THE RSRC SPACE MECHANISM--ANALYSIS BY 3x3 SCREW MATRIX, SYNTHESIS FOR SCREW GENERATION BY VARIATIONAL METHODS

by

CEMIL BAGCI

Bachelor of Science
Oklahoma State University
Stillwater, Oklahoma
1962

Master of Science
Oklahoma State University
Stillwater, Oklahoma
1963

Master of Science
Oklahoma State University
Stillwater, Oklahoma
1964

Submitted to the Faculty of the Graduate College
of the Oklahoma State University
in partial fulfillment of the requirements
for the Degree of
DOCTOR OF PHILOSOPHY
August, 1969

THE RSRC SPACE MECHANISM--ANALYSIS BY 3x3
SCREW MATRIX, SYNTHESIS FOR SCREW
GENERATION BY VARIATIONAL METHODS

Thesis Approved:

Lee Harrisberger
Thesis Adviser

Arunam H. Soni

W.H. Easton

Robert A. Gibson

D. Durham
Dean of the Graduate College

ACKNOWLEDGMENTS

The author wishes to take this opportunity to express his sincere gratitude to Dr. E. Lee Harrisberger for his assuming the responsibility of thesis adviser for the study. Author is indebted to many people, among them Dr. J. H. Boggs, Dr. Lee Harrisberger, Professor W. H. Easton, Dr. J. D. Parker, Dr. Karl Reid, Dr. R. E. Little and Dr. Karl Doss, who supplied the help and encouragement during the years of this study.

The author wishes to express his appreciations to his wife Sarah and his family for their pateince, encouragement and understanding.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

Figure | Page

LIST OF SYMBOLS

$\hat{\overline{A}}$	dual vector
a	fixed link length
$\overline{a}$	unit vector along $\overline{A}$
$\hat{\alpha}$	dual skew angle between the coupler link and the input link
b	dual part of dual input rotation
$\hat{\beta}$	dual skew angle between the pair axes on the coupler link
C	cylinder pair
$\hat{\chi}$	dual angle of rotation of the coupler link relative to the output link
D_1, D_2, D_3	differential operator matrices for rotations about x, y, and z axes, respectively
d_i	length of the i^{th} link
$\hat{\delta}$	dual skew angle between the coupler link and the output link
E	minimized error function
ε	Clifford's screw operator
F	degree of freedom of motion in a mechanism
F_c	number of overclosing constraints in a mechanism
F_0	degree of freedom of motion in a mechanism with no constraint
F_p	number of passive freedoms in a mechanism
f_i	number of passive freedoms on the i^{th} pair
$\hat{\phi}$	dual output rotation; dual Eulerian angle
$\hat{\Phi}$	tangent of $\hat{\phi}/2$

$\Delta\phi_0$	range of output rotation
$\hat{\phi}_{0_1} = \phi_{0_1} - \varepsilon S_1$	position of output link at the origin of the generated screw displacement
H	helical pair
H_s	spiral pair
h_r	error in the r^{th} parameter
i	degree of freedom on the i^{th} class pair
$\bar{i},\bar{j},\bar{k}$	unit vectors along x, y, z axes, respectively
k	number of loops in a mechanism
$\hat{\lambda}$	dual skew angle between the output and the input pair axes
λ	Lagrange multiplier
$\bar{M}$	moment vector
M	total number of general constraints in a mechanism
m_i	number of general constraints in the i^{th} loop
M_R	number of general constraints on rotation
M_T	number of general constraints on translation
N	number of design points
N_i	number of pairs of i freedoms
N_p	number of pairs in a mechanism
N_R	number of rotary freedoms
N_S	number of screw freedoms
N_T	number of translatory freedoms
n	number of links
P	prism pair
P_i	pair of i^{th} class
q_i	parameters of constraints
R	revolute pair

R_ϕ	rotation residual
R_s	translation residual
S	spherical pair, output translation
S_a	axial screw pair
S_p	planar screw pair
S_s	slotted-sphere pair
$\hat{\psi}$	dual Eulerian angle; screw displacement
T	matrix
$\hat{T}_{\hat{\alpha}}, T_{\hat{\alpha}_0}, \hat{T}_{\hat{\alpha}_1}$	dual matrix for dual rotation $\hat{\alpha}$, matrix for rotation α_0, and dual matrix for translation α_1
θ	input parameter
θ_0	input crank rotation
$\Delta\theta_0$	range of input crank rotation
θ_{0_1}	position of input crank at the origin of the screw displacement
τ	slope of the midnormal in finitely separated positions
$\hat{U}$	dual unit vector
X_i, Y_i, Z_i	coordinates of the i^{th} point in the fixed OXYZ system
$\xi_i^{(j)}, \eta_i^{(j)}, \zeta_i^{(j)}$	j^{th} position of the $O_i\ \xi_i\ \eta_i\ \zeta_i$ system
$X_i^{(j)}, Y_i^{(j)}, Z_i^{(j)}$	j^{th} position of the $O_1X_1Y_1Z_1$ body coordinate system

SUBSCRIPTS

0	real part of a dual quantity
1	dual part of a dual quantity
d	desired
g	generated

CHAPTER I

INTRODUCTION

The existance of a large variety of space mechanisms with different functioning characteristics has set forth the need for the development of synthesis techniques for the generation of spatial displacements. The interest in developing synthesis techniques for space mechanisms has occurred only in recent years. Until the late 1950's the main interest in space kinematics was in developing analytical and graphical tools for the kinematic analysis of space mechanisms and in developing criteria to test the existance of mechanisms. Grübler (1,2,3)[1], Malytcheff (4), Kutzbach (5,6) are forerunners in developing mobility criterion for plane mechanisms as well as space mechanisms. Kutzbach's mobility equation was an early attempt to consider the general constraints and passive freedoms of pairs. Kraus (7,8,9) developed a number synthesis technique for plane and space mechanisms with no general constraints, while considering the possibility of having passive freedoms on pairs. However, later Artobolevski and Dobrovolskii (10) reproposed Kutzbach's mobility equation when there is no passive freedoms in pairs. Many used the Kutzbach-Artobolevski-Dobrovolskii mobility equation in developing number synthesis techniques (12,13,14,15).

---

[1]Numbers in brackets refer to similarly numbered references in the bibliography.

Kolchin (16,17) extended Kutzbach's mobility equation by introducing his parameter which considered passive freedoms in pairs, overclosing constraints and general constraints in multiloop mechanisms. Many have conducted studies in recent years in order to determine a rational procedure for determining the number of general constraints (18,19,20,21,22), some being based on the "theory of screws" of Ball (23), and proposed mobility equations in the form of the Kutzbach-Artobolevski-Dobrovolskii mobility equation which does not consider overclosing constraints and passive freedoms in pairs.

Analytical and graphical methods based on vector angebra and partly descriptive geometry were systematically developed by Beyer and his colleague (9,24,25,26,27,28) for the kinematic analysis of space mechanisms. Tavhalidze (29), Kozevnikov (30), Zinovev (31,32), Chace (33, 34,35), Harrisberger (36), Egorov (37), Sieber (38), Trinkl (39,40), Sherwood (41), Hunt (42) used vector and graphical methods for kinematic and dynamic analysis and special cases of synthesis of space mechanisms.

The method of spherical reflections (spherical indicatrix) was applied by Bennett (43) to the analysis of the mechanism named after him. Later Beyer (44) and Dobrovolskii (45,46,47,48) applied the method for the analysis of other space mechanisms.

Kislitsin (49,50) made a major contribution to the field of space mechanisms in 1938, by applying the dual rotation introduced by Kotelnikoff (51) in 1895, to 3x3 screw affinors. Dimentberg (52,53, 54,55), inspired by Kislitsin's work, transformed Rodrigues' formula for finite rotations into a dual formula for finite dual rotations in which real vectors and real rotations were replaced by dual vectors and dual rotations. Thus, the new position of a dual unit vector $\hat{U}_1$ after a dual rotation $\hat{\phi}$ about the axis of a dual unit vector $\hat{\overline{U}}$ is given by

$$\hat{\overline{U}}' = \frac{1}{1 - \hat{\Phi}^2} [(1 - \hat{\Phi}^2)\hat{\overline{U}}_1 + 2(\hat{\overline{U}}\cdot\hat{\overline{U}}_1)\hat{\overline{U}}\,\hat{\Phi}^2 + 2(\hat{\overline{U}}\wedge\hat{\overline{U}}_1)\hat{\Phi}] \qquad (1.1)$$

where $\hat{\Phi} = \tan(\hat{\phi}/2)$. Dimentberg, in his extensive work, studied the RCCC, RS_SSR, RSCR four-bar, and RCRCR, RRCCR, RCSRR five-bar space mechanisms. He did displacement analysis of these mechanisms and studied the conditions for the introduction of passive freedoms on the pairs of the RCCC mechanism.

Denavit (56,57) introduced dual rotations and dual numbers into Cayley-Klein parameters and performed dual transformations by 2x2 screw matrices, Denavit and Uicker (58,59), Beyer (60,9), Mangeron and Dragon (61) introduced 4x4 matrix of homogeneous transformations [4x4 screw matrix] for the analyses of space mechanisms.

Shor (62) studied the nature of the screw axis and its graphical determination by Mayor-Mises construction.

Keler (63) introduced dual vector and dual angles in obtaining spherical reflections of mechanisms on the so-called "dual plane."

Another major contribution to the field of space mechanisms was made by Blaschke (64,65) by applying Dual-Number-Quaternions to kinematic analysis. Inspired with Blaschke's work, Yang (66,67) applied dual-number-quaternions in displacement analysis of space mechanisms. Dual-number-quaternions were originally developed before 1853 by Hamilton (68). Later Clifford (69), Hardy (70) and Blaschke extended its use in geometry and kinematics.

In the analysis and synthesis of spherical mechanisms considerable work has been done, notably by Dobrovolskii (46,71,72), Hein (73), Yang (74,75), Meyer zur Capellen (76,77,78).

All the contributions summarized above were primarily concerned with the adaption of suitable mathematical techniques for defining a space mechanism and have primarily been applicable to the kinematic and dynamic analyses of certain space and spherical mechanisms. In the last decade interest has turned toward developing synthesis techniques for space and spherical mechanisms. Freudenstein's analytical method of approximate synthesis (79,89,81) has been applied by Denavit (57) to synthesize spherical and RCCC space mechanism. Wilson (82) applied the finite position theory along with the use of circle-point and center-point curve functions analogous to those for plane mechanisms to design RSSR space mechanism. Roth (83,84) has initiated studies to develop synthesis technique for space mechanisms. His work is based primarily on the classical work of Schoenflies (85) on the geometry of motion and its synthetic representation and determining the geometry of the locus that may consist the desired positions of the point moving in space. He applied this theory to synthesize space mechanisms for function and spatial path generation. Suh (86,87,88) developed a synthesis technique based on the kinematic inversion of the mechanism along with the use of displacement matrix. He synthesized space mechanisms for function generation and rigid body guidance. Levitskii, Sahbazyan, Sarkisian (89.90), and Chi-Yen (91) have applied variational methods for the approximate synthesis of plane mechanisms for function and coupler-curve generation.

It has become evident that it is virtually impossible to design a mechanism which will generate a specified function exactly in a specified domain, unless the function generated by the mechanism happens to be identical with the specified function. Hence, the approximate generation of specified function is unavoidable. Approximation is done (a) by generating the function exactly at a number of precision points by satisfying the loop closure equation at each precision point, (b) by generating the function approximately at a number of design points by minimizing the error in approximating the function at the design points. The first case is Freudenstein's method of synthesis. The number of precision points in this case are the same as the number of unknown dimensions. In the second case the variational principle is used. The number of the design points is in general greater than the number of the unknown dimensions of the mechanism. In this case an error in the approximation within the specified limits by the design situations is acceptable at each design point. In this case rather than generating the function exactly at a few points, totally ignoring the magnitude of the deviation of the generated function from the desired one between the design points, the error in the approximation is distributed over the entire domain. The number of the design points may be varied as desired. Many design points will give a better approximation over a domain of continuous interval. Fewer design points will result in smaller root-mean-square-error at the design points. If the number of the design points is the same as the number of unknown dimensions, the function is generated exactly at the design points, as it is in the first case. Synthesis by variational methods results in an optimum mechanism.

Many problems of mechanism design consist of constraining conditions to be satisfied along with the approximate generation of the displacements. The constraining conditions are satisfied easily when the variational principle is used. Either Lagrange multipliers, or parameters of constraints may be used to include the constraining conditions in the optimization process. These constraining conditions may, for example, be that the velocity, acceleration, force or torque must have some specified values at certain values of the input parameter.

The variational method of synthesis provides a versatile method for space mechanism design for a broad range of specific requirements. Because of this versatility, the variational method has been selected for the study of the generation of a screw function by the displacements of the output link of the RSRC space mechanism.

The problem of synthesis of a space mechanism may involve with one or more of the following cases:

1. The generation of a spatial path or motion of a particle on a spatial surface. This requires that the desired functional form of the path or the discrete set of desired design positions of the particle are to be generated by the mechanism. Such a path may be generated by a point on the coupler link, or by a point on the output link. For example, the path on a circular or elliptic torus can be generated by a point on the coupler link of a mechanism in which the output pair and the pair which connects the coupler link to the output link are revolute pairs, as shown in Figure 12. If the output pair in Figure 12 is a helical pair the coupler point generates a path on the surface of helical-circular-torus or helical-elliptic-torus. If the coupler link is connected to the output link by a prism pair the coupler point generates a path on the surface of an hyperboloid of rotation. A path on the surface of a conoid is generated by a point on an output link having a spherical output pair. A path on the surface of a right conoid is generated by a coupler point of a mechanism in which

the output pair is a cylinder pair and the pair connecting the coupler link to the output link is a prism pair with the constraint that axis of the prism pair intersects the axis of the cylinder pair orthogonally, as in the RS_SRC mechanism shown in Figure 29b. There are unlimited number of combinations of links and pairs for the generation of paths on surfaces of constant or varying curvature.

2. The generation of a screw motion to guide a rigid body through specified positions in space; the design requirements for a screw motion are:

(a) the screw axes, needed to displace the rigid body from one position to another, may be positioned arbitrarily relative to each other in space,

(b) the screw axes may be collinear,

(c) the screw axes may intersect at a point.

In the first of these three cases the design problem is defined by specifying the six coordinates needed to seat the rigid body at each desired position. When a rigid body is to be guided through two specified positions, the output link of a mechanism, where the output pair is a cylinder pair, can be used to guide the body. In this case at first the equivalent screw displacement is determined such that the rigid body can be guided through a single dual rotation. That is, the position of the axis of the equivalent screw and the equivalent dual rotation about this axis are determined. Then, the mechanism is designed to generate this dual rotation by the displacements of its output link, where the axis of the output pair is collinear with the axis of the equivalent screw. When the rigid body is to be guided through three or more arbitrary positions, the coupler link displacements of a mechanism may be used to guide the body. For example, the coupler link of the RSRC space mechanism shown in Figure 5 may be utilized to guide a rigid body through three or more specified positions. In order that the RSRC mechanism seats a body coordinate system fixed to the coupler link, at the specified positions in space in a fixed coordinate system, the mechanism itself must be seated in a position in space. The RSRC mechanism can guide a body through four arbitrary positions. This requires the solution of 24 simultaneous equations, six for each position, for the 24 unknowns, six of which position the frame of the mechanism, six of which position the body coordinate system in the coupler frame and the remaining 12 unknowns are the dimensions of the mechanism including the four values of the input parameter to position the input crank. Many solutions are possible for the rigid body guidance through three positions since five dimensions of the mechanism may be chosen as desired.

The second case, when the screw axes for successive displacements of a rigid body are collinear, it becomes a problem of displacing a particle through a specified screw displacement $\hat{\psi}_d = \psi_{d0} + \varepsilon\psi_{d1}$, where the functions for the rotation and translation components or the coordinates of a set of precision positions are specified over the desired domain of displacements. Such a screw displacement may be generated by the output link of a mechanism having a cylinder output pair, as in the RSRC, RSHC, PRSC, RCCC, RS_SSC mechanisms.

The third case, when the screw axes for the successive displacements of a rigid body intersect at a point, becomes a problem of displacing a particle on the surface of a sphere if the particle remains at the same distance from the point of intersection of the axes.

Then the body can be guided through the specified positions by the coupler link of a spherical mechanism, or by the output link of a space mechanism having a spherical output pair, as in the RRCS mechanism. If the distance of the rigid body from the point of intersection of the axes varies, the rigid body can be guided through the specified positions by the displacements of the coupler link of a mechanism having spherical output pair and the coupler link connected to the output link by a cylinder pair, as in the RRCS, RHCS and RPCS mechanisms.

3. The problem of synthesis may require that some specified conditions must be satisfied along with the generation of the displacements. Such specified conditions are the conditions of constraints, expressions specifying these conditions are the equations of constraints. The constraining conditions may, for example, be that the velocity, acceleration, input-output force or torque ratio in the entire domain of displacements or at some specified positions must have some specified values.

This study is concerned with the synthesis of the RSRC space mechanism for the generation of screw displacements of the type $\hat{\psi}_d = \psi_{d_0} + \varepsilon\psi_{d_1}$ or for rigid body guidance through successive screws of collinear axes, or for the generation of paths on the surface of a cylinder of unit radius. Two types of screw displacements are considered in this study. These are screw displacements with no constraint and screw displacements with constraints. The exact generation of vanishing velocities and instantaneous dwells in rotation, and exact generation of the range of output rotation are considered to be the constraining conditions. The variational principle is used to optimize the mechanism by minimizing the error in approximating the specified screw displacement at a discrete set of design points.

The 3x3 screw matrix is used in the entire process of kinematic analysis. The results of the kinematic analysis are the only information needed for the optimization process, besides the specified function.

In Chapter II the screw displacement, the dual rotation, the 3x3 screw matrix and its properties are briefed. In Chapter III is the kinematic analysis of the RSRC mechanism using the 3x3 screw matrix, where the dual loop equation is solved for the displacements, velocities and accelerations. Analysis of the coupler curve coordinates by the 3x3 screw matrix is given in Appendix C. The conditions for the vanishing velociites and instantaneous dwells in the output rotation and translation of the RSRC mechanism are investigated and the results are summarized in Appendix D. The facts discovered in Appendix D are used in determining the parameters of constraints used to eliminate the equations of constraints in generating screw displacements with constraints for instantaneous dwells. The variational principle, used for the synthesis by minimizing the error in approximation, is briefly presented in Chapter IV as related to the type of synthesis subject to this study. In Chapter V, the variational method discussed in Chapter IV is applied to optimize the RSRC mechanism for the approximate generation of screw displacements with and without constraints. Two numerical examples are given for the generation of unconstrained screw displacements. In the second part of Chapter V the parameters of constraints for the exact generation of vanishing velocities and instantaneous dwells are defined and used in the synthesis of constrained screw displacements. The use of the parameters of constraints are illustrated in four numerical examples. The synthesis procedure, including the digital computer programs are directly applicable to the synthesis of the 4R plane mechanism. This is illustrated by a numerical example, where a 4R plane mechanism is synthesized to generate a constrained screw of zero pitch. The RSRC mechanism, especially when the skew angle between the two pairs on the output link is zero, can easily be synthesized by the overlay technique. This is illustrated with two numerical examples.

A brief discussion on the screw calculus is given in Appendix A.

The author experienced the need for a general form of a mobility equation for mechanisms. It has been observed that Kolchin's structural formula (16,17,36) consists the most general form of the mobility equation. In Appendix B the general constraints, passive freedoms, redundant freedoms, overclosing constraints are discussed, Kolchin's parameter is identified, and the generality of Kolchin's mobility equation is illustrated with examples.

CHAPTER II

THE 3x3 SCREW MATRIX AND ITS PROPERTIES

The screw calculus has been the most convenient method of specifying one position of a rigid body with respect to another reference position. So it is becoming more and more the major tool for the analysis and synthesis of space mechanisms, for the displacements in a mechanism are the displacements of rigid bodies (links) with respect to each other. In this study the 3x3 screw matrix is being used in obtaining any information involving the displacements in mechanisms. So presented in this chapter is the brief discussion on the screw displacement, the 3x3 screw matrix and its properties.

Screw Displacement

A body can be displaced from a reference position to a sought position through a simple movement about a certain axis, such that the body is rotated about this axis through a determinate angle θ_0 and translated parallel to this axis for a determinate distance θ_1, as shown in Figure 1. Then the body undergoes a twist [screw displacement] about the axis [screw axis] identified by a rotation θ_0 which bears to the final angle of rotation [amplitude of the screw], and a translation θ_1 which bears to the final distance of travel and is defined as the integral of the product of the pitch and the circular measure of the angle of rotation (23). Then, the dual rotation which

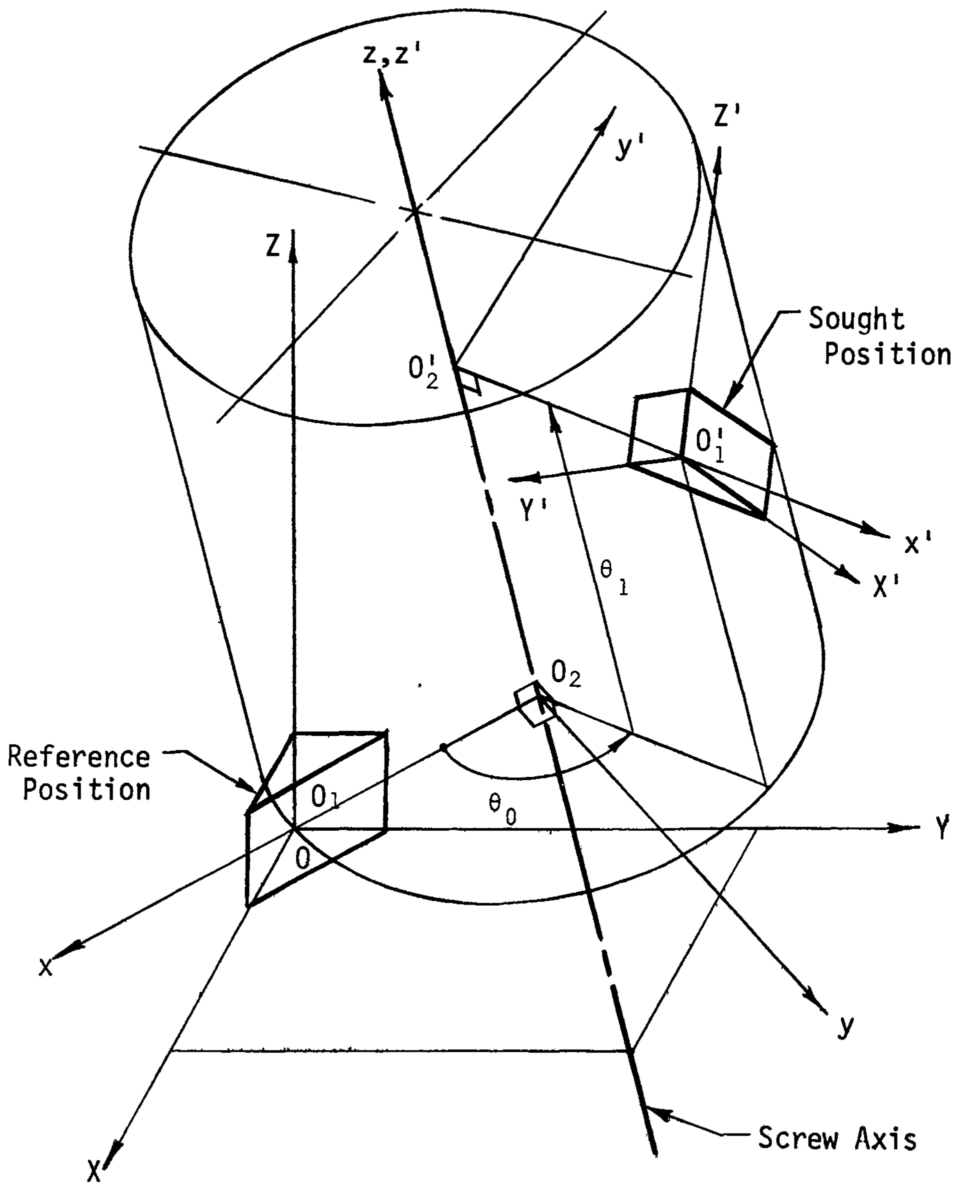


Figure 1. Dual Rotation $\hat{\theta} = \theta_0 + \varepsilon\theta_1$.

identifies the screw displacement is

$$\hat{\theta} = \int_0^{\theta_0} d\hat{\theta} = \int_0^{\theta_0} d\theta_0 + \varepsilon \int_0^{\theta_0} p_\theta \, d\theta_0 = \theta_0 + \varepsilon\theta_1 \qquad (2.1)$$

where $p_\theta = d\theta_1/d\theta_0$ is the pitch of the screw. ε is the Clifford's screw operator having the property of $\varepsilon^2 = 0$ (69). See Appendix A for the brief discussion on the screw calculus. If the pitch of the screw is constant, the dual rotation becomes

$$\hat{\theta} = \theta_0(1 + \varepsilon p_\theta) \qquad (2.2)$$

An infinitesimal dual rotation $d\hat{\theta}$ about an axis is identified as a dual vector in the reference system by

$$d\hat{\overline{\theta}} = d\theta_0(1 + \varepsilon p_\theta)\hat{\overline{U}} \qquad (2.3)$$

where $\hat{\overline{U}} = \overline{U}_0 + \varepsilon\overline{U}_1$ is the dual unit vector which identified the position of the screw axis. $\overline{U}_0$ is the unit vector at the origin of the reference frame and is parallel to the screw axis. Then the x,y,z components of $\overline{U}_0$ define the direction angles of the screw axis. The moment part $\overline{U}_1$ is given by $\overline{U}_1 = \overline{U}_2 \wedge \overline{U}_0$ where $\overline{U}_2$ is the vector extending from the origin of the reference frame to the screw axis. In the case of that the pitch is constant and the screw axis is stationary in the reference frame, the screw vector is

$$\hat{\overline{\theta}} = \theta_0(1 + \varepsilon p_\theta)\hat{\overline{U}} \qquad (2.4)$$

From Equation (2.3) one has

$$\dot{\hat{\overline{\theta}}} = \dot{\theta}_0(1 + \varepsilon p_\theta)\hat{\overline{U}} \qquad (2.5)$$

where $\dot{\hat{\theta}} = d\hat{\theta}/dt$ and $\dot{\theta}_0 = d\theta_0/dt$. Equation (2.5) is the instantaneous screw velocity and called velocity screw (44, refer to page 73).

Ball (23) developed his theory of screws considering screws defined by Equation (2.4). The theory of screws of particular pitch has been applied to the study of over-constrained mechanisms having pairs of zero pitch and pairs of constant pitch (92,93,94). However, when a mechanism consists of cylinder pairs or some other pairs which permit a screw displacement of varying pitch for a link with respect to an adjacent one, the theory of screws of a particular pitch does not apply. Then in general one is involved with instantaneous screws in mechanisms, defined by Equation (2.3). The recent studies concerned with the mobility of mechanisms consider the screws of variable pitch or the instantaneous screws (20,95,96).

The displacement of a link in a mechanism relative to an adjacent link is a screw displacement about an axis. A revolute pair permits a screw displacement of zero pitch about the axis of the pair. A prism pair permits a screw displacement of infinite pitch along the pair axis. A helical pair permits a screw displacement of constant pitch about the pair axis. A cylinder pair permits a screw displacement of varying pitch about the pair axis. A spherical pair permits an instantaneous screw displacement about an axis which moves with a fixed point at the center of the spherical pair as the link moves.

However, the problems of kinematic and dynamic analysis and synthesis of mechanisms require defining an arbitrary geometry of the mechanism with respect to a reference geometry, and involve screw displacements of the type given by Equation (2.1). That is, at an arbitrary position of the mechanism, a link is considered to move through a screw displacement of a certain amplitude about the axis of the freedom permitted by the pair and translation along the same axis, with respect to its reference position defined in the frame of the adjacent link. For example, in the RSRC mechanism shown in Figure 5, the output link is displaced through the screw

$$\hat{\phi} = \phi_0 + \varepsilon\phi_1$$

with respect to its reference position in the $O_3\xi_3'\eta_3'\zeta_3'$ system, where $\phi_1 = -S$. When the output link is in its reference position, the common normal d_3 is along the η_3' axis.

Since in a mechanism the links are displaced relative to each other to close the loop, one can also consider that the displacement of a link with respect to the adjacent link permitted by the cylinder pair consists of two successive screw displacements; screw displacement of zero pitch about the pair axis plus the screw displacement of infinite pitch along the pair axis, or vice versa. The displacement of a link with respect to the adjacent link permitted by the spherical pair is considered to take place through three successive screw displacements of zero pitch about three axes. These three screw displacements of zero pitch are preferably identified by the three Eulerian angles.

The 3x3 Screw Matrix

The displacement of a rigid body is defined by the displacement of a coordinate system fixed to the body [the body coordinate system] with respect to its reference position. Let the $O_1X_1Y_1Z_1$ system be the body coordinate system, OXYZ be the fixed reference system. Then consider a rigid body displaced through three dual rotations, $\hat{\alpha}$ about X_1 axis, $\hat{\beta}$ about Y_1' axis and $\hat{\gamma}$ about Z_1'' axis, with respect to the OXYZ system, as shown in Figure 2. Any dual vector $\bar{U}$ in the $O_1''' X_1''' Y_1''' Z_1'''$ system is defined in the OXYZ system as the dual vector by

$$\hat{\bar{U}} = \hat{T}_{\hat{\alpha}} \hat{T}_{\hat{\beta}} \hat{T}_{\hat{\gamma}} \hat{\bar{U}}''' \tag{2.6}$$

where

$$\hat{T}_{\hat{\alpha}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\hat{\alpha} & -\sin\hat{\alpha} \\ 0 & \sin\hat{\alpha} & \cos\hat{\alpha} \end{bmatrix}, \quad \hat{T}_{\hat{\beta}} = \begin{bmatrix} \cos\hat{\beta} & 0 & \sin\hat{\beta} \\ 0 & 1 & 0 \\ -\sin\hat{\beta} & 0 & \cos\hat{\beta} \end{bmatrix},$$

$$\hat{T}_{\hat{\gamma}} = \begin{bmatrix} \cos\hat{\gamma} & -\sin\hat{\gamma} & 0 \\ \sin\hat{\gamma} & \cos\hat{\gamma} & 0 \\ 0 & & 1 \end{bmatrix} \tag{2.6a}$$

are the 3x3 screw matrices for the dual rotations about the X, Y_1' and Z_1'' axes, respectively. It should be recalled that the body can be displaced to its position at $O_1'X_1Y_1'Z_1'$ through two successive screw displacements, first through a screw displacement of infinite pitch [translation through α_1] then through a screw displacement of zero pitch [rotation through α_0], or vice versa. Then, $\hat{T}_{\hat{\alpha}}$ is the product of two screw matrices one for each component of the dual rotation. Thus,

$$\hat{T}_{\hat{\alpha}} = \hat{T}_{\alpha_1} T_{\alpha_0} = T_{\alpha_0} \hat{T}_{\alpha_1}, \quad \hat{T}_{\hat{\beta}} = \hat{T}_{\beta_1} T_{\beta_0} = T_{\beta_0} \hat{T}_{\beta_1} \quad \text{and}$$

$$\hat{T}_{\hat{\gamma}} = \hat{T}_{\gamma_1} T_{\gamma_0} = T_{\gamma_0} \hat{T}_{\gamma_1} \tag{2.7}$$

where T_{α_0}, T_{β_0} and T_{γ_0} are given by Equation (2.6a), when the dual rotations are of zero pitch, $\alpha_1 = \beta_1 = \gamma_1 = 0$, as

$$T_{\alpha_0} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha_0 & -\sin\alpha_0 \\ 0 & \sin\alpha_0 & \cos\alpha_0 \end{bmatrix}, \quad T_{\beta_0} = \begin{bmatrix} \cos\beta_0 & 0 & \sin\beta_0 \\ 0 & 1 & 0 \\ -\sin\beta_0 & 0 & \cos\beta_0 \end{bmatrix},$$

$$T_{\gamma_0} = \begin{bmatrix} \cos\gamma_0 & -\sin\gamma_0 & 0 \\ \sin\gamma_0 & \cos\gamma_0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{2.7a}$$

$\hat{T}_{\alpha_1}$, $\hat{T}_{\beta_1}$ and $\hat{T}_{\gamma_1}$ are given by Equation (2.6a), when the dual rotations are of infinite pitch, $\alpha_0 = \beta_0 = \gamma_0 = 0$, as

$$\hat{T}_{\alpha_1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -\varepsilon\alpha_1 \\ 0 & \varepsilon\alpha_1 & 1 \end{bmatrix}, \quad T_{\beta_1} = \begin{bmatrix} 1 & 0 & \varepsilon\beta_1 \\ 0 & 1 & 0 \\ -\varepsilon\beta_1 & 0 & 0 \end{bmatrix},$$

$$T_{\gamma_1} = \begin{bmatrix} 1 & -\varepsilon\gamma_1 & 0 \\ \varepsilon\gamma_1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{2.7b}$$

If the rigid body is displaced through three linear displacements α_1, β_1 and γ_1 only along the X_1, Y_1', and Z_1'' axes, respectively, the $O_1''' X_1''' Y_1''' Z_1'''$ body coordinate system remains parallel to the OXYZ system. Then any dual vector in the $O_1''' X_1''' Y_1''' Z_1'''$ system is transformed into the OXYZ system by

$$\hat{U} = \hat{T}_L \hat{U}''' \tag{2.8}$$

where

$$\hat{T}_L = \hat{T}_\gamma \ \hat{T}_\beta \ \hat{T}_\alpha = \begin{bmatrix} 1 & -\varepsilon\gamma_1 & \varepsilon\beta_1 \\ \varepsilon\gamma_1 & 1 & -\varepsilon\alpha_1 \\ -\varepsilon\beta_1 & \varepsilon\alpha_1 & 1 \end{bmatrix} \tag{2.9}$$

The screw matrix $\hat{T}_L$ is an orthogonal matrix and transforms a dual vector in a coordinate system into another parallel coordinate system as a dual vector. Note that $(ij)^{th}$ element of $\hat{T}_L$ is simply the sum of the $(ij)^{th}$ elements of the matrices multiplied. Also note that

$$\hat{T}_L = \hat{T}_{\gamma_1} \ \hat{T}_{\beta_1} \ \hat{T}_{\alpha_1} = \hat{T}_{\alpha_1} \ \hat{T}_{\gamma_1} \ \hat{T}_{\beta_1} \text{ , etc.}$$

since the linear transformations are independent of the order of transformations.

The general displacement of a rigid body in space is defined by six coordinates, three linear and three rotational. Consider the three coordinate systems, OXYZ, $O_1\xi\eta\zeta$ and $O_1X_1Y_1Z_1$. The OXYZ system is the stationary system. The $O_1\xi\eta\zeta$ system is fixed to the rigid body at the origin O_1, moves with the body remaining parallel to the OXYZ system. The $O_1X_1Y_1Z_1$ body coordinate system is fixed to the body and rotates about O_1, with respect to the $O_1\xi\eta\zeta$ system. The three coordinates X_{01}, Y_{01}, Z_{01} of the origin O_1 in the OXYZ system are the three coordinates used in defining the linear displacements of the rigid body. The three rotation coordinates are the three angular displacements of the $O_1X_1Y_1Z_1$ system with respect to the $O_1\xi\eta\zeta$ system, preferably the rotations through the three Eulerian angles ϕ_0, θ_0 and ψ_0. Then the general displacement of a rigid body is defined by six successive screw displacements. The three of these are the screw displacements of infinite pitch, εX_{01}, εY_{01} and εZ_{01} along the X, Y and Z axes, respectively. The remaining three are the screw displacements of zero pitch, ϕ_0, θ_0 and ψ_0 about Z_1, X_1' and Z_1'' axes of the body coordinate system, respectively, as shown in Figure 3. Any dual vector $\hat{\bar{U}}'''$ in the body coordinate system is then defined in the OXYZ system as a dual vector by

$$\hat{\bar{U}} = \hat{T}_L \, T_R \, \hat{\bar{U}}''' \tag{2.10}$$

where $T_R = T_{\phi_0} T_{\theta_0} T_{\psi_0}$ is the rotation matrix for the three Eulerian angles, ϕ_0, θ_0 and ψ_0. T_{θ_0} and T_{ψ_0} are the same as T_{γ_0}, and T_{θ_0} is the same as T_{α_0} in Equation (2.7). $\hat{T}_L$ is given by Equation (2.9) with $\alpha_1 = X_{01}$, $\beta_1 = Y_{01}$ and $\gamma_1 = Z_{01}$.

The screw matrix $\hat{T}_L$ provides practical screw transformations by positioning a point in space such as positioning the center of a spherical pair eliminating the rotation parameters when they are not desired, and such as determining the coordinates of a coupler point in space mechanisms.

It should be noted here also that the dual matrix $\hat{T}_{\hat{\alpha}} = T_{\alpha_0} T_{\alpha_1}$ defined by Equation (2.7) may also be written as

$$\hat{T}_{\hat{\alpha}} = T_{\alpha_0} + \varepsilon T_{\alpha_1} \tag{2.11}$$

where

$$T_{\alpha_1} = \alpha_1 \begin{bmatrix} 0 & 0 & 0 \\ 0 & -\sin\alpha_0 & -\cos\alpha_0 \\ 0 & \cos\alpha_0 & -\sin\alpha_0 \end{bmatrix}$$

The matrix T_{α_1} in Equation (2.11) is not an orthogonal matrix, while the dual matrix $\hat{T}_{\alpha_1}$ in Equation (2.7b) is an orthogonal matrix. In

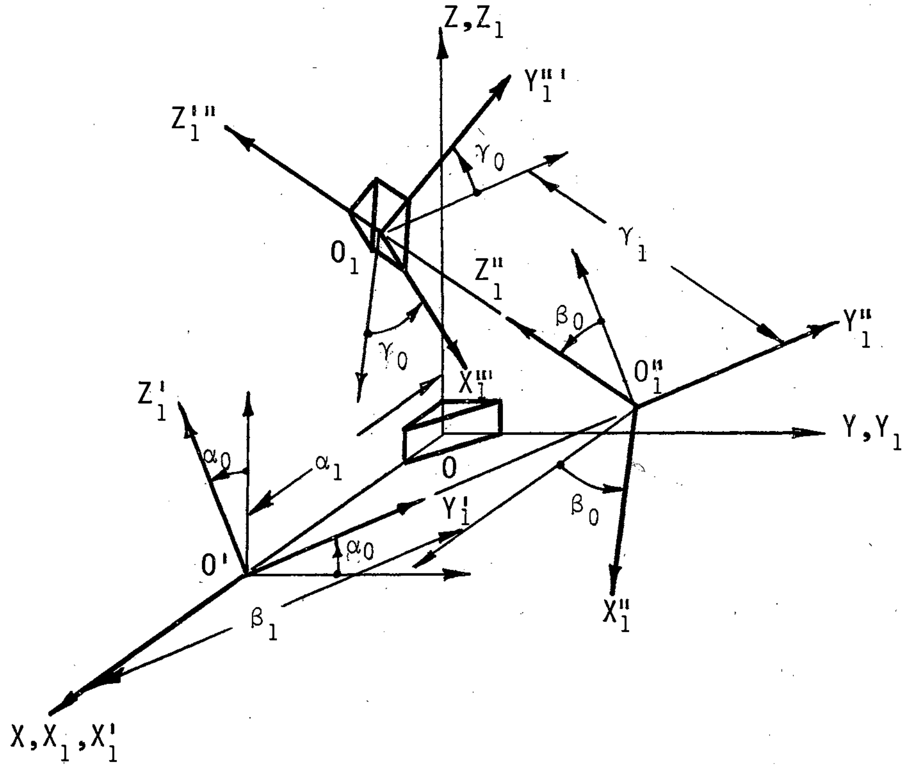


Figure 2. Displacement of a Rigid Body Through Three Dual Rotations $\hat{\alpha}$, $\hat{\beta}$ and $\hat{\gamma}$ About X, Y_1' and Z_1'' Axes, Respectively

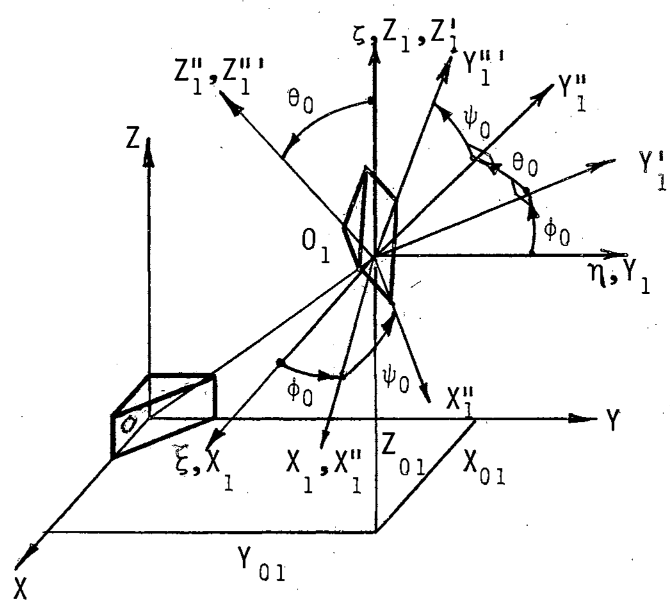


Figure 3. The Six Coordinates of the General Rigid Body Displacement.

this study the dual matrix as defined by Equation (2.7) is preferred since its orthogonal property greatly simplifies the operations with screw matrices.

Dual Eulerian Angles

The rigid body in Figure 2 can be displaced through three dual Eulerian angles $\hat{\phi} = \phi_0 + \varepsilon\phi_1$, $\hat{\theta} = \theta_0 + \varepsilon\theta_1$ and $\hat{\psi} = \psi_0 + \varepsilon\psi_1$, as shown in Figure 4. Then a dual vector in the body coordinate system is defined as a dual vector in the OXYZ system by

$$\hat{U} = \hat{T}_R \hat{U}''' \tag{2.12}$$

where $\hat{T}_R = \hat{T}_{\hat{\phi}} \hat{T}_{\hat{\theta}} \hat{T}_{\hat{\psi}}$ is the equivalent screw matrix for the three screw displacements through the three dual Eulerian angles.

Since Equations (2.10) and (2.12) define the same vector we must have

$$\hat{T}_L T_R = \hat{T}_R \tag{2.13}$$

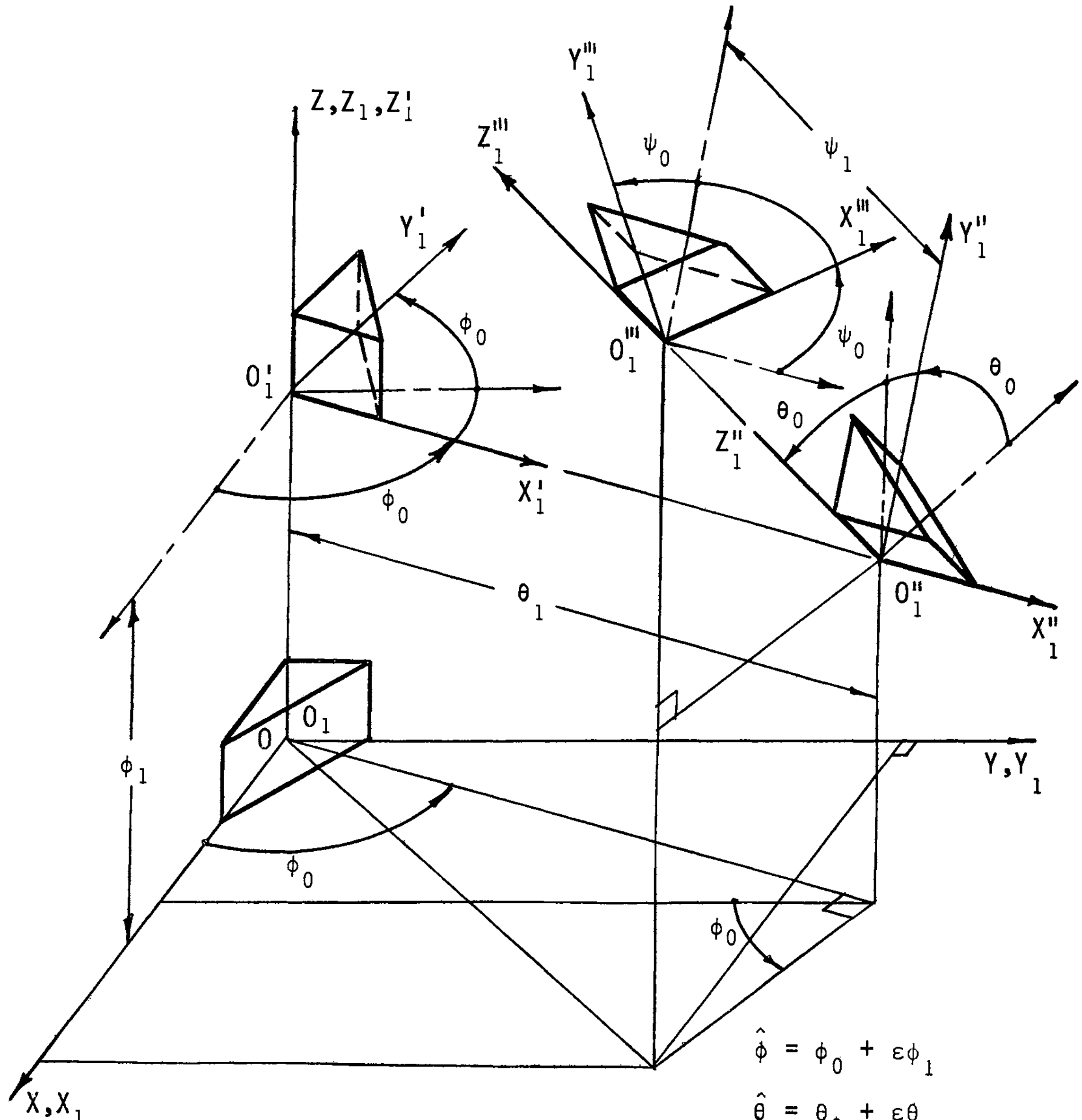
whose solution yields

$$\phi_1 = Z_{01} - \psi_1 \cos \theta_0 \tag{2.14}$$

$$\theta_1 = Y_{01} \sin \phi_0 + X_{01} \cos \phi_0 \tag{2.15}$$

$$\psi_1 = (X_{01} \sin \phi_0 - Y_{01} \cos \phi_0)/\sin \theta_0 \tag{2.16}$$

as the dual parts of the three dual Eulerian angles. Note that when $\theta_0 = 0$ or $180°$, ϕ_0 and ψ_0 are rotations about the Z axis, and $\theta_1 = X_{01}/\cos \phi_0 = Y_{01}/\sin \phi_0$, while $\phi_1 + \psi_1 = Z_{01}$ in the former case, $\phi_1 - \psi_1 = Z_{01}$ in the latter case.



$$\hat{\phi} = \phi_0 + \varepsilon\phi_1$$

$$\hat{\theta} = \theta_0 + \varepsilon\theta_1$$

$$\hat{\psi} = \psi_0 + \varepsilon\psi_1$$

Figure 4. Dual Eulerian Angles

CHAPTER III

KINEMATIC ANALYSIS OF THE RSRC SPACE MECHANISM

The synthesis of the RSRC space mechanism for function generation by variational principles is presented in Chapter V, where the mechanism is optimized by minimizing the error in approximating the specified screw displacement at a discrete set of design points. The error at a design point corresponding to a value of the input parameter is the deviation of the generated displacement function from the function to be generated at that value of the input parameter. Such a process requires the displacement analysis of the mechanism in order to furnish the generated displacement function. The problem of synthesis may involve certain constraining conditions in addition to the displacement function, such as satisfying the velocity or acceleration at certain values of the displacement function, or maintaining certain level of transmissivity in the domain of displacements. In such cases the functional forms of velocities, accelerations, forces, torques and transmissivities are needed in order to furnish the equations of constraints. Furthermore, the RSRC mechanism may be subject to synthesis for coupler curve generation and rigid body guidance in space. If this is the case the coordinates of the coupler points are needed as the generated displacements. Although the constraining conditions considered in this study, in addition to the function to be minimized, are involving velocity only, it is intended here to furnish the complete information regarding the analysis of the RSRC mechanism and methods of obtaining these information for use in further extension of this study on the RSRC mechanism and other space mechanisms. This chapter presents the kinematic analysis of the RSRC mechanism. Coupler curve coordinates for the RSRC mechanism are given in Appendix C.

Displacements in the RSRC Mechanism

The parametric relationships for a mechanism are reduced from the dual loop closure equation, which may be either in 3x3 dual matrix form or in a dual vector form. In general the matrix form is attained when all the parameters of the mechanism are included in the loop equation. However, it is always desirable to eliminate some of the parameters from the loop equation in order to have ease in obtaining expressions for each displacement parameter as functions of the input parameter only. If such is the case the loop equation is written in dual vector form by transforming the unit vector along the axis of the dual rotation, which is to be eliminated from the loop equation, into the fixed frame of reference. In certain mechanisms it may be desirable to position the center of a pair member as of a spherical pair and a sphere-cylinder pair in order that more than one parameters may be eliminated from the loop equation.

Figure 5 shows the RSRC mechanism and its parameters. The angles $-\alpha_0$, β_0 and $-\gamma_0$, which position the coupler link relative to the input link, are the three Eulerian angles. The fixed link of the mechanism is positioned in the XY plane of the OXYZ system. The $O\xi\eta\zeta$ system is initially at the origin of the OXYZ system and is considered to be displaced through the parameters of the mechanism as its geometry shapes.

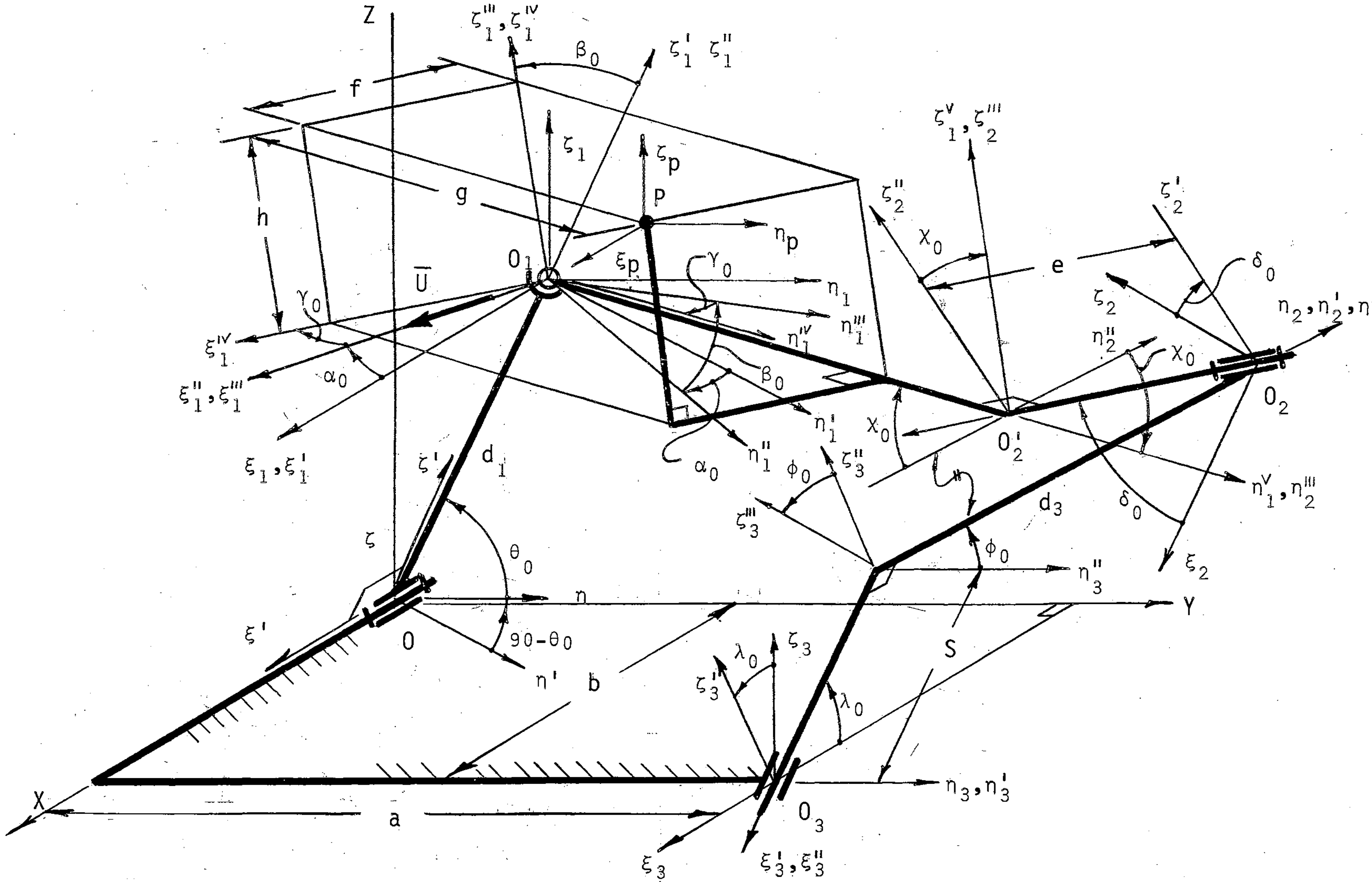


Figure 5. The RSRC Space Mechanism and Its Parameters.

The angles θ_0 and $\hat{\phi} = \phi_0 - \varepsilon S$ are the input and output parameters, respectively. The loop equation is written by transforming a dual vector $\hat{\bar{U}}$ in the $O_1\xi_1''' \eta_1''' \zeta_1'''$ system, whose origin is the center of the spherical pair, into the OXYZ system through two paths O_1-O and O_1-O_2-O_3-O. Consider first that the unit vector is along the ξ_1''' axis, in order that β_0 is eliminated from the loop equation. Thus, the unit vector $\bar{U}$ is defined in the OXYZ system by the dual vector

$$\begin{Bmatrix} \hat{X} \\ \hat{Y} \\ \hat{Z} \end{Bmatrix}_{O_1-O} = T_{90-\theta_0} \hat{T}_{\hat{\alpha}}^{-1} \begin{Bmatrix} 1 \\ 0 \\ 0 \end{Bmatrix} \tag{3.1}$$

and

$$\begin{Bmatrix} \hat{X} \\ \hat{Y} \\ \hat{Z} \end{Bmatrix}_{O_1-O_2-O_3-O} = \hat{T}_b \hat{T}_{\hat{\lambda}} \hat{T}_{\hat{\phi}} \hat{T}_{\hat{\delta}} \hat{T}_{\hat{\chi}} \hat{T}_{d_2} T_{\gamma_0} \begin{Bmatrix} 1 \\ 0 \\ 0 \end{Bmatrix} \tag{3.2}$$

where the subscripts for the matrices indicate the corresponding angles or the linear displacements; $\hat{T}_{\hat{\phi}}$ and $\hat{T}_{\hat{\lambda}}$ are the same as $\hat{T}_{\hat{\alpha}}$ and $\hat{T}_{\hat{\beta}}$ in Equation (2.6), $\hat{T}_{\hat{\alpha}}$, $\hat{T}_{\hat{\delta}}$, $\hat{T}_{\hat{\chi}}$, T_{γ_0} and $T_{90-\theta_0}$ are the same as the inverse of $\hat{T}_{\hat{\gamma}}$, $\hat{T}_{\hat{\beta}}$, $\hat{T}_{\hat{\alpha}}$, T_{β_0} and T_{α_0} in Equation (2.6), respectively, with $\hat{\alpha} = \alpha_1 - \varepsilon d$, $\hat{\lambda} = \lambda_0 + \varepsilon a$, $\hat{\phi} = \phi_0 - \varepsilon S$, $\hat{\delta} = \delta_0 - \varepsilon d_3$ and $\chi = \chi_0 - \varepsilon e$; $\hat{T}_b$ is the same as $\hat{T}_{\alpha_1}$ and $\hat{T}_{d_2}$ is the same as the inverse of $\hat{T}_{\beta_1}$ in Equation (2.7b). Equations (3.1) and (3.2) define the same vector. Therefore, equating them and leaving the variables χ_0 and γ_0 on one side, one has the dual loop vector

$$\hat{T}_e^{-1} \hat{T}_{\hat{\delta}}^{-1} \hat{T}_{\hat{\phi}}^{-1} \hat{T}_{\hat{\lambda}}^{-1} \hat{T}_{90-\theta} \hat{T}_{d_1} T_{\alpha_0}^{-1} \begin{Bmatrix} 1 \\ 0 \\ 0 \end{Bmatrix} = T_{\chi_0} \hat{T}_{d_2} T_{\gamma_0} \begin{Bmatrix} 1 \\ 0 \\ 0 \end{Bmatrix} \tag{3.3}$$

where $\hat{T}_{d_1}$ and $\hat{T}_e$ are the dual parts of $\hat{T}_{\hat{\alpha}}$ and $\hat{T}_{\hat{\chi}}$, and the same as $\hat{T}_{\gamma_1}$ and $\hat{T}_{\alpha_1}$ in Equation (2.7b), $\hat{T}_{90-\hat{\theta}}$ is given by

$$\hat{T}_{90-\hat{\theta}} = \hat{T}_b^{-1}\, T_{90-\theta} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \sin\hat{\theta} & \cos\hat{\theta} \\ 0 & -\cos\hat{\theta} & \sin\hat{\theta} \end{bmatrix}$$

with $\hat{\theta} = \theta_0 - \varepsilon b$. Letting

$$[\hat{D}] = \hat{T}_e\, \hat{T}_{\hat{\delta}}^{-1}\, \hat{T}_{\hat{\phi}}^{-1}\, \hat{T}_{\hat{\lambda}}^{-1}\, \hat{T}_{90-\hat{\theta}}\, \hat{T}_{d_1}^{-1}$$

Equation (3.3) reduces to

$$\begin{Bmatrix} \hat{D}_{11}\cos\alpha_0 - \hat{D}_{12}\sin\alpha_0 \\ \hat{D}_{21}\cos\alpha_0 - \hat{D}_{22}\sin\alpha_0 \\ \hat{D}_{31}\cos\alpha_0 - \hat{D}_{32}\sin\alpha_0 \end{Bmatrix} = \begin{Bmatrix} \cos\gamma_0 \\ \cos\chi_0\sin\gamma_0 + \varepsilon d_2\sin\chi_0\cos\gamma_0 \\ -\sin\chi_0\sin\gamma_0 + \varepsilon d_2\sin\chi_0\cos\gamma_0 \end{Bmatrix} \quad (3.4)$$

where

$$\hat{D}_{11} = \cos\hat{\delta}\cos\hat{\lambda} + \sin\hat{\lambda}\sin\hat{\delta}\cos\hat{\phi} - \varepsilon d_1[\sin\hat{\delta}\sin\hat{\theta}\sin\hat{\phi} + \cos\hat{\theta}(\cos\hat{\lambda}\sin\hat{\delta}\cos\hat{\phi} - \sin\hat{\lambda}\cos\hat{\delta})]$$

$$\hat{D}_{12} = -\sin\hat{\theta}\sin\hat{\delta}\sin\hat{\phi} - \cos\hat{\theta}(\cos\hat{\lambda}\sin\hat{\delta}\cos\hat{\phi} - \sin\hat{\lambda}\cos\hat{\delta}) - \varepsilon d_1(\cos\hat{\delta}\cos\hat{\lambda} + \sin\hat{\lambda}\sin\hat{\delta}\cos\hat{\phi})$$

$$\hat{D}_{13} = -\cos\hat{\theta}\sin\hat{\delta}\sin\hat{\phi} + \sin\hat{\theta}(\cos\hat{\lambda}\sin\hat{\delta}\cos\hat{\phi} - \sin\hat{\lambda}\cos\hat{\delta})$$

$$\hat{D}_{21} = \sin\hat{\lambda}(\sin\hat{\phi} + \varepsilon e\cos\hat{\delta}\cos\hat{\phi}) - \varepsilon e\sin\hat{\delta}\cos\hat{\lambda} + \varepsilon d_1(\sin\hat{\theta}\cos\hat{\phi} - \cos\hat{\theta}\cos\hat{\lambda}\sin\hat{\phi})$$

$$\hat{D}_{22} = -\varepsilon d_1\sin\hat{\lambda}\sin\hat{\phi} + \sin\hat{\theta}(\cos\hat{\phi} - \hat{\varepsilon} e\cos\hat{\delta}\sin\hat{\phi}) - \cos\hat{\theta}[\varepsilon e\sin\hat{\delta}\sin\hat{\lambda} + \cos\hat{\lambda}(\sin\hat{\phi} + \varepsilon e\cos\hat{\delta}\cos\hat{\phi})]$$

$$\hat{D}_{23} = \cos\hat{\theta}(\cos\hat{\phi} - \varepsilon e \cos\hat{\delta} \sin\hat{\phi}) + \sin\hat{\theta}[\varepsilon e \cos\hat{\delta} \sin\hat{\lambda} + \cos\hat{\lambda}(\sin\hat{\phi} + \varepsilon e \cos\hat{\delta} \cos\hat{\phi})]$$

$$\hat{D}_{31} = -\sin\hat{\delta} \cos\hat{\lambda} + \sin\hat{\lambda}(\cos\hat{\delta} \cos\hat{\phi} - \varepsilon e \sin\hat{\phi}) - \varepsilon d_1[\sin\hat{\theta} \cos\hat{\delta} \sin\hat{\phi} + \cos\hat{\theta}(\sin\hat{\delta} \sin\hat{\lambda} + \cos\hat{\lambda} \cos\hat{\delta} \cos\hat{\phi})]$$

$$\hat{D}_{32} = \varepsilon d_1(\sin\hat{\delta} \cos\hat{\lambda} - \sin\hat{\lambda} \cos\hat{\delta} \cos\hat{\phi}) - \sin\hat{\theta}(\varepsilon e \cos\hat{\phi} + \cos\hat{\delta} \sin\hat{\phi}) - \cos\hat{\theta}[\sin\hat{\delta} \sin\hat{\lambda} + \cos\hat{\lambda}(\cos\hat{\delta} \cos\hat{\phi} - \varepsilon e \sin\hat{\phi})]$$

$$\hat{D}_{33} = -\cos\hat{\theta}(\varepsilon e \cos\hat{\phi} + \cos\hat{\delta} \sin\hat{\phi}) + \sin\hat{\theta}[\sin\hat{\delta} \sin\hat{\lambda} + \cos\hat{\lambda}(\cos\hat{\delta} \cos\hat{\phi} - \varepsilon e \sin\hat{\theta})]$$

Separating the real and dual parts of Equation (3.4) and equating the corresponding elements on both sides, one gets the following six equations:

$$D_{110} \cos\alpha_0 - D_{120} \sin\alpha_0 = \cos\gamma_0 \tag{3.5a}$$

$$D_{210} \cos\alpha_0 - D_{220} \sin\alpha_0 = \cos\chi_0 \sin\gamma_0 \tag{3.5b}$$

$$D_{310} \cos\alpha_0 - D_{320} \sin\alpha_0 = -\sin\chi_0 \sin\gamma_0 \tag{3.5c}$$

$$D_{111} \cos\alpha_0 - D_{121} \sin\alpha_0 = 0 \tag{3.5d}$$

$$D_{211} \cos\alpha_0 - D_{221} \sin\alpha_0 = d_2 \sin\chi_0 \cos\gamma_0 \tag{3.5e}$$

$$D_{311} \cos\alpha_0 - D_{321} \sin\alpha_0 = d_2 \cos\chi_0 \cos\gamma_0 \tag{3.5f}$$

where

$$D_{110} = \cos\delta_0 \cos\lambda_0 + \sin\lambda_0 \sin\delta_0 \cos\phi_0$$

$$D_{111} = \cos\phi_0(\delta_1 \sin\lambda_0 \cos\delta_0 + \lambda_1 \cos\lambda_0 \sin\delta_0 - d_1 \cos\theta_0 \cos\lambda_0 \sin\delta_0) - d_1 \sin\phi_0 \sin\delta_0 \sin\theta_0 + S\cdot\sin\delta_0 \sin\lambda_0 \sin\phi_0 + d_1 \sin\lambda_0 \cos\delta_0 \cos\theta_0 - \delta_1 \sin\delta_0 \cos\lambda_0 - \lambda_1 \sin\lambda_0 \cos\delta_0$$

$$D_{120} = \sin\lambda_0 \cos\delta_0 \cos\theta_0 - \cos\lambda_0 \sin\delta_0 \cos\theta_0 \cos\phi_0 - \sin\delta_0 \sin\theta_0 \sin\phi_0$$

$$\begin{aligned} D_{121} = {} & \cos\phi_0(\lambda_1 \sin\lambda_0 \sin\delta_0 \cos\theta_0 + \theta_1 \sin\delta_0 \cos\lambda_0 \sin\theta_0 \\ & - \delta_1 \cos\delta_0 \cos\lambda_0 \cos\theta_0 - d_1 \sin\lambda_0 \sin\delta_0) - \sin\phi_0(\theta_1 \sin\delta_0 \cos\theta_0 \\ & + \delta_1 \cos\delta_0 \sin\theta_0) + S\cdot\sin\delta_0(\cos\phi_0 \sin\theta_0 - \cos\lambda_0 \cos\theta_0 \sin\phi_0) \\ & + \cos\theta_0(\lambda_1 \cos\lambda_0 \cos\delta_0 - \delta_1 \sin\delta_0 \sin\lambda_0) - \theta_1 \sin\lambda_0 \cos\delta_0 \sin\theta_0 \\ & - d_1 \cos\delta_0 \cos\lambda_0 \end{aligned}$$

$$D_{210} = \sin\lambda_0 \sin\phi_0$$

$$\begin{aligned} D_{211} = {} & \cos\phi_0(d_1 \sin\theta_0 + e \sin\lambda_0 \cos\delta_0) + \sin\phi_0 \cos\lambda_0(\lambda_1 - d_1 \cos\theta_0) \\ & - S\cdot\sin\lambda_0 \cos\phi_0 - e \sin\delta_0 \cos\lambda_0 \end{aligned}$$

$$D_{220} = \sin\theta_0 \cos\phi_0 - \cos\lambda_0 \cos\theta_0 \sin\phi_0$$

$$\begin{aligned} D_{221} = {} & \cos\phi_0(\theta_1 \cos\theta_0 - e \cos\theta_0 \cos\lambda_0 \cos\delta_0) + \sin\phi_0(\theta_1 \cos\lambda_0 \sin\theta_0 \\ & - d_1 \sin\lambda_0 - e \cos\delta_0 \sin\theta_0 + S(\sin\phi_0 \sin\theta_0 + \cos\theta_0 \cos\lambda_0 \cos\phi_0) \\ & - e \sin\delta_0 \sin\lambda_0 \cos\theta_0 \end{aligned}$$

$$D_{310} = \sin\lambda_0 \cos\delta_0 \cos\phi_0 - \sin\delta_0 \cos\lambda_0$$

$$\begin{aligned} D_{311} = {} & \cos\phi_0(\lambda_1 \cos\lambda_0 \cos\delta_0 - \delta_1 \sin\delta_0 \sin\lambda_0 - d_1 \cos\delta_0 \cos\lambda_0 \cos\theta_0) \\ & - \sin\phi_0(d_1 \cos\delta_0 \sin\theta_0 + e \sin\lambda_0) + S\cdot\cos\delta_0 \sin\lambda_0 \sin\phi_0 \\ & + \lambda_1 \sin\lambda_0 \sin\delta_0 - \delta_1 \cos\delta_0 \cos\lambda_0 - d_1 \sin\delta_0 \sin\lambda_0 \cos\theta_0 \end{aligned}$$

$$D_{320} = - \cos\delta_0 \cos\lambda_0 \cos\theta_0 \cos\phi_0 - \cos\delta_0 \sin\theta_0 \sin\phi_0 - \sin\delta_0 \sin\lambda_0 \cos\theta_0$$

$$\begin{aligned} D_{321} = {} & \cos\phi_0(\delta_1 \sin\delta_0\cos\lambda_0\cos\theta_0 + \lambda_1 \sin\lambda_0 \cos\delta_0 \cos\theta_0 - d_1 \sin\lambda_0 \cos\delta_0 \\ & - e \sin\theta_0 + \theta_1 \cos\delta_0 \cos\lambda_0 \sin\theta_0) + \sin\phi_0(\delta_1 \sin\delta_0 \sin\theta_0 \\ & - \theta_1 \cos\delta_0 \cos\theta_0 + e \cos\lambda_0 \cos\theta_0) + S(\cos\delta_0 \sin\theta_0 \cos\phi_0 \\ & - \cos\delta_0 \cos\lambda_0 \cos\theta_0 \sin\phi_0) + d_1 \sin\delta_0 \cos\lambda_0 + \theta_1 \sin\delta_0 \sin\lambda_0 \sin\theta_0 \\ & - \delta_1 \cos\theta_0 \cos\delta_0 \sin\lambda_0 - \lambda_1 \cos\lambda_0 \sin\delta_0 \cos\theta_0 \end{aligned}$$

From Equation (3.5) one obtains α_0, γ_0 and χ_0 as functions of the input and output parameters. Thus,

$$\tan \alpha_0 = \frac{D_{111}}{D_{121}} \tag{3.6}$$

$$\cos \gamma_0 = \frac{D_{110} D_{121} - D_{120} D_{111}}{\sqrt{D^2_{121} + D^2_{111}}} \quad \text{or,}$$

$$\tan \gamma_0 = d_2 \frac{D_{210} \cos\alpha_0 - D_{220} \sin\alpha_0}{D_{311} \cos\alpha_0 - D_{321} \sin\alpha_0} \tag{3.7}$$

$$\cos \chi_0 = \frac{D_{311} D_{121} - D_{321} D_{111}}{d_2 (D_{110} D_{121} - D_{120} D_{111})} \quad \text{and}$$

$$\tan \chi_0 = \frac{D_{211} D_{121} - D_{221} D_{111}}{D_{311} D_{121} - D_{321} D_{111}} \tag{3.8}$$

It is to be noted here that the first component of Equation (3.4) is Freudenstein's dual displacement equation for the unconstrainted RSRC mechanism. The real part of this equation, the Equation (3.5a) concurs with the Freudenstein's displacement equation for the spherical six-bar mechanism shown in Figure 6, where the displacements and skew angles in the RSRC mechanism are indicated by the same symbols. This spherical six-bar mechanism is the spherical indicatrix of the RSRC mechanism shown in Figure 5. Note that the spherical indicatrix has varying skew angles α_0 and γ_0 for the input crank and the coupler link. After substituting D_{110} and D_{120}, Equation (3.5a) gives

$$\cos\phi_0(\sin\delta_0 \sin\lambda_0 \cos\alpha_0 + \cos\lambda_0 \sin\delta_0 \sin\alpha_0 \cos\theta_0)$$
$$+ \sin\phi_0 \sin\alpha_0 \sin\delta_0 \sin\theta_0 + \cos\delta_0 \cos\lambda_0 \cos\alpha_0 -$$

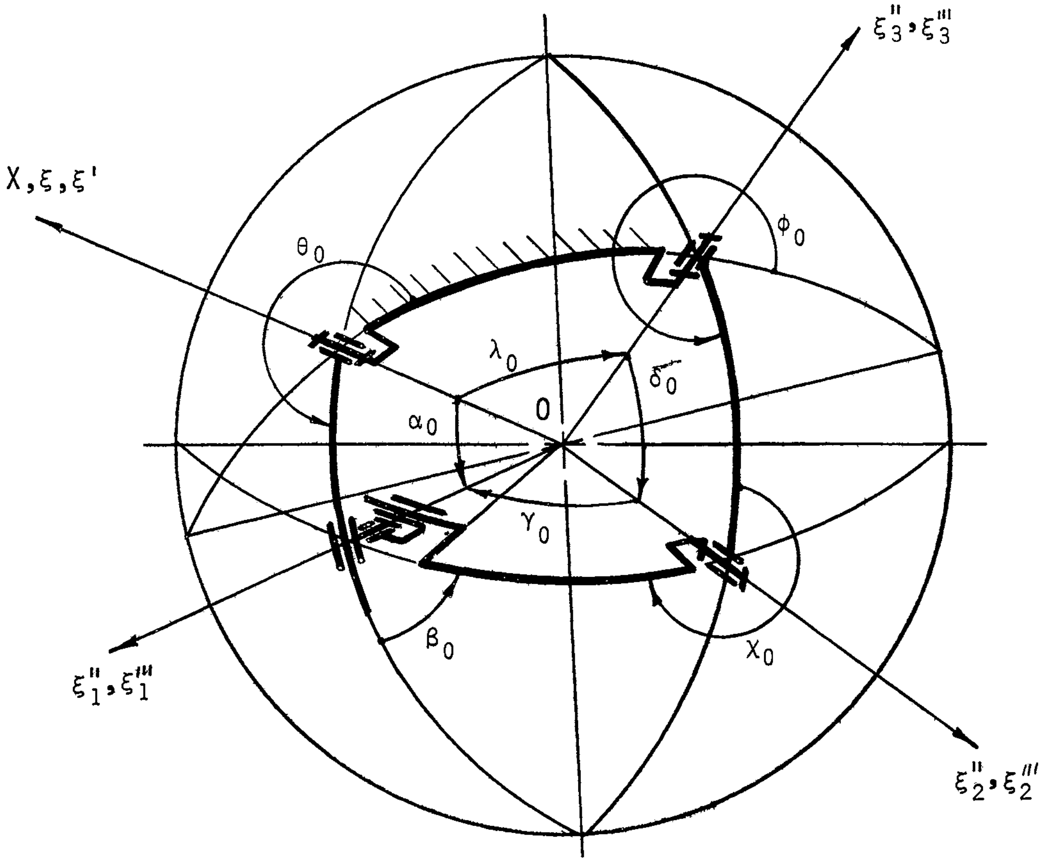


Figure 6. The Spherical Indicatrix of the RSRC Mechanism Shown in Figure 5.

$$- \sin\lambda_0 \cos\delta_0 \cos\theta_0 \sin\alpha_0 = \cos\gamma_0 \tag{3.9}$$

as the displacement equation for the spherical indicatrix of the RSRC mechanism, and so for the unconstrained RSRC mechanism. The output rotation ϕ_0 for a given value of the input rotation, for the unconstrained RSRC mechanism is obtained by solving the quartic equation which results upon substituting α_0 and γ_0 from Equations (3.6) and (3.7) into Equation (3.9). The output translation S for the unconstrained RSRC mechanism is obtained by solving Equation (3.5d) for S after substituting α_0 and γ_0.

Equation (3.5a) and (3.5d) do not lead to input-output relationship for constrained inversions of the RSRC mechanism. For example, the indeterminacy of S can easily be observed in Equation (4.5d) when $\delta_0 = 0$. However, the general expression for the input-output relationship is obtained by eliminating α_0, γ_0 and χ_0 in Equation (3.5). Thus, one has

$$\begin{aligned}(D_{311}D_{121} + D_{321}D_{111})^2 + (D_{211}D_{121} - D_{221}D_{111})^2 = \\ d_2^2(D_{110}D_{121} - D_{120}D_{111})^2\end{aligned} \tag{3.10}$$

as the general form of Freudenstein's displacement equation for the RSRC mechanism. In a 4R plane mechanism $D_{111} = 0$, $D_{110} = 1$. Then, Equations (3.10) and (3.8) reduce to

$$D_{311_p}^2 + D_{211_p}^2 = d_2^2 \tag{3.11}$$

and

$$\tan \chi_0 = \frac{D_{211_p}}{D_{311_p}} \tag{3.12}$$

where

$$D_{211_p} = d_1 \sin\theta_0 \cos\phi_0 + \sin\phi_0(a - d_1 \cos\theta_0)$$

$$D_{311_p} = \cos\phi_0(a - d_1 \cos\theta_0) - d_1 \sin\theta_0 \sin\phi_0 + d_3$$

Substituting D_{211_p} and D_{311_p} into Equation (3.11) one obtains

$$[a \sin\phi_0 - d_1 \sin(\phi_0 - \theta_0)]^2 + [d_3 + a \cos\phi_0 - d_1 \cos(\phi_0 - \theta_0)]^2 = d_2^2 \tag{3.13}$$

which is the well known Freudenstein's displacement equation for 4R plane mechanism.

One should also observe that, when both sides of Equation (3.10) are divided by $d_2^2 \cdot (D_{111}^2 + D_{121}^2)^{1/2}$, the left side becomes $\cos^2 \gamma_0$, as given by Equations (3.5e), (3.5f) and (3.6), while the right side is the square of the left side of Equation (3.5a), indicating that Equation (3.10) also concurs with the displacement equation for its spherical indicatrix given by Equation (3.9).

To obtain an expression for the rotation β_0 the unit vectors in the $O_1\xi_1''' \eta_1''' \zeta_1'''$ system are transformed into the OXYZ system as dual vectors along two paths, O_1-O and O_1-O_2-O_3-O. Thus,

$$T_{90-\theta_0} \hat{T}_{\hat{\alpha}}^{-1} T_{\beta}^{-1} \begin{Bmatrix} 1 \\ 1 \\ 1 \end{Bmatrix} = \hat{T}_{\hat{b}} \hat{T}_{\hat{\lambda}} \hat{T}_{\hat{\phi}} \hat{T}_{\hat{\delta}} \hat{T}_{\hat{\chi}} \hat{T}_{d_2} T_{\gamma_0} \begin{Bmatrix} 1 \\ 1 \\ 1 \end{Bmatrix} \tag{3.14}$$

or

$$[\hat{D}] T_{\alpha_0}^{-1} T_{\beta_0}^{-1} = T_{\chi_0} T_{d_2} \hat{T}_{\gamma_0} \tag{3.15}$$

where the matrices T_{χ_0}, $\hat{T}_{d_2}$, T_{γ_0}, $T_{\alpha_0}^{-1}$ and $[\hat{D}]$ are the same as in

Equations (3.3) and (3.4), T_{β_0} is the same as the inverse of T_{α_0} in Equation (2.7). The coefficients of the matrices on both sides of Equation (3.15) are

$$\begin{bmatrix} (\hat{D}_{11}\cos\alpha_0 - \hat{D}_{12}\sin\alpha_0) & (\hat{D}_{11}\cos\alpha_0 - \hat{D}_{12}\sin\alpha_0)\cos\beta_0 + \hat{D}_{13}\sin\beta_0 & \hat{D}_{13}\cos\beta_0 - (\hat{D}_{11}\sin\alpha_0 + \hat{D}_{12}\cos\alpha_0)\sin\beta_0 \\ (\hat{D}_{21}\cos\alpha_0 - \hat{D}_{22}\sin\alpha_0) & (\hat{D}_{21}\sin\alpha_0 + \hat{D}_{22}\cos\alpha_0)\cos\beta_0 + \hat{D}_{23}\sin\beta_0 & \hat{D}_{23}\cos\beta_0 - (\hat{D}_{21}\sin\alpha_0 + \hat{D}_{22}\cos\alpha_0)\sin\beta_0 \\ (\hat{D}_{31}\cos\alpha_0 - \hat{D}_{32}\sin\alpha_0) & (\hat{D}_{31}\sin\alpha_0 + \hat{D}_{32}\cos\alpha_0)\cos\beta_0 + \hat{D}_{33}\sin\beta_0 & \hat{D}_{33}\cos\beta_0 - (\hat{D}_{31}\sin\alpha_0 + \hat{D}_{32}\cos\alpha_0)\sin\beta_0 \end{bmatrix} =$$

$$= \begin{bmatrix} \cos\gamma_0 & -\sin\gamma_0 & \varepsilon d_2 \\ \begin{pmatrix} \cos\chi_0 \ \sin\gamma_0 \\ + \varepsilon d_2 \ \sin\chi_0 \ \cos\gamma_0 \end{pmatrix} & \begin{pmatrix} \cos\chi_0 \ \cos\gamma_0 \\ - \varepsilon d_2 \ \sin\chi_0 \ \sin\gamma_0 \end{pmatrix} & \sin\chi_0 \\ \begin{pmatrix} \varepsilon d_2 \ \cos\chi_0 \ \cos\gamma_0 \\ - \sin\chi_0 \ \sin\gamma_0 \end{pmatrix} & \begin{pmatrix} - \sin\chi_0 \ \cos\gamma_0 \\ - \varepsilon d_2 \ \cos\chi_0 \ \sin\gamma_0 \end{pmatrix} & \cos\chi_0 \end{bmatrix} \quad (3.16)$$

where the coefficients in the first column on both sides give the loop closure equation, Equation (3.4), in which β_0 was eliminated. β_0 is easily determined from the real part of the (1,3) element of Equation (3.16). Thus,

$$\tan \beta_0 = \frac{D_{130}\ D_{121}}{D_{110}\ D_{111} + D_{120}\ D_{121}} \quad (3.17)$$

β_0 as given by Equation (3.17), is undefined in the case of $\delta_0 = \lambda_0 = 0$. However, the general expression for β_0, as obtained from the real part of the (2,3) and the dual part of the (1,3) elements of Equation (3.16), is

$$\tan\beta_0 = \sqrt{D_{121}^2 + D_{111}^2}\{[D_{131}(D_{211}D_{121} - D_{221}D_{111}) - d_2^2\, D_{230}(D_{110}D_{121} - D_{120}D_{111})]/[(D_{211}D_{121} - D_{221}D_{111})(D_{111}^2 + D_{121}^2) - d_2^2(D_{210}D_{111} + D_{220}D_{121})(D_{110}D_{121} - D_{120}D_{111})] \tag{3.18}$$

One should observe in the spherical indicatrix of the RSRC mechanism shown in Figure 6 that, if ϕ_0 and χ_0 are determined for a given value of θ_0, the spherical indicatrix is defined, and the angles α_0, β_0 and γ_0 may be directly measured from the conformal projection of the spherical indicatrix. Since the spherical indicatrix is formed without knowing the values of α_0, β_0 and γ_0, the loop equation for the RSRC mechanism can be written without the inclusion of any of α_0, β_0 and γ_0. This is done by locating the center of the spherical pair, and presented in the following sections.

