# A MULTI-PREDICTIVE MEASURE TO PREDICT SUCCESS <br> AT TWO LEVELS IN FRESHMAN COLLEGE <br> MATHEMATICS 

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## Thesis Approved:



PREFACE

This dissertation is concerned with the development of certain regression equations based upon use of five variables to predict success in two freshmen mathematics courses at Southern University, Bat on Rouge, Louisiana. The five variables are: $X_{1}$, scores on the Cooperative English Test; $X_{2}$, scores on the Cooperative Mathematics Test form $X ; X_{3}$, scores on the Nelson-Denny Reading Test; $X_{4}$, scores on the American Council on Education Psychological Examination; and $X_{5}$, weighted high school mathematics average.

The problem of placing entering freshmen students in the proper sequence of mathematics courses is of central importance to college officials responsible for placement of freshmen students. The regression equations developed in this study should give some assistance in deciding which of two levels a student can be expected to achieve a desirable measure of success.

Two kinds of equations were used in this study, the simple linear regressions which involve one of the single predictors $X_{1}$, $X_{2}, X_{3}, X_{4}, X_{5}$ to predict grade point average in the freshmen courses and the multiple linear regression equations which consist of a combination of the single predictors formed, by step-wise procedures, to best predict grade point average in one of the courses. The two courses are treated separately in the study, therefore, the equations developed are for use only in the course whose study group was used to develop them.

I would like to take this opportunity to express my appreciation for the assistance and guidance given me by the following members of my committee: Dr. Vernon Troxel, for initial assitance rendered in the development of the proposal for this dissertation; Dr. Milton Berg, for his counsel and his assistance in developing the weighted high school mathematics average; Dr. John Hampton, who was always available for counsel and assisted through suggestions and criticisms to the development of ideas expressed throughout this dissertation; and Dr . Ware Marsden, for his continual interest and assistance.

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## INTRODUCTION

The number of students enrolled in college mathematics courses is increasing. This follows the general trend of the upward surge in college enrollments. In 1961, the United States Office of Education listed the total enrollment in American Colleges at 3,891,230 and one year later, the enrollment was listed at $4,206,672$ students, an increase of eight and one-tenth percent. The Office of Education has projected that enrollments for 1970 will exceed $6,000,000$ students. These figures are probably conservative, but are sufficient for showing a need of wise college planning to meet the demands of the increasing number of students.

The shortage of qualified college mathematics teachers and the strain upon already crowded campuses have made the failure of entering students in mathematics a great problem because they must often be retaught. The frustrations of the failing students, the dissatisfied instructors who have to reteach them, and the postponed entry of these students into a labor force, already short in its supply of college trained personnel, are great prices to be paid in terms of the morale and manpower of the community which the college serves. At the completion of high school less than $15 \%$ of all graduating students are still enrolled in courses in mathematics. However, many colleges have found it necessary to have requi rements that force
these mathematics drop-outs to return to the mathematics classroom. These requirements are usually of two kinds: courses that are prerequisite for the student's major-field, and state or university requirements of a certain minimum number of college credits in mathematics. These developments have forced some colleges to provide sequences of courses that are designed for students with varying abilities and backgrounds in mathematics.

A check of college catalogs shows that a large number of college mathematics department officials have tried to face the problems inherent in such a diverse entering student enrollment. This diversity has made it necessary for college officials to provide levels of placement for students of differing backgrounds. Courses at the various levels created sometimes vary only in rate and/or amount of coverage of subject matter content, depth of coverage of subjects matter content, or beginning and ending points in the sequence of subject matter. Thus, a great deal of over-lap is found in the various courses designed as beginning points in college mathematics.

A check of college catalogs also shows that a one year study of mathematics is often a part of the general education requirements of colleges and state boards of education. The student who enrolls in the most elementary courses may satisfy prerequisites for more advanced courses. However, it is often the desire of departments, including the mathematics department, to have students enroll in the highest level course that they can successfully pass. Many departments encourage the above by allowing no credit toward graduation for courses below a certain course level. Mathematics 110, Elementary Algebra, for example, is the lowest level course offered at Southern University,

Baron Rouge, Louisiana. College Algebra and Trigonometry, Mathematics 160, also serves as a beginning point for entering freshmen.

## Nature of the Problem

A review of the literature reveals that many studies have been completed in which instruments were used for prediction of success in colleges. The authors of some of these studies used single predictors of success, others used multi-predictors. Many college placement officials have found it necessary to research better ways of determining at which level to place the ever increasing numbers of entering freshmen who must be taught by the mathematics staff. The solutions proposed have usually taken into consideration the pertinent characteristics of the student body that is indigenous to that particular college.

Some students placed at certain levels find the work either too elementary to be challenging or too difficult to be successfully passed. The latter of these conditions is serious a; is the former, especially if other levels of placement are available. lt is also true that very often the discovery of the above conrlitions is made too late for registration change. This study is des gned to determine, from the freshmen test battery and other transcript infcrmation, criteria which will assist in predicting the surress of enrering coliege freshmen at the two most frequently ussa placement levels in mathematics at Southern University.

[^0]Description of Entering Freshmen Courses at Southern University

The following is a description of the courses for which the predictions will be made.

Mathematics $\underline{110}$ is an elementary algebra course consisting of such topics as: sets, open-sentences, algebraic operations, equations, identities, elementary inequalities, exponents and radicals, and quadratic equations. (3 hours credit.)

Mathematics 160 is a combined course in college algebra and trigonometry. Some of the topics covered are: exponents and radicals, algebraic polynomials, quadratic equations, systems of quadratics, inequalities, theory of equations, determinants and linear systems of equations and the usual elementary trigonometry through the law of tangents. (5 hours credit)

Statement of the Problem

The proposed study will be concerned with freshmen college students who enroll in freshmen mathematics courses at Southern University, Baton Rouge, Louisiana during the fall semesters of 1965, 1966, and 1967. The problem of this study is: Can some predictors from the freshman test battery, and a special average called weighted high school mathematics average serve as a multi-predictive measure of the success of freshman college students at two levels of placement in the Freshman Mathematics Program? The test battery is composed of the American Council on Education Psychological Examination, 1947 edition; the Ne1son and Denny Reading Test, the Cooperative Mathematics Test, Form X, and the Cooperative English

Test, Form 1-C. It is desired to be shown that the prediction made on the basis of the use of these predictors to form a multi-predictive function will give a strong basis upon which freshmen mathematics placement can be made.

## Hypothesis

A review of the literature, to be given later, leads one to believe that there are three effective predictors of college success. These predictors which are considered to provide most efficient information on college success include an intelligence test, an achievement test and the student's high school average.

As a result of experience in teaching during the last twelve years at Southern University, this writer suggests that, when college placement is considered, knowledge of the level of high school courses and whether or not they are college preparatory courses will be most effective as an aid in predicting success of students. It is, therefore, believed that a weighted high school mathematics average will make a greater contribution to the prediction functions than a consideration of any other single predictor.

The central hypothesis of the present study is that a weighted high school mathematics average combined with other predictors will provide a very effective source of information relative to college success. The survey of the literature suggests that the American Council Examination known as A.C.E. is an appropriate test of intelligence and the Cooperative Mathematics Test, Form $X$, is an appropriate achievement test.

The literature, as well as personal experience, recommends the consideration of other predictors. In the following chapter, as a resultant of surveying the literature additional hypotheses will be developed.

## Operational Definitions

In order that this study be understood and the reader be at all times aware of the framework in which data are being given or interpreted, the following terms will be used as defined throughout this study:

i. N.D.T.: The Neison-Denny Reading Test.
j. Coop. Math. Test: The Cooperative Mathematics Test, Form X
k. Coop. English Test: The Cooperative English Test, Form 1-C.

1. G.P.A.: Grade point average.
m. $\widehat{Y}_{1}$ : Predicted grade point average in Mathematics 110 , - Elementary Algebra.
n. $\widehat{Y}_{2}:$ Predicted grade point average in Mathematics 160 , $\xrightarrow{-}$ College Algebra and Trigonometry.
o. $X_{1}$ : Cooperative English Test Form 1-C, "raw score".
p. $X_{2}$ : Cooperative Mathematics Test "raw score".
q. $X_{3}:$ N.D.T. "raw score".
r. $X_{4}$ : A.C.E. "raw score".
s. $X_{5}$ : Weighted High School Mathematics Average.
t. $Y_{2}:$ Actual grade point average Mathematics 160.
u. $Y_{1}$ : Actual grade point average Mathematics 110 .

## Theoretical Background of the Study

The review of the literature related to the phenomena of predicting college success leads one to believe that it is possible, through use of factors taken from the student's cumulative record, to determine the probability of his college success in mathematics. Researchers in studies based on a single predictor have not obtained as high a coefficient of correlation with college success as those who based their studies upon the use of several predictors.

Although each of two tests, considered separately, might have a low or a moderate correlation with a criterion, the two scores will generally correlate higher or quite significantly with the criterion when treated as a component. This is the case because the two tests in combination have more elements or factors in common with
the criterion than does either test in itself. ${ }^{2}$
The review of studies related to the predictors used in this study leads one to believe that they do correlate to some degree with college scores in mathematics. It is, therefore, assumed they are adequate to predict with some degree of accuracy the success of students in freshman mathematics.

Since predictions have been reported in the literature of both college success and success in a specific subject, it can be safely assumed that there are factors associated with the psychological or achievement backgrounds which, to a large measure, will determine the success of individuals in college. Each of the courses dealt with in this study has been successfully passed by entering freshmen with average grades ranging from $\mathrm{C}-\mathrm{A}$. There must be some student related factors that account for the success differences.

It is further assumed that factors of the high school record such as mathematics average and number of high school courses completed are factors which influence the level of college placement. This is partially true because of the prerequisites necessary to begin performance of a certain task at a higher level. Exposure to as well as mastery of certain concepts and skills are necessary to begin studies at an advanced leve1. Not all high school courses will contain these skills to an equal extent. For example, surveys have shown that general mathematics in most high schools is not intended to serve as background for advanced work in mathematics.
${ }^{2}$ Frank S. Freeman, Theory and Practice of Psychological Testing (New York, 1962), p. 106.

No program in mathematics for general education has as yet been designed for the senior high school. There have been a few efforts in preparing texts which might provide content for a remedial program of sorts at the eleventh or twelfth grade level. The Secondary School Curriculum Committee takes the position that it is desirable for all graduates from the high school to have attained at least a certain minimum degree of mathematical competency and recommends that some means should be provided to determine whether or not prospective graduates have done so. ${ }^{3}$

These general skills referred to will not usually suffice to make success at an advanced level of placement in mathematics possible. College mathematics usually requires special training in certain courses.

College preparatory mathematics should include topics selected from algebra, geometry (demonstrative and coordinate), and trigonometry - all broadly interpreted. The point of view should be in harmony with contemporary mathematical thought; emphasis should be placed upon basic concepts and skills and upon the principles of deductive reasoning regardless of the branch of mathematics from which the topic is chosen. Courses designed for other purposes (e.g., consumer mathematics, business mathematics, shop mathematics) are not acceptable.

The opinion of mathematics experts seems to support the belief that all of these courses cannot have equal weight in determining college success in mathematics courses at the level of Mathematics 160 at Southern University. In this study, courses in algebra, geometry, trigonometry, and advanced hígh school mathematics have been given a heavier weight than those of general mathematics, business mathematics, consumer mathematics and shop mathematics. Those courses which are a part of the regular college preparatory sequence are
${ }^{3}$ Charles $H$. Butler and $F$. Lynwood Wren, The Teaching of Secondary Mathematics (New York, 1960), pp. 47-48.
${ }^{4}$ Charles H. Butler and F. Lynwood Wren, The Teaching of Secondary Mathematics (New York, 1960), p. 19-20.
weighted heavier.
Further, the prediction function used for each of these courses will be confined to those particular variables found to adequately predict. The functions may involve different variables for different courses.

Theoretical Assumptions and Expectations of the Study

The assumptions of this study are that the tests chosen and high school records will form a valid picture of students' potential, and that teachers' grades are consistent and reliable enough to be predicted by a valid measure. Past research supports these assumptions.

An expectation of the study is that one of $X_{1}, X_{2}, X_{3}, X_{4}$, or $X_{5}$, the independent variables or a combination of these variables will upon validation predict to an acceptable level of significance the success of students enrolled in Mathematics 110 or Mathematics 160. A further expectation is that this study will make possible the development of a procedure for placing students at the proper level for success and advising those whose probability of success is low of the dangers present.

This study has great value for three sets of people: the placement officials for mathematics and related areas, the classroom teacher of mathematics, and the potential student in one of the two mathematics courses at the college level. The placement officials will have a guide, which is better than guess work, in advising the student in which courses he should enroll and his probability of success in them. The teacher is likely to receive students who may be
more effectively taught. Teachers will also be more aware of their weaknesses in that certain students who ought to be successful may not be experiencing success and thus, the teacher will be aware that he ought to search for reasons for the lack of it. The person to benefit most will be the student. He can feel at ease knowing that he is where he should be, or at least, he knows the work necessary for success at the level of his placement.
Independent Variables

1. Students' scores on the American Council on Education Psychological Examinations.
2. Students' scores on the NelsonmDenny Reading Test; Vocabulary-Comprehensive-Rates, Form A and B.
3. Students' scores on the Cooperative Mathematics Pre-Test for College Students, Form $\bar{X}$.
4. Students' scores on the Cooperative English Tests, Form 1-C.
5. Weighted High School Mathematics Average of the entering college freshman.

## Dependent Variables

1. Predicted grade point averages in Mathematics 110 , Elementary Algebra.
2. Predicted grade point averages in Mathematics 160 , College Algebra and Trigonometry,

> Limitations of the Study
Although this study is being undertaken for the purpose of aiding in the placement of freshman mathematics students on two levels, it would be impossible to devise a scheme which will function in all individual cases. John G. Darley, writing on the functions of measurement in counseling, makes the following comment:

> By appropriate statistical treatment, the contribution of each separate predictor can be maximized and weighted into a multiple regression equation that gives the best prediction of the criterion measure. This is essentially an actuarial procedure by which the experimenter hopes to improve, but cannot make perfect, his selection for success in the criterion task are, in addition, factors of maturity, motivation, emotional stability, financial support, and personal adjustment, no one of which is ordinarily itemized in the regression equation and any one of which may determine success of failure of the individual student. Thus the counselor finds himself ishading the actuarial prediction one way or the other, depending upon his assessment of the import of these other factors.

In this study no attempt will be made to make application of any findings to students other than those similar to the population from which the data for this study were obtained.

It is also recognized that a large number of studies have been conducted on other factors which should be considered in college success. Factors such as interest, perservance, age, aptitude, emotional factors, attitudes, motivation and differences in the quality of the high schools attended, will not be considered in this study. It is realized, however, they are important and may account for the success or failure of a student. The predictors in this study are all a part of the battery of tests given to entering freshmen at Southern University or are factors of the students' high school records.

[^1]
## CHAPTER II

## REVIEW OF RELATED LITERATURE

College success has been studied from many viewpoints. Studies have been reviewed which have made predictions of college success in all subjects at more than one institution; predictions of college success in all subjects at a single institution; and predictions of college success in a single subject or field, usually at a particular institution. The earliest and probably the most basic predictors of college success were found among the components of the pre-college training of college students. Garrett made it known that the early work toward uniformity of college admissions and the standardization of high school credits or units were procedures aimed at improving the general college population and thus raising the probability of success of college students. No study to this date has established a perfect predictor. The literature reviewed in this section will be largely concerned with predictors related to those with which this study is concerned. The review will be sectioned using the selected predictors.

[^2]Sone studies have tried to use high school average as a predictor of success in any chosen college. Clem ${ }^{2}$ reported in 1922 that the relationship of high school grades to college success was not uniform but was variable. He found the coefficients of correlation to be: Carnegie Institute . 29, Columbia University . 45, Corne1] University .47, and Ohio State University . 38. Cronbach ${ }^{3}$ reported in 1949 that multiple correlations of batteries used to predict an overall index of college performance are usually between . 60 and .70 . This testing was felt to be superior to high school grades as a predictor, Horst ${ }^{4}$, using a technique of isolating students' best abilities in a 1959 study conducted at the University of Washington, reported correlations as high as $r=.89$ and some as low as $r=.13$ between the predictors and college grades. Tribilcock ${ }^{5}$ studied the reconds of 651 high school graduates who had enrolled in a large number of colleges and universities. He reported as result of thos 1938 study evidence which made him conclude that colleges which take students without iegards to high school records perform, in many cases, a valuable service. He felt that while it is wasteful and undesirable to heve the unfit in coliege,
${ }^{2}$ O. M. Clem, Latin Prognosis: A Study of the Detained Factors of the Detailed Factors of Individual Pupils, Contributions to Education, No, $\overline{144 \text { (New York, 1924), p. } 36 . ~}$
${ }^{3}$ Lee J. Cronbach, Essentials of Psychological Testing (New York, 1949), pp. 66-68.
${ }^{4}$ Paul Horst, "Differential Prediction in College Admissions," College Board Review, XXXIII (1957), pp. 19-23.
${ }^{5}$ W. E. Tribilcock, 'Many of the 'Lowest Third' of Our Graduates Are College Material," Clearing House, XII (May, 1938), pp. 544-546.
it is also undesirable to keep the fit out of college. He stated that for many students there is no adequate test of fitness except the actual attempt at college work.

