THEORY AND APPLICATION OF ECONOMICALLY OPTIMUM FARM MACHINERY REPLACEMENT CRITERIA

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PREFACE

The research reported in this dissertation was conducted in the Department of Agricultural Economics, Oklahoma State University. The dissertation includes the development and empirical application of a farm machinery replacement criterion. Special emphasis is placed on tractor replacement but the study also includes combines and automobiles.

I wish to thank my major advisor, Dr. Luther Tweeten, for his guidance and prompt assistance during my graduate study. Thanks are also due to the other members of my committee: Dr. V. R. Eidman, Dr. O. L. Walker, Dr. F. G. Steindl, and Dr. C. E. Marshall.

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TABLE OF CONTENTS

Chapter | Page

LIST OF TABLES

LIST OF FIGURES

CHAPTER I

INTRODUCTION

Some farm management decisions are made so infrequently that a farmer's experience may prove to be of limited value. In such cases, economic analysis can be of real assistance. Machinery management includes problems of this type. Much research has been focused on the optimum level and mix of labor and capital, including machinery, for selected farms. But there has been relatively little work on the economically optimum time to trade machinery. The lack of such knowledge is one of the bottlenecks in machinery analysis. The intent of this study is to develop criteria which farmers can use to make economical yearly replacement decisions. Included in the replacement analysis are many of the variables which affect the optimal replacement interval. Such factors as labor charges, land purchases, capital availability, and other factors that have a profound affect on replacement practices are analyzed.

Objectives

This study is confined to development of optimal replacement procedures for farm machinery, and does not determine what the actual inventory of machinery should be. If the current level of machinery investment is satisfactory and external factors do not change, the problem is maintaining the existing level of machinery services by using optimal replacement procedures. If external factors do change, then the problem is one of adjustment to a new satisfactory level of machinery inventory. Through proper planning, it is possible to determine how and when to move from an existing to a new level of investment.

To formulate replacement procedures, it is first necessary to develop a general theory of replacement. The theory developed should provide general replacement criteria which can be adapted and manipulated for practical application to realistic conditions.

Successful machinery replacement procedures should be usable by farmers. To be usable, the procedures must be convenient to apply and give reliable results. This study includes efforts to condense complex formulas and procedures into convenient tables, graphs, and other easily mastered communication methods.

Besides being useful to individual farmers making replacement decisions, replacement analysis is also crucial for long-term farm planning. The two situations are different. In the short run, the farmer asks the question: Given my current situation and what I expect for the future, should I or should I not replace the machine immediately?

For long range planning, it is necessary to know the optimal expected replacement interval and resulting average cost so that future farm costs and returns can be anticipated. This study is intended to be useful for both short- and long-run situations.

At this point it is necessary to emphasize the place of expected costs in this study. If long-run or expected replacement intervals are being studied, it is obvious that expected costs will be used. When making short-run replacement decisions, expected costs based on past experience of a large number of farmers is used only as a point of departure. The actual replacement decision requires an estimate of future costs made on the basis of actual past costs, experience of other farmers with similar machinery, and firsthand knowledge of the particular machine.

The objective of this replacement study is to assist farm managers in reducing costs of maintaining machinery capability. Toward this end, procedures are developed to analyze the situations outlined in the following paragraphs:

1. The basic situation analyzed is that of a farmer replacing his existing machine with an exact duplicate. A procedure is developed whereby the trading point can be attained with the least possible average yearly cost over time. Usage of the model is then extended so that it is not necessary that the proposed replacement be an exact duplicate.
2. Costs of the currently owned machine may also vary, thereby affecting the optimal replacement pattern. The cost changes may be due to some chronic machine deficiency or due to one large repair bill. Procedures are developed for handling each of these situations.
3. Economists may advise managers who trade either before or after the optimal point of the opportunity costs of such decisions. Such costs are calculated in this study.
4. Specific external factors which may be analyzed with

the model are land acquisitions, changes in labor charges, and changes in interest rates. An objective of this study is to determine how each of these factors affects the optimal trading interval.

5. Another objective involves evaluation of used equipment purchases. Just as optimal trading points can be determined for new equipment, the optimal trading points and associated costs can be determined for used equipment. It is then possible to compare the relative merits of purchasing used or new equipment.
6. Institutional arrangements which affect optimal replacement intervals include investment credit and taxes. Investment credit is a direct saving. There is an indirect tax opportunity cost associated with an older machine as opposed to a newer machine since yearly depreciation can be deducted from taxable income. An objective of the study is to evaluate these institutional arrangements and determine their affect on optimal replacement intervals.

Emphasized in this study are farm tractor replacement procedures. Combine and automobile replacement are also analyzed using the model.

Previous Studies

Many studies have evaluated empirically the costs of owning and operating farm machinery. But few economists have made empirical studies of replacement procedures. Past replacement studies deal principally with theory.

Replacement models developed by industrial engineers deal primarily with situations where numerous similar machines are being operated simultaneously. Farm machinery replacement usually concerns the periodic replacement of one item, for example, a tractor.

Mayer indicates, for an industrial situation, the basic problems of implementing replacement theories.[1] He points out that it is difficult to develop a realistic model which requires relatively simple mathematics and yet presents an accurate picture of the costs involved. It is also difficult to develop accurate estimates of the company's future need for the machines in question. Developing a realistic replacement model is hampered, not by whether or not machinery will be needed, but by inability to anticipate accurately future costs and returns.

In his study Mayer concluded that the primary value of replacement theory is to acquaint industry management personnel with the factors which must be taken into consideration in an equipment replacement decision. He points out that replacement decisions will continue to be made by individuals without intensive economic analysis. This will also hold true for agriculture. But, hopefully, in the future, judgments now based primarily on limited experience will be supplemented with more vigorous economic analysis.

Burt developed a replacement model for a risk situation applicable to both farm and industrial equipment.[2] In his model, equipment may be replaced either because of some random failure or because the minimum

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[1] Raymond R. Mayer, "Problems in the Application of Replacement Theory," Management Science, VI (1959), pp. 303-307.

[2] Oscar R. Burt, "Optimal Replacement Under Risk," Journal of Farm Economics, XLVII (1965), pp. 324-346.

cost interval of ownership is reached. The risk in his model deals with the probability of a random machine failure occurring in any given year, not in the variability of repair costs which may occur for a given machine. To determine the optimal ownership interval, Burt maximizes the present value of the net revenue stream, or alternatively, finds the interval which offers the highest rate of return.

Shaw developed a model which he confined to machinery replacement.[3] After very carefully calculating the total amount of work done by a tractor, he attempts to develop an accurate representation of repairs and other costs. Incorporated in his model are the derivations of optimal repair and maintenance policies. After determining the optimal replacement interval of each part on the machine, he uses the results to find the optimal ownership interval for the entire machine. Shaw's study is completely *a priori*; i.e., the optimal point is determined before the machine is purchased. The model is not designed to assist farmers in making yearly trading decisions. The economically optimum replacement interval is determined by the intersection of the average and marginal cost curves of the machine; which implies the machine is to be replaced by a similar machine. Thus, no allowance is made for purchasing a larger tractor or for other changes which will occur in a realistic situation.

Faris developed a replacement model similar to Shaw's except that he chose to maximize average net revenues rather than to minimize

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[3]H. Russel Shaw, "A Model for Capital Costs," (unpub. manuscript of the California Agricultural Experiment Station.

average costs.[4] If, in Faris' model, cattle were kept in a feedlot past the point when maximum average net revenue over time were reached, the marginal additions to net revenue would be less than the average net revenue anticipated by selling the current lot and buying a new lot. The replacement rule developed is: "The present lot should be carried only to the point where marginal net revenue from it equals maximum average net revenue anticipated from the subsequent lot."[5]

A graphic illustration of the application of the replacement rule is given by Faris.[6] It is possible by analyzing the replacement rule with per-unit cost curves to obtain a more general graphic illustration. The replacement rule requires that when marginal net revenue of the current lot first drops below the maximum average net revenue of the next lot it is time to change lots. In Figure 1, MNR_1 represents marginal net revenue of the current lot and ANR_2 equals the average net revenue of the next lot. ANR_2 reaches a maximum at point "a". Thus, anytime MNR_1 drops below a horizontal line through point "a" it is time to change lots. This occurs in Figure 1 at point "b" which indicates that lot 1 should be sold and lot 2 purchased after "c" units of time.

In addition to analyzing situations where no discounting is required, Faris also analyzes a long term situation for a forestry enterprise. Because of uncertainty and time preference, the replacement rule is altered. "The optimum time to replace is when the marginal net revenue from the present enterprise is equal to the highest

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[4] S. Edwin Faris, "Analytical Techniques Used in Determining the Optimum Replacement Pattern," Journal of Farm Economics, XLII (1960), pp. 755-766.

[5] Ibid., pp. 757-758.

[6] Ibid., pp. 757.

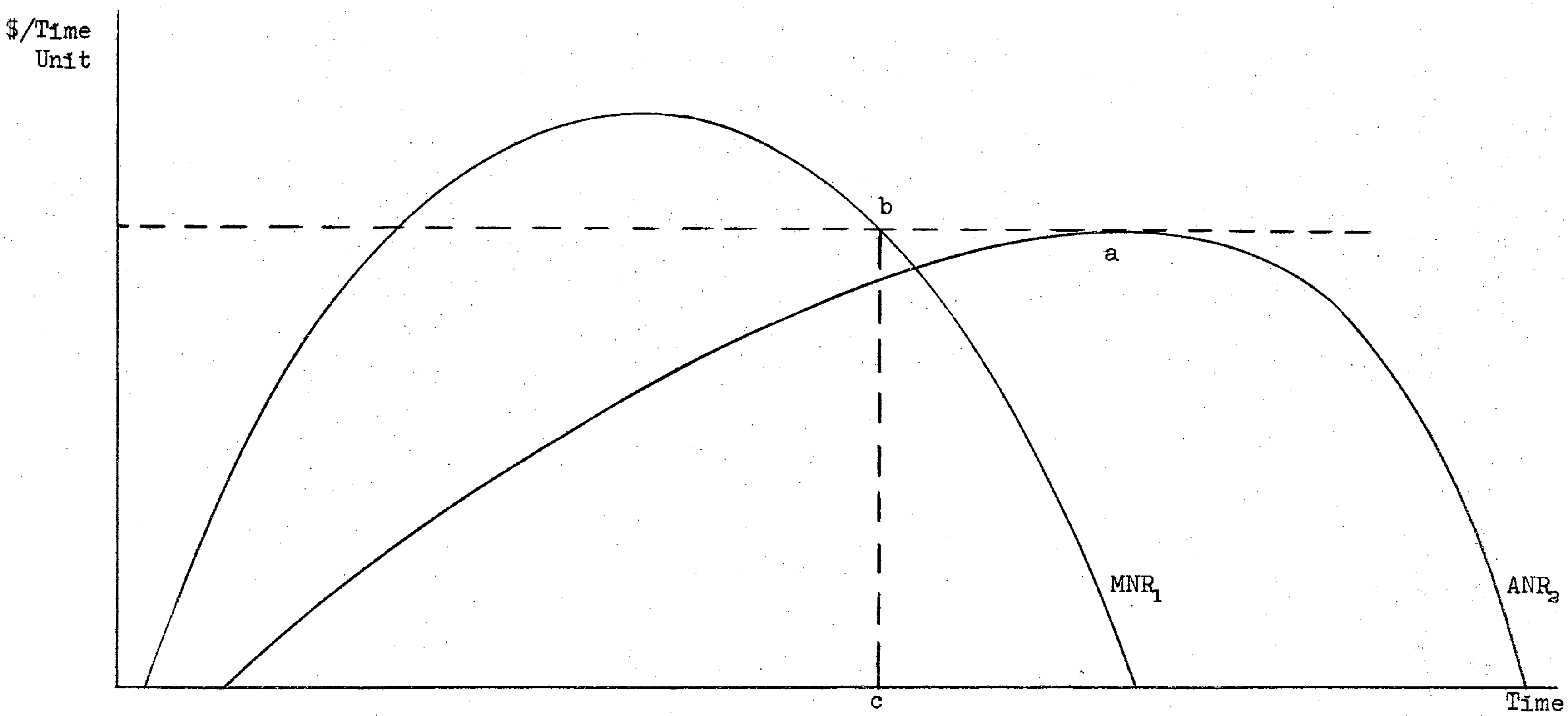

Figure 1. Marginal Net Revenue Curve for Lot One and Average Net Revenue Curve for Lot Two, Feeder Cattle Illustration

amortized present value of anticipated net revenues from the enterprise immediately following."[7]

Faris' model is unique in that it allows the manager to make yearly decisions whether to harvest the timber. This type of decision rule allows for re-evaluation of current and expected future conditions each year. Thus, it is not necessary to rely on some decision made in the past which may or may not be applicable at the present time or in the future.

Faris did not develop a replacement model which considered only costs. But he did develop some very useful concepts and replacement procedures easily applicable to machinery problems.

Shaw's study was a thorough study of a particular tractor and Burt's model would be very difficult to apply to dynamic farm situations. The studies listed above did little toward developing usable machinery replacement policies. This study is designed to develop general replacement policies using many of the concepts which Faris presented.

Outline of Following Chapters

The order of presentation for the remainder of this dissertation is as follows.

Chapter II describes in detail the theory, analytical procedures, and machinery cost components to be used throughout the study. The theoretical effects of land purchase and abnormal costs on the optimal replacement intervals are analyzed. Time preference and uncertainty require discounting of future costs and returns. The analytical

[7]Ibid., pp. 761-762.

procedures for discounting are discussed and the various fixed and variable cost components are delineated.

Chapter III describes the empirical cost equations to be used. Following the equations, the most elementary replacement situation, replacement with a similar machine, is discussed. The analysis then proceeds to more complex problems including replacement with different size machines and replacement if costs are not as expected.

Chapter IV contains additional empirical applications of replacement models developed in Chapter II. Covered are: opportunity costs of not trading at the optimal time, the effect of land acquisition on the optimal replacement interval, purchasing used tractors, effects of investment credit and taxes, and break-even labor charges.

Chapter V contains a simulation model for evaluating the effects of stochastic repair costs on optimal replacement intervals. Because the theoretical rule developed in Chapter II fails to operate effectively in a stochastic situation, alternative methods for implementing the theoretical criteria are proposed and evaluated using simulation.

Chapter VI illustrates the adaptability of the model with examples of automobile and combine replacement. Also developed is a procedure which delineates the linear subjective cost function required to make trading in any particular year optimal.

Chapter VII summarizes the results of the study, presents the conclusions reached, and indicates the need for additional study.

CHAPTER II

THEORY AND PROCEDURE

The first portion of this chapter deals with simple replacement theory. The topics developed are: 1) replacement with a similar machine, 2) replacement when costs are not as expected, 3) replacement with a different machine, and 4) the effect of an abnormal cost on replacement.

The second portion deals with time preference analysis. Machinery replacement studies require that future costs be considered in making decisions today. This will require that appropriate discounting and amortization procedures be used.

The final section of the chapter is concerned with machinery cost components. Many items, both fixed and variable, must be combined to realistically develop an accurate representation of machinery cost.

Replacement Theory

Replacement With a Similar Machine

The economic life of a machine is defined here as the period of time during which that machine will reach its minimum average yearly cost. Depending on the replacement being considered, the economic life may or may not be the optimum ownership interval. At the point of minimum average cost, marginal costs are equal to average costs. (See point "a" in Figure 2.) By considering the horizontal axis to be time in years, the marginal cost curve can be defined as the yearly costs of

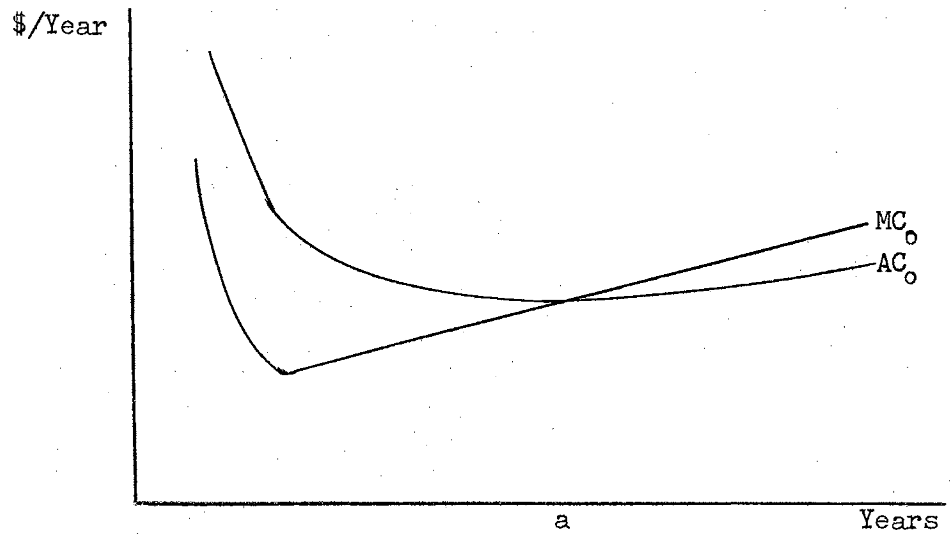

Figure 2. Theoretical Marginal and Average Cost Curves

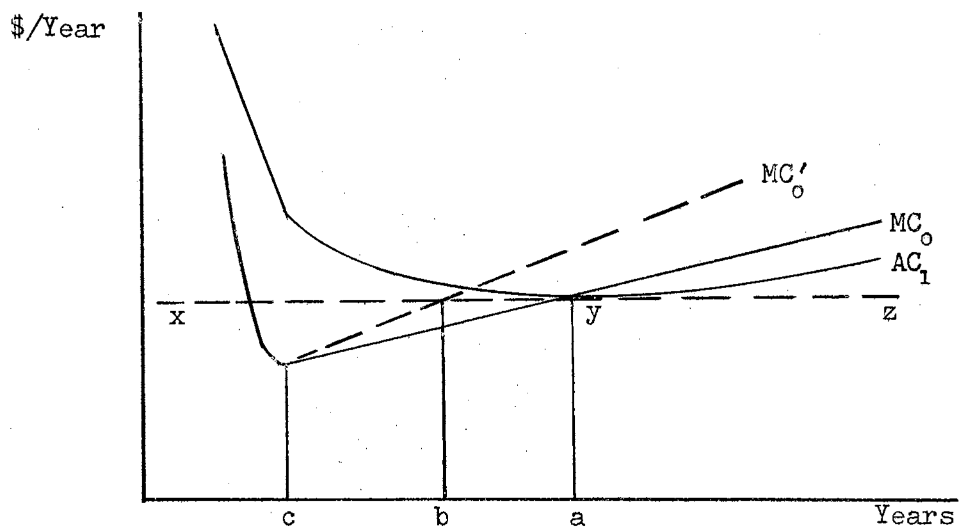

Figure 3. Replacement When Marginal Costs Not as Expected and Proposed Replacement is a Similar Machine

owning and operating a machine. Marginal cost as used here includes all fixed costs. Average cost for any year T is the accumulated total of yearly costs up to and through year T divided by T. The average cost curve, AC_0 in Figure 2, may be found by plotting average cost for each year T, allowing T to range from 1 to n.

The yearly costs vary throughout machine life, and the relative importance of the yearly costs at any point in time must be taken into account using a discounting procedure. A later section of this chapter outlines the procedures used to handle time preference. A timeless environment is assumed for the remaining theory portions of this chapter.

Machinery and vehicles have variable repair costs. The fluctuating outlays for repairs cause less variation in average cost than in marginal cost, which deviates by the entire change occurring in repair costs. Because of the number of years involved, average costs settle to a somewhat stable pattern, but marginal costs continue to fluctuate. The variability of marginal costs makes replacement analysis for each individual machine very crucial, but this same variability limits the ability of a deterministic replacement model to tell an owner exactly when to replace. Smooth marginal and average cost curves will be assumed in this section, leaving repair cost variability to be considered in a future chapter.

In Figure 2, point "a" is the economic life of the machine. If this machine is to be replaced by a machine with duplicate cost and technical capabilities, trading should occur every "a" years. Under the assumptions outlined above, optimum replacement intervals are easily found for machines with cost curves such as those illustrated in Figure 2.

Replacement When Costs Not as Expected

Actual circumstances differ considerably from those depicted in Figure 2. When a tractor is ten years old, a comparable new model is probably no longer available on the market. Also, costs for the older tractor and its proposed replacement will not be the same. Therefore, some kind of generalized replacement criterion is needed.

Replacement, according to Thuesen, should occur under these conditions:

> If the costs associated with an asset continue to rise during the balance of its life, it should be replaced when its costs for the next year will exceed the equivalent annual cost of the prospective replacement.[1]

In other words, replacement should occur if the marginal cost of the older machine is rising and it exceeds the minimum average cost of the proposed replacement.

This criteria may be applied to the situation in Figure 2. Let AC_0 be the average cost of the proposed replacement and MC_0 the marginal cost of the older machine. At "a", the optimum trading interval, marginal cost is rising and is equal to the minimum average cost of the proposed replacement. Thus, the generalized replacement criterion applies to the elementary situation illustrated in Figure 2.

The generalized criterion is next applied to a situation in which costs are not as expected. Assume for this situation, illustrated in Figure 3, that marginal costs of the older machine are higher than expected. The high marginal costs may be caused by an unanticipated chronic machine deficiency that results in large repair costs. Also,

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[1]H. G. Thuesen, Engineering Economy (New York, 1950), pp. 335, 336.

assume that a similar machine will be purchased to replace the existing machine. Point "c" represents the year in which marginal costs first rise above expected yearly costs. After year "c", the marginal cost curve is MC_0'.

The average cost curve of the proposed replacement, AC_1, will not be altered by the increased marginal costs of the presently owned machine. Therefore, the relevant cost curves for making the replacement decision are MC_0' and AC_1. The replacement criterion requires trading machines when the relevant marginal cost first exceeds the minimum average cost of the suggested replacement. The minimum point of AC_1, in Figure 3, occurs at point "y". Therefore, any time the relevant marginal cost curve, MC_0' in this case, crosses the line "xyz" from below and is expected to continue rising, it is time to trade machines. The illustrated example indicates that machines should be traded in year "b".

Replacement With a Different Machine

As mentioned above, a farmer's chances of purchasing an exact duplicate of his present machine are extremely slim. The farmer will usually buy a larger, more efficient machine. In Figure 4, AC_0 and MC_0 are the average and marginal costs of the currently owned machine. A more efficient machine having the average cost curve, AC_1, is the proposed replacement.

Determination of the optimum ownership interval is carried out exactly as above. A line tangent to the minimum point of AC_1 is constructed. When the rising portion of the marginal cost curve, MC_0, crosses the constructed line "xyz", it is time to replace the

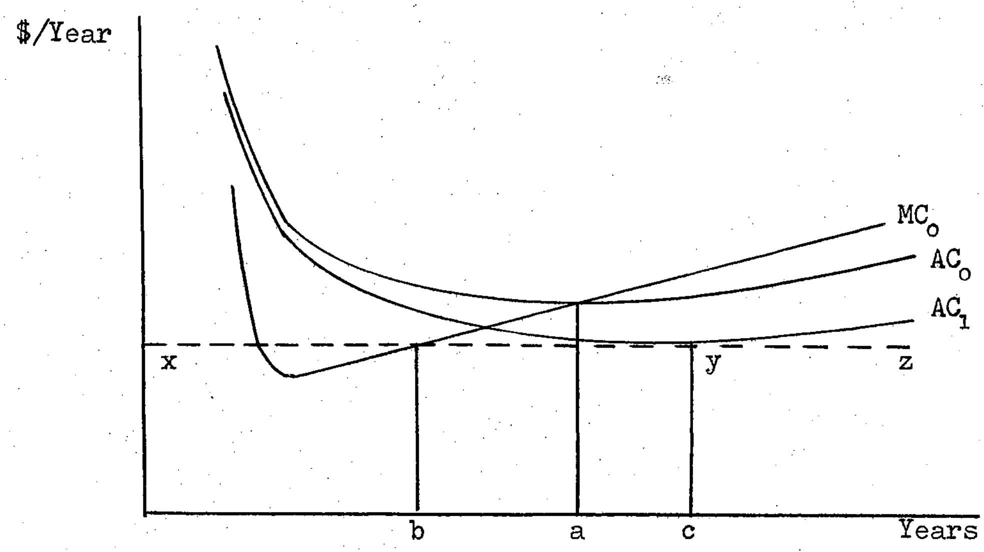

Figure 4. Replacement When Costs Are as Expected But Proposed Replacement is a Machine of a Different Size

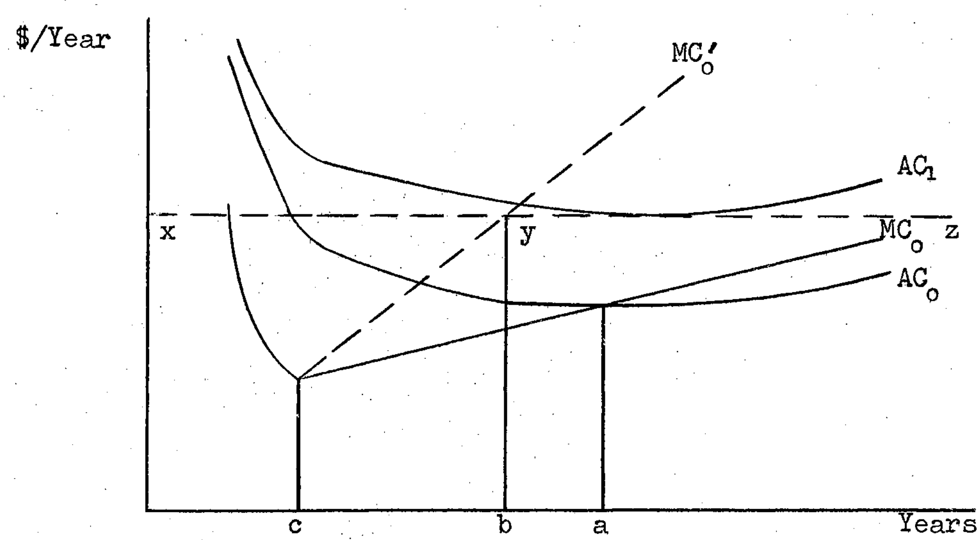

Figure 5. Replacement in a Land Purchase Situation - Higher Marginal Costs and the Proposed Replacement is a Machine of a Different Size

currently owned machine. This crossing occurs at "b"; thus "b" is the optimum year in which to trade.