Displacement Analysis by Iterative Solution to the Dual Loop Closure Equation

The displacement analysis of any mechanism can be carried out by an iterative solution of the dual loop closure equation, Equation (3.14) (59). The dual loop closure equation is rewritten as

$$\hat{T}_{\hat{\alpha}}\ \hat{T}^{-1}_{90-\hat{\theta}}\ \hat{T}_{\hat{\lambda}}\ \hat{T}_{\hat{\phi}}\ \hat{T}_{\hat{\delta}}\ \hat{T}_{\hat{\chi}}\ \hat{T}_{d_2}\ T_{\gamma_0}\ T_{\beta_0} = I \tag{3.19}$$

Letting the loop equation, in general, be

$$\hat{F}_{(\hat{\theta}_i)} = \hat{T}_1\ \hat{T}_2\ \hat{T}_3\ \ldots\ \hat{T}_k = I \tag{3.20}$$

where $\hat{\theta}_i$ is the i^{th} dual variable. Expanding Equation (3.20) in a Taylor series about $\hat{\theta}_{i0} = \hat{\theta}_i - \hat{h}_i$ and neglecting the terms having higher order derivatives Equation (3.20) is approximated by[1]

$$\hat{F}_{(\hat{\theta}_i)} = \hat{F}_{(\hat{\theta}_{i0})} + \sum_{i=1}^{N} \frac{\partial \hat{F}_{(\theta_{i0})}}{\partial \hat{\theta}_i} \hat{h}_i \simeq I \qquad (3.21)$$

where $\hat{h}_i = \hat{\theta}_i - \hat{\theta}_{i0}$ are the dual errors, n is the number of dual variables in addition to the input variable, since the input variable is assumed to be known. Let

$$\hat{F}_{(\hat{\theta}_{i0})} = \hat{T} = \hat{T}_1 \hat{T}_2 \hat{T}_3 \ldots \hat{T}_k \Big|_{\hat{\theta}_i = \hat{\theta}_{i0}} = \hat{G} \qquad (3.22)$$

where $\hat{G}$ is 3x3 dual matrix and is orthogonal.

Note that if $\hat{\theta} = \theta_0 + \varepsilon\theta_1$ is a dual rotation about x axis, the corresponding dual matrix is

$$\hat{T}_{\hat{\theta}} = \hat{T}_{\theta_1} T_{\theta_0}$$

and also

$$\hat{T}_{\hat{\theta}} = T_{\theta_0} + \varepsilon\theta_1[D_1 T_{\theta_0}]$$

where D_1 is the differential operator matrix for rotations about x axis, and given by Equation (A.27), along with the differential operator matrices for rotations about y and z axes. Then the variation of a dual matrix $\hat{T}_{\hat{\theta}}$ is given by

$$d(\hat{T}_{\hat{\theta}}) = d\hat{\theta}[D_1 \hat{T}_{\hat{\theta}}] \quad \text{or} \quad d(\hat{T}_{\hat{\theta}}) = [d(\hat{T}_{\theta_1})]T_{\theta_0} + \hat{T}_{\theta_1}[d(T_{\theta_0})]$$

---

[1]The higher order derivative terms in the Taylor series expansion are neglected since it is an iterative process. By doing so the convergence to the solution is slowed, but the mathematical operations are greatly simplified overcoming the slow convergence.

which take the form

$$d(\hat{T}_{\hat{\theta}}) = [D_1 T_{\theta_0}]d\theta_0 + \varepsilon \; [D_1 T_{\theta_0}]d\theta_1 + \theta_0[D_1^2 T_{\theta_0}]d\theta_0$$

Then we can write

$$\sum_{i=1}^{N} \frac{\partial \hat{F}_{(\theta_{i0})}}{\partial \hat{\theta}_i} \hat{h}_i = \left[[D_1 \hat{T}_1]\hat{T}_2 \hat{T}_3 \ldots \hat{T}_n \right] \hat{h}_i + \left[\hat{T}_1[D_2 \hat{T}_2]\hat{T}_3 \hat{T}_4 \ldots \hat{T}_n \right] \cdot$$

$$\hat{h}_2 + \ldots + \left[\hat{T}_1 \hat{T}_2 \ldots \hat{T}_{n-1}[D_n \hat{T}_n] \right] \hat{h}_n = \sum_{i=1}^{n} \hat{E}_i \hat{h}_i \quad (3.24)$$

where

$$\hat{E}_i = \hat{T}_1 T_2 \ldots \hat{T}_{i-1}[D_i \hat{T}_i]\hat{T}_{i+1} \ldots \hat{T}_n$$

D_i is the differential operator matrix, for the i^{th} dual variable. Note that D_1 is a skew-symmetric matrix where $D_{KJ} = -D_{JK}$ and $D_{JJ} = 0$.

Equation (3.21) can now be written as

$$\sum_{i=1}^{n} \hat{E}_i \hat{h}_i = I - \hat{G} \quad (3.25)$$

Since E_i's are 3x3 dual matrices, Equation (3.25) may be rewritten as

$$\sum_{i=1}^{n} \begin{bmatrix} \hat{E}_{i_{11}}\hat{h}_i & \hat{E}_{i_{12}}\hat{h}_i & \hat{E}_{i_{13}}\hat{h}_i \\ \hat{E}_{i_{21}}\hat{h}_i & \hat{E}_{i_{22}}\hat{h}_i & \hat{E}_{i_{23}}\hat{h}_i \\ \hat{E}_{i_{31}}\hat{h}_i & \hat{E}_{i_{32}}\hat{h}_i & \hat{E}_{i_{33}}\hat{h}_i \end{bmatrix} = \begin{bmatrix} 1-\hat{G}_{11} & -\hat{G}_{12} & -\hat{G}_{13} \\ -\hat{G}_{12} & 1-\hat{G}_{22} & -\hat{G}_{23} \\ -\hat{G}_{31} & -\hat{G}_{32} & 1-\hat{G}_{33} \end{bmatrix} \quad (3.26)$$

Then, equating the corresponding elements on both sides we have nine simultaneous dual equations,

$$\begin{bmatrix} \hat{E}_{111} & \hat{E}_{211} & \hat{E}_{311} & \cdots & \hat{E}_{n11} \\ \hat{E}_{112} & \hat{E}_{212} & \hat{E}_{312} & \cdots & \hat{E}_{n12} \\ \hat{E}_{113} & \hat{E}_{213} & \hat{E}_{313} & \cdots & \hat{E}_{n13} \\ \hat{E}_{121} & \hat{E}_{221} & \hat{E}_{321} & \cdots & \hat{E}_{n21} \\ \hat{E}_{122} & \hat{E}_{222} & \hat{E}_{322} & \cdots & \hat{E}_{n22} \\ \hat{E}_{123} & \hat{E}_{223} & \hat{E}_{323} & \cdots & \hat{E}_{n23} \\ \hat{E}_{131} & \hat{E}_{231} & \hat{E}_{331} & \cdots & \hat{E}_{n31} \\ \hat{E}_{132} & \hat{E}_{232} & \hat{E}_{332} & \cdots & \hat{E}_{n32} \\ \hat{E}_{133} & \hat{E}_{233} & \hat{E}_{333} & \cdots & \hat{E}_{n33} \end{bmatrix} \begin{Bmatrix} \hat{h}_1 \\ \hat{h}_2 \\ \hat{h}_3 \\ \cdot \\ \cdot \\ \cdot \\ \\ \\ \hat{h}_n \end{Bmatrix} = \begin{Bmatrix} 1-\hat{G}_{11} \\ -\hat{G}_{12} \\ -\hat{G}_{13} \\ -\hat{G}_{21} \\ 1-\hat{G}_{22} \\ -\hat{G}_{23} \\ -\hat{G}_{31} \\ -\hat{G}_{32} \\ 1-\hat{G}_{33} \end{Bmatrix} \quad (3.27)$$

or

$$\hat{A}\,\hat{\overline{H}} = \hat{\overline{C}} \quad (3.28)$$

where

$$\hat{A} = A_0 + \varepsilon A_1 = \begin{bmatrix} (E_{111})_0 & (E_{211})_0 & \cdots & (E_{n11})_0 \\ (E_{112})_0 & (E_{212})_0 & & (E_{n12})_0 \\ (E_{113})_0 & (E_{213})_0 & \cdots & (E_{n13})_0 \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ (E_{133})_0 & (E_{233})_0 & \cdots & (E_{n33})_0 \end{bmatrix}$$

$$+ \varepsilon \begin{bmatrix} (E_{111})_1 & (E_{211})_1 & \cdots & (E_{n11})_1 \\ (E_{112})_1 & (E_{212})_1 & \cdots & (E_{n12})_1 \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ (E_{133})_1 & (E_{233})_1 & \cdots & (E_{n33})_1 \end{bmatrix} \quad (3.29)$$

$$\hat{H} = \overline{H}_0 + \varepsilon\overline{H}_1 = \begin{Bmatrix} h_{10} \\ h_{20} \\ \cdot \\ \cdot \\ \cdot \\ h_{n0} \end{Bmatrix} + \varepsilon \begin{Bmatrix} h_{11} \\ h_{21} \\ \cdot \\ \\ \\ h_{n1} \end{Bmatrix} \tag{3.30}$$

$$\overline{C} = \overline{C}_0 + \varepsilon\overline{C}_1 = \begin{Bmatrix} 1-G_{110} \\ -G_{120} \\ -G_{130} \\ \cdot \\ \cdot \\ \cdot \\ 1-G_{330} \end{Bmatrix} + \varepsilon \begin{Bmatrix} -G_{111} \\ -G_{121} \\ -G_{131} \\ \cdot \\ \cdot \\ \cdot \\ -G_{331} \end{Bmatrix} \tag{3.31}$$

separating the real and dual parts in Equation (3.28) gives

$$A_0\ \overline{H}_0 = \overline{C}_0 \tag{3.32}$$

$$A_1\ \overline{H}_0 + A_0\ \overline{H}_1 = \overline{C}_1 \tag{3.33}$$

An initial set of values for the unkonwn parameters are assumed to start the solution, then the dual errors, $\hat{h}_i = h_{i0} + \varepsilon h_{i1}$, in the assumed values of the parameters are computed by Equations (3.32) and (3.33) leading to a new set of improved values of the parameters by $\theta_{i(m+1)} = \hat{\theta}_{im} + \hat{h}_{im}$ for the $(m+1)^{th}$ iteration. The process of iteration is repeated until the dual error vector, $\overline{H}$, may be considered a null vector within the acceptable accuracy limits.

Premultiply both sides of Equations (3.32) and (3.33) by $[A_0^T]_{nX9}$ which is the transpose of the matrix $[A_0]_{9xn}$ and solve for $\overline{H}_0$ and $\overline{H}_1$ to obtain

$$[H_0]_{nx1} = [A_0^T\ A_0]^{-1}_{nxn}\ [A_0]^T_{nx9}\ [C_0]_{9x1} \tag{3.34}$$

and

$$[H_1]_{nx1} = [A_0^T A_0]_{nxn}^{-1} [A_0^T]_{nx9} [C_1 - A_1 H_0]_{9x1} \qquad (3.35)$$

as the real and dual parts of the dual error vector $\hat{\bar{H}}$, respectively.

Note that there are 18 equations for 2xn or less unkonwns where n will not necessarily be nine. In general such a system of equations has no exact solution. It can be solved for the closest approximation of a solution to all 18 equations in the root-mean-square sense.

However, the number of dual equations in Equation (3.28) can be reduced to 6 by utilizing the property of the skew-symmetric differential operator matrix D_i. Thus, if D is an antisymmetric matrix, $\hat{T}^T$ is the transpose of $\hat{T}$, $\hat{T}D\hat{T}^T$ is also an antisymmetric matrix.

Introducing $I = [\hat{T}_1 \hat{T}_2 \ldots \hat{T}_{i-1}]^{-1} [\hat{T}_1 \hat{T}_2 \ldots \hat{T}_{i-1}]$, the unit matrix; rewrite $\hat{E}_i$ given by Equation (3.24) in the form

$$\hat{E}_i = [\hat{T}_1 \hat{T}_2 \ldots \hat{T}_{i-1}]D_i[\hat{T}_1 \hat{T}_2 \ldots \hat{T}_{i-1}]^{-1} [\hat{T}_1 \hat{T}_2 \ldots \hat{T}_{i-1}]\hat{T}_i \hat{T}_{i+1} \ldots \hat{T}_n \qquad (3.36)$$

Letting

$$\hat{Q}_i = [\hat{T}_1 \hat{T}_2 \ldots \hat{T}_{i-1}]D_i[\hat{T}_1\hat{T}_2 \ldots \hat{T}_{i-1}]^{-1} \qquad (3.37)$$

which is an antisymmetric matrix, and since 3x3 screw matrix is orthogonal and its inverse is its transpose. Then

$$\hat{E}_i = [\hat{Q}_i \hat{h}_i]\hat{G} \qquad (3.38)$$

and from Equation (3.25)

$$\left[\sum_{i=1}^{n} \hat{Q}_i \hat{h}_i \right] \hat{G} = I - \hat{G} \qquad (3.39)$$

Post multiplying both sides of Equation (3.39) by $[\hat{G}]^{-1}$, which is easily computed by the transpose of $\hat{G}$ since it is an orthogonal matrix,

$$\sum_{i=1}^{n} \hat{Q}_i \hat{h}_i \simeq [I - \hat{G}][\hat{G}]^{-1} = \hat{R} \qquad (3.40)$$

or

$$\begin{bmatrix} 0 & -\sum_{i=1}^{n} \hat{Q}_{i12} \hat{h}_i & -\sum_{i=1}^{n} \hat{Q}_{i13} \hat{h}_i \\ \sum_{i=1}^{n} \hat{Q}_{i12} \hat{h}_i & 0 & -\sum_{i=1}^{n} \hat{Q}_{i23} \hat{h}_i \\ \sum_{i=1}^{n} \hat{Q}_{i13} \hat{h}_i & \sum_{i=1}^{n} \hat{Q}_{i23} \hat{h}_i & 0 \end{bmatrix} = \hat{R} \qquad (3.41)$$

where

$$\hat{R} = - \begin{bmatrix} \left[-(1 - \hat{G}_{11}) \hat{G}_{11} + \hat{G}_{12}^2 + \hat{G}_{13}^2\right] & \left[-(1 - \hat{G}_{11}) \hat{G}_{21} + \hat{G}_{12}\hat{G}_{22} + \hat{G}_{13}\hat{G}_{33}\right] & \left[-(1 - \hat{G}_{11}) \hat{G}_{31} + \hat{G}_{12}\hat{G}_{32} + \hat{G}_{13}\hat{G}_{33}\right] \\ \left[\hat{G}_{11}\hat{G}_{21} - (1 - \hat{G}_{22}) \hat{G}_{12} + \hat{G}_{23}\hat{G}_{13}\right] & \left[\hat{G}_{21}^2 - (1 - \hat{G}_{22}) \hat{G}_{22} + \hat{G}_{23}^2\right] & \left[\hat{G}_{21}\hat{G}_{31} - (1 - \hat{G}_{22}) \hat{G}_{32} + \hat{G}_{23}\hat{G}_{33}\right] \\ \left[\hat{G}_{31}\hat{G}_{11} + \hat{G}_{32}\hat{G}_{12} - (1 - \hat{G}_{33}) \hat{G}_{13}\right] & \left[\hat{G}_{31}\hat{G}_{21} + \hat{G}_{22}\hat{G}_{32} - (1 - \hat{G}_{33}) \hat{G}_{23}\right] & \left[\hat{G}_{31}^2 + \hat{G}_{32}^2 - (1 - \hat{G}_{33}) \hat{G}_{33}\right] \end{bmatrix} \qquad (3.42)$$

which gives 3 simultaneous dual equations;

$$\begin{bmatrix} -\hat{Q}_{112} & -\hat{Q}_{212} & \cdots & -\hat{Q}_{n12} \\ -\hat{Q}_{113} & -\hat{Q}_{213} & \cdots & -\hat{Q}_{n13} \\ & & & \\ -\hat{Q}_{123} & -\hat{Q}_{223} & \cdots & -\hat{Q}_{n23} \end{bmatrix} \begin{Bmatrix} \hat{h}_1 \\ \hat{h}_2 \\ \vdots \\ \hat{h}_n \end{Bmatrix} = \frac{1}{2} \begin{Bmatrix} \hat{R}_{12} - \hat{R}_{21} \\ \hat{R}_{13} - \hat{R}_{31} \\ \\ \hat{R}_{23} - \hat{R}_{32} \end{Bmatrix} \qquad (3.43)$$

Denoting the matrix on the left by $\hat{B}$, and on the right by $\hat{\overline{P}}$, Equation (3.43) becomes

$$\hat{B}\ \hat{\overline{H}} = \hat{\overline{P}} \qquad (3.44)$$

Separating dual and real parts gives

$$B_0\ \overline{H}_0 = \overline{P}_0 \qquad (3.45)$$

and

$$B_1\ \overline{H}_0 + B_0\ \overline{H}_1 = \overline{P}_1 \qquad (3.46)$$

Then, the real and the dual parts of the error vector are given by

$$\overline{H}_0 = [B_0^T\ B_0]^{-1}\ B_0^T\ \overline{P}_0 \qquad (3.47)$$

and

$$\overline{H}_1 = [B_0^T\ B_0]^{-1}\ B_0^T\ \overline{P}_1 - B_1\ \overline{H}_0 \qquad (3.48)$$

respectively.

The convergence to the solution of Equation (3.28) will be much slower than the convergence to the solution of Equation (3.44).

Equation (3.39) results in nine simultaneous equations. Thus,

$$\sum_{i=1}^{n} \begin{bmatrix} (-\hat{Q}_{i12}\hat{G}_{21} - \hat{Q}_{i13}\hat{G}_{31}) & (-\hat{Q}_{i12}\hat{G}_{22} - \hat{Q}_{i13}\hat{G}_{32}) & (-Q_{i12}G_{23} - Q_{i13}G_{33}) \\ (\hat{Q}_{i12}\hat{G}_{11} - \hat{Q}_{i23}\hat{G}_{31}) & (\hat{Q}_{i12}\hat{G}_{12} - \hat{Q}_{i23}\hat{G}_{32}) & (Q_{i12}G_{13} - Q_{i23}G_{33}) \\ (\hat{Q}_{i13}\hat{G}_{11} + \hat{Q}_{i23}\hat{G}_{21}) & (\hat{Q}_{i13}\hat{G}_{12} + \hat{Q}_{i23}\hat{G}_{22}) & (Q_{i13}G_{13} + Q_{i23}G_{23}) \end{bmatrix} \hat{h}_i = I - \hat{G} \tag{3.49}$$

or

$$\hat{K}\,\hat{\bar{H}} = \hat{\bar{C}} \tag{3.50}$$

where $\hat{\bar{H}}$ and $\hat{\bar{C}}$ are given by Equations (3.30) and (3.31), $\hat{K}$ is the 3x3 dual matrix which forms on the left of Equation (3.49).

In the RSRC mechanism there are five dual variables besides the input rotation. These are $\hat{\theta}_1 = \hat{\alpha} = \alpha_0 - \varepsilon d_1$, $\hat{\theta}_2 = \hat{\phi} = \phi_0 - \varepsilon S$, $\hat{\theta}_3 = \hat{\chi} = \chi_0 - \varepsilon e$, $\hat{\theta}_4 = \hat{\gamma} = \gamma_0$, and $\hat{\theta}_5 = \hat{\beta} = \beta_0$. d_1 and e are constant γ_1 and β_1 are zero. Then the dual error vector in this case is

$$\hat{H} = \begin{Bmatrix} h_{\alpha_0} \\ h_{\phi_0} \\ h_{\chi_0} \\ h_{\gamma_0} \\ h_{\beta_0} \end{Bmatrix} + \varepsilon \begin{Bmatrix} 0 \\ h_s \\ 0 \\ 0 \\ 0 \end{Bmatrix} \tag{3.51}$$

Only one of the equations in Equations (3.33) and (3.51) is used to compute the improved value of S. The error corresponding to the constant part of any dual variable is considered to be zero in the iteration process. Note that $-\alpha_0$ and $-\gamma_0$ are rotations about the z axis, ϕ_0, $-\chi_0$ and β_0 are rotations about the x axis. The number of matrices in Equation (3.20), k, is nine for the RSRC mechanism. Then in Equation (3.25)

$$\hat{E}_1 = [D_3^T \hat{T}_1] \hat{T}_2 \hat{T}_3 \hat{T}_4 \hat{T}_5 \hat{T}_6 \hat{T}_7 \hat{T}_8 \hat{T}_9 \quad \text{where } \hat{T}_1 = \hat{T}_{\hat{\alpha}}$$

$$\hat{E}_2 = \hat{T}_1 \hat{T}_2 \hat{T}_3 [D_1 \hat{T}_4] \hat{T}_5 \hat{T}_6 \hat{T}_7 \hat{T}_8 \hat{T}_9 \quad \text{where } \hat{T}_4 = \hat{T}_{\hat{\phi}}$$

$$\hat{E}_3 = \hat{T}_1 \hat{T}_2 \hat{T}_3 \hat{T}_4 \hat{T}_5 [D_1^T \hat{T}_6] \hat{T}_7 \hat{T}_8 \hat{T}_9 \quad \text{where } \hat{T}_6 = \hat{T}_{\hat{\chi}}$$

$$\hat{E}_4 = \hat{T}_1 \hat{T}_2 \hat{T}_3 \hat{T}_4 \hat{T}_5 \hat{T}_6 \hat{T}_7 [D_3^T \hat{T}_8] \hat{T}_0 \quad \text{where } \hat{T}_8 = \hat{T}_{\hat{\gamma}} = \hat{T}_{\gamma_0}$$

$$\hat{E}_5 = \hat{T}_1 \hat{T}_2 \hat{T}_3 \hat{T}_4 \hat{T}_5 \hat{T}_6 \hat{T}_7 \hat{T}_8 [D_1 \hat{T}_9] \quad \text{where } \hat{T}_9 = \hat{T}_{\hat{\beta}} = \hat{T}_{\beta_0}$$

A digital computer program for IBM 7040 digital computer has been made available to solve Equations (3.28) and (3.44) for any space mechanism. The computer program is not included in the appendix, since the Program A given in Appendix F is seen to be serving the purpose of the study better, especially in separating the geometric inversions of the mechanism. The Program A is prepared using the results of the following section where the rotation parameters α_0, β_0 and γ_0 are eliminated from the loop equation.

It has been observed during the iterative solution to the dual loop equation for the RSRC mechanism that the initially assumed values of α_0, β_0 and γ_0 have considerable effect on the convergence to the solution at a particular value of the input variable. Quite reasonable starting values of α_0, β_0 and γ_0 can be assumed along with the other variables using the conformal projection of the spherical indicatrix of the mechanism shown in Figure 6.

Displacements in the RSRC Mechanism by Eliminating the Parameters for the Rotation Freedoms on the Spherical Pair

All three rotation parameters α_0, β_0 and γ_0 can be eliminated from the dual loop closure equation by positioning the center of the spherical pair in the OXYZ system through two paths. The coordinates of the center of the spherical pair in the OXYZ system are easily obtained by transforming the three unit vectors in the $O_1\xi_1\eta_1\zeta_1$ system into the OXYZ system by using the matrix of linear transformation, $\hat{T}_L$, given in Equation (2.9). The $O_1\xi_1\eta_1\zeta_1$ system is parallel to the OXYZ system. Thus, the three unit vectors in the $O_1\xi_1\eta_1\zeta_1$ system are transformed into the OXYZ system along the path O_1-O by the screw matrix

$$\hat{T}_1 = \begin{bmatrix} 1 & -\varepsilon Z_{01} & \varepsilon Y_{01} \\ \varepsilon Z_{01} & 1 & -\varepsilon X_{01} \\ -\varepsilon Y_{01} & \varepsilon X_{01} & 1 \end{bmatrix}_{O_1-O} = \begin{bmatrix} 1 & -\varepsilon d_1 \sin\theta_0 & \varepsilon d_1 \cos\theta_0 \\ \varepsilon d_1 \sin\theta_0 & 1 & 0 \\ -\varepsilon d_1 \cos\theta_0 & 0 & 1 \end{bmatrix} \tag{3.52}$$

Furthermore the same unit vectors are transformed into the OXYZ system along the path O_1-O_2-O_3-O by the screw matrix

$$\hat{T}_2 = \hat{T}_b \, \hat{T}_a \, \hat{T}_{\bar{s}} \, \hat{T}_{\bar{d}_3} \, \hat{T}_{\bar{e}} \, \hat{T}_{\bar{d}_2} \tag{3.53}$$

where $\hat{T}_b$ is defined with Equation (4.2), $\hat{T}_a$ is the dual part of $\hat{T}_{\hat{\lambda}}$ and is the same as $\hat{T}_{\beta_1}$ in Equation (2.7b),

$$\hat{T}_{\bar{s}} = \begin{bmatrix} 1 & -\varepsilon S\cdot\sin\lambda_0 & 0 \\ \varepsilon S\cdot\sin\lambda_0 & 1 & \varepsilon S\cdot\cos\lambda_0 \\ 0 & -\varepsilon S\cdot\cos\lambda_0 & 1 \end{bmatrix}$$

$$\hat{T}_{\overline{d}_3} = \begin{bmatrix} 1 & -\varepsilon\ d_3A_{32} & \varepsilon\ d_3A_{22} \\ \varepsilon\ d_3A_{32} & 1 & -\varepsilon\ d_3A_{12} \\ -\varepsilon\ d_3A_{22} & \varepsilon\ d_3A_{12} & 1 \end{bmatrix},$$

$$\hat{T}_{\overline{e}} = \begin{bmatrix} 1 & -\varepsilon e\ B_{31} & \varepsilon e\ B_{21} \\ \varepsilon e\ B_{31} & 1 & -\varepsilon e\ B_{11} \\ -\varepsilon e\ B_{21} & \varepsilon e\ B_{11} & 1 \end{bmatrix},$$

$$\hat{T}_{\overline{d}_2} = \begin{bmatrix} 1 & \varepsilon\ d_2B_{32} & -\varepsilon\ d_2B_{22} \\ -\varepsilon\ d_2B_{32} & 1 & \varepsilon\ d_2B_{12} \\ \varepsilon\ d_2B_{22} & -\varepsilon\ d_2B_{12} & 1 \end{bmatrix}$$

A_{iJ} and B_{iJ} are the (iJ) elements of $[A] = T_{\lambda_0}\ T_{\phi_0}$ and $[B] = T_{\lambda_0}\ T_{\phi_0}\ T_{\delta_0}\ T_{\chi_0}$, respectively, where T_{λ_0}, T_{ϕ_0}, T_{δ_0} and T_{χ_0} are defined with Equation (3.2). After multiplying the matrices in Equation (3.53)

$$\hat{T}_2 = \begin{bmatrix} 1 & -\varepsilon\ Z'_{01} & \varepsilon\ Y'_{01} \\ \varepsilon\ Z'_{01} & 1 & -\varepsilon\ X'_{01} \\ -\varepsilon\ Y'_{01} & \varepsilon\ X'_{01} & 1 \end{bmatrix}_{0_1-0_2-0_3-0} \tag{3.54}$$

where

$$X'_{01} = b - S\cdot\cos\lambda_0 + d_3 \sin\lambda_0 \sin\phi_0 + d_2[\sin\chi_0(\sin\lambda_0 \cos\delta_0 \cos\phi_0 - \sin\delta_0\cos\lambda_0) - \sin\lambda_0 \sin\phi_0\cos\chi_0] + e(\cos\lambda_0 \cos\delta_0 + \sin\lambda_0 \sin\delta_0 \cos\phi_0)$$

$$Y'_{01} = a + d_3 \cos\phi_0 - d_2(\cos\phi_0 \cos\chi_0 + \cos\delta_0 \sin\phi_0 \sin\chi_0) - e \sin\delta_0 \sin\phi_0$$

and

$$Z'_{01} = S\cdot\sin\lambda_0 + d_3 \cos\lambda_0 \sin\phi_0 + d_2[\sin\chi_0(\sin\lambda_0 \sin\delta_0 +$$

$+ \cos\lambda_0 \cos\delta_0 \cos\phi_0) - \cos\lambda_0 \sin\phi_0 \cos\chi_0] + e(\cos\lambda_0 \sin\delta_0 \cos\phi_0$

$- \sin\lambda_0 \cos\delta_0)$

Note that, due to the property of the screw matrix of linear transformations, X'_{02}, Y'_{02}, and Z'_{02} are simply the sums of the coefficients of in the (3,2), (1,3) and (2,1) elements of the screw matrices $\hat{T}_b$, $\hat{T}_a$, $\hat{T}_{\overline{s}}$, $\hat{T}_{\overline{d}_3}$, $\hat{T}_{\overline{d}_2}$ and $\hat{T}_{\overline{e}}$, respectively.

Since the screw matrices in Equations (3.52) and (3.54) position the same unit vectors, the dual loop closure equation is

$$\hat{T}_1 = \hat{T}_2 \tag{3.55}$$

Then

$$0 = X'_{02}$$

$$d_1 \cos\theta_0 = Y'_{02} \tag{3.57}$$

$$d_1 \sin\theta_0 = Z'_{02} \tag{3.58}$$

where X'_{02}, Y'_{02} and Z'_{02} are defined with Equation (3.54).

Solve Equations (3.56), (3.57) and (3.58) to obtain

$$\cos\chi_0 = \frac{W_1}{d_2} , \quad \sin\chi_0 = \frac{W_2}{d_2 \cos\delta_0} \tag{3.59}$$

where

$$W_1 = M_0 \sin\phi_0 + N_0 \cos\phi_0 + d_3$$

$$W_2 = N_0 \sin\phi_0 - M_0 \cos\phi_0 - e \sin\delta_0$$

with

$$M_0 = b \sin\lambda_0 - d_1 \cos\lambda_0 \sin\theta_0$$

$$N_0 = a - d_1 \cos\theta_0$$

and

$$S = b \cos\lambda_0 + d_1 \sin\lambda_0 \sin\theta_0 + e \cos\delta_0 - d_2 \sin\delta_0 \sin\chi_0 \quad (3.60)$$

Note that S is a function of $\sin\theta_0$ when $\delta_0 = n\pi$, that is

$$S = C_1 + C_2 \sin\theta_0 \quad (3.60a)$$

where

$$C_1 = e \cos\delta_0 + b \cos\lambda_0$$

$$C_2 = d_1 \sin\lambda_0$$

From Equation (3.59)

$$\cos^2 \delta_0 \, W_1{}^2 + W_2{}^2 = d_2{}^2 \cos^2 \delta_0 \quad (3.61)$$

which relates the output rotation ϕ_0 to the input rotation θ_0. After substituting W_1 and W_2, Equation (3.61) takes the form

$$F_1 \sin^2 \phi_0 + F_2 \sin \phi_0 \cos \phi_0 + F_3 \cos \phi_0 + F_4 \sin \phi_0 = K \quad (3.62)$$

where

$$F_1 = \sin^2 \delta_0 (N_0{}^2 - M_0{}^2)$$

$$F_2 = -2 \, M_0 \, N_0 \sin \delta_0$$

$$F_3 = 2 \, (d_3 \, N_0 \cos^2 \delta_0 + e \, M_0 \sin \delta_0)$$

$$F_4 = 2 \, (d_3 \, M_0 \cos^2 \delta_0 - e \, N_0 \sin \delta_0)$$

$$K = \cos^2 \delta_0 (d_2{}^2 - d_3{}^2 - N_0{}^2) - M_0{}^2 - e^2 \sin^2 \delta_0$$

Equation (3.62) is Freudenstein's displacement equation for the RSRC mechanism. In the case of a 4R plane mechanism $\delta_0 = \lambda_0 = 0$, $b = e = S = 0$, and Equation (3.62) reduces to Freudenstein's displacement equation for plane mechanism given by Equation (3.13).

Using half angle relationships, $\sin\phi_0 = 2\Phi_0/(1 + \Phi_0^2)$ and $\cos\phi_0 = (1 - \Phi_0^2)/(1 + \Phi_0^2)$ where $\Phi_0 = \tan(\phi_0/2)$, Equation (3.62) is transformed into the quartic form

$$F_{(\Phi_0)} = \Phi_0^4 A_4 + \Phi_0^3 A_3 + \Phi_0^2 A_2 + \Phi_0 A_1 + A_0 = 0 \qquad (3.63)$$

where

$$A_4 = -K - 2(d_3 N_0 \cos^2\delta_0 + e M_0 \sin\delta_0)$$

$$A_3 = 4(M_0 N_0 \sin^2\delta_0 + d_3 M_0 \cos^2\delta_0 - e N_0 \sin\delta_0)$$

$$A_2 = 4\sin^2\delta_0 (N_0^2 - M_0^2) - 2K$$

$$A_1 = 4(d_3 M_0 \cos^2\delta_0 - e N_0 \sin\delta_0 - M_0 N_0 \sin^2\delta_0)$$

$$A_0 = 2(d_3 N_0 \cos^2\delta_0 + e M_0 \sin\delta_0) - K$$

Equation (3.63) can easily be solved numerically for the roots of Φ_0 and $\phi_0 = 2\tan^{-1}\Phi_0$ for a given value of the input rotation θ_0. In the iterative solution, $F_{(\Phi_0)}$ is expanded in Taylor series form giving the error at the i^{th} iteration by

$$h_i = -\frac{F_{(\Phi_{0i})}}{\dfrac{\partial F_{(\Phi_{0i})}}{\partial \Phi_0}} \qquad (3.64)$$

where the improved value of Φ_0 is $\Phi_{0_{i+1}} = \Phi_{0i} + h_i$.

In the following, the solution to Equation (3.63) is obtained by using Brown's method of quadratic factors. The quadratic factors of Equation (3.63) are

$$\Phi_0^2 + g_1 \Phi_0 + H_1 = 0 \tag{3.65}$$

$$\Phi_0^2 + g_2 \Phi_0 + H_2 = 0 \tag{3.66}$$

where

$$g_{1,2} = \frac{B_3}{2} \pm \sqrt{\left(\frac{B_3}{2}\right)^2 - B_2 + Y_3} \tag{3.67}$$

$$H_i = \frac{Y_3}{2} \pm \sqrt{\left(\frac{Y_3}{2}\right)^2 - B_0} \quad , \quad i = 1,2 \tag{3.68}$$

and Y_3 is the algebraically largest real root of the cubic equation

$$Y^3 - B_2 Y^2 + C_1 Y + C_0 = 0 \tag{3.69}$$

where

$$C_0 = B_0(4B_2 - B_3^2) - B_1^2$$

$$C_1 = B_3 B_2 - 4B_0$$

and

$$B_J = \frac{A_J}{A_4} \quad , \quad J = 0,1,2,3$$

The sign (+ or -) of the radical corresponding to H_1 (or H_2) in Equation (3.68) is determined by the relationship

$$g_1 H_2 + g_2 H_1 = B_1 \tag{3.70}$$

The four values of Φ_0 for the four geometric inversions (real or imaginary) corresponding to each value of the input rotation θ_0 are given by

$$\Phi_{0_{1,2}} = \frac{1}{2}\left(-g_1 \pm \sqrt{g_1^2 - 4H_1}\right) \tag{3.71}$$

$$\Phi_{0_{3,4}} = \frac{1}{2}\left(-g_2 \pm \sqrt{g_2^2 - 4H_2}\right) \tag{3.72}$$

Then

$$\phi_{0i} = 2 \tan^{-1} (\Phi_{0i}) \quad , \quad i = 1,2,3,4$$

The discriminants in Equations (3.67), (3.68), (3.71) and (3.72) must be real in order that the loop is closed for a given set of dimensions of the mechanism and the value of θ_0.

Note that when $\delta_0 = n\pi$ Equation (3.63) reduces to the quadratic form

$$\Phi_0^2(K + 2\, d_3\, N_0) - 4\, d_3\, M_0\, \Phi_0 + K - 2\, d_3\, N_0 = 0 \tag{3.73}$$

indicating that there are only two geometric inversions of the RSRC mechanism when $\delta_0 = n\pi$. The output rotations for the two geometric inversions are given by

$$\Phi_0 = \frac{2d_3\, M_0 \pm \sqrt{4\, d_3^2\, (M_0^2 + N_0^2) - K^2}}{K + 2\, d_3\, M_0} \tag{3.74}$$

for a given value of θ_0, while χ_0 and S are given by Equations (3.59) and (3.60).

Equation (3.63) also reduces to quadratic form when $\delta_0 = \frac{n + 1}{2}\pi$. Thus,

$$\Phi_0^2\ (M_0 - e \sin \delta_0) + 2\, \Phi_0\, N_0 - M_0 - e \sin \delta_0 = 0 \tag{3.75}$$

and

$$\Phi_0 = \frac{- N_0 \pm \sqrt{N_0^2 + M_0^2 - e^2}}{M_0 - e \sin \delta_0} \tag{3.76}$$

while χ_0 and S are given by Equations (3.59) and (3.60), respectively.

The Program A, prepared for IBM 7040 digital computer and listed in Appendix F, carries out the displacement, velocity and acceleration analyses of the RSRC mechanism. The Program A computes the displacements using Equations (3.59) through (3.76) with the exception of Equation (3.64). In obtaining the algebraically largest real root of the cubic in Equation (3.69), the numerical trisection method is used. The output of this program is utilized in the synthesis of the RSRC mechanism in Chapter V. In the input for the Program A the initial value of θ_0 and increment in θ_0 may be any desired values. Through the use of control cards in the data the subroutines VELCTY and COUPLR may be called to compute and print velocities, accelerations and the coupler curve coordinates. By calling the subroutine PLOT any output may be plotted.

Velocities and Accelerations in the RSRC Mechanism

Expressions for velocities and accelerations in the RSRC mechanism are easily obtained from the time derivatives of the displacement functions given by Equations (3.62), (3.60), and (3.49). Thus,

$$\dot{\phi}_0 = - d_1 \frac{W_2 L_1 + \cos^2 \delta_0\, W_1 L_2}{W_2 M_1 + \cos^2 \delta_0\, W_1 M_2}\ \dot{\theta}_0 \tag{3.77}$$

$$\dot{\chi}_0 = - \frac{M_2 \dot{\phi}_0 + d_1 L_2 \dot{\theta}_0}{d_2 \sin \chi_0} \tag{3.78}$$

and

$$\dot{S} = d_1(\sin \lambda_0 \cos \theta_0)\dot{\theta}_0 - d_2(\sin \delta_0 \cos \chi_0)\dot{\chi}_0 \tag{3.79}$$

as the velocities, where

$$M_1 = N_0 \cos \phi_0 + M_0 \sin \phi_0$$

$$M_2 = M_0 \cos \phi_0 - N_0 \sin \phi_0$$

$$L_1 = \sin \theta_0 \sin \phi_0 + \cos \lambda_0 \cos \theta_0 \cos \phi_0$$

$$L_2 = \sin \theta_0 \cos \phi_0 - \cos \lambda_0 \sin \phi_0 \cos \theta_0$$

and N_0, M_0, W_1, W_2 are identified with Equation (3.59).

Time derivatives of Equations (3.77), (3.78) and (3.79) give the accelerations as

$$\ddot{\phi}_0 = - \frac{1}{Q_5} \{Q_1 (\dot{\phi}_0)^2 + d_1[2 Q_2 \dot{\phi}_0 \dot{\theta}_0 + Q_3 (\dot{\theta}_0)^2 + Q_4 \ddot{\theta}_0]\} \tag{3.80}$$

$$\ddot{\chi}_0 = - \frac{1}{d_2 \sin \chi_0} \{d_2 (\dot{\chi}_0)^2 \cos \chi_0 + M_2 \ddot{\phi}_0 - M_1 (\dot{\phi}_0)^2 - d_1[L_4 (\dot{\theta}_0)^2 - 2 L_1 \dot{\phi}_0 \dot{\theta}_0 + L_2 \ddot{\theta}_0]\} \tag{3.81}$$

and

$$\ddot{S} = d_1 \sin \lambda_0[\ddot{\theta}_0 \cos \theta_0 - (\dot{\theta}_0)^2 \sin \theta_0] + d_2 \sin \delta_0[(\dot{\chi}_0)^2 \sin \chi_0 - \ddot{\chi}_0 \cos \chi_0] \tag{3.82}$$

where

$$Q_1 = M_1{}^2 + W_2 M_2 + \cos^2 \delta_0 (M_2{}^2 - W_1 M_1)$$

$Q_2 = M_1L_1 + W_2L_2 + \cos^2 \delta_0(M_2L_2 - W_1L_1)$

$Q_3 = d_1L_1{}^2 + W_2L_3 + \cos^2 \delta_0(d_1L_2{}^2 + W_1L_4)$

$Q_4 = W_2L_1 + W_1L_2 \cos^2 \delta_0$

$Q_5 = W_2M_1 + W_1M_2 \cos^2 \delta_0$

$L_3 = \sin \phi_0 \cos \theta_0 - \cos \lambda_0 \sin \theta_0 \cos \phi_0$

$L_4 = \cos \phi_0 \cos \theta_0 + \cos \lambda_0 \sin \phi_0 \sin \theta_0$

The computer program for the analysis of the RSRC mechanism, Program A, given in Appendix F computes, prints and plots the velocities $\dot{\phi}_0$, $\dot{\chi}_0$, $\dot{S}$ and accelerations $\ddot{\phi}_0$, $\ddot{\chi}_0$ and $\ddot{S}$ considering constant input velocity $\dot{\theta}_0 = 1$ rad/sec, using Equations (4.77) through (4.82).

In the following four figures, the displacements ϕ_0, S and χ_0, velocities $\dot{\phi}_0$ and $\dot{S}$, and accelerations $\ddot{\phi}_0$ and $\ddot{S}$ in some of the geometric inversions of three different RSRC mechanisms are given. Such plots are useful in assuming the initial set of dimensions of a mechanism to start the optimization process, especially the initial value of the input parameter by which the convergence to the solution is highly effected. Figures 7 and 8 show the displacements ϕ_0, S and χ_0, velocities $\dot{\phi}_0$ and $\dot{S}$ and accelerations $\ddot{\phi}_0$ and $\ddot{S}$ in the existing two real roots of Equation (3.62) for the RSRC mechanism having dimensions $d_1 = 2.0$ in., $d_2 = 4.5$ in., $d_3 = 3.0$ in., a = 3.0 in., b = 3.0 in., e = 1.0 in., $\lambda_0 = 60°$, and $\delta_0 = 18°$. Figure 9 shows the displacements ϕ_0, S and χ_0, velocities $\dot{\phi}_0$ and $\dot{S}$, and accelerations $\ddot{\phi}_0$ and $\ddot{S}$ in the inversion of an RSRC mechanism given by the negative signed radical in Equation (3.74). The dimensions of the mechanism are $\delta_0 = 0$, $d_1 = 2.0$ in., $d_2 = 4.0$ in., $d_3 = 2.925$ in., a = 2.75 in., b = 4.6156 in., e = 0, $\lambda_0 = 60°$. It should be observed in the plots of ϕ_0, $\dot{\phi}_0$, $\ddot{\phi}_0$ that a third order instantaneous dwell in the output rotation occurs

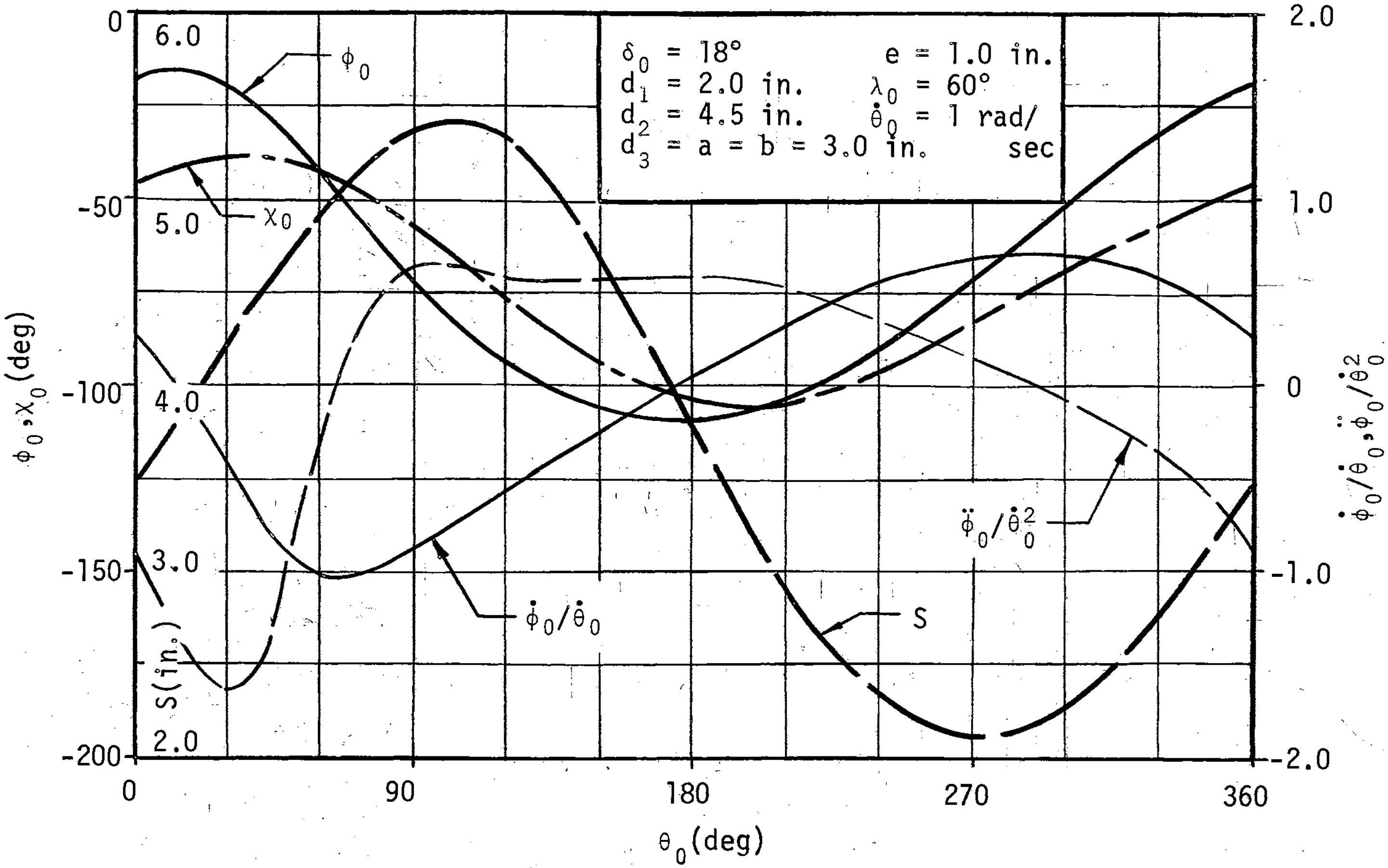


Figure 7. Displacements ϕ_0, S, χ_0, Output Rotation Velocity and Acceleration in the Geometric Inversion of the RSRC Mechanism Given by the First Real Root of Equation (3.62) when δ_0 = 18°

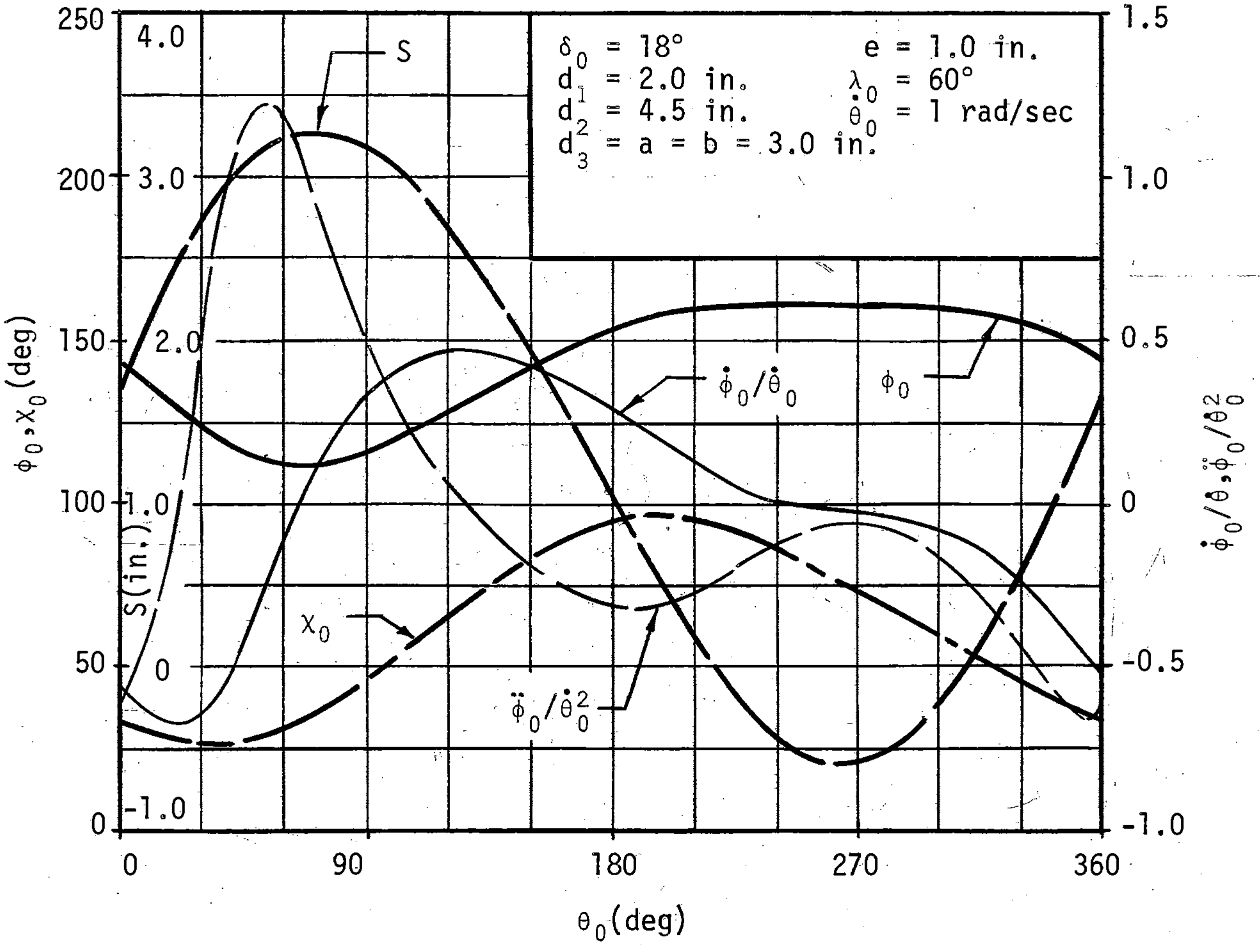


Figure 8. Displacements ϕ_0, S, χ_0, Output Rotation Velocity and Acceleration in the Geometric Inversion of the RSRC Mechanism, Given by the Second Real Root of Equation (3.62) when δ_0 = 18°

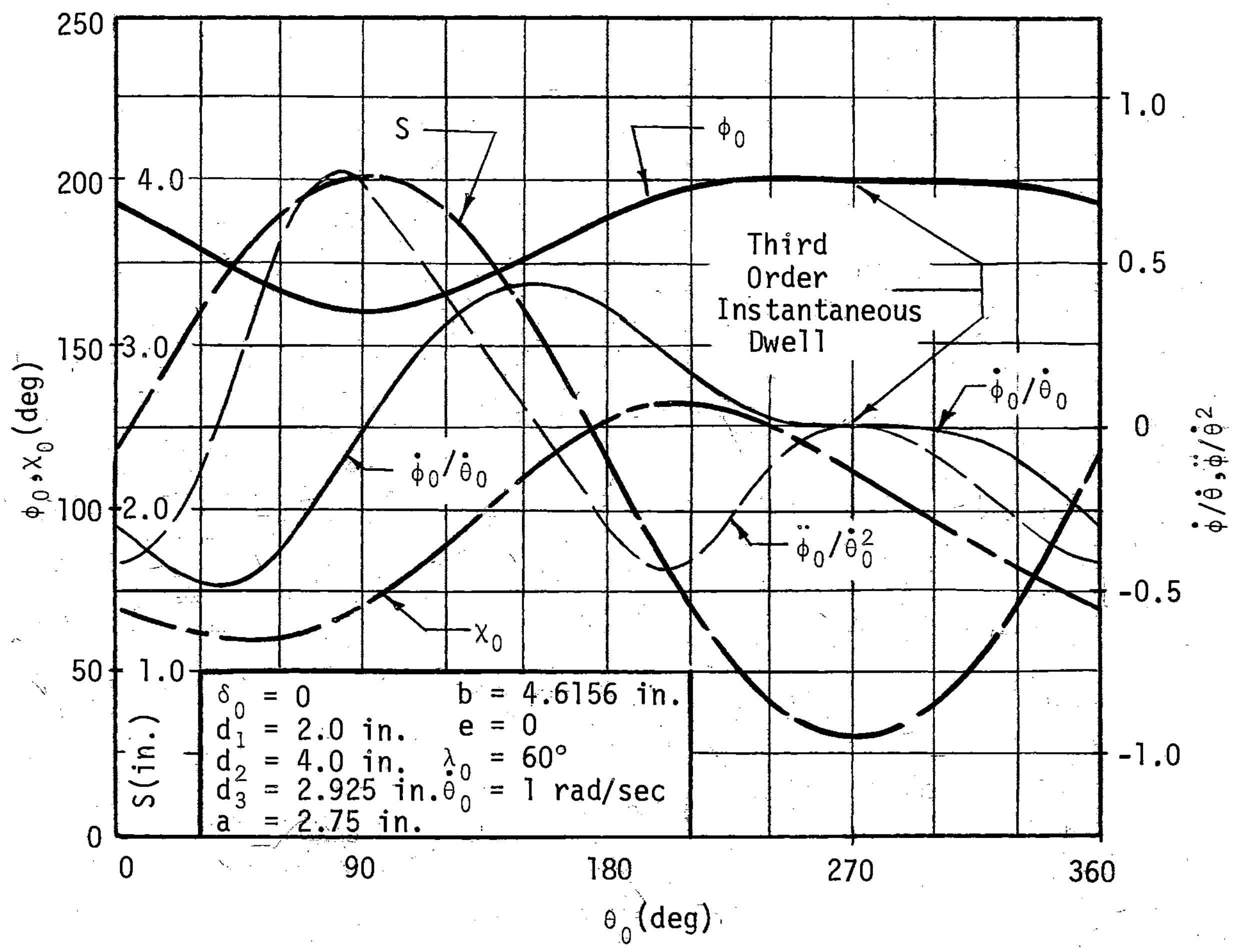


Figure 9. Displacements ϕ_0, S, χ_0, the Output Rotation Velocity and Acceleration in the Geometric Inversion of the RSRC-Crank-Rocker Mechanism Having a Third Order Instantaneous Dwell in the Output Rotation Given by the Negative Signed Radical in Equation (3.74), when δ_0 = 0

at θ_0 = 270°. That is, at that position the output link has zero velocity, zero acceleration and zero jerk in its rotation. The dimensions of this mechanism are chosen to produce higher order dwell and to maintain the simultaneous occurrence of the limit positions of the output rotation and output translation at θ_0 = 90° and 270°. Figure 10 shows the displacements ϕ_0, S and χ_0, velocities $\dot{\phi}_0$ and $\dot{S}$, and accelerations $\ddot{\phi}_0$ and $\ddot{S}$ in the inversion of an RSRC-double-crank mechanism given by the negative signed radical in Equation (3.74). The dimensions of the mechanism are d_1 = 2.0 in., d_2 = 1.7 in., d_3 = 1.5 in., a = 0.75 in., b = 0.4 in., e = 0, λ_0 = 60° and δ_0 = 0. As one observes this mechanism has a second order instantaneous dwell in the output rotation at θ_0 = 220.33°. That is, at that position the output link has zero velocity and zero acceleration, but not zero jerk, in its rotation.

The mechanisms in Figures 9 and 10 are intermittent motion generators. Such a mechanism can instantaneously be coupled with another system at the dwell position since a mass connected to the output link is displaced with no inertia force at this position.

Equations (3.77) and (3.79) are investigated in detail in Appendix D in order to discover the geometric properties for the vanishing derivatives and for the instantaneous dwells in the output displacement components.

Verifying the Displacement Equations

In order to experimentally verify the validity of the displacement equations of the RSRC mechanism, a fully adjustable model of the RSRC mechanism, shown in Figure 11, was prepared. The model is also

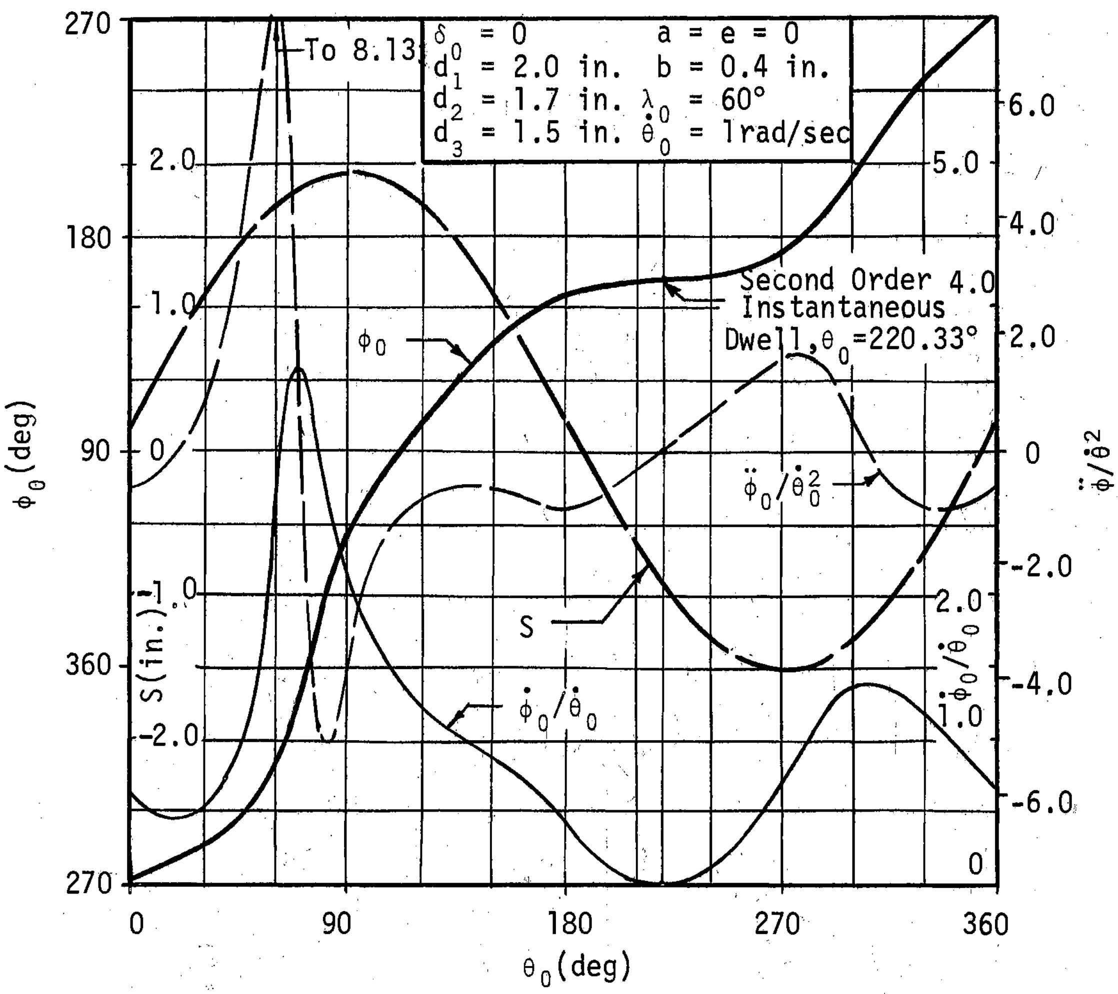


Figure 10. The Output Displacement, Velocity and Acceleration in the Geometric Inversion of the RSRC-Double-Crank Mechanism With a Second-Order-Instantaneous-Dwell in the Output Rotation. Geometric Inversion Given by the Negative Signed Radical in Equation (3.74), when $\delta_0 = 0$

capable of analyzing any of the following mechanisms, RSRC, RSHC, RSPC, RRSC, RPSC, RHSC, RCCC, RSSR, their constrained inversions, and many others.

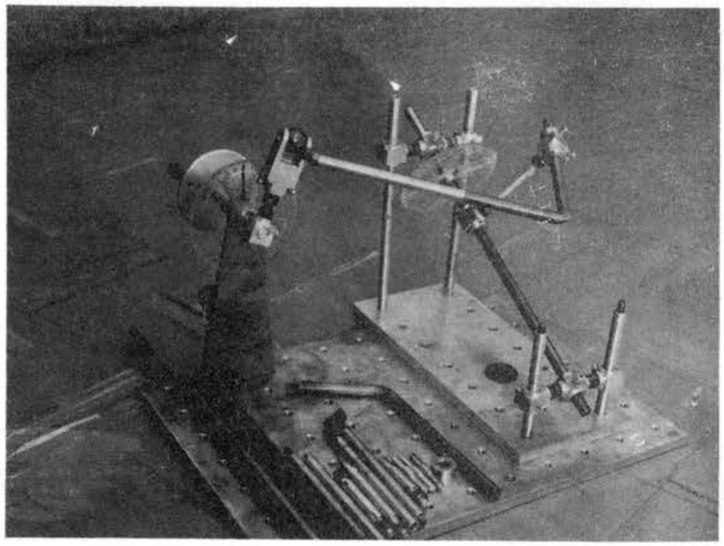


Figure 11. The Mechanical Model

The displacement equations can also be verified graphically. The RSRC mechanism and its constrained inversions can be drawn on a plane and the displacements be observed and measured easily. To do that construct the input crank ellipse using d_1 and λ_0, locate the center of the output crank circle using λ_0, b and a, then draw the output crank-circle of radius d_3, as shown in Figure 34, where the mechanism is projected onto the plane normal to the output pair axis. Then, draw the coupler ellipse on a transparent paper and locate the center of the ellipse relative to the center of the output crank circle using e, δ_0 and d_2. Pin the transparent paper on the previous drawing, at the center of the output crank circle, and rotate it about that center. Any point of intersection of the coupler ellipse and the input crank ellipse is the location of the spherical pair. Then read the angular displacements θ_0, ϕ_0 and χ_0 as shown in Figure 34. Then, determine the output translation S by drawing the input crank ellipse on the plane of a and S, and computing $-d_2 \sin\delta_0 \sin\chi_0$, as shown in Figure 37. No computation for S is necessary if $\delta_0 = n\pi$ and $\delta_0 = \frac{n+1}{2}\pi$. The limit position in the output rotation occurs when the input crank ellipse and the coupler ellipse are tangent together on the plane normal to the output pair axis, while the limit position of the output translation occurs when the input crank ellipse and the coupler ellipse are tangent together on the plane of a and S. The geometric properties at the limit positions are investigated in Appendix D. The facts discovered as related to the limit positions are utilized in defining the parameters of constraints for instantaneous dwells in Chapter V.

Rotation displacements in the RSRC mechanism can also be verified by constructing the conformal projection of the spherical indicatrix of the RSRC mechanism shown in Figure 5, for each value of the input crank rotation θ_0.

CHAPTER IV

APPROXIMATE SYNTHESIS OF MECHANISMS BY VARIATIONAL PRINCIPLES

This chapter briefly presents the variational principles in optimizing mechanism design for the generation of a screw function, a path and for guiding a rigid body through specified positions in space.

Stationary Values of a Function

Optimizing an engineering system is determining the stationary values of a function which characterizes the system. This investigation deals with the minimum of the error function which is defined by the difference of the generated function from the desired function. The function to be minimized may be an algebraic function as

$$E = f(x_1, x_2, \ldots, x_n) \tag{4.1}$$

where n is the number of independent parameters. In the problems of optimum mechanism design by minimizing an algebraic function one is involved with determining all or some of the dimensions of the mechanism so that its output will approximate a specified function, such as displacement, velocity, acceleration, output force, or torque; or its coupler point displacements will approximate a path; or its coupler link will guide a rigid body through specific positions.

The function to be minimized may be an integral as

$$I = \int_{x_1}^{x_2} f[x, y(x), \dot{y}(x), z(x), \dot{z}(x), \ldots]\, dx \qquad (4.2)$$

In this case one needs to determine functions y(x), z(x), etc., which render the integral I a minimum within the limits of the independent parameter x. The integrand may consist of more than one independent parameters. The problems of optimum design of mechanisms involving the minimization of the integral of the type given in Equation (4.2) may be of generating a displacement, doing work, or guiding a rigid body along a specified path and positions. For example, a synthesis of a mechanism, whose coupler point is to trace the shortest path on an elliptic torus within the specified boundaries may be carried on in two successive steps. The first step involves with determining the link and pair combination that will produce the desired elliptic torus, such as shown in Figure 12, then determining the function $\phi_{(\theta)}$ which will render the integral

$$I = \int_{\theta_1}^{\theta_2} f[\theta, \phi(\theta)]\, d\theta \qquad (4.3)$$

a minimum, where the integrand is the differential length on the path of P on the surface of the elliptic torus. The second step of the synthesis involves with determining the type and the size of the mechanism that will drive links A and B in Figure 12, generating the function $\phi_{(\theta)}$ as required for the minimum of the integral in Equation (4.3).

Lagrange Multipliers and Constraints

It is common to have constraining conditions in a physical problem that parameters of the system must satisfy the equations of constraints

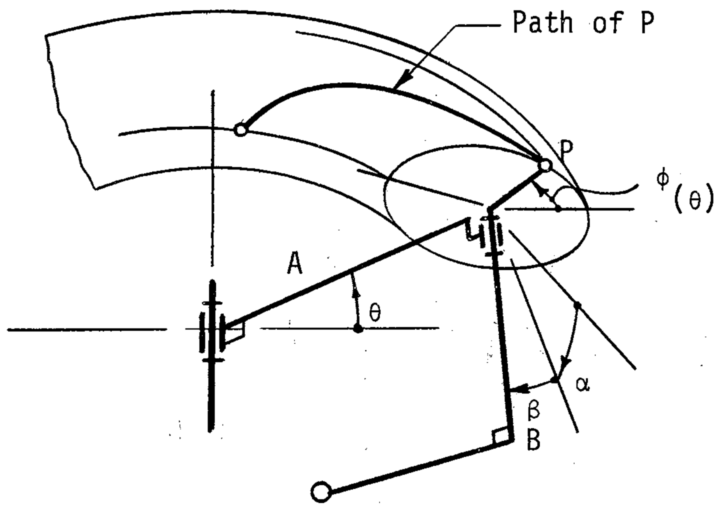


Figure 12. Generation of a Path on an Elliptic Torus

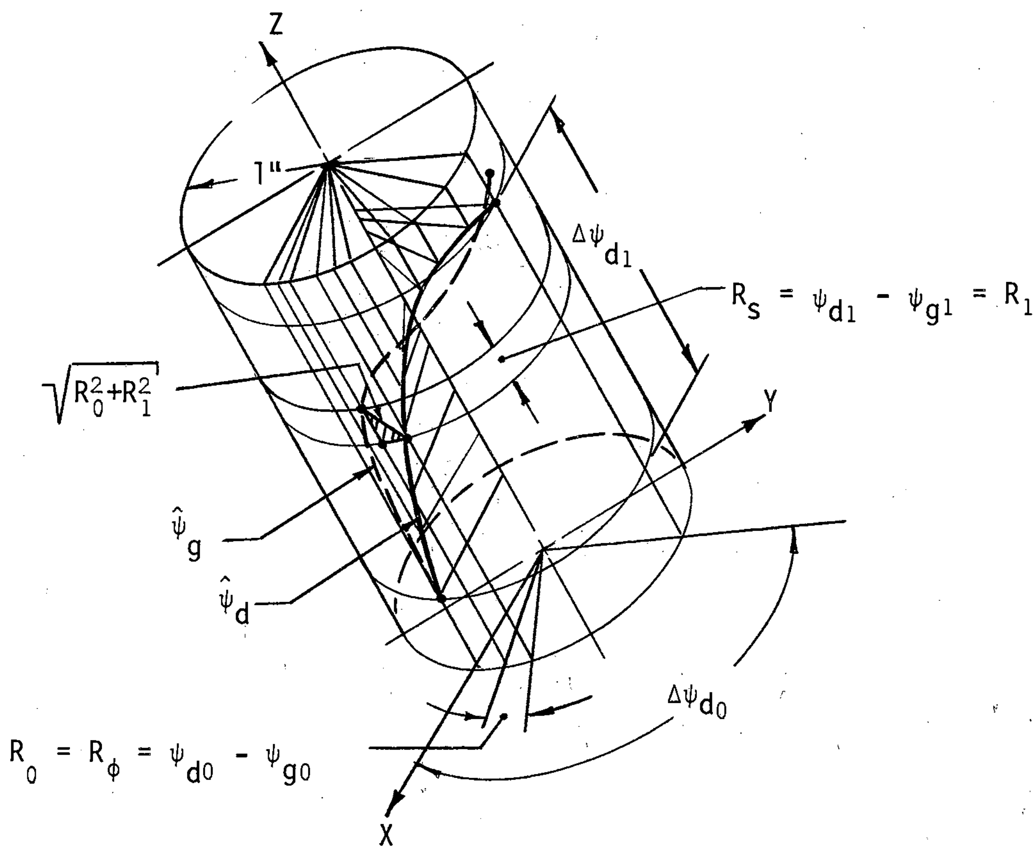


Figure 13. The Error in Generating the Screw Displacement $\hat{\psi}_d$ of Unit Radius by the Screw Displacement $\hat{\psi}_g$

along with rendering minimum for the algebraic function E or the integral I. Let there be m equations of constraints in the system and defined by

$$Q_J(x_1, x_2, \ldots, x_n) = 0 \quad , \quad J = 1, 2, \ldots, m \tag{4.4}$$

By the Lagrange's scheme, in order that the values of the independent parameters $x_1, x_2, \ldots, x_n$ render a stationary value for E and satisfy the equations of constraints given by Equation (4.4), the condition (97, 98, 99)

$$dF = \sum_{r=1}^{n} \frac{\partial F}{\partial x_r} dx_r = 0$$

must be satisfied along with the equations of constraints in Equation (4.4), where

$$F = E + \sum_{J=1}^{m} \lambda_J Q_J$$

and $\lambda_1, \lambda_2, \ldots, \lambda_m$ are the m unknown Lagrange multipliers. Since $dx_1, dx_2, \ldots, dx_n$ are arbitrary and

$$\sum_{J=1}^{m} Q_J \frac{\partial \lambda_J}{\partial x_r} = 0$$

by Equation (4.4),

$$\left.\begin{aligned} &\frac{\partial E}{\partial x_r} + \sum_{J=1}^{m} \lambda_J \frac{\partial Q_J}{\partial x_r} = 0 \;, && r = 1, 2, \ldots, n \\ &Q_J(x_1, x_2, \ldots, x_n) = 0 \;, && J = 1, 2, \ldots, m \end{aligned}\right\} \tag{4.5}$$

The n+m simultaneous equations in Equation (4.5), the <u>equations of</u>

condition, are solved for the n unknown parameters x_1, x_2, ..., x_n and m unknown Lagrange multipliers.

It is possible to eliminate the Lagrange multipliers from the equations of condition, especially when m is small. It is also possible to eliminate an equation of constraint and so the corresponding Lagrange multiplier from the list of unknowns by defining an unknown parameter, the parameter of constraint, which will characterize the condition of constraint imposed by the equation of constraint.

When the optimization is done to determine the set of dependent functions y(x), z(x), etc. to render a stationary value for an integral shown in Equation (4.2) and also satisfy equations of constraints, the Euler-Lagrange equations

$$\left.\begin{aligned} &\frac{\partial F}{\partial y} - \frac{d}{dx}\left(\frac{\partial F}{\partial \dot{y}}\right) = 0 \\ &\frac{\partial F}{\partial z} - \frac{\partial}{\partial x}\left(\frac{\partial F}{\partial \dot{z}}\right) = 0 \\ &\circ \quad \circ \quad \circ \\ \text{and} \quad & \\ &Q_J(x,y,z, \ldots) = 0 \ , \quad J = 1, 2, \ldots, m \end{aligned}\right\} \tag{4.6}$$

must be satisfied over the domain, where

$$F = f + \sum_{J=1}^{m} \lambda_J Q_J \tag{4.7}$$

is the augmented integrand.

In an optimum design problem the limits of the integral in Equation (4.2) may not be predefined. In that case in addition to the equations of condition given in Equation (4.6), the transversality condition

$$\left[\left(F - \frac{\partial y}{\partial x}\frac{\partial F}{\partial \dot{y}} - \frac{\partial z}{\partial x}\frac{\partial F}{\partial \dot{z}} - \ldots\right)dx + \left(\frac{\partial F}{\partial \dot{y}}dy + \frac{\partial F}{\partial \dot{z}}dz + \ldots\right)\right]_2$$

$$- \left[\left(F - \frac{\partial y}{\partial x}\frac{\partial F}{\partial \dot{y}} - \frac{\partial z}{\partial x}\frac{\partial z}{\partial \dot{z}} - \ldots\right)dx + \left(\frac{\partial F}{\partial \dot{y}}dy + \frac{\partial F}{\partial \dot{z}}dz + \ldots\right)\right]_1 = 0 \quad (4.8)$$

must be satisfied at the boundaries of the domain (98, 99). The subscripts 1 and 2 stand for that the corresponding term is computed at the boundary 1 and 2, respectively.

Minimizing the Error

To optimize a mechanism design is to determine its dimensions such that the generated output, say f_g, to give the closest approximation of the desired output, say f_d, over a domain D. That is the generated output must deviate from the desired one within the specified limits of accuracy. The function f_d may define displacement, force, torque, work, velocity or acceleration. Such a variational problem is then involved with determining the set of dimensions of the particular mechanism for that the error, that is the surface area between the desired and the generated output functions, must be minimum in the specified domain of displacements. Any desired function f_d and the generated function f_g can be defined as functions of the input parameter θ. Then $\theta_{01} \leq \theta \leq \theta_{02}$ defines the domain of the displacements.

Considering the fact that the difference $[f_d(\theta) - f_g(\theta)]$ may be positive or negative at a particular value of θ, it is best to minimize the volume whose cross-sectional area at a particular value of θ is $[f_d(\theta) - f_g(\theta)]^2$. Thus, the optimum set of dimensions of the mechanism,

x_1, x_2, ..., x_n, must render a minimum for the error volume integral

$$I = \int_{\theta_1}^{\theta_2} [f_d(\theta) - f_g(\theta)]^2 \, d\theta \qquad (4.9)$$

and satisfy the equations of constraints

$$Q_J(\theta, x_1, x_2, \ldots, x_n) = 0 , \quad J = 1, 2, \ldots, m \qquad (4.10)$$

if there is any. Then, having the augmented integrand

$$F = [f_d(\theta) - f_g(\theta)]^2 + \sum_{J=1}^{m} \lambda_J Q_J \qquad (4.11)$$

the m+n equations of condition given by Equation (4.6) must be solved for the n unknown dimensions and m unknown Lagrange multipliers.

It should be stated here that conditions stated by Equations (4.5) and (4.6) are necessary conditions for a stationary value, but not sufficient for the minimum or maximum. The additional condition, which is required to see if the stationary value is minimum or maximum, is furnished by the second derivatives. That is

$$\frac{\partial^2[f_d - f_g]^2}{\partial x_i \, \partial x_J} - \frac{\partial^2[f_d - f_g]^2}{\partial x_i^2} \cdot \frac{\partial^2[f_d - f_g]^2}{\partial x_J} < 0 \qquad \begin{matrix} i,J = 1, 2, \ldots, n \\ i \neq J \end{matrix}$$

and

$$\frac{\partial^2[f_d - f_g]^2}{\partial x_i^2} > 0 \qquad i = 1, 2, \ldots, n$$

for a minimum,

$$\frac{\partial^2[f_d - f_g]^2}{\partial x_i^2} < 0 \qquad i = 1, 2, \ldots, n$$

for a maximum. However, the matter of sufficiency can usually be determined from the nature of the problem. So the test for the second derivatives of the integrand becomes unnecessary since the result can easily be tested by the conditions present in the problem.

Optimum design of mechanisms having two independent inputs, mechanisms having two degrees of freedom of motion, involves with minimizing a double integral with the intervals $\theta_1 \leq \theta \leq \theta_2$ and $\phi_1 \leq \phi \leq \phi_2$ over the domain of diaplcements. Refer to (98) and (100) for the Euler-Lagrange equations and the transversality conditions to be satisfied for the stationary values of multiple integrals.

The exact generation of some specified function by a mechanism is virtually impossible unless the desired function happens to be identical with the generated function by the mechanism. For example, it is possible to generate a translation exactly as sine function of the input parameter by the output translation of the RSRC mechanism when the skew angle δ_0 is zero. This fact is observed in Equation (3.60).

Since the exact generation of a function by a mechanism is not possible, the method of approximate synthesis by matching the precision points has been well accepted (57, 79, 80, 81), where a number of precision points are selected over the domain and the loop closure equation is satisfied at these precision points. The number of precision points is taken to be the same as the unknown dimensions of the mechanism, in order that exact generation of the desired function is possible at least at the precision points.

In optimizing mechanism design by minimizing the error at a discrete set of design points the error volume integral given in Equation (4.9) may be approximated by

$$I = \sum_{i=1}^{N} [f_{d(\theta_i)} - f_{g(\theta_i)}]^2 \, d_{\theta_i} \tag{4.12}$$

and since $d\theta_1$, $d\theta_2$, ..., $d\theta_N$ are arbitrary, I has minimum when

$$E = \sum_{i=1}^{N} [f_{d(\theta_i)} - f_{g(\theta_i)}]^2 = \min \tag{4.13}$$

where N is the number of design points. The number of design points may be any and selected at any desired values of θ. When there are equations of constraints relating the unknown dimensions of the mechanism, as defined by Equation (4.10), the unknown dimensions of the mechanism, x_1, x_2, ..., x_n and the unknown Lagrange multipliers, λ_1, λ_2, ..., λ_m are determined by solving the n+m equations of condition given by Equation (4.5). The large number of design points distribute the error over the entire domain giving a better approximation over a continuous interval. By using design points it is also possible to exclude the approximation over some portions of the domain in order to improve the efficiency of the approximation in the portions of the domain where higher accuracy is needed.

It should be emphasized here that the desired function $f_d(\theta)$ is generated exactly at the design points, that is, E given by Equation (4.13) becomes zero, if the number of design points, N, is the same as the number of the unknown dimensions of the mechanism, n. When $N < n$ it is possible to have more than one mechanism which generates the desired function exactly at the N design points. In such cases some of the unknown dimensions may be specified as they would be needed by the design situations. The approximating function $f_g(\theta)$ is defined by the geometry of the mechanism, and the approximation is in a domain of finite intervals.

In case the approximating function $f_g(\theta)$ is defined as orthogonal polynomial expansion, a weighing function $w(\theta)$ is introduced transforming the function to be minimized, given by Equations (4.9) and (4.13), into the forms

$$I = \int_{\theta_1}^{\theta_2} w_{(\theta)}[f_{d(\theta)} - f_{g(\theta)}]^2 \, d\theta = \min \qquad (4.14)$$

for the continuous interval, and

$$E = \sum_{i=1}^{n} w_{(\theta_i)} \, [f_{d(\theta_i)} - f_{g(\theta_i)}]^2 = \min \qquad (4.15)$$

for the discrete set of design points. The weighting function has the property

$$w_{(\theta)} \geq 0$$

and

$$\int_{\theta_1}^{\theta_2} w_{(\theta)} \, d\theta > 0$$

over the domain D.

The need for the weighting function arises in order (a) to have ease in evaluating the coefficients of the orthogonal polynomial expansion, (b) to control the frequency with which some functions appear in the sum, particularly when dealing with integrals over infinite and semi infinite intervals, (c) to assure the convergence of the integral of $w_{(\theta)}[f_{d(\theta)} - f_{g(\theta)}]^2$ for infinite and semi infinite intervals, when $f_{g(\theta)}$ is a polynomial of arbitrary degree, and (d) maintain better approximation over some parts of the domain D than it is over other parts (101, 102).

The weighting function $w_{(\theta)} = e^{-\theta}$ is used for semi infinite intervals. Then the sequence of polynomials we require must be orthogonal over $(0, \infty)$ with respect to $e^{-\theta}$. The weighing function $e^{-\theta^2}$ is used for infinite interval. Over finite intervals [-a, a] the weighting function is $w_{(\theta)} = (1 - \theta)^{\alpha} (1 + \theta)^{\beta}$, $\alpha, \beta > -1$. When a = 1, it is Jacobi-Gauss polynomial approximation. When $\alpha = \beta = 0$, it is Legendre-Gauss polynomial approximation. When $\alpha = \beta = -1/2$, $w_{(\theta)} = 1/(1 - \theta^2)^{1/2}$, it is Chebyshev-Gauss polynomial approximation. The Chebyshev-Gauss polynomial approximation is used by Levitskii (89) to synthesize plane Lambda-mechanism for the approximate generation of symmetrical coupler curves.

In this investigation the optimum design of the mechanism is carried on by minimizing E given by Equation (4.13) for a discrete set of design points and giving equal "weight" to the deviation of the generating function from the desired one at each design point in the domain D. That is, the weighting function is $w_{(\theta)} = 1$.

Minimizing the Error in Generating a Screw Displacement

This investigation deals with the generation of a screw displacement $\hat{\psi}_{d(\theta)} = \psi_{d0(\theta)} + \varepsilon\, \psi_{d1(\theta)}$ about an axis by the displacement of the output link of the RSRC space mechanism, $\hat{\psi}_{g(\theta)} = \psi_{g0(\theta)} + \varepsilon\, \psi_{g1(\theta)}$. Since the displacement has two components, rotation about the axis and translation along the axis, it is a problem with two dimensional variation. The dual residual at a particular value of the independent parameter θ, as shown in Figure 13, is given by

$$\hat{R}_{(\theta)} = R_{0(\theta)} + \varepsilon R_{1(\theta)} = \hat{\psi}_{d(\theta)} - \hat{\psi}_{g(\theta)}$$

$$= [\psi_{d0(\theta)} - \psi_{d1(\theta)}] + \varepsilon[\psi_{d1(\theta)} - \psi_{g0(\theta)}] \quad (4.16)$$

where $\theta_1 \leq \theta \leq \theta_2$. The set of dimensions for the optimum mechanism is determined by minimizing

$$E = \sum_{i=1}^{N} \{[R_{0(\theta_i)}]^2 + [R_{1(\theta_i)}]^2\} \quad (4.17)$$

where N is the number of design points, $R_{0(\theta)}$ and $R_{i(\theta)}$ are defined as the real and dual parts of the dual residual, respectively, in Equation (4.16). Note that Equation (4.17) approximates the error volume in approximating a screw of unit radius.

In case of that the approximated screw is of zero pitch or of infinite pitch the second or the first term in Equation (4.17) vanishes, respectively. When the desired displacement is three dimensional, such as the generation of a three dimensional curve by the coupler point displacements the three coordinates of the precision points are used to compute E. For the approximate guidance of a rigid body in space at a series of successive positions, the coordinates of three points in the body can be used to compute E. Then the dimensions of the optimum mechanism must render minimum for

$$E = \sum_{i=1}^{N} \{[x_{d(\theta_i)} - x_{g(\theta_i)}]^2 + [y_{d(\theta_i)} - y_{g(\theta_i)}]^2 + [z_{d(\theta_i)} - z_{g(\theta_i)}]^2\} \quad (4.18)$$

where $x_{d(\theta_i)}$, $y_{d(\theta_i)}$, $z_{d(\theta_i)}$ and $x_{g(\theta_i)}$, $y_{g(\theta_i)}$, $z_{g(\theta_i)}$ are the desired and the generated coordinates of the design points. For the rigid body guidance and coupler curve generation in plane, the last term in

Equation (4.18) vanishes.

