by measurements. A current estimate places it at $\gtrsim 23$ (R1).

†There is another such parameter, α_3, but it is not needed in this thesis.

Relativity (third column) and in the Brans-Dicke-Jordan theory (fourth column), and their present experimental bounds (last column).

The quantities Δ_1, Δ_2, α_1, α_2, and γ are related as follows:

$$\Delta_2 \equiv \alpha_2 - \zeta_1 + 1,$$

$$\Delta_1 \equiv (\alpha_1 - \Delta_2 + 4\gamma + 4)/7, \qquad \text{(I.D.5)}$$

where ζ_1 is another PPN parameter that is needed only here (M3, Box 39.5). The quantity ζ_1 has the value zero in both General Relativity and the Brans-Dicke-Jordan theory.

As an example, Will and Nordtvedt's point-mass metric--rewritten to conform to the standards of this thesis--appears as follows:

$$g_{oo} = -1 + 2U + O(\varepsilon^4),$$

$$g_{oj} = -\frac{7}{2}\Delta_1 \sum_M U_M \frac{V^j_M}{c} - \Delta_2 \sum_M \frac{U_M(\vec{V}_M \cdot \vec{R}_M)}{c|\vec{R}_M|^2} R^j_M - \frac{1}{2}(\alpha_1 - 2\alpha_2)\frac{w^j}{c} U$$

$$-\alpha_2 \sum_M U_M \frac{(\vec{w}\cdot\vec{R}_M)}{c|\vec{R}_M|^2} R^j_M + O(\varepsilon^5),$$

$$g_{ij} = \delta_{ij}(1 + 2\gamma U) + O(\varepsilon^4), \qquad \text{(I.D.6)}$$

where

$$U_M = \frac{G}{c^2}\frac{M_M}{|\vec{R}_M|}, \quad U = \sum_M U_M,$$

$\vec{V}_M$ = the PPN coordinate velocity of the Mth point-mass, V^j_M being the jth component;

M_M = the mass of the Mth point-mass as evaluated in its PPN rest frame;

$\vec{R}_M$ = the PPN coordinate vector separating the field point from the Mth point-mass, R^j_M being the jth component;

and $\sum_M$ indicates summation over the point-masses in the system (W6, Table 4). (The above velocities, divided by c, are of the order $O(\varepsilon^1)$.) For

$M = 1$ and $V_M^j = 0$, the metric reduces to an approximation of the isotropic form of the familiar Schwarzschild metric when the PPN parameters taken on their General Relativity values (see Table I), to first order in U.

I.E The Ringlaser

Although the thesis in general was not written for a particular type of optradich, the ringlaser is discussed in this section because it appears to be a particularly promising one for gravitational experiments, especially terrestrial ones: Bilger and Zavodny (B2, page 591) state "The ringlaser is an extremely sensitive instrument for measuring nonreciprocal phenomena in light propagation." Also, four of the papers reviewed in the next chapter concern ringlasers (D2) (K3) (V3) (V4).

The basic principles of the ringlaser, its potential, and some of the chief practical problems in its application in experiments to test predictions such as in Chapter III are discussed.

I.E.1 The Idealized Ringlaser

The frequency, f, of one of the two contratraveling beams in the idealized ringlaser is related to the trip time of the beam, T:

$$f = \frac{\Pi}{T} , \qquad \text{(I.E.1)}$$

where Π is the "longitudinal mode number" (a large positive integer). Since the difference in frequency between the contratraveling rays, Δf, and the trip time difference, ΔT, are infinitesimal as compared to f and T, respectively, the above can be differentiated, and the result rearranged to this:

$$\Delta f = -\frac{f}{T}\Delta T; \qquad \text{(I.E.2)}$$

Δf is called the beat frequency. Typical values are as follows: $f = 10^{14}$ to 10^{15} Hz; $T = 10^{-9}$ to 10^{-8} s; $\Delta T = 10^{-21}$ to 10^{-16} s; $\Delta f = 10$ to 10^5 Hz. Note the smallness of the values for ΔT as compared to those for Δf.

The Sagnac Effect gives rise to a beat frequency in the idealized ringlaser:

$$\Delta f = 4\frac{A}{\lambda L}\vec{\Omega}\cdot\vec{n}_u, \qquad \text{(I.E.3)}$$

where the loop is assumed planar, $\lambda = c/f$ and $L = cT$, and Equations (I.B.2) and (I.E.2) have been used.

I.E.2 Comparison With the Interferometer

The interferometric optradich requires an external light source as in Figure 2. The information on the trip time difference is given by the shift in the fringes when the contratraveling beams are brought to interference. The shift, in terms of number of fringes, ΔZ, is related to ΔT (ideally):

$$\Delta Z = F\Delta T, \qquad \text{(I.E.4)}$$

where F is the frequency of the external light source.

In contrast, the ringlaser is a "self-oscillating" loop, i.e., it generates its own signals. The plasma tube in Figure 1 contains the lasing system, which "pumps up" or amplifies the signals as they pass through the tube--it can be compared to the amplifier in an electronic oscillator; whereas the loop--actually, it is more often called a cavity in ringlaser work--can be compared to the tank circuit consisting of L and C. The information it gives on the trip time difference is in the form of Δf in Equation (I.E.2).

The performance of an actual interferometric optradich and of an actual ringlaser in measuring the Sagnac Effect due to Earth's rotation are now compared. Michelson and Gale (M2) used an interferometric optradich of loop length, $L \equiv cT = 1{,}900$ m, to obtain a fringe shift, $\Delta Z = .230 \pm .005$. The huge size was needed to obtain this level of accuracy--for one thing, the external light source was not monochromatic (one wonders how much better the performance would be if Michelson and Gale used a laser instead). With a ringlaser of only $L = 3.4$ m, a value of $\Delta f = 48 \pm .6$ Hz was obtained from a 1974 ringlaser experiment with Fresnel drag in moving media (S7), using the same data analysis procedure as in the Michelson-Gale paper (M2).[2] The relative accuracies of measurements were thus $.005/.230 \simeq 2.2\%$ for the interferometer and $.6/48 \simeq 1.2\%$ for the ringlaser. Considering that the ringlaser experiment was not at all designed to measure the Sagnac Effect accurately and that the range in the beat frequencies actually measured (due mainly to the Fresnel drag in the moving medium) was .8 to 53 kHz, the latter error seems quite small.

I.E.3 Practical Considerations

An idea of the ringlaser's bright promise has now been obtained but there are some problems that may limit its usefulness as a tool in gravitational experiments.

First, the relationship between Δf and ΔT in Equation (I.E.2) does not always hold in an actual ringlaser because of frequency pulling between the oppositely traveling signals: The observed beat frequency, denoted Δf_{ob}, is seen to be smaller in magnitude than Δf, when ΔT is small; Δf_{ob} may be zero in fact: frequency locking or synchronization

(A1) (A4) (A5) (A6) (M1). Figure 3 shows a typical relationship between Δf_{ob} and Δf (actual pulling phenomena are usually more complicated than this, but only a simple picture is needed here). In Figure 3, Δf_{ob} vanishes when $|\Delta f| \leq |\Delta f_L|$, where Δf_L is a constant that is dependent on the basic parameters of the ringlaser. The interval $|\Delta f| \leq |\Delta f_L|$ is called the <u>locking</u> or <u>synchronization band</u>; Δf_L is called the <u>lock-in frequency</u>.

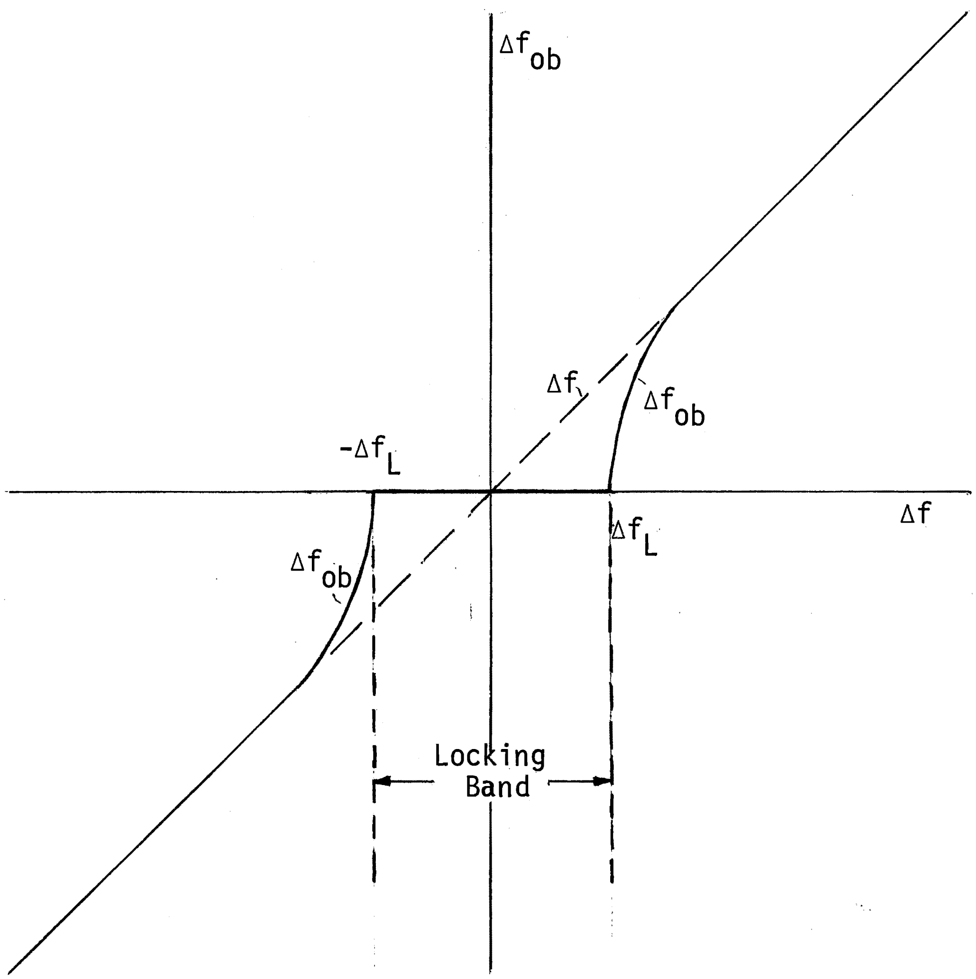

Figure 3. Frequency Pulling

Frequency pulling in the ringlaser exists because the two contra-circulating beams of the ringlaser can be represented by two coupled nonlinear oscillators. The coupling could be due to <u>backscattering</u>, the scattering of a minute fraction, typically 10^{-5}, of a signal's energy into the opposite direction by imperfect mirrors, particles in the optical loop, or something else (A4) (A6).

Frequency pulling could be a serious problem: The Δf that would be expected from the idealized ringlaser measuring gravitational effects as in Chapter III may be much smaller in magnitude than known Δf_L (the ringlaser mentioned in the last subsection had $\Delta f_L \simeq 500$ Hz; see also Pohle's work [P2]). However, one could expect to reduce the coupling and thus Δf_L by decreasing the number of scattering centers (e.g., use no more than three mirrors, or put the entire cavity in a near-vacuum[3]), using better mirrors, changing to a longer wavelength (this reduces Rayleigh scattering, because of the λ^{-4} law), or something else. Increasing L may also decrease Δf_L (A5).

There is yet another solution to the problem of frequency pulling. It may be observed from Figure 3 that Δf_{ob} approaches Δf asymptotically as Δf is increased; in fact, Δf_{ob} may be quite close to Δf when Δf is only a few times Δf_L. An artificial nonreciprocity, called a bias, could be introduced into the ringlaser to unlock it and make Δf_{ob} nearly equal Δf (K1) (W1). Earth's rotation could serve as a good source of bias: it is being monitored with a very high degree of accuracy (S6). By making the ringlaser large, Earth's rotation could easily unlock it (P2).

In practice, however, it may be difficult to separate the effect one wants to measure from the bias, especially when the magnitude of the effect is much smaller than that of the bias. This brings up yet another consideration: It is obvious that a time varying effect would be far easier to detect and identify than a constant effect of about the same magnitude, even in the presence of a large bias, through the distinctiveness of its time variation. Hence, an effort is made in Chapter III to find time varying gravitational effects.[4]

Second, another problem is now discussed: noise. It is convenient to follow Klimontovich, Kovalev, and Landau (K2, page 95) in distinguishing between natural and technical fluctuations. The latter are those which can be decreased by advancing the "state of art," e.g., by controlling the environment or designing a better ringlaser. The former are quantum mechanical effects in the lasing system, the cavity, and the beams, and cannot be minimized, once the basic parameters of the ringlaser have been fixed; they set an insurmountable limit to the sensitivity of the ringlaser to time nonreciprocities. Klimontovich, Kovalev, and Landau (K2, page 108) give a formula to estimate the smallest angular rotation rate that could be measured in a ringlaser that is limited only by natural fluctuations, denoted Ω_s:

$$\Omega_s \equiv \frac{\lambda L}{2A}\sqrt{\frac{\Delta f_{ph}}{T}} , \qquad \text{(I.E.5)}$$

where Δf_{ph} is the "average spread of the beat frequency far from the [locking band]" that is due to natural fluctuations only, T is the observation time, and the rest of the symbols are as before. The complicated formulas for Δf_{ph} are not repeated here. A numerical example is given by Klimontovich et al. (K2, page 109): A/L = .025 m (L ≃ .5 m, if equilateral triangle), $\lambda = .63 \times 10^{-6}$ m, $\Delta f_{ph} = 10^{-2}$ Hz, $T = 10^{2}$ s, so $\Omega_s = 5 \times 10^{-8}$ rad s^{-1}, which is 1,500 times smaller than Earth's rotation rate.

