

STUDENTS' BELIEFS ABOUT THE NATURE OF
MATHEMATICS IN THE CONTEXT OF ONLINE
HOMEWORK

By

OLIVIA WILLHOITE

Bachelor of Science in Mathematics Education

Oral Roberts University

Tulsa, OK

2017

Submitted to the Faculty of the
Graduate College of the
Oklahoma State University
in partial fulfillment of
the requirements for
the Degree of
MASTER OF SCIENCE
July, 2020

STUDENTS' BELIEFS ABOUT THE NATURE OF
MATHEMATICS IN THE CONTEXT OF ONLINE
HOMEWORK

Thesis Approved:

Dr. Allison Dorko

Thesis Adviser

Dr. Melissa Mills

Dr. Michael Oehrtman

ACKNOWLEDGEMENTS

I want to thank my advisor, Dr. Allison Dorko, for your guidance and support throughout the process of conducting this study. You have been an encouragement and a mentor to me during my time at OSU. I also want to thank my committee members, Dr. Melissa Mills and Dr. Michael Oehrtman, for your feedback on previous versions of this study.

Thank you to Adam and Emily, for being like family to me during my time in Stillwater. Thank you to my parents, for your endless love and support. Finally, thank you to my heavenly Father for your provision and strength, without which none of this would have been possible.

Name: OLIVIA WILLHOITE

Date of Degree: JULY, 2020

Title of Study: STUDENTS' BELIEFS ABOUT THE NATURE OF MATHEMATICS
IN THE CONTEXT OF ONLINE HOMEWORK

Major Field: MATHEMATICS

Abstract: It has been reported in the mathematics education literature that many students hold unproductive beliefs about the nature of mathematics, which in turn can negatively affect their mathematics achievement. We do not know, however, how online homework affects students' beliefs about the nature of mathematics or their ability to do mathematics. Because of the growing popularity of online homework and the resources which are offered with it (e.g. multiple tries, hints, 'see similar example' features), it is important to explore whether such resources reinforce unproductive beliefs about mathematics, or if they help to alleviate such beliefs. This study seeks to explore what students believe about the nature of mathematics in the context of online homework, as well as what they believe about mathematics in an outside-of-school context. The latter is of interest because students often see a difference between school mathematics and mathematics in real life. The results of this study, based on quantitative and qualitative survey data coupled with follow-up interviews, indicate that students taking Mathematical Functions and Their Uses (MATH 1483) in the spring 2020 semester hold many productive beliefs about the nature of mathematics both in an online homework and outside-of-school context. Qualitative data from students' short-answer responses on the survey, as well as interview data, give further insight into these beliefs about mathematics in both contexts, and point to ways that online homework platforms can be improved to better support students in developing productive mathematical dispositions.

TABLE OF CONTENTS

Chapter	Page
I. INTRODUCTION	1
1.1 Problem Statement.....	2
II. REVIEW OF LITERATURE	7
2.1 Beliefs About Mathematics.....	7
2.1.2 In Service and Pre-Service Teachers' Beliefs About the Nature of Mathematics	10
2.1.3 Students' Beliefs About the Nature of Mathematics	12
III. THEORETICAL PERSPECTIVE	18
3.1 Defining Beliefs.....	18
3.2 The Structure of Beliefs	20
3.3 A Framework for Students' Mathematical Belief Systems.....	21
IV. METHODOLOGY	23
4.1 Setting and Participants	23
4.2 WebAssign	24
4.3 Data Collection Methods	24
4.3.1 Online Survey	25
4.3.2 Analysis of Survey Data.....	26
4.3.3 Interviews	28
4.3.4 Analysis of Interview Data.....	29
V. FINDINGS.....	31
5.1 Quantitative Results.....	31
5.1.1 Survey Statements About Having Unlimited Tries	36
5.2 Overall Themes Across Survey Statements.....	38
5.3 Formulas Versus Underlying Mathematical Ideas	39
5.3.1 Likert Scale Results for Survey Items 1a and 1b.....	40
5.3.2 Likert Scale Results for Survey Items 12a and 12b.....	41
5.3.3 Themes in Survey Items 1a and 1b.....	43

Chapter	Page
5.3.4 Themes in Survey Items 12a and 12b.....	49
5.4 Underlying Mathematical Ideas Versus Just Getting the Right Answer	53
5.4.1 Likert Scale Results for Survey Items 6a and 6b.....	53
5.4.2 Likert Scale Results for Survey Items 10a and 10b.....	55
5.4.3 Themes in Survey Items 6a and 6b.....	56
5.4.4 Themes in Survey Items 10a and 10b.....	60
5.5 Solving Math Problems in Multiple Ways.....	63
5.5.1 Likert Scale Results for Survey Items 2a and 2b.....	64
5.5.2 Likert Scale Results for Survey Items 8a and 8b.....	65
5.5.3 Themes in Survey Items 2a and 2b.....	66
5.5.4 Themes in Survey Items 8a and 8b.....	70
5.6 The Number of Answers to Math Problems	73
5.6.1 Likert Scale Results for Survey Items 11a and 11b.....	74
5.6.2 Themes in Survey Items 11a and 11b.....	75
5.7 The Amount of Time it Should Take to Solve a Math Problem.....	78
5.7.1 Likert Scale Results for Survey Items 9a and 9b.....	79
5.7.2 Themes in Survey Items 9a and 9b.....	81
5.8 Connections Between Students’ Beliefs and Having Unlimited Tries in Online Homework	84
VI. CONCLUSION.....	86
6.1 Summary of Results and Answers to Research Questions	86
6.2 Implications.....	89
6.3 Limitations	90
6.4 Recommendations for Future Work	91
REFERENCES	93
APPENDICES	101
Appendix A.....	101
Appendix B.....	105
Appendix C.....	109
Appendix D.....	113
Appendix E	116
Appendix F	119
Appendix G.....	122
Appendix H.....	125

LIST OF TABLES

Table	Page
1 Description of the three interview participants.....	29
2 Summary of aggregate mean scores for the 24 beliefs statements on the online survey	32
3 Means for each belief statement/difference between contexts	34
4 Results from survey statements about having unlimited tries	37
5 Descriptive statistics for survey item 1a.....	41
6 Descriptive statistics for survey item 1b	41
7 Descriptive statistics for survey item 12a.....	43
8 Descriptive statistics for survey item 12b.....	43
9 Themes regarding from survey item 1a.....	43
10 Themes regarding from survey item 1b.....	44
11 Themes, criteria, and examples for survey item 1a.....	45
12 Themes, criteria, and examples for survey item 1b.....	46
13 Themes regarding survey item 12a	50
14 Themes regarding survey item 12b	50
15 Descriptive statistics for survey item 6a.....	54
16 Descriptive statistics for survey item 6b.....	55
17 Descriptive statistics for survey item 10a.....	56
18 Descriptive statistics for survey item 10b.....	56
19 Themes regarding survey item 6a	57
20 Themes regarding survey item 6b	57
21 Themes regarding survey item 10a	60
22 Themes regarding survey item 10b	61
23 Descriptive statistics for survey item 2a.....	65
24 Descriptive statistics for survey item 2b.....	65
25 Descriptive statistics for survey item 8a.....	66
26 Descriptive statistics for survey item 8b.....	66
27 Themes regarding survey item 2a	67
28 Themes regarding survey item 2b	67
29 Themes regarding survey item 8a	70
30 Themes regarding survey item 8b	71
31 Descriptive statistics to survey item 11a	75
32 Descriptive statistics to survey item 11b	75
33 Themes regarding survey item 11a	75

Table	Page
34 Themes regarding survey item 11b	76
35 Descriptive statistics to survey item 9a	80
36 Descriptive statistics to survey item 9b	80
37 Codes for survey item 9a	81
38 Codes for survey item 9b.....	81
39 Spread of students' responses to "Having unlimited tries in online homework helps me learn"	84

LIST OF FIGURES

Figure	Page
1. Students' mathematics-related beliefs.....	9
2. Expansion of students' mathematics-related beliefs to include mathematics as a discipline.....	9
3. Relative frequency distribution for the difference in students' aggregate belief scores between contexts.	33
4. Spread of students' responses to survey items 1a and 1b.....	41
5. Spread of students' responses to survey items 12a and 12b.....	42
6. Spread of students' responses to survey items 6a and 6b.....	54
7. Spread of students' responses to survey items 10a and 10b.....	56
8. Spread of students' responses to survey items 2a and 2b.....	64
9. Spread of students' responses to survey items 8a and 8b.....	66
10. Spread of students' responses to survey items 11a and 11b.....	74
11. Spread of students' responses to survey items 9a and 9b.....	80

CHAPTER I

INTRODUCTION

Homework plays a significant role in undergraduate students' learning of mathematics. University Calculus I students spend more time doing homework than they do in class (Ellis et al., 2015; Krause & Putnam, 2016). In upper-division mathematics courses, students are commonly expected to spend considerable time outside of class working on problems (Lew & Zazkis, 2019). Online homework has become increasingly popular in undergraduate mathematics courses, saving instructors and math departments time and resources by providing immediate feedback, instant grading, and individualized assignments. For example, over 1 million students and 2,600 educational institutions use an online homework platform called WebAssign each year (WebAssign.net). Similarly, WebWork, another online homework platform, is used at over 700 colleges and universities (Webwork.maa.org).

Homework is one of students' opportunities to interact with mathematics content. Because students spend a lot of time doing homework, it is important to maximize its potential as a learning opportunity. Few studies have investigated what students learn from homework, how it influences their beliefs about mathematics, or their ability to do mathematics. Both cognitive and affective factors play a critical role in students' mathematical success (e.g., Carlson, 1999; NCTM, 1989; Schoenfeld, 1988). The

technological affordances of online homework, such as immediate feedback, multiple tries, and hints provide unique opportunities to influence students' cognition and affect. The study reported on here follows from the hypothesis that online homework tasks and platforms can be improved to better support students in developing productive beliefs about the nature of mathematics and about their ability to do mathematics.

Students' beliefs regarding mathematics and their ability to do mathematics are important because these beliefs affect the ways in which students engage with mathematical content (e.g., Carlson, 1999; Garofalo, 1989; Kloosterman, 2002; Leder & Forgasz, 2002; Schoenfeld, 1988). The 1989 National Council of Teachers of Mathematics (NCTM) *Standards* emphasized the importance of students' mathematical beliefs, explaining that beliefs "exert a powerful influence on students' evaluation of their own ability, on their willingness to engage in mathematical tasks, and on their ultimate mathematical disposition" (NCTM, 1989, p. 233). Furthermore, the National Research Council (NRC) specified five strands of mathematical proficiency included in the Common Core Standards for Mathematical Practice. The fifth strand that is specified is *productive disposition*, defined as a "habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy" (NRC, 2001, p. 5). Having beliefs about mathematics that lead to a productive disposition is important, but such beliefs are not always held by students. Research that informs how instructional systems can better support students in developing productive mathematical dispositions can contribute to improving success rates in mathematics courses.

1.1 Problem Statement

It has been documented in the literature that many students hold unproductive beliefs

about the nature of mathematics. Schoenfeld (1992) identified the following widely-held beliefs:

1. Mathematics problems have one and only one right answer.
2. There is only one correct way to solve any mathematics problem—usually the rule the teacher has most recently demonstrated to the class.
3. Ordinary students cannot expect to understand mathematics; they expect simply to memorize it, and apply what they have learned mechanically and without understanding.
4. Mathematics is a solitary activity, done by individuals in isolation.
5. Students who have understood the mathematics they have studied will be able to solve any assigned problem in 5 mins or less.
6. The mathematics learned in school has little or nothing to do with the real world.
7. Formal proof is irrelevant to the processes of discovery or invention (Schoenfeld, 1992, numbering added).

Beliefs similar to these have been reported elsewhere in the literature (Hauk & Segalla, 2005; Schoenfeld, 1988; Spangler, 1992). These types of beliefs are unproductive in the sense that they do not support a “habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one’s own efficacy” (NRC, 2001, p. 5), and have the potential to produce undesired consequences. Productive beliefs, on the other hand, support students in persevering in problem-solving (e.g. Carlson, 1999). For example, one typical belief that students hold about the nature of mathematics is that if a student understands the mathematics they have studied, they should be able to solve any assigned problem in five minutes or less

(Schoenfeld, 1992). As a result of holding such a belief, Schoenfeld (1992) explained that students “will give up on a problem after a few minutes of unsuccessful attempts, even though they might have solved it had they persevered” (p. 359). In contrast, Carlson (1999) found graduate students who believed they could solve problems persisted until the problems were solved and found that these students’ beliefs about mathematics played “a prominent role in [their] success and continued mathematical study” (p. 238). Productive beliefs identified by Carlson (1999) include: mathematics is a process involving “many incorrect attempts”; effort is needed when completing difficult problems; and “persistence will eventually result in a solution to a problem” (p. 254-255). As another example of a consequence of students’ beliefs about the nature of mathematics, Garofalo (1989) found that when students believed that almost all math problems can be solved directly by using facts, formulas, and rules provided by the textbook, they spend their time memorizing these facts and formulas rather than trying to understand the concepts.

Within the literature pertaining to online homework, most of the focus has been on student achievement in online platforms versus paper-and-pencil homework, and what students like or dislike about such platforms. Researchers have only recently begun to focus more on homework task design and how students engage with online homework systems (e.g. Breen & O’Shea, 2019; Dorko, in press, 2020a, 2020b; Holdener & Jones, 2019; Morena, Smith, & Tablbert, 2019). We also know that immediate feedback and multiple tries support students in problem solving (Dorko, in press). However, we do not know how online homework affects students’ beliefs about the nature of mathematics or their ability to do mathematics. Because of the growing popularity of online homework,

and the resources which are offered with it (i.e. multiple tries, hints, ‘see similar example’ features), it is important to explore whether such resources reinforce the non-productive beliefs about mathematics as mentioned above, or if they help to alleviate such beliefs.

As well as identifying students’ beliefs about mathematics in the context of online homework, this study also seeks to explore whether these beliefs are similar to or different from their beliefs about mathematics as a discipline and in real life. This is important because students often see a difference between school mathematics and mathematics in real life (Schoenfeld, 1988). Supporting a productive mathematical disposition involves supporting both students’ beliefs about mathematics in a school context as well as an outside-of-school context, and is important specifically because of the focus population of this study.

The population which this study focuses on are students in Mathematical Functions and Their Uses (MATH 1483), an entry level mathematics course replacing College Algebra for students who are non-STEM majors. This course is designed to be more applicable to students’ interests and majors, situating all problems in real-world contexts, and is designed to leverage students’ real-world knowledge. Hence it is reasonable that students might leave the course with an appreciation for mathematics as relevant to their lives and careers. This aligns with the goal of students learning how to reason deeply and think mathematically in order to contribute to advancing society and “compete in today’s and tomorrow’s economy” (NRC, 2001, p. 144).

In an effort to fill the gap in the literature, this study responds to the questions: *What do MATH 1483 students believe about the nature of mathematics in the context of online homework?* and *What do MATH 1483 students believe about the nature of*

mathematics in an outside-of-school context?

CHAPTER II

REVIEW OF LITERATURE

In this chapter, I review the relevant literature pertaining to students' mathematical beliefs. First, I give a definition of beliefs about mathematics. I then discuss research findings concerning in-service and pre-service teachers' beliefs about mathematics, as well as students' beliefs about mathematics.

2.1 Beliefs About Mathematics

While there has been much research over the years focused on students' mathematical beliefs, there is need to clearly define what one means by *beliefs about mathematics*. There is no consensus in the mathematics education literature concerning a definition of students' mathematical beliefs (Leder & Forgasz, 2002; Törner, 2002). This study follows Op'T Eynde et al. (2002):

Students' mathematics-related beliefs are the implicitly or explicitly held subjective conceptions students hold to be true about mathematics education, about themselves as mathematicians, and about the mathematics class context.

These beliefs determine in close interaction with each other and with students' prior knowledge their mathematical learning and problem solving in class (p. 27).

This definition was developed by Op'T Eynde et al. (2002) after they reviewed four categorizations of students' beliefs provided by Underhill (1988), McLeod (1992),

Kloosterman (1996), and Pehkonen (1995). From these four earlier categorizations, Op'T Eynde et al. (2002) developed a framework of students' mathematical beliefs as including

1. **Students' beliefs about mathematics education:** (a) beliefs about mathematics as a subject, (b) beliefs about mathematical learning and problem solving, and (c) beliefs about mathematics teaching in general
2. **Students' beliefs about the self:** (a) self-efficacy beliefs, (b) control beliefs, (c) task-value beliefs, and (d) goal-orientation beliefs
3. **Students beliefs about the social context:** (a) beliefs about the norms in their own class, (a1) the role and functioning of the teacher , (a2) the role and functioning of the students, and (b) beliefs about the socio-mathematical norms in their own class (Op't Eynde et al., 2002, p. 28)

Op'T Eynde et al. (2002) represented these three dimensions of students' mathematical related beliefs as shown in Figure 1. Jankvist (2009) noted a missing dimension to this characterization: students' beliefs about mathematics as a discipline. This dimension was added by Jankvist (2009) due to the fact that "mathematics as a discipline is rather different than mathematics as a subject", which is included under mathematics education in Op'T Eynde's (2002) categorization. Jankvist (2009) defined students' beliefs about mathematics as a discipline (in contrast to their beliefs about mathematics as a school subject) as including "students' view about mathematics as a pure science, an applied science, a system of tools for societal practice, and a platform for aesthetic experiences" (p. 84-85). Jankvist (2015) expanded the triangle figure offered by Op'T Eynde et al. (2002) as depicted in Figure 2, which shows the added dimension of students' beliefs about mathematics as a discipline.

Figure 1

“Constitutive dimensions of students’ mathematics-related belief systems” illustrated by a triangle with the corners: mathematics education, social context (the class), and the self (Op’T Eynde et al., 2002, p.27, Fig. 2).

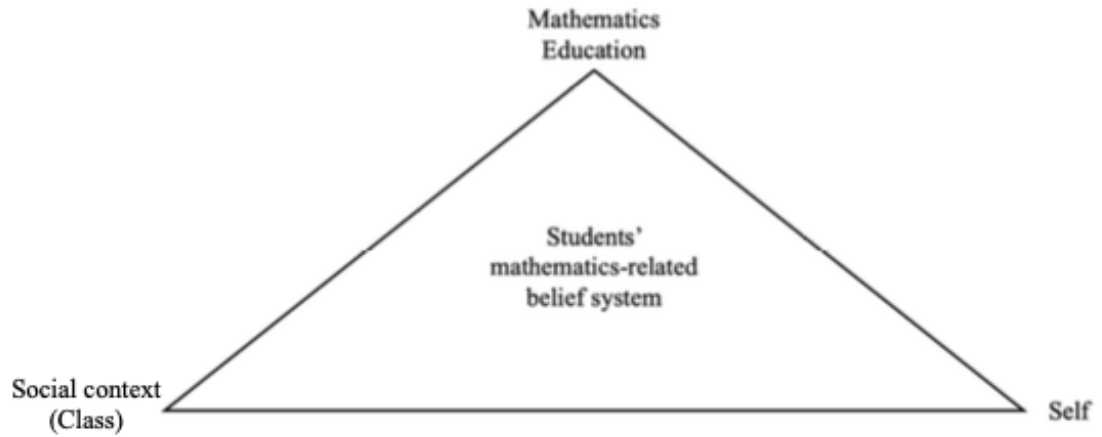
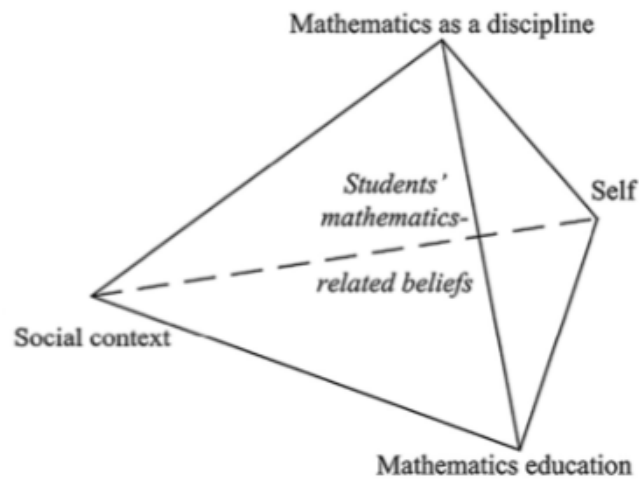


Figure 2:

An expansion of the triangle to a tetrahedron, with the added dimension above the triangle illustrating mathematics as a discipline (Jankvist, 2015, p. 45, Fig. 1).



Because this study seeks to explore what MATH 1483 students believe about mathematics in an online homework context, as well as what they believe about mathematics in an outside-of-school context, the framework provided by Jankvist (2015) will be utilized, with the focus on the two vertices labeled *mathematics education* and *mathematics as a discipline*. I consider mathematics done in an outside of school context to be included within Jankvist's (2015) dimension of mathematics as a discipline. I used the phrasing *outside of school context* within the survey statements (as described in the Methods section) so that students would be more likely to understand what they were being asked, and because I felt like students may not know how to interpret what is meant by *mathematics as a discipline*.

2.1.2 In-Service and Pre-Service Teachers' Beliefs About the Nature of Mathematics

Much of the research on beliefs about mathematics comes from studies of pre-service and in-service teachers. Because the beliefs that teachers have about the nature of mathematics have a powerful influence on students' beliefs, this is not surprising. (See Thompson [1992] and Philipp [2007] for a synthesis on teachers' beliefs, and the influences of teachers' beliefs.) Because pre-service teachers are included in the population of students in this study, this section will discuss work that has been done in examining pre-service teachers' beliefs about the nature of mathematics.

Ernest (1989) described three categories of teachers' beliefs that have been widely reported in studying both in-service and pre-service teachers. The first is *instrumentalism*, which views mathematics as "an accumulation of facts, rules and skills to be used in the pursuance of some external end" (p. 250). According to this view, mathematics is made up of unrelated facts, rules, and procedures. The second is *Platonism*, which views

mathematics as “a static but unified body of certain knowledge” that is “discovered, not created” (p. 250). The third view of mathematics, the *problem-solving* view, considers the subject as a “dynamic, continually expanding field of human creation and invention”, and considers it as a process rather than a product (p. 250).

Alba Thompson (1992) further explained that an individual teacher’s conception of the nature of mathematics may include one or more of the three philosophical views described above. She attributed this to the nature of one’s belief system, and the potential for one to hold seemingly conflicting beliefs. The nature of belief systems is further discussed below when discussing the theoretical perspective.

