A COMPARATIVE ANALYSIS OF FOUR METHODS OF INSTRUCTION IN MATHEMATICS

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THE NATURE OF THE PROBLEM

Introduction

Since World War II, there has been a gradual change in techniques of teaching from the traditional lecture method with its emphasis on the acquisition of facts and rote learning, toward learning experiences in which each individual student plays a more active role. There is general consensus that this change has affected techniques of presenting every subject in the school curriculum. During the past decade, there has been an upsurge of enthusiasm for new ideas in both content and methodology of elementary school mathematics. More attention is now being focused on methods by which children discover mathematical ideas for themselves and develop mathematical skills with understanding.

According to McKeachie (28) research studies regarding teaching methods have been conducted over an extensive period of time. Many of these have been done on such topics as lecture method versus discussion method, distribution of lecture and discussion time, lecture versus automation, student-centered versus instructor-centered teaching and several others. These studies have emphasized the comparison of two organizational schemes. A better question than "Which of the two patterns is superior?" is the question of "Which of these several schemes is superior?" In 1967 Gibbons (22) made a study relative to the latter question, and his research provided a basis for making other comparisons.

Statement of the Problem

A review of recent literature relative to the mathematical sophistication of pre-service and in-service elementary school teachers reveals the general conclusion that they do not have sufficient knowledge to present mathematics effectively at the elementary school level of instruction. On the basis of this deficiency, more stress in teacher training institutions must be placed on finding ways to remedy this situation rather than upon gathering data to re-emphasize that which has already been researched. One effort toward this end was the study at Oklahoma State University reported by Gibbons (22).

During the school year 1966-67, Gibbons (22), investigated (1) potential ways to improve teachers' knowledge and understanding of "modern" mathematics, and (2) whether or not the mastery of this mathematics was affected by the way it was taught at the undergraduate level. Gibbons used three instructional methods called (1) Lecture-Textbook method, (2) the Programmed Lecture Discussion method, and (3) the Lecture-Programmed Discussion method. A control group which consisted of elementary education majors who had not taken the course, Arithmetic for Elementary Teachers (Math 2413), was used for comparison with the experimental groups. The experimental groups were taught the content of the basic course referred to above.

Gibbons found that his lecture-program-discussion and program-lecture-discussion organizations were statistically superior to a lecture-text organization, but were not statistically different from each other. The problem under consideration in this study is the determination of the comparative effectiveness of two additional organizational schemes with two of Gibbons' organizational schemes. It will (1) investigate organizational schemes different from those used in his study, (2) determine whether the mastery of the mathematics of this course is affected by the way it is taught at the undergraduate level, and (3) determine if class size affects mastery of the content. Two instructional methods distinct from those in the Gibbons analysis were used for this study, namely, the program discussion method (P. D.) with approximately 55 students, and the program-lecture-discussion-quiz method (P. L. D. Q.) with approximately 93 students.

Review of Literature

Butler and Wren (9, p. 216) list two important aspects of any true profession, and one of these is "significant knowledge." The careful preparation of prospective teachers in mathematics subject matter is a prerequisite to an improved program in the elementary schools.

Price (33) writing about the adequacy of mathematics for our time commented that we must put forth whatever effort may be required to insure that the education provided by our schools--and in particular, the mathematics education provided by our schools--is adequate for the needs of our times. Price listed three major components of the mathematics education that are adequate for our times as: appropriate course content, qualified teachers, and counselors.

Since this research is not concerned with counselors, no elaboration will be made on that component. Polya (32) and Price (33) say the well qualified teacher must know mathematics, and in addition, he must teach the subject with interest and enthusiasm. Price also believes that mathematics teachers must re-examine their teaching techniques. Some highly effective new techniques have been introduced at the elementary and secondary level. A good example is the "discovery technique" of teaching mathematics, which many teachers have used with much success.

According to the literature (4, p. 296), (5, pp. 4-66), (23, pp. 18-51), the elementary teachers need to improve their basic knowledge and skills in mathematics in order to be prepared to teach elementary mathematics.

In an article written by Garnett (19), many interesting variables were revealed as possible reasons why American students between the age of 13 years and high school graduation ranked so low when compared with students from 12 other countries. Without going into details, it was revealed that, among other things, teacher preparation had its impact.

Melson (29) concluded from a study he conducted with a group of elementary teachers who had successfully completed a course in "modern" mathematics that they were either inadequately prepared for the course or had faulty mastery of it. This conclusion was made on the basis of a test of 33 items for grades one through six in modern elementary mathematics given to 41 elementary teachers in September 1963. The results showed the median score to be 12 out of 33 correct responses (36%); two of the 41 teachers scored above 75%; 27 below 50%; and 12 below 25%.

Garstens (20) feels that an elementary teacher should have a background which is broader and deeper than the level at which the teacher teaches. As in any undertaking, instruction in mathematics aims at certain outcomes or objectives; and as in any other undertaking, the likelihood of attaining the objectives will depend in large measure on how well instruction is planned and on how well the classroom is organized.

McKeachie (28) points out that student learning and memory are closely tied to motivation. Students usually learn what they want to learn, but they often have great difficulty learning material that does not interest them. The primary problem then, after the selection of proper content, is motivating students. McKeachie claims the learning psychologist stops with this point, but to be useful the principle of motivation needs to be accompanied by information about dependable motives of college students.

Polya (32, pp. 605-619), although talking about the high school program, comments "mathematical thinking is not purely formal." He also claims that a teacher should realize that nothing is too good or too bad, too poetical or too trivial to clarify one's abstractions. To motivate students according to Polya, it may be necessary for a little acting or teaching may approach poetry or even may approach profanity.

Langford (26) agrees with McKeachie (28) regarding motivation. Langford says appropriate motivation is essential to effective learning.

Most college teachers could list many ways of motivating students, but consider the case of an important motivational device--grades. McKeachie (28) claims that, whatever a student's motivation for being in college, grades are important to him. If he is genuinely interested in learning, grades represent an expert's appraisal of his success. McKeachie related, however, that most teachers are a little embarrassed by this. Teachers regard grades as one of the necessary evils of teaching. They try to discount grades in discussing the organization of the course; they try to arrive at grades in such a way as to avoid trouble with disappointed students. They also frequently fail to use grades to bring about the sort of learning they desire.

The literature reveals that much research has been done in comparing the lecture method versus the discussion method. In one of the earliest comparisons of lecture and discussion methods, Bane (2) in 1925, found little difference between the methods on measures of immediate recall but a significant superiority for discussion on a measure of delayed recall. In one of the most recent comparisons, Ruja (34) in 1954, found that the lecture was superior to discussion as measured by a test of subject-matter mastery in a general psychology course. Another fairly recent comparison was made by Eglash (14), also in 1954. He found no difference between a discussion class and lecture class in scores on an achievement test administered several weeks after the course had ended, or in scores on a measure of tolerance.

A large number of such comparisons have been made with results similar to the ones mentioned above. McKeachie (28) summed up the situation by saying, "when one is asked whether lecture is better than discussion, the appropriate counter would seem to be, for what goals?" McKeachie did say, however, that since discussion offers the opportunity for a good deal of student activity and feedback, it could, in theory, be more effective than the lecture method in developing concepts and problem solving skills. However, since the rate of transmission of information is slow in discussion classes, we would expect lecture classes to be more efficient in helping students acquire knowledge or information.

The lecture method versus the discussion method has no direct bearing on this research, but indirectly it does. This research is directly concerned with distribution of lecture and discussion time. Some research has been done in this area, but the writer was unable to find any with classes in mathematics. Most of the research has been done in the area of psychology. McKeachie (28) claims that many large colleges and universities divide class meetings between lectures and discussions. This administrative arrangement is supported by a study made in 1956 by Lifson, Rempel, and Johnson (27). In this study, discussion meetings were substituted for one-third of the lectures. It was found that there were no significant differences in achievement, as compared with all lectures, the partial discussion method resulted in more favorable student attitudes, a finding that persisted in a follow up study two years later. Klopper (24) in 1958 at New York University found that most students preferred a combination lecture-discussion method to one employing all lectures or all discussion. Also in 1958, Becker, Murray, and Bechtoldt (3) conducted a study at the State University of Iowa and found that students preferred all group discussion or a combination of lecture and discussion to lectures alone.

McKeachie (28) on the basis of the studies mentioned, concluded that in a course in which the instructors wish not only to give information but also to develop concepts, the use of both lecture and discussions would thus seem to be a logical and popular choice. By participating actively in discussion, the student should not only learn the generalization, but should also begin developing skill in critical thinking.

In searching the literature it is quite apparent that there is a need to develop a method of instruction that will better prepare future elementary teachers in the fundamental concepts of elementary mathematics. One possible solution would be to reinforce the present mathematics course. May (36, p. 445) revealed that students learn by multiple exposure and activity in a repeated cycle of listening, speaking, reading, problem solving, writing, obtaining feedback from answers, etc. One method of such reinforcement that has been suggested is the use of programmed materials.

There has been much written and considerable research on the use of programmed instructional material.

According to Feldman (16), Silberman summarized 15 studies on the use of programmed material. In his summary of the 15 studies, he reported that nine studies reported superior learning for the programmed material, and six reported no difference. In these studies, however, it was not clear whether the comparable material consisted of identical words. In a second summary Silberman reported the results of 12 studies where identical words were used in prompting or in a "confirmation" procedure. In the prompting condition, the programmed material is usually presented in the frames but with the blanks filled in, thus prompting the learner to make the right response. For some of the prompting studies the material was arranged as texts but used the identical words of the program. In the confirmation procedure, the blanks were left for the student to fill in (overtly or covertly), and the correct answer was available to confirm his response.