The ownership interval offering minimum average cost, the economic life, is "a" years for the currently owned machine. Due to the availability of more efficient equipment, the machine with costs as shown in Figure 4 is traded before its economic life expires. By the same token, if the minimum average cost of the proposed replacement is greater than the minimum average cost of the existing machine, the currently owned machine would be kept longer than its economic life. Only if the proposed replacement and the existing machine have the same minimum average cost will the economic life and optimum ownership interval be the same.

Effect of Land Purchase on Replacement

The above two sections deal with changes in the optimum ownership interval caused by unexpected repair costs and by purchasing machines of different sizes. Using a land purchase situation as an example, this section will incorporate and generalize the circumstances depicted in the above two sections. The basic premise of this section is that when land is purchased, marginal or yearly costs of the currently owned machine will increase, and the proposed replacement will probably be a larger tractor.

In Figure 5, AC_o and MC_o are the average and marginal costs for the existing machine before land purchase. The purchase of additional land in year "c" will cause variable costs per year to increase. Therefore, after year "c" the increase in the relevant marginal cost is MC_o'.

Because of the larger farm size after year "c", optimum tractor size is likely to increase. Therefore, the anticipated replacement will have a higher average cost curve, AC_1, than the currently owned machine.

The relevant curves for determining when to replace in a land purchase situation are MC_0' and AC_1. The line "xyz" is constructed tangent to the minimum point at AC_1. The optimum trading point, where MC_0' crosses "xyz", is "b" years. Without knowing actual costs, it is impossible to determine whether "b" is to the right or left of "a". The location of "b" will depend on the relative changes in marginal and average costs. With a large land purchase, it may be optimum to trade during the land purchase year. Or, with a small land purchase the trading interval may be longer than "a" years, the optimum interval if no land had been purchased.

Effect of an Abnormal Cost on Replacement

In any one year, an extremely high cost may be sufficient to make trading economical. The annual cost necessary to justify trading is illustrated in Figure 6. MC_0 and AC_1 are the relevant curves to determine the optimum conventional replacement interval. Year "c" is the optimal replacement interval. Assume the distance from "d" to "c" is one year. For year "d" to be the optimum interval, the marginal cost in year "d" must equal the minimum average cost of the proposed replacement, which in turn is equal to marginal cost in year "c". Thus, the difference between marginal costs in years "c" and "d" is the additional cost required to make trading in year "d" feasible. In the case of the continuous cost curves, this cost is equivalent to the

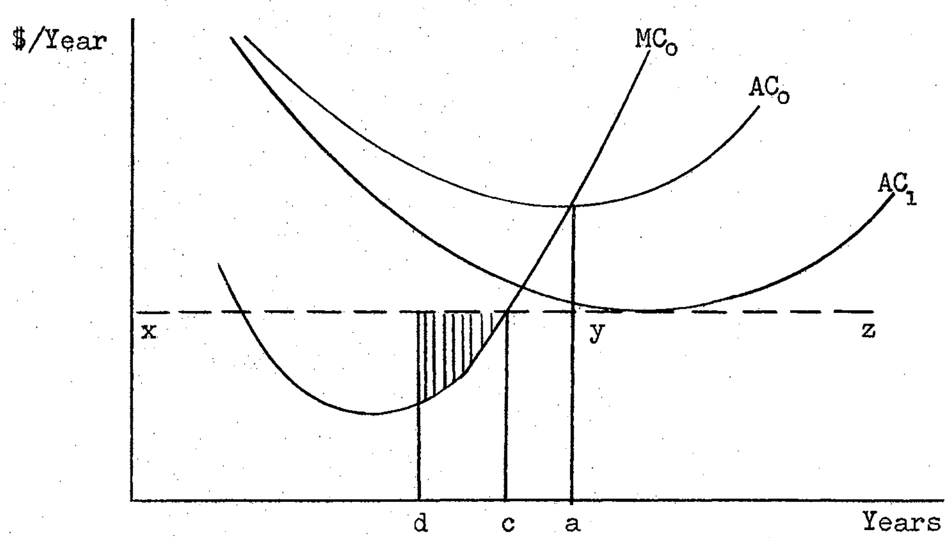

Figure 6. Costs of Not Trading at the Optimal Time

shaded area in Figure 6.

The shaded area remains the additional cost required to trade when the distance from "c" to "d" becomes two years. In the general continuous case, the single cost in any year "d", SC_d, necessary to justify trading is:

$$SC_d = AC_{1,min} \cdot (c - d) - \int_d^c MC_{0t}\, dt. \qquad (2\text{-}1)$$

One condition necessary for SC_d to be the cost in year "d" required to justify trading is that MC_0 be rising throughout the distance from "d" to "c". A final condition required for Equation (2-1) to be true is:

$$MC_0 \geq \min AC_0 \text{ for all values of } T > C.$$

Time Preference Analysis

Most farmers will agree that a dollar currently in the bank is worth more than a dollar to be received one year from today. Uncertainty about receiving the dollar one year from today is one reason the farmer may prefer the dollar now in the bank. One way of handling this uncertainty is to assume some discount rate which adequately reflects the possibility of not receiving the dollar one year from now. A second reason for preferring money now rather than later is the preference of the consumer to buy goods today as opposed to spending a dollar one year from now. The discount rate chosen should appropriately reflect how much the farmer prefers to consume the dollar now.

A final reason for time preference is opportunity cost. The consumer has the option of spending one dollar today or investing it to obtain one dollar plus interest at a later time. At six per cent simple interest, an invested dollar will be worth $1.06 one year from today. By the same token a cost of $1.06 one year from today is equivalent to a cost of one dollar today. One dollar used to pay today's costs can be invested at six per cent interest and used to pay a $1.06 cost next year. Thus, it is necessary to discount future costs to make them comparable to present costs. Opportunity costs are probably the most relevant reasons for discounting in investment situations, since the money is available and the decision is being made on what to do with it.

Selection of the discount rate is very crucial. If the farmer had extra cash lying around, the relevant interest rate is that rate at which he could invest his cash in his own business or some outside activity. On the other hand, if funds must be borrowed, the appropriate discount rate is the interest rate he must pay. If his creditors would allow him to borrow only a certain amount each year, then the appropriate discount rate is that rate which his next best alternative investment will yield, if that rate is above the lending rate.

Present Value Criteria

Bierman and Smidt suggest two criteria, present value and yield, for comparing alternative investment opportunities.[2] Yield is the percentage rate of return over costs and requires that revenues generated be considered. Therefore, in this cost analysis, the yield criteria is ignored and present value criteria are used.

Present values are discounted future values. The sums of present values for two alternative five-year cost streams are directly

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[2] Harold Bierman, Jr. and Semore Smidt, The Capital Budgeting Decision (New York, 1960), Chapter II.

comparable while the sums of the undiscounted streams are not; unless the two streams are equal. When using present values to compare alternatives, each cost stream must supply the same stream of services. The problem in this study is comparing alternative ownership intervals. Since seven and eight year old tractors do not supply the same services, present values per se do not give satisfactory results. Understanding the simple present value formula is necessary however to comprehend the extensions of present value theory used to compare tractors of different ages.

The present value formula given in Equation (2-2) may be used to compare cost streams. The resulting present value sum, PV_T, is the total present value of all costs in years 1 to T.

$$PV_T = \sum_{t=1}^{T} Z_t \; D_t \tag{2-2}$$

where $D_t = 1/(1+r)^t$,

r = discount rate,

t = any year between 1 and T,

T = the final year of costs included,

Z_t = actual dollar cost in any year t,

PV_t = present value of all costs between year 1 and T.

A simplified example of applying Equation (2-2) to the four year cost steam in Table I may be helpful. Assuming a discount rate of eight per cent, the discount factor, D_t, becomes $1/1.08^t$. The discounting factor appropriate for each year t, multiplied by the yearly cost, Z_t, gives the present value of Z_t, $Z_t D_t$. All yearly costs are now evaluated in discounted or present dollars.

TABLE I

SYNTHETIC OPERATING AND OWNERSHIP COSTS FOR A FOUR-YEAR INTERVAL

Year	Operating Costs (dollars)	3-year-old		4-year-old	
		Depreciation (dollars)	Total Costs (dollars)	Depreciation (dollars)	Total Costs (dollars)
1	240	400	640	300	540
2	400	400	800	300	700
3	708	400	1108	300	1008
4	1196	--	--	300	1496

Year (t)	1	2	3	4
Yearly Costs (Z_t) =	540	700	1008	1496
Present Value ($Z_t D_t$) =	500	600	800	1100

By summing the present value, $Z_t D_t$, t = 1, 4, the total present value of incurring the four yearly costs can be found. For the example, the total is $3,000. If $3,000 were placed in an eight per cent investment in year zero, the total undiscounted cost stream of $3,744 could be purchased. At the end of year one, the $3,000 would be worth an additional eight per cent or $3,240. In year one, $540 dollars in expenses were incurred leaving a balance of $2,700. At the end of year two, the $2,700 is worth $2,916. Costs in the second year are $700 leaving a balance of $2,216.37. The eight per cent interest makes $2,216.37 worth $2,393.28. Deducting the $1,008 expense in year three gives $1,385.28 which is exactly enough to cover the $1,496 in expenses at the end of year four. Thus, one could pay off the indicated stream of costs with an income of $3,000 at the start of the period or an income in each year equal to the indicated annual costs. The total present value of a cost stream is the number of dollars now it would take to purchase the entire stream of benefits given the indicated discount rate. It reduces future benefits or costs to present values that can be compared and, hence, can be used to select the best economic alternatives.

When comparing alternative cost streams, the intent of present value analysis is to discover the stream with minimum total present value. The alternative with lowest total present value is preferred since fewer of today's dollars would be required to purchase the entire stream. Also, in a perfect capital market, a cost stream with a lower present value can always be converted into the alternative cost stream with a higher present value with some residual savings remaining.

Discovering the optimal ownership intervals for farm equipment requires cost comparisons for tractors of different ages. For present value analysis to be applicable, alternatives must supply the same stream of service. Tractors used seven years and eight years under the same conditions do not provide the same services. Therefore, some extension must be made in present value theory.

It is not feasible to compare the tractors above by averaging total present values. Present value analysis makes costs in later years much less important than costs in current years. A total present-value for a seven year cost stream is composed of seven yearly present value costs each valued at a smaller percentage of its original value. If an additional yearly cost were added to the seven-year cost stream, the eighth year cost adds proportionately less to total present value than does the seventh year's cost.

Because of the decreased valuation of each additional yearly cost, a seven-year average present value is not comparable to an eight-year average present value. In fact, average present values may continue to decrease even though the marginal increase in yearly costs are quite large.

For average present value, $APV_T = PV_T/T$, to be a minimum, APV_{T+1} must be larger than APV_T. The required increase in yearly cost, Z_t, is computed in Equation (2-3). The yearly cost in year $T+1$ must be greater than or equal to average present value for year T compounded $T+1$ years.

$$APV_{T+1} \geq APV_T \qquad (2\text{-}3)$$

$$\frac{PV_T}{T+1} + \frac{Z_{T+1}\ D_{T+1}}{T+1} \geq \frac{PV_T}{T}$$

$$\frac{Z_{T+1}\, D_{T+1}}{T+1} \geq \frac{PV_T}{T} - \frac{PV_T}{T+1}$$

$$Z_{T+1} \geq \frac{(T+1)}{D_{T+1}} \left[\frac{PV_T}{T} - \frac{PV_T}{T+1}\right]$$

$$Z_{T+1} \geq \frac{(T+1)PV_T - TPV_T}{T \cdot D_{T+1}}$$

$$Z_{T+1} \geq \frac{PV_T \cdot D_{T+1}^{-1}}{T}$$

$$Z_{T+1} \geq APV \cdot D_{T+1}^{-1} .$$

If D_T is the discount factor, the compounding factor is D_T^{-1}. For ten years at eight per cent interest, the compounding factor is 2.16. For a minimum average present value in year nine, the costs in year ten must equal 2.16 times the average present value cost for year nine.

Another incorrect criteria often proposed in the comparison of 7/8 of an eight-year old tractor's present value with the present value cost of a seven-year old tractor. This method is incorrect because finding 7/8 of a present value sum implies each year is considered of equal importance. But a present value total evaluates each succeeding year with decreasing weight and if 1/8 of the total present value were compounded eight years it would be much larger than the actual cost in year eight.

Comparison Criteria for Cost Streams of Different Lengths

Two accurate methods of analyzing cost streams of various lengths are presented below. There are two concepts particularly helpful in replacement studies: the minimum cost ownership interval and an average cost usable in replacement analysis. The first discounting procedure considered will give the first of these concepts. The second procedure will provide both concepts.

It is possible to use present values directly in comparing cost streams of different lengths. But the method requires comparing of, for example, three four-year old tractors and four three-year old tractors. Each of these alternatives provides the same stream of services, twelve years, thus they are directly comparable. Table II gives the total present value of the two twelve year streams computed from data in Table I. The ownership interval preferred is the one offering the lowest present value cost for the twelve year period. The sum of present values for the service of three-year old machines is $6,310.55, while the present value total for service of four-year old tractors is $6,825.68. Thus, the preferred alternative is to purchase a new tractor and then trade every three years. For future reference, a ratio of the two present values is 6825.68/6310.55 = 1.08. The ratio may be interpreted to mean that in the long run keeping tractors four years is eight per cent more expensive than keeping tractors three years.

Another correct way of analyzing alternatives is the uniform annual or amortized average cost criterion. This criterion is also an extension of simple present value analysis. To understand this method

TABLE II

COMPUTATIONS REQUIRED FOR FINDING TOTAL PRESENT VALUE OF OWNING FOUR THREE-YEAR OLD OR THREE FOUR-YEAR OLD TRACTORS

Year	Discount Factor	3-year old series		4-year old series	
		Yearly Cost (dollars)	Present Value (dollars)	Yearly Cost (dollars)	Present Value (dollars)
1	.9259	640	592.58	540	500.00
2	.8573	800	685.84	700	600.00
3	.7938	1108	879.53	1008	800.15
4	.7350	640	470.40	1496	1099.56
5	.6806	800	544.48	540	367.52
6	.6302	1108	698.26	700	441.14
7	.4835	640	373.44	1008	588.17
8	.5403	800	432.24	1496	808.29
9	.5002	1108	554.22	540	270.11
10	.4632	640	296.45	700	324.24
11	.4289	800	343.12	1008	432.33
12	.3971	1108	439.99	1496	594.06
			6310.55		6825.68

it is first necessary to understand the concept of amortization.

Amortization can be explained with an example. Assume \$3,000 is placed in the bank by a high school senior to be used for his college education. The student will pay for each year of education at the end of the school year. The student wishes to divide the \$3,000 in such a manner that he will have an equal amount to spend each year. If the \$3,000 were incapable of earning interest, he could split the \$3,000 into four equal amounts of \$750. If the \$3,000 is capable of drawing interest, the student can withdraw more than \$750 each year. How much more depends on the interest rate.

The process of finding the student's equal yearly allowances that will exhaust his \$3,000 plus interest in a given period of time is called amortization. The formula for the amortization factor is:

$$AF = \frac{r(1+r)^t}{(1+r)^t - 1}.$$

When the amortization factor, AF, is multiplied by the total sum to be divided, the amortized average is determined. The amortization factor depends both on the interest rate, r, and the number of years, t, over which the sum is to be split.

For the example the interest rate is assumed to be eight per cent and the number of years involved, four. Inserting these values into the amortization factor formula gives a factor of .302. When .302 is taken times \$3,000 the product is \$906. The student will be able to spend a sum of \$906 each year.

This may be checked. After one year, the \$3,000 is worth \$3,240. The student then spends \$906 and has left \$2,334. After the second year \$2,334 is worth an additional eight per cent or \$2,520.72. The student spends another \$906 leaving \$1,614.72 in savings. The \$1,614.72 is worth \$1,743.90 after another year. The third \$906 is deducted leaving \$837.90 in savings. After the fourth year, the \$837.90 is worth \$906. Thus, the \$3,000 is capable of providing the college student \$906 during each of his four years of college.

It can be seen from the example that amortization is a procedure whereby a sum of money capable of earning interest can be divided into a series of uniform amounts over a given period of time. The uniform series may be called an average of the original sum corrected for time preference or the earning power of money.

The principle of amortization can now be applied in replacement studies. In the example, the sum of money invested and to be divided into four equal sums had a present value of \$3,000. Just as the \$3,000 return stream above can be converted to a uniform annual return series, a cost stream with a present value of \$3,000 can be converted to a uniform annual cost series. At eight per cent interest the amortized average cost for the four year series is \$906 as above.

The present value total cost of owning one three-year old tractor is \$2,157.95. The amortization factor for three years at eight per cent interest is .38803. Multiplying 2157.95 times .38803 gives \$837.35, the uniform annual cost of owning a tractor three years. The uniform annual cost of owning the tractor traded each four years is \$905.75; thus, the three-year replacement pattern is most economical.

The ratio of the two uniform annual costs, $905.75/837.35 = 1.08$, is the same as that found using present values for a twelve year ownership sequence. It is apparent that each of the two methods, total present value of a series of machines and uniform annual cost, gives the same result when comparing alternatives.

The uniform annual cost criterion gives the annual average cost corrected for time preference. Also, the amortized averages are relatively easy to compute. If the amortized average cost is computed for each year of tractor life, a time-preference corrected, average cost curve can be constructed. The replacement procedures discussed earlier in this chapter may be readily applied to this average cost curve.

Earlier, replacement theory was explained using simple average costs. Amortized averages alter the shape of the average cost curves only slightly. But, more important, the theory applied to the simple average cost curve is just as relevant for amortized average cost curves.

Another criteria, while not used in this analysis, can be employed to make replacement decisions. The series cost criterion requires the same information as the present value and uniform annual cost criteria. It is sometimes easier to use than the present value criterion. It does not provide annual cost information.

To use the series cost criterion, it is necessary to first estimate the present value of owning a given tractor for a certain number of years, PV_T. This is the same information that was required to compute the uniform annual cost. The present value cost, PV_T, for any year T is multiplied by a factor, SF_T, (see Equation (2-5)). The product found, TSC_T, in (2-6) is the total present value of all future costs for a series of machines each exactly alike and each used T years. Total series cost, TSC_T, may be defined as the sum of money a farmer must set aside in a machinery fund today if the machinery fund is to provide a new, duplicate machine every T years and pay all normally expected costs for an infinite period of time. By examining series costs for machines having different life expectations,

$$\text{Series Factor } (SF_T) = \frac{(1+r)^T}{(1+r)^T - 1} \qquad (2\text{-}5)$$

$$\text{Series Cost } (TSC_T) = PV_T \cdot SF_T$$

the alternative can be chosen which promises the lowest expected cost.

Machinery Cost Components

To this point procedures to be used in studying replacement have been discussed. The remainder of this chapter will cover the cost components to be included in the previously discussed models. Machinery costs are usually divided into two portions, fixed and variable. Fixed costs are those costs which occur whether or not usage of equipment is taking place. Variable costs vary with the amount of machine usage per unit of time. Normally, fixed costs are associated with ownership while variable costs include operating expenses.

A subjectively evaluated opportunity cost also will be discussed. As machines age they become less dependable and break down more often than in their earlier life. With each breakdown is associated an opportunity cost. Various assumptions will be made regarding the size and composition of the subjectively evaluated dependability and prestige costs.

Fixed Costs

Components of fixed costs are depreciation, taxes, housing, insurance, and interest on investment. The fixed cost most difficult to evaluate is depreciation. Depreciation will be discussed first, since, besides being a cost itself, it is used in determining the other fixed costs which depend on remaining machine value.

Depreciation. Tractor services typically do not significantly decline the first few years of tractor life. Thus, there is little depreciation, viewed as the decline in ability of the tractor input to contribute to output. Alternatively, depreciation may be defined as the investment required to maintain machine services at their initial level. This investment includes the cost of repairs and preventive maintenance practices so employed.

A third concept of depreciation is measured by the change in market value of the machine. For least-cost ownership, it is the change in market value which is relevant, thus, this concept of replacement is used in the replacement models in this study. Depreciation is determined by subtracting current machine value from its value the preceding year. Of course, estimates of the yearly machine price must be available.

Typically, market depreciation is large the first year and then declines over time. If depreciation is plotted with time on the horizontal axis, the characteristic shape of the market depreciation curve is downward and to the right.

Taxes, Housing, Insurance, and Interest on Investment. Costs associated with investment, taxes, housing, and insurance all depend in varying degrees upon depreciation. All are treated as percentages of the remaining value of the machine in this study. Interest on investment depends directly on the remaining farm value of the machine. Interest on investment is included since it is a measure of the opportunity cost of having capital tied up in machinery. If depreciation is small, the portion of purchase price not deducted as a cost is large, thereby causing interest on investment to remain large.

Tax rates on machinery are often a stable percentage of remaining farm value. Thus, a curve showing taxes over time would also slope downward and to the right, providing the machine never depreciated from one year to the next. Insurance costs vary but they may also be figured as a constant percentage of remaining farm value.

Machinery which is not housed will have higher depreciation and repair costs. Therefore, a cost for housing should be included whether or not the machine is housed.[2] Costs for housing will be considered a constant percentage of remaining farm value.

Fixed costs can be divided into two portions, actual and accounting. Actual fixed costs are those which must be paid during the year. Included are taxes and insurance. Interest on investment is also a real fixed cost if the capital for purchasing the machine was borrowed. If the capital was not borrowed, the interest on investment is an opportunity cost. Accounting fixed costs are those which occur in one lump and must be apportioned over time. Included in accounting fixed cost are housing, depreciation, and interest on investment if it was unnecessary to borrow capital to buy the machine.

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[2] Wendell Bowers, Costs of Owning and Operating Farm Machinery, University of Illinois College of Agriculture, Cooperative Extension work, Bulletin AENG-867 (Urbana, 1966), p. 3.

Variable Costs

As opposed to fixed costs, variable costs are a function of the amount the machines are operated. If a machine is not used, variable costs will not be incurred. It was mentioned earlier that a portion of depreciation may be a variable cost attributed to use. Difficulties arise, however, when one attempts to separate depreciation into its components. One way of delineating the two portions would be to find the market value of a machine which has never been used. The portion attributable to variable cost would probably be small. Martin found such things as accumulated hours, repairs, and service time do not significantly affect the trade-in value.[3]

Repairs. The largest and most unpredictable of the variable costs is repairs. Repair costs are the primary stumbling blocks in replacement interval determination. Repairs cannot accurately be predicted for individual tractors, but they do have some distinctive group characteristics. An old tractor used the same amount as a new tractor will usually have a larger repair bill. Repair costs vary directly with hours of use and size of tractor. But to a lesser degree, skill of operator, climate and type of tractor also affect repair costs. Based on these general characteristics, it may be possible to compute average repair cost as a function of machine age, use per year, and machine size.

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[3]William E. Martin, Farm Machinery Costs In The Western States, University of Arizona, Agricultural Experiment Station, College of Agriculture, Technical Bulletin No. 154 (Tucson, 1964), p. 58.

Functions depicting yearly repair costs are usually assumed to slope upward and to the right, increasing at a decreasing rate. Ultimately, a constant level of repairs would be reached which would keep the machine in a steady level of serviceability. This amount of repairs could be total replacement of all parts in the extreme case. If the constant yearly repair bill were twenty per cent of new cost, then conceptually, one-fifth of the machine would be replaced each year to maintain the machine's state of repair in perpetuity.

Fuel, Oil, Grease, and Labor. Fuel, oil, grease, and labor depend mostly on machine size and yearly hours of use. By assuming hours of use per year to be constant, these costs will remain the same each year. Machines, as they become older may require more gas, oil, and grease per hour due to machine wear, but the marginal change in these costs is so small that it is usually ignored. Increases in labor costs per year could also be anticipated due to declining machine efficiency. Because of the assumption that the machine will be maintained in a constant state of repair, changes in machine efficiency will be nil. Other factors affecting fuel and lubricant costs are machine load, speed, and starts and stops. The importance of these factors depend on the machine. A tractor will usually have varying loads but operate at full throttle and have comparatively few starts and stops. Automobiles, on the other hand, will usually travel at varying speeds and make many starts and stops.

Subjectively Evaluated Costs

All actual out-of-pocket machine charges are included in fixed and variable costs. One additional very important cost consideration is machine dependability. Old machines are not as dependable as new machines and, therefore, require a considerable amount of repair time. If machines are idle, being repaired during crucial use periods, an opportunity cost occurs. The opportunity cost associated with down-time is considered to be an arbitrary amount in this study, since measurement of income lost due to machine breakdown would be very difficult.

There are two distinct characteristics of a dependability function. First, if charges per year are determined by machine age, the function slopes upward and to the right. Second, since the cost is somewhat subjective, each farmer may have a unique dependability function suited to his particular circumstances.

Several factors must be considered when selecting the dependability cost function. Machine breakdown is much more crucial for a farmer with one tractor rather than two. Also, for some crops, timeliness is very important. The loss of a tractor for several days during haying or planting could be costly.

It has been argued that dependability charges should not be considered in economic studies since they are not out-of-pocket costs. The costs are real, however, as opportunity costs representing lost income. The decrease in hay returns due to tractor failure is an example of a large opportunity cost. In some cases, the opportunity costs may be small. Moisture lost to weeds because of a one day delay in working the wheat land may reduce wheat yields, but not significantly.

There is also a subjectively evaluated cost associated with prestige. Conspicuous consumption is not usually considered in optimizing formulae, but it may be rational depending on the utility gained from prestige. Once the individual has decided that the new car is worth the extra costs, the role of the economic replacement model is altered. The economist must now use the model to tell the new car owner how much he has paid for his luxury. Conspicuous consumption is not confined to car owners. Many farmers are willing to incur some extra costs to own machinery they can be proud of.

There are various procedures for handling dependability and prestige costs. A dependability cost function could be chosen and optimum replacement intervals determined based on it. Alternatively, dependability costs needed to justify a cost minimizing trade each year of machine life could be found. The individual could then observe how much he is paying for dependability and prestige.

Summary

Three primary areas relevant to analysis in this thesis have been presented. Initially, optimum replacement strategies were considered for several situations. The basic replacement rule revolved around the marginal cost curve of the currently owned machine and the minimum average cost of the proposed replacement. Trading machines was dictated when the marginal cost of the current machine first exceeded the minimum average cost of the proposed replacement.

Since the models as developed are deterministic, they are applicable only to replacement decisions occurring in a short time period. The models as developed are applicable to replacement decisions occurring in a short time period since they made no allowance for time preference. To allow time preference in the models, it was necessary to discuss discounting and amortization procedures which could be incorporated into the models. The three discount criteria discussed give the same results when making comparisons. But, the amortization procedure allows the computation of average costs which are useful for planning purposes and making yearly decisions. This will become apparent in the simulation model presented later.