The problem of optimizing a mechanism for the generation of the screw displacement $\hat{\psi}_{d(\theta)}$ may consist of constraining conditions which must be satisfied along with the minimum for E in Equation (4.17). The constraining conditions may be to maintain vanishing velocities, instantaneous dwells in rotation or in translation or in both at some specified values of the input parameter. The equations of such constraints for the RSRC mechanism are given by Equations (3.77), (3.79), (3.80) and (3.82) when they are set equal to zero. If for example zero jerk in one of the components of the displacement at a specified value of the input parameter is needed, the vanishing third derivative of the displacement component with respect to the input parameter will be the additional equation of constraint. The dimensions of the optimum mechanism which will generate such a constrained screw displacement must satisfy the equations of condition given by Equation (4.5) best.

Linearizing the Equations of Condition

The equations of condition given by Equations (4.5) and (4.6) are nonlinear in general. The analytical solutions cannot ordinarily be found except in very simple situations. Iterative solutions especially step by step solution by relaxation becomes necessary. The equations of condition that result, when the equations of constraints are eliminated either by forward substitution or by introducing parameters of constraints, or when there exist no equation of constraint, are however relatively easy to solve by linearizing them in terms of errors using Taylor's theorem. The resulting linear simultaneous equations are then solved for errors in the previously assumed values of the unknown parameters. If the previously assumed value of the r^{th} unknown is x_{r0} and its deviation from its true value is h_r [the error], the true value of the unknown is

$$x_r = x_{r0} + h_r \tag{4.19}$$

Let the n equations of condition given by Equation (4.5), when there are no equations of constraints, be defined by

$$F_r(x_1, x_2, \ldots, x_n) = \frac{\partial E}{\partial x_r} = 0 , \qquad r = 1, 2, \ldots, n \tag{4.20}$$

where

$$E = \sum_{i=1}^{N} [f_{d(\theta_i)} - f_{g(\theta_i)}]^2$$

and n is the number of unknown parameters. The unknown parameters may be the unknown dimensions of the mechanism or it may include the parameters of constraints. Using Taylor series expansion, neglecting higher order derivative terms, and considering that the desired function $f_{d(\theta)}$ is independent of the unknown parameters, we have

$$F_r(x_1, x_2, \ldots, x_n) = F_r(x_{10}, x_{20}, \ldots, x_{n0}) + \sum_{t=1}^{n} h_t \cdot$$

$$\frac{F_r(x_{10}, x_{20}, \ldots, x_{n0})}{\partial x_t} = 0 \tag{4.21}$$

Letting F_{r0} denote $F_r(x_{10}, x_{20}, \ldots, x_{n0})$, the value of F_r computed using the previously assumed values of the unknown parameters, we write

$$\sum_{t=1}^{n} h_t \frac{\partial F_{r0}}{\partial x_t} = - F_{r0} \qquad r, t = 1, 2, \ldots, n \tag{4.22}$$

where

$$F_{r0} = -2 \sum_{i=1}^{N} \frac{\partial f_{g(\theta_i)}}{\partial x_r} \left[f_{d(\theta_i)} - f_{g(\theta_i)}\right] \qquad (4.23)$$

and

$$\frac{\partial F_{r0}}{\partial x_t} = -2 \sum_{i=1}^{N} \frac{\partial^2 f_{g(\theta_i)}}{\partial x_r \, \partial x_t} \left[f_{d(\theta_i)} - f_{g(\theta_i)}\right] - \frac{\partial f_{g(\theta_i)}}{\partial x_t} \frac{\partial f_{g(\theta_i)}}{\partial x_r} ,$$

$$r, t = 1, 2, \ldots, n \qquad (4.24)$$

The n simultaneous equations given by Equation (4.22) are the linearized forms of the nonlinear simultaneous equations of condition given by Equation (4.20). The unknowns in the linearized equations of condition are the n errors, h_1, h_2, ..., h_n. Equation (4.22) can be written in matrix form

$$A \overline{H} = \overline{C} \qquad (4.25)$$

where

$$\overline{H} = \left\{ \begin{matrix} h_1 \\ h_2 \\ \cdot \\ \cdot \\ \cdot \\ h_n \end{matrix} \right\} , \quad \text{the error vector,} \qquad (4.26)$$

$$\overline{C} = - \left\{ \begin{matrix} F_{10} \\ F_{20} \\ \cdot \\ \cdot \\ \cdot \\ F_{n0} \end{matrix} \right\} \qquad (4.27)$$

and

$$A = [a_{rt}] = \left[\left(\frac{\partial F_{r0}}{\partial x_t}\right)_{rt}\right] = \begin{bmatrix} \frac{\partial F_{10}}{\partial x_1} & \frac{\partial F_{10}}{\partial x_2} & \cdots & \frac{\partial F_{10}}{\partial x_n} \\ \frac{\partial F_{20}}{\partial x_1} & \frac{\partial F_{20}}{\partial x_2} & \cdots & \frac{\partial F_{20}}{\partial x_n} \\ \frac{\partial F_{n0}}{\partial x_1} & \frac{\partial F_{n0}}{\partial x_2} & \cdots & \frac{\partial F_{n0}}{\partial x_n} \end{bmatrix}_{n\times n} \tag{4.28}$$

Then

$$\overline{H} + A^{-1}\,\overline{C} \tag{4.29}$$

provided that $|A| \neq 0$. The improved values of the parameters are given by

$$\begin{Bmatrix} x_1 \\ x_2 \\ \cdot \\ \cdot \\ \cdot \\ x_n \end{Bmatrix} = \begin{Bmatrix} x_{10} \\ x_{20} \\ \cdot \\ \cdot \\ \cdot \\ x_{n0} \end{Bmatrix} + \begin{Bmatrix} h_1 \\ h_2 \\ \cdot \\ \cdot \\ \cdot \\ h_n \end{Bmatrix} \tag{4.30}$$

Substituting these improved values of the unknown parameters into Equation (4.29) yields new values of errors and new improved values of the parameters. The process is repeated until the error terms become negligible, or $\overline{H}$ may be considered a null vector within the specified limits of accuracy.

When E is defined for two and three dimensional variations as given by Equations (4.17) and (4.18), Equations (4.23) and (4.24) consist additional terms corresponding to the variation in the additional direction. Thus for the three dimensional variation, for which E is given by Equation (4.18), Equations (4.23) and (4.24) become

$$F_{r0} = -2 \sum_{i=1}^{N} \left\{ \frac{\partial x_{g(\theta_i)}}{\partial x_r} [x_{d(\theta_i)} - x_{g(\theta_i)}] + \frac{\partial y_{g(\theta_i)}}{\partial x_r} [y_{d(\theta_i)} - y_{g(\theta_i)}] + \frac{\partial z_{g(\theta_i)}}{\partial x_r} [z_{d(\theta_i)} - z_{g(\theta_i)}] \right\} \quad , \quad r = 1, 2, \ldots, n \qquad (4.31)$$

and

$$\frac{\partial F_{r0}}{\partial x_t} = -2 \sum_{i=1}^{N} \left\{ \frac{\partial^2 x_{g(\theta_i)}}{\partial x_t \, \partial x_r} [x_{d(\theta_i)} - x_{g(\theta_i)}] + \frac{\partial^2 y_{g(\theta_i)}}{\partial x_t \, \partial x_r} [y_{d(\theta_i)} - y_{g(\theta_i)}] + \frac{\partial z_{g(\theta_i)}}{\partial x_t \, \partial x_r} [z_{d(\theta_i)} - z_{g(\theta_i)}] - \frac{\partial x_{g(\theta_i)}}{\partial x_t} \frac{\partial x_{g(\theta_i)}}{\partial x_r} - \frac{\partial y_{g(\theta_i)}}{\partial x_t} \frac{\partial y_{g(\theta_i)}}{\partial x_r} - \frac{\partial z_{g(\theta_i)}}{\partial x_t} \frac{\partial z_{g(\theta_i)}}{\partial x_r} \right\} \quad , \quad r, t = 1, 2, \ldots, n \qquad (4.32)$$

In this study the variational problem is of two dimensional, as defined by Equation (4.17), and the terms corresponding to the third coordinate, z, in Equations (4.31) and (4.32) vanish.

Efficiency of the Approximation

The smallness of the root-mean-square-error (RMSE) in the approximation over the domain D is used to test the efficiency of the approximation of a function by the displacements of the mechanism, relative to the exact generation of the function. Over a domain of a continuous interval, the RMSE is

$$\text{RMSE} = \left[\frac{1}{|\Delta\theta|} \int_{\theta_1}^{\theta_2} [f_{d(\theta)} - f_{g(\theta)}]^2 \, d\theta \right]^{1/2} \qquad (4.33)$$

where $\Delta\theta = \theta_2 - \theta_1$. Over a domain of discrete set of design points the RSME is given by

$$\text{RMSE} = \sqrt{\frac{E}{N}} \tag{4.34}$$

where E is defined by Equations (4.13), (4.17), and (4.18) for the one, two and three dimensional variations.

It is virtually impossible to reduce the RMSE to zero for the generation of an arbitrary function over a domain of a continuous interval, or discrete set of design points where the number of design points, N, is greater than the number of unknown dimensions of the mechanism. However, it is possible to reduce RMSE to zero if N is the same or less than the number of unknown dimensions of the mechanism. Therefore, when there are fewer design points the efficiency in approximating the function at the design points will be higher, that is the RMSE will be smaller. It should also be noted that any constraining condition tends to reduce the efficiency of the approximation. Thus, the optimized mechanism is the one which affords the best approximation in the least squares sense.

The results summarized in this chapter are used in Chapter V for the approximate synthesis of the RSRC mechanism for the generation of the screw displacement $\hat{\psi}_{d(\theta)} = \psi_{d0(\theta)} + \varepsilon\, \psi_{d1(\theta)}$ by the displacements of its output link. It is considered that the specified screw displacement $\hat{\psi}_{d(\theta)}$ (a) has no constraint, (b) has constraints such that instantaneous dwells of odd or even orders must be generated at specified values of the input parameter θ, besides the approximate generation of the displacement itself, in the specified domain of the input parameter.

CHAPTER V

APPROXIMATE SYNTHESIS OF THE RSRC SPACE MECHANISM FOR THE GENERATION OF SPECIFIED SCREW DISPLACEMENTS BY VARIATIONAL PRINCIPLES

Screw Displacement With No Constraint

The variational principles discussed in Chapter IV are applied in this chapter to determine the optimum set of dimensions of the RSRC space mechanism so that the generated output displacement of the mechanism approximates the specified screw displacement in the specified limits of accuracy.

The RSRC mechanism that is to generate the screw displacement is shown in Figure 14. Such a mechanism generates screw displacement of the form

$$\hat{\phi}_d(\theta) = \psi_{d_0}(\theta) + \varepsilon S_d(\theta) \tag{5.1}$$

where ψ_{d_0} and S_d are the rotation and translation components of the desired screw displacement, θ is the input parameter, the independent variable, whose origin is θ_{01}. Then $\theta = \theta_0 - \theta_{01}$, where θ_0 positions the input crank. θ_{01} may be one of the unknown dimensions of the mechanism if not specified. Corresponding to θ_{01} is the initial displacement of the output link, $\hat{\phi}_{01} = \phi_{01} + \varepsilon S_1$ from which the desired screw displacement originates. Then the desired output displacement of the mechanism is

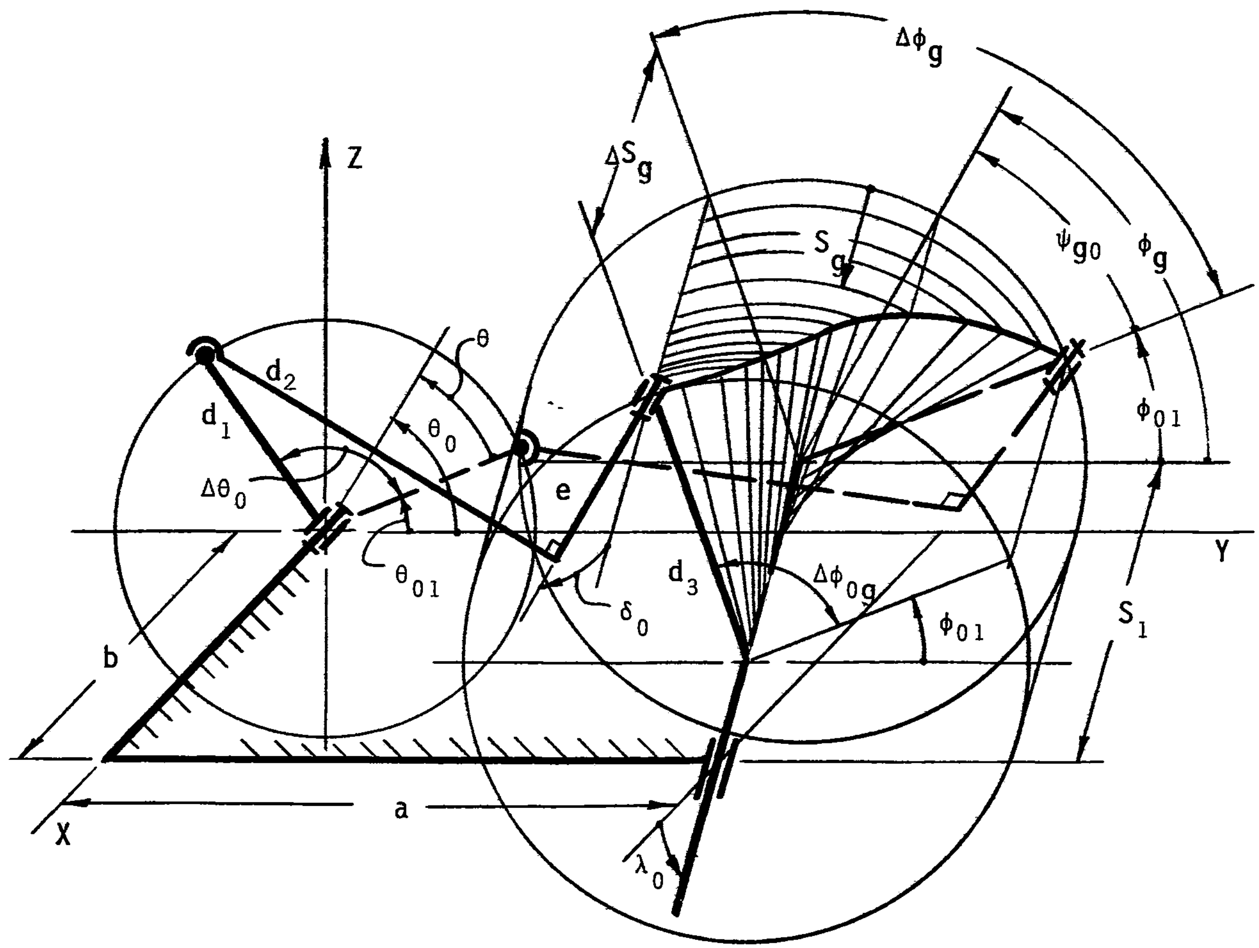


Figure 14. The Screw Displacement Generated by the RSRC Mechanism

$$\hat{\phi}_d(\theta) = [\phi_{01} + \psi_{d_0}(\theta)] + \varepsilon[S_1 + S_d(\theta)] \tag{5.2}$$

while the generated output displacement of the mechanism is

$$\hat{\phi}_g(\theta) = \phi_g(\theta) + \varepsilon[S_1 + S_g(\theta)] \tag{5.3}$$

where $\theta = \theta_0 - \theta_{01}$. $\phi_g \equiv \phi_0$ and $S_1 + S_g = \overline{S}_g \equiv S$ are given by Equations (3.62) and (3.60) respectively. As θ varies from zero to $\Delta\theta_0$, ψ_{d_0} varies from zero to $\Delta\phi_0$ and S_d varies from zero to ΔS_g, as shown in Figure 14.

This is a problem of two dimensional variation. When a discrete set of design points is used the function to be minimized is given by Equation (4.17). Thus

$$E = E_\phi + E_s \tag{5.4}$$

where

$$E_\phi = \sum_{i=1}^{N} [R_\phi(\theta_i)]^2$$

$$E_s = \sum_{i=1}^{N} [R_s(\theta_i)]^2$$

and

$$R_\phi(\theta) = \phi_{01} + \psi_{d_0}(\theta) - \phi_g(\theta)$$

$$R_s(\theta) = S_1 + S_d(\theta) - \overline{S}_g(\theta)$$

N is the number of design points considered in the domain of output displacement. $\psi_d(\theta)$ and $S_d(\theta)$ are some specified functions of θ and independent of the unknown dimensions of the mechanism. The unknown dimensions of the RSRC mechanism are $X_r \equiv e, d_1, d_2, d_3, a, b, \delta_0$,

λ_0, θ_{01}. ϕ_g, $\overline{S}_g$, ϕ_{01} and S_1 are functions of these dimensions. The equations of condition, when there are no equations of constraints, are given by Equation (4.20) as

$$F_r = 2 \sum_{i=1}^{N} \left\{ R_\phi(\theta_i) \left[\frac{\partial \phi_{01}}{\partial x_r} - \frac{\partial \phi_g(\theta_i)}{\partial x_r} \right] + R_s(\theta_i) \left[\frac{\partial S_1}{\partial x_r} - \frac{\partial \overline{S}_g(\theta_i)}{\partial x_r} \right] \right\} = 0 \ , \quad r = 1, 2, \ldots, n \qquad (5.5)$$

The n nonlinear simultaneous equations in Equation (5.5) are solved for the n unknown dimensions of the mechanism, after linearizing these equations in n unknown error terms by Taylor's theorem. An initial value is assigned to each unknown dimension, x_{r_0} to the r^{th} unknown dimension x_r. Then the errors in these assumed values are determined by solving Equation (4.29) for the error vector $\overline{H}$. The improved values of the parameters are then given by Equation (4.30).

The coefficients in the column vector $\overline{C}$ are computed by $-F_r$ in Equation (5.5) when the assumed values of the dimensions, x_{10}, x_{20}, ..., x_{n0} are used. The (r,t) coefficient of the matrix A is given by

$$\left(\frac{\partial F_r}{\partial x_t}\right)_{rt} = 2 \sum_{i=1}^{N} \left\{ \left[\frac{\partial \phi_{01}}{\partial x_t} - \frac{\partial \phi_g(\theta_i)}{\partial x_t}\right] \left[\frac{\partial \phi_{01}}{\partial x_r} - \frac{\partial \phi_g(\theta_i)}{\partial x_r}\right] + R_\phi(\theta_i) \left[\frac{\partial^2 \phi_{01}}{\partial x_r \partial x_t} - \frac{\partial^2 \phi_g(\theta_i)}{\partial x_r \partial x_t}\right] + R_s(\theta_i) \left[\frac{\partial^2 S_1}{\partial x_r \partial x_t} - \frac{\partial \overline{S}_g(\theta_i)}{\partial x_r \partial x_t}\right] + \left[\frac{\partial S_1}{\partial x_t} - \frac{\partial \overline{S}_g(\theta_i)}{\partial x_t}\right] \left[\frac{\partial S_1}{\partial x_r} - \frac{\partial \overline{S}_g(\theta_i)}{\partial x_r}\right] \right\} , \quad r,t = 1, 2, \ldots, n \qquad (5.6)$$

The first and the second partial derivatives of the output displacement components of the RSRC mechanism, with respect to the dimensions of the mechanism are listed in Appendix E. They are used to compute $-F_{r_0}$, the coefficients of the vector $\bar{C}$, given by Equation (5.5), and to compute $\left(\frac{\partial F_{r_0}}{\partial x_t}\right)_{rt}$, the coefficients of the matrix A, given by Equation (5.6).

One should expect that there will be more than one solution to a problem of optimum mechanism design. This is due to the fact that there exists more than one geometric inversion of the mechanism. Then each geometric inversion of the mechanism must be optimized. The one which renders the smallest root-mean-square-error (RMSE) is the one which approximates the desired displacement best. However, the optimized inversions must be compared for their dimensions and the efficiency of the transmission.

Sometimes an exact minimum of E tends to cause some of the dimensions to become infintely large as seen in some of the following examples. In such cases it is necessary to compromize on the magnitude of the RMSE to maintain smaller dimensions and favorable transmission, or choose a different mechanism.

Design situations may require that different accuracies may be needed for the approximation of each of the components of the screw displacement. Then, the RMSE for each component must be tested separately as well as their sum. The RMSE for the rotation and the translation are given by

$$(RMSE)_\phi = \sqrt{\frac{E_\phi}{N_\phi}} \tag{5.7}$$

and

$$(RMSE)_s = \sqrt{\frac{E_s}{N_s}} \tag{5.8}$$

respectively, where E_ϕ and E_s are defined with Equation (5.4), N_ϕ and

N_s are the numbers of the design points selected over the domains of rotation and translation, respectively.

Design situations may also require that either the rotation or the translation component may be of interest. The functional form for the desired displacement ψ_{d_0} or S_d may be specified requiring that the unspecified component takes place in a desirable range of motion. In such cases either the mechanism is optimized only for the specified component by minimizing E given by Equation (5.4), in which only the residual for the specified component remains, then the range of the unspecified component is tested; or a few design points may be specified for the unspecified component in order that it follows a desirable path.

It should be observed in Equation (5.4) that both ψ_{d_0} and S_d originate at $\theta = 0$ or at $\theta_0 = \theta_{01}$. Then the desired screw displacement $\hat{\psi}_d$ is generated exactly at the origin of the independent parameter θ. The origin $\theta_0 = \theta_{01}$ may not necessarily define one of the limits of the domain of displacements. It may be chosen anywhere in the domain of input parameter θ. Then θ may be measured from θ_{01} in any direction as it suits the specified problem. In case of constrained screw displacement the origin may be preferred at a point where the constraint is to be introduced.

The speed of convergence depends on the initially assumed mechanism geometry. A mechanism, in which the respective directions of the rotation and translation components of the output displacement can initially be observed, is the best choice. For example an RSRC mechanism with $\delta_0 = 0$, $e = 0$ is a good starting mechanism.

In a design problem some of the dimensions may be considered as design constants and specified, such as the zero value for the parameters e and δ_0 in order that some undesirable force and torque distributions may be eliminated. The fixed link length a and the skew angle λ_0 may also be specified since they position the screw axis relative to the frame of the mechanism. In the following illustrative examples λ_0 is taken to be a design constant.

The Program B, in Fortran IV language, prepared for IBM 7040 Digital Computer and listed in Appendix F carries out the optimization of the RSRC mechanism. The computer program solves the equations of condition given by Equation (5.5). By solving the linearized form in terms of errors given by Equation (4.29). The input data include the initially assumed values for the unknown dimensions, the number of design points and the corresponding values of the input parameter θ. The input crank position is defined by $\theta_0 = \theta_{01} + \theta$, where θ_{01} is one of the unknown dimensions. For each function to be generated define the number IV in the data, then add the cards for the desired rotation and translation functions ψ_{d_0} and S_d at the proper locations for the specified value of IV in the SUBROUTINE DSIRED. The Program B is directly applicable to synthesize 4R plane mechanism for function generation, by taking $\delta_0 = \lambda_0 = 0$ and $b = e = S = 0$. Such an optimum design of plane mechanism is the computerized form of the overlay technique for function generation, since it is also a method of finding the mechanism which renders the minimum RMSE for the selected design points.

<u>Example 1</u>: Design an RSRC mechanism which will approximate the screw displacement

$$\hat{\psi}_d(\theta) = (0) + \varepsilon\Delta S \left[\left(1 - \frac{2\theta}{\Delta\theta_0}\right)^2 - 1 \right]$$

where $\Delta S = 0.45$ in. and $\Delta\theta_0 = 80°$. The required skew angle between the input and output shafts is to be $\lambda_0 = 60°$. The desired screw displacement is of infinite pitch, that is, the designed mechanism is to approximate a pure translation in the specified range of motion, or the mechanism is to function as an RSRP mechanism.

It is to be noted that, once the dimensions of a mechanism are determined for a certain value of ΔS, the linear dimensions of the mechanism can be scaled to generate different values of ΔS. Then the error in the translation component is varied in the same proportion. The problem was run in the digital computer, IBM 7040, using Program B, with initially assumed dimensions $d_1 = 1.00$ in., $d_2 = 3.00$ in., $d_3 = 2.00$ in., $a = 2.5$ in., $b = 1.5$ in., $e = 0$, $\delta_0 = 0$ and $\theta_{01} = 200°$. Nine design points were used in the process of optimization on both rotation and translation components. The mechanism dimensions are

$$d_1 = 1.97551 \text{ in.} \qquad b = 2.78177 \text{ in.}$$

$$d_2 = 3.98095 \text{ in.} \qquad e = -0.89040 \text{ in.}$$

$$d_3 = 2.79436 \text{ in.} \qquad \delta_0 = 13.83421°$$

$$a = 2.78421 \text{ in.} \qquad \theta_{01} = 228.60602°$$

This solution required 0.08 hours for 11 iterations leading to the root-mean-square error

$$\text{RMSE} = \left\{ \frac{1}{9} \sum_{i=1}^{9} [R_\phi(\theta_i)]^2 + [R_s(\theta_i)]^2 \right\}^{1/2} =$$

$$= \left[\frac{1}{9} (0.00029773 + 0.00008) \right]^{1/2}$$

$$= 0.00623 \text{ in.}$$

for the screw of unit radius. The root-mean-square error for the rotation and the translation are

$$(RMSE)_\phi = (0.00029773/9)^{1/2} = 0.005752 \text{ rad} = 0.33°$$

and

$$(RMSE)_S = (0.00008/9)^{1/2} = 0.002981 \text{ in.}$$

which indicate the average deviation of the generated function from the desired one at the design points.

The error terms, the coefficients of the error vector $\overline{H}$ in Equation (4.29) were less than 0.000005.

The components of the desired screw displacement $\hat{\psi}_d(\theta) = \psi_{d_0}(\theta) + S_d(\theta)$ and the generated screw displacement $\hat{\psi}_g(\theta) = \psi_{g_0}(\theta) + S_g(\theta)$ are listed in Table I as printed out by the Program B. The components of the generated screw are defined by

$$\psi_{g_0}(\theta) = \phi_g(\theta) - \phi_{01}$$

and

$$S_g(\theta) = S(\theta) - S_1$$

as defined in the residuals R_ϕ and R_S in Equation (5.4). The angle χ_0 is also listed in Table I.

The maximum deviation in the rotation is 0.529667° and -0.006291 in. in the translation.

Figure 15 shows the desired and the generated screw components plotted from the results given in Table I.

Example 2: Consider the optimum design of an RSRC mechanism for the generation of the screw displacement

TABLE I

THE OUTPUT OF PROGRAM B FOR THE MECHANISM OPTIMIZED IN EXAMPLE 1

i	θ_0 (deg)	ϕ_g (deg)	ψ_{g_0} (deg)	ψ_{d_0} (deg)	$\psi_{d_0}-\psi_{g_0}$ (deg)
1	228.606020	174.359554	-0.000000	0.000000	0.000000
2	238.606020	174.672424	0.312870	0.000000	-0.312870
3	248.606020	174.783409	0.423409	0.000000	-0.423855
4	258.606018	174.781048	0.421494	0.000000	-0.421494
5	268.606018	174.724985	0.365431	0.000000	-0.365431
6	278.606018	174.639650	0.280096	0.000000	-0.280096
7	288.606018	174.509344	0.149790	0.000000	-0.149790
8	298.606018	174.274517	-0.085037	0.000000	0.085037
9	308.606018	173.829887	-0.529667	0.000000	0.529667

i	χ_0 (deg)	S (in.)	S_g (in.)	S_d (in.)	S_d-S_g (in.)
1	104.053877	-1.680532	0.000000	0.000000	-0.000000
2	100.134561	-1.871116	-0.190584	-0.196875	-0.006291
3	95.669849	-2.013882	-0.333350	-0.337500	-0.004150
4	90.859954	-2.102601	-0.422069	-0.421875	0.000194
5	85.875113	-2.133454	-0.452922	-0.450000	0.002922
6	80.851623	-2.105055	-0.424523	-0.421875	0.002648
7	75.891116	-2.018294	-0.337762	-0.337500	0.000262
8	71.061774	-1.876061	-0.195529	-0.196875	-0.001346
9	66.401508	-1.682926	-0.002394	0.000000	0.002394

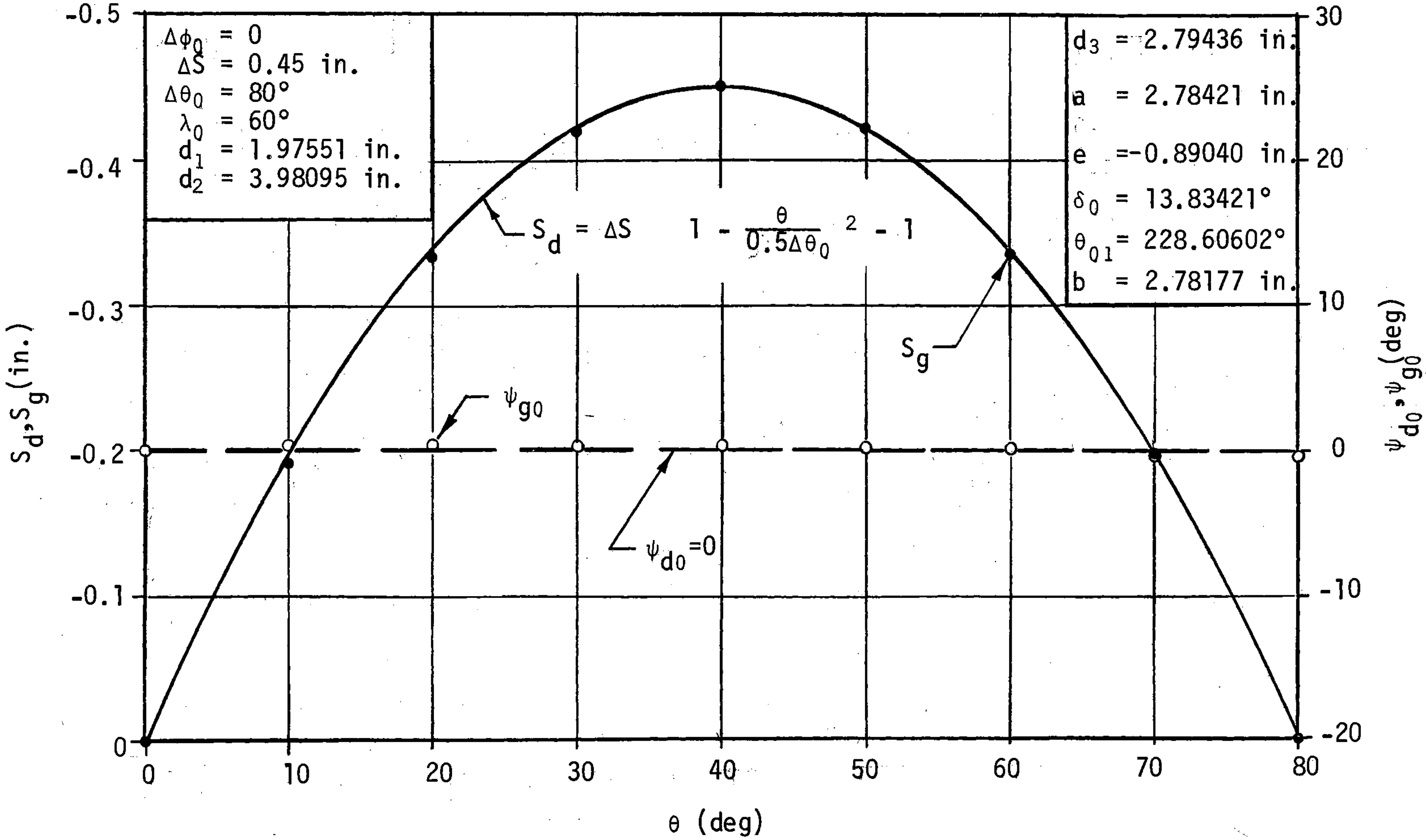


Figure 15. Desired and Generated Output Displacements of the RSRC Mechanism Optimized in Example 1

$$\hat{\psi}_d(\theta) = \Delta\phi_o\left(\frac{\theta}{\Delta\theta_o}\right)^2 + \varepsilon\Delta S\left[1 - \left(1 - \frac{\theta}{\Delta\theta_o}\right)^2\right]$$

where $\Delta\phi_o = -60°$, $\Delta S = 1.5$ in., $\Delta\theta_o = 80°$, $\lambda_o = 60°$. The number of design points for the translation is $N_s = 17$, one for every 5° increment in θ, and for the rotation is $N_\phi = 3$, one for every 40° increment in θ.

The geometric inversions of the RSRC mechanism were optimized starting with the initially assumed mechanism dimensions $d_1 = 1.00$ in. $d_2 = 3.00$ in., $d_3 = 2.00$ in., $a = 2.5$ in., $b = 1.5$ in., $e = 0$, $\delta_o = 0$ and $\theta_{01} = 30°$. After 14 iterations the mechanism dimensions were

$d_1 = 2.00216$ in.	$b = 2.45422$ in.
$d_2 = 3.56840$ in.	$e = 1.67500$ in.
$d_3 = 2.60782$ in.	$\delta_o = -8.20841$
$a = 2.66609$ in.	$\theta_{01} = 4.57613°$

The root-mean-square error for translation and rotation are

$$(RMSE)_s = \sqrt{\frac{E_s}{17}} = (0.00533435)^{1/2} = 0.0001771 \text{ in.}$$

and

$$(RMSE)_\phi = \sqrt{\frac{E_\phi}{3}} - (0.00012462/3)^{1/2} = 0.006445 \quad \text{rad} = 0.3691°$$

The coefficients of the error vector in Equation (4.29) were less than 0.000005. The output of Program B is given in Table II. Figure 16 shows the desired and generated screw displacements plotted from the results given in Table II.

Screw Displacement with Donstraints for Instantaneous Dwells

The variational problems of synthesizing the RSRC mechanism for the

TABLE II

THE OUTPUT OF PROGRAM B FOR THE MECHANISM OPTIMIZED IN EXAMPLE 2

i	θ_0 (deg)	ϕ_g (deg)	ψ_{g_0} (deg)	ψ_{d_0} (deg)	$\psi_{d_0}-\psi_{g_0}$ (deg)
1	4.576130	-6.952345	0.000000	-0.000000	-0.000000
2	9.576130	-6.323050	0.629296	-0.234375	
3	14.576130	-6.300361	0.651985	-0.937500	
4	19.576130	-6.971284	-0.018938	-2.109375	
5	24.576130	-8.418514	-1.466169	-3.750000	
6	29.576130	-10.706198	-3.753853	-5.859375	
7	34.576130	-13.861247	-6.908902	-8.437500	
8	39.576130	-17.854822	-10.902477	-11.484375	
9	44.576130	-22.591880	-15.639534	-15.000000	0.639535
10	49.576130	-27.916303	-20.963958	-18.984375	
11	54.576130	-33.632833	-26.680488	-23.437500	
12	59.576130	-39.538372	-32.586026	-28.359375	
13	64.576130	-45.450965	-38.498620	-33.750000	
14	69.576130	-51.227860	-44.275515	-39.609375	
15	74.576130	-56.770682	-49.818336	-45.937500	
16	79.576130	-62.021196	-55.068851	-52.734375	
17	84.576130	-66.952420	-60.000075	-60.000000	0.000075

i	χ_0 (deg)	S (in.)	S_g (in.)	S_d (in.)	S_d-S_g (in.)
1	-32.017321	2.753177	-0.000000	-0.000000	0.000000
2	-30.339425	2.916055	0.162877	0.181641	0.018763
3	-28.827226	3.075666	0.322488	0.351563	0.029074
4	-27.525998	3.230462	0.477284	0.509766	0.032481
5	-26.488373	3.378857	0.625680	0.656250	0.030570
6	-25.771132	3.519270	0.766093	0.791016	0.024923
7	-25.429707	3.650181	0.897004	0.914063	0.017058
8	-25.510973	3.770214	1.017036	1.025391	0.008354
9	-26.046030	3.878207	1.125029	1.125000	-0.000029
10	-27.045438	3.973272	1.220095	1.212891	-0.007205
11	-28.498644	4.054809	1.301632	1.289063	-0.012569
12	-30.377496	4.122476	1.369299	1.353516	-0.015783
13	-32.642101	4.176147	1.422969	1.406250	-0.016719
14	-35.246849	4.215853	1.462676	1.447266	-0.015410
15	-38.145079	4.241744	1.488567	1.476563	-0.012004
16	-41.292014	4.254054	1.500877	1.494141	-0.006736
17	-44.646154	4.253086	1.499909	1.500000	0.000091

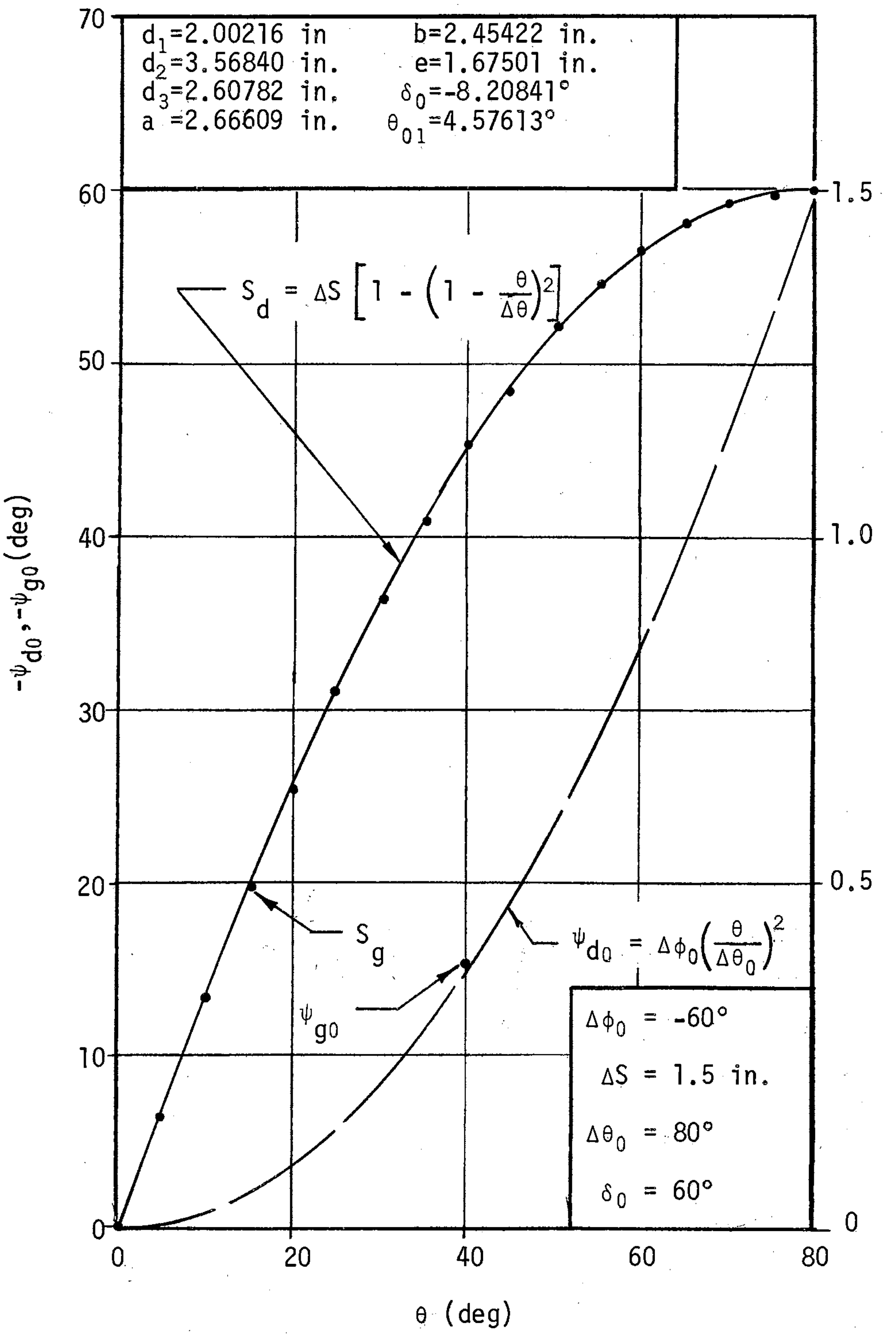


Figure 16. Desired and Generated Output Displacements of the RSRC Mechanism Optimized in Example 2

generation of screw displacements with constraints are being presented below. The constraining conditions are the generation of instantaneous dwells, or the generation of approximate dwells. It is virtually impossible to generate an exact dwell by a mechanism, since an exact dwell in a displacement component requires that all the consecutive derivatives of the displacement component with respect to the input parameter must vanish. A mechanism can satisfy this condition only when the freedom of the pair which permits this displacement component becomes a passive freedom. Order of the instantaneous dwell determines the length of the duration that the instantaneous dwell takes place. The higher the order of the instantaneous dwell, the longer is the duration of the dwell. A first order instantaneous dwell in a displacement component occurs when the first derivative of this displacement component with respect to the input parameter vanishes. At such an instantaneous dwell position the link experiencing this displacement is at rest with inertia force. Vanishing higher derivatives of the displacement provides instantaneous dwells of longer duration, with no inertia force. The highest order of the vanishing derivative is the order of the instantaneous dwell.

The necessary condition for the existance of all orders of instantaneous dwells in the output rotation and output translation of the RSRC mechanism are given by Equations (D.2) and (D.3), respectively. However, these conditions are sufficient for the first order instantaneous dwells. Therefore, the equations of constraints for the first order instantaneous dwells in the output rotation of the RSRC mechanism, at the p values of the input parameter are

$$Q_{\phi_J}(\theta_J) = W_2(\theta_J)L_1(\theta_J) + W_1(\theta_J)L_2(\theta_J)\cos^2 \delta_0 = 0, \quad J = 1,2,\ldots,p \qquad (5.9)$$

while the equations of constraint for the first order instantaneous dwells in the output translation, at the s values of the input parameter are

$$Q_{s_k}(\theta_k) = M_2(\theta_k)[W_2(\theta_k)L_1(\theta_k) + \cos^2 \delta_0 \; W_1(\theta_k)L_2(\theta_k)] +$$
$$+ L_2(\theta_k)[W_2(\theta_k)M_1(\theta_k) + \cos^2 \delta_0 \; W_1(\theta_k)M_2(\theta_k)]$$
$$- \sin\lambda_0 \; \cos\theta_{0_k} \; \tan\chi_{0_k} \; [W_2(\theta_k)M_1(\theta_k) + \cos^2 \delta_0 \; W_1(\theta_k)M_2(\theta_k)]$$
$$= 0 \; , \qquad k = 1, 2, \ldots, s \qquad (5.10)$$

where M_1, M_2, L_1 and L_2 are defined with Equation (3.77), W_1 and W_2 are defined with Equation (3.59). If the constraining conditions are for the first order instantaneous dwells in the components of the desired screw displacement, the optimum set of dimensions for the mechanism must satisfy the n equations of condition

$$\frac{\partial E}{\partial x_r} + \sum_{J=1}^{p} \lambda_J \frac{\partial Q_{\phi_J}}{\partial x_r} + \sum_{k=1}^{s} \lambda_k \frac{\partial Q_{s_k}}{\partial x_r} = 0 \; , \qquad r = 1, 2, \ldots, n \qquad (5.11)$$

along with the p+s equations of constraints given by Equations (5.9) and (5.10). The instantaneous dwells in rotation and translation may occur at the same values of the input parameter.

When higher order instantaneous dwells in the desired screw components are needed one must introduce additional equations of constraints by setting the consecutive derivatives of the displacement components equal to zero. Each equation of constraint added into the system introduces a new unknown Lagrange multiplier.

Parameters of Constraints

In the following the solution to the equations of condition given by Equations (5.11), (5.9) and (5.10) is greatly simplified by introducing parameters of constraints utilizing the facts stated in Appendix D regarding the instantaneous dwells in the components of the output dis-placement of the RSRC mechanism. All orders of instantaneous dwells in any component of the output displacement occur at the limits of this displacement component. The limits may be at maxima, minima or at inflection points. The necessary condition for each of these conditions is that the first derivative of the displacement component must vanish. The order of the instantaneous dwell is determined by the degree of conformity between the curvatures of the input-crank-ellipse and the coupler-ellipse at the point of tangency, where the limit position occurs. If some generalized parameters can be defined to maintain the limit positions in the displacement component at some specified value of the independent parameter during the optimization process, the necessary condition for all orders of instantaneous dwells is satisfied. The conditions for the sufficiency for the higher order instantaneous dwells may be left to be taken care of by the propensity of the variational principle by choosing the initial dimensions of the mechanism especially the value of θ_{01} in the zone such that the value of $\theta_0 = \theta_{01} + \theta_i$ will produce higher order instantaneous dwell. For example, the mechanism of Figure 10 with $\theta_{01} = 220°$ may be a good starting mechanism for the generation of rotation component having even order instantaneous dwell. The mechanism of Figure 9 with $\theta_{01} = 270°$ and the mechanism of Figure 7 with $\theta_{01} = 170°$ may be good starting mechanisms for the odd order instantaneous dwell in rotation.

By introducing such generalized parameters, the parameters of constraints, the equations of constraints and so the Lagrange multipliers may be eliminated from the process. The parameters of constraints in general consist some of the dimensions and variable parameters of the mechanism. However they enter the process as new parameters, they may exclude some of the dimensions of the mechanism from the process. The excluded dimensions are then determined after the parameters of constraints are determined to render the optimum mechanism.

In the following the parameters of constraints for instantaneous dwells in the output rotation are defined and used to design optimum RSRC mechanisms to generate screw displacements with instantaneous dwells. As it is proven in Appendix D, Equation (5.9) is satisfied when the input-crank-ellipse is tangent to the coupler-ellipse, where the ellipses are the projections of the input-crank-circle and the coupler-circle upon the plane normal to the output pair axis.

Figure 17 shows the RSRC space mechanism in two finitely separated positions on the plane normal to the output pair axis. F_1', F_2' and F_1'', F_2'' are the focal centers of the coupler-ellipse in the two positions, and located at $d_2 \cdot \sin\delta_0$ from the center of the ellipse. Let q_i be the slope of the normal to the coupler ellipse at the location of the spherical pair in the i^{th} position. q_i is given by

$$q_i = \tan^{-1} \frac{W_{1i} \cos^2\delta_0 \sin\phi_{0i} - W_{2i} \cos\phi_{0i}}{W_{2i} \sin\phi_{0i} + W_{1i} \cos^2\delta_0 \cos\phi_{0i}} \quad , \quad i = 1, 2, \ldots, p \qquad (5.12)$$

where W_1 and W_2 are identified with Equation (3.59). The parameters q_1, q_2, ..., q_p defined by Equation (5.12) are the parameters of constraints for the first and higher order instantaneous dwells. However, for the higher order instantaneous dwells they only satisfy the

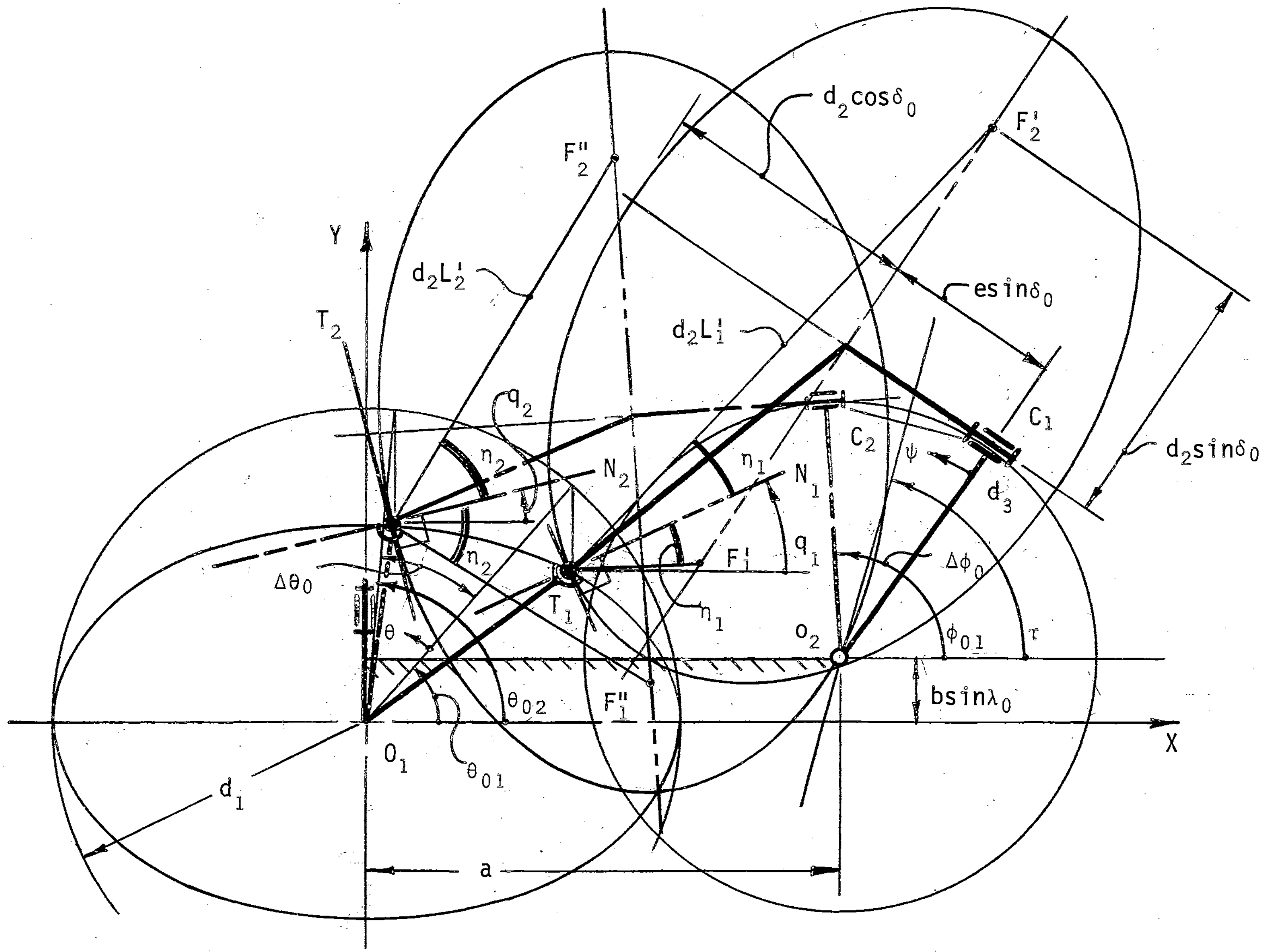


Figure 17. The RSRC Mechanism at Two Finitely Separated Positions and the Corresponding Parameters of Constraints q_1 and q_2

necessary condition. They are used as unknown parameters during the minimizing process. As one observes in Equation (3.59), they are functions of the dimensions $d_1, d_3, a, b, e, \delta_0, \lambda_0$ and θ_{01}. However, the introduction of the parameters of constraints for instantaneous dwells in the output rotation eliminates only the dimensions a, b and d_1 from the minimizing process.

When the value of the parameter of constraint q_i is set to be

$$q_i = \tan^{-1}\left[\frac{\tan(\theta_{01} + \theta_i)}{\cos\lambda_0}\right] \tag{5.13}$$

it becomes the slope of the normal to the coupler-ellipse when it is tangent to the input-crank-ellipse at θ_i. If Equation (5.13) is satisfied during the process of optimization the output link is at the limit position of its output rotation at θ_i. This satisfies the necessary condition for all orders of instantaneous dwells in the output rotation at θ_i.

An RSRC-quick-return mechanism can be synthesized to generate the exact return-to-advance-time-ratio in the output rotation, if two parameters of constraints are defined at the limits of the domain of the output rotation and Equation (5.13) is satisfied at these two limits at $\theta = 0$ and $\theta = \Delta\theta_0$.

Let the two positions of the RSRC mechanism shown in Figure 17 correspond to the two limits of the domain of the output rotation, $\phi_{01} \leq \phi_0 \leq \phi_{02}$. Meanwhile the input crank rotates within $\theta_{01} \leq \theta \leq \theta_{02}$. Then the desired output rotation is given by Equation (5.2), with

$$\phi_{01} = \tau - \frac{\Delta\phi_0}{2} \tag{5.14}$$

where $\Delta\phi_0 = \phi_{02} - \phi_{01}$, and τ is the slope of the bisector of $\Delta\phi_0$, as shown in Figure 17 and is defined by

$$\tau = \tan^{-1}\left[\frac{X_{c_1} - X_{c_2}}{Y_{c_2} - Y_{c_1}}\right] \quad (5.15)$$

where X_{c_i} and Y_{c_i} are the coordinates of the Point C in the i^{th} position, and given by

$$Y_{c_i} = d_1 \sin\theta_{0i} \cos\lambda_0 + d_2 L_i' \sin[q_i + \tan^{-1}(\tan\delta_0 \sin\chi_{0i})]$$

$$+ \sin\delta_0 (e \cos\phi_{0i} - d_2 \sin\phi_{0i}) \quad (5.16)$$

$$X_{c_i} = d_1 \cos\theta_{0i} + d_2 L_i' \cos[q_i + \tan^{-1}(\tan\delta_0 \sin\chi_{0i})]$$

$$+ (e - d_2) \sin\delta_0 \sin\phi_{0i} \quad (5.17)$$

where $\theta_{02} = \tau + \frac{\Delta\phi_0}{2}$, $\theta_{02} = \theta_{01} + \Delta\theta_0$ and

$$L_i' = \frac{1}{d_2} \sqrt{(d_2 \sin\delta_0 + W_{1_i})^2 + (W_{2_1})^2}$$

$$= \sqrt{1 + \sin\delta_0 \cos\chi_{0i} (2 + \sin\delta_0 \cos\chi_{0i})} \quad (5.18)$$

X_{0i} is the value of χ_0 at $\theta_{0i} = \theta_{01} + \theta_i$ and given by Equation (3.59)

The unknown dimensions d_1, d_2, δ_0, λ_0, θ_{01} and the parameters of constraints are determined by minimizing E given by Equation (5.4), by solving the equations of condition given by Equation (5.5) either by the matrix method of iteration using Equation (4.29) or by the relaxation methods. ϕ_{01} in Equation (5.4) is replaced by $\tau - \Delta\phi_0/2$, where $\Delta\phi_0$ is specified. After determining the unknown dimensions, the dimensions a, b and d_3 are determined by

$$a = \frac{1}{2}(X_{c_1} + X_{c_2}) + \frac{1}{2}(Y_{c_1} - Y_{c_2}) \cot\left(\frac{\Delta\phi_0}{2}\right) \quad (5.19)$$

$$b = \frac{1}{\sin\lambda_0}\left[\frac{1}{2}(Y_{c_1} + Y_{c_2}) + \frac{1}{2}(X_{c_2} - X_{c_1}) \cot\left(\frac{\Delta\phi_0}{2}\right)\right] \quad (5.20)$$

$$d_3 = \frac{1}{2\left|\sin\left(\frac{\Delta\phi_0}{2}\right)\right|} \sqrt{(X_{c_1} - X_{c_2})^2 + (Y_{c_2} - Y_{c_1})^2} \qquad (5.21)$$

The introduction of the parameters of constraints makes it possible to generate instantaneous dwells of the first or higher orders excluding the equations of constraints for the necessary condition from the process. Whenever a parameter of constraint is used to maintain instantaneous dwell at the i^{th} value of the input parameter, its value is defined by Equation (5.13) and it is excluded from the list of unknowns. However, its value varies during the minimizing process, since the value of θ_{01} varies.

It should be called to attention that the $\Delta\phi_0$ defined above may not be limiting the domain of the output rotation. For example, C_1 and C_2 shown in Figure 17 may be within the domain rather than being at the boundaries of the domain and the rotation ψ may originate at any position of the output crank and may be read in both C.W. and C.C.W. directions. This makes it possible to generate instantaneous dwells defined by inflection points, maxima or minima within the domain.

Figure 18 shows the parameters of constraints for the RSRC mechanism when $\delta_0 = n\pi$, where the parameters of constraints are the slopes of the coupler link at the corresponding position. Note in Figure 18 and in Equation (5.21) that both geometric inversions are included in the process by introducing the parameters of constraints. Both inversions should be optimized and compared.

It should also be recalled that in case of $\delta_0 = n\pi$, the output translation is a function of $\sin\theta$, whose amplitude is defined by $d_1 \cdot \sin\lambda_0$; and e becomes a redundant dimension which has no effect on the generated output displacement and is excluded from the list of

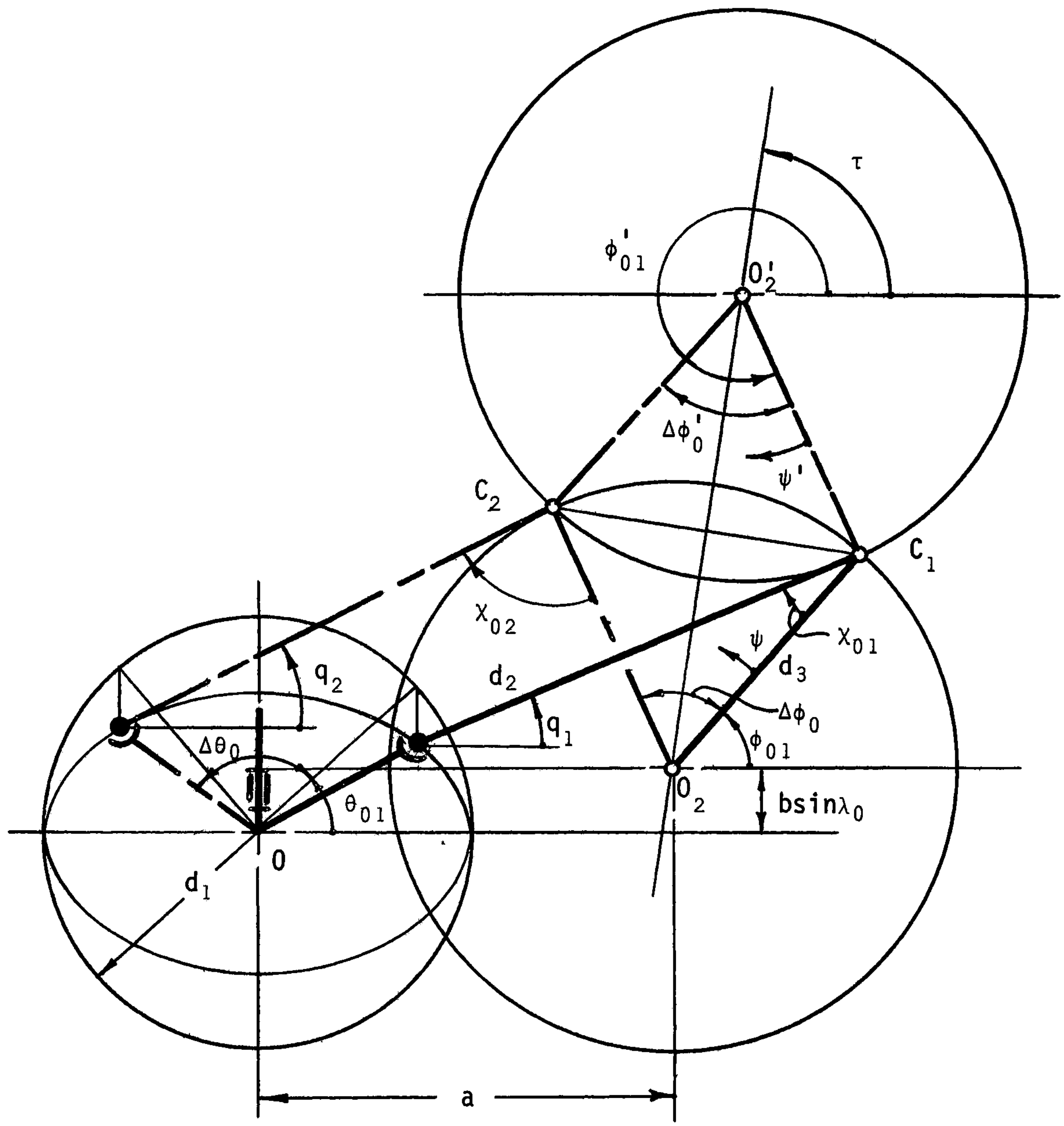


Figure 18. Parameters of Constraints and the Two Geometric Inversions of the RSRC Mechanism when $\delta_0 = n\pi$

unknown parameters. However the functional form of the output translation may be given additional variation by introducing an axial screw pair in place of the revolute pair making e a function of χ_0. In this case the output translation is given by

$$S = b\ \cos\lambda_0 + d_1\ \sin\lambda_0\ \sin\theta_0 + \cos\delta_0\ (e_0 \pm p\chi_0) \qquad (5.22)$$

where p is the pitch of the axial screw pair and is one of the unknown parameters, and e_0 is the value of e when $\chi_0 = 0$.

The following examples are to illustrate the use of the parameters of constraints in synthesizing the RSRC mechanism for the generation of screw displacements having constraints for first and higher order instantaneous dwells in the rotation component.

The Program C, in Fortran IV language, prepared for IBM 7040 digital computer and listed in Appendix F carries out the optimization of the RSRC mechanism when the parameters of constraints are introduced. This program solves the equations of condition given in Equation (5.5) by the relaxation method of Gauss. The input data include the initially assumed values of the undetermined parameters, starting values of increments in the parameters, $\Delta\theta_0$, $\Delta\theta_0$ and ΔS. The increments are reduced as the solution is approached. The functional form for the desired displacement is defined in the SUBROUTINE DSIRED as it is done in Program B. When the parameters of constraints are defined at the values of θ within the domain of displacements, the values of the desired screw displacement at the design points must be put in a successive order by the SUBROUTINE DSIRED. This is necessary to compute the errors at the design points in consecutive order in the main program, when E is being computed.

The Program C is also directly applicable to synthesize a 4R plane mechanism for the generation of screw displacements of zero pitch, which may, or may not, have constraints for odd order instantaneous dwells in rotation. The RSRC mechanism reduces to a 4R plane mechanism when $\delta_0 = \lambda_0 = 0$ and $S = e + b =$ constant. When the mechanism is projected onto the plane normal to the output pair axis, the input-crank-ellipse and the coupler-ellipse become circles of radii d_1 and d_2, respectively. Figure 19 shows the two geometric inversions and the parameters of constraints for the 4R plane mechanism, where the parameters of constraints are the slopes of the coupler link at the corresponding positions. The primed dimensions correspond to the second inversion. When the parameters of constraints define both, the positions at which output rotation velocity vanishes and the limits of the output rotation, a quick-return 4R plane mechanism is optimized, which generates the return-to-advance-time-ratio exactly.

It should be noted here that an even order instantaneous dwell in the output rotation of a 4R plane mechanism and multi-loop plane mechanisms, in which the output link of one loop is the driving link of the next loop, can never be generated. This is because of that an even order instantaneous dwell occurs at an inflection point, and the inflection point in the output rotation of a 4R plane mechanism occurs at the dead center position of the output link.

Example 3: The method of optimizing the RSRC mechanism by use of the parameters of constraints, just discussed above is applied to design a 4R plane mechanism to generate the screw displacement of zero pitch

$$\hat{\psi}_d = \Delta\phi_0\left(\frac{\theta}{\Delta\theta_0}\right)^2 + \varepsilon(0)$$

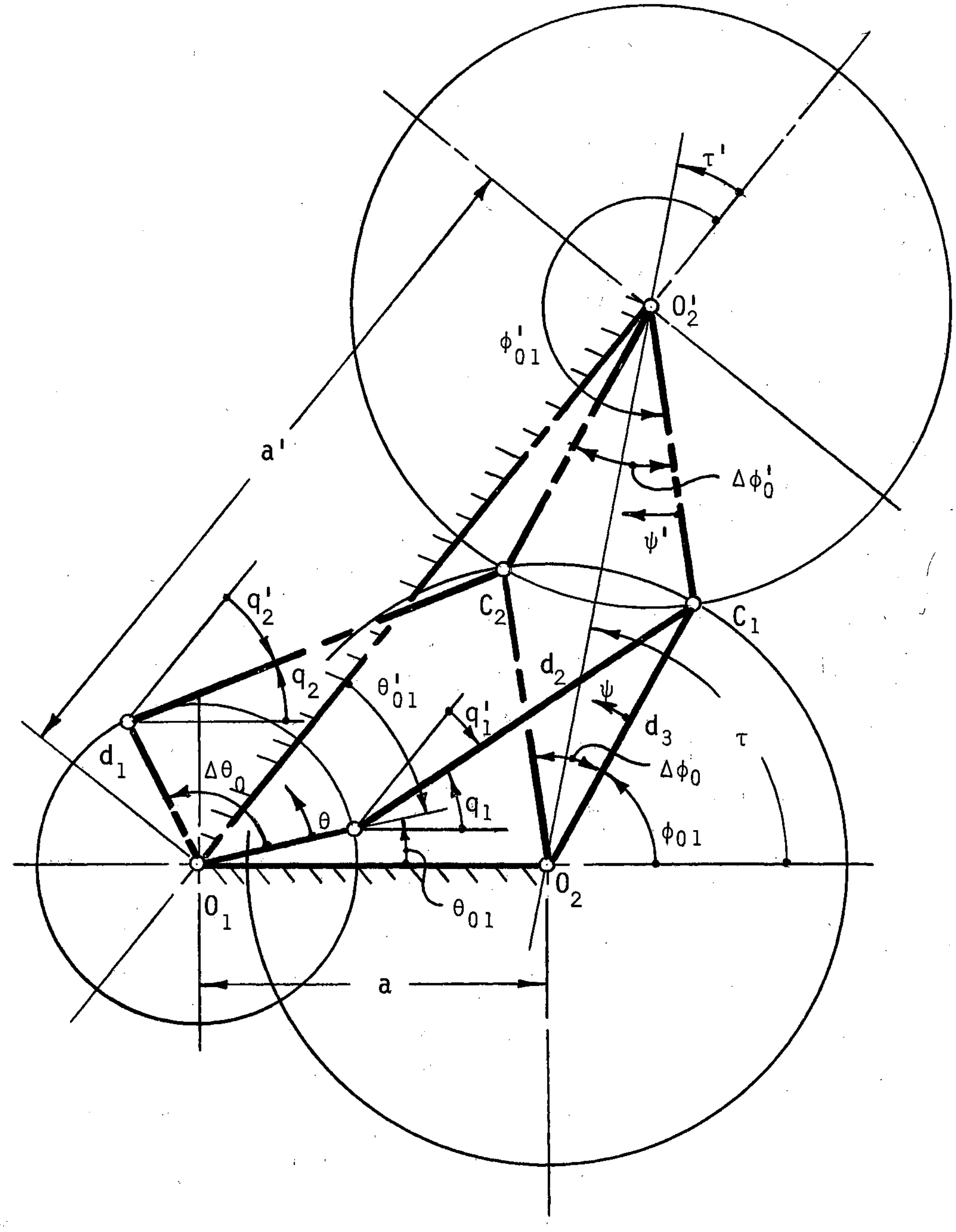


Figure 19. Parameters of Constraint and the Two Geometric Inversions of a 4R Plane Mechanism.

with $\Delta\phi_0 = 60°$ and $\Delta\theta_0 = 90°$. Exact generation of $\Delta\phi_0$ is desired.

The specified output rotation has first order instantaneous dwell when $\theta = 0$. Program C is used to design the optimum mechanism starting with the initially assumed values of mechanism dimensions; $d_1 = 1.0$ in., $d_2 = 2.0$ in., $q_2 = 45°$ and $\theta_{01} = 27.5°$. The number of design points is 10. The solution reached the optimum mechanism within 1 minute. The changes in dimensions during the last relaxation on each dimension were less than 0.000002. The dimensions of the optimized mechanism are

$$d_1 = 1.0 \text{ in.}$$

$$d_2 = 8.099989 \text{ in.}$$

$$\theta_{01} = 7.374689°$$

$$q_1 = 7.374688°$$

$$q_2 = 10.09969294°$$

resulting

$$a = 9.204072 \text{ in.}$$

$$d_3 = 1.181742 \text{ in.}$$

The output of Program C for these dimensions is given in Table III. The desired and the generated displacements are plotted in Figure 20a. Figure 20b shows the mechanism at the limits of the rotation.

Example 4: Let us now consider the design of an RSRC mechanism with a predefined value of $\delta_0 = 0$ to generate a screw displacement. As it is observed in Equation (3.60) the output displacement of such an RSRC mechanism is a predefined sine function of the input parameter. The output translation consists of the unknown dimensions d_1, λ_0 and θ_{01}. The redundant dimension e may be taken zero in this case. Note in Appendix B that the RSRC mechanism has one general constraint on

TABLE III

THE OUTPUT OF PROGRAM C FOR THE PLANE MECHANISM OPTIMIZED IN EXAMPLE 3

i	θ_0 (deg)	ϕ_g (deg)	ψ_{g0} (deg)	ψ_{d0} (deg)	$\psi_{d0}-\psi_{g0}$ (deg)
1	7.374689	98.729754	-0.000000	0.000000	0.000000
2	-2.625311	99.556449	0.826694	0.740741	-0.085955
3	-12.625311	102.001689	3.271935	2.962963	-0.308972
4	-22.625311	105.968099	7.238344	6.666667	-0.571678
5	-32.625311	111.319002	12.589248	11.851852	-0.737396
6	-42.625311	117.918717	19.188963	18.518518	-0.670445
7	-52.625311	125.686106	26.956351	26.666666	-0.289685
8	-62.625311	134.672544	35.942790	36.296296	0.353506
9	-72.625311	145.234989	46.505235	47.407407	0.902172
10	-82.625311	158.729752	59.999997	60.000000	0.000002

i	χ_0 (deg)	S (in.)	S_g (in.)	S_d (in.)	S_d-S_g (in.)
1	91.355067	0.000000	0.000000	0.000000	0.000000
2	90.957079	0.000000	0.000000	0.000000	0.000000
3	92.231908	0.000000	0.000000	0.000000	0.000000
4	95.146055	0.000000	0.000000	0.000000	0.000000
5	99.637464	0.000000	0.000000	0.000000	0.000000
6	105.648691	0.000000	0.000000	0.000000	0.000000
7	113.176216	0.000000	0.000000	0.000000	0.000000
8	122.351875	0.000000	0.000000	0.000000	0.000000
9	133.638693	0.000000	0.000000	0.000000	0.000000
10	148.630058	0.000000	0.000000	0.000000	0.000000

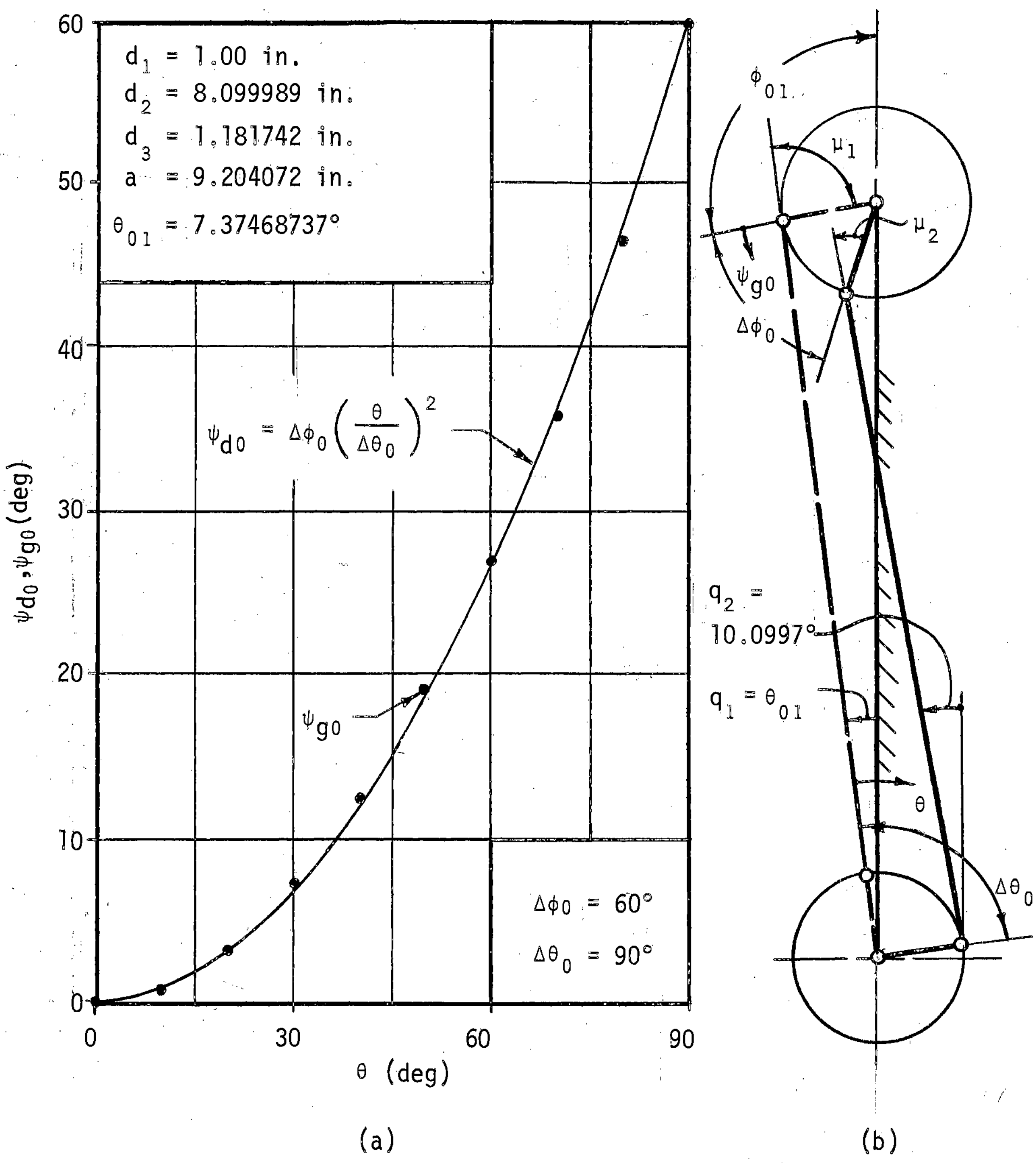


Figure 20. (a) Desired and Generated Displacements of the 4R Plane Mechanism Optimized in Example 3
(b) The Plane Mechanism Designed

rotation when $\delta_0 = 0$.

Certain portions of the output translation, the sine function of $\theta_0 = \theta_{01} + \theta$, may be used to approximate the translation component of the desired screw displacement, determining the dimensions d_1, λ_0 and θ_{01}. The dimensions d_2, d_3, a and b may be left to be determined when optimizing the mechanism to approximate the rotation component. This gives the designer a chance to design the mechanism in two independent steps, one for each component of the desired screw displacement. For example

$$S = C_1 + d_1 \sin\lambda_0 \sin\theta_0 \tag{5.23}$$

approximates the straight line

$$S_s = C_1 + 0.017111\, d_1\, \theta_0 \sin\lambda_0 \quad \text{for} \quad -30° < \theta_0 < 30° \tag{5.24a}$$

and

$$S_s = C \;\; - 0.017111\, d_1\, \theta_0 \sin\lambda_0 \quad \text{for} \quad 150° < \theta_0 < 210° \tag{5.24b}$$

in the least-squares sense. When $30 < \theta_0 < 150°$ and $210 < \theta_0 < 330°$ the sine function may be used to approximate an exponential function.

Let the desired screw displacement

$$\hat{\psi}_d = \Delta\phi_0\left(\frac{\theta}{\Delta\theta_0}\right)^2 + \varepsilon\Delta S\left(\frac{\theta}{\Delta\theta_0}\right)$$

to be generated as the input crank rotates counterclockwise, where $\Delta\phi_0 = \pm 40°$, $\Delta\theta_0 = 47.5°$, $\Delta S = -2.0$ in. The rotation component of this displacement has a first order instantaneous dwell when $\theta = 0$. The dimensions d_1, λ_0 and θ_{01} may be determined by either minimizing $E = E_\phi + E_S$, where E_ϕ and E_S are defined with Equation (5.4), or by minimizing E_ϕ only and using Equation (5.24). By considering $\lambda_0 = 60°$

as a design constant and $\theta_{01} = 25°$, $d_1 = 1.0$ in., $-30° < \theta_0 < 30°$, the translation component of the desired screw is generated. However, the displacement of the input crank from $\theta_{01} = 25°$ to $\theta_{02} = 25 - 47.5 = 22.5°$ with $d_1 = 1.0$ in. does not produce the desired ΔS, the linear dimensions of the mechanism is scaled up to give the desired value of ΔS after determining the remaining dimensions by minimizing E_ϕ by any of the programs, Program B or Program C. Since the rotation component of the desired screw has a first order instantaneous dwell at $\theta = 0$, Program C can easily be used leaving d_2 and q_2 as the undetermined parameters. q_1 is determined by Equation (5.13) at $\theta = 0$ where the instantaneous dwell occurs. q_2 is defined at $\theta = \Delta\theta_0$. The optimization for the rotation component was done by using 20 design points. It was observed that as the solution approached the optimum mechanism, the dimension d_2 approached to infinity. Both geometric inversions defined by Equation (3.74) were tested. The negative signed inversion had smaller (RMSE) for the same value of d_2 when d_2 was very large. The output of the inversions, defined by the negative signed radical in Equation (3.74), is given in Table IV for $\Delta\phi_0 = 40°$, when $d_2 = 28.734519$ in., $q_2 = 43.5°$, $q_1 = 43.003°$, $d_1 = 1.00$ in., $d_3 = 0.3933$ in., a = 22.1475 in., b = 22.5032 in. and $(RMSE)_\phi = 0.0019274$ rad = 0.1104°. The maximum deviation of the generated rotation from the desired one is 0.231348°. Table V shows the output of the same inversion when $d_2 = 3.7602$ in., $d_1 = 1.00$ in., $d_3 = 0.4220$ in., a = 3.95641 in., b = 2.86290 in., $q_1 = 43.003°$ and $q_2 = 46°$, and $(RMSE)_\phi = 0.0029285$ rad = 0.1621°. The maximum deviation of the generated rotation from the desired one is 0.356234° in this case.

Certainly the designer would prefer the second mechanism which

TABLE IV

THE OUTPUT OF PROGRAM C FOR THE GEOMETRIC INVERSION WITH NEGATIVE SIGNED RADICAL IN EXAMPLE 4. d_2 = 28.734519 in.

i	θ_0 (deg)	ϕ_g (deg)	ψ_{d_0} (deg)	ψ_{g_0} (deg)	$\lvert\psi_{d_0}-\psi_{g_0}\rvert$ (deg)
1	25.000000	125.287534	0.000000	0.000133	0.000133
2	22.500000	125.402253	0.110803	0.114587	0.003784
3	20.000000	125.745815	0.443213	0.458149	0.014936
4	17.500000	126.316768	0.997230	1.029101	0.031871
5	15.000000	127.112867	1.772853	1.825201	0.052348
6	12.500000	128.131393	2.770083	2.843727	0.073644
7	10.000000	129.369194	3.988919	4.081528	0.092608
8	7.500000	130.823021	5.429363	5.535355	0.105992
9	5.000000	132.489727	7.091413	7.202061	0.110648
10	2.500000	134.366556	8.975069	9.078890	0.103821
11	-0.000000	136.451567	11.080332	11.163900	0.083568
12	-2.500000	138.743885	13.407202	13.456219	0.049017
13	-5.000000	141.244236	15.955678	15.956570	0.000892
14	-7.500000	143.955454	18.725761	18.667788	0.057974
15	-10.000000	146.882946	21.717451	21.595280	0.122171
16	-12.500000	150.035753	24.930747	24.748087	0.182660
17	-15.000000	153.427565	28.365650	28.139898	0.225752
18	-17.500000	157.078478	32.022160	31.790812	0.231348
19	-20.000000	161.017622	35.900276	35.729956	0.170320
20	-22.500000	165.287489	39.999999	39.999823	0.000176

i	χ_0 (deg)	S (in.)	S_1+S_d (in.)	$\lvert S_1+S_d-S\rvert$ (in.)
1	82.284462	11.617584	11.622052	0.004468
2	82.345959	11.582999	11.585006	0.002006
3	82.637760	11.547784	11.547959	0.000175
4	83.158567	11.512005	11.510912	0.001092
5	83.906330	11.475730	11.473866	0.001864
6	84.878547	11.439028	11.436819	0.002209
7	86.072315	11.401970	11.399772	0.002197
8	87.484665	11.364625	11.362726	0.001899
9	89.112753	11.327065	11.325679	0.001386
10	90.954149	11.289361	11.288632	0.000729
11	93.007258	11.251586	11.251586	0.000000
12	95.271581	11.213810	11.214539	0.000729
13	97.748234	11.176107	11.177492	0.001386
14	100.440474	11.138547	11.140446	0.001899
15	103.354163	11.101202	11.103399	0.002197
16	106.498846	11.064144	11.066353	0.002209
17	109.888763	11.027442	11.029306	0.001864
18	113.544657	10.991167	10.992259	0.001092
19	117.496418	10.955388	10.955213	0.000175
20	121.787486	10.920172	10.918166	0.002006

TABLE V

THE OUTPUT OF PROGRAM C FOR THE GEOMETRIC INVERSION WITH NEGATIVE SIGNED RADICAL IN EXAMPLE 4. d_2 = 3.7602 in.

i	θ_0 (deg)	ϕ_g (deg)	ψ_{d0} (deg)	ψ_{g0} (deg)	$\lvert\psi_{d_0}-\psi_{g_0}\rvert$ (deg)
1	25.000000	135.376221	0.000000	0.000010	0.000010
2	22.500000	135.493334	0.110803	0.117104	0.006300
3	20.000000	135.842621	0.443213	0.466391	0.023177
4	17.500000	136.420662	0.997230	1.044432	0.047202
5	15.000000	137.223680	1.772853	1.847450	0.074597
6	12.500000	138.247589	2.770083	2.871359	0.101276
7	10.000000	139.488180	3.988919	4.111950	0.123030
8	7.500000	140.941299	5.429363	5.565069	0.135707
9	5.000000	142.603146	7.091413	7.226915	0.135503
10	2.500000	144.470566	8.975069	9.094336	0.119267
11	-0.000000	146.541426	11.080332	11.165195	0.084863
12	-2.500000	148.815058	13.407202	13.438828	0.031626
13	-5.000000	151.292820	15.955678	15.916590	0.039088
14	-7.500000	153.978758	18.725761	18.602528	0.123234
15	-10.000000	156.880491	21.717451	21.504261	0.213190
16	-12.500000	160.010536	24.930747	24.634306	0.296441
17	-15.000000	163.388130	28.365650	28.011900	0.353750
18	-17.500000	167.042156	32.022160	31.665926	0.356234
19	-20.000000	171.015995	35.900276	35.639765	0.260511
20	-22.500000	175.376215	39.999999	39.999985	0.000014

i	χ_0 (deg)	S (in.)	S_1+S_d (in.)	$\lvert S_1+S_d-S\rvert$ (in.)
1	92.373151	1.797457	1.801926	0.004468
2	92.085680	1.762873	1.764879	0.002006
3	92.045843	1.727657	1.727832	0.000175
4	92.251410	1.691878	1.690786	0.001092
5	92.700054	1.655603	1.653739	0.001864
6	93.389382	1.618901	1.616692	0.002209
7	94.317107	1.581843	1.579646	0.002197
8	95.481208	1.544498	1.542599	0.001899
9	96.880193	1.506938	1.505552	0.001386
10	98.513394	1.469235	1.468506	0.000729
11	100.381339	1.431459	1.431459	0.000000
12	102.486183	1.393684	1.394413	0.000729
13	104.832316	1.355980	1.357366	0.001386
14	107.427055	1.318420	1.320319	0.001899
15	110.281624	1.281075	1.283273	0.002197
16	113.412588	1.244017	1.246226	0.002209
17	116.843906	1.207315	1.209179	0.001864
18	120.610190	1.171040	1.172133	0.001092
19	124.762126	1.135261	1.135086	0.000175
20	129.376213	1.100046	1.098039	0.002006

results $(RMSE)_\phi$ being 0.05° different than what the first mechanism results in which d_2, a and b are about seven times larger than the dimensions of the second mechanism. As this example demonstrates the designer may have to compromize on the magnitude of RMSE in order to maintain smaller dimensions.