Of the 12 studies summarized, seven showed superior learning for the prompted condition, two for the confirmation procedure; and three showed no difference for the two conditions. Filling in the blanks or program-induced activity, as opposed to self-imposed activity which a student habitually uses when reading, does not always seem to produce better learning.

In another case, Ripple (37) reported the results of a study at Cornell University that compared learning by program with what was called "comparable" text material or "conventional" instruction. The groups tested in this study were selected from sophomores enrolled in the beginning psychology course at Ithaca College, New York, for the fall semester of the 1963-64 school year. Since these groups were carefully selected, no difference was expected or found between the groups in the pretest. Finally, no difference on gains was found between the two formats.

May (36) says that programmed materials offer more detailed guidance than text but have few of the many features that make texts so handy for preview, summary, and review. Above all, they lack extended problems and connected exposition.

Brown and Mayor (4) reported in their study that much research is needed on methods of instruction, improvement of teaching aids, and learning. Educators need to know a great deal more about developing special courses for teachers.

Although the literature revealed at least one essential fact; namely, that students do learn from programmed material, there is no conclusive evidence that students learn significantly more or with greater efficiency. However, programmed material has been found to have other important attributes, and many predictions have been made concerning its usefulness for instruction.

Stolorou (35, p. 85) says,

> These devices (automated instruction) are here to stay. Future research will concern itself with important characteristics of the developments, a theory of teaching will emerge. The devices of the future will be either books (programmed or scrambled) or computer based machines, small devices will drop out. The results of the experiments in programmed instruction suggest an impressive contribution to education; and if the right programs can be developed and combined with an economical and effective means of presentations, the application of programmed instruction will be widespread.

Coulson (39) writes that we must consider programmed instruction in proper perspective among other educational techniques and attempt to discover what combinations of methods will lead to most efficient learning under specified conditions. Future research must be directed toward the discovery of optimal combinations of educational techniques for specific students and task characteristics. Reynard (38) says much research and experimentation with techniques other than the questionnaire surveys are needed in relation to all aspects of teacher education programs. Coulson's and Reynard's remarks have pointed implications for the aims of this study.

In summary, the literature reviewed was concerned with mathematical sophistication of teachers, organizational patterns of instruction, and programmed learning materials. The literature emphasized the need for the improvement of elementary teachers' mathematical background. It also emphasized that additional research should be directed toward discovering more effective combinations of educational techniques that would lead the student to maximum understanding. Much literature is devoted to research in programmed instruction. The prediction made by Stolorou (35, p. 85) summarized the relevant literature for this method of instruction.

Theoretical Design

According to Bruner (6) a theory of instruction is "prescriptive" in the sense that it sets forth rules concerning the most effective way of achieving knowledge or skill. He also says, "a theory of instruction is a normative theory." It sets up criteria and states the conditions for meeting them. Another way of saying the same thing, is, a theory of instruction is concerned with how best to help others learn what one wishes to teach, with improving, rather than describing learning.

According to Gage (17), theories of teaching have been neglected. In comparison with learning, teaching goes almost unmentioned in the theoretical writings of psychologists. Bruner (6) claims this is not to say that learning and developmental theories are irrelevant to a theory of instruction. In fact, a theory of instruction must be concerned with both learning and development as well as with the nature of particular subject matter.

For maximum effective learning the following four parts are essential (6, p. 203).

(a) The instructional situation should specify the experiences of the student. It must be motivation-producing, perception-directing, response-eliciting, and reinforcement-providing (11, p. 276). These are the stages through which the student will pass. They flow directly from the following axioms of learning theory: (1) pre-instruction procedures do produce greater learning in a given situation (10, p. 640), (2) active response on the part of the student is more effective than passive listening (10, p. 638), (3) a wide range of stimulating materials increases learning (30, p. 300), and (4) immediate continuous reinforcement facilitates learning (21, p. 541).

The organizational schemes described in this study meet the specifications described through programmed instructional material, informal discussion groups and weekly quizes.

(b) The instruction should specify the way in which a body of knowledge should be structured so that it can be most readily grasped by the learner. This concept of full understanding through facts and relationships has had its foundations in the following postulates: (1) The size of the steps in learning must be varied. If they are too small, general principles are not understood. If they are too large, specific facts are overlooked or underestimated (10, p. 626), and (2) learning is a developmental process in which earlier learning greatly influenced later learning (1, p, 504). The organizational schemes in this study meet this specification through programmed instruction, practice sheets over each chapter and a knowledge of prior assignments.

(c) The instruction should specify the most effective sequence in which to present the material to be learned. This proper sequence of topics or methods of instruction is essential to the logical and psychological development of a body of knowledge. These sequences of topics or methods must be in direct relation with the following axioms of learning theory: (1) new material should not be introduced until prior material in a sequence is thoroughly consolidated (1, p. 506), (2) new materials or methods should have a derivative relationship with prior material and methods for maximum learning (1, p. 508), (3) maintaining and improving desired responses increase learning (17,

p. 542), (4) a mixture of prompted and unprompted trials is more effective than using complete prompting throughout (4, p. 345), and (5) practicing responses in varied conditions facilitates their establishment (18, p. 57). The organizational schemes in the study meet the specification because material has been used experimentally over several semesters.

(d) The instruction must provide for the proper emphasis and spacing of rewards and punishment. No instructional situation is complete without proper evaluation. The evaluation should be both comprehensive and individualistic. The following axioms are guideposts for evaluation: (1) a knowledge of results should come at a point when the learner is comparing the results of his tryout with some criterion of what he seeks to achieve (6, p. 315), (2) rewards should be given periodically and frequently for effective learning (21, p. 355), (3) individual differences must be taken into account when evaluating an instructional situation (17, p. 208), and (4) immediate feedback of results aids length of retention and transfer of learning to new situations (21, p. 378). The organizational schemes meet this specification through weekly quizes and discussion sessions.

The points mentioned above and literature not cited imply that an optimal instruction situation must provide many phases for learning. It must provide an introduction and a motivation. It must contain small steps which culminate as a "principle" which is enriched by the large step sequence. It must attempt to evoke, reinforce, maintain and improve desired responses. Finally, it must consider the learner as an individual within a group.

In summary, effective instruction must provide many stages for learning. The sequence must provide an introduction and a motivation. There must be small steps which culminate as a "principle" which is enriched by the large step sequence. The total instructional program must attempt to evoke, maintain, supplement, and improve desired responses.

Hypothesis and Rational

Using the points and considerations given in the Theoretical Design, the following rational is presented:

(a) The programmed lecture discussion quiz group (hereafter denoted the P. L. D. Q. group) received the following method of instruction. This method introduced mathematical concepts in a logical sequential manner by use of programmed material. These concepts were then supplemented and enlarged upon by a related lecture. Finally, the programmed material, related homework assignment, and lecture were discussed during the first part of the last weekly session. The last ten to fifteen minutes of this session were devoted to a quiz over the material discussed during the previous portion of the class session. This weekly cycle was repeated throughout the course.

The total method of instruction applied to this group best fitted the theoretical design of this study for the following reasons: (1) the student has a chance to discover before being told by the instructor, (2) the topic is then reinforced through a lecture in a general structural manner, (3) the understanding of the topic is improved by a very informal discussion, (4) the motivation is compounded by the weekly ten to fifteen minute quizes, (5) the use of four distinct stages of instruction provides a wide range of materials and situations for the learner,

(6) the discussion provided a situation in which the instructor could evaluate the class' general understanding, and make recommendations to the lecture instructor for a possible short review or a different approach in presenting the concept, (7) the programmed materials provided an opportunity for the learner to continuously evaluate his understanding of the materials presented. This immediate feedback of results enhanced the length of retention, and (8) the programmed material allowed for some individualization with respect to pacing.

(b) Gibbons (22) described the Program Lecture Discussion group (hereafter denoted the P. L. D. group) in the following manner.

> Each new concept, or set of concepts, was first introduced through programmed materials. The learner read these materials prior to attending a given lecture. These concepts were then supplemented and enlarged upon by a related lecture. Finally, the programmed materials and lecture were then discussed at the next class meeting. This cycle was then repeated throughout the entire course.

The P. L. D. method has many of the characteristics of the P. L. D. Q. method. However, it did not appear to be as complete for the following reasons: (1) one important motivational aspect (test) is not included, and (2) discussion sessions are not as informal as in P. L. D. Q.

(c) Gibbons (22) described the Lecture Program Discussion group (hereafter denoted the L. P. D. group) in the following manner.

> Each new concept, or set of concepts, was first introduced through a lecture that was supplemented by a homework assignment that consisted of reading a certain number of frames from related programmed materials. The concept was then discussed in details, by both student and instructor, at the next class meeting. This cycle was repeated throughout the entire course.

This method falls short of the above two methods for the following reasons: (1) it does not purposely allow students to discover concepts for themselves, (2) one important motivational aspect (weekly quiz) is not included.