Garrett ${ }^{6}$ reported, in 1944, on 200 graduates of Warren G. Harding High School in Ohio enrolled in fifty-two different colleges. He found a coefficient of correlation of .67 between high school grades and first semester averages.

Boon ${ }^{7}$, in his study, recommended two variables as having significance in the prediction of grades of freshmen engineering students. These two predictors were high school grades and the SAT total score. Dresse $11^{8}$, in a comparative study of fifteen large Michigan high schools, found that grades represented a wide variation of achievement in different schools even though the fifteen were a fairly homogeneous group of schools. He suggested that knowledge of specific differences in high schools could help in predicting college grades. Emme ${ }^{9}$ reviewed forty-four studies dealing with prediction of college success in 1942. He discussed seven criteria for predicting it and concluded that rank in high school class or high school performance seemed to

[^3]be the best single criterion for predicting college success.
Based upon the above studies it seems that if uniform standards of high schools and colleges were available it would be desirable to try to predict college success in general; it is apparent, however, that this is not the case. A large number of studies have been made concerning the prediction of success at a single institution. These studies take advantage of the fact that at one institution the grading and academic expectations are usually more uniform. Buckton and Dappelt ${ }^{10}$ in a 1950 study of a test battery and high school averages at Brooklyn College, found a correlation, $\mathrm{r}=.41$, for the test battery and one of $r=.63$ for high school averages. Although Treumann ${ }^{11}$, in a 1949 study of success of engineering students found that aptitude tests were the best predictors, she reported that they were very closely followed by high school percentile rank, the second best predictor.

Laughton ${ }^{12}$, in a 1961 study at Pennsylvania State University, reported that the high school index was superior for prediction of first semester college grade point averages at Pennsylvania State University.
${ }^{10}$ LaVerne Buckton and Jerome E. Dappelt, "The Use of Selective Tests at Bradley College," Occupations, XXVIII (March, 1950), pp. 35760.
${ }^{11}$ Mildred J. Treumann and Ben A. Sullivan, "Use of the Engineering and Physical Science Aptitude Test as a Predictor of Academic Achievement of Freshman Engineering Students," Journal of Education Research, XLIII (October, 1949), pp. 129-37.
${ }^{12}$ James W. Laughton, "College First Semester Academics Achievement as Related to Characteristics of a High School Graduating Class," (unpub. doctoral dissertation, Pennsylvania State University, 1961), p. 116.

Brim ${ }^{13}$ wrote in a 1961 study at the University of Illinois that intrahigh school variability is a better predictor than either high school performance or academic aptitude. Engin ${ }^{14}$ studied 380 selected students who enrolled at the University of South Dakota in the fall of 1957. His finding was that the best predictor of college success was high school performance.

Flora ${ }^{15}$ in a study involving long-range prediction and first year college achievement found that junior high school average was the best predictor found in the early records. Other researchers have been concerned with many phases of high school scholarships. Among these phases are high school percentile rank, high school decile rank, the pattern of high school subjects and the number of high school subjects taken.

Gebhardt ${ }^{16}$ reported in 1922 that there was no relation between number of high school credits or units and college scholarship. He studied the relationships of mathematics among the variables. Bolenbaugh

[^4]and Proctor ${ }^{17}$, in 1926, stated that high school averages for students with less than three vocational subjects correlated only .28 with their college average, while those with three units or more of vocational subjects had a correlation coefficient of .49 between their high school and college averages. Douglass ${ }^{18}$ studied the relation of the number of unit credits in various high school subjects to average college marks while holding intelligence and industry constant. He reported that all coefficients were practically zero except that for foreign language which was .17. Norton ${ }^{19}$ used both teacher and peer rating in high school as a predictor of success. He found teachers' rating to be negatively associated and peer ratings for boys were more closely associated with grades than were aptitudes.

The foregoing studies all deal with predictors based upon some aspect of students' high school records. It seems that in each case, the correlation coefficients giving the degree of relationship between the college mathematics grade point average and the predicted grade point average have been less than desirable, though usually significant. The opinion held by this writer is that the grade point average in the related high school subject as well as whether or not it is college
${ }^{17}$ Lawrence Bolengaugh and W. M. Proctor, "Relation of the Subjects Taken in High School to Success in College," Journal of Educational Research, XV (February, 1927), pp. 87-92.
${ }^{18}$ Harl R. Douglass, "The Relation of High School Preparation and Certain Other Factors to Academic Success at the University of Oregon," University of Oregon Publication, Education Series III (September, 1931), p. 61.
${ }^{19}$ Daniel P. Nortion, "The Relationship of Study Habits and Other Measures of Achievement in Ninth Grade General Science," Journal of Experimental Education, XXVII (1959), pp. 211-217.
prepatory should be given consideration. It is, therefore, proposed in this study that a weighted high school mathematics average be used as the high school average in the desire for a high correlation between the predicted G.P.A. and the actual G.P.A. of the students.

The A.C.E. as a Predictor of College Success

The most popular factor studied in its relation to college success has been intelligence. The A.C.E. has been widely used in prediction of college success. Harston ${ }^{20}$ in a study at Oberlin College in 1928, observed that the A.C.E. predicted college grades of women correlated .50 with the actual grades received while the predicted grades for men correlated .53. In 1929, Drake ${ }^{21}$ found a correlation of $\mathrm{r}=.51$ between the A.C.E. score and college grades at Adelphi Women's College. Gerberich ${ }^{22}$, in a 1930 study of 1,000 high school seniors, reported that the A.C.E. scores correlated at $\mathrm{r}=.58$ with college grades.

In 1931, Nelson ${ }^{23}$ in a study conducted at Iowa State Teachers College reported that the A.C.E. correlated .67 with college grades.
${ }^{20}$ L. D. Harston, "The Most Valid Combination of Twenty-Three Tests for Predicting Freshman Scholarship at Oberlin College," Oberlin College Association Bulletin, (Columbus, Ohio, 1928), p. 63.
${ }^{21}$ C. A. Drake, A Study of an Interest Test and an Affectivity Test in Forecasting Freshman Success in College (Teachers College Contribution to Education, No. 504 [New York, 1931]), p. 60.
${ }^{22}$ J. B. Gerberich, A Personnel Survey of 1000 Iowa High School Seniors, (Studies in Education, No. 3 [Iowa City, 1929-30]), pp. 1-62.
${ }^{23}$ M. J. Nelson, "Study in the Value of Entrance Requirements for Iowa State Teachers College," School and Society, XXXVII (February, 1933), pp. 262-264.

Flemming ${ }^{24}$, in a study conducted over a two year period at seven colleges, found a correlation coefficient of . 56 between women's grades and A.C.E. scores and one of . 46 between men's grades and the A.C.E. A 1939 study by Dubois ${ }^{25}$ reported a correlation coefficient of .44 at New Mexico University. Weber ${ }^{26}$, in a study at Wells College in 1944, reported a correlation coefficient of .45, while Smith ${ }^{27}$ at Fresno State, found a correlation coefficient of . 42. Both of these studies used the A.C.E. as the predictor. Sege1 ${ }^{28}$, in a 1934 study of prediction, reported that he found the mean coefficient of correlation between general scholarship and achievement on A.C.E. was .39 with a low of 27 . Prediction of success in mathematics ranged from . 59 to 28 .

Brown ${ }^{29}$ noted, in his 1950 study, that the A.C.E. serves as a differential predictor and that there is no significant difference

[^5]in using all parts or the part in arithmetic reasoning alone to predict college grades. Hoerres and odea ${ }^{30}$, in a 1959 study, reported very low correlations. They reported a correlation of . 29 but it was significant at the .01 level. Henderson and Melveg. ${ }^{31}$ reported in a 1959 study a correlation coefficient of .58 between the A.C.E. and college grades. Goodstein ${ }^{32}$ noted in his 1963 article that aptitude and achievement accounted for approximately thirty-five percent of the total variance of the criterion of college achievement as represented by grade point average.

The A.C.E., as established by the foregoing review of literature, seems to aid in the prediction of success in college. It is not the most popular predictor in use in colleges today, but appears to be quite adequate to fulfill the goal herein intended. It is the test which is presently in use as a part of the Freshmen Test Battery at Southern University. In as much as the prevailing belief is that an intelligence test aids greatly in the prediction of college success, this test has been included as one of the predictors investigated. It is believed that the A.C.E. will be a significant predictor of freshman college student success in mathematics.

[^6]
## The Nelson-Denny Reading Test as a Predictor

From a review of the literature, it appears that the NelsonDenny Reading Test has not been used as widely in predictive studies as some other tests. The limited use of this test as a predictor may be partially accounted for by the fact that many researchers have used linguistics rather than measured reading skills. Nelson ${ }^{33}$, in a 1930 study conducted at Iowa State Teachers College, using 757 students evaluated for a period of one year, reported that he obtained a corre1ation coefficient between the N.D.T. and college grades of . 45 . In a study one year later, using 157 students, he obtained a coefficient of .67. Davis ${ }^{34}$, in a study at the University of Arizona reported in 1938 that the Nelson-Denny Reading Test, along with an English test and the Carnegie Intelligence Test could serve as worthwhile predictors.

Roy ${ }^{35}$ reported in a 1939 study that the Nelson-Denny Test and the Cooperative Survey Test in Mathematics gave predictions of which the coefficient of correlation was .39 but after he applied a correction formula he obtained a coefficient of correlation of .44. Lawrence ${ }^{36}$, in a 1939 study at Louisiana State University, used the

[^7]Nelson-Denny Test, the A.C.E., $^{\text {N }}$, an English test, and rank in high school class. He suggested that each of these could significantly predict college success. Smith ${ }^{37}$, in a 1959 study of 19 variables affecting college achievements, noted that the Nelson-Denny Test was the highest single contributor to verbal ability but it was low on scientific creativity versus aesthetic creativity, both of which are necessary for modern mathematics.

Lott ${ }^{38}$ in a 1938 study at Louisiana State University reported that out of five predictors used, the $A_{0} C_{0} E$. was the first and the Nelson-Denny the second best of the predictors of success at Louisiana State University. This study helped in placement of freshmen students at Louisiana State University.

There may be questions as to why the N.D.T. was chosen as a predictor in this study. It is a test which includes reading skills which are very important in the interpretation and comprehension of modern problems. This author considers that with the present nature of modern textbooks and their great emphasis upon precise definitions and concepts, the student who reads well has a definite advantage in achieving success in his college mathematics courses. This test is a part of the Freshmen Test Battery at Southern University. It should aid greatly in determining the probability of success by entering freshmen students.

[^8]The Cooperative General Mathematics Test has been used many times as a part of college test batteries to screen and place college freshmen. It has also been used as a predictor of college success. Moor ${ }^{39}$, in a 1949 summary of efforts to predict success in engineering schools, reported that high school mathematics was one of the best predictors. He has also compared the correlation of grades in some high schools with the entrance mathematics test and concluded that mathematics was the best predictor of success in the survey of studies that he conducted.

Brownley and Carter ${ }^{40}$, in a 1950 study at the University of Illinois, found that the Cooperative General Achievement Test correlated with college mathematics grades, $\mathrm{r}=.35$, while the correlation with rank in high school class was . 40. Seigle ${ }^{41}$, in a study to predict success in college mathematics at Washburn University in 1.954 , reported a mathematics entrance test was the best predictor while high school grade point average was the second best predictor if this prediction were made before any mathematics courses were taken.

[^9]Allgood ${ }^{42}$, in a study of academic success at Virginia State College found that of eight variables studied, high school rank was the highest predictor. One of his post-admission variables was the Cooperative Mathematics Pre-test for College Students. Barnette ${ }^{43}$, in his 1967 study at North Texas State University, used 214 students of the 1964 class. He reported that of two test batteries, one of which was largely composed of Cooperative Tests and the other test from the American College Testing Program, no significant difference was found in the ability to predict academic achievement.

The value of an achievement test in the subject matter area in which the prediction was to be made has been cited many times in the literature. The background of the student is expected to be closely related to his standing on this test; however, his ability to transfer information to new settings not directly related to classroom performance might be measured here. It is believed that the particular test chosen is closely related to the purposes for which the study is intended as the two courses have high algebraic content and the Cooperative Mathematics Test is basically an algebra test. It is also believed that the Cooperative Mathematics Test will aid significantly in determining students whose probability of success is high at either of the two levels under discussion.
${ }^{42}$ E. V. Allgood, "Prediction of Academic Success at Virginia State College," (unpub, doctoral dissertation, Pennsylvania State University, 1964), p. 124.
${ }^{43}$ T. M. Barnette, "The Predictive Validities as Measured by Multiple Correlation of Two Batteries Using Academic Achievement As Criterion," (unpub. doctoral dissertation, North Texas State University, 1967), p. 108.

Since English is the modern language in which all of the mathematics teaching at Southern University is conducted, the ability to use the language correctly and to understand it when it is used correctly should be a decided advantage. The Cooperative English Test is herein used to measure that ability. It is believed, and the review of the literature supports the belief, that the scores on this test will show as a significant predictor.

## Studies Using Multi~Predictors

Many studies have investigated college success using multipredictors. These studied, in general, report higher coefficients of correlation than those of the single predictors. Harston ${ }^{44}$ found a high three-variable coefficient of correlation by combining high school marks, Ohio State University Psychological Test and a study performance test to get .75. Douglass and Lovegren ${ }^{45}$ combined the following variables: high school percentile ranks, Wesley College Test of Social Terms, American Council Test, and Minnesota College Aptitude Test percentile rank. They found a high of $r=.709$. They obtained this high with four variables but obtained $r=0707$ using three variables. In Douglass' study the diminishing returns obtained when combining more than two or three of the best prediction variables was shown.

[^10][^11]Durflinger ${ }^{46}$, in 1943, after reporting a group of studies made since the year 1934, using multiple coefficients of correlation, noted that multiple coefficients of correlation were rarely higher than . 80 regardless of the variables used. He also reported that an intelligence test, a good achievement test and high school averages used together usually bring the highest multiple correlations. Hanna ${ }^{47}$, in his 1939 study, found that scores on Cooperative Tests in Mathematics and French are better for prediction of college grades than marks in high school. Stone ${ }^{48}$ used more than 20 measures of ability, interest, personality and temperament to report that for male college seniors majoring in the physical sciences, the battery that best predicted academic performance included measures of general intelligence, mechanical interest, morale, stability and activity levels. The addition of personality factors more than doubled the efficiency of prediction using ability measures alone. Wallace ${ }^{49}$, in his 1950 study at the University of Michigan, used A.C.E., Cooperative English, Social Studies Vocabulary, Science Vocabulary, Iowa Foreign Language,

[^12]and Mathematics Placement Test. In this study, he developed predictors for the first semester grades, and the coefficient of correlation was . 554 between the predictors and first semester grades.

According to a review of the literature, it has been observed that multiple correlation coefficients are generally higher than those obtained by the use of a single predictor. The present study intends to review this supposition in terms of the student population under study. The writer has observed seeming inconsistencies in grade point average in the freshmen courses where any single predictor is used. It is believed that peculiar strengths in any of these areas will help the student achieve success. This proposition will be examined in light of the present student population. It is also believed that the multiple regression equations based upon the use of several of these predictor variables will indeed prove to be a better predictor than any singie predictor.

## Summary

The number of predictive studies has been large. These studies have been made using many variables and combinations of variables. The applications made from these studies are important. They must not be misused but usually can be used to improve the probability of predicting the success of the students. Every college should make the maximum use of the test batteries given its entering freshmen for guidance and placement, but no matter how prestigeous the college making the study, any other college applying the resuits must do so with great caution. Wallace warns in closing his study:

Generalization of the present results and conclusions should not be made to institutions other than those of the type represented in these data without investigation to establish its own set of validities so that it may be aware of the meaning of test scores as applied to its curricula and students. 50

In the search of the related literature, it has been discovered that high school average, achievement test, intelligence test, and many other factors rank in the order listed when classified according to coefficients of correlation with college grades. It, therefore, seems that the predictors chosen in the present study could possibly give a high coefficient of correlations with each of the two courses used. The researcher was not able to find any studies using high school average weighted in the form used in the present study.

## Summary of Literary Hypotheses

It was stated in Chapter I that further hypotheses would be developed in Chapter II as a result of the review of the literature. The hypotheses stated below have been listed at the end of each of the sections of the review of the literature related to the particular predictor. They are:

1. The weighted high school average will aid significantly in the prediction of success in freshman college mathematics as measured by teachers' grades.
2. The A.C.E. wiIl aid significantly in the prediction of success in freshman college mathematics as measured by teachers' grades.
3. The N.D.T. will aid significantly in the prediction of success in freshman college mathematics as measured by teachers' grades.

[^13]4. The Cooperative Mathematics Test will aid significantly in the prediction of success in freshman college mathematics as measured by teachers' grades.
5. The Cooperative English Test will aid significantly in the prediction of success in freshman college mathematics as measured by teachers' grades.
6. The multiple-regression found by use of the single predictors $X_{1}, X_{2}, X_{3}, X_{4}, X_{5}$ will significantly predict the freshman college mathematics grade point average of the student population.