It has been reported that pre-service teachers hold non-productive beliefs about the nature of mathematics similar to the beliefs that Schoenfeld (1992) mentioned (Ambrose, 2004; Viholainen et al., 2014; Watson, 2019). For example, the pre-service teachers in Viholainen et al. (2014) viewed mathematics as a “ready-made, static system” (p. 169), and did not view it as a subject involving problem solving. In Watson’s (2019) study, pre-service teachers viewed mathematics as static body of knowledge, which is discovered, not created.

There have also been studies exploring whether the unproductive beliefs of pre-service teachers could be altered (e.g. Maasepp & Bobis, 2015; McGinnis et al., 2002; Szydlik et al., 2003). These studies focus on changes in pre-service teachers’ beliefs following courses emphasizing problem-solving and teacher education programs designed to specifically encourage more productive mathematical beliefs. For example, Szydlik et al. (2003) found that it is possible for elementary pre-service teachers’ beliefs about mathematics to change as a result of a number systems course that encouraged

students to problem-solve and which fostered “sense-making rather than memorization” (p. 256). The change in the pre-service teachers’ beliefs about the nature of mathematics changed dramatically, as seen from survey and interview data. The interviews at the end of the course revealed that students attributed their change in beliefs to norms of the classroom culture, namely that the course emphasized challenging problems, the teacher provided very little assistance in answering problems, and that the students worked as a community to solve the problems. Similar to Szydlik et al. (2003), McGinnis et al. (2002) found that pre-service math and science teachers’ beliefs shifted to be more aligned with a productive mathematical disposition after participation in a teacher education program which emphasized reformed-based recommendations such as inquiry-based courses, and field experiences integrating science and mathematics.

2.1.3 Students’ Beliefs About the Nature of Mathematics

As well as studies focusing on in-service and pre-service teachers, researchers have studied beliefs about mathematics held by STEM students, remedial mathematics students, and academically gifted high school students (Dewar, 2008; Frank 1988; Stage & Kloosterman, 1995). These studies reported similar findings to Schoenfeld (1992), agreeing that many students hold unproductive beliefs about the nature of mathematics. Along with literature documenting these unproductive beliefs that students have about mathematics, there has been research which has explored whether these unproductive beliefs can be challenged or altered (Hauk & Segalla, 2005; Jankvist, 2015; Jankvist & Niss, 2018; Rupnow, 2018; Szydlik, 2013; Taylor, 2009).

Several researchers have found that exposing students to mathematics courses involving problem solving, implementation of specially designed teaching modules, and

targeted interventions can help shift students' mathematical beliefs from less productive to more productive (Szydlik, 2013; Jankvist, 2015; Jankvist & Niss, 2018; Rupnow, 2018; Taylor, 2009). For example, after a five-week summer school program which emphasized difficult, open-ended mathematics problems, Taylor (2009) found that some of the non-productive beliefs held by students had changed. These changes were seen after giving students a pre-/post-survey which measured beliefs. The most significant change was found in students' beliefs about the importance of understanding mathematics. The statement on the survey corresponding to this belief was:

Understanding concepts is important in mathematics. The difference in the mean scores related to this statement were found to be statistically significant after the program compared to at the beginning ($p < 0.01$). There were other beliefs, however, that did not change as a result of the program (i.e. *there are some mathematics problems which cannot be solved with simple, step-by-step procedures*). Taylor (2009) explained that this may have been because of the small sample size ($n=23$), as well as the short time frame of the program.

In another study, Szydlik (2013) sought to find out if general education students' beliefs about the nature of mathematics could be affected by taking a one-semester course which emphasized problem-solving. The course was designed so that students were working in groups and engaged in "authentic mathematical experience[s]" (p. 98). Szydlik (2013) used the Mathematics Beliefs Instrument (MBI) both at the beginning and the end of the course to measure changes in students' beliefs. The quantitative data from the MBI revealed that students' beliefs at the beginning of the course were found to be aligned with the beliefs identified in Schoenfeld (1992), viewing mathematics as mostly

having to do with rules and procedures. These beliefs changed drastically by the end of the course, with students expressing the value of the discipline of mathematics, and a smaller percentage of students emphasizing that mathematics has to do with procedures and formulas.

Similar to Szydlik (2013), Jankvist (2015) also found that students' unproductive beliefs about the nature of mathematics could be challenged. This was seen after implementation of two teaching modules which exposed students to applications, history, and philosophy of mathematics. Through giving students questionnaires and interviewing students, one major shift in students' beliefs was identified, and was related to mathematics being discovered or invented. After the teaching modules, the majority of students believed that mathematics was invented rather than discovered. The researchers related this change to the fact that the teaching modules exposed students to mathematical advances such as Hamming's development of error-correcting codes, and the historical development of negative numbers. One student, Gloria, first expressed that "You can't come up with some brilliant mathematical thing now – because so much have already been created" (p. 52). This belief changed by the second teaching module, as she realized the possibility for new mathematical developments even today. Gloria expressed that this change was due to the modules causing her to reflect on the nature of mathematics.

Jankvist and Niss (2018) implemented a targeted approach to alter students' misconceptions and myths they believed about mathematics as a discipline. Math counselors identified students who held unproductive views of mathematics and designed activities to alter these beliefs "into more constructive ones", thus "dispelling, at least partly, some of the myths that governed their view of mathematics" (Jankvist & Niss,

2018, p. 2). One student, Christian, believed that mathematics was mostly about memorization, and that it “makes no sense from a logical point of view” (Jankvist & Niss, 2018, p. 12). The math counselors focused on the topic of algebra, a subject that Christian had negative feelings about, and through showing Christian a few algebraic conventions and axioms, they led Christian to realize that he could actually arrive at mathematical conclusions on his own through reasoning and carrying out deductions. This dispelled his original belief that mathematics was just about memorizing procedures and caused him to have increased confidence in doing mathematics. The teaching modules also caused another student, Rikke, to realize that she did not have to be restricted to using only rules and formulas while doing mathematics, but that she could be successful at mathematics through using various methods such as experimentation, problem solving, and performing calculations.

In a fifth study, Rupnow (2018) explored abstract algebra students’ beliefs about the nature of mathematics from two types of classes. One was an inquiry-oriented class and the other was a class taught using two days of lecture and one day in which students discussed problems in groups. Rupnow (2018) found that students’ beliefs about the nature of mathematics did not noticeably change in either class. This was attributed to the fact that students’ beliefs may be quite established by the time they take an upper-level mathematics course. Though students’ beliefs about the nature of mathematics did not change, their beliefs about the possibilities of college mathematics instruction did. In particular, students were more open to non-lecture methods of instruction, in line with what they experienced in their class.

Lastly, this literature review located one study that reported on students’

mathematical beliefs in the context of online homework. Hauk and Segalla (2005) found that online homework challenged students' beliefs about the nature of mathematics which inhibited self-regulation. The beliefs that the students in this study held were similar to commonly held beliefs of college students identified by Spangler (1992). These four main beliefs were:

(1) mathematics is *computation* that does not involve reflection during task engagement; (2) mathematics must be done *quickly*, or, spending little time is a more important task goal than sense-making; (3) mathematics problems have *one right answer* and no further action or evaluation is required once an answer is found; and (4) the *teacher is the agent* of mathematical learning, not the student (i.e., only intentional acts on the part of the teacher lead to learning, no intentionality on the part of the student is necessary) (Spangler, 1992 as cited in Hauk & Segalla, 2005, p. 241)

Hauk and Segalla (2005) found that some students felt like they “weren’t really doing math” (p. 242), because the online homework program could perform computations such as $(7-1)/3$, which challenged Belief #1. Challenges to Belief #2 were seen as students’ comments expressed a tone of complaint related to having multiple tries in online homework, saying “math homework shouldn’t take so long” (p. 242). Belief #3 was challenged, in that 11% of students reported concern that WebWork problems could have more than one right answer. It is noted that the online homework platform accepted $(7-1)/3$, $6/3$, and 2 as all being correct. Lastly, Belief #4 was challenged in that students wanted hints about why an answer was wrong, viewing the WebWork program as a “surrogate teacher” which fails to be “active” (p. 242-243).

A commonality in the studies that found a shift toward more productive beliefs was that students' unproductive beliefs about the nature of mathematics were challenged and altered as a result of them taking courses emphasizing problem solving, as well as more targeted interventions to dispel mathematical myths. It is important to note that students' beliefs tend to be tenacious and resistant to change (Pajares, 1992). This resistance is seen in Rupnow's (2018) study, as students' beliefs about the nature of mathematics did not seem to change, explained to be the result of their beliefs being quite established by the time they took an upper-level mathematics course. Jankvist (2015) expressed that reflection is an important component of altering students' beliefs, and that "the higher the level of reflection associated with changes in beliefs, the larger the probability that the changes may last" (p. 44). It is important to note that the studies described above in which students' unproductive mathematical beliefs were challenged or changed all provided and exposed students to experiences which encouraged beliefs that were aligned with a more productive mathematical disposition. This is important to keep in mind as this current study seeks to extend upon this literature, as well as fill a gap, in exploring what beliefs MATH 1483 students have about the nature of mathematics in an online homework context, as well as in an outside-of-school context.

A key takeaway from the studies described above is that students' mathematical beliefs can be altered by engaging students in particular mathematical experiences. However, with the exception of Hauk and Segalla's (2005) work, this has been done primarily through classroom experiences. By focusing on students' mathematical beliefs in online homework, my research lays the groundwork to leverage online homework in developing productive mathematical beliefs.

CHAPTER III

THEORETICAL PERSPECTIVE

The underlying educational philosophy of this study is constructivist. In this theoretical perspective, beliefs are viewed as psychologically held and constructed by individuals through their experience in the world (von Glasersfeld, 1995). I will first define what I mean by beliefs, and I will then discuss the structure of beliefs and belief systems. I will conclude with discussing a framework for students' mathematical beliefs.

3.1 Defining Beliefs

This section serves to ground the current study in an analysis of the nature of beliefs and the structure of beliefs as they function within belief systems. Before doing so, it is important to clearly define what is meant by the term *belief*. According to Philipp (2007), this term is widely used in the literature, but “many who write about beliefs do so without defining the term” (p. 259). Alba Thompson (1992) agreed, noting, “for the most part, researchers have assumed that readers know what beliefs are” (p. 129). Instead of making this assumption, I follow Philipp's (2007) definition of beliefs. Based on an extensive literature review on the construct, Philipp (2007) defined beliefs as “*psychologically held understandings, premises, or propositions about the world that are thought to be true*” (p. 259).

Similar to Thompson (1992), this study views beliefs as a subset of conceptions,

where conceptions encompass “beliefs, meanings, concepts, propositions, rules, mental images, preferences, and the like” (p. 130). In using the term conception, Thompson (1992) pointed out the futility in looking for a definite characterization between beliefs and knowledge, and often used the terms interchangeably. Similarly, Beswick (2011) reviewed several ways in which people have distinguished beliefs and knowledge, but argued that such distinctions “are not at all clear cut”, and “for all practical purposes, [...] are neither helpful nor necessary” (p. 48). The practicality of equating beliefs and knowledge is explained further by both Thompson (1992) and Beswick (2011), who both examined the beliefs of teachers, as they pointed out that teachers act as if their beliefs are true. This is consistent with the definition of beliefs from Philipp (2007) that this study adheres to, in which one’s beliefs are “thought to be true” (p. 259). In this way, beliefs in this study are seen as constructed in the same way that knowledge is constructed.

It is important to note that because this study follows from a radical constructivist perspective, with students’ beliefs being psychologically held, such beliefs must be inferred from what students say or do (Pajares, 1992). Because such inferences must be made, having multiple data sources is important to get a more complete picture of students’ mathematical beliefs. As outlined in the Methods section, this is why student interviews were conducted, in addition to having students’ responses to an online survey.

Because this study views students’ beliefs as being constructed from their experiences in the world, both students’ experiences in online homework, as well as the experiences they are provided with in their mathematics courses, are important and can indicate ways in which students are supported (or not supported) in constructing

productive mathematical beliefs.

3.2 The Structure of Beliefs

Thompson (1992), drawing upon Green's (1971) work, discussed the structure of beliefs, in that they reside in belief systems, and defines a belief system as "a metaphor for examining and describing how an individual's beliefs are organized" (p. 130).

Thompson (1992) goes on to explain three dimensions of belief systems. The first dimension is that a belief is never totally isolated from other beliefs. The structure of beliefs is quasi-logical, as some beliefs are foundational to others. Green (1971) distinguishes these beliefs as being either primary beliefs or derivative beliefs. For example, Thompson (1992) gives the example of a teacher holding the (primary) belief that presenting mathematics "clearly" is important. The teacher would then believe it is important "(a) to prepare lessons thoroughly, to ensure a clear, sequential presentation, and (b) to be prepared to answer readily any questions posed by the students" (p. 130). These are derivative beliefs.

The second dimension that Green (1971) gives about beliefs systems is that beliefs are held with differing convictions. Central beliefs are those which are held most strongly, and peripheral beliefs are those which are "most susceptible to change or examination" (Thompson, 1992, p. 130). In the example given above, the teachers' derivative belief of being prepared to answer questions may be more central due to the teacher wanting to "maintain authority and credibility" than for providing clarification for students (Thompson, 1992, p. 130). The third dimension of beliefs systems offered by Green (1971) is that beliefs are held in clusters "more or less in isolation from other clusters and protected from any relationship with other sets of beliefs" (p. 47-48). This

makes it possible for a person to hold, from an outsider's perspective, contradictory beliefs, as beliefs in different clusters arise in independent contexts. Holding beliefs in this way allows for people to "avoid confrontation between belief structures" (Philipp, 2007, p. 260).

It is important to note, however, that "people always strive for a coherent belief system" (Op'T Eynde et al., 2002, p. 25). Leatham (2006) described belief systems as sensible systems, emphasizing the fact that even though an observer might conclude that one is holding contradictory beliefs, "the person holding those beliefs finds a way to resolve the conflict" and further says that "our incredulity does not diminish another's coherence" (p. 95).

The more central a belief is, the harder it is to change. There are beliefs which are derived from other more central beliefs, thus forming a belief system which is a very connected structure. This means that in order for a more central belief to change, the beliefs which are derivative to it must also change in order for the individual to maintain coherence in their belief system.

3.3 A Framework for Students' Mathematical Belief Systems

In the literature review, I discussed Op'T Eynde et al.'s (2002) categorization of student beliefs, which Jankvist (2009) extended to include students' beliefs about mathematics as a discipline. Represented by the tetrahedron in Figure 2, this view of students' mathematical belief systems will be utilized in this study. Jankvist (2015) intentionally made the figure a tetrahedron instead of a square to emphasize that students' beliefs about mathematics as a discipline develop through "the interplay between their social context (class), their mathematics education, and their self, which is to say the

triangle making up the base of the tetrahedron” (p. 45).

Each vertex of the tetrahedron should be seen as a cluster included in students’ mathematical belief system. Because each vertex of the tetrahedron represents a cluster, there is the possibility for a student to hold, for instance, a belief about school mathematics that is seemingly contradictory to another belief about mathematics as a discipline. For example, Schoenfeld (1985) reported data on students’ beliefs and found that “on the one hand, there was a tendency to regard mathematics learning largely as a matter of memorization. On the other hand, the students expressed significant support for the idea that mathematics is interesting and challenging, allowing a great deal of room for discovery” (p. 14). The nature of beliefs being held in clusters within a belief system provides a way to explain the potential contradictions or discrepancies between students’ beliefs about mathematics in the context of online homework and their beliefs about mathematics in an outside-of-school context.

CHAPTER IV

METHODOLOGY

4.1 Setting and Participants

The participants in this study were students taking Mathematical Functions and Their Uses (MATH 1483) at a large public university in the US in Spring 2020. MATH 1483 is an entry-level mathematics course which is part of the Oklahoma Math Pathways project. As an alternative to College Algebra, the course which STEM students take, this course is designed to be more applicable to non-STEM students' areas of interest, and serves as a prerequisite both to Business Calculus and statistics courses for the social sciences. The textbook used for MATH 1483 presents problems in real-world contexts, and is designed so that "students see that mastery of basic mathematical thinking can be a powerful tool for success in many [...] areas" (Crauder, Evans, & Noell, 2017).

The fact that this course presents all problems in real-world contexts is important because the type of problems that students work on can impact their mathematical beliefs in different ways. In many mathematics courses, students are commonly given procedural-type problems. It has been reported that post-secondary Calculus 1 final exams mainly ask students to complete problems which are procedural in nature and seldom include questions which are situated in real-world contexts (Tallman, 2016). In addition to situating all problems in real-world contexts, MATH 1483 also encourages

students in solving problems using a variety of solution methods. This includes utilizing technology such as a graphing calculator to solve problems, as well as solving problems algebraically, or by hand. For example, when learning how to solve quadratics equations, MATH 1483 students are shown both how to solve them algebraically, and also how to solve them by graphing.

There were five total sections of this course, including two corequisite sections. The corequisite model is designed to get students into a college-level math course and out of remedial instruction faster by providing students with additional support, including added class meeting times which typically involve additional exposure to the course material and opportunities for working on problems in small groups.

4.2 WebAssign

Students taking MATH 1483 complete both online and written homework weekly. As a coordinated course, every student completes the same homework assignments. The online homework platform utilized for this course is WebAssign. This platform offers students immediate feedback on whether they answered a question correctly or not. If answered incorrectly, students have unlimited tries to answer it correctly, up until the due date. This excludes multiple choice questions, in which students have $n-1$ or $n-2$ tries (where n represents the number of possible answers). As well as having unlimited tries, the Practice Another Version feature is also enabled for students, which gives them the option to work on a different randomized version of the problem.

4.3 Data collection methods

I will now discuss the data collection methods of the study, including a

description of the online survey that was given to students, how the survey data was analyzed, a description of the interview procedures, and how the interview data was analyzed.

4.3.1 Online survey

MATH 1483 students were given an online survey during week 8 of the Spring 2020 semester which assessed their beliefs about mathematics in the context of online homework and their beliefs about mathematics in an outside-of-school context¹. The survey was given midway through the semester so that the students had adequate time to use and get familiar with the online homework platform.

To address the research questions, the survey consisted of 24 statements which were adapted from a modified version of the Fennema-Sherman Mathematics Attitude Scales (Fennema & Sherman, 1976). Many of the statements were similar to Schoenfeld's (1992) list of commonly held beliefs of students. The 24 statements were made up of 12 pairs of parallel statements, in which students would respond to a certain belief both in the context of online homework and in an outside-of-school context. For example, the first two statements in the survey were: *While doing online homework, knowing the underlying mathematical ideas is more important than the formulas*; and, *When math is done outside of a school context, knowing the underlying mathematical ideas is more important than the formulas*. Students were asked to respond to each statement on a five-point Likert scale ('Strongly Agree' to 'Strongly Disagree'). Half of the statements were phrased positively, reflecting a productive mathematical disposition, and half were phrased negatively, reflecting an unproductive mathematical disposition.

¹ The online survey was piloted during the Fall 2019 semester with one MATH 1483 student. This student also participated in a follow-up interview.

Each statement was scored from 1 to 5, with 5 reflecting a productive mathematical disposition. Whether the statement was positively or negatively worded determined how the statement was scored. For example, the first statement on the survey (*‘While doing online homework, knowing the underlying mathematical ideas is more important than the formulas’*) was positively worded, so a response of ‘Strongly Agree’ was scored 5 and a response of ‘Strongly Disagree’ was scored 1. The twelve pairs of belief statements on the survey resulted in each student having two ‘Belief Scores’, one pertaining to an online homework context and one pertaining to an outside of school context, each score ranging from 12 to 60. The survey also asked students to respond to five statements pertaining to having unlimited tries in online homework, to explore whether this added feature influenced their views of the nature of mathematics in this context.

Along with responding to each statement with their level of agreement, students were also asked to give a brief explanation of why they responded as they did. Because of the nature of Likert-type surveys and some of the weaknesses identified with such instruments (i.e. difficulty in knowing how respondents are interpreting items, and items not being contextualized [Ambrose et al., 2003]), students were asked to provide an explanation after indicating their level of agreement. This helped to ensure the reliability of the survey, and to be able to capture students’ nuanced responses to the statements.

4.3.2 Analysis of survey data

Quantitative Data:

To analyze the Likert-scale responses from the online survey, the descriptive statistics for each survey statement were computed. Since the 24 belief statements on the survey were made up of 12 sets of parallel statements in which students would respond to

a certain belief both in the context of online homework and in an outside-of-school context, each student had two “Belief Scores”, one for each context. The mean scores for each statement were also examined, as were the differences in mean scores for each statement between the two contexts. This allowed me to determine in what context students were exhibiting a more productive mathematical disposition in (i.e. what context they were scoring higher in). Also, in looking at the differences in the means for each belief statement across the two contexts, I was able to identify which beliefs students were scoring highest in, and which beliefs had the greatest difference in scores between the two contexts.

Qualitative Data:

After responding to each statement on the Likert-scale, students were then asked to provide a short explanation as to why they responded as they did. To analyze these written responses, we employed the constant comparative method (Strauss & Corbin, 1994). The constant comparative method involves the researcher identifying themes in the data, and then creating categories to sort and describe the data. This is an iterative process where the researcher uses a subset of the data to develop initial categories as well as criteria for the categories, and then codes the entire data set to test those categories and refine them. This process continues until the categories describe the entire data set.

The constant comparative method in this study involved several iterations of developing codes, refining them, re-coding data, and refining the codes for each survey item. It is important to note that each survey item was coded individually; for example, I developed codes for survey item 1a, *While doing online homework, knowing the mathematical ideas is more important than the formulas* and then began the coding

process anew for item 1b, *When math is done outside of a school context, knowing the mathematical ideas is more important than the formulas*. While in many cases students' responses to the survey items were similar, it was important to capture nuances in how students thought about the two contexts because such nuances are at the crux of the research question of exploring MATH 1483 students' beliefs about the nature of mathematics in an online homework context as well as an outside-of-school context.