In both mechanisms above ΔS = 0.703886 in. since d_1, λ_0 and θ_{01} remain the same. In order to obtain ΔS = 2.0 in. the dimensions of the mechanism are multiplied by the factor 2.0/0.703886 = 2.840374.

The generated and desired displacements along with χ_0, for the mechanism with d_2 = 3.7602 in. and q_2 = 46° are shown plotted in Figure 21.

The output of the inversion, defined by the positive signed radical in Equation (3.74), is given in Table VI, for $\Delta\phi_0$ = -40°, when d_2 = 3.011968 in., q_2 = 50°, d_1 = 1.00 in., d_3 = 0.42480 in., a = 2.77892 in., b = 2.9248 in., q_1 = 43.003°, ΔS = 0.703886 in. and $(RMSE)_\phi$ = 0.0029395 rad = 0.16849°.

The inversion defined by the negative signed radical in Equation (3.74) was optimized using 6 design points. The output of one mechanism is given in Table VII, when d_2 = 3.7665 in., q_2 = 46°, d_3 = 1.00 in., d_3 = 0.42182 in., a = 3.96072 in., b = 2.86783 in., ΔS = 0.703886 in., and $(RMSE)_\phi$ = 0.0001419 rad = 0.008131°.

The RSRC mechanism with δ_0 = 0 is also optimized to generate the same screw function given in Example 4 using $\Delta\theta_0$ = 40°, $\Delta\phi_0$ = 40°, and ΔS = -0.7 in. by minimizing $E = E_\phi + E_S$ and considering d_1, d_2, d_3, a, b, q_2 and θ_{01} as unknown dimensions. λ_0 = 60° is considered a design constant. Ten design points are used. The optimum mechanism approaches an unacceptable size. The output of one of the small size mechanisms

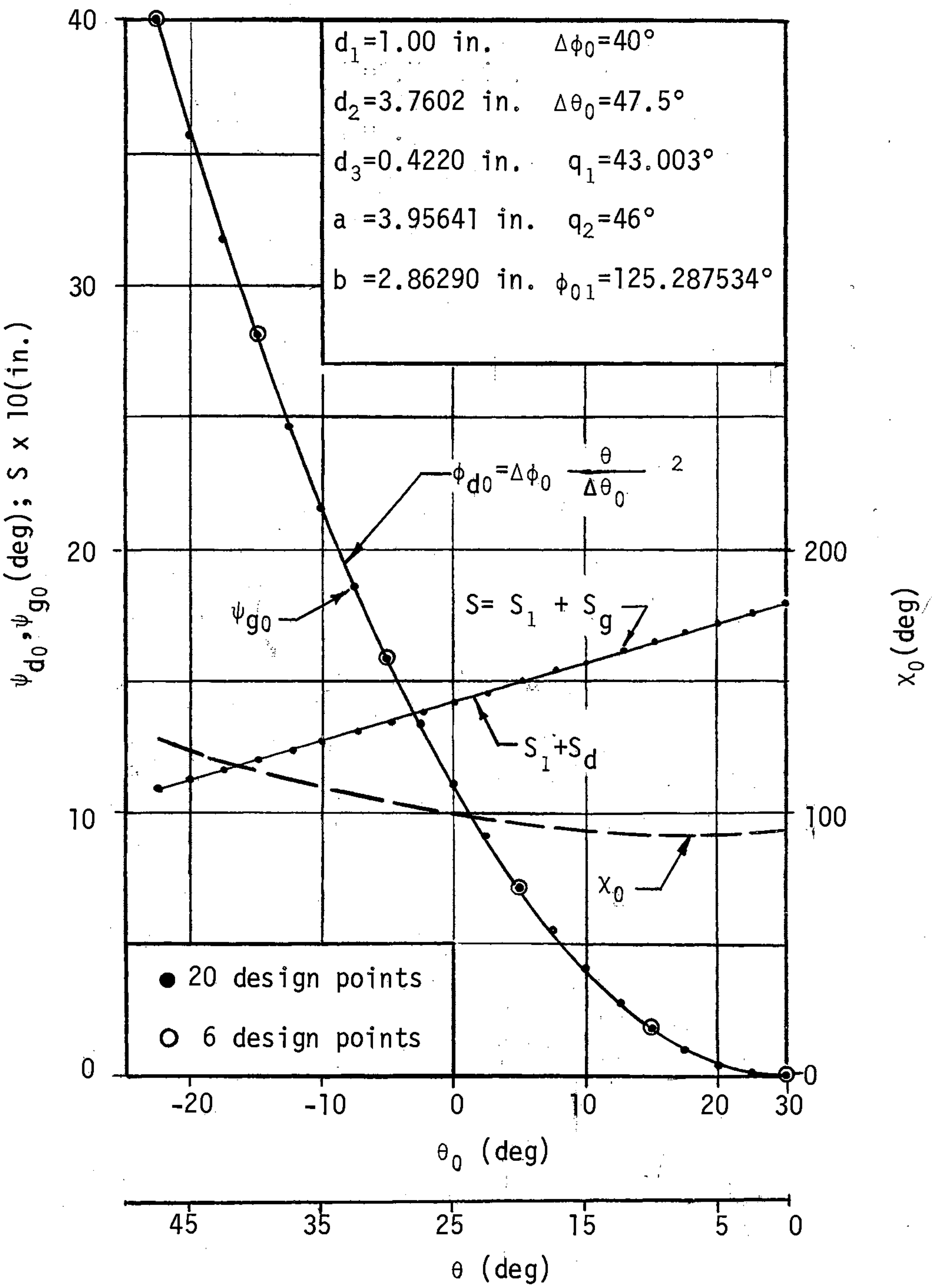


Figure 21. Desired and Generated Displacements of the Geometric Inversion with Negative Signed Radical in Example 4. q_2 = 46°, d_2 =3.7602 in. 20 Design Points

TABLE VI

THE OUTPUT OF PROGRAM C FOR THE GEOMETRIC INVERSION WITH POSITIVE SIGNED RADICAL IN EXAMPLE 4. d_2 = 3.011968 in.

i	θ_0 (deg)	ϕg (deg)	ψd_0 (deg)	ψg_0 (deg)	$\lvert \psi d_0 - \psi g_0 \rvert$ (deg)
1	25.000000	-39.005967	0.000000	0.000011	0.000011
2	22.500000	-39.126394	0.110803	0.120416	0.009613
3	20.000000	-39.484516	0.443213	0.478539	0.035325
4	17.500000	-40.075136	0.997230	1.069159	0.071929
5	15.000000	-40.892664	1.772853	1.886686	0.113833
6	12.500000	-41.931355	2.770083	2.925377	0.155294
7	10.000000	-43.185620	3.988919	4.179643	0.190723
8	7.500000	-44.650286	5.429363	5.644309	0.214946
9	5.000000	-46.320910	7.091413	7.314932	0.223520
10	2.500000	-48.194057	8.975069	9.188079	0.213010
11	-0.000000	-50.267625	11.080332	11.261648	0.181316
12	-2.500000	-52.541230	13.407202	13.535252	0.128050
13	-5.000000	-55.016610	15.955678	16.010633	0.054955
14	-7.500000	-57.698237	18.725761	18.692260	0.033501
15	-10.000000	-60.594017	21.717451	21.588039	0.129412
16	-12.500000	-63.716424	24.930747	24.710446	0.220300
17	-15.000000	-67.084121	28.365650	28.078143	0.287507
18	-17.500000	-70.724548	32.022160	31.718571	0.303589
19	-20.000000	-74.678090	35.900276	35.672112	0.228163
20	-22.500000	-79.005965	39.999999	39.999988	0.000011

i	χ_0 (deg)	S (in.)	S_1+S_d (in.)	$\lvert S_1+S_d-S \rvert$ (in.)
1	82.009036	1.828390	1.832859	0.004468
2	82.632886	1.793806	1.795812	0.002006
3	83.472299	1.758590	1.758765	0.000175
4	84.521888	1.722811	1.721719	0.001092
5	85.776280	1.686536	1.684672	0.001864
6	87.230319	1.649835	1.647626	0.002209
7	88.879354	1.612776	1.610579	0.002197
8	90.719460	1.575431	1.573532	0.001899
9	92.747746	1.537871	1.536486	0.001386
10	94.962619	1.500168	1.499439	0.000729
11	97.364115	1.462392	1.462392	0.000000
12	99.954289	1.424617	1.425346	0.000729
13	102.737679	1.386913	1.388299	0.001386
14	105.721955	1.349353	1.351252	0.001899
15	108.918736	1.312009	1.314206	0.002197
16	112.344876	1.274950	1.277159	0.002209
17	116.024318	1.238248	1.240112	0.001864
18	119.991070	1.201973	1.203066	0.001092
19	124.294019	1.166194	1.166019	0.000175
20	129.005960	1.130979	1.128973	0.002006

TABLE VII

THE OUTPUT OF PROGRAM C FOR THE GEOMETRIC INVERSION WITH NEGATIVE SIGNED RADICAL IN EXAMPLE 4. d_2 = 3.7665

i	θ_0 (deg)	ϕ_g (deg)	ψ_{d_0} (deg)	ψ_{g_0} (deg)	$\lvert\psi_{d_0}-\psi_{g_0}\rvert$ (deg)
1	25.000000	135.313141	0.000000	0.000013	0.000013
2	15.000000	137.161003	1.772900	1.847849	0.074949
3	5.000000	142.541656	7.091400	7.228502	0.137102
4	-5.000000	151.232758	15.955700	15.919603	0.036097
5	-15.000000	163.328279	28.365700	28.015125	0.350575
6	-22.500000	175.313133	40.000000	39.999979	0.000021

i	χ_0 (deg)	S (in.)	S_1+S_d (in.)	$\lvert S_1+S_d-S\rvert$ (in.)
1	92.310071	1.799898	1.804366	0.004468
2	92.639771	1.658044	1.656180	0.001864
3	96.822508	1.509379	1.507993	0.001386
4	104.776271	1.358421	1.359806	0.001386
5	116.786704	1.209756	1.211620	0.001864
6	129.313129	1.102486	1.100480	0.002006

is given in Table VIII, when $d_2 = 3.66660$ in., $d_1 = 1.05792$ in., $d_3 = 0.402608$ in., $a = 3.944825$ in., $b = 2.762847$ in., $\theta_{01} = 24.49992°$, $q_1 = 42.3475776°$, $q_2 = 45.83350182°$, $\phi_{01} = 132.553783°$, $S_1 = 1.761359$ in., $\Delta S = -0.700791$ in., $(RMSE)_\phi = 0.0023793$ rad $= 0.13615°$, $(RMSE)_S = 0.0029025$ in., and RMSE = 0.003616 in. for a screw of unit radius.

Example 5: Now consider the design of an RSRC quick-return mechanism having 2 to 1 advance-to-return-time-ratio generating the displacement specified below. The translation component is

$$S_d = C_1 \sin\theta_0$$

the rotation is a linear function of θ_0 for $30° \leq \theta_0 \leq 210°$ during the advance stroke (C.C.W.) and for $270° \leq \theta_0 \leq 330°$ during the return stroke (C.W.). It is an exponential function of θ in the rest of the domain as shown by the solid lines for ψ_{d_0} in Figure 22. The input for the desired screw displacement for Program C was prepared as data at 24 design points rather than defining the functional forms in the SUBROUTINE DSIRED. The design points are taken with 15° increments in the input crank rotation. The values of the desired rotation displacement ψ_{d_0} at 24 design points are given in the fourth column in Table IX. The additional constraining condition in the problem is that the limit position of the rotation is to correspond to the value of $\theta = 0$ at which the limit position for the maximum value of the translation [first order instantaneous dwell in translation] takes place. $\lambda_0 = 60°$ is a design constant.

The desired translation component suggests that the RSRC mechanism with $\delta_0 = 0$ is the best choice, which generates the translation displacement exactly. In this mechanism the upper limit of translation

TABLE VIII

AN RSRC MECHANISM OPTIMIZED TO GENERATE THE SCREW DISPLACEMENT IN EXAMPLE 4, HAVING θ_{01} AS ONE OF THE UNKNOWN DIMENSIONS

i	θ_0 (deg)	ϕ_g (deg)	ψ_{g_0} (deg)	ψ_{d_0} (deg)	$\psi_{d_0}-\psi_{g_0}$ (deg)
1	24.499920	132.553783	-0.000000	0.000000	0.000000
2	19.499920	133.077570	0.523787	0.493827	-0.029961
3	14.499920	134.627676	2.073893	1.975309	-0.098585
4	9.499920	137.166204	4.612421	4.444444	-0.167978
5	4.499920	140.652363	8.098579	7.901235	-0.197346
6	-0.500080	145.052898	12.499115	12.345679	-0.153437
7	-5.500080	150.357382	17.803598	17.777777	-0.025822
8	-10.500080	156.600681	24.046898	24.197531	0.150633
9	-15.500080	163.902224	31.348440	31.604938	0.256496
10	-20.500080	172.553782	39.999998	40.000000	0.000002

i	S (in.)	S_g (in.)	S_d (in.)	χ_0 (deg)
1	1.761359	-0.000000	0.000000	90.206206
2	1.687252	-0.074107	-0.077778	89.872369
3	1.610817	-0.150541	-0.155556	90.633061
4	1.532637	-0.228772	-0.233333	92.465133
5	1.453306	-0.308053	-0.311111	95.347013
6	1.373427	-0.387931	-0.388889	99.268359
7	1.293610	-0.467749	-0.466667	104.245034
8	1.214461	-0.546898	-0.544444	110.342483
9	1.136583	-0.624776	-0.622222	117.718403
10	1.060568	-0.700791	-0.700000	126.720280

d_1 = 2.00 in.
d_2 = 9.886017 in.
d_3 = 10.219801
a = 3.835109 in.
b = 1.631712 in.
δ_0 = 0°
λ_0 = 60°
$\Delta\phi_0$ = 35°
$\Delta\phi_0$ = 360°
ΔS = 3.464101
q_1 = θ_{Q1} = 90°
q_2 = 130.893°

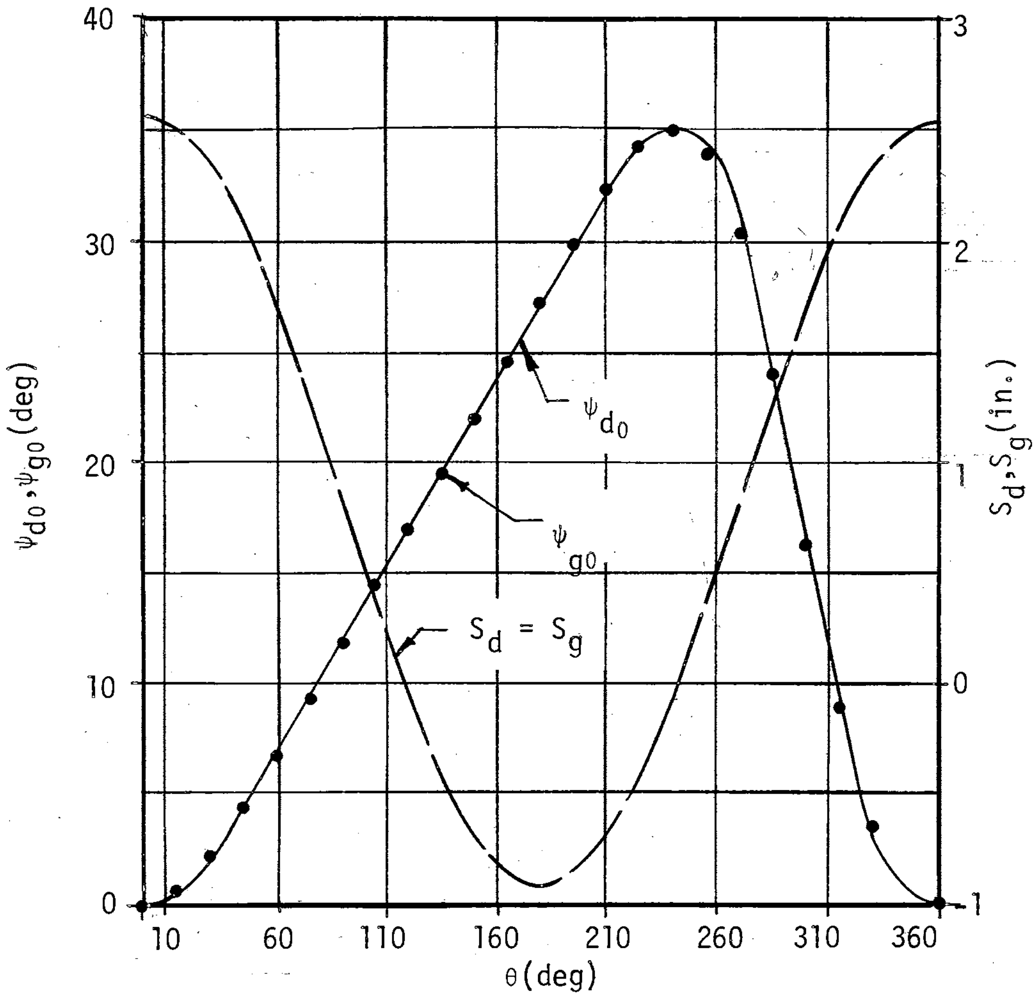


Figure 22. Desired and Generated Displacements of the RSRC-Quick-Return Mechanism Optimized in Example 5

occurs when $\theta_0 = 90°$. Since one of the limit positions of the output rotation must occur when the upper limit of the translation occurs, $\theta_{01} = \theta_0 = 90°$ is where the coupler ellipse must be tangent to the input crank ellipse. Then from Equation (5.13) it follows that

$$q_1 = 90°$$

Since the advance-to-return-time-ratio is 2/1, the value of θ_{02} corresponding to the second limit position of the output crank rotation is given by

$$\theta_{02} = \frac{2}{3}(360) + \theta_{01} = 330°$$

then from Equation (5.13) we have

$$q_2 = 130.8931°$$

d_2 is the only unknown dimension left to be determined by minimizing E_ϕ, since the dimensions a, b and d_3 are determined by Equations (5.19), (5.20), and (5.21) after determining d_2. Let $d_1 = 2.0$ in.

Thus, the optimum value of d_2, for which E_ϕ is minimum, is $d_2 =$ 9.886017 in. Then, a = 3.835109 in., b = 1.631712 in. and $d_3 =$ 10.219801 in., $(RMSE)_s = 0$ and $(RMSE)_\phi = 0.255711°$. The maximum deviation of generated rotation from the desired one at the design points is 1.093482°. The displacements in this mechanism are given in Table IX, and plotted in Figure 22.

It is to be noted that smaller dimensions could be obtained by fewer design points. Another RSRC quick-return mechanism is optimized using 6 design points at $\theta = 0°, 75°, 165°, 210°, 240°$ and $255°$. The optimum dimensions of this mechanism are $d_2 = 8.416473$ in.,

TABLE IX

THE OUTPUT OF PROGRAM C FOR THE RSRC-QUICK-RETURN MECHANISM OPTIMIZED IN EXAMPLE 5. 24 DESIGN POINTS

i	θ_0 (deg)	ϕ_g (deg)	ψ_{d_0} (deg)	S (in.)	χ_0 (deg)	ψ_{g_0} (deg)	$\lvert\psi_{d_0}-\psi_{g_0}\rvert$ (deg)
1	0.000000	142.3735395	31.000000	0.815856	13.093168	30.334782	0.665218
2	15.000000	136.092590	24.000000	1.264144	12.570049	24.051977	0.051977
3	30.000000	128.265175	17.000000	1.681882	12.957454	16.224562	0.775438
4	45.000000	120.947131	10.000000	2.040601	14.284992	8.906518	1.093482
5	60.000000	115.693986	3.000000	2.315856	16.404535	3.653373	0.653373
6	75.000000	112.857127	0.700000	2.488889	19.074224	0.816514	0.116514
7	90.000000	112.040612	0.000000	2.547907	22.040613	0.000001	0.000001
8	105.000000	112.670526	0.500000	2.488889	25.069034	0.629912	0.129912
9	120.000000	114.246048	2.000000	2.315856	27.947970	2.205435	0.205435
10	135.000000	116.390727	4.500000	2.040601	30.490191	4.350114	0.149886
11	150.000000	118.838938	7.000000	1.681882	32.537152	6.798325	0.201675
12	165.000000	121.412871	9.500000	1.264144	33.966106	9.372258	0.127742
13	180.000000	124.004066	12.000000	0.815856	34.697705	11.963452	0.036548
14	195.000000	126.559879	14.500000	0.367569	34.701591	14.519266	0.019266
15	210.000000	129.072105	17.000000	-0.050169	33.998079	17.031492	0.031492
16	225.000000	131.564783	19.500000	-0.408888	32.654871	19.524170	0.024170
17	240.000000	134.078751	22.000000	-0.684144	30.778931	22.038137	0.038137
18	225.000000	136.650759	24.500000	-0.857176	28.504699	24.610146	0.110146
19	270.000000	139.284864	27.000000	-0.916194	25.980267	27.244251	0.244251
20	285.000000	141.914398	29.500000	-0.857176	23.353583	29.873785	0.373785
21	300.000000	144.354401	32.000000	-0.684144	20.761158	32.313787	0.313787
22	315.000000	146.250225	34.000000	-0.408889	18.322808	34.209612	0.209612
23	330.000000	147.040613	35.000000	-0.050169	16.147225	35.000000	0.000000
24	345.000000	145.985077	34.500000	0.367569	14.352112	33.944464	0.555536

$d_1 = 2.0$ in., $a = 3.747413$ in., $b = 1.903521$ in., $d_3 = 8.624605$ in., $q_1 = 90°$, $q_2 = 130.8931°$. The maximum deviation of the generated rotation from the desired one at the precision points is 0.091816°. $(RMSE)_S = 0$ and $(RMSE)_\phi = 0.04789°$. The displacements in this mechanism are given in Table X.

In the examples given above the screw displacements have first order instantaneous dwells in rotation, and the instantaneous dwells occur when the exteriors of the input-crank-ellipse and the coupler-ellipse are tangent, that is the curvatures of the input-crank-ellipse and the coupler-ellipse are of opposite signs. When the desired rotation displacement is a function of θ^n with instantaneous dwells, where $n \geq 3$ the solution will exist where the curvature of the input-crank-ellipse and the coupler-ellipse conform, that is when the interior of one ellipse is tangent to the exterior of the other ellipse. As it is discussed in the early part of this chapter, the designer may consider the mechanisms given in Figures 7, 8, 9 and 10 as the starting mechanisms when synthesizing the RSRC mechanism for the generation of screw displacements having higher order instantaneous dwells in rotation. The following examples illustrate the use of the parameters of constraints in synthesizing the RSRC mechanism for the generation of screw displacements having even order instantaneous dwells in rotation.

<u>Example 6</u>: Design an RSRC mechanism to generate the screw displacement

$$\hat{Y} = \frac{x^3}{4} + \varepsilon S_g \qquad -4 \leq x \leq 4$$

where S_g stands for that the generated output translation is accepted as it is. $\delta_0 = 0$, $e = 0$ and $\lambda_0 = 60°$ are specified design constants.

TABLE X

THE OUTPUT OF PROGRAM C FOR THE RSRC - QUICK - RETURN MECHANISM OPTIMIZED IN EXAMPLE 5. 6 DESIGN POINTS.

i	θ_0 (deg)	ϕ_g (deg)	ψ_{d0} (deg)	S (in.)	χ_0 (deg)	χ_{g0} (deg)	$\lvert\psi_{d0}-\psi_{g0}\rvert$ (deg)
1	15.000000	140.287519	24.500000	1.400050	15.357441	24.534338	0.034338
2	90.000000	115.753181	0.000000	2.683813	25.753181	0.000000	0.000000
3	165.000000	125.720345	10.000000	1.400050	40.108031	9.967164	0.032836
4	255.000000	140.844997	25.000000	-0.721270	34.114264	25.091816	0.091816
5	300.000000	148.197788	32.500000	-0.548237	25.209724	32.444607	0.055393
6	330.000000	150.753181	35.000000	0.085737	19.859791	35.000000	0.000000

The function is to be generated within $\Delta\theta_0 = \Delta\phi_0 = 120°$.

Let the design points be at x = -4, -3, -2, -1, 0, 1, 2, 3, 4, where the real part of the desired function assumes Y_0 = -16, -6.75, -2, -0.25, 0, 0.25, 2, 6.75, 16, respectively. The corresponding values of the input and output crank rotations are θ' = -60°, -45°, -30°, -15°, 0°, 15°, 30°, 45°, 60° and ψ'_{d_0} = -60°, -25.29°, -7.5°, -0.93779°, 0°, 0.93779°, 7.5°, 25.29°, 60°, respectively. However these rotations are put in a consecutive order by introducing $\theta = \theta' + 60$ and $\psi_{d_0} = \psi'_{d_0} + 60$. Thus, the input parameter and the desired screw displacement at the design points are θ = 0°, 15°, 30°, 45°, 60°, 75°, 90°, 105°, 120° and ψ_{d_0} = 0°, 34.71°, 54.5°, 59.06219°, 60°, 60.93779°, 67.5°, 85.29°, 120°, respectively.

This is a screw displacement having a second order instantaneous dwell in the rotation component at x = 0, or $\theta = 60°$. The necessary condition for the second order instantaneous dwell must be satisfied at $\theta = 60°$, where the desired rotation has an inflection point. An RSRC mechanism whose output rotation has inflection point may be used as the starting mechanism in Program C. Dimensions and displacements of such a mechanism are given in Figure 10. One may find RSRC mechanisms with inflection points in the output rotation in four zones of the input crank rotation. These zones are $10° < \theta_0 < 60°$, $120° < \theta_0 < 170°$, $190° < \theta_0 < 240°$ and $330° < \theta_0 < 350°$. An RSRC mechanism was optimized starting with the dimensions of the RSRC mechanism in Figure 10. Two parameters of constraints were introduced at $\theta = 0°$ and $\theta = 60°$. Since the constraint for the instantaneous dwell is at $\theta = 60°$ the parameter of constraint q_2 at $\theta = 60°$ is defined by Equation (5.13). q_1 at $\theta = 0°$ was one of the unknown parameters. Noting that $\theta = 60°$ must correspond to $\theta_0 \approx 220°$ in the starting mechanism, the starting value for θ_{01} was taken 160° in the input data. $q_1 = 5°$ was assumed as the starting value. The desired screw displacement was read as data. One of the parameters of constraints is defined within the domain of output rotation. Then the data for the Program C must have $\Delta\theta_0 = 60°$ and $\Delta\phi_0 = 60°$ which are defined by $\theta = 0°$ and 60°. The remaining dimensions of the optimum mechanism are $d_1 = 3.457402$ in., $d_2 = 2.280365$ in., $d_3 = 0.86211$ in., $a = -0.734927$ in., $b = 0.543187$ in., $\theta_{01} = 150.52356°$, $q_1 = 12.16845°$, $q_2 = 49.70775°$, and $(RMSE)_\phi = 0.011091$ rad $= 0.63513°$. The desired and the generated displacements are given in Table XI and plotted in Figure 23. The generated output rotation velocity, $\dot{\phi}_0/\dot{\theta}_0$, and the rotation acceleration, $\ddot{\phi}_0/\dot{\theta}_0^2$, at the precision points, when $\dot{\theta}_0 = 1$ rad/sec, are also shown in Figure 23 to clarify the properties of the generated instantaneous dwell.

Example 7: Design a double-crank-RSRC mechanism whose output link is to rotate as a linear function of the input parameter during the 50% of the cycle, and must have an approximate dwell during the 10% of the cycle, as shown by the straight lines for ψ_{d_0} in Figure 24.

Note that such an instantaneous dwell in the output rotation of a double-crank-4R plane mechanism can not be obtained since the inflection point in the output rotation occurs at the dead center position.

A double-crank-RSRC mechanism with $\delta_0 = 0$, $e = 0$ and $\lambda_0 = 60°$ was optimized using 17 design points, 4 at the linear portion, 13 at the dwell portion. The mechanism in Figure 10 was used as the starting mechanism, with $\theta_{01} = 220°$, $q_2 = 230°$ at $\theta_{01} = 150°$. q_1 at $\theta = 0$ is defined by Equation (5.13). The dimensions of the optimized mechanism are $d_1 = 1.488489$ in., $d_2 = 2.13769$ in., $a = 0.737225$ in.,

TABLE XI

THE OUTPUT OF PROGRAM C FOR THE RSRC MECHANISM OPTIMIZED IN EXAMPLE 6. 8 DESIGN POINTS

i	θ_0 (deg)	ϕ_g (deg)	ψ_{g_0} (deg)	ψ_{d_0} (deg)	$\psi_{d_0}-\psi_{g_0}$ (deg)
1	150.52356	93.04567	0.0	0.0	0.0
2	180.52356	145.20401	52.15823	52.50000	0.34177
3	195.52356	151.31146	58.26579	59.06219	0.79640
4	210.52356	153.04556	59.99989	60.00000	0.00011
5	225.52356	153.51276	60.46709	60.93779	0.47070
6	240.52356	159.66833	66.62267	67.50000	0.87733
7	255.52356	178.98608	85.94041	85.28999	-0.65042
8	270.52344	213.14404	120.09837	120.00000	-0.09837

i	χ_0 (deg)	S_1+S_g (in.)	$S_d=S_g$ (in.)	S_1+S_d (in.)	S_d-S_g (in.)
1	80.87721	1.74494	0.0	1.74494	0.0
2	115.68513	0.24424	-1.50070	0.24424	0.0
3	115.10745	-0.52975	-2.27468	-0.52975	0.0
4	103.34473	-1.24913	-2.99407	-1.24913	0.0
5	87.19028	-1.86487	-3.60981	-1.86487	0.0
6	73.64011	-2.33502	-4.07996	-2.33502	0.0
7	70.24660	-2.62753	-4.37247	-2.62753	0.0
8	82.40030	-2.72247	-4.46741	-2.72247	0.0

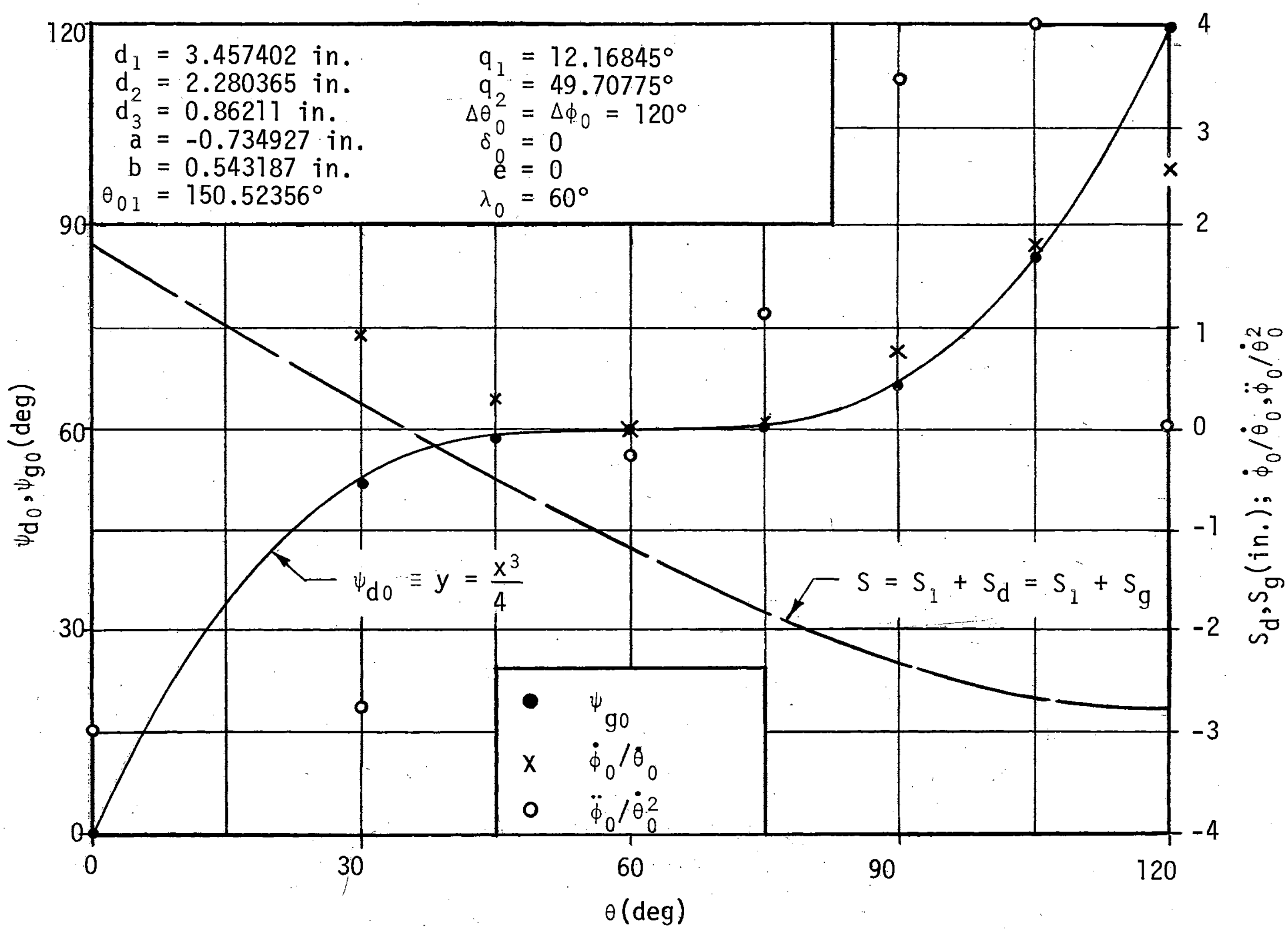


Figure 23. Desired and Generated Output Displacements, Output Rotation Velocity, Output Rotation Acceleration of the RSRC Mechanism Optimized in Example 6

$b = -0.148267$ in., $\theta_{01} = 215,35207°$, $q_1 = 54.823044°$, $q_2 = 233.961044°$, and $(\text{RMSE})_\phi = 0.03553074$ rad $= 2.03576183°$.

The desired and the generated output displacements are given in Table XII and plotted in Figure 24. The output rotation velocity, $\dot{\phi}_0/\dot{\theta}_0$, and acceleration, $\ddot{\phi}/\dot{\theta}_0^2$ at the design points in the dwell zone, when $\dot{\theta}_0 = 1$ rad/sec, are $\dot{\phi}_0/\dot{\theta}_0 = 0.11401, 0.07253, 0.03443, 0.0,$ $-0.03087, -0.05813, -0.08187$, $\ddot{\phi}_0/\dot{\theta}_0^2 = -0.49422, -0.45595, -0.41574,$ $-0.37447, -0.33296, -0.29199, -0.25226$. The corresponding values of θ are 345°, 350°, 355°, 0°, 5°, 10°, and 15°.

Synthesis of the RSRC Mechanism by the Overlay Technique

Overlay technique used in synthesizing plane mechanisms for function generation is a graphical way of optimizing the mechanism by minimizing the error by sight, where the efficiency of the approximation depends largely on the accuracy of the drawing.

One can use the overlay technique to synthesize an RSRC mechanism by drawing a series of ellipses and more than one overlay. This may not be a desirable method of solution since it will consume a lot of time. However, the overlay technique is relatively simple when $\delta_0 = 0$. Then it requires drawing only the input-crank-ellipse, a series of coupler-circles, and an overlay for the output crank rotation. This is illustrated in two examples in the following.

<u>Example 8</u>: Consider the design of a constrained RSRC mechanism having $\delta_0 = 0$ to generate the screw displacement

$$\hat{\psi} = \Delta\phi_0 \left(\frac{\theta}{\Delta\theta_0}\right) + \varepsilon\Delta S \left(\frac{\theta}{\Delta\theta_0}\right)$$

TABLE XII

THE OUTPUT OF PROGRAM C FOR THE DOUBLE-CRANK-RSRC MECHANISM OPTIMIZED IN EXAMPLE 7

i	θ_0 (deg)	ϕ_g (deg)	ψ_{g0} (deg)	ψ_{d0} (deg)	$\psi_{d0}-\psi_{g0}$ (deg)
1	215.35207	116.47552	0.0	0.0	0.0
2	220.35207	116.39694	-0.07858	0.0	0.07858
3	225.35207	116.17291	-0.30261	0.0	0.30261
4	230.35207	115.82146	-0.65407	0.0	0.65407
5	235.35207	115.35995	-1.11557	0.0	1.11557
6	240.35207	114.80469	-1.67084	0.0	1.67084
7	245.35207	114.17113	-2.30440	0.0	2.30440
8	335.35205	205.07916	88.60364	90.00000	1.39636
9	5.35205	251.47572	135.00020	135.00000	-0.00020
10	65.35205	338.50366	222.02814	225.00000	2.97186
11	125.35205	76.79199	320.31641	315.00000	-5.31641
12	185.35205	112.91147	356.43579	360.00000	3.56421
13	190.35205	114.06790	357.59229	360.00000	2.40771
14	195.35205	114.97932	358.50366	360.00000	1.49634
15	200.35205	115.65984	359.18408	360.00000	0.81592
16	205.35205	116.12483	359.64917	360.00000	0.35083
17	210.35205	116.39082	359.91528	360.00000	0.08472
18	215.35207	116.47552	0.0	0.0	0.0

i	χ_0 (deg)	S_1+S_g (in.)	$S_d=S_g$ (in.)	S_1+S_d (in.)	S_d-S_g (in.)
1	61.65247	-0.81999	0.0	-0.81999	0.0
2	59.06410	-0.90878	-0.08880	-0.90878	0.0
3	56.24515	-0.99123	-0.17124	-0.99123	0.0
4	53.22261	-1.06669	-0.24670	-1.06669	0.0
5	50.02332	-1.13460	-0.31461	-1.13460	0.0
6	46.67352	-1.19444	-0.37445	-1.19444	0.0
7	43.19914	-1.24575	-0.42576	-1.24575	0.0
8	11.48880	-0.61173	0.20825	-0.61173	0.0
9	17.51471	0.04610	0.86609	0.04610	0.0
10	19.28972	1.09748	1.91747	1.09748	0.0
11	53.76271	0.97725	1.79723	0.97725	0.0
12	70.99617	-0.19437	0.62562	-0.19437	0.0
13	70.27577	-0.30577	0.51421	-0.30577	0.0
14	69.19917	-0.41541	0.40457	-0.41541	0.0
15	67.77969	-0.52245	0.29753	-0.52245	0.0
16	66.03461	-0.62608	0.19390	-0.62608	0.0
17	63.98434	-0.72551	0.09447	-0.72551	0.0
18	61.65247	-0.81999	0.0	-0.81999	0.0

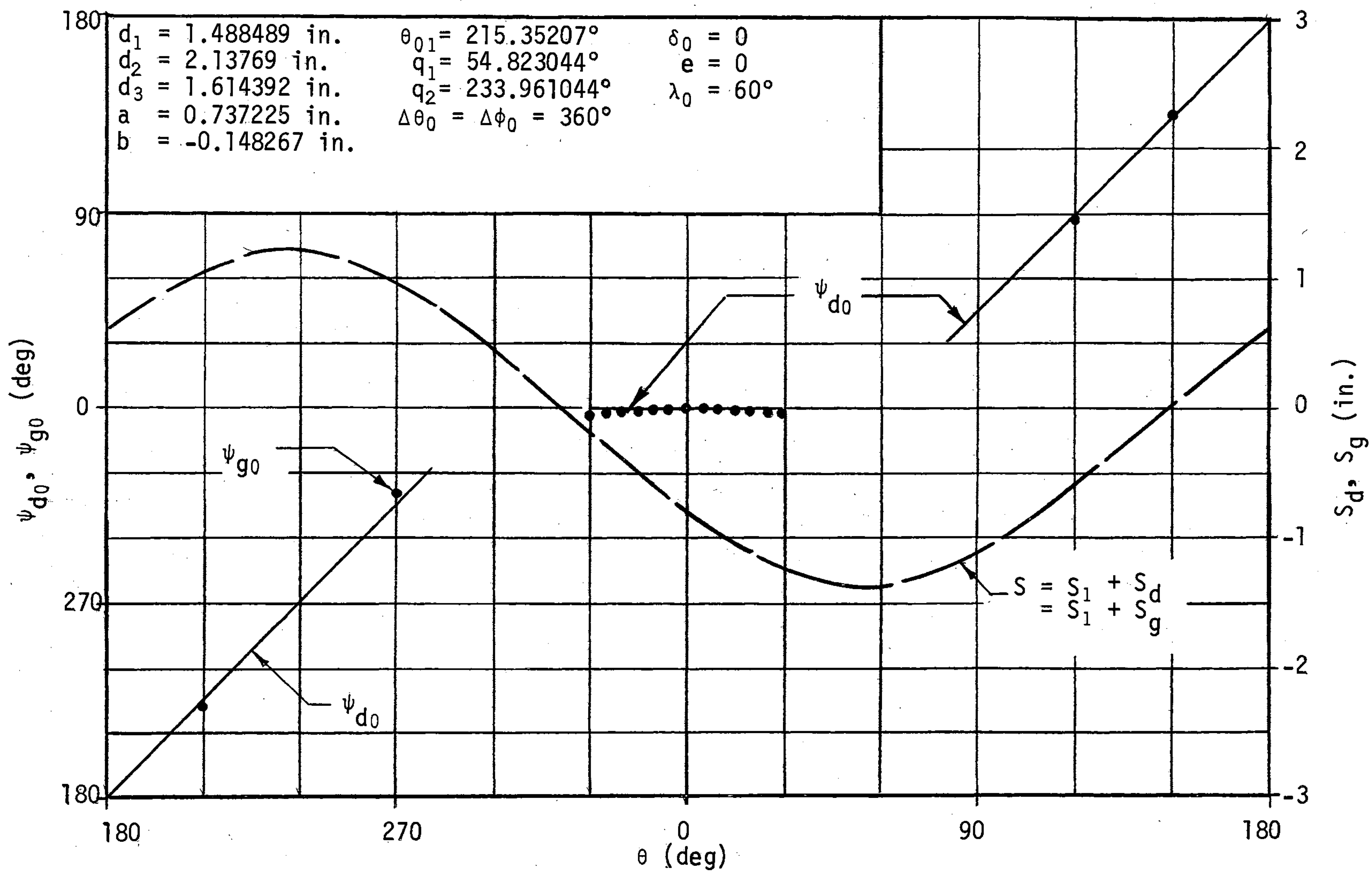


Figure 24. Desired and Generated Output Displacements of the Double-Crank-RSRC Mechanism Optimized in Example 7.

about an axis which forms the skew angle $\lambda_0 = 60°$ with the axis of the input shaft, where $\Delta\phi_0 \pm 45°$, $\Delta\theta_0 = 90°$ and $\Delta S = -10$ in. This axial screw is a helical displacement of pitch $\Delta S/\Delta\phi_0 = 0.22222$ in./deg. Then the designed mechanism is to replace a space mechanism with a helical output pair of the same pitch. Note that if a helical pair having a 30° pressure angle were used to generate the desired output, it would require a pitch diameter of 44.1 in.

Steps in the overlay solution:

1. Since the output translation will approximate a straight line when $-45° < \theta_{01} < 45°$, compute d_1 using the displacements of the mechanism designed in Example 5, where λ_0 was also 60°. Thus, $\Delta S = 2.44949$ in. for $d_1 = 2$ in. in the domain defined. Then, $d_1 = 10/1.224745 = 8.162$ in. The remaining dimensions, d_2, d_3, a and b are determined to approximate the output rotation.

2. The overlay solution is constructed on the plane normal to the output pair axis. Construct the input-crank-ellipse, considering the input crank of unit length. See Figure 25.

3. Define the minimum number of precision points on the rotation component and the corresponding values of θ. Locate them on the input-crank-ellipse. Seven design points were taken with 15° intervals in Figure 25.

4. Dimension d_2 may be chosen arbitrarily. Draw a family of circles of radii d_2, with centers at the design points on the input-crank-ellipse.

5. Construct a transparent overlay for the output rotation. This overlay has a series of radial lines indicating the position of the output-crank at each design point, and a series of circles with centers

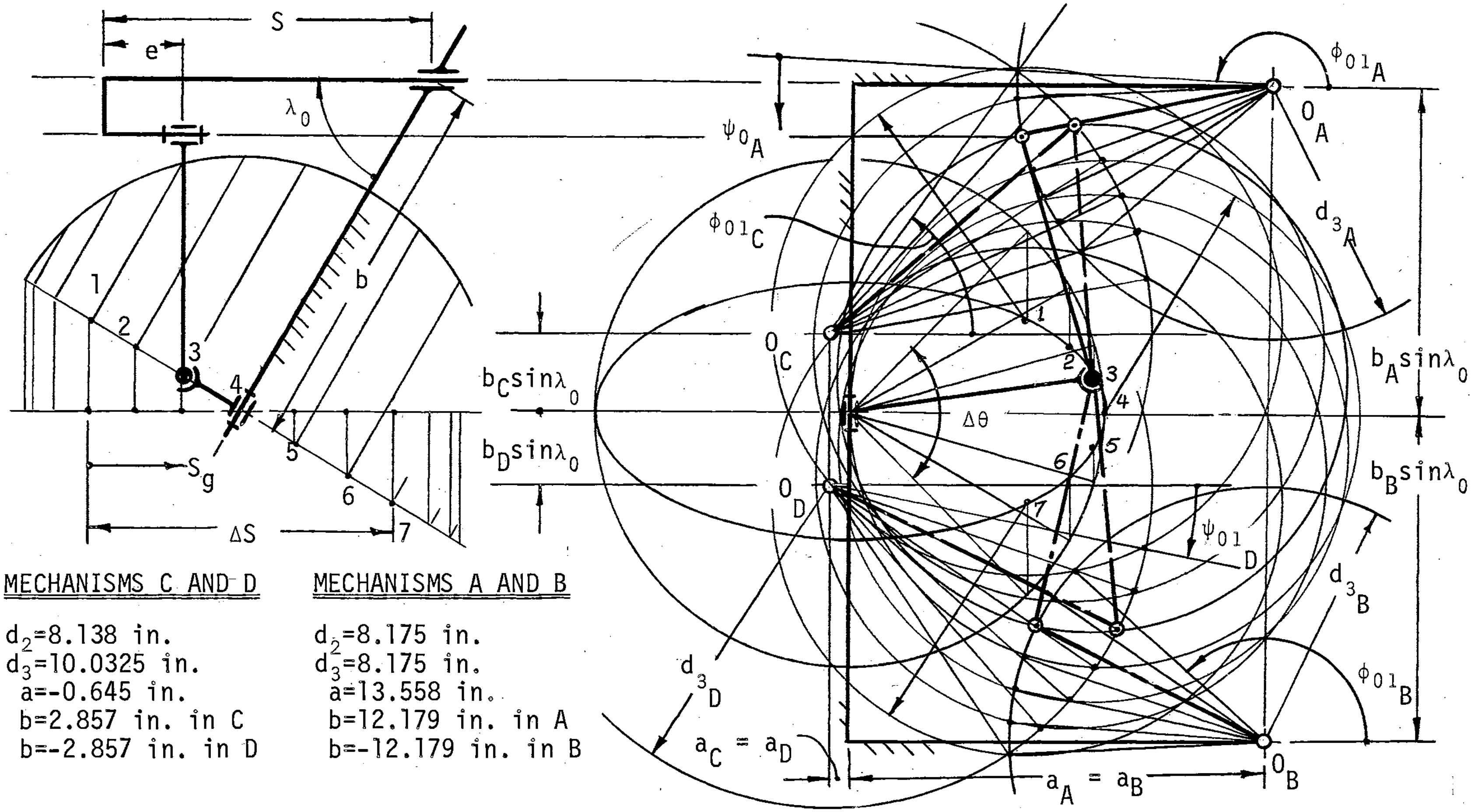


Figure 25. The RSRC Mechanism Optimized by the Overlay Technique in Example 8, for the Generation of the Screw Displacement $\psi_d = \Delta\phi_0(\theta/\Delta\theta_0) + \varepsilon\Delta S(\theta/\Delta\theta_0)$, a helix. $\delta_0 = 0°$, $\lambda_0 = 60°$, $d_1 = 8.162$ in., $\Delta S = -10$ in., $\Delta\theta_0 = 90°$, $\Delta\phi_0 = 45°$

at the center of the radial lines. The intersection of a circle with the radial lines are the design points on the output rotation.

6. Match the design points on one of the circles on the overlay to the family of circles drawn in Step 4.

7. The center of the overlay circles determine the dimensions a and b. The radius of the matching overlay circle determines d_3.

In the overlay solution constructed in Figure 25 $d_2 = d_1$ is used. Four constrained RSRC mechanisms which generate the desired output rotation are shown. Mechanisms A and B are for counterclockwise output rotation, and Mechanisms C and D are for clockwise output rotation. The side view of the Mechanism A shows the output translation at the design points.

Example 9: Approximate dwells in the output rotation can also be generated when using the overlay technique, if the geometric properties of the mechanism at the limit positions are utilized during the process. The translation component is approximated first by the portion of a sine function determining d_1, λ_0 and θ_{01}. In order to generate an instantaneous dwell at a specified value of θ a normal is drawn to the input-crank-ellipse at the point defined by θ. The design point on the overlay for the value of θ at which the instantaneous dwell is to take place, must be placed on that normal during the overlay process. At the point where a higher order instantaneous dwell is to be generated the radius of curvature of the input crank ellipse must be approximated by the coupler link length d_2. A rest in the rotation of the output link for a 120° rotation of the input crank is possible. In order to obtain first order instantaneous dwells at specified values of the input parameter, the normal to the input-crank-ellipse is drawn to maintain a point of tangency of the exteriors of the input-crank-ellipse and the coupler-circle at the specified value of the input parameter. As an example let us design an RSRC mechanism with $\delta_0 = 0$ and $\lambda_0 = 60°$ to generate the same dual function considered in Example 6

$$\hat{y} = \frac{x^3}{4} + \varepsilon S_g , \qquad -4 \leq x \leq 4$$

where S_g stands for that the output translation is accepted as generated by the mechanism. The real part of this function has a second order instantaneous dwell when $x = 0$. Let $\Delta\theta_0 = \Delta\phi_0 = 120°$, and $x = -4, -3, -2, -1, 0, 1, 2, 3, 4$ be the design points. The values of the real part of the dual function at the design points are $y_0 = -16, -6.75, -2, -0.25, 0, 0.25, 2, 6.75, 16$. The corresponding values of the input parameter and the desired output crank rotation are $\theta = -60°, -45°, -30°, -15°, 0°, 15°, 30°, 45°, 60°$ and $\psi_{d_0} = -60°, -25.29°, -7.5°, -0.9378°, 0°, 0.9378°, 7.5°, 25.29°, 60°$. Let us consider that the translation displacement is acceptable if it does not have a limit position within the domain of displacements defined. The value of θ_{01} which corresponds to $x = 0$ or $\theta = 0$ is a very important dimension. In this example $\theta_{01} = 210°$. Since the desired output rotation has an even order instantaneous dwell at θ_{01}, at first the coupler-circle must be drawn tangent to the input-crank-ellipse at θ_{01}, such that the center of the coupler-circle must be rotating about a center in the same direction as the spherical pair moves along the input-crank-ellipse when passing the position at θ_{01}. That is, an inflection point in the output rotation must be determined. Since the value of θ_{01} is defined, $q_1 = 49.06°$ at θ_{01} by Equation (5.13). Let $d_1 = 3.0$ in. $d_2 = 1.96$ in. approximates the radius of curvature of the input-crank-ellipse at $\theta_{01} = 210°$. One may vary d_2 according to the order of the instantaneous dwell at θ_{01}.

Figure 26 shows the overlay construction and an optimized mechanism.

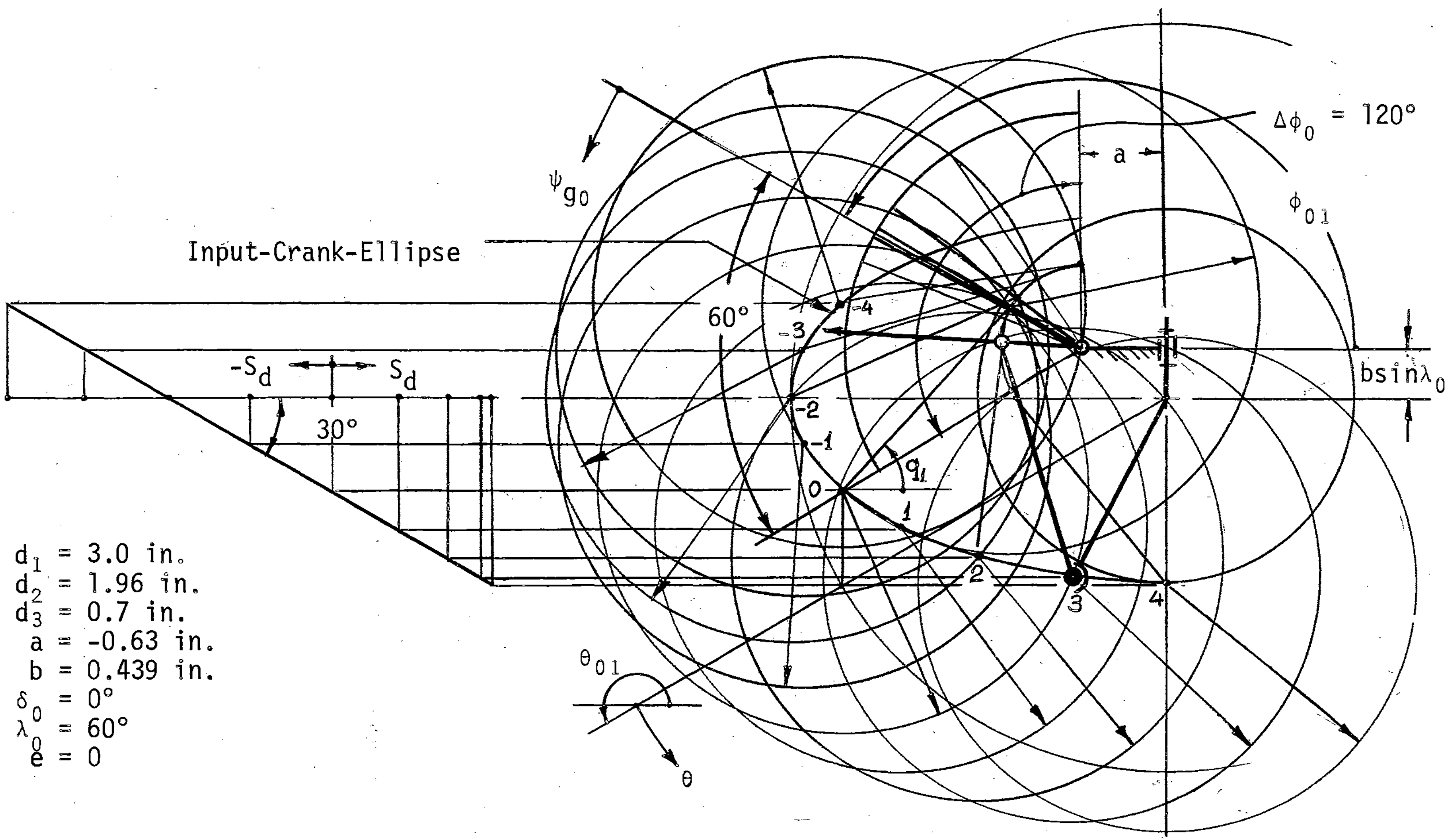


Figure 26. The RSRC Mechanism Optimized by the Overlay Technique to Generate the Dual Function $\hat{y} = x^3/4 + \varepsilon S_g$

CHAPTER VI

SUMMARY AND CONCLUSIONS

This investigation was made to develop a technique using the variational principle to synthesize a space mechanism having cylinder output pair for the generation of screw displacements by the displacements of its output link. The desired screw is of the form $\hat{\psi}_d(\theta) = \psi_{d_0}(\theta) + \varepsilon\, \psi_{d_1}(\theta)$, where $\psi_{d_0}(\theta)$ and $\psi_{d_1}(\theta)$ are the rotation and the linear components, θ is the independent input parameter. Such a screw displacement is to guide a rigid body through successive screws of common axis. The technique is used to synthesize the RSRC space mechanism to generate screw displacements within the specified limits of input and output link displacements. By the variational method the optimum set of dimensions of the mechanism is obtained by minimizing the error function

$$E = \sum_{i=1}^{N} \{[\psi_{d_0}(\theta_i) - \psi_{g_0}(\theta_i)]^2 + [\psi_{d_1}(\theta_i) - \psi_{g_1}(\theta_i)]^2\}$$

for the screw of unit radius, where ψ_{g_0} and ψ_{g_1} are the rotation and linear components of the screw displacement generated by the mechanism, N is the number of design points chosen along the screw function at the N values of the independent parameter θ. The number of design points may be as many as desired. The large number of design points provide approximation closer to the approximation within a continuous interval. However, the large number of design points reduce the efficiency in approximating the desired function at the design points, since a mechanism has a limited number of dimensions. The screw function is generated exactly at the design points if the sum of the number of the design points on the rotation and the linear components is the same or less than the number of the unknown dimensions of the mechanism. In this study all the dimensions of the RSRC mechanism were considered unknown parameters with the exception of the skew angle between the input pair and the output pair axes, which was taken to be a design constant since it defines the screw axis relative to a stationary frame of reference.

The screw displacements with no constraint, and screw displacements with constraints were considered in this investigation. The RSRC mechanism is optimized to generate unconstrained screw displacements where only the displacement is generated approximately at the design points. The nonlinear equations of condition which result in the minimizing process were linearized by Taylor's theorem. The optimum set of dimensions of the mechanism were obtained by solving the linearized equations of condition by the matrix method of iteration and also by solving the nonlinear equations of condition by the relaxation method of Gauss. The relaxation method proved to be an efficient method for solving the equations of condition even when the initial values of the unknown dimensions were too far from those of the optimum mechanism. The convergence was fast, and the method provided information on the effect of each parameter on the convergence and the efficiency in approximating the function at the design points.

The constrained screw displacements are those, which have conditions to be satisfied besides the approximate generation of the displacement. These constraints may be on velocity, acceleration, jerk, force and torque distributions, either in the entire domain of the displacements or at some specified values of the input parameter. The constraining conditions considered in this investigation are the generation of first and higher order instantaneous dwells in the rotation component of the screw displacement at some specified values of the independent parameter within the domain of displacements. The necessary condition for the introduction of any order instantaneous dwell in any component of the screw displacement at a specified value of the independent parameter is that the condition for the limit position for that component of the screw displacement must be satisfied at that value of the input parameter. That is, the first order derivative of the displacement component with respect to the independent parameter must vanish at that value of the independent parameter. The necessary and sufficient conditions for the generation of an n^{th} order instantaneous dwell in any screw component at a specified value of the independent parameter are satisfied when the n consecutive derivatives of that component with respect to the independent parameter vanish at that value of the independent parameter. Generation of exact dwell in any component is impossible, except when a passive freedom is generated. The set of dimensions of a mechanism, which renders a minimum for a function subject to constraints, must satisfy the equations of condition for the displacements and the equations of constraints. In such a case the equations of condition are usually combined with the equations of constraints by the method of Lagrange multipliers. In this investigation the equations of constraints for the necessary conditions for all orders of instantaneous dwells are eliminated from the process by introducing parameters of constraints.

These parameters maintain the geometry in the mechanism that will satisfy the equations of constraints at the specified values of the independent parameter during the minimizing process. These parameters are in general functions of the dimensions of the mechanism. However, they enter the process as unknown parameters, their introduction may exclude some other dimensions from the process. Thus, introducing the parameters of constraints for the generation of instantaneous dwells in the output rotation of the RSRC mechanism three linear dimensions are excluded from the process, and they are computed after the parameters of constraints are determined. The parameters of constraints defined in this study provide that the necessary condition for the generation of any order instantaneous dwell is satisfied exactly. It is shown that higher order instantaneous dwell in any component of the generated screw displacement at a specified value of the input parameter is generated when the radii of curvatures of the input-crank-ellipse and the coupler-ellipse conform, when the entire mechanism is projected onto a plane in which that component of the screw displacement is seen normal. Since the generation of exact dwell in a displacement within a specified interval means that exact generation of the displacement at infinite number of design points, and it is not possible, inclusion of the equations of constraints for a higher order instantaneous dwell is not necessary if the necessary condition is satisfied along with the minimum of the error function for the displacement. So the sufficient conditions for the higher order instantaneous dwells may be left to be satisfied, or approximated, by the propensity of the variational method, rather than introducing time consuming and error bearing operations.

The RSRC mechanism is synthesized to generate the screw displacements

$$\hat{\psi}_d = (0) + \varepsilon\, \Delta S \left[\left(1 - \frac{2\theta}{\Delta\theta_0}\right)^2 - 1 \right] \qquad 0 \leq \theta \leq 80°$$

and

$$\hat{\psi}_d = \Delta\phi_0 \left(\frac{\theta}{\Delta\theta_0}\right)^2 + \varepsilon\Delta S \left[1 - \left(1 - \frac{\theta}{\Delta\theta_0}\right)^2 \right] \quad 0 \leq \theta \leq 80°$$

with no constraints, and the screw displacement

$$\hat{\psi}_d = \Delta\phi_0 \left(\frac{\theta}{\Delta\theta_0}\right)^2 + \varepsilon\Delta S \left(\frac{\theta}{\Delta\theta_0}\right) \qquad 0 \leq \theta \leq 47.5°$$

with a first order instantaneous dwell in rotation at $\theta = 0$, and the dual function

$$\hat{y} = \frac{x^3}{4} + \varepsilon\, S_g \qquad -4 \leq x \leq 4$$

with $\Delta\theta_0 = \Delta\phi_0 = 120°$, where S_g stands for that the linear displacement is accepted as generated by the mechanism. This dual displacement has a second order instantaneous dwell in the real part when $x = 0$. A double-crank-RSRC mechanism is designed to have linear output rotation with a higher even order instantaneous dwell. It should be noted here that an even order instantaneous dwell can never be generated by a single loop 4R plane mechanism and multi-loop plane mechanisms in which the output link of one loop is the driver of the next loop.

An RSRC-quick-return mechanism is designed to illustrate the use of the parameters of constraints in obtaining more than one instantaneous dwells in rotation, and generating exact return-to-advance-time-ratio.

The technique and the computer programs are directly applicable to synthesize 4R plane mechanism for the generation of screw displacements of zero pitch, having no constraints, or having constraints for odd order instantaneous dwells. A 4R plane mechanism is optimized to generate the screw displacement

$$\hat{\psi}_d = \Delta\phi_0\left(\frac{\theta}{\Delta\theta_0}\right)^2 + \varepsilon(0) \qquad 0 \le \theta \le 90°$$

with a constraint for a first order instantaneous dwell at $\theta = 0$, and the requirement that $\Delta\phi_0$ must be exact.

It has been shown that the RSRC and other space mechanisms can be synthesized by a graphical overlay technique and illustrated by synthesizing a constrained RSRC mechanism for the approximate generation of the screw displacement

$$\hat{\psi}_d = \Delta\phi_0\left(\frac{\theta}{\Delta\theta_0}\right) + \varepsilon\Delta S\left(\frac{\theta}{\Delta\theta_0}\right) \qquad 0 \le \theta \le 90°$$

and the dual function

$$\hat{y} = \frac{x^3}{4} + \varepsilon\, S_g \qquad -4 \le x \le 4$$

with $\Delta\theta_0 = \Delta\phi_0 = 120°$.

The process of optimization by variational methods requires the kinematic analysis of the mechanism to be synthesized in order to provide the function for the generated screw displacement. The 3x3 screw matrix was used to determine the displacements, velocities, accelerations and coupler curve coordinates for the basic mechanisms being synthesized.

It can briefly be stated that the outcome of this investigation was the development of

(1) a synthesis technique for the RSRC mechanism by variational methods for the generation of constrained and unconstrained screw displacements,

(2) parameters of constraints for instantaneous dwells, and so a technique to eliminate the equations of constraints for instantaneous dwells and the corresponding Lagrange multipliers,

(3) the grahpical overlay technique to synthesize the constrained RSRC mechanism for screw generation,

(4) the method of analysis of space, spherical and plane mechanisms by the 3x3 screw matrix,

(5) the general form of the mobility equation by redefining Kolchin's parameter in terms of the number of general constraints, the number of passive freedoms and the number of overclosing constraints.

(6) digital computer program for the complete analysis of the RSRC mechanism, and

(7) digital computer programs for the synthesis of the RSRC mechanism for the generation of constrained and unconstrained screw displacements.

The results of the present study offer the following as subjects for further study:

(1) Synthesis of other space mechanisms for the generation of screw displacements by variational methods.

(2) Developing the parameters of constraints for instantaneous dwells in the linear component of the screw output of the RSRC mechanism and so developing a technique to synthesize the RSRC mechanism for the generation of screw displacements having constraints for instantaneous dwells in the linear component or in both components at some specified values of the independent parameter.

(3) Developing the parameters of constraints for the vanishing

second and third order derivatives of the output displacement components of the RSRC mechanism, or other space mechanisms, for use in synthesizing those mechanisms for instantaneous dwells with no inertia force and no jerk.

(4) Synthesis of the RSRC and other space mechanisms for the generation of spatial path with and without constraints by variational methods.

(5) Synthesis of the RSRC and other space mechanisms for rigid body guidance in space by variational methods.

(6) Synthesis of plane and space mechanisms for the generation of specified force and torque distributions, or velocity and acceleration distributions by variational methods.

(7) Dynamic analysis of space mechanisms by 3x3 screw matrix.

(8) Gross motion analysis of the RSRC and other space mechanisms.

(9) Effect of elastic deformations in the links of space mechanisms on the output displacements.

(10) Developing a mathematical procedure to determine the number of overclosing constraints in mechanisms.

(11) Developing a mathematical procedure to determine the axes of the three basic screws, the existing components of these screws, so the number and types of general constraints in mechanisms.

(12) Number synthesis technique for space mechanisms considering general constraints, passive freedoms and overclosing constraints.

BIBLIOGRAPHY

(1) Grübler, M. "Das Kriterium der Zwanglaufigkeit der schraubenketler." Festschrift, O. Muhr Zum. 80, Gubertstag, Berlin, 1916.

(2) Grübler, M. Getriebelehre. Eine Theorie des Zwanglaufes und der ebenen Mechanismen. Berlin: Springer, 1917.

(3) Grübler, M. "Uber raumliche kinematische ketten kleinster Gliederzahl, sprach Geheitmrat." Z. VDI, Bd. 71 (1927) p. 165.

(4) Malytcheff, A. P. "Analysis and Synthesis of Mechanisms with the View Point of Their Structure." Izvestya Tomskoro of Technological Institute, 1923.

(5) Kutzbach, K. "Mechanische Leitungsverzweigung, ihre Gesetze und Anwendungen." Masch-Bau, Betrieb, Bd. 8 (1929) pp. 710-716.

(6) Kutzbach, K., "Quer-und winkelbewegliche Gleichganggelenke für Wellenleitungen." Z. VDI., Bd. 81 (1937) n 30.

(7) Kraus, R. Grundlagen des Systematischen Getriebeaufbaus. Berlin: Verlag Tech., 1952.

(8) Kraus, R. "Zur Zahlsynthese der räumlichen Mechanizmen." Masch-Bau. Betrieb, Bd. 10 (1940) pp. 33-39.

(9) Beyer, R. Technische Raumkinematik. Berlin: Springer-Verlag, 1963.

(10) Artobolevskii, I. I. Teoria Mehanismov i Masin. Gosudarstv: Izdatl Tehn-Teori. Lit, Moscow, 1953.

(11) Dobrovolskii, V. V. Teoria Mehanismov. Maschgis: Moskau, 1953.

(12) Popov, A. F. "Bases of the Theory of Contour Construction of Kinematic Chains and Their Applications for the Determination of the Degree of Mobility." Nauk. Zap., L'vovsk. Politekihn. In-ta., No. 43 (1956), pp. 158-166.

(13) Pisarev, M. N. "Regarding the Number of Links in Mechanisms Relating to Simple Closed Kinematic Chains." Trudi Gor'kovsk Politekihn. In-ta., Vol. 14, No. 1 (1958), pp. 88-91.

(14) Lifshits, Y. G. "Theory of the Structure and the Classification of Plane and Spatial Groups of Mechanisms." Trudi Rostovsk. na-Danu. In-ta S. kh mashinostr, No. 6 (1954), pp. 47-62.

(15) Bugaievski, Bogdan, and Pelecudi. "Contribution to the Classification of Spatial Mechanisms." Acad. Repub. Pop. Romane, Rev. Mecan. Appl., Vol. 2 (1957), pp. 157-170.

(16) Kolchin, N. I. "An Attempt to Construct an Expanded Structural Classification on Mechanisms and a Structural Table Based on it." Transactions of the 2nd All-Union Conference on the Basic Problems of the Theory of Machines and Mechanisms, Moscow (1960), pp. 85-97.

(17) Harrisberger, L. "A Number Synthesis Survey of Three Dimensional Mechanisms." Transactions of the ASME, Vol. 87 (May 1965), pp. 213-220.

(18) Monolescu, N. and Manafu, V. "On the Determination of the Degree of Mobility of Mechanisms." Bulletin of Polytechnic Institute, Bucharest, Vol. 25, n5 (1963), pp. 45-66.

(19) Moroshkin, I. F. "On the Geometry of Compounded Kinematic Chains." Soviet Phys.-Doklady, Vol. 3, n2 (1958), pp. 269-272.

(20) Voinea, R. P. and Atanasiu, M. C. "Geometrical Theory of Screws and Some Applications to the Theory of Mechanisms." Revue de Mécanique Appliqué, Vol. 7, n4 (1962), pp. 845-860.

(21) Soni, A. H. and Harrisberger, Lee. "Existance Criteria of Mechanisms." ASME, 10th Mechanisms Conference, Paper No. 68-MECH-33.

(22) Soni, A. H., "The Existance Criteria of One-General Constraint Mechanisms." (Ph.D. Dissertation, Oklahoma State Univeristy, May 1967).

(23) Ball, R. S. "A Treatise on the Theory of Screws." Cambridge University Press, 1900.

(24) Beyer, R. "Zur Geometrie und Synthese Eingentlicher Raumkurbelgetriebe." VDI-Berichte, Vol. 5 (1955), pp. 5-10.

(25) Beyer, R. "Zur Synthese und Analyse Von Raumkurbelgetriebe." VDI-Berichte, 12 (1955), pp. 5-20.

(26) Beyer, R. "Wissenschaftliche Hilfsmittel und Verfahren zur Untersuchung Raumlicher Gelenkgetriebe." Z. Konstruktion (1957), pp. 224-230, 285-290.

(27) Beyer, R. "A Survey of Techniques for Analyzing Motion Properties of All Types of 3-D Mechanisms." Transactions of the 5th Conference on Mechanisms, Panton Publishing Company, Cleveland, Ohio (1958), pp. 141-163.

(28) Beyer, R. "A Survey of Techniques for Analyzing Statics and Dynamics in Different Types of 3-D Mechanisms." Transactions of the 6th Conference of Mechanisms, Panton Publishing Company, Cleveland, Ohio, 1960.

(29) Tavhelidze, D. S. "Concerning the Existence of a Crank or Two Cranks in Space Mechanisms." Akad. Nauk. SSSR. Trudy Sem. Teorii Masin i Mehanizmov, 3, No. 9 (1947), pp. 5-17.

(30) Kozevnikov, S. N. "On the Kinematic and Design of Spatial Crank-Level Mechanism." Akad. Nauk. SSSR. Trudy Sem. Teorii Masin i Mehanizmov, 4, No. 14 (1948), pp. 32-63.

(31) Zinovev, V. A. "Kinematic Analysis of Spatial Four-Bar Mechanisms." Akad. Nauk. SSSR. Trudy Sem. Teorii Masin i Mehanizmov, 7, No. 28 (1949), pp. 78-98.

(32) Zinovev, V. A. "Kinematic Analysis of Spatial Mechanisms." Akad. Nauk. SSSR. Trudy. Sem. Teorii Masin i Mehanizmov, Vol. 10, No. 42 (1951), pp. 52-99.

(33) Chace, M. A. "Development and Application of Vector Mathematics for Kinematic Analysis of Three-Dimensional Mechanisms." (Ph.D. Diss., The University of Michigan, Ann Arbor, Michigan, 1965).

(34) Chace, M. A. "Solutions to the Vetor Tetrahedron Equation." Transactions of the ASME, Journal of Engineering for Industry Vol. 87, No. 2, Series B (1965), pp. 228-234.

(35) Chace, M. A. "Vector Analysis of Linkages." Trans. of the ASME, Journal of Eng. for Industry, Vol. 84, Series B (1963), pp. 289-296.

(36) Harrisberger, E. L. "Gross Motions of Space Mechanisms." (Ph.D. Diss. Purdue University, 1963).

(37) Egorov, V. V. "A Graphical Method for the Determination of the Positions of Spatial Mechanisms." Akad. Nauk. SSSR. Trudy Sem. Teorii Masin i Mehanizmov, 7, No. 25 (1949), pp. 5-58.

(38) Sieber, V. H. "Analytische und Graphische Verfahren zur Statik und Dynamik Raumlicher Kurbelgetriebe." (Diss. T. H. Muchen, 1959).

(39) Trinkl, F. "Analytische und Zeichnerische Verfahren zur Untersuchung Eigentlicher Raumkurbelgetriebe." Konstruktion, Vol. 11, No. 9 (1959), pp. 349-359.

(40) Trinkl, F. "Analytische und Zeichnerisches Verfahren zur Untersuchung Eigentlicher Raumkurbelgetriebe, dargestellt an einem Raumlichen Viergelenkgetriebe mit einem Kugelgelenk, einem Drehschubgelenk und Zwei dem Kugelgelenk Benachbarten Drehgelenken." (Diss. T. H. Muchen, 1958).

(41) Sherwood, A. A. "The Mechanical Generation of Simple Harmonic Motion by Three-Dimensional Linkages." Aust. J. Appl. Sci., No. 9 (1957), pp. 96-104.

(42) Hunt, K, H. "Exact Linear Simple Harmonic Motion by a Space-Linkage." Aust. J. Appl. Sci., No. 10 (1958), pp. 332-336.

(43) Bennett, G. T. "The Skew Isogram Mechanism." Proceedings of London Mathematical Society, 2nd Series, Vol. 13 (1913-1914), pp. 151-173.

(44) Beyer, R. "Technische Kinematic." Leipzig: J. A. Barth, 1931.

(45) Dobrovolskii, V. V. "The Construction of the Relative Positions of the Links of Spatial Seven-Bar Linkages by the Method of Spherical Representation." Akad. Nauk. SSSR. Trudy Sem. Teorii Masin i Mehanizmov 12, No. 47 (1952), pp. 52-62.

(46) Dobrovolskii, V. V. "Spherical Representation of Three Dimensional Four-Bar Linkages." Akad. Nauk. SSSR. Trudy Sem. Teorii Masin i Mehanizmov, 2 (1947), pp. 111-126.

(47) Dobrovolskii, V. V. "The Method of Spherical Representation in the Theory of Spatial Mechanisms." Akad. Nauk. SSSR. Trudy Sem. Teorii Masin i Mehanizmov, 3, No. 11 (1947), pp. 5-37.

(48) Dobrovolskii, V. V. "On the Statically Indeterminate Mechanisms." Akad. Nauk. SSSR. Trudy Sem. Teorii Masin i Mehanizmov, 26, No. 2 (1960), pp. 37-47.

(49) Kislitsin, S. G. "Helical Affinors and Certain Applications on the Question of Kinematics of Rigid Bodies." Ucheningradskogo Gosundarstvennogo Delagogicheskogo Instituta Km. A. I. Gertsena T. X., 1938.

(50) Kislitsin, S. G. "Application of Tensors to Space Mechanisms." Akad. Nauk. SSSR. Trudy Sem. Teorii Masin i Mechanizmov, 14 (1954), pp. 51-75.

(51) Kotelnikoff, A. P. "Screw Calculus and Some Applications of the Same to Geometry and Mechanics." Annals of the Imperial University of Kazan, 1895.

(52) Dimentberg, F. M. "A General Method for Finite Displacements of Spatial Mechanisms and on Certain Passive Constraints," Akad. Nauk. SSSR. Trudy Sem. Teorii Masin i Mehanizmov, 5, No. 17 (1948), pp. 5-39.

(53) Dimentberg, F. M. "The Determination of Positions of Spatial Mechanisms. Application of the Method of 'Screws' to the Investigation of the Displacements of Spatial Mechanisms." Izdat. Akad. Nauk. SSSR, Moscow, 1950, 142 pages.

(54) Dimentberg, F. M. "An Analogy Between Finite Motion of Plane and Spatial Four-Bar Linkages." Akad. Nauk. SSSR. Izvestiia. Otdelenie Teknickes. Nauk., 2 (1959), pp. 181-185.

(55) Dimentberg, F. M. and Kislitsin, S. G. "Application of Screw Calculus to the Analysis of Spatial Mechanisms." Akad. Nauk. SSSR. Izvestiia. Otdelenie Teknickes. Nauk. 8 (1965), pp. 55-65.

(56) Denavit, J. "Displacement Analysis of Mechanisms Based on (2x2) Matrices of Dual Numbers. VDI-Berichte, 29 (1958), pp. 81-88.

(57) Denavit, J., and Hartenberg, R. S. "Approximate Synthesis of Spatial Linkages." Transactions of the ASME, Journal of Applied Mechanics, Vol. 81, Series E (1960), pp. 201-206.

(58) Denavit, S., Hartenberg, R. S. "A Kinematic Notation for Lower-Pair Mechanisms Based on Matrices." ASME, Journal of Applied Mechanics, Vol. 76, Series E (1955), pp. 215-221.

(59) Uicker, J. J., Denavit, J., Hartenberg, K. S. "An Iterative Method for the Displacement Analysis of Spatial Mechanisms." Transactions of the ASME, Journal of Applied Mechanics, Vol. 85 (1964), pp. 309-314.

(60) Beyer, R. "Das Matrizenkalkul als Hilfsmittel Zur Untersuchung Raumlicher Gelenkgetriebe." Z. Feinwerktecknik (1957), pp. 317-327.

(61) Mangeron, D., and Dragon, K. "Kinematic Study with New-Matrix-Tensor Methods for Four-Bar Spatial Mechanisms." Revue de Mecanique Appliquee, Vol. 7 (1962), pp. 539-551.

(62) Shor, J. B. "On the Determination of Screw Axes in Spatial Mechanisms." Prikladnaia Matematika i Mekhanika. Leningrad. Vol. 5 (1941), pp. 267-276.

(63) Keler, M. L. "Analyse and Synthese der Raumkurbel Getriebe mittels Raum Liniengeometrie und Dualer Grossen." Forsch. Ing., Vol. 25 (1959), pp. 26-32 and 55-63.

(64) Blaschke, W. "Anwendung Dualer Quaternionen Auf Kinematik." Annates Academie Scientiorum Fennicae, Helsinki, Series A 250/3 (1958), pp. 1-13.

(65) Blaschke, W. "Kinematik and Quaternionen." Math. Monographien, 4. VEB Deutscher Verlag der Wissenschafton, Berlin, 1960.

(66) Yang, A. T. "Application of Quaternion Algebra and Dual Numbers to the Analysis of Spatial Mechanisms." (Ph.D. Diss., Columbia University, New York, N. Y., 1963).

(67) Yang, A. T., and Freudenstein, F. "Application of Dual-Number Quaternion Algebra to the Analysis of Spatial Mechanisms." Transactions of the ASME, Journal of Applied Mechanics, Vol. 85 (1964), pp. 300-308.

(68) Hamilton, R. "Lectures on Quaternions." Hodges and Smith, Dublin: McMillan Co., 1853.

(69) Clifford, M. A. "Preliminary Sketch of Biquaternions." Proceedings of the London Math. Soc., Nos. 64, 65, Vol. IV (June 12, 1873), pp. 381-395.

(70) Hardy, A. S. "Elements of Quaternions." Boston: Ginn Co., 1887.

(71) Dobrovolskii, V. V. "Spherical Three-Bar Curve." Prikladnaia Mathematika i Mekhanika. Akad. Nauk. SSSR. Vol. 8 (1944), pp. 475-477.

(72) Dobrovolskii, V. V. "Burmester's Points in Spherical Motion." Prikladnaia Matematika i Mekhanika. Akad. Nauk. SSSR, Vol. 9 (1945), pp. 489-491.

(73) Hein, G. "Analytische und Zeichnerische Unterlagen fur Zwei und Drei Lagen eines Spharisch Bewegten Getriebegliedes als Hilfsmittel zur Synthese Spharischer Kurbelgetriebe." (Diss. T. H. Munchen, 1958).

(74) Yang, A. T. "Harmonic Analysis of Spherical Four-Bar Mechanisms." Transactions of ASME, Journal of Applied Mechanics, Vol. 83, Series E (1962), pp. 683-688.

(75) Yang, A. T. "Static Force and Torque Analysis of Spherical Four-Bar Mechanisms." Transactions of ASME, Journal of Engineering for Industry, Vol. 87, Series B (1965), pp. 221-227.

(76) Meyer zur Capellen, W. "Kinematik der spharrischen Schubkurbel." Forschungsberichte des Wirtschafts und Verkehrsministeriums Nordrhein Westfalen, No. 873, Westdeutcher Verlag, Cologne, Germany, 1960.

(77) Meyer zur Capellen, W. "Der spharische Doppelschieber als kinematische Unkchrung des Kreuzgelenks." Indistrie Anzeiger, No. 65 (1962), pp. 1591-1595.

(78) Meyer zur Capellen, W., Dittrich, G., and Janssen. "Systematik und kinematik ebener und spherische Viergelenkgetriebe." Forschungsbericht des Landes Nordrhein-Westfalen, Kohn, West Deutscher, Verlag, No. 1911, 1965.

(79) Freudenstein, F. "Approximate Synthesis of Four-Bar Linkages." Transactions of the ASME, Vol. 76 (August, 1955), pp. 853-861.

(80) Freudenstein, F. "Synthesis of Path-Generating Mechanisms by Means of a Programmed Digital Computer." Transactions of the ASME, Journal of Engineering for Industry, Vol. 81 (May 1959), pp. 159-168.