(d) The Program Discussion Group (hereafter denoted the P. D. group) received the following method of instruction. The class was assigned a set of frames to study and complete before attending class. During the class session the instructor encouraged group discussions. The instructor was sometimes involved with entire class discussions, small group discussions, and individual discussion. The sessions were very informal with emphasis on active participation by both student and instructor. This weekly cycle was repeated throughout the entire course. The P. D. method falls short of the above methods for the following reasons: (1) discussion groups of the size of this section cannot be sufficiently informal to induce learning, (2) one instructor is not adequate to properly supervise and evaluate problem areas, and (3) it is difficult for programmed materials consisting of small steps to give a complete structural introduction to a set of concepts.

The following hypothesis was deduced from the theoretical design and rationale.

Those students involved in the Program Lecture Discussion Quiz organizational scheme exhibit a significantly greater level of achievement and understanding in mathematics than those students involved in the other organizational schemes.

CHAPTER II

THE EXPERIMENT

Introduction

The experiment was conducted at Oklahoma State University, Stillwater, Oklahoma during the first semester of the 1966-67 school year and during the first semester of the 1967-68 school year. The purpose of the study was to evaluate the impact of various organizational schemes of instruction on achievement and understanding in mathematics for elementary teachers.

Five sections of Mathematics 2413 were involved in this study. No attempt was made to control enrollment in any of these sections. Two sections of the experimental groups were used in a study investigated by Gibbons (22) thus accounting for the two academic school years involved.

The instructors involved in the experiment were interested in the mathematical preparation of elementary teachers and were experienced classroom teachers.

The pretest, The Structure of the Number System (Form A) was administered to each group during the first week of the semester in September. The posttest, The Structure of the Number System (Form B), was administered to each group during the last week of the semester in January. All statistical analysis related to the experiment was completed by using the adjusted posttest results.

Subject Matter

The subject matter involved in the study is commonly referred to as modern mathematics for elementary teachers. Topics covered included language of sets, the whole numbers, systems of numeration, fractions, the integers, the number line and its uses, and the rational numbers.

In the unit on set theory the following concepts were developed: set, set membership, set notation (including set-builder notation), set measurement (empty set, finite set, and infinite set), set relationships (equality, equivalence, nonequivalence, greater than, less than, disjointedness, subset, proper subset), universal set, complement set, set operations (union, intersection, complementation, cross-product, and partition), and set operation properties (closure, commutativity, associativity, identity, and distributivity).

In the unit on whole numbers the following concepts were developed: number, number names, counting numbers, place-value, expanded notation, addition, subtraction, multiplication, division, order, and ordinal numbers. The properties for the four operations (addition, subtraction, multiplication, and division) were also developed. These included closure, commutativity, associativity, identity, cancellation, and distributivity. Understanding each property was reinforced by applying it in the solution of problems and mathematical proofs. All the above concepts were developed by relating them to an appropriate concept from set theory. For example, the foundations of addition were developed using the union of disjoint sets. Finally, the algorithms for each operation were developed in great detail.

In the unit on systems of numeration the important concepts from base ten were reviewed. During this review base ten was presented as a mathematical system consisting of ten basic symbols, a place-value principle, two primary operations (addition and multiplication), and two secondary operations (subtraction and division). The concept of grouping was developed and then used to illustrate that a given number idea may have many different symbolizations. The operations (addition, subtraction, multiplication, and division) were presented through the use of expanded notation and regrouping. This method added much to the meaning of each operation, and served to reinforce the understanding of the grouping procedure. Following each of these detailed presentations, the given algorithm was introduced and explained. For example, in base five (23 + 14) was presented in the following manner: $23 + 14 = (20 + 3) + (10 + 4) = (20 + 10) + (3 + 4) = 30 + (10 + 2) = (30 + 10) + 2 = 40 + 2 = 42$. Finally, the properties for each operation were discussed, and it was pointed out that these properties are independent of any given system of numeration.

Fractions were introduced by carefully defining a fraction through the use of set partitions. Following this, the concepts of unit fraction, ordered pairs, and equivalent fractions were developed by diagram and definition. The operations of addition, subtraction, multiplication, and division were illustrated by diagrams and then defined by mathematical equations. The properties for these operations (closure, commutativity, associativity, identity, multiplication inverse, and distributivity) were proved as theorems, which were based on previous definitions and whole number properties. For example, given that a, b, c, and d were whole numbers with b and d not equal to zero, commutativity for the addition of fractions was developed in the following manner: $a/b + c/d = (ad + bc)/bd = (da + cb)/db = (cb + da)/db = c/d + a/b$. Order was introduced ($a/b < c/d$ if and only if $ad < bc$) in such a manner as to enable the student to determine simple inequality and direction. Although not stated directly, this chapter introduced the student to the basic concepts involved in mathematical proofs.

The integers were developed by using ordered pairs of whole numbers. The concepts of equivalence, addition, and multiplication were defined and developed through the use of these ordered pairs. Also, the properties of addition and multiplication (closure, commutativity, associativity, identity, inverse, and distributivity) were proved as theorems based on ordered pairs. Subtraction and division were developed from the additive and multiplicative points of view. Next, the ordered pairs were defined in such a way, (a, b) is equivalent to $+(a - b)$ if $a > b$, 0 if $a = b$, $-(b - a)$ if $a < b$, as to enable the student to interpret them as signed numbers. Finally, the various properties for signed numbers were proved by using the ordered pair notation. For example, the proof that a negative integer multiplied by a negative integer is a positive integer was developed in the following manner: (o, x) and (o, y) are considered as negative x and negative y, and $(o, x) \cdot (o, y) = (o \cdot o + x \cdot y, o \cdot x + o \cdot y) = (xy, o)$ which is considered as positive xy.

The number line was introduced at this time as an aid in understanding ideas presented in the first five units. It was used as a model to illustrate number facts, not to prove them. The number line was presented as an arbitrary line (usually horizontal) with an arbitrary point as the origin and an arbitrary unit of length for determining the position of each integer. Each of the four operations (addition, subtraction, multiplication, and division) was explained using whole numbers, integers, and fractions. Also, the properties for each of these operations were illustrated using both integers and fractions.

The unit on rational numbers was introduced by defining a rational number as an ordered pair of integers with the second element being positive. This definition was then used in defining an equivalence relation, addition, subtraction, multiplication, and division. The properties for these operations (closure, commutativity, associativity, identity, inverse, and distributivity) were developed as theorems based on the above definitions and the related properties from the integers. Definitions for order and density were given, and many related theorems were proved. For example, it was shown that if $a/b < c/d$ then $(a/b + c/d)/2$ was between a/b and c/d by showing $a/b < (a/b + c/d)/2$ and $(a/b + c/d)/2 < c/d$. The final topic in this unit was decimals. Included under this topic were the following concepts: numerator, denominator, basic units, place-value, expanded notation, exponents and the rules for operating with exponents, converting rational numbers to terminating or repeating decimals, and converting terminating or repeating decimals to rational numbers.

Methods of Instruction

Four methods of instruction were employed in the experiment. They were (1) the Lecture Program Discussion method, (2) the Program Lecture Discussion method, (3) the Program Discussion method, and (4) the Program Lecture Discussion Quiz method.

Gibbon (22) described the Lecture Program Discussion method and the Program Lecture Discussion method in the following manner.

The L. P. D. method was a three step method of instruction. Each new concept, or set of concepts, was first introduced through a lecture. The number of concepts developed in a given period varied in relation to the complexity of the given concepts. The lecture was then supplemented by a homework assignment that consisted of reading a certain number of frames from related programmed materials. The concepts were then thoroughly discussed at the next class meeting. This cycle was repeated throughout the entire course.

The lecture presented essentially the same content as was to be assigned in the programmed materials. Each lecture began with a brief overview of the concepts to be presented. Then, the individual facts, principles, and examples were structured in such a way as to put them in proper perspective with regard to the total unit. The lecture was then summarized by reviewing the concepts just presented. Finally, the instructor concluded by making suggestions that would aid the student in his reading of the programmed materials.

The programmed materials were structured to add the small-step logic and sequence that was necessary for developing more complete understanding of concepts presented in the lecture. The number of frames needed to develop a given concept depended upon the complexity of the concept. There were approximately forty to forty-five frames assigned for each class meeting.

The discussion period provided time for each student to ask questions, make comments, and attempt generalizations whenever possible. It also provided an opportunity for the instructor to make comments, ask probing questions, and pass subjective judgement on general class understanding.

Once the cycle (lecture, programmed materials, and discussion) was set in motion it appeared that fifteen to twenty minutes was sufficient for each discussion period. Therefore, each class meeting consisted of fifteen to twenty minutes of discussion and thirty to thirty-five minutes of lecture. This is illustrated by the following diagram:

LECTURE→FRAMES→DISCUSSION, LECTURE→FRAMES→

The actual subject matter was contained in a programmed text consisting of seven chapters. Each chapter was completed in approximately two weeks. There were one hour examinations at the end of Chapters two, four, and six. The last examination was two hours, and it was cumulative. There were no unannounced quizzes. The distribution of class periods for each of the first three examination intervals was (i) ten periods for discussion and lecture, (ii) one period for review, (iii) one period for the examination, and (iv) one period for explaining the examination. The last examination interval consisted of six discussion-lecture periods, two review periods (one for Chapter 7, and one cumulative), and one final examination period.

The P. L. D. (Program Lecture Discussion) method was also a three step method of instruction. Each new concept, or set of concepts, was first introduced through programmed materials that were read prior to attending a given lecture. Again the number of concepts developed varied in relation to the complexity of the given concepts. These programmed materials were then supplemented by a related lecture. The concepts were then thoroughly discussed at the next class meeting. This cycle was repeated throughout the entire course.