Hypotheses To Be Tested

Since the present study is concerned with two distinct populations and includes two separate courses, the hypotheses to be tested must be separately stated in the manner in which they will be tested. They are stated below in the form in which they will be tested. $A_{1}$ and $B_{1}$ are considered the two major hypotheses of this study. Mathematics 160
$A_{1}$ The F-Value for the multiple linear regression does not differ significantly from zero.
$A_{2}$ There is no significant difference between the multiple correlation coefficient obtained by use of $X_{5}$ and the one obtained by use of $X_{1}$.
$A_{3}$ There is no significant difference between the multiple correlation coefficient obtained by use of $X_{5}$ and the one obtained by use of $\mathrm{X}_{2}$ 。
$A_{4}$ There is no significant difference between the multiple correlation coefficient obtained by use of $X_{5}$ and the one obtained by use of $\mathrm{X}_{3}$.
$A_{5}$ There is no significant difference between the multiple correlation coefficient obtained by use of $X_{5}$ and the one obtained by use of $\mathrm{X}_{4}$.
$A_{6}$ The correlation coefficient between the actual student grades and the grades predicted by the multiple-regression equation for the validation group used in this study will not differ significantly from zero.
$A_{7}$ Using the multiple linear regression, the correlation between predicted and actual grades for the validation group does not differ significantly from the correlation between predicted and actual grades for the study group.

Mathematics 110
$B_{1}$ The F-Value for the multiple linear regression does not differ significantly from zero.
$B_{2}$ There is no significant difference between the multiple correlation coefficient obtained by use of $X_{5}$ and the one obtained by use of $X_{1}$ as a predictor.
$B_{3}$ There is no significant difference between the multiple correlation coefficient obtained by use of $X_{5}$ and the one obtained by use of $X_{2}$ as a predictor.
$B_{4}$ There is no significant difference between the multiple correlation coefficient obtained by use of $X_{5}$ and the one obtained by use of $X_{3}$ as a predictor.
$B_{5}$ There is no significant difference between the multiple correlation coefficient obtained by use of $X_{5}$ and the one obtained by use of $X_{4}$ as a predictor。

B6 The correlation coefficient between the actual student grades and the grades predicted by the multiple-regression equation for the validation group used in this study wilil not differ significantly from zero.
$B_{7}$ Using the multiple linear regression, the correlation between predicted and actual grades for the validation group does not differ significantly from the correlation between predicted and actual grades for the study group.

## CHAPTER III

## PROCEDURES

The purpose of this chapter is to acquaint the reader with the general procedures used in this study. These considerations, such as the conditions under which the subjects of the study were chosen, the instruments used in the study, the weighted high school mathematics average and the statistical procedures used to analyze the data, are each explained in this chapter.

Subjects

The subjects used in this study were randomly chosen from the 1965 and 1966 entering freshman classes at Southern University. Of these students, less than twenty per cent were graduates of high schools located in other states. There were 1,491 students enrolled in Mathematics 110 during the 1965 fall semester, Of this number 331 were repeating the course because of previous failures and 95 students either dropped the course or received the grade of incomplete. There were 1,871 students enrolled in Mathematics 110 during the 1966 fall semester. Of this number 466 were repeating the course because of previous failures and 110 students either dropped the course or received the grade of incomplete. The total number of entering freshmen enrolled and completing the course with a grade of $A, B, C$, D, or $F$ during the first semester of the two years was 2,370 . It was
possible to secure the complete test records and high school transcripts of 1,936 of the above students. These 1,936 students composed the population from which the sample for Mathematics 110 was selected. The procedure of selection will be explained later in this chapter.

A total of 101 students were enrolled in Mathematics 160 during the 1965 fall semester. Of this number, five were repeating the course because of previous failures and 13 students either dropped the course or received the grade of incomplete. There were 142 students enrolled in Mathematics 160 during the 1966 fall semester. Of this number 14 were repeating the course because of previous failures and 26 students either dropped the course or received the grade of incomplete. The total number of entering freshmen enrolled and completing the Mathematics 160 course with a grade of A, B, C, D or F during the first semester of the two years was 185 . It was possible to secure complete tests results and high school transcripts of 171 of these students. These 171 students composed the population from which the sample for Mathematics 160 was selected. The procedure of selection will be explained later in this section.

Forty-four sections of Mathematics 110 were taught during the first semester 1965 and 55 sections were taught during the first semester 1966. The average class size for both years was 34 students. There were 17 teachers engaged in teaching at least one section of the course during the 1965 semester and 21 teachers engaged in teaching at least one section during the 1966 semester.

The validation groups for both Mathematics 110 and Mathematics 160 were randomly chosen from the students enrolling in the particular course during the first semester of the 1967-68 school year. First
semester students were chosen in all cases because student populations are felt by the writer to be more representative during the fall semester. A two-year period was used in order to include more teacher, student and high school representation in the study group. The validation group was chosen from a year different from the ones involving the study group in order to more objectively evaluate the predictive quality of the measures derived.

## Selection of Study Group and Validation Group

The selection of the study groups was made by partitioning the populations of each course, as previously discussed, into three cells. These cells were composed of those student achieving different levels of success in the courses. The cells of each course were as follows: (1) students receiving the grade of $A$ or $B$, who were considered to be very successful; (2) students receiving the grade of C who were considered successful and (3) students receiving the grade of $D$ or $F$ who were considered not successful.

From each of these three cells of the Mathematics 110 population, 75 students were randomly chosen and from each cell of the Mathematics 160 population, 25 students were randomly chosen. The students chosen by the methods outlined above constituted the study group. Differences in study group.size is due to differences in class population size and thus a difference in available subjects.

The validation groups were randomly chosen from the entire class enrollments of the 1967 entering freshman class. Forty students compose the validation group for Mathematics 110 and 25 students compose the validation group for Mathematics 160.

Table I below presents the exact grade distribution of the students of the study groups.

> TABLE I
> GRADES OF SUBJECTS OF STUDY GROUPS

| Course |  | Grades |  |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | A | B | C | D | F | TOTALS |  |  |
|  |  |  |  |  |  |  |  |  |
| Mathematics 110 | 19 | 56 | 75 | 48 | 27 | 225 |  |  |
| Mathematics 160 | 8 | 17 | 25 | 8 | 17 | 75 |  |  |
| TOTALS | 27 | 73 | 100 | 56 | 44 | 300 |  |  |

Table II below gives the exact grade distribution of the validation groups chosen from the 1967 student population.

TABLE II
GRADES OF SUBJECTS OF VALIDATION GROUPS

| Course |  | Grades |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | A | B | C | D | F | TOTALS |  |
| Mathematics 110 | 10 | 10 | 7 | 11 | 2 | 40 |  |
| Mathematics 160 | 5 | 7 | 4 | 6 | 3 | 25 |  |
| TOTALS | 15 | 17 | 11 | 17 | 5 | 65 |  |

## Sources of Data

The data secured from tests used in this study were obtained from the Test Bureau at Southern University. The director of testing, Dr. E. E. Johnson, and his staff are responsible for administering the test battery to each entering freshman before registering. Tests used in this study are a part of that battery. High school records and teachers grade sheets were obtained from the Registrar's Office at Southern University. Weighted high school mathematics average and grade point average in the freshman course were computed from these records by methods to be later explained.

The writer is pleased in that the sources of the data were not obtained from a group designed only for the purpose of the study, but was a group extracted from class enrollments and teacher population. No effort was made to distinguish one teacher's grades from another. Freshman classes at Southern University are graded largely on a set of five common examinations and although the writer is aware that teachers will vary in methods of grading, these examinations should add some measure of uniformity in the evaluation of students. Because of the unawareness of both the student and teacher populations that these results would ever be used for such a study, criterion contamination and 'Hawthorne Effect' should be eliminated.

Description of Instruments and Average Used

The independent variables $X_{1}, X_{2}, X_{3}, X_{4}$ were obtained as the raw-scores from tests and $X_{5}$, weighted high school mathematics average explained below, was computed by the writer from the high
school transcripts. The following is a description of the tests and methods used to compute the weighted high school mathematics average to form the predictor variables.

The A.C.E. has been used in a number of predictive studies of which the prediction of a grade in mathematics was an objective. Some of these studies are reported in the review of the literature. The A.C.E. is a good test of intelligence, and although it is not one of the latest available, opinions of it, as surveyed by this writer, are very high. Many colleges still use it for entering freshmen and Southern University is one of them.

In a review of the A.C.E., W. D. Commins, Associate Professor of Psychology, Catholic University of America, Washington, D. C. wrote the following:

This is perhaps the test that one is likely to recommend to anyone who is looking for a "good" intelligence test to give a group of college freshmen ... Although the data
on the norms and the relative ranking of different types of colleges are not available for each yearly edition until well into the school year, the authors always try before hand to make the scores experimentally equivalent. ${ }^{1}$

The fact that the test has been reviewed many times, seemingly each time for the better, is in its favor. Relative to this, Commins wrote:

The later yearly editions have improved in general over the early ones in a number of mechanical features as well as in the dropping of the artificial language test. It would seem, however, that the study of the individual items might be carried further. This might be in the direction of the "mental functions" that are supposedly tested by the items constituting each subtest. The psychologist would like the test material to be homogeneous in this respect and is not always satisfied with
${ }^{1}$ Oscar K. Buros, Third Mental Measurements Yearbook, Highland Parks, N.J.: Gryphon Press $\overline{\text { 1949), }} \overline{\text { pp. 296-297. }}$
the exclusive use of a "factor analysis" approach and the disregarding of some kind of "qualitative" analysis. Thus, some items in the completion test of the present edition seem to plumb one's familiarity with relatively uncommon words, as "gill" and "gobbler", while other items seem aimed more ${ }_{2}$ at an understanding of the object whose name is sought. ${ }^{2}$

Many reports support the use of the A.C.E. as a predictor. Hunter ${ }^{3}$ cautioned about making any comparisons other than for entering college freshmen which is the group used for standardization purposes. The test is divided into two parts, quantitative (arithmetic and spatial) and linguistic. There are three scores available for use: the quantitative score, called the Q-score; the linguistic score, called the L-score and the total score which is the sum of the $Q$ and $L$ scores. Some factors seem to influence the test. $S^{2}$ mith $^{4}$ found the factor of rural versus urban living influences while Barnes ${ }^{5}$ found that two years of college mathematics had no appreciable effect on the Q-score.

The authors of the test, L. L. and T. G. Thurstone report oddeven reliabilities of .95 for the total score and of .87 and .95 for the $Q$ and $L$ scores respectively. The reported validity coefficients range from .30 to .65 with a median of .36 in attempts to measure relatively distinct components of intelligence.
${ }^{2}$ Oscar K. Buros, Third Mental Measurements Yearbook (Highland Parks, 1949), pp. 296-2 $\overline{97}$.
${ }^{3}$ E. C. Hunter, "Changes in Scores of College Students on the A.C.E. Psychological Examination at Yearly Intervals", Journal of Educational Research, XXXVI (1942), pp. 284-291.
${ }^{4}$ M. Smith, "University Student Intelligence and Occupation of Father", American Sociology Review, VII (1942), pp. 764-771.
${ }^{5}$ M. W. Barnes; "Relationships of the Study of Mathematics to Q-scores on the A.C.E. Psychological Examination", School Science Mathematics XLIII (1943), pp. 581-582.

The A.C.E. usually correlates very high with other intelligence tests. For example, $\mathrm{Kohn}^{6}$ reported in 1938 that it correlated at . 69 with the 1916 Binet and Traxler ${ }^{7}$ reported coefficients of .78 and .82 with the Otis S. A. Higher Forms.

Opinions of the A.C.E., with respect to entering college freshmen, are very high, as the foregoing information supports, and the following table taken from information given by Super ${ }^{8}$ supports high correlations with mathematics grades.

TABLE III
RELATIONSHIPS OF A.C.E. PART-SCORE TO OTHER ABILITIES $\mathrm{N}=123$

| A.C.E. | Reading | Mathematics | Names <br> Checks | Number <br> Checks | T | Q | L |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| TOTAL | .66 | .65 | .62 | .26 | .- | .75 | .92 |
| Q | .37 | .56 | .91 | .18 | .75 | - | .87 |
| L | .80 | .56 | .58 | .22 | .92 | .87 | - |

${ }^{6}$ H. A. Kohn, "Achievement and Intelligence Examinations Correlated with Each Other and with Teacher's Rankings," Journal Genetic Psychology LII (1938), pp. 433-437.
${ }^{7}$ A. E. Traxler, "The Correlation Between Two Tests of Academic Aptitude," School and Sociology LXI (1945), pp. 383-384.

8D. E. Super, "The A.C.E. Psychological Examination and Special Abilities," Journal of Psychology IX (1940), pp. 221-226.

The Nelson-Denny Reading Test published by the Houghton-Mifflin Company has enjoyed wide use. Not too many of the studies located included mathematics predictions; however, correlations with well known tests range generally high. Garrett ${ }^{9}$ reported 57 correlations with academic success and gave a range from .10 through .70 with a median of .40 . The test manual ${ }^{10}$ gives a great deal of information on validity and difficulty data. The following table is taken from the test manual.

TABLE IV
VALIDITY AND DIFFICULTY DATA FOR VOCABULARY TEST ITEMS NELSON-DENNY FORMS A AND B

| Form | No. of Items | Validity |  | Difficulty |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | Mean | Range | Mean |  |  |
| Original A | 100 | $-12-67$ | 39.7 | $12-96$ | 57.1 |
| Original B | 100 | $-3-71$ | 38.2 | $15-97$ | 55.5 |
| Revised A | 100 | $31-71$ | 47.5 | $27-96$ | 62.3 |
| Revised B | 100 | $31-75$ | 47.4 | $26-96$ | 62.3 |

$9^{\text {Harley F. Garrett, "A Review and Interpretation of Investigation }}$ of Factors Related to Scholastic Success in Colleges of Arts and Sciences and Teachers Colleges;" Journal of Experimental Education (December, 1949), p. 130.
${ }^{10}$ Manual of Directions for Nelson-Denny Reading Test, (Boston, 1956), p. 16 .

In the above table difficulty indices are approximations of the items-total score correlations obtained by means of the Flanagan Table. The difficulty values for each item were obtained by averaging the per cent passing each item in the upper and lower 27 per cent of the cases used for the item analysis. The standard error of measurements for the total tests are 7.67 for the Form $A$ and 7.84 for Form B.

The standardization group for Form B, obtained in fall of 1955, consisted of 3,205 students of which 3,027 student results were used. These students represented a wide cross section of the American freshman college population. Grade equivalents are listed in the manual; for example, a total score of 46 gives a grade equivalent of 9.7 on Form $A$ and 9.6 on Form B. In general an advance of one point in raw score gives an advance for a grade equivalent of one-tenth.

In the test handbook, studies are cited which report a correlation of .730 with the Cooperative English Test and a correlation of .830 with the A.C.E. John O. Crites, in a review of the Nelson-Denny Reading Test reported in the Journal of Counsel Psychology during the summer of 1963, writes the following:

Unusually complete normative data are given for the test, which was standardized upon large numbers of Ss. Reliabilities for the test ... based upon a carefully conducted study of 110 college students seem to be adequate for both general screening purposes with the total scale and diagnostic work with the subscales ... . With respect to the latter, the validity data on the test, which consist primarily of item analyses indicates that it can be used to identify differential difficulties in vocabulary and comprehension ... . Although the Manual attempts to convey the impression that the Nelson-Denny usually correlates with scholastic achievement in the .60's, the data which are cited are far from conclusive. The correlaplant situation, many items being expressed in terms relevant to an industrial environment.

[^14]The Nelson-Denny Reading Test is considered a good reading test. This is evidently the reason for its inclusion in the test battery at Southern University and certainly the reason it is included among the tests used to give predictive scores in this study.

The Cooperative Mathematics Test, Form $X$ is a test in which a great deal of emphasis is placed upon reasoning rather than routine computation according to the publisher's Handbook, ${ }^{12}$ Forms $X$ and $Y$ are adopted from the experimental Forms $A$ and $B$. Forty items to be completed in 40 minutes, provide a sampling of elementary and intermediate algebra and geometry (limited to mensuration). Norms for entering college freshmen are available.

The Handbook ${ }^{13}$ places a great deal of emphasis upon percentile rank and scaled scores. No emphasis will be given to the above in this section as they are not pertinent to the purposes for which the test will be used in the present study. Dunlap ${ }^{14}$, in a 1955 study, cited reliability coefficients of .90 or higher and correlations with college grades ranging from $r=.30$ to $r=.50$ for appropriate subjects. The standard error of measurements is given as 2.65 . The medium score for college freshmen when given in terms of translated score is 150 , this represents about 18 correct answers in terms of "raw score". The standard deviation is 10 . The Handbook cites reliability coefficients in the middle $.80^{\prime} \mathrm{s}$.

[^15]The courses of the present study are very heavily weighted in algebra and the Cooperative Mathematics Test gives the primary place to algebra. E. P. Starke, Professor of Mathematics, Rutgers University, New Brunswick, New Jersey, made the following observations in a review of the Cooperative Tests.

This test was designed by the Committee on Tests of the Mathematical Association of America 'to furnish a supplementary means of checking on classification in appropriate mathematics courses" ... . The test can be used to eliminate those who are unprepared for college science and mathematics but it will be of little use for predicting success in more advanced work.

In general, the items are carefully worded and unambiguous, although Item 9 of Form X misses its purpose: "If $a / b=3 / 2$ and $b / c=2 / 7$, what does $a / c$ equal?" The correct answer is obtained by equating $a$ and $c$ to 3 and 7 respectively, with no knowledge of operations with fractions. 15

Some of the available information on scores from this test, when used as a predictor, is given in the review of the literature. This test served the intended purpose of including a content test closely related to the area to be predicted in the present study.