In terms of the specific steps in the constant comparative analysis, I first selected responses from ten students (chosen randomly), to develop initial codes for the short answer responses. I then coded the full data set using those initial codes, making adjustments to them as needed, which included refining certain codes and creating new codes. For each code, I listed the necessary criteria that must be present for a statement to fit that code and also included example statements for each code. Once I coded the full set of short answer responses, the other member of the research team then coded the full data set using those codes. They then identified any refinements or additions that needed to be made to the codes, which resulted in updated codes after we discussed and agreed on any changes. Both researchers independently re-coded all the data using the refined codes. We then compared our coding to each other's, and where any discrepancies were present, we discussed and came to an agreement as to what codes would best fit each short answer response.

4.3.3 Interviews

The last question on the online survey asked students to provide their name if they were willing to be interviewed to further discuss their responses. The interviews served to ask students to further explain their responses from the online survey, especially any

responses that were surprising or needed clarification. The interviews also served to provide context to and reasons for students’ beliefs, and gave further insight into how such beliefs were constructed. A common type of question that was asked in each interview was “In the online survey, you said _____. Could you talk a little bit more about why you said that?”

Because of COVID-19, which caused classes to be transitioned online and did not allow for in-person activities on campus to continue, only three interviews were able to be conducted. The interviewees were chosen because they were the first to respond as willing to be interviewed and time was of the essence due to the pandemic. The interviews were audio- and video-recorded. Table 1 below displays the gender, major, and length of the interview for these three students.

Table 1

Description of the three interview participants

Name (pseudonym)	Self-reported Gender	Major	Interview Length
Evan	Male	Sports Management	28 minutes
Abbi	Female	Elementary Education	34 minutes
Taylor	Female	Interior Design	40 minutes

4.3.4 Analysis of interview data

Once all the interviews were transcribed, each transcript was analyzed to identify students’ responses that further shed light on the themes which were identified in the short-answer responses of the online survey. As will be seen in the Results section, responses from the interviews are included after discussing the themes found in the

online survey. These responses from the interviewees provide more detailed explanations and provide context to the identified themes from the online survey data.

CHAPTER V

FINDINGS

In this section I discuss the results of the study. These results describe MATH 1483 students' beliefs about the nature of mathematics both in the context of online homework and in an outside of school context. I first discuss the quantitative results of the study which come from students' Likert-scale responses. I then discuss the qualitative results, including broadly identified themes found in students' short answer response to the online survey items, as well as specific themes identified within selected survey items. Within this, I include interview data to give insight into the specific themes of students' short answer responses.

5.1 Quantitative Results

As outlined in the Methods section, the quantitative results of this study come from analyzing Likert-scale data from an online survey. The survey questions included 12 statements pertaining to beliefs about math in the context of online homework, 12 statements pertaining to beliefs about math in an outside of school context, and five statements pertaining to having unlimited tries in online homework. Each belief statement was scored on a 5-point Likert scale, where 1 reflected an unproductive mathematical disposition and 5 reflected a productive mathematical disposition. Therefore, each student had a two cumulative belief scores, one for each context, which could range between 12

and 60 points. Descriptive statistics for selected belief statements are reported throughout Sections 5.3-5.7 as the qualitative results from those items are discussed. Table 2 shows a summary of students' aggregate mean scores for the 24 beliefs statements on the online survey.

Table 2

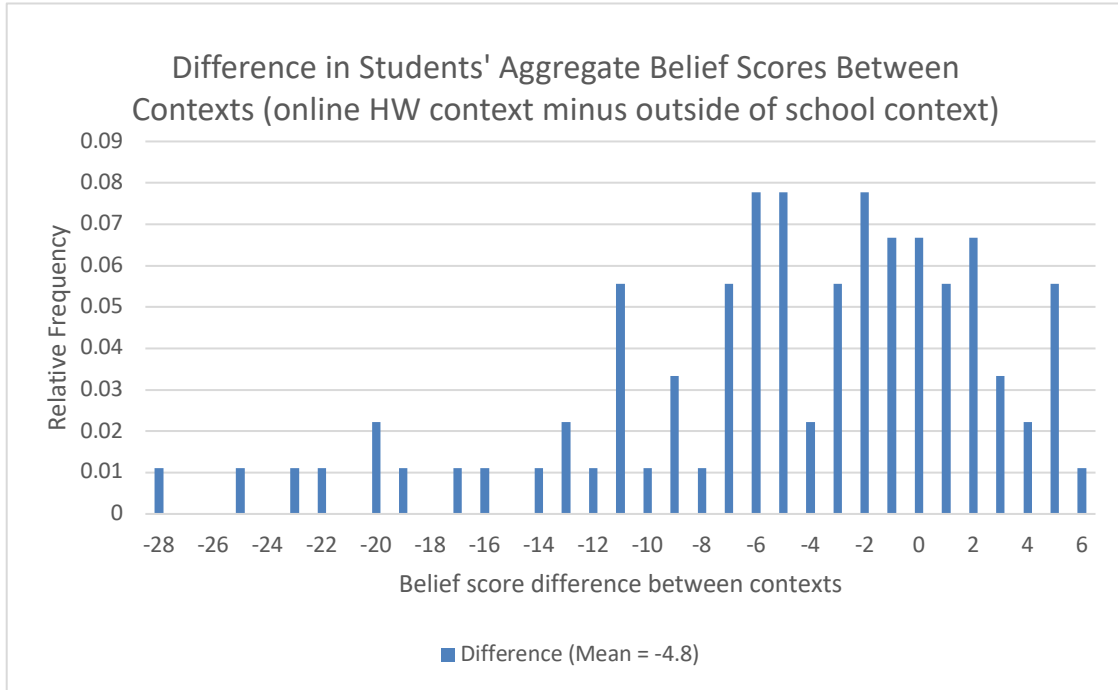
Summary of aggregate mean scores for the 24 beliefs statements on the online survey

Belief Score	Minimum Score	Maximum Score	Mean	Standard Deviation
Online Homework Context (n=90)	23	54	37.64	5.84
Outside of school context (n=90)	29	54	42.44	5.31

The MATH1483 student aggregate belief scores in the context of online homework ranged from 23 to 54 out of a possible range of 12 to 60, with the mean being 37.64. The aggregate belief scores in an outside of school context ranged from 29 to 54, with a mean of 42.44. The smallest difference between scores across the two contexts was 0 points, the largest difference was 28 points, and the mean difference was 4.8 points, which indicates that on average, MATH 1483 students have a more productive mathematical disposition in an outside of school context than they do in an online homework context. Figure 3 shows a graph of the relative frequency distribution for the differences in students' belief scores across the two contexts.

Figure 3

Relative frequency distribution for the difference in students' aggregate belief scores between contexts.



Of the 90 students who completed the online survey, 62 of them had a higher belief score in an outside-of-school context, 22 of them had a higher belief score in an online homework context, and 6 students had equal belief scores across both contexts. Table 3 below shows a summary of mean scores for the individual items on the online survey.

Table 3*Means for each belief statement/difference between contexts*

Belief Statement	Context		Mean Difference
	(a) Online Homework	(b) Outside of School	
1. The underlying mathematical ideas are more important than the formulas.	3.69	3.83	-0.14
2. There are often several different ways to solve a math problem.	3.97	4.31	-0.34
3. Time used to investigate why a solution to a math problem works is usually time well spent.	3.63	3.67	-0.04
4. In math, you can be creative and discover things by yourself.	3.27	3.97	-0.7
5. The math I learn in school is thought provoking.	3.32	3.74	-0.42
6. In addition to getting a right answer to a math problem, it is important to understand why the answer is correct.	4.44	4.17	0.27
7. Just about everything important about math is already known by mathematicians.	2.07	2.9	-0.83
8. To solve math problems, you have to know the exact procedure for each problem.	2.5	3.02	-0.52
9. Students who understand the math they have studied will be able to solve any assigned problem in five minutes or less.	2.8	3.18	-0.38
10. It doesn't really matter if you understand a math problem, as long as you can get the right answer.	3.7	3.66	0.04
11. Math problems have one and only one right answer.	1.83	2.99	-1.16

Belief Statement	Context		Mean Difference
	(a) Online Homework	(b) Outside of School	
12. Math is mostly a matter of memorizing formulas and procedures.	2.42	3.01	-0.59

As seen in Table 3 (above), on all but two statements, students scored lower when responding in the context of online homework than they did in an outside of school context, signifying a less productive mathematical disposition in an online homework context. The first exception was statement #6, *“In addition to getting a right answer to a math problem, it is important to understand why the answer is correct”*, which had a mean of 4.44 in an online homework context, and a mean of 4.17 in an outside of school context, a difference of +0.27. The second exception was statement #10, *“It doesn’t really matter if you understand a math problem, as long as you can get the right answer”*, which had a mean of 3.7 in an online homework context, and a mean of 3.66 in an outside of school context, a difference of +0.04.

The belief statement that had the biggest difference between contexts was *“Math problems have one and only one answer”*. In an online homework context, students generally agreed with the statement ($\mu=1.83$), and in an outside of school context, students aligned more with “neither agree nor disagree” ($\mu=2.99$), a 1.16-point difference. The statement that had the second biggest difference in mean scores across contexts was *“Just about everything important about math is already known by mathematicians”*, with students generally agreeing to the statement in an online homework context ($\mu=2.07$), but neither agreeing or disagreeing in an outside of school context ($\mu=2.9$). Perhaps this result is not surprising, as the math in online homework is clearly known by

mathematicians, while math that could be done outside of school is much more extensive. The third largest difference in mean scores across contexts was 0.59 points, for the statement “*Math is mostly a matter of memorizing formulas and procedures*”. Again, students generally agreed to this statement in an online homework context ($\mu=2.42$), but neither agreed nor disagreed in an outside of school context ($\mu=3.01$).

Overall, these quantitative results are encouraging in that students showed alignment with certain beliefs which reflect a more productive mathematical disposition than what is generally reported in much of the literature (c.f. Hauk & Segalla, 2005; Schoenfeld, 1992; Spangler, 1992). In particular, students agree to statements about understanding the underlying ideas as important both in an online homework context and in an outside of school context (statement #1, #6, and #10). Also, the commonly held belief that there are not many different ways to solve math problems is challenged here, with students expressing agreement with statement #2. The mean scores for statement #9 are also encouraging, as it has been reported in the literature that many students believe math problems should be done quickly. The mean scores for this statement indicate that MATH 1483 students generally neither agree nor disagree that math problems should be solved in five minutes or less. Section 5.2 describes results from the qualitative data that yield insight into these quantitative results, reporting reasons that students give for their beliefs about mathematics in each context.

5.1.1 Survey statements about having unlimited tries

The last five statements of the online survey focused on unlimited tries in online homework. The results of students’ responses for these statements are recorded in Table 4 below. Both the number of responses, and the corresponding percentage of students who

selected each level of agreement are shown.

Table 4

Results from survey statements about having unlimited tries

Statement	Strongly disagree	Somewhat disagree	Neither agree nor disagree	Somewhat agree	Strongly agree
I like having unlimited tries in online homework.	3 3.33%	2 2.22%	1 1.11%	7 7.78%	77 85.56%
Having unlimited tries in online homework helps me learn.	0 0.00%	2 2.22%	3 3.33%	7 7.78%	78 86.67%
Being able to work on a problem until I get it correct has increased my overall confidence in my ability to do math.	1 1.11%	0 0.00%	5 5.56%	23 25.56%	61 67.78%
Having unlimited tries in online homework means I am likely to guess answers.	11 12.22%	25 27.78%	21 23.33%	26 28.89%	7 7.78%
I often will try an online homework problem multiple times.	1 1.11%	2 2.22%	9 10.00%	30 33.33%	48 53.33%

As seen in the table above, the majority of students agree that they like having unlimited tries, that having unlimited tries helps them learn, that having unlimited tries increases their confidence in their ability to do math, and that they try online homework problems multiple times. Students' responses to whether or not having unlimited tries makes them more likely to guess answers were more varied. These statements on the survey, and particularly students' short-answer responses explaining why they agreed/disagreed to the statements, give some insight into their beliefs about mathematics in an online homework context. This is discussed further below.

5.2 Overall themes across survey statements

Students' short answer responses revealed some prominent themes across multiple survey items, which will be discussed in more detail below. These themes include students' belief that having an underlying understanding of the mathematics they do is important, both in an online homework and outside of school context. Reasons that students gave for the importance of having this understanding were varied, and will be discussed later as we look at particular themes within selected survey items. Next, the short answer responses also revealed that many students believe that there are multiple ways to solve problems, both in online homework and outside of school. This belief, along with their belief that having an underlying understanding of math problems is important, is encouraging and reveals that MATH 1483 students do hold some beliefs that are aligned with a productive mathematical disposition.

Another noteworthy belief that was seen in students' short answer responses was that math problems in online homework have only one right answer, whereas math problems done outside of school could have multiple right answers. As noted above in the discussion about the quantitative results, students' belief that math problems have one and only one right answer showed the biggest mean difference across an online homework vs. an outside of school context. The specific themes identified within these survey items, as well as specific students' responses, will be discussed in more detail below, and provide more insight into the numerical data.

Lastly, students' short answer responses also revealed their belief that doing a math problem in five minutes or less depends on the person doing the problem, or on the problem itself. This belief was prominent both in an online homework an outside of

school context. Again, this belief is perhaps more encouraging than what is commonly reported in the literature, that students believe that math problems *should* be able to be solved quickly (Schoenfeld, 1992).

Below I present the individual themes from selected survey items. These items were selected because they explain in more detail the broadly identified themes discussed above and most clearly give insight into and answer the research questions.

5.3 Formulas versus underlying mathematical ideas

Findings from the literature indicate that students often believe mathematics is formula-based. For example, Schoenfeld (1992) found many students believe there is only one correct way to solve any problem (usually the rule the teacher has most recently demonstrated) and that ordinary students cannot expect to understand mathematics, but rather must memorize and apply what they have learned mechanically and without understanding. Taken together, these findings suggest that students' conception of the nature of mathematics may be that the important content of the discipline is formulas, not underlying ideas. The instructional approach and curricular materials in MATH 1483 seek to promote ideas over formulas, in part by having students work exclusively on real-world problems. To investigate what 1483 students believed about the importance of formulas and underlying ideas, I asked students to respond to the following four survey items:

Survey item 1a: *While doing online homework, knowing the mathematical ideas is more important than the formulas.*

Survey item 1b: *When math is done outside of a school context, knowing the mathematical ideas is more important than the formulas.*

Survey item 12a: The math in online homework is mostly a matter of memorizing formulas and procedures.

Survey item 12b: Math that is done outside of a school context is mostly a matter of memorizing formulas and procedures.

As detailed in the Methods section, students responded to these statements on a five-point Likert scale and completed a fill-in field asking them to explain why they answered in the way that they did. In the sections that follow, I describe the numerical data and then the themes from their short-answer responses.

5.3.1 Likert scale results for survey items 1a and 1b

Descriptive statistics for the first two online survey statements are shown below in Tables 5 and 6, along with Figure 4 which show the spread of students' responses to the Likert-scale for both contexts. Both means show that students generally agree that knowing the underlying mathematical ideas is more important than the formulas, both in an online homework and an outside of school context ($\mu=3.69$ and $\mu=3.83$, respectively). Seventy-one percent of students agreed (either strongly agreed or somewhat agreed) to the statement referring to an online homework context, while 70% of students agreed to the statement referring to an outside of school context.

Figure 4

Spread of students' responses to survey items 1a and 1b.

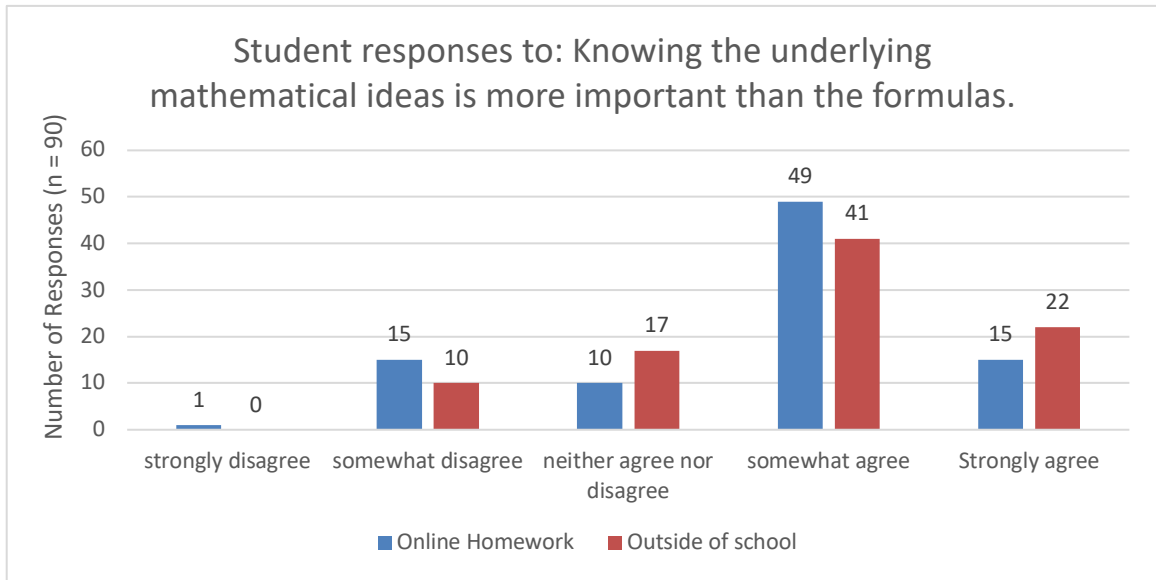


Table 5

Descriptive statistics for survey item 1a: “When doing online homework, knowing the underlying mathematical ideas is more important than the formulas”

Mean	Standard Deviation	Variance	Count
3.69	0.97	0.95	90

Table 6

Descriptive statistics for survey item 1b: “When math is done outside of a school context, knowing the underlying mathematical ideas is more important than the formulas”

Mean	Standard Deviation	Variance	Count
3.83	0.92	0.85	90

5.3.2 Likert scale results for survey items 12a and 12b

Below are the descriptive statistics for the next two survey statements focusing on underlying understanding versus memorizing formulas (survey items 12a and 12b), shown in Tables 7 and 8, along with Figure 5 showing the spread of students' responses to the Likert-scale for both contexts. These numerical results show that students generally agreed that math in online homework is mostly a matter of memorizing formulas and procedures, with survey item 12a having a mean of 2.42. When responding to whether math outside of school is mostly a matter of memorizing formulas and procedures, the mean was slightly higher ($\mu=3.01$), signifying that students generally neither agreed nor disagreed with the statement. The graph, however, shows that students generally either agreed or disagreed that math outside of school is mostly a matter or memorizing formulas and procedures. Next, the themes within students' short answer responses to these statements will be discussed, and will offer further insight into these numerical results.

Figure 5

Spread of students' responses to survey items 12a and 12b.

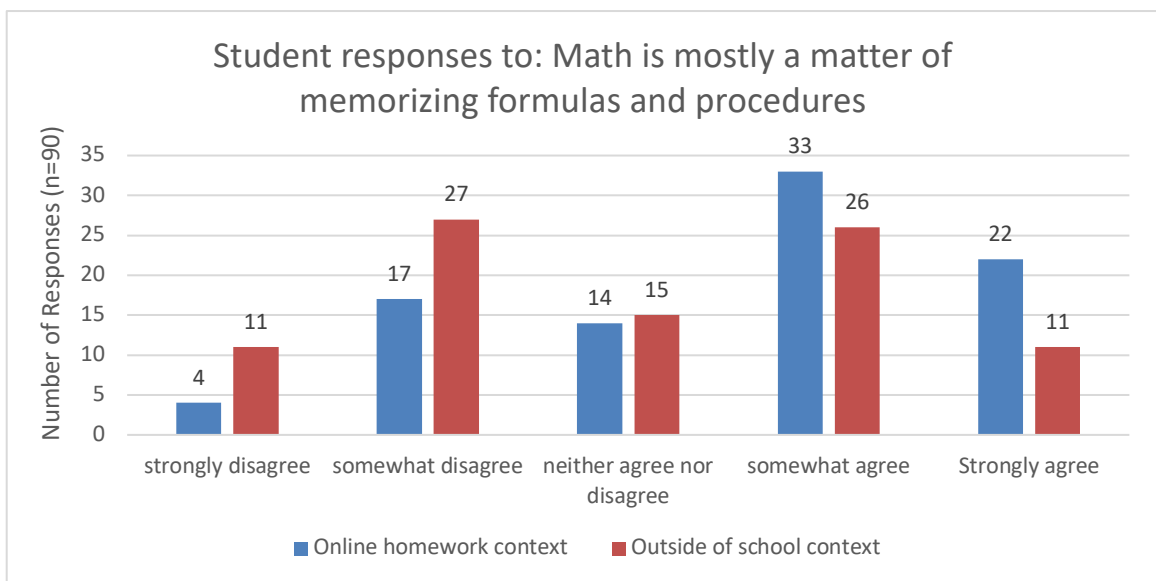


Table 7

Descriptive statistics for survey item 12a: “The math in online homework is mostly a matter of memorizing formulas and procedures”

Mean	Standard Deviation	Variance	Count
2.42	1.17	1.38	90

Table 8

Descriptive statistics for survey item 12b: “Math that is done outside of a school context is mostly a matter of memorizing formulas and procedures”

Mean	Standard Deviation	Variance	Count
3.01	1.25	1.57	90

5.3.3. Themes in survey items 1a and 1b

The constant-comparative analysis (Strauss & Corbin, 1994) detailed in the Methods section resulted in the following themes for the first two survey items (Tables 9 and 10), which are not mutually exclusive. These tables allow the reader to obtain a brief overview of the themes, many of which are similar.

Table 9

Themes regarding from survey item 1a: “While doing online homework, knowing the mathematical ideas is more important than the formulas.”

Theme	Frequency (88 total responses)
1. Knowing how to use/understand the formula is important (the underlying idea and formulas are equally important)	31 (35%)
2. Understanding what’s going on, the underlying idea, is important/helpful, not memorization	31 (35%)

3. Knowing the formulas is what is important, they are what get the right answer	20 (23%)
4. Formulas are not always needed, there are multiple ways to solve problems	4 (5%)
5. Knowing the underlying idea is important because of tests/future problems	3 (3%)
6. I'm not sure	3 (3%)
7. In online homework, the only thing that matters is getting the answers and moving on, not the underlying understanding	4 (5%)

Table 10

Themes regarding from survey item 1b: "When math is done outside of a school context, knowing the mathematical ideas is more important than the formulas."