The programmed materials, having been read before the lecture, not only provided for the student the small-step logic and sequence, but they also provided a thorough preview of the succeeding lecture.

Each lecture was prepared in advance and presented essentially the same content as was contained in the programmed materials. However, the students were allowed to present questions and reactions prior to the actual lecture. This was done in order to enable the instructor to adjust his lecture in such a way as to satisfy existing questions and reactions. If no questions or reactions were presented, the instructor presented a few of his own in order to motivate the students toward the succeeding lecture. For example, he (the instructor) might motivate the students toward the properties of addition in fractions by reviewing the properties of addition in the whole numbers. Each lecture was presented in the following pattern: (i) a brief overview of the topics contained in the programmed materials, (ii) a structured presentation in which the individual facts, principles, and examples were put in proper perspective with regard to the total unit, and (iii) a summary that attempted to completely interrelate the lecture and the programmed materials.

The discussion period again provided time for the students to ask further questions, make comments, and attempt generalizations whenever possible. It also provided time for the instructor to make comments, ask probing questions, and pass subjective judgement on general class understanding.

Once the cycle was set in motion it was found that thirty to thirty-five minutes was sufficient for each lecture. Therefore, each class meeting consisted of thirty to thirty-five minutes of lecture, and fifteen to twenty minutes of discussion devoted to interrelating the programmed materials and the lecture. This is illustrated by the following diagram.

FRAMES→LECTURE, DISCUSSION→FRAMES→LECTURE, DISCUSSION

The subject matter and programmed text for this method were the same as that of the L. P. D. method. Each chapter was completed in approximately two weeks. There were one hour examinations at the end of Chapters two, four, and six. The last examination was two hours, and it was cumulative. There were no unannounced quizzes. The distribution of class periods for each of the first three examination intervals was (i) ten periods for lecture and discussion, (ii) one period for review, (iii) one period for the examination, and (iv) one period for explaining the examination. The examination interval consisted of six lecture-discussion periods (one for Chapter 7, and one cumulative), and one final examination period.

The P. D. (Program Discussion) was a two step method of instruction. Each new concept, or set of concepts, was first introduced through programmed materials that were read prior to attending a given lecture. The number of concepts developed in this organizational scheme also varied in relation to the complexity of the given concepts. The programmed materials were discussed very informally among the students themselves and also with the instructor involved.

The programmed material having been read prior to coming to class prompted immediate questions regarding concepts or a particular concept.

The instructor was always prepared to present the concepts of the immediate assignment from a different point of view, or he was able to direct the attention to previously studied concepts for the student that lead to a better understanding on the part of the student. For example, the instructor might influence the student toward the discovery of addition of rationals by reviewing the properties of integers.

Students were allowed to ask questions concerning any pervious assignment thus allowing for a certain amount of review at any given class meeting. If, on occasion, the discussion became dull, the instructor was always prepared to have students perform operations related to the frames discussed in the particular assignment, but perhaps of a more sophisticated nature, also to ask probing questions relating to the material being studied.

Once the cycle was set in motion it was found that the fifty minute class period provided, in most cases, sufficient time for most students to become involved either with the instructor or another student. This cycle is illustrated by the following diagram.

FRAMES → DISCUSSION, FRAMES → DISCUSSION,

The subject matter and programmed text for this method were the same as that of the L. P. D. and the P. L. D. methods. Each chapter was completed in approximately two weeks. There were one hour examinations at the end of Chapters 2, 4, and 6. The final examination was a two hour comprehensive examination. There were no unannounced quizzes. The distribution of class periods for each of the first three examination intervals was identical to that of the P. L. D. group.

The P. L. D. Q. (Program Lecture Discussion Quiz) method was a four step method of instruction. Each new concept or set of concepts was first introduced through programmed material prior to attending a given lecture. Again, the number of concepts developed varied in relation to the complexity of the given concepts. These programmed materials were then supplemented by a related lecture. The programmed material, related homework assignment, and lecture were discussed during the first part of the discussion session, and finally, the last ten to fifteen minutes of this session were devoted to a quiz over that portion of the material covered during the previous week. This weekly cycle was repeated throughout the course.

Each lecture was prepared in advance and included essentially the same content as was contained in the programmed material. Each lecture was presented in the following pattern: (i) a brief overview of the topics contained in the programmed materials, (ii) a structured presentation in which the individual facts and examples were put in proper perspective with regard to the total unit, and (iii) a summary that attempted to completely interrelate the lecture and the programmed materials.

The discussion-quiz session was divided into two parts. The first 35 to 40 minutes of the 50 minute session were devoted to informal discussion of the material covered in the two previous lectures, the corresponding program, and related homework assignment. Discussion was carried on between groups of students, and an instructor was available for consultation with these groups; and when feasible, the instructor worked with students individually. The remaining 10 to 15 minutes of the 50 minute period were devoted to a quiz over the material covered during the preceding week.

The instructors for all discussion quiz sessions were graduate assistants pursuing the doctorate degree with an interest in the training of teachers in mathematics. The instructors for the lectures were regular, full-time college mathematics professors.

The cycle used for this group (P. L. D. Q.) consisted of two 50 minute organized lectures, one 35 to 40 minute informal discussion, and a 10 to 15 minute quiz session. This is illustrated by the following diagram.

FRAMES → LECTURE, FRAMES → LECTURE, DISCUSSION →QUIZ

The subject matter and programmed text for this scheme were the same as that of the three previous schemes discussed. Each chapter was completed in approximately two weeks. Each student was given a printed course outline at the beginning of the semester containing: (i) information as to the number of frames to be covered by a particular date, (ii) the dates of all quizzes and the frames over which the quizzes would be taken, and (iii) the date of the comprehensive final examination.

The programmed material employed in the above mentioned organizational schemes (L. P. D., P. L. D., P. D., P. L. D. Q.) was Basic Mathematics, A Programmed Introduction by Berg and Goff, which is quite unique. It is a hybrid form of programming that combines both the structural and discriminatory forms. This combination was accomplished in the following manner: (i) a series of structural-type frames that are single response, completion statements (these statements usually require less thinking on the part of the reader than the discriminatory-type frames), (ii) a discriminatory-type frame which is a multiple choice statement (this frame usually requires some thinking or generalizing on the part of the reader), and (iii) a repetition of parts (i) and (ii). The number of structural-type frames between discriminatory-type frames ranged from five to fifteen.

Evaluation Instruments

The instruments that were used to measure the levels of achievement that resulted from the various methods of instruction were: (i) American College Test in Mathematics (A. C. T.), (ii) The Structure of the Number System (Form A), and (iii) The Structure of the Number System (Form B).

The A. C. T. Mathematics Test was developed by the American College Testing Program. It is a mathematical aptitude test that is considered to be a good predictor of future achievement in college mathematics (8, p. 9). The test consisted of 36 multiple choice questions that sampled aptitudes related to precollege mathematics. The results of this test were used as one of the two covariates in the statistical analysis of the posttest results.

The Structure of the Number System (Form A) was produced by Educational Testing Service, Cooperative Mathematics Tests Division. This test is an achievement test that measures understanding of the real number system up to the rational numbers. The test consisted of forty multiple choice questions that sampled the following topics: arithmetic judgement, operational properties (closure, commutative, associative, and distributive), inverses and identities, properties of the integers, place value, (factors, divisors, and multiples), prime numbers, number lines, zero denominator, number systems (bases other than ten), modular arithmetic, and Roman numerals. This test was used as a pretest in the experiment, and the results were used as one of the two covariates in the statistical analysis of the posttest results.

The Structure of the Number System (Form B) was also produced by Educational Testing Service, Cooperative Mathematics Tests Division. It is also an achievement test that measures understanding of the real number system up to the rational numbers. The test consisted of 40 multiple choice questions and was used as the posttest in the experiment. Form B is considered an alternate form of Form A, and thus covered the same topics as Form A.

The two number systems tests were designed by the Educational Testing Service staff and some 46 high school and college mathematics teachers. The tests were pretested throughout the country in May, 1960. After analyzing the results, they were revised in May, 1961 and pretested again in May, 1962. The results from the second pretesting indicated the tests were appropriate for the intended population.

These two tests were selected because they were the only commercially produced tests directly related to the objectives of the experiment. They stress understanding of facts, principles, and relationships; and do not emphasize computational skills. Furthermore, the tests are measures of developed abilities, and thus their content validity is very important. Educational Testing Service feels (13, p. 62) they have insured this by entrusting test construction to persons well-qualified to judge the relationship of test content to teaching objectives. The reliabilities reported by E. T. S. are measures of internal consistency, computed by using the Kuder-Richardson Formula 20. The reliability of Form A was .86 with a standard error of measurement of 2.73. The reliability of Form B was .84 with a standard error of measurement of 2.75. The correlation of Form A with the SCAT-Quantitative Test was .78, and that of Form B was .74.

Educational Testing Service pointed out (13, p. 64) that this was lower than expected, but this was due to the fact that Forms A and B measure understanding, while the SCAT-Quantitative emphasizes computational skills. Form A had an item-total score discrimination correlation of .50, and that of Form B was .48. These results indicate that the tests are effective in discriminating between high and low ability students (13, p. 64). Finally, the equivalence of these two alternate forms was very good. The converted raw scores differed by no more than two at all levels of performance. These results are tabulated in the Educational Testing Service mathematics booklet (13, p. 67).