The Cooperative English Test measures achievement in two general areas: written expression and reading. The questioned validity of multiple-choice English tests as a substitute for more tedious evaluation procedures based on students' themes has forced the author, in the test manual, to reassure users that evidence suggests that ability to do well on this test is related to ability to write well in "essay" situations. The material in the reading section of the test is well chosen and the sections are varied in content and in

[^16]length. In the Manual of Directions ${ }^{16}$, the author states as one of the purposes of the test, "to establish meaningful and objective standards for admission, placement, promotion, certification and graduation, and for transfer and advanced standing relations with other institutions; and to maintain such standards uniformly from year to year". The above statement certainly associates this test with the purposes of the present study.

An objective view on the technical data of the test was given by Leonard S. Feldt, Professor of Education, State University of Iowa, Iowa City, Iowa, as follows:

Technical Data. The two manuals which accompany these tests provide a wealth of technical data on validity, reliability, scaling, and norming. In addition to information bearing on content validity, the manual includes a summary of the results of about twenty predictive validity studies primarily against grade criteria. All but one of these involve earlier forms of the reading comprehension test. The median coefficient is in the . $40-.45$ range, a value quite consistent with other research in this field. Reliability data are reported for grades 10 and 12 only, a deficiency to be lamented. Since the standard error of measurement plays an important role in the interpretive techniques suggested by the publisher, one might wonder how the standard error values were arrived at for grades 9, 11 and $13 .{ }^{17}$

This test has been used for a number of years at Southern University, and according to Dr. Carl Marshall, English Department Chairman, and Dr. E. E. Johnson, director of testing, has correlated very highly with the English language abilities of the entering freshmen. This test has been reported to correlate highly with the Nelson-Denny

[^17]${ }^{17}$ Oscar K. Buros, Sixth Mental Measurement Yearbook, (Highland Park, 1965), p. 347.

Reading Test, previously cited, and with the A.C.E. in the Super studies also previously cited.

The Weighted high school mathematics average was computed from the students' high school transcript as follows:

General Mathematics I (.5 x G.P.A. in General Mathematics I
General Mathematics II (. 5 x G.P.A. in General Mathematics II for each half unit of credit)

Algebra I
(l x G.P.A. in Algebra I. for each half unit of credit)

Algebra II
(1 x G.P.A. in Algebra II for each half unit of credit)

Trigonometry (l x G.P.A. in Trigonometry for each half unit of credit)

Plane Geometry
(1 x G.P.A. in Plane Geometry for each half unit of credit)

Averages in any other college preparatory courses such as Advanced High School Mathematics, Solid Geometry and Analytic Geometry are computed in the same manner as that of Algebra II. Averages in Business Mathematics, Senior Mathematics, Consumers Mathematics, and Shop Mathematics are computed in the same manner as that for General Mathematics II. The numbers by which the grade point average, for each half unit of each course, are multiplied were arbitrarily chosen in such manner as to give a higher weight to college preparatory courses. The half unit was used in computing the weighted averages of each course in order to make easy consideration of students who enrolled in a course for a single semester. The weighted high school mathematics average for each student was computed by taking the sum of the weighted averages of each course. Every grade appearing in mathematics on the transcript was considered in computing the average for any
particular course. The G.P.A. used for letter grades was $A=4$, $B=3, C=2, D=1, F=0$.

Tests discussed above form the test battery at Southern University and will be available for future use in order to apply the results of this study. The weighted high school mathematics average is the high school average used in this study.

## Procedures

Reported in this section are the steps which were taken in processing the data obtained from tests and high school records for the subjects.

After the data were collected. the next step was processing. A portion of the analysis was a step-wise procedure for multiple regression analysis ${ }^{18}$. In the step-wise procedure, one variable was entered at a time into the regression equation. The potential variance reduction of all remaining variables was considered and the next variable was selected which reduced the variance the most in a single iteration.

This portion of the analysis was written in two parts. The first step was to give the raw sums, means, sums of sequences and crossproducts, and simple correlation coefficients for each pair of variables. The second phase was the step-wise procedure of writing regression equations; selecting for each equation the next independent

[^18]variable which reduced the variance most when used with previously selected variables. This procedure was followed with the data for each of the two courses. For each step of the program, the regression equation was written with standard error of the predicted variable; standard error of the regression coefficients and the F-level of the reduction of variance for the predictive variables entered.

Since the regression equations were given with only the regression coefficients, beta weights for each independent variable were computed by the writer. The beta weights were calculated so that the contribution to the explained variance of each significant variable could be shown in terms of the beta coefficients as well as the simple correlation coefficients. The equation containing the optimum combination of predictor variables determined by $F=0.001$ for variable entry and $F=0.000$ for variable removal was used on the validation group.

The simple correlation coefficient between predicted and actual grades of the validation group was computed, because of the "shrinkage" problems faced in multiple regression. The belief is held that the correlation coefficient of the validation group is a better measure of future success in prediction than the one gotten with the study group ${ }^{19}$. A table of scores were set up using the scores of the study group to predict cut off scores for success in the two courses. This was done to assist future counselors in making rapid decisions as to student placement for success. The validity of these tables was checked by using the validation groups.
${ }^{19}$ Robert M. W. Travers, An Introduction to Educational Research, (New York, 1964), pp. 376-380.

Simple linear regressions were written using each of the five variables $X_{1}, X_{2}, X_{3}, X_{4}, X_{5}$ as predictors of $Y$. This was done to test the effectiveness of a single predictor as compared to the multiple linear regression. The effectiveness of each of these single predictors was tested to see if weighted high school mathematics average was the most effective single predictor. The standard error of estimate and T-value was given for each of the simple linear regression. The F-value attributed to regressions was also developed by the program. This procedure made it possible to test the level of significance of the single predictors. The multiple-regressions were examined to see if weighted high-school average was the variable with the greatest variance reducing potential.

Formulas and specific tests used to test the hypotheses of this study will be given in the Analysis which is presented in Chapter IV.

## CHAPTER IV

## ANALYSIS AND TREATMENT OF DATA

This chapter is divided into three major parts. In Part I information is given pertinent to the testing of hypotheses dealing with the single predictors $X_{1}, X_{2}, X_{3}, X_{4}, X_{5}$. The simple linear regressions are of chief concern in Part I. Part II gives information pertinent to the testing of hypotheses dealing with the multiple linear regression involving $X_{1}, X_{2}, X_{3}, X_{4}, X_{5}$. Part III gives information pertinent to the testing of hypotheses dealing with the validation groups.

Each of the three major parts listed above is further divided into two sections. The first of these sections gives information on the Mathematics 110 study group and the second section gives information on the Mathematics 160 study group. Since two different study groups are treated, it is necessary that information on each study group pertinent to the testing of hypotheses concerning the expectations for the particular course be separately treated.

Part I, section one, gives information pertinent to the testing of hypotheses $B_{2}, B_{3}, B_{4}, B_{5}$ while Part $I$, section two, analyzes data pertinent to hypotheses $A_{2}, A_{3}, A_{4}, A_{5}$. Part II, section one, gives information which is pertinent to the testing of hypotheses involving the multiple linear regression developed to predict grade point average in Mathematics 110 , while Part II, section two, analyzes
data from the multiple linear regression pertinent to testing hypotheses for Mathematics 160. The hypotheses tested in Part II are the major hypotheses $A_{1}$ and $B_{1}$. Part III, section one, deals with the validation groups for Mathematics 110 while section two deals with the validation group for Mathematics 160.

## Part I. The Single Predictors

This part is concerned with the predictors $X_{1}, X_{2}, X_{3}, X_{4}, X_{5}$ and the study groups of both Mathematics 110 and Mathematics 160. The first section will deal with Mathematics 110 and the second with Mathematics 160 .

Mathematics 110

The Mathematics 110 study group was composed of 225 students. The method of selection of the students from the general freshman student population at Southern University was explained in Chapter III. The five independent variables which composed the source of data for this study were $X_{1}$, scores on Cooperative English Test; $X_{2}$, scores on Cooperative Mathematics Test; $X_{3}$, scores on Nelson-Denny Reading Test; $X_{4}$, scores on the A.C.E.; and $X_{5}$, the weighted high school mathematics average. $Y_{1}$, the grade point average in mathematics was also collected as part of the data for this study group.

Table I gives the summary of data for each of the independent variables and $Y_{1}$.

TABLE V
SUMMARY OF DATA ON PREDICTOR VARIABLES
FOR STUDY GROUP
$\mathrm{N}=225$

|  | INDEPENDENT VARIABLE |  |  |  | DEPENDENT <br> VARIABLE |  |
| :--- | ---: | :---: | :---: | :---: | :---: | :---: |
|  | $\mathrm{X}_{1}$ | $\mathrm{x}_{2}$ | $\mathrm{x}_{3}$ | $\mathrm{X}_{4}$ | $\mathrm{X}_{5}$ |  |
| Mean | 143.71 | 5.10 | 7.80 | 55.76 | 12.88 | 1.96 |
| Standard Deviation | 10.61 | 4.94 | 1.87 | 21.07 | 6.27 | 1.13 |

Since each of the independent variables had some connection with the academic background or ability of the students, it was necessary that the close relationship between each of the independent variables be computed. The results of the determination of intercorrelations among all variables is shown in Table VI.

TABLE VI
INTERCORRELATIONS AMONG ALL VARIABLES FOR STUDY GROUP
$\mathrm{N}=225$

| $\mathrm{X}_{1}$ | .348 | .632 | .570 | .252 | .409 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathrm{X}_{2}$ |  | .515 | .523 | .472 | .563 |
| $\mathrm{X}_{3}$ |  |  | .780 | .347 | .505 |
| $\mathrm{X}_{4}$ |  |  | .322 | .518 |  |
| $\mathrm{X}_{5}$ |  |  | .539 |  |  |
|  |  |  |  |  |  |
|  | $\mathrm{r}=.138$ at the $5 \%$ level |  |  |  |  |

Based upon the data given in Table VI, it can be seen that all of the r's were significant at both the $5 \%$ and $1 \%$ levels. Worthy of special note is the fact that $X_{3}$, Nelson-Denny Reading Test scores and $X_{4}$, A.C.E. scores correlated with $r=.78$. Also notable is the fact that the simple correlation coefficient between $X_{2}$ and $Y_{1}$, $r=.563$, was the highest for any single independent variable with the dependent variable $Y_{1}$. It was followed closely by $X_{5}$ with the $r$ between $X_{5}$ and $Y_{1}$ being $r=.539$.

The predictive ability of the single predictors was of great interest in this study. Simple linear regression, in which only one predictor was used along with the dependent variable, $Y_{1}$, were written. Table VI summarizes the results of the simple linear regression for the Mathematics 110 study group.

SUMMARY OF LINEAR REGRESSIONS INVOLVING SINGLE INDEPENDENT VARIABLES
$N=225$

| INDEPENDENT VARIABLE | REGRESSION EQUATION | $\begin{gathered} \text { TOTAL } \\ \text { Df } \end{gathered}$ | R | SE est | t | $P(t)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{X}_{1}$ | $\widehat{Y}_{1}=.04368 X_{1}-4.31353$ | 224 | . 409 | 1.04 | 6.69 | . 01 |
| $\mathrm{X}_{2}$ | $\widehat{Y}_{1}=.12926 X_{2}+1.30496$ | 224 | . 563 | . 939 | 10.18 | . 01 |
| $X_{3}$ | $\widehat{Y}_{1}=.30642 \mathrm{X}_{3}-0.42757$ | 224 | . 505 | . 980 | 8.74 | . 01 |
| $\mathrm{X}_{4}$ | $\widehat{Y}_{1}=.02785 \mathrm{X}_{4} \nsucc .41169$ | 224 | . 518 | . 972 | 9.04 | . 01 |
| $\mathrm{X}_{5}$ | $\widehat{Y}_{1}=.09757 \mathrm{X}_{5}+.70735$ | 224 | . 539 | . 960 | 9.574 | . 01 |

Formula 1, in Appendix A, was used to calculate t-values for each simple linear regression. The result of these calculations are also shown in Table VII. All of the simple linear regressions were highly significant. The $P(t)$, the probability of obtaining a certain $t-v a l u e$, was less than .01 in each case. The t-values ranged from a high of 10.18 for the regression of $Y_{1}$ on $X_{2}$, to a low of 6.68 for the regression of $Y_{1}$ on $X_{1}$. Table VII shows that $R_{Y_{1}} X_{2}$, the multiple correlations coefficient obtained as a result of the regression of $Y_{1}$ on $X_{2}$ is . 563, thus $R_{Y_{1} X_{2}}^{2}=.317 . X_{2}$ was thus shown to be able to account for approximately $32 \%$ of the variance. This was the highest per cent of the variance for which any single independent variable was able to account
in its regression. However, $R_{Y_{1} X_{5}}$ was .539 and $R_{Y_{1} X_{5}}=.291$. Thus $X_{5}$ was able to account for the second highest amount of the variance; approximately $29 \%$ of the variance of $Y_{1}$.

In simple regression equations such as regression $\bar{Z}$ on $Z_{X}$, the regression coefficient is equal to the slope. The regression equations in this study are given in terms of original measures. Since the program did not give coefficients for use with standard scores, the writer calculated those coefficients which are given in Table VIII and are called Beta Weights.

TABLE VIII
BETA WEIGHTS FOR SIMPLE LINEAR REGRESSION MATHEMATICS 110
$\mathrm{N}=225$

| INDEPENDENT <br> VARIABLE | DEPENDENT <br> VARIABLE | bi | Bi |
| :---: | :---: | :---: | :---: |
| $\mathrm{X}_{2}$ | $\mathrm{Y}_{1}$ | .12926 | .5650 |
| $\mathrm{X}_{5}$ | $\mathrm{Y}_{1}$ | .09757 | .5414 |
| $\mathrm{X}_{4}$ | $\mathrm{Y}_{1}$ | .02785 | .5193 |
| $\mathrm{X}_{3}$ | $\mathrm{Y}_{1}$ | .30642 | .5068 |
| $\mathrm{X}_{1}$ | $\mathrm{Y}_{1}$ | .04368 | .4101 |

In the case of simple linear regressions, where standard scores are used, the Beta weight is equal to the correlation coefficient. Table VIII also shows that each $B_{i}$ is approximately equal to $R_{Y_{1}} X_{1}$. In Table VIII it can be seen that the highest contribution to prediction is made by the regression of $Y_{1}$ on $X_{2}$. The other regressions from highest to lowest are $Y_{1}$ on $X_{5}, Y_{1}$ on $X_{4}, Y_{1}$ on $X_{3}$ and $Y_{1}$ on $X_{1}$.

The predictive efficiency, E , of a regression is the proportionate reduction in the error of estimate from the maximum error characteristic of zero correlations. E was calculated by using formula 3 in Appendix A. In Table IX, E ranges from a low of $8.7 \%$ to a high of $17.5 \%$. The low index of efficiency is for the regression of $Y_{1}$ on $X_{1}$ while the high is for the regression of $Y_{1}$ on $X_{2}$.

TABLE IX
THE INDEX OF PREDICTIVE EFFICIENCY, E

$$
\begin{gathered}
\text { FOR VALUES OF } \mathrm{r}_{\mathrm{X}_{i} \mathrm{Y}_{1}} \\
\mathrm{~N}=225
\end{gathered}
$$

| $x_{i}$ | $r_{X_{i} Y_{1}}$ | $E$ |
| :--- | :---: | :---: |
| $X_{1}$ | .409 | $8.9 \%$ |
| $X_{2}$ | .563 | $17.5 \%$ |
| $X_{3}$ | .505 | $13.6 \%$ |
| $x_{4}$ | .518 | $14.6 \%$ |
| $X_{5}$ | .539 | $15.8 \%$ |

It should be noted in Table IX that although there is a range of approximately 9 percentage points in the spread of the E's, the top four E's have a range of only approximately 4 points.

The Analysis of Variance is the procedure used to test the significance of regressions. The Analysis of Variance for the five simple linear regressions developed for the Mathematics 110 study group are given in Tables $X$ through Table XIV. The F-value, by which the tests is performed is given in each table. Table XLIII in Appendix B gives the symbolic method used for calculation of data shown in Tables $X$ through XIV. In each of these tables, the level of the significance of the regression is indicated. ${ }^{1}$

Table $X$ shows the results involving the regression of $Y_{1}$ on $X_{1}$. The F-value, calculated by formula 10 in Appendix A, is 44.75 .

```
l * Significant (.05 level)
    ** Highly Significant (.01 level)
```

TABLE X
ANALYSIS OF VARIANCE
$\underset{N}{\text { FOR }}=225$

| Source of Variation | Df | SS | Mean <br> Square | F-Value |
| :--- | ---: | ---: | ---: | ---: |
| Attributed to regression | 1 | 48.08324 | 48.08324 | $44.75 * *$ |
| Deviation from regression | 223 | 239.63231 | 1.07458 |  |
| TOTAL | 224 | 287.71555 |  |  |

The probability of an F-value larger than $\mathrm{F}=44.75$ when $\mathrm{N}=225$ is less than .01 . The fact that the probability is so small would indicate significance for the regression at both the $5 \%$ and $1 \%$ levels. The regression equation for the regression of $\mathrm{Y}_{1}$ on $\mathrm{X}_{1}$ is

$$
\widehat{Y}_{1}=.0438 X_{1}-4.31353
$$

Table XI shows the Analysis of Variance for the regression of $\mathrm{Y}_{1}$ on $X_{2}$. The F -value which determines the significance of this regression is $\mathrm{F}=103.54$.

