GENERATION RECOMBINATION NOISE SPECTRA OF DOUBLE-INJECTION DIODES

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CHAPTER I

INTRODUCTION

1.1 Statement of the Problem. Noise, or random electrical fluctuation set a lower limit to the current and voltage handling capabilities of a solid state device. Also, it has been shown in the past, that the study of fluctuations of a system provides additional insight into the physical phenomena underlying the operation of a system or device.

There are, among others, four distinct sources of noise: thermal noise, shot noise, generation-recombination noise (g-r noise), and flicker noise. The thermal noise which gives rise to a random voltage across a resistor due to the thermal velocity of the current carriers, has been calculated by Nyquist (1928). Shot noise has been calculated by Schottky (1918); g-r noise has been shown to exist by Herzog and van der Ziel (1951). Flicker noise has been shown to exist by Williams and Thatcher (1932) and has been thoroughly studied by Bernamont (1937). This dissertation deals with the g-r noise in a double-injection diode. Double-injection is the simultaneous injection of holes and electrons from opposite contacts on a semiconductor. In a p-π-n (π means slightly p doped) double-injection diode holes are injected into the p-π junction and electrons are injected into the π-n junction. Since the injected holes and electrons can neutralize each other, a large amount of current can be obtained. Bilger et al. (1968) experimentally established and also made plausible that at high frequencies (at $f >> \frac{1}{2\pi\tau}$, τ = carrier

lifetime) Nyquist noise (thermal noise) prevails, i.e.,

$$S_i(f) = 4kTg(f) \qquad (1.1.1)$$

$S_i(f) = \frac{\overline{i^2}}{\Delta f}$ = noise current spectral density (A^2s)

k = Boltzmann constant = $1.38 \cdot 10^{-23}$ $(\frac{VAs}{K})$

T = Temperature ($^\circ K$)

$g(f)$ = high-frequency conductance (real part of admittance) (Ω^{-1})

The MKS system of units is used in this dissertation, unless it is stated otherwise. The major effort in this dissertation is, therefore, devoted to calculating the low-frequency portion of the noise in a double-injection diode. Two problems arise here: a.) to find a formulation which adequately describes the "microscopic cause" of noise, b.) to integrate the noise contributions of a volume element over the volume of the device. This latter problem is made difficult by the fact that such a device is nonlinear.

1.2 Scope of Investigation. The basic equations are presented which lead to the evaluation of the concentration profile of electrons and holes, the field intensity, and the I-V characteristic. The solutions are given which were obtained by a numerical method and by an analytical method.

The theory of local fluctuations in the case of non-equilibrium is slightly extended. Based on a method of van der Ziel (1959), an equation for the g-r noise spectrum of long double-injection diodes is derived.

This equation predicts the magnitude and the spectral shape of the noise of realizable double-injection diodes. The theory includes the effects of both electric field and carrier diffusion in the ohmic and in the semiconductor regime. Numerical solutions have been obtained through a computer program which solves a highly non-linear third order differential equation with three boundary conditions. These solutions are compared with simple analytical approximations, which neglect some of the fine details of the boundaries. The numerical results are compared with experimentally obtained I-V characteristics. Special solutions have been obtained in the ohmic regime with small injection and in the semiconductor regime.

CHAPTER II

THEORY OF HOLE AND ELECTRON FLUCTUATION IN A SEMICONDUCTOR

2.1 Introduction. This chapter is concerned with the development of a mathematical model of hole and electron fluctuation in a semiconductor. In order to calculate the generation-recombination noise, it is necessary to know the fluctuation in the number of mobile charge carriers. Burgess (1954, 1955, 1956) used both thermodynamical and statistical approaches to calculate the extent of the fluctuation in the numbers of mobile charge carriers (i.e. electrons in the conduction band and holes in the valence band) due to the randomness in the process of generation and recombination. He indicated that the thermodynamical treatment applies to those cases in which thermal equilibrium is established; the statistical approach is capable of application to systems in which equilibrium does not exist. In this chapter the statistical method of Burgess is presented. A modifying factor is suggested in the result of hole and electron fluctuation in the case of non-equilibrium.

2.2 Fluctuation in the Number of Charge Carriers in the Case of Thermal Equilibrium. As stated in Burgess' papers (1954, 1955, 1956) it is assumed that there is only one independently fluctuating variable, which may be either the number of holes P in the valence band or the number of electrons N in the conduction band. When there are N electrons in the conduction band, let the probability that another electron enters during the time dt be $g_e(N)dt$, and let the probability that an electron

leaves the band during dt be $r_e(N)dt$. It is assumed that the generation and recombination rate g_e and r_e depend on only one fluctuating variable, in this case N. If W(N) is the steady state probability distribution for the number of electrons in the conduction band, we have:

$$0 = \frac{dW(N)}{dt} = r_e(N+1)W(N+1) + g_e(N-1)W(N-1) - W(N)[g_e(N) + r_e(N)] \quad . \tag{2.2.1}$$

The above equation exists for N equal to every positive integer. From

$$\frac{dW(0)}{dt} = r_e(1)W(1) - W(0)g_e(0) = 0 \tag{2.2.2}$$

we have

$$\frac{W(1)}{W(0)} = \frac{g_e(0)}{r_e(1)} \quad , \tag{2.2.3}$$

and from

$$\frac{dW(1)}{dt} = r_e(2)W(2) + g_e(0)W(0) - W(1)\,[g_e(1) + r_e(1)] = 0 \quad , \tag{2.2.4}$$

we have

$$\frac{W(2)}{W(1)} = \frac{g_e(1)}{r_e(2)} \qquad \text{i.e.} \quad \frac{W(2)}{W(0)} = \frac{g_e(0)\ g_e(1)}{r_e(1)\ r_e(2)} \tag{2.2.5}$$

By induction from N = 1 to N it follows:

$$\frac{W(N)}{W(0)} = \frac{\prod_{\nu=0}^{N-1} g_e(\nu)}{\prod_{\nu=1}^{N} r_e(\nu)} \quad . \tag{2.2.6}$$

The logarithmic differentiation of W(N) at the most probable value of N, i.e. the steady state value N_0 of N is:

$$\frac{d}{dN} \ln W(N) \Big|_{N=N_0} = \frac{\Delta \ln W(N)}{\Delta N} \Big|_{N=N_0} = \frac{\ln W(N+1) - \ln W(N)}{(N+1) - N} \Big|_{N=N_0} = 0 \quad . \tag{2.2.7}$$

From Equations 2.2.6 and 2.2.7 we have:

$$\frac{d \ln W(N)}{dN} = \ln \frac{\prod_{\nu=0}^{N} g_e(\nu)}{\prod_{\nu=1}^{N+1} r_e(\nu)} W(0) - \ln \frac{\prod_{\nu=0}^{N-1} g_e(\nu)}{\prod_{\nu=1}^{N} r_e(\nu)} W(0)$$

$$= \ln g_e(N) - \ln r_e(N+1) \tag{2.2.8}$$

it follows:

$$\frac{d \ln W(N_0)}{dN} = \ln g_e(N_0) - \ln r_e(N_0 + 1) = 0 \quad . \qquad (2.2.9)$$

It is assumed throughout that all the numbers N_0, P_0 are large compared with unity. From the above equations we have:

$$g_e(N_0) = r_e(N_0 + 1) \simeq r_e(N_0) \qquad (2.2.10)$$

In order to make a Taylor expansion of ln W(N) up to the quadratic term, we evaluate d ln $W(N_0)/dN = 0$ (See Equation 2.2.9) and d^2 ln $W(N_0)/dN^2$ as follows (Use Equation 2.2.8):

$$\frac{d^2}{dN^2} \ln W(N)\Big|_{N=N_0} = \frac{\frac{d}{dN} \ln W(N+1) - \frac{d}{dN} \ln W(N)}{(N+1) - N}\Bigg|_{N=N_0}$$

$$= \left[\ln g_e(N+1) - \ln r_e(N+2) \right]\Big|_{N=N_0} - \left[\ln g_e(N) - \ln r_e(N+1) \right]\Big|_{N=N_0}$$

$$= \frac{\ln g_e(N+1) - \ln g_e(N)}{(N+1) - N}\Bigg|_{N=N_0} - \frac{\left[\ln r_e(N+2) - \ln r_e(N+1) \right]}{(N+2) - (N+1)}\Bigg|_{N=N_0}$$

$$= \frac{d \ln g_e(N)}{dN}\bigg|_{N=N_0} - \frac{d \ln r_e(N+1)}{dN}\bigg|_{N=N_0}$$

$$= \frac{g_e'(N_0)}{g_e(N_0)} - \frac{r_e'(N_0+1)}{r_e(N_0+1)}$$

$$\simeq \frac{g_e'(N_0)}{g_e(N_0)} - \frac{r_e'(N_0)}{r_e(N_0)} \quad , \tag{2.2.11}$$

where the prime denotes differentiation with respect to N. Thus, the Taylor expansion yields:

$$\ln W(N) = \ln W(N_0) - \frac{1}{2}(N - N_0)^2 \left[\frac{r_e'(N_0)}{r_e(N_0)} - \frac{g'(N_0)}{g(N_0)}\right] \tag{2.2.12}$$

or

$$W(N) = W(N_0) \exp\left[-\frac{(N - N_0)^2}{2(N - N_0)^2}\right] \tag{2.2.13}$$

where

$$\overline{(N - N_0)^2} = \left[\frac{r_e'(N_0)}{r_e(N_0)} - \frac{g_e'(N_0)}{g_e(N_0)}\right]^{-1} \quad . \tag{2.2.14}$$

From Equation 2.2.10 it follows that

$$\overline{\Delta N^2} = \overline{(N - N_0)^2} = \frac{g_e(N_0)}{r_e'(N_0) - g_e'(N_0)} \quad . \qquad (2.2.15)$$

If we are interested only in extrinsic semiconductors with fully ionized impurity atoms, e.g. donor atoms N_D, the equilibrium number of electrons in the conduction band is $N = N_D + P \approx N_D$. We, then, consider only transitions between the valence band and conduction band as producing fluctuations where now (Burgess, 1956)

$$g_e = \text{constant} \qquad r_e = \rho\, NP = \rho\, N(N - N_D) \qquad (2.2.16)$$

and where ρ is a proportionality constant. From Equation 2.2.15 and 2.2.16 we have

$$\overline{\Delta N^2} = \frac{g_e(N_0)}{\rho(2N_0 - N_D)}$$

$$= \frac{g_e(N_0)}{\rho(N_0 + P_0)} \quad . \qquad (2.2.17)$$

From Equation 2.2.10 and 2.2.16 we have

$$g_e(N_0) = \rho\ N_0 P_0 \quad . \tag{2.2.18}$$

It follows immediately that

$$\overline{\Delta N^2} = \frac{N_0 P_0}{N_0 + P_0} \quad . \tag{2.2.19}$$

2.3 Fluctuation in the Number of Charge Carriers in the Case of Non-Equilibrium. The development of the mean square fluctuation in Section 2.2 of this chapter by Burgess is a good approximation, if N_0 and P_0 are in the neighborhood of thermal equilibrium. In the steady state case where the values of N_0 and P_0 may be quite different from the thermal equilibrium values, we should not omit higher order terms in the expansion of ln W(N), i.e. we should have more than the two terms shown in Equation 2.2.11. It is reasonable to assume that the existence of space charges due to excess electrons in the conduction band and excess holes in the valence band will affect the fluctuation of charge carriers. Since in a double-injection diode the steady state values of N_0 and P_0 in the case of non-equilibrium are always greater than the steady state values in the case of thermal equilibrium, we will tentatively assume Equation 2.2.12 for N less than N_0, i.e. we rewrite it as follows:

$$W(N) = W(N_0) \exp\left[-\frac{(N - N_0)^2}{2\ \overline{\Delta N_t^2}}\right] \text{ for } N < N_0 \qquad (2.3.1)$$

where

$$\overline{\Delta N_t^2} = \frac{N_0 P_0}{N_0 + P_0} \quad . \qquad (2.3.2)$$

If N_0 is much greater than unity, Equation 2.3.1 can be considered as a continuous distribution function rather than a discrete distribution function in the neighborhood of N_0. We can shift the zero point of N_0 by transformation $X = N - N_0$, and Equation 2.3.1 becomes then

$$W(X) = W(0) \exp\left[-\frac{X^2}{2\overline{\Delta N_t^2}}\right] \text{ for } X < 0 \quad . \qquad (2.3.3)$$

For a distribution function where N is greater than N_0, we retain the same probability for $X > 0$ as for $X < 0$ i.e. for $N > N_0$ as for $N < N_0$. We will tentatively introduce a suppresion factor k_1 of the simplest kind (i.e. k_1 is a constant to be determined but is independent of N) due to the existence of space charge such that

$$Y = k_1 X \qquad \text{where} \qquad 0 < k_1 \leq 1 \quad .$$

and

$$W(Y)YdY = W(X)XdX \quad . \tag{2.3.4}$$

From the above relationship we have

$$W(Y) = \frac{W(0)}{k_1^2} \exp\left(-\frac{Y^2}{2k_1^2\,\overline{\Delta N_t^2}}\right) \qquad \text{for } 0 < Y \quad . \tag{2.3.5}$$

By changing the dummy variable from Y to X in Equation 2.3.4 and using Equation 2.3.2

$$W(X) = W(0) \exp\left(-\frac{X^2}{2\,\overline{\Delta N_t^2}}\right) \qquad \text{for } X < 0$$

$$= \frac{W(0)}{k_1^2} \exp\left(-\frac{X^2}{2k_1^2\,\overline{\Delta N_t^2}}\right) \text{ for } 0 < X \tag{2.3.6}$$

where

$$\overline{\Delta N_t^2} = \frac{N_0 P_0}{N_0 + P_0} \qquad \text{and} \qquad 0 < k_1 \leq 1 \quad .$$

From the distribution of Equation 2.3.6, it is easy to find that the mean is equal to zero and the variance is

$$\overline{\Delta N_s^2} = \left(\frac{k_1 + 1}{2}\right) \frac{N_0 P_0}{N_0 + P_0} \qquad 0 < k_1 \leq 1$$

$$= c \frac{N_0 P_0}{N_0 + P_0} \qquad \frac{1}{2} < c \leq 1 \qquad (2.3.7)$$

where

$$c = \frac{1 + k_1}{2} \quad .$$

It is obvious that in thermal equilibrium the "suppression factor" k_1 is equal to 1 because Equation 2.3.6 has to converge to the Burgess theory (Equation 2.2.12). When N_0 is much higher than the steady state value of thermal equilibrium, k_1 may be near zero. By the preceding argument together with $c = (k_1 + 1)/2$, we obtain a result which is one half that of Burgess when operating in very high carrier density level.