Sample

The sample for this study consisted of 290 students, all of whom were enrolled in Mathematics 2413 at Oklahoma State University, Stillwater, Oklahoma. The experimental groups were distributed in five sections in the following manner: (i) forty-seven students in L. P. D., (ii) thirty-three students in P. L. D., (iii) fifty in P. D., and (iv) two sections with 160 in P. L. D. Q. Any student who was repeating the course, or who withdrew, or on whom related data were unavailable, was not included in the sample analysis. In the P. L. D. group eleven dropped the course, and four were discarded due to lack of data. In the L. P. D. group nine dropped the course, and four were discarded due to lack of related data. In the P. D. group five dropped the course, and fourteen were discarded due to lack of data. In the P. L. D. Q. group two were repeating the course for the second time, seven dropped the course, and eighteen were discarded due to lack of data. Most students involved in the study were elementary education majors.

Gibbons (22) revealed the following data.

> The L. P. D. group had a mean score of 19.32 on the A. C. T. mathematics test. This test had a possible score of 36. This group also had a mean score of 19.32 on the pretest. This test had a possible score of 40.
>
> The P. L. D. group had a mean score of 18.30 on the A. C. T. mathematics test and a mean score of 18.30 on the pretest.

The P. D. group had a mean score of 16.00 on the A. C. T. mathematics test. This test had a possible score of 36. This group also had a mean score of 17.10 on the pretest. This test had a possible score of 40.

The P. L. D.Q. group had a mean score of 18.66 on the A. C. T. mathematics test and a mean score of 17.85 on the pretest.

The abbreviated doolittle (40) test indicated an analysis of covariance was a valid test.

Analysis

Each group was administered the pretest, The Structure of the Number System (Form A), during the first week of the semester in which the data were collected. The posttest, The Structure of the Number System (Form B), was administered during the last week of the respective semester. The data that were used to test the hypothesis were the A. C. T. mathematics test scores, the pretest scores, and the posttest scores.

Analysis of covariance was employed in comparing the groups on the posttest results. The regression coefficients were tested for homogeneity, thus constituting a valid assumption. Therefore, it was possible to compare adjusted means. Several authors (21), (12), (15), in explaining the application of the analysis of covariance, let the covariate be a pretest score. In this analysis, the pretest score was used as one covariable, but the A. C. T. mathematics test was also used for the dependent variable, the posttest score.

Garrett (21, p. 225) explains the use of analysis of covariance for this situation when he states:

> Analysis of covariance represents an extension of the analysis of variance to allow for the correlation between initial and final scores. Covariance analysis is especially useful for experiments in the behavioral sciences where for various reasons it is impossible or quite difficult to equate control and experimental groups at the start, a situation which one often obtains in actual experiments. Through covariance analysis one is able to affect adjustment in final or terminal scores which will allow for differences in some initial variable.

A model of the form $y = a_0 + a_1x_1 + a_2x_2 + \epsilon$ was used where

y = posttest

x_1 = A. C. T. mathematics test

x_2 = pretest

ϵ = random error

The model was hypothesized for each organizational scheme. (L. P. D., P. L. D., P. D., P. L. D. Q.) The abbreviated doolittle method was used to solve these equations for the various constants as suggested by Steel and Torie (40).

If the analysis of covariance does not reveal any significant difference, Winer (41) suggests using Tukey's procedure for comparing individual means which consisted of testing for a significant gap. If a significant gap does not exist, it will be useless to test for a "straggler" and for excessive variability.

CHAPTER III

ANALYSIS OF THE DATA

Introduction

This chapter contains the findings of the statistical data used to determine the validity of the test and the significance of the results of this investigation. The .05 level of probability was used to judge the significance of all statistical data. The rejection of any hypothesis was directed; therefore, one-tailed tests of significance were employed. The major statistical analyses were (i) abbreviated doolittle--four equations, (ii) multiple analysis of covariance--four groups, and (iii) test for a significant gap.

The presentation of the statistical analysis will be followed by a summary of the results. Information such as average gain for the four groups, an opinionated survey, and an analysis of the groups two at a time, is included although unrelated to the original hypothesis.

Abbreviated Doolittle--Four Equations

This statistical technique determines whether the four planes represented by the following four equations are parallel

$$y_1 = a_0 + a_1x_1 + a_2x_2 + \epsilon$$
$$y_2 = b_0 + b_1x_1 + b_2x_2 + \epsilon$$
$$y_3 = c_0 + c_1x_1 + c_2x_2 + \epsilon$$
$$y_4 = d_0 + d_1x_1 + d_2x_2 + \epsilon$$

The reduction in sums of squares attributable to regression was tested for significance by F.

The data for the four experimental groups were prepared for an IBM 7040 computer system at the Oklahoma State University Computing Center under the direction of Gary Lance and Francis Hajek. The abbreviated doolittle program of Steele and Torie (41, p. 289) was utilized. This program calculated an F value.

The findings concerning these four groups (L. P. D., P. L. D., P. D., P. L. D. Q.) are presented in Table I.

TABLE I

ABBREVIATED DOOLITTLE -- FOUR GROUPS

Source of Variation	Sum of Squares	df	Mean Sum of Squares	F
Total	200684.11	n		
$R(\beta_0)$	195416.50	1		
$R(\beta_{1c}/\beta_0)$	2883.88	1		
$R(\beta_{2c}/\beta_{1c}, \beta_0)$	2334.55	1		
$R(\beta_{11}, \beta_{12}, \beta_{13}, \beta_{14}, \beta_{22}, \beta_{23}, \beta_{24}, /\beta_{2c}, \beta_{1c}, \beta_0)$	49.19	8	8.197	.531
Error	4301.90	n-11	15.420	

From Table I, the calculated F value is shown as .531 correct to three decimal places. The critical F value, for the given degrees of freedom was less than one. These results disclose the fact that the planes are sufficiently parallel, and the analysis of covariance--four groups is a valid test.

Multiple Analysis of Covariance--Four Groups

This statistical technique was a combination of analysis of variance and multiple regression techniques. The method enabled the writer to statistically equate the means of the groups with respect to the covariates before drawing conclusions about treatment effects. Further, this statistic allowed the writer to control the differences in A. C. T. mathematics test scores while comparing the differences exhibited on the posttest scores.

The data for the four experimental groups were prepared for an IBM 7040 computer system at the Oklahoma State University Computer Center. The Fortran program for analyzing the data was provided by Mr. Francis Hajek. The multiple analysis of covariance program of Winer (41, p. 618) was utilized. This program calculated the F ratio for the adjusted treatment means, the Beta coefficients and their standard errors, and the adjusted treatment means with their accompanying standard errors. The findings concerning these four groups (L. P. D., P. L. D., P. D., P. L. D. Q.) are presented in Table II.

TABLE II

ANALYSIS OF COVARIANCE--FOUR GROUPS

Source of Variation	Adjusted Sum of Squares	df	Mean Sum of Square	F
Treatment	44.29	3	14.76	
Error	4306.78	284	15.16	.974
Total*	4351.07	287		

*Covariates were the A. C. T. mathematics test scores and the pretest (Structure of the Number System (E. T. S.), Form A) scores.

From Table II, the calculated $\underline{F}$ value was shown as .974 correct to three decimal places. The critical $\underline{F}$ value, for the given degrees of freedom was 2.61. These results disclosed the fact that no significant differences existed among the four groups on the adjusted posttest results. Steele and Torie (40) suggested that under similar circumstance, there is a possibility of a gap between adjacent means when they are arranged in order of magnitude.

The method selected for analyzing the adjusted posttest results was Tukey's procedure for comparing individual means (41, p. 330). This method classifies the means into groups that are alike among themselves but differ from each other.

Test for a Significant Gap

The first step in this test was to arrange the adjusted posttest means for the four groups in order of magnitude as shown in Table III.

TABLE III

ADJUSTED MEANS ARRANGED IN ORDER OF MAGNITUDE

	Experimental Conditions			
	P. D.	P. L. D.	L. P. D.	P. L. D. Q.
Adjusted $\overline{Y}$	24.46	25.67	26.89	26.90

The statistic used in this test was given by the formula

$$\text{Significant gap} = (t_{.05})(\sqrt{2})(S_{\overline{x}}),$$

where $S_{\bar{x}}$ was the standard error of the mean, and $t_{.05}$ was the tabulated value of t at the 5 per cent level for the degrees of freedom associated with the mean square of the error from Table II.

For the data of Table II, t at the 5 per cent level for 284 degrees of freedom was 1.97. $S_{\bar{x}}$ was 1.59, and the number was calculated by the following formula:

$$S_{\bar{x}} = \sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2} + \frac{S_3^2}{n_3} + \frac{S_4^2}{n_4}}$$

where S_i^2 was the adjusted mean square of the error for each group, and n_i was the number of subjects in each group. Substituting in the first formula with appropriate values, it was found that

$$\text{Significant gap} = (1.97)\ (1.41)\ (1.59) = 4.41.$$

Inspecting the differences between the adjacent pairs of means from Table III, it was found that there was no significant gap.

Although Tukey's (41, p. 330) method for analyzing posttest results was divided into two additional subdivisions--testing for a "straggler" and testing for excessive variability, Winer (42) suggested that the latter two methods be used only if a significant gap exist among the adjusted means.

Summary of the Results

Included in this section is a summary of the results of the statistical analysis used in conjunction with the given hypothesis. The final conclusions and recommendations are presented in Chapter IV.