It might be rather optimistic to expect that the total fluctuations in the beat frequency can be controlled to the level of the numerical value given for Δf_{ob} above within this century.[5] But, the ringlaser in the example above is a small one; if a value of A/L = 100 m were used instead, $\Omega_s = 1.2 \times 10^{-11}$ rad s^{-1}, which could be achievable someday.

Third, some advantages of large size have been recounted (see also Pohle's thesis [P2]), but there is at least one problem. Heretofore single mode excitation--the beams in the ringlaser have a single value of Π--had been implicitly assumed. Single mode excitation (denoted SE) is obviously preferable to multi-mode excitation (ME): the latter may mean much noisier output, with frequency pullings between the various excited modes, obscuring any effect one may want to measure. The excited modes are those "cold-cavity" modes which happen to fall within the bandwidth of the lasing energy-level transition (it is assumed that each mode resonates much more narrowly than the transition does). The frequency difference between adjacent modes (Π and $\Pi + 1$), denoted f_D, can be found from Equation (I.E.1):

$$f_D \equiv \frac{1}{T} \equiv \frac{c}{L} . \qquad \text{(I.E.6)}$$

It can now be seen that a small ringlaser may have only a few excitable modes because f_D is then large; hence, SE may be easy to enforce in it. At least five modes have been observed in the ringlaser mentioned in the last subsection. However, SE may be quite difficult to enforce in a giant ringlaser: there could be over a thousand excitable modes.

There is a possible solution: One could form an auxiliary loop, e.g., as in Figure 4. According to an analysis by Kutin and Troshin (K4), this arrangement may enforce single-mode excitation (or a few modes). However, it has not yet been proven for giant ringlasers, and it could also introduce additional problems such as increased scattering in the cavity.

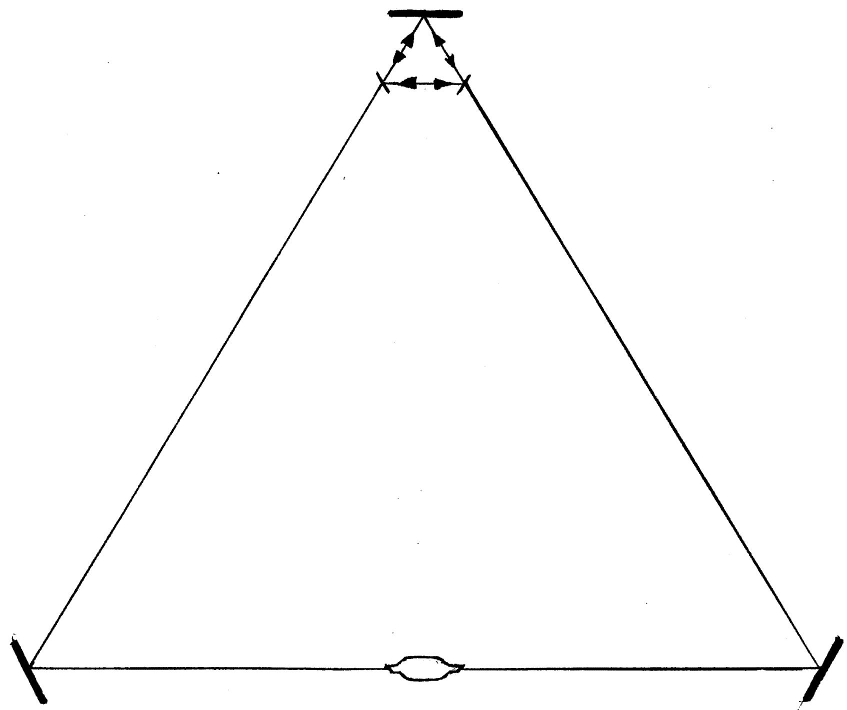

Figure 4. Kutin and Troshin's Diagram

Much work obviously remains to improve the sensitivity of the ringlaser. Only a glimpse into the complexities of the ringlaser has been given.

ENDNOTES

[1]Table I was based for the most part on Box 39.2 in Misner, Thorne, and Wheeler's book (M3), and Table 1 in Will's paper (W4). The experimental bounds on α_1 and α_2 were taken from Equations (40.16) and (40.49) in Misner, Thorne, and Wheeler's book (M3); the experimental bound on γ was taken from Fomalont and Sramek's work (Fl, page 754). Those for Δ_1 and Δ_2 were calculated roughly from the experimental bounds for α_1, α_2, and γ above.

[2]The thesis author wants to thank Dr. H. R. Bilger for providing him this result.

[3]Dr. H. R. Bilger has suggested this (Oklahoma State University, 1975).

[4]The thesis author wants to thank Dr. H. R. Bilger for pointing out the importance of time varying effects; c.f. R. Dicke's search for time varying effects in his Eötvös-type experiment--see Figure 1.6 in the book by Misner, Thorne, and Wheeler (M3).

[5]This opinion is based on discussions with Dr. H. R. Bilger at Oklahoma State University in 1975.

CHAPTER II

THE GENERALIZED SAGNAC EFFECT

II.A Introduction

The central question of the next two sections is this: how to compute the difference in the trip times of the optradich, ΔT, given a Riemannian spacetime and the motion of the optradich through that spacetime. Four general procedures to do this are presented in Section II.C, after some major assumptions are stated and partly discussed in Section II.B. As a result of the study of these procedures, a theoretical viewpoint is developed and presented in Section II.D: formulas are given, the Sagnac Effect is generalized, and it is shown how the optradich can measure the "dragging of inertial frames" effect.

For uniformity and consistency, the notations, conventions, etc., of a reviewed paper have been changed, wherever necessary, to conform to the standards given in Section I.C. Some other changes have also been made. Reasons for them are given either where they occur or in Subsection II.C.5. Some differences other than the changes above are also discussed there.

Some abbreviation of Synge's work (S8, Chapter XI, Section 7) seems necessary because of its length and detail, but the other works reviewed in Section II.C are presented with reasonable completeness.

A paper, "The Sagnac Effect in General Relativity," has only very recently been noticed. It is not reviewed here because of that lateness, and because it employs mathematical concepts and techniques that appear advanced (e.g., Killing vectors, n-forms, and homology groups) (A7).

Some other works perhaps should be mentioned here even though they did not treat gravitational effects in detail. Silberstein (S5, footnote on page 306) gave a rough estimate of gravitational effects on Earth-bound optradiches. Volkov and Kiselev (V5) used a procedure similar to that reviewed in Subsection II.C.4; see also the works cited in their footnotes 1, 2, and 4. Lianis (L2) developed a general procedure that could be applied to gravitational effects without modification. All of these works dealt with the Sagnac Effect.

II.B Assumptions

Some major assumptions are explicitly stated and partly discussed here. This section is based for the most part on Synge's discussion of his assumptions (S8, pages 401-403). (Each assumption is provided with an asterisked number for later reference.)

1*. The region of spacetime enclosing the world history of the optradich has small dimensions for the duration of the experiment.[1]

2*. At any particular instant of time, whatever the motion of the optradich, there is an inertial frame of reference (IFR) in which a particular point on the optradich is at rest (momentarily, as the case may be). Then at the same instant the velocities of the other points of the optradich with respect to the IFR are small as compared to c.

3*. The geometry of spacetime is not affected by optradich experiments.

4*. The angular velocity of the optradich relative to inertial guidance gyroscopes can be assumed to be constant over the duration of the trip times of the optradich signals. This assumption need not be unduly restrictive, for the trip times can be very short, as seen in Subsection I.E.1; thus, angular velocities that vary slowly with time are permitted.

5*. Let A be an accelerated frame. Do the standard clocks or rods in A measure the same time intervals or the same lengths as those that the standard ones measure in an IFR, when the relative velocity between A and the IFR is zero at a particular instant?

Misner, Thorne, and Wheeler (M3) say:

> One need not--and indeed must not!--postulate that proper length s is measured by a certain type of rod (e.g. platinum meter stick), or that proper time τ is measured by a certain type of clock (e.g. hydrogen-maser clock). Rather, one must ask the laws of physics themselves what types of rods and clocks will do the job. Put differently, one defines an "ideal" rod or clock to be one which measures proper length as given by $ds = (g_{\alpha\beta}dx^{\alpha}dx^{\beta})^{1/2}$ or proper time as given by $[cd\tau] = (-g_{\alpha\beta}dx^{\alpha}dx^{\beta})^{1/2}$. . . . One must then determine the accuracy to which a given rod or clock is ideal under given circumstances by using the laws of physics to analyze its behavior (page 393).

In the rest of this thesis ideal rods and clocks are used.

6*. Synge (S8, page 401) wished to avoid any assumption on the rigidity of the optradich. However, on account of assumptions 1* to 4*, it is possible and convenient to assume that without incurring serious error: Consider two triangles, one a perfect equilateral triangle of side ℓ and the other one like it but with a side increased by a relative amount $\Delta\ell/\ell$ (they are small so that Euclidean geometry can be assumed). Compute the Sagnac Effect--Equation (I.B.1)--for both triangles and thereby obtain the result that the strain $\Delta\ell/\ell$--if small--only introduces a relative change in ΔT of the order of $\Delta\ell/\ell$. The strain can be smaller than those induced in Earth's body by the Moon and the Sun, and hence negligible.

7*. If reflectors are used to define the paths of the signals, instantaneous reflection is assumed.

8*. The signals travel through vacuum only. However, travel through material media is considered in Subsection II.C.5.

9*. The loop is assumed approximately planar for convenience and simplicity.

II.C Literature Review

(Note that conclusions which otherwise would have come at the ends of the first four subsections have instead been summarized in Section II.D, and that the accelerated observer's proper frame, as in the appendix, is often referred to as just the proper frame.)

II.C.1 Møller's Approach

For convenience, equation numbers prefixed by, and page numbers suffixed by, M refer to the same respective numbers in Møller's paper (M5). Note that assumption 8* is suspended here.

Møller considered the effect of stationary gravitational fields on the velocity of light. The metric is rewritten:

$$g_{\mu\nu}dx^{\mu}dx^{\nu} = d\sigma^2 - (c^* \, dt - \gamma_i \, dx^i)^2, \qquad \text{(II.C.1)}$$

where

$$d\sigma^2 = (g_{ik} + \gamma_i\gamma_k) \, dx^i dx^k, \qquad \text{(II.C.2)}$$

the "spatial metric,"

$$c^* = c\sqrt{-g_{oo}}, \qquad \text{(II.C.3)}$$

and

$$\gamma_i = \frac{g_{oi}}{\sqrt{-g_{oo}}} , \qquad \text{(II.C.4)}$$

what Møller called the vector potential. From this he derived the "coordinate speed of light":

$$w = \frac{c^*}{\frac{c}{\overset{\circ}{w}} + \gamma_i e^i} , \qquad \text{(II.C.5)}$$

where $\overset{\circ}{w}$ is the speed of light as measured in a Lorentz frame, and e^i is the unit vector in the direction of light propagation (see also pages 270-271, M4).

The passage from pages 386-387M appears particularly quotable:

> The difficulty in checking the formula [II.C.5] by terrestrial experiments lies in the fact that only differences in the velocity [II.C.5] for different space points give rise to observable effects. This follows at once from the principle of equivalence; for inside a region of essentially constant potentials we may treat the phenomena used in the experiment from the point of [an IFR], where the gravitational effects disappear. It is therefore clear that the experimental arrangements must cover large areas. Further there is in general a danger that uncontrollable variations in the properties of the medium (i.e. in $\overset{\circ}{w}$) will overshadow the weak effects due to the gravitational field. There is one arrangement, however, in which this difficulty is eliminated. Consider two signals which, starting from the same point P, are going along a closed loop but in opposite directions. The time intervals T_+ and T_- needed for the signals to make one turn are then, according to Equation [II.C.5],
>
> $$T_{\pm} = \oint_{(\pm)} \frac{d\sigma}{w} . \qquad \text{(II.C.6)}$$
>
> The time intervals between the arrivals at P of the two signals after one turn is then completely independent of the properties of the medium traversed and equal to
>
> $$\Delta T = T_+ - T_- = \frac{2}{c} \oint_{(+)} \frac{\gamma_i}{\sqrt{-g_{oo}}} e^i \, d\sigma . \qquad \text{(II.C.7)}$$

(Some changes have been made in the equations above.) See also page 394 in Møller's book (M4). Thus, one is led to consider optradiches, especially large ones.

For reasons given in Subsection II.C.5, the proper frame is used in the place of the frame that Møller used. After putting the spatial origin at P and rotating the frame so that every point of the loop has constant spatial coordinates in the frame, one can derive from Equations (II.C.4) and (App. 4) the following:

$$\frac{\gamma_k}{\sqrt{-g_{oo}}} = [k\ a\ b]\ \frac{\omega^{\hat{a}}}{c}\ x^{\hat{b}}, \qquad \text{(II.C.8)}$$

where $a^{\hat{j}}x^{\hat{j}}/c^2$ is neglected on account of assumption 1*. Thus, Equation (II.C.7) becomes, when the integration is performed on the basis of assumption 9*,

$$\Delta T = 4\ \frac{\vec{\omega}\cdot\vec{n}_u}{c^2}\ A, \qquad \text{(II.C.9)}$$

where A and $\vec{n}_u$ are as before. The significance of this is made more clear in Section II.D.

II.C.2 Synge's Approach

For convenience, equations and page numbers, etc., prefixed by S refer to the same respective numbers in Synge's book (S8, Chapter XI, Section 7).

In the preface to his book, Synge stated his aim to put General Relativity on a more operational footing than previously. Also, he wished to demonstrate the utility of his "world-function," W (he actually used the notation Ω but that is reserved for use to denote angular speed in this thesis).

The necessary machinery for working with W is developed in Chapter SII, in particular Section S14, "The World-Function in Terms of Fermi Coordinates for Two Points on Adjacent Timelike Curves." The Fermi coordinates are just the coordinates of the proper frame when $\omega^{\alpha} \equiv 0$. He called such a frame a Fermi frame.