(81) Freudenstein, F. "Synthesis of Path-Generating Mechanisms by Numerical Methods." Transactions of the ASME, Journal of Engineering for Industry, Vol. 85 (August 1963), pp. 298-304.

(82) Wilson, J. T. "Analytical Kinematic Synthesis by Finete Displacements." ASME Paper No. 64-Mech-13, 9 pages, 1964.

(83) Roth, B. "The Kinematics of Motion Through Finitely Separated Positions." Transactions of the ASME, Journal of Applied Mechanics, Vol. 88 (September, 1967), pp. 591-598.

(84) Roth, B. "Finite Position Theory Applied to Mechanism Synthesis,"T Transactions of the ASME, Journal of Applied Mechanics, Vol. 88 (September, 1967), pp. 599-605.

(85) Schoenflies, A. "Geometrie der Bewegung in Synthtischer Darstellung." Leipzig, Germany, 194 pages, 1886.

(86) Suh, C. H. "Design of Space Mechanisms for Rigid Body Guidance." ASME Paper No. 67-DE-F.

(87) Suh, C. H. "Design of Space Mechanisms for Function Generation," Transactions of the ASME, Journal of Engineering for Industry, Vol. 90 (August, 1968), pp. 499-596.

(88) Suh, C. H., and Radcliffe, C. W. "Synthesis of Spherical Linkages with Use of the Displacement Matrix." Transactions of the ASME, Journal of Engineering for Industry, Series B, Vol. 89, No. 2 (1967), pp. 215-222.

(89) Levitskii, N. I. "Application of the Least-Square Method to Mechanism Design," Akademiia Nauk. SSSR. Institut Mashinovedeiia. Trudy Seminar Po theorii Mashin i Makhanizmov, Vol. 5, No. 17 (1948), pp. 40-68.

(90) Levitskii, N. I. and Sahbazyan, K. H. "The Synthesis of Spatial Four-Link Mechanisms with Lower Pairs." Akad. Nauk. SSSR. Trudy Sem. Teorii Masin i Mehanizmov, Vol. 14, No. 54 (1954), pp. 5-24.

(91) Chi-Yeh, H. "A General Method for the Optimum Design of Mechanisms." Journal of Mechanisms, Vol. 1 (1966), pp. 301-313.

(92) Plucker, J. "Gesammelte Wissenschaftliche Abhandungen." Teubner, Leipzig, Vol. 1, 1895.

(93) Bricard, J. "Leçons de Cinématique." Gauthier-Villars Paris, Vol. 2 (1927), pp. 7-12.

(94) Waldron, K. J. "Hybrid Overconstrained Linkages." Journal of Mechanisms, Vol. 3 (1968), No. 3.

(95) Hunt, K. H. "Screw Axes and Mobility in Spatial Mechanisms via the Linear Complex." Journal of Mechanisms, Vol. 2, No. 3 (1967), pp. 307-327.

(96) Waldron, K. J. "Symmetric Overconstrained Linkages." ASME Paper 68-Mech-20, 10th Mechanisms Conference, November 12, 1968.

(97) Halfman, R. L. Dynamics, Vol. II: Systems Variational Methods, and Relativity. Addison-Wesley Publication, pp. 434-506, 1962.

(98) Weinstock, R. The Calculus of Variations. Dover Publication, New York, 397 pages, 1967.

(99) Sokolnikoff, I. S. "Mathematics of Physics and Modern Engineering." McGraw-Hill Book Company, 1958.

(100) Langhaar, H. L. "Energy Methods in Applied Mechanics." John Wiley and Sons, Inc., 1962.

(101) Ralston, A. "A First Course in Numerical Analysis." McGraw-Hill Book Company, 1965.

(102) Isaacson, E., and Keller, H. B. "Analysis of Numerical Methods." John Wiley and Sons, Inc., 1966.

(103) Goldberg, M. "New Five-Bar and Six-Bar Linkages in Three Dimensions," Transactions of the ASME, Vol. 64 (August 1943), pp. 649-661.

(104) Bruevich, N. G. "Kinetostatika Prostranstvennykh Mechanizmov." Trudy Voennoi Vozdushnoi Akademii, im. Zukovskogo, No. 22, 1937.

(105) Verkhovskii, A. V. "Chetyrekhzvennyi Prostranstvennyi Mechanism s Tsilindricheskimi Sharnirami, Osi Kotorykh ne Parallel'ny i ne Peresekaiutsya v Odnoi Tochke, i ego Issledovanie." Izvestigy Tomoskogo Tekhnologichekogo Instituta. Vol. VI, No. 2, 1925.

(106) Bushgens, C. C. "Mechanism Benneta-Verkouskago," Priklandnaya Matematika i Mekhanika, Vol. II, No. 4, 1939.

(107) Altman, F. G. "Sonderformen Raumlieher Koppelgetriebe und Grenzen Ihrer Verwendbarkeit." Konstruktion, Werkstoffe Versuchswesen, No. 4 (1952), pp. 97-106.

(108) Harding, B. L. "Hesitation." ASME Paper No. 64-Mech-11, 8th Mechanisms Conference, Lafayette, Ind., October 19-21, 1964, 8 pages.

APPENDIX A

BRIEF DISCUSSION ON THE SCREW CALCULUS

Following are some of the basic operations in screw calculus.

Dual Vector

The vector $\overline{A}$, shown in Figure 27, is defined as a dual vector [bivector] in the OXYZ system by

$$\hat{\overline{A}} = \overline{A}_0 + \varepsilon\overline{A}_1 \tag{A.1}$$

where $\overline{A}_0$ is the vector at the origin of the coordinate system, parallel to the vector $\overline{A}$ and $|\overline{A}_0| = |\overline{A}|$. The vector $\overline{A}_1$ is the moment of the vector about the origin of the coordinate system, and is given by $\overline{A}_1 = \overline{A}_2 \wedge \overline{A}_0$. The symbol $\wedge$ signifies the vector product of two vectors. $\overline{A}_2$ is the vector which positions a point on the line of action of the vector $\overline{A}$. The symbol ε is the Clifford's screw operator which transforms the moment of a vector into a geometrically equivalent vector (69). Repeated operations with ε results zero operation. Then $\varepsilon^2 = 0$. The caret over $\overline{A}$ identifies a dual quantity.

If $\overline{a}$ and $\overline{a}_0$ are the unit vectors along $\overline{A}$ and $\overline{A}_0$, we can define $\overline{a}_1 = \overline{A}_2 \wedge \overline{a}_0$ as the vector of moment arm. Then

$$\hat{\overline{A}} = |\overline{A}_0|\ \overline{a}_0 + \varepsilon\ \overline{a}_1\ |\overline{A}| = |\overline{A}_0|\ (\overline{a}_0 + \varepsilon\overline{a}_1)$$

or

$$\hat{\overline{A}} = |\overline{A}_0|\hat{\overline{a}} \tag{A.2}$$

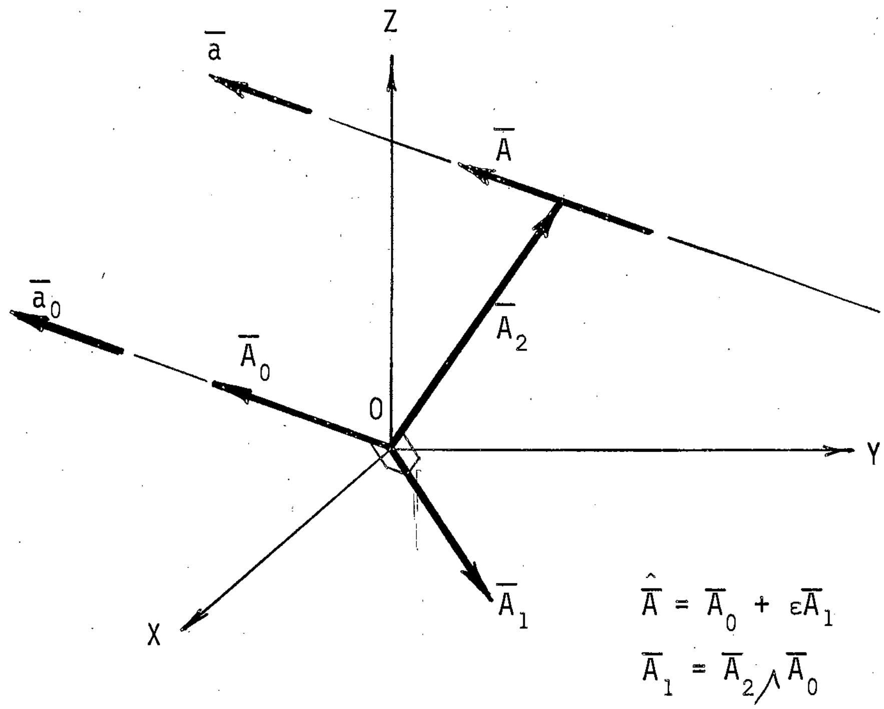


Figure 27. The Dual Vector

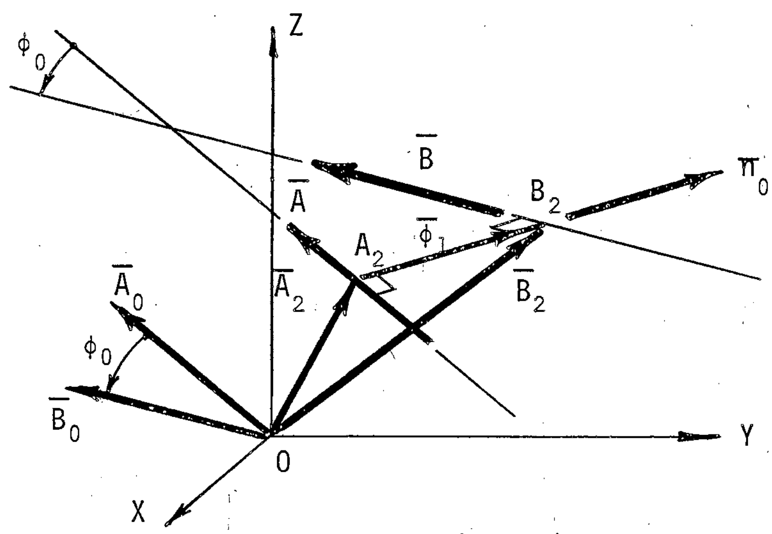


Figure 28. The Dual Vectors $\hat{A}$ and $\hat{B}$

where $\hat{\bar{a}} = \bar{a}_0 + \varepsilon\bar{a}_1$ is the dual unit vector, and $|\bar{a}_1| = |\bar{A}_2 \wedge \bar{a}_0|$ is the distance of the vector $\bar{A}$ from the origin of the OXYZ system.

Dual Number

A dual number is of the form

$$\hat{d} = d_0 + \varepsilon d_1 \tag{A.3}$$

where d_0 and d_1 are real numbers, d_0 is the real part, d_1 is the dual part, or moment part. Algebra of the dual numbers is the same as the algebra of complex numbers with imaginary parts. Thus, if $\hat{d} = d_0 + \varepsilon d_1$ and $\hat{e} = e_0 + \varepsilon e_1$ are two dual numbers, in virtue of the property of the operator ε, we have the following rules (51,53,9):

$$\hat{d} = 0 \quad \text{if} \quad d_0 = d_1 = 0 \tag{A.4}$$

$$\hat{d} = \hat{e} \quad \text{if} \quad d_0 = e_0 \quad \text{and} \quad d_1 = e_1 \tag{A.5}$$

$$\hat{d} \pm \hat{e} = (d_0 \pm e_0) + \varepsilon(d_1 \pm e_1) \tag{A.6}$$

$$\hat{d} \cdot \hat{e} = d_0 e_0 + \varepsilon(d_1 e_0 + d_0 e_1) \tag{A.7}$$

$$\frac{\hat{d}}{\hat{e}} = \frac{d_0}{e_0}\left[1 + \varepsilon\left(\frac{d_1}{d_0} - \frac{e_1}{e_0}\right)\right] \tag{A.8}$$

$$\hat{d}^n = d_0^n + \varepsilon d_1 \, n \, d_1^{n-1} \tag{A.9}$$

$$\sqrt[n]{\hat{d}} = \sqrt[n]{d_0} + \varepsilon \frac{d_1}{\sqrt[n]{d_0^{n-1}}} \tag{A.10}$$

$$e^{\hat{d}} = e^{d_0} e^{d_1} = e^{d_0}(1 + \varepsilon d_1) \tag{A.11}$$

$$\ell n \hat{d} = \ell n\left(d_0 + \varepsilon \frac{d_1}{d_0}\right) = \ell n \, d_0 + \varepsilon \frac{d_1}{d_0} \tag{A.12}$$

Dual Angle

The displacement of a coordinate system through a rotation θ_0

about an axis and translation θ_1 along the same axis, with respect to its initial position can be defined by the dual angle

$$\hat{\theta} = \theta_0 + \varepsilon\theta_1 \tag{A.13}$$

as discussed in Chapter II. θ_0 and θ_1 are positive in the right hand screw direction. Frequently used dual trigonometric relationships are

$$\sin\hat{\theta} = \sin\theta_0 + \varepsilon\theta_1 \cos\theta_0 \tag{A.14}$$

$$\cos\hat{\theta} = \cos\theta_0 - \varepsilon\theta_1 \sin\theta_0 \tag{A.15}$$

$$\sin^2\hat{\theta} + \cos^2\hat{\theta} = 1 \tag{A.16}$$

$$\tan\hat{\theta} = \tan\theta_0 + \varepsilon\theta_1 (1 + \tan^2\theta_0) \tag{A.17}$$

$$\cot\hat{\theta} = \cot\theta_0 - \varepsilon\theta_1 (1 + \cot^2\theta_0) \tag{A.18}$$

Scalar and Vector Products of Two Dual Vectors

The operations with dual vectors are not distinguished from the operations with the ordinary vectors. If $\hat{\overline{A}} = \overline{A}_0 + \varepsilon\overline{A}_1$ and $\hat{\overline{B}} = \overline{B}_0 + \varepsilon\overline{B}_1$ are two dual vectors, shown in Figure 28, the scalar product of the two vectors is

$$\hat{\overline{A}}\cdot\hat{\overline{B}} = \overline{A}_0\cdot\overline{B}_0 + \varepsilon[(\overline{A}_2 \wedge \overline{A}_0)\cdot\overline{B}_0 + \overline{A}_0\cdot(\overline{B}_2 \wedge \overline{B}_0)] = \overline{A}_0\cdot\overline{B}_0$$

$$- \varepsilon(\overline{B}_2 - \overline{A}_2) \cdot (\overline{A}_0 \quad \overline{B}_0) \tag{A.19}$$

Let $\overline{A}_2$ and $\overline{B}_2$ position the points A_2 and B_2 on the vectors $\overline{A}$ and $\overline{B}$, where A_2B_2 is the common normal of the two vectors, and introduce

$$\overline{\phi}_1 = \overline{B}_2 - \overline{A}_2 = \phi_1 \overline{n}_0$$

where $\phi_1 = |\overline{B}_2 - \overline{A}_2|$ is the length of the common normal, $\overline{n}_0$ is the unit vector along the common normal A_2B_2.

If the skew angle between the two vectors $\overline{A}_0$ and $\overline{B}_0$ is ϕ_0, we have $\overline{A}_0 \cdot \overline{B}_0 = |\overline{A}|\ |\overline{B}| \cos\phi_0$, $\overline{A}_0 \wedge \overline{B}_0 = |\overline{A}|\ |\overline{B}| \sin\phi_0 \overline{n}_0$. Then Equation (A.19) becomes

$$\hat{\overline{A}} \cdot \hat{\overline{B}} + |\overline{A}|\ |\overline{B}| \cos\hat{\phi} \tag{A.20}$$

The vector product of the dual vectors $\hat{\overline{A}}$ and $\hat{\overline{B}}$ is

$$\hat{\overline{A}} \wedge \hat{\overline{B}} = |\overline{A}|\ |\overline{B}| \sin\phi_0 \overline{n}_0 + \varepsilon \overline{C}_1 \tag{A.21}$$

where

$$\overline{C}_1 = (\overline{A}_2 \wedge \overline{A}_0) \wedge \overline{B}_0 + \overline{A}_0 \wedge (\overline{B}_2 \wedge \overline{B}_0)$$

and noting that $\overline{B}_2 = \overline{A}_2 + \phi_1 \overline{n}_0$

$$\overline{C}_1 = |\overline{A}|\ |\overline{B}|\ \phi_1 \cos\phi_0 \overline{n}_0 + \overline{A}_2 \wedge (\overline{A}_0 \wedge \overline{B}_0)$$

Let $\overline{A}_2 \wedge \overline{n}_0 = \overline{n}_1$, where $\overline{n}_1$ is normal to the plane OA_2B_2 and $|\overline{n}_1|$ is the distance of $\overline{n}_0$ from the origin O. Then

$$\overline{C}_1 = |\overline{A}|\ |\overline{B}|\ \phi_1 \cos\phi_0 \overline{n}_0 + \sin\phi_0 \overline{n}_1$$

and

$$\hat{\overline{A}} \wedge \hat{\overline{B}} = |\overline{A}|\ |\overline{B}|\ (\sin\hat{\phi}\ \overline{n}_0 + \varepsilon \sin\phi_0\ \overline{n}_1) \tag{A.22}$$

Derivative of a Dual Matrix

The derivative of a dual variable $\hat{\alpha}$ with respect to t, let t be time, is

$$\frac{d\hat{\alpha}}{dt} = \frac{d\alpha_0}{dt} + \varepsilon \frac{d\alpha_1}{dt} \tag{A.23}$$

Consider a dual matrix

$$\hat{T}_{\hat{\alpha}} = \hat{T}_{\alpha_1} T_{\alpha_0} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\hat{\alpha} & -\sin\hat{\alpha} \\ 0 & \sin\hat{\alpha} & \cos\hat{\alpha} \end{bmatrix}$$

for the dual rotation $\hat{\alpha}$ about the x axis, where T_{α_0} and $\hat{T}_{\alpha_1}$ are given by Equation (2.7). The derivative of $\hat{T}_{\hat{\alpha}}$ with respect to t is

$$\frac{d\hat{T}_{\hat{\alpha}}}{dt} = \left(\frac{d}{dt}\hat{T}_{\alpha_1}\right) T_{\alpha_0} + \hat{T}_{\alpha_1}\left(\frac{d}{dt} T_{\alpha_0}\right) \tag{A.24}$$

noting that

$$\frac{d}{dt} T_{\alpha_0} = D_1 T_{\alpha_0} \frac{d\alpha_0}{dt} \tag{A.25}$$

and

$$\frac{d}{dt}\hat{T}_{\alpha_1} = \varepsilon D_1 \frac{d\alpha_1}{dt} \tag{A.26}$$

where

$$D_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \tag{A.27a}$$

is the differential operator matrix for the rotation about x axis. The differential operator matrices for the rotations about y and z axes are

$$D_2 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix} \quad \text{and} \quad D_3 = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{A.27b}$$

Then,

$$\frac{d}{dt}\hat{T}_{\hat{\alpha}} = \varepsilon D_1 T_{\alpha_0} \frac{d\alpha_1}{dt} + \hat{T}_{\alpha_1} D_1 T_{\alpha_0} \frac{d\alpha_0}{dt} \tag{A.28}$$

also noting that

$$\hat{T}_{\hat{\alpha}} = T_{\alpha_0} + \varepsilon\, \alpha_1\, D_1\, T_{\alpha_0}$$

$$\frac{d}{dt}\hat{T}_{\hat{\alpha}} = D_1\, T_{\alpha_0}\frac{d\alpha_0}{dt} + \varepsilon\left(D_1\, T_{\alpha_0}\frac{d\alpha_1}{dt} + \alpha_1\, D_1^2\, T_{\alpha_0}\frac{d\alpha_0}{dt}\right) \tag{A.29}$$

and if $\hat{T}^T_{\hat{\alpha}}$ is the transpose of $\hat{T}_{\hat{\alpha}}$, we have

$$\frac{d}{dt}\hat{T}^T_{\hat{\alpha}} = D_1^T\, \hat{T}^T_{\hat{\alpha}}\frac{d\hat{\alpha}}{dt} \tag{A.30}$$

and

$$\frac{d^2}{dt^2}\hat{T}^T_{\hat{\alpha}} = D_1^T\, \hat{T}^T_{\hat{\alpha}}\frac{d^2\hat{\alpha}}{dt^2} + D_1^T\, D_1^T\, \hat{T}^T_{\hat{\alpha}}\frac{d\hat{\alpha}}{dt} \tag{A.31}$$

where D_1^T is the transpose of D_1.

Taylor Series Expansion of a Dual Function

Let $\hat{f}(\hat{a},\hat{b},\hat{c},\ldots)$ be defined in some intervals of the dual variables $\hat{a},\hat{b},\hat{c},\ldots$, and let $\hat{a}_i$, $\hat{b}_i$, $\hat{c}_i,\ldots$ define a point within the intervals. If all derivatives of $\hat{f}(\hat{a},\hat{b},\hat{c},\ldots)$ exist at $(\hat{a}_i, \hat{b}_i, \hat{c}_i,\ldots)$, the Taylor series

$$\hat{f}(\hat{a},\hat{b},\hat{c},\ldots) = \sum_{n=0}^{n}\left[\frac{\partial^n\hat{f}(\hat{a}_i,\hat{b}_i,\ldots)}{\partial\hat{a}^n}\cdot\frac{(d\hat{a})^n}{n!} + \frac{\partial^n\hat{f}(\hat{a}_i,\hat{b}_i,\ldots)}{\partial\hat{b}^n}\cdot\frac{(d\hat{b})^n}{n!} + \frac{\partial^n\hat{f}(\hat{a}_i,\hat{b}_i,\ldots)}{\partial\hat{c}^n}\cdot\frac{(d\hat{c})^n}{n!} + \ldots\right] \tag{A.30}$$

will converge to $\hat{f}(\hat{a},\hat{b},\hat{c},\ldots)$ for all $\hat{a},\hat{b},\hat{c},\ldots$, or else in an interval with $\hat{a}_i,\hat{b}_i,\hat{c}_i,\ldots$ as the midpoints of the respective intervals; where $\hat{a}_i = \hat{a} - d\hat{a}$, $\hat{b}_i = \hat{b} - d\hat{b}$, ...

In the special case when $\hat{a}_i = \hat{b}_i = \hat{c}_i = . . . = 0$, the Taylor series expansion given by Equation (A.30) becomes

$$\hat{f}(\hat{a},\hat{b},\hat{c},\ldots) = \hat{f}(a_0,b_0,c_0,\ldots) + \varepsilon \left[a_1 \frac{\partial \hat{f}(a_0,b_0,\ldots)}{\partial a_0} + b_1 \frac{\partial \hat{f}(a_0,b_0,\ldots)}{\partial b_0} + c_1 \frac{\partial \hat{f}(a_0,b_0,\ldots)}{\partial c_0} + \ldots \right] \quad (A.31)$$

since, for example $\partial\hat{f}/\partial\hat{a}$ evaluated at $\hat{a} = \hat{b} = \hat{c} = \ldots = 0$ becomes $\frac{\partial \hat{f}(a_0,b_0,c_0,\ldots)}{\partial a_0}$, where $\hat{f}(a_0,b_0,c_0,\ldots)$ means that in the dual function each independent dual variable is replaced by its real part. One should observe the Taylor series expansions for $\hat{d}^n$, $e^{\hat{d}}$, $\sin\hat{\theta}$ and $\cos\hat{\theta}$ in Equations (A.9), (A.11), (A.14) and (A.15).

APPENDIX B

MOBILITY CRITERIA

However the subject of mobility in mechanisms is not in the scope of this study, the author experienced the need for a mobility equation which predicts the degree of freedom of motion in mechanisms in general. Among the proposed ones, so far, Kolchin's structural formula has the general form of the mobility equation (16,17,36). However his parameter is not fully defined. In the following the general form of the mobility equation is developed by which the number of degrees of freedom of motion in a mechanism having general constraints, overclosing constraints, passive and redundant freedoms is predicted.

Malytsheff's Mobility Equation

When a mechanism has no constraints the number of degrees of freedom of motion in the mechanism is given by

$$F_0 = 6(n-1) - \sum_{i=1}^{5} (6-i)N_i \qquad \text{(B.1)}$$

where n is the number of links in the mechanism, i is the class of a pair, N_i is the number of the i^{th} class pairs in the mechanism. A pair of Class i connecting two links permits i freedoms of motion of one link relative to the other. Equation (B.1) is Malytsheff's mobility equation (4). However, it is not a sufficient criterion in general, since it does not consider the effect of constraints. This was noted first by Bricard (93) when it failed to predict the degree of freedom of motion in Bricard's six-bar space mechanism.

In fact geometric constraints imposed on the mechanism has the total effect of producing passive freedoms, redundant freedoms, general constraints and overclosing constraints.

General Constraints

When there is no general constraint in a mechanism the motion of the links relative to the fixed link is described by three basic screws; three rotations and three translations about three screw axes, orthogonal or not. General constraints have the effect of destroying some of the components of the three basic-screws, such that none of the links has components of its motion in the direction of the destroyed screw components. The remaining components describe the entire motion of the mechanism. The number of the nonexisting components of the basic-screws is the number of general constraints in the mechanism. Let m define the number of general constraints;

$$m(M_{R_1} M_{R_2} M_{R_3}, M_{T_1} M_{T_2} M_{T_3}) = \sum_{J=i}^{3} (M_{R_J} + M_{T_J})$$

where M_{R_J} and M_{T_J} are the general constraints on rotation and translation about the J^{th} Basic Screw Axis (BSA) describing the general displacement of the mechanism, respectively. M_{R_J} and M_{T_J} are zero or one.

The number of the general constraints in a mechanism can at most be 5, where m = 5 implies Class I pair. There are 19 possible types of general constraint combinations. In case of plane mechanism $m_{(110,001)} = 3$ states that there is no rotation about BSA1 and BSA2 while there is no translation along BSA3 . $m_{(000,111)} = 3$ indicates a spherical mechanisms. $m_{(111,000)} = 3$ indicates a mechanism in which all the pairs are prism pairs and at least one link has a displacement out of the plane of the two of the three Basic Screw Axes.

In Figure 29 several mechanisms having different numbers of general constraints from 0 to 4 are shown. The RSPC mechanism, shown in Figure 29a, functions as the RS_SPC mechanism when the skew angle is zero. Then one of the rotation freedoms of the spherical pair becomes passive, and the pair can be replaced by the slotted sphere pair (S_S), as shown in Figure 29b. In this mechanism none of the links can have a rotation component of its motion about one of the three basic screw axes, and the mechanism has one general constraint on rotation, $m_{(100,000)} = 1$. When the skew angle α_0 between the input link and the coupler link of the RCCC mechanism shown in Figure 29c is zero, the rotation freedom on the two cylinder pairs on the output link are passive. Then the mechanism reduces to an RCPP mechanism having the general constraints $m_{(011,000)} = 2$, as shown in Figure 29d. When the revolute pair in the RCCC mechanism is replaced by a prism pair, the mechanism reduces to 4P mechanism of $m_{(111,000)} = 3$, as shown in Figure 29e. One note worthy observation regarding the effect of the geometry on the mobility of the RCCC mechanism is that when the skew angle δ_0 is zero the mechanism functions as a prism pair. However, this does not mean that the RCCC mechanism does not reduce to a mechanism when δ_0 is zero. A constrained inversion of the RCCC mechanism, the RCRC or RCCR mechanism, is formed when $\delta_0 = \alpha_0 = 0$. If, in

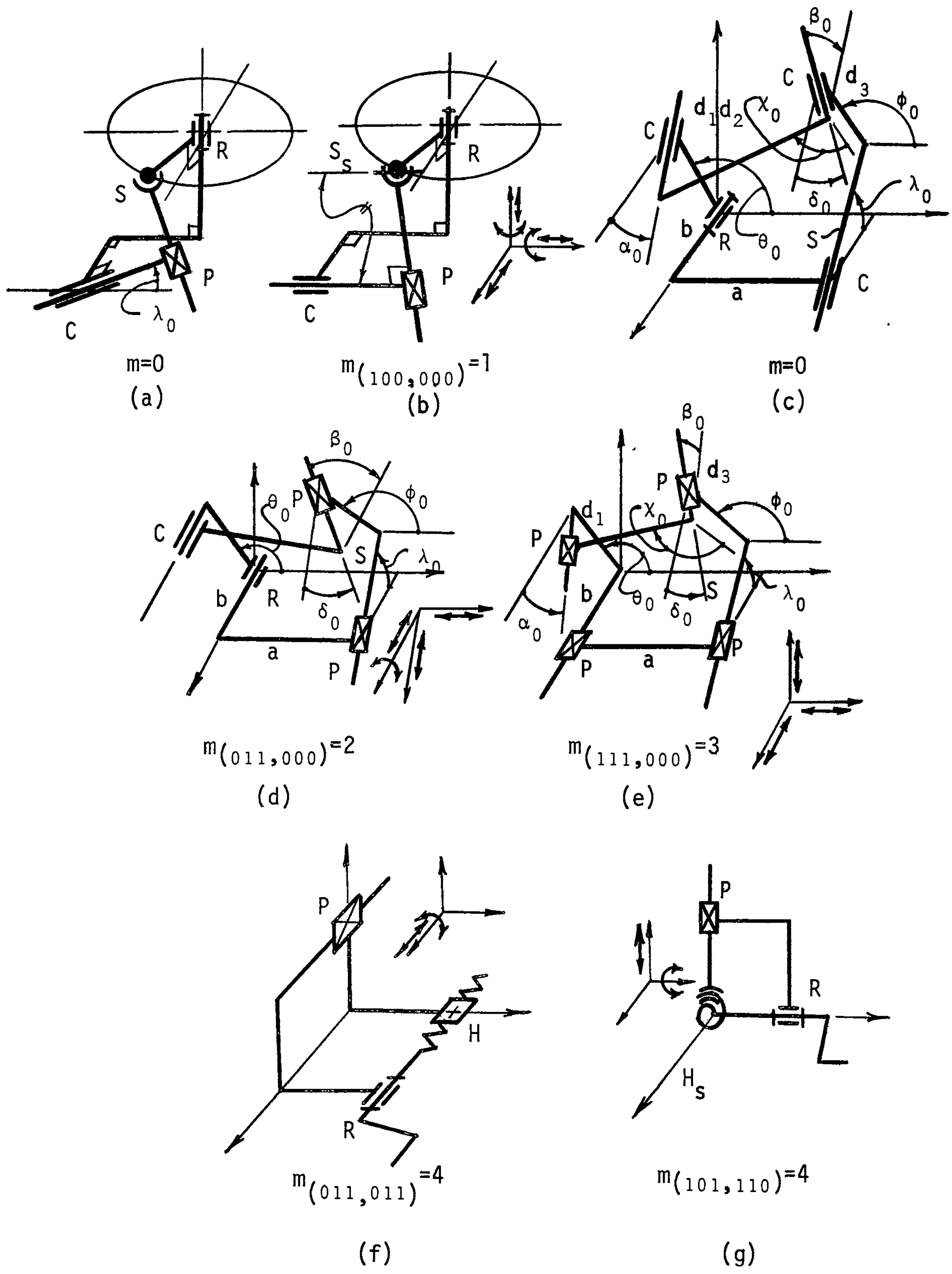


Figure 29. Mechanisms With General Constraints Varying from 1 to 4

the RCCC mechanism, $d_1 = d_2 = d_3 = 0$ and the pair axes intersect at one point, a spherical mechanism results, while if $|d_1/\sin\alpha_0| = |d_2/\sin\beta_0|$ and $\alpha_0 = \delta_0$, $\beta_0 = \lambda_0$ translation freedoms on the pairs become passive resulting in the well known Bennett mechanism. The components of the dual loop closure equation for a mechanism may be used to predict its motion. In case of the Bennett mechanism both the dual and the real parts of the dual loop closure equation concur with the displacements in its spherical indicatrix. So there exists three translational general constraints in its motion, $m_{(000,111)} = 3$. Due to the symmetry in its geometry, the displacement of the coupler link is rotation about an axis in space which moves to infinity as the geometry of the mechanism approaches the geometry of plane mechanism [parallelogram]. Then the axes of the three basic screws are parallel, and the rotation about the axis at infinity is defined by two translation freedoms. Goldberg's five-bar and six-bar mechanisms (103), which are Bennett mechanisms in series, then have general constraints $m_{(000,111)} = 3$ in each loop.

Figures 29f and 29g show the RHP and RH_sP mechanisms having the general constraints $m_{(011,011)} = 4$ and $m_{(101,110)} = 4$, respectively.

Redundant Freedoms

Two spherical pairs in a single-loop mechanism introduce an additional degree of freedom into the mechanism. Under certain geometric conditions it requires a second input for the transmission, as in the RSRS mechanism. This second degree of freedom of motion is a redundant freedom in the case of the RSSR mechanism. In this case, however, the redundant freedom has no effect on the input-output transmission, but it can be used to derive a second loop. Such a redundant freedom can be eliminated by replacing one of the spherical pairs by slotted sphere pair, provided that the axis of the pin in the slotted-sphere pair is not along the common axes of the two pairs on the coupler link.

Redundant rotation freedom will also result when a spherical pair is on the axis of a cylinder pair on a link. In this case the cylinder pair could be replaced by a prism pair. Redundant translation freedom will result whenever the axes of two cylinder or two prism or one prism and one cylinder pair are parallel, as in the RCCC mechanism when $\delta_0 = 0$, or $\beta_0 = 0$. If both pairs are cylinder pairs the redundant freedom is eliminated by replacing one of the pairs by revolute pair. If one of the pairs is a prism pair either prism pair is eliminated or cylinder pair is replaced by revolute pair.

In case the axis of a prism or a cylinder pair is parallel to the axis of an axial-screw-pair or to the axis of translation freedom of a planar-screw-pair, the screw freedom is redundant. In such cases the screw pair functions as a revolute pair in a single loop, however, it can function as a screw pair in a second loop. Redundant screw freedom will also result whenever the axis of a cylinder or a revolute pair is in line with the axis of an axial-screw-pair, and with the axis of the rotation freedom of a planar-screw-pair. Then the screw pair functions as a prism pair.

Artobolevskii-Dobrovolskii Mobility Equation

Since an i^{th} class pair destroys 6-m-i degrees of freedom in a mechanism having m general constraints, Equation (B.1) becomes

$$F_m = (6-m)\ (n-1) - \sum_{i=1}^{6-m-1} (6-m-i)\ N_i \tag{B.2}$$

This is Kutbach's mobility equation which predicts the degree of freedom of motion in a mechanism in which all the loops has the same number of general constraints and there exists no passive freedoms and over-closed constraints (5,6). Equation (B.2) was reproposed by Artobolevskii, (10) and Dobrovolskii (11).

Let k be the number of the loops in a mechanism. Noting that

$$(n-1) + k = \sum_{i-1}^{5} N_i \tag{B.3}$$

and substituting into Equation (B.2) and rearranging we have

$$F_m = 6(n-1) - \sum_{i=1}^{6-m-1} (6-i)N_i + mk \tag{B.4a}$$

or

$$F_m = F_0 + mk \tag{B.4b}$$

In the case of the multi-loop mechanisms having different numbers of general constraints in different loops, Equation (B.4) takes the form

$$F_m = F_0 + M \tag{B.5}$$

where

$$M = \sum_{J=1}^{k} m_J$$

and m_J is the number of general constraints in the J^{th} loop.

Passive Freedoms

Passive freedoms are destroyed, or idled, freedoms of the pairs, due to certain geometric constraints [passive constraints]. Passive freedoms can never be utilized. However, in practice the passive freedoms and also the redundant freedoms are kept in the mechanisms rather than eliminating them by replacing the pairs with lower class pairs. This is preferred for ease in design, operation and lubrication. For example, the spherical pair is preferred in place of the slotted-sphere pair in the RSSR mechanism, the cylinder pair is preferred in place of a prism pair in the mechanisms of Figures 29a and d.

Any geometric constraint which produces passive freedoms in pairs is prone to introduce general constraints. Bruevich (104), Verhovskii (105), Bushgens (106) and Dimentberg (52,53) formulated general schemes for determining the conditions for the introduction of passive freedoms in the individual pairs of mechanisms and in particular the RCCC mechanism.

Passive freedoms have no effect on the motion transmission by the mechanism and are deductable from the total degree of freedom of motion given by Equation (B.5). Then the equation of mobility becomes

$$F = F_0 + M - F_p \qquad \text{(B.6)}$$

where

$$F_p = \sum_{i=1}^{N_p} f_i$$

and f_i is the number of passive freedoms on the i^{th} pair, N_p is the number of pairs in the mechanism.

In the case of that $M = km_k$, Equation (B.6) may be written as

$$F = (6-m)(n-1) - \sum_{i=1}^{N_p} [6 - m - (i - f_i)] \qquad (B.7a)$$

or

$$F = b(n-1) - \sum_{i=1}^{N_p} (b - h_i) \qquad (B.7b)$$

where i is the number of freedoms on the i^{th} pair, b = 6-m is the existing general motion components in the displacements of the mechanism, and $h_i = i - f_i$ is the number of the active freedoms on the i^{th} pair. Equation (B.7) is the Kutzbach's mobility equation, and is applicable when all the loops in the mechanism have the same number of general constraints.

Kraus (7,8,9) developed his number synthesis scheme for plane and space mechanisms using Equation (B.7).

As an example, consider the two-loop six-bar mechanism shown in Figure 30, where $m_1 = 2$, $m_2 = 1$, $N_1 = 4$, $N_2 = 2$, and $N_3 = 1$. Note that geometric constraints introduce one rotational passive freedom on the spherical pair as shown, $F_p = 1$. Using Equation (B.6), the degree of freedom of motion of the mechanism is

$$F = 6(6-1) - 5(4) - 4(2) - 3(1) + 2 + 1 - 1 = 1$$

The RSRC mechanism shown in Figure 5 has a passive freedom on the spherical pair when $\delta_0 = 0$. This passive freedom introduces one general constraint on the rotation. In this case the RSRC mechanism functions as the RS_sRC mechanism shown in Figure 31, where the axis of the rotation freedom of the slotted-sphere pair about the pin axis traces a cone surface which has slant angle λ_0 and axis parallel to the input

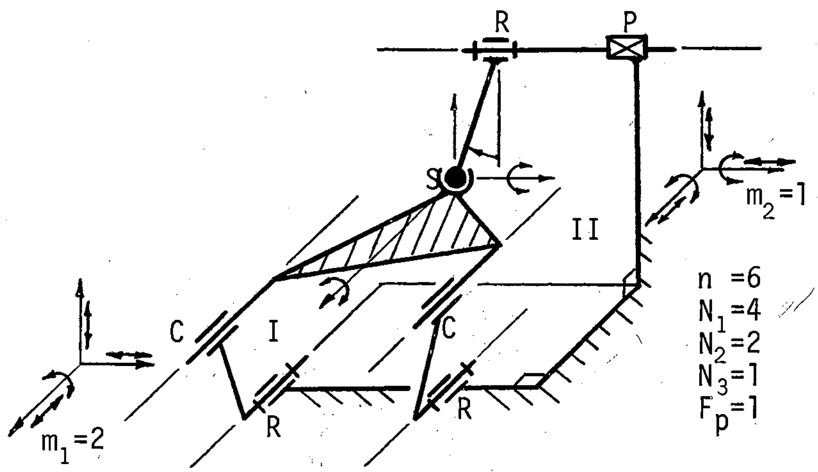


Figure 30. Two-Loop Six-Bar Space Mechanism with $m_1 = 2$, $m_2 = 1$, $F_p = 1$.

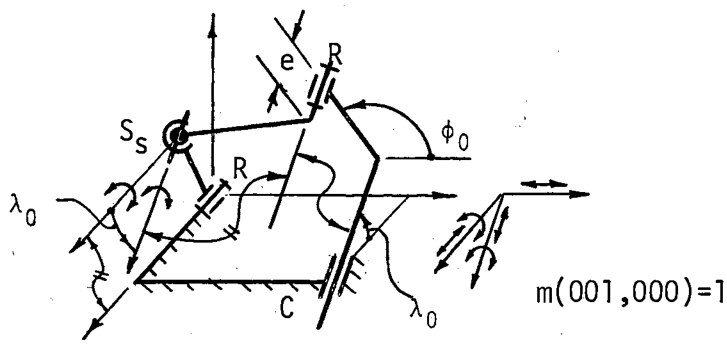


Figure 31. The RS_sRC Mechanism with m=1, which is an RSRS Mechanism Having $\delta_o = 0°$ with a Passive Rotation Freedom on the Spherical Pair.

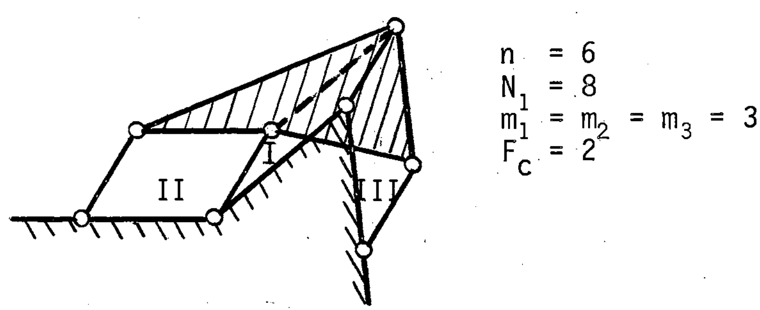


Figure 32. Three-Loop Plane Parallelogram Mechanism of $F_c = 2$.

pair axis.

Overclosing Constraints

In multi-loop mechanisms certain geometric conditions, i.e., symmetry in the geometry and dimensions, produce overclosing constraints. If the number of overclosing constraints is defined by $F_c = \sum_{J=1}^{k} q_J$, where q_J is the number of overclosing constraints in the J^{th} loop, the mobility equation in general becomes

$$F = F_0 + M - F_p + F_c \tag{B.8}$$

In a k-loop series mechanisms $F_c = k - 1$. A k-loop plane parallelogram mechanism, such as the one shown in Figure 32 and Roberts' ten-bar plane mechanism have $F_c = k - 1$.

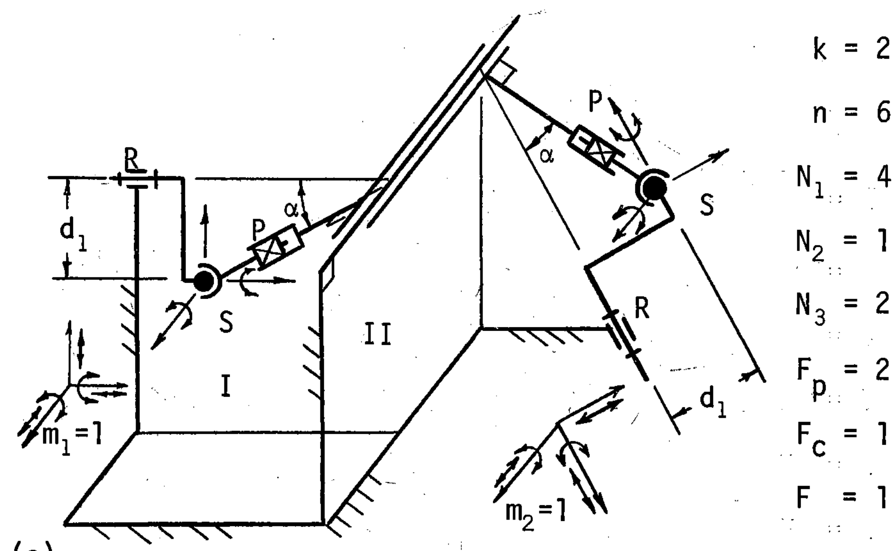
Figure 33 shows two multi-loop series space mechanisms having the overclosing constraints defined by $F_c = k - 1$. The two-loop six-bar mechanism in Figure 33a has one overclosing constraint and two passive rotation freedoms, one on each spherical pair. The three-loop eight-bar mechanism in Figure 33b has two overclosing constraints. Each loop in both mechanisms has one general constraint on rotation. These two mechanisms were derived from Altman's mechanisms (107, refer to Figures 26, 27, 28, 29 , 43, 50).

Equation (B.8) is the most general form of the mobility equation. Kolchin (16,17,36) used Equation (B.8) in his classification scheme, by writing it in the form

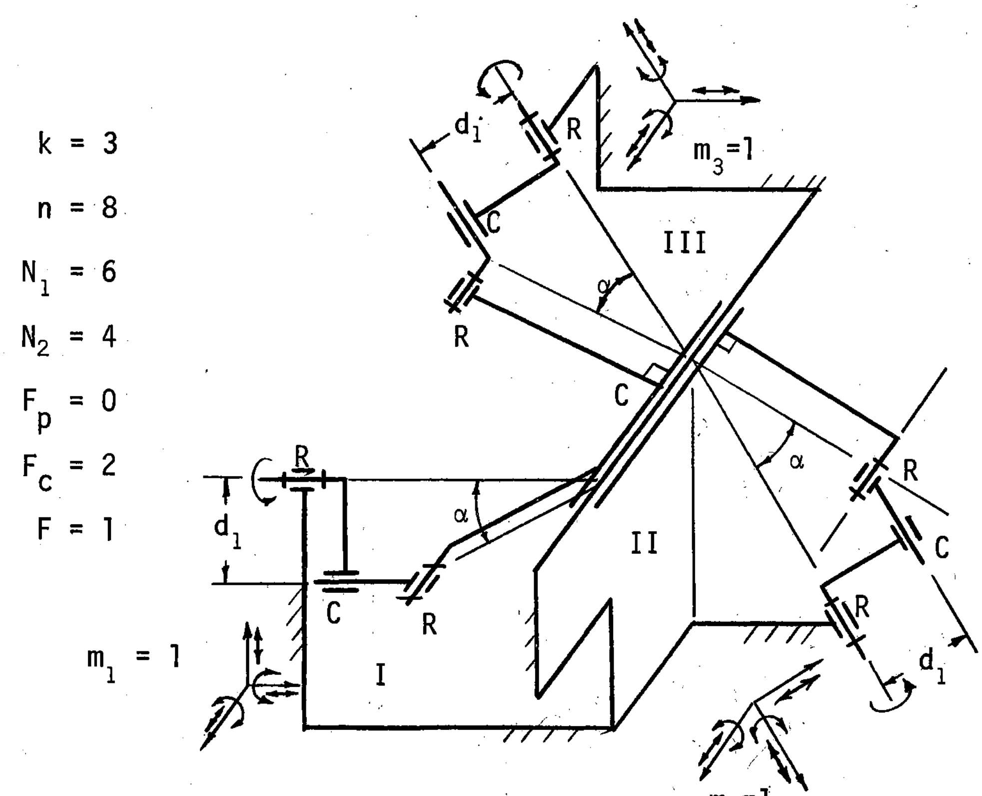
$$F = F_0 + H \tag{B.9}$$

where

$$H = M - F_p + F_c \tag{B.10}$$



(a)



(b)

Figure 33. Multi-Loop Space Mechanisms With Overclosing Constraints, (a) Two-Loop Six-Bar Mechanism with F_c=1, (b) Three-Loop Eight-Bar Mechanism with F_c=2.

is the Kolchin's parameter.

Classification of Mechanisms

Kolchin classified mechanisms according to the number of general constraints. A mechanism with m general constraints is of Class m mechanism. He grouped the mechanisms according to the group symbol.

$$N_g = \frac{6 - \frac{H}{k}}{M/k} \qquad (B.11)$$

and named mechanisms in which $F_p = F_c = 0$ or $H = m$ as basic mechanisms, mechanisms in which $H < M$ as "unlimited mechanisms," mechanisms in which $H > M$ as "special mechanisms" (16,17,21,22). However, the latter two may be extended by naming the mechanisms in which $F_p = 0$, $F_c \neq 0$ and $H > M$ as overclosed mechanisms; mechanisms in which $F_p \neq 0$, $F_c = 0$ and $H < M$ as mechanisms with passive freedoms; mechanisms in which $F_c \neq 0$, $F_p \neq 0$, where $F_p > F_c$ and $H < M$ or $F_p = F_c$ and $H = M$ or $F_p < F_c$ and $H > M$, as overclosed mechanisms with passive freedoms. The RSPC, RS_sPC, RCCC, RCPP, 4P, RHP and RH_sP mechanisms shown in Figure 29 and the RS_sRC mechanism shown in Figure 31 are Basic Mechanisms. The two-loop six-bar mechanism shown in Figure 30 is a mechanism with passive freedom, the three-loop eight-bar mechanism shown in Figure 33b is an overclosed mechanism while the two-loop six-bar mechanism shown in Figure 33a is an overclosed mechanism with passive freedoms.

It is evident that a complete parametric study is needed to identify the number of passive freedoms, redundant freedoms, number of general constraints, number of overclosing constraints, and the axes of the three basic-screws along with their existing motion components. In the mechanisms having relatively simple geometry of motion, identification of these freedoms and constraints is relatively easy. However, developing mathematical tools to predict the components of the three basic-screws, the number of general constraints and the number of overclosing constraints in a mechanism having a complex geometry of motion is subject for further study.

APPENDIX C

COUPLER CURVE COORDINATES

The synthesis of the RSRC or any other mechanism for path generation and rigid body guidance in space using the variational principles requires the coordinates of the coupler point as the generated coordinates, as discussed in Chapter IV. However, the synthesis for path generation and rigid body guidance are not in the scope of this study, the coupler point coordinates for the RSRC mechanism are given below in order to furnish information for further extension of this study, and illustrate how the coordinates of a point are determined by using the 3x3 screw matrix.

A coupler point P has the coordinates f, g and h in the coupler frame $O_1\xi_1^{iv}\eta_1^{iv}\zeta_1^{iv}$, which is parallel to the $O_2'\xi_2'''\eta_2'''\zeta_2'''$ system in which the coordinates of the coupler point P are f, $-(d_2 - g)$ and h, as shown in Figure 5. The coordinates of the coupler point P in the OXYZ system are easily determined by transforming the unit vectors in the $P\xi_p\eta_p\zeta_p$ system into the OXYZ system through the path P - O_2 - O_3 - O by using the screw matrix $\hat{T}_L$ given in Equation (2.9), where the $P\xi_p\eta_p\zeta_p$ system is parallel to the OXYZ system. Thus,

$$\begin{bmatrix} 1 & -\varepsilon Z_p & \varepsilon Y_p \\ \varepsilon Z_p & 1 & -\varepsilon X_p \\ -\varepsilon Y_p & \varepsilon X_p & 1 \end{bmatrix} \begin{Bmatrix} 1 \\ 1 \\ 1 \end{Bmatrix} = \hat{T}_b \hat{T}_a \hat{T}_{\bar{s}} \hat{T}_{\bar{d}_3} \hat{T}_{e'} \hat{T}_{d_2'} \hat{T}_{h'} \begin{Bmatrix} 1 \\ 1 \\ 1 \end{Bmatrix} \qquad \text{(C.1)}$$

where X_p, Y_p and Z_p are the coordinates of the coupler point P in the OXYZ system; $\hat{T}_b$, $\hat{T}_a$, $\hat{T}_{\overline{S}}$ and $\hat{T}_{\overline{d}_3}$ are defined with Equation (3.53);

$$\hat{T}_{e'} = \begin{bmatrix} 1 & -\varepsilon(e+f)B_{31} & \varepsilon(e+f)B_{21} \\ \varepsilon(e+f)B_{31} & 1 & -\varepsilon(e+f)B_{11} \\ -\varepsilon(e+f)B_{21} & \varepsilon(e+f)B_{11} & 1 \end{bmatrix}$$

$$\hat{T}_{d_2'} = \begin{bmatrix} 1 & \varepsilon(d_2-g)B_{32} & -\varepsilon(d_2-g)B_{22} \\ -\varepsilon(d_2-g)B_{32} & 1 & \varepsilon(d_2-g)B_{12} \\ \varepsilon(d_2-g)B_{22} & -\varepsilon(d_2-g)B_{12} & 1 \end{bmatrix}$$

and

$$\hat{T}_h = \begin{bmatrix} 1 & -\varepsilon hB_{33} & \varepsilon hB_{23} \\ \varepsilon hB_{33} & 1 & -\varepsilon hB_{13} \\ -\varepsilon hB_{23} & \varepsilon hB_{13} & 1 \end{bmatrix}$$

where

$$[B] = T_{\lambda_0} \; T_{\phi_0} \; T_{\delta_0} \; T_{\chi_0}$$

as defined with Equation (3.53). The coordinates of the coupler point P are simply the sum of the corresponding elements in the matrices on the right side of Equation (C.1). Then,

$$\begin{aligned} X_p = {} & b - S\cos\lambda_0 + d_3 \sin\lambda_0 \sin\phi_0 + (d_2-g) [\sin\chi_0 (\sin\lambda_0 \cos\delta_0 \cos\phi_0 \\ & - \sin\delta_0 \cos\lambda_0) - \sin\lambda_0 \sin\phi_0 \cos\chi_0] + (e+f) (\cos\lambda_0 \cos\delta_0 \\ & + \sin\lambda_0 \sin\delta_0 \cos\phi_0) + h[\sin\lambda_0 \sin\phi_0 \sin\chi_0 \\ & + \cos\chi_0 (\sin\lambda_0 \cos\delta_0 \cos\phi_0 - \sin\delta_0 \cos\lambda_0] \end{aligned} \qquad (C.2)$$

$$Y_p = a + d_3 \cos\phi_0 - (d_2-g)(\cos\phi_0 \cos\chi_0 + \cos\delta_0 \sin\phi_0 \sin\chi_0) -$$
$$- (e+f) \sin\delta_0 \sin\phi_0 + h(\cos\phi_0 \sin\chi_0 - \cos\delta_0 \sin\phi_0 \cos\chi_0) \quad (C.3)$$

and

$$Z_p = S \cdot \sin\lambda_0 + d_3 \cos\lambda_0 \sin\phi_0 + (d_2-g)[\sin\chi_0(\sin\lambda_0 \sin\delta_0$$
$$+ \cos\lambda_0 \cos\delta_0 \cos\phi_0) - \cos\lambda_0 \sin\phi_0 \cos\chi_0]$$
$$+ (e+f)(\cos\lambda_0 \sin\delta_0 \cos\phi_0 - \sin\lambda_0 \cos\delta_0)$$
$$+ h[\cos\chi_0 (\cos\lambda_0 \cos\delta_0 \cos\phi_0 + \sin\lambda_0 \sin\delta_0)$$
$$+ \cos\lambda_0 \sin\phi_0 \sin\chi_0] \quad (C.4)$$

In many design problems the path of the coupler point relative to a coordinate system on the axis of a screw displacement, such as the $O_3\xi_3'\eta_3'\zeta_3'$ system whose ξ_3' axis is the axis of the output pair, may be desired. The coordinates of the coupler point P, ξ_{3p}', η_{3p}', ζ_{3p}', relative to the $O_3\xi_3'\eta_3'\zeta_3'$ system are given by Equation (C.1) with $\hat{T}_b$, $\hat{T}_a$ and T_{λ_0} as being unit matrices, $\hat{T}_{\bar{S}}$ is as defined with Equation (3.53) when $\lambda_0 = 0$. Thus,

$$\xi_{3p}' = -S - (d_2-g) \sin\delta_0 \sin\chi_0 + (e+f) \cos\delta_0 - h \sin\delta_0 \cos\chi_0 \quad (C.5)$$

$$\eta_{3p}' = d_3 \cos\phi_0 - (d_2-g)(\cos\phi_0 \cos\chi_0 + \cos\delta_0 \sin\phi_0 \sin\chi_0$$
$$- (e+f) \sin\delta_0 \sin\phi_0 + h(\cos\phi_0 \sin\chi_0 - \cos\delta_0 \sin\phi_0 \cos\chi_0) \quad (C.6)$$

$$\zeta_{3p}' = d_3 \sin\phi_0 + (d_2-g)(\cos\delta_0 \cos\phi_0 \sin\chi_0 - \sin\phi_0 \cos\chi_0)$$
$$+ (e+f) \sin\delta_0 \cos\phi_0 + h(\cos\delta_0 \cos\phi_0 \cos\chi_0 + \sin\phi_0 \sin\chi_0) \quad (C.7)$$

The coordinates of the coupler point P given by Equations (C.2) through (C.7) are valid for a series of mechanisms such as the RSRC, RSPC, RSHC, RSCR, RSCP, RSCH, and RCCC spatial mechanisms, keeping in mind that χ_0 is a constant in the RSPC mechanism, while e is constant in the RSRC mechanism, and δ_0 is $-\delta_0$ in the RCCC mechanism and its constrained inversions shown in Figure 29.

Coupler point coordinates can be computed by the SUBROUTINE COUPLR in the digital computer program, Program A, given in Appendix F. The program computes the coupler point coordinates as given by Equations (C.2) through (C.7) and prints out. The coupler curves can be plotted, if desired, by calling the PLOT subroutine. The three projections of a coupler curve on the X - Y, X - Z and Y - Z planes are plotted on the same page by unfolding the planes of projection onto the Y - Z plane, where the +X axes of the X - Z plane and the X - Y plane overlie the +Y axis and the -Z axes of the Y - Z plane, respectively. A similar plot is given for the projections of the coupler curve in the $O_3\xi_3'\eta_3'\zeta_3'$ system.

APPENDIX D

GEOMETRIC PROPERTIES FOR THE LIMIT POSITIONS AND INSTANTANEOUS DWELLS IN THE OUTPUT DISPLACEMENT OF THE RSRC SPACE MECHANISM

The exact generation of vanishing velocities and exact generation of instantaneous dwells at specified values of the input parameter are considered to be the constraining conditions in the variational problems of the synthesis of the RSRC mechanism for screw generation by the displacements of its output link investigated in Chapter V, where parameters of constraints are defined to replace the equations of constraints utilizing the facts summarized in this appendix. These parameters of constraints are also used to generate the range of the displacement upon which they impose constraints for instantaneous dwells.

Exact dwell in a displacement component of a link is virtually impossible. Exact dwell for a certain duration occurs when the derivatives of the displacement of all orders with respect to the input parameter vanish. This means having partial passive freedom in the pair in the duration of the dwell, and discontinuities in the higher order derivatives. Hence, an exact dwell in any component of the output displacement occurs when the freedom of the output pair permitting this component of the output displacement is passive. Then the resulting mechanism is a constrained inversion.

Instantaneous dwells are merely approximate dwells. Instantaneous dwells in the displacements of a link occur at the limits of its displacements, where the link stops instantaneously. The link may rest at such a position with or without inertia force. If the second derivative of the displacement with respect to the input parameter vanishes at the limit position, the link rests with no inertia force at that position. Here it is considered that input crank has constant speed.

The limit positions in a displacement are defined by the vanishing derivatives of the displacement with respect to the input parameter. The derivative will vanish at the maxima, maxima and at the points of inflection along the displacement. A maximum or minimum in the displacement occurs when the odd order derivatives (1^{st}, 3^{rd}, etc.) vanish. An inflection in the displacement occurs when the even order derivatives (2^{nd}, 4^{th}, etc.) vanish.

As defined by Harding (108) a link is at the state of the first degree hesitation [first order instantaneous dwell] at a limit position if only the first derivative of the displacement of the link vanishes at that position. At such a position the link rests with inertia force. If n consecutive derivatives of the displacement vanish at the limit position it is an n^{th} degree hesitation [n^{th} order instantaneous dwell]. Exact dwell, then, is an infinite order instantaneous dwell. The double-crank-RSRC mechanism whose displacements are given in Figure 10 has a second order instantaneous dwell in the output rotation when $\theta_0 = 220.33°$. The RSRC mechanism whose displacements are given in Figure 9 has a third order instantaneous dwell in the output rotation when $\theta_0 = 270°$.

The order of the instantaneous dwell depends on the degree of conformity between the curvature of the input-crank-ellipse and the coupler-ellipse as it is shown in the following discussion.

The limit positions for the dual output displacement of the RSRC mechanism must satisfy the conditions

$$\frac{d\hat{\phi}}{d\theta_0} = 0 \qquad \text{or} \qquad \frac{d\phi_0}{d\theta_0} = 0 \ , \ \frac{ds}{d\theta_0} = 0 \tag{D.1}$$

If both of these conditions are satisfied simultaneously, the limits of the rotation and translation take place simultaneously. From Equations (3.77) and (3.79) it follows that at the limit positions of the output rotation

$$\begin{aligned} W_2L_1 + W_1L_2 \cos^2\delta_0 &= 0 \\ W_2M_1 + W_1M_2 \cos^2\delta_0 &\neq 0 \end{aligned} \tag{D.2}$$

where M_1, M_2, L_1, L_2 are identified with Equation (3.78), W_1 and W_2 are identified with Equation (3.59). The second condition states that the input crank must not have a limit position at the same time as the output crank rotation. That is, the mechanism must not be at the locking position. At the limit positions of the output translation, the geometry satisfies

$$[M_2(W_2L_1 + \cos^2\delta_0 W_1L_2) + L_2(W_2M_1 + \cos^2\delta_0 W_1M_2]\sin\delta_0 =$$

$$\sin\lambda_0 \cos\theta_0 \tan\chi_0 (W_2M_1 + \cos^2\delta_0 W_1M_2) \tag{D.3}$$

along with the second condition in Equation (D.2). After necessary substitutions, Equation (D.2) reduces to

$$\frac{\tan\theta_0}{\cos\lambda_0} = -\frac{W_2 \cos\phi_0 - W_1 \cos^2\delta_0 \sin\phi_0}{W_2 \sin\phi_0 + W_1 \cos^2\delta_0 \cos\phi_0} \tag{D.4a}$$

or

$$\frac{\tan\theta_0}{\cos\lambda_0} = - \frac{[N_0 \sin^2\delta_0 \cos\phi_0 \sin\phi_0 - M_0(\cos^2\phi_0 + \cos^2\delta_0 \sin^2\phi_0) - e \sin\delta_0 \cos\phi_0 - d_3 \cos^2\delta_0 \sin\phi_0]}{[N_0(\sin^2\phi_0 + \cos^2\delta_0 \cos^2\phi_0) - M_0 \sin^2\delta_0 \cos\phi_0 \sin\phi_0 - e \sin\delta_0 \sin\phi_0 + d_3 \cos^2\delta_0 \cos\phi_0]} \quad (D.4)$$

Equation (D.4) is the slope, $\tan \beta_{ni}$, of the normal N_i to the input-crank-ellipse at the location of the spherical pair, as shown in Figure 34, where the entire mechanism is projected onto a plane normal to the output pair axis.

Now consider the position of the coupler ellipse relative to the position of the output link as shown in Figure 35. The point P is the location of the spherical pair on the coupler-ellipse. The coordinates of the point P in the $O_2'x'y'$ system are

$$x_p' = -d_2 \cos\chi_0$$
$$y_p' = d_2 \cos\delta_0 \sin\chi_0$$

The slope of the normal N_c to the coupler-ellipse at point P, in the $O_2'x'y'$ system is

$$\tan \beta_{nc}' = - \frac{d_2^2 \, y_p'}{(d_2 \cos\delta_0)^2 \, x_p'} = - \frac{\tan\chi_0}{\cos\delta_0} \quad (D.5)$$

then the slope of this normal relative to the X axis is

$$\tan \beta_{nc} = - \frac{1}{\tan(\phi_0 + \gamma_{tc}')} = - \frac{\cos\phi_0 \sin\chi_0 - \sin\phi_0 \cos\delta_0 \cos\chi_0}{\sin\phi_0 \sin\chi_0 + \cos\delta_0 \cos\chi_0 \cos\phi_0} \quad (D.6a)$$

or

$$\tan \beta_{nc} = - \frac{W_2 \cos\phi_0 - W_1 \cos^2\delta_0 \sin\phi_0}{W_2 \sin\phi_0 + W_1 \cos^2\delta_0 \cos\phi_0} \quad (D.6)$$

This is also the slope, $\tan \beta_{ni}$, of the normal N_i to the input-crank-

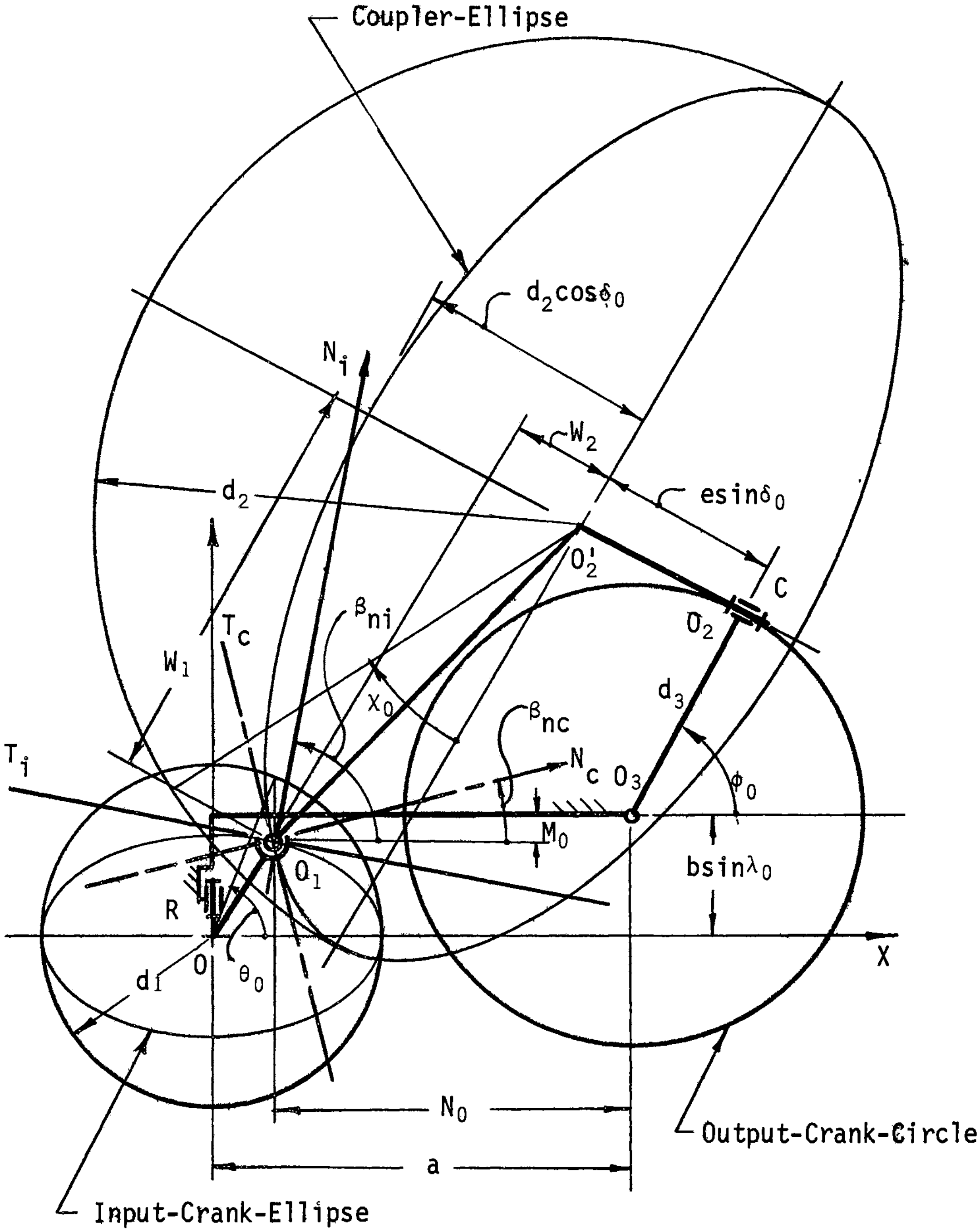


Figure 34. The RSRC Space Mechanism Projected onto a Plane Normal to the Output Pair Axis

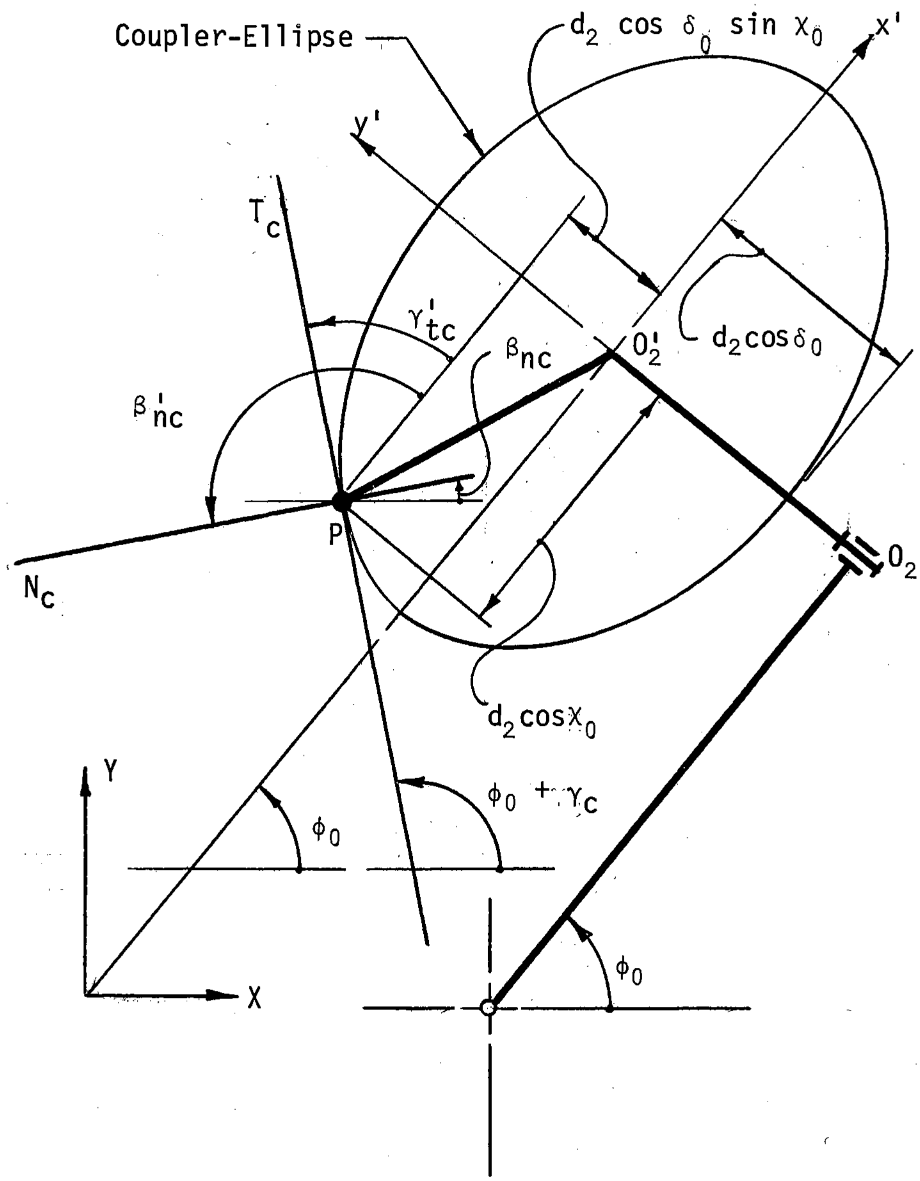


Figure 35. The Slope of the Normal to the Coupler-Ellipse at the Spherical Pair Location

ellipse as given by Equation (D.4).

Therefore, the output crank is at its rotation limits when the input-crank-ellipse and the coupler-ellipse, each obtained when the entire mechanism is projected onto a plane normal to the output pair axis, are tangent to each other.

Examining Equations (D.4) and (3.63) it can be stated that an RSRC mechanism may have up to eight limit positions in its output rotation, and so eight values of input parameter at which first and higher order instantaneous dwells occur. The number of the limit positions depends on the dimensions of the mechanism. Figure 36a shows an RSRC mechanism at four limit positions of its rotation. When $\delta_0 = n\pi$, the coupler-ellipse is a circle of radius d_2, where the normal N_c is along the coupler link. Then the condition for the limit positions on the output rotation given by Equation (D.4) reduce to

$$\frac{\tan\theta_0}{\cos\lambda_0} = \tan(\phi_0 - \chi_0) \qquad \text{(D.7)}$$

resulting in at most four limit positions on rotation when the circumference of the input-crank-ellipse and the coupler-circle are tangent, as shown in Figure 36b. When $\delta_0 = \frac{n+1}{2}\pi$ the coupler ellipse is a plane parallel to the output pair axis. Then Equation (D.4) reduces to

$$\frac{\tan\theta_0}{\cos\lambda_0} = -\cot\phi_0 \qquad \text{(D.8)}$$

resulting in at most four limit positions on rotation, when the plane of the coupler-ellipse is tangent to the input-crank-ellipse as shown in Figure 36c.

The duration of the rest that output link experiences in its

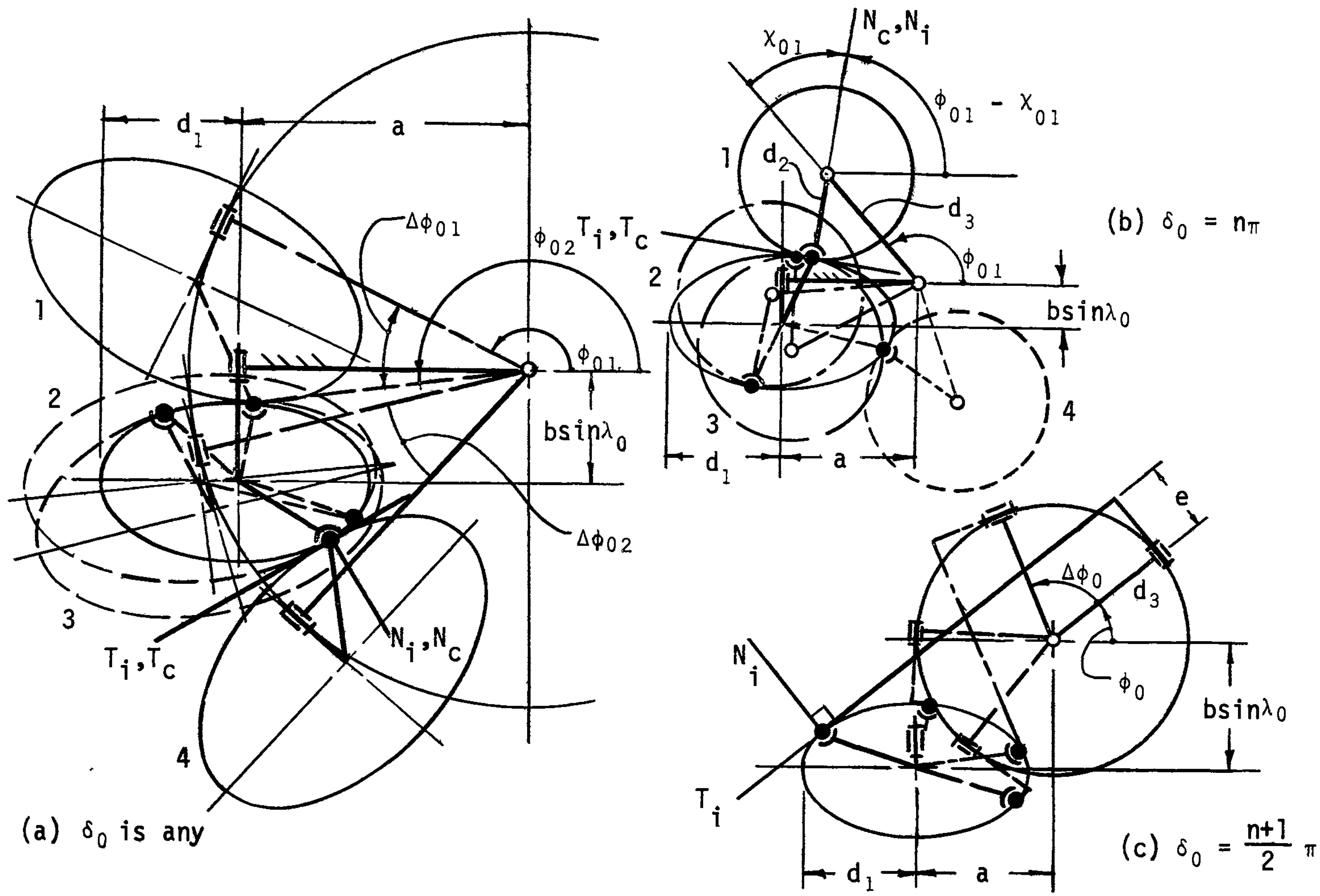


Figure 36. The RSRC Mechanisms at the Limit Positions of the Output Rotation; (a) δ_0 is any, (b) $\delta_0 = n\pi$, (c) $\delta_0 = (n+1)\pi/2$

rotation displacement at the limit positions depends on the degree of conformity between the radii of curvatures of the input-crank-ellipse and the coupler-ellipse. When the exteriors of the two ellipses are tangent at the limit position, that is, when the two ellipses have radii of curvatures of opposite sign at the point of contact the output link has a first order instantaneous dwell in rotation such as defined by the coupler-ellipses numbered 1 and 4 in Figure 36a and circles numbered 1 and 4 in Figure 36b. Higher order instantaneous dwells in the output rotation occur when the interior of one ellipse is tangent to the exterior of the other, that is, when the two ellipses have radii of curvatures of the same sign at the point of contact. One should observe that the output link in Figure 36a has an even order instantaneous dwell at the limit position determined by the coupler-ellipse numbered 2. Note that if the coupler link length d_2 in the RSRC mechanism shown in Figure 36b is chosen so that it concurs with the radius of curvature of the input-crank-ellipse at $\theta_0 = 270°$ a third order instantaneous dwell in the output rotation occurs as shown in Figure 9.

Generation of an exact dwell in the output rotation requires that the input-crank-ellipse and the coupler-ellipse must be of the same size and one overlies the other. Such a geometric condition [passive constraint] destroys the rotation freedom of the output pair and the output link experiences translation only. In this case the RSRC mechanism has one general constraint on rotation and functions as RS_RSP mechanism.

The geometry corresponding to the limit positions of the output translation is defined by the condition stated in Equation (D.3). Thus, after necessary substitutions

$$-\sin\lambda_0 \cot\theta_0 = \sin\delta_0 \cot\chi_0(\cos\phi_0 - \cos\lambda_0 \sin\phi_0 \cot\theta_0) \quad (D.9)$$

where the left side of Equation (D.9) is the slope of the tangent to the input-crank-ellipse, tan γ_t, relative to the X axis. The input-crank-ellipse, in this case, is the projection of the input-crank-circle onto the plane of the fixed link and the output pair axis, as shown in Figure 37. The right side of Equation (D.9) is the slope of the tangent to the coupler ellipse which is the projection of the coupler-link-circle onto the forementioned plane. Then, the limits of the output translation take place at the geometry when the input-crank-ellipse and the coupler-ellipse, each obtained when the entire mechanism is projected onto the plane of the fixed link and the output pair axis, are tangent to each other.

The right side of Equation (D.9) indicates a zero slope for the common tangent when $\delta_0 = n\pi$, and the limit positions for the output translation will be loacted at $\theta_0 = \pm 90°$. This is also observed in the displacement function given by Equation (3.60), since it reduces to a function of $\sin\theta_0$ when $\delta_0 = n\pi$.

The same conditions stated for the order of the instantaneous dwells in the output rotation can be stated for the order of the instantaneous dwells in the output translation. The set of dimensions of the mechanism which will cause the coupler ellipse to remain tangent to the input crank ellipse as the output link rotates will destroy the translation freedom of the output pair.

The limit positions of the output rotation and translation take place simultaneously when the conditions given by Equations (D.2) and (D.3) are satisfied simultaneously.

In the case of the 4R plane mechanism the higher order instantaneous

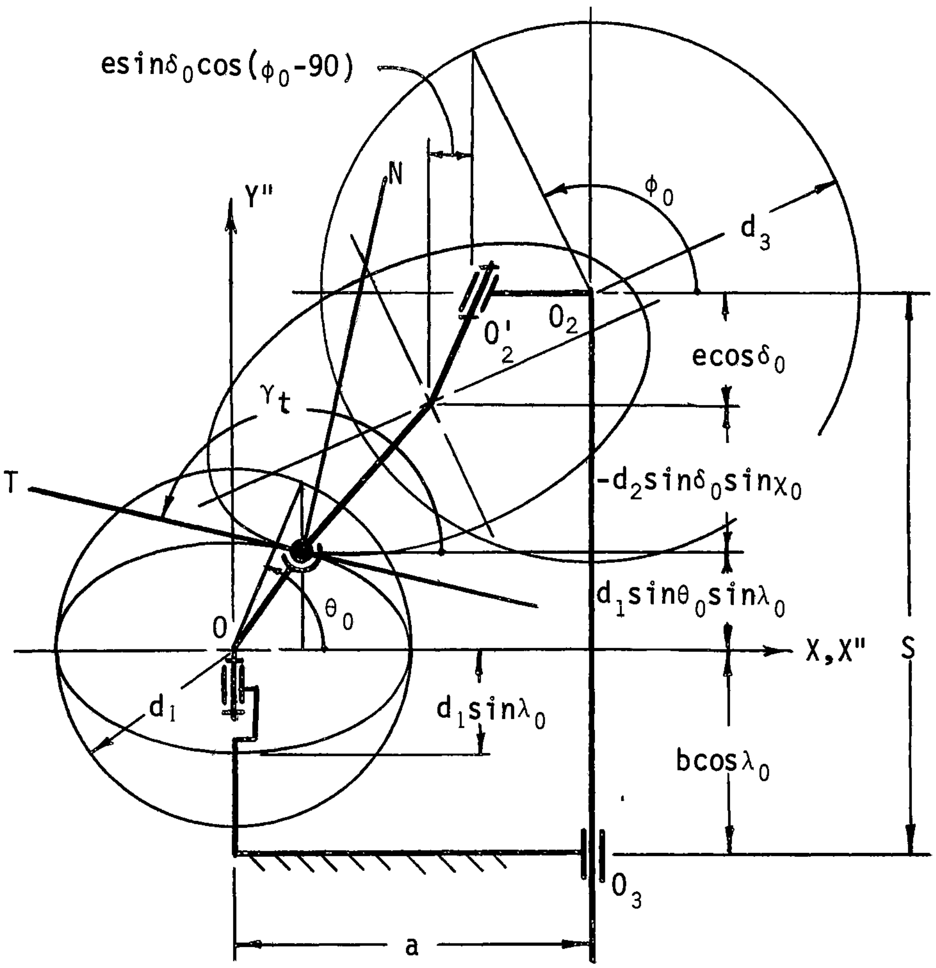


Figure 37. The RSRC Space Mechanism at the Limit Position of the Output Translation

dwells in the output rotation are generated when the curvatures of the input-crank-circle and the coupler-circle have the same sign. This occurs when the input crank is at its dead center position. Infinite order instantaneous dwell in the output rotation occurs when the input crank and the coupler link, and the fixed link and the output link are of the same size. When the output crank of this mechanism overlies the fixed link, the coupler-circle overlies the input-crank-circle, and input crank and the coupler link rotate about the input pair axis. Then the output link may be kept at rest for the whole cycle.