Theme	Frequency (92 total responses)
1. Knowing how to use/understand the formulas outside of school is important, both formulas and understanding are important	18 (20%)
2. Having a general understanding of the underling idea is what is needed/important for outside of school math	39 (43%)
3. Outside of school I don't do much math, or I only do simple math	4 (4%)
4. Outside of school, formulas are what is important what gets you the answer	7 (8%)
5. Outside of school, your answer doesn't have to be exactly correct, there's more than one way to find answers, it's 'more casual'	16 (17%)
6. It depends on the problem/context if underlying idea is important outside of school or not	4 (4%)
7. Outside of school, getting the answer is what matters	6 (7%)
8. I'm unsure	4 (4%)

Tables 11 and 12 provide the criterion for an element of data to be coded as a particular theme as well as the frequency with which a particular theme occurred. These codes, as well as codes for all other survey items that will be discussed, are not mutually exclusive. We coded a total of 88 responses for survey item 1a and 92 responses for survey item 1b.

Table 11

Themes, criteria, and examples for survey item 1a: “While doing online homework, knowing the mathematical ideas is more important than the formulas.”

1a: While doing online homework, knowing the underlying mathematical ideas is more important than the formulas.	Criteria:	Example:	Frequency (88 total responses)
1. Knowing how to use/understand the formula is important (the underlying idea and formulas are equally important)	1. student explains that both the importance of the formulas and being able to understand, OR 2. they mention the importance of understanding how to use, and apply the formulas.	“Remembering how to use the formula and its way is a bit more helpful than knowing just the formula.”	31 (35%)
2. Understanding what’s going on, the underlying idea, is important/helpful, not memorization	1. student mentions how understanding the underlying idea is important, OR 2. they mention how knowing the underlying idea makes it easier to do the problem; OR 3. they mention that memorization is not as helpful	“It is better to know the basis of the problem, then just know the actual formula”	31 (35%)
3. Knowing the formulas is what is important, they are what gets the right answer	1. student explains that one just needs to formula to solve the problem, OR 2. student explains that knowing the formula is more important than the underlying idea	“I feel it is more important to know the formulas than the ideas”	20 (23%)
4. Formulas are not always needed, there are multiple ways to solve problems	1. student explains that there are multiple ways to solve the problems, OR 2. they explain that you can solve the problems without use of a	“Sometimes you can do the questions without using a formula”	4 (5%)

	formula.		
5. Knowing the underlying idea is important because of tests/future problems	1. student explains that having an understanding is important – for the sake of doing well on tests or homework problems	“I feel like knowing the mathematical ideas will be more important for tests.”	3 (3%)
6. I’m not sure	1. student explains that they are not sure, or explains that they don’t understand the statement.	“I’m not quite sure what you meant by mathematical ideas”	3 (3%)
7. In online homework, the only thing that matters is getting the answers and moving on, not the underlying understanding	1. student expresses that they don’t care about the understanding; they just want the answer, the good grade.	“Sometimes i feel i dont need all the whys it just is what it is this is how you do it”	4 (5%)

Table 12

Themes, criteria, and examples for survey item 1b: “when math is done outside of a school context, knowing the mathematical ideas is more important than the formulas.”

1b: When math is done outside of a school context, knowing the underlying mathematical ideas is more important than the formulas.	Criteria:	Example:	Frequency (92 total responses)
1. Knowing how to use/understand the formulas outside of school is important, both formulas and understanding are important	1. student mentions both the importance of the formulas and being able to understand, OR 2. they mention the importance of understanding how to use, and apply, the formulas	“Knowing the underlying idea helps you know what formula to use.”	18 (20%)
2. Having a general understanding of the underlying idea is what is needed/important for	1. student mentions how understanding the underlying idea is important, OR	“It is important to understand what you are doing”	39 (43%)

outside of school math	<p>2. they mention how knowing the underlying idea makes it easier to do the problem; include mentions of knowing how to use 'everyday math', OR</p> <p>2. they mention that memorization is not as helpful, OR</p> <p>3. they simply respond with agreement</p>		
3. Outside of school I don't do much math, or I only do simple math	<p>1. student mentions that they don't do much math outside of school, or that they only do 'simple math'</p>	<p>"The math iv had to do in the real world seems simpler"</p>	4 (4%)
4. Outside of school, formulas are what is important what gets you the answer	<p>1. student mentions that one just needs to formula to solve the problem, OR</p> <p>2. student mentions that knowing the formula is more important than the underlying idea</p>	<p>"There are formulas that can be used in the real world to help make things easier"</p>	7 (8%)
5. Outside of school, your answer doesn't have to be exactly correct, there's more than one way to find answers, it's 'more casual'	<p>1. student explains that there are different ways to solve problems, and that there doesn't have to be an exact answer.</p> <p>2. student describes math outside of school as 'casual', and less objective</p>	<p>"Outside of school there are different ways to solve math problems on the smaller scale"</p>	16 (17%)
6. It depends on the problem/context if underlying idea is important outside of school or not	<p>1. student says "it depends", OR</p> <p>2. student explains how the underlying ideas are important in some settings, and not important in others.</p>	<p>"In my degree that is sometimes true, however in other areas they are not as prevalent in a day to day basis.</p>	4 (4%)
7. Outside of school, getting the answer is what matters	<p>1. student explains that getting the answer is all that matters.</p>	<p>"In real world problems, more significance is often focused on how to</p>	6 (7%)

		get the right answer.”	
8. I'm unsure	1. student explains that they're not sure, or explains that they don't understand the statement, OR 2. Student neither agrees or disagrees.	“I don't agree or disagree”	4 (4%)

Regarding the particular themes, many students agreed that knowing how to use the formula, or understanding the formulas, was important regardless of whether they were doing math in online homework or outside of school. Thirty-five percent of students believed this was true while doing online homework, and 20% believed it was true while doing mathematics outside of school. Similarly, data for both questions indicate many students believe the underlying mathematical idea is more important than memorizing a formula (35% for doing mathematics in an online homework context, 42% for doing mathematics in an outside-of-school context). One interviewee, Abbi, elaborated on this when talking about doing mathematics in online homework:

Abbi: “I just feel like if you know the formula, like, I, if you know the formulas you're, you're gonna get it down, but if you need to, if you want to know more why that formula's like that, you need to go more in depth. My high school math teacher, she taught us by giving us the formula and then giving us the answers to our homework and expected us to know the formulas ... didn't really teach much after that. And I'm like ‘why does this work this way?’ I'm not, I'm just, I would like to know why things work like they do.”

Interviewer: “mm-hmm. So, would you say the formulas are still important?”

Abbi: “Yes. They are still important but knowing the reasons behind the formulas to me are important, is a little more important.”

When asked about doing mathematics outside of school, Abbi again explained that having an underlying understanding was important particularly because of her job

working at a hotel. She explained that when she is finding the occupancy, she needs to understand the mathematics of how to do it so that she does not get “wacky numbers” and calculate the occupancy incorrectly.

These results are encouraging, and indicate that some students see underlying ideas as important. However, some students seemed to value the underlying idea because it would help them on a future assignment rather than for the sake of understanding itself. For example, Student 76 and Student 22’s response to survey item 1a were:

Student 76: “If I understand the idea then I can do better for written questions”

Code for quote: 5. Knowing the underlying idea is important because of tests/future problems

Student 22: “Just worried about the grade not the knowledge”

Code for quote: 7. In online homework, the only thing that matters is getting the answers and moving on, not the underlying understanding

Evan, another interviewee, elaborated on this as he explained the important of understanding the mathematics he does in online homework because “you’re gonna see those problems pop up again on tests and stuff.” This belief was contrasted to his belief about whether or not understanding the underlying idea was important when he does mathematics outside of school:

Evan: “Outside of school, just, it’s all about getting the right answer and moving on with your life. I think I don’t think you really need to know why you got it.”

Code for quote: 7. Outside of school, getting the answer is what matters

It is important to keep in mind that Evan explained that when he does do mathematics outside of school, he mainly does “simple math” (i.e. subtraction).

5.3.4 Themes in survey items 12a and 12b

Tables 13 and 14 below show the individual themes, along with the frequency of each, that were identified in survey items 12a and 12b. The themes are not mutually

exclusive. We coded a total of 79 responses for survey item 12a and a total of 71 responses for survey item 12b. The criteria for a statement to be coded a certain theme is reported in Appendix A. Many of the themes for these two survey items are similar to the ones discussed above concerning items 1a and 1b.

Table 13

Themes regarding survey item 12a: “The math in online homework is mostly a matter of memorizing formulas and procedures.”

Theme	Frequency (79 total responses)
1. Online homework is about knowing how to use/understand the formula (both the formulas and understanding is important)	13 (16%)
2. Online homework is about understanding what’s going on; it's not just about memorizing; the underlying idea is important	16 (20%)
3. Online homework does not involve many formulas	3 (4%)
4. In online homework, memorizing formulas and procedures is what it's about, and what gets you the answer	38 (48%)
5. Sometimes/It depends on the online homework problem whether or not it's all about memorizing formulas/procedures	8 (10%)
6. In online homework, I'm just looking for the answer, the one specific answer	3 (4%)

Table 14

Themes regarding survey item 12b: “Math that is done outside of a school context is mostly a matter of memorizing formulas and procedures.”

Theme	Frequency (71 total responses)
1. Math outside of school is about knowing how to use/understand formulas (formulas and understanding equally important)	5 (7%)
2. No- Math outside of school is mainly about understanding what's going on, not just memorizing formulas	20 (28%)
3. Math outside of school does not involve many formulas	5 (7%)
4. Yes- math outside of school is mainly about memorizing formulas and procedures, they are what help get you the answer	24 (34%)

5. It depends on the math outside of school whether it is mostly about memorizing procedures and formulas	11 (15%)
6. I'm unsure	1 (1%)
7. Math outside of school is more flexible, you can solve problems in various ways, 'it's subjective'	10 (14%)
8. Math outside of school is about "winging it", or doing it without thinking.	2 (3%)

Regarding the particular themes in these two survey items, many students expressed that math is mostly about memorizing formulas and procedures (48% expressing this in an online homework context and 34% in an outside of school context). For example, Student 47, who strongly agreed to the statement, said "That is a solid definition of what the online homework is like." Another student, Student 53, when responding to the statement in an outside of school context, said, "You will not be able to solve the math problems unless you know the procedures or formulas that go [with] it."

Some students did, however, express the belief that understanding the underlying idea of the math is what is important (20% in an online homework context, and 28% in an outside of school context). Other students expressed the belief that the underlying understanding and the formulas are equally important, with 16% expressing this in an online homework context, and 7% expressing this in an outside of school context. One interviewee, Abbi, explained her belief that both understanding the underlying idea and knowing the formulas is important, motivated by the fact that she is studying to become a teacher:

Abbi: "[...] if I don't understand it but I know the formulas and everything, what happens if I've got a student ask me "why does it work that way?" and I'm like "I don't know". I'm gonna have to answer "I don't know it's just like that." So I just feel like I won't be an effective teacher."

Interviewer: "So in online homework you're not mostly memorizing formulas and procedures, or at least it's not your goal?"

Abbi: “My goal is to understand behind that. Yes, I do need, do need to memorize those but I do want to know the reasons why behind.”

Another interviewee, Evan, who on the survey said “no need for formulas just use your brain”, explained his belief that formulas are not needed when doing math outside of school:

Interviewer: “[...] so you don't need formulas outside of school, why, why so?”

Evan: “[...] I just don't feel like you need those [formulas, such as $y=mx+b$] outside of school, when, I don't think they'll come in handy in five years, and unless you're at a certain field of course, but I just don't find them very useful. I've never run into a situation where I've needed something, a certain formula, if that makes sense.”

Evan further explained that he does not use formulas for math outside of school because he mostly does simple math, which can be solved by “just thinking through it” and using “common sense”. Evan’s original survey response, “no need for formulas just use your brain”, was coded 2. *No- Math outside of school is mainly about understanding what's going on, not just memorizing formulas*, but his response in the interview explaining that he just does simple math outside of school gives more context to this belief.

The data from these two survey items, and particularly in item 12a which refers to an online homework context, show that many students do hold the conception that math is mainly about memorizing formulas and procedures. This result along with the themes identified in the previous two survey items (1a and 1b) reveal that although students believe that math is mostly about memorizing formulas and procedures, many students believe that knowing the underlying ideas behind the formulas and procedures is important, which is an encouraging result.

Next, we will discuss students’ beliefs regarding the importance of understanding underlying mathematical ideas versus just getting a correct answer.

5.4 Underlying mathematical ideas versus just getting the right answer

Following the previous discussion of the importance of the underlying mathematical idea versus just knowing the formulas, we will now examine students' beliefs about the importance of the underlying idea versus just getting a correct answer. The four survey items which give insight into this belief are:

Survey item 6a: In addition to getting a right answer to an online homework problem, it is important to understand why the answer is correct.

Survey item 6b: When math is done outside of a school context, it is important to understand why the answer is correct, in addition to getting the right answer.

Survey item 10a: It doesn't really matter if you understand an online homework problem, as long as you can get the right answer.

Survey item 10b: When math is done outside of a school context, it doesn't really matter if it is understood, as long as the answer is right.

The numerical data for these two items is reported below, followed by a discussion of the themes found within students' short answer responses.

5.4.1. Likert scale results for survey items 6a and 6b

Descriptive statistics for survey items 6a and 6b are shown in Tables 15 and 16 below, along with Figure 6 showing the spread of students' responses to the survey statements, which asked students to respond to the importance of the underlying mathematical idea versus just getting a correct answer. Both in an online homework context and an outside of school context, students generally agreed that it is important to understand why an answer is correct, with the mean in an online homework context being 4.44 and the mean in an outside of school context being 4.17. Overall, these numerical

results are encouraging, showing that most students agree that it is important to understand why an answer to a math problem is correct, rather than just being concerned about correct answers.

Figure 6

Spread of students' responses to survey items 6a and 6b.

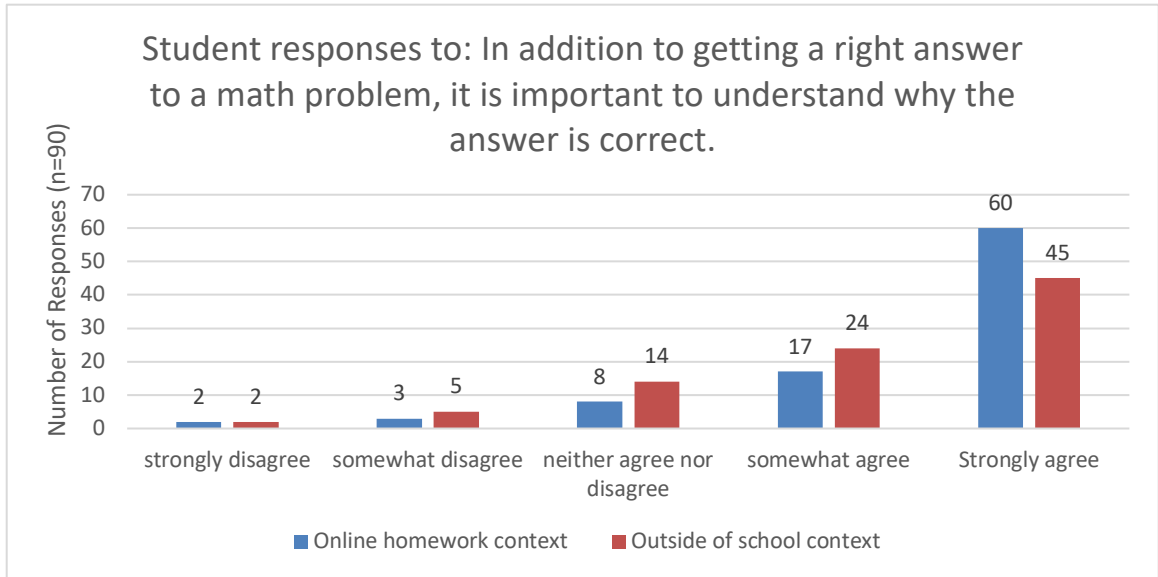


Table 15

Descriptive statistics for survey item 6a: "In addition to getting a right answer to an online homework problem, it is important to understand why the answer is correct."

Mean	Standard Deviation	Variance	Count
4.44	0.94	0.89	90

Table 16

Descriptive statistics for survey item 6b: “When math is done outside of a school context, it is important to understand why the answer is correct, in addition to getting the right answer.

Mean	Standard Deviation	Variance	Count
4.17	1.02	1.05	90

5.4.2 Likert scale results for survey items 10a and 10b

Descriptive statistics for survey items 10a and 10b are shown in Tables 17 and 18, along with Figure 7 which show the spread of students’ Likert-scale responses to whether it matters if understanding a math problem is important. The mean score of survey item 10a was 3.7, and the mean score of survey item 10b was 3.66. This shows that the students generally disagreed (either somewhat disagreed or strongly disagreed) to the statement both in an online homework context as well as an outside of school context, revealing that students believe that it is important to have an understanding of the math problem they are doing, rather than solely being worried about getting a correct answer. The themes in students’ short answer responses discussed next give more insight into these numerical results.

Figure 7

Spread of students' responses to survey items 10a and 10b.

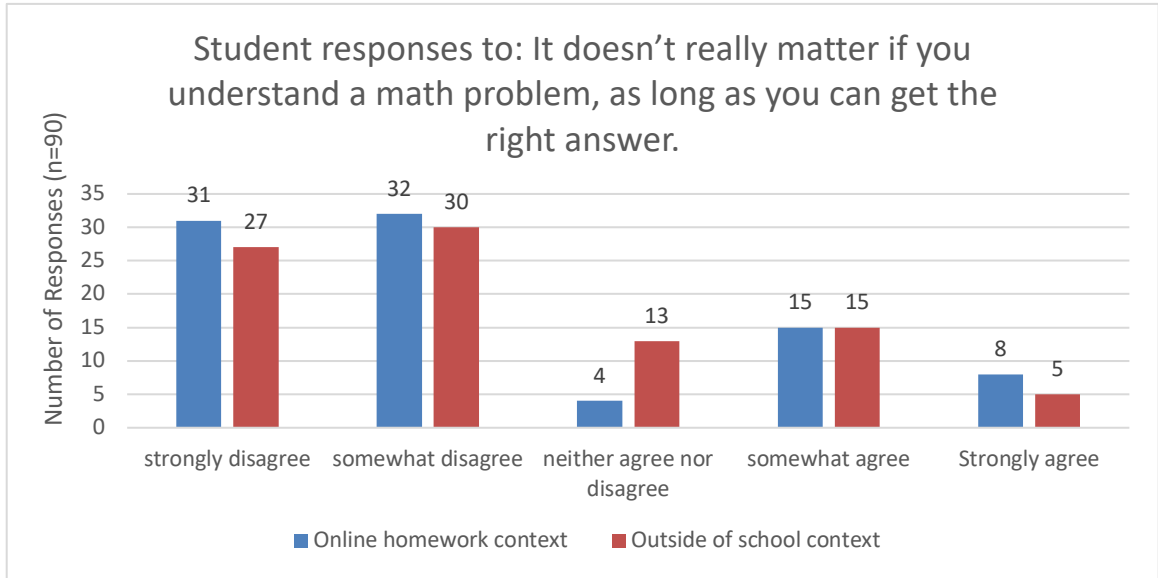


Table 17

Descriptive statistics for survey item 10a: "It doesn't really matter if you understand an online homework problem, as long as you can get the right answer"

Mean	Standard Deviation	Variance	Count
3.70	1.33	1.77	90

Table 18

Descriptive statistics for survey item 10b: "When math is done outside of a school context, it doesn't really matter if it is understood, as long the answer is right"

Mean	Standard Deviation	Variance	Count
3.66	1.22	1.49	90

5.4.3. Themes in survey items 6a and 6b

Tables 19 and 20 below show the themes which were identified within students' short

answer responses to survey items 6a and 6b, along with the frequency of each theme. The themes are not mutually exclusive. A total of 83 responses were coded for survey item 6a and a total of 75 responses for survey item 6b. The criteria for a response to be coded a certain theme are reported in Appendix B.

Table 19

Themes regarding survey item 6a: “In addition to getting a right answer to an online homework problem, it is important to understand why the answer is correct”

Theme	Frequency (83 total responses)
1a. Yes, understanding why an answer is correct is important for the sake of future problems or assignments or tests.	32 (39%)
1b. Yes understanding why an answer is correct is important for the future (in a general future sense)	4 (5%)
1c. Yes understanding why an answer is correct is important for the sake of a future career or job	1 (1%)
2. Yes understanding why an answer is correct is important just for the sake of understanding, the underlying idea is important	37 (45%)
3. Sometimes the online homework problem is hard, so sometimes I don't understand it	2 (2%)
4. Understanding in online homework is not necessarily important, I just want to get an answer and move on	10 (12%)
5. It depends on the person or the problem	2 (2%)
6. If you understand, you can be confident your answer is correct or you will be alert to wrong answers	1 (1%)

Table 20

Themes regarding survey item 6b: “When math is done outside of a school context, it is important to understand why the answer is correct, in addition to getting the right answer”

Theme	Frequency (75 total responses)
1a. Understanding why an answer is correct outside of school is important so you can solve other similar problems in the future.	13 (17%)

1b. Yes understanding why an answer is correct is important outside of school because of the future	6 (8%)
1c. Yes understanding why an answer is correct outside of school is important for the sake of a future career or job	2 (3%)
2. Understanding why the answer is correct outside of school is important just for the sake of understanding, knowing the underlying ideas is important	24 (32%)
3. Understanding why an answer is correct outside of school is not important, you just need the answer and don't need to explain it	12 (16%)
4. Understanding why an answer is correct outside of school is important because there could be serious consequences if you don't and/or knowing that something will work the way you think it does is important	14 (19%)
5. It depends on the question whether it's important to understand why the answer is correct	5 (7%)
6. I'm not sure	2 (3%)
7. I don't use math outside of school	1 (1%)
8. If you understand it you can explain it to others (need to justify reasoning)	1 (1%)

Regarding the individual themes for these two survey items, many students expressed agreement that understanding why an answer is correct is important, but for different reasons. In an online homework context, 39% of students expressed the belief that understanding why an answer is correct is important for the sake of being able to do future problems, assignments, or tests, while 45% of students explained the importance of understanding why an answer was correct just for understanding's sake. For example, Student 5 and Student 34's responses were:

Student 5: "If you don't understand how you got the correct answer, it won't help you during future tests or in future problems."

Code for quote: 1a. Yes, understanding why an answer is correct is important for the sake of future problems or assignments or tests.