The analysis comparing the four experimental groups, when considering the total number of subjects, disclosed the fact that no significant difference existed among the four groups on the adjusted posttest results. The calculated F value was .974 while the critical F value for the given degrees of freedom was 2.08. Although there was no significant difference, there was the possibility that a significant gap existed. Tukey's procedure for comparing individual means was utilized. The test for a significant gap consisted of finding a gap that could be used in determining whether gaps between the adjacent adjusted posttest means (Table II) were large enough to be considered. The significant gap was found to be 4.41. Upon inspecting the differences between adjacent pairs of means in Table III, it was found that there was no significant gap between the means. These results suggested that it was useless to investigate Tukey's other two basic subdivisions.

In summary, these tests indicated that the groups were not significantly different. These findings allowed the writer to reject the hypothesis given in Chapter I. (Those students involved in P. L. D. Q. groups exhibit a significantly greater level of achievement and understanding in mathematics than those students involved in other organized schemes.)

The following points, though not included in the statistical analysis or related to the hypothesis, may be of interest to the reader. The average gain for the four groups on the posttest over the pretest was (i) 7.17 (L. P. D.), (ii) 7.46 (P. D.), (iii) 7.63 (P. L. D.) and (iv) 8.58 (P. L. D. Q.).

An opinionated survey regarding the course was given to each student enrolled in the P. L. D. Q. group during the last week of the semester. Students were asked not to use their names on the survey and to underline the most appropriate answers. (See Appendix A.)

Listed below are the results of a few responses which substantiated the theoretical design of Chapter I.

82% felt that their understanding of mathematics had increased significantly.

79% felt that their attitudes toward mathematics had improved greatly.

76.8% indicated that the course had provided enough individual attention and individual work, while 10.2% felt that the course had not provided enough individual attention.

When asked about selecting another course in mathematics theory as a consequence of having had this course, 18.2% said yes, while 58% said maybe, and 9.4% said no.

As a consequence of having had this course would you select any course that used these same methods of instruction? To this question, 60.2% indicated yes; 8.4% indicated no; while the remaining 31.4% chose a response somewhere between these extremes. In regard to computational skills, 86.4% indicated great improvement, while .04% indicated no change.

Although the hypothesis was to compare the four organizational schemes, the writer was curious as to what statistical implication one might find if the groups were analyzed two at a time. The following table reveals the findings of the calculated F for the groups.

From Table IV, the calculated F value is shown correct to two decimal places. These results disclosed the fact that no significant differences existed among any two groups at the .05 level of probability. It might be well to point out, however, that the critical F value for the two groups, P. D. and P. L. D. Q., was 2.71 at the .10 level of probability, and calculated F value was 2.78. These results indicate that significant differences existed for the P. D. group and the P. L. D. Q. group at the .10 level of probability.

TABLE IV

ANALYSIS OF COVARIANCE--TWO GROUPS

	P. L. D.	L. P. D.	P. D.	P. L. D. Q.
P. L. D.		.254	.292	1.690
L. P. D.	.254		.196	.612
P. D.	.292	.196		2.780
P. L. D. Q.	1.690	.612	2.780	

CHAPTER IV

SUMMARY, LIMITATIONS, AND CONCLUSIONS

Summary

The purpose of this study was to investigate experimentally the comparative effectiveness of four methods of mathematics instruction at the undergraduate level.

Two of the four experimental methods (the L. P. D. method and the P. L. D. method) were used in a study by Gibbons. These two experimental methods were three-step methods that employed lecture, programmed materials, and discussions. Gibbons found in his study that students showed a significantly greater level of achievement and understanding in mathematics than those students involved in the Lecture Text method.

The other two methods (the P. D. method and the P. L. D. Q. method) were used for the first time in this study. The experimental method (P. D.) was a two-step method that employed programmed materials and discussions only. The experimental method (P. L. D. Q.) was a four-step method that employed programmed material, lecture, discussion and quiz. The P. L. D. Q. method was selected as it seemed to best satisfy the many assumptions considered necessary for effective learning. These assumptions were selected from a review of the writings of various psychologists who are considered to be authorities in the field of learning theory.

The same programmed material, Basic Mathematics, A Programmed Introduction by Berg and Goff, was used with each of the four experimental groups.

A total of 364 undergraduate students were involved in this experiment; however, only 290 participated to the extent that they were included in the statistical analysis. Seventy-four students were deleted from the experiment either because of missing data, dropping the course, or because they were repeaters in the course. The entire population included in this analysis enrolled and completed Mathematics 2413 at Oklahoma State University.

The basic design of the study was pretest--treatment--posttest. The pretest was administered to all subjects during the first week of the semester in which the study occurred. The treatments (the method of instruction) were applied three times per week for the entire semester for three groups (P. L. D., L. P. D., P. D.), and for one group (P. L. D. Q.) the treatment was program lecture twice per week and discussion quiz once per week. The posttest was administered to all subjects who completed the course during the last week of the semesters of the school years 1966-67 and 1967-68.

The independent variables were the four methods of instruction: the L. P. D. method, the P. L. D. method, the P. D. method and the P. L. D. Q. method. The dependent variables were the adjusted scores of these groups on the posttest.

Evaluation of the instruction was accomplished through the use of commercially made tests. The pretest and the posttest (The Structure of the Number System, Forms A and B) were produced by the Educational Testing Service, Cooperative Mathematics Tests Division.

These tests included alternate forms and were used to measure the achievement of the subjects after one semester of mathematics for elementary teachers. The A. C. T. mathematics tests were produced by the American College Testing Program. These tests are aptitude tests, and the results were used as one of the two covariates in the statistical analysis. The pretest results were used as the other covariate.

There were three major statistical analyses in the experiment, the abbreviated doolittle, the analysis of covariance, and Tukey's procedure for comparing individual means.

The abbreviated doolittle test was used to determine whether the analysis of covariance was a valid test under the given circumstances. The analysis of covariance was used in analyzing all four groups in order to determine if there were significant differences between the groups. This statistic was selected as it allowed the writer to draw conclusions about treatment effect that affected the observation after variables were adjusted statistically. Tukey's procedure for comparing individual means was selected as it allowed the writer to determine if a significant gap existed among the four groups.

It may also be of interest to mention in this summary that the results of this study seem to indicate that the professorial staff in departments of mathematics may be more effectively utilized by using a model similar to the one described for the P. L. D. Q. group. In the P. L. D. Q. group two professors in the department of mathematics lectured two hours each per week, thus constituting one-quarter time each, or one-half time for professors. Two graduate assistants conducted three discussion-quiz sessions each per week, thus constituting one-quarter time each or one-half time for graduate assistants. The normal teaching load for a full time staff member is twelve semester hours; however, using this model, the equivalent of eighteen semester hours was being taught by the equivalent of one full time staff member, thus, in effect, reducing the teaching staff required by one-third.

Limitations

It is important to point out some conditions that may cast limitations on the findings. The reader should be aware of these limitations so that any tendency to overinterpret or overgeneralize may be minimized.

The reader should keep in mind that the population came from Oklahoma State University elementary education majors, and the students were not randomly selected; thus the sample may not necessarily be a representative sample of the general college population. It should also be mentioned that the sample subgroups were different with respect to the A. C. T. mathematics test scores and the pretest scores. However, these differences were statistically controlled by employing the analysis of covariance.

The writer recognizes the limitations introduced by collecting data from three of the subgroups during the 1966-67 academic year and for one of the subgroups during the 1967-68 academic year.

Another consideration in interpreting the results of this investigation is the Hawthorn effect. The experimental groups realized they were part of a study, and this may have affected the results.

The pretest and posttest were the only standardized tests given during the semester; consequently, the effects of taking the pretest may have affected the posttest results.

Conclusions

The results from the analysis of the data revealed the following conclusions:

First, there were no significant differences among the adjusted posttest means of the four groups. This conclusion was accepted as a result of the analysis of covariance--four groups.

Second, the students involved in the P. L. D. Q. group did show, as predicted, a greater level of achievement and understanding in mathematics than did the students in the other three groups. However, this level of achievement was not significantly greater. Therefore, the hypothesis of Chapter I was not accepted. (Those students involved in the P. L. D. Q. organizational scheme exhibited a significantly greater level of achievement and understanding in mathematics than those students involved in the other organizational schemes.)

Third, since learning theory does not exist as a unified science, any application to a particular method is difficult (11, p. 25). Furthermore, the fact that two methods (P. L. D. and L. P. D.) were highly similar and differed only with respect to the order of motivation and supplementation made predicting their relative effectiveness even more difficult.

For maximum effective learning the following four parts are essential (6, p. 203).

(a) The instructional situation should specify the experience of the student. It must be motivation-producing, perception-directing, response-eliciting, and reinforcement-providing (11, p. 276). These are the stages through which the student will pass. They flow directly from the following axioms of learning theory: (1) preinstruction procedures do not produce greater learning in a given situation (10, p. 640), (2) active response on the part of the student is more effective than passive listening (10, p. 638), (3) a wide range of stimulating materials increases learning (30, p. 300), and (4) immediate continuous reinforcement facilitates learning (21, p. 541).

The P. L. D. Q. group described in this study meets the specifications described through programmed instructional material, formal lecture, informal discussion groups and weekly quizzes.