APPENDIX E

PARTIAL DERIVATIVES OF THE GENERATED SCREW DISPLACEMENT

The first and second partial derivatives of the rotation and translation components of the generated screw, ϕ_g and $\overline{S}_g$, with respect to the unknown dimensions x_r and x_t, needed to compute F_{r_0} and $\frac{\partial F_{r_0}}{\partial xt}$, and so to determine the matrices C and A in Equation (4.29), are given below. These derivatives are obtained by differentiating the dispalcement functions for χ_g, $\overline{S}_g$ and ϕ_g given by Equations (3.59), (3.60) and (3.62), with respect to the unknown dimensions. $x_r \equiv e$, d_1, d_2, d_3, a, b, δ_0, λ_0, θ_{01} for the unconstrained screw generation, x_r = e, d_1, d_2, q_1, q_2, δ_0, λ_0, θ_{01} for the constrained screw generation. Thus, letting $\phi_g = \phi_0$, $\overline{S}_g = S$,

$$\frac{\partial\phi_g}{\partial x_r} = \frac{\frac{\partial K}{\partial x_r} - \sin^2\phi_g \frac{\partial F_1}{\partial x_r} + \sin\phi_g \cos\phi_g \frac{\partial F_2}{\partial x_r} - \cos\phi_g \frac{\partial F_3}{\partial x_r} - \sin\phi_g \frac{\partial F_4}{\partial x_r}}{F_1 \sin 2\phi_g - F_2(\cos^2\phi_g - \sin^2\phi_g) - F_3 \sin\phi_g + F_4 \cos\phi_g} \tag{E.1}$$

$$F \cdot \frac{\partial^2\phi_g}{\partial x_t \partial x_r} = F\left\{\frac{\partial^2 K}{\partial x_t \partial x_r} - \sin^2\phi_g \frac{\partial^2 F_1}{\partial x_t \partial x_r} + \sin\phi_g \cos\phi_g \frac{\partial^2 F_2}{\partial x_t \partial x_r}\right.$$

$$- \cos\phi_g \frac{\partial^2 F_3}{\partial x_t \partial x_r} - \sin\phi_g \frac{\partial^2 F_4}{\partial x_t \partial x_r} + \frac{\partial\phi_g}{\partial x_t}\left[(\cos^2\phi_g - \sin^2\phi_g) \frac{\partial F_2}{\partial x_r}\right.$$

$$- \sin^2\phi_g \frac{\partial F_1}{\partial x_r} + \sin\phi_g \frac{\partial F_3}{\partial x_r} - \cos\phi_g \frac{\partial F_4}{\partial x_r} \Bigg] \Bigg\} - \left(\frac{\partial K}{\partial x_r} - \sin^2\phi_g \frac{\partial F_1}{\partial x_r} \right.$$

$$\left. + \sin\phi_g \cos\phi_g \frac{\partial F_2}{\partial x_r} - \cos\phi_g \frac{\partial F_3}{\partial x_r} - \sin\phi_g \frac{\partial F_4}{\partial x_r} \right) \Bigg\{ \sin^2\phi_g \frac{\partial F_1}{\partial x_t}$$

$$- (\cos^2\phi_g - \sin^2\phi_g) \frac{\partial F_2}{\partial x_t} - \sin\phi_g \frac{\partial F_3}{\partial x_t} + \cos\phi_g \frac{\partial F_4}{\partial x_t}$$

$$+ \frac{\partial \phi_g}{\partial x_t} \left[2F_1(\cos^2\phi_g - \sin^2\phi_g) + 2F_2 \sin 2\phi_g - F_3 \cos\phi_g \right.$$

$$\left. - F_4 \sin\phi_g \right] \Bigg\} \qquad r,t = 1,2,3,\ldots,n \qquad \text{(E.2)}$$

$F = F_1 \sin^2\phi_g - F_2(\cos^2\phi_g - \sin^2\phi_g) - F_3 \sin\phi_g + F_4 \cos\phi_g$
and F_1, F_2, F_3, F_4 are defined with Equation (3.62). Then,

$$\frac{\partial F_1}{\partial x_r} = (N_0^2 - M_0^2) \frac{\partial(\sin^2\delta_0)}{\partial x_r} + 2 \sin^2\delta_0 \left(N_0 \frac{\partial N_0}{\partial x_r} - M_0 \frac{\partial M_0}{\partial x_r} \right) \qquad \text{(E.3)}$$

$$\frac{\partial^2 F_1}{x_t \partial x_r} = (N_0^2 - M_0^2) \frac{\partial^2(\sin^2\delta_0)}{\partial x_t \partial x_r} + 2 \left(N_0 \frac{\partial N_0}{\partial x_r} - M_0 \frac{\partial M_0}{\partial x_t} \right) \frac{\partial(\sin^2\delta_0)}{\partial x_r}$$

$$+ 2 \left(N_0 \frac{\partial N_0}{\partial x_r} - M_0 \frac{\partial M_0}{\partial x_r} \right) \frac{\partial(\sin^2\delta_0)}{\partial x_t} + 2 \sin^2\delta_0 \left(\frac{\partial N_0}{\partial x_t} \frac{\partial N_0}{\partial x_r} \right.$$

$$\left. + N_0 \frac{\partial^2 N_0}{x_t \partial x_r} - M_0 \frac{\partial^2 M_0}{\partial x_t \partial x_r} - \frac{\partial M_0}{\partial x_t} \frac{\partial M_0}{\partial x_r} \right) \qquad \text{(E.4)}$$

$$\frac{\partial F_2}{\partial x_r} = 2 M_0 N_0 \frac{\partial(\sin^2\delta_0)}{\partial x_r} + 2 \sin^2\delta_0 \left(N_0 \frac{\partial M_0}{\partial x_r} + M_0 \frac{\partial N_0}{\partial x_r} \right) \qquad \text{(E.5)}$$

$$\frac{\partial^2 F_2}{x_t \partial x_r} = 2 M_0 N_0 \frac{\partial^2 \sin^2\delta_0}{\partial x_t \partial x_r} + 2 \left(N_0 \frac{\partial M}{\partial x_t} + M_0 \frac{\partial N_0}{\partial x_t} \right) \frac{\partial(\sin^2\delta_0)}{\partial x_r} + 2 \left(N_0 \frac{\partial M_0}{\partial x_r} \right.$$

$$\left. + M_0 \frac{\partial N_0}{\partial x_r} \right) \frac{\partial(\sin^2\delta_0)}{\partial x_t} + 2 \sin^2\delta_0 \left(\frac{\partial N_0}{\partial x_t} \frac{\partial M_0}{\partial x_r} + \frac{\partial M_0}{\partial x_t} \frac{\partial N_0}{\partial x_r} \right.$$

$$+ N_0 \frac{\partial^2 M_0}{\partial x_t \partial x_r} + M_0 \frac{\partial^2 N_0}{\partial x_t \partial x_r}\Bigg) \tag{E.6}$$

$$\frac{\partial F_3}{\partial x_r} = 2\Bigg[N_0 \cos^2\delta_0 \frac{\partial d_3}{\partial x_r} + d_3\left(\cos^2\delta_0 \frac{\partial N_0}{\partial x_r} + N_0 \frac{\partial(\cos^2\delta_0)}{\partial x_r}\right) + M_0 \sin\delta_0 \frac{\partial e}{\partial x_r}$$

$$+ e\left(\sin\delta_0 \frac{\partial M_0}{\partial x_r} + M_0 \frac{\partial(\sin\delta_0)}{\partial x_r}\right)\Bigg] \tag{E.7}$$

$$\frac{\partial^2 F_3}{\partial x_t \partial x_r} = 2\Bigg[N_0 \cos^2\delta_0 \frac{\partial^2 d_3}{\partial x_t \partial x_r} + \left(\cos^2\delta_0 \frac{\partial N_0}{\partial x_t} + N_0 \frac{\partial(\cos^2\delta_0)}{\partial x_t}\right)\frac{\partial d_3}{\partial x_r}$$

$$+ \left(\cos^2\delta_0 \frac{\partial N_0}{\partial x_r} + N_0 \frac{\partial(\cos^2\delta_0)}{\partial x_r}\right)\frac{\partial d_3}{\partial x_t} + d_3\Bigg(\frac{\partial N_0}{\partial x_r} \frac{\partial(\cos^2\delta_0)}{\partial x_t}$$

$$+ \cos^2\delta_0 \frac{\partial^2 N_0}{\partial x_t \partial x_r} + \frac{\partial N_0}{\partial x_t} \frac{\partial(\cos^2\delta_0)}{\partial x_r} + N_0 \frac{\partial^2(\cos^2\delta_0)}{\partial x_t \partial x_r}\Bigg)$$

$$+ e\Bigg(\frac{\partial M_0}{\partial x_r} \frac{\partial(\sin\delta_0)}{\partial x_t} + \frac{\partial M_0}{\partial x_t} \frac{\partial(\sin\delta_0)}{\partial x_r} + M_0 \frac{\partial^2(\sin\delta_0)}{\partial x_t \partial x_r}$$

$$+ \sin\delta_0 \frac{\partial^2 M_0}{\partial x_t \partial x_r}\Bigg) + \left(\sin\delta_0 \frac{\partial M_0}{\partial x_r} + M_0 \frac{\partial(\sin\delta_0)}{\partial x_r}\right)\frac{\partial e}{\partial x_t}\Bigg] \tag{E.8}$$

$$\frac{\partial F_4}{\partial x_r} = 2\Bigg[M_0 \cos^2\delta_0 \frac{\partial d_3}{\partial x_r} + d_3\left(\cos^2\delta_0 \frac{\partial M_0}{\partial x_r} + M_0 \frac{\partial(\cos^2\delta_0)}{\partial x_r}\right)$$

$$- N_0 \sin\delta_0 \frac{\partial e}{\partial x_r} - e\left(\sin\delta_0 \frac{\partial N_0}{\partial x_r} + N_0 \frac{\partial(\sin\delta_0)}{\partial x_r}\right)\Bigg] \tag{E.9}$$

$$\frac{\partial^2 F_4}{\partial x_t \partial x_r} = 2\Bigg[M_0 \cos^2\delta_0 \frac{\partial^2 d_3}{\partial x_t \partial x_r} + \left(\cos^2\delta_0 \frac{\partial M_0}{\partial x_t} + M_0 \frac{\partial(\cos^2\delta_0)}{\partial x_t}\right)\frac{\partial d_3}{\partial x_r}$$

$$+ \left(\cos^2\delta_0 \frac{\partial M_0}{\partial x_r} + M_0 \frac{\partial(\cos^2\delta_0)}{\partial x_r}\right)\frac{\partial d_3}{\partial x_t} + d_3\Bigg(\frac{\partial M_0}{\partial x_r} \frac{\partial(\cos^2\delta_0)}{\partial x_t}$$

$$+ \cos^2\delta_0 \frac{\partial^2 M_0}{\partial x_t \partial x_r} + \frac{\partial M_0}{\partial x_t} \frac{\partial(\cos^2\delta_0)}{\partial x_r} + M_0 \frac{\partial^2(\cos^2\delta_0)}{\partial x_t \partial x_r}\Bigg)$$

$$- \left(\sin \delta_0 \frac{\partial N_0}{\partial x_t} + N_0 \frac{\partial(\sin\delta_0)}{\partial x_t} \right) \frac{\partial e}{\partial x_r} - \left(\sin\delta_0 \frac{\partial N_0}{\partial x_r} \right.$$

$$\left. + N_0 \frac{\partial(\sin\delta_0)}{\partial x_r} \right) \frac{\partial e}{\partial x_t} - e \left(\frac{\partial N_0}{\partial x_r} \frac{\partial(\sin\delta_0)}{\partial x_t} + \frac{\partial N_0}{\partial x_t} \frac{\partial \sin\delta_0}{\partial x_r} \right.$$

$$\left. \left. + \sin\delta_0 \frac{\partial^2 N_0}{\partial x_t \partial x_r} + N_0 \frac{\partial^2(\sin\delta_0)}{\partial x_t \partial x_r} \right) \right] \qquad \text{(E.10)}$$

$$\frac{\partial K}{\partial r} = (d_2^2 - d_3^2 - N_0^2) \frac{\partial(\cos^2\delta_0)}{\partial x_r} - e^2 \frac{\partial(\sin^2\delta_0)}{\partial x_r} + 2 \cos^2\delta_0 \left(d_2 \frac{\partial d_2}{\partial x_r} \right.$$

$$\left. - d_3 \frac{\partial d_3}{\partial x_r} - N_0 \frac{\partial N_0}{\partial x_r} \right) - 2 M_0 \frac{\partial M_0}{\partial x_r} - 2 e \sin^2\delta_0 \frac{\partial e}{\partial x_r} \qquad \text{(E.11)}$$

$$\frac{\partial^2 K}{\partial x_t \partial x_r} = (d_2^2 - d_3^2 - N_0^2) \frac{\partial^2(\cos^2\delta_0)}{\partial x_t \partial x_r} + 2 \left(d_2 \frac{\partial d_2}{\partial x_t} - d_3 \frac{\partial d_3}{\partial x_t} \right.$$

$$\left. - N_0 \frac{\partial N_0}{\partial x_t} \right) \frac{\partial(\cos^2\delta_0)}{\partial x_r} - 2 e \frac{\partial e}{\partial x_t} \left(\frac{\partial(\cos^2\delta_0)}{\partial x_r} + \frac{\partial e}{\partial x_r} \frac{\partial(\sin^2\delta_0)}{\partial x_t} \right)$$

$$- e^2 \frac{\partial^2(\sin^2\delta_0)}{\partial x_t \partial x_r} + 2 \left(d_2 \frac{\partial d_2}{\partial x_r} - d_3 \frac{\partial d_3}{\partial x_r} - N_0 \frac{\partial N_0}{\partial x_r} \right) \frac{\partial \cos^2\delta_0}{\partial x_t}$$

$$+ 2 \cos^2\delta_0 \left(\frac{\partial d_2}{\partial x_t} \frac{\partial d_2}{\partial x_r} - \frac{\partial d_3}{\partial x_t} \frac{\partial d_3}{\partial x_r} - d_3 \frac{\partial^2 d_3}{\partial x_t \partial x_r} - \frac{\partial N_0}{\partial x_t} \frac{\partial N_0}{\partial x_r} \right.$$

$$\left. - N_0 \frac{\partial^2 N_0}{\partial x_t \partial x_r} \right) - 2 \left(\frac{\partial M_0}{\partial x_t} \frac{\partial M_0}{\partial x_r} + M_0 \frac{\partial^2 M_0}{\partial x_t \partial x_r} \right.$$

$$\left. + \sin^2\delta_0 \frac{\partial e}{\partial x_t} \frac{\partial e}{\partial x_r} \right) \qquad \text{(E.12)}$$

$$\frac{\partial \overline{S}_g}{\partial x_r} = \cos\lambda_0 \frac{\partial b}{\partial x_r} + \sin\lambda_0 \frac{\partial}{\partial r} (d_1 \sin\theta_0) + \frac{\partial}{\partial r} (e \cos\delta_0)$$

$$- \sqrt{d_2^2 - W_1^2} \frac{\partial \sin\delta_0}{\partial x_r} - \frac{\sin\delta_0}{2\sqrt{d_2^2 - W_1^2}} \frac{\partial}{\partial x_r} (d_2^2 - W_1^2) \qquad \text{(E.13)}$$

$$\frac{\partial^2 \overline{S}_g}{\partial x_t \partial x_r} = \cos\lambda_0 \frac{\partial^2 b}{\partial x_t \partial x_r} + \sin\lambda_0 \frac{\partial^2 (d_1 \sin\theta_0)}{\partial x_t \partial x_r} + \frac{\partial^2 (e \cos\delta_0)}{\partial x_t \partial x_r}$$

$$- \sqrt{d_2^2 - W_1^2} \frac{\partial^2 (\sin\delta_0)}{\partial x_t \partial x_r} - \frac{1}{2\sqrt{d_2^2 - W_1^2}} \left[\frac{\partial (\sin\delta_0)}{\partial x_r} \frac{\partial (d_2^2 - W_1^2)}{\partial x_t} \right.$$

$$\left. + \frac{\partial (\sin\delta_0)}{\partial x_t} \frac{\partial (d_2^2 - W_1^2)}{\partial} \right] - \frac{\sin\delta_0}{4(d_2^2 - W_1^2)^{3/2}} \left[2(d_2^2 - W_1^2) \right.$$

$$\left. \cdot \frac{\partial (d_2^2 - W_1^2)}{\partial x_t \partial x_r} - \frac{\partial (d_1^2 - W_2^2)}{\partial x_t} \frac{\partial (d_2^2 - W_1^2)}{\partial x_r} \right] \qquad (E.14)$$

where

$$\frac{\partial W_1}{\partial x_r} = \sin\phi_g \frac{\partial M_0}{\partial x_r} + \cos\phi_g \frac{\partial N_0}{\partial x_r} + \frac{\partial d_3}{\partial x_r} + (M_0 \cos\phi_g - N_0 \sin\phi_g) \frac{\partial \phi_g}{\partial x_r} \qquad (E.15)$$

$$\frac{\partial^2 W_1^2}{\partial x_t \partial x_r} = \sin\phi_g \frac{\partial^2 M_0}{\partial x_t \partial x_r} + \cos\phi_g \frac{\partial^2 N_0}{\partial x_t \partial x_r} + \left(\cos\phi_g \frac{\partial M_0}{\partial x_t} - \sin\phi_g \frac{\partial N_0}{\partial x_t} \right) \frac{\partial \phi_g}{\partial x_r}$$

$$- (\sin\phi_g M_0 + \cos\phi_g N_0) \frac{\partial \phi_g}{\partial x_t} \frac{\partial \phi_g}{\partial x_r} + \left(\cos\phi_g \frac{\partial M_0}{\partial x_r} - \sin\phi_g \frac{\partial N_0}{\partial x_r} \right) \cdot$$

$$\frac{\partial \phi_g}{\partial x_t} + (M_0 \cos\phi_g - N_0 \sin\phi_g) \frac{\partial^2 \phi_g}{\partial x_t \partial x_r} \qquad (E.16)$$

$\frac{\partial (\cos\delta_0)}{\partial x_r} = \frac{\partial (\sin\delta_0)}{\partial x_r} = 0$ if $x_r \neq \delta_0$, $\frac{\partial x_r}{\partial x_t} = 0$ if $t \neq r$ and M_0 and N_0 are defined with Equation (3.59).

When $\delta_0 = 0$ the relationships given above are greatly simplified as listed below;

$$\frac{\partial \phi_g}{\partial x_r} = \left[\frac{\partial K}{\partial x_r} - 2(N_0 \cos\phi_g + M_0 \sin\phi_g) \frac{\partial d_3}{\partial x_r} - 2 d_3 \left(\cos\phi_g \frac{\partial N_0}{\partial x_r} \right. \right.$$

$$\left. \left. + \sin\phi_g \frac{\partial M_0}{\partial x_r} \right) \right] \Big/ 2 d_3 (M_0 \cos\phi_g - N_0 \sin\phi_g) \qquad (E.17)$$

$$\frac{\partial^2\phi_g}{\partial x_t \partial x_r} = \frac{1}{F}\left\{ \frac{\partial^2 K}{\partial x_t \partial x_r} - 2\,\frac{\partial d_3}{\partial x_r}\left(\cos\phi_g\,\frac{\partial N_0}{\partial x_t} + \sin\phi_g\,\frac{\partial M_0}{\partial x_t} + F\,\frac{\partial \phi_g}{\partial x_t}\right)\right.$$

$$- 2\,\frac{\partial d_3}{\partial x_t}\left(\cos\phi_g\,\frac{\partial N_0}{\partial x_r} + \sin\phi_0\,\frac{\partial M_0}{\partial x_r} + F\,\frac{\partial \phi_g}{\partial x_r}\right) - 2\,d_3\left[\cos\phi_g\,\frac{\partial^2 N_0}{\partial x_t \partial x_r}\right.$$

$$+ \sin\phi_g\,\frac{\partial^2 M_0}{\partial x_t \partial x_r} + \left(\cos\phi_g\,\frac{\partial M_0}{\partial x_r} - \sin\phi_g\,\frac{\partial N_0}{\partial x_r}\right)\frac{\partial \phi_g}{\partial x_t} + \left(\cos\phi_g\,\frac{\partial M_0}{\partial x_t}\right.$$

$$\left.- \sin\phi_g\,\frac{\partial N_0}{\partial x_t}\right)\frac{\partial \phi_g}{\partial x_r} - (M_0\,\sin\phi_g + N_0\,\cos\phi_g)\,\frac{\partial \phi_g}{\partial x_t}\,\frac{\partial \phi_g}{\partial x_r}\Bigg]$$

$$\left. - 2(N_0\,\cos\phi_g + M_0\,\sin\phi_g)\,\frac{\partial^2 d_3}{\partial x_t \partial x_r}\right\} \qquad \text{(E.18)}$$

where e is a predefined dimension and

$$F = M_0\,\cos\phi_0 - N_0\,\sin\phi_0 \qquad \text{(E.19)}$$

$$\frac{\partial K}{\partial x_r} = 2\left(d_2\,\frac{\partial d_2}{\partial x_r} - d_3\,\frac{\partial d_3}{\partial x_r} - N_0\,\frac{\partial N_0}{\partial x_r} - M_0\,\frac{\partial M_0}{\partial x_r}\right) \qquad \text{(E.20)}$$

$$\frac{\partial^2 K}{\partial x_t \partial x_r} = 2\left[\left(\frac{\partial d_2}{\partial x_r}\right)^2 - d_3\,\frac{\partial^2 d_3}{\partial x_t \partial x_r} - \frac{\partial d_3}{\partial x_t}\,\frac{\partial d_3}{\partial x_r} - N_0\,\frac{\partial^2 N_0}{\partial x_t \partial x_r} - \frac{\partial N_0}{\partial x_t}\,\frac{\partial N_0}{\partial x_r}\right.$$

$$\left. - M_0\,\frac{\partial^2 M_0}{\partial x_t \partial x_r} - \frac{\partial M_0}{\partial x_t}\,\frac{\partial M_0}{\partial x_r}\right] \qquad \text{(E.21)}$$

$$\frac{\partial \overline{S}_g}{\partial x_t} = \cos\lambda_0\,\frac{\partial b}{\partial x_r} + \sin\lambda_0\,\frac{\partial}{\partial x_r}\,(d_1\,\sin\theta_0) \qquad \text{(E.22)}$$

$$\frac{\partial^2 \overline{S}_g}{\partial x_t \partial x_r} = \cos\lambda_0\,\frac{\partial^2 b}{\partial x_t \partial x_r} + \sin\lambda_0\,\frac{\partial^2 (d_1\,\sin\theta_0)}{\partial x_t \partial x_r} \qquad \text{(E.23)}$$

APPENDIX F

DIGITAL COMPUTER PROGRAMS

The three digital computer programs used during this investigation are listed in the following pages.

The Program A is for the kinematic analysis of the RSRC mechanism. It computes the displacements ϕ_0, S, x_0, the velocities $\dot{\phi}_0$, $\dot{S}$, $\dot{x}_0$, the accelerations $\ddot{\phi}_0$, $\ddot{S}$, $\ddot{x}_0$, and the coupler curve coordinates X_p, Y_p, Z_p and ξ'_{3p}, η'_{3p}, ζ'_{3p} for the existing geometric inversions for a given set of dimensions. The computer plots any of these output if needed and specified in the data required by the card having statement number 50. The input data for the Program A are the dimensions of the mechanism, starting value of θ_0, increment in θ_0, angular input velocity if velocities and accelerations are needed, the coupler point coordinates, f, g, and h, in the coupler frame, if the coupler curve coordinates are needed, and the scale limits for the plots if plots are needed. Refer to the comment cards for the description of the input data.

The Program B is for the synthesis of the RSRC mechanism for the generation of unconstrained screw displacements by variational methods. The linearized equations of condition are solved for the errors in the initially assumed values for the unknown dimensions, by the matrix method of iteration. The desired screw displacement is either read as data or it is computed in the SUBROUTINE DSIRED. The correct function in the subroutine is picked by the parameter IV in the input data. The input data for the Program B are the initially assumed dimensions of the mechanism, the specified values for $\Delta\theta_0$, $\Delta\phi_0$, ΔS, the increment in θ to compute the values of θ at the design points if they are not read in as data, desired accuracy in the dimensions, number of iterations, and the characteristic number for the inversion which is $I_8 = I_9 = 1$ for the inversions defined by the positive signed radicals in the displacement equations, $I_8 = I_9 = 2$ for the inversions defined by the negative signed radicals. Read the comment cards for the description of the input data.

The Program C is for the synthesis of the RSRC mechanism for the generation of screws having constraints for instantaneous dwells, or no constraints. In this program the parameters of constraints are introduced as discussed in Section B of Chapter V, and the minimum for E, E_ϕ or E_s are obtained by the relaxation method of Gauss. The desired screw function is either defined in the SUBROUTINE DSIRED by the parameter IV, or it is read as data. Input requires an initial set of dimensions of the mechanism, initial increment in the dimensions, value of dividers for the increments after each iteration, the number of reductions to be done in the increments, characteristic number for the inversion, and the number of iterations for each undefined dimension. Any number of dimensions may be predefined reducing the number of unknowns. When the number of iterations for all the dimensions are set equal to zero the program is used for the kinematic analysis. The detailed information regarding the input and output of the Program C is given in the comment cards in the SUBROUTINE DSIRED.

The following is the list of key parameters and the corresponding symbols used in the computer programs.

The symbol used in the computer program	The description or the corresponding symbol used in the equations
TETO (deg.), TETOR (rad.)	θ_0
CELO (deg.), CELOR (rad.)	λ_0
DELO (deg.), DELOR (rad.)	δ_0
FI01, FI02, FI03, FI04, (deg.); FI01R, RI02R, FI03R, FI04R (rad.)	ϕ_0 for the four inversions
S1, S2, S3, S4	S for the four inversions
X01, X02, X03, X04, (deg.); X01R, X02R, X03R, X04R (rad.)	χ_0 for the four inversions
D1, D2, D3, DA, DB, DE (in.)	d_1, d_2, d_3, a, b, e
XC1, YC1, ZC1 (in.)	X_p, Y_p, Z_p coordinates of the coupler curve for the first inversion
XP1, YP1, ZP1 (in.)	ξ_{3p}, η_{3p}, ζ_{3p} coordinates of the coupler curve for the first inversion
DF, DG, DH (in.)	f, g, h coordinates of the coupler point in the coupler frame
TT01, TT02, TT03, TT04 (deg.)	values of the input parameter put in order within the mobile regions of the four inversions
VF1, VX1, VS1	$\dot{\phi}_0/\dot{\theta}_0$, $\dot{\chi}_0/\dot{\theta}_0$ and $\dot{S}/\dot{\theta}_0$
AF1, AX1, AS1	$\ddot{\phi}_0/\dot{\theta}_0^2$, $\ddot{\chi}_0/\dot{\theta}_0^2$ and $\ddot{S}/\dot{\theta}_0^2$
TET (deg.)	θ, the independent parameter
SY0 (deg.)	ψ_{d_0}
SF (in.)	ψ_{d_1}
SY0G (deg.)	ψ_{g_0}
SFG (in.)	ψ_{g_1}
TET01 (deg.)	θ_{01}
FY01 (deg.)	ϕ_{01}

Q1 (deg.), Q1R (rad.)	q_1
Q2 (deg.), Q2R (rad.)	q_2
SLOPE1 (rad.), SLOPE2 (rad.)	q_1, q_2
DELSLP (rad.)	τ
FG01 (deg.), FG01R (rad.)	$\phi_{01} + \psi_{g_0}$
RF	E_ϕ
RS	E_s
R2	E
RMSEFO (deg.), RMSEFR (rad.)	$(RMSE)_\phi$
RMSES (in.)	$(RMSE)_s$
RMSE2 (in.)	RMSE
H	error vector
PQ	the matrix A in equation (4.29)
PQINV	inverse of the matrix A
SMQO	the matrix C in Equation (4.29) in Program B, equation of condition in Program C as given by Equation (5.5)
DTET12 (deg.)	$\Delta\theta_0$
DTET (deg.)	increment in θ
DFY12 (deg.)	$\Delta\phi_0$
DLS12 (in.)	ΔS

PROGRAM A

```
C       PROGRA RSRC-A
C       CEMIL BAGCI-KINEMATIC ANALYSIS OF THE RSRC SPACE
C       MECHANISM.DISPLACEMENTS,VELOCITIES,ACCELERATIONS,
C       AND COUPLER CURVE COORDINATES ARE COMPUTED AND PLOTTED
C       FOR EACH EXISTING REAL GEOMETRIC INVERSION.
        DIMENSION TETOR(36 ),TETO(36 ),FIO1(36 ),FIO1R(36 )
        DIMENSION AF1(36)          ,VF1(36 ),VX1(36 ),VS1(36 )
        DIMENSION AX1(36 ),AS1(36 )
        DIMENSION XO2R(36 ),FIO3R(36 ),XO3R(36 ),FIO4R(36 )
        DIMENSION XO1R(36 ) ,FIO2(36 ),XO2(36 ),S2(36 )
        DIMENSION XO1(36 ),XO4R(36),FIO2R(36),FIO3(36),S1(36)
        DIMENSION XO3(36 ),S3(36 ),FIO4(36 ),XO4(36 ),S4(36 )
        DIMENSION XC1(36 ),YC1(36 ),ZC1(36 ),XP1(36 ),YP1(36 )
        DIMENSION VF2(36 ),VF3(36 ),VF4(36 ),VX2(36 ),VX3(36 )
        DIMENSION VS2(36 ),VS3(36 ),VS4(36 ),AF2(36 ),AF3(36 )
        DIMENSION AX2(36 ),AX3(36 ),AX4(36 ),AS2(36 ),AS3(36 )
        DIMENSION XC2(36 ),YC2(36 ),ZC2(36 ),XP2(36 ),YP2(36 )
        DIMENSION XC3(36 ),YC3(36 ),ZC3(36 ),XP3(36 ),YP3(36 )
        DIMENSION XC4(36 ),YC4(36 ),ZC4(36 ),XP4(36 ),YP4(36 )
        DIMENSION ZP1(36),VX4(36),AF4(36),AS4(36),ZP2(36)
        DIMENSION ZP3(36),ZP4(36)
        DIMENSION X(216),Y(216)
        DIMENSION TTO1(36 ),TTO2( 36),TTO3( 36),TTO4( 36)
1       FORMAT(5X,22HRSRC-SPATIAL MECHANISM///)
  103   FORMAT(7F10.4)
8       FORMAT(5X,14HIMAGINARY ROOT////)
10      FORMAT(10X,9F10.5)
101     FORMAT(///12X,2HDF,8X,2HDG,8X,2HDH/)
102     FORMAT(8X,3F10.4/)
11      FORMAT(10X,4F11.6)
111     FORMAT(I4)
112     FORMAT(6I4)
113     FORMAT(4F8.3)
114     FORMAT(F8.3)
115     FORMAT(3F10.5)
116     FORMAT(6F10.7)
117     FORMAT(7F10.7)
118     FORMAT(20X,I5)
119     FORMAT(2X,12F10.5)
12      FORMAT( 6X,8F11.6)
120     FORMAT(F10.7)
122     FORMAT(8F8.1)
18      FORMAT(13X,4HTETO,7X,4HFIO4,8X,2HS4,8X,3HXO4///)
```

```
19     FORMAT(13X,4HTETO,7X,4HFIO3,8X,2HS3,8X,3HXO3///)
20     FORMAT(8X,6HLAMBDA,6X,5HDELTA,10X,2HD1,9X,2HD2,9X,2HD3
      1,10X,1HA,10X,1HB,10X,1HE/)
21     FORMAT(13X,4HTETO,7X,4HFIO1,8X,2HS1,8X,3HXO1///)
22     FORMAT(13X,4HTETO,7X,4HFIO2,8X,2HS2,8X,3HXO2///)
23     FORMAT(1H1,10X,25HVELOCITY AND ACCELERATION////)
24     FORMAT(1H1,10X,13HDISPLACEMENTS////)
25     FORMAT(13X,4HTETO,7X,6HVELFI1,6X,4HVELS1,6X,5HVELX1,2X
      1,8HACC.FIO1,4X,6HACC.S1,4X,7HACC.XO1///)
  250 FORMAT(/6X,'INPUT ANGULAR VELOCITY=',F8,4,1X,'RAD PER
      1SEC.',4X,'INPUT ANG.ACCEL.=',F10.4,1X,' RAD PER SEC**2'
      2/)
26     FORMAT(13X,4HTETO,7X,6HVELFI2,6X,5HVELS2,6X,5HVELX2,2X
      1,8HACC.FIO2,4X,6HACC.S2,4X,7HACC.XO2///)
14    FORMAT(13X,4HTETO,7X,6HVELFI3,6X,5HVELS3,6X,5HVELX3,2X
     1,8HACC.FIO3,4X,6HACC.S3,4X,7HACC.XO3///)
15    FORMAT(13X,4HTETO,7X,6HVELFI4,6X,5HVELS4,6X,5HVELX4,2X
     1,8HACC.FIO4,4X,6HACC.S4,4X,7HACC.XO4///)
27    FORMAT(1H1,10X,25HCOUPLER POINT COORDINATES////)
28    FORMAT(13X,4HTETO,6X,4HFIO1,8X,2HS1,8X,3HXC1,7X,3HYC1,
     17X,3HZC1,7X,3HXP1,7X,3HYP1,7X,3HZP1//)
29    FORMAT(13X,4HTETO,6X,4HFIO2,8X,2HS2,8X,3HXC2,7X,3HYC2,
     17X,3HZC2,7X,3HXP2,7X,3HYP2,7X,3HZP2//)
16    FORMAT(13X,4HTETO,6X,4HFIO3,8X,2HS3,8X,3HXC3,7X,3HYC3,
     17X,3HZC3,7X,3HXP3,7X,3HYP3,7X,3HZP3//)
17    FORMAT(13X,4HTETO,6X,4HFIO4,8X,2HS4,8X,3HXC4,7X,3HYC4,
     17X,3HZC4,7X,3HXP4,7X,3HYP4,7X,3HZP4//)
30    FORMAT(/9X,29HROOT1,SIGN OF RADICAL IS PLUS//)
31    FORMAT(/9X,33HROOT2,SIGN OF RADICAL IS NEGATIVE//)
36    FORMAT(/9X,57HROOT1,ROOT1 OF QUADRATIC FACTOR 1, SIGN O
     1F RADICAL IS PLUS//)
37    FORMAT(/9X,61HROOT2,ROOT2, OF QUADRATIC FACTOR 1,SIGN O
     1F RADICAL IS NEGATIVE//)
38    FORMAT(/9X,57HROOT3,ROOT1 OR QUADRATIC FACTOR 2, SIGN O
     1F FADICAL IS PLUS//)
39    FORMAT(/9X,61HROOT4,ROOT2 OF QUADRATIC FACTOR 2, SIGN O
     1F RADICAL IS NEGATIVE//)
  41  FORMAT(10X,7F12,4)
      READ(5,111)KNMECH
C     KNMECH IS THE NUMBER OF MECHANISMS FOR WHICH DATA
C     EXIST.
      KM=1
50    READ(5,112)NTYPE,NVEL,NCOUPL,NPLOTD,NPLOTC,NPLOTV
C     NTYPE IS 1 FOR THE RSRC MECHANISM.
C     IF NVEL IS NOT ZERO IT COMPUTES VELOCITIES AND
C     ACCELERATIONS.
C     IF NCOUPL IS NOT ZERO IT COMPUTES COUPLER CURVE
C     COORDINATES.
C     IF NPLOTD, NPLOTC, NPLOTV ARE NOT ZERO IT PLOTS
C     DISPLACEMENTS
C     COUPLER CURVES, AND VELOCITIES AND ACCELERATIONS,
C     RESPECTIVELY.
```

```
      READ(5,111) NI
C     NI IS THE NUMBER OF INCREMENTS IN THE INPUT PARAMETER
C     TETA.
      READ(5,113)TETIN,DTET,CELO,DELO
C     TETIN IS THE INITIAL VALUE OF THE INPUT PARAMETER.
C     DTET IS THE INCREMENT IN THE INPUT PARAMETER.CELO IS
C     THE ANGLE LAMBDA (DEG).
C     DELO IS THE ANGLE DELTA (DEG.)
      IF(NVEL.NE.0) GO TO 40
      GO TO 410
   40 READ(5,113)VWIN,AWIN
C     VWIN AND AWIN ARE THE INPUT ANGULAR VEL.AND ACCEL.IN
C     RAD/SEC.THEY ARE NEEDED IF NVEL IS NOT ZERO.
410   IF(NCOUPL .NE.0) GO TO 42
      GO TO 43
42    READ(5,115)DF,DG,DH
C     DF, DG, DH ARE THE COORDINATES OF THE COUPLER POINT
43    IF(NPLOTD.NE.0) GO TO 421
      IF(NPLOTC.NE.0) GO TO 421
      IF(NPLOTV.NE.0) GO TO 421
      GO TO 44
421   READ(5,122) STMI,STMX,SDMI,SDMX,SCMI,SCMX,SVMI,SVMX
C     STMI, STMX ARE THE MINIMUM AND MAXIMUM VALUES FOR THE
C     INPUT PARAMETER IN THE PLOT.
C     SDMI, SDMX ARE THE MINIMUM AND MAXIMUM VALUES FOR THE
C     OUTPUT ROTATION IN THE PLOT.
C     SCMI, SCMX ARE THE MINIMUM AND MAXIMUM VALUES FOR
C     THE COUPLER CURVE COORDINATES AND THE OUTPUT TRANSLAT.
C     IN THE PLOT.
C     SVMI, SVMX ARE THE MINIMUM AND MAXIMUM VALUES FOR THE
C     VELOCITY AND ACCELERATION RATIOS IN THE PLOT.
44    PI=3.14159265
      TROP=PI/180.0
      DO 45 I=1,NI
      IF(I.EQ.1) GO TO 46
      GO TO 47
46    TETO(I)=TETIN
      GO TO 48
47    I1=I-1
      TETO(I)=TETO(I1)&DTET
48    TETOR(I)=TETO(I)*TROP
45    CONTINUE
53    READ(5,116)D1,D2,D3,DA,DB,DE
C     D1, D2, D3 ARE THE LENGTHS OF THE INPUT, COUPLER AND
C     OUTPUT LINKS.DA IS THE LENGTH OF THE FIZED LINK.DB IS
C     THE DISTANCE OF THE INPUT PAIR FROM THE FIXED LINK.
C     DE IS,E,THE DUAL PART OF THE TRANSMISSICN ANGLE.
      WRITE(6,1)
      CALL       RXXC(NTYPE,NI,TETO,CELO,DELO,D1,D2,D3,DA,DB
     1,DE,0.,FIO1,FIO2,XO1,XO2,S1,S2,KA4,FIO3,FIO4,XO3,XO4,S
     23,S4,MJ1,MJ2,ML1,ML2,VF1,VF2,VF3,VF4,XC1,YC1,ZC1,XC2,Y
     3C2,ZC2,XP1,YP1,ZP1,XP2,YP2,ZP2,XP3,YP3,ZP3,XP4,YP4,ZP4
```

```
   4)
      M1=MJ1
      M2=MJ2
      M3=ML1
      M4=ML2
14000 WRITE(6,24)
      WRITE(6,20)
      WRITE(6,12)CELO,DELO,D1,D2,D3,DA,DB,DE
      WRITE(6,118) MJ1
      WRITE(6,118) MJ2
      WRITE(6,118) ML1
      WRITE(6,118) ML2
      IF(M1.NE.0) GO TO 601
      IF(M2.NE.0) GO TO 601
      IF(M3.NE.0) GO TO 6010
      IF(M4.NE.0) GO TO 6010
141   WRITE(6,30)
      WRITE(6,31)
      WRITE(6,36)
      WRITE(6,37)
      WRITE(6,38)
      WRITE(6,39)
      WRITE(6,8)
      GO TO 830
601   IF(DELO.EQ.0.0) GO TO 602
      IF(ABS(DELO).EQ.180.0) GO TO 602
      IF(ABS(DELO).EQ.90.0) GO TO 602
6010  IF(KA4.EQ.5) GO TO 6011
      IF(M3.NE.0) GO TO 62
      IF(M4.NE.0) GO TO 62
      IF(M1.NE.0) GO TO 602
      IF(M2.NE.0) GO TO 602
      GO TO 141
6011  WRITE(6,118) KA4
      GO TO 830
602   WRITE(6,30)
      WRITE(6,21)
      IF(M1.EQ.0) GO TO 603
      KD=11
6001  DO 6020 J=1,M1
      TT01(J)=VF1(J)
      FI01(J)=XC1(J)
      X01(J)=YC1(J)
      S1(J)=ZC1(J)
6020  CONTINUE
60201 WRITE(6,11 )(TT01(I),FI01(I),S1(I),X01(I),I=1,M1)
      MJ=M1
      J2N=2*MJ
      J3N=3*MJ
      J4N=4*MJ
      J5N=5*MJ
      J6N=6*MJ
```

```
      J1N1=MJ&1
      J2N1=2*MJ&1
      J3N1=3*MJ&1
      J4N1=4*MJ&1
      J5N1=5*MJ&1
      IF(NPLOTD.EQ.0) GO TO 60231
6002  CALL DSPLOT(M1,J1N1,J2N,J2N1,J3N,TTO1,FIO1,S1,XO1,STMI
     1,STMX,SDMI,SDMX)
C     SUBROUTINE DSPLOT PLOTS THE DISPLACEMENTS.
60231 IF(KD.EQ.11) GO TO 603
      IF(KD.EQ.21) GO TO 62025
603   WRITE(6,31)
      WRITE(6,22)
      KD=12
      IF(M2.EQ.0) GO TO 611
60300 DO 60301 J=1,M2
      TTO2(J)=VF2(J)
      FIO2(J)=XC2(J)
      XO2(J)=YC2(J)
      S2(J)=ZC2(J)
60301 CONTINUE
60302 MJ=M2
      K2N=2*MJ
      K3N=3*MJ
      K4N=4*MJ
      K5N=5*MJ
      K6N=6*MJ
      K1N1=MJ&1
      K2N1=2*MJ&1
      K3N1=3*MJ&1
      K4N1=4*MJ&1
      K5N1=5*MJ&1
      WRITE(6,11 )(TTO2(I),FIO2(I),S2(I),XO2(I),I=1,M2)
      IF(NPLOTD.EQ.0) GO TO 60331
6003  CALL DSPLOT(M2,K1N1,K2N,K2N1,K3N,TTO2,FIO2,S2,XO2,STMI
     1,STMX,SDMI,SDMX)
60331 IF(KD.EQ.12) GO TO 611
      IF(KD.EQ.22) GO TO 6205
611   IF(NCOUPL .EQ.0) GO TO 612
      IF(M1.EQ.0) GO TO 6117
      KC=11
61100 CALL COUPLR(NTYPE,M1,TTO1,CELO,DELO,D1,D2,D3,DA,DB,DE,
     10.0,FIO1,XO1,S1,DF,DG,DH,XC1,YC1,ZC1,XP1,YP1,ZP1)
      WRITE(6,27)
      WRITE(6,20)
      WRITE(6,12)CELO,DELO,D1,D2,D3,DA,DB,DE
      WRITE(6,101)
      WRITE(6,102) DF,DG,DH
      WRITE(6,30)
      WRITE(6,28)
      WRITE(6,10)(TTO1(I),FIO1(I),S1(I),XC1(I),YC1(I),ZC1(I)
     1,XP1(I),YP1(I),ZP1(I),I=1,M1)
```

```
      IF(NPLOTC.EQ.0) GO TO 61161
 6110 CALL COPPLT(M1,J1N1,J2N,J2N1,J3N,XC1,YC1,ZC1,XP1,YP1,Z
     1P1,SCMI,SCMX)
C     SUBROUTINE COPPLT PLOTS THE COUPLER CURVE PROJECTIONS.
61161 IF(KC.EQ.11) GO TO 6117
      IF(KC.EQ.21) GO TO 6218
6117  IF(M2.EQ.0) GO TO 612
      KC=12
61171 CALL COUPLR(NTYPE,M2,TTO2,CELO,DELO,D1,D2,D3,DA,DB,DE,
     10.0,FIO2,XO2,S2,DF,DG,DH,XC2,YC2,ZC2,XP2,YP2,ZP2)
      WRITE(6,101)
      WRITE(6,102) DF,DG,DH
      WRITE(6,31)
      WRITE(6,29)
      WRITE(6,10)(TTO2(I),FIO2(I),S2(I),XC2(I),YC2(I),ZC2(I)
     1,XP2(I),YP2(I),ZP2(I),I=1,M2)
      IF(NPLOTC.EQ.0) GO TO 61261
61170 CALL COPPLT(M2,K1N1,K2N,K2N1,K3N,XC2,YC2,ZC2,XP2,YP2,Z
     1P2,SCMI,SCMX)
61261 IF(KC.EQ.12) GO TO 612
      IF(KC.EQ.22) GO TO 6211
612   IF(NVEL.EQ.0) GO TO 830
      IF(M1.EQ.0) GO TO 6129
      KV=11
61200 CALL VELCTY(NTYPE,M1,TTO1,CELO,DELO,D1,D2,D3,DA,DB,DE,
     10.0,FIO1,XO1,S1,VWIN,AWIN,VF1,VX1,VS1,AF1,AX1,AS1)
      WRITE(6,23)
      WRITE(6,30)
      WRITE(6,20)
      WRITE(6,12)CELO,DELO,D1,D2,D3,DA,DB,DE
      WRITE(6,250)VWIN,AWIN
      WRITE(6,25)
      WRITE(6,41)(TTO1(I),VF1(I),VS1(I),VX1(I),AF1(I),AS1(I)
     1,AX1(I),I=1,M1)
      DO 2511 I=1,M1
 2511 PUNCH 103, TTO1(I),VWIN,AWIN,FIO1(I),VF1(I),AF1(I),S1(
     1I),VS1(I),AS1(I),XO1(I),VX1(I),AX1(I)
      IF(NPLOTV.EQ.0) GO TO 61283
6120  CALLVELPLT(M1,J1N1,J2N,J2N1,J3N,J3N1,J4N,J4N1,J5N,J5N1
     1,J6N,TTO1,VF1,VS1,VX1,AF1,AS1,AX1,STMI,STMX,SVMI,SVMX)
C     SUBROUTINE VELPLT PLOTS THE VELOCITIES AND
C     ACCELERATIONS.
61283 IF(KV.EQ.11) GO TO 6129
      IF(KV.EQ.21) GO TO 6229
6129  IF(M2.EQ.0) GO TO 613
      KV=12
61284 CALL VELCTY(NTYPE,M2,TTO2,CELO,DELO,D1,D2,D3,DA,DB,DE,
     10.0,FIO2,XO2,S2,VWIN,AWIN,VF2,VX2,VS2,AF2,AX2,AS2)
      WRITE(6,31)
      WRITE(6,250)VWIN,AWIN
      WRITE(6,26)
      WRITE(6,41)(TTO2(I),VF2(I),VS2(I),VX2(I),AF2(I),AS2(I)
```

```
     1,AX2(I),I=1,M2)
      DO 2512 I=1,M2
 2512 PUNCH 103, TTO2(I),VWIN,AWIN,FIO2(I),VF2(I),AF2(I),S2(
     1I),VS2(I),AS2(I),XO2(I),VX2(I),AX2(I)
      IF(NPLOTV.EQ.0) GO TO 61297
61290 CALL VELPLT(M2,K1N1,K2N,K2N1,K3N,K3N1,K4N,K4N1,K5N,K5N
     11,K6N,TTO2,VF2,VS2,VX2,AF2,AS2,AX2,STMI,STMX,SVMI,SVMX
     2)
61297 IF(KV.EQ.12) GO TO 613
      IF(KV.EQ.22) GO TO 6221
613   GO TO 830
62    WRITE(6,24)
      WRITE(6,20)
      WRITE(6,12)CELO,DELO,D1,D2,D3,DA,DB,DE
      IF(M1.EQ.0) GO TO 62025
      DO 6201 J=1,M1
      TTO1(J)=VF1(J)
      FIO1(J)=XP1(J)
      XO1(J)=YP1(J)
      S1(J)=ZP1(J)
6201  CONTINUE
6202  WRITE(6,36)
      WRITE(6,21)
      KD=21
      GO TO 60201
62025 IF(M2.EQ.0) GO TO 6205
      DO 62021 J=1,M2
      TTO2(J)=VF2(J)
      FIO2(J)=XP2(J)
      XO2(J)=YP2(J)
      S2(J)=ZP2(J)
62021 CONTINUE
      KD=22
      WRITE(6,37)
      WRITE(6,22)
      GO TO 60302
6205  IF(M3.EQ.0) GO TO 62055
      DO 62022 J=1,M3
      TTO3(J)=VF3(J)
      FIO3(J)=XP3(J)
      XO3(J)=YP3(J)
      S3(J)=ZP3(J)
62022 CONTINUE
      WRITE(6,38)
      WRITE(6,19)
      WRITE(6,11 )(TTO3(I),FIO3(I),S3(I),XO3(I),I=1,M3)
      ML=M3
      I1M1=ML&1
      I2M1=2*ML&1
      I3M1=3*ML&1
      I4M1=4*ML&1
      I5M1=5*ML&1
```

```
      I2M=2*ML
      I3M=3*ML
      I4M=4*ML
      I5M=5*ML
      I6M=6*ML
      IF(NPLOTD.EQ.0) GO TO 62055
      CALL DSPLOT(M3,I1M1,I2M,I2M1,I3M,TT03,FI03,S3,X03,STMI
     1,STMX,SDMI,SDMX)
62055 IF(M4.EQ.0) GO TO 621
      DO 62050 J=1,M4
      TT04(J)=VF4(J)
      FI04(J)=XP4(J)
      X04(J)=YP4(J)
      S4(J)=ZP4(J)
62050 CONTINUE
      ML=M4
      J1M1=ML&1
      J2M1=2*ML&1
      J3M1=3*ML&1
      J4M1=4*ML&1
      J5M1=5*ML&1
      J2M=2*ML
      J3M=3*ML
      J4M=4*ML
      J5M=5*ML
      J6M=6*ML
      WRITE(6,39)
      WRITE(6,18)
      WRITE(6,11 )(TT04(I),FI04(I),S4(I),X04(I),I=1,M4)
      IF(NPLOTD.EQ.0) GO TO 621
      CALL DSPLOT(M4,J1M1,J2M,J2M1,J3M,TT04,FI04,S4,X04,STMI
     1,STMX,SDMI,SDMX)
621   IF(NCOUPL .EQ.0) GO TO 622
      IF(M1.EQ.0) GO TO 6218
      KC=21
      GO TO 61100
6218  IF(M2.EQ.0) GO TO 6211
      KC=22
      GO TO 61171
6211  IF(M3.EQ.0) GO TO 6238
      CALL COUPLR(NTYPE,M3,TT03,CELO,DELO,D1,D2,D3,DA,DB,DE,
     10.0,FI03,X03,S3,DF,DG,DH,XC3,YC3,ZC3,XP3,YP3,ZP3)
      WRITE(6,101)
      WRITE(6,102) DF,DG,DH
      WRITE(6,38)
      WRITE(6,16)
      WRITE(6,10)(TT03(I),FI03(I),S3(I),XC3(I),YC3(I),ZC3(I)
     1,XP3(I),YP3(I),ZP3(I),I=1,M3)
      IF(NPLOTC.EQ.0) GO TO 6238
      CALL COPPLT(M3,I1M1,I2M,I2M1,I3M,XC3,YC3,ZC3,XP3,YP3,Z
     1P3,SCMI,SCMX)
6238  IF(M4.EQ.0) GO TO 622
```

```
      CALL COUPLR(NTYPE,M4,TT04,CELO,DELO,D1,D2,D3,DA,DB,DE,
     10.0,FI04,X04,S4,DF,DG,DH,XC4,YC4,ZC4,XP4,YP4,ZP4)
      WRITE(6,101)
      WRITE(6,102) DF,DG,DH
      WRITE(6,39)
      WRITE(6,17)
      WRITE(6,10)(TT04(I),FI04(I),S4(I),XC4(I),YC4(I),ZC4(I)
     1,XP4(I),YP4(I),ZP4(I),I=1,M4)
      IF(NPLOTC.EQ.0) GO TO 622
      CALL COPPLT(M4,J1M1,J2M,J2M1,J3M,XC4,YC4,ZC4,XP4,YP4,Z
     1P4,SCMI,SCMX)
622   IF(NVEL.EQ.0) GO TO 830
      IF(M1.EQ.0) GO TO 6229
      KV=21
      GO TO 61200
6229  IF(M2.EQ.0) GO TO 6221
      KV=22
      GO TO 61284
6221  IF(M3.EQ.0) GO TO 6267
      CALL VELCTY(NTYPE,M3,TT03,CELO,DELO,D1,D2,D3,DA,DB,DE,
     10.0,FI03,X03,S3,VWIN,AWIN,VF3,VX3,VS3,AF3,AX3,AS3)
      WRITE(6,38)
      WRITE(6,250)VWIN,AWIN
      WRITE(6,15)
      WRITE(6,41)(TT03(I),VF3(I),VS3(I),VX3(I),AF3(I),AS3(I)
     1,AX3(I),I=1,M3)
      DO 2513 I=1,M3
 2513 PUNCH 103, TT03(I),VWIN,AWIN,FI03(I),VF3(I),AF3(I),S3(
     1I),VS3(I),AS3(I),X03(I),VX3(I),AX3(I)
      IF(NPLOTV.EQ.0) GO TO 6267
      CALL VELPLT(M3,I1M1,I2M,I2M1,I3M,I3M1,I4M,I4M1,I5M,I5M
     11,I6M,TT03,VF3,VS3,VX3,AF3,AS3,AX3,STMI,STMX,SVMI,SVMX
     2)
6267  IF(M4.EQ.0) GO TO 70
      CALL VELCTY(NTYPE,M4,TT04,CELO,DELO,D1,D2,D3,DA,DB,DE,
     10.0,FI04,X04,S4,VWIN,AWIN,VF4,VX4,VS4,AF4,AX4,AS4)
      WRITE(6,39)
      WRITE(6,250)VWIN,AWIN
      WRITE(6,14)
      WRITE(6,41)(TT04(I),VF4(I),VS4(I),VX4(I),AF4(I),AS4(I)
     1,AX4(I),I=1,M4)
      DO 2514 I=1,M4
 2514 PUNCH 103, TT04(I),VWIN,AWIN,FI04(I),VF4(I),AF4(I),S4(
     1I),VS4(I),AS4(I),X04(I),VX4(I),AX4(I)
      IF(NPLOTV.EQ.0) GO TO 70
      CALL VELPLT(M4,J1M1,J2M,J2M1,J3M,J3M1,J4M,J4M1,J5M,J5M
     11,J6M,TT04,VF4,VS4,VX4,AF4,AS4,AX4,STMI,STMX,SVMI,SVMX
     2)
70    CONTINUE
80    CONTINUE
830   KM=KM&1
      IF(KM.GT.KNMECH) GO TO 831
```

```
      GO TO 50
831   STOP
      END

      SUBROUTINE RXXC(NTYPE,NI,TETO,CELO,DELO,D1,D2,D3,DA,DB
     1,DE,HE,FIO1,FIO2,XO1,XO2,S1,S2,KA4,FIO3,FIO4,XO3,XO4,S
     23,S4,MJ1,MJ2,ML1,ML2,VF1,VF2,VF3,VF4,XC1,YC1,ZC1,XC2,Y
     3C2,ZC2,XP1,YP1,ZP1,XP2,YP2,ZP2,XP3,YP3,ZP3,XP4,YP4,ZP4
     4)
      DIMENSION TETOR(36 ),TETO(36 ),FIO1(36 ),FIO1R(36 ),XO
     11(36 )
      DIMENSION XO1R(36 ) ,FIO2(36 ),XO2(36 ),S2(36 ),FIO3(3
     16 ),S1(36 )
      DIMENSION XO2R(36 ),FIO3R(36 ),XO3R(36 ),FIO4R(36 ),XO
     14R(36 )
      DIMENSION XO3(36 ),S3(36 ),FIO4(36 ),XO4(36 ),S4(36 ),
     1FIO2R(36 )
      DIMENSION XC1(36 ),YC1(36 ),ZC1(36 ),XP1(36 ),YP1(36 )
     1,ZP1(36 )
      DIMENSION XC2(36 ),YC2(36 ),ZC2(36 ),XP2(36 ),YP2(36 )
     1,ZP2(36 )
      DIMENSION XP3(36 ),YP3(36 ),ZP3(36 ),XP4(36 ),YP4(36 )
     1,ZP4(36 )
      DIMENSION VF2(36 ),VF3(36 ),VF4(36 ),VF1(36)
      PI=3.14159265
      TROP=PI/180.0
      CELOR=CELO*TROP
      DELOR=DELO*TROP
      SDEL=SIN(DELOR)
      CDEL=COS(DELOR)
      SLAM=SIN(CELOR)
      CLAM=COS(CELOR)
      DO 101 I=1,NI
      TETOR(I)=TETO(I)*TROP
101   CONTINUE
      IF(DELO.EQ.0.0) GO TO 110
      IF(ABS(DELO).EQ.180.0) GO TO 110
      IF(ABS(DELO).EQ.90.0) GO TO 1202
      GO TO 130
110   MJ1=0
      MJ2=0
      DO 1101 I=1,NI
      GNO=DA-D1*COS(TETOR(I))
      GMO=DB*SLAM-D1*CLAM*SIN(TETOR(I))
      AO=2.0*D3*GNO
      BO=2.0*D3*GMO
      CO=D3**2-D2**2&GNO**2&GMO**2
      RADI=BO**2&AO**2-CO**2
      IF(RADI    .LT.0.0) GO TO 1101
      ROOT=SQRT(RADI)
      FIO1R(I)=2.0*ATAN((-BO&ROOT)/(CO-AO))
```

```
      FIO1(I)=FIO1R(I)/TROP
      T1=             ((GMO   *SIN(FIO1R(I))&GNO   *COS(FIO1R(I
     1))&D3)/D2)
      IF(ABS(T1).GT.1.0) GO TO 11013
      MJ1=MJ1&1
      J1=MJ1
      TS1=(GNO*SIN(FIO1R(I))-GMO*COS(FIO1R(I))-DE*SDEL)/(D2*
     1CDEL)
      XO1R(I)=ARCOS(ABS(T1))
      IF(T1.GT.0.0) GO TO 99011
      IF(TS1.GT.0.0) GO TO 99012
      XO1R(I)=XO1R(I)-PI
      GO TO 99014
99012 XO1R(I)=PI-XO1R(I)
      GO TO 99014
99011 IF(TS1.GT.0.0) GO TO 99014
      XO1R(I)=-XO1R(I)
99014 XO1(I)=XO1R(I)/TROP
      S1(I)=DE*CDEL&DB*CLAM&D1*SLAM*SIN(TETOR(I))
      VF1(J1)=TETO(I)
      XC1(J1)=FIO1(I)
      YC1(J1)=XO1(I)
      ZC1(J1)=S1(I)
11013 FIO2R(I)=2.0*ATAN((-BO-ROOT)/(CO-AO))
      FIO2(I)=FIO2R(I)/TROP
      T2=             ((GMO   *SIN(FIO2R(I))&GNO   *COS(FIO2R(I
     1))&D3)/D2)
      IF(ABS(T2).GT.1.0) GO TO 1101
      MJ2=MJ2&1
      J2=MJ2
      TS2=(GNO*SIN(FIO2R(I))-GMO*COS(FIO2R(I))-DE*SDEL)/(D2*
     1CDEL)
      XO2R(I)=ARCOS(ABS(T2))
      IF(T2.GT.0.0) GO TO 99021
      IF(TS2.GT.0.0) GO TO 99022
      XO2R(I)=XO2R(I)-PI
      GO TO 99024
99022 XO2R(I)=PI-XO2R(I)
      GO TO 99024
99021 IF(TS2.GT.0.0) GO TO 99024
      XO2R(I)=-XO2R(I)
99024 XO2(I)=XO2R(I)/TROP
      S2(I)=DE*CDEL&DB*CLAM&D1*SLAM*SIN(TETOR(I))
      VF2(J2)=TETO(I)
      XC2(J2)=FIO2(I)
      YC2(J2)=XO2(I)
      ZC2(J2)=S2(I)
1101  CONTINUE
      GO TO 140
1202  MJ1=0
      MJ2=0
      DO 1201 I=1,NI
```

```
      GNO=DA-D1*COS(TETOR(I))
      GMO=DB*SLAM-D1*CLAM*SIN(TETOR(I))
      AO=DE*SDEL
      BO=GNO
      CO=GMO
      RADI=BO**2&CO**2-AO**2
      IF(RADI    .LT.0.0) GO TO 1201
      ROOT=SQRT(RADI)
      FIO1R(I)=2.0*ATAN((-BO&ROOT)/(CO-AO))
      FIO1(I)=FIO1R(I)/TROP
      T1=            ((GMO    *SIN(FIO1R(I))&GNO    *COS(FIO1R(I
     1))&D3)/D2)
      IF(ABS(T1).GT.1.0) GO TO 12013
      MJ1=MJ1&1
      J1=MJ1
      XO1R(I)=ARCOS((GMO    *SIN(FIO1R(I))&GNO    *COS(FIO1R(I
     1))&D3)/D2)
      XO1(I)=XO1R(I)/TROP
      S1(I)=DB*CLAM&D1*SLAM*SIN(TETOR(I))-D2*SDEL*SIN(XO1R(I
     1))
      VF1(J1)=TETO(I)
      XC1(J1)=FIO1(I)
      YC1(J1)=XO1(I)
      ZC1(J1)=S1(I)
12013 FIO2R(I)=2.0*ATAN((-BO-ROOT)/(CO-AO))
      FIO2(I)=FIO2R(I)/TROP
      T2=            ((GMO    *SIN(FIO2R(I))&GNO    *COS(FIO2R(I
     1))&D3)/D2)
      IF(ABS(T2).GT.1.0) GO TO 1201
      MJ2=MJ2&1
      J2=MJ2
      XO2R(I)=ARCOS((GMO    *SIN(FIO2R(I))&GNO    *COS(FIO2R(I
     1))&D3)/D2)
      XO2(I)=XO2R(I)/TROP
      S2(I)=DB*CLAM&D1*SLAM*SIN(TETOR(I))-D2*SDEL*SIN(XO2R(I
     1))
      VF2(J2)=TETO(I)
      XC2(J2)=FIO2(I)
      YC2(J2)=XO2(I)
      ZC2(J2)=S2(I)
1201  CONTINUE
      GO TO 140
130   MJ1=0
      MJ2=0
      ML1=0
      ML2=0
      DO 1301 I=1,NI
      NEO=0
      GNO=DA-D1*COS(TETOR(I))
      GMO=DB*SLAM-D1*CLAM*SIN(TETOR(I))
      GKO=(    CDEL**2)*(D2**2-D3**2-GNO    **2)-GMO    **2-(DE
     1*SDEL)**2
```

```
      A0=    2.0*(D3*GNO    *(CDEL**2)&DE*GMO    *SDEL)-GKO
      A1=    4.0*(D3*GMO    *(CDEL**2)-DE*GNO    *SDEL-GMO*GNO*
     1(SDEL**2))
      A2=    4.0*(SDEL**2)*(GNO     **2-GMO     **2)-2.0*GKO
      A3=   4.0*(GMO    *GNO    *(SDEL**2)&D3*GMO    *(CDEL**2)-
     1DE*GNO*SDEL)
      A4=    -GKO    -2.0*(D3*GNO    *(CDEL**2)&DE*GMO*SDEL)
      IF(A4    .EQ.0.0) GO TO 12011
      GO TO 12012
12011 KA4=5
      GO TO 140
12012 B0=A0/A4
      B1=A1/A4
      B2=A2/A4
      B3=A3/A4
      C0=B0*(4.0*B2-B3**2)-B1**2
      C1=B3*B1-4.0*B0
      E0=(2.0*(B2**3)-9.0*B2*C1-27.0*C0)/27.0
      E1=(-3.0*C1&B2**2)/3.0
      IF(E0    .LT.0.0) GO TO 1306
      GO TO 13061
1306  EM0=-E0
      NE0=NE0&1
      E0=EM0
13061 CHE0=27.0*(E0**2)
      CHE1=4.0*(E1**3)
      IF(E0.EQ.0.0) GO TO 1307
      IF(E1    .EQ.0.0) GO TO 1308
      IF(CHE0    .GT.CHE1    ) GO TO 1302
      IF(CHE0    .EQ.CHE1    ) GO TO 1303
      IF(CHE0    .LT.CHE1    ) GO TO 1304
1302  IF(E1    .GT.0.0) GO TO 13022
      SAYA1=    0.5*E0    /((-E1    /3.0)**1.5)
      SAYRT=    ALOG(SAYA1    &SQRT(SAYA1    **2&1.0))
      IF(NE0.GT.0) GO TO 13064
      Y3=    B2    /3.0&2.0*SQRT(-E1    /3.0)*SINH(SAYRT    /3.0
     1)
      GO TO 131
13064 Y3=    B2    /3.0-2.0*SQRT(-E1    /3.0)*SINH(SAYRT    /3.0
     1)
      GO TO 131
13022 SAYA2=    0.5*E0    /((E1    /3.0)**1.5)
      IF(SAYA2    .LT.1.0) GO TO 1301
      SAYRT=    ALOG(SAYA2    &SQRT(SAYA2    **2-1.0))
13024 IF(NE0.GT.0) GO TO 13065
      Y3=    B2    /3.0&2.0*SQRT(E1    /3.0)*COSH(SAYRT    /3.0)
      GO TO 131
13065 Y3=    B2    /3.0-2.0*SQRT(E1    /3.0)*COSH(SAYRT    /3.0)
      GO TO 131
1303  IF(NE0.GT.0) GO TO 13062
      GO TO 13031
13062 ROOT1=    -2.0*((E0    /2.0)**(1.0/3.0))&B2    /3.0
```

```
      ROOT2=   (EO    /2.0)**(1.0/3.0)&B2    /3.0
      GO TO 13032
13031 ROOT1=   2.0*((EO    /2.0)**(1.0/3.0))&B2/3.0
      ROOT2=   -(EO    /2.0)**(1.0/3.0)&B2    /3.0
13032 IF(ROOT1.GT.ROOT2) GO TO 1305
      Y3=ROOT2
      GO TO 131
1305  Y3=ROOT1
      GO TO 131
1304  SAYRT=   ARCOS(0.5*EO    /((E1    /3.0)**1.5))
      IF(NEO.GT.0) GO TO 13066
      GO TO 13040
13066 ROOT1=   -2.0*SQRT(E1    /3.0)*COS(SAYRT    /3.0)&B2    /
     13.0
      ROOT2=   2.0*SQRT(E1    /3.0)*COS((PI-SAYRT    )/3.0)&B2
     1    /3.0
      ROOT3=   2.0*SQRT(E1    /3.0)*COS((PI&SAYRT    )/3.0)&B2
     1/3.0
      GO TO 13041
13040 ROOT1=   2.0*SQRT(E1    /3.0)*COS(SAYRT    /3.0)&B2/3.0
      ROOT2=   -2.0*SQRT(E1    /3.0)*COS((PI-SAYRT    )/3.0)&B
     12/3.0
      ROOT3=   -2.0*SQRT(E1    /3.0)*COS((PI&SAYRT    )/3.0)&B
     12/3.0
13041 IF(ROOT1    .GT.ROOT2    ) GO TO 13042
      IF(ROOT2    .GT.ROOT3    ) GO TO 13043
      Y3=ROOT3
      GO TO 131
13042 IF(ROOT1    .GT.ROOT3    ) GO TO 13044
      Y3=ROOT3
      GO TO 131
13044 Y3=    ROOT1
      GO TO 131
13043 Y3=ROOT2
      GO TO 131
1307  IF(E1    .LE.0.0) GO TO 13071
      ROOT1=B2/3.0
      ROOT2=   SQRT(E1    )&B2/3.0
      ROOT3=   -SQRT(E1    )&B2/3.0
      IF(ROOT1    .GE.ROOT2    ) GO TO 13073
      IF(ROOT2    .GE.ROOT3    ) GO TO 13074
      Y3=ROOT3
      GO TO 131
13073 IF(ROOT1    .GE.ROOT3    ) GO TO 13075
      Y3=ROOT3
      GO TO 131
13074 Y3=ROOT2
      GO TO 131
13075 Y3=ROOT1
      GO TO 131
13071 Y3=B2/3.0
      GO TO 131
```

```
1308  IF(NEO.NE.0) GO TO 13081
      Y3=    B2    /3.0&(EO    **(1.0/3.0))
      GO TO 131
13081 Y3=    B2    /3.0-(EO    **(1.0/3.0))
131   IF((((0.5*B3) **2)-B2    &Y3    ).LT.0.0) GO TO 1301
      QS1=    0.5*B3    &SQRT((0.5*B3    )**2-B2    &Y3)
      QS2=    0.5*B3    -SQRT((0.5*B3    )**2-B2    &Y3)
      IF((((0.5*Y3    )**2)-BO    ).LT.0.0) GO TO 1301
      HS1=    0.5*Y3    &SQRT((0.5*Y3    )**2-BO)
      HS2=    0.5*Y3    -SQRT((0.5*Y3    )**2-BO)
      QH1=    QS1    *HS2    &QS2    *HS1
      QH2=    QS1    *HS1    &QS2    *HS2
      IF(ABS(QH1    -B1    ).LE.0.0001) GO TO 1311
      IF(ABS(QH2    -B1    ).LE.0.0001) GO TO 1312
      GO TO 1301
1311  H1=HS1
      H2=HS2
      GO TO 1313
1312  H1=HS2
      H2=HS1
1313  RADI1=    QS1    **2-4.0*H1
      RADI2=QS2**2-4.0*H2
      IF(RADI1    .LT.0.0) GO TO 1316
      IF(RADI2.GE.0.0) GO TO 1318
      FIO1R(I)=2.0*ATAN(0.5*(-QS1    -SQRT(RADI1)))
      FIO1(I)=FIO1R(I)/TROP
      T1=            ((GMO    *SIN(FIO1R(I))&GNO    *COS(FIO1R(I
     1))&D3)/D2)
      IF(ABS(T1).GT.1.0) GO TO 13131
      MJ1=MJ1&1
      J1=MJ1
      TS1=(GNO*SIN(FIO1R(I))-GMO*COS(FIO1R(I))-DE*SDEL)/(D2*
     1CDEL)
      XO1R(I)=ARCOS(ABS(T1))
      IF(T1.GT.0.0) GO TO 99031
      IF(TS1.GT.0.0) GO TO 99032
      XO1R(I)=XO1R(I)-PI
      GO TO 99034
99032 XO1R(I)=PI-XO1R(I)
      GO TO 99034
99031 IF(TS1.GT.0.0) GO TO 99034
      XO1R(I)=-XO1R(I)
99034 XO1(I)=XO1R(I)/TROP
      S1(I)=DB*CLAM&D1*SLAM*SIN(TETOR(I))-D2*SDEL*SIN(XO1R(I
     1))&DE*CDEL
      VF1(J1)=TETO(I)
      XC1(J1)=FIO1(I)
      YC1(J1)=XO1(I)
      ZC1(J1)=S1(I)
13131 FIO2R(I)=2.0*ATAN(0.5*(-QS1    &SQRT(RADI1)))
      FIO2(I)=FIO2R(I)/TROP
      T2=            ((GMO    *SIN(FIO2R(I))&GNO    *COS(FIO2R(I
```

```
     1))&D3)/D2)
      IF(ABS(T2).GT.1.0) GO TO 1301
      MJ2=MJ2&1
      J2=MJ2
      TS2=(GNO*SIN(FIO2R(I))-GMO*COS(FIO2R(I))-DE*SDEL)/(D2*
     1CDEL)
      XO2R(I)=ARCOS(ABS(T2))
      IF(T2.GT.0.0) GO TO 99041
      IF(TS2.GT.0.0) GO TO 99042
      XO2R(I)=XO2R(I)-PI
      GO TO 99044
99042 XO2R(I)=PI-XO2R(I)
      GO TO 99044
99041 IF(TS2.GT.0.0) GO TO 99044
      XO2R(I)=-XO2R(I)
99044 XO2(I)=XO2R(I)/TROP
      S2(I)=DB*CLAM&D1*SLAM*SIN(TETOR(I))-D2*SDEL*SIN(XO2R(I
     1))&DE*CDEL
      VF2(J2)=TETO(I)
      XC2(J2)=FIO2(I)
      YC2(J2)=XO2(I)
      ZC2(J2)=S2(I)
      GO TO 1301
1316  IF(RADI2   .LT.0.0) GO TO 1301
      FIO1R(I)=2.0*ATAN(0.5*(-QS2   -SQRT(RADI2)))
      FIO1(I)=FIO1R(I)/TROP
      T1=           ((GMO   *SIN(FIO1R(I))&GNO   *COS(FIO1R(I
     1))&D3)/D2)
      IF(ABS(T1).GT.1.0) GO TO 13161
      MJ1=MJ1&1
      J1=MJ1
      TS1=(GNO*SIN(FIO1R(I))-GMO*COS(FIO1R(I))-DE*SDEL)/(D2*
     1CDEL)
      XO1R(I)=ARCOS(ABS(T1))
      IF(T1.GT.0.0) GO TO 99051
      IF(TS1.GT.0.0) GO TO 99052
      XO1R(I)=XO1R(I)-PI
      GO TO 99054
99052 XO1R(I)=PI-XO1R(I)
      GO TO 99054
99051 IF(TS1.GT.0.0) GO TO 99054
      XO1R(I)=-XO1R(I)
99054 XO1(I)=XO1R(I)/TROP
      S1(I)=DB*CLAM&D1*SLAM*SIN(TETOR(I))-D2*SDEL*SIN(XO1R(I
     1))&DE*CDEL
      VF1(J1)=TETO(I)
      XC1(J1)=FIO1(I)
      ZC1(J1)=S1(I)
      YC1(J1)=XO1(I)
13161 FIO2R(I)=2.0*ATAN(0.5*(-QS2   &SQRT(RADI2)))
      FIO2(I)=FIO2R(I)/TROP
      T2=           ((GMO   *SIN(FIO2R(I))&GNO   *COS(FIO2R(I
```

```
     1))&D3)/D2)
      IF(ABS(T2).GT.1.0) GO TO 1301
      MJ2=MJ2&1
      J2=MJ2
      TS2=(GNO*SIN(FIO2R(I))-GMO*COS(FIO2R(I))-DE*SDEL)/(D2*
     1CDEL)
      XO2R(I)=ARCOS(ABS(T2))
      IF(T2.GT.0.0) GO TO 99061
      IF(TS2.GT.0.0) GO TO 99062
      XO2R(I)=XO2R(I)-PI
      GO TO 99064
99062 XO2R(I)=PI-XO2R(I)
      GO TO 99064
99061 IF(TS2.GT.0.0) GO TO 99064
      XO2R(I)=-XO2R(I)
99064 XO2(I)=XO2R(I)/TROP
      S2(I)=DB*CLAM&D1*SLAM*SIN(TETOR(I))-D2*SDEL*SIN(XO2R(I
     1))&DE*CDEL
      VF2(J2)=TETO(I)
      XC2(J2)=FIO2(I)
      YC2(J2)=XO2(I)
      ZC2(J2)=S2(I)
      GO TO 1301
1318  FIO1R(I)=2.0*ATAN(0.5*(-QS1   -SQRT(RADI1)))
      FIO1(I)=FIO1R(I)/TROP
      T1=           ((GMO   *SIN(FIO1R(I))&GNO   *COS(FIO1R(I
     1))&D3)/D2)
      IF(ABS(T1).GT.1.0) GO TO 13181
      MJ1=MJ1&1
      J1=MJ1
      TS1=(GNO*SIN(FIO1R(I))-GMO*COS(FIO1R(I))-DE*SDEL)/(D2*
     1CDEL)
      XO1R(I)=ARCOS(ABS(T1))
      IF(T1.GT.0.0) GO TO 99071
      IF(TS1.GT.0.0) GO TO 99072
      XO1R(I)=XO1R(I)-PI
      GO TO 99074
99072 XO1R(I)=PI-XO1R(I)
      GO TO 99074
99071 IF(TS1.GT.0.0) GO TO 99074
      XO1R(I)=-XO1R(I)
99074 XO1(I)=XO1R(I)/TROP
      S1(I)=DB*CLAM&D1*SLAM*SIN(TETOR(I))-D2*SDEL*SIN(XO1R(I
     1))&DE*CDEL
      VF1(J1)=TETO(I)
      XP1(J1)=FIO1(I)
      YP1(J1)=XO1(I)
      ZP1(J1)=S1(I)
13181 FIO2R(I)=2.0*ATAN(0.5*(-QS1   &SQRT(RADI1)))
      FIO2(I)=FIO2R(I)/TROP
      T2=           ((GMO   *SIN(FIO2R(I))&GNO   *COS(FIO2R(I
     1))&D3)/D2)
```

```
      IF(ABS(T2).GT.1.0) GO TO 13182
      MJ2=MJ2&1
      J2=MJ2
      TS2=(GNO*SIN(FIO2R(I))-GMO*COS(FIO2R(I))-DE*SDEL)/(D2*
     1CDEL)
      XO2R(I)=ARCOS(ABS(T2))
      IF(T2.GT.0.0) GO TO 99081
      IF(TS2.GT.0.0) GO TO 99082
      XO2R(I)=XO2R(I)-PI
      GO TO 99084
99082 XO2R(I)=PI-XO2R(I)
      GO TO 99084
99081 IF(TS2.GT.0.0) GO TO 99084
      XO2R(I)=-XO2R(I)
99084 XO2(I)=XO2R(I)/TROP
      S2(I)=DB*CLAM&D1*SLAM*SIN(TETOR(I))-D2*SDEL*SIN(XO2R(I
     1))&DE*CDEL
      VF2(J2)=TETO(I)
      XP2(J2)=FIO2(I)
      YP2(J2)=XO2(I)
      ZP2(J2)=S2(I)
13182 FIO3R(I)=2.0*ATAN(0.5*(-QS2   -SQRT(RADI2)))
      FIO3(I)=FIO3R(I)/TROP
      T3=          ((GMO   *SIN(FIO3R(I))&GNO   *COS(FIO3R(I
     1))&D3)/D2)
      IF(ABS(T3).GT.1.0) GO TO 13183
      ML1=ML1&1
      L1=ML1
      TS3=(GNO*SIN(FIO3R(I))-GMO*COS(FIO3R(I))-DE*SDEL)/(D2*
     1CDEL)
      XO3R(I)=ARCOS(ABS(T3))
      IF(T3.GT.0.0) GO TO 99091
      IF(TS3.GT.0.0) GO TO 99092
      XO3R(I)=XO3R(I)-PI
      GO TO 99094
99092 XO3R(I)=PI-XO3R(I)
      GO TO 99094
99091 IF(TS3.GT.0.0) GO TO 99094
      XO3R(I)=-XO3R(I)
99094 XO3(I)=XO3R(I)/TROP
      S3(I)=DB*CLAM&D1*SLAM*SIN(TETOR(I))-D2*SDEL*SIN(XO3R(I
     1))&DE*CDEL
      VF3(L1)=TETO(I)
      XP3(L1)=FIO3(I)
      YP3(L1)=XO3(I)
      ZP3(L1)=S3(I)
13183 FIO4R(I)=2.0*ATAN(0.5*(-QS2   &SQRT(RADI2)))
      FIO4(I)=FIO4R(I)/TROP
      T4=          ((GMO   *SIN(FIO4R(I))&GNO   *COS(FIO4R(I
     1))&D3)/D2)
      IF(ABS(T4).GT.1.0) GO TO 1301
      ML2=ML2&1
```

```
      L2=ML2
      TS4=(GNO*SIN(FIO4R(I))-GMO*COS(FIO4R(I))-DE*SDEL)/(D2*
     1CDEL)
      XO4R(I)=ARCOS(ABS(T4))
      IF(T4.GT.0.0) GO TO 99101
      IF(TS4.GT.0.0) GO TO 99102
      XO4R(I)=XO4R(I)-PI
      GO TO 99104
99102 XO4R(I)=PI-XO4R(I)
      GO TO 99104
99101 IF(TS4.GT.0.0) GO TO 99104
      XO4R(I)=-XO4R(I)
99104 XO4(I)=XO4R(I)/TROP
      S4(I)=DB*CLAM&D1*SLAM*SIN(TETOR(I))-D2*SDEL*SIN(XO4R(I
     1))&DE*CDEL
      VF4(L2)=TETO(I)
      XP4(L2)=FIO4(I)
      YP4(L2)=XO4(I)
      ZP4(L2)=S4(I)
1301  CONTINUE
 140  RETURN
      END


      SUBROUTINE COUPLR(NTYPE,NI,TETO,CELO,DELO,D1,D2,D3,DA,
     1DB,DE,HE,FIO1,XO1,S1,DF,DG,DH,XC1,YC1,ZC1,XP1,YP1,ZP1)
      DIMENSION TETOR(36 ),TETO(36 ),FIO1(36 ),FIO1R(36 ),XO
     11(36 )
      DIMENSION XO1R(36 ),S1(36 )
      DIMENSION XC1(36 ),YC1(36 ),ZC1(36 ),XP1(36 ),YP1(36 )
     1,ZP1(36 )
      PI=3.14159265
      TROP=PI/180.0
      CELOR=CELO*TROP
      DELOR=DELO*TROP
      SDEL=SIN(DELOR)
      CDEL=COS(DELOR)
      SLAM=SIN(CELOR)
      CLAM=COS(CELOR)
      DO 409 I=1,NI
      TETOR(I)=TETO(I)*TROP
      FIO1R(I)=FIO1(I)*TROP
      XO1R(I)=XO1(I)*TROP
409   CONTINUE
      IF(NTYPE.GT.3) GO TO 410
      DO 400 I=1,NI
      XC1(I)=DB-S1(I)*CLAM&D3*SLAM*SIN(FIO1R(I))&(DE&DF)*(CL
     1AM*CDEL&SLAM*SDEL*COS(FIO1R(I)))&SLAM*SIN(FIO1R(I))*(D
     2H*SIN(XO1R(I))-(D2-DG)*COS(XO1R(I)))&(SLAM*COS(FIO1R(I
     3))*CDEL-SDEL*CLAM)*(DH*COS(XO1R(I))&(D2-DG)*SIN(XO1R(I
     4)))
      YC1(I)=DA&D3*COS(FIO1R(I))-(DE&DF)*SDEL*SIN(FIO1R(I))&
```

```
     1COS(FIO1R(I))*(DH*SIN(XO1R(I))-(D2-DG)*COS(XO1R(I)))-C
     2DEL*SIN(FIO1R(I))*(DH*COS(XO1R(I))&(D2-DG)*SIN(XO1R(I)
     3))
      ZC1(I)=S1(I)*SLAM&D3*CLAM*SIN(FIO1R(I))&(DE&DF)*(CLAM*
     1SDEL*COS(FIO1R(I))-SLAM*CDEL)&CLAM*SIN(FIO1R(I))*(DH*S
     2IN(XO1R(I))-(D2-DG)*COS(XO1R(I)))&(CLAM*COS(FIO1R(I))*
     3CDEL&SLAM*SDEL)*(DH*COS(XO1R(I))&(D2-DG)*SIN(XO1R(I)))
      XP1(I)=-S1(I)&(DE&DF)*CDEL-SDEL*(DH*COS(XO1R(I))&(D2-D
     1G)*SIN(XO1R(I)))
      YP1(I)=D3*COS(FIO1R(I))-(DE&DF)*SDEL*SIN(FIO1R(I))&COS
     1(FIO1R(I))*(DH*SIN(XO1R(I))-(D2-DG)*COS(XO1R(I)))-SIN(
     2FIO1R(I))*CDEL*(DH*COS(XO1R(I))&(D2-DG)*SIN(XO1R(I)))
      ZP1(I)=D3*SIN(FIO1R(I))&(DE&DF)*COS(FIO1R(I))*SDEL&SIN
     1(FIO1R(I))*(DH*SIN(XO1R(I))-(D2-DG)*COS(XO1R(I)))&COS(
     2FIO1R(I))*CDEL*(DH*COS(XO1R(I))&(D2-DG)*SIN(XO1R(I)))
400   CONTINUE
      GO TO 430
410   IF(NTYPE.GT.6) GO TO 430
      GO TO 430
430   RETURN
      END


      SUBROUTINE VELCTY(NTYPE,NI,TETO,CELO,DELO,D1,D2,D3,DA,
     1DB,DE,HE,FIO1,XO1,S1,VWIN,AWIN,VF1,VX1,VS1,AF1,AX1,AS1
     2)
      DIMENSION TETOR(36 ),TETO(36 ),FIO1(36 ),FIO1R(36 ),XO
     11(36 )
      DIMENSION XO1R(36 ),VF1(36 ),VX1(36 ),VS1(36 ),AF1(36
     1)
      DIMENSION AX1(36 ),AS1(36 )
      PI=3.14159265
      TROP=PI/180.0
      CELOR=CELO*TROP
      DELOR=DELO*TROP
      SDEL=SIN(DELOR)
      CDEL=COS(DELOR)
      SLAM=SIN(CELOR)
      CLAM=COS(CELOR)
      DO 509 I=1,NI
      TETOR(I)=TETO(I)*TROP
      FIO1R(I)=FIO1(I)*TROP
      XO1R(I)=XO1(I)*TROP
509   CONTINUE
      DO 500 I=1,NI
      IF(NTYPE.GT.3) GO TO 520
      GNO=    DA-D1*COS(TETOR(I))
      GMO=    DB*SLAM-D1*CLAM*SIN(TETOR(I))
      GM1=    GNO    *COS(FIO1R(I))&GMO    *SIN(FIO1R(I))
      GM2=    GMO    *COS(FIO1R(I))-GNO    *SIN(FIO1R(I))
      W1=    GMO    *SIN(FIO1R(I))&GNO    *COS(FIO1R(I))&D3
      W2=    GNO    *SIN(FIO1R(I))-GMO    *COS(FIO1R(I))-DE*SDE
```

```
1L
 GL1=    SIN(TETOR(I))*SIN(FIO1R(I))&CLAM*COS(TETOR(I))*
1COS(FIO1R(I))
 GL2=    SIN(TETOR(I))*COS(FIO1R(I))-CLAM*SIN(FIO1R(I))*
1COS(TETOR(I))
 GL3=    COS(TETOR(I))*SIN(FIO1R(I))-CLAM*SIN(TETOR(I))*
1COS(FIO1R(I))
 GL4=    CLAM*SIN(TETOR(I))*SIN(FIO1R(I))&COS(TETOR(I))*
1COS(FIO1R(I))
 IF(ABS(DELO).EQ.90.0) GO TO 512
 VF1(I)=-D1*(W2    *GL1    &(CDEL**2)*W1*GL2)*VWIN /(W2
1 *GM1    &(CDEL**2)*W1    *GM2)
 GO TO 513
512    VF1(I)=-D1*GL1*VWIN/GM1
513    VX1(I)=-(GM2*VF1(I)+D1*GL2*VWIN )/(D2*SIN(XO1R(I)))
 VS1(I)=D1*SLAM*COS(TETOR(I))*VWIN-D2*SDEL*COS(XO1R(I))
1*VX1(I)
 Q1=    GM1    **2&W2    *GM2    &(CDEL**2)*(GM2    **2-W1
1 *GM1)
 Q2=    GM1    *GL1    &W2    *GL2    &(CDEL**2)*(GM2    *GL2
1    -W1*GL1)
 Q3=    D1*(GL1    **2)&W2    *GL3    &(CDEL**2)*(D1*(GL2**
12)&W1*GL4)
 Q4=    W2    *GL1    &(CDEL**2)*W1    *GL2
 Q5=    W2    *GM1    &(CDEL**2)*W1    *GM2
 IF(ABS(DELO).EQ.90.0) GO TO 514
 AF1(I)=-(Q1    *(VF1(I)**2)&2.0*D1*Q2*VWIN*VF1(I)+D1*Q3
1*(VWIN**2)+D1*Q4*AWIN)/Q5
 GO TO 515
514    AF1(I)=-(GM2    *(VF1(I)**2)&2.0*D1*GL2*VWIN*VF1(I)+D1*
1GL3*(VWIN**2)+D1*GL1*AWIN)/GM1
515    AX1(I)=-(D2*COS(XO1R(I))*(VX1(I)**2)&GM2    *AF1(I)-GM1
1*(VF1(I)**2)+D1*(GL4*(VWIN**2)-2.0*GL1*VF1(I)*VWIN+GL2
2*AWIN))/(D2*SIN(XO1R(I)))
 AS1(I)=D2*SDEL*(SIN(XO1R(I))*VX1(I)**2-COS(XO1R(I))*AX
11(I))+D1*SLAM*(AWIN*COS(TETOR(I))-(VWIN**2)*SIN(TETOR(
2I)))
 GO TO 500
520    GO TO 500
500    CONTINUE
 RETURN
 END

 SUBROUTINE DSPLOT(LE,L1M1,L2M,L2M1,L3M,VF1,VF2,VF3,VF4
1,STMI,STMX,SDMI,SDMX)
 DIMENSION VF1(36),VF2(36),VF3(36),VF4(36),X(216),Y(216
1)
 DO 62051 I=1,LE
 X(I)=VF1 (I)
 Y(I)=VF2 (I)
62051 CONTINUE
```

```
      DO 62052 I=L1M1,L2M
      I1NI=I-LE
      X(I)=VF1 (I1NI)
      Y(I)=VF3(I1NI)*20.0
62052 CONTINUE
      DO 62053 I=L2M1,L3M
      I2NI=I-2*LE
      X(I)=VF1 (I2NI)
      Y(I)=VF4(I2NI)
62053 CONTINUE
      CALL PLOT(X,STMI, STMX,0,Y,SDMI ,SDMX,0,0.,0.,0.,0,L3M
     1,3,1,3,2)
      RETURN
      END


      SUBROUTINE COPPLT(LE,L1M1,L2M,L2M1,L3M,      VF1,VF2,VF
     13,AF1,AF2,AF3,SCMI,SCMX)
      DIMENSION VF1(36),VF2(36),VF3(36),AF1(36),AF2(36),AF3(
     136)
      DIMENSION X(216),Y(216)
      DO 6251 I=1,LE
      X(I)=VF2(I)
      Y(I)=-VF1(I)
6251  CONTINUE
      DO 6252  I=L1M1,L2M
      I1NI=I-LE
      X(I)=VF2(I1NI)
      Y(I)=VF3(I1NI)
6252  CONTINUE
      DO 6253  I=L2M1,L3M
      I2NI=I-2*LE
      X(I)=VF1(I2NI)
      Y(I)=VF3(I2NI)
6253  CONTINUE
      CALL PLOT(X,SCMI ,SCMX,0,Y, SCMI,SCMX,0,0.,0.,0.,0,L3M
     1,3,1,3,2)
      DO 6254 I=1,LE
      X(I)=AF2(I)
      Y(I)=-AF1(I)
6254  CONTINUE
      DO 6255  I=L1M1,L2M
      I1NI=I-LE
      X(I)=AF2(I1NI)
      Y(I)=AF3(I1NI)
6255  CONTINUE
      DO 6256 I=L2M1,L3M
      I2NI=I-2*LE
      X(I)=AF1(I2NI)
      Y(I)=AF3(I2NI)
6256  CONTINUE
      CALL PLOT(X,SCMI ,SCMX,0,Y, SCMI,SCMX,0,0.,0.,0.,0,L3M
```

```
      1,3,1,3,2)
       RETURN
       END


       SUBROUTINE VELPLT(LE,L1M1,L2M,L2M1,L3M,L3M1,L4M,L4M1,L
      15M,L5M1,L6M,XP1,YP1,YP2,YP3,ZP1,ZP2,ZP3,STMI,STMX,SVMI
      2, SVMX)
       DIMENSION X(216),Y(216),XP1(36),YP1(36),YP2(36),YP3(36
      1),ZP1(36)
       DIMENSION ZP2(36),ZP3(36)
       DO 6271  I=1,LE
       X(I)=XP1 (I)
       Y(I)=YP1(I)
 6271  CONTINUE
       DO 6272  I=L1M1,L2M
       I1NI=I-LE
       X(I)=XP1 (I1NI)
       Y(I)=YP2(I1NI)
 6272  CONTINUE
       DO 6273  I=L2M1,L3M
       I2NI=I-2*LE
       X(I)=XP1 (I2NI)
       Y(I)=YP3(I2NI)
 6273  CONTINUE
       DO 6274  I=L3M1,L4M
       I3NI=I-3*LE
       X(I)=XP1 (I3NI)
       Y(I)=ZP1(I3NI)
 6274  CONTINUE
       DO 6275  I=L4M1,L5M
       I4NI=I-4*LE
       X(I)=XP1 (I4NI)
       Y(I)=ZP2(I4NI)
 6275  CONTINUE
       DO 6276  I=L5M1,L6M
       I5NI=I-5*LE
       X(I)=XP1 (I5NI)
       Y(I)=ZP3(I5NI)
 6276  CONTINUE
       CALL PLOT(X,STMI, STMX,0,Y,SVMI ,SVMX,0,0.,0.,0.,0,L6M
      1,6,1,3,2)
       RETURN
       END


       SUBROUTINEPLOT(X,XMIN,XMAX,LX,Y,YMIN,YMAX,LY,Z,ZMIN,ZM
      1AX,LZ,NPT,NPLOT,NCOPY,NCD,NDIM)
C      THIS IS THE STANDARD PLOT SUBROUTINE FOR IBM-7040
C      DIGITAL COMPUTER.
```