Student 34: "It is always important that you understand why the answer is the answer because if your just putting answers in without understanding why, then you aren't really learning anything"

Code for quote: 2. Yes understanding why an answer is correct is important just for the sake of understanding, the underlying idea is important

One interviewee, Abbi, elaborated on this as she explained the importance of knowing why the answer to online homework questions are correct so that she will be able to do similar problems on the test: “I’ve learned that if I don’t understand, [...] I’m not going to be able to function and do that problem on that test.” She further explained that she had recently taken a math test, and did not know how to do one of the problems, saying, “I didn’t understand the problem so I completely blanked and left those two problems blank on my math test.” In her response to the survey, Abbi also explained the importance of having an understanding of why an answer is correct because of her future career as a teacher, and wanting to be able to teach the math to her students.

In an outside of school context, similar reasons were expressed by students for why it is important to understand why an answer is correct. Seventeen percent of students explained that it was important because of future problems they may have to solve, and 32% of students explained that it was important simply for the sake of understanding. Additional reasons students gave for the importance of understanding why an answer is correct include them explaining that there could be serious consequences if you do not. Nineteen percent of students’ responses included this reasoning. For example, Student 71 said, “you need to know what you’re doing in the real world when your actions have consequences.”

In addition to these encouraging themes which were identified in these two survey items, there were also some students who expressed the belief that understanding why an answer to a math problem is not important, and that they are mainly concerned with just getting the answer and moving on. Twelve percent of students expressed this within an online homework context, and 16% of students expressed this within an outside of school

context. When responding to an online homework context, Student 50 expressed this belief, saying, “If I can get the right answer in a way I normally just leave it at that.”

The next two survey items which contained similar themes within students’ responses are reported next.

5.4.4 Themes in survey items 10a and 10b

Tables 21 and 22 below show the themes which were identified within students’ short answer responses to survey items 10a and 10b, along with the frequency of each theme. The themes are not mutually exclusive. A total of 79 responses were coded for survey item 10a and a total of 69 responses for survey item 10b. The criteria for a response to be coded a certain theme are reported in Appendix C.

Table 21

Themes regarding survey item 10a: “It doesn’t really matter if you understand an online homework problem, as long as you can get the right answer”

Theme	Frequency (79 total responses)
1a. understanding how you got the right answer to an online homework problem is important for the sake of future problems or assignments or tests	35 (44%)
1b. Understanding how you got the right answer to an online homework problem is important for the sake of the future (in a general sense)	3 (4%)
2. In online homework, all you need is the right answer, not understanding, for the sake of a good grade	17 (22%)
3. Understanding how you got that right answer to an online homework problem; the underlying idea is important	30 (38%)

Table 22

Themes regarding survey item 10b: “When math is done outside of a school context, it doesn’t really matter if it is understood, as long as the answer is right”

Theme	Frequency (69 total responses)
1a. Outside of school, understanding how you got the right answer is important for the sake of future problems, or tasks that are similar	13 (19%)
1b. Yes understanding is important outside of school for the sake of a future career or job	2 (3%)
2. What matters outside of school is just getting the right answer and moving on, not worrying about understanding	14 (20%)
2a. Understanding isn’t as important because problems outside of school have more leeway with how to solve things	1 (1%)
3a. Outside of school, understanding how you got the right answer; the underlying idea is important outside of school	23 (33%)
3b. Understanding how you got the right answer is important outside of school to avoid potential consequences	2 (3%)
3c. Understanding how you got the right answer is important outside of school so that you know your answer is correct	3 (4%)
3d. Understanding is important outside of school because you need to be able to justify your answer	4 (6%)
3e. Understanding is important outside of school so that you can be able to leave a trail for others	4 (6%)
4. It depends (if understanding the problem is important)	10 (14%)
5. I’m not sure	2 (3%)

Similar to the themes identified in the last two survey items, the themes for survey statements 10a and 10b reveal that many students believe that understanding how you got a right answer to a math problem is important for reasons such as being able to do future problems that are similar, and also for the sake of understanding the underlying idea. In an online homework context, 44% explained the importance of the underlying idea for the sake of being able to do future problems on assignments and tests, and 38% explained the importance of the underlying understanding just for understanding’s sake. In an outside of school context, 19% expressed the belief that understanding the math problems

are important in order to be able to do future problems, and 33% expressed that just simply understanding the problem is what is important. Responding to an online homework context, Student 63 and Student 73 said the following:

Student 63: “It is important to understand because it could be on a test.”

Code for quote: 1a. understanding how you got the right answer to an online homework problem is important for the sake of future problems or assignments or tests

Student 73: “If you do not understand then you are not learning.”

Code for quote: 3. Understanding how you got that right answer to an online homework problem; the underlying idea is important

One interviewee, Abbi, expressed her belief that understanding how you got a right answer outside of school is important for the sake of being able to do future problems, or tasks that are similar. This belief is seen to have been constructed from her experiences at her job working at a hotel. She explained that it is important for her to understand how to correctly calculate the occupancy because if she calculates it one day, she may need to be able to calculate it again the next day:

Abbi: “[...] I know from day to day it varies so if I did one this way one day and then the next day I had to do it, I know it's important to know how to do that from time to time.”

In addition to the two themes mentioned above, responses to survey item 10b, which focused on an outside of school context, revealed more varied themes than the responses concerning an online homework context. For example, two students' responses expressed the belief that understanding a math problem outside of school was important so that the answer can be justified, and also so that others will be able to also understand it. One such response is as follows:

Student 76: “If you're a boss you need to be able to explain to your employees how things work.”

Codes for quote: 3d. Understanding is important outside of school because you need to be able to justify your answer, and 3e. Understanding is important outside of school so that you can be able to leave a trail for others

Like with the previous two survey statements which were discussed (items 6a and 6b), there was also some students that expressed the belief that just getting the right answer is what is important, not understanding the problem. Twenty-two percent of students expressed this belief in an online homework context, and 20% expressed this in an outside of school context. Student 77 expressed this belief by saying, “you get a good grade for clicking the right answer, not if you understand why it is right.”

We will now examine the survey statements which revealed students’ beliefs related to being able to solve math problems in multiple ways.

5.5 Solving math problems in multiple ways

Another commonly-held student belief is that there is one correct way to solve math problems. Schoenfeld (1992) described that often this one correct way is usually the rule that the teacher has most recently shown in class. This kind of belief is not aligned with a productive mathematical disposition in that it does not encourage students to be problem solvers, but instead leads them to be focused on memorizing procedures and looking for one exact way to solve problems. One goal of MATH 1483 is to promote ideas over formulas by working exclusively on real-world problems. To investigate what 1483 students believed about being able to solve math problems in multiple ways, I asked students to respond to the following four survey items:

***Survey item 2a:** There are often several different ways to solve an online homework problem.*

***Survey item 2b:** There are often several different ways to solve math problems that are*

done outside of a school context.

Survey item 8a: *To solve math problems in online homework, you have to know the exact procedure for each problem.*

Survey item 8b: *To solve math problems outside of a school context, you have to know the exact procedure for each problem.*

The next sections describe the numerical results from students' Likert-responses to these four items, as well as the themes found in their short answer responses.

5.5.1 Likert scale results for survey items 2a and 2b

Descriptive statistics for survey items 2a and 2b are shown in Tables 23 and 24, along with Figure 8 which shows the spread of students' responses to whether or not there are different ways to solve math problems. Both in an online homework context and an outside of school context, students generally agreed that there are often several different ways to solve a math problem ($\mu=3.97$ and $\mu=4.31$, respectively).

Figure 8

Spread of students' responses to survey items 2a and 2b.

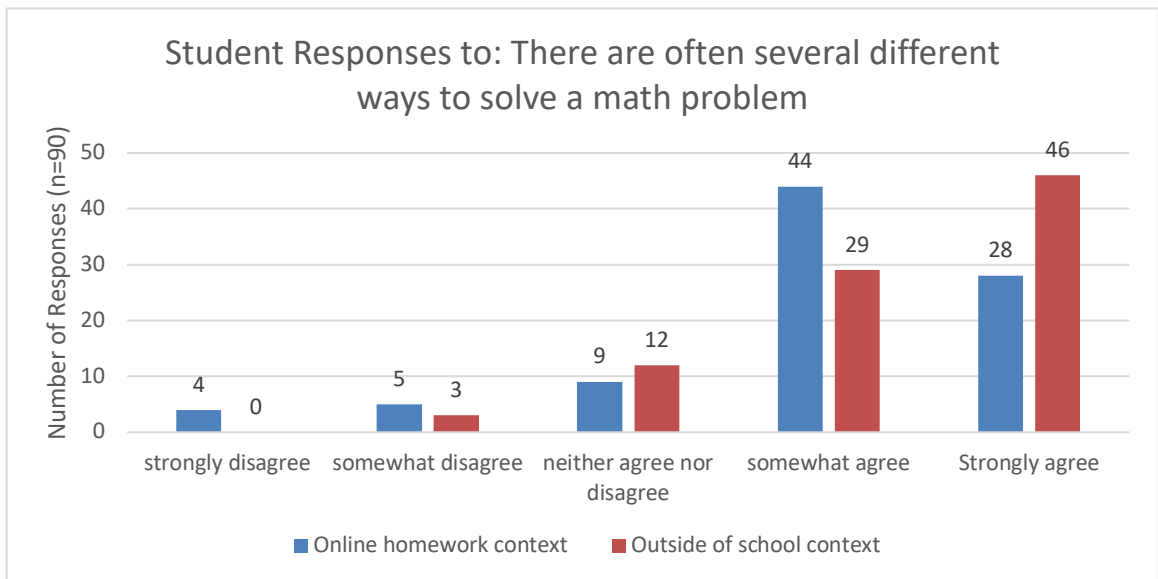


Table 23

Descriptive statistics for survey item 2a: “There are often several different ways to solve an online homework problem”

Mean	Standard Deviation	Variance	Count
3.97	1.02	1.03	90

Table 24

Descriptive statistics for survey item 2b: “There are often several different ways to solve math problems that are done outside of a school context”

Mean	Standard Deviation	Variance	Count
4.31	0.83	0.68	90

5.5.2 Likert scale results for survey items 8a and 8b

Descriptive statistics for survey items 8a and 8b are shown in Tables 25 and 26, along with Figure 9 which shows the spread of students’ responses to whether or not the exact procedures need to be known to solve math problems. The mean for item 8a was 2.5 and the mean for item 8b was 3.02. These means indicate that, in general, students neither agreed nor disagreed to whether or not an exact procedure must be known in order to solve a math problem. The graph below, however, tells a different, more interesting story. The graph indicates that students fell into two camps: those that generally believe you need to know the exact procedures and those that generally believe you do not need to know the exact procedures. Students’ short answer responses to these statements are discussed below, and provide further insight into these numerical results.

Figure 9

Spread of students' responses to survey items 8a and 8b.

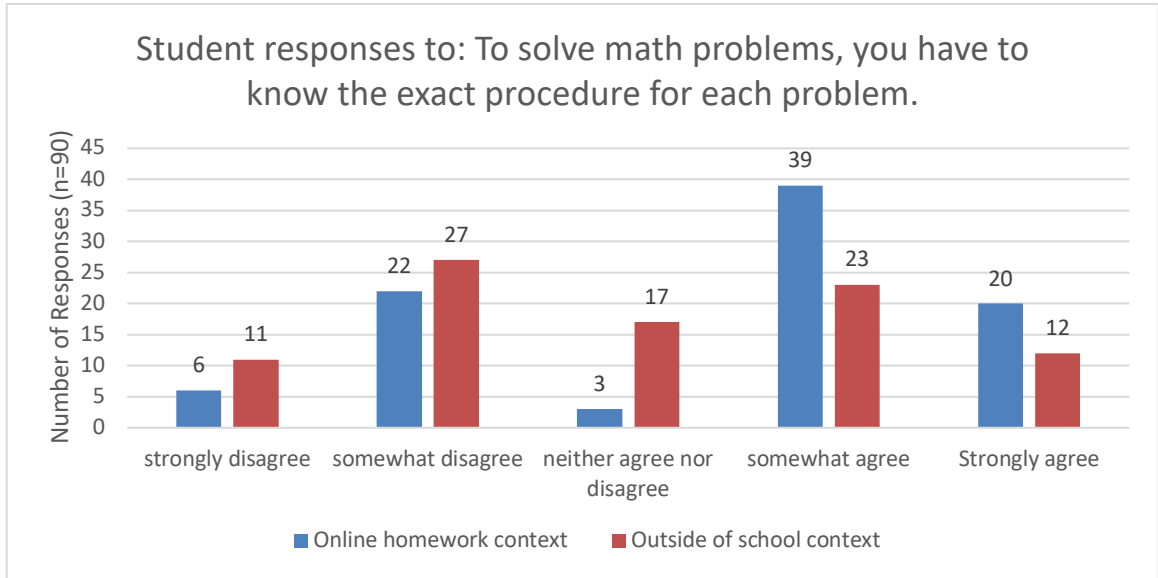


Table 25

Descriptive statistics for survey item 8a: "To solve math problems in online homework, you have to know the exact procedure for each problem"

Mean	Standard Deviation	Variance	Count
2.50	1.26	1.58	90

Table 26

Descriptive statistics for survey item 8b: "To solve math problems outside of a school context, you have to know the exact procedure for each problem"

Mean	Standard Deviation	Variance	Count
3.02	1.26	1.58	90

5.5.3 Themes in survey items 2a and 2b

The themes which were found in students' short answers responses to survey statements 2a and 2b, along with the frequency of each, are reported in Tables 27 and 28. The themes are not mutually exclusive. A total of 88 responses were coded from survey item 2a and 82 responses from survey item 2b. The criterion for each theme is reported in Appendix D.

Table 27

Themes regarding survey item 2a: "There are often several different ways to solve an online homework problem."

Theme	Frequency (88 total responses)
1. There might be multiple ways to solve an online homework problem, but most the time there are not (or I choose one way)	6 (7%)
2. There is more than one way to solve problems in online homework	51 (58%)
3. There is one distinct answer in online homework	11 (13%)
4. Sometimes/It depends on the online homework problem whether or not there are multiple ways to solve it	25 (28%)
5. There is one way to solve online homework problems, I stick to the formulas or one way of solving them	6 (7%)
6. I'm not sure whether there are multiple ways to solve online homework problems	2 (2%)

Table 28

Themes regarding survey item 2b: "There are often several different ways to solve math problems outside of a school context."

Theme	Frequency (82 total responses)
1. There are multiple ways to solve problems outside of school	62 (76%)
2. Sometimes/it depends on the problem if there are multiple ways to solve it	14 (17%)
3. Math problems are easier outside of school	7 (9%)
4. I'm not sure	3 (4%)
5. The answers outside of school don't have to be specific	1 (1%)
6. In school there is one way to solve problems	3 (4%)

7. There is one answer to outside of school problems, getting the answer is the goal 2 (2%)

Regarding the particular themes to these two survey items, the majority of students agreed that there are multiple ways to solve math problems, both in an online homework and outside of school context. Fifty-eight percent of students agreed to this in an online homework context, and 76% in an outside of school context. Student 26 and Student 61 expressed this belief, saying:

Student 26: “In my own experience, I may not understand how to get the answer one way [in online homework]. However, taking an alternative approach often does.”

Code for quote: 2. There is more than one way to solve problems in online homework

Student 61: “People can do real world problems many different ways and they all can get done”

Code for quote: 1. There are multiple ways to solve problems outside of school

When asked about the reason she believed that online homework problems could be solved in multiple ways, one interviewee, Abbi, explained that her husband had shown her other ways to solve problems that made more sense than what she was taught in class: “he taught me completely opposite of what she [her instructor] did and it still clicked either way.”

Other students expressed that being able to solve math problems in multiple ways depended on the problem (28% in an online homework context and 17% in an outside of school context). For example, when responding to an outside of school context, Student 11 said, “For many problems there are other ways to work through them, but there are still some that require formulas”, and Student 12 said, “Each problem is different.”

On the other hand, a small percentage of students explained that there is only one way to do online homework problems. Seven percent expressed this belief when

responding to doing math in online homework, and interestingly, 4% expressed this belief even when responding to the statement in an outside of school context. The reason for this can be seen in Student 48's response, who compared the contexts:

Student 48: "Outside of school there are more then [sic] one way. In school we know of one way, the way we learn in class"

Codes for quote: 1. There's multiple ways to solve problems outside of school, and 6. In school there is one way to solve problems

In the interview, Evan further expressed his belief that there is only one way to solve online homework problems. Evan explained that when solving online homework problems, there is a set formula that he needs to follow, and "very specific ways you need to solve the problems". Interestingly, the reason that he gave for having to solve the problem in a certain way was because the online homework program only accepts a specific answer: "Because it's like, it only takes a certain type of answer and even though there could be multiple answers to the problem, the online homework only takes one answer so you kinda gotta go that certain route." It is also important to note that within the short answer responses to survey item 2a, which asked about being able to solve online homework problems in more than one way, 13% of students noted there is one specific answer to the problems. Not all of these responses, however, used this as a reason to explain why online homework problems should be solved in one certain way, as Evan had. For example, Student 34 explained that even though online homework problems have a distinct answer, there could still be multiple ways to solve the problems: "While math has a distinct answer for most problems, there is never one way to solve a problem, just like in real life." This response was coded 2. *There is more than one way to solve problems in online homework*, and 3. *There is one distinct answer in online homework*.

Responses such as Evan’s demonstrate how, for some students, online homework programs requiring one specific answer lead students to believe that the problems should be solved one certain way.

5.5.4 Themes in survey items 8a and 8b

The themes which were found in students’ short answers responses to survey statements 8a and 8b, along with the frequency of each, are reported in Tables 29 and 30 below. The themes are not mutually exclusive. A total of 80 responses were coded from survey item 8a and 77 responses from survey item 8b. The criterion for each theme is reported in Appendix E.

Table 29

Themes regarding survey item 8a: “To solve math problems in online homework, you have to know the exact procedure for each problem.”

Theme	Frequency (80 total responses)
1. In online homework you don’t need to know the exact procedure, there are multiple ways to solve problems	31 (39%)
2. In online homework you should know the exact way to do the problem, that’s how you get it right, it makes it easier	33 (41%)
3. You don’t need the exact procedure in online homework, you can just guess	3 (4%)
4. You don’t always need exact procedures in online homework, you can figure it out having a basic understanding	7 (9%)
5. It depends on the online homework problem if you need an exact procedure	11 (14%)
6. Online homework needs a specific answer	12 (15%)

Table 30

Themes regarding survey item 8b: “To solve math problems outside of a school context, you have to know the exact procedure for each problem.”

Theme	Frequency (77 total responses)
1. Outside of school, one procedure is not always needed, there are different ways to solve math problems outside of school	43 (56%)
2. I don't have an opinion, or am unsure	2 (3%)
3. To solve math problems outside of school, you can guess or make it up as you go	5 (6%)
4. When doing math outside of school, you need to know the correct procedures to get it right, it makes it easier	16 (21%)
5. It depends on the problem outside of school whether you need an exact procedure	9 (12%)
6. just getting the right answer is what is important outside of school, they don't need explanation	2 (3%)
7. For math problems outside of school, you can figure it out having a basic understanding	10 (13%)
8. Math problems outside of school have one right answer	1 (1%)
9. math problems outside of school may be more complex than those that can be solved with a simple formula	1 (1%)

Regarding the particular themes for these two survey items, many students expressed the belief that math problems can be solved in more than one way, similar to the themes discussed in the previous two survey items. Thirty-nine percent of students explained that online homework problems can be solved in multiple ways, and that an exact procedure is not needed. Fifty-six percent explained this same belief in an outside of school context. For example, Student 27 explained, “I sometimes figure it out a completely different way.”

Evan, when interviewed and asked about doing math outside of school, explained his belief that exact procedures are not needed, and that there are multiple ways one can solve problems:

Interviewer: “So, outside of school is there need to follow exact procedures?”

Evan: “No. You can do it however you want as long as you get the right answer.”

Evan went on to explain that these other ways of solving problems involve multiplication, division, and “just a lot of mental math.” It is important to note that Evan explained he mostly just does simple, everyday math outside of school. This belief is similar to 13% of students’ responses to survey item 8b, which were coded 7. *For math problems outside of school, you can figure it out having a basic understanding.*

While students expressed that there are multiple ways to solve online homework problems, many also explained that one should know the exact procedure and that knowing it makes solving the problem easier. For example, Student 32 said, “I think so because we learn exact formulas and procedures... it at least definitely makes it easier.”

One interviewee, Abbi, responded on the online survey by saying “at the beginning of this semester I understood how to get things and didn’t know the correct procedures, but a rule of thumb is to know the correct way to solve problems.” This was a seemingly contradictory response, whereas she had previously explained that online homework problems could be solved in multiple ways. When asked if she believed that most of the time there were correct ways to solve problems, Abbi explained: “there isn’t most of the time as long as you know the formula and you know the procedures behind it and stuff.” This response helped to clear up the seemingly contradictory responses she was giving, and further shows Abbi’s belief that even though she feels that one should know how the formulas and procedures work, this does not mean that she has to solve the problem in one certain way.

Also, similar to the themes discussed in the last pair of survey items, 15% of

students explained that online homework problems need a specific answer. As Evan's interview response earlier revealed, some students agree that there are certain ways online homework problems should be solved because of the fact that they require one specific answer. Student 77's response is an example of this. Student 77 strongly agreed to survey item 8a, that online homework problems need exact procedures in order to be solved, and explained, "the computer wants a specific answer." This result provides insight into the ways in which online homework platforms could be improved so that they better support students in having a productive mathematical disposition.

Next, students' beliefs about how many answers math problems have will be discussed.

5.6 The number of answers to math problems

Another common belief that students hold is that there is one and only one correct answer to math problems. As seen above, this belief has the potential to also encourage students in believing that math problems must be solved using exact formulas and procedures. When students believe that their job is to look for the "one correct answer", they might focus on memorizing formulas rather than focusing on problem solving, the latter of which is an instructional goal of MATH 1483. To investigate what MATH 1483 students believed about math problems having one right answer, I asked them to respond to the following two survey items:

Survey item 11a: *Online homework problems have one and only one right answer.*

Survey item 11b: *Math problems that are done outside of a school context have one and only one right answer.*

The numerical results to these two survey items, as well as the themes in students' short

answer responses will be discussed next.