Finally, the machinery cost components were discussed. In addition to the usual fixed and variable costs considered, an opportunity cost due to decreased machine dependability and prestige was added. As machines age they become less dependable and breakdown more often, resulting in lost production. The value of production lost is an opportunity cost that must be included in determining optimum machinery replacement.

In the following chapters, the theory presented in this chapter is empirically applied.

CHAPTER III

EMPIRICAL ANALYSIS

The first section of this chapter contains empirical estimates of cost equations for tractors that are later used to determine optimal replacement patterns. Particular emphasis is placed on the sources and characteristics of the cost measures. The latter part of the chapter contains empirical estimates of optimum replacement intervals for tractors based on the cost equations in this chapter and the replacement criteria presented in the previous chapter.

Empirical Cost Equations

The prediction of tractor operation and ownership costs depend on many factors, including tractor size, use per year, and tractor age. These latter factors are used as independent variables in equations developed to predict tractor operation and ownership cost. Previously, costs were separated into fixed and variable with an added subjective dependability cost. However, to express the empirical cost equation, it is also useful to classify costs by their movement over time. By assuming a constant hourly machine usage per year, costs may be separated into decreasing, constant, and increasing components. Fixed costs become decreasing costs. Variable costs which include labor, fuel, and lubricants, are considered constant costs per year while repairs along with subjective dependability are increasing costs.

Decreasing Costs

Ownership costs depend primarily on machine age and size. The effects of use on changes in machine market value are difficult to measure, but are believed to be small. All decreasing costs are assumed to be determined by age and machine size. Machine market values decline at a decreasing rate over time, thus, depreciation is less each year. Since interest, taxes, housing, and insurance are assumed to be a constant percentage of market value, they are less each year. While age determines the appropriate yearly percentages, machine size is the base figure upon which the percentages are used to determine yearly costs.

Empirically, two decreasing cost functions may be delineated: depreciation in one function, and interest, taxes, housing, and insurance in the other. Prediction of decreasing costs is based on three elements: the tractor's list price, X_1; the interest rate, r; and machine age, t. Used for expressing machine market value in any year is the equation:[1]

$$W_{1t} = .675\ X_1\ .933^t \qquad (3\text{-}1)$$

where W_{1t} = tractor market value after t years.

The data used in finding Equation (3-1) was taken from the Official Tractor and Farm Equipment Guide.[2] The guide gives market prices for

[1] Wendell Bowers, University of Illinois College of Agriculture, Cooperative Extension Work, Costs of Owning and Operating Farm Machinery, Bulletin AENG-867 (Urbana, 1966), p. 2.

[2] National Farm and Power Equipment Dealers Association, Official Tractor and Farm Equipment Guide (St. Louis: NRFEA Publication, Inc., 1967).

tractors and farm equipment of all ages. Equation (3-1) depicts a first year depreciation of thirty-seven per cent of machine list price. Data from the Official Tractor and Farm Equipment Guide corroborate the large first year depreciation.

Given yearly market values from Equation (3-1), depreciation may be found. Depreciation is the decrease in market value from one year to the next.

$$Y_{1t} = W_{1(t-1)} - W_{1t} \tag{3-2}$$

where Y_{1t} = depreciation during the year t.

The second decreasing cost function includes interest on investment, taxes, housing, and insurance. Empirically, this function is:

$$Y_{2t} = (r + .045)W_{1t} \tag{3-3}$$

where r = interest rate,

and Y_{2t} = interest, housing, taxes, and insurance for year t.

These costs are all percentages of the remaining tractor value. For computational purposes, the percentages are summed, and, in this case, except for interest, the sum is four and one-half per cent. Taxes in Oklahoma are approximately equal to two per cent of machine value each year.[3] Housing charges should be made whether or not the machine is housed because depreciation and maintenance will be higher if machines are not housed. The charge made for housing is approximately two

[3] Personal Property Schedule, Oklahoma 1964, prepared by Oklahoma Tax Commission (Oklahoma, 1964).

per cent of current machine value.[4] Insurance costs for any year are assumed to be one-half of one per cent of the machine's remaining value. The total of the above three cost components, the coefficient in Equation (3-3), is four and one-half per cent.

Constant Costs

Three yearly costs remain constant if the machine is used an equal number of hours each year. Constant costs are for labor, fuel, and lubricants. Primary information required for computation of constant costs are: yearly use, X_2; labor charge, X_3; and tractor cost, X_1. Tractor age is not necessary for determining yearly labor and operating costs. An exception would arise if as the machine aged it would no longer be able to perform a given task with a given input of labor, fuel, and lubricant. If a deteriorating machine were used on a farm to perform a given task, then more labor, fuel, and lubricants would be required each year. Higher requirements would mean higher costs, and the constant costs would become increasing costs.

For computational purposes, the three constant cost components could be combined. However, to allow cost comparisons, they are kept separate. Labor cost per year is:

$$Y_{3t} = X_2 X_3 \qquad (3\text{-}4)$$

where Y_{3t} = labor cost in year t.

Labor costs equal machine use, in hours per year, multiplied by the hourly labor charge. The labor charge will be specified each time the

[4]Bowers, p. 3.

equation is used.

The second constant cost equation includes both fuel and lubricants. Adequate estimates on tractor fuel and lubricant consumption have been made.[5] Fuel and lubricant costs are:

$$Y_{4t} = .000158\, X_1 X_2 \qquad (3\text{-}5)$$

where Y_{4t} = fuel and lubricant cost in year t.

Fuel and lubricant costs are the product of hourly usage per year, tractor list price, and a constant. The constant incorporates fuel cost and a fuel consumption multiplier. The multiplier for gas tractors, the one used in Equation (3-5), is .000158.[6] Twenty cents per gallon was the fuel price used. The basic fuel multiplier which was taken times the fuel price is .00079. Multipliers for diesel and L.P.G. tractors are .00051 and .00087, respectively. The above multipliers are fifteen per cent higher than actual fuel consumption multipliers to include lubricant requirements.

Increasing Costs

Two costs vary directly with machine age, repairs, and subjective charges. Increasing costs are considered to be functions of: (1) tractor age, t; (2) yearly use, X_2; (3) list price, X_1; and (4) marginal increase in yearly dependability cost, X_4. In a latter portion of this study, replacement will be analyzed using a repair cost distribution.

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[5]Ibid., p. 3.

[6]The multiplier is an index of fuel and lubricant consumption per hour. The values given by Bowers, Ibid., p. 4, are divided by 1000 to obtain values per dollar of list price as opposed to $1000 of list price.

For current analysis expected repair costs will be used. The estimated cumulative repair cost function is:

$$W_{2t} = .00000913\ X_1\ (t \cdot X_2)^{1.5} \qquad (3\text{-}6)$$

where W_{2t} = total repair cost from year 1 to t.[7]

Appendix I contains the assumptions and conditions used to construct Equation (3-6). Repair costs for any year may be found by substracting cumulative repair costs in year t-1 from those in year t as in Equation (3-7). Equation (3-6) is specified such that the yearly repair costs, given in Equation (3-7), will increase throughout the entire life span.

$$Y_{5t} = W_{2t} - W_{2(t-1)} \qquad (3\text{-}7)$$

where Y_{5t} = repair costs in year t.

The second increasing cost takes into account the subjective costs of decreased dependability and prestige. Since the method of calculating dependability and prestige costs is arbitrary, many alternative procedures could be developed. It might be argued that a machine's dependability varies directly with repairs. If this is the case, then yearly dependability costs may be computed as some percentage of yearly repairs.

Alternatively, it may be argued that as machines get older more decapacitating breakdowns occur. Also, parts may have to be ordered

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[7]See Appendix I for the derivation of Equation (3-6). Equation (3-6) is an altered form of an equation constructed in W. E. Larsen and W. Bowers, "Engineering Analysis of Machinery Costs," Presented at 1965 meeting American Society of Agricultural Engineers (June, 1965), Appendix p. 2.

and more time required for their replacement. Yearly dependability costs, under these circumstances, might be assumed to increase by a constant amount each year. In this study, dependability costs are assumed to increase linearly with no charge the first year. The most often used dependability cost increment is $25.00. However, other alternative assumptions regarding X_4 are included later. The equation used is:

$$Y_{6t} = (t-1)\, X_4 \qquad t \geq 1 \tag{3-8}$$

where Y_{6t} = dependability or prestige cost.

The yearly cost increment, X_4, can be viewed as arbitrarily determined by the farmer or other user of the model. Alternatively, several different values could be assumed, allowing the user to pick the yearly cost increment relevant to his situation. The cumulative cost from Equation (3-8) increases at an increasing rate, indicating that the importance of dependability charges increase considerably over time.

Cost Function Summary

The six cost functions used to predict yearly tractor costs are presented above. The six functions depend on six parameters: X_1 through X_4, r, and t.

X_1 = tractor list price

X_2 = yearly use in hours

X_3 = labor charge per hour

X_4 = yearly increment in dependability cost

r = interest rate

t = age of tractor.

Based on these parameters, two prerequisite cumulative values can be found:

Value of tractor in year t: $W_{1t} = .675\ X_1\ (.933)^t$

Cumulative repairs to year t: $W_{2t} = .00000913\ X_1\ (tX_2)^{1.5}$

Given the parameters and the two intermediate values, the relevant costs for any year t are:

Depreciation: $Y_{1t} = W_{1(t-1)} - W_{1t}$

Interest, taxes, housing, and insurance: $Y_{2t} = (r + .045)\ W_{1t}$

Labor: $Y_{3t} = X_2 X_3$

Fuel and lubricants: $Y_{4t} = .000158\ X_1 X_2$

Repair cost: $Y_{5t} = W_{2t} - W_{2(t-1)}$

Dependability and/or prestige cost: $Y_{6t} = (t - 1)\ X_4$.

<u>Determining Average and Marginal Costs</u>

The total cost in year t is:

$$Z_{1t} = \sum_{j=1}^{6} Y_{jt} \tag{3-9}$$

where Z_{1t} = total cost in year t,

and j = index of the 6 costs listed above.

Actual expenses incurred in year t are Z_{1t} minus Y_{6t}, the subjectively charged dependability cost.

The cumulative total cost for a life of T years is:

$$Z_{1T} = \sum_{t=1}^{T} \sum_{j=1}^{6} Y_{jt} . \qquad (3\text{-}10)$$

The cumulative total cost is the basis for the simple average cost which is:

$$A_{1t} = Z_{1t}/T. \qquad (3\text{-}11)$$

As discussed previously, a simple average has relatively little use since time affects the value of money.

Amortized averages, discussed in the previous chapter, are used to take time into account. To compute average costs in this manner it is first necessary to find the discounted present value of all costs as illustrated in Equation (2-2). Since Equation (3-9) is total cost in any year, the present value of costs occurring in year t is:

$$Z_{2t} = \sum_{j=1}^{6} Y_{jt} / (1+r)^t . \qquad (3\text{-}12)$$

The amortized average cost computation involves finding the total present value of all costs from year 1 through T. The total present value of all costs is:

$$Z_{2T} = \sum_{t=1}^{T} \frac{Z_{1t}}{(1+r)^t} = \sum_{t=1}^{T} \frac{\sum_{j=1}^{6} Y_{jt}}{(1+r)^t} . \qquad (3\text{-}13)$$

Referring to Equations (2-5) and (2-6), amortized average cost may be constructed as given in Equation (3-14).

$$A_{aT} = Z_{aT} \frac{r(1+r)^T}{(1+r)^T - 1}. \quad (3\text{-}14)$$

A_{aT} is an average amortized cost for any length of T years. By determining A_{aT}, T = 1 to N, an amortized average cost curve for N years may be traced. The minimum point of this average cost curve is the minimum cost interval of ownership. At the minimum average cost point, average and marginal costs are equal. Marginal costs may be computed for N years by plotting the yearly costs, Z_{1t}, allowing t to range between 1 and N.

Empirical Results

It is now possible to integrate the six equations of this chapter into the replacement models developed in the previous chapter. There are several applications of these models given in this and subsequent chapters.

Computer techniques are useful to empirically implement the theory presented in the previous chapters. The computer quickly estimates the marginal and amortized average costs. Given the six previously discussed parameters for the present machine and the proposed replacement, optimum replacement intervals can be determined.

Replacement With a Similar Machine

Managers replacing an existing machine with a similar machine need only an average cost curve to determine the optimum ownership intervals. Upon reaching the minimum point on the average cost curve, it is time to trade machines. Similar machine replacement is illustrated in Figure 2.

Cost curves associated with a $6,100, sixty horsepower tractor are shown in Figures 7 and 8. Tabulated values used in these figures are given in Table III. Also incorporated into the figures and table is a dependability cost function which has a cost increment of twenty-five dollars per year. The curve depicting amortized average cost flattens out quickly and then remains flat for a considerable period of time. In the case of the sixty horsepower tractor, Figure 8, the amortized average cost becomes relatively flat in ten years and stays flat for an indefinite time, well over thirty years. The minimum average cost occurs in year seventeen. It is apparent, however, that a trading interval of over 17 years would be about equally as profitable.

The marginal cost curve is relevant only when it is rising. Marginal and average costs are by definition the same for year one. In year two, marginal or yearly costs are at a minimum for most tractors. Thus, beginning in year two, marginal costs for the sixty horsepower tractor are relevant for use in the replacement models.

In Figure 7 costs presented in Table III are combined into four categories. These are decreasing costs, constant costs, and increasing costs divided into two portions, dependability charges and repair costs. Figure 7 is useful in illustrating the relationship of the costs. For instance, not until sometime after thirty years are repairs increasing more than fixed costs are decreasing. Therefore, if no dependability costs are charged, the optimum replacement interval is somewhat longer than thirty years. Constant yearly costs for fuel, lubricants, and labor are by far the largest cost component in yearly charges.

TABLE III

COST COMPONENTS, ANNUAL AND AMORTIZED AVERAGE COSTS FOR A $6100 TRACTOR. USE PER YEAR: 600 HOURS; LABOR: $1.50 PER HOUR; DEPENDABILITY INCREMENT: $25.00 PER YEAR; INTEREST RATE: 8 PER CENT

Year	Amortized Average	Total Annual Cost	Repair Cost	Depreciation Charge	Dependability Charge	Taxes, Housing Insurance	Gas and Lubricants	Interest	Labor
1	4298.71	4298.71	81.85	2258.37	0.00	172.87	578.28	307.33	900.00
2	3365.85	2358.36	149.66	257.39	25.00	161.29	578.28	286.74	900.00
3	3062.25	2380.24	193.80	240.14	50.00	150.48	578.28	267.53	900.00
4	2914.58	2396.84	229.50	224.05	75.00	140.40	578.28	249.60	900.00
5	2828.83	2411.91	260.32	209.04	100.00	130.99	578.28	232.88	900.00
6	2773.87	2425.65	287.84	195.04	125.00	122.22	578.28	217.28	900.00
7	2736.45	2439.94	312.95	181.97	150.00	114.03	578.28	202.72	900.00
8	2709.96	2454.76	336.18	169.78	175.00	106.39	578.28	189.14	900.00
9	2690.77	2470.31	357.90	158.40	200.00	99.26	578.28	176.46	900.00
10	2676.68	2486.70	378.38	147.79	225.00	92.61	578.28	164.64	900.00
11	2666.31	2503.99	397.81	137.89	250.00	86.41	578.28	153.61	900.00
12	2658.72	2522.19	416.33	128.65	275.00	80.62	578.28	143.32	900.00
13	2653.25	2541.30	434.06	120.03	300.00	75.22	578.28	133.72	900.00
14	2649.46	2561.29	451.09	111.99	325.00	70.18	578.28	124.76	900.00
15	2646.98	2582.14	467.50	104.48	350.00	65.47	578.28	116.40	900.00
16	2645.55	2603.81	483.35	97.48	375.00	61.09	578.28	108.60	900.00
17	2644.98	2626.26	498.71	90.95	400.00	56.99	578.28	101.32	900.00
18	2645.10	2649.45	513.60	84.86	425.00	53.18	578.28	94.54	900.00
19	2645.78	2673.34	528.07	79.17	450.00	49.61	578.28	88.20	900.00
20	2646.92	2697.88	542.15	73.87	475.00	46.29	578.28	82.29	900.00
21	2648.43	2723.05	555.89	68.92	500.00	43.19	578.28	76.78	900.00
22	2650.24	2748.79	569.28	64.30	525.00	40.29	578.28	71.63	900.00
23	2652.29	2775.07	582.37	59.99	550.00	37.59	578.28	66.83	900.00

TABLE III (Continued)

Year	Amortized Average	Total Annual Cost	Repair Cost	Depreciation Charge	Dependability Charge	Taxes, Housing, Insurance	Gas and Lubricants	Interest	Labor
24	2654.53	2801.86	595.17	55.97	575.00	35.08	578.28	62.36	900.00
25	2656.92	2829.12	607.71	52.22	600.00	32.73	578.28	58.18	900.00
26	2659.42	2856.80	619.99	48.72	625.00	30.53	578.28	54.28	900.00
27	2662.00	2884.90	632.03	45.46	650.00	28.49	578.28	50.64	900.00
28	2664.64	2913.37	643.84	42.41	675.00	26.58	578.28	47.25	900.00
29	2667.31	2942.18	655.44	39.57	700.00	24.80	578.28	44.09	900.00
30	2669.99	2971.31	666.84	36.92	725.00	23.14	578.28	41.13	900.00

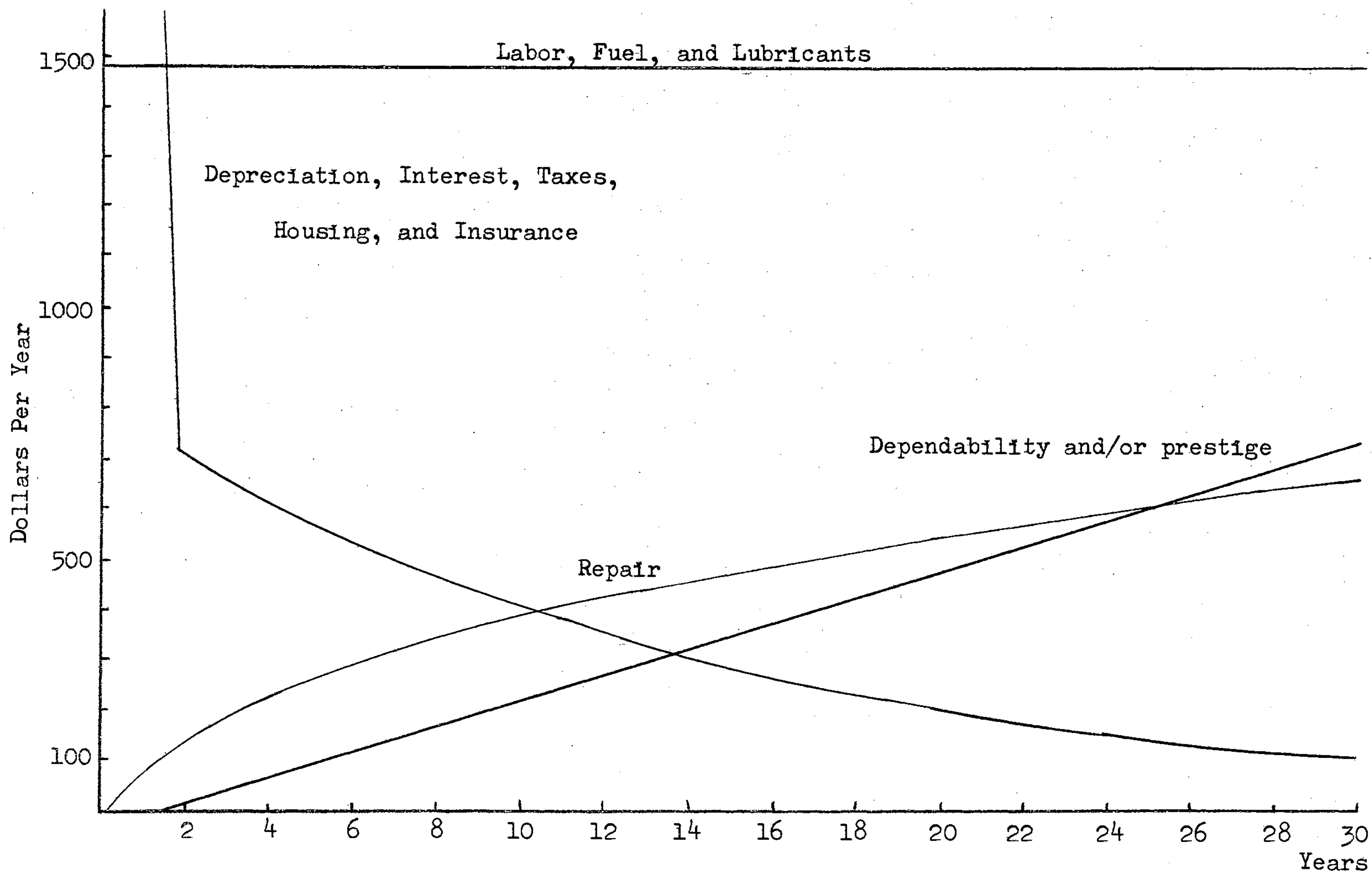

Figure 7. Annual Cost Components for a $6100 Tractor:
Tabular Data Given in Table III

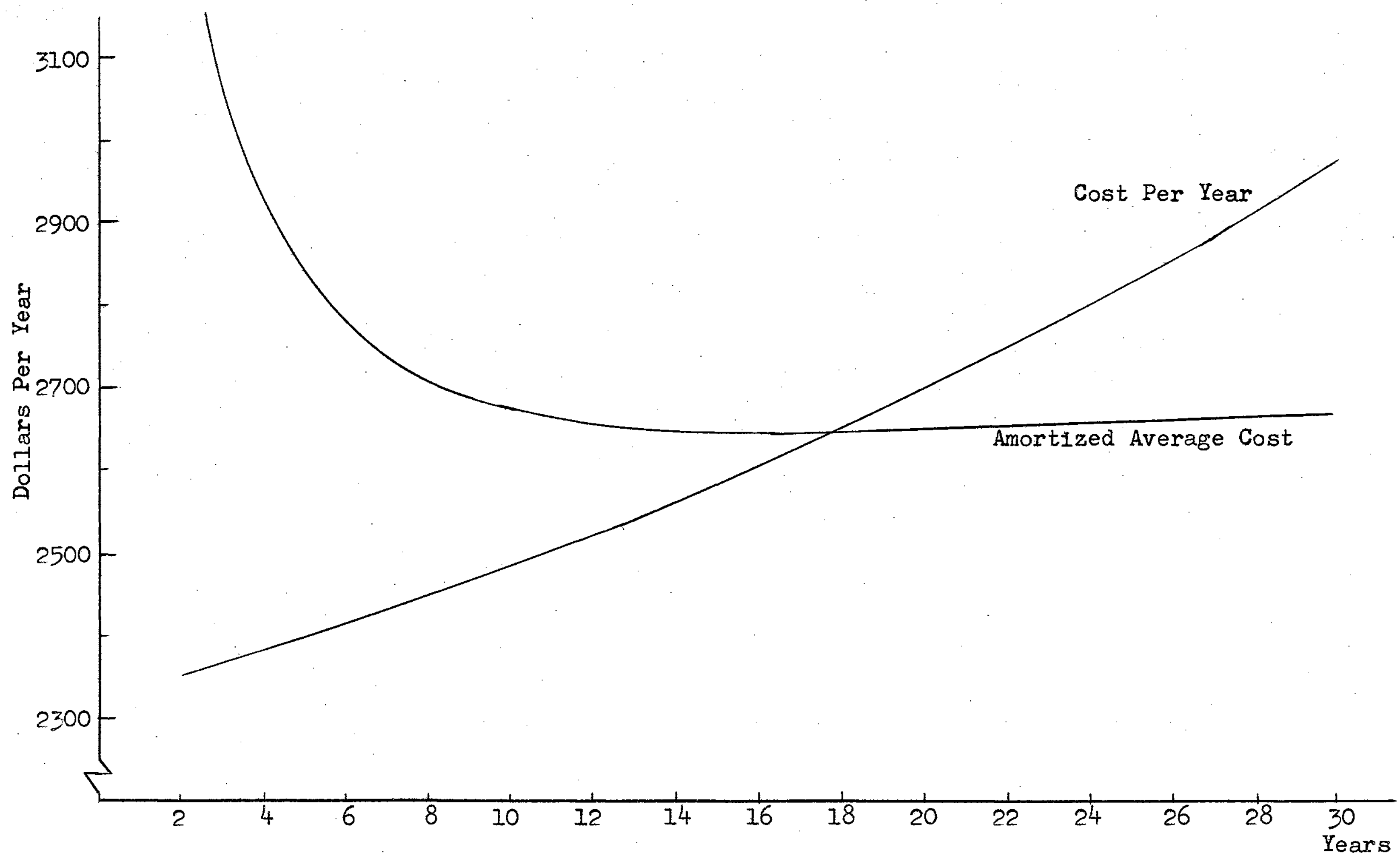

Figure 8. Marginal and Amortized Average Cost Curves for a $6100 Tractor: Tabular Data Given in Table III

Replacement if Costs Not as Expected

Table III and Figure 7 are developed assuming costs will be exactly as expected. Often costs are either greater or less than was anticipated. The effects of greater than normal costs on replacement are shown in Figure 3. In Figure 3 because of high costs, MC_0 is shifted up and to the left of MC_0'. The shift results in decreasing the optimal replacement interval from 'a' to 'b', because the new marginal cost curve crosses the minimum average cost line earlier.

Empirically, the effect of unexpected high costs on replacement is illustrated by assuming repair costs are fifteen per cent higher than average. As in Figure 3, the higher yearly costs move marginal cost up and to the left, thus decreasing the optimum replacement interval. For a sixty horsepower tractor, minimum average cost is \$2,644.98. (See Table III.) As soon as marginal costs exceed this minimum, it is time to replace the tractor. Costs per year, with repair costs increased fifteen per cent, are given in Table IV. In year fifteen, marginal cost exceeds \$2,644.98; therefore, the tractor should be replaced. The effect of the "unexpectedly" high marginal costs is to shift the replacement interval from seventeen to fifteen years.

Deciding when to replace is a yearly decision. If in any year marginal costs are expected to be above the minimum average cost of the proposed replacement, trading should be considered. At the time marginal cost first exceeds average cost, marginal cost should be rising and be expected to continue to rise.