CHAPTER III

NOISE SPECTRA DENSITIES

3.1 Introduction. In Chapter II the fluctuation of the number of charge carriers in a microscopic volume was discussed. In this chapter based on a method of van der Ziel (1966), we will discuss thermal noise of a single-injection Space-Charge-Limited (SCL) diode. But we will obtain a different result than van der Ziel, namely,

$$\overline{i^2} = 4kT\, g\Delta f \quad . \tag{3.1.1}$$

Theoretical support to the equation which was first suggested by Webb and Wright (1962), will be given. Then the equations for the generation-recombination noise of double-injection devices will also be developed.

3.2 Thermal Noise in a Single-Injection SCL Diode. If we assume thermal noise for a single-injection diode in a microscopic volume, there are several possibilities to get the noise spectra of the whole device. Van der Ziel (1966) showed that if the "thermal noise hypothesis" (namely the Nyquist Equation) is applied rigorously; the following result is obtained

$$\overline{i^2} = 8kT\, g\Delta f, \tag{3.2.1}$$

which is a factor two different of Equation 3.1.1 and g is the differential conductance of the device.

There are experimental results (Liu, et al., 1967) which seem to support Equation 3.2.1 for single-injection diodes. However, Klassen (1968) derived through a different argument the result

$$\overline{i^2} = \frac{8}{3}kTg\ (\Delta f) \qquad (3.2.2)$$

which is 1/3 the value suggested by van der Ziel.

For further discussion on this subject, it is worthwhile to present a method which leads to Equation 3.1.1. In a single-injection n - ν - n SCL diode with ν (slightly n doped) material (See Figure 1) it is well known that as long as drift predominates over diffusion:

$$I_a = q\mu_n n AE \qquad \text{(Drift Current)} \qquad (3.2.3)$$

$$-\frac{d^2V}{dx^2} = \frac{dE}{dx} = \frac{qn}{\epsilon\epsilon_0} \qquad \text{(Poisson's Equation)} \qquad (3.2.4)$$

where

I_a = electric current (A)

q = electric charge (coulombs)

μ_n = electron mobility $(\frac{m^2}{Vs})$

n = electron density (m^{-3})

A = area of cross section of the device (m^2)

E = electric field intensity ($\frac{V}{m}$)

ϵ = relative dielectric constant

ϵ_0 = dielectric constant ($\frac{As}{Vm}$).

(In this dissertation I_a is called positive when it flows from right to left, and we assumed $V = 0$ at $x = 0$ and $V = V_L$ at $x = L$.) The field intensity and the electron density in the device is obtained from the above two equations with appropriate boundary conditions (i.e. $E(0) = 0$) as

$$E(x) = \left(\frac{2I_a}{\epsilon\epsilon_0 \mu_n A}\right)^{\frac{1}{2}} x^{\frac{1}{2}} \qquad 0 \leq x \leq L \qquad (3.2.5)$$

$$n(x) = \frac{\epsilon\epsilon_0}{2q} \left(\frac{2I_a}{\epsilon\epsilon_0 \mu_n A}\right)^{\frac{1}{2}} x^{-\frac{1}{2}} \qquad 0 \leq x \leq L \qquad (3.2.6)$$

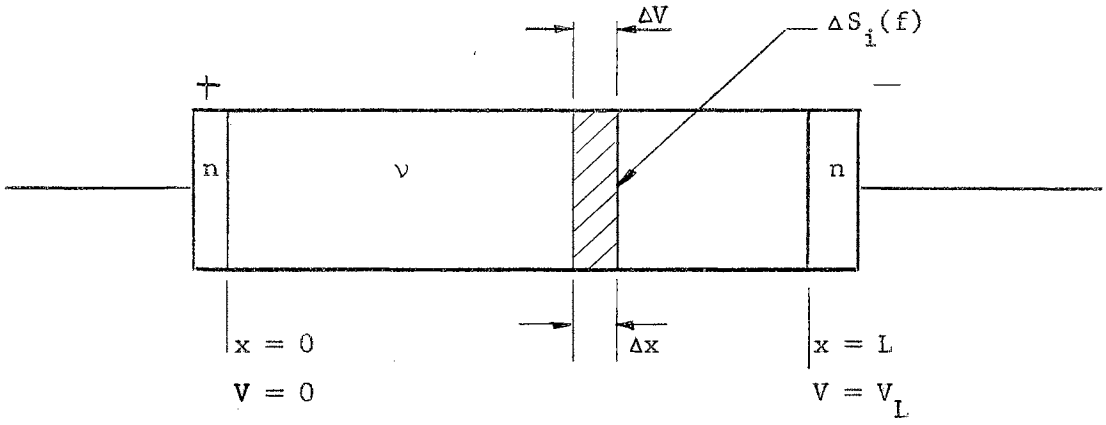

Figure 1. Noise in a Column Element of an n - ν - n Device

As shown in Figure 1 the voltage across the section Δx is ΔV where ΔV is obtained from Equation 3.2.5 as

$$\Delta V = -E\,\Delta x = -\left(\frac{2I_a}{\epsilon\epsilon_0 \mu_n A}\right)^{\frac{1}{2}} x^{\frac{1}{2}}\,\Delta x \quad . \tag{3.2.7}$$

In order to apply the thermal noise hypothesis, we should use the incremental resistance as the one responsible for thermal noise rather than the D.C. value suggested by van der Ziel (1966), since the current fluctuates around its steady state value I_a. Instead of using

$$\Delta R(x) = \frac{\Delta V(x)}{I_a} \quad , \tag{3.2.8}$$

we use

$$\Delta R_i(x) = \frac{\partial \Delta V(x)}{\partial I_a} \quad . \tag{3.2.9}$$

From Equation 3.2.7 and 3.2.9 it follows that

$$\Delta R_i(x) = \left(\frac{I}{2\epsilon\epsilon_0 \mu_n A I_a}\right)^{\frac{1}{2}} x^{\frac{1}{2}}\,\Delta x \quad . \tag{3.2.10}$$

If we assume thermal noise in a microscopic volume $A\Delta x$ the thermal noise spectrum is given by

$$\Delta \overline{v^2} = 4kT \, \Delta R_i \, \Delta f \quad . \tag{3.2.11}$$

Furthermore, if we assume no position correlation of the thermal noise it follows that

$$\overline{v^2} = \Sigma \, 4kT \, \Delta R_i(x) \, \Delta f \quad . \tag{3.2.12}$$

If we define R_n as the sum of incremental resistance as follows together with Equation 3.2.10, we have,

$$R_n = \Sigma \, \Delta R_i(x) = \int_0^L \Delta R_i(x) = \frac{1}{3}\left(\frac{2}{I_a \epsilon \epsilon_0 \mu_n A}\right)^{\frac{1}{2}} L^{\frac{3}{2}} \quad , \tag{3.2.13}$$

where L is the length of the device.

To relate Equations 3.2.12 and 3.2.13 to the I-V characteristic, we integrate Equation 3.2.7 between $V_{(x = 0)} = 0$ and $V_{(x = L)} = V_L$ to get

$$\int_0^{V_L} \Delta V = -\int_0^{L} \left(\frac{2I_a}{\epsilon\epsilon_0 \mu_n A}\right)^{\frac{1}{2}} x^{\frac{1}{2}} \Delta x \qquad (3.2.14)$$

and it follows

$$I_a = \frac{9}{8} \epsilon\epsilon_0 \mu_n A \frac{V_L^2}{L^3} \quad . \qquad (3.2.15)$$

In Equation 3.2.13 replace I_a by its value from Equation 3.2.15, we have

$$R_n = \frac{1}{\left(\frac{9}{4}\right) \epsilon\epsilon_0 \mu_n A \frac{(-V_L)}{L^3}} = \frac{1}{g} \qquad (3.2.16)$$

where

$$g = \frac{\partial I_a}{\partial V_L} \quad .$$

If the noise is represented by a current generator $\overline{i^2}$ in parallel with the device. Thus, from the equation

$$\overline{i^2} = \overline{v^2}\, g^2 \qquad (3.2.17)$$

we obtain

$$\overline{i^2} = (4kTR_n \Delta f)\, g^2 = 4kTg\Delta f \quad . \qquad (3.2.18)$$

Although this type of solution was first suggested by Webb and Wright (1962), this section gives a theoretical support of their argument.

If we have position correlation as indicated by Sergiescu (1966), we expect a value somewhat lower than the value shown in Equation 3.2.16. However, future experiments about noise in such devices will decide which of the approaches is more nearly true.

3.3 Generation Recombination Noise Spectra in a Double-Injection Device. Let us assume that the generation-recombination noise theory as given by van der Ziel (1959) for a linear element holds also in a small section Δx of a nonlinear element and together with the following:

$$\Delta N_0 = An\Delta x$$

$$\Delta P_0 = Ap\Delta x$$

$$\overline{\Delta N_s^2} = c\,\frac{\Delta N_0\,\Delta P_0}{\Delta N_0 + \Delta P_0} \qquad \frac{1}{2} < c \leq 1 \quad ,$$

we have

$$\Delta S_i(f) = c \frac{4I_a^2 (b+1)^2 np\tau}{(bn+p)^2 (n+p) A(1+\omega^2\tau^2) \Delta x} \qquad (3.3.1)$$

where

$\Delta S_i(f)$ = the current noise spectra in a section Δx

μ_n = electron mobility

μ_p = hole mobility

$b = \mu_n/\mu_p$

n = electron density

p = hole density

ΔN_0 = number of electron in a section Δx

ΔP_0 = number of hole in a section Δx

τ = average charge carrier lifetime

ω = angular frequency.

In all following calculations c has been tentatively set equal to one, and we have

$$\Delta S_i(f) = \frac{4I_a^2 (b+1)^2 np\tau}{(bn+p)^2 (n+p) A(1+\omega^2\tau^2) \Delta x} \quad . \qquad (3.3.2)$$

We consider a double-injection device as a quasi-linear device in a a section Δx, and therefore, we may apply the following equation to each section of the device

$$\Delta S_V(f) = \Delta S_i(f)(\Delta R)^2 = \Delta S_i(f)\left(\frac{\Delta V(x)}{I_a}\right)^2 \qquad (3.3.3)$$

where $\Delta S_V(f)$ = voltage noise spectra in a section Δx. Along with Equation 3.3.2 we have

$$\Delta S_V(f) = \frac{4(b+1)^2\, np\, \tau\, (\Delta V)^2}{(bn+p)^2\, A(n+p)(1+\omega^2\tau^2)\, \Delta x} \quad . \qquad (3.3.4)$$

It follows that

$$S_V(f) = \frac{4(b+1)^2\tau}{A(1+\omega^2\tau^2)} \int_0^L \frac{n(x)\, p(x)\, E(x)^2\, dx}{[n(x)+p(x)]\,[bn(x)+p(x)]^2} \quad . \qquad (3.3.5)$$

Finally, from the equation

$$S_i(f) = S_V(f) g^2 = S_V(f)\left[\frac{\partial I_a}{\partial V_L}\right]^2 \qquad (3.3.6)$$

we have

$$S_i(f) = \left(\frac{\partial I_a}{\partial V_L}\right)^2 \frac{4(b+1)^2\tau}{A(1+\omega^2\tau^2)} \int_0^L \frac{n(x)\, p(x)\, E^2(x)\, dx}{[n(x)+p(x)]\,[bn(x)+p(x)]^2} \quad . \qquad (3.3.7)$$

Calculations on g-r noise in a somewhat different nonlinear diode have apparently been done in a thesis by Barrera (1966) (communication, Hans R. Bilger). This is the basic equation for the calculation of g-r noise spectra. In order to calculate the g-r noise in a double-injection diode, we need to know the I-V characteristic as well as the properties $E(x)$, $n(x)$ and $p(x)$.