TABLE XI

$$
\begin{gathered}
\text { ANALYSIS OF VARIANCE FOR REGRESSION OF } \mathrm{Y}_{1} \text { ON } X_{2} \\
\mathrm{~N}=225
\end{gathered}
$$

| Source of Variation | Df | SS | Mean <br> Square | F-Value |
| :--- | ---: | ---: | ---: | ---: |
| Attributed to regression | 1 | 91.23069 | 91.23019 | $103.54203 * *$ |
| Deviation from regression | 223 | 196.48487 | 0.88110 |  |
| TOTAL | 224 | 287.71556 |  |  |

The F-value for the regressions of $Y_{1}$ on $X_{2}$ is large and for this population would indicate significance far beyond the $1 \%$ level. In this study, the testing is at the $5 \%$ and $1 \%$ levels and $Y_{1}$ on $X_{2}$ met the criterion for significance at both of these levels.

The $F$-value for the regression of $Y_{1}$ on $X_{3}$ given in Table XII is $F=76.47$. This F-value denotes a highly significant regression. The regression $Y_{1}$ on $X_{2}$ is represented by $\widehat{Y}_{1}=.12926 X_{2}+1.30496$.

TABLE XII
$\begin{array}{rl}\text { ANALYSIS OF VARIANCE FOR REGRESSION OF } \\ \mathrm{N} & \mathrm{Y} \\ =225\end{array}$

$$
N=225
$$

| Source of Variation | Df | SS | Mean <br> Square | F-Value |
| :--- | ---: | ---: | ---: | ---: |
| Attributed to Regression | 1 | 73.46513 | 73.46513 | $76.47 * *$ |
| Deviation from Regression | 223 | 214.25043 | 0.96076 |  |
| TOTAL | 224 | 287.71556 |  |  |

The regression of $Y_{1}$ on $X_{3}$ is significant at both the $5 \%$ and $1 \%$ levels. Table XII gives the Analysis of Variance for the regression of $Y_{1}$ on $X_{3}$. The F-value for the regression in Table XIII is 81.70 The regression equation is $\widehat{Y}_{1}=.30642 \mathrm{X}_{3}-0.42757$.

## TABLE XIII

ANALYSIS OF VARIANCE FOR REGRESSION OF $\mathrm{Y}_{1}$ ON $\mathrm{X}_{4}$
$\mathrm{N}=225$

| Source of Variation | Df | SS | Mean <br> Square | F-Value |
| :--- | ---: | ---: | ---: | ---: |
| Attributed to Regression | 1 | 77.14775 | 77.14375 | $81.70^{* *}$ |
| Deviation from Regression | 223 | 210.57180 | 0.94427 |  |
| TOTAL | 224 | 287.71556 |  |  |

Since the F-value in Table XIII exceeds that for Table XII and the populations are identical, it was easily determined that the regression of $Y_{1}$ on $X_{3}$ was also highly significant. The regression equation is $\widehat{Y}_{1}=.02785 \mathrm{X}_{4}+.41169$.

Table XIV contains the results of calculations necessary for the Analysis of Variance of the regression of $Y_{1}$ on $X_{5}$. The $F$-value, $\mathrm{F}=91.67$, also exceeds that for Table XII:

TABLE XIV

$$
\begin{aligned}
& \text { ANALYSIS OF VARIANCE FOR REGRESSION OF } Y_{1} \text { ON } X_{5} \\
& N=225
\end{aligned}
$$

| Source of Variation | Df | SS | Mean <br> Square | F-Value |
| :--- | ---: | ---: | ---: | ---: |
| Attributed to Regression | 1 | 83.81728 | 83.81728 | $91.67 * *$ |
| Deviation from Regression | 223 | 203.89827 | 0.91434 |  |
| TOTAL | 224 | 287.71555 |  |  |

The F -value, $\mathrm{F}=91.67$ is reasonably close to the F -value, $\mathrm{F}=103.54$ in terms of the significance of the regression, thus the regression is really significant at a level which suppasses the $1 \%$ level. This would indicate significance at both the $5 \%$ and $1 \%$ levels which are the levels at which the regressions were tested. The equation representing the regression is $\widehat{\mathrm{Y}}_{1}=.09757 \mathrm{X}_{5}+.70735$. Hypotheses Related to Mathematics 110 and Single Predictors

Listed below are the hypotheses related to Mathematics 110 and the single predictors.
$B_{2} \quad$ There is no significant difference between the multiple correlation coefficient obtained by use of $\mathrm{X}_{5}$ and the one obtained by use of $X_{1}$ as a predictor.
$B_{3} \quad$ There is no significant difference between the multiple correlation coefficient obtained by use of $X_{5}$ and the one obtained by use of $X_{2}$ as a predictor.
$B_{4} \quad$ There is no significant difference between the multiple correlation coefficient obtained by use of $X_{5}$ and the one obtained by use of $X_{3}$ as a predictor.
$B_{5} \quad$ There is no significant difference between the multiple correlation coefficient obtained by use of $X_{5}$ and the one obtained by use of $X_{4}$ as a predictor.

The correlation coefficients for the two regressions which were compared in each hypothesis came from bi-variate samples with one array, the $Y_{1}$ variable, in common. It was therefore necessary to use a test designed for this purpose. The standard error of the difference between correlation coefficients, for the cases involving the common array, were calculated by use of formulas 4 and 5 in Appendix A. The resulting $T$-values which were calculated by use of formula 6 in Appendix A are shown in Table XV.

TABLE XV
TESTS FOR SIGNIFICANCE OF DIFFERENCE BETWEEN CORRELATION COEFFICIENT OF $Y_{1}$ ON $X_{5}$ AND $Y_{1}$ ON $X_{i}$
$N=225$


In view of the information given in Table $X V$, the hypotheses were treated as follows:
$B_{2} \quad$ There is no significant difference between the multiple correlation coefficient obtained by use of $X_{5}$ and the one obtained by use of $X_{1}$ as a predictor.

The writer failed to reject $\mathrm{B}_{2}$ at the .05 level and failed to accept it at the .01 level since in. Table XI $P(t)=.0274$ which is less than .05 but is greater than . 01 .
\# \# \# \# \#
$B_{3} \quad$ There is no significant difference between the multiple correlation coefficient obtained by use of $X_{5}$ and the one obtained by use of $X_{2}$ as a predictor.

The writer, based upon the information in Table XI, failed to reject $B_{3}$ at either the .05 or .01 levels. Since $P(T)=.3264$, there is an indication of doubt of the superiority of $X_{5}$ as a predictor over $X_{2}$ or vice versa. Since it has been previously shown that $X_{2}$ may be a better predictor than $X_{5}$, the information given in Table $X I$, upon which the test is based, suggests a question as how much better a prediction based on $X_{2}$ would be than one based on the use of $X_{5}$ since $P(T)$ shows the difference is not significant at the minimal . 05 level.

```
# # # # #
```

$B_{4} \quad$ There is no significant difference between the multiple correlation coefficient obtained by use of $X_{5}$ and the one obtained by use of $X_{3}$ as a predictor.

The writer failed to reject $\mathrm{B}_{4}$ at either the .05 level or the .01 level. $P(T)=.2742$ indicates that there is a question as to the superiority of $X_{5}$ over $X_{3}$ as a predictor.
\# \# \# \# \#
$B_{5} \quad$ There is no significant difference between the multiple correlation coefficient obtained by use of $X_{5}$ and the one obtained by use of $X_{4}$ as a predictor.

By use of Table $X V$, it is shown that $P(T)=.3632$. The writer failed to reject $B_{4}$ at the .05 level or at the .01 level since $.05<.3632$ and $.01<.3632$.
\# \# \# \# \#

Mathematics 160

This section deals with Mathematics 160 . The study group for Mathematics 160 was composed of 75 students. The procedures for selection from the general freshman student population at Southern University were given in Chapter III. Table XVI below gives a summary of data for each of the independent variables $X_{1}$, scores on Cooperative English Test; $X_{2}$, scores on the Cooperative Mathematics Test; $X_{3}$, scores on the Ne1son-Denny Reading Test; $X_{4}$, scores on the A.C.E.; and $X_{5}$, the weighted high school mathematics average. The summary of data also includes the data for $Y_{2}$, the grade point average in Mathematics 160.

TABLE XVI
SUMMARY OF DATA ON PREDICTOR VARIABLES FOR STUDY GROUP $\mathrm{N}=75$

|  | INDEPENDENT VARIABLE |  |  |  |  | DEPENDENT VARIABLE |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\mathrm{x}_{1}$ | $\mathrm{X}_{2}$ | $\mathrm{X}_{3}$ | $\mathrm{X}_{4}$ | $\mathrm{X}_{5}$ | $Y_{2}$ |
| Mean | 145.52 | 14.85 | 8.73 | 67.69 | 20.25 | 1.88 |
| S.D. | 9.62 | 11.67 | 1.73 | 16.58 | 6.46 | 1.29 |

There was a great possibility that the variables used in this study would be highly related. The results of the determination of intercorrelations among all variables are shown in Table XVII. The table shows that with the exception of $r_{15}, r_{12}, r_{23}$ and $r_{35}$ all of r's were significant at the $1 \%$ level. The correlation coefficient $r_{12}$ and $r_{35}$ were, however, significant at the $5 \%$ level. Further it was found that all of the independent variables when correlated with $Y_{2}$ gave r's that were significant at the $5 \%$ level.

TABLE XVII
INTERCORRELATIONS AMONG ALL VARIABLES
USED FOR STUDY GROUP

$$
N=75
$$

|  | $\mathrm{x}_{2}$ | $\mathrm{x}_{3}$ | $\mathrm{X}_{4}$ | $\mathrm{X}_{5}$ | $\mathrm{Y}_{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{X}_{1}$ | . 367 | . 621 | . 542 | . 128 | . 293 |
| $\mathrm{X}_{2}$ |  | . 215 | . 423 | . 325 | . 291 |
| $X_{3}$ |  |  | . 590 | . 253 | . 364 |
| $\mathrm{X}_{4}$ |  |  |  | . 395 | . 502 |
| $\mathrm{X}_{5}$ |  | $r=$ | $\begin{aligned} & \text { the } 5 \% \\ & \text { the } 1 \% \end{aligned}$ |  | . 552 |

Simple linear regressions were written to provide information on the predictive ability of the single predictors. In Table XVIII the results are summarized. Using Formula 1 in Appendix A, t-values were calculated for each of the simple linear regressions.

Table XVIII is a summary of the simple linear regressions involving the single predictors.

TABLE XVIII
SUMMARY OF LINEAR REGRESSIONS INVOLVING SINGLE INDEPENDENT VARIABLES
$\mathrm{N}=75$

| INDEPENDENT <br> VARIABLE | REGRESSION <br> EQUATION | TOTAL <br> Df | R | SE <br> est | t | $\mathrm{P}(\mathrm{t})$ |
| :--- | :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{X}_{1}$ | $\hat{Y}_{2}=.03937 \mathrm{X}_{1}-3.84842$ | 74 | .409 | 1.25 | 2.61 | .0045 |
| $\mathrm{X}_{2}$ | $\hat{\mathrm{Y}}_{2}=.03230 \mathrm{X}_{2}+1.40026$ | 74 | .563 | 1.25 | 2.60 | .0047 |
| $\mathrm{X}_{3}$ | $\hat{\mathrm{Y}}_{2}=.27155 \mathrm{X}_{3}-.49044$ | 74 | .505 | 1.21 | 3.34 | .0014 |
| $\mathrm{X}_{4}$ | $\hat{\mathrm{Y}}_{2}=.03920 \mathrm{X}_{4}-.77381$ | 74 | .518 | 1.13 | 4.962 | .001 |
| $\mathrm{X}_{5}$ | $\hat{\mathrm{Y}}_{2}=.11044 \mathrm{X}_{5}-.35678$ | 74 | .539 | 1.08 | 5.65 | .0001 |

In Table XVIII, it can be seen that the regression containing the predictor $X_{5}$ and $X_{4}$, weighted high school mathematics average and A.C.E. test scores are significant at the $1 \%$ level. Regressions
containing are other independent variables were significant at the $1 \%$ level. All of the regressions were significant at the $5 \%$ level. Further, Table XVIII shows that $R_{Y_{2}} X_{5}$, the multiple correlation coefficient obtained as a result of the regression, $Y_{2}$ on $X_{5}$ is. .552, therefore $R^{2}=.293$. This implies that the single independent variables $X_{5}$ was able to account for $29.3 \%$ of the variance. This was the highest per cent of the variance for which any single predictor was able to account although $X_{4}$ followed closely with $R=.502$ thus accounting for $25.2 \%$ of the variance.

In order to be able to quickly judge the highest contributor to its regression equation, standard regression coefficients often called beta weights were computed by using formula 2 in the list of formulas found in the appendix. Table XIX below gives the results of the calculations from the highest contributing $X$ to the lowest.

TABLE XIX
BETA WEIGHTS FOR SIMPLE LINEAR
REGRESSIONS
$\mathrm{N}=75$

| INDEPENDENT <br> VARIABLE | DEPENDENT <br> VARIABLE | bi | Bi |
| :---: | :---: | :---: | :---: |
| $\mathrm{X}_{5}$ | $\mathrm{y}_{2}$ | .1104 | .553 |
| $\mathrm{X}_{4}$ | $\mathrm{Y}_{2}$ | .0392 | .503 |
| $\mathrm{X}_{3}$ | $\mathrm{Y}_{2}$ | .2716 | .363 |
| $\mathrm{X}_{1}$ | $\mathrm{y}_{2}$ | .0393 | .293 |
| $\mathrm{X}_{2}$ | $\mathrm{y}_{2}$ | .0323 | .246 |

It is shown in Table XIX that $X_{5}$, weighted high school mathematics average, makes the highest contribution and $X_{2}$, Cooperative Mathematics Test, makes the lowest contribution to prediction. This fact could be readily seen if standard scores were used, however, Table XVIII does not give the regression coefficients in standard form, but in terms of original measures.

The proportionate reduction in the error of estimate from the maximum error characteristic of zero correlations is given in Table XX. Table $X X$ gives the predictive efficiency of the single predictors.

This index was calculated by using formula 3 in Appendix A. In Table $X X$, E ranges from a low of $4.3 \%$ for $X_{2}$ and $X_{1}$ to a high of $16.5 \%$ for $x_{5}$ 。

TABLE XX
THE INDEX OF PREDICTIVE EFFICIENCY, E

$$
\text { FOR VALUES OF } r_{X_{i}} Y_{2}
$$

$$
\mathrm{N}=75
$$

| $X_{i}$ | $r_{X_{i} Y_{2}}$ | $E$ |
| :---: | :--- | :--- |
| $X_{1}$ | .29 | $4.3 \%$ |
| $X_{2}$ | .29 | $4.3 \%$ |
| $X_{3}$ | .36 | $6.7 \%$ |
| $X_{4}$ | .50 | $13.4 \%$ |
| $X_{5}$ | .55 | $16.5 \%$ |

Table $X X$ shows that $X_{5}$ has the highest predictive efficiency but is closely followed by $X_{4}$.

The Analysis of Variance for the five simple linear regressions is given in Tables XXI through Table XXV. The F-value is given in each table. The symbolic method for calculation of these data is shown in Table XLIII in Appendix B.

In Table XXI, the results of the computations involving the regression of. $Y_{2}$ on $X_{1}$ is shown. The F-value of the regression, defined by formula 10 in Appendix A, is 6.84 .

TABLE XXI
ANALYSIS OF VARIANCE FOR REGRESSION
OF $\mathrm{Y}_{2} \mathrm{ON} \mathrm{X}_{1}$
$\mathrm{N}=75$

| Source of <br> Variation | Df | SS | Mean <br> Squares | F-Value |
| :---: | :---: | :---: | :---: | :---: |
| Attributed to Regression | 1 | 10.61601 | 10.61601 | $6.83973^{*}$ |
| Deviation from Regression | 73 | 113.30399 | 1.55211 |  |
| TOTAL | 74 | 123.9200 |  |  |

The probability of a large $F$ than $F=6.84$, for this population; is smaller than .024 . This implies that F is significant at the $5 \%$ level but not at the $1 \%$ level. The regression equation is

$$
\widehat{Y}_{2}=.3937 X_{1}-3.84842
$$

Table XXI shows the computational results of the analysis of the regression of $Y_{2}$ on $X_{2}$. The F-value, which determines the significance of this regression, is $F=6.77$.

TABLE XXII
ANALYSIS OF VARIANCE FOR REGRESSION OF $\mathrm{Y}_{2} \mathrm{ON} \mathrm{X}_{2}$ $\mathrm{N}=75$

| Source of <br> Variation | Df | SS | Mean <br> Squares | F-Value |
| :---: | ---: | :---: | ---: | :---: | :---: |
| Attributed to Regression | 1 | 10.51903 | 10.51903 | $6.77185 *$ |
| Deviation from Regression | 73 | 113.40097 | 1.55344 |  |
| TOTAL | 74 | 123.9200 |  |  |

Since the $F$-value for the regression of $Y_{2}$ on $X_{2}$ is so close to the $F$-value from the regression of $Y_{2}$ on $X_{1}$, we apply the same argument and get the results that $F$ is significant at the $5 \%$ but not at the $1 \%$ level. The regression of $Y_{2}$ on $X_{2}$ is given by the equation

$$
\widehat{Y}_{2}=.03230 X_{2}+1.40026
$$

From analyzing the variance from the regression of $Y_{2}$ on $X_{3}$, we get an F-value of 11.13. Table XXIII is the result of analyzing the regression of $Y_{2}$ on $X_{3}$.