Replacement With a Different Machine

In the above illustration, a shift in the marginal cost curve

TABLE IV

ANNUAL AND AMORTIZED AVERAGE COSTS FOR A $6100 TRACTOR. USE PER YEAR: 600 HOURS; LABOR: $1.50 PER HOUR; DEPENDABILITY INCREMENT: $25.00 PER YEAR; INTEREST RATE: 8 PER CENT; REPAIR COSTS ARE FIFTEEN PER CENT HIGHER THAN ANTICIPATED

Year	Amortized Average (dollars)	Cost in Year (dollars)
1	4310.99	4310.99
2	3383.02	2380.81
3	3083.08	2409.31
4	2938.43	2431.26
5	2855.27	2450.56
6	2802.59	2468.83
7	2767.21	2486.89
8	2742.58	2505.19
9	2725.07	2524.00
10	2712.54	2543.46
11	2703.59	2563.66
12	2697.32	2584.64
13	2693.09	2606.41
14	2690.44	2628.95
15	2689.04	2652.26
16	2688.62	2676.31
17	2688.99	2701.06
18	2689.99	2726.49
19	2691.50	2752.55
20	2693.41	2779.21
21	2695.66	2806.43
22	2698.15	2834.19
23	2700.85	2862.43
24	2703.70	2891.14
25	2706.66	2920.27
26	2709.71	2949.80
27	2712.80	2979.70
28	2715.91	3009.94
29	2719.03	3040.50
30	2722.14	3071.34
31	3133.80	2728.27
32	3165.39	2731.27
33	3197.17	2734.20
34	3229.14	2737.08
35	3261.28	2739.88
36	3293.57	2742.60
37	3326.01	2745.25
38	3358.57	2747.82
39	3391.24	2750.30

altered the replacement pattern. Since in the following example the proposed machine is a different tractor, there will be a different average cost curve as shown in Figure 4 but the marginal cost curve need not necessarily shift.

A different tractor size may be chosen for several reasons. First, the proposed replacement may be more efficient than the presently owned machine. Second, the operator may have purchased more land, which may necessitate a larger tractor to do the field work in the required time. Third, the farmer may desire to do his fieldwork in fewer hours. Arguments could also be made for a smaller tractor as the proposed replacement.

The ability to perform the same job in fewer hours is the reason for the larger proposed replacement in the example below. An Oklahoma panhandle farm situation illustrates the model.[8] Assume, as an example, a 640 acre panhandle farm presently using a $6,100 tractor. If the proposed replacement is a $7,200 tractor, the farmer should trade when marginal costs of the presently owned machine equal or exceed the minimum expected average costs of the proposed machine. Table V gives the relevant marginal and average costs for the panhandle situation. The $7,200 machine's minimum average cost is $3,086.94 in year eighteen. In year twelve, marginal costs of the older machine exceed this figure. Therefore, the farmer should plan to keep the $6,100 tractor until it is twelve years old and then trade for the $7,200 machine. The shortened trading interval is explained mainly by the lower labor requirement of the large tractor.

[8]See Appendix II for computations necessary to find the hours each size tractor will require on the assumed farm.

TABLE V

ANNUAL AND AMORTIZED AVERAGE COSTS FOR A $6100 TRACTOR AND AMORTIZED AVERAGE COSTS FOR A $7200 TRACTOR ON A 640-ACRE FARM WITH THE $6100 AND $7200 TRACTOR REQUIRING 645 AND 761 HOURS, RESPECTIVELY. LABOR: $1.50 PER HOUR; DEPENDABILITY INCREMENT: $25.00 PER YEAR; INTEREST RATE: 8 PER CENT

Year	Amortized Average $6100 Tractor (dollars)	Cost in Year $6100 Tractor (dollars)	Amortized Average $7200 Tractor (dollars)
1	4729.72	4729.72	5039.93
2	3810.80	2818.37	3941.04
3	3517.65	2859.13	3583.85
4	3378.59	2890.99	3410.20
5	3300.22	2918.85	3309.30
6	3251.76	2944.75	3244.52
7	3220.16	2969.78	3200.29
8	3198.95	2994.54	3168.85
9	3184.57	3019.38	3145.94
10	3174.90	3044.53	3128.98
11	3168.61	3070.12	3116.35
12	3164.79	3096.24	3106.96
13	3162.85	3122.93	3100.06
14	3162.32	3150.20	3095.10
15	3162.90	3178.07	3091.68
16	3164.34	3206.52	3089.50
17	3166.45	3235.54	3088.31
18	3169.09	3265.09	3087.94
19	3172.13	3295.16	3088.23
20	3175.49	3325.74	3089.07
21	3179.08	3356.78	3090.34
22	3182.85	3388.25	3091.98
23	3186.75	3420.14	3093.90
24	3190.73	3452.39	3096.06
25	3194.75	3485.01	3098.41
26	3198.80	3517.95	3100.90
27	3202.83	3551.19	3103.49
28	3206.84	3584.71	3106.16
29	3210.79	3618.49	3108.88
30	3214.69	3652.50	3111.64
31	3218.52	3686.72	3114.40
32	3222.27	3721.14	3117.16
33	3225.92	3755.72	3119.89
34	3229.48	3790.48	3122.60
35	3232.94	3825.37	3125.27

The analysis is carried out using economic life for the proposed replacement. This approach gives results that may appear contradictory. For the assumed panhandle situation, the larger machine has a lower minimum average cost. Therefore, if a farmer is comparing machines over their economic life, the larger machine is optimum. However, if $6,100 and $7,200 machines are compared for a short ownership interval, the smaller machine is more economical. For the panhandle situation, the breakeven point occurs in year seven. If the planning horizon is less than seven years, the smaller machine incurs a lower average cost. The converse is true for a longer interval.

Replacement of a Very Large Tractor

During the last decade, very large tractors have come into use. These tractors, some above one hundred horsepower, may cost more than $10,000. The size of investment required makes a thorough study of replacement practices much more important. Because large tractors have been on the market a relatively short period of time, very little *ex post* cost information is available. The cost equations used in this dissertation were computed for tractors with between thirty and seventy horsepower. Therefore, any application of these equations to a 100 horsepower, $10,000 tractor is an extrapolation.

Table III indicates that the minimum cost interval for owning a $6,100 tractor is seventeen years. The cost for 600 hours of operation per year is $2,644.98. For a $10,000 tractor used 600 hours per year, the minimum cost interval is 24 years and the average cost per year $3,648.98, or about $1,000 per year more. The larger tractor will do much more work per hour, but the farmer must decide if the additional cost is worthwhile. The opportunity cost of labor becomes very important. If labor charges are sufficiently high, the larger tractor may provide a lower tractor cost for the whole farm. Also, if labor is scarce, the larger tractor may reduce tractor requirements from two machines to one. Another disadvantage of the large tractor is that it must be kept a longer period of time to reach its minimum cost point. If the farmer uses a planning horizon shorter than 24 years, the relative cost of the large tractor increases. Therefore, farmers should analyze their situation carefully before purchasing a large tractor.

Generalized Replacement Decision Tables

Replacement to this point has been considered in a restricted framework as only two tractor size and hourly use situations have been discussed. However, it is possible to develop tables which could be applicable to most replacement conditions. Information, other than his own records, that must be supplied to a farmer making a replacement decision is the minimum average cost of the proposed replacement. The variables affecting costs in these tables include size, use per year, interest rate, fuel type, and dependability among others. Table VI is an example of a minimum average cost table.

Information necessary for the presently owned machine may all be found in the farmer's records and includes all operating and fixed costs plus any subjective charge the farmer may wish to make for dependability and prestige. Often, farmers have some notion of likely repair costs the following year. If such expectations have sufficient reliability, they can reduce the machine's cost through more optimal trading patterns.

TABLE VI

MINIMUM AVERAGE ANNUAL COST AND YEAR IN WHICH IT OCCURS FOR A $6100 TRACTOR UNDER ALTERNATIVE HOURS OF USE PER YEAR, DEPENDABILITY INCREMENTS AND INTEREST RATES. LABOR CHARGE: $1.50 PER HOUR

Interest Rate (Per Cent)	Dependability Increment ($ Per Year)	Hours Per Year							
		400		600		800		1000	
		Cost (Dollars)	Years	Cost (Dollars)	Years	Cost (Dollars)	Years	Cost (Dollars)	Years
4	10	1740.10	31	2404.04	22	3071.50	15	3734.46	11
	25	1879.78	18	2511.22	14	3151.83	11	3795.71	9
	50	2022.97	11	2630.66	10	3249.67	8	3876.31	7
8	10	1875.82+	40+	2532.11	29	3201.38	20	3870.55	13
	25	2011.92	22	2644.98	17	3289.33	13	3937.45	10
	50	2162.97	13	2772.30	11	3394.20	9	4023.34	8
12	10	2041.93+	40+	1679.45	38	3339.27	26	4008.14	17
	25	2156.79	28	2785.35	22	3429.26	16	4079.66	12
	50	2306.16	15	2916.14	13	3539.60	11	5178.48	9
24	10	2610.74+	40+	3210.72	40+	3830.45	40	4467.19	33
	25	2673.13	40	3273.11	36	3892.65	28	4528.15	21
	50	2776.27	24	3375.02	23	3990.81	16	4619.29	13

The procedure for use of these tables might be as follows: Each year after the farmer computes his machine cost for the year, he may anticipate what costs he expects for the following year. Armed with the cost information of his machine and replacement Table VI he must now make his yearly decision whether to trade or not to trade. If his yearly costs equal the minimum average cost of the proposed replacement, and are expected to rise, he should consider trading. Assume his costs are $2,500 this year and he anticipates costs of $2,800 for next year. Also assume his proposed replacement is a $6,100 tractor to be used 600 hours per year and the interest rate is eight per cent. If he uses a dependability increment of fifty dollars per year, should he or should he not trade?

His decision is still somewhat subjective. If the farmer anticipates increasing marginal costs, he should trade since his yearly costs are above the tabular value, $2,772.30. On the other hand, if he anticipates a lower repair cost the following year, perhaps he could lower his tractor costs over time by keeping the older machine. The farmer must also consider credit availability and other intangibles not considered in the model.

CHAPTER IV

FURTHER EMPIRICAL APPLICATION

Additional uses and variations of the empirical replacement models will be presented in this chapter. Initially, costs of not trading at the optimal time will be discussed. Costs of trading too soon or too late will be evaluated. The second section deals with the effect of land acquisition on replacement decisions. Due to financial considerations, some farmers purchase only used tractors. In the third section an evaluation of purchasing used tractors and their effect on the optimal replacement interval is made. The final portion of the chapter dwells on the effect of investment credit and taxes on the replacement interval. Investment credit shortens the optimal trading interval as does a tax opportunity cost associated with the small depreciation of an old tractor.

Costs of Not Trading at the Optimal Time

Just as there are costs associated with buying the wrong tractor size, there are opportunity costs connected with not trading at the optimal time. The difference between the minimum average cost of a proposed replacement and the marginal cost of the present machine is the cost of not trading at the optimal time. If the optimal trading period were seventeen years, the cost of trading in the sixteenth year is the difference between the proposed replacement's minimum average cost and marginal cost in the sixteenth year. Allow 'c' in Figure 6 to be equivalent to year seventeen and 'd' equivalent to year sixteen. If the cost equations were continuous, the shaded area is the opportunity cost (savings foregone) of trading in year sixteen.

It is optimal to trade a \$6100 tractor used 600 hours per year after seventeen years if it is then replaced by a similar machine. In Table III the relevant marginal and average cost information is given. The minimum average cost, occurring in year seventeen, is \$2644.98. Marginal costs in year sixteen are \$2603.81. The difference in the two costs is \$41.17. The sum \$41.17 is the additional cost incurred by trading in year sixteen as opposed to year seventeen.

The cost of trading two years prematurely is the sum of the differences for the two years. The cost of trading in year fifteen in addition to that incurred in year sixteen is \$62.84, \$2644.98 - \$2582.00. To find in year fifteen the total cost of trading in year fifteen, it is necessary to consider time preference. Time may be considered by discounting one year the trading cost incurred if the machine were traded in year sixteen. The discounted sixteenth year cost is then added to the fifteenth year total. The total of the two costs is the cost of trading in year fifteen. Table VII gives the costs of trading before the optimal trading interval of seventeen years.

Table VII also may be used to determine whether to replace because of an abnormally high cost. For example, if expected costs were \$270 above tabulated "typical" costs for year thirteen, it would be profitable to trade tractors in year thirteen. Trading is advantageous since \$270 is larger than the present value of all costs associated with trading prematurely, \$267.74. The fact that \$270 is larger than the

TABLE VII

COSTS OF TRADING PREMATURELY FOR A $6100 TRACTOR. INTEREST RATE: 8 PER CENT; DEPENDABILITY INCREMENT: $25.00 PER YEAR; USE: 600 HOURS PER YEAR; OPTIMUM INTERVAL: 17 YEARS

Tractor Age	Cost in Year (Dollars)	Total Cost (Dollars)	Discounted Total Cost (Dollars)
16	41.17	41.17	41.17
15	62.84	104.02	100.97
14	83.69	187.71	177.18
13	103.68	291.39	267.74
12	122.79	414.18	370.70
11	140.99	555.17	484.23
10	158.28	713.45	606.64
9	174.67	888.12	736.37
8	190.22	1078.34	872.04
7	205.04	1283.37	1012.48
6	219.33	1502.70	1156.81
5	233.47	1736.17	1304.59
4	248.14	1984.31	1456.10
3	264.74	2249.05	1612.98
2	286.62	2535.68	1780.12

$267.74 indicates that if marginal costs were as expected until year seventeen, the average of marginal costs between years thirteen and seventeen would be larger than the minimum average cost of the proposed replacement. Therefore, the tractor should be traded in year thirteen.

There are also costs associated with keeping a tractor longer than the optimal period of time. The procedure for computing the cost is essentially the same except the minimum average cost is now subtracted from the higher marginal costs. To find the total cost for year nineteen, it is necessary to compound the excess cost incurred in year eighteen for one year and add the total to the excess cost in year nineteen. Table VIII gives calculations for years eighteen through twenty-five for a $6100 tractor used 600 hours per year. Results in Figure 8 show that costs of trading one or two years after the optimal are small, but then begin increasing.

Another use of Table VIII would be to indicate the out-of-pocket cost for keeping the money required for the purchase of a new tractor. For example, in year twenty the cost of not having traded in year seventeen is $52.90. The additional investment required to buy a new $6100 tractor is $5071.34, the rest of the new tractor cost being covered by the trade-in. The $52.90 is slightly more than one per cent of $5071.34. Thus, if the $5071.34 is earning over nine per cent in other uses, it should not be used to purchase a new tractor.

This section of the chapter has dealt with costs of not trading at the optimal time. Tables VII and VIII are based on the assumption that the proposed replacement is a similar machine but tables could also be constructed for situations where the proposed replacement is of a different size.

TABLE VIII

COSTS OF TRADING LATE. $6100 TRACTOR; INTEREST RATE: 8 PER CENT; DEPENDABILITY INCREMENT: $25.00; USE: 600 HOURS PER YEAR; OPTIMUM INTERVAL: 17 YEARS

Tractor Age (Years)	Cost in Year (Dollars)	Total Cost (Dollars)	Compounded Total Cost (Dollars)	Investment Required* (Dollars)	Per Cent Return Required**
18	4.47	4.47	4.47	4918.30	8.09
19	28.36	32.83	33.19	4997.47	8.58
20	52.90	85.73	88.74	5071.34	9.04
21	78.07	163.80	173.90	5140.26	9.52
22	103.81	267.61	291.62	5204.56	9.99
23	130.09	397.70	445.02	5264.55	10.47
24	156.88	554.58	637.50	5320.52	11.00
25	184.14	738.72	872.64	5372.74	11.43
26	211.82	950.54	1154.27	5421.46	11.91
27	239.92	1190.46	1486.53	5466.92	12.39
28	268.39	1458.85	1873.84	5509.33	12.87
29	297.20	1756.05	2320.95	5548.90	13.36
30	326.33	2082.38	2832.96	5585.82	13.84

*Investment required to obtain a new $6100 tractor. This is equal to: 6100 minus the total depreciation from year 1 to t.

**If money required to purchase new machine is earning at least the given percentage return on investment in other uses, it is better to not trade machines.

Effect of Land Acquisition on Replacement

Land purchases place additional burdens on existing farm machinery. The added work load will, in many cases, lead to replacement with more efficient and larger equipment. This discussion gives the optimal replacement decisions under several land purchase situations. The conditions relevant to the analysis are illustrated in Figure 9.

The optimal trading pattern following land purchase depends among other things on the resource situation and machinery requirements. The panhandle farm situation discussed earlier is used.[1] Even when restricted to the panhandle resource situation, there are many possible farm, present tractor, proposed tractor, and land purchase situations which could be considered. It is hoped that a sufficient number is covered so that general inferences can be drawn as to the effect of changes in selected variables.

The analysis procedure involves computing the costs of the present tractor both before and after land purchase. It is assumed that a new, larger tractor will not be purchased until after the land is bought. Therefore, costs for the proposed replacement will be computed assuming the land has been purchased. Additional yearly machine usage changes labor, fuel, and repair costs.

To illustrate use of the model, assume current ownership of 480 acres of land and a $4800 tractor. After a land purchase of 160 acres, assume the optimum size tractor costs $6100. For a 640-acre panhandle farm, the minimum average annual cost for a $6100 tractor is $3162.32.

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[1] See Appendix II for a discussion of the panhandle resource situation. The optimal size tractor is determined independently of the replacement decision.

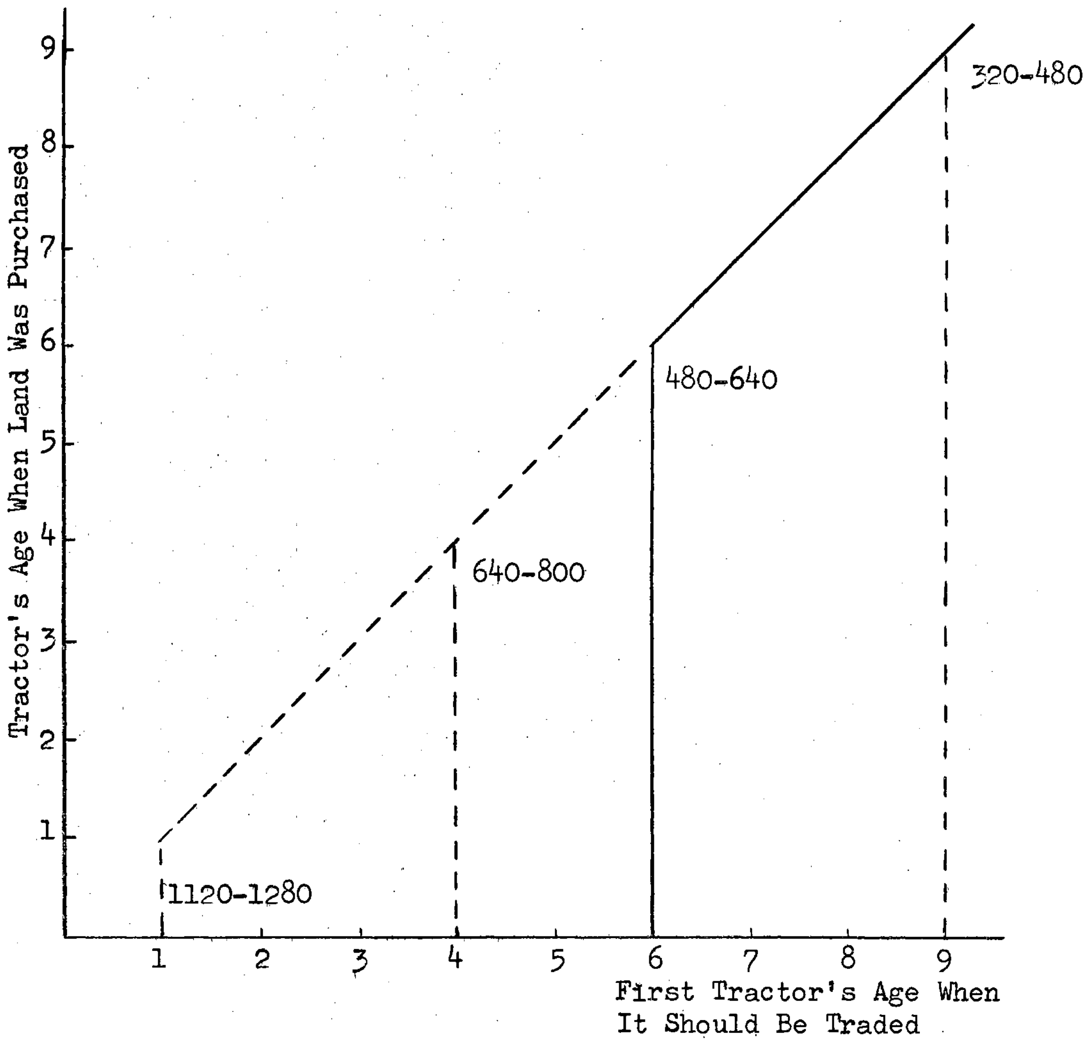

Figure 9. Optimal Years for Trading a $4800 for a $6100 Tractor Because of the Purchase of 160 Acres of Land. Land May be Purchased in Any of First Nine Years of Tractor Life With Various Beginning Farm Sizes

The marginal costs of owning a $4800 tractor both before and after land purchase are given in Table IX.

As soon as marginal costs equal or exceed the minimum average cost of the proposed replacement, $3162.32, it is time to trade. Assume that land purchase occurs in year four. Therefore, for the $4800 tractor in Table IX, the column giving costs for 480 acres is relevant for years one, two, and three, the columns giving costs for 640 acres are relevant for years four and after. In year four, the year of land purchase, marginal costs are $3110.89. Since $3110.89 does not exceed $3162.32, tractors should not be traded in year four. As can be seen in Table IX, not until year six do the marginal costs exceed $3162.32.

The marginal cost stream for a 480-acre farm is relevant until the land is purchased. The relevant marginal cost stream is then found in the column for a 640-acre farm. If the 160 acres is purchased before year six, the $4800 tractor is kept until year six and then traded. If land purchase occurs after year six, the tractors are traded in the land purchase year.

Table IX is applicable to a 160-acre land purchase in any year, but farm size must shift from 480 to 640 acres and tractor size from $4800 to $6100. Figure 9 generalizes Table IX to additional farm size situations. The tractor size and land purchase assumptions are the same as those in Table IX. There are two possible alternatives. The tractor will be replaced in the year of land purchase or in some following year. Consider first the solid line in Figure 9 which gives the optimal replacement years if present farm size is 480 acres and after land purchase farm size is 640 acres. To use Figure 9, find on the vertical axis the land purchase year. For example, choose year four.

TABLE IX

MARGINAL COSTS OF A $4800 TRACTOR ON 480 AND 640-ACRE PANHANDLE FARMS. INTEREST RATE: 8 PER CENT; DEPENDABILITY INCREMENT: $25.00 PER YEAR

Year	Cost for 480 Acres (Dollars)	Cost for 640 Acres (Dollars)
1	3878.01	4470.01
2	2374.03	3004.28
3	2407.98	3063.14
4	2435.60	3110.89
5	2460.44	3153.11
6	2484.01	3192.20
7	2507.07	3229.43
8	2530.06	3265.52
9	2553.32	3300.96

Go across from year four until the vertical, solid 480-640 acre line is reached. The horizontal axis gives the optimal replacement year. If land were purchased in year four, the present tractor should be kept until year six and then traded for a larger tractor. When presently owned tractors are relatively older, the vertical line is no longer used for making replacement decisions. If the land were purchased in year seven, Figure 9 indicates that trading for the larger tractor should occur immediately. The solid line is for a farm size shift from 480 to 640 acres. The sample procedure can be used for other farm size shifts. In Figure 9, the dotted lines give optimal replacement points for farm size shifts of 1120-1280 acres, 640-800 acres, and 320-480 acres.

Table IX and Figure 9 illustrate only a few of the many decision guides that could be constructed. It would take a great number of tables to cover all possible farm size, land purchase, and tractor size situations. Table X is an example of one approach to the problem.

The column headings are alternative tractor size shifts. For example, the first heading 3900-4800 means that the tractor owned before land purchase cost $3900. The proposed replacement is a $4800 tractor. The row headings are the farm size before and after land purchase. If, when land is purchased, the current tractor's age is less than the tabular amount, the smaller tractor should be kept until it reaches the tabular age, then it should be traded. In all cases, the larger tractor is ultimately the more economical. If land is purchased and the smaller tractor's age is greater than the tabular value, trade immediately.

For a farmer who currently owns a $6100 tractor and a 480-acre

TABLE X

OPTIMUM TRADING AGE (YEARS) FOR VARIOUS TRACTOR SIZE AND FARM SIZE SHIFTS

Acreage Shift	Tractor Size Shift			
	$3900-4800	$4800-6100	$6100-7200	$7200-8200
320-480	6	9	18	24
480-640	4	6	12	18
640-800	3	4	9	13
800-960	Immediately	3	7	10

farm, when should he trade tractors if he buys an additional 160 acres? If he proposes buying a $7200 tractor as a replacement, the 6100-7200 column and the 480-640 row are appropriate. The number given in the table is 12. If his $6100 tractor is less than 12 years old, he should not trade tractors until it is twelve years of age. If, when the land is purchased, the tractor is more than twelve years old, he should trade immediately.

Purchasing Used Tractors

Many farmers consider used equipment untrustworthy. Their fears are often well-founded. Not many farmers trade every year, and place "quality" one-year-old machinery on the market. When one or two-year-old equipment is traded, it is often because of some inherent deficiency or unsatisfactory service the machine has given.

If relatively good quality, adequately guaranteed used machinery is available, it is often a good buy. This fact is borne out in the following analysis. Using cost equations presented earlier and some basic assumptions concerning costs of used equipment, optimum replacement patterns for used tractors may be illustrated.

The first major assumption concerns the purchase price of a used tractor. In the model, a remaining farm value for each year of tractor life can be found. This is not, however, the price at which this tractor can be bought. It is the price (wholesale) which the tractor will bring when sold. A dealer would add some amount of markup (reconditioning and marketing cost plus profit) to the wholesale price to obtain the price farmers must pay. For purposes of this analysis, marketing costs of twenty per cent of wholesale tractor value are assumed. As with a new tractor, the trading cost or markup is charged to the machine as depreciation during its first year of use. This is plausible since farmers cannot recover the trading cost once the tractor has been purchased.