CHAPTER IV

GENERATION-RECOMBINATION NOISE SPECTRUM OF A DOUBLE-INJECTION DIODE OPERATING IN THE OHMIC REGIME WITHOUT INJECTED CARRIERS

4.1 Introduction. This chapter is concerned with a demonstration showing how to apply the theory we developed in Chapter III to a realizable double-injection diode (See Figure 2). The simplest operation of the double-injection diode will occur when there are no externally injected holes and electrons. This case occurs when no voltage is applied to the double-injection diode. Calculating the g-r noise in this condition will give us an example of how to use Equation 3.3.7 and will give us the approximate g-r noise when the current is small. (In Chapter VII with injected carriers we will use the same method to find the g-r noise spectra of a double-injection diode when it is operating in the ohmic regime with injected carriers and in the Lampert semiconductor regime.)

4.2 Ohmic Regime Without Injected Carriers. In a double-injection diode without injected carriers the current flow equations are given by

$$J_n = q \mu_n n_T E \qquad \text{(Ohm's law for electrons)} \qquad (4.2.1)$$

$$J_p = q \mu_p p_T E \qquad \text{(Ohm's law for holes)} \qquad (4.2.2)$$

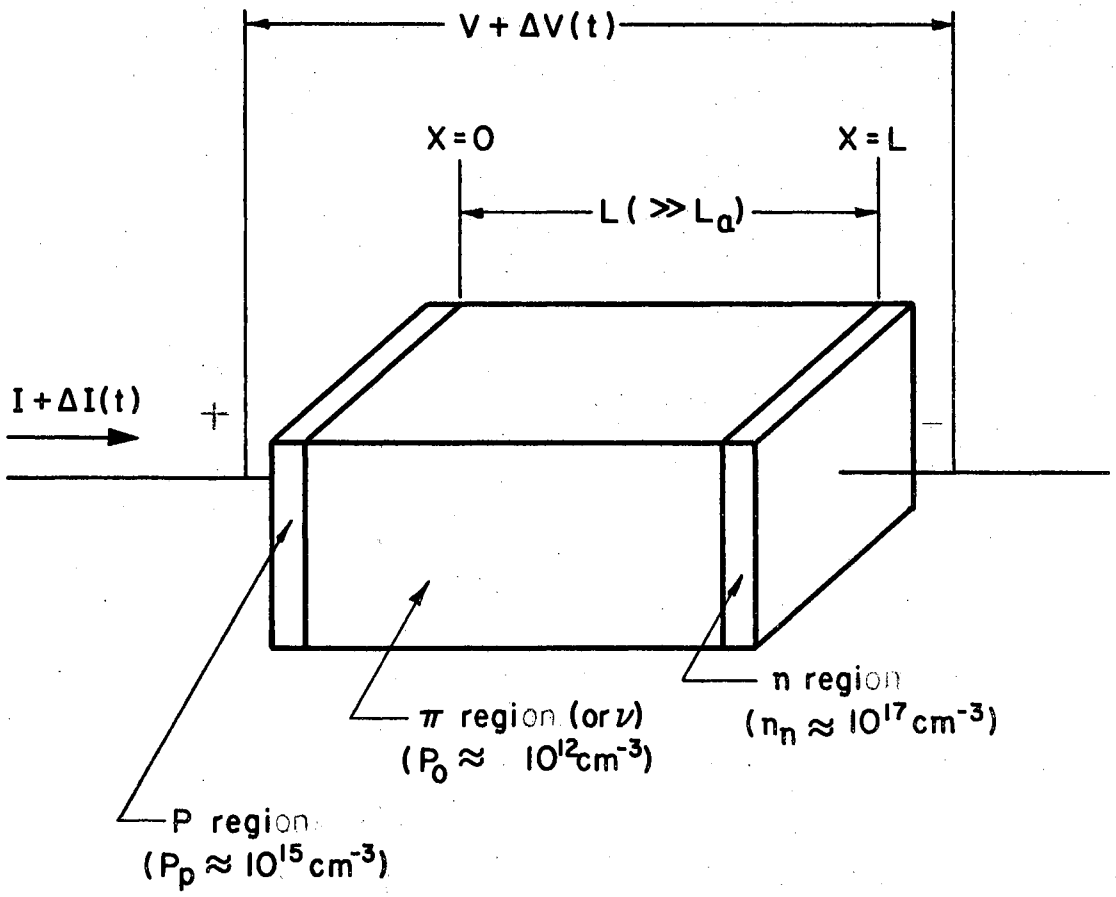

Figure 2. Schematic Representation of a Double-Injection Diode

where

J_n = electron current density

J_p = hole current density

n_T = electron density in thermal equilibrium (assumed independent of x)

p_T = hole density in thermal equilibrium (assumed independent of x).

The addition of Equations 4.2.1 and 4.2.2 gives the total current density as

$$J = J_n + J_p$$

$$= q(\mu_n n_T + \mu_p p_T) E \qquad (4.2.3)$$

where J = total current density.

The device is assumed to have a constant cross section A. In this chapter we assume no space charge exists, therefore

$$I_a = - \frac{q(\mu_n n_T + \mu_p p_T) A V_L}{L} \qquad (4.2.4)$$

where I_a = total current passing through the device, and this case we have:

$$g = \frac{\partial I_a}{\partial V_L} = \frac{I_a}{V_L} \quad .$$

From Equation 3.3.7 in Chapter III the equation of g-r noise is then

$$S_i(f) = \left(\frac{\partial I_a}{\partial V_L}\right)^2 \frac{4(b+1)^2 \tau}{A(1+\omega^2\tau^2)} \int_0^L \frac{n_T\, b_T\, E^2\, dx}{(n_T + p_T)(bn_T + p_T)^2}$$

$$= \left[\frac{q(bn_T + p_T) A \mu_p}{L}\right]^2 \frac{4(b+1)^2 \tau\, n_T\, p_T\, V_L^2}{A(1+\omega^2\tau^2)(n_T + p_T)(bn_T + p_T)^2 L}$$

$$= \frac{4(b+1)^2 \tau\, n_T\, p_T\, I_a^2}{(1+\omega^2\tau^2)\, A(n_T + p_T)(bn_T + p_T)^2 L} \quad . \qquad (4.2.5)$$

Representing the spectral density by the equivalent shot noise current

$$I_{eq}(f) = \frac{S_i(f)}{2q} \qquad (4.2.6)$$

where I_{eq} = equivalent shot noise current, we find a g-r noise spectrum of the form

$$I_{eq} \propto I_a^2 \qquad (4.2.7)$$

in the ohmic regime of a double-injection diode without injected carriers. Results similar to Equation 4.2.7 have been obtained by Fazakas and Friedman (1968) with different approaches.

CHAPTER V

THE DOUBLE-INJECTION PROCESSES IN A SEMICONDUCTOR AND AN APPROXIMATE METHOD OF ANALYSIS

5.1 Introduction. The theory of double-injection in a semiconductor has been developed by Lampert (1959, 1962), Lampert et al. (1961), Baron (1965, 1968), Mayer et al. (1965, 1968) and will again be discussed in Chapter XI. The nonlinear ordinary differential equation which defines the problem has no closed form solutions. This chapter will give an approximate analytical solution to the problem. This approximate solution will give a physical picture of the variation of the quantities: $n(x)$, $p(x)$, and $E(x)$, and the I-V characteristic. These quantities are needed in order to calculate the g-r noise of Equation 3.3.7 in such a nonlinear device.

5.2 The Equations of a Double-Injection Diode. As shown in the paper by Baron (1968), we start with the following set of equations which completely defines the problem.

(1.) The current flow equations (with drift and diffusion terms).

$$J_n = q\mu_n \left[(n_T + \delta n)\, E + \beta \frac{d\delta n}{dx} \right] \qquad (5.2.1)$$

$$J_p = q\mu_p \left[(p_T + \delta p)\, E - \beta \frac{d\delta p}{dx} \right] \qquad (5.2.2)$$

$$J = J_n + J_p \tag{5.2.3}$$

where $\beta = kT/q$ and δn, δp are excess injected electron and excess injected hole densities.

(2.) The particle conservation equations.

$$\frac{1}{q}\frac{dJ_n}{dx} = r \qquad \text{(in three dimensions } r = \frac{1}{q} \text{ div } J_n) \tag{5.2.4}$$

$$-\frac{1}{q}\frac{dJ_p}{dx} = r \qquad \text{(in three dimensions } r = -\frac{1}{q} \text{ div } J_p) \tag{5.2.5}$$

where the recombination rate r shall be given by

$$r = -\frac{\partial n}{\partial t} = -\frac{\partial p}{\partial t} = \frac{\delta n}{\tau} \quad . \tag{5.2.6}$$

(In this dissertation we consider only a one dimensional problem.)

(3.) Poisson's equation is:

$$\left(\frac{\epsilon\epsilon_0}{q}\right)\frac{dE}{dx} = \delta p - \delta n \tag{5.2.7}$$

and

$$E = -\frac{dV}{dx} \qquad (5.2.8)$$

(4.)

where V is the electrostatic potential. Equations 5.2.1, 5.2.2 and 5.2.3 give

$$J = q\mu_p \left[(bn_T + p_T)\, E + (b\delta n + \delta p)\, E + \beta\left(b\frac{d\delta n}{dx} - \frac{d\delta p}{dx}\right)\right] \quad . \qquad (5.2.9)$$

Multiplying 5.2.4 by $1/\mu_n$ and 5.2.5 by $1/\mu_p = b/\mu_n$ and then adding the results gives, using 5.2.1 and 5.2.2

$$\frac{(b+1)}{\mu_n} r = (n_T - p_T)\frac{dE}{dx} - \frac{d}{dx}\left[(\delta p - \delta n)\, E\right] + \beta\frac{d^2}{dx^2}(\delta n + \delta p) \quad . \qquad (5.2.10)$$

We will only consider π type material, i.e. slightly p doped, which has $n_T << p_T$, $n = \delta n$ and $p = p_T + \delta p$. Furthermore, we can assume the current is due to drift mainly, i.e. the last term in Equation 5.2.9 is negligible compared with the other terms. In this case Equations 5.2.9 and 5.2.10 can be replaced by

$$J = q\mu_p\,(p_T + b\delta n + \delta p)\, E \qquad (5.2.11)$$

$$\frac{(b+1)}{\mu_n} r = - p_T \frac{dE}{dx} - \frac{\epsilon\epsilon_0}{q} \frac{d}{dx} \left(E \frac{dE}{dx}\right) + \beta \frac{d^2}{dx^2} (\delta n + \delta p) \quad , \quad (5.2.12)$$

where Equation 5.2.7 has been used to eliminate $\delta_p - \delta_n$ in the second term of 5.2.10. The solutions for the Lampert semiconductor regime, the Lampert insulator regime, and the diffusion dominated regime are obtained by retaining respectively only the first, the second, or the third term in 5.2.12. This solution has been carried out with the following results (Baron, 1965):

$$J = \left(\frac{9}{8}\right) q \mu_n \mu_p p_T \tau \frac{V_L^2}{L^3} \text{ for the Lampert semiconductor regime} \quad (5.2.13)$$

$$J = - \left(\frac{125}{18}\right) \epsilon\epsilon_0 \mu_n \mu_p \tau \frac{V_L^3}{L^5} \text{ for the Lampert insulator regime.} \quad (5.2.14)$$

In the remainder of this chapter, we always assume that $\delta n \tilde{\sim} \delta p$. Thus, the second term in 5.2.10 is small compared with the other two terms, consequently is neglected.

5.3 The Diffusion Dominated Sections. We will not solve the remaining differential equation for the whole device. Instead this chapter will provide a method for finding out which part of the device is in the Lampert semiconductor region, i.e. where the first term on the right hand side of Equation 5.2.12 dominates, and which part is in the diffusion dominated region, i.e. where the last term on the right hand side of Equation 5.2.12 dominates. If we apply a voltage across the double-injection device, the holes will inject from the left and the electrons will inject from the right (See Figure 2). As shown in Figure 3, we

define x_{DO} and x_{DL} by the following conditions:

For $0 \leq x \leq x_{DO}$ and $L - x_{DL} \leq x \leq L$, the diffusion term, i.e. the last term on the right hand side of Equation 5.2.12, shall dominate.

For $x_{DO} \leq x \leq x_{DL}$, the first term on the right hand side of Equation 5.2.12 shall dominate.