TABLE XXIII
ANALYSIS OF VARIANCE FOR REGRESSION OF.

$$
\begin{array}{r}
Y_{2} \mathrm{ON} \mathrm{X}_{3} \\
\mathrm{~N}=75
\end{array}
$$

| Source of <br> Variation | Df | SS | Mean <br> Squares | F-Value |
| :---: | ---: | ---: | ---: | ---: |
| Attributed to Regression | 1 | 16.39175 | 16.39175 | $11.12822^{* *}$ |
| Deviation from Regression | 73 | 107.52825 | 1.47299 |  |
| TOTAL | 74 | 123.92000 |  |  |

The F-value 11.13 is highly significant. The probability of getting a larger $F$-value than 8.49 would be less than .005 , thus the F-value 11.13 is significant at both the $5 \%$ and $1 \%$ levels. The regression is given by $\hat{\mathrm{Y}}_{2}=.27155 \mathrm{X}_{3}-.49044$.

Table XXIV is the summary of the analysis of variance for the regression of $Y_{2}$ on $X_{4}$ and Table XXV is the summary of the results from the regression of $Y_{2}$ on $X_{5}$.

TABLE XXIV
ANALYSIS OF VARIANCE FOR REGRESSION OF

$$
\begin{array}{r}
Y_{2} \text { ON } X_{4} \\
\mathrm{~N}=75
\end{array}
$$

| SOURCE OF VARIATION | Df | SS | MEAN SQUARES | F-Value |
| :--- | ---: | ---: | ---: | ---: | :--- |
| Attributed to Regression | 1 | 31.25459 | 31.25459 | $24.62175^{* *}$ |
| Deviation from Regression | 73 | 92.66541 | 1.26939 |  |
| TOTAL | 74 | 123.92000 |  |  |

The regression of $Y_{2}$ on $X_{4}$ is given by the equation

$$
Y_{2}=.0390 X_{4}-.77381
$$

TABLE XXV
ANALYSIS OF VARIANCE FOR REGRESSION OF

$$
\mathrm{Y}_{2} \mathrm{ON} \mathrm{X}_{5}
$$

$$
N=75
$$

| SOURCE OF VARIATION | Df | SS | MEAN SQUARES | V-Value |
| :---: | :---: | :---: | :---: | ---: | :---: |
| Attributed to Regression | 1 | 37.69094 | 37.69094 | $31.90848^{* *}$ |
| Deviation from Regression | 73 | 86.22906 | 1.18122 |  |
| TOTAL | 74 | 123.92000 |  |  |

Table XXIV with an F-value of 24.62 and Table XXV with an F-value of 31.91 have very highly significant $F$-values. The probability of a larger F-value is less than .001. Thus, they are both significant at the $5 \%$ and $1 \%$ levels.

The $F$-value for $Y_{2}$ on $X_{5}$ is higher than any other regression. The regression equation is $Y_{2}=.11044 X_{5}-.35678$. Hypotheses Related to Mathematics 160 and Single Predictors

Listed below are the hypotheses related to Mathematics 160 and the single predictors.
$A_{2} \quad$ There is no significant difference between the multiple correlation coefficient obtained by use of $X_{5}$ and the one obtained by use of $X_{1}$.
$A_{3} \quad$ There is no significant difference between the multiple correlation coefficient obtained by use of $X_{5}$ and the one obtained by use of $X_{2}$.
$A_{4} \quad$ There is no significant difference between the multiple correlation coefficient obtained by use of $X_{5}$ and the one obtained by use of $X_{3}$.
$A_{5} \quad$ There is no significant difference between the multiple correlation coefficient obtained by use of $X_{5}$ and the one obtained by use of $X_{4}$.

In order to test these hypotheses, it was necessary to determine the form of the distributions from which the correlation coefficients for the two regression equations were obtained. These correlation coefficients came from bi-variate samples with one array, the $Y_{2}$ variable, in common. It was, therefore, necessary to compute a quantity $\quad \sigma\left(r_{Y_{2} X_{5}}-r_{Y_{2} X_{i}}\right)$. This quantity, called the standard error of the difference between correlation coefficients, was calculated by formulas 4 and 5 in Appendix A. The resulting T-value calculated by use of formula 6 in Appendix $A$ and the probability of getting the T-values, are shown in Table XXVI,

TABLE XXVI
TESTS FOR SIGNIFICANCE OF DIFFERENCES BETWEEN CORRELATION COEFFICIENTS OF $\mathrm{Y}_{2}$ ON $\mathrm{X}_{5}$

$$
\begin{gathered}
\text { AND } Y_{2} \text { ON } X_{i} \\
N=75
\end{gathered}
$$

| CORRELATION COEFFICIENTS <br> OF REGRESSION |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| $Y_{2}$ on $X_{5}$ | $Y_{2}$ on $X_{i}$ | Difference of <br> R's | T-Value | $P(T)$ |
| $\mathrm{X}_{5}$ | $\mathrm{X}_{1}$ |  |  |  |
| .552 | .283 | .259 | 2.06 | .0197 |
| $\mathrm{X}_{5}$ | $\mathrm{X}_{2}$ |  |  |  |
| .552 | .291 | .261 | 2.30 | .0107 |
| $\mathrm{X}_{5}$ | $\mathrm{X}_{3}$ |  |  |  |
| .552 | .364 | .188 | 1.63 | .0516 |
| $\mathrm{X}_{5}$ | $\mathrm{X}_{4}$ |  |  |  |
| .552 | .502 | .050 | .490 | .4801 |

In view of the information in Table XXVI, the hypotheses were treated as follows:
$A_{2} \quad$ There is no significant difference between the multiple correlation coefficient obtained by use of $X_{5}$ and the one obtained by use of $X_{1}$.

The writer failed to reject $A_{6}$ at the point of .05 level and failed to accept it at the .01 level since $P(T)=.0197$. \# \# \# \# \#
$A_{3} \quad$ There is no significant difference between the multiple correlation coefficient obtained by use of $X_{5}$ and the one obtained by use of $X_{2}$.

The writer failed to reject $A_{7}$ at the .05 level and failed to accept the hypothesis at the . Ol level since $.05>.0107$ and $.01<.0107$.
\# \# \# \# \#
$A_{4} \quad$ There is no significant difference between the multiple correlation coefficient obtained by use of $X_{5}$ and the one obtained by use of $X_{3}$.

The writer failed to accept the hypotheses at either the .05 level or .01 level since .01 is less than .0516 and and .05 is less than .0516 .

> \# \# \# \# \#
$A_{5} \quad$ There is no significant difference between the multiple correlation coefficient obtained by use of $X_{5}$ and the one obtained by use of $X_{4}$.

The writer failed to accept the hypotheses at either the .05 level or .01 level since $.01<.4801$ and $.05<.4801$.

The $P(T)$ in $T a b l e$ XXIV is .4801.
\# \# \# \# \#

Part II. The Multiple Linear Regressions

The chief considerations in this part are the two multiple linear regressions. The regression for Mathematics 110 is analyzed first and is followed by the analysis of the regression for Mathematics 160.

Mathematics 110

A program involving step-wise procedures was used on the computer at Oklahoma State University, Stillwater, Oklahoma, to develop the multiple linear regression for Mathematics 110. The procedure was
designed to derive the best prediction equation possible through use of the independent variables $X_{1}, X_{2}, X_{3}, X_{4}, X_{5}$. Table I, given earlier in this chapter gives the means and standard deviations of test scores, weighted high school mathematics average, and grade point averages in Mathematics 110. Table VI, which was also given earlier, contains the intercorrelation among all variables used with the study group.

In Table XXVII, a summary of the determination of the multiple regression for Mathematics 110 is given. It also contains the multiple correlation coefficiencies and standard error of the predicted grades developed by use of the computer program.

## TABLE XXVEI

SUMMAKY OF CORRELATION COEFFICIENTS WITH STANDARDS
ERROR OF $Y$, AND REGRESSION
EQUATIONS
$\mathrm{N}=225$


The computer was instructed to use $\mathrm{F}=.001$ for entry of the variable into the regression and $F=.000$ for refusal of entry of the variable. The program was designed so that it introduced at each step the variable which contributed the greatest amount to the explained variance of the dependent variable taking into account the variables already introduced and their intercorrelations with the variables which had not been introduced.

The final multiple regression equation for the Mathematics 110 study group was:

$$
\begin{aligned}
& Y_{1}= .01063 X_{1}+.05836 X_{2}+.03935 X_{3}+ \\
& .00946 X_{4}+.05706 X_{5}-1.43031
\end{aligned}
$$

Beta weights were calculated so that the contribution to the explained variance of each significant variable could be shown in terms of the beta coefficients as well as simple correlation coefficients. Table XXVIII gives the result of using formula 2, presented in Appendix A, to calculate the beta weights for the independent variables in the multiple linear regression involving $Y_{1}$,

TABLE XXVIII
PREDICTORS IN ORDER OF SELECTION WITH BETA NUMBERS AND BETA WEIGHTS

MATHEMATICS 110
$\mathrm{N}=225$

| PREDICTORS SELECTED | BETA NUMBER | BETA |
| :---: | :---: | :---: |
| $\mathrm{X}_{2}$ | $\mathrm{~B}_{2}$ | .3551 |
| $\mathrm{X}_{5}$ | $\mathrm{~B}_{5}$ | .3166 |
| $\mathrm{X}_{4}$ | $\mathrm{~B}_{4}$ | .1764 |
| $\mathrm{X}_{1}$ | $\mathrm{~B}_{1}$ | .0998 |
| $\mathrm{X}_{3}$ | $\mathrm{~B}_{3}$ | .0651 |

The order in which the variable entered the regression was $X_{2}$, $X_{5}, X_{4}, X_{1}, X_{3}$. The highest contribution to the explanation of the variance was made by $X_{2}$ closely followed by $X_{5}$.

The standard error of the predicted scores furnish an interval $\mathrm{Y} \pm$ (standard error of Y ). An individual whose predicted score was less than $Y \pm$ (standard error of $Y$ ) would be considered as a probable unsuccessful student and one with a predicted score greater than $Y \pm$ (standard error of Y ) would be considered a probable success. In Table XXVII, a multiple $R$ of .691 was obtained as a result of the multiple regression. This implies $R^{2}=.477$ and thus the predictors in combination account for approximately $48 \%$ of the variance of $\mathrm{Y}_{1}$. All five of the independent variables were included in the final
regression equation. When the simple linear regressions were compared with the multiple regression, it was found that the highest amount of variance accounted for by a simple regression was approximately $32 \%$ and the $48 \%$ accounted for by the multiple regression was greater than the $32 \%$ accounted for by $X_{2}$.

Hypothesis for Multiple Linear Regression for Mathematics 110

The hypothesis $B_{1}$, one of the major hypotheses, will now be considered.
$B_{1}$ The $F$-value for the multiple linear regression does not differ significantly from zero.

Table XXIX presents the summary of the analysis of the multiple regression from Mathematics 110. The form of this table is that of Table XLIV in Appendix B, which is the form table used to perform an analysis of variance for multiple linear regressions.

TABLE XXIX
MULTIPLE REGRESSION ANALYSIS
MATHEMATICS 110
$\mathrm{N}=225$

| SOURCE OF VARIATION | Df | SUM OF SQUARES | MS | F |
| :--- | :---: | :---: | :---: | :---: |
| Regression | 5 | 198.87 | 39.77 | $99.3 * *$ |
| Error | 219 | 88.84 | .4006 |  |
| TOTAL | 224 | 287.71 |  |  |

In Table XXIX, it is shown that the regression contains all five independent variables and is highly significant. The significance of a multiple linear regression is determined by the F-value. This value, when significant, indicates the absence of chance having determined the observed reductions in the total sum of squares.

Formula 10, in Appendix A, was used to calculate the F-value in Table XXIX. The F-value obtained in Table XXIX was $F=99.3$. This F-value is very highly significant. The probability of an F-value larger than 9.07 would be .01 and $F=99.3$ is considerably larger than 9.07.

In view of the highly significant F-value obtained in Table XXIX, the writer fails to accept the major hypothesis $B_{1}$ at either the $5 \%$ or $1 \%$ levels.

Mathematics 160

The multiple linear regression for Mathematics 160 was written by a step-wise procedure on the computer at Oklahoma State University, Stillwater, Oklahoma. The purpose for which the procedure was designed was to develop the best prediction equation possible through use of the five independent variables $X_{1}, X_{2}, X_{3}, X_{4}, X_{5}$. Table XVI, given earlier in this chapter, gives the means and standard deviations of test scores, weighted high school mathematics averages and grade point averages. Simple correlations among all variables are found in Table XVII which also was given earlier.

Table XXX summarizes the determination of the multiple regression equation for Mathematics 160 . In Table $X X X$, the multiple correlation coefficients and standard error of the predicted grades are given. Multiple correlations were calculated by the computer program.

TABLE XXX
SUMMARY OF CORRELATION COEFFICIENTS WITH STANDARD ERROR OF $\mathrm{Y}_{2}$ AND REGRESSION EQUATIONS
$\mathrm{N}=755$

| MULTIPLE R OR r | STANDARD ERROR OF $\mathrm{Y}_{2}$ | F-Value | REGRESSION EQUATION |
| :---: | :---: | :---: | :---: |
| $\mathrm{r}_{\mathrm{Y}_{2} 2}=.552$ | 1.0868 | 31.909 | $\widehat{Y}_{2}=.11044 X_{5}-0.35678$ |
| $\mathrm{R}_{\mathrm{Y} 2 \text { (54) }}=.632$ | 1.0162 | 11.4993 | $\widehat{Y}_{2}=.02630 X_{4}+.08379 X_{5}-1.59761$ |
| $\mathrm{R}_{\mathrm{Y} 2(543)}=.637$ | 1.0188 | . 6813 | $\begin{aligned} \widehat{Y}_{2} & =.06733 X_{3}+.02221 X_{4}+.08336 X_{5} \\ & -1.89971 \end{aligned}$ |
| $\mathrm{R}_{\mathrm{Y} 2 \text { (5431) }}=.638$ | 1.0247 | . 1838 | $\hat{\mathrm{Y}}_{2}=.00712 \mathrm{X}_{1}+.04889 \mathrm{X}_{3}+.02093 \mathrm{X}_{4}$ |
|  |  |  | $\not \subset .08456 X_{5}-2.71193$ |
| $\mathrm{R}_{\mathrm{Y} 2(54312)}=.638$ | 1.0321 | . 0082 | $\begin{aligned} \widehat{Y}_{2} & =.00671 x_{1}+.00108 x_{2}+.05038 x_{3} \\ & +.02071 x_{4}+.08412 x_{5}-2.65804 \end{aligned}$ |

The results in Table $X X X$ were obtained by instructing the computer to give the best combination of variables for predicting success in the Mathematics 160 course. The computer was further instructed to use $\mathrm{F}=.001$ for entry of the variable into the regression and $\mathrm{F}=.000$ for refusal of entry of the variable. The order of entry of the variable was from the variable contributing most highly to the explained variance of $Y_{2}$ to the variable contributing least to the variance of $Y_{2}$ so long as $F=.000$ did not result. The resulting multiple regression equation for the Mathematics 160 study group was:

$$
\begin{gathered}
Y_{2}=.00671 X_{1}+.00108 X_{2}+.05038 X_{3}+ \\
.02071 X_{4}+.08412 X_{5}-2.65804
\end{gathered}
$$

Beta weights for use when the scores are given in standard measure were not given by the computer. It is possible by use of beta coefficients to determine the highest contributing variable to the regression when standard scores are used. Table XXXI gives the results of using formula 2 presented in Appendix A to calculate the beta weights for the independent variables in the multiple regression involving $Y_{2}$.

TABLE XaXI
PREDICTOR IN ORDER OF SELECTION WITH BETA NUMBERS
AND BETA WEIGHTS

| PREDICTOR SELECTED | BETA NUMBER | BETA WEIGHT |
| :---: | :---: | :---: |
| $\mathrm{X}_{5}$ | $\mathrm{~B}_{5}$ | 421 |
| $\mathrm{X}_{4}$ | $\mathrm{~B}_{4}$ | 266 |
| $\mathrm{X}_{3}$ | $\mathrm{~B}_{3}$ | .067 |
| $\mathrm{X}_{1}$ | $\mathrm{~B}_{1}$ | .050 |
| $\mathrm{X}_{2}$ | $\mathrm{~B}_{2}$ | .009 |

Checking the beta weights, it is possible to see the order in which the variables entered the step-wise regression. The ordex was $x_{5}, X_{4}, x_{3}, X_{1}, x_{2}$. Thus, the highest contribution to explanation of the variance was made by $X_{5}$.

The standard error of the predicted scores furnished an interval $Y \pm$ (standard error of $Y$ ). An individual whose predicted score was less than $Y \pm$ (standard error of $Y$ ) would be considered as a probable unsuccessful student while one with a predicted score greater than $Y \pm$ (standard error of $Y$ ) would be considered a probable success.