Repair costs are based on machine age and new tractor cost. Therefore, when a tractor is two years old, the second owner of the machine has the same repair cost that the first owner would have expected had he kept the machine. The assumption is made that the tractor is used the same number of hours per year regardless of whether the first or second owner has possession.

Dependability charges are based on actual tractor age, while taxes, housing, and insurance costs are based on the used tractor price. Interest charges are a constant, equal to labor charge per hour times the hours of use per year.

By computing tractor costs using the previously presented equations and the above assumptions, cost patterns of purchasing tractors of various ages can be found. Optimal replacement intervals can then be determined. Table XI contains costs for a $6100 tractor purchased when one-year-old. Table XI is typical of most tractor size, age, and use conditions and may be compared to Table III, a parallel tabulation of costs for a similar new machine.

Several cost comparisons can be made between purchasing a new and one-year-old tractor. The optimal ownership interval for a new tractor is seventeen years, while a tractor purchased when one-year-old should be kept eleven years and sold when twelve years old. The average cost per year for the optimal ownership interval decreases from $2644.98 to $2538.64, a saving of about $115.00 per year. The savings are due to

TABLE XI

YEARLY COST COMPONENTS FOR PURCHASING A ONE-YEAR-OLD TRACTOR. NEW TRACTOR COST: $6100; ONE-YEAR-OLD COST: $4609.96; USE: 600 HOURS PER YEAR; LABOR CHARGE: $1.50 PER HOUR; DEPENDABILITY INCREMENT: $25.00 PER YEAR; INTEREST RATE: 8 PER CENT

Age (Year)	Annual Average (Dollars)	Marginal Cost (Dollars)	Repairs (Dollars)	Depreciation (Dollars)	Dependability (Dollars)	Taxes, Housing Insurance (Dollars)	Fuel Lubricants (Dollars)	Interest (Dollars)	Labor (Dollars)
1	3126.68	3126.68	149.66	1025.71	25.00	161.29	578.28	286.74	900.00
2	2767.82	2380.24	193.80	240.14	50.00	150.48	578.28	267.53	900.00
3	2653.54	2396.84	229.50	224.05	75.00	140.40	578.28	249.60	900.00
4	2599.83	2411.51	260.32	209.04	100.00	130.99	578.28	232.88	900.00
5	2570.14	2524.65	287.84	195.04	125.00	122.22	578.28	217.28	900.00
6	2552.39	2439.94	312.95	181.97	150.00	114.03	578.28	202.72	900.00
7	2541.45	2454.76	336.18	169.78	175.00	106.39	578.28	189.14	900.00
8	2534.76	2470.31	357.90	158.40	200.00	99.26	578.28	176.46	900.00
9	2530.91	2486.70	378.38	147.79	225.00	92.61	578.28	164.64	900.00
10	2529.06	2503.99	397.81	137.89	250.00	86.41	578.28	153.61	900.00
11	2528.64	2528.64	416.33	128.65	275.00	80.62	578.28	143.32	900.00
12	2529.06	2541.30	434.06	120.03	300.00	75.22	578.28	133.72	900.00
13	2530.80	2561.29	451.09	111.99	325.00	70.18	578.28	124.76	900.00
14	2532.92	2582.14	467.50	104.48	350.00	65.47	578.28	116.40	900.00
15	2535.53	2603.81	483.35	97.48	375.00	61.09	578.28	108.60	900.00
16	2538.52	2626.26	498.71	90.95	400.00	56.99	578.28	101.32	900.00
17	2541.81	2649.45	513.60	84.86	425.00	53.18	578.28	94.54	900.00
18	2545.32	2673.34	528.07	79.17	450.00	49.61	578.28	88.20	900.00
19	2549.00	2697.88	542.15	73.87	475.00	46.29	578.28	82.29	900.00
20	2552.80	2723.05	555.89	68.92	500.00	43.19	578.28	76.78	900.00
21	2556.69	2748.79	569.28	64.30	525.00	40.29	578.28	71.63	900.00
22	2560.63	2775.07	582.37	59.99	550.00	37.59	578.28	66.83	900.00
23	2564.59	2801.86	595.17	55.97	575.00	35.08	578.28	62.36	900.00
24	2568.55	2829.12	607.71	52.22	600.00	32.73	578.28	58.18	900.00
25	2572.50	2856.80	619.99	48.72	625.00	30.53	578.28	54.28	900.00
26	2576.40	2884.90	632.03	45.46	650.00	28.49	578.28	50.64	900.00
27	2580.26	2913.37	643.84	42.41	675.00	26.58	578.28	47.25	900.00
28	2584.06	2942.18	655.44	39.57	700.00	24.80	578.28	44.09	900.00
29	2587.78	2971.31	666.84	36.92	725.00	23.14	578.28	41.13	900.00
30	2591.43	3000.74	678.05	34.45	750.00	21.59	578.28	38.38	900.00

TABLE XII

MINIMUM AVERAGE ANNUAL COSTS AND OPTIMAL OWNERSHIP INTERVALS FOR ALTERNATIVE PURCHASE AGES, SIZES, AND HOURS USED PER YEAR. INTEREST RATE: 8 PER CENT; LABOR CHARGE: $1.50 PER HOUR; DEPENDABILITY INCREMENT: $25.00 PER YEAR

Hours Used Per Year	New Cost (Dollars)	Purchase Age: New		1		2		3		4		5		6		7		8		9	
		Cost	Age	Cost	Age	Cost	Age	Cost	Age	Cost	Age	Cost	Age	Cost	Age	Cost	Age	Cost	Age	Cost	Age
		(Cost in Dollars, Age in Years)																			
400	4800	1748.80	18	1656.24	12	1659.93	11	1664.17	11	1669.04	10	1674.89	9	1681.66	9	1689.25	8	1697.94	8	1707.36	7
	6100	2011.92	22	1901.02	15	1900.90	14	1901.42	13	1902.87	12	1905.36	11	1908.83	11	1913.35	10	1919.04	9	1925.77	9
	7200	2230.08	25	2104.01	18	2100.91	17	2098.41	15	2097.15	14	2096.95	13	2097.94	12	2100.17	11	2103.48	11	2107.97	10
	8400	2466.82	28	2322.35	20	2315.98	19	2310.64	18	2306.32	18	2303.33	15	2301.69	14	2301.40	13	2302.43	12	2304.79	11
600	4800	2304.48	15	2207.27	9	2219.46	9	2231.17	8	2342.97	8	2255.53	8	2268.21	7	2281.58	7	2296.35	6	2311.19	6
	6100	2644.78	17	2528.64	11	2538.44	10	2547.67	10	2557.74	9	2567.30	9	2578.06	8	2589.72	8	2602.39	7	2615.55	7
	7200	2930.08	19	2797.89	12	2805.77	12	2813.12	11	2820.93	10	2829.17	10	2838.36	9	2848.85	9	2859.50	8	2871.56	8
	8400	3238.92	21	3089.48	14	3095.38	13	3100.89	12	3106.75	12	3113.20	11	3120.70	10	3129.27	10	3138.75	9	3159.67	9
800	4800	2869.36	12	3766.37	7	2790.38	7	2812.47	7	2833.64	6	2854.28	6	2875.19	6	2896.10	6	2918.44	5	2930.16	5
	6100	3289.33	13	3164.70	8	3188.80	8	3210.70	8	3231.20	7	3251.53	7	3987.34	7	3293.29	6	3314.49	7	3336.54	6
	7200	3642.72	14	3500.51	8	3524.45	8	3545.79	8	3566.31	8	3586.80	7	3606.85	7	3627.62	7	3649.26	8	3670.98	6
	8400	4026.74	15	3865.25	9	3889.40	9	3910.83	9	3931.11	8	3950.89	8	3971.20	8	3991.82	7	4012.91	8	4035.11	7
1000	4800	3438.31	9	3331.08	6	3370.15	6	3404.96	5	3437.23	5	3468.35	5	3498.91	5	3529.34	5	3559.86	5	3590.63	5
	6100	3937.45	10	3804.51	6	3847.70	6	3885.33	6	3920.36	6	3954.66	6	3987.15	5	4019.05	5	4051.07	5	4083.40	5
	7200	4358.60	11	4205.66	6	4251.78	6	4291.69	6	4328.52	6	4363.79	6	4398.35	6	4432.72	6	4466.71	5	4500.36	5
	8400	4817.37	11	4642.51	7	4691.79	7	4234.55	7	4773.79	6	4810.77	6	4846.92	6	4882.86	6	4918.94	6	4955.33	5

*Figures enclosed in boxes denote the optimum age to buy a used tractor of a given size used the same amount each year when the optimum age is other than one-year old.

Figures enclosed in circles denote used tractor age in which the yearly expected cost is first above that expected for a new tractor.

the decrease in capital required because of the significantly smaller initial tractor cost.

Table XII contains the minimum average annual costs of owning used tractors under 160 different purchase age, size, and use situations. Also given is the economic life of each tractor. Minimum average annual cost is relevant if the machine is to be considered as a proposed replacement. The economic life is the number of years the tractor should be kept if it is to be replaced by a similar machine.

In many cases, the most economical time to purchase a machine is when it is one year old. In Table XII, the most economical tractor of each size and use group is enclosed in a square. Smaller tractors used a large number of hours per year, purchased when relatively old, may be more expensive than new machines. The circled costs in Table XII indicate the year in which costs of used tractors first exceed new tractor costs for a particular size and use situation.

Analysis in this section has indicated quality used equipment, if available, is an economical purchase. The dependability increment used is twenty-five dollars per year. Dependability charges are considered a function of machine age, not purchase year. Other tables such as XII could be constructed based on alternative dependability charges, interest rates, and other factors. Since farmers may consider used equipment untrustworthy, higher dependability increments may be applicable. If larger dependability charges were made, used machinery would lose some of its appeal.

Effects of Investment Credit and Taxes

Investment credit is reputed to have a large affect on replacement. Investigation shows, however, that because of the long optimum replacement intervals for tractors, investment credit has little affect on either replacement intervals or costs. Investment credit is a tax concession granted to those who make capital investments. To qualify for investment credit, purchased capital equipment must be kept longer than three years and meet other specific requirements.

For eligible equipment, Table XIII indicates the amount of investment credit allowed under various replacement intervals. When the average farmer purchases a tractor, he usually does not know precisely how long the machine will be kept. He may, nevertheless, take the entire amount of investment credit allowed. If he trades before the planned time period has elapsed, he must return a portion of the claimed investment credit. Even if some money must be returned, the farmer has gotten the use of interest free money for a considerable period of time. In the model used here, trading intervals are known, therefore, exact determination of investment credit can be made. Perfect knowledge of replacement intervals eliminates computation of the adjusted balances actually required when tractors are traded before the end of the planned time period.

Table XIV contains the average cost, marginal cost, and investment credit for a $6100 tractor. The table shows fifteen years to be the optimal replacement interval when a similar machine is the proposed replacement. Average and marginal costs in Table XIV can be compared with those in Table III where no investment credit is considered. The optimal replacement interval is two years less when investment credit is taken. Investment credit lowers the minimum average annual cost

TABLE XIII

PORTIONS AND PERCENTAGES OF ELIGIBLE INVESTMENTS THAT MAY BE USED IN CALCULATING INVESTMENT CREDIT

Planned Replacement Interval (Years)	Portion Eligible for Credit	Percentage of Portion Deductable (Per Cent)
1 to 3	0	0
4 and 5	1/3	7
6 and 7	2/3	7
8 or more	All	7

[1]U.S. Treasury Department, Internal Revenue Service, Farmer's Tax Guide 1967 Edition, Publication No. 225, p. 14.

TABLE XIV

AMORTIZED AVERAGE AND MARGINAL COSTS CORRECTED FOR INVESTMENT CREDIT. TRACTOR SIZE: $6100; USE: 600 HOURS PER YEAR; INTEREST RATE: 8 PER CENT; LABOR CHARGE: $1.50 PER HOUR; DEPENDABILITY INCREMENT: $25.00 PER YEAR

Year	Amortized Average (Dollars)	Marginal Cost (Dollars)	Investment Credit (Dollars)
1	4298.71	4298.71	0.00
2	3365.85	2358.36	0.00
3	3062.25	2380.24	0.00
4	2874.83	2396.84	142.19
5	2795.85	2411.51	142.19
6	2716.82	2425.65	284.81
7	2685.79	2439.94	284.81
8	2641.16	2454.76	427.00
9	2627.48	2470.31	427.00
10	2617.76	2486.70	427.00
11	2610.93	2503.99	427.00
12	2606.25	2522.19	427.00
13	2603.23	2541.30	427.00
14	2601.50	2561.29	427.00
15	2600.79	2582.14	427.00
16	2600.88	2603.81	427.00
17	2601.64	2626.26	427.00
18	2602.91	2649.45	427.00
19	2604.61	2673.34	427.00
20	2606.65	2697.88	427.00
21	2608.96	2723.05	427.00
22	2611.48	2748.79	427.00
23	2614.17	2775.07	427.00
24	2616.98	2801.86	427.00
25	2619.88	2829.12	427.00
26	2622.84	2856.80	427.00
27	2625.84	2884.90	427.00
28	2628.86	2913.37	427.00
29	2631.87	2942.18	427.00
30	2634.87	2971.31	427.00

from $2644.98 to $2600.79. The actual out-of-pocket costs occurring in any year are the same, whether or not investment credit is considered since the investment credit is taken only in the first year.

In Table XIV, there is no investment credit allowed for replacement intervals of one, two, and three years. The $142.19 in year four is equivalent to seven per cent of one-third the eligible investment. Eligible investment is the purchase price, $6100. The large jumps in investment credit between the five and six year replacement intervals and the seven and eight year replacement intervals are caused by the increases in eligible investment. Eligible investment increases from one-third to two-thirds between years five and six and from two-thirds to the entire amount between years seven and eight. After year eight, investment credit is a constant $427.

As stated above, the primary effects of investment credit are a $44.19 per year reduction in costs and a reduction in the optimal replacement interval from seventeen to fifteen years. If a farmer were to trade machines every eight years, the savings because of investment credit average $68.80 per year. Because year eight is the first year of maximum eligible investment, it is also the year when investment credit gives the maximum reduction in amortized average cost.

In addition to investment credit, another tax concession is available to purchasers of eligible investments. Depreciation may be deducted from taxable income. If a tractor is kept a great number of years, depreciation will average only a small amount each year. But if tractors are traded frequently, average depreciation will be larger and the relative decrease in taxes also large. This being the case, there is an opportunity cost associated with not trading tractors every few years.

In this study, market depreciation has been used for replacement analysis. However, depreciation for tax purposes is usually computed by using either a straight line, sum-of-digits, or declining balance method. Because it allows the fastest depreciation, the declining balance method is used. Depreciation is assumed at a rate of twenty per cent per year. An additional twenty per cent depreciation is allowed the first year and is included. It is assumed for the purposes of this study that the farmer is in a sixteen per cent tax bracket. The higher the tax bracket, the more important are the savings from trading relatively often.

There are various alternatives when considering the opportunity cost associated with taxes. It was decided that since investment credit savings were a maximum in year eight, the opportunity costs associated with taxes would also be computed from year eight. The procedure used was to compute the tax savings each year for the first eight years. These tax savings were discounted and summed to year one. The total of tax savings, discounted and summed, were then amortized for the eight year period. Resulting was the average saving in taxes for the first eight years of tractor ownership. For all years past eight, the opportunity costs can be computed for not attaining the level of tax savings averaged the first eight years. This was done by subtracting the tax savings in year nine and each subsequent year from the first eight year average. It is not necessary to use the eight year time period; any interval could be chosen. The maximum opportunity costs would occur if the base interval were trading every year rather than every eight years.

Table XV gives the marginal and amortized average cost curves that result when the tax opportunity cost is considered. The optimum ownership interval is fourteen years with an amortized average cost of $2680.60 per year. Tax opportunity costs reduce the optimum replacement interval three years. If, instead of an eight year base interval, a one-year interval had been used, the optimum replacement interval is still fourteen years but the amortized average cost increases to $2912.26 per year.

To this point investment credit and a tax opportunity cost have been considered independently. By considering both, the optimum replacement interval is thirteen years and the amortized average cost $2630.93. It should be pointed out that the average cost is only decreased about $14.00. The small change results because the tax opportunity cost is added to the cost stream while investment credit is deducted from the first year's cost.

Breakeven Labor Charges

A small tractor being used on a 640-acre farm has relatively low fixed costs and high operating costs, whereas a large tractor has large fixed costs and relatively low operating costs per year. Because of the cost relationships between small and large tractors, it is possible to find breakeven yearly costs for small and large tractors on a given farm.

Assume for a given farm size that costs other than labor for a small and large farm are C_1 and C_2, respectively. Also, assume hours

TABLE XV

AMORTIZED AVERAGE AND MARGINAL COSTS CORRECTED FOR THE TAX SAVINGS GIVEN UP UNDER LONG OWNERSHIP INTERVALS. BASE INTERVAL: 8 YEARS; NEW TRACTOR COST: $6100.00; USE: 600 HOURS PER YEAR; INTEREST RATE: 8 PER CENT; LABOR CHARGE: $1.50 PER HOUR; DEPENDABILITY INCREMENT: $25.00 PER YEAR. (COSTS THE SAME AS IN TABLE III THE FIRST EIGHT YEARS.)

Replacement Interval (Years)	Amortized Average (Dollars)	Marginal Cost (Dollars)	Tax Savings Given Up (Dollars)
9	2698.26	2563.82	93.51
10	2690.47	2585.45	98.75
11	2685.45	2606.93	102.94
12	2682.45	2628.48	106.29
13	2680.95	2650.27	108.97
14	2680.60	2672.41	111.12
15	2681.13	2694.97	112.84
16	2682.35	2718.02	114.21
17	2684.10	2741.57	115.31
18	2686.28	2765.64	116.19
19	2688.79	2790.23	116.89
20	2691.55	2815.34	117.45
21	2694.52	2840.96	117.90
22	2697.63	2867.06	118.26
23	2700.85	2893.63	118.55
24	2704.14	2920.64	118.78
25	2707.47	2948.08	118.97

of labor required are H_1 and H_2 for the small and large farm, respectively. If one wishes to find a labor charge, LC, such that the average yearly costs, AYC, for the two tractors is the same, then one has a system of two equations with two unknowns, LC and AYC.

$$C_1 + H_1 LC = AYC$$

$$C_2 + H_2 LC = AYC.$$

Since these two equations are both equal to AYC, they are equal to each other. The resulting labor charge which will make the two average yearly costs equal is $LC = \frac{C_1 - C_2}{H_2 - H_1}$.

Table XVI gives breakeven labor charges for an Oklahoma panhandle farm situation. The column headings give the tractor sizes being compared and the row headings give the size of farm being considered. Alternative planning horizons are also given. The tractors are kept the optimum length of time or to the end of the planning horizon, whichever is shorter. The optimal ownership interval is enclosed in parenthesis when it is shorter than the planning horizon.

The table may be used as follows. If the planning horizon is ten years, and the farm size 480 acres, what size of tractor should the farmer buy if he purchases labor (or values his own time) at $1.50 per hour. The table indicates that between labor charges $1.01 and $1.58 per hour, the optimal tractor size is $6100. Therefore, the farmer should buy a $6100 tractor. If on the other hand, the farmer values labor at more than $2.18 per hour, he should buy $8200 tractor.

Table XVI clearly indicates the relationship between labor costs and optimal tractor size. A relatively small change in the labor charge can make a big difference in the optimal tractor size.

TABLE XVI

BREAKEVEN LABOR CHARGES BETWEEN ALTERNATIVE TRACTOR SIZES FOR A GIVEN OKLAHOMA PANHANDLE RESOURCE SITUATION. INTEREST RATE: 8 PER CENT; DEPENDABILITY INCREMENT: $25.00 PER YEAR

Planning Horizon*	Farm Size	Tractor size (Dollars) 3900		4800		6100		7200		8200
			(Dollars)		(Dollars)		(Dollars)		(Dollars)	
25 years	320		.66		1.12		1.76		2.43	
	480	(24)	.34		.63		1.05		1.48	
	640	(12)	.27	(20)	.38		.67		.99	
	800	(8)	.26	(12)	.34	(21)	.45		.68	
	960	(6)	.25	(8)	.33	(13)	.41	(20)	.49	(25)
15 years	320		.83		1.37		2.13		2.91	
	480		.47		.82		1.31		1.83	
	640	(12)	.29		.53		.89		1.27	
	800	(8)	.26	(12)	.36		.62		.92	
	960	(6)	.25	(8)	.33	(13)	.45		.68	
10 years	320		1.01		1.64		2.51		3.42	
	480		.60		1.01		1.58		2.18	
	640		.38		.68		1.10		1.55	
	800	(8)	.27		.48		.81		1.16	
	960	(6)	.25	(8)	.35		.61		.89	
8 years	320		1.12		1.80		2.76		3.75	
	480		.68		1.13		1.75		2.41	
	640		.45		.78		1.24		1.73	
	800		.30		.56		.93		1.31	
	960	(5)	.25		.41		.71		1.02	

TABLE XVI (Continued)

Planning Horizon*	Farm Size	Tractor size (Dollars)			
		3900 4800	4800 6100	6100 7200	7200 8200
		(Dollars)	(Dollars)	(Dollars)	(Dollars)
5 years	320	1.39	2.22	3.38	4.57
	480	.87	1.42	2.18	2.98
	640	.61	1.01	1.58	2.17
	800	.44	.76	1.21	1.67
	960	.33	.59	.96	1.34

*If optimal interval less than planning horizon, optimum interval used and enclosed in parenthesis.

Therefore, it is imperative that any farmer seriously consider the current and anticipated labor charges before he purchases a tractor.

Summary

The current chapter and part of the previous chapter have been devoted to developing empirical applications of the replacement models. If a machine is to be replaced by a similar machine, the year in which the minimum amortized average cost occurs indicates the economic life and optimum ownership interval for the tractor.

The generalized replacement criteria is: The current machine's marginal cost must equal the proposed replacement's minimum average cost for it to be economical to trade. If either the marginal (current) cost of the currently owned machine or the minimum average cost of the proposed replacement are altered, the optimal ownership interval will shift. Reasons why farmers may purchase larger machines are: Larger machines are more efficient, additional land has been purchased, or labor has become more expensive. Farmers purchasing large tractors should be aware that they have longer optimal ownership intervals, have higher yearly costs, but require substantially less labor for a given farm size.

Tables which contain the economic life and minimum amortized average cost for a number of tractors can be valuable replacement aids. With appropriate tables and adequate records plus a working knowledge of their tractors, farmers can make a cost minimizing replacement decision each year.

Often, farmers wish to trade machines at other than the optimal time. The opportunity costs (savings foregone) associated with this practice were shown in this chapter. Alternatively, a large anticipated repair cost may make it economical to trade earlier than the optimal interval found a priori, by methods used in this study. If a farmer had a table of minimum average costs for proposed replacements and a table of expected yearly costs for his present machine, he could decide if trading were feasible because of a large repair cost.

Land acquisition has a profound affect on the optimal ownership interval. It greatly increases costs and alters the optimal machine size. For a farmer to decide whether to trade, it would be necessary to consider the size of land purchase, the value of his labor, and the other relevant variables. Again, tables applicable to an individual farmer's situation would be helpful.

The analysis showed that, if available, quality used tractors are a good buy. If the farm size is large and the chosen tractor size relatively small, the wisdom of purchasing used tractors is questionable. Relatively large tractors, one year old, may be a good buy because during the first year the machine depreciated considerably, thereby allowing the second owner to get by on a smaller fixed cost per year.

Taxes are also important in replacement decisions. Tax concessions available include investment credit and the deduction of depreciation from taxable income. Since the influence of these two concessions occurs primarily during the first few years of tractor ownership, they reduce the optimal ownership interval. Of the two concessions, depreciation deductions from taxable income shorten the optimal interval the most. This occurs primarily because investment credit is deducted in only the first year of ownership.

The final portion of this chapter was devoted to illustrating the profound importance of labor costs. If farmers use relatively long planning horizons and labor charges are above a dollar per hour, farmers are justified in purchasing large tractors. If a farmer's time has a small opportunity cost, he should purchase a relatively small tractor and take longer to farm his land. The farmer should not, however, buy so small a tractor that the opportunity cost of not getting jobs done at the correct time is prohibitive.

Relating the results of this chapter to a $6100 tractor allows several conclusions to be drawn. The optimum trading interval is 17 years, but if trading occurs anytime between years 14 and 21, the total additional cost will be less than $200. If a one-year old $6100 tractor is purchased, the expected minimum average cost is $2528.64 rather than the $2644.78 for a new tractor. Consideration of investment credit shortens the optimal replacement interval from 17 to 15 years and decreases amortized average costs $44.98 per year. The tax savings of trading every 8 years rather than periods longer than 8 years is about $100. For a 640-acre farm with an 8-year planning horizon, the breakeven labor charge between a $6100 and $7200 tractor is 89 cents. If the farmer's labor is worth more than 89 cents per hour, he should purchase the larger tractor.

CHAPTER V

SELECTION OF A REPLACEMENT PROCEDURE USING SIMULATION

In an earlier chapter, an optimal replacement criterion was developed. The theoretical criterion, as developed, requires yearly costs to behave in an orderly manner. However, in the real world costs fluctuate making the theoretical model of limited value. The purpose of this chapter is to select from among several alternative rules of thumb the best method of implementing the replacement criterion. Simulation will be used to evaluate the alternative rules and select the one offering the lowest average cost over time. Of particular interest is the impact of a stochastic repair distribution on the optimal replacement interval.

Theoretical expectations may be used to determine optimal tractor size, observe expected repair costs, and determine single valued optimal replacement intervals. But, developing a usable replacement procedure for year-to-year decisions requires that actual conditions and short run expectations be used.

Several alternative rules of thumb may be suggested. First, the machine can be replaced at the theoretical optimum replacement interval. Second, the farmer may replace when some average of marginal costs exceeds the minimum average cost of the proposed replacement. Third, replacement may occur when marginal costs in any year are sufficiently high. The size of repair cost required will be discussed later.

Distribution of Repair Costs

The most unpredictable farm tractor cost is repair and before simulation can take place a distribution must be constructed from which yearly repair costs can be drawn at random. Because repair costs fluctuate widely, collection of a large number of observations is necessary to determine with some degree of confidence the distribution's shape. Data collection poses a problem since it is difficult to obtain data from a large number of tractors which are the same age, size, and which are used the same amount. This problem was overcome by constructing a generalized distribution. Repair cost data were collected on tractors of various sizes, ages, and use levels. Given the size, age, and use, the repair cost equation presented in Chapter III can be used to determine <u>expected</u> repair costs for the tractor. Each repair cost observation was divided by the repair cost expected for the machine. The ratios found were then tabulated giving a frequency distribution of actual repairs as a per cent of expected repairs. The expected value of the frequency distribution should be one.