Consider three adjacent timelike world lines of three observers, C_0, C_A, and C_B, with C_0 transporting a Fermi frame (Figure 5). Then, after studying the Appendix and keeping in mind that $\omega^{\alpha} \equiv 0$ for the Fermi frame, one can write, with reference to Figure 5, the Fermi coordinates of the point, P_A on C_A, as $F_A^0 = c\tau_A$, $F_A^1 = \sigma_A n_A^1$, $F_A^2 = \sigma_A n_A^2$, and $F_A^3 = \sigma_A n_A^3$; likewise for P_B on C_B (replace the subscript A with B). Here, τ is the proper time of C_0, and $c\tau_A$, $c\tau_B$, σ_A, and σ_B are "small, of first order" (assumption 1*).

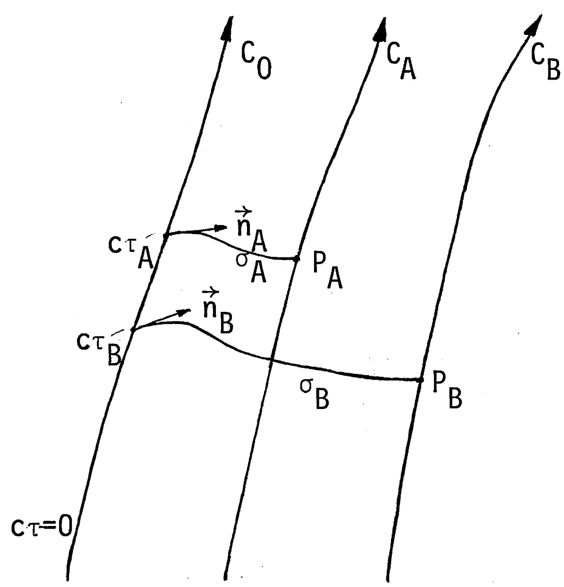

Figure 5. Synge's Diagram

The world-function of P_A and P_B are expanded in terms of the Fermi coordinates, the four-acceleration of C_0 (a_μ/c^2, as in the Appendix), the "curvature of spacetime," etc.:

$$W(P_A, P_B) = M_2 + M_3 + N_3 + M_4 + N_4 + O_5, \qquad \text{(II.C.10)}$$

where

$$M_2 = -\frac{1}{2} c^2 (\tau_A - \tau_B)^2 + \frac{1}{2} r_{AB}{}^2,$$

$$M_3 = -\frac{1}{2} c^2 (\tau_A - \tau_B)^2 (F_A^b + F_A^b) a^b / c^2,$$

$$N_3 = (F_A^b - F_B^b) \left(\tau_A \frac{d\, F_A^b}{d\tau} - \tau_B \frac{d\, F_B^b}{d\tau}\right),$$

and M_4 contains terms in which the components of the Riemann (or curvature) tensor appear, N_4 is a complicated expression in the four-acceleration, and $d/d\tau$ denotes ordinary differentiation with respect to τ. Further,

$$r_{AB} \equiv r_{BA} \equiv [(F_A^i - F_B^i)(F_A^i - F_B^i)]^{1/2},$$

the "Fermi distance" between P_A and P_B, and O_5 contains terms of the fifth order in $c\tau_A$, $c\tau_B$, σ_A, and σ_B. Note that the numbers on the M's, N's, and O are the numbers of the orders of $c\tau_A$, $c\tau_B$, σ_A, and σ_B.

A goal stated at the beginning of Chapter SXI is to devise experiments to measure the "curvature of acceleration" of the observer and the "curvature of spacetime." To this end, Synge set up a tetrahedral array of point mirrors--each mirror at a vertex of the tetrahedron. Here, it is enough to consider just three mirrors, denoted, respectively, C_0, C_A, and C_B, with C_0 carrying a Fermi frame. The Fermi coordinates of P_A and P_B are considered functions of τ, the proper time on C_0's world line. Suppose that a signal leaves C_0 at τ_0, arrives at C_A at τ_A, C_B at τ_B, and finally returns to C_0 at $\overline{\tau_0}$. This circuit, and also the trip time, are denoted OABO; OBAO is the same circuit taken in the opposite sense.

Since he wished to avoid making any assumption on the rigidity of the tetrahedron, Synge used signals to monitor the dimensions of the tetrahedron, as is made more clear later.

As the signals pursue null geodesics (vacuum travel), $W(P_A, P_B) = 0$, e.g. Thus, approximative manipulation of Equation (II.C.10) yields this:

$$c(\tau_A - \tau_B) = r_{AB} + \tau_A \frac{d\, r_{AB}}{d\tau} - (F^i_A - F^i_B) \frac{d\, F^i_B}{cd\tau} + \phi_{BA} + \psi_{AB}, \tag{II.C.11}$$

with an unwritten error term, O_4, and where ϕ contains terms from N_3 and M_4, and ψ contains some terms from M_4 (note that $\phi_{AB} = \phi_{BA}$ and $\psi_{AB} = -\psi_{BA}$).

In his analysis Synge dropped N_4 but kept M_4 in order to explore the effects of the Riemann tensor. But, there is almost no hope that such could be discerned with optradiches, especially when it is still doubtful that the gravitational effects considered here could be measured by optradiches (see Section IV.A). So, M_4, and hence ψ, are dropped as well.

Manipulation of the subscripts in Equation (II.C.11)--e.g., substituting 0 for A--will yield the other equations needed subsequently. Further manipulation for Equation (II.C.11) lead to these equations:

$$\frac{c}{2}\, OAO = r_{OA} \frac{d\, r_{OA}}{cd\tau} + \phi_{OA}, \tag{II.C.12}$$

$$\frac{c}{2}\, OBO = r_{OB} \frac{d\, r_{OB}}{cd\tau} + \phi_{OB}, \tag{II.C.13}$$

$$\frac{c}{2}\,OABOBAO = (r_{OA} + r_{AB} + r_{BO})\left(1 + \frac{1}{2}\frac{d\,r_{OA}}{cd\tau} + \frac{1}{2}\frac{d\,r_{BO}}{cd\tau}\right)$$

$$+ \frac{1}{2}\,r_{OA}\,\frac{d\,r_{OA}}{cd\tau} + \frac{1}{2}\,r_{OB}\,\frac{d\,r_{OB}}{cd\tau} + \frac{1}{2}\,(r_{OA} + r_{OB})\,\frac{d\,r_{AB}}{cd\tau}\;. \qquad \text{(II.C.14)}$$

Although assumption 6* is kept even here, it is interesting to see what could be done if it were necessary to take into account the nonrigidity of the optradich. In principle, at least, after the trip time and the four-acceleration of C_0 are continuously recorded as functions of τ, the Fermi coordinates of the mirrors, C_A and C_B, can then be solved for in terms of the trip times. The dimensions of the optradich are thus effectively monitored.

The terms in $dr/cd\tau$ can be dropped on the basis of assumption 5*. So, according to Equation (II.C.11),

$$\Delta T = OABO - OBAO = \frac{1}{c^2}\left(F^i_B\,\frac{d\,F^i_A}{d\tau} - F^i_A\,\frac{d\,F^i_B}{d\tau}\right). \qquad \text{(II.C.15)}$$

But, the optradich is rotating relative to the Fermi frame with the same angular velocity, $\omega^{\hat{j}}$, as in the Appendix; consideration of the right hand side of Equation (II.C.15) does indeed show this. Hence, Equation (II.C.15) is equivalent to Equation (II.C.9).

II.C.3 Dehnen's Approach

For convenience, equation numbers prefixed by, and page numbers suffixed by, D refer to the respective numbers in Dehnen's work (D2). The convention with the Latin indices is relaxed in this subsection: i and k, and only these, range over A, B, and C; moreover, Einstein's summation convention is suspended for i and k also. Two changes made in Dehnen's work, aside from those necessary to maintain consistency in notations, conventions, etc., are pointed out: ΔT is used instead of his δT and vice versa, and three mirrors are used instead of his four (mostly to avoid copying his work too closely). Apart from these changes, the first part of the presentation here may differ somewhat from what Dehnen had in mind, but it is hoped that these differences are not too large.[2]

For the first time, an author calculates gravitational effects on a ringlaser, but his results are applicable to other optradiches as well.[3]

The three mirrors are labeled A, B, and C. In addition, an observer, O, comoves with the mirrors, carrying a clock with him which measures his proper time; this clock is denoted the O clock.

A special coordinate system, y^α, is used here. Each element of the optradich (not including the signals) has a unique triplet, y^j (j = 1, 2, and 3), which remains constant throughout the world history of the element. The fourth coordinate (timelike), y^o, is determined as follows: draw a spacelike geodesic orthogonal to O's world line at the point where the O clock reads time = T, to the world line of the element; at that intersection, $y^o = T$ also. The coordinates can be arbitrary otherwise, except that smoothness, in the usual mathematical sense, may be convenient. Another coordinate system, x^α, can be introduced in terms of y^α via coordinate transformations.

O's four-velocity is u^μ, as in the Appendix. Pick one of the mirrors, i = A, B, or C. Draw a spacelike geodesic orthogonal to O's world line at the point, P, where the O clock reads time = T to the mirror's world line. Let σ_{0i} denote the proper length of the geodesic between O and i; and let $(n_{0i})^\mu$ denote the tangent to the geodesic at P[$(n_{0i})^\mu (n_{0i})_\mu = 1$ and $(n_{0i})^\mu u_\mu = 0$]. Definitions:

$$\delta_{0i}\ x^{\mu *} = \sigma_{0i}(n_{0i})^{\mu} \tag{II.C.16}$$

and

$$\delta_{ik}\ x^{\mu *} = \sigma_{0k}(n_{0k})^{\mu} - \sigma_{0i}(n_{0i})^{\mu}, \tag{II.C.17}$$

where $i \neq k$, here and henceforth.

Dehnen assumes the signals in the optradich travel on null geodesics, as Synge did:

$$ds = 0. \tag{II.C.18}$$

He changes this to

$$cdT = dS, \tag{II.C.19}$$

where T is the time on the 0 clock; and, assuming the light signal is traveling from the ith mirror to the kth mirror, S is given by

$$S = S_{ik} \equiv \sqrt{\delta_{ik}\ x^{\mu *}\ \delta_{ik}\ x^{\mu *}} \equiv \ell_{ik}. \tag{II.C.20}$$

Integration of Equation (II.C.19) yields

$$cT_{ik} = \int_{x_{(i)}(T_i)}^{x_{(k)}(T_k)} dS, \qquad x = \{x^{\mu}\}, \tag{II.C.21}$$

where T_{ik} is the trip time of the signal from the ith mirror at time T_i to the kth mirror at T_k. Since the trip times from mirror to mirror are small (assumption 1*),

$$x^{\mu}_{(k)}(T_k) = x^{\mu}_{(k)}(T_i) + \delta x^{\mu}_{(k)}. \tag{II.C.22}$$

Equations (II.C.21) and (II.C.22) combine to form

$$cT_{ik} = S_{ik}(T_i) + \frac{\partial S_{ik}}{\partial x^{\mu}_{(k)}}(T_k)\delta x^{\mu}_{(k)}, \tag{II.C.23}$$

which is a Taylor series that is truncated after the linear term. Since

$$\delta x^{\mu}_{(k)} = \frac{x^{\mu}_{(k)}(T_k) - x^{\mu}_{(k)}(T_i)}{T_k - T_i}(T_k - T_i) = \frac{\partial x_{(k)}}{c\partial T} c\delta T, \qquad \text{(II.C.24)}$$

where δT is the trip time $T_k - T_i$ in the first approximation, the second term on the right hand side of Equation (II.C.23) becomes

$$\frac{\partial S_{ik}}{\partial x^{\mu}_{(k)}}(T_i)\delta x^{\mu}_{(k)} = \frac{\partial S_{ik}}{\partial x^{\mu}_{(k)}} \frac{\partial x^{\mu}_{(k)}}{\partial cT} c\delta T = \frac{\partial S_{ik}}{\partial T_{(k)}} \delta T, \qquad \text{(II.C.25)}$$

in the first approximation, where the symbol $\partial T_{(k)}$ means to carry out the time differentiation at the kth mirror only. So, to first approximation,

$$cT_{ik} = S_{ik}(T_i) + \frac{\partial S_{ik}}{c\partial T_{(k)}}(T_i)\ c\delta T, \qquad \text{(II.C.26)}$$

Figure 6 diagrams the world lines of the mirrors A, B, and C (light, nearly vertical lines), and of two signals leaving A at time T_1 (heavy, more horizontal lines, one solid and the other dashed). T_2 is the approximate time at which both signals arrive at their second mirrors, respectively; T_3, their third mirrors; $T_{4'}$ is the time the solid-line signal arrives at A; $T_{4''}$ is the time the dashed-line signal arrives at A. From Figure 6, it can be seen that

$$c\Delta T = c(T_{4'} - T_{4''}) = c\{(T_{AB} - T_{BA''}) + (T_{CA'} - T_{AC}) + (T_{BC} - T_{CB})\}. \qquad \text{(II.C.27)}$$

So, according to Equation (II.C.26) and Figure 6, Equation (II.C.27) becomes

$$\begin{aligned} c\Delta T = {} & [S_{AB}(T_1) - S_{BA''}(T_3)] + [S_{CA'}(T_3) - S_{AC}(T_1)] \\ & + [S_{BC}(T_2) - S_{CB}(T_2)] + \delta T\{[\frac{\partial S_{AB}}{\partial T_{(B)}} - \frac{\partial S_{BA''}}{\partial T_{(A'')}}(T_3)] \end{aligned}$$

$$+ [\frac{\partial S_{CA'}}{\partial T_{(A')}} (T_3) - \frac{\partial S_{AC}}{\partial T_{(C)}} (T_1)] + [\frac{\partial S_{BC}}{\partial T_{(C)}} (T_2) - \frac{\partial S_{CB}}{\partial T_{(B)}} (T_2)]\}. \tag{II.C.28}$$