Table XXVIII gives a muitiple $R=.638, R^{2}=.397$ which indicates that the predictors in combination account for approximately $40 \%$ of the variance of $Y$. All five of the independent variables axe included in the final multiple linear regression equation Comparing the multiple
$R$ of the simple regressions led to the information that the linear regression of $Y_{2}$ on $X_{5}$ accounted for more variance than any other of the simple linear regressions. The multiple regressions accounted for a larger percentage of the variance than $X_{5}$ did, thus it surpassed every simple linear regression in its efficiency of prediction.

Hypothesis for Multiple Linear Regression for Mathematics 160

It was stated earlier in this chapter that hypothesis $A_{1}$ would be considered one of the two major hypotheses of this study. The hypothesis $A_{1}$ is concerned with the regression for Mathematics 160.

A $_{1} \quad$ The F-value for the multiple linear regression does not differ significantly from zero.

Table XXXII presents the summary of the analysis of the multiple regression from Mathematics 160 . The form of this table with the source of its entries is given as Table XLIV in Appendix B.

TABLE XXXII
MULTIPLE REGRESSION ANALYSIS

$$
N=75
$$

| SOURCE OF VARIATION | Df | SUM OF SQUARES | MS | F |
| :--- | :---: | :---: | :---: | :---: |
| Regression | 5 | 78.88 | 15.78 | $24.3^{* *}$ |
| Error | 69 | 44.75 | .648 |  |
| TOTAL | 74 | 123.63 |  |  |

In the preceding tables, it is shown that the regression which is shown by Table $\operatorname{XXX}$ to contain all five independent variables is very highly significant. The significance of a multiple linear regression is determined by the F -value which, when significant, indicates that in the total sum of squares of the dependent variable the reduction due to the combined effects of $X_{1}, X_{2}, X_{3}, X_{4}, X_{5}$ are not likely the result of chance.

The formula for calculating the F-value is given in Appendix A as formula 10. The F-value for the multiple linear regression for Mathematics 160 was $F=24.3$. This $F$-value is very highly significant. An F-value of 9.17 would be a $1 \%$ point, it could be expected to be exceeded only $1 \%$ of the time by chance, for the distribution of F and $F=24.3$ is even larger than $F=9.17$. The writer, therefore, fails to accept the major hypothesis, $A_{1}$, at either the $5 \%$ or $1 \%$ levels.

## Part III. The Validation Groups

After the multiple regression equation for the study groups were developed, the test data for the members of the validation groups were substituted into their respective "course group" equations and grade point averages were predicted. Coefficients of correlation between the predicted grades and the actual grades were computed by the Pearson product-moment method. Probable successful or unsuccessful performances were calculated using the regression equation. This was checked against their actual grades to evaluate the efficiency of
prediction. An assertion by Vineyard may help to clarify the importance of the above procedures.

However, when a researcher finds that relationships found between variables within one group or sample tend to hold fairly constant in a subsequent sample from the same population, he feels much more confident about his findings. If it is found that the coefficient of correlation between actual and predicted grades for the validation group does not differ significantly from the coefficient of multiple correlation between the test variables and the criterion, then we feel that we are dealing with relationships which remain fairly stable from sample to sample within the population. If the two coefficients of correlation differ significantly, then we may assume that we are dealing with relationships which vary, for reasons which may be known, suspected, or unknown from sample to sample within the same population. ${ }^{2}$

The ultimate purpose for which the findings of the study will be used is to assist teachers, placemement officials, and students in determining the proper course sequence in which a student should begin his study. An analysis of predicted grades was made to see which predicted score gave the highest percentage of efficiency in predicting successful students as well as the highest percentage in identifying the unsuccessful. The study groups were used to make a chart with this information given. The validation group for each course was analyzed on the basis of the appropriate chart and the percentage of error determined. This procedure was used to make it easier for future use of the study and to determine the two levels, successful and unsuccessful, on which it would be necessary to consider student performance.

[^19]Mathematics 110

The validation group for Mathematics 110 consisted of 40 students. Table XXXIII gives the data on the independent variables as well as $Y_{1}$ for this group.

TABLE XXXIII
TEST DATA FOR THE VALIDATION GROUP

$$
N=40
$$

| VARIABLE | $\mathrm{X}_{1}$ | $\mathrm{X}_{2}$ | $\mathrm{X}_{3}$ | $\mathrm{X}_{4}$ | $\mathrm{X}_{5}$ | $\mathrm{Y}_{1}$ |
| :--- | ---: | ---: | :---: | :---: | :---: | :---: |
| Means | 141.40 | 4.83 | 7.91 | 57.90 | 15.08 | 2.40 |
| S. D. | 10.73 | 4.66 | 1.28 | 21.30 | 6.57 | 1.22 |

The data presented in Table XXXIII were examined in their relation to the comparable data for the Mathematics 110 study group. Table XXXIV gives the confidence interval limits for the mean and standard deviation for the population of the study group.

TABLE XXXIV
CONFIDENCE INTERVALS FOR POPULATION MEANS AND S.D.'S OF MATHEMATICS 110 STUDY GROUP

$$
N=225
$$

| $\begin{gathered} \text { PREDICTOR } \\ X_{i} \end{gathered}$ | $\begin{gathered} \text { MEAN } \\ X_{i} \end{gathered}$ | S.D. | 5\% LEVEL LIMITS | 1\% LEVEL LIMITS |
| :---: | :---: | :---: | :---: | :---: |
| $\mathrm{X}_{1}$ | 143.40 | 10.61 | $\begin{array}{r} 142.32<\mu_{1}<145.10 \\ 10.59<\sigma_{1}<11.63 \end{array}$ | $\begin{array}{r} 141.88<\mu_{1}<145.54 \\ 9.32<\sigma_{1}<11.90 \end{array}$ |
| $\mathrm{X}_{2}$ | 5.10 | 4.94 | $\begin{aligned} & 4.45<\mu_{2}<5.75 \\ & 4.48<\sigma_{2}<5.40 \end{aligned}$ | $\begin{aligned} & 4.25<\mu_{2}<5.95 \\ & 4.34<\sigma_{2}<5.55 \end{aligned}$ |
| $\mathrm{X}_{3}$ | 7.80 | 1.87 | $\begin{aligned} & 7.55<\mu_{2}<8.05 \\ & 1.70<\sigma_{3}<2.04 \end{aligned}$ | $\begin{aligned} & 7.48<\mu_{3}<8.12 \\ & 1.44<\sigma_{3}<2.10 \end{aligned}$ |
| $\mathrm{X}_{4}$ | 55.76 | 21.07 | $\begin{aligned} & 53.00<\mu_{4}<58.52 \\ & 19.12<\sigma_{4}<23.02 \end{aligned}$ | $\begin{aligned} & 52.12<\mu_{4}<59.40 \\ & 18.51<\sigma_{4}<23.63 \end{aligned}$ |
| $\mathrm{X}_{5}$ | 12.88 | 6.27 | $\begin{array}{r} 12.86<\mu_{5}<13.70 \\ 5.69<\sigma_{5}<6.85 \end{array}$ | $\begin{array}{rr} 12.80 & <\mu_{5}<13.96 \\ 5.51<\sigma_{5}<7.03 \end{array}$ |

$\mu_{i}=$ population mean with respect to the $i-t h$ predictor
$\sigma_{i}=$ the standard deviation of the population with respect to the i-th predictor.

The means of $X_{2}, X_{3}$, and $X_{4}$ for the validation group fell within the $5 \%$ limits and thus within the $1 \%$ limits also. The mean of $X_{1}$ and $X_{5}$ did not fall within the $1 \%$ limits and, therefore, did not fall within the $5 \%$ level limits. The S.D. of $X_{1}, X_{2}, X_{4}$ and $X_{5}$ fell within both the $1 \%$ and $5 \%$ limit levels. The S.D. of $X_{3}$ did not fall within the $5 \%$ or $1 \%$ level limits.

The Pearson Product-Moment correlation was obtained by using the actual and predicted grade of the Mathematics 110 validation group. A multiple R of .797 was obtained for the validation group while $R=.691$ had been obtained from the study group. The significance of the difference between these two R's will be examined in the hypotheses at the end of this section.

It has been mentioned earlier that students with a grade point average of 2.0 were considered to be successful. A predicted grade point average of 2.0 was considered as indicative of success. However, since, as usual, there was no perfect predictions made, some consideration was given to the standard error of $Y_{1}$ in making predictions. In Table XXXV, we have a comparative distribution of grades of the study group when 2.0 is used as a measure of success.

TABLE XXXV

> DISTRIBUTION OF GRADES OF THE STUDY GROUP BY SUCCESS OR NON-SUCCESS CLASSIFICATION USING A CRITERION OF 2.0 AS A MEASURE OF SUCCESS $N=225$

|  | SUCCESSFUL | UNSUCCESSFUL |
| :--- | :---: | :---: |
| Predicted <br> Successful | 90 | 8 |
| Predicted <br> Unsuccessful | 60 | 67 |

Error of Prediction $\frac{68}{225}=.30$ or $30 \%$

In Table XXXV the error rate for prediction of successful students was $\frac{60}{150}$ or $40 \%$. The error rate for predicting the unsuccessful was $\frac{8}{75}$ or approximately $11 \%$. The error rate over-all was $E R=\frac{68}{225}$ or $30 \%$.

Table XXXVI gives the distribution of errors when the standard error of $Y_{1}$ was considered. Using the standard error of $Y_{1}$, which was 1.13, a student was considered a probable success if his predicted grade point exceeded or equaled $2 \pm(1.13)$.

TABLE XXXVI
DISTRIBUTION OF GRADES OF STUDY GROUP BY SUCCESS
OR NON-SUCCESS CLASSIFICATION USING A CRITERION OF 2 - (STANDARD ERROR OF $Y_{1}$ ) AS A MEASURE OF SUCCESS $\mathrm{N}=225$

SUCCESSFUL
UNSUCCESSFUL

Predicted
Successful
140
41

Predicted Unsuccessful

10
34

Error of Prediction $=\frac{51}{225}=22.6$ or $22.6 \%$

Referring to Table XXXVI, the error rate for a prediction rate of the successful was $\frac{10}{150}$ or $6.6 \%$. The error rate for an unsuccessful prediction was $\frac{41}{75}$ or $54.7 \%$. The overall error rate was $E R=\frac{51}{225}$ or $22.6 \%$.

The error rate of prediction shown in Table XXXVII is that of the Mathematics 110 validation group. This table was constructed using the results of Table XXXVI.

TABLE XXXVII

DISTRIBUTION OF GRADES OF VALIDATION GROUP BY SUCCESS
OR NON-SUCCESS CLASSIFICATION USING A CRITERION OF 2 - (STANDARD ERROR OF Y ${ }_{1}$ ) AS A MEASURE OF SUCCESS ${ }^{1}$
$\mathrm{N}=40$

|  | SUCCESSFUL | UNSUCCESSFUL |
| :--- | :---: | :---: |
| Predicted <br> Successful | 25 | 7 |
| Predicted <br> Unsuccessful | 2 | 6 |
|  | Error of Prediction $=\frac{9}{40}=.225$ |  |
|  | or $22.5 \%$ |  |

The error rate of a successful prediction was $\frac{2}{27}$ or $7.4 \%$. The error rate for an unsuccess prediction was $\frac{7}{13}$ or $53.8 \%$. The overall error rate was $E R=\frac{9}{40}$ or $22.5 \%$. This was very close to the results for the study group found in Table XXXVI.

Hypotheses for Mathematics 110 Validation Group

Two hypotheses were concerned with the validation group for Mathematics 110. The first was $B_{6}$ which was concerned with information relative to expected shrinkage of $R$, and the significance of the regression for future populations. The second hypothesis was $B_{7}$ which dealt with the problem of whether the validation group was significantly different from the study group.

B6 The correlation coefficient between the actual student grades and the grades predicted by the multiple-regression equation for the validation group used in this study will not differ significantly from zero.

Given much earlier the correlation coefficient between the predicted and actual grades of the validation group was $R=.80$, The number of students in the validation group was 40 . Using formula 1 in the appendix to calculate $t$, a t-value of 8.23 was obtained. The probability of a $t$-value larger than 2.75 was .01 since $F=t^{2}$, the $F$-value was 67.73. This value is much greater than the 13.83 which would be required for significance at the .01 level. The writer failed to accept the hypothesis $B_{6}$ based upon the above information.

$$
\# \# \# \# \# \#
$$

$B_{7} \quad$ Using the multiple linear regression, the correlation between predicted and actual grades for the validation group does not differ significantly from the correlation between predicted and actual grades for the study group.

The coefficients of correlation between the predicted and actual grades of the validation group was $r=.80$. The coefficient for the study group was $r=$.69. A test was made to determine if $r=.80$ when $N=40$ differed significantly from $r=.69$ when $N=225$. Formulas 8 and 9 in the appendix were used to obtain a value $z=1.40$. Since $-1.96<1.40<1.96$ and $-2.58<1.40<2.58$, the writer failed to reject $A_{7}$ at either the $5 \%$ or $1 \%$ levels.

Mathematics 160

The validation group for Mathematics 160 consisted of 25 students. Table XXXVIII gives the test data on this group.

## TABLE XXXVIII

TEST DATA FOR THE VALIDATION GROUP
$\mathrm{N}=25$

| VARIABLE | $\mathrm{X}_{1}$ | $\mathrm{X}_{2}$ | $\mathrm{X}_{3}$ | $\mathrm{X}_{4}$ | $\mathrm{X}_{5}$ | $\mathrm{Y}_{2}$ |
| :--- | ---: | ---: | ---: | :---: | :---: | :---: |
| Mean | 143.30 | 12.56 | 7.91 | 55.4 | 20.2 | 2.16 |
| S. D. | 9.95 | 7.74 | 1.74 | 20.69 | 8.54 | 1.39 |

The test data presented in Table XXXVIII were examined in their relation to the test data for the Mathematics 160 study group. Table XXXIX gives the confidence interval limits for both the mean and S.D. for the population of the study group.

TABLE XXXIX
CONFIDENCE INTERVAL ESTIMATES FOR POPULATION MEANS AND S.D.'S OF STUDY GROUP

$$
N=75
$$

| $\begin{gathered} \text { PREDICTOR } \\ \mathrm{X}_{\mathbf{i}} \end{gathered}$ | $\begin{gathered} \text { MEAN } \\ X_{i} \end{gathered}$ | $\begin{gathered} \text { S.D. } \\ S_{i} \end{gathered}$ | 5\% LEVEL LIMITS | 1\% LEVEL LIMITS |
| :---: | :---: | :---: | :---: | :---: |
| $\mathrm{X}_{1}$ | 145.52 | 9.62 | $\begin{array}{r} 143.3<\mu_{1}<147.70 \\ 8.08<\sigma_{1}<11.16 \end{array}$ | $\begin{array}{r} 142.65<\mu_{1}<148.39 \\ 7.60<\sigma_{1}<11.65 \end{array}$ |
| $\mathrm{X}_{2}$ | 14 | 11.67 | $\begin{array}{r} 11.36<\mu_{2}<16.64 \\ 9.80<\sigma_{2}<13.54 \end{array}$ | $\begin{aligned} 10.52 & <\mu_{2}<17.48 \\ 9.21 & <\sigma_{2}<14.13 \end{aligned}$ |
| $\mathrm{X}_{3}$ | 8.73 | 1.73 | $\begin{aligned} & 8.34<\mu_{3}<9.11 \\ & 1.41<\sigma_{3}<2.01 \end{aligned}$ | $\begin{aligned} & 8.21<\mu_{3}<9.25 \\ & 1.37<\sigma_{3}<2.09 \end{aligned}$ |
| $\mathrm{X}_{4}$ | 67.69 | 16.50 | $\begin{aligned} & 63.94<\mu_{4}<71.44 \\ & 18.86<\sigma_{4}<19.14 \end{aligned}$ | $\begin{aligned} & 62.45<\mu_{4}<72.92 \\ & 11.03<\sigma_{4}<21.98 \end{aligned}$ |
| $\mathrm{X}_{5}$ | 20.25 | 6.46 | $\begin{aligned} 18.79 & <\mu_{5}<21.71 \\ 5.43 & <\sigma_{5}<7.49 \end{aligned}$ | $\begin{aligned} 18.33 & <\mu_{5}<22.17 \\ 5.10 & <\sigma_{5}<7.82 \end{aligned}$ |

$\mu_{i}=$ population mean with respect to the $i-t h$ predictor
$\sigma_{i}=$ the standard deviation of the population with respect to the i-th predictor

All of the means of the validation group, except $X_{3}$, fell within the $5 \%$ confidence level. The mean for $X_{3}$ did not fall within either the $5 \%$ or $1 \%$ level. The standard deviation of $X_{1}$ and $X_{3}$ fell within the $5 \%$ level while the standard deviation of $X_{1}, X_{3}, X_{4}$ fell within the $1 \%$ level. The standard deviation of $X_{2}$ and $X_{5}$ did not fall within either the $5 \%$ or $1 \%$ level of confidence limits.

The Pearson Product-Moment correlation was obtained by using the predicted and actual grades of the validation group. A multiple $R$ of .866 was obtained while $R=.638$ had been obtained from the study group. The significance of the difference between these two R's will be examined later in the hypotheses.

A grade point average of 2.0 was considered to be indicative of probable success. It was necessary, however, to give some consideration to the standard error of $Y_{2}$ in making predictions. Table XL gives the distribution when the consideration of success is based wholly upon a 2.0 or better predicted score.