5.6.1 Likert scale results for survey items 11a and 11b

Descriptive statistics for survey items 11a and 11b are shown below in Tables 31 and 32, along with Figure 10 showing the spread of students' responses to these items. The mean for survey item 11a was 1.83, signifying that students generally agreed that online homework problems have one and only one right answer. Seventy-six percent of students either somewhat agreed or strongly agreed to this statement. On the other hand, survey item 11b had a mean of 2.99, signifying that students generally did not agree not disagree with the statement. As was discussed earlier when presenting the quantitative results, this belief statements had the greatest mean difference across the two contexts of online homework and outside of school. The following section will discuss students short answer responses to these statements, which give further insight into these numerical results.

Figure 10

Spread of students' responses to survey items 11a and 11b.

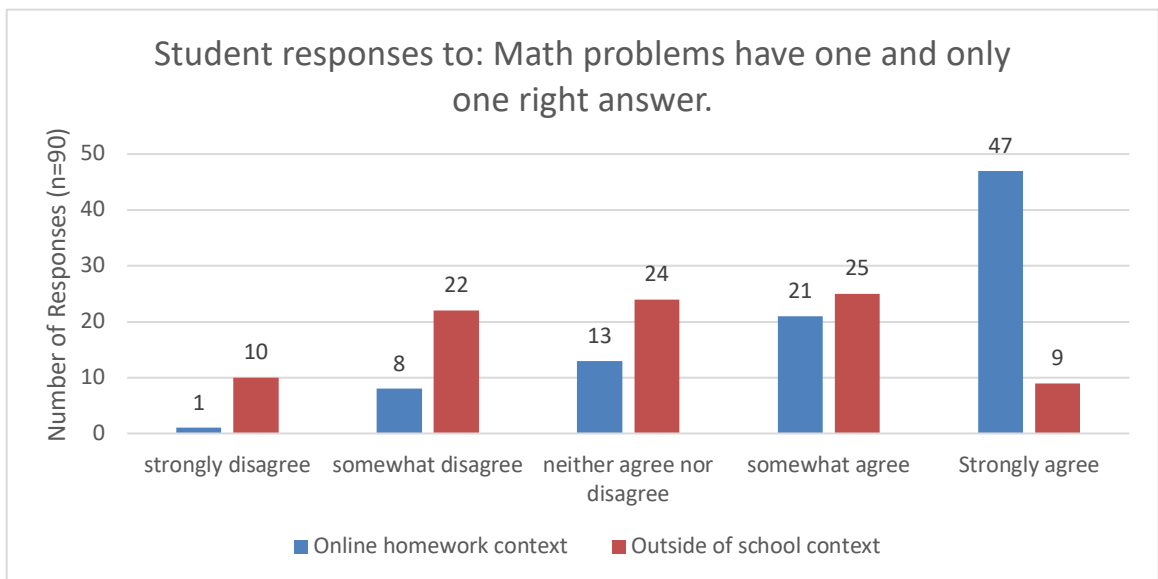


Table 31

Descriptive statistics to survey item 11a: “Online homework problems have one and only one right answer.”

Mean	Standard Deviation	Variance	Count
1.83	1.05	1.09	90

Table 32

Descriptive statistics to survey item 11b: “Math problems that are done outside of school have one and only one right answer.”

Mean	Standard Deviation	Variance	Count
2.99	1.17	1.37	90

5.6.2 Themes in survey items 11a and 11b

The themes which were found in students’ short answers responses to survey statements 11a and 11b, along with the frequency of each, are reported in Tables 33 and 34 below. The themes are not mutually exclusive. A total of 79 responses were coded from survey item 11a and 73 responses from survey item 11b. The criterion for each theme is reported Appendix F.

Table 33

Themes regarding survey item 11a: “Online homework problems have one and only one right answer.”

Theme	Frequency (79 total responses)
1. Online homework problems have one exact right answer (the program is picky about entering answers).	55 (70%)
2. Online homework problems could have multiple answers (including	13 (16%)

things like rounding differently or reordering an expression).	
3. It depends on the problem if it has exactly one answer.	12 (15%)
4. I'm not sure whether online homework problems have online one right answer	3 (4%)
5. You can solve online homework problems in multiple ways.	3 (4%)

Table 34

Themes regarding survey item 11b: "Math problems that are done outside of school have one and only one right answer."

Theme	Frequency (73 total responses)
1. Math problems outside of school have exactly one right answer.	16 (22%)
2. It depends on the problem/context whether there is only one answer.	27 (37%)
3. Math problems outside of school can have multiple answers; outside of school the math is more flexible.	25 (34%)
4. Math problems outside of school can be solved multiple ways.	9 (12%)
5. I'm not sure whether problems outside of school have only one right answer.	7 (10%)

Like what is indicated from the Likert results for these survey items, most students agreed that online homework problems have only one answer (70%). This is contrasted to the 22% of responses which expressed that math problems outside of school have only one answer. When responding to whether online homework problems have one right answer, many students expressed agreement, describing how the online homework program was picky about answers being inputted. For example, Student 83 and Student 25 responded:

Student 83: "Online problems have to be in the correct format and answer or it is counted wrong."

Code for quote: 1. Online homework problems have one exact right answer (the program is picky about entering answers).

Student 25: "It will count the answer as incorrect unless it is exactly what their answer is."

Code for quote: 1. Online homework problems have one exact right answer (the program is picky about entering answers).

In contrast, when responding to whether math problems outside of school have only one right answer, students' beliefs were more aligned with disagreement, and expressed that there could be more than one right answer. For example, Student 34 used the example of fashion design to give context to this belief:

Student 34: "In fashion design, we may all have the same project and math, but others have to change that math so the garment fits them since all our proportions are different so sometimes one result or answer is not the same as the other."

Code for quote: 3. Math problems outside of school can have multiple answers; outside of school the math is more flexible.

Still, 37% of students expressed that it depended whether math problems outside of school have one right answer: "It depends on the context of the problem." (Student 26).

All three interviewees expressed that WebAssign was very picky about answers, and that online homework problems have only one right answer. For example, Evan explained this belief when he said, "online homework is so programmed and you have to submit it a certain way, certain right answer." Another interviewee, Abbi, explained the pickiness of the online homework platform, and how her answer could be wrong simply because she did not capitalize a letter:

"It is very picky. So I knew last, last week, I was doing the homework problem I'm like, alright, I know this formula is correct, what am I doing wrong? It was the difference of a capital letter... it was supposed to be, supposed to be lowercase, and I was putting capital, and it's that picky."

Abbi's belief that online homework problems have only one correct answer is seen to have been constructed from her experience of the 'pickiness' of the online homework platform. In contrast, when asked about whether or not there could be multiple

answers to math problems outside of school, Abbi agreed, and gave an example from her experience working at a hotel, and giving back change to her customers in multiple ways:

“If I’ve got somebody giving me a dollar bill and they’re needing change, I can give them four quarters or ten dimes or, I think it’s 20 nickels, or 100 pennies. And just, you’ve got multiple right answers of ways of giving them change from a dollar bill”.

When interviewed, Taylor and Evan also both expressed their belief that math problems outside of school can have multiple right answers, because human interpretation is involved:

Evan: “yeah like outside of school there could be definitely different answers because [...] you’re bringing like humans into play, so humans interpret things a lot differently than you know a computer can.”

Taylor: “I think there’s definitely just a set selection of answers they’ll accept [in online homework], and there’s no human forgiveness. [...] in general I think there’s a lot more forgiveness with your answer outside of online homework. [...] because it’s just a person looking at it.”

Students’ beliefs about whether math problems have only one right answer are contrasted between the two contexts of online homework and math that is done outside of school. As was seen in the Likert results of these survey items, these two belief statements had the greatest difference of mean scores across the two contexts. This difference in belief across contexts is highlighted by the selected students’ responses above.

Next, we will look at students’ beliefs about how quickly math problems should be done.

5.7 The amount of time it should take to solve a math problem

Another common belief that students hold about mathematics is that these problems should be done quickly. Schoenfeld (1992) described this belief as being prominent among students, and explained that it can have negative consequences:

“Students with the belief will give up on a problem after a few minutes of unsuccessful attempts, even though they might have solved it had they persevered” (p. 359). To investigate what MATH 1483 students believed about doing mathematics problems quickly, I asked them to respond to the following two survey items:

***Survey item 9a:** Students who understand the math they have studied will be able to solve any assigned online homework problem in five minutes or less.*

***Survey item 9b:** Math problems that are done outside of a school context can be solved in five minutes or less.*

The numerical results to these two survey items, as well as themes from students’ short answer responses are reported next.

5.7.1 Likert scale results for survey items 9a and 9b

Descriptive statistics to survey items 9a and 9b are shown below in Tables 35 and 36, as well as Figure 11 showing the spread of students’ Likert responses to whether or not students should be able to solve math problems in five minutes or less. The means and distribution of the two items show that students generally neither agree nor disagree that math problems should be solved in five minutes or less, both in an online homework and an outside of school context ($\mu=2.8$ and $\mu=3.18$, respectively). Students’ short answer responses to these two survey items give further insight into these numerical results.

Figure 11

Spread of students' responses to survey items 9a and 9b.

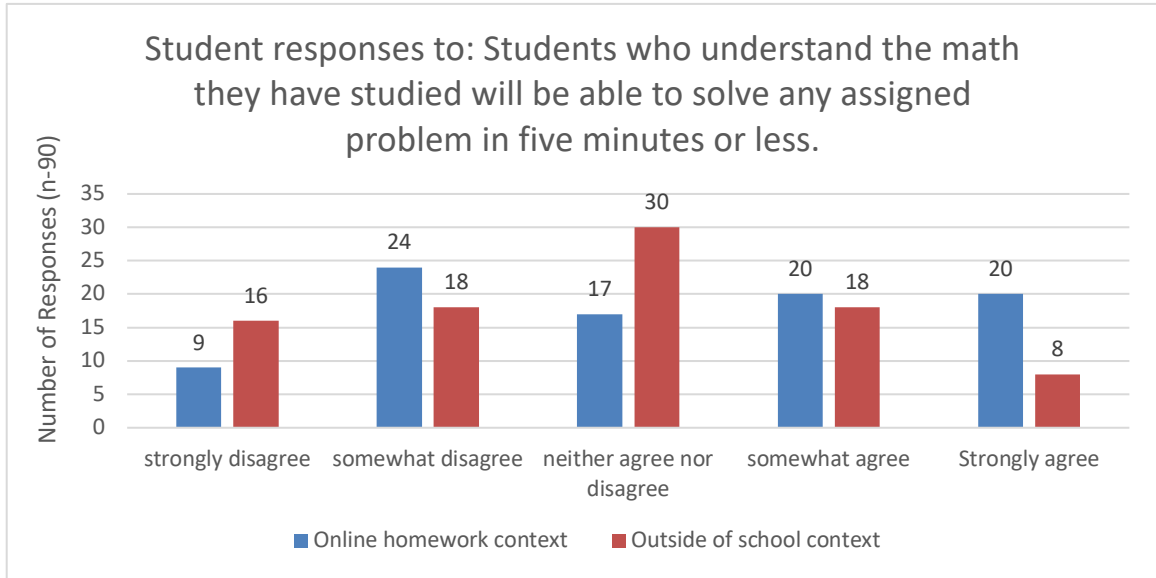


Table 35

Descriptive statistics to survey item 9a: "Students who understand the math they have studied will be able to solve any assigned online homework problem in five minutes or less"

Mean	Standard Deviation	Variance	Count
2.80	1.32	1.74	90

Table 36

Descriptive statistics to survey item 9b: "Math problems that are done outside of a school context can be solved in five minutes or less"

Mean	Standard Deviation	Variance	Count
3.18	1.20	1.44	90

5.7.2 Themes in survey items 9a and 9b

Tables 37 and 38 below display the themes which were identified from survey items 9a and 9b, as well as the frequency of each. The themes are not mutually exclusive. A total of 81 responses were coded from item 9a, and 76 responses from item 9b. The criteria for a response to be coded a certain theme is outlined in Appendix G.

Table 37

Codes for survey item 9a: “Students who understand the math they have studied will be able to solve any assigned online homework problem in five minutes or less.”

Theme	Frequency (81 total responses)
1. Finishing an online homework problem in 5 minutes or less depends on the person	18 (22%)
2. Online homework problems should be done in 5 minutes or less, these problems are easy if you understand the math	26 (32%)
3. Even if you understand, that doesn’t mean you have to do the online homework problem in 5 minutes or less (understanding doesn’t mean doing quickly)	10 (12%)
4. It depends on the problem, there are some online homework problems that could take longer than 5 minutes, some problems are harder	32 (40%)
5. It takes me longer to solve online homework problems because I like to check my answers	1 (1%)

Table 38

Codes for survey item 9b: “Math problems that are done outside of a school context can be solved in five minutes or less.”

Theme	Frequency (76 total responses)
1. Solving a math problem outside of school in 5 minutes or less depends on the person	17 (22%)
2. It depends on the problem, there are some problems that take longer than 5 minutes (like if the problem is complex, or if it’s in a new context)	50 (66%)
3. Problems outside of school could be done in 5 minutes or less, they are	12 (16%)

easy if you understand the math, it's just simple math	
4. Even if you understand, that doesn't mean you have to do problems outside of school in 5 minutes or less (understanding doesn't mean doing quickly)	1 (1%)
5. I don't know	2 (3%)
6. Some math problems take longer than five minutes because accuracy is very important	3 (4%)

The majority of students' responses to these survey items expressed the belief that finishing a math problem in five minutes or less, both in an online homework context as well as an outside of school context, depends either on the person doing the problem, or the problem itself. In an online homework context, 22% of students said it depended on the person, while 40% said it depended on the problem. For example, Student 34 explained:

Student 34: "Everyone works at their own pace. They may be able to solve the same problem as others but you should never expect everyone to solve the same problem at the same time and as quick as others."

Code for quote: 1. Finishing an online homework problem in 5 minutes or less depends on the person

Explaining that some online homework problems are harder than others, Student 43 said, "Sometimes problems are more difficult to solve than others and will take more time."

Similarly, when responding to whether math problems done in an outside of school context should be able to be solved in five minutes or less, 22% of students said it depended on the person solving it, and 66% said that it depended on the problem. For example, Student 63 said, "It takes people different lengths of time based on who they are and how they function", and Student 31 said, "Depending on the situation of the problem the time it takes to solve could vary."

This type of belief was verbalized by one interviewee, Taylor, as she explained that doing mathematics problems outside of school in under five minutes depended on the

situation, and also upon what might be expected of someone at their job. She gave an example pertaining to her academic major, interior design:

Taylor: “I think it definitely depends on what you’re doing, how quickly they’re expecting a response from you. Because like, if you’re working on the floor plan and they’re asking like, “okay we need to know how much, what’s the distance between each of these lights? And how many like can we put in?” It’s gonna take you a little while to calculate that and make sure you’re in compliance with everything.”

When asked about doing mathematics in five minutes or less in online homework, Taylor verbalized a different belief, explaining that “once I go over the problem multiple times, learning how to do it, it becomes just, in no time I can solve it”. She then added that it depends on the problem she is doing, but “in general” finishing the problems in five minutes or less is doable. This belief is aligned with 32% of students’ responses on survey item 9a which were coded 2. *Online homework problems should be done in 5 minutes or less, these problems are easy if you understand the math.*

Perhaps the belief that being able to doing mathematics problems in 5 minutes or less depends on the person solving the problem or on the problem itself is more encouraging than the commonly held belief reported in the literature – that all math problems *should* be done in five minutes or less. Even more encouraging are the responses from 12% of students who expressed the belief that understanding an online homework problem does not equate to being able to do it quickly. One such response is from Student 3, who said, “Understanding how to do something does not mean you can solve it within a certain period of time.” This belief was also expressed by another interviewee, Abbi. After first explaining that doing a math problem in five minutes or less depended on the problem, Abbi further explained that she likes to “break things down individually”, and that even though she understands the mathematics, it will take her

more than five minutes to solve the problem. Responses such as Abbi’s and Taylor’s give further insight and context to their beliefs. Taylor’s belief can be seen to be constructed from her experience and what she knows about doing math outside of school related to interior design. Abbi’s belief is seen to follow from her experiences doing online homework, and “breaking things down individually”.

5.8 Connections between students’ beliefs and having unlimited tries in online homework

As reported in the Methods section, as well as responding to belief statements in an online homework and outside of school context, students also were asked to respond to statements having to do with having unlimited tries in online homework to explore whether this influenced their beliefs about the nature of mathematics and their ability to do mathematics. Data from students’ short answer responses to the statement *Having unlimited tries in online homework helps me learn* did reveal some connections to students’ beliefs about the nature of mathematics their ability to do mathematics in an online homework context. The spread of students’ responses to the Likert-scale are reported in Table 39.

Table 39

Spread of students’ responses to “Having unlimited tries in online homework helps me learn”

	Strongly disagree	Somewhat disagree	Neither agree nor disagree	Somewhat agree	Strongly agree
Having unlimited tries in online homework helps me learn.	0 0.00%	2 2.22%	3 3.33%	7 7.78%	78 86.67%

Close to half of students' short-answer responses (49%) to this statement explained that having unlimited tries in online homework makes them more willing to work on problems, and also to try different methods to solve the problems. For example, consider three students' responses below:

Student 37: "I can try different ways to solve them and see what works and what comes easiest to me"

Student 63: "I can learn from my mistakes and try new methods."

Student 14: "I can backtrack and try different things"

These results indicate that having unlimited tries may help to encourage students in believing that math problems can be solved in multiple ways, and further encourage the conception that problem solving is an important part of doing mathematics. In contrast, the short-answer responses from the two students who disagreed to this survey statement are as follows:

Student 41: "If it's a problem I'm not understanding, unlimited tries doesn't really fix that"

Student 53: "Having unlimited tries can make it difficult to know if you really go[t] the right answer"

The responses from these two students revealed their belief that having unlimited tries does not help them learn or encourage them to problem solve.

CHAPTER VI

CONCLUSION

6.1 Summary of results and answers to research questions

This study sought to answer the following research questions: *What do MATH 1483 students believe about the nature of mathematics in the context of online homework?* and *what do MATH 1483 students believe about the nature of mathematics in an outside of school context?* Overall, data from the online survey, as well as interviews with students, revealed encouraging results as students expressed many beliefs which were aligned with a productive mathematical disposition.

Concerning an online homework context, many MATH 1483 students expressed the belief that understanding the underlying mathematical ideas is more important than just getting right answers. Some students expressed this belief explaining that the underlying idea is important simply for understanding's sake, and others explained that it is important so that they would be able to solve future problems on assignments or tests. This highlights that many MATH 1483 students are motivated by a performance goal orientation, focusing on the outcome of a good grade, rather than a mastery goal orientation, which focuses on learning and understanding (Pintrich & Zusho, 2007).

Similarly, many students expressed the importance of the underlying mathematical ideas in an outside of school context. Many of these students explained that simply having the underlying understanding of the math was important, while some

students explained the importance of understanding to be able to solve problems they might encounter in the future, or in order to avoid potential consequences in the real world.

Another productive belief which was identified in the data in both contexts was the importance of understanding the ideas behind mathematical formulas and procedures. Though many students expressed the belief that doing mathematics was mainly about memorizing formulas and procedures (items 12a and 12b), many students expressed the importance of knowing how to use and apply those formulas and procedures, or that having an underlying understanding, rather than just memorizing formulas and procedures, was important.

Another encouraging result identified in the data concerning students' mathematical beliefs, both in an online homework and outside of school context, was that many students believed there are multiple ways to solve math problems. Although a higher percentage of students believed this was true when doing math outside of school, there was still 39% of responses on survey item 8a and 58% of responses on survey item 2a which explained how there are multiple ways to solve online homework problems. Furthermore, the survey data also showed that many students explained that having unlimited tries in online homework makes them more willing to work on problems and try different methods to solve the problems, signifying that this supports at least some students in constructing productive mathematical beliefs, and countering beliefs such as "math problems must be solved one certain way". The data from this study also reveals another encouraging result, that many MATH 1483 students do not necessarily believe that math problems in either context should be solved in five minutes or less, as most

students' responses expressed that it depends either on the problem being solved or the person solving the problem.

Along with much of the data revealing student beliefs which align with a productive mathematical disposition, there were many students who held the belief that online homework problems have one and only one right answer, which was attributed by many to the strictness of WebAssign in accepting only specific answers. Some students also used this as a reason to explain why math problems must be solved one certain way. This was contrasted to students expressing a different belief outside of a school context, that math problems could have multiple right answers.

The decision to ask students to explain why they responded as they did on the Likert portion of the survey was due to some of the weaknesses which have been identified with instruments only utilizing Likert-responses, including difficulty in knowing how respondents are interpreting items, as well as items not being contextualized (Ambrose et al., 2003). Asking students to include these short answer responses proved to be important, as students' provided reasons for and gave context to their beliefs.

Interviewing students also gave further insight into the numerical results of the study as well as students' short answer responses. The interview data presented in this study displays the ways in which students' beliefs are constructed from their experiences working within the WebAssign platform, as well as their experiences in doing mathematics outside of school. For example, all three interviewees held the belief that online homework problems have one and only one right answer. Abbi explained how for her, this belief was informed by her experiences using WebAssign, including times where

her answer is incorrect simple because of not capitalizing a letter. Both Abbi and Taylor also expressed beliefs which were informed by what their future careers would be. In Abbi's case, because she was going to be a future teacher, she held the conception that having an understanding of the underlying mathematical ideas was important because she wants to effectively be able to teach her students math. For Taylor, who is studying interior design, being able to solve math problems quickly depends on the type of problem being done, because certain mathematical tasks in her field of study are more complex in nature.

In addition to providing context to students' constructed beliefs, the interview data also illustrates ways in which students' beliefs about mathematics in an online homework context lie in a separate cluster within their belief system from their beliefs about mathematics in an outside of school context, as discussed in the theoretical perspective. This clustering of beliefs is seen, for example, in the way Evan explained being able to solve math problems outside of school in multiple ways, but believing that online homework problems needed to be solved in one certain way. Beliefs which lie in separate clusters arise in independent contexts (Green, 1971), which may explain Evan's different beliefs for the different contexts.