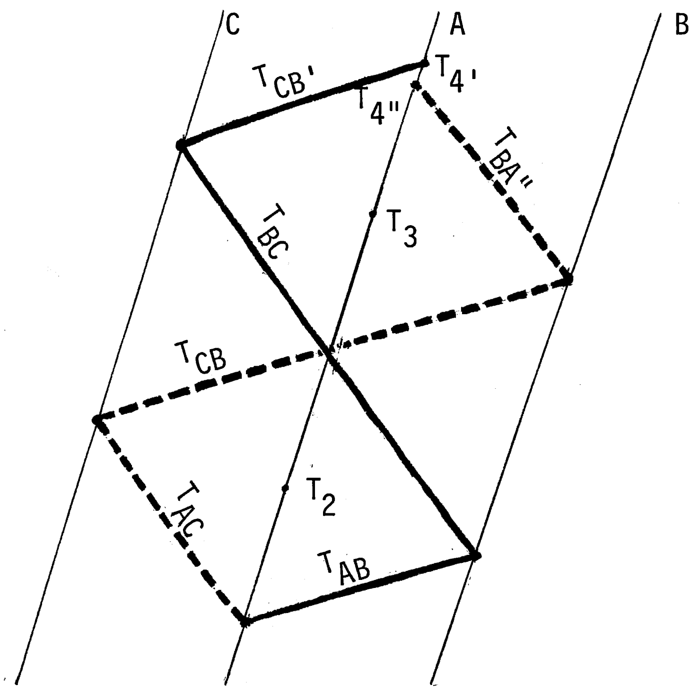

Figure 6. Dehnen's Diagram

In the first approximation,

$$T_2 = T_1 + \delta T, \qquad T_3 = T_1 + 2\delta T. \tag{II.C.29}$$

So, Equation (II.C.28) changes to

$$c\Delta T = \{2 \frac{dS_{CA}}{dT} - 2 \frac{dS_{BA}}{dT} + \frac{\partial S_{AB}}{\partial T_{(B)}} - \frac{\partial S_{BA}}{\partial T_{(A)}} + \frac{\partial S_{CA}}{\partial T_{(A)}} - \frac{\partial S_{AC}}{\partial T_{(C)}} + \frac{\partial S_{BC}}{\partial T_{(C)}} - \frac{\partial S_{CB}}{\partial T_{(B)}}\} \delta T, \tag{II.C.30}$$

in the first approximation where all terms are evaluated at T_1.

Differentiation of Equation (II.C.20) gives

$$\frac{\partial S_{ik}}{\partial T_{(k)}} = \frac{c}{\ell_{ik}} \delta_{ik}\, x_{\mu^*}\, (\delta_{0k}\, x^{\mu^*})^{\bullet} \tag{II.C.31}$$

and

$$\frac{dS_{ik}}{dT} = \frac{\partial S_{ik}}{\partial T_{(k)}} + \frac{\partial S_{ik}}{\partial T_{(i)}}\,, \tag{II.C.32}$$

where $(\ )^{\bullet}$ denotes covariant differentiation with respect to T (T is the time on the 0 clock) along the world lines (E1, page 798). It can be shown that

$$(\delta_{0i}\, x^{\mu^*})^{\bullet} = u^{\mu}{}_{;\nu}\, \delta_{0i}\, x^{\nu^*} \tag{II.C.33}$$

(E1, page 799).

It is convenient to have at this point

$$\ell_{AB} = \ell_{BA} = \ell_{AC} = \ell_{CA} = \ell_{BC} = \ell_{CB} = \ell \tag{II.C.34}$$

(equilateral triangle).

Thus, Equations (II.C.30, 31, 33, and 34) combine to form

$$cT = \{3(\frac{\partial S_{CA}}{\partial T_{(A)}} - \frac{\partial S_{BA}}{\partial T_{(A)}}) + \frac{\partial S_{AB}}{\partial T_{(B)}} + \frac{\partial S_{BC}}{\partial T_{(C)}} - \frac{\partial S_{AC}}{\partial T_{(C)}} - \frac{\partial S_{CB}}{\partial T_{(B)}}\}\ \delta T$$

and then

$$\begin{aligned} cT = c\, \frac{\delta T}{\ell}\, u_{\mu;\nu} [&3\delta_{0B} x^{\mu^*}\ \delta_{0A} x^{\nu^*} + \delta_{0A} x^{\mu^*}\ \delta_{0B} x^{\nu^*} \\ &- 3\delta_{0C} x^{\mu^*}\ \delta_{0A} x^{\nu^*} - \delta_{0A} x^{\mu^*}\ \delta_{0C} x^{\nu^*} \\ &+ 2(\delta_{0C} x^{\mu^*}\ \delta_{0C} x^{\nu^*} - \delta_{0B} x^{\mu^*}\ \delta_{0B} x^{\nu^*}) \\ &+ \delta_{0C} x^{\mu^*}\ \delta_{0B} x^{\nu^*} - \delta_{0B} x^{\mu^*}\ \delta_{0C} x^{\nu^*}]. \end{aligned} \tag{II.C.35}$$

It has been shown that $u_{\mu;\nu}$ can be decomposed in this way:

$$u_{\mu;\nu} = \omega_{\mu\nu} + \sigma_{\mu\nu} + \frac{1}{3}\theta h_{\mu\nu} - a_{\mu} u_{\nu}/c^2, \qquad \text{(II.C.36)}$$

where $\omega_{\mu\nu}$ measures the rotation, $\sigma_{\mu\nu}$ the shear, and θ the expansion; a_{μ} is as before; and $h_{\mu\nu}$ is the projection tensor (El, page 800).

It can be assumed that $\sigma_{\mu\nu} = \theta = 0$ in view of assumption 6*, and the term in $a_{\mu}u_{\nu}$ is of no consequence since $u_{\mu}\delta_{0i}x^{\mu*} = 0$ (see Equation (II.C.16) and the sentences just above it). Further simplifications of Equation (II.C.35) can be had by imposing

$$\delta_{0C}x^{\mu*} = -\delta_{0B}x^{\mu*}. \qquad \text{(II.C.37)}$$

(See Figure 7.)

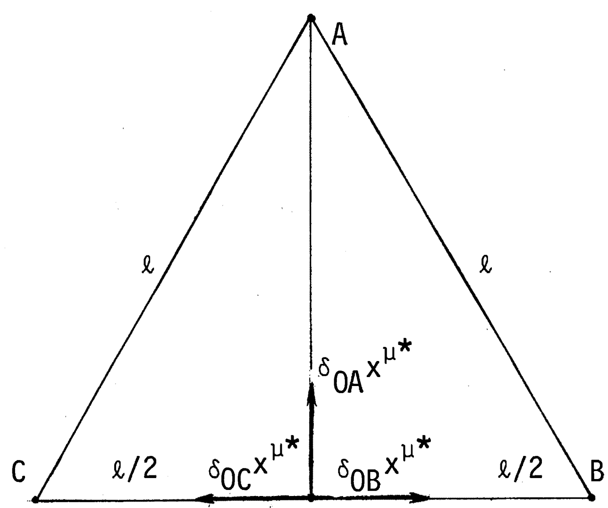

Figure 7. Optradich Vectors

So, in view of these facts,

$$\omega_{\mu\nu} \equiv -\omega_{\nu\mu}, \qquad c\delta T \simeq \ell, \qquad \text{(II.C.38)}$$

this can be seen to hold:

$$c\Delta T = 4\omega_{\mu\nu}\delta_{OB}x^{\mu*}\ \delta_{OA}x^{\nu*}. \tag{II.C.39}$$

It can also be shown that ω^{μ}, as in the Appendix, is related to $\omega_{\mu\nu}$:

$$\omega_{\mu\nu} = \varepsilon_{\mu\nu\alpha\beta}\omega^{\alpha}u^{\beta}/c \tag{II.C.40}$$

(E1, page 800).

So,

$$c\Delta T = \frac{4}{c}\,\varepsilon_{\mu\nu\alpha\beta}\omega^{\alpha}u^{\beta}\ \delta_{OB}x^{\mu*}\ \delta_{OA}x^{\nu*}. \tag{II.C.41}$$

This can be shown to be the general expression of Equation (II.C.9). In the proper frame of the observer, $u^{\mu} = (\ 1,\ 0,\ 0,\ 0\ )$, $\omega^{0} = 0$, and $\varepsilon_{\mu\nu\alpha\beta} = [\mu\nu\alpha\beta]$ along the world line of the observer; so,

$$c^{2}\Delta T = 4\ \vec{\omega}\cdot\vec{n}_{u}A, \tag{II.C.42}$$

where $\vec{n}_{u}A$ has the components,

$$n_{u}^{m}\ A = [m\ a\ b]\ \delta_{OB}x^{\hat{a}*}\ \delta_{OA}x^{\hat{b}*}, \tag{II.C.43}$$

and $\vec{\omega}$ has the components $\omega^{\hat{j}}$, as in the Appendix.

By writing of the "'Mitführung' des Inertialsystems" and of the "Fokker-Präzession" on pages 820 and 821D, Dehnen anticipated most of the content of Section II.D.

II.C.4 The Approach by Volkov et al.

On the next page, Table II shows several versions of Maxwell's electromagnetic field equations (E2) (H2) (M3) (P1) (V3) (V4) (V5). System I in Table II is in the usual form (rationalized MKS units). $\vec{E}$ is the electric field; $\vec{B}$, the magnetic field; $\vec{D}$, the electric flux density; $\vec{H}$, the magnetic intensity; $\vec{J}$, the current density; and ρ, the charge density.

TABLE II

MAXWELL'S ELECTROMAGNETIC FIELD EQUATIONS IN VARIOUS FORMS

System I

The Source-Independent Equations

$$\vec{\nabla} \times \vec{E} + \partial\vec{B}/\partial t = 0, \quad \vec{\nabla}\cdot\vec{B} = 0.$$

The Source-Dependent Equations

$$\vec{\nabla} \times \vec{H} - \partial\vec{D}/\partial t = \vec{J}, \quad \vec{\nabla}\cdot\vec{D} = \rho.$$

The Constitutive Equations

(In general, tensor relationships among the components of the four electromagnetic vectors.)

System II

The Source-Independent Equations

$$F_{\alpha\beta;\delta} + F_{\beta\delta;\alpha} + F_{\delta\alpha;\beta} = 0.$$

The Source-Dependent Equations

$$H^{\alpha\beta}{}_{;\beta} = J^{\alpha}.$$

The Constitutive Equations

$$H^{\alpha\beta} = S^{\mu\nu\alpha\beta} F_{\mu\nu}.$$

System III

The Source-Independent Equations

$$F_{\alpha\beta;\delta} + F_{\beta\delta;\alpha} + F_{\delta\alpha;\beta} = 0.$$

The Source-Dependent Equations

$$F^{\alpha\beta}{}_{;\beta} = 4\pi J^{\alpha}.$$

The Constitutive Equations

$$F^{\alpha\beta} = g^{\alpha\mu} g^{\beta\nu} F_{\mu\nu}.$$

System I is valid only in an IFR; however, as is known, General Relativity requires equations valid generally in other frames (principle of covariance; equations having only "geometric objects" in them [M3, Section 12.5]). The equations in System II are such equations. $F_{\alpha\beta}$, which may be called the electromagnetic field tensor, has the following components in a Lorentz frame:

$$\|F_{\alpha\beta}\| = \begin{Vmatrix} 0 & -\frac{E_x}{c} & -\frac{E_y}{c} & -\frac{E_z}{c} \\ \frac{E_x}{c} & 0 & B_z & -B_y \\ \frac{E_y}{c} & -B_z & 0 & B_x \\ \frac{E_z}{c} & B_y & -B_x & 0 \end{Vmatrix} \qquad \text{(II.C.44)}$$

(M3, a modification of Equation [3.7]), where E_x = the x-component of $\vec{E}$, B_y = the y-component of $\vec{B}$, and so on. $H^{\alpha\beta}$ may be called the electromagnetic intensity tensor, analogous to the pair of $\vec{D}$ and $\vec{H}$. J^{α} could be termed the four-current density--it might have components, ρu^{α}, where u^{α} is the four-velocity of the source element having rest-frame charge density ρ. $S^{\mu\nu\alpha\beta}$ could be called the constitutive tensor, analogous to the permittivity and permeability of the medium.

System III is another version of System II, being written in Gaussian units such that $\vec{E} = \vec{D}$ and $\vec{B} = \vec{H}$ in vacuum; and it is valid for vacuum only, aside from electrical sources. A reason for introducing it here is that its use seems quite prevalent in the literature (H2) (M3) (P1) (V3) (V4) (V5). The reader may find it convenient to have II and III listed together. III is used in this subsection because it is more convenient: Volkov et al. uses it, and the constitutive tensor has a simpler form in III than in II, at least in vacuum.

The more recent of the papers by Volkov et al. is reviewed here (V3).