TABLE XL
DISTRIBUTION OF GRADES OF THE STUDY GROUP BY SUCCESS OR NON-SUCCESS CLASSIFICATION USING A CRITERION OF 2.0 AS A MEASURE OF SUCCESS

$$
N=75
$$

|  | SUCCESSFUL | UNSUCCESSFUL |
| :--- | :---: | :---: |
| Predicted <br> Successful | 27 | 5 |
| Predicted <br> Unsuccessful | 23 | 20 |
|  | Error of Prediction $=\frac{28}{75}=.373$ or $37.3 \%$ |  |

In Table XL, the error rate for prediction of successful students was $23 / 50$ or $46 \%$. The error rate for predicting the unsuccessful was $\frac{5}{25}$ or $20 \%$. The error rate over-all was $E R=\frac{28}{75}$ or $37.3 \%$.

Table XLI gives the distribution when the standard error of Y is taken into consideration. Using the standard error of Y , which is 1.03 , a student was considered a probable success if his predicted grade point exceeded or equaled $2 \pm$ (1.03).

TABLE XLI
DISTRIBUTION OF GRADES OF STUDY GROUP BY SUCCESS OR NON-SUCCESS CLASSIFICATION USING A CRITERION OF 2 - (STANDARD ERROR OF Y 2 ) AS A MEASURE OF SUCCESS

$$
N=75
$$

|  | SUCCESSFUL | UNSUCCESSFUL |
| :--- | :---: | :---: |
| Predicted <br> Successful | 48 | 17 |
| Predicted <br> Unsuccessful | 2 | 8 |
| Error of Prediction $=\frac{19}{75}=.253$ or $25.3 \%$ |  |  |

Referring to Table XLI, the error rate for a prediction rate of the successful was $\frac{2}{50}$ or $4 \%$. The error rate for prediction of the successful was $\frac{17}{25}$ or $85 \%$. The over-all error rate was $E R=\frac{19}{75}$ or $25.3 \%$.

The error rate of prediction shown in Table XLII is that of the validation group. This table was calculated using the results of Table XLI.

TABLE XLII

> DISTRIBUTION OF GRADES OF VALIDATION GROUP BY SUCCESS OR NON-SUCCESS CLASSIFICATION USING A CRITERION OF
> 2 - (STANDARD ERROR OF Y ${ }_{2}$ ) AS A MEASURE OF SUCCESS
> $\mathrm{N}=25$

|  | SUCCESSFUL | UNSUCCESSFUL |
| :--- | :---: | :---: |
| Predicted <br> Successful | 16 | 7 |
| Predicted <br> Unsuccessful | 1 | 1 |
|  | Error of Prediction $=\frac{8}{25}=.32$ or $32 \%$ |  |

The error rate of prediction shows in Table XXXVIII is $32 \%$. This is slightly higher than that for Table XXXVII but lower than for Table XXXVI.

Hypotheses for Mathematics 160 Validation Group

There were two hypotheses which were concerned with the Mathematics 160 validation group. The first one was $A_{6}$, which was to give information relative to the shrinkage problems and the significance of the regression for future population, and $A_{7}$, which dealt with the
problem of whether the correlations obtained for the validation group were significantly different from those for the study group.

A6 The correlation coefficient between the actual student grades and the grades predicted by the multipleregression equation for the validation group used in this study will not differ significantly from zero.

The correlation coefficient for the validation group was $R=.866$. The number of students in the validation
group was 25 . Using formula 1 in the appendix to calculate $t$, a $t$-value of $t=10.2$ was obtained. The probability of a t-value larger than 2.57 is .01 . Since $F=t^{2}$, the $F$-value was $F=104.04$. This

F-value is decidedly greater than the F-value 14.02
which would be required for significance at the . 01
level. The writer failed to accept the hypothesis
$A_{6}$ based upon the above information.
\# \# \# \# \#
$A_{7} \quad$ Using the multiple linear regression, the correlation between predicted and actual grades for the validation group does not differ significantly from the correlation between predicted and actual grades for the study group.

The coefficient of correlation obtained by using the predicted and actual grades of the validation group was $r=.866$. The coefficient for the study group was $\mathrm{r}=.638$. A test was made to see if $\mathrm{r}=.866$ when $N=25$ differed significantly from $r=.638$ when $N=75$. Formulas 8 and 9 in the appendix were used to obtain a $z$ value $z=.254$. Since $-1.96<.254<1.96$
and $-2.58<.254<2.58$, the writer failed to
reject $A_{7}$ at either the $5 \%$ or $1 \%$ levels.

## CHAPTER V

## SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

When we consider the rapidly expanding college enrollment; the changing complexion of the freshman college enrollment; the changing function of college freshman mathematics courses; and the shortage in the supply of well trained college mathematics teachers; the necessity for being able to choose the correct course sequence in which to begin a student's mathematics training becomes apparent. Southern University, Baton Rouge, Louisiana, has two courses into which the majority of the entering freshman class is enrolled. The first of these courses, Mathematics 110, is quite elementary, Both Mathematics 110 and Mathematics 160 have been described in Chapter I. The work in Mathematics 160 is more advanced and for students who do not have the proper background is often extremely difficult to successfully pass.

The unifying theme of this study is that the two courses on which the study has been based are the two major beginning points for freshman students at Southern University. A remedial course is provided in Southern University's Bureau of Developmental Service, but it is opinion of this writer that any student found to be able to do so should enter the regular freshman sequence most closely associated with his major field of study at the university. The course most valuable for the student to pursue will have already been chosen for him by the officials of his college or major department.

The big question facing placement officials, students and mathematics teachers in general is, 'What is the proper background which would enable a student to experience success in his freshman mathematics courses?" This study has tried to shed some light upon this question. The writer has developed two multiple regression equations that can be used to give assistance in placing students in the proper beginning sequence as well as to help in appraising students of their probability of success in these beginning courses.

It must be clearly understood and the writer is aware that no scheme designed to predict success or failure will be correct in all cases and that the person making the decision must be able to weigh many other circumstances before making a decision, and in particular the student's attitude toward his work must be considered. No measure of attitude or interest is included in the freshman test battery, which in the writer's opinion is a serious omission.

It is in the spirit of this study that as many factors as the placement official find pertinent to the case under consideration be brought to bear. The following course summaries are given in the light of the foregoing discussion.

Summary: Mathematics 110

The equations developed for Mathematics 110 are believed to be of greater assistance to the placement official or student, in deciding his direction, than a mere guess. The writer had expected that the independent variable $X_{5}$, weighted high school average, would have proven to be the best single predictor.

In the light of the findings of the study, no such claim for $X_{5}$ can be made. The Cooperative Mathematics Test was found to account for more variance than any of the other single predictors. The weighted high school average was second in accounting for variance. It is perhaps not surprising that in an elementary course such as Mathematics 110 very rudimentary knowledge would be important and that ability would play a large role. The A.C.E. was significant in its ability to predict success in Mathematics 110.

It should be noted that each of the predictors occurred in the multiple regression equation. It must then be assumed that each had something to contribute to the determination of success or failure of the student. The multiple regression, as was expected, proved to be the best predictor and is the measure which accounted for a higher percentage of the variance than any other predictor.

It is asserted here that weighted high school mathematics average, although not the best single predictor, made a worthwhile contribution in the determination of the best predictor. It must also be remembered that no superiority could be shown for the Cooperative Mathematics Test over weighted high school mathematics average as a predictor of success. It must also be remembered that the second variable selected in the step-wise regression was $X_{5}$.

The group used for cross validation of the multiple regression satisfied the condition necessary to be representative of the same student population on the study group. This should remove most of the fear that as these equations are used from year to year there may be a great shrinkage in their ability to predict. It should, of course, be understood that with a great amount of change in student
preparation at the high school level some shrinkage will occur with the passage of time.

In view of the findings with the validation group, it would be wise that the standard error of $Y$ be considered whenever a decision on the possibility of success is to be made. It is also true that no predictor should be used to keep an eager and interested student, even if he is poorly prepared, from attempting a course which he desires to attempt. However, the idea of pursuing non-credit courses is not a popular one, so care must be taken to assure that what seems to be eagerness to pursue a certain course is not merely an attempt to omit a prolonged stay in mathematics courses. The following conclusions were reached relative to the Mathematics 110 course.

1. Any of the simple regressions involving either of the independent variables, $X_{1}, X_{2}, X_{3}, X_{4}, X_{5}$, can assist in making proper decisions on placement or non-placement in Mathematics 110.
2. The best of the simple linear regressions for prediction involves $X_{2}$, scores on Cooperative Mathematics Test and the second best involves $X_{5}$, weighted high school mathematics average.
3. The multiple linear regression equation developed in this study will significantly aid in predicting success in Mathematics 110.
4. The equations developed in this study can be used on similar student populations without too much loss of applicability.

Summary: Mathematics 160

The successful completion of Mathematics 160 usually requires more than the minimum of ability and former training on the part of the student. A perplexing problem has been the determination of just how much ability and/or former training is necessary for the desired
success. The writer's opinion was that the independent variable $X_{5}$, weighted high school mathematics, would prove to be a considerably better single predictor than any other measure.

In the light of the findings of this study, it can be assumed that although $X_{5}$ surpasses $X_{1}$, scores on Cooperative English Test, $X_{2}$, scores on Cooperative Mathematics Test; and $X_{3}$, scores on NelsonDenny Reading Test, the independent variable $X_{4}$, scores on A.C.E., must be given equal impact in the determination of success. The independent variable, $X_{5}$, accounted for the greatest amount of variance of any single predictor, but was closely followed by $X_{4}$ in the amount of variance accounted for by the regression and in predictive efficiency.

It should be noted that each of the single predictors occurred in the multiple regression. This implies that they each had a contribution to make in the determination of probable success. The multiple regression was a decidedly more significant predictor than any of the single predictors as is shown by the much greater amount of variance in $Y_{2}$ for which it was able to account.

It is asserted here that weighted high school mathematics average proved to be a worthwhile addition to the test battery in determining success. The second measure of very great importance was the independent variable $X_{4}, A^{A} C . E$ test scores.

It would seem that both prior training and ability play an important role in the determination of success in Mathematics 160. One measure missing in the test battery, and which in the writer's opinion would greatly enhance its placement value, is a measure of student interest. In courses above the most elementary levels, the student's interest
and his understanding of the future usefulness of the material plays an important part in his desire to work for success.

In the multiple regression, the first variable selected by the step-wise procedure was $X_{5}$. This implies that $X_{5}$ accounted for more variance in the dependent variable than any other single predictor. The second variable selected was $X_{4}$. This implies that once the variability accounted for by $X_{5}$ has been determined, $X_{4}$ was the next highest accountor for the remaining variance.

A surprising result was that of all the single predictors in the study; $X_{2}$, scores on Cooperative Mathematics Test, was the poorest. It may be that the level of response required in this test is not in keeping with the content level of the Mathematics 160 course.

The group used for cross validation of the multiple regression satisfied the requirements necessary to be considered a part of the same student population. No great shrinkage in the correlation coefficient was observed, and the correlation coefficient found between actual and predicted students grades from the validation group did not differ significantly from the correlation coefficient for the study group. It should be noted that with the changing high school programs and better prepared students the result must be closely observed for future shrinkage effects.

In view of the findings for the validation group, it is necessary that the standard error of $Y$ be considered in determining the possibility of success or failure of the entering students. It is also cautioned that the placement officials be aware that these are factors which may account for success or failure other than the ones included in this study.

1. Any simple regression determined as a result of this study will significantly aid in determining the probability of the success or failure of a student in Mathematics 160.
2. The best single predictor in determining the probability of the success of a student in Mathematics 160 is $X_{5}$, weighted high school mathematics average, but it is closely followed by $X_{4}$, A.C.E. test scores.
3. The multiple linear regression equation developed in this study will significantly aid in determining the feasibility of enrolling a student in Mathematics 160 .
4. The equations developed in this study are of enough generality to be applied to the freshman student population at Southern University.
5. Some study should be conducted relative to the Cooperative Mathematics Test, Form $X$ being used as the mathematics test in the freshman test battery when advanced placement is to be considered.

## Recommendations

It is recommended that the equations, especially the multiple linear regression developed in this study, be used by the counselors of students in the entering freshmen class at Southern University and other similar student bodies entering similar courses. It should be realized, however, that these results must not be used alone but in conjunction with other factors including fomner teachers' estimates of the student's ability, motivation, and the emotional maturity of the student. It is also recommended that occasional validity checks of these techniques be made with different student groups.

It is recommended that some measure of student interest be added to the freshmen test battery at Southern University. Prior research supports the opinion of this writer that the inclusions of
sưch a measure would be of great value in determining freshmen placement, especially in Mathematics 160.

More research of the same nature as that of the present study is needed and recommended. The weighted high school mathematics average seemed to be a worthy addition to the prediction variables, but other weightings should be tried to determine their general effectiveness.

The possibility of finding a more applicable mathematics test for advanced placement than the Cooperative Mathematics Test Form X should be explored. A test to be constructed by the Marhematics Department at Southern University should be considered and developed.

Such studies as the present one might prove to be of value in other academic areas of the university. The inclusion of measures such as interests, attitude, and personality traits, might enhance the placement. value of the freshmen test battery.

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APPENDIX A
i. $t=r \sqrt{\frac{N-2}{1-r^{2}}} \quad N-2$ degrees of freedom
2. $B_{i}=b_{i} \frac{S_{i}}{S_{y_{k}}}$
$i=1,2,3,4 ; 5 ; k=1,2$
3. $E=100 \%\left(1-\sqrt{1-r^{2}{ }_{x_{i} y_{k}}}\right) \quad k=1,2 ; i=1,2,3,4,5$
4. $\left.\int_{\left(r_{y_{k} x_{i}}\right.}-r_{y_{k} x_{1}}\right)=\sqrt{6 r_{y_{k} x_{i}}^{2}+\sqrt{r_{y_{k} x_{1}}^{2}}-2 r_{r_{y_{k} x_{i}} r_{x_{1} y_{k}} \sigma r_{y_{k} x_{i}} \sigma r_{y_{k} x_{1}}}}$
$\mathrm{k}=1,2$; $\mathrm{i}=1,2,3,4,5$;
$1=1,2,3,4,5$
i $\neq 1$

$i=1,2,3,4,4$
$k=1,2$
$1=1,2,3,4,5$
i $\neq 1$.
6. $T=\frac{r_{y_{k} x_{5}}-r_{y_{k} x_{i}}}{\left(r_{y_{k} x_{5}}-r_{y_{k} x_{i}}\right)}$
$i=1,2,3,4$
$k=1,2$
7. $\sigma_{\sigma}=\frac{\sigma}{2\left(\mathrm{~N}_{\mathrm{s}}\right)}$
8. $S=.5 \ln \left(\frac{1+r}{1-r}\right)$
9. $z=\frac{\int 1-\int 2}{\sqrt{\frac{1}{N_{1}-3}+\sqrt{N_{2}-3}}}$
10. $\mathrm{F}=\frac{\text { additional reduction mean square }}{\text { residual mean square }}$

APPENDIX B

TABLE XLIII

$$
\begin{gathered}
\text { ANALYSIS OF VARIANCE OF LINEAR } \\
\text { REGRESSION } \\
n=\mathrm{N}
\end{gathered}
$$

| Source of Variation | df | Symbolic SS | M.S. |
| :---: | :---: | :---: | :---: |
| Residual | 1 | $(\Sigma x y)^{2} / \Sigma x^{2}$ | $(\Sigma x y)^{2} / \Sigma x^{2}$ |
| TOTAL | $n-2$ | by subtraction | $\frac{\text { Residual SS for } Y}{n-2}$ |

TABLE XLIV
FORM OF MULTIPLE REGRESSION ANALYSIS
( $\mathrm{n}=$ number of multiple observations; $\mathrm{k}=$ number of independent variables)


VITA<br>Vernon Williams<br>Candidate for the Degree of<br>Doctor of Education

Thesis: A MULTI-PREDICTOR MEASURE FOR PREDICTION OF SUCCESS AT TWO LEVELS OF PLACEMENT IN FRESHMEN MATHEMATICS

Major Field: Higher Education
Biographical:
Personal Data: Born in Augusta, Georgia, November 10, 1926, the son of Mr. Julius and Mrs. Clara B. Williams.

Education: Graduated from Augustus R. Johnson High School (now Lucy C. Laney High School), Augusta, Georgia, in June, 1945; received Bachelor of Arts degree from Paine College, Augusta, Georgia in 1949, with a major in Natural Science; attended Atlanta University, Atlanta, Georgia, for summers of 1949, 1950; received the Master of Arts degree in Mathematics from University of Michigan in 1953; attended University of Michigan summer 1956; Michigan State University for summer of 1960 on a National Science Grant for College Mathematics Teachers; Teaching Associate in Mathematics, Wayne State University 1960-1961; completed requirements for the Doctor of Education degree at Oklahoma State University in May, 1969.

Professional organizations: Alpha Kappa Mu; Pi Mu Epsilon; Phi Delta Kappa; Mathematics Association of America; Louisiana Education Association.

Professional experience: Instructor of Mathematics, Paine College, Augusta, Georgia, 1949-1954; Instructor of Mathematics, Florida A and M University, Tallahassee, Florida, 1954-1956; Assistant Professor of Mathematics, Southern University, Baton Rouge, Louisiana, 1956-1962; Associate Professor of Mathematics, Southern University, 1962-present.


[^0]:    Mathematics 130, College Algebra, was originally included in this study but was exciuded because of the changed composition of the student population and its being assigned to serve a cognste area at the sophomore level. It has as its prerequisite Mathematics 110 on its equival-

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