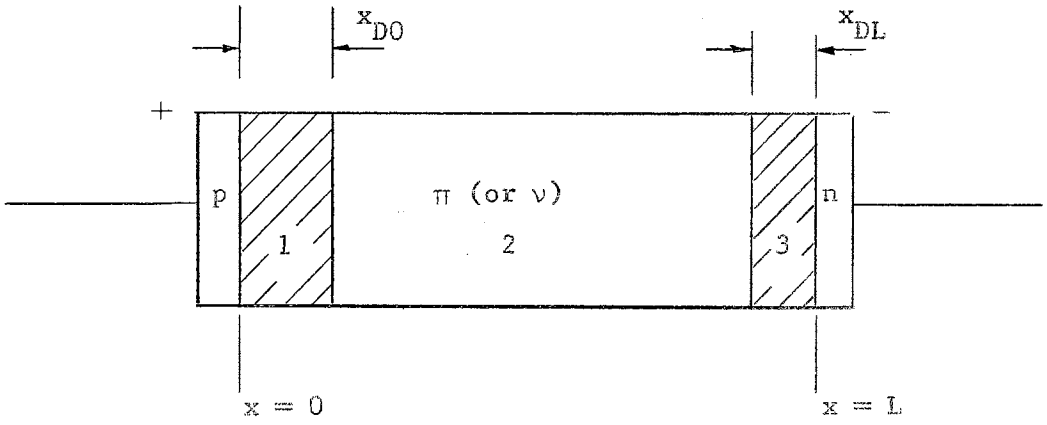

Figure 3. Diffusion Dominated Sections of a Device (Sections 1 and 3)

If we assume high level conditions, i.e. $\delta n \approx \delta p >> p_T$, and neglect the space charge term, i.e. neglect the second term on the right hand side of Equation 5.2.12, Equations 5.2.11 and 5.2.12 become together with $n = \delta n$ and $\delta p = \delta p + p_T$

$$J = q \mu_p (b + 1) nE \tag{5.3.1}$$

$$\frac{(b + 1)}{\mu_n} r = - p_T \frac{dE}{dx} + 2\beta \frac{d^2 n}{dx^2} \quad . \tag{5.3.2}$$

With Equation 5.3.1 and $r = n/\tau$ Equation 5.3.2 becomes

$$\frac{(b+1)\,n}{\mu_n \tau} = \frac{p_T J}{q\,\mu_p (b+1)\,n^2}\frac{dn}{dx} + 2\,\beta\,\frac{d^2n}{dx^2} \quad . \tag{5.3.3}$$

If the diffusion term dominates, Equation 5.3.3 becomes

$$\frac{(b+1)\,n}{\mu_n \tau} = 2\,\beta\,\frac{d^2n}{dx^2} \quad . \tag{5.3.4}$$

The general solution is

$$n(x) = c_1\, e^{+\frac{x}{L_a}} + c_2\, e^{-\frac{x}{L_a}} \tag{5.3.5}$$

where

$$L_a = \sqrt{\frac{2\,\beta\,\mu_n\,\tau}{(b+1)}} \quad \text{is the ambipolar diffusion length,} \tag{5.3.6}$$

and c_1, c_2 are arbitrary constants.

In the long devices which we studied, the total length is given as $L >> L_a$. In this kind of problem $c_1 e^{-x/L_a}$ must dominate in section 1

of Figure 3 and $c_2 e^{x/L_a}$ must dominate in section 3. Since the first term of Equation 5.3.3 was neglected, we then will have in Section 1

$$\left| 2 \beta \frac{d^2 n}{dx^2} \right| >> \left| \frac{p_T J}{q \mu_p (b + 1) n^2} \frac{dn}{dx} \right| \tag{5.3.7}$$

where

$$n(x) \simeq c_1 e^{-\frac{x}{L_a}} . \tag{5.3.8}$$

Substituting Equation 5.3.8 into Equation 5.3.7, we have the following condition for a diffusion dominated region:

$$n(x) >> \left[\frac{p_T J L_a}{2 \beta q \mu_p (b + 1)} \right]^{\frac{1}{2}} . \tag{5.3.9}$$

From Equation 5.3.6 it follows that

$$n(x) >> \left[\frac{p_T^2 J^2 \tau b}{2 q^2 (b + 1)^3 \beta \mu_p} \right]^{\frac{1}{4}} . \tag{5.3.10}$$

Since at $x = 0$, the boundary condition is (Baron, 1965)

$$n(0) = \frac{JL_a}{2\, q\, \mu_p\, \beta} \quad , \tag{5.3.11}$$

and from Equations 5.3.8 and 5.3.11, Equation 5.3.9 becomes

$$\frac{JL_a}{2q\,\mu_p\,\beta}\, e^{-\frac{x}{L_a}} >> \left[\frac{p_T J\, L_a}{2\,\beta\, q\,\mu_p (b+1)}\right]^{\frac{1}{2}} \quad . \tag{5.3.12}$$

It follows that

$$x << \frac{L_a}{2} \ln \left(\frac{L_a\, J(b+1)}{2q\,\beta\,\mu_p\, p_T}\right) \quad . \tag{5.3.13}$$

x_{DO} (See Figure 3) is defined to be equal to the right hand side of Equation 5.3.13 and is given by

$$x_{DO} = \frac{L_a}{2} \ln \left(\frac{L_\varepsilon\, J(b+1)}{2q\,\beta\,\mu_p\, p_T}\right) \quad . \tag{5.3.14}$$

For Section 3 of Figure 3, by using the boundary condition (Baron, 1965):

$$n(L) = \frac{JL_a}{2q\mu_n\beta} \quad , \tag{5.3.15}$$

and in a similar way we get the result

$$x_{DL} = \frac{L_a}{2} \ln\left(\frac{L_a J(b+1)}{2bq\beta\mu_p p_T}\right) \quad , \tag{5.3.16}$$

which is by a factor $L_a/2 \ln(b)$ smaller than x_{DO}.

5.4 The Lampert Semiconductor Sections. If the first term on the right hand side of Equation 5.3.3 dominates, we have

$$\frac{(b+1)\,n}{b\mu_p\tau} = \frac{p_T J}{q\mu_p(b+1)n^2}\frac{dn}{dx} \quad . \tag{5.4.1}$$

Solving the above equation, we have

$$n(x) = \frac{1}{\sqrt{\frac{-x2q(b+1)^2}{b\tau p_T J} + c_3}} \quad , \tag{5.4.2}$$

where c_3 is an arbitrary constant.

The range of validity of Equation 5.4.2 is determined by assuming that the first term on the right hand side of 5.3.3 is large compared to the second term. This condition gives:

$$\left| 2\beta \frac{d^2n}{dx^2} \right| << \left| \frac{p_T J}{q\mu_p (b+1)\, n^2} \frac{dn}{dx} \right| \qquad (5.4.3)$$

where n(x) is shown in 5.4.2. Substituting Equation 5.4.2 into Equation 5.4.3, we may put the above condition into the form

$$\left[\frac{p_T^2 J^2 \tau b}{6q^2 (b+1)^3 \beta \mu_p}\right]^{\frac{1}{4}} >> n(x) \quad , \qquad (5.4.4)$$

where the left side of Equation 5.4.4 is a lower limit for n. This formula is roughly the same quantity as in Equation 5.3.10 where it represents a lower limit for n in the diffusion dominated region. So we define a new quantity M as follows:

$$M = \left[\frac{p_T^2 J^2 \tau b}{4q^2 (b+1)^3 \beta \mu_p}\right]^{\frac{1}{4}} \quad , \qquad (5.4.5)$$

where the condition

$n(x) >> M$ determines the diffusion dominated region, and

$n(x) << M$ determines the Lampert semiconductor region.

5.5 An Approximate Solution to the Differential Equation. From Section 5.3, i.e. Equations 5.3.12 and 5.3.14 ($b = \mu_n/\mu_p > 1$), we derive that in a p-π-n diode, diffusion is more prominant on the left side of the π section. Since we consider a long diode ($L >> L_a$), an approximate solution of Equation 5.3.3 can be found by neglecting the diffusion

effect on the right hand side of the π section but by including the diffusion effect on the left hand side. We start with a linear superposition of the separate solutions of Equations 5.3.8 and 5.4.2 as follows:

$$n(x) = c_1 e^{-\frac{x}{L_a}} + \frac{1}{\sqrt{\frac{-x2q\,(b+1)^2}{Jb\,\tau\,p_T} + c_2}} \quad , \qquad (5.5.1)$$

where c_1 and c_2 can be evaluated through the boundary conditions of n(x), i.e. $n(x)_{x=0} = n(0)$ and $n(x)_{x=L} = n(L)$ as given by Equations 5.3.10 and 5.3.13.

In addition, if we neglect the diffusion term on the left hand side of the π section, Equation 5.5.1 becomes

$$n(x) = \frac{1}{\sqrt{\frac{-x2q\,(b+1)^2}{Jb\,\tau\,p_T} + c_3}} \quad , \qquad (5.5.2)$$

where c_3 is an arbitrary constant and can be evaluated through the boundary condition $n(x)_{x=L} = n(L)$. Then Equation 5.5.2 becomes

$$n(x) = \frac{1}{\sqrt{(L-x)\,\frac{2q\,(b+1)^2}{Jb\,\tau\,p_T} + \frac{1}{n^2(L)}}} \quad , \qquad (5.5.3)$$

when the device is operating in the regime where $J \propto V^2$, i.e. which

means that 10 $A/m^2 \leq J \leq 1000\ A/m^2$ (See Section 6.4). We may neglect the term $1/n^2(L)$ over most of the diode length. As an example we take the lowest current (worse case because $n^2(L)$ increases with J^2), $J = 10\ A/m^2$, $L - x = L_a$. The following numerical quantities are assumed (Bilger, et al., 1968): $\mu_n = 0.135\ m^2/Vs$, $\mu_p = 0.048\ m^2/Vs$, $P_T = 1.1 \times 10^{18} m^{-3}$, $\tau = 40\ \mu$ sec., $T = 300^\circ$ K. Then we have:

$$(L - x)\frac{2q(b+1)^2}{Jb\tau p_T} = L_a \frac{2q(b+1)^2}{Jb\tau p_T} = 1.01 \times 10^{-36}\ m^6 \quad , \qquad (5.5.4)$$

and (from Equation 5.3.13)

$$\frac{1}{n^2(L)} = \left(\frac{2q\mu_n \beta}{JL_a}\right)^2 = 1.71 \times 10^{-37}\ m^6 \quad . \qquad (5.5.5)$$

From the results of Equations 5.5.4 and 5.5.5, we justified the approximation

$$n(x) = \frac{1}{\sqrt{(L - x)\frac{2q(b+1)^2}{Jb\tau p_T}}} \quad . \qquad (5.5.6)$$

This result is within a factor $1/\sqrt{2}$ of the result given as Equation 12 by Bilger, et al., (1968). Comparing Equation 5.5.6 with the numerical results derived in Section 6.4, we see that except for the small

diffusion dominated part on the left hand side of the π section, it is a a good approximation for J in the regime where $J \propto V^2$, i.e.

$$10\ A/m^2 \leq J \leq 1000\ A/m^2 \quad .$$

CHAPTER VI

A COMPUTER SOLUTION OF THE DIFFERENTIAL EQUATION OF DOUBLE-INJECTION IN A SEMICONDUCTOR

6.1 Introduction. In this chapter the equations of double-injection are solved as a two boundary value problem (split boundary value problem). A computer program is written in such a way that we guess the the unknown initial condition on one side, for instance, the left side of the device. It then becomes a one boundary value problem. The solution that we have by guessing the unknown initial condition on the left side will give us a new boundary value on the right hand side. The new boundary value on the right hand side is compared with the specified boundary condition on the right hand side. If the difference of the two is not tolerable the computer will automatically select a new boundary value on the left hand side. The process will reiterate until the difference between the calculated and the known boundary values on the right hand side is within the desired preset limit. One set of computer results will give us E(x), n(x) at every point of the device, as well as the voltage corresponding to the assumed constant current density. Several sets of computer results will give us the I-V characteristic which, togethor with E(x) and n(x), are needed to calculate the g-r noise spectra of Equation 3.3.7. Compared with the experimental data by Bilger, et al., (1968), the calculated I-V characteristics are within 35%. The results are also compared with the analytic methods given in

Chapter V.

6.2 Master Equations and Boundary Conditions. The master equations are derived as follows:

(1.) Master differential equations - From Equations 5.2.11 and 5.2.12, assuming $n \approx \delta n \approx \delta p$ and $r = n/\tau$ as given by Equation 5.2.6, we have:

$$\frac{d^2E}{dx^2} = \frac{-\frac{JE^2}{\mu_n \mu_p \tau} + \frac{qp_T E^3}{\mu_n \tau} - qp_T E^3 \frac{dE}{dx} + \left[-\epsilon\epsilon_0 E^3 + \frac{4\beta J}{\mu_p (b+1)}\right] \left(\frac{dE}{dx}\right)^2}{\left[\epsilon\epsilon_0 E^4 + \frac{2\beta J E}{\mu_p (b+1)}\right]} \tag{6.2.1}$$

Written as a function of V, with the x coordinate normalized by $y = x/L_a$, we have:

$$\frac{d^3V}{dy^3} = \frac{\frac{JL_a^5 \left(\frac{dV}{dy}\right)^2}{\mu_n \mu_p \tau} + \frac{qp_T L_a^4 \left(\frac{dV}{dy}\right)^3}{\mu_n \tau} + qp_T L_a^2 \left(\frac{dV}{dy}\right)^3 \frac{d^2V}{dy^2} - \left[\epsilon\epsilon_0 \left(\frac{dV}{dy}\right)^3 + \frac{4\beta J L_a^3}{\mu_p (b+1)}\right] \left(\frac{d^2V}{dy^2}\right)^2}{\left[\epsilon\epsilon_0 \left(\frac{dV}{dy}\right)^4 - \frac{2\beta J L_a^3 \frac{dV}{dy}}{\mu_p (b+1)}\right]} \tag{6.2.2}$$

This equation results in a third order differential equation.