Volkov et al. introduce fictitious "vectors," E_k, B_k, D_k, and H_k, as functions of the components of $F^{\alpha\beta}$ and $H_{\alpha\beta}$:

$$\frac{E_k}{c} = F_{ko}, \qquad B_k = \frac{1}{2}[k\ n\ m]\, F_{nm},$$

$$D_k = (-g)^{1/2}\, H^{ok}, \qquad \frac{H_k}{c} = \frac{1}{2}\, \varepsilon_{oknm}\, H^{nm}. \tag{II.C.45}$$

These can be substituted in System III--assuming source-free vacuum, $J^{\alpha} \equiv 0$. The result is

$$[k\ n\ m]\, \frac{E_{m,n}}{c} + B_{k,0} = 0, \qquad B_{k,k} = 0,$$

$$[k\ n\ m]\, \frac{H_{m,n}}{c} - D_{k,0} = 0, \qquad D_{k,k} = 0,$$

$$D_k = \varepsilon_{kn}\, \frac{E_n}{c} - [k\ n\ m]\, g_n\, \frac{H_m}{c},$$

$$B_k = \varepsilon_{kn}\, \frac{H_n}{c} + [k\ n\ m]\, g_n\, \frac{E_m}{c}, \tag{II.C.46}$$

where

$$\varepsilon_{kn} = \frac{(g)^{1/2}}{-g_{oo}}\, g^{kn}, \qquad g_k = \frac{g_{ko}}{-g_{oo}}, \tag{II.C.47}$$

and some minor changes have been made in the equations of Volkov et al.

Equation (II.C.46)--henceforth referred to as System IV--is identical in form with System I when the latter is written in a Lorentz frame for the case of a source-free medium characterized by constitutive relationships of the form of the last two equations of Equation (II.C.46), when factors of c are inserted where appropriate. As Heer (H2, page A800) puts it, "a noncovariant notation is found more convenient and consistent with conventional techniques."

It appears that it can be assumed that the signals in an optradich are of such a nature as to permit a classical geometric-optics solution to System IV. If this is so, and if ε_{kn} is diagonal, then

$$\frac{(N_1 - g_1)^{1/2}}{\varepsilon_{22}\varepsilon_{33}} + \frac{(N_2 - g_2)^{1/2}}{\varepsilon_{11}\varepsilon_{33}} + \frac{(N_3 - g_3)^{1/2}}{\varepsilon_{11}\varepsilon_{22}} - 1 = 0 \qquad \text{(II.C.48)}$$

(V2, a modification of its Equation [4]), where N_1 is the first component of $N\vec{e} = \vec{N}$, and so on ($\vec{e}$ is the unit vector in the direction of light propagation, and N is the index of refraction in that direction).

For reasons that are given in the next subsection, the proper frame is used in the place of the frame that Volkov et al. used. Then from Equation (II.C.47), in the proper frame,

$$\varepsilon_{kn} \simeq \frac{1}{\sqrt{-g_{\hat{0}\hat{0}}}}\,\delta_{kn}, \qquad g_k = [k\ n\ m]\,\frac{\omega^{\hat{k}}\ x^{\hat{n}}}{-g_{\hat{0}\hat{0}}}, \qquad \text{(II.C.49)}$$

where terms such as $\omega^{\hat{1}}\ x^{\hat{2}}\ \omega^{\hat{2}}\ x^{\hat{3}}/c^2$ are neglected on account of assumption 2*. Hence, Equation (II.C.48), in the proper frame, becomes

$$(N_j - g_j)\ (N_j - g_j) = \frac{1}{-g_{\hat{0}\hat{0}}}\ . \qquad \text{(II.C.50)}$$

The solution is

$$N_k = \frac{e_k + g_k\sqrt{-g_{\hat{0}\hat{0}}}}{\sqrt{-g_{\hat{0}\hat{0}}}}\ , \qquad \text{(II.C.51)}$$

where e_j is the jth component of $\vec{e}$. To find the trip time of one signal, the following is taken

$$T = \oint N_k\ e_k\ d\sigma, \qquad \text{(II.C.52)}$$

which can be recognized as Equation (II.C.6).

II.C.5 Differences With Previous Papers

Some major differences with the works of Møller (M5), Volkov et al. (V3) (V4), and Kuriyagawa et al. (K3) are stated, and sometimes explained or discussed here. Henceforth, V3, V4, and K3 are used to refer to the respective papers.

V3 and V4 give the covariant source-free Maxwell's equations as

$$F_{\alpha\beta;\delta} + F_{\beta\delta;\alpha} + F_{\delta\alpha;\beta} = 0, \qquad H^{\alpha\beta}{}_{;\beta} = 0, \tag{II.C.53}$$

as before. Next, V4 gives the "covariant" constitutive equations as

$$\sqrt{-g}\, H^{\alpha\beta} = \sqrt{-g}\, g^{\alpha\mu}\, g^{\beta\nu}\, S^{\sigma\tau}{}_{\mu\nu}\, F_{\sigma\tau}, \tag{II.C.54}$$

whereas, V3's version, valid for vacuum only, is

$$\sqrt{-g}\, H^{\alpha\beta} = \sqrt{-g}\, g^{\alpha\mu}\, g^{\beta\nu}\, F_{\mu\nu}. \tag{II.C.55}$$

In this thesis, they are of course given as

$$H^{\alpha\beta} = S^{\mu\nu\alpha\beta}\, F_{\mu\nu}. \tag{II.C.56}$$

Since both sides of each of Equations (II.C.54 and 55) apparently transform as tensor densities, it perhaps would be preferable to call only Equation (II.C.56) covariant.

K3 differs from V4 in using $F^{\alpha\beta}$ in the place of $H^{\alpha\beta}$ in Equation (II.C.54). In this thesis, $F^{\alpha\beta}$ has always the meaning $F^{\alpha\beta} = g^{\alpha\mu}g^{\beta\nu}F_{\mu\nu}$. Since non-vacuum processes were examined in K3, apparently $F^{\alpha\beta} \neq H^{\alpha\beta}$--the inequality seems to hold even in vacuum if the SI system of units is used--this thesis does not conform to the practice of K3.

Next, V4 uses a second-rank tensor to characterize an isotropic medium:

$$S^{\mu\nu\alpha\beta} = g^{\alpha\delta} g^{\beta\tau} S^{\mu}{}_{\delta} S^{\nu}{}_{\tau}, \tag{II.C.57}$$

with

$$S^{o}_{o} = \varepsilon\sqrt{\mu},\ S^{i}_{j} = (1/\sqrt{\mu})\delta^{i}_{j},\ \text{rest zero}, \tag{II.C.58}$$

where ε and μ are "the values of the dielectric constant and the magnetic permeability of the medium, as measured by local observers coupled to the medium" (V4, page 996). K3 uses, instead,

$$S^{o}_{o} = \varepsilon_{s}\mu_{s},\ S^{i}_{j} = \delta^{i}_{j},\ \text{rest zero}, \tag{II.C.59}$$

where ε_s and μ_s are "the relative permittivity and the relative permeability of a medium measured by local observers coupled to the medium" (K3, page 2955).

To resolve this apparent discrepancy, calculations in the SI units are made for the isotropic-medium case. As a medium cannot be isotropic except when at rest in an IFR, a Lorentz frame is used.

First, factors of c are inserted in System I so that ct can be used in the place of t, as in Special Relativity. There are two general ways to do this: (1) divide the curl equation ($\vec{\nabla}\times\vec{E}$ or $\nabla\times\vec{H}$ . . .) and use $\vec{E}/c$ or $\vec{H}/c$ in the place of $\vec{E}$ or $\vec{H}$; (2) multiply the time partial derivative term, $\partial/\partial t$, by c/c and the divergence equation by c, and use $c\vec{B}$ or $c\vec{D}$ instead of $\vec{B}$ or $\vec{D}$. $F^{\alpha\beta}$, as given by Equation (II.C.44), has been calculated via way (1) above; $H^{\alpha\beta}$, via the same way, is given by

$$\|H^{\alpha\beta}\| = \begin{Vmatrix} 0 & D_x & D_y & D_x \\ -D_x & 0 & \frac{H_z}{c} & -\frac{H_y}{c} \\ -D_y & -\frac{H_z}{c} & 0 & H_x \\ -D_z & \frac{H_y}{c} & -\frac{H_x}{c} & 0 \end{Vmatrix} \tag{II.C.60}$$

If way (2) were wanted instead, the right hand side of Equation (II.C.44) or (II.C.60) can be multiplied by c. Way (1) could even be applied to the source-independent Maxwell's equations in System I and way (2) to the source-dependent ones, or vice versa. In any case,

$$H^{\alpha\beta} = \eta^{\alpha\mu} \eta^{\alpha\nu} S^{\delta}_{\mu} S^{\tau}_{\nu} F_{\delta\tau}, \qquad \text{(II.C.61)}$$

where

$$S^{o}_{o} = a, \; S^{i}_{j} = b\delta^{i}_{j}, \text{ rest zero,}$$

in which a and b are absolute scalars whose values depend on which of the following four possibilities are used: (A) way (1) is applied to both the source-independent equations and the source-dependent equations; (B) way (1) to the source-independent equations, but (2) to the other pair; (C) way (2) and then (1); (D) way (2) to both pairs. Table III gives the values that have been calculated for these possibilities.

TABLE III

VALUES FOR a AND b

Possibility	a	b
A	$c\varepsilon\sqrt{c\mu}$	$1/\sqrt{c\mu}$
B	$c^2\varepsilon\sqrt{\mu}$	$1/\sqrt{\mu}$
C	$c\varepsilon\sqrt{\mu}$	$1/\sqrt{c^2\mu}$
D	$c\varepsilon\sqrt{\mu}$	$1/\sqrt{c\mu}$

Comparing the values in Table III with those in Equations (II.C.58) and (II.C.59), it seems that the only difference with V4's values is that c does not appear in them. Possibly V4 used units such that c should not appear, although it apparently is not clear what units are used. K3's values do not even seem to resemble those above, and there seems to be no obvious reason for this disagreement.

Equation (II.C.61) appears to give $H^{\alpha\beta}$ correctly in terms of $F_{\alpha\beta}$, but $H^{\alpha\beta}$ is antisymmetric; therefore, $S^{\mu\nu\alpha\beta}$ should also be antisymmetric in α and β as well--Yildiz and Tang (Y1) note further antisymmetric properties. The following appears to have the correct antisymmetric properties:

$$S^{\mu\nu}{}_{\alpha\beta} = \frac{1}{2}\,(S^{\mu}_{\alpha}\, S^{\nu}_{\beta} - S^{\mu}_{\beta}\, S^{\nu}_{\alpha}). \qquad \text{(II.C.62)}$$

V4 and K3 next proceed to apply Equations (II.C.57, 58 and 59) to the ringlaser even though it is generally in a NIFR: it appears as if V4 and K3 assume the ringlaser were accelerating relative to the media inside its cavity. They make no attempt to justify this procedure. Some reasons are now given to show why one might have to be careful to proceed as above. It is well known that many media change in their electromagnetic properties on being stressed, as by accelerating them (cf. photoelasticity): some otherwise isotropic media become anisotropic in fact. But even if the stresses in the medium could be neglected, its acceleration may still enter into its constitutive properties (H2) (L2) (V2). It is true that the effects of acceleration may be small in most practical cases, but they could be even larger than the gravitational effects considered here.

It appears prudent, convenient, and even practical to consider only vacuum processes in this thesis. It may be practical because it appears feasible as well as simple. Michelson and Gale's interferometric optradich (see Subsection I.E.2) was put inside a partial vacuum system; and it may be possible to construct a practical ringlaser inside a (nearly) vacuum system also (see Subsection I.E.3). It would seem that the experimentalist would seek to design an optradich experiment that is as simple and clean (i.e., free of extraneous or peripheral influences) as possible. For example, if he needs a bias source to unlock the ringlaser (see Subsection I.E.3), Earth's rotation appears better than any most nonreciprocal medium because it can be monitored with a high degree of precision, it does not require additional optical elements in the ringlaser cavity, and the ringlaser can be firmly attached to the surface of the spinning Earth.

Now, the issue of what coordinate frame to use is considered. First, for background it is described how Møller (M5) obtained the form of the metric tensor for his rotating frame. He started from a frame, x^{α}, in which the optradich does not have constant spatial coordinates, and used coordinate transformations to go to $x^{\alpha'}$, in which the optradich does have constant spatial coordinates. It so happens that x^{α} approximates the Lorentz-frame coordinates, i.e., $g_{\alpha\beta} \simeq \eta_{\alpha\beta}$, where $g_{\alpha\beta}$ is the metric tensor in the x^{α} frame. This seems to have motivated Møller to rigidly rotate x^{α} in space to the new frame, $x^{\alpha'}$: the coordinate transformations are

$$x^{o} = x^{o'},$$
$$x^{j} = \Lambda^{j}_{j'}\, x^{j'}, \qquad \text{(II.C.63)}$$

where $\Lambda^{j}_{j'}$ is the usual rotation matrix in Euclidean space, which is parametrized by three Euler angles (A3, pages 178-180), one of which is a function of time ($\Omega t + a_0$ where a_0 is a constant and Ω is the angular speed, as used by V4 and K3). (These equations are more general than Møller's, but they are more convenient here.)

V3, V4, and K3 may also have obtained their forms of the metric tensor essentially in this way, although no details were given. It is assumed that they did proceed essentially as above.

Some reasons are now given for using the proper frame in the place of Møller's, V3's, V4's, and K3's frames. First, it appears that it is not easy or simple to generalize the coordinate-transformation procedure to cover arbitrary motion of the optradich (e.g., translation about Earth as well as rotation). There is no such problem with the proper frame: it can follow the motion of the observer, whatever it may be. Next, there could be problems with the coordinate-transformation approach, according to Lianis (L2, Section 5). From his discussion, it would seem desirable to investigate the physical meaning of Ω; however, ω^j of the proper frame has a well-defined meaning: it is the angular velocity relative to inertial guidance gyroscopes. Now, along the world line of the observer, the coordinate time in the proper frame is also the observer's proper time, when the observer is at the origin. This is not generally so with the other frames: the optradich may be some distance from the origin of the frame.[4] Further, the forms of the metric tensor as used by Møller and the others could be somewhat more complicated and less easy to use than the proper frame form in many cases. The latter form also seems to make more transparent the physics of the gravitational field as "seen" by the optradich, and is written entirely in terms of coordinate-independent quantities that can be measured by the observer in the proper frame: $\omega^{\hat{j}}$ and $a^{\hat{j}}$. Last, but not least, the proper frame apparently makes it easier to bring the general results of Møller and others into line with those of Synge and Dehnen: Synge uses a frame like the proper frame but with $\omega^j \equiv 0$, and while Dehnen does not seem to have used any special frame, he does use the rotation tensor, $\omega^{\alpha\beta}$, which, as seen earlier, is connected with the angular velocity, ω^γ (Equation (II.C.40)).