PROGRAM B.

```
C      PROGRAM-RSRC  B
C      CEMIL BAGCI-SYNTHESIS OF THE RSRC SPACE MECHANISM FOR
C      UNCONSTRAINED SCREW GEGERATION BY MATRIX ITERATION.
       DIMENSION TET(25),          SMQO(8)          ,V(66),VW(66),
      1PQ(8,8)
       DIMENSION TETOR(25 ),TETO(25 ),FIO1(25 ),FIO1R(25 ),XO
      11(25 )
       DIMENSION XO1R(25 ) ,FIO2(25 ),XO2(25 ),S2(25 ),FIO2R(
      125),S1(25)
       DIMENSION PQINV(8,8),H(8),CHECK(8),SYO(25),SF(25),XO2R
      1(25)
10050  FORMAT(5F10.3)
10051  FORMAT(8F9.5)
10052  FORMAT(3I5)
10053  FORMAT( 8F7.2)
10054  FORMAT(9F7.2)
10055  FORMAT(8F7.2)
10056  FORMAT(3F10.4)
10057  FORMAT(I5)
10037  FORMAT(1H1///5X, 95HRSRC MECHANISM WHICH APPROXIMATES
      1SPECIFIED SCREW DISPLACEMENT BEST IN THE LEAST SQUARES
      2 SENSE.///)
10041  FORMAT(12X,4HR**2,12X,5HRF**2,12X,5HRS**2/)
10042  FORMAT(5X,3F15.8)
10044  FORMAT(5X,10F12.6)
10043  FORMAT(///10X,2HDE,10X,2HD1,10X,2HD2,10X,2HD3,10X,2HDA
      1,10X,2HDB,8X,5HDELTA,6X,5HTETO1,6X,6HLAMBDA/)
10045  FORMAT(///8X,9HINPUT RO.,2X,10HOUTPUT RO.,5X,7HGEN.SAY
      1,5X,7HDES.SAY,2X,10HOUTPUT TR.,5X,7HGEN.TR.,5X,7HDES.T
      2R.,4X,8HANGLE XO,2X,10HDIF.IN RO.,2X,10HDIF.IN TR./)
10046  FORMAT(5X,8F12.6)
10038  FORMAT(///5X,42HSOLUTION CORRESPONDING TO THE MECHANIS
      1M OF/)
10039  FORMAT(22X,I2)
       READ(5,10057)NDIM
C      NDIM IS THE NUMBER OF MECHANISMS OPTIMIZED.
       NMECH=1
39997  READ(5,10050) DTET12,DFY12,CELO,DTET,DLS12
C      DTET12 IS THE RANGE OF INPUT CRANK ROTATION.
C      DFY12 IS THE RANGE OF OUTPUT CRANK ROTATION.
C      CELO IS THE SKEW ANGLE LAMBDA.
C      DTET IS THE INCREMENT IN THE INDEPENDENT PARAMETER
C      TETO.DLS12 IS THE RANGE OF THE OUTPUT TRANSLATION.
```

```
      READ(5,10057) NITLMX
C     NITLMX IS THE NUMBER OF INITIAL SET OF DIMENSIONS FOR
C     A MECHANISM.
      NINITL=1
      PI=3.14159265
      TROP=PI/180.0
      CLAM=COS(CELO*TROP)
      SLAM=SIN(CELO*TROP)
      READ(5,10051)(CHECK(I),I=1,8)
C     CHECK (I) ARE THE DESIRED ACCURACY IN THE DIMENSIONS
      READ(5,10052) I8,I9,IV
C     I8=I9=1 DEFINES THE INVERSION WITH POSITIVE SIGNED
C     RADICAL IN THE DISPLACEMENT EQUATIONS,I8=I9=2 DEFINES
C     THE INVERSION WITH NEGATIVE SIGNED RADICAL.IV DEFINES
C     FUNCTION BEING GENERATED IN THE SUBROUTINE 'DESIRED'.
      READ(5,10057)NPR
C     NPR IS THE NUMBER OF PRECISION POINTS.
      READ(5,10057)NITMX
C     NITMX IS THE MAXIMUM NUMBER OF ITERATIONS FOR EACH SET
C     OF INITIAL DIMENSIONS.
      READ(5,10057)NCHPNT
C     IF NCHPNT IS NOT ZERO IT PRINTS THE DISPLACEMENTS
C     AFTER EACH ITERATION.
      READ(5,10056)R2MX,RF2MX,RS2MX
C     R2MX, RF2MX, RS2MX DEFINE THE DESIRABLE LIMITS FOR SUM
C     OF THE SQUARED ERRORS IN BOTH ROTATION AND TRANS.,
C     ROTATION ONLY,AND IN TRANSLATION ONLY.
      READ(5,10057)NTDATA
C     IF NTDATA IS NOT ZERO, A SET OF DATA FOR THE DESIRED
C     FUNCTION IS NECESSARY.
      DO 50060 KDRCT=I8,I9
      IF(NTDATA.NE.0) GO TO 39998
      CALL DSIRED(      DTET,DFY12,DTET12,DLS12,NPR,KDRCT,IV
     1,TET,SYO,SF)
      GO TO 39999
39998 READ(5,10056)(TET(I),SYO(I),SF(I),I=1,NPR)
39999 READ(5,10053) DE,D1,D2,D3,DA,DB,DELO,TETO1
      DEIN=DE
      D1IN=D1
      D2IN=D2
      D3IN=D3
      DAIN=DA
      DBIN=DB
      NITER=0
      WRITE(6,10044)DE,D1,D2,D3,DA,DB,DELO,TETO1
40008 CDEL=COS(DELO*TROP)
      SDEL=SIN(DELO*TROP)
      IF(ABS(DE).GT.10.0)DE=DEIN
      IF(ABS(DA).GT.30.0) DA=DAIN
      IF(ABS(DB).GT.30.0) DB=DBIN
      IF(D1.LE.0.0) D1=D1IN
      IF(D2.LE.0.0) D2=D2IN
```

```
      IF(D3.LE.0.0) D3=D3IN
      DEIN=DE
      D1IN=D1
      D2IN=D2
      D3IN=D3
      DBIN=DB
      DAIN=DA
      DO 10024 I=1,NPR
      TETO(I)=TETO1&TET(I)
      TETOR(I)=TETO(I)*TROP
10024 CONTINUE
      NI=NPR
100   NEO=0
      IF(DELO.EQ.0.0) GO TO 110
      IF(ABS(DELO).EQ.180.0) GO TO 110
      IF(ABS(DELO),EQ.90.0) GO TO 1202
      GO TO 130
110   MJ1=0
      MJ2=0
      DO 1101 I=1,NI
      GNO=DA-D1*COS(TETOR(I))
      GMO=DB*SLAM-D1*CLAM*SIN(TETOR(I))
      AO=2.0*D3*GNO
      BO=2.0*D3*GMO
      CO=D3**2-D2**2&GNO**2&GMO**2
      RADI=BO**2&AO**2-CO**2
      IF(RADI    .LT.0.0) GO TO 1101
      ROOT=SQRT(RADI)
      FIO1R(I)=2.0*ATAN((-BO&ROOT)/(CO-AO))
      FIO1(I)=FIO1R(I)/TROP
      T1=  (GMO   *SIN(FIO1R(I))&GNO   *COS(FIO1R(I))&D3)/D2
      IF(ABS(T1).GT.1.0) GO TO 11013
      MJ1=MJ1&1
      J1=MJ1
      TS1=(GNO*SIN(FIO1R(I))-GMO*COS(FIO1R(I))-DE*SDEL)/(D2*
     1CDEL)
      XO1R(I)=ARCOS(ABS(T1))
      IF(T1.GT.0.0) GO TO 99011
      IF(TS1.GT.0.0) GO TO 99012
      XO1R(I)=XO1R(I)-PI
      GO TO 99014
99012 XO1R(I)=PI-XO1R(I)
      GO TO 99014
99011 IF(TS1.GT.0.0) GO TO 99014
      XO1R(I)=-XO1R(I)
99014 XO1(I)=XO1R(I)/TROP
      S1(I)=DE*CDEL&DB*CLAM&D1*SLAM*SIN(TETOR(I))
11013 FIO2R(I)=2.0*ATAN((-BO-ROOT)/(CO-AO))
      FIO2(I)=FIO2R(I)/TROP
      T2=  (GMO   *SIN(FIO2R(I))&GNO   *COS(FIO2R(I))&D3)/D2
      IF(ABS(T2).GT.1.0) GO TO 1101
      MJ2=MJ2&1
```

```
      J2=MJ2
      TS2=(GNO*SIN(FIO2R(I))-GMO*COS(FIO2R(I))-DE*SDEL)/(D2*
     1CDEL)
      XO2R(I)=ARCOS(ABS(T2))
      IF(T2.GT.0.0) GO TO 99021
      IF(TS2.GT.0.0) GO TO 99022
      XO2R(I)=XO2R(I)-PI
      GO TO 99024
99022 XO2R(I)=PI-XO2R(I)
      GO TO 99024
99021 IF(TS2.GT.0.0) GO TO 99024
      XO2R(I)=-XO2R(I)
99024 XO2(I)=XO2R(I)/TROP
      S2(I)=DE*CDEL&DB*CLAM&D1*SLAM*SIN(TETOR(I))
1101  CONTINUE
      GO TO 140
1202  MJ1=0
      MJ2=0
      DO 1201 I=1,NI
      GNO=DA-D1*COS(TETOR(I))
      GMO=DB*SLAM-D1*CLAM*SIN(TETOR(I))
      AO=DE*SDEL
      BO=GNO
      CO=GMO
      RADI=BO**2&CO**2-AO**2
      IF(RADI    .LT.0.0) GO TO 1201
      ROOT=SQRT(RADI)
      FIO1R(I)=2.0*ATAN((-BO&ROOT)/(CO-AO))
      FIO1(I)=FIO1R(I)/TROP
      T1=  (GMO    *SIN(FIO1R(I))&GNO    *COS(FIO1R(I))&D3)/D2
      IF(ABS(T1).GT.1.0) GO TO 12013
      MJ1=MJ1&1
      J1=MJ1
      XO1R(I)=ARCOS((GMO    *SIN(FIO1R(I))&GNO    *COS(FIO1R(I
     1))&D3)/D2)
      XO1(I)=XO1R(I)/TROP
      S1(I)=DB*CLAM&D1*SLAM*SIN(TETOR(I))-D2*SDEL*SIN(XO1R(I
     1))
12013 FIO2R(I)=2.0*ATAN((-BO-ROOT)/(CO-AO))
      FIO2(I)=FIO2R(I)/TROP
      T2=  (GMO    *SIN(FIO2R(I))&GNO    *COS(FIO2R(I))&D3)/D2
      IF(ABS(T2).GT.1.0) GO TO 1201
      MJ2=MJ2&1
      J2=MJ2
      XO2R(I)=ARCOS((GMO    *SIN(FIO2R(I))&GNO    *COS(FIO2R(I
     1))&D3)/D2)
      XO2(I)=XO2R(I)/TROP
      S2(I)=DB*CLAM&D1*SLAM*SIN(TETOR(I))-D2*SDEL*SIN(XO2R(I
     1))
1201  CONTINUE
      GO TO 140
130   MJ1=0
```

```
      MJ2=0
      DO 1301 I=1,NI
      GNO=DA-D1*COS(TETOR(I))
      GMO=DB*SLAM-D1*CLAM*SIN(TETOR(I))
      GKO=(    CDEL**2)*(D2**2-D3**2-GNO    **2)-GMO    **2-(DE
     1*SDEL)**2
      AO=   2.0*(D3*GNO    *(CDEL**2)&DE*GMO    *SDEL)-GKO
      A1=   4.0*(D3*GMO    *(CDEL**2)-DE*GNO    *SDEL-GMO*GNO*
     1(SDEL**2))
      A2=   4.0*(SDEL**2)*(GNO    **2-GMO    **2)-2.0*GKO
      A3=  4.0*(GMO    *GNO    *(SDEL**2)&D3*GMO    *(CDEL**2)-
     1DE*GNO*SDEL)
      A4=   -GKO    -2.0*(D3*GNO    *(CDEL**2)&DE*GMO*SDEL)
      IF(A4    .EQ.0.0) GO TO 51043
12012 BO=AO/A4
      B1=A1/A4
      B2=A2/A4
      B3=A3/A4
      CO=BO*(4.0*B2-B3**2)-B1**2
      C1=B3*B1-4.0*BO
      EO=(2.0*(B2**3)-9.0*B2*C1-27.0*CO)/27.0
      E1=(-3.0*C1&B2**2)/3.0
      IF(EO    .LT.0.0) GO TO 1306
      GO TO 13061
1306  EMO=-EO
      NEO=NEO&1
      EO=EMO
13061 CHEO=27.0*(EO**2)
      CHE1=4.0*(E1**3)
      IF(EO.EQ.0.0) GO TO 1307
      IF(E1    .EQ.0.0) GO TO 1308
      IF(CHEO    .GT.CHE1    ) GO TO 1302
      IF(CHEO    .EQ.CHE1    ) GO TO 1303
      IF(CHEO    .LT.CHE1    ) GO TO 1304
1302  IF(E1    .GT.0.0) GO TO 13022
      SAYA1=    0.5*EO    /((-E1    /3.0)**1.5)
      SAYRT=    ALOG(SAYA1    &SQRT(SAYA1    **2&1.0))
      IF(NEO.GT.0) GO TO 13064
      Y3=    B2    /3.0&2.0*SQRT(-E1    /3.0)*SINH(SAYRT    /3.0
     1)
      GO TO 131
13064 Y3=    B2    /3.0-2.0*SQRT(-E1    /3.0)*SINH(SAYRT    /3.0
     1)
      GO TO 131
13022 SAYA2=    0.5*EO    /((E1    /3.0)**1.5)
      IF(SAYA2    .LT.1.0) GO TO 1301
      SAYRT=    ALOG(SAYA2    &SQRT(SAYA2    **2-1.0))
13024 IF(NEO.GT.0) GO TO 13065
      Y3=    B2    /3.0&2.0*SQRT(E1    /3.0)*COSH(SAYRT    /3.0)
      GO TO 131
13065 Y3=    B2    /3.0-2.0*SQRT(E1    /3.0)*COSH(SAYRT    /3.0)
      GO TO 131
```

```
1303   IF(NEO.GT.0) GO TO 13062
       GO TO 13031
13062 ROOT1=    -2.0*((EO    /2.0)**(1.0/3.0))&B2    /3.0
       ROOT2=    (EO    /2.0)**(1.0/3.0)&B2    /3.0
       GO TO 13032
13031 ROOT1=    2.0*((EO    /2.0)**(1.0/3.0))&B2/3.0
       ROOT2=    -(EO    /2.0)**(1.0/3.0)&B2    /3.0
13032 IF(ROOT1.GT.ROOT2) GO TO 1305
       Y3=ROOT2
       GO TO 131
1305   Y3=ROOT1
       GO TO 131
1304   SAYRT=    ARCOS(0.5*EO    /((E1    /3.0)**1.5))
       IF(NEO.GT.0) GO TO 13066
       GO TO 13040
13066 ROOT1=    -2.0*SQRT(E1    /3.0)*COS(SAYRT    /3.0)&B2    /
      13.0
       ROOT2=    2.0*SQRT(E1    /3.0)*COS((PI-SAYRT    )/3.0)&B2
      1    /3.0
       ROOT3=    2.0*SQRT(E1    /3.0)*COS((PI&SAYRT    )/3.0)&B2
      1/3.0
       GO TO 13041
13040 ROOT1=    2.0*SQRT(E1    /3.0)*COS(SAYRT    /3.0)&B2/3.0
       ROOT2=    -2.0*SQRT(E1    /3.0)*COS((PI-SAYRT    )/3.0)&B
      12/3.0
       ROOT3=    -2.0*SQRT(E1    /3.0)*COS((PI&SAYRT    )/3.0)&B
      12/3.0
13041 IF(ROOT1    .GT.ROOT2    ) GO TO 13042
       IF(ROOT2    .GT.ROOT3    ) GO TO 13043
       Y3=ROOT3
       GO TO 131
13042 IF(ROOT1    .GT.ROOT3    ) GO TO 13044
       Y3=ROOT3
       GO TO 131
13044 Y3=    ROOT1
       GO TO 131
13043 Y3=ROOT2
       GO TO 131
1307   IF(E1    .LE.0.0) GO TO 13071
       ROOT1=B2/3.0
       ROOT2=    SQRT(E1    )&B2/3.0
       ROOT3=    -SQRT(E1    )&B2/3.0
       IF(ROOT1    .GE.ROOT2    ) GO TO 13073
       IF(ROOT2    .GE.ROOT3    ) GO TO 13074
       Y3=ROOT3
       GO TO 131
13073 IF(ROOT1    .GE.ROOT3    ) GO TO 13075
       Y3=ROOT3
       GO TO 131
13074 Y3=ROOT2
       GO TO 131
13075 Y3=ROOT1
```

```
      GO TO 131
13071 Y3=B2/3.0
      GO TO 131
1308  IF(NEO.NE.0) GO TO 13081
      Y3=   B2   /3.0&(EO   **(1.0/3.0))
      GO TO 131
13081 Y3=   B2   /3.0-(EO   **(1.0/3.0))
131   CONTINUE
      IF((((0.5*B3) **2)-B2   &Y3   ).LT.0.0) GO TO 1301
      QS1=   0.5*B3   &SQRT((0.5*B3   )**2-B2   &Y3)
      QS2=   0.5*B3   -SQRT((0.5*B3   )**2-B2   &Y3)
      IF((((0.5*Y3   )**2)-B0   ).LT.0.0) GO TO 1301
      HS1=   0.5*Y3   &SQRT((0.5*Y3   )**2-B0)
      HS2=   0.5*Y3   -SQRT((0.5*Y3   )**2-B0)
      QH1=   QS1   *HS2   &QS2   *HS1
      QH2=   QS1   *HS1   &QS2   *HS2
      IF(ABS(QH1   -B1   ).LE.0.0001) GO TO 1311
      IF(ABS(QH2   -B1   ).LE.0.0001) GO TO 1312
      GO TO 1301
1311  H1=HS1
      H2=HS2
      GO TO 1313
1312  H1=HS2
      H2=HS1
1313  RADI1=   QS1   **2-4.0*H1
      RADI2=QS2**2-4.0*H2
      IF(RADI1   .LT.0.0) GO TO 1316
      IF(RADI2.GE.0.0) GO TO 51043
13130 FIO1R(I)=2.0*ATAN(0.5*(-QS1   -SQRT(RADI1)))
      FIO1(I)=FIO1R(I)/TROP
      T1=  (GMO   *SIN(FIO1R(I))&GNO   *COS(FIO1R(I))&D3)/D2
      IF(ABS(T1).GT.1.0) GO TO 13131
      MJ1=MJ1&1
      J1=MJ1
      TS1=(GNO*SIN(FIO1R(I))-GMO*COS(FIO1R(I))-DE*SDEL)/(D2*
     1CDEL)
      XO1R(I)=ARCOS(ABS(T1))
      IF(T1.GT.0.0) GO TO 99031
      IF(TS1.GT.0.0) GO TO 99032
      XO1R(I)=XO1R(I)-PI
      GO TO 99034
99032 XO1R(I)=PI-XO1R(I)
      GO TO 99034
99031 IF(TS1.GT.0.0) GO TO 99034
      XO1R(I)=-XO1R(I)
99034 XO1(I)=XO1R(I)/TROP
      S1(I)=DB*CLAM&D1*SLAM*SIN(TETOR(I))-D2*SDEL*SIN(XO1R(I
     1))&DE*CDEL
13131 FIO2R(I)=2.0*ATAN(0.5*(-QS1   &SQRT(RADI1)))
      FIO2(I)=FIO2R(I)/TROP
      T2=  (GMO   *SIN(FIO2R(I))&GNO   *COS(FIO2R(I))&D3)/D2
      IF(ABS(T2).GT.1.0) GO TO 1301
```

```
      MJ2=MJ2&1
      J2=MJ2
      TS2=(GNO*SIN(FIO2R(I))-GMO*COS(FIO2R(I))-DE*SDEL)/(D2*
     1CDEL)
      XO2R(I)=ARCOS(ABS(T2))
      IF(T2.GT.0.0) GO TO 99041
      IF(TS2.GT.0.0) GO TO 99042
      XO2R(I)=XO2R(I)-PI
      GO TO 99044
99042 XO2R(I)=PI-XO2R(I)
      GO TO 99044
99041 IF(TS2.GT.0.0) GO TO 99044
      XO2R(I)=-XO2R(I)
99044 XO2(I)=XO2R(I)/TROP
      S2(I)=DB*CLAM&D1*SLAM*SIN(TETOR(I))-D2*SDEL*SIN(XO2R(I
     1))&DE*CDEL
      GO TO 1301
1316  IF(RADI2   .LT.0.0) GO TO 1301
      FIO1R(I)=2.0*ATAN(0.5*(-QS2   -SQRT(RADI2)))
      FIO1(I)=FIO1R(I)/TROP
      T1=  (GMO   *SIN(FIO1R(I))&GNO   *COS(FIO1R(I))&D3)/D2
      IF(ABS(T1).GT.1.0) GO TO 13161
      MJ1=MJ1&1
      J1=MJ1
      TS1=(GNO*SIN(FIO1R(I))-GMO*COS(FIO1R(I))-DE*SDEL)/(D2*
     1CDEL)
      XO1R(I)=ARCOS(ABS(T1))
      IF(T1.GT.0.0) GO TO 99051
      IF(TS1.GT.0.0) GO TO 99052
      XO1R(I)=XO1R(I)-PI
      GO TO 99054
99052 XO1R(I)=PI-XO1R(I)
      GO TO 99054
99051 IF(TS1.GT.0.0) GO TO 99054
      XO1R(I)=-XO1R(I)
99054 XO1(I)=XO1R(I)/TROP
      S1(I)=DB*CLAM&D1*SLAM*SIN(TETOR(I))-D2*SDEL*SIN(XO1R(I
     1))&DE*CDEL
13161 FIO2R(I)=2.0*ATAN(0.5*(-QS2   &SQRT(RADI2)))
      FIO2(I)=FIO2R(I)/TROP
      T2=  (GMO   *SIN(FIO2R(I))&GNO   *COS(FIO2R(I))&D3)/D2
      IF(ABS(T2).GT.1.0) GO TO 1301
      MJ2=MJ2&1
      J2=MJ2
      TS2=(GNO*SIN(FIO2R(I))-GMO*COS(FIO2R(I))-DE*SDEL)/(D2*
     1CDEL)
      XO2R(I)=ARCOS(ABS(T2))
      IF(T2.GT.0.0) GO TO 99061
      IF(TS2.GT.0.0) GO TO 99062
      XO2R(I)=XO2R(I)-PI
      GO TO 99064
99062 XO2R(I)=PI-XO2R(I)
```

```
      GO TO 99064
99061 IF(TS2.GT.0.0) GO TO 99064
      XO2R(I)=-XO2R(I)
99064 XO2(I)=XO2R(I)/TROP
      S2(I)=DB*CLAM&D1*SLAM*SIN(TETOR(I))-D2*SDEL*SIN(XO2R(I
     1))&DE*CDEL
1301  CONTINUE
140   CONTINUE
10005 R2=0.0
      RF2=0.0
      RS2=0.0
      DO 11005 I=1,8
      SMQO(I)=0.0
      DO 11006 J=1,8
      PQ(I,J)=0.0
11006 CONTINUE
11005 CONTINUE
      DO 10000 I=1,NPR
      IF(KDRCT.EQ.2) GO TO 10006
      FIYDR=(FIO1(1)&SYO(I))*TROP
      SO1=S1(1)
      GO TO 10009
10006 FIYDR=(FIO2(1)&SYO(I))*TROP
      SO1=S2(1)
      GO TO 10010
10009 RF=FIYDR-FIO1(I)*TROP
      RS=SO1&SF(I)-S1(I)
      CFIY=COS(FIO1(I)*TROP)
      SFIY=SIN(FIO1(I)*TROP)
      GO TO 10020
10010 RF=FIYDR-FIO2(I)*TROP
      RS=SO1&SF(I)-S2(I)
      SFIY=SIN(FIO2(I)*TROP)
      CFIY=COS(FIO2(I)*TROP)
10020 RF2=RF2&RF**2
      RS2=RS2&RS**2
      R=SQRT(RF**2&RS**2)
      R2=R**2&R2
      CT=COS(TETO(I)*TROP)
      ST=SIN(TETO(I)*TROP)
      GMO=DB*SLAM-D1*CLAM*ST
      GNO=DA-D1*CT
      W=GMO*SFIY&GNO*CFIY&D3
      D2W=D2**2-W**2
      IF(D2W.LE.0.0) GO TO 51043
      PMB=SLAM
      PMD1=-CLAM*ST
      PMT=-D1*CLAM*CT
      PNA=1.0
      PND1=-CT
      PNT=D1*ST
      PMTT=D1*CLAM*ST
```

```
      PMD1T=-CLAM*CT
      PND1T=ST
      PNTT=D1*CT
      F1=(SDEL**2)*(GNO**2-GMO**2)
      F2=2.0*GMO*GNO*(SDEL**2)
      F3=2.0*(D3*GNO*(CDEL**2)&DE*GMO*SDEL)
      F4=2.0*(D3*GMO*(CDEL**2)-DE*GNO*SDEL)
      GKO=(CDEL**2)*(D2**2-D3**2-GNO**2)-GMO**2-(DE*SDEL)**2
      GMNN=GMO*CFIY-GNO*SFIY
      GMNP=GMO*SFIY&GNO*CFIY
      PF1A =2.0*(SDEL**2)*(GNO*PNA)
      PF1D1=2.0*(SDEL**2)*(GNO*PND1-GMO*PMD1)
      PF1B =2.0*(SDEL**2)*(          -GMO*PMB)
      PF1DL=2.0*SDEL*CDEL*(GNO**2-GMO**2)
      PF1T =2.0*(SDEL**2)*(GNO*PNT -GMO*PMT )
      PF1D1A=2.0*(SDEL**2)*(PND1*PNA)
      PF1D1B=2.0*(SDEL**2)*(-PMD1*PMB)
      PF1D11=2.0*(SDEL**2)*(PND1**2-PMD1**2)
      PF1DLL=2.0*(CDEL**2-SDEL**2)*(GNO**2-GMO**2)
      PF1TT =2.0*(SDEL**2)*(PNT **2&GNO*PNTT  -PMT **2-GMO*P
     1MTT  )
      PF1DLA=4.0*SDEL*CDEL*(GNO*PNA)
      PF1TA =2.0*(SDEL**2)*(PNT *PNA)
      PF1D1L=4.0*SDEL*CDEL*(GNO*PND1-GMO*PMD1)
      PF1D1T=2.0*(SDEL**2)*(PNT *PND1&GNO*PND1T -PMT *PMD1-G
     1MO*PMD1T )
      PF1DLB=-4.0*SDEL*CDEL*GMO*PMB
      PF1TB=-2.0*(SDEL**2)*PMT*PMB
      PF1DLT=4.0*SDEL*CDEL*(GNO*PNT -GMO*PMT )
      PF2A= 2.0*(SDEL**2)*GMO*PNA
      PF2D1=2.0*(SDEL**2)*(GNO*PMD1&GMO*PND1)
      PF2B =2.0*(SDEL**2)*GNO*PMB
      PF2DL=4.0*CDEL*SDEL*GNO*GMO
      PF2T =2.0*(SDEL**2)*(GNO*PMT &GMO*PNT )
      PF2D11=2.0*(SDEL**2)*(2.0*PND1*PMD1)
      PF2DLL=4.0*GNO*GMO*(CDEL**2-SDEL**2)
      PF2TT =2.0*(SDEL**2)*(2.0*PNT *PMT &GNO*PMTT  &GMO*PNT
     1T  )
      PF2D1A=2.0*(SDEL**2)*PMD1*PNA
      PF2AB =2.0*(SDEL**2)*PNA*PMB
      PF2DLA=4.0*SDEL*CDEL*GMO*PNA
      PF2TA=2.0*(SDEL**2)*PMT*PNA
      PF2D1B=2.0*(SDEL**2)*PMB*PND1
      PF2D1L=4.0*SDEL*CDEL*(GNO*PMD1&GMO*PND1)
      PF2D1T=2.0*(SDEL**2)*(PNT *PMD1&GNO*PMD1T &PMT *PND1&G
     1MO*PND1T )
      PF2DLB=4.0*SDEL*CDEL*(GNO*PMB)
      PF2TB =2.0*(SDEL**2)*(PNT *PMB)
      PF2DLT=4.0*SDEL*CDEL*(GNO*PMT &GMO*PNT )
      PF3E=2.0*SDEL*GMO
      PF3D1=2.0*((CDEL**2)*D3*PND1               &DE*SDEL*PMD1)
      PF3T =2.0*((CDEL**2)*D3*PNT                &DE*SDEL*PMT )
```

```
      PF3DL=2.0*(-2.0*SDEL*CDEL*D3*GNO&DE*GMO*CDEL)
      PF3B=2.0*DE*PMB*SDEL
      PF3A=2.0*D3*(CDEL**2)*PNA
      PF3D3=2.0*(CDEL**2)*GNO
      PF3DLL=2.0*(-2.0*D3*GNO*(CDEL**2-SDEL**2)-DE*GMO*SDEL)
      PF3DLA=-4.0*D3*SDEL*CDEL*PNA
      PF3DLB=2.0*DE*CDEL*PMB
      PF3TT =2.0*((CDEL**2)*D3*PNTT&DE*SDEL*PMTT)
      PF3D1L=2.0*(-2.0*CDEL*SDEL*D3*PND1&DE*CDEL*PMD1)
      PF3D1T=2.0*((CDEL**2)*D3*PND1T&DE*SDEL*PMD1T)
      PF3DLT=2.0*(-2.0*CDEL*SDEL*D3*PNT&DE*CDEL*PMT)
      PF3EB=2.0*SDEL*PMB
      PF3EDL=2.0*CDEL*GMO
      PF3ET=2.0*SDEL*PMT
      PF3ED1=2.0*SDEL*PMD1
      PF3D3A=2.0*(CDEL**2)*PNA
      PF3TD3=2.0*(CDEL**2)*PNT
      PF3DL3=-4.0*GNO*SDEL*CDEL
      PF3D13=2.0*(CDEL**2)*PND1
      PF4E=-2.0*GNO*SDEL
      PF4D3=2.0*(CDEL**2)*GMO
      PF4B=2.0*D3*(CDEL**2)*PMB
      PF4A=-2.0*DE*SDEL*PNA
      PF4D1=2.0*((CDEL**2)*D3*PMD1-DE*SDEL*PND1)
      PF4DL=2.0*(-2.0*CDEL*SDEL*D3*GMO-DE*CDEL*GNO)
      PF4T =2.0*((CDEL**2)*           D3*PMT   -DE*SDEL*PNT )
      PF4ET=-2.0*SDEL*PNT
      PF4ED1=-2.0*SDEL*PND1
      PF4EA=-2.0*SDEL*PNA
      PF4EDL=-2.0*GNO*CDEL
      PF4D3B=2.0*(CDEL**2)*PMB
      PF4DL3=-4.0*SDEL*CDEL*GMO
      PF4TD3=2.0*(CDEL**2)*PMT
      PF4D13=2.0*(CDEL**2)*PMD1
      PF4DLA=-2.0*DE*CDEL*PNA
      PF4DLB=-4.0*CDEL*SDEL*D3*PMB
      PF4DLL=2.0*(-2.0*D3*GMO*(CDEL**2-SDEL**2)&DE*SDEL*GNO)
      PF4TT =2.0*((CDEL**2)*D3*PMTT-DE*SDEL*PNTT)
      PF4D1L=2.0*(-2.0*CDEL*SDEL*D3*PMD1-DE*CDEL*PND1)
      PF4D1T=2.0*((CDEL**2)**D3*PMD1T-DE*SDEL*PND1T)
      PF4DLT=2.0*(-2.0*CDEL*SDEL*D3*PMT-DE*CDEL*PNT)
      PKD1=-2.0*((CDEL**2)*GNO*PND1&GMO*PMD1)
      PKB=-2.0*GMO*PMB
      PKA=-2.0*(CDEL**2)*GNO*PNA
      PKD3=-2.0*(CDEL**2)*D3
      PKE=-2.0*DE*(SDEL**2)
      PKD2=2.0* (CDEL**2)*D2
      PKDL=2.0*(-CDEL*SDEL*(D2**2-D3**2-GNO**2)-(DE**2)*SDEL
     1*CDEL)
      PKT =2.0*((CDEL**2)*(          -GNO*PNT )-GMO*PMT )
      PKD2DL=-4.0*D2*SDEL*CDEL
      PKD2D2=2.0*(CDEL**2)
```

```
      PKD1DL=4.0*SDEL*CDEL*GNO*PND1
      PKD1T=-2.0*((CDEL**2)*(PNT*PND1&GNO*PND1T)-PMT*PMD1-GM
     1O*PMD1T)
      PKD1B=-2.0*PMB*PMD1
      PKD1A=-2.0*((CDEL**2)*PNA*PND1)
      PKD1D1=-2.0*((CDEL**2)*(PND1**2)&PMD1**2)
      PKEDL=-4.0*DE*SDEL*CDEL
      PKEE=-2.0*(SDEL**2)
      PKDLT =-4.0*CDEL*SDEL*(         -GNO*PNT )
      PKDLA=4.0*SDEL*CDEL*GNO*PNA
      PKDLD3=4.0*CDEL*SDEL*D3
      PKDLDL=-(D2**2&DE**2-D3**2-GNO**2)*2.0*(CDEL**2-SDEL**
     12)
      PKTT  =2.0*((CDEL**2)*(-PNT**2-GNO*PNTT)-PMT**2-GMO*PM
     1TT)
      PKTA=-2.0*(CDEL**2)*PNA*PNT
      PKTB=-2.0*PMB*PMT
      PKAA=-2.0*(CDEL**2)*(PNA**2)
      PKBB=-2.0*(PMB**2)
      PKD3D3=-2.0*(CDEL**2)
      FDN1=     2.0*F1*SFIY*CFIY-F2*(CFIY**2-SFIY**2)-F3*SFIY
     1&F4*CFIY
      IF(FDN1.EQ.0.0) GO TO 51043
      FDN=1.0/FDN1
      IF(FDN.EQ.0.0) GO TO 51043
      PFGE= FDN*(PKE                              -CFIY*PF3E -
     1SFIY*PF4E)
      PFGD1=FDN*(PKD1-(SFIY**2)*PF1D1&SFIY*CFIY*PF2D1-CFIY*P
     1F3D1-SFIY*PF4D1)
      PFGD2=FDN* PKD2
      PFGDL=FDN*(PKDL-(SFIY**2)*PF1DL&SFIY*CFIY*PF2DL-CFIY*P
     1F3DL-SFIY*PF4DL)
      PFGT =FDN*(PKT-(SFIY**2)*PF1T&SFIY*CFIY*PF2T-CFIY*PF3T
     1 -SFIY*PF4T)
      PFGA =FDN*(PKA-(SFIY**2)*PF1A&SFIY*CFIY*PF2A-CFIY*PF3A
     1 -SFIY*PF4A)
      PFGB =FDN*(PKB-(SFIY**2)*PF1B&SFIY*CFIY*PF2B-CFIY*PF3B
     1 -SFIY*PF4B)
      PFGD3=FDN*(PKD3-CFIY*PF3D3-SFIY*PF4D3)
      PFGEE =(FDN**2)*((PKEE &PFGE*(SFIY*PF3E-CFIY*PF4E))/FD
     1N-(PKE  -CFIY*PF3E -SFIY*PF4E )*(-SFIY*PF3E &CFIY*PF4E
     2 &PFGE *(2.0*F1*(CFIY**2-SFIY**2)&4.0*F2*CFIY*SFIY-CFI
     3Y*F3-SFIY*F4)))
      PFGD11=(FDN**2)*((PKD1D1&SFIY*CFIY*PF2D11-(SFIY**2)*PF
     11D11                    &PFGD1*(PF2D1*(CFIY**2-SFIY**2)-2
     22.0*SFIY*CFIY*PF1D1&SFIY*PF3D1-CFIY*PF4D1))/FDN-(PKD1&S
     3FIY*CFIY*PF2D1-(SFIY**2)*PF1D1-CFIY*PF3D1-SFIY*PF4D1)*
     4(2.0*SFIY*CFIY*PF1D1-(CFIY**2-SFIY**2)*PF2D1-SFIY*PF3D
     51&CFIY*PF4D1&PFGD1*(2.0*F1*(CFIY**2-SFIY**2)&4.0*F2*CF
     6IY*SFIY-CFIY*F3-SFIY*F4)))
      PFGD22=(FDN**2)*((PKD2D2)/FDN-PKD2*(PFGD2*(2.0*F1*(CFI
     1Y**2-SFIY**2)&4.0*F2*CFIY*SFIY-CFIY*F3-SFIY*F4)))
```

```
 PFGDLL=(FDN**2)*((PKDLDL&SFIY*CFIY*PF2DLL-(SFIY**2)*PF
11DLL-CFIY*PF3DLL -SFIY*PF4DLL&PFGDL*(PF2DL*(CFIY**2-SF
2IY**2)-2.0*SFIY*CFIY*PF1DL&SFIY*PF3DL-CFIY*PF4DL))/FDN
3-(PKDL&SFIY*CFIY*PF2DL-(SFIY**2)*PF1DL-CFIY*PF3DL-SFIY
4*PF4DL)*(2.0*SFIY*CFIY*PF1DL-(CFIY**2-SFIY**2)*PF2DL-S
5FIY*PF3DL&CFIY*PF4DL&PFGDL*(2.0*F1*(CFIY**2-SFIY**2)&4
6.0*F2*CFIY*SFIY-CFIY*F3-SFIY*F4)))
 PFGTT =(FDN**2)*((PKTT  &SFIY*CFIY*PF2TT -(SFIY**2)*PF
11TT -CFIY*PF3TT  -SFIY*PF4TT &PFGT *(PF2T *(CFIY**2-SF
2IY**2)-2.0*SFIY*CFIY*PF1T &SFIY*PF3T -CFIY*PF4T ))/FDN
3-(PKT &SFIY*CFIY*PF2T -(SFIY**2)*PF1T -CFIY*PF3T -SFIY
4*PF4T )*(2.0*SFIY*CFIY*PF1T -(CFIY**2-SFIY**2)*PF2T -S
5FIY*PF3T &CFIY*PF4T &PFGT *(2.0*F1*(CFIY**2-SFIY**2)&4
6.0*F2*CFIY*SFIY-CFIY*F3-SFIY*F4)))
 PFGED1=(FDN**2)*((      -CFIY*PF3ED1-SFIY*PF4ED1&PFGD1*
1(   SFIY*PF3E -CFIY*PF4E ))/FDN-(PKE -CFIY*PF3E -SFIY*P
2F4E )*(2.0*SFIY*CFIY*PF1D1-(CFIY**2-SFIY**2)*PF2D1-SFI
3Y*PF3D1&CFIY*PF4D1&PFGD1*(2.0*F1*(CFIY**2-SFIY**2)&4.0
4*F2*CFIY*SFIY-CFIY*F3-SFIY*F4)))
 PFGED2=(FDN**2)*((PFGD2*(SFIY*PF3E-CFIY*PF4E))/FDN-(PK
1E -CFIY*PF3E -SFIY*PF4E )*(PFGD2*(2.0*F1*(CFIY**2-SFIY
2**2)&4.0*F2*CFIY*SFIY-CFIY*F3-SFIY*F4)))
 PFGEDL=(FDN**2)*((PKEDL-CFIY*PF3EDL-SFIY*PF4EDL&PFGDL*
1(   SFIY*PF3E -CFIY*PF4E ))/FDN-(PKE -CFIY*PF3E -SFIY*P
2F4E )*(2.0*SFIY*CFIY*PF1DL-(CFIY**2-SFIY**2)*PF2DL-SFI
3Y*PF3DL&CFIY*PF4DL&PFGDL*(2.0*F1*(CFIY**2-SFIY**2)&4.0
4*F2*CFIY*SFIY-CFIY*F3-SFIY*F4)))
 PFGET =(FDN**2)*((      -CFIY*PF3ET-SFIY*PF4ET&PFGT*(
1SFIY*PF3E -CFIY*PF4E ))/FDN-(PKE -CFIY*PF3E -SFIY*PF4E
2 )*(2.0*SFIY*CFIY*PF1T -(CFIY**2-SFIY**2)*PF2T -SFIY*P
3F3T &CFIY*PF4T &PFGT *(2.0*F1*(CFIY**2-SFIY**2)&4.0*F2
4*CFIY*SFIY-CFIY*F3-SFIY*F4)))
 PFGD12=(FDN**2)*((                    PFGD2*(PF2D1*(CFIY*
1*2-SFIY**2)-2.0*SFIY*CFIY*PF1D1&SFIY*PF3D1-CFIY*PF4D1)
2)/FDN-(PKD1&SFIY*CFIY*PF2D1-(SFIY**2)*PF1D1-CFIY*PF3D1
3-SFIY*PF4D1)*(PFGD2*(2.0*F1*(CFIY**2-SFIY**2)&4.0*F2*C
4FIY*SFIY-CFIY*F3-SFIY*F4)))
 PFGD13=(FDN**2)*((
1    -CFIY*PF3D13-SFIY*PF4D13&PFGD3*(PF2D1*(CFIY**2-SFI
2Y**2)-2.0*SFIY*CFIY*PF1D1&SFIY*PF3D1-CFIY*PF4D1))/FDN-
3(PKD1&SFIY*CFIY*PF2D1-(SFIY**2)*PF1D1-CFIY*PF3D1-SFIY*
4PF4D1)*(    -SFIY*PF3D3&CFIY*PF4D3&PFGD3*(2.0*F1*(CFIY*
5*2-SFIY**2)&4.0*F2*CFIY*SFIY-CFIY*F3-SFIY*F4)))
 PFGD1L=(FDN**2)*((PKD1DL&SFIY*CFIY*PF2D1L-PF1D1L*(SFIY
1**2)-CFIY*PF3D1L-SFIY*PF4D1L&PFGDL*(PF2D1*(CFIY**2-SFI
2Y**2)-2.0*SFIY*CFIY*PF1D1&SFIY*PF3D1-CFIY*PF4D1))/FDN-
3(PKD1&SFIY*CFIY*PF2D1-(SFIY**2)*PF1D1-CFIY*PF3D1-SFIY*
4PF4D1)*(2.0*SFIY*CFIY*PF1DL-(CFIY**2-SFIY**2)*PF2DL-SF
5IY*PF3DL&CFIY*PF4DL&PFGDL*(2.0*F1*(CFIY**2-SFIY**2)&4.
60*F2*CFIY*SFIY-CFIY*F3-SFIY*F4)))
 PFGD1T=(FDN**2)*((PKD1T &SFIY*CFIY*PF2D1T-PF1D1T*(SFIY
1**2)-CFIY*PF3D1T-SFIY*PF4D1T&PFGT *(PF2D1*(CFIY**2-SFI
```

```
2Y**2)-2.0*SFIY*CFIY*PF1D1&SFIY*PF3D1-CFIY*PF4D1))/FDN-
3(PKD1&SFIY*CFIY*PF2D1-(SFIY**2)*PF1D1-CFIY*PF3D1-SFIY*
4PF4D1)*(2.0*SFIY*CFIY*PF1T -(CFIY**2-SFIY**2)*PF2T -SF
5IY*PF3T &CFIY*PF4T &PFGT *(2.0*F1*(CFIY**2-SFIY**2)&4.
60*F2*CFIY*SFIY-CFIY*F3-SFIY*F4)))
 PFGD2L=(FDN**2)*((PKD2DL)/FDN-(PKD2)*(
1          2.0*SFIY*CFIY*PF1DL-(CFIY**2-SFIY**2)*PF2DL-
2SFIY*PF3DL&CFIY*PF4DL&PFGDL*(2.0*F1*(CFIY**2-SFIY**2)&
34.0*F2*CFIY*SFIY-CFIY*F3-SFIY*F4)))
 PFGD2T=(FDN**2)*(-PKD2*(  2.0*SFIY*CFIY*PF1T -(CFIY**2
1-SFIY**2)*PF2T -SFIY*PF3T &CFIY*PF4T &PFGT *(2.0*F1*(C
2FIY**2-SFIY**2)&4.0*F2*CFIY*SFIY-CFIY*F3-SFIY*F4)))
 PFGDLT=(FDN**2)*((PKDLT &SFIY*CFIY*PF2DLT-PF1DLT*(SFIY
1**2)-CFIY*PF3DLT-SFIY*PF4DLT&PFGT *(PF2DL*(CFIY**2-SFI
2Y**2)-2.0*SFIY*CFIY*PF1DL&SFIY*PF3DL-CFIY*PF4DL))/FDN-
3(PKDL&SFIY*CFIY*PF2DL-(SFIY**2)*PF1DL-CFIY*PF3DL-SFIY*
4PF4DL)*(2.0*SFIY*CFIY*PF1T -(CFIY**2-SFIY**2)*PF2T -SF
5IY*PF3T &CFIY*PF4T &PFGT *(2.0*F1*(CFIY**2-SFIY**2)&4.
60*F2*CFIY*SFIY-CFIY*F3-SFIY*F4)))
 PFGAA =(FDN**2)*((PKAA                    &PFGA *(PF2A *(
1CFIY**2-SFIY**2)-2.0*SFIY*CFIY*PF1A &SFIY*PF3A -CFIY*P
2F4A ))/FDN-(PKA &SFIY*CFIY*PF2A -(SFIY**2)*PF1A -CFIY*
3PF3A -SFIY*PF4A )*(2.0*SFIY*CFIY*PF1A -(CFIY**2-SFIY**
42)*PF2A -SFIY*PF3A &CFIY*PF4A &PFGA *(2.0*F1*(CFIY**2-
5SFIY**2)&4.0*F2*CFIY*SFIY-CFIY*F3-SFIY*F4)))
 PFGEA =(FDN**2)*((-SFIY*PF4EA&PFGA*(  SFIY*PF3E -CFIY*
1PF4E ))/FDN-(PKE -CFIY*PF3E -SFIY*PF4E )*(2.0*SFIY*CFI
2Y*PF1A -(CFIY**2-SFIY**2)*PF2A -SFIY*PF3A &CFIY*PF4A &
3PFGA *(2.0*F1*(CFIY**2-SFIY**2)&4.0*F2*CFIY*SFIY-CFIY*
4F3-SFIY*F4)))
 PFGD1A=(FDN**2)*((PKD1A &SFIY*CFIY*PF2D1A-PF1D1A*(SFIY
1**2)                    &PFGA *(PF2D1*(CFIY**2-SFIY**2)-2.
20*SFIY*CFIY*PF1D1&SFIY*PF3D1-CFIY*PF4D1))/FDN-(PKD1&SF
3IY*CFIY*PF2D1-(SFIY**2)*PF1D1-CFIY*PF3D1-SFIY*PF4D1)*(
42.0*SFIY*CFIY*PF1A -(CFIY**2-SFIY**2)*PF2A -SFIY*PF3A
5&CFIY*PF4A &PFGA *(2.0*F1*(CFIY**2-SFIY**2)&4.0*F2*CFI
6Y*SFIY-CFIY*F3-SFIY*F4)))
 PFGD2A=(FDN**2)*(-PKD2*(  2.0*SFIY*CFIY*PF1A -(CFIY**2
1-SFIY**2)*PF2A -SFIY*PF3A &CFIY*PF4A &PFGA *(2.0*F1*(C
2FIY**2-SFIY**2)&4.0*F2*CFIY*SFIY-CFIY*F3-SFIY*F4)))
 PFGDLA=(FDN**2)*((PKDLA &SFIY*CFIY*PF2DLA-PF1DLA*(SFIY
1**2)-CFIY*PF3DLA-SFIY*PF4DLA&PFGA *(PF2DL*(CFIY**2-SFI
2Y**2)-2.0*SFIY*CFIY*PF1DL&SFIY*PF3DL-CFIY*PF4DL))/FDN-
3(PKDL&SFIY*CFIY*PF2DL-(SFIY**2)*PF1DL-CFIY*PF3DL-SFIY*
4PF4DL)*(2.0*SFIY*CFIY*PF1A -(CFIY**2-SFIY**2)*PF2A -SF
5IY*PF3A &CFIY*PF4A &PFGA *(2.0*F1*(CFIY**2-SFIY**2)&4.
60*F2*CFIY*SFIY-CFIY*F3-SFIY*F4)))
 PFGTA =(FDN**2)*((PKTA  &SFIY*CFIY*PF2TA -PF1TA *(SFIY
1**2)                    &PFGA *(PF2T *(CFIY**2-SFIY**2)-2.
20*SFIY*CFIY*PF1T &SFIY*PF3T -CFIY*PF4T ))/FDN-(PKT &SF
3IY*CFIY*PF2T -(SFIY**2)*PF1T -CFIY*PF3T -SFIY*PF4T )*(
42.0*SFIY*CFIY*PF1A -(CFIY**2-SFIY**2)*PF2A -SFIY*PF3A
```

```
5&CFIY*PF4A &PFGA *(2.0*F1*(CFIY**2-SFIY**2)&4.0*F2*CFI
6Y*SFIY-CFIY*F3-SFIY*F4)))
 PFGD3A=(FDN**2)*((-CFIY*PF3D3A&PFGA*(SFIY*PF3D3-CFIY*P
1F4D3))/FDN-(PKD3-CFIY*PF3D3-SFIY*PF4D3)*(2.0*SFIY*CFIY
2*PF1A-(CFIY**2-SFIY**2)*PF2A-SFIY*PF3A&CFIY*PF4A&PFGA
3*(2.0*F1*(CFIY**2-SFIY**2)&4.0*F2*CFIY*SFIY-CFIY*F3-SF
4IY*F4)))
 PFGD3B=(FDN**2)*((-SFIY*PF4D3B&PFGB*(SFIY*PF3D3-CFIY*P
1F4D3))/FDN-(PKD3-CFIY*PF3D3-SFIY*PF4D3)*(2.0*SFIY*CFIY
2*PF1B-(CFIY**2-SFIY**2)*PF2B-SFIY*PF3B&CFIY*PF4B&PFGB
3*(2.0*F1*(CFIY**2-SFIY**2)&4.0*F2*CFIY*SFIY-CFIY*F3-SF
4IY*F4)))
 PFGTB =(FDN**2)*((PKTB  &SFIY*CFIY*PF2TB -PF1TB *(SFIY
1**2)                    &PFGB *(PF2T *(CFIY**2-SFIY**2)-2.
20*SFIY*CFIY*PF1T &SFIY*PF3T -CFIY*PF4T ))/FDN-(PKT &SF
3IY*CFIY*PF2T -(SFIY**2)*PF1T -CFIY*PF3T -SFIY*PF4T )*(
42.0*SFIY*CFIY*PF1B -(CFIY**2-SFIY**2)*PF2B -SFIY*PF3B
5&CFIY*PF4B &PFGB *(2.0*F1*(CFIY**2-SFIY**2)&4.0*F2*CFI
6Y*SFIY-CFIY*F3-SFIY*F4)))
 PFGD33=(FDN**2)*((PKD3D3&PFGD3*(SFIY*PF3D3-CFIY*PF4D3)
1)/FDN-(PKD3  -CFIY*PF3D3-SFIY*PF4D3)*(    -SFIY*PF3D3&
2CFIY*PF4D3&PFGD3*(2.0*F1*(CFIY**2-SFIY**2)&4.0*F2*CFIY
3*SFIY-CFIY*F3-SFIY*F4)))
 PFGEDB=(FDN**2)*((PFGD3*(SFIY*PF3E-CFIY*PF4E))/FDN-(PK
1E -CFIY*PF3E -SFIY*PF4E )*(    -SFIY*PF3D3&CFIY*PF4D3&P
2FGD3*(2.0*F1*(CFIY**2-SFIY**2)&4.0*F2*CFIY*SFIY-CFIY*F
33-SFIY*F4)))
 PFGD23=(FDN**2)*(-(PKD2)*(    -SFIY*PF3D3&CFIY*PF4D3&PF
1GD3*(2.0*F1*(CFIY**2-SFIY**2)&4.0*F2*CFIY*SFIY-CFIY*F3
2-SFIY*F4)))
 PFGTD3=(FDN**2)*((-CFIY*PF3TD3-SFIY*PF4TD3&PFGD3*(PF2T
1 *(CFIY**2-SFIY**2)-2.0*SFIY*CFIY*PF1T &SFIY*PF3T -CFI
2Y*PF4T ))/FDN-(PKT &SFIY*CFIY*PF2T -(SFIY**2)*PF1T -CF
3IY*PF3T -SFIY*PF4T )*(    -SFIY*PF3D3&CFIY*PF4D3&PFGD3*
4(2.0*F1*(CFIY**2-SFIY**2)&4.0*F2*CFIY*SFIY-CFIY*F3-SFI
5Y*F4)))
 PFGDL3=(FDN**2)*((PKDLD3-CFIY*PF3DL3-SFIY*PF4DL3&PFGD3
1*(PF2DL*(CFIY**2-SFIY**2)-2.0*SFIY*CFIY*PF1DL&SFIY*PF3
2DL-CFIY*PF4DL))/FDN-(PKDL&SFIY*CFIY*PF2DL-(SFIY**2)*PF
31DL-CFIY*PF3DL-SFIY*PF4DL)*(    -SFIY*PF3D3&CFIY*PF4D3&
4PFGD3*(2.0*F1*(CFIY**2-SFIY**2)&4.0*F2*CFIY*SFIY-CFIY*
5F3-SFIY*F4)))
 PFGBB =(FDN**2)*((PKBB                          &PFGB *(PF2B *(
1CFIY**2-SFIY**2)-2.0*SFIY*CFIY*PF1B &SFIY*PF3B -CFIY*P
2F4B ))/FDN-(PKB &SFIY*CFIY*PF2B -(SFIY**2)*PF1B -CFIY*
3PF3B -SFIY*PF4B )*(2.0*SFIY*CFIY*PF1B -(CFIY**2-SFIY**
42)*PF2B -SFIY*PF3B &CFIY*PF4B &PFGB *(2.0*F1*(CFIY**2-
5SFIY**2)&4.0*F2*CFIY*SFIY-CFIY*F3-SFIY*F4)))
 PFGEB =(FDN**2)*((-CFIY*PF3EB&PFGB*( &SFIY*PF3E -CFIY*
1PF4E ))/FDN-(PKE -CFIY*PF3E -SFIY*PF4E )*(2.0*SFIY*CFI
2Y*PF1B -(CFIY**2-SFIY**2)*PF2B -SFIY*PF3B &CFIY*PF4B &
3PFGB *(2.0*F1*(CFIY**2-SFIY**2)&4.0*F2*CFIY*SFIY-CFIY*
```

```
4F3-SFIY*F4)))
 PFGD1B=(FDN**2)*((PKD1B &SFIY*CFIY*PF2D1B-PF1D1B*(SFIY
1**2)                    &PFGB *(PF2D1*(CFIY**2-SFIY**2)-2.
20*SFIY*CFIY*PF1D1&SFIY*PF3D1-CFIY*PF4D1))/FDN-(PKD1&SF
3IY*CFIY*PF2D1-(SFIY**2)*PF1D1-CFIY*PF3D1-SFIY*PF4D1)*(
42.0*SFIY*CFIY*PF1B -(CFIY**2-SFIY**2)*PF2B -SFIY*PF3B
5&CFIY*PF4B &PFGB *(2.0*F1*(CFIY**2-SFIY**2)&4.0*F2*CFI
6Y*SFIY-CFIY*F3-SFIY*F4)))
 PFGD2B=(FDN**2)*(-PKD2 *( 2.0*SFIY*CFIY*PF1B -(CFIY**2
1-SFIY**2)*PF2B -SFIY*PF3B &CFIY*PF4B &PFGB *(2.0*F1*(C
2FIY**2-SFIY**2)&4.0*F2*CFIY*SFIY-CFIY*F3-SFIY*F4)))
 PFGDLB=(FDN**2)*((        SFIY*CFIY*PF2DLB-PF1DLB*(SFIY
1**2)-CFIY*PF3DLB-SFIY*PF4DLB&PFGB *(PF2DL*(CFIY**2-SFI
2Y**2)-2.0*SFIY*CFIY*PF1DL&SFIY*PF3DL-CFIY*PF4DL))/FDN-
3(PKDL&SFIY*CFIY*PF2DL-(SFIY**2)*PF1DL-CFIY*PF3DL-SFIY*
4PF4DL)*(2.0*SFIY*CFIY*PF1B -(CFIY**2-SFIY**2)*PF2B -SF
5IY*PF3B &CFIY*PF4B &PFGB *(2.0*F1*(CFIY**2-SFIY**2)&4.
60*F2*CFIY*SFIY-CFIY*F3-SFIY*F4)))
 PFGAB =(FDN**2)*((        SFIY*CFIY*PF2AB
1  &PFGB *(PF2A *(CFIY**2-SFIY**2)-2.0*SFIY*CFIY*PF1A &
2SFIY*PF3A -CFIY*PF4A ))/FDN-(PKA &SFIY*CFIY*PF2A -(SFI
3Y**2)*PF1A -CFIY*PF3A -SFIY*PF4A )*(2.0*SFIY*CFIY*PF1B
4 -(CFIY**2-SFIY**2)*PF2B -SFIY*PF3B &CFIY*PF4B &PFGB *
5(2.0*F1*(CFIY**2-SFIY**2)&4.0*F2*CFIY*SFIY-CFIY*F3-SFI
6Y*F4)))
 PWE=GMNN*PFGE
 PWD1=GMNN*PFGD1&SFIY*PMD1&CFIY*PND1
 PWD2=GMNN*PFGD2
 PWDL=GMNN*PFGDL
 PWT= GMNN*PFGT &SFIY*PMT &CFIY*PNT
 PWA=GMNN*PFGA&CFIY*PNA
 PWB=GMNN*PFGB&SFIY*PMB
 PWD3=GMNN*PFGD3&1.0
 PWEE  =-GMNP*(PFGE **2)&GMNN*PFGEE
 PWD1D1=-GMNP*(PFGD1**2)&CFIY*2.0*PMD1*PFGD1&GMNN*PFGD1
11 -SFIY*2.0*PND1*PFGD1
 PWD2D2=-GMNP*(PFGD2**2)&GMNN*PFGD22
 PWDLDL=-GMNP*(PFGDL**2)&GMNN*PFGDLL
 PWTT  =-GMNP*(PFGT **2)&CFIY*2.0*PMT *PFGT &GMNN*PFGTT
1 &SFIY*PMTT -SFIY*2.0*PNT *PFGT &CFIY*PNTT
 PWD3D3=-GMNP*(PFGD3**2)&GMNN*PFGD33
 PWAA=-GMNP*(PFGA**2)&GMNN*PFGAA-SFIY*2.0*PNA*PFGA
 PWBB=-GMNP*(PFGB**2)&GMNN*PFGBB&2.0*CFIY*PMB*PFGB
 PWED1 =-GMNP*PFGD1*PFGE &CFIY*(PMD1*PFGE            )&
1GMNN*PFGED1              -SFIY*(PND1*PFGE)
 PWED2 =-GMNP*PFGD2*PFGE &GMNN*PFGED2
 PWEDL =-GMNP*PFGDL*PFGE &GMNN*PFGEDL
 PWET  =-GMNP*PFGT *PFGE &CFIY*(PMT *PFGE            )&
1GMNN*PFGET               -SFIY*(PNT *PFGE  )
 PWD1D2=-GMNP*PFGD2*PFGD1&CFIY*(           &PFGD2*PMD1)&
1GMNN*PFGD12              -SFIY*(          &PFGD2*PND1)
 PWD1DL=-GMNP*PFGDL*PFGD1&CFIY*(           &PFGDL*PMD1)&
```

```
1GMNN*PFGD1L                 -SFIY*(              &PFGDL*PND1)
 PWD1T =-GMNP*PFGT *PFGD1&CFIY*(PMT *PFGD1&PFGT *PMD1)&
1GMNN*PFGD1T&SFIY*PMD1T -SFIY*(PNT *PFGD1&PFGT *PND1)&C
2FIY*PND1T
 PWD2DL=-GMNP*PFGDL*PFGD2&GMNN*PFGD2L
 PWD2T =-GMNP*PFGT *PFGD2&CFIY*(PMT *PFGD2              )&
1GMNN*PFGD2T                 -SFIY*(PNT *PFGD2)
 PWDLT =-GMNP*PFGT *PFGDL&CFIY*(PMT *PFGDL              )&
1GMNN*PFGDLT                -SFIY*(PNT *PFGDL)
 PWEA=-GMNP*PFGA*PFGE&GMNN*PFGEA-SFIY*PNA*PFGE
 PWEB=-GMNP*PFGB*PFGE&GMNN*PFGEB&CFIY*PMB*PFGE
 PWED3=-GMNP*PFGD3*PFGE&GMNN*PFGED3
 PWD1D3=-GMNP*PFGD3*PFGD1&CFIY*PFGD3*PMD1&GMNN*PFGD13-S
1FIY*PFGD3*PND1
 PWD1A=-GMNP*PFGA*PFGD1&CFIY*PFGA*PMD1&GMNN*PFGD1A-SFIY
1*(PNA*PFGD1&PFGA*PND1)
 PWD1B=-GMNP*PFGB*PFGD1&CFIY*(PMB*PFGD1&PFGB*PMD1)&GMNN
1*PFGD1B-SFIY*PFGB*PND1
 PWD2A=-GMNP*PFGA*PFGD2&GMNN*PFGD2A-SFIY*PNA*PFGD2
 PWD2B=-GMNP*PFGB*PFGD2&GMNN*PFGD2B&CFIY*PMB*PFGD2
 PWD2D3=-GMNP*PFGD3*PFGD2&GMNN*PFGD23
 PWDLD3=-GMNP*PFGD3*PFGDL&GMNN*PFGDL3
 PWDLA =-GMNP*PFGA *PFGDL&GMNN*PFGDLA-SFIY*PNA*PFGDL
 PWDLB =-GMNP*PFGB *PFGDL&GMNN*PFGDLB&CFIY*PMB*PFGDL
 PWTD3=-GMNP*PFGT*PFGD3&GMNN*PFGTD3&CFIY*PMT*PFGD3-SFIY
1*PNT*PFGD3
 PWTA =-GMNP*PFGT*PFGA &GMNN*PFGTA &CFIY*PMT*PFGA -SFIY
1*(PNT*PFGA&PFGT*PNA)
 PWTB =-GMNP*PFGT*PFGB &GMNN*PFGTB&CFIY*(PMT*PFGB&PFGT*
1PMB)-SFIY*PNT*PFGB
 PWD3A=-GMNP*PFGA*PFGD3&GMNN*PFGD3A-SFIY*PNA*PFGD3
 PWD3B=-GMNP*PFGB*PFGD3&GMNN*PFGD3B&CFIY*PMB*PFGD3
 PWAB=-GMNP*PFGB*PFGA&CFIY*PMB*PFGA&GMNN*PFGAB-SFIY*PFG
1B*PNA
 PSGE=          CDEL&SDEL*W*PWE/SQRT(D2W)
 PSGD1=           SLAM*ST&SDEL*W*PWD1/SQRT(D2W)
 PSGD2=          -SDEL*(D2-W*PWD2)/SQRT(D2W)
 PSGT=          D1*SLAM*CT&SDEL*W*PWT/SQRT(D2W)
 PSGDL=          -DE*SDEL-CDEL*SQRT(D2W)&SDEL*W*PWDL/SQR
1T(D2W)
 PSGA=SDEL*W*PWA/SQRT(D2W)
 PSGB=CLAM&SDEL*W*PWB/SQRT(D2W)
 PSGD3=SDEL*W*PWD3/SQRT(D2W)
 PSGEE =SDEL*((PWE **2&W*PWEE  )*D2W&(W**2)*(PWE **2))/
1(D2W**1.5)
 PSGD11=SDEL*((PWD1**2&W*PWD1D1)*D2W&(W**2)*(PWD1**2))/
1(D2W**1.5)
 PSGD22=               -SDEL*((1.0-PWD2**2-W*PWD2D2)*D2W-(D
12-W*PWD2)**2)/(D2W**1.5)
 PSGDLL=               -DE*CDEL&SDEL*SQRT(D2W)&2.0*CDEL*W*P
1WDL/SQRT(D2W)&SDEL*((PWDL**2&W*PWDLDL)*D2W&(W**2)*(PWD
2L**2))/(D2W**1.5)
```

```
 PSGTT=            -D1*SLAM*ST&SDEL*((PWT**2&W*PWTT)*D2W&(
1W**2)*(PWT**2))/(D2W**1.5)
 PSGAA=SDEL*((PWA**2&W*PWAA)*D2W&(W**2)*(PWA**2))/(D2W*
1*1.5)
 PSGBB=SDEL*((PWB**2&W*PWBB)*D2W&(W**2)*(PWB**2))/(D2W*
1*1.5)
 PSGD33=SDEL*((PWD3**2&W*PWD3D3)*D2W&(W**2)*(PWD3**2))/
1(D2W**1.5)
 PSGED1= SDEL*((PWD1*PWE&W*PWED1)*D2W&(W**2)*PWE*PWD1)/
1(D2W**1.5)
 PSGED2=SDEL*((PWD2*PWE&W*PWED2)*D2W-W*PWE*(D2-W*PWD2))
1/(D2W**1.5)
 PSGEDL=-SDEL      &CDEL*W*PWE/SQRT(D2W)&SDEL*((PWDL*PWE
1&W*PWEDL)*D2W&(W**2)*PWE*PWDL)/(D2W**1.5)
 PSGET=SDEL*((PWT*PWE&W*PWET)*D2W&(W**2)*PWE*PWT)/(D2W*
1*1.5)
 PSGD12=                SDEL*((PWD2*PWD1&W*PWD1D2)*D2W-W*PW
1D1*(D2-W*PWD2))/(D2W**1.5)
 PSGD1L=                CDEL*W*PWD1/SQRT(D2W)&SDEL*((PWDL*P
1WD1&W*PWD1DL)*D2W&(W**2)*PWD1*PWDL)/(D2W**1.5)
 PSGD1T=               SLAM*CT&SDEL*((PWT*PWD1&W*PWD1T)*D2W
1&(W**2)*PWT*PWD1)/(D2W**1.5)
 PSGD2L=              -CDEL*(D2-W*PWD2)/SQRT(D2W)-SDEL*((-
1PWDL*PWD2-W*PWD2DL)*D2W&(D2-W*PWD2)*W*PWDL)/(D2W**1.5)
 PSGD2T=              -SDEL*((-PWT*PWD2-W*PWD2T)*D2W&(D2-W*
1PWD2)*W*PWT)/(D2W**1.5)
 PSGDLT=               CDEL*W*PWT/SQRT(D2W)&SDEL*((PWDL*PWT
1&W*PWDLT)*D2W&(W**2)*PWT*PWDL)/(D2W**1.5)
 PSGEA = SDEL*((PWA *PWE&W*PWEA )*D2W&(W**2)*PWE*PWA )/
1(D2W**1.5)
 PSGEB = SDEL*((PWB *PWE&W*PWEB )*D2W&(W**2)*PWE*PWB )/
1(D2W**1.5)
 PSGED3= SDEL*((PWD3*PWE&W*PWED3)*D2W&(W**2)*PWE*PWD3)/
1(D2W**1.5)
 PSGTA = SDEL*((PWA *PWT&W*PWTA )*D2W&(W**2)*PWT*PWA )/
1(D2W**1.5)
 PSGTB = SDEL*((PWB *PWT&W*PWTB )*D2W&(W**2)*PWT*PWB )/
1(D2W**1.5)
 PSGTD3= SDEL*((PWD3*PWT&W*PWTD3)*D2W&(W**2)*PWT*PWD3)/
1(D2W**1.5)
 PSGD1A= SDEL*((PWA*PWD1&W*PWD1A)*D2W&(W**2)*PWD1*PWA)/
1(D2W**1.5)
 PSGD1B= SDEL*((PWB*PWD1&W*PWD1B)*D2W&(W**2)*PWD1*PWB)/
1(D2W**1.5)
 PSGD13=SDEL*((PWD3*PWD1&W*PWD1D3)*D2W&(W**2)*PWD1*PWD3
1)/(D2W**1.5)
 PSGD2A=              -SDEL*((-PWA*PWD2-W*PWD2A)*D2W&(D2-W*
1PWD2)*W*PWA)/(D2W**1.5)
 PSGD2B=              -SDEL*((-PWB*PWD2-W*PWD2B)*D2W&(D2-W*
1PWD2)*W*PWB)/(D2W**1.5)
 PSGD23=-SDEL*((-PWD3*PWD2-W*PWD2D3)*D2W&(D2-W*PWD2)*W*
1PWD3)/(D2W**1.5)
```

```
      PSGDLA=              CDEL*W*PWA/SQRT(D2W)&SDEL*((PWDL*PWA
     1&W*PWDLA)*D2W&(W**2)*PWA*PWDL)/(D2W**1.5)
      PSGDLB=              CDEL*W*PWB/SQRT(D2W)&SDEL*((PWDL*PWB
     1&W*PWDLB)*D2W&(W**2)*PWB*PWDL)/(D2W**1.5)
      PSGDL3=CDEL*W*PWD3/SQRT(D2W)&SDEL*((PWDL*PWD3&W*PWDLD3
     1)*D2W&(W**2)*PWD3*PWDL)/(D2W**1.5)
      PSGD3A= SDEL*((PWA*PWD3&W*PWD3A)*D2W&(W**2)*PWD3*PWA)/
     1(D2W**1.5)
      PSGD3B= SDEL*((PWB*PWD3&W*PWD3B)*D2W&(W**2)*PWD3*PWB)/
     1(D2W**1.5)
      PSGAB = SDEL*((PWB *PWA&W*PWAB )*D2W&(W**2)*PWA*PWB )/
     1(D2W**1.5)
      IF(I.GT.1) GO TO 10C49
      PDLE=PFGE
      PDLD1=PFGD1
      PDLD2=PFGD2
      PDLDL=PFGDL
      PDLT =PFGT
      PDLD3=PFGD3
      PDLA =PFGA
      PDLB =PFGB
      PDLEE=PFGEE
      PDLD11=PFGD11
      PDLD22=PFGD22
      PDLDLL=PFGDLL
      PDLTT=PFGTT
      PDLD33=PFGD33
      PDLAA=PFGAA
      PDLBB=PFGBB
      PDLED1=PFGED1
      PDLED2=PFGED2
      PDLED3=PFGED3
      PDLEDL=PFGEDL
      PDLET =PFGET
      PDLEA =PFGEA
      PDLEB =PFGEB
      PDLD12=PFGD12
      PDLD13=PFGD13
      PDLD1L=PFGD1L
      PDLD1T=PFGD1T
      PDLD1A=PFGD1A
      PDLD1B=PFGD1B
      PDLD2L=PFGD2L
      PDLD23=PFGD23
      PDLD2T=PFGD2T
      PDLD2A=PFGD2A
      PDLD2B=PFGD2B
      PDLDLT=PFGDLT
      PDLDL3=PFGDL3
      PDLDLA=PFGDLA
      PDLDLB=PFGDLB
      PDLTD3=PFGTD3
```

```
      PDLTA =PFGTA
      PDLTB =PFGTB
      PDLD3A=PFGD3A
      PDLD3B=PFGD3B
      PDLAB =PFGAB
      PSOE =PSGE
      PSOD1=PSGD1
      PSOD2=PSGD2
      PSOD3=PSGD3
      PSODL=PSGDL
      PSOT =PSGT
      PSOA =PSGA
      PSOB =PSGB
      PSOED1  =PSGED1
      PSOED2  =PSGED2
      PSOED3  =PSGED3
      PSOEDL  =PSGEDL
      PSOET   =PSGET
      PSOEA   =PSGEA
      PSOEB   =PSGEB
      PSOEE   =PSGEE
      PSOAA   =PSGAA
      PSOBB   =PSGBB
      PSOTT   =PSGTT
      PSOD11  =PSGD11
      PSOD22  =PSGD22
      PSOD33  =PSGD33
      PSODLL  =PSGDLL
      PSOD12  =PSGD12
      PSOD13  =PSGD13
      PSOD1L  =PSGD1L
      PSOD1A  =PSGD1A
      PSOD1B  =PSGD1B
      PSOD1T  =PSGD1T
      PSOD23  =PSGD23
      PSOD2L  =PSGD2L
      PSOD2T  =PSGD2T
      PSOD2B  =PSGD2B
      PSOD2A  =PSGD2A
      PSODL3  =PSGDL3
      PSODLT  =PSGDLT
      PSODLA  =PSGDLA
      PSODLB  =PSGDLB
      PSOTD3=PSGTD3
      PSOTA =PSGTA
      PSOTB =PSGTB
      PSOD3A  =PSGD3A
      PSOD3B  =PSGD3B
      PSOAB   =PSGAB
10049 QE=RF*(PDLE-PFGE)&RS*(PSOE-PSGE)
      QD1=RF*(PDLD1-PFGD1)&RS*(PSOD1-PSGD1)
      QD2=RF*(PDLD2-PFGD2)&RS*(PSOD2-PSGD2)
```

```
 QDL=RF*(PDLDL-PFGDL)&RS*(PSODL-PSGDL)
 QT=RF*(PDLT      -PFGT)&RS*(PSOT      -PSGT)
 QD3=RF*(PDLD3-PFGD3)&RS*(PSOD3-PSGD3)
 QA =RF*(PDLA -PFGA )&RS*(PSOA -PSGA )
 QB =RF*(PDLB -PFGB )&RS*(PSOB -PSGB )
 SMQO(1)=QE &SMQO(1)
 SMQO(2)=QD1&SMQO(2)
 SMQO(3)=QD2&SMQO(3)
 SMQO(4)=QDL&SMQO(4)
 SMQO(5)=QT &SMQO(5)
 SMQO(6)=SMQO(6)&QD3
 SMQO(7)=SMQO(7)&QA
 SMQO(8)=SMQO(8)&QB
 PQEE  =(PDLE -PFGE)**2&RF*(PDLEE -PFGEE )&(PSOE -PSGE)
1**2&RS*(PSOEE -PSGEE )
 PQD1D1=(PDLD1-PFGD1)**2&RF*(PDLD11-PFGD11)&(PSOD1-PSGD
11)**2&RS*(PSOD11-PSGD11)
 PQD2D2=(PDLD2-PFGD2)**2&RF*(PDLD22-PFGD22)&(PSOD2-PSGD
12)**2&RS*(PSOD22-PSGD22)
 PQDLDL=(PDLDL-PFGDL)**2&RF*(PDLDLL-PFGDLL)&(PSODL-PSGD
1L)**2&RS*(PSODLL -PSGDLL)
 PQTT=(PDLT      -PFGT)**2&RF*(PDLTT       -PFGTT)&(PSOT
1      -PSGT)**2&RS*(PSOTT       -PSGTT)
 PQD3D3=(PDLD3-PFGD3)**2&RF*(PDLD33-PFGD33)&(PSOD3-PSGD
13)**2&RS*(PSOD33-PSGD33)
 PQED1 =(PDLD1-PFGD1)*(PDLE -PFGE )&RF*(PDLED1-PFGED1)&
1(PSOD1-PSGD1)*(PSOE -PSGE )&RS*(PSOED1-PSGED1)
 PQED3 =(PDLD3-PFGD3)*(PDLE -PFGE )&RF*(PDLED3-PFGED3)&
1(PSOD3-PSGD3)*(PSOE -PSGE )&RS*(PSOED3-PSGED3)
 PQED2 =(PDLD2-PFGD2)*(PDLE -PFGE )&RF*(PDLED2-PFGED2)&
1(PSOD2-PSGD2)*(PSOE -PSGE )&RS*(PSOED2-PSGED2)
 PQEDL =(PDLDL-PFGDL)*(PDLE -PFGE )&RF*(PDLEDL-PFGEDL)&
1(PSODL-PSGDL)*(PSOE -PSGE )&RS*(PSOEDL-PSGEDL)
 PQET =(PDLT      -PFGT)*(PDLE -PFGE )&RF*(PDLET -PFGET )
1&RS*(PSOET -PSGET )&(PSOT      -PSGT)*(PSOE -PSGE )
 PQD1D2=(PDLD2-PFGD2)*(PDLD1-PFGD1)&RF*(PDLD12-PFGD12)&
1(PSOD2-PSGD2)*(PSOD1-PSGD1)&RS*(PSOD12-PSGD12)
 PQD1DL=(PDLDL-PFGDL)*(PDLD1-PFGD1)&RF*(PDLD1L-PFGD1L)&
1(PSODL-PSGDL)*(PSOD1-PSGD1)&RS*(PSOD1L-PSGD1L)
 PQD1T=(PDLT      -PFGT)*(PDLD1-PFGD1)&RF*(PDLD1T-PFGD1T)
1&RS*(PSOD1T-PSGD1T)&(PSOT      -PSGT)*(PSOD1-PSGD1)
 PQD2DL=(PDLDL-PFGDL)*(PDLD2-PFGD2)&RF*(PDLD2L-PFGD2L)&
1(PSODL-PSGDL)*(PSOD2-PSGD2)&RS*(PSOD2L-PSGD2L)
 PQD1D3=(PDLD3-PFGD3)*(PDLD1-PFGD1)&RF*(PDLD13-PFGD13)&
1(PSOD3-PSGD3)*(PSOD1-PSGD1)&RS*(PSOD13-PSGD13)
 PQD2D3=(PDLD3-PFGD3)*(PDLD2-PFGD2)&RF*(PDLD23-PFGD23)&
1(PSOD3-PSGD3)*(PSOD2-PSGD2)&RS*(PSOD23-PSGD23)
 PQDLD3=(PDLD3-PFGD3)*(PDLDL-PFGDL)&RF*(PDLDL3-PFGDL3)&
1(PSOD3-PSGD3)*(PSODL-PSGDL)&RS*(PSODL3-PSGDL3)
 PQTD3 =(PDLD3-PFGD3)*(PDLT -PFGT )&RF*(PDLTD3-PFGTD3)&
1(PSOD3-PSGD3)*(PSOT -PSGT )&RS*(PSOTD3-PSGTD3)
 PQD2T=(PDLT      -PFGT)*(PDLD2-PFGD2)&RF*(PDLD2T-PFGD2T)
```

```
1&RS*(PSOD2T-PSGD2T)&(PSOT      -PSGT)*(PSOD2-PSGD2)
 PQDLT=(PDLT     -PFGT)*(PDLDL-PFGDL)&RF*(PDLDLT-PFGDLT)
1&RS*(PSODLT-PSGDLT)&(PSOT      -PSGT)*(PSODL-PSGDL)
 PQEA  =(PDLA -PFGA )*(PDLE -PFGE )&RF*(PDLEA -PFGEA )&
1(PSOA -PSGA)*(PSOE -PSGE )&RS*(PSOEA -PSGEA )
 PQD1A =(PDLA -PFGA )*(PDLD1-PFGD1)&RF*(PDLD1A-PFGD1A)&
1(PSOA -PSGA)*(PSOD1-PSGD1)&RS*(PSOD1A-PSGD1A)
 PQD3A =(PDLA -PFGA )*(PDLD3-PFGD3)&RF*(PDLD3A-PFGD3A)&
1(PSOA -PSGA)*(PSOD3-PSGD3)&RS*(PSOD3A-PSGD3A)
 PQD2A =(PDLA -PFGA )*(PDLD2-PFGD2)&RF*(PDLD2A-PFGD2A)&
1(PSOA -PSGA)*(PSOD2-PSGD2)&RS*(PSOD2A-PSGD2A)
 PQDLA =(PDLA -PFGA )*(PDLDL-PFGDL)&RF*(PDLDLA-PFGDLA)&
1(PSOA -PSGA)*(PSODL-PSGDL)&RS*(PSODLA-PSGDLA)
 PQTA =(PDLT     -PFGT)*(PDLA -PFGA )&RF*(PDLTA -PFGTA )
1&RS*(PSOTA -PSGTA )&(PSOT      -PSGT)*(PSOA -PSGA )
 PQAA  =(PDLA -PFGA )**2&RF*(PDLAA -PFGAA )&(PSOA -PSGA
1 )**2&RS*(PSOAA -PSGAA )
 PQEB  =(PDLB -PFGB )*(PDLE -PFGE )&RF*(PDLEB -PFGEB )&
1(PSOB -PSGB)*(PSOE -PSGE )&RS*(PSOEB -PSGEB )
 PQD1B =(PDLB -PFGB )*(PDLD1-PFGD1)&RF*(PDLD1B-PFGD1B)&
1(PSOB -PSGB)*(PSOD1-PSGD1)&RS*(PSOD1B-PSGD1B)
 PQD3B =(PDLB -PFGB )*(PDLD3-PFGD3)&RF*(PDLD3B-PFGD3B)&
1(PSOB -PSGB)*(PSOD3-PSGD3)&RS*(PSOD3B-PSGD3B)
 PQD2B =(PDLB -PFGB )*(PDLD2-PFGD2)&RF*(PDLD2B-PFGD2B)&
1(PSOB -PSGB)*(PSOD2-PSGD2)&RS*(PSOD2B-PSGD2B)
 PQDLB =(PDLB -PFGB )*(PDLDL-PFGDL)&RF*(PDLDLB-PFGDLB)&
1(PSOB -PSGB)*(PSODL-PSGDL)&RS*(PSODLB-PSGDLB)
 PQTB =(PDLT     -PFGT)*(PDLB -PFGB )&RF*(PDLTB -PFGTB )
1&RS*(PSOTB -PSGTB )&(PSOT      -PSGT)*(PSOB -PSGB )
 PQAB =(PDLA     -PFGA)*(PDLB -PFGB )&RF*(PDLAB -PFGAB )
1&RS*(PSOAB -PSGAB )&(PSOA      -PSGA)*(PSOB -PSGB )
 PQBB  =(PDLB -PFGB )**2&RF*(PDLBB -PFGBB )&(PSOB -PSGB
1 )**2&RS*(PSOBB -PSGBB )
 PQ(1,1)=PQ(1,1)&PQEE
 PQ(1,2)=PQ(1,2)&PQED1
 PQ(1,3)=PQ(1,3)&PQED2
 PQ(1,4)=PQ(1,4)&PQEDL
 PQ(1,5)=PQ(1,5)&PQET
 PQ(1,6)=PQ(1,6)&PQED3
 PQ(1,7)=PQ(1,7)&PQEA
 PQ(1,8)=PQ(1,8)&PQEB
 PQ(2,2)=PQ(2,2)&PQD1D1
 PQ(2,3)=PQ(2,3)&PQD1D2
 PQ(2,4)=PQ(2,4)&PQD1DL
 PQ(2,5)=PQ(2,5)&PQD1T
 PQ(2,6)=PQ(2,6)&PQD1D3
 PQ(2,7)=PQ(2,7)&PQD1A
 PQ(2,8)=PQ(2,8)&PQD1B
 PQ(3,3)=PQ(3,3)&PQD2D2
 PQ(3,4)=PQ(3,4)&PQD2DL
 PQ(3,5)=PQ(3,5)&PQD2T
 PQ(3,6)=PQ(3,6)&PQD2D3
```

```
      PQ(3,7)=PQ(3,7)&PQD2A
      PQ(3,8)=PQ(3,8)&PQD2B
      PQ(4,4)=PQ(4,4)&PQDLDL
      PQ(4,5)=PQ(4,5)&PQDLT
      PQ(4,6)=PQ(4,6)&PQDLD3
      PQ(4,7)=PQ(4,7)&PQDLA
      PQ(4,8)=PQ(4,8)&PQDLB
      PQ(5,5)=PQ(5,5)&PQTT
      PQ(5,6)=PQ(5,6)&PQTD3
      PQ(5,7)=PQ(5,7)&PQTA
      PQ(5,8)=PQ(5,8)&PQTB
      PQ(6,6)=PQ(6,6)&PQD3D3
      PQ(6,7)=PQ(6,7)&PQD3A
      PQ(6,8)=PQ(6,8)&PQD3B
      PQ(7,7)=PQ(7,7)&PQAA
      PQ(7,8)=PQ(7,8)&PQAB
      PQ(8,8)=PQ(8,8)&PQBB
10000 CONTINUE
      NA=8
50040 DO 50041 I=1,NA
      SMQO(I)=-SMQO(I)
      DO 50042 J=1,NA
      IF(J.GE.I) GO TO 50042
      PQ(I,J)=PQ(J,I)
50042 CONTINUE
50041 CONTINUE
      V(1)=NA
      V(2)=NA
      DO 50043 K=1,NA
      I2=K*NA&2
      I1=(K-1)*NA&3
      DO 50044 I=I1,I2
      J=I-I1&1
      V(I)=PQ(K,J)
50044 CONTINUE
50043 CONTINUE
      CALL INVERX(V,VW,DET,IE)
C     THIS IS THE STANDARD PROGRAM TO INVERT A MATRIX BY THE
C      IBM -7040
C     DIGITAL COMPUTER.
      IF(DET.NE.0.0) GO TO 50045
51043 D2=1.1*D2IN
      GO TO 40008
50045 DO 50046 K=1,NA
      I1=(K-1)*NA&3
      I2=K*NA&2
      DO 50047 I=I1,I2
      J=I-I1&1
      PQINV(K,J)=VW(I)
50047 CONTINUE
50046 CONTINUE
      DO 50048 K=1,NA
```

```
      SUMP=0.0
      DO 50049 L=1,NA
      SUMP=SUMP&PQINV(K,L)*SMQO(L)
50049 CONTINUE
      H(K)=SUMP
      WRITE(6,10044)H(K)
50048 CONTINUE
      NCHECK=0
      DO 50050 K=1,NA
      IF(ABS(H(K)).GT.CHECK(K)) GO TO 50051
      GO TO 50050
50051 NCHECK=NCHECK&1
50050 CONTINUE
      DE=DE&H(1)
      D1=D1&H(2)
      D2=D2&H(3)
      DELOR=DELOR&H(4)
      DELO=DELOR/TROP
      TETO1R=TETO1R&H(5)
      TETO1=TETO1R/TROP
      D3=D3&H(6)
      DA=DA&H(7)
      DB=DB&H(8)
      NITER=NITER&1
      IF(NITER.GT.NITMX) GO TO 50065
      IF(NCHECK.EQ.0) GO TO 50065
      IF(D3.LE.0.0) GO TO 40008
      IF(D2.LE.0.0) GO TO 40008
      IF(D1.LE.0.0) GO TO 40008
      IF(ABS(DB).GT.30.0) GO TO 40008
      IF(ABS(DA).GT.30.0) GO TO 40008
      IF(ABS(DE).GT.10.0)GO TO 40008
      WRITE(6,10039)KDRCT
      WRITE(6,10039)NITER
      WRITE(6,10042)R2,RF2,RS2
      RMSEFR=SQRT(RF2/FLOAT(NPR))
      RMSEFO=RMSEFR/TROP
      RMSES=SQRT(RS2/FLOAT(NPR))
      RMSE2=SQRT(R2/FLOAT(NPR))
      WRITE(6,10044)RMSE2,RMSEFR,RMSEFO,RMSES
      WRITE(6,10044)DE,D1,D2,D3,DA,DB,DELO,TETO1
      NFINPT=0
      IF(NCHPNT.NE.0) GO TO 52064
      GO TO 40008
50065 WRITE(6,10037)
      NFINPT=1
      WRITE(6,10038)
      WRITE(6,10039)KDRCT
      WRITE(6,10041)
      WRITE(6,10042)R2,RF2,RS2
      WRITE(6,10043)
51065 WRITE(6,10044)DE,D1,D2,D3,DA,DB,DELO,TETO1,CELO
```

```
      WRITE(6,10045)
      GO TO 52065
52064 IF(RF2.LE.RF2MX) GO TO 52065
      IF(RS2.LE.RS2MX) GO TO 52065
      IF(R2.GT.R2MX) GO TO 40008
52065 DO 50066 I=1,NPR
      IF(KDRCT.EQ.2) GO TO 50067
      SYOG=FIO1(I)-FIO1(1)
      SFG=S1(I)-SO1
      DIFROT=FIO1(1)&SYO(I)-FIO1(I)
      DIFTRN=SO1&SF(I)-S1(I)
      WRITE(6,10044) TETO(I),FIO1(I),SYOG,SYO(I),S1(I),SFG,S
     1F(I),XO1(I),DIFROT,DIFTRN
      GO TO 50066
50067 SYOG=FIO2(I)-FIO2(1)
      SFG=S2(I)-SO1
      DIFROT=FIO2(1)&SYO(I)-FIO2(I)
      DIFTRN=SO1&SF(I)-S2(I)
      WRITE(6,10044) TETO(I),FIO2(I),SYOG,SYO(I),S2(I),SFG,S
     1F(I),XO2(I),DIFROT,DIFTRN
50066 CONTINUE
      IF(NFINPT.NE.0)GO TO 50060
      GO TO 40008
50060 CONTINUE
51066 NINITL=NINITL&1
      IF(NINITL.GT.NITLMX)GO TO 51067
      GO TO  39999
51067 NMECH=NMECH&1
      IF(NMECH.GT.NDIM) GO TO 50070
      GO TO 39997
50070 STOP
      END


      SUBROUTINE DSIRED(DTET,DFY12,DTET12,DLS12,NPR,KDRCT,IV
     1,TET,SYO,SF)
      DIMENSION TET(25),SYO(25),SF(25)
      PI=3.14159265
      TROP=PI/180.0
      DO 100 I=1,NPR
      IF(I.GT.1) GO TO 101
      TET(I)=0.0
      GO TO 102
101   I1=I-1
      TET(I)=TET(I1)&DTET
102   IF(KDRCT.EQ.1) GO TO 103
      IF(KDRCT.EQ.2) GO TO 104
103   IF(IV.EQ.1) GO TO 1031
      IF(IV.EQ.2) GO TO 1032
1031  SYO(I)=-DFY12*      ((TET(I)/DTET12)**2)
      SF(I)=DLS12*(1.0-(1.0-TET(I)/DTET12)**2)
      GO TO 100
```

```
1032  SYO(I)=DFY12*(1.0-(1.0-TET(I)/(0.5*DTET12))**2)*(-1.0)
      SF(I)=DLS12 *(1.0-(1.0-TET(I)/(0.5*DTET12))**2)
      GO TO 100
104   IF(IV.EQ.1) GO TO 1041
      IF(IV.EQ.2) GO TO 1042
1041  SYO(I)=0.0
      SF(I)=DLS12*(((TET(I)-0.5*DTET12)/(0.5*DTET12))**2-1.0
     1)
      GO TO 100
1042  SYO(I)=DFY12*(1.0-(1.0-TET(I)/(0.5*DTET12))**2)*(-1.0)
      SF(I)=DLS12 *(1.0-(1.0-TET(I)/(0.5*DTET12))**2)
100   CONTINUE
      RETURN
      END
```