(2.) High level regime - We assume $n \approx \delta n \approx \delta p$, $r = n/\tau$, and $(b+1)\,\delta n >> p_T$. From Equations 5.2.11 and 5.2.12, we get the following result:

$$\frac{d^2E}{dx^2} = \frac{-\frac{JE^2}{\mu_n \mu_p \tau} - q\,p_T\,E^3 \frac{dE}{dx} + \left[-\epsilon\epsilon_0 E^3 + \frac{4\beta J}{\mu_p (b+1)}\right] \left(\frac{dE}{dx}\right)^2}{\left[\epsilon\epsilon_0 E^4 + \frac{2\beta J E}{\mu_p (b+1)}\right]} \tag{6.2.3}$$

Written as a function of V, we have:

$$\frac{d^3V}{dy^3} = \frac{\frac{JL_a^5(\frac{dV}{dy})^2}{\mu_n \mu_p \tau} + qp_T L_a^2 (\frac{dV}{dy})^3 \frac{d^2V}{dy^2} - [\epsilon\epsilon_0 (\frac{dV}{dy})^3 + \frac{4\beta J L_a^3}{\mu_p (b+1)}] (\frac{d^2V}{dy^2})^2}{[\epsilon\epsilon_0 (\frac{dV}{dy})^4 - \frac{2\beta J L_a^3 \frac{dV}{dy}}{\mu_p (b+1)}]} \qquad (6.2.4)$$

where $y = x/L_a$.

In order to solve Equations 6.2.2 and 6.2.4, we use the two boundary conditions as given by Equations 5.3.10 and 5.3.13. The initial conditions of E are obtained by Equation 5.2.11 for Equation 6.2.2 and by Equation 5.3.1 for Equation 6.2.4. The other initial condition for Equations 6.2.2 and 6.2.4 is $V = 0$ at $x = 0$. We write the initial conditions for Equation 6.2.2 such that

$$V(0) = 0 \qquad V(L) = V_L \quad \text{(to be calculated)}$$

$$\frac{dV(0)}{dx} = \frac{-2\beta J}{(2q\mu_p \beta p_T + (b+1) L_a J)} \qquad \frac{dV(L)}{dx} = \frac{-2qb\beta J}{(2qb\mu_p \beta p_T + (b+1) L_a J)} \qquad (6.2.5)$$

$$\frac{d^2V(0)}{dx^2} = \text{to be initially estimated} \qquad \frac{d^2V(L)}{dx^2} = \text{to be calculated.}$$

Similarly, we write the initial conditions for Equation 6.2.4 such that

$$V(0) = 0 \qquad V(L) = V_L \quad \text{(to be calculated)}$$

$$\frac{dV(0)}{dx} = \frac{-2\beta}{(b+1)L_a} \qquad \frac{dV(L)}{dx} = \frac{-2\beta}{(b+1)L_a} \qquad (6.2.6)$$

$$\frac{d^2V(0)}{dx^2} = \text{to be initially estimated} \qquad \frac{d^2V(L)}{dx^2} = \text{to be calculated.}$$

As stated in Equation 6.2.5 or 6.2.6, we guess the unknown value of $d^2V(0)/dx^2$. The computer program then becomes a one boundary value problem. The computer solution that we have by guessing $d^2V(0)/dx^2$ will give us a set of values of V(L), dV(L)/dx and $d^2V(L)/dx^2$. If the difference of the new value of dV(L)/dx with the specified dV(L)/dx given by Equation 6.2.5 or 6.2.6 is not tolerable, the computer will automatically select a new value of $d^2V(0)/dx^2$. The process will reiterate until the difference between the calculated dV(L)/dx, and the specified dV(L)/dx is within the desired preset limit.

In his paper Baron (1968) stated two more approximate boundary conditions:

$$\frac{dn(0)}{dx} = -\frac{J}{2q\mu_p\beta} \qquad \text{and} \qquad (6.2.7)$$

$$\frac{dn(L)}{dx} = \frac{J}{2q\mu_n\beta} \quad . \qquad (6.2.8)$$

The initial conditions of d^2V/dx^2 are obtained by Equations 5.2.11, 5.3.10, and 5.3.13 for Equation 6.2.2, together with the others such

that:

$$V(0) = 0 \qquad V(L) = V_L \quad \text{(to be calculated)}$$

$$\frac{dV(0)}{dx} = \frac{-2\beta J}{[2q\mu_p \beta p_T + (b+1)L_a J]} \qquad \frac{dV(L)}{dx} = \frac{-2qb\beta J}{[2q\mu_p \beta p_T + (b+1)L_a J]} \tag{6.2.9}$$

$$\frac{d^2V(0)}{dx^2} = \frac{2q\mu_p \beta (J - q\mu_p p_T)}{(b+1) L_a^2 J} \qquad \frac{d^2V(L)}{dx^2} = \frac{-2q\mu_n \beta (J - q\mu_p p_T)}{(b+1) L_a^2 J}$$

Similarly, the initial conditions of d^2V/dx^2 are obtained by Equations 5.3.1, 5.3.10, and 5.3.13 for Equations 6.2.4, together with the others such that:

$$V(0) = 0 \qquad V(L) = V_L \quad \text{(to be calculated)}$$

$$\frac{dV(0)}{dx} = \frac{-2\beta}{(b+1)L_a} \qquad \frac{dV(L)}{dx} = \frac{-2\beta b}{(b+1)L_a} \tag{6.2.10}$$

$$\frac{d^2V(0)}{dx^2} = \frac{2q\mu_p \beta}{(b+1)L_a^2} \qquad \frac{d^2V(L)}{dx^2} = \frac{-2q\mu_n \beta}{(b+1)L_a^2}$$

For example, in solving Equation 6.2.4, if the boundary conditions

at x = 0 of Equation 6.2.10 are used, we do not necessarily get dV(L)/dx and $d^2V(L)/dx^2$ of Equation 6.2.10. Knowing the approximate initial condition $d^2V(0)/dx^2$ of Equation 6.2.6 will save a lot of computer time because the solution will converge faster by guessing approximately the correct one. We do not know whether Baron (1968) used the split boundary value conditions as we do, but if he does not, we have reason to believe that our results are more accurate because his results are the first iterations of our computer program.

6.3 I-V Characteristic. In order to compare the theoretical results (i.e. by solving Equations 6.2.2 and 6.2.4 with appropriate boundary conditions as discussed in Section 6.2) with the experimental data of Bilger et al. (1968), we use the following numerical quantities:

$\mu_n = 0.135\ m^2/Vs$

$\mu_p = 0.048\ m^2/Vs$

$b = \mu_n/\mu_p = 2.813$

$p_T = 1.1 \times 10^{18}\ m^{-3}$

$q = 1.6 \times 10^{-19}$ coulombs

$\tau = 40\ \mu$ sec

$T = 300^\circ$ K

$A = 10 \times 10^{-6}\ m^2$

$L = 6 \times 10^{-3}$ m.

The variable V of Equations 6.2.2 and 6.2.4 is a function of the normalized distance, y, where $y = x/L_a$. The ambipolar diffusion length L_a is given by Equations 5.3.6 and, in this case, is given as $L_a = 2.7 \times 10^{-4}$ m, and $L \simeq 22\ L_a$.

For each constant current density, J, by solving Equation 6.2.2 we get a corresponding voltage difference V_L across the device. The I-V

characteristic is plotted in Figure 4. We found $I \propto V^{1.243}$ in the ohmic regime, except for the transition regime (between the ohmic regime and the Lampert semiconductor regime). These results are within 35% of the experimental data. It is to be noted that each of the parameters used in the experiments may be uncertain by 30%. If we also consider the contact voltage drop of the p-π and π-n junction, we expect that the theoretical values will be closer to the experimental ones.

Similarly, by solving Equation 6.2.4 we found that $I \propto V^{2.315}$ is in the Lampert semiconductor regime. Comparing Equation 6.2.4 with the experimental data in this regime where $I \propto V^2$, we also found an accuracy of 35%.

In the transition regime the results of Equation 6.2.2 and Equation 6.2.4 are different. This problem remains to be further investigated.

6.4 Carrier Densities and Field Intensities. The field intensities for various assumed current densities are plotted in Figure 5 as functions of the normalized distance, y. The field intensity profile of the dashed line, i.e. for J = 2 Amps/m^2, is an example of the current density in the ohmic regime. In the Lampert semiconductor regime we plot three curves, one each at J = 10, 100, and 1000 Amps/m^2. For each curve, the current passing through the device is constant. In a given curve the diffusion effects (third term in Equation 5.2.12) become less important as the field intensity is larger. From studying the curves in Figure 5, we see the drift effect is predominant over most of the central part of the device.

The carrier densities can be obtained from the field intensities by Equation 5.2.11 for the ohmic regime and can be obtained from the field intensities by Equation 5.3.1 for the Lampert semiconductor regime.