The data used to find a distribution using the above procedure are the same data used to construct Equation (3-6).[1] Since the tractors surveyed varied in age and there has been a large amount of inflation since many of them had been purchased, it is necessary to inflate the tractor prices to a 1966 equivalent. The index of prices paid by farmers was used to inflate the tractor prices. The data used in

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[1]The data was collected in Illinois and Indiana and is analyzed in William E. Larsen, and Wendell Bowers, "Engineering Analysis of Machinery Costs" for presentation at the 1965 annual meeting of the American Society of Agricultural Engineers.

finding the repair cost distribution was collected on tractors between two and twenty-six years old with at least a $3000 inflated purchase price and used a minimum of 400 hours per year. Tractors one year old were excluded because of the large amount of warranty work. Also, many of the one-year-old tractors were probably used only part of a year and at the time of the survey were yet to be repaired. By eliminating first year data, the expected value of the repair cost distribution was increased. Before elimination of the first year data, the distribution was more skewed than the one shown in Figure 10.

Observations on 475 tractors were used in the construction of the repair cost distribution presented in Table XVII and illustrated in Figure 10. The frequency distribution shown in Figure 10 was adjusted for two reasons. First, to facilitate the simulation procedure, a distribution with a more regular shape than provided by the raw data was desired. Second, the distribution was adjusted so that its expected value would be one. To accomplish these objectives, several components of the distribution were arbitrarily increased or decreased. The expected value of the raw frequency distribution was .875. After adjustment, the expected value was .996. The adjustments altered the distribution towards a normal curve, but it is still significantly skewed. Because repair costs tend to occur in lumps every several years, the mode of the distribution is considerably less than the expected value. The distribution allows repair costs to vary from five to 495 per cent of the expected value. If expected repair costs for a year were $100, then the possible range of repair costs would be from $5 to $495. As expected costs increase, the possible range of repair costs increase also. If expected repair costs were $200, then the

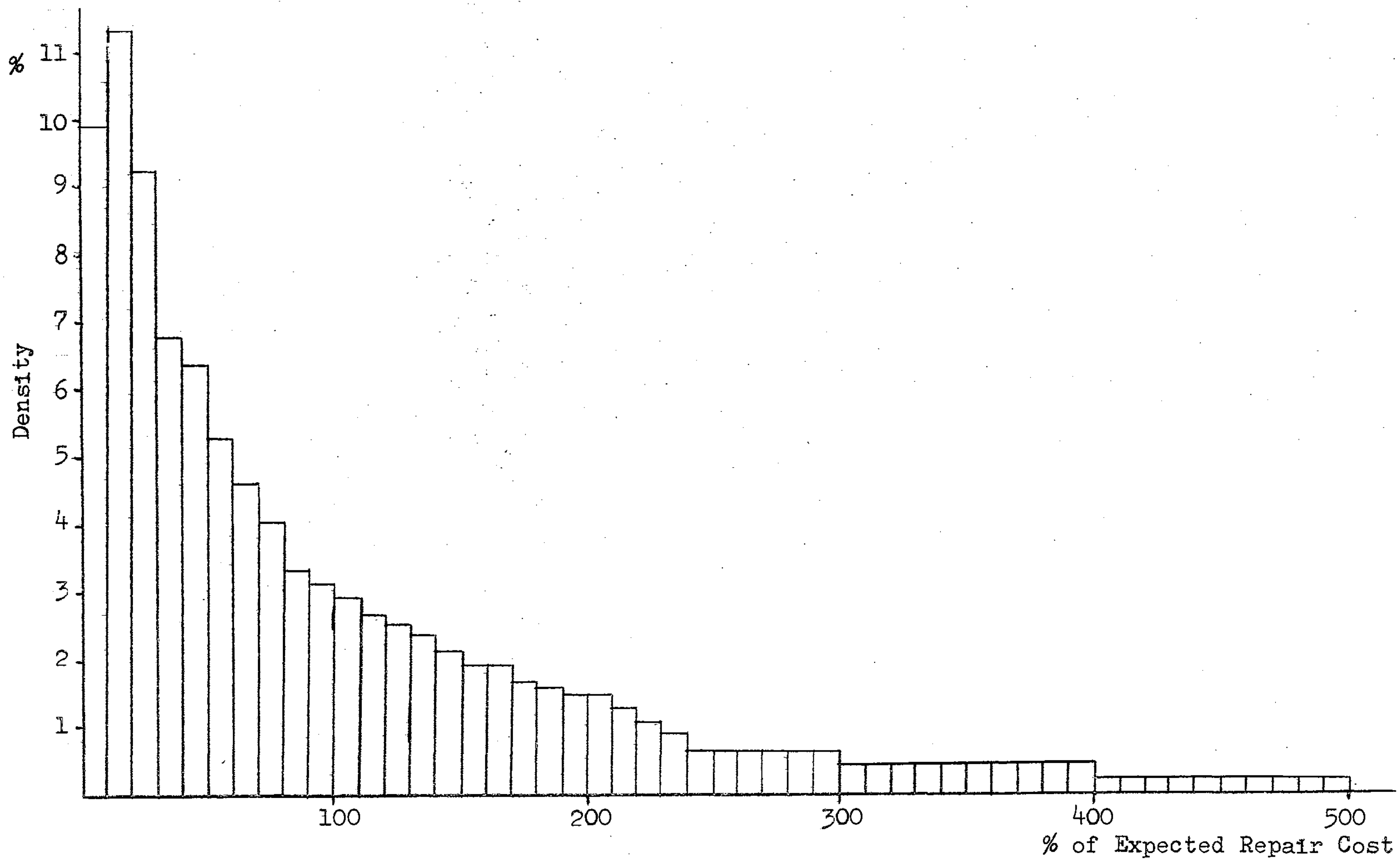

Figure 10. Density Distribution of Repair Cost as Per Cent of Expected Repair Cost
for a $6100 Machine

TABLE XVII

PROBABILITY AND CUMULATIVE DISTRIBUTION OF TRACTOR REPAIR COSTS AS A PER CENT OF EXPECTED REPAIR COSTS

Proportion of Expected Repair Cost	Probability of the Proportion Occurring	Cumulative Distribution	Proportion of Expected Repair Cost	Probability of the Proportion Occurring	Cumulative Distribution
.05	.09895	.09895	2.55	.00632	.91160
.15	.11368	.21263	2.65	.00632	.91792
.25	.09263	.30526	2.75	.00632	.92424
.35	.06736	.37262	2.85	.00632	.93056
.45	.06315	.43577	2.95	.00632	.93688
.55	.05263	.48840	3.05	.00421	.94109
.65	.04632	.53472	3.15	.00421	.94530
.75	.04000	.57472	3.25	.00421	.94951
.85	.03368	.60840	3.35	.00421	.95372
.95	.03158	.63998	3.45	.00421	.95793
1.05	.02947	.66945	3.55	.00421	.96214
1.15	.02737	.69682	3.65	.00421	.96635
1.25	.02526	.72208	3.75	.00421	.97056
1.35	.02316	.74524	3.85	.00421	.97477
1.45	.02105	.76629	3.95	.00421	.97898
1.55	.01895	.78524	4.05	.00211	.98109
1.65	.01895	.80419	4.15	.00211	.98320
1.75	.01687	.82106	4.25	.00211	.98531
1.85	.01684	.83780	4.35	.00211	.98742
1.95	.01474	.85264	4.45	.00211	.98953
2.05	.01474	.86738	4.55	.00211	.99164
2.15	.01263	.88001	4.65	.00211	.99375
2.25	.01053	.89054	4.75	.00211	.99586
2.35	.00842	.89896	4.85	.00211	.99797
2.45	.00632	.90528	4.95	.00203	1.00000

possible range of repair costs would be from $10 to $990. In Table III the expected repair cost for a thirty year old tractor is above $600. Using the repair cost distribution, the highest possible repair cost in year thirty is above $3000. Clearly, repair costs of this magnitude are not conceivable in normal everyday operations and available data do not indicate that they would ever be that high. Since the distribution gives unsatisfactory results when expected repair costs are high, an arbitrary limit of $1300 is placed on the repair cost size which could occur in any year.

The high percentage of low costs indicate to what extent the distribution is skewed. Over fifty per cent of the time, simulated repair costs will be less than sixty-five per cent of the expected repair cost. About sixty-four per cent of the time simulated repair costs will be less than their expected value. On the other end of the distribution, only ten per cent of the repair costs will be more than 2.45 times the expected cost.

In the simulation procedure, the repair cost density function is used to determine yearly repair costs. Random numbers are used to select from the cumulative distribution given in Table XVII the proportion of expected repair costs to be used for the year. The repair cost proportion obtained is then multiplied times the expected repair cost to procure the simulated repair charge. By securing thirty random numbers, finding the corresponding proportion of expected repair cost in Table XVII, and multiplying the proportion by the appropriate thirty expected repair costs, thirty years of tractor repair costs can be simulated.

The Simulation Procedure

The simulation procedure used for replacement criteria evaluation is as follows: First, the minimum amortized average cost of the proposed replacement is found. The replacement's minimum average cost is the pivotal variable in trading decisions. Except for the repair portion, marginal costs are computed for the existing machine exactly as they were in the theoretical model. A sample simulation procedure is given in Table XVIII. A random number and the cumulative distribution are used to select a repair cost proportion in Table XVII. Simulated repair costs are found by multiplying expected repair cost by the appropriate portion of expected costs. Yearly simulated costs are equal to expected costs plus the difference between simulated and expected repairs.

Once the simulated yearly cost is obtained, the procedure used to implement the replacement criterion is applied. For expositional purposes, the replacement procedure used in Table XVIII is a three-year average of marginal (annual actual) costs. An average of marginal costs is used to implement the replacement criterion because of marginal cost variability. By using an average of marginal costs, it is hoped that premature replacement due to one large repair cost can be prevented. When the three-year average of marginal costs exceed the minimum expected average cost of the proposed replacement, it is time to trade. Other replacement procedures will be considered and evaluated later but the analytic procedure is the same as for the three-year average. It was previously pointed out that only when marginal costs are rising is the replacement model relevant. In Table XVIII, expected yearly costs begin rising in year two. Thus, not until year four is it

TABLE XVIII

ILLUSTRATION OF SIMULATION PROCEDURE USING THREE-YEAR-AVERAGE CRITERION ON A $6100 TRACTOR WHICH HAS A MINIMUM AVERAGE COST OF $2644.98

Tractor Age (Years)	Random Number	Repair Cost Factor	Expected Repair (Dollars)	Simulated Repair (Dollars)	Expected Yearly Cost (Dollars)	Simulated Yearly Cost (Dollars)	3 Yr. Avg. of Simulated Cost (Dollars)	Is Replacement Criteria Met?
1	42365	.45	81.85	36.83	4298.71	4253.69		
2	92667	2.85	149.66	426.63	2358.36	2635.33		
3	22746	.25	193.80	48.45	2350.24	2234.89		
4	29222	.45	229.50	103.27	2396.84	2270.61	2380.27	No
5	98762	4.45	260.32	1158.42	2411.51	3309.61	2605.03	No
6	20159	.15	287.84	43.18	2425.65	2180.99	2588.90	No
7	95497	3.45	312.92	1079.57	2439.94	3206.59	2899.06	Yes
1	88460	2.25	81.85	184.16	4298.71	4401.02		
2	47195	.55	149.66	82.31	2358.36	2291.01		
3	53963	.75	193.80	145.35	2380.24	2331.79		
4	68423	1.15	229.50	263.93	2396.84	2431.27	2351.35	No
5	43590	.55	260.32	143.18	2411.51	2294.37	2352.47	No
6	39020	.45	287.84	129.53	2425.65	2267.34	2370.99	No
7	30866	.35	312.95	109.53	2439.94	2236.52	2266.07	No
8	18813	.15	336.18	50.43	2454.76	2169.01	2224.29	No
9	29888	.25	357.90	89.48	2470.31	2201.89	2202.47	No
10	19141	.15	378.32	56.76	2486.70	2165.08	2178.66	No
11	67205	1.15	397.81	457.48	2503.99	2563.66	2310.21	No
12	74732	1.45	416.33	603.68	2522.19	2709.54	2479.42	No
13	53695	.75	434.06	325.55	2541.30	2422.79	2568.66	No
14	15578	.15	451.09	67.66	2561.29	2177.86	2440.06	No
15	56432	.75	467.50	350.62	2582.14	2465.21	2538.63	No

TABLE XVIII (Continued)

Tractor Age (Years)	Random Number	Repair Cost Factor	Expected Repair (Dollars)	Simulated Repair (Dollars)	Expected Yearly Cost (Dollars)	Simulated Yearly Cost (Dollars)	3 Yr. Avg. of Simulated Cost (Dollars)	Is Replacement Criteria Met?
16	15578	.45	483.35	217.51	2603.81	2337.97	2327.03	No
17	56432	.85	498.71	423.90	2626.26	2551.45	2451.56	No
18	80571	1.75	513.60	898.80	2649.45	3034.65	2642.12	No
19	91216	2.65	528.07	1300.00	2673.34	3445.27	4010.45	Yes

possible to have a three year average which can be tested against the minimum average cost of the proposed replacement.

In Table XVIII, all that is done for the first three years of tractor life is to find the simulated yearly cost. In year four, a three-year average of marginal costs is found. This average is checked against the minimum amortized average cost of the proposed replacement. If the three-year average is larger, the tractor is traded. Otherwise, the tractor is kept and the simulation of year five begun.

The procedure outlined above continues until the tractor is replaced. In Table XVIII, two tractor lives are simulated. One machine is kept seven years; the next is kept nineteen. The way in which the simulation procedure is used to evaluate various replacement criteria is the topic of the following section.

Evaluation of Replacement Procedures

The purpose of simulating tractor ownership intervals is to have some means of evaluating alternative replacement procedures. In theory, there is no problem -- as soon as marginal cost exceeds the minimum average cost of the proposed replacement, it is time to trade. Also when marginal cost exceeds minimum average cost, it is necessary that it remain above average cost. This condition will not be met in real life as yearly costs fluctuate considerably, especially the repairs component. When large repair costs occur early in machine life, the farmer may either trade or keep the machine. If he follows the theory directly, he will trade. If he trades, he may forego the subsequent low marginal costs expected on the current machine for the relatively high average yearly cost of the proposed replacement.

The objective of effective tractor management is the minimization of long-run costs. Minimum average cost is the norm chosen to compare alternative replacement strategies. The replacement procedure which provides for minimum average cost over time is preferred.

The simulation procedure presented provides a means of determining, with a reasonable degree of accuracy, the average costs associated with each procedure. A large number of tractor lives are simulated using a given rule of thumb for determining when to replace. The total costs associated with each tractor can then be summed and divided by the number of years to give an average cost over time. The replacement procedure offering the lowest average cost over time is the most economical choice.

In this simulation of tractor lives, it is assumed that the farmer can correctly anticipate costs for the following year. Using a three-year average rule of thumb, the simulation results presented in Table XVIII imply that the first tractor would actually be traded in six years. The high repair cost in year seven would have been anticipated and the farmer would have traded machines before the cost occurred.

As mentioned earlier, procedures proposed for implementing the replacement criteria fall into three groups. The first requires keeping each tractor its economic life and then trading. For a $6100 machine, the expected minimum amortized average cost is $2644.98 and the corresponding economic life, 17 years. This is based on single-valued expected annual costs with no provision for cost variability.

The second rule of thumb involves averages of marginal cost. Two, three, four, five, seven, nine, and twelve year averages are considered. If a twelve year average of marginal costs is used, it means that no machine could be replaced before year thirteen. Therefore, an alteration is made in the average cost criteria. In year four, a three-year marginal cost average is tested against the minimum average cost of the proposed replacement. In year five, a four-year average is used. The averaging process is continued until a maximum twelve-year average is found. Thus, replacement based on (say) twelve year averages can occur as early as year four.

The third rule of thumb is based on the occurrence of a very large repair cost. Required to cause replacement is a repair cost which, when added to the sum of marginal costs between the large cost year and the expected optimal year, would yield an average of marginal costs greater than the minimum average cost of the proposed replacement. Also considered in the simulation analysis were combinations of the large cost replacement rule and the average of marginal costs rule.

Table XIX gives the simulation results. The procedures marked with asterisks offer the lowest average costs over time. The large cost criterion, averaged over 1000 trials offers an average cost over time twenty dollars per year less than other methods tested. The average replacement interval using the large cost method is 13.7 years, whereas the economic life of the machine is seventeen years. The expected simple average cost of owning a $6100 tractor seventeen years is $2592. The averages in Table XIX and $2592 are comparable figures. Several of the procedures have average costs above $2592, which indicates that trading in a set pattern of every seventeen years would be preferred to using such methods.

The large cost procedure provides a saving of about fifty dollars per year over the arbitrary decision rule of trading every seventeen

TABLE XIX

EXPECTED VALUE OF REPLACEMENT INTERVALS AND AVERAGE COSTS FOR ALTERNATIVE REPLACEMENT CRITERIA. 1000 TRACTOR LIVES SIMULATED USING EACH CRITERIA**

Criteria	Average Cost (Dollars)	Expected Replacement Interval (Years)
Large Cost	*2540.96	13.7
2-year-average	2620.63	11.3
3-year-average	2591.54	14.7
5-year-average	2603.54	17.3
9-year-average	2617.75	21.3
12-year-average	2614.50	24.0
2-year-avg. + Large Cost	2595.69	10.6
3-year-avg. + Large Cost	2572.42	12.0
4-year-avg. + Large Cost	*2562.59	12.8
5-year avg. + Large Cost	2564.79	12.9
7-year avg. + Large Cost	2567.97	13.2
9-year avg. + Large Cost	2566.96	13.4
12-year avg. + Large Cost	2568.99	13.8

**The minimum amortized average cost of the proposed replacement, $2644.98, is equal to a simple average cost in year seventeen of $2592. The difference between $2592 and the average costs above are measures of the savings per year.

years. The large cost method used in conjunction with the average of marginal costs provide lower costs than the average of marginal costs criterion used alone.

The simulation results indicate that over a long period of time, the various replacement procedures tested offer only very small cost reductions compared to trading every seventeen years. However, a long period of time is many times the farmer's age. Therefore, it may be argued that during a farmer's lifespan utilization of rules two and three may be very important. If the rule of trading every seventeen years were followed for a $6100 tractor, the typical farmer would own no more than three tractors during his life. Using rules two and three may not always save much, but, if a "lemon" were purchased, savings could be considerable.

Distribution of Replacement Intervals

Once the optimum replacement procedure is selected, it is possible to construct a replacement interval distribution based on the chosen method. The density distribution of replacement intervals for the large cost procedure is given in Figure 11 and the final column of Table XX. The data used for construction of this distribution were obtained from the simulation results. Each time a tractor life was simulated, the replacement year was recorded. Figure 11 is based on the results of 1000 simulated tractor lives. The expected value of the distribution is 13.74. In the simulation, no machines were replaced before year eight because the cost equations used made it impossible to have a sufficiently large cost.

For the large cost procedure, it is possible to construct a

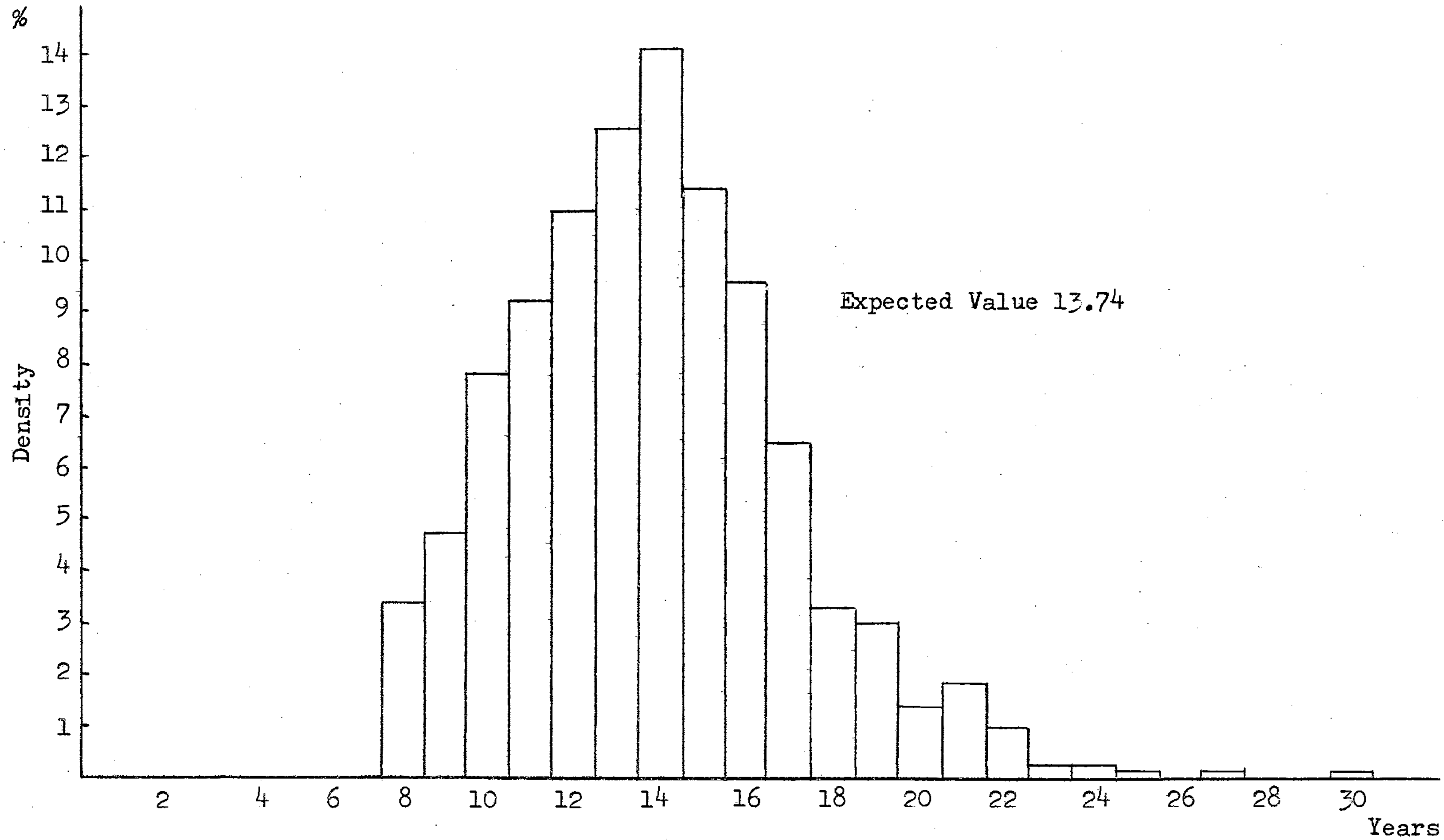

Figure 11. Tractor Replacement Interval Distribution Found Using Large Cost Criteria and 1000 Repititions

TABLE XX

A PRIORI DISTRIBUTION OF REPLACEMENTS USING LARGE COST CRITERIA. EXPECTED VALUE: 13.74 YEARS

Replacement Year	High Cost[A] Required $	% of Expected[B] Repair Cost Required %	Probability[C] of Getting	Density Distribution %	% of Dist. Remaining %	Simulated Density Distribution %
4	1456	738	0		100.00	
5	1305	601	0		100.00	
6	1157	502	0		100.00	
7	1012	423	.016	1.60	98.40	0
8	872	359	.038	3.73	94.67	3.3
9	736	306	.061	5.77	88.90	4.7
10	607	260	.088	7.82	81.08	7.8
11	484	222	.117	9.49	71.59	9.2
12	371	189	.164	11.74	59.85	10.9
13	268	162	.211	12.63	47.22	12.5
14	177	139	.257	12.14	35.08	13.9
15	101	122	.303	10.62	24.46	11.3
16	41	108	.331	8.10	16.36	9.5
17	0	100	.360	5.89	10.47	6.4
18	0	100	.360	3.77	6.70	3.2
19	0	100	.360	2.41	4.29	2.8
20	0	100	.360	1.54	2.75	1.3
21	0	100	.360	.99	1.76	1.6
22	0	100	.360	.63	1.13	.9
23	0	100	.360	.41	.72	.2
24	0	100	.360	.26	.46	.2
25	0	100	.360	.17	.29	.1
26	0	100	.360	.10	.19	0

TABLE XX (Continued)

Replacement Year	High Cost[A] Required $	% of Expected[B] Repair Cost Required %	Probability[C] of Getting	Density Distribution %	% of Dist. Remaining %	Simulated Density Distribution %
27	0	100	.360	.07	.12	.1
28	0	100	.360	.04	.08	0
29	0	100	.360	.03	.05	0
30	0	100	.360	.05	0.00	.1

[A]Taken from Table VII.

[B]High cost required, divided by expected repairs given in Table III.

[C]Taken from Table XVII.

density distribution without resorting to simulation as is done in Table XX. From Table VII the large repair cost required to make trading economical can be found for each year of tractor life. Table III gives the expected repair cost for each year. The high cost divided by expected repair cost gives the size of cost required. The required cost is given as a per cent of expected cost. The probability of getting a sufficient or larger cost can be found in Table XVII. Given the probability of getting the required cost in any year, a density distribution of replacement intervals can be found.

Through six years no tractors are replaced. In year seven, the probability of getting a sufficient cost is .016. Therefore, over a number of years 1.6 per cent of the tractors will be replaced in year seven. If 1.6 per cent are replaced in year seven, 98.4 per cent (100 - 1.6) are not replaced. In year eight, the probability of getting a sufficiently large cost is .038. Therefore, over a number of years, .038 of the eight year old tractors composing 98.4 per cent of the original number of tractors will be replaced. The percentage of the original number of tractors replaced in year eight is .038 x 98.4 = 3.73 per cent. If 3.73 per cent of the original tractors are replaced in year eight, after year eight there are 94.67 per cent (98.4 - 3.73) of the tractors to be replaced. By continuing the procedure, a density distribution of replacement intervals can be found. Also, included in Table XX is the simulated density distribution which can be compared with the constructed distribution. The two distributions are almost identical.

It is also possible to derive density functions using the other replacement criteria but some restricting assumptions must be made.