Another matter is now taken up. K3 says that $d\vec{l}$ on the left side of K3's Equation (15) is "proper length" and thus must be replaced by "coordinate length"; this does not seem to be in accord with the thinking of Heer (see the quote in Subsection II.C.4) and Møller (see Subsection II.C.1). In this thesis, $d\vec{l}$ would be treated as "coordinate length."

Further, on page 2956, K3 neglected the angular momentum of Earth. However, V3 (page 412), V4 (page 998), and Dehnen (D2, page 820) do not. Møller did seem to neglect Earth's angular momentum. (Synge [S8, Section 7, Chapter XI] did not consider special cases.) The results in Chapter III incidate that the angular momentum should not be neglected.

Finally, K3 (page 2958) says, "the beat frequency [of the ringlaser] . . . is not influenced by the static gravitational field." The third term in Equation (II.E.1) seems to indicate that when an optradich is in motion through the static part of a gravitational field, it may be influenced by that part (M3, page 1119).

II.D The Sagnac Effect Generalized

The main conclusion of the preceding section, reached in four different ways, is that an optradich in arbitrary motion through spacetime having an arbitrary Riemann geometry measures ΔT as given by

$$\Delta T = 4 \frac{\vec{\omega}\cdot\vec{n}_u}{c^2} A, \qquad (II.D.1)$$

where $\vec{\omega}$ is the angular velocity of the optradich relative to a frame which comoves with the optradich and whose spatial axes are attached to inertial guidance gyroscopes--Synge's Fermi frame (Subsection II.C.2)--A and $\vec{n}_u$ are as before, and assumptions 1* to 9* are in effect here and henceforth. The Fermi frame above is referred to henceforth as the inertial guidance frame (IGF) of the optradich.

When the optradich is in free fall, the IGF becomes an IFR, so Equation (II.D.1) describes the classical Sagnac Effect for a plane optradich: from Equation (I.B.2),

$$\Delta T = 4 \frac{\vec{\Omega}\cdot\vec{n}_u}{c^2} A. \qquad (II.D.2)$$

Moreover, Equations (II.D.1) and (II.D.2) are identical in form. Thus it is proposed that the effect as represented by Equation (II.D.1) be referred to as the Generalized Sagnac Effect.

How does an optradich measure gravitational effects? Let "distant stars frame" (DSF) denote the frame which comoves with the optradich or the IGF and whose orthonormal basis vectors are related to the PPN coordinate frame basis vectors by a pure Lorentz boost (see M3, Box 2.4) plus renormalization (M3, Equation [39.41]). Since the PPN basis vectors behave as if they were attached to a Lorentz frame far from the gravitationally bound system, the DSF could thus appear to be attached to the "distant stars" (M3, page 1117). Now, it is convenient here and henceforth to consider just $\vec{\omega}\cdot\vec{n}_u$ in Equation (II.D.1): let

$$\mathcal{S} \equiv \vec{\omega}\cdot\vec{n}_u. \qquad (II.D.3)$$

In the DSF this can be rewritten:

$$S = (\vec{\omega}_D - \vec{\Omega}_D)\cdot\vec{n}_u, \qquad \text{(II.D.4)}$$

where $\vec{\omega}_D$ and $\vec{\Omega}_D$ are the respective angular velocities of the optradich and the IGF relative to the DSF. In curved spacetime the IGF rotates relative to the DSF: the well-known "dragging of inertial frames" effect, which is reviewed in the next section. Thus, by attaching the optradich to its DSF, i.e., $\vec{\omega}_D \equiv 0$, or by orienting the optradich so that $\vec{\omega}_D\cdot\vec{n}_u \equiv 0$, the dragging of inertial frames could thus be directly measured. Of course, if $\vec{\omega}_D\cdot\vec{n}_u$ were known with sufficient accuracy, then by subtracting out this from S, the drag effect could thus be estimated. Some special cases with numerical results are presented in Chapter III.

II.E Dragging of Inertial Frames

Misner, Thorne, and Wheeler (M3, Section 40.7) show how the angular velocity of precession of an inertial guidance gyroscope relative to its DSF, Ω_D, can be calculated at the post-Newtonian level of approximation in the PPN Formalism, in complete generality:

$$\vec{\Omega}_D = -\frac{1}{2}\,\vec{v} \times \frac{\vec{a}}{c^2} - \frac{1}{2}\,\vec{\nabla} \times \vec{g} + (\gamma + \frac{1}{2})\,\vec{v} \times \vec{\nabla}U, \qquad \text{(II.E.1)}$$

where $\vec{v}$ is the PPN coordinate velocity, and $\vec{a}$ is the acceleration, of the gyroscope; γ and U are as in Section I.D; and $\vec{g}$ has the components g_{oj}, where g_{oj} is as in Equation (I.D.4).

The first term in the above equation represents the Thomas precession of Special Relativity (S1). It exists even in the absence of gravity. Partly because Special Relativity is already one of the most well established theories empirically, this term does not seem to hold much interest for the gravitation experimentalist.

The second term is the most interesting one here, apparently. Leaving aside preferred frame effects--see Section I.D--for the moment, the term could be thought of as a "mass-current" effect, analogous (at least in a rough way) to effects on a magnet from a magnetic field set up by an electric current. It is the only one to come from the off-diagonal part of the metric tensor and also the only one to include preferred frame effects. It perhaps should be pointed out that no off-diagonal effect has ever been knowingly detected in any experiment performed so far, as far as it is known.

The third term is also interesting. It arises from the motion of the gyroscope through the static part of spacetime (M3, page 1119). It is analogous to the effects on a magnet moving through an electric field.

For convenience later, the three terms are named: the first one is called the Thomas term; the second one, the Lense-Thirring term; and the third one, the de Sitter term.[5]

ENDNOTES

[1]This note elaborates on the text. Let time be multiplied by and placed on an equal footing with the spatial dimensions; let ℓ be a typical dimension of the region.

If the optradich is accelerated ("feels a weight"), and if a is the magnitude of the acceleration, then the "smallness" condition can be restated:

$$\ell << \frac{c^2}{a}$$

(M3, Chapter 6 and Section 13.6). For an optradich attached to Earth's surface, $c^2/a \simeq$ one light year. If, however, the optradich is in free fall ($a \equiv 0$), then another form of the "smallness" condition is given: ℓ is so small that gravitational tide-producing effects are negligible (M3, Section 1.6). Both forms of the condition may be taken together: a nearby IFR can be used to determine the numerical limit set by the latter form.

[2]The thesis author does not feel that his understanding of the first part of Dehnen's development is sufficiently reliable. Hence, he rewrote that part in his own words to ensure that he would not make serious errors through his lack of comprehension. For example, the concepts of <u>Durchstosspunkte</u> and <u>Verbindungsvektoren</u> (D2, page 818) are not clear. Language difficulties--his paper is in German--may be responsible.

[3]Rosenthal (R2), however, appears to have been the first to propose using ringlasers in gravitational physics.

[4]This seems to require some elaboration. If point P has constant spatial coordinates in a frame, and has coordinate time, T_c, and proper time, T, then

$$\frac{dT_c}{dT} = \sqrt{\frac{(dx^o)^2}{-(ds)^2}} = \sqrt{\frac{1}{-g_{oo}}}.$$

However, in the frames of Møller and others, $\sqrt{-g_{oo}} \simeq 1$, so that one could use either T_c or T since $\Delta T_c \simeq \Delta T$. Thus the reason given in the text is probably of minor importance here.

[5]This nomenclature was suggested to the thesis author by such papers on Schiff's gyroscope experiment as O'Connell's paper (O1), and page 1119 of Misner, Thorne, and Wheeler's book (M3).

CHAPTER III

SPECIAL CASES IN THE SOLAR SYSTEM

III.A Introduction

The main purpose of this chapter is to use Equations (II.D.4) and (II.E.1) to obtain some rough ideas on the largest gravitational effects on an optradich that could be found in the Solar System by analyzing some simple cases. An effort is made to find time varying effects since they are more interesting and generally much easier to detect and identify than constant ones (see Subsection I.E.3).

The cases of an optradich attached to an isolated astronomical body, and of optradiches orbiting the same body are considered in Section III.B; the influences of the Sun on optradiches attached to, and orbiting, Earth are considered in Section III.C. Numerical results are given.

III.B The Isolated Body

III.B.1 Preliminaries

An idealized astronomical body is used in this section. It is isolated, rigid, homogeneous, spherical, and of mass M, radius R, angular velocity $\vec{\omega}_B$, and angular momentum $\vec{J} = \frac{2}{5} M R^2 \vec{\omega}_B$. The following quantities from Equation (II.E.1) are computed for this case:

$$\vec{\nabla} U = - \frac{G M}{c^2 r^3} \vec{r}, \qquad \text{(III.B.1)}$$

where $r = |\vec{r}|$, and $\vec{r}$ is the position vector of the field point in the PPN frame; and

$$\frac{1}{2}\vec{\nabla}\times\vec{g} = \left(\frac{4\gamma + 4 + \alpha_1}{8}\right)\frac{G}{c^2}\frac{1}{r^3}\left(3\,\frac{\vec{J}\cdot\vec{r}}{r^2}\,\vec{r} - \vec{J}\right) + \alpha_1\,\vec{w}\times\frac{G\,M}{c^2\,r^3}\,\vec{r}. \tag{III.B.2}$$

(Some details of the derivation are given. Misner, Thorne, and Wheeler [M3, Exercise 40.7] show how the terms V_j and W_j in Equation [I.D.4] for g_{oj} reduce for the above case to this:

$$\frac{1}{c}\,\vec{g} = \left(\frac{7}{4}\,\Delta_1 + \frac{1}{4}\,\Delta_2\right)\frac{G}{c^3}\,\frac{\vec{J}\times\vec{r}}{r^3}\,. \tag{III.B.3}$$

Note that the terms in α_2 in Equation [I.D.4] vanish identically when the curl of these terms is taken [see M3, Equations (39.34 a and g)].)

The PPN coordinate frame is so placed and oriented that its spatial origin is at the center of mass of the body and that $\vec{\omega}_B = \omega_B\ (\,0,\,0,\,1\,)$ in the frame, where $\omega_B = |\vec{\omega}_B|$. Let $\vec{\omega}_u \equiv \vec{\omega}_B/\omega_B$, $\vec{r}_u \equiv \vec{r}/r$, and $\vec{R} = R\vec{r}_u$.

III.B.2 The Earth-Bound Optradich

The optradich partakes of the body's rotation, so

$$\vec{\omega}_D = \vec{\omega}_B, \qquad \vec{v} = \vec{\omega}_B\times\vec{R}, \tag{III.B.4}$$

for Equations (II.D.4) and (II.E.1); its acceleration is approximately

$$\frac{\vec{a}}{c^2} = -\vec{\nabla}U. \tag{III.B.5}$$

Hence, Equation (II.D.4) specializes to the case of an optradich attached to the surface of the body:

$$S = \vec{\omega}_B\cdot\vec{n}_u + \frac{4}{5}\left(\frac{\gamma+1}{2}\right)\frac{G}{c^2}\frac{M}{R}\,\omega_B\,[(\vec{\omega}_u\cdot\vec{R}_u)\,\vec{R}_u - 2\vec{\omega}_u]\cdot\vec{n}_u$$

$$+ \frac{\alpha_1}{4}\,\frac{G}{c^2}\frac{M}{R}\left[\frac{1}{5}\,\vec{\omega}_B - \frac{\vec{w}}{R}\times\vec{R}_u - \frac{3}{5}\,(\vec{\omega}_B\cdot\vec{R}_u)\vec{R}_u\right]\cdot\vec{n}_u. \tag{III.B.6}$$

Let for later $\vec{Q} = (\vec{\omega}_u \cdot \vec{R}_u)\vec{R}_u - 2\vec{\omega}_u$.

The effect represented by the first term in Equation (III.B.6) could be used as a bias in a ringlaser (see Subsection I.E.3).

The maximum value that the second term in the last equation can have is now computed. Q has magnitude

$$Q \equiv |\vec{Q}| = \sqrt{4 - 3\cos^2\theta}, \tag{III.B.7}$$

where $\cos\theta \equiv \vec{\omega}_u \cdot \vec{R}_u$. After Q is differentiated with respect to θ and the derivative set to zero, the extremum condition is $\cos\theta \sin\theta = 0$; $\sin\theta = 0$ leads to $Q = 1$, while $\cos\theta = 0$ leads to $Q = 2$. Hence, at the equator, $\vec{Q}$ has the maximum magnitude of 2 and points "due South."[1] When $\vec{n}_u$ is oriented "Southward" also, the term has the value at the equator:

$$S = \frac{8}{5} \left(\frac{\gamma+1}{2}\right) \frac{G}{c^2} \frac{M}{R} \omega_B. \tag{III.B.8}$$

For Earth, it is $\left(\frac{\gamma+1}{2}\right) \times 8.15 \times 10^{-14}$ rad s^{-1}.