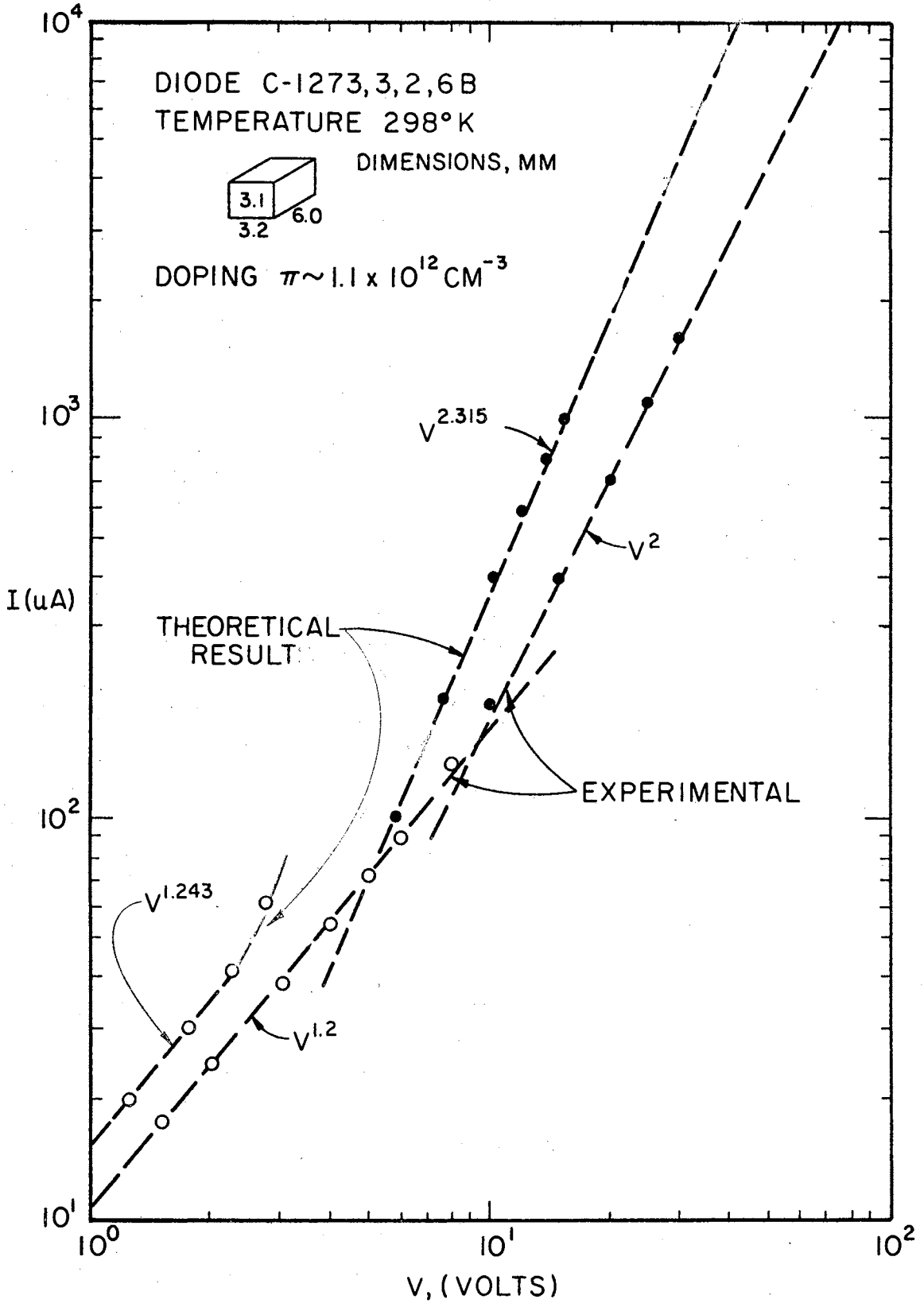

Figure 4. I-V Characteristics of an Experimental Device

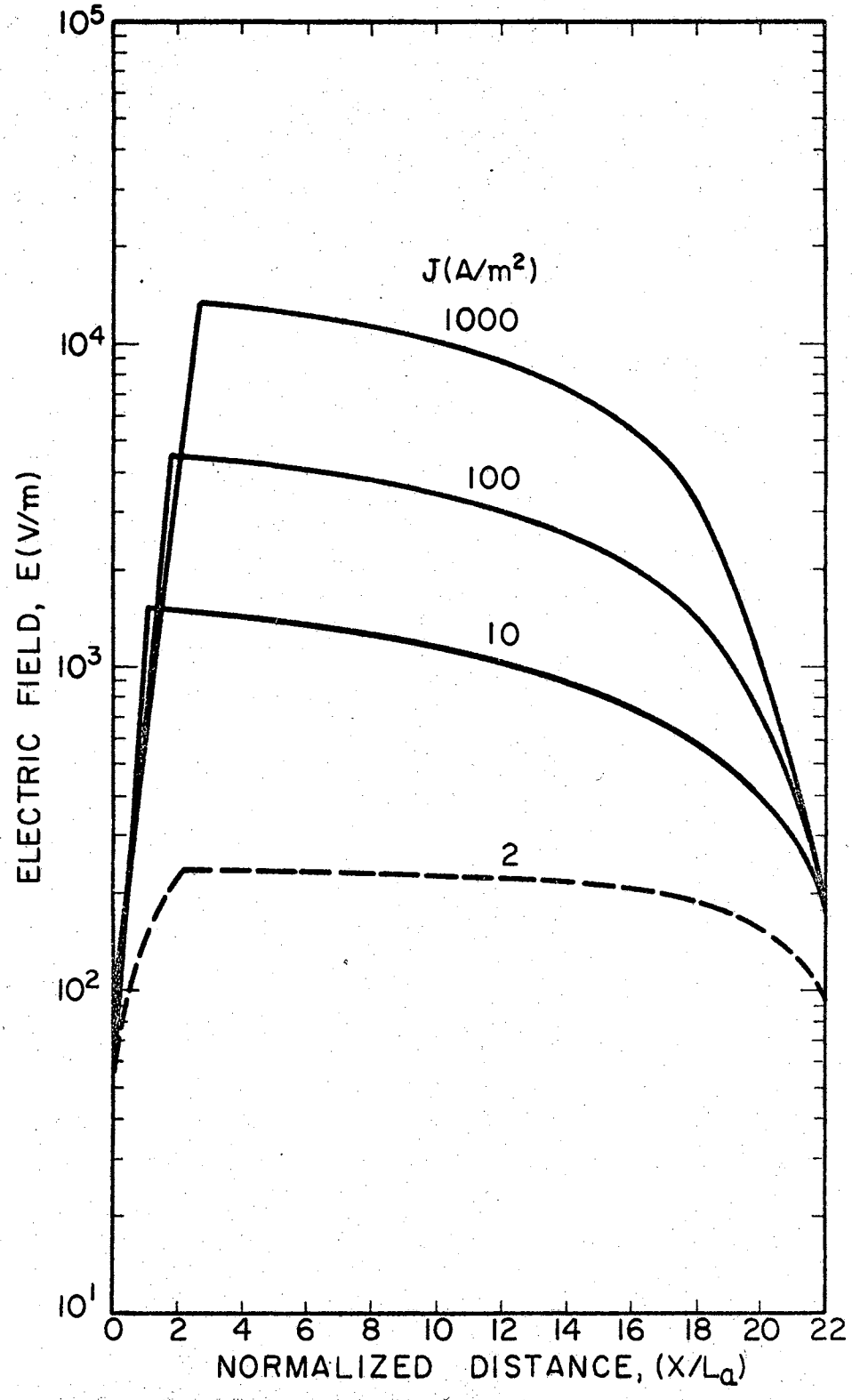

Figure 5. Field Intensity Profiles in a Device for Various Current Densities

The carrier density curves for several current densities corresponding to the field intensity curves in Figure 5 are plotted in Figure 6. The diffusion effect is proportional to the gradient of the carrier density (see the second tern on the right hand side of Equation 5.2.1 or 5.2.2). From the curves in Figure 6, we see that the diffusion effect is predominant in the neighborhood of contacts (i.e. in the neighborhood where $x = 0$ and $x = L$). The diffusion effect is more prominent on the left hand side than on the right hand side as is also shown by Equations 5.3.12 and 5.3.14, because $b \approx 2.813$ and $x_{DO} > x_{DL}$.

In order to compare the carrier densities in the device as a function of normalized distance with the analytical method list in Chapter V, i.e. Equation 5.5.6, we rewrite Equation 5.4.5 as follows:

$$M = \left[\frac{p_T^2 J^2 \tau b}{4q^2 (b+1)^3 \beta \mu_p}\right]^{\frac{1}{4}}$$

$$= 3.7 \times 10^{17} J^{\frac{1}{2}} \quad . \qquad (6.4.1)$$

We rewrite Equation 5.5.6 as follows:

$$n(x) = \frac{1}{\sqrt{(L - x)\frac{2q (b+1)^2}{J b \tau p_T}}}$$

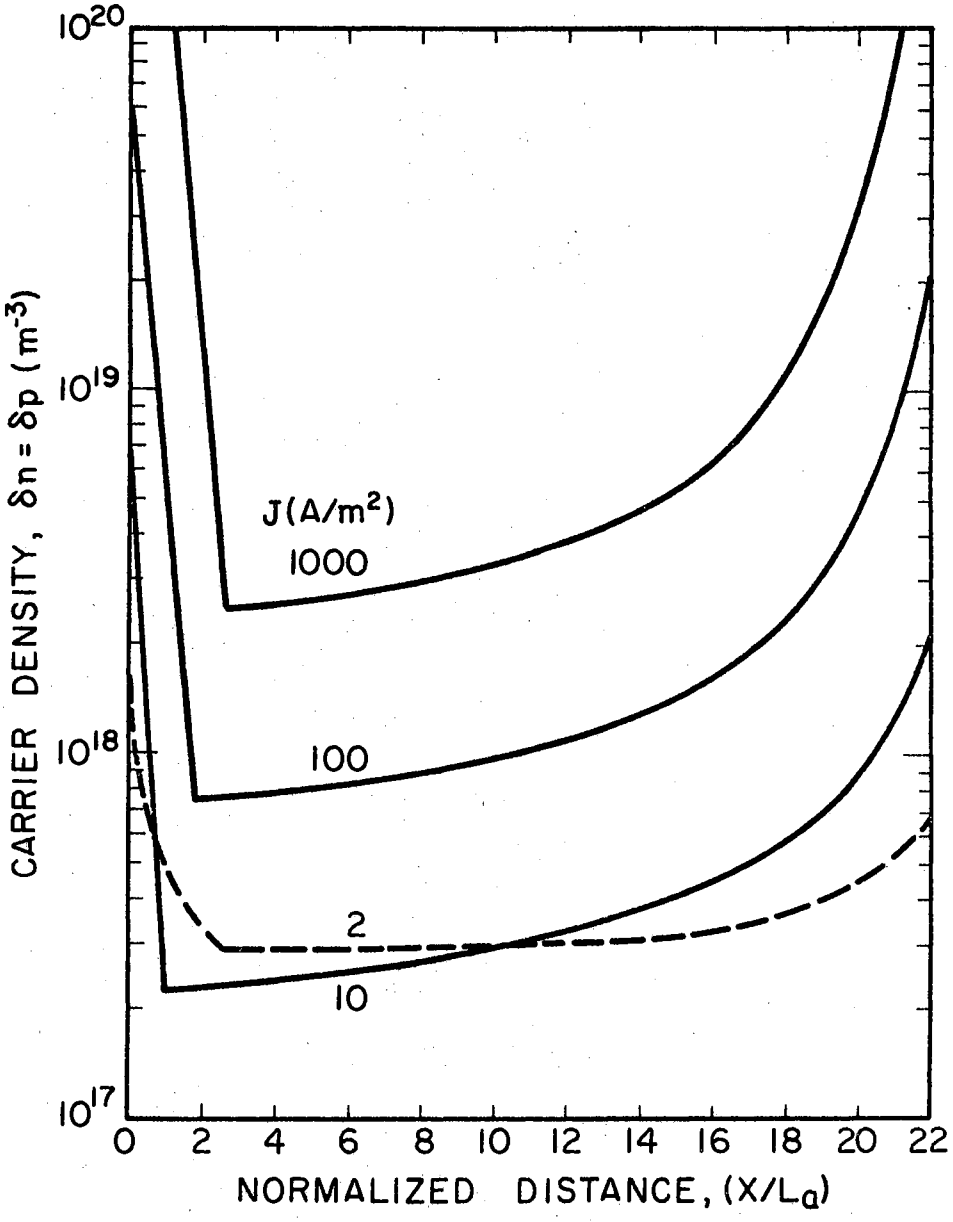

Figure 6. Numerical Results of the Carrier Density Profiles for Various Current Densities

$$= \frac{1}{\sqrt{(22\ L_a - x)\ 3.79 \times \frac{10^{-32}}{J}}} \quad . \qquad (6.4.2)$$

The numerical quantities are taken from Section 6.3. Equation 6.4.1 is plotted as a dashed line in Figures 7, 8, and 9. For $n(x) > > M$ (above the dashed line), we have the diffusion dominated region. On the other hand for $n(x) < < M$ (below the dashed line), we have the Lampert semiconductor region.

In Figure 7 by solving Equation 6.2.2 with the boundary conditions of Equation 6.2.5, the numerical results of $J = 2$ Amps/m^2 are plotted together with Equation 6.4.2 at the same current density. It is seen that Equation 6.4.2 is not a good approximation because the curve of the numerical results does not satisfy the condition $n(x) < < M$, i.e. most of the curve is not in the Lampert semiconductor region.

In Figure 8, we plotted Equation 6.4.2 together with the numerical results by solving Equation 6.2.4 with the boundary conditions of Equation 6.2.6 at $J = 10$ Amps/m^2. We found that Equation 6.4.2 is a good approximation except for the diffusion part on the left hand side of the device. These results are true because, in Figure 8, most of the curve of the numerical results differ from $M = 3.7 \times 10^{17} J^{1/2}$ by a much larger amount than in Figure 7. Thus, most of the curve is in the Lampert semiconductor region.

In Figure 9, the approximation of the results of the two curves is tolerable but is not as close as those in Figure 8. The reason for this difference is that the diffusion effect on the right hand side of the device becomes a factor that we should consider as current density increases. This effect is shown by Equation 5.3.14. From this argument,

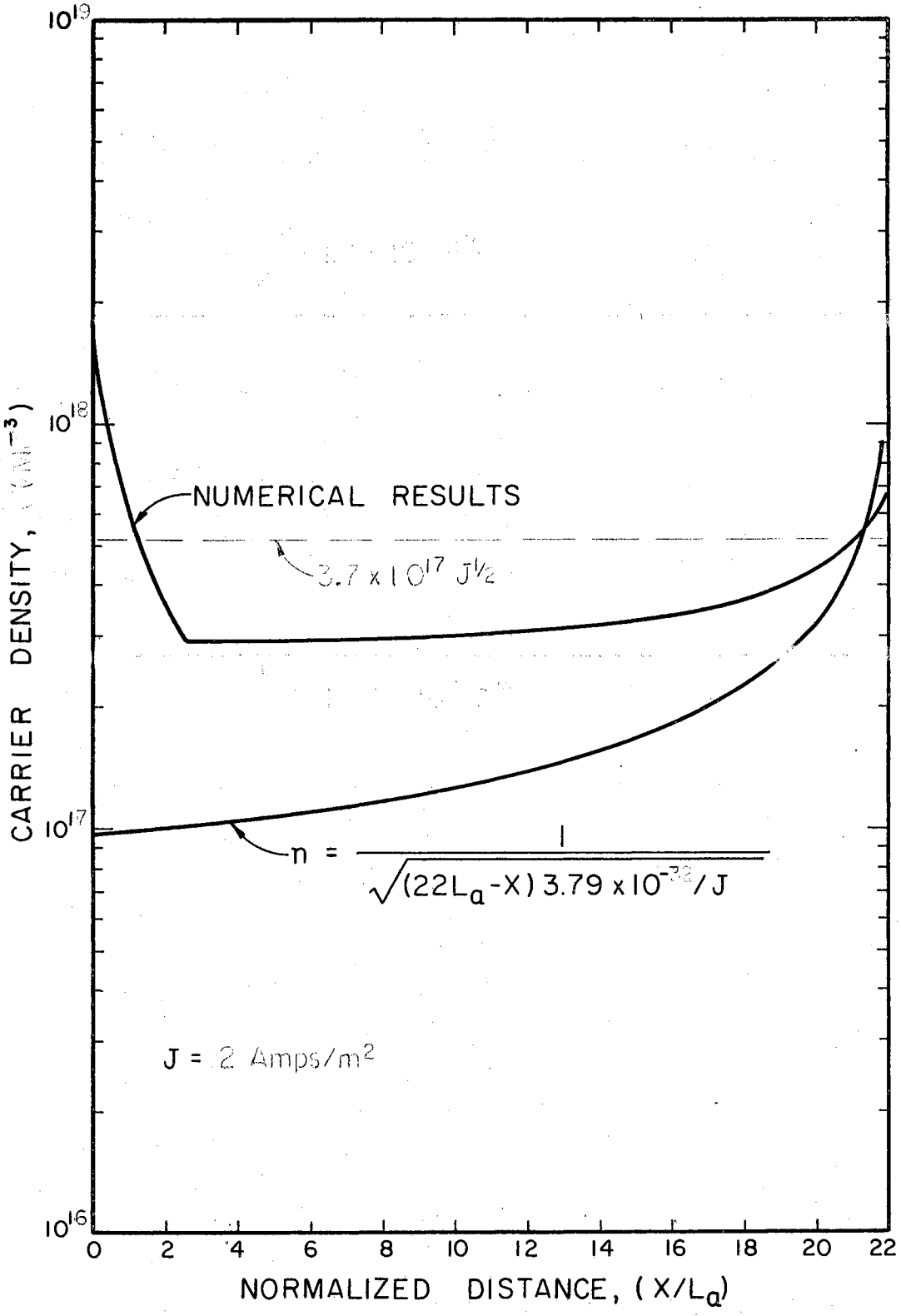

Figure 7. Carrier Density Profile Calculated Numerically and Through an Analytical Approximation at J = 2 Amps/m^2