6.2 Implications

The results of this study inform ways in which online homework platforms can be utilized and improved in order to better foster more productive mathematical dispositions for students. As seen in this study, the use of unlimited tries may encourage students in constructing the productive belief that math problems can be solved using a variety of solution methods, and may support them in problem solving. Also, WebAssign could be

improved such that it accepts a wider range of answers, such as accepting rounded answers within a range of values. This may encourage students to be more focused on problem solving and understanding the problem rather than just being worried about appeasing a picky system by typing in an exact answer. Students' complaints about programs being "picky", both in this study and in the literature more broadly (Heenehan & Khorami, 2016; Leong & Alexander, 2014; Roth, Ivanchenko, & Record, 2008; Yushau & Khan, 2014), may indicate places in which students are missing an important component of instruction. For example, instructors may not have done a good enough job explaining why capitalization (or lack thereof) matters in mathematics.

Also, these results show that MATH 1483 students have a variety of beliefs about doing math outside of a school context. For some, doing math in the real world means only doing "simple math", but for others it means using the subject in their workplace figuring out the occupancy of a hotel or designing floor plans that comply with certain building regulations. Intentionally implementing opportunities for MATH 1483 students to reflect upon how mathematics is relevant and useful to their lives, including through things such as projects or reflection questions, could be useful to students in seeing mathematics as more meaningful outside of school, encouraging a productive mathematical disposition both in school and outside of it.

6.3 Limitations

One limitation of this study lies in the fact that the beliefs of students had to be inferred from how they responded on the online survey, or from what they said in interviews. The nature of beliefs results in these inferences having to be made from what students say or do (Pajares, 1992). Though students' short-answer responses and

interview data did provide valuable context to students' professed beliefs, I remain aware that some students' responses may have been informed by reasons which were not explicitly stated, resulting in an incomplete picture of students' beliefs. Conducting more interviews, as well as potentially observing students as they work on online homework could provide further insight into students' beliefs, which often times are subconsciously held.

6.4 Recommendations for future work

Like mentioned at above, in building upon the current study, observations of students working on online homework could be conducted, to see if what they believe about the nature of mathematics in an online homework context influences their practice in any way, or if any connections between their stated beliefs and practice can be identified. Particularly for students such as Evan, who held the belief that since online homework problems require one specific answer, they should be solved one certain way, observing how such a belief effects students as they work on online homework could prove to be insightful. Also, intentionally implementing opportunities for MATH 1483 students to reflect on their mathematical beliefs concerning an outside of school context could encourage these students to see the subject as being more useful and applicable to their lives. I mention reflection upon beliefs because, as Jankvist (2015) noted in his study, the higher level of reflection, the more likely any change in such beliefs will last. The literature review identified many instances in which students' unproductive mathematical beliefs were altered and challenged, but remembering that reflection is a key part in lasting belief change is important, as beliefs are known to be resilient and resistant to change, developed throughout years of a students' school experience.

Another potential area of research would be to look into how other online homework platforms affect students' beliefs about mathematics, including how enabling other features such as hints, or limiting the number of tries, helps to foster or discourage students in having productive mathematical beliefs. Continuing to explore how online homework platforms (including the features we enable within them) can be better improved to encourage students in having a more productive mathematical disposition is an important endeavor.

REFERENCES

- Ambrose, R. (2004). Initiating change in prospective elementary school teachers' orientations to mathematics teaching by building on beliefs. *Journal of Mathematics Teacher Education*, 7(2), 91-119.
- Ambrose, R., Philipp, R., Chauvot, J., & Clement, L. (2003). A web-based survey to assess prospective elementary school teachers' beliefs about mathematics and mathematics learning: An alternative to Likert scales. In N. A. Pateman, B.J. Dougherty, & J.T. Zilliox (Eds.), *Proceedings of the 27th Conference of the International Group for the Psychology of Mathematics Education held jointly with the 25th Conference of PME-NA* (Vol. 2, pp. 33-40). Hawaii.
- Beswick, K. (2011). Knowledge/beliefs and their relationship to emotion. In K. Kislenko (Ed.), *Current State of Research on Mathematical Beliefs XVI: Proceedings of the MAVI-16 Conference* (pp. 43-59). Tallinn, Estonia.
- Breen, S., & O'Shea, A. (2019). Designing mathematical thinking tasks. *PRIMUS*, 29(1), 9-20.
- Carlson, M. P. (1999). The mathematical behavior of six successful mathematics graduate students: Influences leading to mathematical success. *Educational Studies in Mathematics*, 40(3), 237-258.

- Crauder, B., Evans, B., & Noell, A. (2017). *Functions and change: A modeling approach to college algebra*. Cengage Learning.
- Dewar, J. (2008). What is mathematics: student and faculty views. *Electronic Proceedings for the Eleventh Special Interest Group of the Mathematical Association of America on Research in Undergraduate Mathematics*. San Diego, CA. Retrieved from <http://sigmaa.maa.org/rume/crume2008/Proceedings/Proceedings/Proceedings.html>
- Dorko, A. (in press). Red X's and green checks: A model of how students engage with online homework. *The International Journal of Research in Undergraduate Mathematics Education*.
- Dorko, A. (2020a). What do we know about student learning from online mathematics homework? J.P. Howard and J.F. Rivers (Eds.), *Teaching and Learning Mathematics Online* (pp. 17-24). CRC Press.
- Dorko, A. (2020b). How do students engage with 'practice another version' in online homework? In (Eds.) XXXXX, *Proceedings of the 23rd Annual Conference on Research in Undergraduate Mathematics Education* (pp. XXXX). Boston, MA.
- Ellis, J., Hanson, K., Nuñez, G., & Rasmussen, C. (2015). Beyond plug and chug: An analysis of calculus I homework. *International Journal of Research in Undergraduate Mathematics Education*, 1(2), 268-287.
- Ernest, P. (1989). *Mathematics teaching: The state of the art*. The Falmer Press, Taylor & Francis, Inc., 1900 Frost Rd., Suite 101, Bristol, PA 19007.
- Fennema, E., & Sherman, J. A. (1976). Fennema-Sherman mathematics attitudes scales: Instruments designed to measure attitudes toward the learning of mathematics by

- females and males. *Journal for Research in Mathematics Education*, 7(5), 324-326.
- Frank, M. L. (1988). Problem solving and mathematical beliefs. *Arithmetic Teacher*, 35(5), 32-34.
- Garofalo, J. (1989). Beliefs and their influence on mathematical performance. *The Mathematics Teacher*, 82(7), 502-505.
- Green, T. F. (1971). *The activities of teaching*. New York: McGraw-Hill.
- Hauk, S., & Segalla, A. (2005). Student perceptions of the web-based homework program WeBWorK in moderate enrollment college algebra classes. *Journal of Computers in Mathematics and Science Teaching*, 24(3), 229-253.
- Heenehan, M. E., & Khorami, M. (2016). Students' reactions to the homework assessment system WeBWorK. *Mathematics and Computer Education*, 50(1), 42-51.
- Holdener, J. A., & Jones, B. D. (2019). Calculus homework: A storied approach. *PRIMUS*, 29(1), 21-42.
- Jankvist, U. T. (2009). A categorization of the “whys” and “hows” of using history in mathematics education. *Educational Studies in Mathematics*, 71(3), 235-261.
- Jankvist, U. T. (2015). Changing students' images of “mathematics as a discipline”. *The Journal of Mathematical Behavior*, 38, 41-56.
- Jankvist, U., & Niss, M. (2018). Counteracting destructive student misconceptions of mathematics. *Education Sciences*, 8(2), 1-17.
- Kloosterman, P. (1996). Students' beliefs about knowing and learning mathematics: Implications for motivation. *Motivation in Mathematics*, 131-156.

- Kloosterman, P. (2002). Beliefs about mathematics and mathematics learning in the secondary school: Measurement and implications for motivation. In G. C. Leder, E. Pehkonen, & G. Törner (Eds.), *Beliefs: A hidden variable in mathematics education?* (pp. 247-269). Springer, Dordrecht.
- Krause, A., & Putnam, R. (2016). Online calculus homework: The student experience. In T. Fukawa-Connelly, N. Infante, M. Wawro, and S. Brown (Eds.), *Proceedings of the 19th Annual Conference on Research in Undergraduate Mathematics Education* (pp. 266-280). Pittsburgh, PA: West Virginia University.
- Leatham, K. R. (2006). Viewing mathematics teachers' beliefs as sensible systems. *Journal of Mathematics Teacher Education*, 9(1), 91-102.
- Leder, G. C., & Forgasz, H. J. (2002). Measuring mathematical beliefs and their impact on the learning of mathematics: A new approach. In G. C. Leder, E. Pehkonen, & G. Törner (Eds.), *Beliefs: A hidden variable in mathematics education?* (pp. 95-113). Springer, Dordrecht.
- Leong, K. E., & Alexander, N. (2014). College students attitude and mathematics achievement using web based homework. *Eurasia Journal of Mathematics, Science & Technology Education*, 10(6), 609-615.
- Lew, K., & Zazkis, D. (2019). Undergraduate mathematics students' at-home exploration of a prove-or-disprove task. *The Journal of Mathematical Behavior*, 54, 1-15.
- Maasepp, B., & Bobis, J. (2015). Prospective primary teachers' beliefs about mathematics. *Mathematics Teacher Education and Development*, 16(2), 89-107.
- McGinnis, J. R., Kramer, S., Shama, G., Graeber, A. O., Parker, C. A., & Watanabe, T. (2002). Undergraduates' attitudes and beliefs about subject matter and pedagogy

- measured periodically in a reform-based mathematics and science teacher preparation program. *Journal of Research in Science Teaching*, 39(8), 713-737.
- McLeod, D. B. (1992). Research on affect in mathematics education: A reconceptualization. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 575-596). New York: MacMillan.
- Morena, M. A., Smith, S., & Talbert, R. (2019). Video made the calculus star. *PRIMUS*, 29(1), 43-55.
- National Council of Teachers of Mathematics. (1989). *Curriculum and evaluation standards for school mathematics*. Reston, VA: NCTM.
- National Research Council. (2001). *Adding it up: Helping children learn mathematics*. Washington, DC: National Academy Press.
- Op't Eynde, P., De Corte, E., & Verschaffel, L. (2002). Framing students' mathematics-related beliefs. In G. C. Leder, E. Pehkonen, & G. Törner (Eds.), *Beliefs: A hidden variable in mathematics education?* (pp. 13- 37). Springer, Dordrecht.
- Pajares, M. F. (1992). Teachers' beliefs and educational research: Cleaning up a messy construct. *Review of Educational Research*, 62(3), 307-332.
- Pehkonen, E. (1995). *Pupils' View of Mathematics: Initial Report for an International Comparison Project. Research Report 152*. University of Helsinki, Department of Teacher Education, PO Box 38 (Ratakatu 6A), Helsinki 00014, Finland. Retrieved from <https://files.eric.ed.gov/fulltext/ED419712.pdf>
- Philipp, R. (2007). Mathematics teachers' beliefs and affect. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 257–315). Charlotte, NC: Information Age Publishing.

- Pintrich, P. R., & Zusho, A. (2007). Student motivation and self-regulated learning in the college classroom. In R. P. Perry and J. C. Smart (Eds.), *The scholarship of teaching and learning in higher education: An evidence-based perspective* (pp. 731-810). Springer.
- Roth, V., Ivanchenko, V., & Record, N. (2008). Evaluating student response to WeBWorK, a web-based homework delivery and grading system. *Computers & Education, 50*(4), 1462-1482.
- Rupnow, R. L. (2019). *Examining connections among instruction, conceptual metaphors, and beliefs of instructors and students* [Doctoral dissertation, Virginia Tech].
- Schoenfeld, A. (1988). When good teaching leads to bad results: The disasters of “well-taught” mathematics courses. *Educational Psychologist, 23*(2), 145-166.
- Schoenfeld, A. H. (1985). *Students' beliefs about mathematics and their effects on mathematical performance: A questionnaire analysis*. Paper presented at the annual meeting of the American Educational Research Association, Chicago, Ill.
- Schoenfeld, A. H. (1992). Learning to think mathematically: Problem solving, metacognition, and sense making in mathematics. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning*, (pp. 334-370). New York: Macmillan.
- Spangler, D. A. (1992). Assessing students' beliefs about mathematics. *The Mathematics Educator, 3*(1), 19-23.
- Stage, F. K., & Kloosterman, P. (1995). Gender, beliefs, and achievement in remedial college-level mathematics. *The Journal of Higher Education, 66*(3), 294-311.

- Strauss, A. & Corbin, J. (1994). Grounded theory methodology: An overview. In N.K. Denzin & Y.S. Lincoln (Eds.), *Handbook of qualitative research* (pp. 273 – 285). Thousand Oaks, CA: Sage Publications.
- Szydlik, J. E., Szydlik, S. D., & Benson, S. R. (2003). Exploring changes in pre-service elementary teachers' mathematical beliefs. *Journal of Mathematics Teacher Education*, 6(3), 253-279.
- Szydlik, S. D. (2013). Beliefs of liberal arts mathematics students regarding the nature of mathematics. *Teaching Mathematics and its Applications: An International Journal of the IMA*, 32(3), 95-111.
- Taylor, M. W. (2009). Changing students' minds about mathematics: Examining short-term changes in the beliefs of middle-school students. In S. L. Swars, D. W. Stinson, & S. Lemons-Smith (Eds.), *Proceedings of the 31st annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (pp. 105-112). Atlanta, Georgia.
- Tallman, M. A., Carlson, M. P., Bressoud, D. M., & Pearson, M. (2016). A characterization of calculus I final exams in US colleges and universities. *International Journal of Research in Undergraduate Mathematics Education*, 2(1), 105-133.
- Thompson, A. G. (1992). Teachers' beliefs and conceptions: A synthesis of research. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 127-146). New York: Macmillan.
- Törner, G. (2002). Mathematical beliefs—A search for a common ground: Some theoretical considerations on structuring beliefs, some research questions, and

- some phenomenological observations. In G. C. Leder, E. Pehkonen, & G. Törner (Eds.), *Beliefs: A hidden variable in mathematics education?* (pp. 73-94). Springer, Dordrecht.
- Underhill, R. (1988). Focus on research into practice in diagnostic and prescriptive mathematics: Mathematics learners' beliefs: A review. *Focus on Learning Problems in Mathematics*, 10(1), 55-69.
- Viholainen, A., Asikainen, M., & Hirvonen, P. E. (2014). Mathematics student teachers' epistemological beliefs about the nature of mathematics and the goals of mathematics teaching and learning in the beginning of their studies. *Eurasia Journal of Mathematics, Science and Technology Education*, 10(2), 159-171.
- von Glasersfeld, E. (1995). *Radical constructivism: A way of knowing and learning* (Studies in Mathematics Education Series (Book 6)). London: Routledge Falmer.
- Watson, L. A. (2019). *Elementary prospective teachers and the nature of mathematics: An explanatory phenomenological study*. [Doctoral dissertation, Middle Tennessee State University].
- WebAssign.net. (n.d.). Retrieved May 23, 2020, from <https://webassign.com/corporate/about-us/>
- WebWork.maa.org. (n.d.). Retrieved May 30, 2020, from <https://webwork.maa.org/intro.html>
- Yushau, B., & Khan, M. A. (2014). Student perceptions of online homework in preparatory year pre-calculus courses. *International Journal of Mathematics Trends and Technology*, 8(1), 12-17.

APPENDICES

Appendix A

Table A1

Themes, criteria, examples, and frequency for survey item 12a: “The math in online homework is mostly a matter of memorizing formulas and procedures.”

12a: The math in online homework is mostly a matter of memorizing formulas and procedures.	Criteria:	Example:	Frequency (79 total responses)
1. Online homework is about knowing how to use/understand the formula (both the formulas and understanding is important)	1. student mentions both the importance of the formulas and being able to understand, OR 2. they mention the importance of understanding how to use, and apply, the formulas, OR 3. they explain that the formulas and procedures helps them understand the problem.	“Its applying those procedures to help you understand”	13 (16%)
2. Online homework is about understanding what’s going on; it’s not just about memorizing; the underlying idea is important	1. student mentions how understanding the underlying idea is important; they mention how knowing the underlying idea makes it easier to do the problem; OR 2. they mention that memorization is not as helpful, OR	“No, you should know how to solve it”	16 (20%)

	3. they simply express disagreement with the statement.		
3. Online homework does not involve many formulas	1. student explains that there are not many formulas in online HW	“There is not a lot of formulas”	3 (4%)
4. In online homework, memorizing formulas and procedures is what it’s about, and what gets you the answer	1. student mentions that one just needs to formula to solve the problem, OR 2. student mentions that knowing the formula is more important than the underlying idea, OR 3. they simple express agreement with the statement	“That’s all math really is to me. If I have a formula I will be able to workout the problem”	38 (48%)
5. Sometimes/It depends on the online homework problem whether or not it’s all about memorizing formulas/procedures	1. student says “sometimes”, “it depends”, OR 2. they explain that sometimes you need formulas and procedures, and other times you don’t	“Sometimes you need to know the formulas and procedures, while other times you can figure it out without.”	8 (10%)
6. In online homework, I’m just looking for the answer, the one specific answer	1. student explains that they are just looking for the right answer when doing online homework	“It’s about getting the right answer”	3 (4%)

Table A2

Themes, criteria, examples, and frequency for survey item 12b: “Math that is done outside of a school context is mostly a matter of memorizing formulas and procedures.”

12b: Math that is done outside of a school context is mostly a matter of memorizing formulas and procedures.	Criteria:	Example:	Frequency (71 total responses)
1. Math outside of school is about knowing how to use/understand formulas (formulas and understanding equally important)	1. student mentions both the importance of the formulas and being able to understand, OR 2. they mention the importance of	“You have to understand how to apply the formulas correctly”	5 (7%)

	understanding how to use, and apply, the formulas, OR		
2. No- Math outside of school is mainly about understanding what's going on, not just memorizing formulas	1. student mentions how understanding the underlying idea is important; they mention how knowing the underlying idea makes it easier to do the problem; OR 2. they mention that memorization is not as helpful, OR 3. they simply express disagreement with the statement	"I think it really relies on knowing the ideas"	20 (28%)
3. Math outside of school does not involve many formulas	1. student explains that math problems outside of school do not involve many formulas	"Formulas are not often use outside of school"	5 (7%)
4. Yes- math outside of school is mainly about memorizing formulas and procedures, they are what help get you the answer	1. student explains that one just needs to formula to solve the problem, OR 2. student explains that knowing the formula is more important than the underlying idea, OR 3. they simple express agreement with the statement	"It'll help steer you in the right direction" (and responded 'strongly agree')	24 (34%)
5. It depends on the math outside of school whether it is mostly about memorizing procedures and formulas	1. student says that "it depends", OR 2. they explain that some problems don't need formulas while others do	"Not always, depends on the question"	11 (15%)
6. I'm unsure	1. student says "n/a", or not sure	"Unsure"	1 (1%)
7. Math outside of school is more flexible, you can solve problems in various ways, 'it's subjective'	1. student explains that you can solve problems multiple ways outside of school, or you can find multiple ways to solve them, OR 2. they explain that outside of school, there is	"Kind of but its more fluid" "Calculators"	10 (14%)

more flexibility and
subjectivity

8. Math outside of school is about “winging it”, or doing it without thinking.	1. student explains that they just “wing it”, or do mathematics outside of school quickly without thinking.	“You just do it quick without thinking”	2 (3%)
---	---	---	--------

Appendix B

Table B1

Themes, criteria, examples, and frequency for survey item 6a: “In addition to getting a right answer to an online homework problem, it is important to understand why the answer is correct”

6a: In addition to getting a right answer to an online homework problem, it is important to understand why the answer is correct.	Criteria:	Example:	Frequency (83 total responses)
1a. Yes, understanding why an answer is correct is important for the sake of future problems or assignments or tests.	1. student mentions how having an understanding is important – for the sake of doing well on tests or homework problems, OR 2. they mention understanding is important in order to be successful in the class	“You need to know how you did it for quizzes and tests”	32 (39%)
1b. Yes understanding why an answer is correct is important for the future	1. student explains that having an understanding is important for the sake of the future (in a general sense, not specifically mentioning tests, problems, assignments)	“For the future”	4 (5%)
1c. Yes understanding why an answer is correct is important for the sake of a future career or job	1. students explains that having an underlying understanding is important because of their future career or job	“If you cannot even understand it. How are your students supposed to understand it.”	1 (1%)
2. Yes understanding why an answer is correct is important just for the sake of understanding, the underlying idea is important	1. student explains that understanding the underlying idea is important, OR 2. mention of “understanding why”, “knowing how”, OR 3. they simply express agreement with the statement.	“It’s important to know why you’re getting an answer”	37 (45%)

3. Sometimes the online homework problem is hard, so sometimes I don't understand it	1. student explains that they sometimes don't understand the OHW problems, or that the problems are hard	"Sometimes I do not understand why I got an answer right"	2 (2%)
4. Understanding in online homework is not necessarily important, I just want to get an answer and move on	1. student explains that they are not worried about understanding the problem, OR 2. they explain they just want to get the answer and move on, OR 3. they explain that often times they are not trying to understand why an answer is correct	"I typically care about the answer and getting it right more"	10 (12%)
5. It depends on the person or the problem	1. student explains that it depends on the person, or the problem	"Depends on the student but I think so but others may not"	2 (2%)
6. If you understand, you can be confident your answer is correct or you will be alert to wrong answers	1. student explains that understanding is important so that you know your answer is correct or you will be alert to wrong answers.	"Yes that way you can understand if a problem is wrong"	1(1%)

Table B2

Themes, criteria, examples, and frequency for survey item 6b: "When math is done outside of a school context, it is important to understand why the answer is correct, in addition to getting the right answer"

6b: When math is done outside of a school context, it is important to understand why the answer is correct, in addition to getting the right answer.	Criteria:	Example:	Frequency (75 total responses)
1a. Understanding why an answer is correct outside of school is important so you can solve other similar problems in the future.	1. student explains how having an understanding is important – for the sake of future problems, or similar situations	"So you don't have to look it up everytime"	13 (17%)