For any of the criteria requiring averaging, it must be assumed that all components of the average except for the final value must equal their expected values. While calculations for finding the derived density functions may be tedious, they do lend credence to the distributions found using simulations.

Summary

In this chapter, a simulation routine was devised for evaluating alternative rules of thumb which could be used to implement the theoretical replacement criterion. The replacement criterion is the equating of current machine marginal cost and the proposed replacement's minimum amortized average cost. In a real world situation, costs do not behave in an orderly manner, causing application of the theoretical model to lead to costly replacement decisions.

Rules of thumb tested using simulation were: First, trading only when expected economic life expires. Second, trading when a selected average of marginal costs is greater than the minimum average cost of the proposed replacement. Third, trading when a sufficiently large cost occurs.

Simulation results indicate that over the lives of a number of tractors, use of economic life as the replacement procedure offers nearly as low an average cost as any other rule of thumb. However, other replacement rules offer advantage to farmers who own few tractors in a lifetime.

Other replacement procedures might be proposed and evaluated using simulation. Although a $6100 tractor was used in the simulation analysis, any tractor size could be used. In addition, it is not necessary that the proposed replacement be a duplicate of the existing machine.

CHAPTER VI

OPTIMAL REPLACEMENT INTERVALS FOR AUTOMOBILES AND COMBINES

The empirical applications so far have dealt only with farm tractors. However, the theory developed is applicable to most farm machinery, trucks, and automobiles. In this chapter, the model is applied to cars and combines.

Automobile Replacement

Cost Components

Efficient use of replacement models in a dynamic situation requires much information. As with tractors, repairs comprise the most unpredictable cost and have a major part in determining optimum automobile ownership intervals. Because of data limitations, only a deterministic replacement model using discreet cost data for a specific automobile will be considered.

The situation chosen for analysis is a $3000 automobile being driven 12,000 miles per year. Since a specific automobile is being considered, only the discreet data given in Table XXI are required. As with machinery, automobile costs are divided into fixed and variable portions. The primary component of fixed cost is depreciation. The depreciation schedule given in Table XXI varied only slightly from a

TABLE XXI

YEARLY COST COMPONENTS FOR A $3000.00 AUTOMOBILE DRIVEN 1000 MILES PER MONTH

Age	Automobile Value After Yr. 1 (Dollars)	Depreciation (Dollars)	Repair (Dollars)	Interest (Dollars)	Tag (Dollars)	Insurance (Dollars)	Housing, Fuel (Dollars)
1	1999.00	1001.00[1]	117.59	159.92	55.50	181.00	429.80
2	1410.00	589.00	173.26	112.80	50.00	170.00	429.80
3	990.00	420.00	256.46	79.20	45.05	163.50	429.80
4	710.00	280.00	304.60	56.80	40.60	157.00	429.80
5	480.00	230.00	401.21	38.40	36.59	150.50	429.80
6	310.00	170.00	430.00	24.80	32.98	116.00	429.80
7	180.00	130.00	445.00	14.40	29.73	116.00	429.80
8	90.00	90.00	460.00	7.20	26.81	116.00	429.80
9	40.00	50.00	475.00	3.20	24.18	116.00	429.80
10	20.00	30.00	485.00	1.60	21.81	116.00	429.80
11	0.00	20.00	495.00	0.00	21.24	116.00	429.80
12	0.00	0.00	500.00	0.00	21.24	116.00	429.80
13	0.00	0.00	505.00	0.00	21.24	116.00	429.80
14	0.00	0.00	510.00	0.00	21.24	116.00	429.80
15	0.00	0.00	515.00	0.00	21.24	116.00	429.80
16	0.00	0.00	520.00	0.00	21.24	116.00	429.80
17	0.00	0.00	525.00	0.00	21.24	116.00	429.80
18	0.00	0.00	530.00	0.00	21.24	116.00	429.80
19	0.00	0.00	535.00	0.00	21.24	116.00	429.80
20	0.00	0.00	540.00	0.00	21.24	116.00	429.80

[1]First year depreciation includes an excise tax of $159.00.

schedule presented in a Department of Transportation publication.[1] Although taken from a different source, little difference was found when the depreciation data in Table XXI was compared with a depreciation schedule constructed of information taken from the N.A.D.A. Official Used Car Guide.[2] The depreciation schedule approached an average of market depreciation schedules using wholesale and retail prices.

The depreciation schedule has both direct and indirect effects on automobile costs. Depreciation charges directly affect yearly costs. Interest on investment is charged on undepreciated investment and fast depreciation leaves a smaller undepreciated balance upon which to charge interest. By year eleven the car was depreciated out, indicating that after year eleven there were no depreciation or interest charges included in yearly costs.

Also considered as fixed costs are tags, taxes, housing, and insurance. Of these costs, housing charges are assumed constant each year, the tax is a one-time cost, and tags and insurance are decreasing costs. Housing costs include indirect charges for the owner's garage, parking fees, and toll charges. In Table XXI, housing costs comprise $134 of the $429.80 charged for fuel and housing. The tax is a one-time Federal Manufacturer's excise tax paid when the car is purchased. For computational ease, the $159 tax charge is included as a component of first year depreciation. Actual depreciation the first year is $842,

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[1] U.S. Department of Transportation, Federal Highway Commission, Cost of Operating an Automobile, by E. M. Cope and L. L. Liston (Washington: 1968), p. 9.

[2] N.A.D.A. Official Used Car Guide. (Washington: National Automobile Dealers Used Car Guide Co., 1967).

(1001 - 159). Tag charges are taken from an Oklahoma tax rate sheet for automobiles.[3] The first tag costs $55 and in year eleven the minimum charge of $21.24 is reached. Insurance coverage includes a $50,000 combined public liability, property damage, and comprehensive for the entire car life. In addition, $50 deductible collison insurance is included for the first five years.

Variable costs, in contrast to the fixed costs discussed above, are a function of the amount of use. Included as variable costs are repairs, fuel, and lubricants. The repair cost schedule was extrapolated from information obtained from the U. S. Department of Transportation.[4]

Usually, older automobiles are relegated to use as second cars and second cars are usually not used for long trips requiring an extremely dependable automobile. Therefore, it is usually not necessary to maintain the car at an exceedingly high level. However, for the purposes of this analysis, it is assumed that the car is to provide an identical service each year throughout its life. Maintaining an older car at a high level requires large maintenance and repair expenditures. Also, because of the inclusion of a $40 annual charge for tire replacement and accessories, repair costs may appear to be excessive.

Fuel and lubricant costs vary with the number of miles driven. However, it is assumed in this study that the automobile is driven the same number of miles each year. Therefore, fuel and lubricant costs

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[3]Oklahoma Tax Commission Rate Sheet for Automobiles.

[4]U.S. Department of Transportation, p. 9.

are constant each year. The car is assumed to require a gallon of gas for each 14.3 miles, with each gallon of gas costing 31.9 cents. Each year $28.15 is spent on oil and grease, pushing the yearly total cost for fuel and lubricants to $295.80. The addition of the $134 housing charge enlarges the total fuel, lubricant, and housing charge recorded in Table XXI to $429.80.

Of costs discussed to this point, only repairs increase over time and repairs have a marginal increase of only five dollars per year after year ten. The small increase is nearly equivalent to saying that repair costs have reached a steady state. Therefore, for it to be possible to have a minimum marginal cost, the combined total of other yearly costs must at some point decrease by less than five dollars per year. It is impossible for average cost to reach a minimum until marginal cost starts rising. It is apparent that if dependability and prestige considerations are ignored, the economic optimum interval of ownership is a considerable period of time. In addition, if repair costs reached a certain level and became constant, the optimum ownership interval considering only out-of-pocket costs, would be infinite.

There are, however, some non-quantifiable increasing costs which should be considered. As with tractors, there are dependability and prestige factors. It may be argued that dependability is a real cost subjectively evaluated while prestige is a subjective cost, subjectively evaluated. Dependability charges may be considered real in the sense that old cars are more likely to break down, causing time and monetary loss. Future automobiles may contain more intricate working parts, allowing fewer operators to have the mechanical knowledge to handle breakdowns. Therefore, car owners of the future may place higher values on dependability.

Prestige is defined as a reputation based on high achievement. Some car owners feel new high-powered automobiles illuminate their level of economic attainment. Therefore, those individuals desiring to conspicuously consume, incur larger than necessary costs in order to frequently purchase a new car. Prestige is a non-economic consideration and it is impossible to determine an exact function evaluating prestige for any or all individuals. If, however, an individual car owner is willing to specify a particular prestige function which he is careful to follow, then it is possible to determine an optimum replacement interval for that individual.

Figure 12 illustrates cost components for automobiles. Along with the real costs, six linear subjective cost functions are illustrated. Beginning with A which is a $25 increment, the functions progress to F which is a $150 increment per year. From Figure 12 some perspective may be gained as to the size relationship between the subjective and all other costs. After several years, the subjective cost becomes dominant regardless of the yearly increment used, therefore, selection of a yearly cost increment is crucial for determining optimum replacement intervals.

It is possible to determine a breakeven yearly prestige or dependability cost increment associated with optimally trading any age of car, if the general form of the prestige or dependability function is known or assumed. Coefficients for the function can be determined such that in any year the amortized average cost will be a minimum. The procedure can be used to determine how much car owners who trade in any given year are implicitly paying for dependability and prestige.

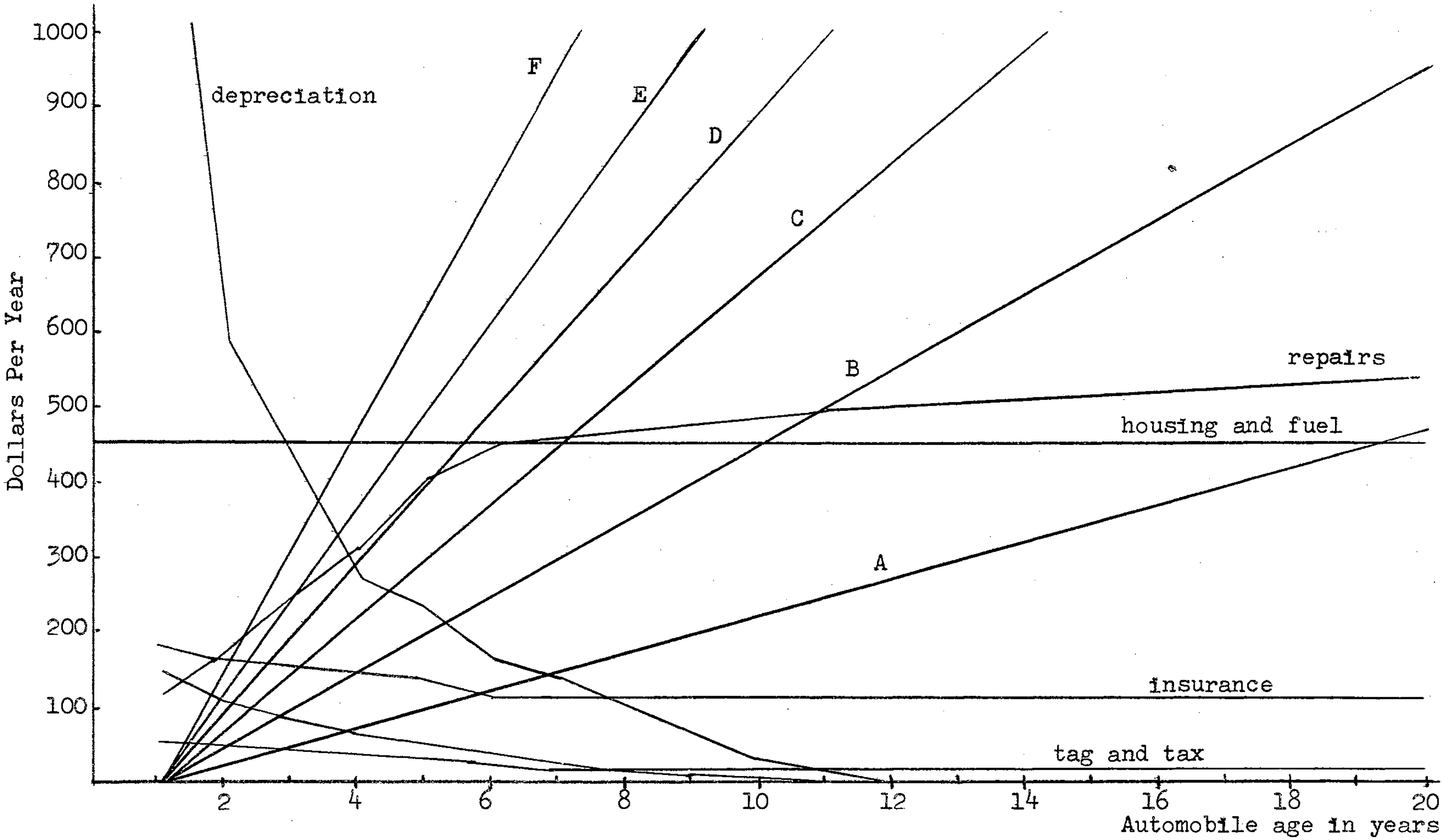

Figure 12. Cost Components for a $3000 Automobile Driven 12,000 Miles Per Year With Various Subjective Charges

Since prestige and dependability are not actual costs, these costs may be excluded and alternative intervals compared by using amortized average annual out-of-pocket costs. The minimum amortized average cost for a $3000 automobile is $1250.15 in year forty-one. The amortized average cost of trading cars every ten years is shown in Table XXII as $1359.60. The difference between $1359.60 and $1250.15, $109.45, is the cost per year of trading every ten years as opposed to every forty-one years. Over a period of ten years, the additional cost of trading in year ten as opposed to the economic optimum is $1094.50. Trading every five years, as opposed to the forty-one year optimum, causes the car owner to incur an extra $258.31, (1508.46 - 1250.15) per year.

Optimal Ownership Intervals

The analytical procedures used for finding marginal costs and time corrected average costs were outlined in Chapter II and are again used in constructing Table XXII. Table XXII includes the simple average, marginal, and amortized average costs for years one through twenty. No subjective costs are included in the table, therefore, the cost figures presented may be considered out-of-pocket costs.

Repairs, the only increasing cost, increase so slightly after the first several years that there is no minimum average or amortized average cost in the twenty year span considered in Table XXII. If only quantifiable costs are considered, the results suggest extremely long ownership intervals. Where subjective costs do not play an important part, such as for second cars, Table XXII may provide usable results.

Also illustrated in Table XXII is the computational difference between simple and amortized averages. Amortized averages are determined

TABLE XXII

SIMPLE AVERAGE, MARGINAL, AND AMORTIZED AVERAGE COSTS FOR YEARS ONE THROUGH TWENTY FOR A $3000 AUTOMOBILE DRIVEN 1000 MILES PER MONTH

Automobile Age (Years)	Simple Average Cost (Dollars)	Cost in Year (Dollars)	Amortized Average Cost (Dollars)
1	1944.81	1944.81	1944.81
2	1734.83	1524.86	1742.91
3	1621.23	1394.01	1635.44
4	1533.12	1268.80	1554.07
5	1483.80	1286.50	1508.46
6	1437.09	1203.58	1466.90
7	1398.21	1164.93	1433.06
8	1364.66	1129.81	1404.55
9	1335.05	1098.18	1380.02
10	1309.97	1084.21	1359.60
11	1289.25	1082.04	1342.92
12	1270.73	1067.04	1328.38
13	1255.45	1072.04	1316.46
14	1242.70	1077.04	1306.57
15	1231.99	1082.04	1298.30
16	1222.93	1087.04	1291.34
17	1215.23	1092.04	1285.43
18	1208.67	1097.04	1280.40
19	1203.06	1102.04	1276.10
20	1198.25	1107.04	1272.40

by the discount rate, whereas simple averages are in no way affected by the discount rate. An amortized average is a series of equal yearly costs having a cumulative present value equal to the the total present value of the series of marginal costs. When finding the cumulative present value of a marginal cost series, costs for early years have a much larger effect on the total than do later costs. Since the cumulative present value of the marginal cost series is required to determine the amortized average cost, the large first year marginal cost is very important. Large marginal costs in the early years cause the amortized average cost to be larger than the simple average, whereas if early marginal costs had been small, the simple average would tend to lie above the amortized average.

As stated above, the optimum replacement interval is well over twenty years if no subjective cost is considered. Table XXIII contains optimal replacement intervals under various alternative linear subjective cost functions. (To clearly understand Table XXIII in its proper perspective, refer to Figure 12 which illustrates the relationship of alternative subjective costs to the other costs.) Table XXIII also includes amortized average out-of-pocket costs corresponding to the optimal replacement interval year. The difference between amortized costs of including and excluding subjective costs may be considered the average yearly cost of the linear subjective cost. Only if an individual is willing to accept a particular linear subjective cost function and the other costs are as predicted may the optimal replacement intervals in Table XXIII indicate when the car should be traded.

TABLE XXIII

OPTIMUM AUTOMOBILE REPLACEMENT INTERVALS UNDER VARIOUS LINEAR SUBJECTIVE COST ASSUMPTION

Subjective Cost Per Year (Dollars)	Optimum Ownership Interval (Years)	Amortized Cost Including Subjective Cost (Dollars)	Amortized Cost With No Subjective Cost (Dollars)
00	41	1250.15	
25	15	1438.16	1298.30
50	10	1553.16	1359.60
75	7	1635.09	1433.06
100	5	1693.11	1508.46
125	4	1729.57	1554.07
150	4	1764.67	1554.07

Subjective Costs of Trading in Any Year

By successive approximation, it is possible to determine for any year the subjective cost function which is sufficient to make that year the optimal replacement point. Table XXIV contains the yearly cost increments for a linear subjective cost function along with other transformations of the costs. A non-linear function could be used and would only make computations more difficult.

The amortized average cost in year one is \$1944.81. Therefore, for year one to be the optimal trading interval, it is necessary that amortized average cost in year two be greater than \$1944.81. A subjective cost of \$420 in year two causes the amortized average in year two to be \$1944.83, making one year the optimal ownership interval. If in year two the appropriate subjective cost were \$420 or more, then it would be optimal to trade every year. How the subjective cost is split between dependability and prestige is not important unless there is a different functional form for each.

In year one the subjective cost is zero by definition. In year two the subjective cost is $(t-1)x$ or $1x$. It is relatively easy to determine x such that the amortized average in year two is greater than the average for year one. However, to make year two the optimal interval it is necessary to take subjective costs in years two and three into account. If ten dollars is added to marginal costs in year two, twenty dollars must be added in year three. Therefore, determination of the subjective cost increment in year three sufficient to make year two the optimal ownership interval requires successive approximations.

The subjective cost increment which makes it optimal to trade every two years is \$230. If \$230 were added to marginal costs in year

TABLE XXIV

AUTOMOBILE SUBJECTIVE COST INCREMENTS AND RESULTING COSTS WHICH WOULD BE SUFFICIENT FOR TRADING IN ANY YEAR

Car Age (Years)	Amortized Average Without Dependability (Dollars)	Subjective Cost Increment (Dollars)	Amortized Cost In Year (Dollars)	Amortized Cost in Previous Year (Dollars)	Marginal Cost (Dollars)	Cost/Mile Including Subjective Cost (Dollars)	Cost/Mile Without Subjective Cost (Dollars)
2	1742.91	420.00	1944.83	1944.81	1524.86	0.1621	0.1452
3	1635.44	230.00	1853.65	1853.49	1394.31	0.1545	0.1363
4	1554.07	179.00	1805.38	1805.26	1268.80	0.1504	0.1295
5	1508.46	104.00	1700.00	1700.09	1286.50	0.1417	0.1257
6	1466.90	97.00	1687.71	1687.57	1203.58	0.1406	0.1222
7	1433.06	82.00	1653.94	1653.56	1164.93	0.1378	0.1194
8	1404.55	71.00	1624.55	1624.31	1129.81	0.1354	0.1170
9	1380.02	63.00	1599.95	1599.76	1098.18	0.1333	0.1150
10	1359.60	54.00	1568.65	1568.53	1084.21	0.1307	0.1133
11	1342.92	46.00	1537.94	1537.68	1082.04	0.1282	0.1119
12	1328.38	41.00	1516.81	1516.74	1067.04	0.1264	0.1107
13	1316.46	35.00	1489.37	1489.24	1072.04	0.1241	0.1097
14	1306.57	30.00	1464.76	1464.67	1077.04	0.1221	0.1089
15	1298.30	26.00	1443.76	1443.67	1082.04	0.1203	0.1082
16	1291.34	23.00	1427.14	1426.98	1087.04	0.1189	0.1076
17	1285.43	20.00	1409.51	1409.43	1092.04	0.1175	0.1071
18	1280.40	18.00	1397.26	1397.10	1097.04	0.1164	0.1067
19	1276.10	16.00	1384.41	1384.27	1102.04	0.1154	0.1063
20	1272.40	14.00	1370.92	1370.87	1107.04	0.1142	0.1060

two the amortized cost for year two is $1853.49. Using the linear subjective cost assumption requires an addition of $230 to marginal costs in year two and $460 to marginal cost in year three. The amortized average cost for year three is $1853.65. Therefore, trading in year two is optimal. The same procedure is used to find the subjective cost required for any year to be the optimal trading point.

The final two columns of Table XXIV are costs per mile assuming the car is driven 12,000 miles per year. For trading yearly, the inclusion of subjective costs cause an increase in costs per mile of 1.49 cents, from 14.52 to 16.21 cents. If cars were traded every twenty years, the $14.00 subjective cost increment adds .82 cents per mile to the cost. If a car were driven for forty-one years, the per mile cost is 10.41 cents, only .19 cents less than the cost for a twenty year trading interval. If one were willing to spend one cent per mile for dependability and prestige, trading could occur every ten years with a total cost of 11.33 cents per mile.

Purchasing Used Automobiles

Good used cars are thought to be wise purchases because of lower investment. When subjective costs are low or nil (Subjective cost level 00 in Table XXV), the purchase of used automobiles lowers yearly costs. In Table XXV, the purchase of a three-year-old car saves $77.09 (1250.15 - 1173.06) per year. The reason for the small savings is the assumed trading cost. The difference between wholesale and retail car values, $350, is added to the purchase price of the used car.[5] Also

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[5]This value was computed as the difference between wholesale and retail values in NADA Official Used Car Guide (Washington: National Automobile Dealers Used Car Guide Co., 1967).

TABLE XXV

MINIMUM AMORTIZED AVERAGE COSTS AND OPTIMAL REPLACEMENT INTERVALS FOR USED AUTOMOBILES

Subjective Cost Levels	Car Age When Purchased							
	New		One		Two		Three	
	Cost[1]	Age[2]	Cost[1]	Age[2]	Cost[1]	Age[2]	Cost[1]	Age[2]
00	1250.15	41	1221.25	41	1193.13	36	1173.06	33
25	1438.16	15	1418.61	14	1397.22	13	1386.08	13
50	1553.16	10	1546.50	10	1536.69	10	1539.51	10
75	1635.09	7	1643.80	7	1647.79	7	1667.97	8
100	1693.11	5	1719.52	6	1739.64	6	1780.77	7
125	1729.57	4						
150	1764.67	4						

[1]Minimum amortized average annual cost in dollars.

[2]Optimal car age at time sold in years.

noteworthy in Table XXV is the optimal replacement intervals for used cars. When a three-year-old car is purchased, it is kept until 33 years old, whereas a new car optimum replacement interval is 41 years.

When subjective costs are included the amortized average costs begin to rise. When the subjective cost increment is somewhere between \$50 and \$75 per year, the amortized average costs are the same for both a new and a three-year-old automobile. Therefore, if subjective costs are considered to be of importance, purchasing used cars loses much of its cost advantage.

With a fifty dollar subjective cost increment, minimum expected marginal cost occurs in year four. Therefore, if a three-year-old car is purchased, the low fourth year marginal cost is supplanted by a relatively large cost. The large cost is caused by the inclusion of the trading cost in first year depreciation. Since the minimum marginal cost was replaced by the maximum marginal cost of the series, minimum amortized average costs for buying a three-year-old car are greater than those for buying a two-year-old car.

When using a fifty dollar subjective cost increment, cars should be traded when they are ten years old regardless of their age when purchased. However, for a seventy-five dollar subjective cost increment, minimum amortized costs increase as car purchase age increases from new to three-years-old and the optimum ownership interval increases from seven to eight years.

Combine Replacement

For the amount of time used per year, combines cost farmers more than most other farm machinery. Seldom does a farmer operate a combine on his own land more than 200 hours per year. Since a machine breakdown during harvest could prove costly, combines have extremely high dependability costs. This is especially true if no custom machines are readily available.

Cost Equations

Equations, similar to those used for tractors, are available for combines. To determine optimal combine replacement patterns, appropriate cost equations from previous studies are inserted into the replacement model.[6] Initially, the cost equations are presented, then the resulting replacement intervals and costs are discussed.

Combines depreciate more rapidly than do tractors. This faster depreciation may also reflect the rapid development of technology in grain harvesting and may indirectly reflect farmers' feelings that old combines are undependable and should be avoided. Equation (6-1) indicates that

$$W_{1t} = .651(.900)^t X_1 \tag{6-1}$$

depreciation the first year is 41.4 per cent, whereas for tractors the first year depreciation was 37 per cent.

Equation (6-1) gives the remaining farm value for any year. Depreciation is the change in remaining farm value from one year to the

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[6] Repairs and depreciation equations were taken from William E. Larsen and Wendell Bowers, "Engineering Analysis of Machinery Costs," presented at the 1965 Annual Meeting of the American Society of Agricultural Engineers, University of Georgia. Fuel, lubricants, housing, and insurance cost factors were taken from University of Illinois College of Agriculture, Cooperative Extension work, Costs of Owning and Operating Farm Equipment, by Wendell Bowers (Urbana, 1966), pp. 3-4.

next, and is given in Equation (6-2).

$$Y_{1t} = W_{1(t-1)} - W_{1t}. \qquad (6\text{-}2)$$

Interest, taxes, housing, and insurance costs are computed just as they are for tractors. These costs are a percentage of the remaining farm value. (For the actual calculation, see Equation (3-3).)

Two costs, fuel and labor, remain constant year-after-year under the assumptions of this model. They have no affect on the replacement interval, but are necessary for accurate cost representation. These equations are exactly the same as for tractors given in Equations (3-4) and (3-5).

Before a minimum average cost can be attained, it is necessary that marginal costs be increasing. Marginal costs do not begin increasing until the total of the two increasing costs is larger than the total of all decreasing costs. The large combine ownership costs extend the optimal ownership interval for combines by contributing a large decreasing cost.