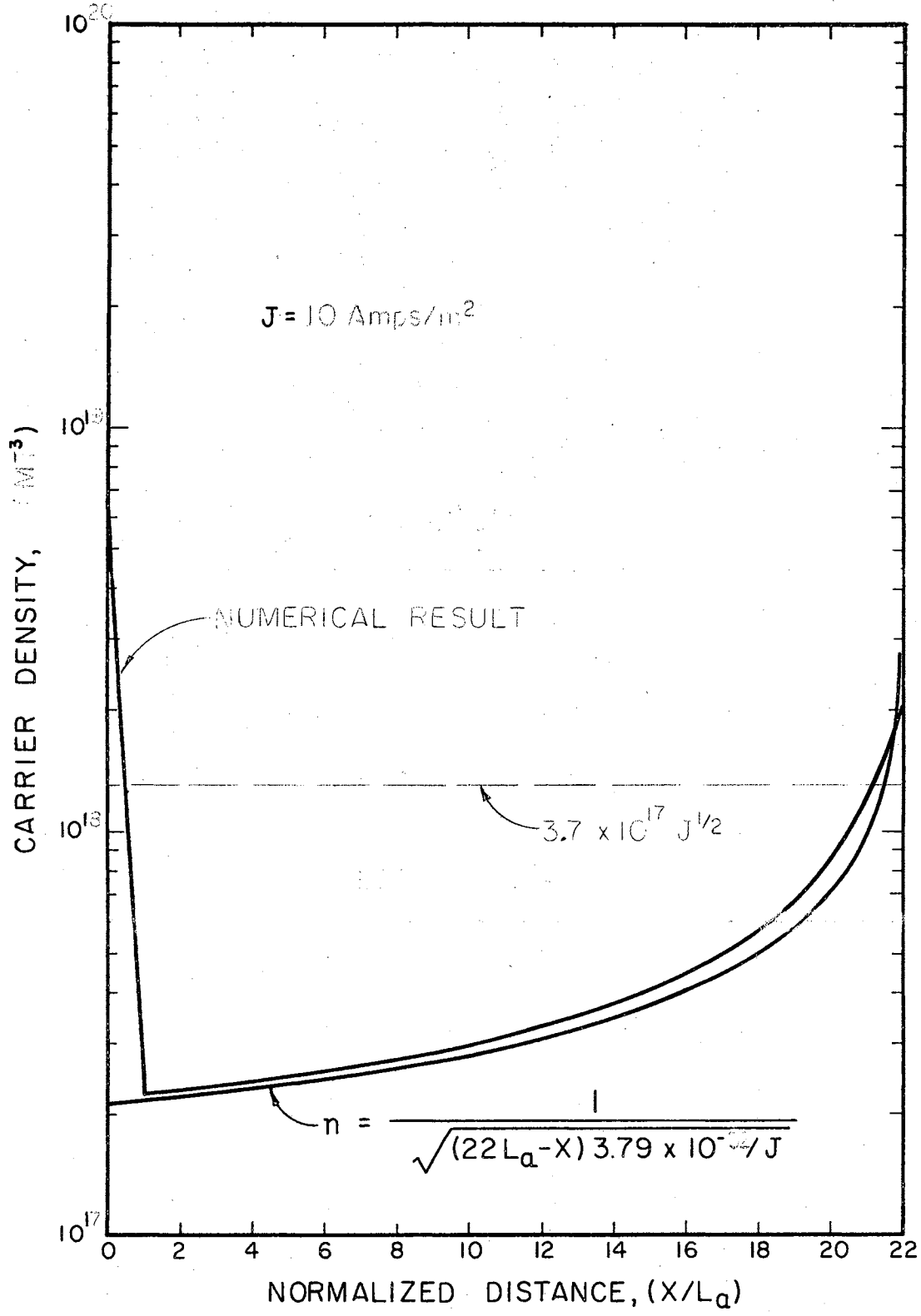

Figure 8. Carrier Density Profile Calculated Numerically and Through an Analytical Approximation at J = 10 Amps/m^2

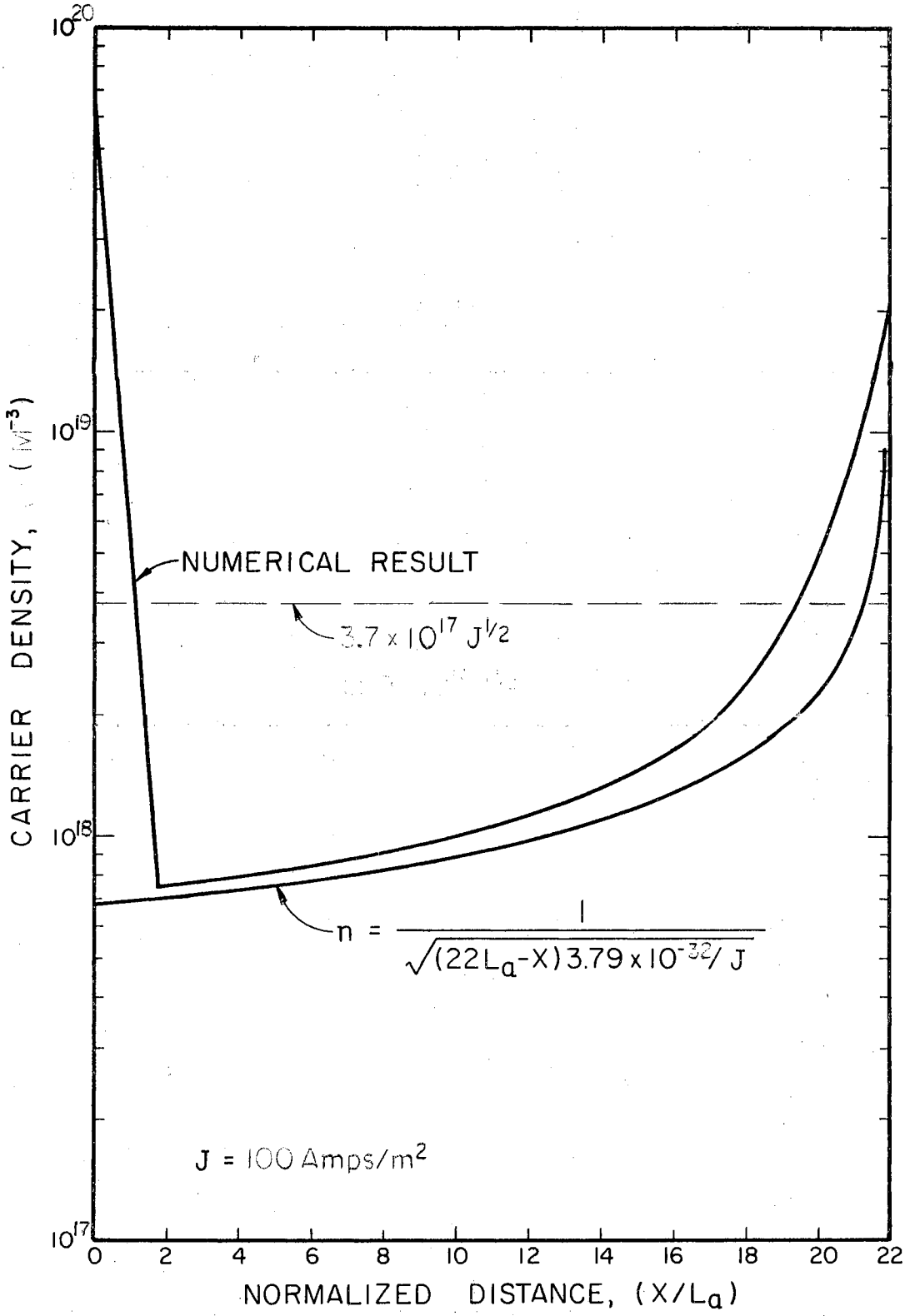

Figure 9. Carrier Density Profile Calculated Numerically and Through an Analytical Approximation at J = 100 Amps/m^2

we see Equation 5.5.6, i.e. in this case Equation 6.4.2, is a good approximation for the current density in the Lampert semiconductor regime.

CHAPTER VII

G-R NOISE SPECTRAL DENSITIES OF A DOUBLE-INJECTION DIODE

7.1 Introduction. Using the equation developed in Chapter III and the results of Chapters V and VI, the noise spectrum was calculated by both the analytical method and the numerical method. It has been found that at low frequencies the generation-recombination noise spectra have the dependence

$$S_i(f) \propto I_a^m \qquad \text{where} \qquad \frac{3}{2} < m < 2 \quad . \qquad (7.1.1)$$

m should be near 2 when the current is small and should be near 3/2 for current in the Lampert semiconductor regime.

7.2 G-R Noise Spectrum by the Analytical Method. As shown in Chapter III, the generation-recombination noise spectrum is given by

$$S_i(f) = \left(\frac{\partial I_a}{\partial V_L}\right)^2 \frac{4(b+1)^2 \tau}{A(1+\omega^2\tau^2)} \int_0^L \frac{np\, dx}{(n+p)(bn+p)^2}\, dx \quad . \quad (7.2.1)$$

From Equation 5.2.11, Equation 7.2.1 becomes:

$$S_i(f) = \left(\frac{\partial I_a}{\partial V_L}\right)^2 \frac{4\,(b+1)^2\,\tau\,J^2}{q^2\,\mu_p^{\,2}\,A(1+\omega^2\tau^2)} \int_0^L \frac{np\,dx}{(n+p)(bn+p)^2(p_T+bn+\delta p)^2} \tag{7.2.2}$$

where, in this case, $n = \delta n$, $p = p_T + \delta p$. In the Lampert semiconductor regime $\delta n \approx \delta p >> p_T$, from Equation 7.2.2 we have

$$S_i(f) = \left(\frac{\partial I_a}{\partial V_L}\right)^2 \frac{2\,\tau\,J^2}{q^2\,\mu_p^{\,2}\,(b+1)^2\,(1+\omega^2\,\tau^2)A} \int_0^L \frac{1}{n^3(x)}\,dx \quad . \tag{7.2.3}$$

The approximate value of $n(x)$ is given by Equation 5.5.6, substituting $n(x)$ into Equation 5.3.1, we have

$$E(x) = \sqrt{\frac{(L-x)\,2\,J}{b\,\tau\,p_T\,q\,\mu_p^{\,2}}} \quad . \tag{7.2.4}$$

It follows from $(E = -dV/dx)$ that

$$dV\Big|_0^{V_L} = -\sqrt{\frac{2\,J}{b\,\tau\,p_T\,q\,\mu_p^{\,2}}} \int_0^L \sqrt{L-x}\,dx \tag{7.2.5}$$

and

$$V_L = -\sqrt{\frac{8}{9}\,\frac{8\,J\,L^3}{q\,\mu_p^{\,2}\,b\,\tau\,p_T}} \quad . \tag{7.2.6}$$

Rewriting Equation 7.2.6 we have

$$J = \frac{9}{8} q \mu_p^2 b \tau p_T \frac{V_L^2}{L^3} \tag{7.2.7}$$

(which is exactly the same as given by Equation 5.2.13). From Equations 5.5.6 and 7.2.7, Equation 7.2.3 becomes

$$S_i(f) = \frac{36\sqrt{2}\, A\, q^{\frac{1}{2}} (b+1)\, \tau^{\frac{1}{2}} J^{\frac{3}{2}}}{5\, b^{\frac{1}{2}} L^{\frac{1}{2}} p_T^{\frac{1}{2}} (1 + \omega^2 \tau^2)} \quad . \tag{7.2.8}$$

From Equation 4.2.6 we have

$$I_{eq} = \frac{18\sqrt{2}\, A\, (b+1)\, \tau^{\frac{1}{2}} J^{\frac{3}{2}}}{5 q^{\frac{1}{2}} b^{\frac{1}{2}} L^{\frac{1}{2}} p_T^{\frac{1}{2}} (1 + \omega^2 \tau^2)} \quad . \tag{7.2.9}$$

7.3 G-R Noise From the Numerical Results. Using the numerical results (i.e. n(x), p(x), and E(x) as well as I-V characteristics of Chapter VI) and numerically integrating Equation 7.2.1, we found that

$$I_{eq} \propto I_a^{1.91} \tag{7.3.1}$$

in the ohmic regime, and

$$I_{eq} \propto I_a^{1.512} \qquad (7.3.2)$$

in the Lampert semiconductor regime.

Using the numerical quantities as given in Section 6.3, Equation 7.2.9 is plotted in Figure 10 together with the numerical results of Equations 7.3.1 and 7.3.2. Comparing the results of Equations 7.2.9 and 7.3.2, it is seen that 7.2.9 is a good approximation. The noise spectra as a function of ω at several currents, are shown in Figure 11. From the results of Equations 7.3.1 and 7.3.2, we know the noise spectra as a function of current should be in the range as shown in Equation 7.1.1.