Now the third term in Equation (III.B.6)--a preferred frame effect --is investigated. A definitive value for $\vec{w}$ is not available yet; however, on the basis of available evidence, it seems reasonable to assume $|w| \simeq 6 \times 10^{-4}$ c (W4, page 95). As this is over fourteen times the rotational speed of a point on Jupiter's equator, the terms in $\alpha_1\omega_B$ are henceforth ignored. Although $\vec{w}$ is fixed in the PPN frame with respect to time, at least approximately over a period of several years, $\vec{R}$ and $\vec{n}_u$ vary sinusoidally with time. Hence, $\vec{w} \times \vec{R} \cdot \vec{n}_u$ should have a complex time variation which should be quite easy to identify in optradich measurements--if the effect is ever discernible. If $\vec{w} \times \vec{R}_u \cdot \vec{n}_u = |\vec{w}|$ at some time, then the maximum magnitude is

$$S = \frac{\alpha_1}{4} \frac{G}{c^2} \frac{M}{R^2} |\vec{w}|. \tag{III.B.9}$$

If indeed $|\vec{w}| = 6 \times 10^{-4}$ c, then for Earth it is $\alpha_1 \times 5 \times 10^{-12}$ rad s^{-1}. Note that it does not appear in General Relativity and the Brans-Dicke-Jordan theory (see Table I).

III.B.3 Orbiting Optradiches

It is assumed that the acceleration of the orbiting optradich as seen in a nearby Lorentz frame is negligible: the Thomas term is ignored here. Further, for simplicity and convenience, the optradich is attached to its DSF, so $\vec{\omega}_D \equiv 0$ in Equation (II.D.4). Apparently, classical celestial mechanics can be assumed here, as effects from departures from it are probably negligible; a general result from that theory is that

$$\vec{r} \times \vec{v} = \vec{H}, \qquad \text{(III.B.10)}$$

where $\vec{H}$ is a constant--the moment of momentum vector of the orbit (S3).

The preferred frame effects have already been adequately discussed in the last subsection, except that it is noted that they have a $1/r^2$ dependence; and that, since r could vary here, the effects may have even more complex time variations. Henceforth, $\alpha_1 \equiv 0$ is assumed.

In accordance with the preceding paragraphs, Equation (II.D.4) specializes to the case of the orbiting optradich:

$$\mathcal{S} = \{(\frac{\gamma+1}{2}) \frac{G}{c^2} \frac{J}{r^3} [\vec{\omega}_u - 3 (\vec{\omega}_u \cdot \vec{r}_u) \vec{r}_u] + (\frac{2\gamma+1}{3}) \frac{3}{2} \frac{G}{c^2} \frac{M}{r} \frac{\vec{H}}{r^2}\} \cdot \vec{n}_u, \qquad \text{(III.B.11)}$$

where $J = |\vec{J}|$.

Note that it is possible to orient the optradich so that $\vec{H} \cdot \vec{n}_u \equiv 0$ but that the terms in J do not vanish. Hence, it is possible to cleanly separate the effects stemming from the off-diagonal terms of the PPN metric tensor from those from the diagonal terms in the case of the orbiting optradich.

The following type of orbit, from an infinite variety, appears to be of especial interest: a near-body, circular and polar orbit; i.e., $\vec{H}\cdot\vec{J} = 0$; $\vec{\omega}_u\cdot\vec{r}_u = \cos\psi$ where $\psi = 2\pi/P$, P being the period of the orbit; and $r \simeq R$. Further, the optradich is oriented so that $\vec{H}\cdot\vec{n}_u = 0$, and $\vec{n}_u = \vec{\omega}_u$. In that case, Equation (III.B.11) specializes further to

$$\mathcal{S} = \left(\frac{\gamma+1}{2}\right) \frac{2}{5} \frac{G}{c^2} \frac{M}{R} \omega_B \,(1 - 3\cos^2\psi). \qquad \text{(III.B.12)}$$

Table IV is developed as an aid in obtaining numerical results for the Solar System. It lists the values of some relevant physical quantities for three of the Solar System bodies that seem to be of especial interest.

TABLE IV

SOME SOLAR SYSTEM PHYSICAL QUANTITIES

Body	$\frac{GM}{c^2}$ (meters)	R (meters)	ω_B (rad s^{-1})
Sun	1.5×10^3	7.0×10^8	3×10^{-7} to 7×10^{-5}
Jupiter	1.4	7.0×10^7	1.8×10^{-4}
Earth	4.4×10^{-3}	6.4×10^6	6.1×10^{-5}

Some things need to be said about Table IV. Since Earth, Jupiter, and the Sun, as is well known, are not rigid, homogeneous, and spherical, the values given in Table IV for ω_B are only "educated guesses" of the "effective values," which, when put in Equation (III.B.12), would give correct values for S. Haas and Ross (H1) say that the angular momenta of the Sun and Jupiter are not known very well. Barker and O'Connell (B1, footnote 10) say that Earth's angular momentum is about 17% smaller than what it would be if Earth were homogeneous; thus, the value for ω_B in Table IV is reduced by 17% from Earth's sidereal rate. The values for ω_B for the Sun and Jupiter are based on data from Haas and Ross' paper (H1).

Table V is developed from Equation (III.B.12) and Table IV: The second column lists the values of $\frac{2}{5}\frac{GM}{c^2}\frac{M}{R}\omega_B$ for the respective bodies (however, an orbit of 10 solar radii is assumed in the case of the Sun, instead, as smaller orbits may not be practical [H1, page 9]).

TABLE V

SOME VALUES OF THE LENSE-THIRRING TERM

Body	S (rad s^{-1})
Sun	2×10^{-14} to 6×10^{-12}
Jupiter	1×10^{-12}
Earth	1×10^{-14}

It is now of interest to see what $\vec{H}$ could contribute. The optradich is reoriented so that $\vec{H}\cdot\vec{n}_u = |\vec{H}|$ but is allowed the same orbit. If the other terms in Equation (III.B.11) are ignored, then

$$S = \left(\frac{2\gamma+1}{3}\right) \frac{3}{2} \frac{c}{R} \left(\frac{G}{c^2} \frac{M}{R}\right)^{3/2} . \qquad \text{(III.B.13)}$$

Table VI is developed from the above equation and Table IV: the second column presents the values of $\frac{3}{2} \frac{c}{R} \left(\frac{G}{c^2} \frac{M}{R}\right)^{3/2}$ for the respective bodies (again, an orbit of 10 solar radii is assumed).

TABLE VI

SOME VALUES OF THE DE SITTER TERM

Body	S (rad s^{-1})
Sun	6×10^{-12}
Jupiter	1×10^{-11}
Earth	1×10^{-12}

III.C The Influence of the Sun on Earth Optradiches

III.C.1 Introduction

The influences of the Sun on optradiches close to Earth are considered in this section.

The Earth and the Sun are now idealized as point-masses; even Earth's angular momentum is neglected. So, Will and Nordtvedt's point-mass metric (Equation I.D.6) can be used here.

The origin of the PPN coordinate frame is now placed at the center of the Sun.

The PPN velocity of the optradich, $\vec{v}$, is split into two parts:

$$\vec{v} = \vec{v}_E + \vec{v}_S, \qquad \text{(III.C.1)}$$

where $\vec{v}_E$ is the velocity of the optradich relative to Earth in the PPN frame and $\vec{v}_S$ is the PPN velocity of Earth. The gravitational potential, U, is likewise split into two parts:

$$U = U_E + U_S, \qquad \text{(III.C.2)}$$

where

$$U_E = \frac{G\, M_\oplus}{c^2\, R_E}, \qquad U_S = \frac{G\, M_\odot}{c^2\, R_S}, \qquad \text{(III.C.3)}$$

$M_\oplus$ and $M_\odot$ are the respective masses of Earth and the Sun, and R_E and R_S are the respective distances of the optradich from Earth and the Sun.

Hence, if preferred frame effects are neglected ($\alpha_1 = \alpha_2 = 0$),

$$-\frac{1}{2}\,\vec{\nabla} \times \vec{g} = (\gamma + 1)\,\vec{v}_S \times \vec{\nabla} U_E. \qquad \text{(III.C.4)}$$

The specialized expression for S is thus

$$S = \vec{\omega}_D \cdot \vec{n}_u - [-\frac{1}{2}\,\vec{v} \times \frac{\vec{a}}{c^2} + (\gamma + 1)\,\vec{v}_S \times \vec{\nabla} U_E$$

$$+ (\gamma + \frac{1}{2})\,\vec{v} \times \vec{\nabla} U] \cdot \vec{n}_u. \qquad \text{(III.C.5)}$$

In the next subsection, the case of an Earth-bound optradich is now considered; in the last subsection, an Earth-orbiting optradich is considered.

III.C.2 The Earth-Bound Optradich

Equations (III.B.4) and (III.B.5) are used to specialize further Equation (III.C.5) to

$$\mathcal{S} = \vec{\omega}_B \cdot \vec{n}_u - \{2\left(\frac{\gamma+1}{2}\right) \vec{v}_E \times \vec{\nabla}\mathcal{U}_E + \left(\gamma + \frac{1}{2}\right) \vec{v} \times \vec{\nabla}\mathcal{U}_S\} \cdot \vec{n}_u. \qquad \text{(III.C.6)}$$

The first term inside the braces has already been considered in Section III.B; so now the magnitude of the second term inside the braces (which can be considered to be a de Sitter term due to the optradich's motion through the static part of the Sun's field) is estimated: $\left(\frac{\gamma+1}{2}\right) 2 \times 10^{-16}$ rad s^{-1} for the Earth-bound optradich.

Neglect of the Sun's influence would seem justified in this case, except possibly for optradiches located at the poles of Earth.

III.C.3 The Earth-Orbiting Optradich

As in Subsection III.B.3, the Thomas term is ignored here and the optradich is to be attached to its DSF. Thus, from Equation (III.C.5),

$$\mathcal{S} = -\{\left(\gamma + \frac{1}{2}\right)[\vec{v}_E \times \vec{\nabla}\mathcal{U}_E + \vec{v} \times \vec{\nabla}\mathcal{U}_S] - \frac{1}{2} \vec{v}_S \times \vec{\nabla}\mathcal{U}_E\} \cdot \vec{n}_u. \qquad \text{(III.C.7)}$$

Whatever the distance of the optradich from Earth may be, the second term within the square brackets is essentially the same as that considered in the last subsection, even though the orbiting optradich's velocity relative to Earth can be somewhat greater than that of an optradich on Earth's surface in magnitude. The term is thus henceforth ignored.

Accordingly, Equation (III.C.7) is rewritten:

$$\mathcal{S} = -\frac{1}{2}\{[\left(\frac{2\gamma+1}{3}\right) 3\, \vec{v}_E - \vec{v}_S] \times \vec{\nabla}\mathcal{U}_E\} \cdot \vec{n}_u. \qquad \text{(III.C.8)}$$

Since $\vec{v}_E$, $\vec{v}_S$, and $\vec{\nabla}U_E$ all vary with time in the PPN frame, it is obvious that S has a complex time dependence (and therefore is easily identifiable). Furthermore, the maximum possible magnitude of S here can be much larger than the value of the de Sitter term given for Earth in Table VI, since the maximum possible value of $3|\vec{v}_E| + |\vec{v}_S|$ is about 1.8×10^{-4} c, more than twice that of $3\,|\vec{v}_E|$ alone.

It would seem, then, that the influence of the Sun on an Earth-orbiting optradich should always be taken into account, in contrast to the conclusion in the last subsection.

ENDNOTES

[1]See pages 880-881 in Schiff's paper (S2) for a discussion of latitude effects on gyroscopes. See also page 1119 in Misner, Thorne, and Wheeler's book (M3).

CHAPTER IV

CONCLUDING DISCUSSIONS

IV.A The Outlook for Optradich Experiments

Experiments with optradiches based on Equations (II.D.4) and (II.E.1) are considered in this chapter.

Assuming General Relativity values for γ and α_1 (see Table I), the largest numerical value found in Chapter III is of the order of 10^{-12} rad s^{-1}. Although larger values than that could be found, they are expected to be of the same order of magnitude in the Solar System. These values are about seven decades smaller than Earth's rotation rate and are just below the limit of the ringlaser's sensitivity as calculated in the second speculative example in Subsection I.E.3. Another example: Let Ω_s be 10^{-13} rad s^{-1} (so that the ringlaser could be sensitive enough), λ = .63 x 10^{-6} m (as before), and $\sqrt{\Delta f_{ph}/T} = 10^{-3}$ Hz (ten times smaller than before); then, A/L = 3 x 10^3 m. Most likely, giant ringlasers would have to be considered.

Thus, prospects for optradich experiments appear quite bleak and seem likely to remain so in the near future. No experience with ringlasers of L > 40 m has been obtained, as far as is known (P2). However, the ultimate limit of the ringlaser is not yet definitely known, and some other type of optradich might be developed in the future with the needed sensitivity: e.g., the Michelson-Gale interferometer (Subsection I.E.2) could be improved by replacing the light source with a highly monochromatic and coherent laser. So, it is assumed in the rest of the thesis that a practical optradich able to measure down to 10^{-15} rad s^{-1} will be available in the future.

IV.B The Optradich Experiment Versus the Gyroscope Experiment

It has been seen in Section II.D that the optradich experiment measures the same basic effect--the "dragging of inertial frames" effect--as does Schiff's gyroscope experiment (M3, Section 40.7). However, the optradich does not necessarily merely duplicate the function of the gyroscope: there seems to be three points of difference at least.

First, the optradich experiment offers an opportunity to investigate some aspects of the interactions between gravitational fields and electromagnetic fields. (It may possibly be of some value in unified-field physics.)

Second, assume that $\vec{\Omega}_D$ (see Section II.E) is constant, for simplicity and convenience. To determine both the magnitude and direction of $\vec{\Omega}_D$ on a gyroscope, one needs to measure at least two shifts of the gyroscope axis--if the axis is initially nonparallel to $\vec{\Omega}_D$--but does not need to otherwise disturb the gyroscope. On the other hand, to determine the magnitude and direction of $\vec{\Omega}_D$ unambiguously on an optradich, he must vary the spatial orientation of the optradich relative to its DSF, where the DSF is as in Section II.D.