All costs other than repairs are reasonably easy to predict, thus repairs with their highly random nature cloud cost and replacement calculations. The expected accumulated repairs equation for combines is:

$$W_{2t} = .014345\ (X_2 t)^{1.4}\ X_1\ (.001). \qquad (6\text{-}3)$$

Repairs in any year t given tractor cost and use per year are:

$$Y_{5t} = W_{2t} - W_{2(t-1)}. \qquad (6\text{-}4)$$

The above costs are tabulated in Table XXVI.

TABLE XXVI

COST COMPONENTS, MARGINAL, AND AMORTIZED AVERAGE COSTS FOR A $10,000 COMBINE USED 200 HOURS PER YEAR

Combine Age (Years)	Depreciation (Dollars)	Repairs (Dollars)	Tax, Housing Insurance (Dollars)	Labor (Dollars)	Fuel (Dollars)	Interest (Dollars)	Cost in Year (Dollars)	Amortized Average Cost (Dollars)	Cost Per Hour, 4½A/hr. (Dollars)
1	4141.00	238.86	263.65	300.00	316.00	468.72	5728.23	5728.23	6.36
2	585.90	391.49	237.29	300.00	316.00	421.85	2252.53	4057.22	4.51
3	527.31	481.67	213.56	300.00	316.00	379.66	2218.20	3490.74	3.88
4	474.58	551.49	192.20	300.00	316.00	341.70	2175.98	3198.97	3.55
5	427.12	610.02	172.98	300.00	316.00	307.53	2133.65	3017.38	3.35
6	384.41	661.11	155.69	300.00	316.00	276.77	2093.98	2891.50	3.21
7	345.97	706.86	140.12	300.00	316.00	249.10	2058.04	2798.10	3.11
8	311.37	748.54	126.11	300.00	316.00	224.19	2026.21	2725.53	3.03
9	280.23	787.01	113.49	300.00	316.00	201.77	1998.50	2667.31	2.96
10	252.21	822.83	102.15	300.00	316.00	181.59	1974.78	2619.50	2.91
11	226.99	856.46	91.93	300.00	316.00	163.43	1954.81	2579.57	2.87
12	204.29	888.21	82.74	300.00	316.00	147.09	1938.33	2545.78	2.83
13	183.86	918.35	74.46	300.00	316.00	132.38	1925.05	2516.90	2.80
14	165.48	947.07	67.02	300.00	316.00	119.14	1914.70	2592.03	2.77
15	148.93	974.53	60.32	300.00	316.00	107.23	1907.01	2470.49	2.74
16	134.04	1000.89	54.28	300.00	316.00	96.51	1901.71	2451.73	2.73
17	120.63	1026.24	48.86	300.00	316.00	86.85	1898.58	2435.34	2.71
18	108.57	1050.68	43.97	300.00	316.00	78.17	1897.39	2420.98	2.69
19	97.71	1074.30	39.47	300.00	316.00	70.35	1897.94	2408.36	2.68
20	87.94	1097.17	35.62	300.00	316.00	63.32	1900.04	2397.25	2.66
21	79.15	1119.34	32.05	300.00	316.00	56.99	1903.53	2387.46	2.65
22	71.23	1140.87	28.85	300.00	316.00	51.29	1908.24	2378.82	2.64
23	64.11	1161.81	25.96	300.00	316.00	46.16	1914.04	2371.18	2.63
24	57.70	1182.20	23.37	300.00	316.00	41.54	1920.80	2364.44	2.63
25	51.93	1202.07	21.03	300.00	316.00	37.39	1928.42	2358.47	2.62

The final cost is the arbitrarily determined dependability cost. As with automobiles, the subjective cost is very important but for a different reason. For automobiles it is considered mainly a prestige cost while for combines it is mainly a dependability cost. Farmers dislike taking into the field a machine in which they do not have confidence. No matter how well maintenance is carried out, most farmer's feel there is a larger chance for breakdown with an older machine. Because of the importance of the dependability charge for combines, results of several different dependability assumptions are used. They are all linear with the marginal dependability charge increasing at rates varying between 0 and $150 per year.

Empirical Results

Table XXVI gives each individual cost component as well as total cost per year and amortized average cost per year excluding any dependability cost. These costs are for combines used 200 hours per year. The minimum average cost per year occurs sometime after the 25 years listed. This means the dependability costs are crucial in replacement interval determination. Costs per acre assuming four and one-half acres per hour are also given.

Table XXVII lists optimum replacement interval and amortized costs per year for four different rates of use and various dependability charges. Per acre costs for situations both including and excluding the dependability charge are given. The assumption is made that four and one-half acres per hour can be harvested.

There has been considerable discussion of whether it is profitable for a farmer to employ custom machines or harvest his own grain. The

TABLE XXVII

OPTIMAL REPLACEMENT INTERVALS, COSTS PER YEAR, AND PER ACRE COSTS BOTH INCLUDING AND EXCLUDING SUBJECTIVE COSTS FOR A $10,000 COMBINE FOR SEVERAL USE SITUATIONS. ASSUME 4.5 ACRES CUT PER HOUR

Hours Per Year (Acres)	Subjective Cost Increment (Dollars)	Optimum Interval (Years)	Amortized Average Cost (Dollars)	Per Acre Cost Including Subjective Cost (Dollars)	Per Acre Cost Excluding Subjective Cost (Dollars)
80 (360)	00	25+	1463.20	4.06	4.06
	25	25+	1668.83	4.63	4.06
	50	24	1874.25	5.21	4.09
	75	17	2053.65	5.70	4.41
	100	14	2198.42	6.11	4.64
	200	8	2599.19	7.22	5.50
120 (540)	00	25+	1740.49	3.22	3.22
	25	25+	1946.12	3.60	3.22
	50	22	2148.66	3.98	3.28
	75	16	2316.44	4.29	3.47
	100	13	2452.60	4.54	3.63
	200	8	2832.73	5.25	4.10
160 (720)	00	25+	2040.12	2.83	2.83
	25	25+	2245.74	3.12	2.83
	50	20	2440.83	3.39	2.90
	75	15	2595.86	3.61	3.02
	100	12	2722.36	3.78	3.14
	200	7	3080.22	4.28	3.53

TABLE XXVII (Continued)

Hours Per Year (Acres)	Subjective Cost Increment (Dollars)	Optimum Interval (Years)	Amortized Average Cost (Dollars)	Per Acre Cost Including Subjective Cost (Dollars)	Per Acre Cost Excluding Subjective Cost (Dollars)
200	00	25+	2358.47	2.62	2.62
(900)	25	25+	2564.11	2.85	2.62
	50	17	2745.53	3.05	2.71
	75	13	2887.42	3.20	2.80
	100	11	3003.52	3.34	2.87
	200	7	3336.83	3.71	3.11

per acre costs given in Tables XXVI, XXVII, and XXVIII, may shed some light on the problem. In Table XXVI the cost per acre drops considerably the first few years and continues to drop even if the machine is kept longer than 25 years. If the custom rate is $3.50 per acre, it is profitable for him to own his own machine and trade no more often than every five years. If the custom rate is $3 per acre, it is profitable for him to trade no more often than nine years. It must be remembered that this assumes harvesting four and one-half acres per hour for 200 hours or 900 acres per year with the machine.

A cursory analysis of the per acre costs in Table XXVII indicate that if combines are kept an optimum length of time, costs per acre may be reasonably small. This is true even with a rather large dependability charge, especially if the machine is operated 160 or 200 hours per year. The difference between the two per acre costs for each situation is a measure of the dependability cost per acre sufficient to induce trading in the indicated year.

The average farmer may not consider owning a combine longer than ten years. Because of this, Table XXVIII is included. Only if the machine is kept nine or ten years and operated 200 hours per year is it profitable to own a machine if the custom rate is $3 per acre.

Table XXVIII and other per acre cost figures are constructed assuming four and one-half acres harvested per hour. If five acres are harvested per hour, costs will be substantially lower. For 200 hours operation per year, costs will drop nearly 50 cents per acre the first few years and 25 cents per acre near the end of the 25 year period. A farmer would not realize all the benefits of a shift from $4\frac{1}{2}$ to 5 acres per hour because it would then take him fewer hours to harvest a

TABLE XXVIII

PER ACRE AVERAGE COSTS FOR SELECTED HOURS USE PER YEAR AND VARIOUS MACHINE LIFE'S. NO DEPENDABILITY COST. LABOR CHARGE: $1.50 PER HOUR. ASSUME 4.5 ACRES CUT PER HOUR

Combine Age (Years)		Use per year			
	Acres:	360A	540A	720A	900A
	Hours:	80	120	160	200
2		9.62	6.76	5.35	4.51
4		7.04	5.08	4.12	3.55
6		6.06	4.45	3.67	3.21
8		5.50	4.10	3.42	3.03
10		5.12	3.86	3.26	2.91

given acreage. Costs will, however, be lower.

Summary

For a $3000 automobile driven 12,000 miles per year, the optimal replacement interval is 41 years when no dependability or prestige costs are included. A subjective cost increment of $100 per year shortens the optimal replacement interval to 5 years. It is possible to find a subjective cost increment which makes it optimal to trade for a new or used car in any year. For example, to make it optimal to trade every year, a subjective cost increment of $420 per year or $1.15 per day must be used. If an owner feels it is worth an additional $1.15 per day to have a new car every year, then trading every year is the optimal pattern.

If subjective costs are not a consideration, then used cars are a good buy. But, the savings accrued will probably not be more than $100 per year over the optimum replacement interval. When subjective costs are considered, purchasing used cars may not yield any savings. Therefore, before purchasing used cars, buyers should consider the size of subjective cost they are going to charge against the car.

Combines used a large number of years can greatly decrease per acre costs. This occurs because of the large ownership costs associated with combines. The optimum ownership interval for a 10,000 combine used 200 hours per year is 18 years with an average cost of $1897.39 per year. The per acre cost of owning a $10,000 combine to harvest 360 acres per year is $4.06 per acre. If farm size is increased to 900 acres, the per acre cost is lowered to $2.62 per hour.

Because many farmers have too little land to economically own their own combine, they are forced to make a decision. Should he accept the larger per acre cost for the convenience of owning his own machine or should he hire a custom cutter? In order to have their own machine, many farmers will accept the higher costs. There is another alternative. The farmer may buy the machine, cut his own grain, and then become a custom cutter. In this way he effectively lowers per acre costs for his home farm, but at the same time has the convenience of owning his own machine.

CHAPTER VII

SUMMARY AND CONCLUSIONS

The objective of this study was to develop a generalized machinery replacement model which could be altered to handle designated situations. With the use of the model, general implications concerning the effects of various factors on the optimal replacement interval were determined. Replacement studies are relevant for two basic reasons. First, farm planning of optimum resource levels requires some knowledge of machinery investment over time. The optimal replacement interval provides a means of finding investment and average yearly costs. The second problem involves a farmer's yearly decision whether or not to replace his machine. The decision depends on machinery repairs, other costs, and external factors. The optimal replacement intervals are first based on expected values of repair costs, with other costs and external conditions held constant. Since costs and external conditions do not remain static in the real world, farmers must be provided some framework for making economically sound decisions in a dynamic environment. The second basic objective of this study was to provide farmers a rule of thumb which they can apply each year to their particular situation.

Results

It was found that with minor alterations in the replacement model, both of the above objectives could be achieved. Initially, the analysis dealt with optimal replacement intervals in a long-run planning situation. This required computing optimal replacement intervals and costs using expected costs. In a latter portion of the study, yearly replacement decision rules using uncertain repair costs were evaluated. Emphasized was the development of a criteria which could be applied to a stochastic situation.

Empirical implementation of the developed replacement model required estimates of machinery costs for repairs, taxes, insurance, and depreciation. The cost equations were generally taken from secondary sources. Also included were subjective costs which allowed for decreased machine dependability and/or loss of prestige. The subjective costs were assumed to be a linear function of machine age.

The optimal replacement interval occurs during the year of minimum amortized average cost if a machine is to be supplanted by a duplicate machine. For a $6100 tractor used 600 hours per year, the economic optimum is in year seventeen. This is true only if all external conditions such as farm size and fuel costs remain the same. If and when any of these factors do change, the optimal replacement interval changes also. The difficult part of this study is not the determination of an economic optimum, but ascertaining the effects of various external conditions on the economic optimum.

The replacement criterion used in handling changes in external conditions is as follows: When marginal costs of the presently owned machine first exceed the minimum amortized average cost of the proposed replacement, it is time to replace. The first external condition altered was yearly costs. When marginal costs were altered, the year in which marginal costs first rise above the minimum average cost was changed; therefore, the optimum replacement interval was altered. If marginal costs for a $6100 tractor are increased by 15 per cent, the optimal replacement interval is shortened from seventeen to fifteen years.

Deciding when to replace machinery is a yearly decision. Managers should take costs which have occurred, anticipate costs which will occur, and then analyze these costs to determine whether or not replacement is in order. In any given year, costs may jump above the minimum amortized average cost of the proposed replacement, but this does not necessarily call for a trade. In the following year, costs may drop considerably, and the average of the two years may be below the minimum amortized average cost of the proposed replacement.

Farmers face constantly changing costs and other external conditions and should, therefore, maintain flexibility in their decision making. Each year farmers should evaluate available information and make a replacement decision. Information required for making a decision each year whether to replace a machine includes the minimum amortized average cost of the proposed replacement. This figure is difficult for farmers to determine and can be provided by tables such as found in this study. Additional information required is the yearly marginal costs of the currently owned machine. Hopefully, managers will be able to make reasonably accurate estimates of repair costs for the following year. If the expected repair costs are high, the manager may be able to save money by trading before incurring the large cost. For the stochastic repair cost situation analyzed using simulation, one replacement procedure which could be used is averaging last year's marginal cost, this year's marginal cost, and next year's anticipated marginal cost. If the three-year average marginal cost figure were found to be greater than the minimum amortized average cost of the proposed replacement, the farmer should trade tractors.

For any one of a number of reasons, a farmer may wish to consider a replacement of a different size. This will, of course, alter the optimal interval. In the yearly decision framework just described, the analysis is not changed but the minimum average cost of the proposed replacement is altered.

Much has been made of trading at the optimal time. For various reasons, farmers may wish to trade either before or after the minimum cost point. It is possible to determine the opportunity costs associated with such trading patterns. If little value is placed on machine dependability, then the optimal interval is long. For a $6100 tractor with a $10 annual increment in the dependability charge, the optimal interval is twenty-nine years. But the opportunity costs of trading after only a ten year period are relatively small. If a high value were placed on dependability, say a $50 increment for the $6100 tractor, the optimal interval is eleven years and the opportunity cost of trading several years before or after this time is relatively large.

Purchase or sale of land affects the optimal farm tractor size. Land purchase places higher requirements on currently owned machinery and raises labor requirements. When both a different proposed replacement and higher marginal costs are inserted into the replacement model, it is impossible to say whether the optimal replacement interval will be longer or shorter, but it will most likely change. The model is still applicable, however, and can be used in making the yearly replacement decision.

It was possible to determine optimal replacement intervals for used machinery on a given farm, and ascertain the effects of investment credit and taxes on optimal replacement patterns. Additional uses of the model included studies of replacement strategies for automobiles and combines. Limited data on automobiles placed severe restrictions on the analysis. General cost equations were available for combines, and the combine replacement result can be applied to a considerable range of combine sizes and farm situations.

With low dependability cost increments per year, the optimal combine replacement interval was found to be long. Dependability costs are conceivably much higher for combines than for other farm machinery. The result of using a high dependability increment is a considerably shorter optimal replacement interval.

General Conclusions

Management of machinery so that the least possible cost is incurred implies few trades--long ownership intervals. Over time there is little lost by using the interval computed from single valued costs. However, with the knowledge farmers have of their particular situations, it is possible for them to make economical, yearly replacement decisions. Yearly decisions require adequate records, a knowledge of the tractor, and a table of minimum amortized average costs for proposed replacements.

In general, machinery average cost curves have relatively long segments for which the average cost is not far from the minimum. Thus, the opportunity costs of not trading machines at the precise optimal trading point are not large. Correspondingly, it was found that the opportunity cost of purchasing too large a tractor for a given farm is relatively small.

If small value is placed on dependability and prestige, the optimal ownership interval is long. Alternatively, high subjective costs shorten the optimal interval considerably. If, in addition, the benefits of tax savings opportunity costs and investment credit are considered, the optimal interval will be further shortened: How much depends on the particular situation.

When analyzing a farm, the most important factor in determining the optimal tractor is usually the labor charge. If a farmer has a full-time job off the farm and as a result his labor has a high opportunity cost, a relatively large tractor will be his optimal choice. The higher the charge made for labor, the larger the cost-minimizing tractor size for the farm. Thus, even though wage rates do not affect the optimal ownership interval when replacing a given tractor with an exact duplicate, they are very important for determining the optimal tractor size for a given farm.

Suggestions for Future Research

To make machinery replacement models more realistic and useful, considerable work should be done to improve machinery cost estimates. Tractors will be used as an example, but the points are applicable to most machinery.

It may be helpful to introduce additional variables into the cost equations. Such things as type of fuel and tractor model affect the cost of fuel required to do a given job. Also, tractor manufacturers and dealers provide guarantees of various types. The effect of such non-price considerations on machinery cost could be considerable. Thus, more exacting cost equations would enhance the development of accurate replacement procedures. However, if the desire is to develop truly generalized tables of minimum amortized average cost, there is a limit to how far it is practical to go in estimating costs. Perhaps at the present time it would be more appropriate to take existing cost equations and develop a set of generalized cost tables.

Declining efficiency is considered subjectively in the dependability charge. As a tractor becomes old, it requires more time to do a given job, thereby increasing labor costs. Another factor implicitly considered in this model is the effect of machinery improvements and innovations by manufacturers. When such technological improvements arise, there may develop an opportunity cost of not owning the improved equipment. Perhaps by directly including declining efficiency and improving technology, it would be possible to improve the predictive power of the cost equations.

In this study, when alternative tractor sizes were compared for a particular resource situation, a typical Panhandle farm was used. To make the results more generally applicable, it may be beneficial to include several typical resource situations.

When studying the optimal machinery inventory for a given farm, timeliness needs to be considered. Studying out-of-pocket costs without regard for subjective or opportunity costs may lead to purchasing too small a tractor. If done, farm income may suffer from not getting all fieldwork done at approximately the right time. It would be useful to develop some procedure for accurately evaluating timeliness.

In this study only tractors, combines, and automobiles were discussed. In order to analyze other machinery replacement problems, cost equations need to be developed and incorporated into a replacement model. Perhaps an ultimate goal of replacement studies is to develop a total replacement model that would at once determine the optimum total inventory of machinery for the farm and the optimum replacement pattern for the components thereof.

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APPENDIX A

TRACTOR REPAIR COST EQUATIONS

Bowers estimates total accumulated repairs as a per cent of new machine cost.[1]

$$TAR = .0012 \times D^{1.5} \qquad \text{(A-1)}$$

where TAR = total accumulated repairs as a per cent of new cost,

and D = total accumulated hours as a per cent of lifetime hours.

The use of Equation (A-1) requires estimates and assumptions about machine life. Bowers estimated tractor life as 12,000 hours. Studies also indicate that during these 12,000 hours total accumulated repairs equal 120 per cent of tractor list price. To arrive at Equation (3-6) given Equation (A-1) and the above assumptions, it is necessary to go through the procedure given below.

Equation (3-6) is a cumulative cost function dependent on total accumulated hours. The form of Equation (3-6) is given in Equation (A-2).

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[1]All assumptions and Equation (A-1) are taken from W. E. Larsen and W. Bowers, "Engineering Analysis and Machinery Costs." Paper presented at 1965 meeting American Society of Agricultural Engineers (June 1965), Appendix p. 2.

$$TAR' = C'D'^{1.5} \qquad (A\text{-}2)$$

where TAR′ = total accumulated repairs per 1,000 dollars of list price,

where D′ = total accumulated hours,

and C′ = a constant.

The problem arises in altering Equation (A-1) which requires percentage information, into Equation (A-2), which uses actual hours.

Use of Equation (A-2) requires determination of the constant C′. Given the assumption that during the first 12,000 hours of life repairs will accumulate to 120 per cent of list price, the constant can be found. Assume a 1,000 dollar tractor is purchased. When the tractor is used for 12,000 hours, the accumulated repairs will be $1200. Inserting these values into Equation (A-2) gives:

$$1200 = C' \times 12{,}000^{1.5} \qquad (A\text{-}3)$$

where C′ = .00913.

Equation (A-2) becomes Equation (A-4) when the constant found in (A-3) is used.

$$TAR = .000913\ D^{1.5}. \qquad (A\text{-}4)$$

Several identities are necessary to make Equation (4) equal to Equation (3-6). Total accumulated hours, D′, equals hours per year, X_2, multiplied by the number of years, t. Equation (A-4) gives repairs per 1000 dollars of list price. By multiplying Equation (A-4) by the list price, X_1, and then dividing the Equation by 1000, repair costs for any size tractor, X_1, can be found. Equation (3-6) is copied here as

Equation (A-5) for comparison purposes.

$$TAR = W_{2t} = .00000913\ X_1\ (tX_2)^{1.5}. \qquad \text{(A-5)}$$

Another use of Equation (A-5) is to find the expected rate of repair for any hour. Substitute D′ for tX_2 and take a derivative of Equation (A-5) with respect to D′. Resubstituting D′ for tX gives:

$$R_X = .0000137\ X_1\ (tX_2)^{.5}. \qquad \text{(A-6)}$$

The rate of repairs may be useful in a study of costs per hour at different times in machine life.

APPENDIX B

MACHINERY USAGE FOR A TYPICAL ACRE OF OKLAHOMA PANHANDLE FARM LAND

A typical acre of land in the Oklahoma Panhandle is assumed to be composed of .208 acres of wheat land fallow, .260 acres of wheat stubble, .324 acres of sorghum, and .208 acres of sorghum land fallow.[1] The operations and their frequency per year are given in Table XXIX.

TABLE XXIX

MACHINERY PRACTICES FOR CROPS IN THE OKLAHOMA PANHANDLE[2]

	Chisel	Oneway	Lister	Harrow	Drill	Plant	Cultivator
Wheat on Fallow	1	4	0	0	1	0	0
Wheat on Stubble	1	3	0	0	1	0	0
Sorghum	0	2	1	1	0	1	2
Fallow	1	4	0	0	1	0	0

[1]Oklahoma Experiment Station, Machinery Combinations for Oklahoma Panhandle Grain Farms, Bulletin B-630, by Odell Walker (Stillwater, 1964), p. 5.

[2]Ibid., p. 5.

It is necessary to find an equation which for any tractor will give the number of hours required per year to farm one acre of the above land. The first step in finding the equation is to determine the time required per acre for three tractor sizes: a three plow, a four plow, and a five plow. The size of machinery used with each tractor size is given in Table XXX.

TABLE XXX

COMPOSITION OF MACHINERY SETS FOR ALTERNATIVE TRACTORS[3]

	3 plow tractor	4 and 5 plow tractor
Oneway	12 foot	15 foot
Chisel	12 foot	15 foot
Cultivator	2 row	4 row
Lister	2 row	4 row
Harrow	4 section	4 section
Drill	16-10	16-10

To compute the number of acres which can be covered in one hour, it is necessary to know the speed the tractor is going to travel. Figures were obtained from The Official Tractor and Farm Equipment Guide and then averaged to give the tractor speeds in Table XXXI.[4]

[3]Ibid., p. 7.

[4]National Farm and Power Equipment Dealers Association, Official Tractor and Farm Equipment Guide (St. Louis: NRFEA Publication, Inc., 1967).

TABLE XXXI

AVERAGE SPEEDS TRAVELED BY VARIOUS TRACTOR SIZES

Gear	3 plow	4 plow	5 plow
2nd	3.70	3.70	4.20
3rd	5.20	5.20	5.80
4th	7.30	7.30	7.90

Chiseling and harrowing are done in third gear, all other operations are done in second gear.

From the above information, the theoretical capacity for each type of tractor with each implement can be found using the formula[5]:

Theoretical capacity (acres per hour) =

$$\frac{\text{speed (mph)} \times \text{width (feet)}}{8.25}.$$

If there were no overlap and no turning necessary in farm operations equipment might approach its theoretical capacity. Since it is impossible to perform at the theoretical capacity, it is necessary to multiply the theoretical capacity for each piece of equipment by a field efficiency factor which will give the actual number of acres covered in an hour. All equipment except the drill have an 85 per cent

[5]University of Illinois College of Agriculture, Cooperative Extension Work, Costs of Owning and Operating Farm Machinery, Bulletin AEng-867, by Wendell Bowers (Urbana, 1966), p. 7.

field efficiency rating. The drill has a rating of 75 per cent.[6] Table XXXII gives the acres covered in one hour for three tractor sizes and all implements.

TABLE XXXII

ACTUAL HOURS PER ACRE FOR THREE TRACTOR SIZES

Implement	3 plow	4 plow	5 plow
Chisel	4.57	5.72	6.49
Oneway	2.47	3.09	3.25
Lister	1.23	2.47	2.59
Cultivator	1.23	2.47	2.59
Planter	1.09	2.18	2.29
Harrow	12.85	12.85	14.34
Drill	5.38	5.38	6.11

By using the composition of one acre of land, the information in Table XXIX, and the information in Table XXXII, it is possible to find the length of time required to farm one acre of Panhandle farmland. Acreage per hour is computed for each tractor size giving three points corresponding to the three tractor sizes. The number of hours which it takes to farm one acre by a 3 plow, 4 plow, and 5 plow tractor are 1.561, 1.122, and .981, respectively.

[6] Ibid., p. 8.

For the information to be useful in the programming procedures, it is necessary to be able to find the hours per year necessary to farm an acre no matter what size tractor is assumed. Since all tractor sizes are allowed, a continuous equation is required. An index of tractor size is original cost. The assumed original costs of the 3 plow, 4 plow, and 5 plow tractors are \$3800, \$5200, and \$6100, respectively. A geometric curve was fitted to the tractor cost and hours per acre data. The resulting equation is:

$$Y = 60.42 \times X^{-1}$$

where Y = hours per acre

and X = machine cost in hundreds of dollars.

The above equation is used to estimate the number of hours required to work an acre. The assumption is made that as a tractor increases in size the speed and/or farm implements increase in size. The assumption is necessary to be able to estimate hours of farm work per acre on a continuous basis. It should be pointed out that the coefficient found is the number of hours required per acre per year. It is also necessary to assume a given farm area, such as Oklahoma Panhandle, because different areas require different tillage practices.

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