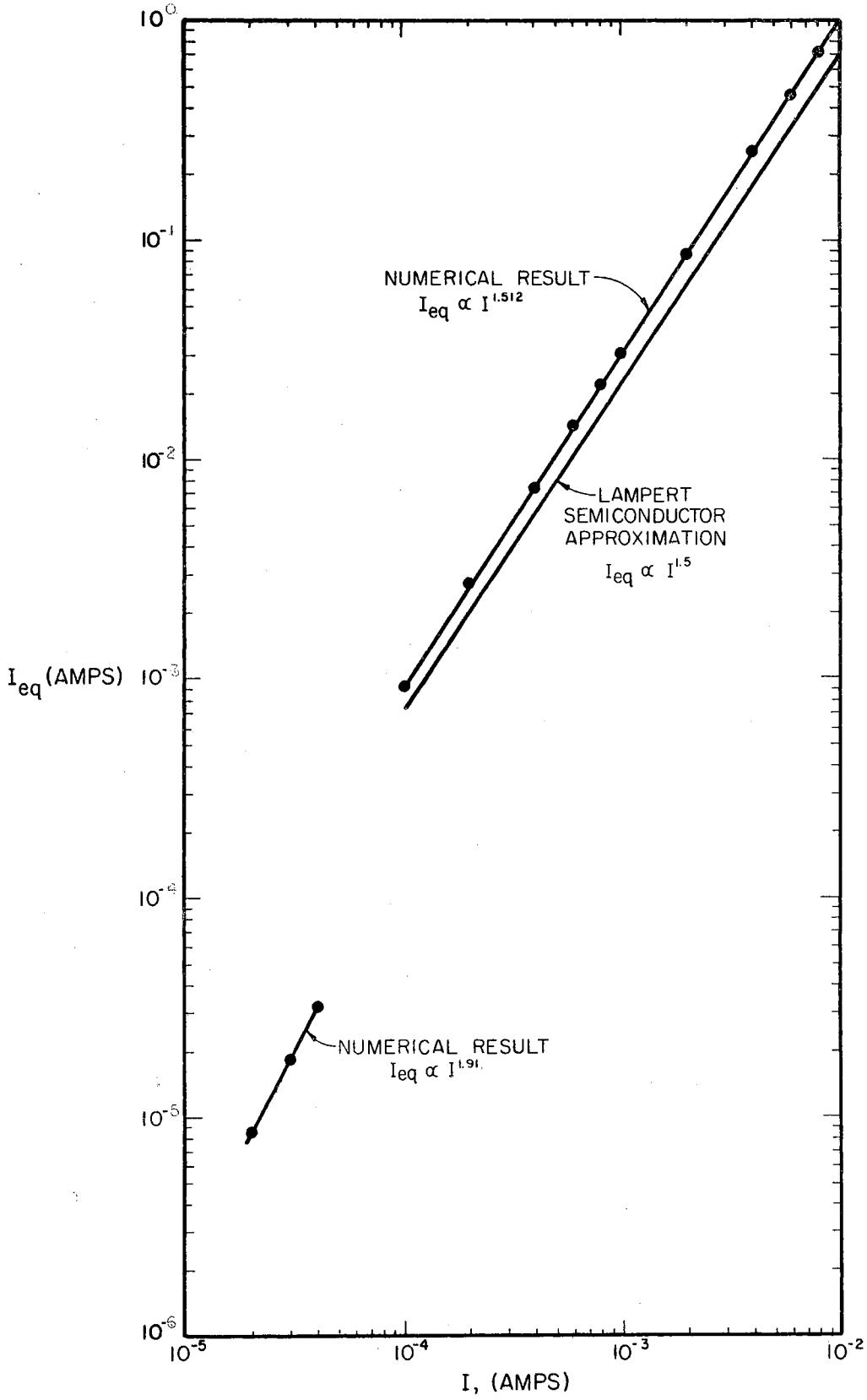

Figure 10. $I_{eq}(0)$ as a Function of Current Density

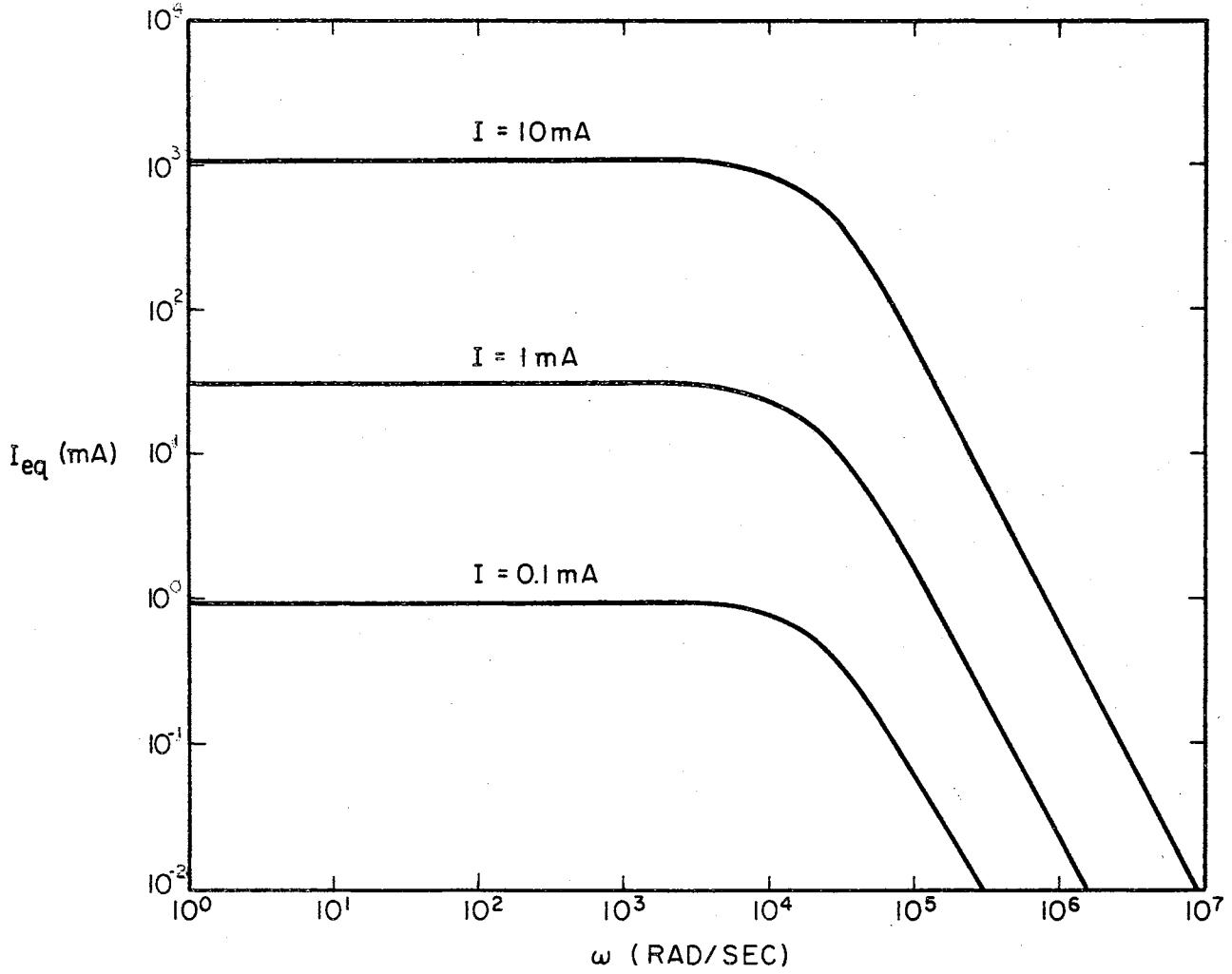

Figure 11. Calculated G-R Noise Spectra for Various Currents

CHAPTER VIII

SUMMARY AND CONCLUSIONS

8.1 Summary. A theory of the generation-recombination noise spectra was developed from a microscopic viewpoint which extended over the entire device. In order to get the g-r noise, the I-V characteristic of double-injection had to be thoroughly studied. From these results theoretical predictions of g-r noise as a function of current were made.

In Chapter II the theory of carrier density fluctuation in a semiconductor was discussed and was extended for the case of large departures from thermal equilibrium. Using the results in Chapter III, the g-r noise spectrum was developed once the local carrier fluctuation was known.

In Chapter IV a simple example was given to show how to apply the theory to the double-injection device operating in the linear mode.

In Chapter V and VI the physical properties of double-injection in a semiconductor were solved both analytically and numerically with the aid of a computer.

In Chapter VII the g-r noise of double-injection was solved by both an analytical method and a numerical method.

8.2 Conclusion. The following equation for g-r noise has been derived:

$$S_i(f) = c \left(\frac{\partial I_a}{\partial V_L}\right)^2 \frac{4 (b+1)^2 \tau}{A(1+\omega^2 \tau^2)} \int_0^L \frac{np\, E^2\, dx}{(n+p)(bn+p)^2} \qquad (8.2.1)$$

where $1/2 < c \leq 1$.

In the ohmic regime, the results were found that $I_a \propto V_L^{1.2}$ and $I_{eq} \propto I_a^{1.91}$.

In the Lampert semiconductor regime, the results were found that $I_a \propto V_L^{2.315}$ and $I_{eq} \propto I_a^{1.512}$.

If we neglect the diffusion part from the p-π junction, the electron density in the Lampert semiconductor regime is

$$n(x, J) = \frac{1}{\sqrt{(L - x)\frac{2q\,(b+1)^2}{b\,\tau\,p_T\,J} + \frac{1}{\left(\frac{J\,L_a}{2q\,\beta\,\mu_n}\right)^2}}} \qquad (8.2.2)$$

In general the g-r noise spectrum should be

$$S_i(f) \propto I_a^{\,m} \qquad \text{where} \qquad \frac{3}{2} < m < 2,$$

m should be near 2 when the current is small and should be near 3/2 when the current is in the Lampert semiconductor regime.

8.3 Recommendation for Further Study. The most interesting and fruitful extension of this dissertation would be a program of experimental measurements to probe the predictions made in this dissertation.

A second extension would be to look into the transition regime between the ohmic and the Lampert semiconductor regime and to determine how the g-r noise changes from $I_{eq} \approx I_a^{1.91}$ to $I_{eq} \approx I_a^{1.512}$.

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APPENDIX A

LIST OF SYMBOLS

A = cross section area of a device (m^2)

$b = \mu_n/\mu_p$

$\beta = kT/q$ (volt)

$c = (1 + k_1)/2$

E = electric field intensity (V/m)

ϵ = relative dielectric constant

ϵ_0 = dielectric constant = 8.854×10^{-12} (As/Vm)

f = frequency (H_z)

$g(f)$ = high frequency conductance (real part of admittance) (Ω^{-1})

$g_e(N)$ = the probability that an electron enters the conduction band

I_a = the DC current passing through the device (A)

$I_{eq}(f)$ = equivalent shot noise current (A)

$\overline{i^2}$ = the variance of current fluctuation (A^2)

J = total current density (A/m^2)

k = Boltzmann constant = 1.38×10^{-23} (VAs/m^2)

k_1 = a suppression constant due to the existence of space-charge

L = length of the device (m)

$$M = \left[\frac{p_T^2 J^2 \tau b}{4q^2(b+1)^3 \beta \mu_p}\right]^{\frac{1}{4}} \quad (m^{-3})$$

where the conditions $n(x) >> M$ determines the diffusion dominated region

$n(x) << M$ determines the Lampert semiconductor region

μ_n = electron mobility

μ_p = hole mobility

$$L_a = \sqrt{\frac{2 \beta \mu_n \tau}{1 + b}} \quad \text{(m)}$$

= the ambipolar diffusion length

N = number of electrons

n = electron density (m^{-3})

N_D = donor atoms

N_0 = the steady state value of N

$\overline{\Delta N^2}$ = variance of the variable N

$$\overline{\Delta N_t^2} = \frac{N_0 p_0}{N_0 + p_0}$$

= variance of N at thermal equilibrium

$$\overline{\Delta N_s^2} = c \frac{N_0 p_0}{N_0 + p_0} \quad \text{where } \frac{1}{2} < c \leq 1$$

= variance of N in the case of non-equilibrium

ΔN_0 = number of electrons in a section Δx

n_T = electron density in thermal equilibrium (m^{-3})

δn = excess electron density (m^{-3})

P = number of holes

p = hole density (m^{-3})

P_0 = the steady state value of P

ΔP_0 = number of holes in a section Δx

p_T = hole density in thermal equilibrium (m^{-3})

δ_p = excess hole density (m^{-3})

q = electron charge = 1.60×10^{-19} (As)

r = recombination rate of electron and hole ($m^{-3}s^{-1}$)

$r_e(N)$ = the probability that an electron leaves the conduction band

$\Delta R(x)$ = D.C. resistance in a section Δx of a device (Ω)

$\Delta R_i(x)$ = incremental resistance in a section x of a device (Ω)

R_n = total resistance responsible for thermal noise in a device (Ω)

$S_i(f)$ = noise current spectral density (A^2s)

$S_v(f)$ = noise voltage spectral density (V^2s)

T = absolute temperature ($^\circ K$)

V = voltage difference (V)

V_L = voltage difference applied to a device (V)

$\overline{v^2}$ = variance of voltage (V^2)

$W(N)$ = the steady state probability distribution for the number of electrons in the conduction band

ω = angular frequency (Hz)

$X = N - N_0$

x = length coordinate of a device (m)

$$x_{D0} = \frac{L_a}{2} \ln \left(\frac{L_a J(b+1)}{2q \beta \mu_p p_T}\right) \quad (m)$$

where $x << x_{D0}$ is the diffusion dominated region adjacent to the p-π contact

$$x_{DL} = \frac{L_a}{2} \ln \left(\frac{L_a J(b+1)}{2bq \beta \mu_p p_T}\right) \quad (m)$$

where $L - x << x_{DL}$ is the diffusion dominated region adjacent to the π-n contact

$Y = k_1 \cdot x$

$y = x/L_a$ normalized length coordinate of a device

V = electrostatic potential (V)

τ = carrier average life time (s)

VITA

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