Third, there apparently are no practical methods available to measure directly the instantaneous value of $\vec{\Omega}_D$--especially small ones--on a gyroscope (H1) (O1) (E3). Hence, one could say that a gyroscope gives directly only the time-integrated value of $\vec{\Omega}_D$. A ringlaser can be made to give directly the time-integrated value of S, where S is as in Section II.D, by means of its nature (K1), but there may be some types of optradiches which can give directly the instantaneous value of S. If the optradich experiment is adapted to measure the time-integrated value, it would essentially compete with the gyroscope experiment (if unified-field effects are negligible). Then, the comparison between these experiments could simply become technological in nature: relative expense in design, construction, and operation; relative long-term stability; relative sensitivity; and so on. This aspect is, however, outside the scope of this thesis.

Another aspect is then considered. Take Equation (III.B.12) as an example; rewrite it as

$$S = A + B \cos 2\psi, \tag{IV.B.1}$$

where

$$A = -\left(\frac{\gamma+1}{2}\right) \frac{1}{5} \frac{G}{c^2} \frac{M}{R} \omega_B$$

and

$$B = 3A.$$

This can be assumed to apply to the angular shift of the gyroscope's spin as well, for convenience and simplicity. In the gyroscope, the first term will eventually dominate the other; apparently, actual gyroscopes at present cannot detect the second term (H1) (O1). On the other hand, the second term is much more readily detected on those types of optradiches which detect the instantaneous value of S than the first one. Another example: Misner, Thorne, and Wheeler (M3, page 1120) say that preferred frame effects in the Solar System (if any) are too small for present-day gyroscopes to measure. However, the super-optradich (see the end of the last section) could help set experimental bounds on these putative effects that are much smaller than what present-day gyroscopes can set, partly because the effects may have distinctive time variations (see Subsection III.B.2).

In conclusion, if a sufficiently sensitive optradich could be developed, it need not compete with Schiff's gyroscope, but rather complement it, when set up to detect instantaneous values of S.[1]

IV.C Orbiting Optradiches Versus Land Optradiches

As Earth appears to be the best place in the Solar System for land-based optradiches, only Earth is considered here for such optradiches. Preferred frame effects are ignored here, as they are essentially the same for both orbiting and land optradiches, and they may be nonexistent or small anyway.

The orbiting optradich offers the experimentalist the largest effects and a much wider variety in them, as can be seen in Chapter III. The Earth-attached optradich offers essentially the same kind of effect: Equation (III.B.6). The off-diagonal effects are inextricably bound with the diagonal effect in the land optradich. On the other hand, the orbiting optradich allows the experimentalist to cleanly separate the off-diagonal effects from the diagonal ones; that is not a trivial advantage. There are no time varying effects for the optradich on Earth to measure, as there are for orbiting ones.

There are irregularities in Earth's rotation which can be accounted for by Newton's gravitational theory and which are expected to appear along with the post-Newtonian effects in the Earth-based optradich. The prime one is the precession of the equinoxes which appear to contribute a time varying effect of amplitude of the order of 10^{-12} rad s^{-1}, and may thus interfere considerably with the measurement of terrestrial post-Newtonian gravitational effects.

It has been suggested in Subsections I.E.3 and II.C.5 that the loop of an optradich be put in a vacuum system to minimize complications from material media filling parts of the loop. This is obviously expensive for giant optradiches on Earth, but for orbiting optradiches there is a vacuum "harder" than any man-made ones and which costs nothing to maintain.

While an Earth-based optradich's size is obviously ultimately limited by the sphericity of Earth, there is no theoretical limit to the size of an orbiting optradich. Figure 8 shows an example of an optradich of size $L = 1.1 \times 10^{11}$ m formed by three satellites orbiting the Sun.

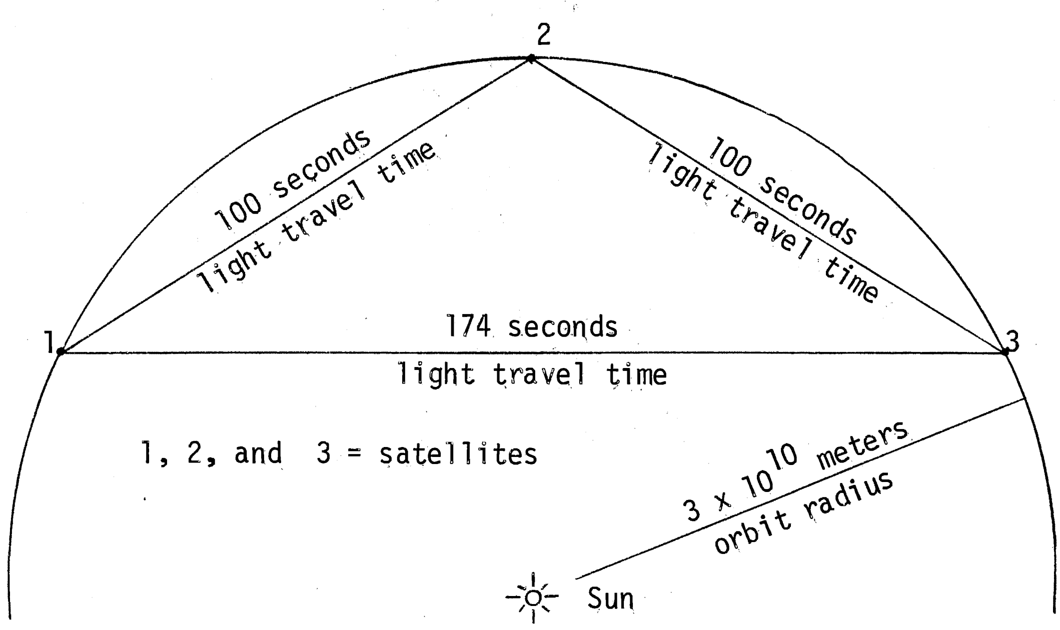

Figure 8. The Three-Satellite Optradich

Each satellite, at a vertex of the triangular loop, may contain a laser amplifier plus a system to keep it aligned with the others.

Davies (D1) appears to be the first to suggest an experiment as in Figure 8. He uses Earth as one of the "satellites" in his scheme to measure the angular momentum of the Sun. It appears that the assumptions in Section II.B might still apply here; if so, the results of Subsection III.B.3 are applicable except that a term, $\vec{\omega}_0 \cdot \vec{n}_u$, where $\vec{\omega}_0$ is the orbital angular velocity of the satellites relative to the distant stars, must be added to S. As noted by him, the satellites can be used for other purposes at the same time. His discussion of various experimental details may interest the reader.

On the other hand, there are at least two problems with orbiting optradiches. Either the orbiting optradich's rotation relative to its DSF would have to be known with a sufficient degree of accuracy or be kept nearly zero by attaching the optradich to a frame attached to star-pointing telescopes. The telescope frame rotates with respect to the DSF in general, due perhaps to the aberration of light by the motion of the telescope (W2). Also, the orbiting optradich may be more expensive and difficult to design, construct, and operate than land optradiches, although the steady progresses of space technology may reduce some of the problems in the future.

It would seem that orbiting optradiches and land optradiches are complementary, that the choice between them depends on what the experimentalist wants to do specifically. Further study may have to be made on this question, perhaps after more is known about ringlasers.

IV.D The Scientific Worth of Optradich Experiments

Some experiments or observations in gravitational physics only test the foundations of gravitational theories: for example, the Eötvös-type experiments and the gravitational red-shift ones (M3, Chapter 38). Provided that the foundations are tested and found to be sound or valid, these experiments do not distinghish between various gravitational theories, nor put constraints on them, as long as the theories are based on the parts of the foundations that the experiments test. Some others, however, have some capability (in varying degrees) of distinguishing between gravitational theories, or of testing some significant (as opposed to fundamental or basic) aspect of a theory: a prime example is the "excess" in the perihelion shift of Mercury (A2, Section 6.5).

The optradich experiment (and also the gyroscope experiment) appears to belong to the latter class above. The "dragging of inertial frames" effect is a significant (in the sense above) aspect of present gravitational theories, and the results of optradich experiments may be quite useful in disproving some gravitational theories.[2]

Moreover, effects stemming from the off-diagonal part of the metric tensor do not seem to have been found in experiments yet, although the Schiff gyroscope experiment may fly soon (H1, page 3) (E3) (M3, page 1120). Hence, the dragging of inertial frames, which involves some off-diagonal effects, is a new (i.e., not yet experimentally tested) and significant effect, and would seem to furnish an excellent reason to do optradich experiments.

IV.E Conclusion

Although it seems hopeless at present, efforts should be made to develop optradiches of sufficient sensitivity to measure Solar System gravitational effects because of the great scientific knowledge that could be obtained (see the above section).

ENDNOTES

[1]In practice, the output from a ringlaser is Δf_{ob} (see Section I.E.3), typically less than 1 M Hz. One of the best methods in practice to measure such frequencies is to count the number of cycles over a known finite interval of time, according to Dr. Bilger (the thesis adviser). Thus, in effect the ringlaser gives the time-integrated value of S, not the instantaneous value of S. Equation (I.E.5) also implies this, for if T was zero (which is required for the ringlaser to give the instantaneous value of S), then Ω_s would be infinite and so the ringlaser could not measure anything.

However, it could be possible to develop ringlasers that are able to measure down to 10^{-15} rad s^{-1} even for, say, T = 100 s (cf. the last paragraph of Section IV.A). In contrast, gyroscopes at present would require observation times of more than a few months to achieve such sensitivities (H1) (E3). As far as measurement of a $\vec{\Omega}_D$ (see Section II.E) which is essentially constant over any given interval of time less than 100 s but which varies significantly over any given interval of, say, a few months and whose magnitude is not much greater than 10^{-15} rad s^{-1} at any time is concerned, a ringlaser such as those putative ringlasers above can be regarded as an instrument that gives the instantaneous value of S, in comparison with any of the present-day gyroscopes.

[2]See Chapter 40 in Misner, Thorne, and Wheeler's book (M3) for some examples in which results from PPN experiments are used to rule out some theories.

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APPENDIX

THE PROPER FRAME OF AN ACCELERATED OBSERVER

This summarizes Section 13.6 of the book by Misner, Thorne, and Wheeler (M3).

Let P denote the accelerated observer's world line, parametrized by τ, the proper time of the observer; his four-velocity is $u^\mu \equiv dx^\mu/cd\tau$, and his four-acceleration is $a^\mu/c^2 = u^\mu{}_{;\beta}u^\beta$ ($a^\mu a_\mu$ = the magnitude of the acceleration as measured by the observer's accelerometers in units of m s^{-2}). Let the observer carry a tetrad of basis vectors, $(e_{\hat{\alpha}})$, with $(e_{\hat{0}})^\mu \equiv u^\mu$ (the index within parentheses only denotes which vector, not which component; the hats [^] signify that the quantity is expressed in the proper frame). The vectors are orthonormal:

$$g_{\mu\nu}(e_{\hat{\alpha}})^\mu (e_{\hat{\beta}})^\nu = \eta_{\alpha\beta}. \qquad \text{(App. 1)}$$

Further, they obey the transport law given by

$$(e_{\hat{\alpha}})^\mu{}_{;\nu}u^\nu = -\Omega^{\mu\xi}(e_{\hat{\alpha}})_\xi, \qquad \text{(App. 2)}$$

in which

$$\Omega^{\mu\nu} = -\Omega^{\nu\mu} = \frac{a^\mu u^\nu}{c^2} - \frac{a^\nu u^\mu}{c^2} + u_\alpha \frac{\omega_\beta}{c}\varepsilon^{\alpha\beta\mu\nu}, \qquad \text{(App. 3)}$$

where ω^β is the angular velocity of the spatial basis vectors relative to inertial-guidance gyroscopes.

The coordinate system of the proper frame is constructed as follows: Pass a geodesic that is orthogonal to P at a point, P, to an event near

P, E. The proper length of the geodesic from P to E is σ, and the unit vector tangent to the geodesic at P is n^μ. See Figure 9. Let $n^1 = n^\mu (e_{\hat{1}})_\mu$, $n^2 = n^\mu(e_{\hat{2}})_\mu$, and $n^3 = n^\mu(e_{\hat{3}})_\mu$. The coordinates of E are then $x^{\hat{0}} = c\tau$, $x^{\hat{1}} = \sigma n^1$, $x^{\hat{2}} = \sigma n^2$, and $x^{\hat{3}} = \sigma n^3$.

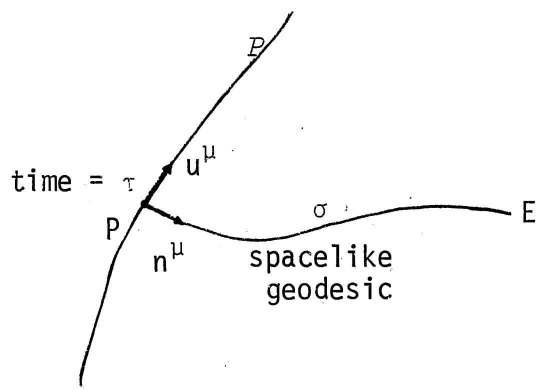

Figure 9. Diagram for the Proper Frame

The proper frame metric at E is

$$ds^2 = -(1 + 2\,\frac{a_{\hat{j}}\, x^{\hat{j}}}{c^2})\,(dx^{\hat{0}})^2 - 2([j\ k\ n]\, x^{\hat{k}}\, \frac{\omega^{\hat{n}}}{c})\, dx^{\hat{0}}\, dx^{\hat{j}}$$

$$+ \delta_{jk} dx^{\hat{j}} dx^{\hat{k}} + O(|x^{\hat{j}}|^2)\, dx^{\hat{\alpha}} dx^{\hat{\beta}}. \qquad \text{(App. 4)}$$

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