(b) The instruction should specify the way in which a body of knowledge should be structured so that it can be most readily grasped by the learner. This concept of full understanding through facts and relationships has its foundations in the following postulates: (1) The size of the steps in learning must be varied. If they are too small, general principles are not understood. If they are too large, specific facts are overlooked or underestimated (10, p. 626), and (2) learning is a developmental process in which earlier learning greatly influences later learning (10, p. 504). The organizational schemes in this study meet this specification through programmed instruction, practice sheets over each chapter, and a knowledge of prior assignment.

(c) The instruction should specify the most effective sequence in which to present the material to be learned. This proper sequence of topics or methods of instruction is essential to the logical and psychological development of a body of knowledge. These sequences of topics or methods must be in direct relation with the following axioms of learning theory: (1) new material should not be introduced until prior material in a sequence is thoroughly consolidated (1, p. 506), (2) new materials or methods should have a derivative relationship with prior material and method for maximum learning (1, p. 508), (3) maintaining and improving desired responses increase learning (16, p. 542), (4) a mixture of prompted and unprompted trials is more effective than using complete prompting throughout (4, p. 345), and (5) practicing responses in varied conditions facilitates their establishment (18, p. 57). The organizational schemes in the study meet the specification because material has been used experimentally over several semesters.

(d) The instruction must provide for the proper emphasis and spacing of rewards and punishment. No instructional situation is complete without proper evaluation. The evaluation should be both comprehensive and individualistic. The following axioms are guideposts for this evaluation: (1) a knowledge of results should come at a point when the learner is comparing the results of his tryout with some criterion of what he seeks to achieve (6, p. 315), (2) rewards should be given periodically and frequently for effective learning (21, p. 355), (3) individual differences must be taken into account when evaluating an instructional situation (17, p. 208), (4) immediate feedback of results aids length of retention and transfer of learning to new situations (21, p. 378). The organizational schemes meet this specification through weekly quizzes and discussion sessions.

The points mentioned above imply that an optimal instructional situation must provide an introduction and a motivation. It must contain small steps which culminate as a "principle" which is enriched by the large step sequence. It must attempt to evoke, reinforce, maintain, and improve desired responses. Finally, it must consider the learner as an individual within a group.

In summary, effective instruction must provide many stages of learning. The sequence must provide an introduction and a motivation. There must be small steps which culminate as a "principle" which is enriched by the large step sequence. The total instructional program must attempt to evoke, maintain, supplement, and improve desired responses.

In summary, the writer found no evidence for rejecting the theoretical design, and under the conditions of this experiment, he could accept the statement that the P. L. D. Q. method was the best fit to the theoretical design.

Recommendations

Gibbon's study revealed that the L. P. D. group and the P. L. D. group achieved a significantly higher level of achievement and understanding than the L. T. group, and this study revealed that the P. L. D. Q. group showed a greater level of achievement and understanding than any of the other experimental groups. However, in no case was the advantage significantly greater. These results seemed to indicate that a method of instruction with the greater number of phases was more effective than a method of instruction that consisted of fewer phases. This encourages the writer to recommend that additional research be carried on using a variety of organizational schemes, perhaps with four or even five step methods of instruction.

A second recommendation is to suggest an experiment identical in nature to the P. L. D. Q. group in which the size of the lecture sections could be increased, perhaps decidely; and both lecture and discussion would be conducted by the same professor.

A third recommendation is that a theoretical design be developed which would provide for periodic measures of student growth during the progress of the experiment as compared to this study, which measured only final changes in behavior.

A further recommendation would be to apply the theoretical design, or one highly similar to it, to other areas of mathematics. If it has success in these areas, then experimentation might be carried on in other subject matter areas.

Finally, it is recommended that other methods of instruction be developed and investigated with emphasis on large lecture sessions and small discussion sessions.

The above research is recommended as it might enable future research to make conclusions concerning the feasibility of adding an additional step to the instructional sequence.

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APPENDIX A

INDIVIDUAL SCORES OF SUBJECTS PARTICIPATING IN THE STUDY

and

AN OPINIONATED SURVEY

P. L. D.			L. P. D.			P. D.			P. L. D. Q.		
ACTM	Prt	Pst	ACTM	Prt	Pst	ACTM	Prt	Pst	ACTM	Prt	Pst
14	13	18	20	19	26	22	13	19	24	22	31
23	26	35	23	22	30	20	10	24	18	12	21
18	20	25	18	15	26	12	22	29	22	30	32
17	16	24	30	28	34	18	14	18	19	22	29
14	13	19	27	34	35	17	23	29	26	20	27
14	22	28	25	25	30	21	25	31	21	23	24
17	15	21	17	15	20	11	6	23	15	10	22
20	28	32	17	18	25	25	14	28	27	30	39
21	32	33	18	12	22	8	17	22	26	23	30
17	15	25	16	12	25	10	15	24	19	14	27
16	21	26	21	20	29	16	8	19	12	21	21
10	14	19	25	22	31	14	11	17	17	15	25
22	22	30	29	26	31	18	10	19	17	20	26
24	22	29	21	30	31	18	22	32	12	23	27
18	12	23	25	32	34	20	21	32	16	15	18
14	14	17	22	26	36	22	14	31	15	23	30
24	21	32	13	8	13	18	10	25	19	24	30
21	20	25	19	13	24	18	21	25	19	21	27
14	15	25	18	14	28	18	12	21	20	18	26
3	14	21	12	14	20	19	21	35	21	14	26
20	19	28	24	24	25	16	18	25	20	18	27
23	21	28	24	18	28	12	16	12	11	15	23
15	15	18	18	18	27	14	29	31	19	20	29
25	15	30	19	13	24	17	13	16	14	17	23
17	13	18	14	18	28	14	10	16	22	21	30
21	23	29	7	16	14	27	22	28	18	16	27
15	10	15	14	23	24	16	16	21	21	20	29
19	18	25	27	21	21	16	18	30	26	14	28
20	13	28	17	20	26	12	13	16	12	14	21
21	16	33	34	26	38	20	21	23	15	20	32
23	32	37	18	24	25	16	19	25	16	18	22
17	16	23	18	19	31	16	15	21	13	13	23
27	18	28	3	12	24	8	9	16	13	14	20
			14	11	21	21	17	23	14	12	19
			26	24	30	18	30	37	21	12	28
			11	13	31	12	27	33	12	9	19
			17	17	26	12	19	29	16	18	26
			22	24	32	16	25	31	14	10	23
			17	23	24	18	20	27	20	28	37
			18	15	21	30	24	34	16	22	30
			16	16	25	11	21	23	18	13	23
			18	19	32	19	17	29	12	9	20
			14	14	30	14	14	10	16	10	18
			18	16	22	17	15	27	16	29	34
			28	22	29	11	7	17	20	13	25
			18	17	24	16	10	25	20	25	35
			18	20	32	14	10	17	8	14	23
						12	18	22	6	17	26
						8	27	26	14	10	17
						11	26	30	6	14	19

P. L. D. Q.

ACTM	Prt	Pst	ACTM	Prt	Pst	ACTM	Prt	Pst
20	20	33	18	13	26	15	14	27
12	17	21	21	21	33	16	18	30
16	16	31	24	23	35	6	11	15
16	23	29	14	20	19	18	12	24
12	12	24	18	15	27	18	19	22
16	23	18	12	14	21	18	26	27
16	14	25	28	27	33	16	18	25
6	12	11	23	26	33	18	23	26
14	8	15	19	7	18	15	12	22
16	13	28	13	9	15	22	26	34
12	12	16	24	24	37	17	12	22
18	20	33	20	16	18	23	20	32
10	14	23	17	23	33	20	18	30
14	20	25	19	19	25	12	25	28
14	14	19	15	9	25			
18	20	38	15	17	29			
19	21	27	4	15	25			
21	17	26	20	16	22			
27	11	29	18	14	20			
16	14	30	19	24	32			
25	27	28	21	17	35			
10	10	29	24	24	26			
20	22	28	13	13	20			
16	18	28	24	24	31			
24	28	32	25	22	30			
8	22	32	25	24	27			
17	12	30	22	8	21			
26	16	26	25	25	34			
18	17	22	13	10	12			
21	21	23	24	27	37			
17	18	23	27	30	34			
25	21	29	15	14	24			
20	22	32	25	30	34			
27	26	33	24	20	30			
12	12	12	17	17	23			
16	12	16	18	12	28			
23	24	34	17	20	21			
28	24	31	24	18	27			
14	14	23	18	16	31			
27	20	33	13	7	11			
19	22	29	14	17	33			
18	15	24	14	25	31			
15	13	30	20	18	27			
6	14	20	29	24	27			
19	21	26	20	22	27			
22	20	25	23	16	28			
18	23	31	15	9	17			
15	13	16	24	7	29			

TO BE TAKEN WITH POSTTEST

PLEASE ANSWER THE FOLLOWING QUESTIONS BY UNDERLINING YOUR CHOICE.

1. To what extent do you feel your <u>understanding</u> of arithmetic has been affected by this course?

   (has increased significantly, has increased slightly, has remained the same, has decreased)

2. To what extent do you feel your <u>attitude</u> toward arithmetic has been affected by this course?

   (has improved greatly, has improved some, has not changed, has been affected unfavorably)

3. Do you feel the course has provided enough individual attention and individual work?

   (more than enough, enough, not enough)

4. As a consequence of having had this course, would you select another such course in arithmetic theory?

   (yes, maybe, no)

5. As a consequence of having had this course, would you select any course that used these same methods of instruction?

   (yes, maybe, no)

6. As a result of this course do you feel you have improved your computational skills in arithmetic?

   (improved greatly, improved some, no change, decreased)

7. Do you feel you are able to use any of the concepts learned in this course in other courses?

   (yes, no) If yes, please state the courses.