1b. Yes understanding why an answer is correct is important outside of school because of the future	1. student explains that having an understanding is important for the sake of the future (in a general sense, not specifically mentioning specific circumstances or problems)	“For the future”	6 (8%)
1c. Yes understanding why an answer is correct outside of school is important for the sake of a future career or job	1. student explains that understanding the underlying idea is important for the sake of their career or job	“I would agree based on a certain career in order to have knowledge when it is needed in the future”	2 (3%)
2. Understanding why the answer is correct outside of school is important just for the sake of understanding, knowing the underlying ideas is important	1. student explains that understanding the underlying idea is important, OR 2. mention of “understanding why”, “knowing how”, OR 3. they simply express agreement with the statement.	“You need to know how you got your answer.”	24 (32%)
3. Understanding why an answer is correct outside of school is not important, you just need the answer and don’t need to explain it	1. student explains that understanding is not important, OR 2. they are just worried about getting the answer and moving on, OR 3. the simply express disagreement with the statement.	“It doesn’t really matter to me outside of school”	12 (16%)
4. Understanding why an answer is correct outside of school is important because there could be serious consequences if you don’t and/or knowing that something will work the way you think it does is important	1. student explains that understanding is important outside of school, because of consequences that could happen if you don’t, or because it is a real-life situation, OR 2. they explain the importance of knowing that something WILL work the way you think it does	“It can be a matter of life or death in some cases, while using medications”	14 (16%)
5. It depends on the question whether it’s important to	1. student explains that it depends, OR	“Sometimes it matters sometimes it	5 (7%)

understand why the answer is correct	2. they explain that sometimes understanding does matter, but other times it doesn't	doesn't"	
6. I'm not sure	1. student explains that they're not sure, or explains that they don't know how to explain it	"Eh don't really think about it"	2 (3%)
7. I don't use math outside of school	1. student explains that they do not use math outside of a school context	"Don't really use math outside of school"	1 (1%)
8. If you understand it you can explain it to others (need to justify reasoning)	1. student explains that understanding is important so that you can explain it to other people.	"If you understand why it is correct, you can explain it somebody else "	1 (1%)

Appendix C

Table C1

Themes, criteria, examples, and frequency for survey item 10a: “It doesn’t really matter if you understand an online homework problem, as long as you can get the right answer”

10a: It doesn’t really matter if you understand an online homework problem, as long as you can get the right answer.	Criteria:	Example:	Frequency (79 total responses)
1a. Understanding how you got the right answer to an online homework problem is important for the sake of future problems or assignments or tests	1. student mentions how having an understanding is important – for the sake of doing well on tests or homework problems, or “for the future”, OR 2. they mention that understanding is important in order to “be successful”	“You need to understand it for the exam”	35 (44%)
1b. Understanding how you got the right answer to an online homework problem is important for the sake of the future (in a general sense)	1. student explains that having an understanding is important for the sake of the future (in a general sense), does not explicitly mention future problems, or tests	“You have to know how to do well in the future”	3 (4%)
2. In online homework, all you need is the right answer, not understanding, for the sake of a good grade	1. student explains that just getting the answer, or the grade, is what matters, OR 2. they explain that sometimes they don’t have time to worry about the understanding, OR 3. there is no need to demonstrate the understanding in OHW, OR 4. they simply express agreement with the statement	“It’s all about getting a good grade”	17 (22%)
3. Understanding how you got that right answer to an	1. student explains that understanding the	“You should always know how you got the	30 (38%)

online homework problem; the underlying idea is important

underlying idea is important (just as much as getting the answer), OR
 2. mention of “understanding why”, “knowing how”, OR
 3. they explain that if you got the answer, then you understand how to do it, OR
 4. they simply express disagreement with the statement.

answer”

Table C2

Themes, criteria, examples, and frequency for survey item 10b: “When math is done outside of a school context, it doesn’t really matter if it is understood, as long as the answer is right”

10b: When math is done outside of a school context, it doesn’t really matter if it is understood, as long as the answer is right.	Criteria:	Example:	Frequency (69 total responses)
1a. Outside of school, understanding how you got the right answer is important for the sake of future problems, or tasks that are similar	1. student explains how having an understanding is important – for the sake of future problems, or similar situations, “for the future”, OR 2. they explain how understanding is important in order to improve or be successful	“You have to understand how to work the problem so it’ll help with future problems”	13 (19%)
1b. Yes understanding is important outside of school for the sake of a future career or job	1. student explains that having an understanding is important because of a job or career they will have in the future	“If your job requires this math, you have to know”	2 (3%)
2. What matters outside of school is just getting the right answer and moving	1. student explains that having an understanding is not important, OR	“As long as it worked it does not really matter”	14 (20%)

on, not worrying about understanding	2. they explain that they are just worried about getting the answer and moving on, OR 3. they simply express agreement with the statement		
2a. Understanding isn't as important because problems outside of school have more leeway with how to solve things	1. student explains that having an understanding is not as important because outside of school there is more leeway with how to solve things.	"I disagree less because I think math outside school has more leeway with how to solve things"	1 (1%)
3a. Outside of school, understanding how you got the right answer; the underlying idea is important outside of school	1. student explains that understanding the underlying idea is important (just as much as getting the answer), OR 2. mention of "understanding why", "knowing how", OR 3. they explain that if you got the answer, then you understand how to do it, OR 4. they simply express disagreement with the statement.	"You should always know how you solved the problem at hand"	23 (33%)
3b. Understanding how you got the right answer is important outside of school to avoid potential consequences	1. student explains that having an understanding of the mathematics is important to avoid potential consequences	"I think it could be detrimental if you don't know what's going on"	2 (3%)
3c. Understanding how you got the right answer is important outside of school so that you know your answer is correct	1. student explains that having an understanding of the mathematics is important so that they know the answer is correct.	"Right answers with no way to show what you know are useless"	3 (4%)
3d. Understanding is important outside of school because you need to be able to justify your answer	1. student explains the importance of being able to justify their answer	"You have the right answer, but you may need to justify it"	4 (6%)

3e. Understanding is important outside of school so that you can be able to leave a trail for others	1. student explains the importance of having an understanding of the problem so that you can explain it to others; so that others will understand	“It matters if you need to explain it to another person”	4 (6%)
4. It depends (if understanding the problem is important)	1. students says “It depends” OR 2. they explain that sometimes understanding the problem is important, and other times it is not.	“Sometimes true but sometimes not true”	10 (14%)
5. I’m not sure	1. student says “not sure”, or explains that they don’t know	“Idk”	2 (3%)

Appendix D

Table D1

Themes, criteria, examples, and frequency for survey item 2a: “There are often several different ways to solve an online homework problem.”

2a: There are often several different ways to solve an online homework problem.	Criteria:	Example:	Frequency (88 total responses)
1. There might be multiple ways to solve an online homework problem, but most the time there’s not (or I choose one way)	1. student mentions that multiple ways of solving exist, but they stick to certain ways of solving.	“I know there are multiple ways sometimes, but I tend to stick to formulas for simplicity“	6 (7%)
2. There is more than one way to solve problems in online homework	1. student explains there is multiple ways to solve problems (including ways such as using calculator, solving by hand, using guess & check, other help sources), OR 2. mention of some students being able to solve a problem or think in different ways, OR 3. simply stating that they agree with the statement	“You can do it many different ways usually”	51 (58%)
3. There is one distinct answer in online homework	1. student explain that the OHW has one specific, distinct answer	“Sometimes you must give the exact answer the computer wants even though there could be others”	11 (13%)
4. Sometimes/it depends on the online homework problem whether or not there is multiple ways to solve it	1. student says “it depends”, “sometimes”, OR 2. student explains how in some settings problems can be solved	“On some problems you can solve it multiple ways on others you cant”	25 (28%)

	multiple ways, but not in other settings.		
5. There is one way to solve online homework problems, I stick to the formulas or one way of solving them	1. student mentions that OHW problems need to be solved one way, or that most of the time they solve them using a formula, or one way, OR 2. they mention that OHW is made to be solved one way	“Most of the time there is one right way to get the answer”	6 (7%)
6. I’m not sure whether there are multiple ways to solve online homework problems	1. student explains that they are not sure, or don’t know	“I’m not sure”	2 (2%)

Table D2

Themes, criteria, examples, and frequency for survey item 2b: “There are often several different ways to solve an math problems that are done outside of school.”

2b: There are often several different ways to solve math problems that are done outside of a school context.	Criteria:	Example:	Frequency (82 total responses)
1. There’s multiple ways to solve problems outside of school	1. student explains that there are multiple ways to solve problems, mention of not needing to use a certain formula or method, mention alternative ways to solve the problems, OR 2. they explain how math outside of school is not as formal, is “unexpected”, OR 3. they simply say that they agree with the statement	“There are often multiple ways to solve problems”	62 (76%)
2. Sometimes/it depends on the problem if there are multiple ways to solve it	1. student says “it depends”, “sometimes”, OR	“The number of answers will depend on the type of problem	14 (17%)

	2. explains how each problem/situation is different	that you could be doing. “	
3. Math problems are easier outside of school	1. along with agreeing with the statement, the student explains that they mostly do basic or easy math outside of school, OR 2. along with agreeing with the statement, the student gives outside of school math examples, such as “shopping”	“I normally am doing very basic math outside of school”	7 (9%)
4. I’m not sure	1. students expresses that they are not sure, or don’t know	“Not quite sure”	3 (4%)
5. The answers outside of school don’t have to be specific	1. student explains how answers outside of school are more “broad”	“The answer can be more broad”	1 (1%)
6. In school there is one way to solve problems	1. student mentions that in school, there is one way to solve problems (they include this to contrast with outside of school)	“In school we know of one way, the way we learn in class”	3 (4%)
7. There is one answer to outside of school problems, getting the answer is the goal	1. student specifically mentions that outside of school there is “one answer” to problems, OR 2. explains how getting the answer as the goal	“As long as you get the answer”	2 (2%)

Appendix E

Table E1

Themes, criteria, examples, and frequency for survey item 8a: “To solve math problems in online homework, you have to know the exact procedure for each problem.”

8a: To solve math problems in online homework, you have to know the exact procedure for each problem.	Criteria:	Example:	Frequency (80 total responses)
1. In online homework you don't need to know the exact procedure, there are multiple ways to solve problems	1. student explains that there are multiple ways to solve a problem, or that you can find multiple ways to solve a problem, OR 2. they explain how you don't have to solve problem a certain way, OR 3. they express disagreement with the statement	“There's not always a “right” way to solve a problem”	31 (39%)
2. In online homework you should know the exact way to do the problem, that's how you get it right, it makes it easier	1. student explains that you have to know the exact way to solve a problem, OR 2. they explain that knowing the exact procedures is what gets the right answer, and makes it easier, and helps you do well on exams, OR 3. they mention how the OHW program needs you to solve the problems in a certain way, OR 4. they simply express agreement with the statement	“I think so because we learn exact formulas and procedures... it at least definitely makes it easier”	33 (41%)
3. You don't need the exact procedure in online homework, you can just guess	1. student explains that you can just guess on OHW problems.	“You can sort of just guess”	3 (4%)
4. You don't always need exact procedures in online homework, you can figure it	1. student explains that you can answer OHW problems by just figuring it out,	“As long as you know the brief understanding you	7 (9%)

out having a basic understanding	having a basic understanding, or a general idea	are fine”	
5. It depends on the online homework problem if you need an exact procedure	1. students says “it depends”, OR 2. they explains how sometimes you do need exact procedures, while other times you don’t.	“Some do some don’t”	11 (14%)
6. Online homework needs a specific answer	1. student explains that the OHW requires one specific, right answer, OR 2. they mention that the computer is picky about answers.	“It is looking for a precise answer”	12 (15%)

Table E2

Themes, criteria, examples, and frequency for survey item 8b: “To solve math problems outside of a school context, you have to know the exact procedure for each problem.”

8b: To solve math problems outside of a school context, you have to know the exact procedure for each problem.	Criteria:	Example:	Frequency (77 total responses)
1. Outside of school, one procedure is not always needed, there are different ways to solve math problems outside of school	1. student explains that there are multiple ways to solve problems outside of school, or that you can find multiple ways to solve the problem, OR 2. they explain that you don’t have to solve a problem one certain way, OR 3. they simply express disagreement with the statement	“Different ways to solve” “There’s not always a right way to solve a problem”	43 (56%)
2. I don’t have an opinion, or am unsure	1. student says that they don’t have an opinion, or that they are unsure	“I do not have an opinion on this”	2 (3%)
3. To solve math problems outside of school, you can guess	1. student explains that you can just guess to solve the	“Sometimes you might have to	5 (6%)

or make it up as you go	problems, or that you can just make it up as you go.	make it up as you go” “You can sort of guess”	
4. When doing math outside of school, you need to know the correct procedures to get it right, it makes it easier	1. student explains that you have to know the exact way to solve a problem, OR 2. they explain that knowing the exact procedures is what gets the right answer, and makes it easier, OR 3. they simply express agreement with the statement	“If you want to get the answer correct, then yes you should know the procedures”	16 (21%)
5. It depends on the problem outside of school whether you need an exact procedure	1. student says “it depends”, “sometimes”, OR 2. they explain that sometimes problems might require exact procedures, and other times not.	“Depends on the problem” “I think its possible to learn outside of the classroom with exact procedures”	9 (12%)
6. Just getting the right answer is what is important outside of school, they don’t need explanation	1. student explains that the goal is just to get the right answer, OR 2. they explain that there is no need for explanation outside of school	“Nope just get the right answer” “Math doesn’t need explanation outside of school”	2 (3%)
7. For math problems outside of school, you can figure it out having a basic understanding	1. student explains that you can answer OHW problems by just figuring it out, having a basic understanding, or a general idea	“Basic knowledge helps finding answers” “You can figure it out”	10 (13%)
8. Math problems outside of school have one right answer	1. student explains that math problems have one set, right answer.	“Math problems have a set solution”	1 (1%)
9. Math problems outside of school may be more complex than those that can be solved with a simple formula	1. student explains that math problems outside of school might not be able to be solved with a simple procedure or formula.	“It might not be that simple”	1 (1%)

Appendix F

Table F1

Themes, criteria, examples, and frequency for survey item 11a: “Online homework problems have one and only one right answer.”

11a: Online homework problems have one and only one right answer.	Criteria:	Example:	Frequency (79 total responses)
1. OHW problems have one exact right answer (the program is picky about entering answers)	1. student explains that there is only one right answer to OHW problems, OR 2. they explain that the OHW system is picky, only accepts one (perfect) answer, OR 3. they simply express agreement with the statement	“Because you have to type in the exact answer” “math usually has only one correct answer”	55 (70%)
2. OHW problems could have multiple answers – including things like rounding differently or reordering an expression	1. student explains that OHW problems could have multiple answers, OR 2. they mention that the OHW could accept a range of answers, like if you rounded differently , OR 3. they simply express disagreement with the statement	“Sometimes the decimals allow different answers”	13 (16%)
3. It depends on the problem if it has exactly one answer	1. student says “it depends”, OR 2. they explain that sometimes there is only one right answer, and sometimes there is more than one right answer.	“Depends on the problem” “Sometimes there is more than one way to type something in and sometimes not”	12 (15%)
4. I’m not sure	1. student says “I’m not sure”, OR	“Not really sure”	3 (4%)

	2. they explain than they neither agree nor disagree		
5. You can solve OHW problems in multiple ways	1. student explains that there can be more than one way to solve a problem	“Math has many ways to solve the problem”	3 (4%)

Table F2

Themes, criteria, examples, and frequency for survey item 11b: “Math problems that are done outside of a school context have one and only one right answer.”

11b: Math problems that are done outside of a school context have one and only one right answer.	Criteria:	Example:	Frequency (73 total responses)
1. Math problems outside of school have exactly one right answer	1. student explains that math problems outside of school has exactly one right answer, OR 2. they simply express agreement with the statement	“They only have one right answer, its either right or wrong”	16 (22%)
2. It depends on the problem/context whether there is only one answer	1. students says that “it depends”, they explain that it depends on the problem or the context, OR 2. they explain that sometimes problems have only one right answer, while other times they can have more than one	“The problems can vary”	27 (37%)
3. Math problems outside of school can have multiple answers; outside of school the math is more flexible	1. student explains that math problems can have multiple right answers, OR 2. they explain that outside of school there is more flexibility in the answers, OR 3. they simply express disagreement with the statement	“There’s more freedom outside of school conformity”	25 (34%)
4. Math problems outside of school can be solved multiple ways	1. Student explains that math problems outside of school can be solved in multiple ways.	“There can be more than one way to solve a math problem”	9 (12%)
5. I’m not sure whether problems outside of school	1. student says “not sure”, “n/a”, or they explain that	“Unsure”	7 (10%)

have only one right answer they neither agree not disagree.

Appendix G

Table G1

Codes, criteria, examples, and frequency for survey item 9a: “Students who understand the math they have studied will be able to solve any assigned online homework problem in five minutes or less.”

9a: Students who understand the math they have studied will be able to solve any assigned online homework problem in five minutes or less.	Criteria:	Example:	Frequency (81 total responses)
1. Finishing an OHW problem in 5 minutes or less depends on the person	1. student explains that it depends on the person whether or not you should be able to solve the problem in five minutes or less, OR 2. they explain that sometimes they can solve a problem in five minutes or less, but other times they can't	“You may take longer to work the problem out it depends”	18 (22%)
2. OHW problems should be done in 5 minutes or less, these problems are easy if you understand the math	1. student explains that you should be able to solve the OHW problems in five minutes or less, OR 2. they explain how the OHW problems are easy, so they don't take much time if you understand the math, OR 3. they simply express agreement with the statement.	“They will be better at problem solving when they fully understand” “I'm sure of it”	26 (32%)
3. Even if you understand, that doesn't mean you have to do the OHW problem in 5 minutes or less (Understanding doesn't mean doing quickly)	1. student explains that even if you understand the problem, that does not mean that you should solve the problem in five minutes or less.	“Even though they know how, it can take longer”	10 (12%)
4. It depends on the problem, there are some problems that could take	1. student explains that it depends on the problem whether or not they should be	“It could take you longer than 5 minutes”	32 (40%)

longer than 5 minutes, some problems are harder	able to solve it in five minutes or less, OR 2. they explains that some problems could take longer than 5 minutes, OR 3. they explain that some problems are harder, so they may take more time.	“The problems in class are really easy but the online can be really hard”	
5. It takes me longer because I like to check my answers	1. student explains that it takes them longer than five minutes to solve a problem because they like to check their answers	“Some students like to check their answers”	1 (1%)

Table G2

Codes, criteria, examples, and frequency for survey item 9b: “Math problems that are done outside of a school context can be solved in five minutes or less.”

9b: Math problems that are done outside of a school context can be solved in five minutes or less.	Criteria:	Example:	Frequency (76 total responses)
1. Solving a problem outside of school in 5 minutes or less depends on the person	1. student explains that it depends on the person whether or not you should be able to solve the problem in five minutes or less, OR 2. they explain that sometimes they can solve a problem in five minutes or less, but other times they can't	“It depends on the person. Not everyone is the same” “Not true for everyone”	17 (22%)
2. It depends on the problem, there are problems that take longer than 5 minutes, like if the problem is complex, or if it's in a new context	1. student explains that it depends on the problem whether or not you should be able to solve it in five minutes or less, OR 2. student explains that some problems could take longer than 5 minutes, OR 2. they explain that some problems are harder, more	“Some problems are harder than others” “No I think they can be solved in more than 5 minutes”	50 (66%)

	complex, so they may take more time		
3. Problems outside of school could be done in 5 minutes or less, they are easy if you understand the math, it's just simple math	1. student explains that you should be able to solve the math problems outside of school in five minutes or less, OR 2. they explain how the math problems are easy, or it's just simple math, OR 3. they simply express agreement with the statement.	“Most math problems done outside of school are simple math” “If you know it, you can solve it”	12 (16%)
4. Even if you understand, that doesn't mean you have to do problems outside of school in 5 minutes or less (understanding doesn't mean doing quickly)	1. student explains that even if you understand the problem, that does not mean that you should solve the problem in five minutes or less.	“Some students may understand what they are solving but at times they might have to think a little longer to solve it”	1 (1%)
5. I don't know	1. students says, “I don't know”	“I don't know”	2 (3%)
6. Some problems take longer than five minutes because accuracy is very important	1. student explains that problems could take longer than five minutes because being accurate, and getting the answer right, is very important	“Because it has to be exactly right”	3 (4%)

Appendix H



Oklahoma State University Institutional Review Board

Date: 09/25/2019
Application Number: AS-19-107
Proposal Title: Online Homework in Undergraduate Mathematics

Principal Investigator: Allison Dorko
Co-Investigator(s): Olivia Willhoite
Faculty Adviser:
Project Coordinator:
Research Assistant(s):

Processed as: Exempt
Exempt Category:

Status Recommended by Reviewer(s): Approved

The IRB application referenced above has been approved. It is the judgment of the reviewers that the rights and welfare of individuals who may be asked to participate in this study will be respected, and that the research will be conducted in a manner consistent with the IRB requirements as outlined in 45CFR46.

This study meets criteria in the Revised Common Rule, as well as, one or more of the circumstances for which continuing review is not required. As Principal Investigator of this research, you will be required to submit a status report to the IRB triennially.

The final versions of any recruitment, consent and assent documents bearing the IRB approval stamp are available for download from IRBManager. These are the versions that must be used during the study.

As Principal Investigator, it is your responsibility to do the following:

1. Conduct this study exactly as it has been approved. Any modifications to the research protocol must be approved by the IRB. Protocol modifications requiring approval may include changes to the title, PI, adviser, other research personnel, funding status or sponsor, subject population composition or size, recruitment, inclusion/exclusion criteria, research site, research procedures and consent/assent process or forms.
2. Submit a request for continuation if the study extends beyond the approval period. This continuation must receive IRB review and approval before the research can continue.
3. Report any unanticipated and/or adverse events to the IRB Office promptly.
4. Notify the IRB office when your research project is complete or when you are no longer affiliated with Oklahoma State University.

Please note that approved protocols are subject to monitoring by the IRB and that the IRB office has the authority to inspect research records associated with this protocol at any time. If you have questions about the IRB procedures or need any assistance from the Board, please contact the IRB Office at 405-744-3377 or irb@okstate.edu.

Sincerely,
Oklahoma State University IRB

VITA

Olivia Willhoite

Candidate for the Degree of

Master of Science

Thesis: STUDENTS' BELIEFS ABOUT THE NATURE OF MATHEMATICS IN
THE CONTEXT OF ONLINE HOMEWORK

Major Field: Mathematics

Biographical:

Education:

Completed the requirements for the Master of Science in Mathematics at
Oklahoma State University, Stillwater, Oklahoma in July, 2020.

Completed the requirements for the Bachelor of Science in Mathematics
Education at Oral Roberts University, Tulsa, Oklahoma in 2017.

Experience:

Graduate Teaching Assistant, Department of Mathematics, Oklahoma State
University, January 2019 – May 2020. Courses taught include: Mathematical
Functions and Their Uses and Geometric Structures

6th Grade Math Teacher, Glenpool Middle School, August 2017 – May 2018.