8. In what ways would you change and/or improve this course? Please list these. (Use the back of the sheet if necessary.)

APPENDIX B

STATISTICAL EQUATIONS USED IN THE ANALYSIS OF COVARIANCE--FOUR GROUPS

Statistical Equations

1. For adjusted sum of squares of error: Value

	Value
$E_{xx} = \sum_{j} (E_{xxj}) = \sum_{j}\sum_{i} (X_{ij} - \overline{X}_j)^2$	7531.31
$E_{zz} = \sum_{j} (E_{zzj}) = \sum_{j}\sum_{i} (Z_{ij} - \overline{Z}_j)^2$	9230.78
$E_{yy} = \sum_{j} (E_{yyj}) = \sum_{j}\sum_{i} (Y_{ij} - \overline{Y}_j)^2$	9403.00
$E_{xy} = \sum_{j} (E_{xyj}) = \sum_{j}\sum_{i} (X_{ij} - \overline{X}_j)(Y_{ij} - \overline{Y}_j)$	4549.83
$E_{zy} = \sum_{j} (E_{zyj}) = \sum_{j}\sum_{i} (Z_{ij} - \overline{Z}_j)(Y_{ij} - \overline{Y}_j)$	6474.94
$E_{xz} = \sum_{j} (E_{xzy}) = \sum_{j}\sum_{i} (X_{ij} - \overline{X}_j)(Z_{ij} - \overline{Z}_j)$	3914.31
$d = E_{xx}E_{zz} - (E_{xz})^2$	54,197,957.
$b_{xy} = \dfrac{E_{zz}E_{xy} - E_{xz}E_{zy}}{d}$	0.31

$$b_{yz} = \frac{E_{xx}E_{zy} - E_{xz}E_{xy}}{d} \qquad 0.57$$

$$E'_{yy} = E_{yy} - b_{yx}E_{xy} - b_{yz}E_{zy} \qquad 4306.78$$

2. For adjusted sum of squares of treatment:

$$T_{xx} = \sum_j n_j \, (\bar{X}_j - \bar{X})^2 \qquad 247.71$$

$$T_{zz} = \sum_j n_j \, (\bar{Z}_j - \bar{Z})^2 \qquad 129.21$$

$$T_{yy} = \sum_j n_j \, (\bar{Y}_j - \bar{Y})^2 \qquad 166.51$$

$$T_{xy} = \sum_j n_j \, (\bar{X}_j - \bar{X})\,(\bar{Y}_j - \bar{Y}) \qquad 186.60$$

$$T_{zy} = \sum_j n_j \, (\bar{Z}_j - \bar{Z})\,(\bar{Y}_j - \bar{Y}) \qquad 116.14$$

$$T_{xz} = \sum_j n_j \, (\bar{X}_j - \bar{X})\,(\bar{Z}_j - \bar{Z}) \qquad 169.95$$

$$T'_{yy} = S'_{yy} - E'_{yy} \qquad 44.29$$

3. For total sum of squares:

S_{xx}	$= T_{xx} + E_{xx}$	7779.02
S_{zz}	$= T_{zz} + E_{zz}$	9359.99
S_{yy}	$= T_{yy} + E_{yy}$	9569.51
S_{xy}	$= T_{xy} + E_{xy}$	4736.43
S_{zy}	$= T_{zy} + E_{zy}$	6591.08
S_{xz}	$= T_{xz} + E_{xz}$	4084.26
d''	$= S_{xx}S_{zz} - (S_{xz})^2$	56,130,336.50
b''_{yx}	$= \frac{S_{zz}S_{xy} - S_{xz}S_{zy}}{d''}$	0.31
b''_{yz}	$= \frac{S_{xx}S_{zy} - S_{xz}S_{xy}}{d''}$	0.57
S'_{yy}	$= S_{yy} - b''_{yx}S_{xy} - b''_{yz}S_{zy}$	4351.08

APPENDIX C

STATISTICAL EQUATIONS RELATED TO TUKEY'S PROCEDURE FOR COMPARING INDIVIDUAL MEANS

1. Program Lecture Discussion Group: Values

$E_{xx} = \sum X_i^2 - \frac{\left(\sum X_i\right)^2}{n_1}$		734.97
$E_{zz} = \sum Z_i^2 - \frac{\left(\sum Z_i\right)^2}{n_1}$		966.97
$E_{yy} = \sum Y_i^2 - \frac{\left(\sum Y_i\right)^2}{n_1}$		1007.33
$E_{xy} = \sum X_i Y_i - \frac{\sum X_i \sum Y_i}{n_1}$		595.33
$E_{zy} = \sum Z_i Y_i - \frac{\sum Z_i \sum Y_i}{n_1}$		784.33
$E_{xz} = \sum X_i Z_i - \frac{\sum X_i \sum Z_i}{n_1}$		413.97
$E'_{yy} = E_{yy} - b_{xy}E_{xy} - b_{yz}E_{zy}$		250.36
$\overline{Y}_{A1} = \overline{Y}_1 - b''_{yx}(\overline{X}_1 - \overline{X}) - b''_{yz}(\overline{Z}_1 - \overline{Z})$		25.67

2. Lecture Program Discussion Group:

$$E_{xx} = \sum X_i^2 - \frac{\left(\sum X_i\right)^2}{n_2} \qquad 1598.21$$

$$E_{zz} = \sum Z_i^2 - \frac{\left(\sum Z_i\right)^2}{n_2} \qquad 1542.21$$

$$E_{yy} = \sum Y_i^2 - \frac{\left(\sum Y_i\right)^2}{n_2} \qquad 1264.47$$

$$E_{xy} = \sum X_i Y_i - \frac{\sum X_i \sum Y_i}{n_2} \qquad 859.60$$

$$E_{zy} = \sum Z_i Y_i - \frac{\sum Z_i \sum Y_i}{n_2} \qquad 945.60$$

$$E_{xz} = \sum X_i Z_i - \frac{\sum X_i \sum Z_i}{n_2} \qquad 1092.21$$

$$E'_{yy} = E_{yy} - b_{yx}E_{xy} - b_{yz}E_{yz} \qquad 640.95$$

$$\overline{Y}_{A2} = \overline{Y}_2 - b''_{yx}(\overline{X}_2 - \overline{X}) - b''_{yz}(\overline{Z}_2 - \overline{Z}) \qquad 26.89$$

3. Program Discussion Group:

$$E_{xx} = \sum X_i^2 - \frac{\left(\sum X_i\right)^2}{n_3} \qquad 1063.38$$

$$E_{zz} = \sum Z_i^2 - \frac{\left(\sum Z_i\right)^2}{n_3} \qquad 1800.50$$

$$E_{yy} = \sum Y_i^2 - \frac{\left(\sum Y_i\right)^2}{n_3} \qquad 1908.42$$

$$E_{xy} = \sum X_i Y_i - \frac{\sum X_i \sum Y_i}{n_3} \qquad 506.86$$

$$E_{zy} = \sum Z_i Y_i - \frac{\sum Z_i \sum Y_i}{n_3} \qquad 1308.70$$

$$E_{xz} = \sum X_i Z_i - \frac{\sum X_i \sum Z_i}{n_3} \qquad 150.10$$

$$E'_{yy} = E_{yy} - b_{xy}E_{xy} - b_{zy}E_{zy} \qquad 807.16$$

$$\overline{Y}_{A3} = \overline{Y}_3 - b''_{yx}(\overline{X}_3 - \overline{X}) - b''_{yz}(\overline{Z}_3 - \overline{Z}) \qquad 24.46$$

4. Program Lecture Discussion Group:

$$E_{xx} = \sum X_i^2 - \frac{\left(\sum X_i\right)^2}{n_4} \qquad 4134.74$$

$$E_{zz} = \sum Z_i^2 - \frac{\left(\sum Z_i\right)^2}{n_4} \qquad 4921.09$$

$$E_{yy} = \sum Y_i^2 - \frac{\left(\sum Y_i\right)^2}{n_4} \qquad 5222.78$$

$$E_{xy} = \sum X_i Y_i - \frac{\sum X_i \sum Y_i}{n_4} \qquad 2588.04$$

$$E_{zy} = \sum Z_i Y_i - \frac{\sum Z_i \sum Y_i}{n_4} \qquad 3436.31$$

$$E_{xz} = \sum X_i Z_i - \frac{\sum X_i \sum Z_i}{n_4} \qquad 2557.03$$

$$E'_{yy} = E_{yy} - b_{xy}E_{xy} - b_{zy}E_{zy} \qquad 2492.85$$

$$\overline{Y}_{A4} = \overline{Y}_4 - b''_{yx}(\overline{X}_4 - \overline{X}) - b''_{yz}(\overline{Z}_4 - \overline{Z}) \qquad 26.21$$

5. Mean Square of the Error (Significant Gap Test)

$$S_{\bar{x}} = \sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2} + \frac{S_3^2}{n_3} + \frac{S_4^2}{n_4}} , \qquad 1.59$$

S_i^2 = mean square of error for the i group.

VITA

3

William Percy Hytche

Candidate for the Degree of

Doctor of Education

Thesis: A COMPARATIVE ANALYSIS OF FOUR METHODS OF INSTRUCTION IN MATHEMATICS

Major Field: Higher Education

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