PROGRAM C

```
C      PROGRAM-RSRC  C
C      CEMIL BAGCI-SYNTHESIS OF THE RSRC SPACE MECHANISM FOR
C      CONSTRAINED AND UNCONSTRAINED SCREW GENERATION.
C      PARAMETERS OF CONSTRAINTS ARE INCLUDED.DESIGN
C      EQUATIONS ARE SOLVED BY RELAXATION.
       DIMENSION TETOR(30 ),TETO(30 ),FIO1(30 ),FIO1R(30 ),XO
      11(30 )
       DIMENSION XO1R(30 ) ,FIO2(30 ),XO2(30 ),S2(30 ),FIO2R(
      130),S1(30)
       DIMENSION XO2R(30)
       DIMENSION VX1(30),VS1(30),VF1(30),AF1(30),AS1(30),AX1(
      130)
       DIMENSION D(8),DDD(8),NIT(8),SMQO(8),ACCRCY(8)
       DIMENSION TET(30),SYO(30),SF(30)
       DIMENSION SYOG(30),SFG(30),DIFROT(30),DIFTRN(30)
       DIMENSION FGO1(30),FGO2(30),FGO1R(30),FGO2R(30)
       CALL ERRSET (259,0,0,2,0,0)
10037 FORMAT(1H1///5X, 95HRSRC MECHANISM WHICH APPROXIMATES
      1SPECIFIED SCREW DISPLACEMENT BEST IN THE LEAST SQUARES
      2 SENSE.////)
10038 FORMAT(///5X,42HSOLUTION CORRESPONDING TO THE MECHANIS
      1M OF/)
10039 FORMAT(22X,I2)
10041 FORMAT( 12X,4HR**2,12X,5HRF**2,12X,5HRS**2/)
10042 FORMAT(5X,9F13.8)
10043 FORMAT(         ///11X,2HD1,12X,2HD2,12X,2HD3,12X,2HDA,1
      12X,2HDB,12X, 2HDE,11X,5HDELTA, 6X,6HLAMBDA,6X,5HALFA1,
      27X,5HALFA2/)
10044 FORMAT(5X,10F12.6)
10045 FORMAT(///8X,9HINPUT RO.,2X,10HOUTPUT RO.,5X,7HGEN.SAY
      1,5X,7HDES.SAY,2X,10HOUTPUT TR.,5X,7HGEN.TR.,5X,7HDES.T
      2R.,4X,8HANGLE XO,2X,10HDIF.IN RO.,2X,10HDIF.IN TR./)
10046 FORMAT(5X,8F12.6)
10047 FORMAT(4F10.5)
10048 FORMAT(1H1,5X,2I5///////////////)
10049 FORMAT(2I5)
10050 FORMAT(5F10.3)
10051 FORMAT(6F10.5)
10052 FORMAT(3I5)
10053 FORMAT(8F9.5)
10054 FORMAT(3F10.5)
10056 FORMAT(6F10.5)
10057 FORMAT(I5)
```

```
10058 FORMAT(8I5)
10059 FORMAT(3F10.8)
10060 FORMAT(2X,I5,5X,F10.5)
10061 FORMAT(4I5)
10062 FORMAT(5X,12I5)
10063 FORMAT(2I5,I10)
10064 FORMAT(10X,5F15.5)
10065 FORMAT(1H1,///15X,4HTETO,7X,12HROT.VELOCITY,3X,13HTRAN
     1.VELOCITY,10X,10HROT.ACCEL.,8X,11HTRAN.ACCEL.///)
10066 FORMAT(1H1,///38HINVERSION WITH POSITIVE SIGNED RADICA
     1L///)
10067 FORMAT(1H1,///38HINVERSION WITH NEGATIVE SIGNED RADICA
     1L///)
10068 FORMAT(1H1,5X,6HKDRCT=,I3,5X,3HIV=,I3///)
10069 FORMAT(////////5X,10F12.6)
10070 FORMAT(10X,3F15.5)
10071 FORMAT(18X,3HTET,12X,3HSYO,12X,2HSF/)
10072 FORMAT(10X,7HRMSEFR=,F15.8,5X,7HRMSEFO=,F15.8)
10073 FORMAT(10X,6HRMSES=,F15.8)
10074 FORMAT(10X,6HRMSE2=,F15.8)
10075 FORMAT(     5X,6HKDRCT=,I3,5X,3HIV=,I3///)
10076 FORMAT(//////5X,I2,3X,5F11.5)
10077 FORMAT(1X,I2,2X,5F11.5)
      PI=3.14159265
      TROP=PI/180.0
      NCOUT=1
      READ(5,10057)NDIM
900   READ(5,10050) DTET12,DFY12,CELO,DTET,DLS12
      READ(5,10061)I8,I9,IV,KD3
      READ(5,10057) NTRANS
      READ(5,10057)NCLOCK
      READ(5,10057)NFNPNT
      READ(5,10057)NPRINT
      READ(5,10057)NVLPNT
      READ(5,10057) NITMOR
      READ(5,10058)NFSYM,NSSYM,NFO,NSO
      READ(5,10049) NQ1,NQ2
      READ(5,10052)KR2CH,KRFCH,KRSCH
      READ(5,10049) NPR,NPRL
      READ(5,10056)R2MX,RF2MX,RS2MX,R2SEE,DIFFMX,DIFSMX
      READ(5,10053)(ACCRCY(I),I=1,8)
      READ(5,10057)NTDATA
      READ(5,10057) MDLZRO
      READ(5,10057) NSLP
      CLAM=COS(CELO*TROP)
      SLAM=SIN(CELO*TROP)
      NREDO=0
      NSKPD=0
90001 DO50060 KDRCT=I8,I9
      WRITE(6,10075)KDRCT,IV
      IF(NSKPD.GT.0) GO TO 39080
      IF(NTDATA.NE.0) GO TO 39998
```

```
      CALL DSIRED(      DTET,DFY12,DTET12,DLS12,NPR,KORCT,IV
     1,TET,SYO,SF)
      WRITE(6,10071)
      WRITE(6,10070)(TET(I),SYO(I), SF(I),I=1,NPR)
      GO TO 39999
39998 READ(5,10054)(TET(I),SYO(I),SF(I),I=1,NPR)
      WRITE(6,10071)
      WRITE(6,10070)(TET(I),SYO(I), SF(I),I=1,NPR)
39999 READ(5,10053)D(1),D(4),D(5),D(6),D(7),D(8),D(3),D(2)
      READ(5,10053)DDD(1),DDD(4),DDD(5),DDD(6),DDD(7),DDD(8)
     1,DDD(3),DDD(2)
      READ(5,10058)NIT(1),NIT(4),NIT(5),NIT(6),NIT(7),NIT(8)
     1,NIT(3),NIT(2)
      DE=D(1)
      TETO1=D(2)
      DELO=D(3)
      D1=D(4)
      D2=D(5)
      DB=D(8)
      IF(MDLZRO.EQ.0) GO TO 39069
      IF(NSLP.LT.3) GO TO 39070
39069 D3=D(6)
      DA=D(7)
      GO TO 39080
39070 Q1=D(6)
      Q2=D(7)
39080 NCONE=0
      L1=0
39081 IF(NIT(1).NE.0) GO TO 39091
39082 IF(NIT(2).NE.0) GO TO 39092
39083 IF(NIT(3).NE.0) GO TO 39093
39084 IF(NIT(4).NE.0) GO TO 39094
39085 IF(NIT(5).NE.0) GO TO 39095
39086 IF(NIT(6).NE.0) GO TO 39096
39087 IF(NIT(7).NE.0) GO TO 39097
39088 IF(NIT(8).NE.0) GO TO 39098
      IF(L1.NE.0) GO TO 51066
      NCONE=1
      L1=8
      GO TO 90002
39091 L1=1
      GO TO 90002
39092 L1=2
      GO TO 90002
39093 L1=3
      GO TO 90002
39094 L1=4
      GO TO 90002
39095 L1=5
      GO TO 90002
39096 L1=6
      GO TO 90002
```

```
39097 L1=7
      GO TO 90002
39098 L1=8
90002 DO 60000 L=L1,L1
      NR21=0
      NR22=0
      N2=NIT(L)
      NPLPNT=0
      DO 60001 N=1,N2
      IF(NCONE.EQ.1) GO TO 40004
      IF(N.EQ.1) GO TO 40004
      IF(DDD(L).EQ.0.0) GO TO 60000
      IF(L.EQ.1) GO TO 60011
      IF(L.EQ.2) GO TO 60012
      IF(L.EQ.3) GO TO 60013
      IF(L.EQ.4) GO TO 60014
      IF(L.EQ.5) GO TO 60015
      IF(L.EQ.6) GO TO 60C16
      IF(L.EQ.7) GO TO 60017
      IF(L.EQ.8) GO TO 60C18
      GO TO 60001
60011 DE=D(1)&DDD(1)
      GO TO 98797
60014 D1=D(L)&DDD(L)
      GO TO 98797
60015 D2=D(L)&DDD(L)
      GO TO 98797
60016 IF(MDLZRO.NE.0) GO TO 61016
      D3=D(L)&DDD(L)
      GO TO 98797
61016 Q1=D(L)&DDD(L)
      GO TO 98797
60017 IF(MDLZRO.NE.0) GO TO 61017
      DA=D(L)&DDD(L)
      GO TO 98797
61017 Q2=D(L)&DDD(L)
      GO TO 98797
60018 DB=D(L)&DDD(L)
      GO TO 98797
60013 DELO=D(L)&DDD(L)
      GO TO 98797
60012 TETO1=D(L)&DDD(L)
      IF(TETO1.LT.360.0) GO TO 98797
      TETO1=TETO1-360.0
98797 D(L)=D(L)&DDD(L)
40004 CDEL=COS(DELO*TROP)
      SDEL=SIN(DELO*TROP)
      DO 10024 I=1,NPR
      TETO(I)=TETO1&TET(I)
      IF(TETO(I).LT.360.0) GO TO 45056
      TETO(I)=TETO(I)-360.0
45056 TETOR(I)=TETO(I)*TROP
```

```
10024 CONTINUE
      IF(MDLZRO.NE.0) GO TO 39909
      IF(NSLP.EQ.0) GO TO 39909
      CALL       RSRC(NPR,TETO,D1,D2,D3,DA,DB,DE,DELO,CELO,F
     1IO1,XO1,S1,FIO2,XO2,S2,FIO1R,FIO2R,XO1R,XO2R,FGO1,FGO2
     2,A4)
      IF(A4.EQ.0.0) GO TO 900
      CALL       Q1Q2(TETO,TETOR,FIO1,FIO1R,FIO2,FIO2R,XO1,X
     1O1R,XO2,XO2R,NSLP,KDRCT,NQ1,NQ2,D1,D2,D3,DE,DB,DELO,CE
     2LO,Q1,Q1R,Q2,Q2R)
39909 IF(NSLP.EQ.0) GO TO 142
      IF(NSLP.EQ.1) GO TO 1411
      IF(NSLP.EQ.2) GO TO 1412
      GO TO 1413
1411  SLOPE1=Q1*TROP
      Q1R=Q1*TROP
      GO TO 1441
1412  SLOPE2=Q2*TROP
      Q2R=Q2*TROP
      GO TO 142
1413  SLOPE1=Q1*TROP
      SLOPE2=Q2*TROP
      Q1R=Q1*TROP
      Q2R=Q2*TROP
      GO TO 146
142   TAB1A=ABS(ABS(TETO1*TROP)-PI/2.0)
      TAB1B=ABS(ABS(TETO1*TROP)-1.5*PI)
      IF(TAB1A.LT.0.00001) GO TO 14202
      IF(TAB1B.LT.0.00001) GO TO 14202
      GO TO 14203
14202 SLOPE1=PI/2.0
      GO TO 144
14203 SLOPE1=ATAN(TAN(TETO1*TROP)/CLAM)
      IF(COS(TETO1*TROP).LT.0.0) GO TO 14204
      IF(SIN(TETO1*TROP).LT.0.0) GO TO 14205
      GO TO 144
14205 SLOPE1=PI+SLOPE1
      GO TO 144
14204 IF(SIN(TETO1*TROP).LT.0.0) GO TO 14206
      GO TO 14205
14206 SLOPE1=ABS(SLOPE1)
  144 Q1R=SLOPE1
      Q1=Q1R/TROP
      IF(NSLP.EQ.0) GO TO 1441
      GO TO 146
1441  TAB2A=ABS(ABS((TETO1&DTET12)*TROP)-PI/2.0)
      TAB2B=ABS(ABS((TETO1&DTET12)*TROP)-1.5*PI)
      IF(TAB2A.LT.0.00001) GO TO 14402
      IF(TAB2B.LT.0.00001) GO TO 14402
      GO TO 14403
14402 SLOPE2=PI/2.0
      GO TO 14604
```

```
14403 SLOPE2=ATAN(TAN((TETO1&DTET12)*TROP)/CLAM)
      IF(COS((TETO1+DTET12)*TROP).LT.0.0) GO TO 14404
      IF(SIN((TETO1+DTET12)*TROP).LT.0.0) GO TO 14405
      GO TO 14604
14405 SLOPE2=PI+SLOPE2
      GO TO 14604
14404 IF(SIN((TETO1+DTET12)*TROP).LT.0.0) GO TO 14406
      GO TO 14405
14406 SLOPE2=ABS(SLOPE2)
14604 Q2R=SLOPE2
      Q2=Q2R/TROP
  146 IF(MDLZRO.EQ.0) GO TO 14609
      YC1=D1*CLAM*SIN(TETC1*TROP)&D2*SIN(SLOPE1)
      YC2=D1*CLAM*SIN((TETO1&DTET12)*TROP)&D2*SIN(SLOPE2)
      XC1=D1*COS(TETO1*TROP)&D2*COS(SLOPE1)
      XC2=D1*COS((TETO1&DTET12)*TROP)&D2*COS(SLOPE2)
      GO TO 14605
14609 CALL        XCYC(TETO,TETOR,FIO1,FIO1R,FIO2,FIO2R,XO1,X
     1O1R,XO2,XO2R,NSLP,KDRCT,NQ1,NQ2,D1,D2,D3,DE,DB,DELO,CE
     2LO,Q1,Q1R,Q2,Q2R,TETO1,DTET12,QL1,QL2,XC1,XC2,YC1,YC2)
14605 Y2Y1=YC2-YC1
      X1X2=XC1-XC2
      Q1=SLOPE1/TROP
      Q2=SLOPE2/TROP
      DELSLP=ATAN(X1X2/Y2Y1)
      IF(DELSLP.LT.0.0) GO TO 1501
      GO TO 1502
1501  DELSLP=PI&DELSLP
      GO TO 1504
1502  IF(DELSLP.GT.PI) GO TO 1503
      GO TO 1504
1503  DELSLP=DELSLP-PI
1504  DELSLO=DELSLP/TROP
      SHAFFY=SIN(DFY12*0.5*TROP)
      CHAFFY=COS(DFY12*0.5*TROP)
      GL3=(Y2Y1**2&X1X2**2)/(4.0*(SHAFFY**2))
      D3=SQRT(GL3)
      IF(KD3  .EQ.2) GO TO 15101
      YO4 =YC1&0.5*(YC2-YC1)&D3*SIN(DELSLP)*CHAFFY
      XO4 =XC1-0.5*(XC1-XC2)&D3*CHAFFY*COS(DELSLP)
      GO TO 15102
15101 YO4 =YC1&0.5*(YC2-YC1)-D3*SIN(DELSLP)*CHAFFY
      XO4 =XC1-0.5*(XC1-XC2)-D3*CHAFFY*COS(DELSLP)
15102 DA=XO4
      IF(CELO.EQ.0.0) GO TO 151
      IF(CELO.EQ.180.0) GO TO 151
      GO TO 152
151   DB=0
      NPLPNT=1
      HPL=D1*SIN(TETO1*TROP)&D2*SIN(SLOPE1)-D3*SIN(DELSLP-.5
     1*DFY12*TROP)
      FOPLNR=ATAN(HPL/DA)
```

```
      DA1=DA
      IF(FOPLNR.EQ.0.0) GO TO 15197
      IF(FOPLNR.EQ.PI ) GO TO 15197
      DA=HPL/SIN(FOPLNR)
15197 FOPLNO=FOPLNR/TROP
      TETO1P=TETO1
      TETO1=TETO1-FOPLNO
      GO TO 40005
152   DB=YO4/SLAM
40005 CALL        RSRC(NPR,TETO,D1,D2,D3,DA,DB,DE,DELO,CELO,F
     1IO1,XO1,S1,FIO2,XO2,S2,FIO1R,FIO2R,XO1R,XO2R,FGO1,FGO2
     2,A4)
      IF(A4.EQ.0.0) GO TO 60001
40001 IF(KDRCT.EQ.2) GO TO 40031
      FYO1=FIO1(1)
      XOL1=XO1(1)
      FYO2=FIO1(NPR)
      XOL2=XO1(NPR)
      IF(NFSYM.NE.0) GO TO 85990
      DFY12G=FIO1(1)-FIO1(NPR)
      GO TO 85991
85990 DFY12G=FIO1(1)-FIO1(NFO)
85991 SO1=S1(1)
      IF(NSSYM.NE.0) GO TO 86990
      DS=S1(1)-S1(NPR)
      GO TO 86991
86990 DS=S1(1)-S1(NSO)
86991 DIFFCH=ABS(ABS(DFY12G)-ABS(DFY12))
      DIFSCH=ABS(ABS(DS)-ABS(DLS12))
      GO TO 10005
40031 FYO1=FIO2(1)
      XOL1=XO2(1)
      FYO2=FIO2(NPR)
      XOL2=XO2(NPR)
      IF(NFSYM.NE.0) GO TO 87990
      DFY12G=FIO2(1)-FIO2(NPR)
      GO TO 87991
87990 DFY12G=FIO2(1)-FIO2(NFO)
87991 SO1=S2(1)
      IF(NSSYM.NE.0) GO TO 88990
      DS=S2(1)-S2(NPR)
      GO TO 88991
88990 DS=S2(1)-S2(NSO)
88991 DIFFCH=ABS(ABS(DFY12G)-ABS(DFY12))
      DIFSCH=ABS(ABS(DS)-ABS(DLS12))
10005 R2=0.0
      RF2=0.0
      RS2=0.0
      DO 11005 I=1,8
      SMQO(I)=0.0
11005  CONTINUE
      DO 10000 I=1,NPRL
```

```
      IF(KORCT.EQ.2) GO TO 10006
      IF(NCLOCK.NE.0) GO TO 12206
      SYOG(I)=FG01(I)-FY01
      GO TO 12205
12206 SYOG(I)=FY01-FG01(I)
12205 SFIY=SIN(FI01R(I))
      CFIY=COS(FI01R(I))
      RS=SO1&SF(I)-S1(I)
      SFG(I)=S1(I)-SO1
      GO TO 10020
10006 IF(NCLOCK.NE.0) GO TO 12306
      SYOG(I)=FG02(I)-FY01
      GO TO 12305
12306 SYOG(I)=FY01-FG02(I)
12305 SFIY=SIN(FI02R(I))
      CFIY=COS(FI02R(I))
      RS=SO1&SF(I)-S2(I)
      SFG(I)=S2(I)-SO1
10020 DIFROT(I)=SYO(I)-SYOG(I)
      RF=DIFROT(I)*TROP
      RF2=RF2&RF**2
      RS2=RS2&RS**2
      R=SQRT(RF**2&RS**2)
      R2=R**2&R2
      CT=COS(TETO(I)*TROP)
      ST=SIN(TETO(I)*TROP)
      GMO=DB*SLAM-D1*CLAM*ST
      GNO=DA-D1*CT
      W=GMO*SFIY&GNO*CFIY&D3
      D2W=D2**2-W**2
      IF(D2W.LE.0.0) GO TO 60001
      PMB=SLAM
      PMD1=-CLAM*ST
      PMT=-D1*CLAM*CT
      PNA=1.0
      PND1=-CT
      PNT=D1*ST
      F1=(SDEL**2)*(GNO**2-GMO**2)
      F2=2.0*GMO*GNO*(SDEL**2)
      F3=2.0*(D3*GNO*(CDEL**2)&DE*GMO*SDEL)
      F4=2.0*(D3*GMO*(CDEL**2)-DE*GNO*SDEL)
      GKO=(CDEL**2)*(D2**2-D3**2-GNO**2)-GMO**2-(DE*SDEL)**2
      GMNN=GMO*CFIY-GNO*SFIY
      GMNP=GMO*SFIY&GNO*CFIY
      PF1A =2.0*(SDEL**2)*(GNO*PNA)
      PF1D1=2.0*(SDEL**2)*(GNO*PND1-GMO*PMD1)
      PF1B =2.0*(SDEL**2)*(          -GMO*PMB)
      PF1DL=2.0*SDEL*CDEL*(GNO**2-GMO**2)
      PF1T =2.0*(SDEL**2)*(GNO*PNT -GMO*PMT )
      PF2A= 2.0*(SDEL**2)*GMO*PNA
      PF2D1=2.0*(SDEL**2)*(GNO*PMD1&GMO*PND1)
      PF2B =2.0*(SDEL**2)*GNO*PMB
```

```
 PF2DL=4.0*CDEL*SDEL*GNO*GMO
 PF2T =2.0*(SDEL**2)*(GNO*PMT &GMO*PNT )
 PF3E=2.0*SDEL*GMO
 PF3D1=2.0*((CDEL**2)*D3*PND1              &DE*SDEL*PMD1)
 PF3T =2.0*((CDEL**2)*D3*PNT               &DE*SDEL*PMT )
 PF3DL=2.0*(-2.0*SDEL*CDEL*D3*GNO&DE*GMO*CDEL)
 PF3B=2.0*DE*PMB*SDEL
 PF3A=2.0*D3*(CDEL**2)*PNA
 PF3D3=2.0*(CDEL**2)*GNO
 PF4E=-2.0*GNO*SDEL
 PF4D3=2.0*(CDEL**2)*GMO
 PF4B=2.0*D3*(CDEL**2)*PMB
 PF4A=-2.0*DE*SDEL*PNA
 PF4D1=2.0*((CDEL**2)*D3*PMD1-DE*SDEL*PND1)
 PF4DL=2.0*(-2.0*CDEL*SDEL*D3*GMO-DE*CDEL*GNO)
 PF4T =2.0*((CDEL**2)*            D3*PMT  -DE*SDEL*PNT )
 PKD1=-2.0*((CDEL**2)*GNO*PND1&GMO*PMD1)
 PKB=-2.0*GMO*PMB
 PKA=-2.0*(CDEL**2)*GNO*PNA
 PKD3=-2.0*(CDEL**2)*D3
 PKE=-2.0*DE*(SDEL**2)
 PKD2=2.0* (CDEL**2)*D2
 PKDL=2.0*(-CDEL*SDEL*(D2**2-D3**2-GNO**2)-(DE**2)*SDEL
1*CDEL)
 PKT =2.0*((CDEL**2)*(           -GNO*PNT )-GMO*PMT )
 FDN1=     2.0*F1*SFIY*CFIY-F2*(CFIY**2-SFIY**2)-F3*SFIY
1&F4*CFIY
 IF(FDN1.EQ.0.0) GO TO 60001
 FDN=1.0/FDN1
 IF(FDN.EQ.0.0) GO TO 60001
 PFGE= FDN*(PKE                                -CFIY*PF3E -
1SFIY*PF4E)
 PFGD1=FDN*(PKD1-(SFIY**2)*PF1D1&SFIY*CFIY*PF2D1-CFIY*P
1F3D1-SFIY*PF4D1)
 PFGD2=FDN* PKD2
 PFGDL=FDN*(PKDL-(SFIY**2)*PF1DL&SFIY*CFIY*PF2DL-CFIY*P
1F3DL-SFIY*PF4DL)
 PFGT =FDN*(PKT-(SFIY**2)*PF1T&SFIY*CFIY*PF2T-CFIY*PF3T
1 -SFIY*PF4T)
 PFGA =FDN*(PKA-(SFIY**2)*PF1A&SFIY*CFIY*PF2A-CFIY*PF3A
1 -SFIY*PF4A)
 PFGB =FDN*(PKB-(SFIY**2)*PF1B&SFIY*CFIY*PF2B-CFIY*PF3B
1 -SFIY*PF4B)
 PFGD3=FDN*(PKD3-CFIY*PF3D3-SFIY*PF4D3)
 PWE=GMNN*PFGE
 PWD1=GMNN*PFGD1&SFIY*PMD1&CFIY*PND1
 PWD2=GMNN*PFGD2
 PWDL=GMNN*PFGDL
 PWT= GMNN*PFGT &SFIY*PMT &CFIY*PNT
 PWA=GMNN*PFGA&CFIY*PNA
 PWB=GMNN*PFGB&SFIY*PMB
 PWD3=GMNN*PFGD3&1.0
```

```
      PSGE=           CDEL&SDEL*W*PWE/SQRT(D2W)
      PSGD1=            SLAM*ST&SDEL*W*PWD1/SQRT(D2W)
      PSGD2=           -SDEL*(D2-W*PWD2)/SQRT(D2W)
      PSGT=           D1*SLAM*CT&SDEL*W*PWT/SQRT(D2W)
      PSGDL=           -DE*SDEL-CDEL*SQRT(D2W)&SDEL*W*PWDL/SQR
     1T(D2W)
      PSGA=SDEL*W*PWA/SQRT(D2W)
      PSGB=CLAM&SDEL*W*PWB/SQRT(D2W)
      PSGD3=SDEL*W*PWD3/SQRT(D2W)
      IF(I.GT.1) GO TO 19949
      PDLE=PFGE
      PDLD1=PFGD1
      PDLD2=PFGD2
      PDLDL=PFGDL
      PDLT =PFGT
      PDLD3=PFGD3
      PDLA =PFGA
      PDLB =PFGB
      PSOE =PSGE
      PSOD1=PSGD1
      PSOD2=PSGD2
      PSOD3=PSGD3
      PSODL=PSGDL
      PSOT =PSGT
      PSOA =PSGA
      PSOB =PSGB
19949 QE=RF*(PDLE-PFGE)&RS*(PSOE-PSGE)
      QD1=RF*(PDLD1-PFGD1)&RS*(PSOD1-PSGD1)
      QD2=RF*(PDLD2-PFGD2)&RS*(PSOD2-PSGD2)
      QDL=RF*(PDLDL-PFGDL)&RS*(PSODL-PSGDL)
      QT=RF*(PDLT      -PFGT)&RS*(PSOT      -PSGT)
      QD3=RF*(PDLD3-PFGD3)&RS*(PSOD3-PSGD3)
      QA =RF*(PDLA -PFGA )&RS*(PSOA -PSGA )
      QB =RF*(PDLB -PFGB )&RS*(PSOB -PSGB )
      SMQO(1)=QE &SMQO(1)
      SMQO(2)=QD1&SMQO(2)
      SMQO(3)=QD2&SMQO(3)
      SMQO(4)=QDL&SMQO(4)
      SMQO(5)=QT &SMQO(5)
      SMQO(6)=SMQO(6)&QD3
      SMQO(7)=SMQO(7)&QA
      SMQO(8)=SMQO(8)&QB
10000 CONTINUE
      IF(KRFCH.EQ.0)GO TO 19101
      RMSEFR=SQRT(RF2/FLOAT(NPRL))
      RMSEFO=RMSEFR/TROP
      GO TO 19001
19101 IF(KRSCH.EQ.0) GO TO 19102
      RMSES=SQRT(RS2/FLOAT(NPRL))
      GO TO 19002
19102 IF(KR2CH.EQ.0 ) GO TO 19103
      RMSE2=SQRT(R2/FLOAT(NPRL))
```

```
      IF(R2.GT.R2MX) GO TO 60020
      GO TO 52065
19103 NR2SEE=0
      IF(NCONE.NE.0) GO TO 52066
      IF(NFNPNT.NE.0) GO TO 52066
      IF(R2.GT.R2SEE) GO TO 60020
      NR2SEE=1
      GO TO 52066
19001 IF(RF2.LE.RF2MX) GO TO 52065
      GO TO 19103
19002 IF(RS2.LE.RS2MX) GO TO 52065
      GO TO 19103
52065 IF(DIFSCH.GT.DIFSMX) GO TO 60020
      IF(DIFFCH.GT.DIFFMX) GO TO 60020
52066 WRITE(6,10044)DE,D1,D2,D3,DA,DB,DELO,TETO1
      WRITE(6,10042)R2,RF2,RS2,DIFFCH,DIFSCH
      IF(KRFCH.EQ.0) GO TO 19104
      WRITE(6,10072) RMSEFR,RMSEFO
19104 IF(KRSCH.EQ.0) GO TO 19105
      WRITE(6,10073) RMSES
19105 IF(KR2CH.EQ.0)GO TO 19106
      WRITE(6,10074) RMSE2
19106 WRITE(6,10042)Q1,Q2,DELSLO
      IF(NPLPNT.EQ.0) GO TO 52766
      WRITE(6,10042) TETO1P,FOPLNO
52766 WRITE(6,10042)(SMQO(I),I=1,8)
60020 IF(NPLPNT.EQ.0) GO TO 69421
      TETO1=TETO1P
      DA=DA1
69421 IF(NFNPNT.NE.0) GO TO 52268
      IF(NCONE.NE.0) GO TO 52165
60021 IF(NR2SEE.EQ.1) GO TO 60060
      IF(N.EQ.1) GO TO 60060
      IF(KR2CH.NE.0) GO TO 60030
      IF(KRFCH.NE.0) GO TO 60040
      IF(KRSCH.NE.0) GO TO 60050
60030 IF(R21 .GT.R2 ) GO TO 60060
60036 IF(NR22.GT.0) GO TO 60035
      D(L)=D(L)-DDD(L)
      DDD(L)=-DDD(L)
      NR22=1
      RS2=RS21
      R2=R21
      RF2=RF21
      GO TO 60060
60035 D(L)=D(L)-DDD(L)
      WRITE(6,10060) L,D(L)
      R2=R21
      RF2=RF21
      RS2=RS21
      GO TO 60000
60040 IF(RF21.GT.RF2) GO TO 60060
```

```
      GO TO 60036
60050 IF(RS21.GT.RS2) GO TO 60060
      GO TO 60036
60060 R21=R2
      RF21=RF2
      RS21=RS2
      IF(NR2SEE.EQ.1) GO TO 52165
52068 IF(NPRINT.EQ.0) GO TO 60001
52268 CONTINUE
52165 IF(NFNPNT.NE.0) WRITE(6,10068) KDRCT,IV
      IF(NPRINT.EQ.0) GO TO 60001
      DO 50066 I=1,NPRL
      IF(KDRCT.EQ.2) GO TO 50067
      IF(NTRANS.NE.0) GO TO 55541
      DIFTRN(I)=0
      SF(I)=S1(I)
      GO TO 55542
55541 DIFTRN(I)=SO1&SF(I)-S1(I)
55542 IF(NFNPNT.EQ.0) GO TO 55543
      WRITE(6,10069) TETO(I),FIO1(I),SYOG(I),SYO(I),S1(I),SF
     1G(I),SF(I),XO1(I),DIFROT(I),DIFTRN(I)
      GO TO 50066
55543 WRITE(6,10044) TETO(I),FIO1(I),SYOG(I),SYO(I),S1(I),SF
     1G(I),SF(I),XO1(I),DIFROT(I),DIFTRN(I)
      GO TO 50066
50067 IF(NTRANS.NE.0) GO TO 55551
      DIFTRN(I)=0
      SF(I)=S2(I)
      GO TO 55552
55551 DIFTRN(I)=SO1&SF(I)-S2(I)
55552 IF(NFNPNT.EQ.0) GO TO 55553
      WRITE(6,10069) TETO(I),FIO2(I),SYOG(I),SYO(I),S2(I),SF
     1G(I),SF(I),XO2(I),DIFROT(I),DIFTRN(I)
      GO TO 50066
55553 WRITE(6,10044) TETO(I),FIO2(I),SYOG(I),SYO(I),S2(I),SF
     1G(I),SF(I),XO2(I),DIFROT(I),DIFTRN(I)
50066 CONTINUE
      IF(NFNPNT.EQ.0) GO TO 60001
      IF(KDRCT.EQ.2) GO TO 55554
      WRITE(6,10077)(I,TETO(I),FIO1(I),SYOG(I),SYO(I),DIFROT
     1(I),I=1,NPRL)
      WRITE(7,10077)(I,TETO(I),FIO1(I),SYOG(I),SYO(I),DIFROT
     1(I),I=1,NPRL)
      WRITE(6,10077)(I,XO1(I),S1(I),SFG(I),SF(I),DIFTRN(I),I
     1=1,NPRL)
      WRITE(7,10077)(I,XO1(I),S1(I),SFG(I),SF(I),DIFTRN(I),I
     1=1,NPRL)
      GO TO 60001
55554 WRITE(7,10077)(I,TETO(I),FIO2(I),SYOG(I),SYO(I),DIFROT
     1(I),I=1,NPRL)
      WRITE(6,10077)(I,TETO(I),FIO2(I),SYOG(I),SYO(I),DIFROT
     1(I),I=1,NPRL)
```

```
      WRITE(6,10077)(I,XO2(I),S2(I),SFG(I),SF(I),DIFTRN(I),I
     1=1,NPRL)
      WRITE(7,10077)(I,XO2(I),S2(I),SFG(I),SF(I),DIFTRN(I),I
     1=1,NPRL)
60001 CONTINUE
60000 CONTINUE
      IF(L1.EQ.1) GO TO 39082
      IF(L1.EQ.2) GO TO 39083
      IF(L1.EQ.3) GO TO 39084
      IF(L1.EQ.4) GO TO 39085
      IF(L1.EQ.5) GO TO 39086
      IF(L1.EQ.6) GO TO 39087
      IF(L1.EQ.7) GO TO 39088
      IF(L1.EQ.8) GO TO 51066
50060 CONTINUE
51066 IF(NITMOR.EQ.0) GO TO 50070
      NSKPD=1
      NREDO=NREDO&1
      IF(NREDO.GT.NITMOR) GO TO 50070
      DO 50770 L=1,8
      DDD(L)=DDD(L)/ACCRCY(L)
50770 CONTINUE
      GO TO 90001
50070 IF(NVLPNT.EQ.0) GO TO 50078
      DO 50077 KDRCT=I8,I9
      IF(KDRCT.EQ.2) GO TO 50075
      CALL VELCTY(1,NPR          ,TETO,CELO,DELO,D1,D2,D3,DA,
     1DB,DE,.0,FIO1,XO1,S1,VF1,VX1,VS1,AF1,AX1,AS1)
      WRITE(6,10066)
      WRITE(6,10077)(I,TETO(I),FIO1(I),SYOG(I),SYO(I),DIFROT
     1(I),I=1,NPRL)
      WRITE(6,10077)(I,XO1(I),S1(I),SFG(I),SF(I),DIFTRN(I),I
     1=1,NPRL)
      WRITE(6,10065)
      WRITE(6,10064)(TETO(I),VF1(I),VS1(I),AF1(I),AS1(I),I=1
     1,NPR)
      GO TO 50077
50075 CALL VELCTY(1,NPR          ,TETO,CELO,DELO,D1,D2,D3,DA,
     1DB,DE,.0,FIO2,XO2,S2,VF1,VX1,VS1,AF1,AX1,AS1)
      WRITE(6,10067)
      WRITE(6,10077)(I,TETO(I),FIO2(I),SYOG(I),SYO(I),DIFROT
     1(I),I=1,NPRL)
      WRITE(6,10077)(I,XO2(I),S2(I),SFG(I),SF(I),DIFTRN(I),I
     1=1,NPRL)
      WRITE(6,10065)
      WRITE(6,10064)(TETO(I),VF1(I),VS1(I),AF1(I),AS1(I),I=1
     1,NPR)
50077 CONTINUE
50078 NCOUT=NCOUT&1
      IF(NCOUT.GT.NDIM) GO TO 50080
      GO TO 900
50080 STOP
```

```
      END

      SUBROUTINE RSRC(NPR,TETO,D1,D2,D3,DA,DB,DE,DELO,CELO,F
     1IO1,XO1,S1,FIO2,XO2,S2,FIO1R,FIO2R,XO1R,XO2R,FGO1,FGO2
     2,A4)
      DIMENSION TETOR(30 ),TETO(30 ),FIO1(30 ),FIO1R(30 ),XO
     11(30 )
      DIMENSION XO1R(30 ) ,FIO2(30 ),XO2(30 ),S2(30 ),FIO2R(
     130),S1(30)
      DIMENSION XO2R(30)
      DIMENSION FGO1(30),FGO2(30),FGO1R(30),FGO2R(30)
      PI=3.14159265
      TROP=PI/180.0
      CLAM=COS(CELO*TROP)
      SLAM=SIN(CELO*TROP)
      CDEL=COS(DELO*TROP)
      SDEL=SIN(DELO*TROP)
      DO 101 I=1,NPR
      TETOR(I)=TETO(I)*TROP
101   CONTINUE
100   NEO=0
      NI=NPR
      IF(DELO.EQ.0.0) GO TO 110
      IF(ABS(DELO).EQ.180.0) GO TO 110
      IF(ABS(DELO).EQ.90.0) GO TO 1202
      GO TO 130
110   MJ1=0
      MJ2=0
      A4=1.0
      DO 1101 I=1,NI
      GNO=DA-D1*COS(TETOR(I))
      GMO=DB*SLAM-D1*CLAM*SIN(TETOR(I))
      AO=2.0*D3*GNO
      BO=2.0*D3*GMO
      CO=D3**2-D2**2&GNO**2&GMO**2
      RADI=BO**2&AO**2-CO**2
      IF(RADI   .LT.0.0) GO TO 1101
      ROOT=SQRT(RADI)
      FIO1R(I)=2.0*ATAN((-BO&ROOT)/(CO-AO))
      IF(FIO1R(I).GT.0.) GO TO 11911
      FIO1R(I)=2.0*PI&FIO1R(I)
11911 FIO1(I)=FIO1R(I)/TROP
      IF(I.NE.1) GO TO 11970
      FGO1R(I)=FIO1R(I)
      GO TO 11980
11970 IM=I-1
      IF(FIO1R(I).LT.PI) GO TO 11971
      IF(FIO1R(IM).LT.PI) GO TO 11972
      GO TO 11973
11972 IF(COS(FIO1R(I)).LT.0.0) GO TO 11973
      IF(ABS(FIO1R(I)-FIO1R(IM)).LT.PI) GO TO 11973
```

```
      FG01R(I)=FG01R(IM)-FI01R(IM)-(2.0*PI-FI01R(I))
      GO TO 11980
11971 IF(FI01R(IM).LT.PI) GO TO 11973
      IF(COS(FI01R(I)).LT.0.0) GO TO 11973
      IF(ABS(FI01R(IM)-FI01R(I)) .LT.PI) GO TO 11973
      FG01R(I)=FG01R(IM)&FI01R(I)&2.0*PI-FI01R(IM)
      GO TO 11980
11973 FG01R(I)=FG01R(IM)&FI01R(I)-FI01R(IM)
11980 FG01(I)=FG01R(I)/TROP
      T1=((GMO   *SIN(FI01R(I))&GNO   *COS(FI01R(I))&D3)/D2)
      IF(ABS(T1).GT.1.0) GO TO 11013
      MJ1=MJ1&1
      J1=MJ1
      TS1=(GNO*SIN(FI01R(I))-GMO*COS(FI01R(I))-DE*SDEL)/(D2*
     1CDEL)
      X01R(I)=ARCOS(ABS(T1))
      IF(T1.GT.0.0) GO TO 99011
      IF(TS1.GT.0.0) GO TO 99012
      X01R(I)=X01R(I)-PI
      GO TO 99014
99012 X01R(I)=PI-X01R(I)
      GO TO 99014
99011 IF(TS1.GT.0.0) GO TO 99014
      X01R(I)=-X01R(I)
99014 X01(I)=X01R(I)/TROP
      S1(I)=DE*CDEL&DB*CLAM&D1*SLAM*SIN(TETOR(I))
11013 FI02R(I)=2.0*ATAN((-BO-ROOT)/(CO-AO))
      IF(FI02R(I).GT.0.) GO TO 11912
      FI02R(I)=2.0*PI&FI02R(I)
11912 FI02(I)=FI02R(I)/TROP
      IF(I.NE.1) GO TO 12970
      FG02R(I)=FI02R(I)
      GO TO 12980
12970 IM=I-1
      IF(FI02R(I).LT.PI) GO TO 12971
      IF(FI02R(IM).LT.PI) GO TO 12972
      GO TO 12973
12972 IF(COS(FI02R(I)).LT.0.0) GO TO 12973
      IF(ABS(FI02R(I)-FI02R(IM)).LT.PI) GO TO 12973
      FG02R(I)=FG02R(IM)-FI02R(IM)-(2.0*PI-FI02R(I))
      GO TO 12980
12971 IF(FI02R(IM).LT.PI) GO TO 12973
      IF(COS(FI02R(I)).LT.0.0) GO TO 12973
      IF(ABS(FI02R(IM)-FI02R(I)) .LT.PI) GO TO 12973
      FG02R(I)=FG02R(IM)&FI02R(I)&2.0*PI-FI02R(IM)
      GO TO 12980
12973 FG02R(I)=FG02R(IM)&FI02R(I)-FI02R(IM)
12980 FG02(I)=FG02R(I)/TROP
       T2=((GMO   *SIN(FI02R(I))&GNO   *COS(FI02R(I))&D3)/D2
     1)
      IF(ABS(T2).GT.1.0) GO TO 1101
      MJ2=MJ2&1
```

```
      J2=MJ2
      TS2=(GNO*SIN(FIO2R(I))-GMO*COS(FIO2R(I))-DE*SDEL)/(D2*
     1CDEL)
      XO2R(I)=ARCOS(ABS(T2))
      IF(T2.GT.0.0) GO TO 99021
      IF(TS2.GT.0.0) GO TO 99022
      XO2R(I)=XO2R(I)-PI
      GO TO 99024
99022 XO2R(I)=PI-XO2R(I)
      GO TO 99024
99021 IF(TS2.GT.0.0) GO TO 99024
      XO2R(I)=-XO2R(I)
99024 XO2(I)=XO2R(I)/TROP
      S2(I)=DE*CDEL&DB*CLAM&D1*SLAM*SIN(TETOR(I))
1101  CONTINUE
      GO TO 140
1202  MJ1=0
      MJ2=0
      A4=1.0
      DO 1201 I=1,NI
      GNO=DA-D1*COS(TETOR(I))
      GMO=DB*SLAM-D1*CLAM*SIN(TETOR(I))
      AO=DE*SDEL
      BO=GNO
      CO=GMO
      RADI=BO**2&CO**2-AO**2
      IF(RADI    .LT.0.0) GO TO 1201
      ROOT=SQRT(RADI)
      FIO1R(I)=2.0*ATAN((-BO&ROOT)/(CO-AO))
      IF(FIO1R(I).GT.0.) GO TO 12911
      FIO1R(I)=2.0*PI&FIO1R(I)
12911 FIO1(I)=FIO1R(I)/TROP
      IF(I.NE.1) GO TO 17970
      FGO1R(I)=FIO1R(I)
      GO TO 17980
17970 IM=I-1
      IF(FIO1R(I).LT.PI) GO TO 17971
      IF(FIO1R(IM).LT.PI) GO TO 17972
      GO TO 17973
17972 IF(COS(FIO1R(I)).LT.0.0) GO TO 17973
      IF(ABS(FIO1R(I)-FIO1R(IM)).LT.PI) GO TO 17973
      FGO1R(I)=FGO1R(IM)-FIO1R(IM)-(2.0*PI-FIO1R(I))
      GO TO 17980
17971 IF(FIO1R(IM).LT.PI) GO TO 17973
      IF(COS(FIO1R(I)).LT.0.0) GO TO 17973
      IF(ABS(FIO1R(IM)-FIO1R(I)) .LT.PI) GO TO 17973
      FGO1R(I)=FGO1R(IM)&FIO1R(I)&2.0*PI-FIO1R(IM)
      GO TO 17980
17973 FGO1R(I)=FGO1R(IM)&FIO1R(I)-FIO1R(IM)
17980 FGO1(I)=FGO1R(I)/TROP
      T1=((GMO    *SIN(FIO1R(I))&GNO    *COS(FIO1R(I))&D3)/D2)
      IF(ABS(T1).GT.1.0) GO TO 12013
```

```
      MJ1=MJ1&1
      J1=MJ1
      X01R(I)=ARCOS((GMO    *SIN(FI01R(I))&GNO    *COS(FI01R(I
     1))&D3)/D2)
      X01(I)=X01R(I)/TROP
      S1(I)=DB*CLAM&D1*SLAM*SIN(TETOR(I))-D2*SDEL*SIN(X01R(I
     1))
12013 FI02R(I)=2.0*ATAN((-BO-ROOT)/(CO-AO))
      IF(FI02R(I).GT.0.) GO TO 12912
      FI02R(I)=2.0*PI&FI02R(I)
12912 FI02(I)=FI02R(I)/TROP
      IF(I.NE.1) GO TO 18970
      FG02R(I)=FI02R(I)
      GO TO 18980
18970 IM=I-1
      IF(FI02R(I).LT.PI) GO TO 18971
      IF(FI02R(IM).LT.PI) GO TO 18972
      GO TO 18973
18972 IF(COS(FI02R(I)).LT.0.0) GO TO 18973
      IF(ABS(FI02R(I)-FI02R(IM)).LT.PI) GO TO 18973
      FG02R(I)=FG02R(IM)-FI02R(IM)-(2.0*PI-FI02R(I))
      GO TO 18980
18971 IF(FI02R(IM).LT.PI) GO TO 18973
      IF(COS(FI02R(I)).LT.0.0) GO TO 18973
      IF(ABS(FI02R(IM)-FI02R(I)) .LT.PI) GO TO 18973
      FG02R(I)=FG02R(IM)&FI02R(I)&2.0*PI-FI02R(IM)
      GO TO 18980
18973 FG02R(I)=FG02R(IM)&FI02R(I)-FI02R(IM)
18980 FG02(I)=FG02R(I)/TROP
       T2=((GMO    *SIN(FI02R(I))&GNO    *COS(FI02R(I))&D3)/D2
     1)
      IF(ABS(T2).GT.1.0) GO TO 1201
      MJ2=MJ2&1
      J2=MJ2
      X02R(I)=ARCOS((GMO    *SIN(FI02R(I))&GNO    *COS(FI02R(I
     1))&D3)/D2)
      X02(I)=X02R(I)/TROP
      S2(I)=DB*CLAM&D1*SLAM*SIN(TETOR(I))-D2*SDEL*SIN(X02R(I
     1))
1201  CONTINUE
      GO TO 140
130   MJ1=0
      MJ2=0
      DO 1301 I=1,NI
      GNO=DA-D1*COS(TETOR(I))
      GMO=DB*SLAM-D1*CLAM*SIN(TETOR(I))
      GKO=(    CDEL**2)*(D2**2-D3**2-GNO    **2)-GMO    **2-(DE
     1*SDEL)**2
      AO=    2.0*(D3*GNO    *(CDEL**2)&DE*GMO    *SDEL)-GKO
      A1=    4.0*(D3*GMO    *(CDEL**2)-DE*GNO    *SDEL-GMO*GNO*
     1(SDEL**2))
      A2=    4.0*(SDEL**2)*(GNO    **2-GMO    **2)-2.0*GKO
```

```
      A3=  4.0*(GMO    *GNO    *(SDEL**2)&D3*GMO    *(CDEL**2)-
     1DE*GNO*SDEL)
      A4=   -GKO   -2.0*(D3*GNO   *(CDEL**2)&DE*GMO*SDEL)
      IF(A4   .EQ.0.0) GO TO 140
12012 B0=A0/A4
      B1=A1/A4
      B2=A2/A4
      B3=A3/A4
      C0=B0*(4.0*B2-B3**2)-B1**2
      C1=B3*B1-4.0*B0
      E0=(2.0*(B2**3)-9.0*B2*C1-27.0*C0)/27.0
      E1=(-3.0*C1&B2**2)/3.0
      IF(E0   .LT.0.0) GO TO 1306
      GO TO 13061
1306  EMO=-E0
      NEO=NEO&1
      E0=EMO
13061 CHE0=27.0*(E0**2)
      CHE1=4.0*(E1**3)
      IF(E0.EQ.0.0) GO TO 1307
      IF(E1   .EQ.0.0) GO TO 1308
      IF(CHE0   .GT.CHE1    ) GO TO 1302
      IF(CHE0   .EQ.CHE1    ) GO TO 1303
      IF(CHE0   .LT.CHE1    ) GO TO 1304
1302  IF(E1   .GT.0.0) GO TO 13022
      SAYA1=   0.5*E0   /((-E1   /3.0)**1.5)
      SAYRT=   ALOG(SAYA1   &SQRT(SAYA1    **2&1.0))
      IF(NEO.GT.0) GO TO 13064
      Y3=    B2   /3.0&2.0*SQRT(-E1   /3.0)*SINH(SAYRT   /3.0
     1)
      GO TO 131
13064 Y3=    B2   /3.0-2.0*SQRT(-E1   /3.0)*SINH(SAYRT   /3.0
     1)
      GO TO 131
13022 SAYA2=   0.5*E0   /((E1   /3.0)**1.5)
      IF(SAYA2   .LT.1.0) GO TO 1301
      SAYRT=   ALOG(SAYA2   &SQRT(SAYA2    **2-1.0))
13024 IF(NEO.GT.0) GO TO 13065
      Y3=    B2   /3.0&2.0*SQRT(E1   /3.0)*COSH(SAYRT   /3.0)
      GO TO 131
13065 Y3=    B2   /3.0-2.0*SQRT(E1   /3.0)*COSH(SAYRT   /3.0)
      GO TO 131
1303  IF(NEO.GT.0) GO TO 13062
      GO TO 13031
13062 ROOT1=    -2.0*((E0   /2.0)**(1.0/3.0))&B2   /3.0
      ROOT2=   (E0   /2.0)**(1.0/3.0)&B2   /3.0
      GO TO 13032
13031 ROOT1=   2.0*((E0   /2.0)**(1.0/3.0))&B2/3.0
      ROOT2=   -(E0   /2.0)**(1.0/3.0)&B2   /3.0
13032 IF(ROOT1.GT.ROOT2) GO TO 1305
      Y3=ROOT2
      GO TO 131
```

```
1305  Y3=ROOT1
      GO TO 131
1304  SAYRT=   ARCOS(0.5*EO   /((E1   /3.0)**1.5))
      IF(NEO.GT.0) GO TO 13066
      GO TO 13040
13066 ROOT1=   -2.0*SQRT(E1   /3.0)*COS(SAYRT   /3.0)&B2   /
     13.0
      ROOT2=   2.0*SQRT(E1   /3.0)*COS((PI-SAYRT   )/3.0)&B2
     1   /3.0
      ROOT3=   2.0*SQRT(E1   /3.0)*COS((PI&SAYRT   )/3.0)&B2
     1/3.0
      GO TO 13041
13040 ROOT1=   2.0*SQRT(E1   /3.0)*COS(SAYRT   /3.0)&B2/3.0
      ROOT2=   -2.0*SQRT(E1   /3.0)*COS((PI-SAYRT   )/3.0)&B
     12/3.0
      ROOT3=   -2.0*SQRT(E1   /3.0)*COS((PI&SAYRT   )/3.0)&B
     12/3.0
13041 IF(ROOT1   .GT.ROOT2   ) GO TO 13042
      IF(ROOT2   .GT.ROOT3   ) GO TO 13043
      Y3=ROOT3
      GO TO 131
13042 IF(ROOT1   .GT.ROOT3   ) GO TO 13044
      Y3=ROOT3
      GO TO 131
13044 Y3=   ROOT1
      GO TO 131
13043 Y3=ROOT2
      GO TO 131
1307  IF(E1   .LE.0.0) GO TO 13071
      ROOT1=B2/3.0
      ROOT2=   SQRT(E1   )&B2/3.0
      ROOT3=   -SQRT(E1   )&B2/3.0
      IF(ROOT1   .GE.ROOT2   ) GO TO 13073
      IF(ROOT2   .GE.ROOT3   ) GO TO 13074
      Y3=ROOT3
      GO TO 131
13073 IF(ROOT1   .GE.ROOT3   ) GO TO 13075
      Y3=ROOT3
      GO TO 131
13074 Y3=ROOT2
      GO TO 131
13075 Y3=ROOT1
      GO TO 131
13071 Y3=B2/3.0
      GO TO 131
1308  IF(NEO.NE.0) GO TO 13C81
      Y3=   B2   /3.0&(EO   **(1.0/3.0))
      GO TO 131
13081 Y3=   B2   /3.0-(EO   **(1.0/3.0))
131   CONTINUE
      IF((((0.5*B3) **2)-B2   &Y3   ).LT.0.0) GO TO 1301
      QS1=   0.5*B3   &SQRT((0.5*B3   )**2-B2   &Y3)
```

```
      QS2=   0.5*B3   -SQRT((0.5*B3   )**2-B2   &Y3)
      IF((((0.5*Y3   )**2)-B0   ).LT.0.0) GO TO 1301
      HS1=  0.5*Y3   &SQRT((0.5*Y3   )**2-B0)
      HS2=   0.5*Y3   -SQRT((0.5*Y3   )**2-B0)
      QH1=   QS1   *HS2   &QS2   *HS1
      QH2=   QS1   *HS1   &QS2   *HS2
      IF(ABS(QH1   -B1   ).LE.0.0001) GO TO 1311
      IF(ABS(QH2   -B1   ).LE.0.0001) GO TO 1312
      GO TO 1301
1311  H1=HS1
      H2=HS2
      GO TO 1313
1312  H1=HS2
      H2=HS1
1313  RADI1=   QS1   **2-4.0*H1
      RADI2=QS2**2-4.0*H2
      IF(RADI1   .LT.0.0) GO TO 1316
      IF(RADI2.GE.0.0) GO TO 140
13130 FI01R(I)=2.0*ATAN(0.5*(-QS1   -SQRT(RADI1)))
      IF(FI01R(I).GT.0.) GO TO 13911
      FI01R(I)=2.0*PI&FI01R(I)
13911 FI01(I)=FI01R(I)/TROP
      IF(I.NE.1) GO TO 13970
      FG01R(I)=FI01R(I)
      GO TO 13980
13970 IM=I-1
      IF(FI01R(I).LT.PI) GO TO 13971
      IF(FI01R(IM).LT.PI) GO TO 13972
      GO TO 13973
13972 IF(COS(FI01R(I)).LT.0.0) GO TO 13973
      IF(ABS(FI01R(I)-FI01R(IM)).LT.PI) GO TO 13973
      FG01R(I)=FG01R(IM)-FI01R(IM)-(2.0*PI-FI01R(I))
      GO TO 13980
13971 IF(FI01R(IM).LT.PI) GO TO 13973
      IF(COS(FI01R(I)).LT.0.0) GO TO 13973
      IF(ABS(FI01R(IM)-FI01R(I)) .LT.PI) GO TO 13973
      FG01R(I)=FG01R(IM)&FI01R(I)&2.0*PI-FI01R(IM)
      GO TO 13980
13973 FG01R(I)=FG01R(IM)&FI01R(I)-FI01R(IM)
13980 FG01(I)=FG01R(I)/TROP
      T1=((GMO   *SIN(FI01R(I))&GNO   *COS(FI01R(I))&D3)/D2)
      IF(ABS(T1).GT.1.0) GO TO 13131
      MJ1=MJ1&1
      J1=MJ1
      TS1=(GNO*SIN(FI01R(I))-GMO*COS(FI01R(I))-DE*SDEL)/(D2*
     1CDEL)
      X01R(I)=ARCOS(ABS(T1))
      IF(T1.GT.0.0) GO TO 99031
      IF(TS1.GT.0.0) GO TO 99032
      X01R(I)=X01R(I)-PI
      GO TO 99034
99032 X01R(I)=PI-X01R(I)
```

```
      GO TO 99034
99031 IF(TS1.GT.0.0) GO TO 99034
      XO1R(I)=-XO1R(I)
99034 XO1(I)=XO1R(I)/TROP
      S1(I)=DB*CLAM&D1*SLAM*SIN(TETOR(I))-D2*SDEL*SIN(XO1R(I
     1))&DE*CDEL
13131 FIO2R(I)=2.0*ATAN(0.5*(-QS1   &SQRT(RADI1)))
      IF(FIO2R(I).GT.0.) GO TO 13912
      FIO2R(I)=2.0*PI&FIO2R(I)
13912 FIO2(I)=FIO2R(I)/TROP
      IF(I.NE.1) GO TO 14970
      FGO2R(I)=FIO2R(I)
      GO TO 14980
14970 IM=I-1
      IF(FIO2R(I).LT.PI) GO TO 14971
      IF(FIO2R(IM).LT.PI) GO TO 14972
      GO TO 14973
14972 IF(COS(FIO2R(I)).LT.0.0) GO TO 14973
      IF(ABS(FIO2R(I)-FIO2R(IM)).LT.PI) GO TO 14973
      FGO2R(I)=FGO2R(IM)-FIO2R(IM)-(2.0*PI-FIO2R(I))
      GO TO 14980
14971 IF(FIO2R(IM).LT.PI) GO TO 14973
      IF(COS(FIO2R(I)).LT.0.0) GO TO 14973
      IF(ABS(FIO2R(IM)-FIO2R(I)) .LT.PI) GO TO 14973
      FGO2R(I)=FGO2R(IM)&FIO2R(I)&2.0*PI-FIO2R(IM)
      GO TO 14980
14973 FGO2R(I)=FGO2R(IM)&FIO2R(I)-FIO2R(IM)
14980 FGO2(I)=FGO2R(I)/TROP
       T2=((GMO   *SIN(FIO2R(I))&GNO   *COS(FIO2R(I))&D3)/D2
     1)
       IF(ABS(T2).GT.1.0) GO TO 1301
       MJ2=MJ2&1
       J2=MJ2
       TS2=(GNO*SIN(FIO2R(I))-GMO*COS(FIO2R(I))-DE*SDEL)/(D2*
     1CDEL)
       XO2R(I)=ARCOS(ABS(T2))
       IF(T2.GT.0.0) GO TO 99041
       IF(TS2.GT.0.0) GO TO 99042
       XO2R(I)=XO2R(I)-PI
       GO TO 99044
99042 XO2R(I)=PI-XO2R(I)
      GO TO 99044
99041 IF(TS2.GT.0.0) GO TO 99044
      XO2R(I)=-XO2R(I)
99044 XO2(I)=XO2R(I)/TROP
      S2(I)=DB*CLAM&D1*SLAM*SIN(TETOR(I))-D2*SDEL*SIN(XO2R(I
     1))&DE*CDEL
      GO TO 1301
1316  IF(RADI2   .LT.0.0) GO TO 1301
      FIO1R(I)=2.0*ATAN(0.5*(-QS2   -SQRT(RADI2)))
      IF(FIO1R(I).GT.0.) GO TO 13913
      FIO1R(I)=2.0*PI&FIO1R(I)
```

```
13913 FIO1(I)=FIO1R(I)/TROP
      IF(I.NE.1) GO TO 15970
      FGO1R(I)=FIO1R(I)
      GO TO 15980
15970 IM=I-1
      IF(FIO1R(I).LT.PI) GO TO 15971
      IF(FIO1R(IM).LT.PI) GO TO 15972
      GO TO 15973
15972 IF(COS(FIO1R(I)).LT.0.0) GO TO 15973
      IF(ABS(FIO1R(I)-FIO1R(IM)).LT.PI) GO TO 15973
      FGO1R(I)=FGO1R(IM)-FIO1R(IM)-(2.0*PI-FIO1R(I))
      GO TO 15980
15971 IF(FIO1R(IM).LT.PI) GO TO 15973
      IF(COS(FIO1R(I)).LT.0.0) GO TO 15973
      IF(ABS(FIO1R(IM)-FIO1R(I)) .LT.PI) GO TO 15973
      FGO1R(I)=FGO1R(IM)&FIO1R(I)&2.0*PI-FIO1R(IM)
      GO TO 15980
15973 FGO1R(I)=FGO1R(IM)&FIO1R(I)-FIO1R(IM)
15980 FGO1(I)=FGO1R(I)/TROP
      T1=((GMO    *SIN(FIO1R(I))&GNO    *COS(FIO1R(I))&D3)/D2)
      IF(ABS(T1).GT.1.0) GO TO 13161
      MJ1=MJ1&1
      J1=MJ1
      TS1=(GNO*SIN(FIO1R(I))-GMO*COS(FIO1R(I))-DE*SDEL)/(D2*
     1CDEL)
      XO1R(I)=ARCOS(ABS(T1))
      IF(T1.GT.0.0) GO TO 99051
      IF(TS1.GT.0.0) GO TO 99052
      XO1R(I)=XO1R(I)-PI
      GO TO 99054
99052 XO1R(I)=PI-XO1R(I)
      GO TO 99054
99051 IF(TS1.GT.0.0) GO TO 99054
      XO1R(I)=-XO1R(I)
99054 XO1(I)=XO1R(I)/TROP
      S1(I)=DB*CLAM&D1*SLAM*SIN(TETOR(I))-D2*SDEL*SIN(XO1R(I
     1))&DE*CDEL
13161 FIO2R(I)=2.0*ATAN(0.5*(-QS2    &SQRT(RADI2)))
      IF(FIO2R(I).GT.0.) GO TO 13914
      FIO2R(I)=2.0*PI&FIO2R(I)
13914 FIO2(I)=FIO2R(I)/TROP
      IF(I.NE.1) GO TO 16970
      FGO2R(I)=FIO2R(I)
      GO TO 16980
16970 IM=I-1
      IF(FIO2R(I).LT.PI) GO TO 16971
      IF(FIO2R(IM).LT.PI) GO TO 16972
      GO TO 16973
16972 IF(COS(FIO2R(I)).LT.0.0) GO TO 16973
      IF(ABS(FIO2R(I)-FIO2R(IM)).LT.PI) GO TO 16973
      FGO2R(I)=FGO2R(IM)-FIO2R(IM)-(2.0*PI-FIO2R(I))
      GO TO 16980
```

```
16971 IF(FIO2R(IM).LT.PI) GO TO 16973
      IF(COS(FIO2R(I)).LT.0.0) GO TO 16973
      IF(ABS(FIO2R(IM)-FIO2R(I)) .LT.PI) GO TO 16973
      FGO2R(I)=FGO2R(IM)&FIO2R(I)&2.0*PI-FIO2R(IM)
      GO TO 16980
16973 FGO2R(I)=FGO2R(IM)&FIO2R(I)-FIO2R(IM)
16980 FGO2(I)=FGO2R(I)/TROP
       T2=((GMO   *SIN(FIO2R(I))&GNO   *COS(FIO2R(I))&D3)/D2
     1)
      IF(ABS(T2).GT.1.0) GO TO 1301
      MJ2=MJ2&1
      J2=MJ2
      TS2=(GNO*SIN(FIO2R(I))-GMO*COS(FIO2R(I))-DE*SDEL)/(D2*
     1CDEL)
      XO2R(I)=ARCOS(ABS(T2))
      IF(T2.GT.0.0) GO TO 99061
      IF(TS2.GT.0.0) GO TO 99062
      XO2R(I)=XO2R(I)-PI
      GO TO 99064
99062 XO2R(I)=PI-XO2R(I)
      GO TO 99064
99061 IF(TS2.GT.0.0) GO TO 99064
      XO2R(I)=-XO2R(I)
99064 XO2(I)=XO2R(I)/TROP
      S2(I)=DB*CLAM&D1*SLAM*SIN(TETOR(I))-D2*SDEL*SIN(XO2R(I
     1))&DE*CDEL
1301  CONTINUE
140   RETURN
      END


      SUBROUTINE Q1Q2(TETO,TETOR,FIO1,FIO1R,FIO2,FIO2R,XO1,X
     1O1R,XO2,XO2R,NSLP,KDRCT,NQ1,NQ2,D1,D2,D3,DE,DB,DELO,CE
     2LO,Q1,Q1R,Q2,Q2R)
      DIMENSION TETO(30),TETOR(30),FIO1(30),FIO1R(30),FIO2(3
     10),FIO2R(30)
      DIMENSION XO1(30),XO1R(30),XO2(30),XO2R(30)
      PI=3.14159265
      TROP=PI/180.0
      SLAM=SIN(CELO*TROP)
      CLAM=COS(CELO*TROP)
      CDEL=COS(DELO*TROP)
      SDEL=SIN(DELO*TROP)
      IF(KDRCT.EQ.2) GO TO 39901
      IF(NSLP.EQ.1) GO TO 39902
      W11=D2*COS(XO1R(NQ1))
      W21=D2*CDEL*SIN(XO1R(NQ1))
      Q11D=W21*SIN(FIO1R(NQ1))+W11*(CDEL**2)*COS(FIO1R(NQ1))
      IF(Q11D.NE.0.0) GO TO 39903
      Q1R=PI/2.0
      GO TO 39904
39903 Q1R=ATAN((W11*(CDEL**2)*SIN(FIO1R(NQ1))-W21*COS(FIO1R(
```

```
     1NQ1)))/(W21*SIN(FIO1R(NQ1))+W11*(CDEL**2)*COS(FIO1R(NQ
     21))))
39904 Q1=Q1R/TROP
      IF(NSLP.EQ.2) GO TO 41014
39902 W12=D2*COS(XO1R(NQ2))
      W22=D2*CDEL*SIN(XO1R(NQ2))
      Q22D=W22*SIN(FIO1R(NQ2))+W12*(CDEL**2)*COS(FIO1R(NQ2))
      IF(Q22D.NE.0.0) GO TO 39905
      Q2R=PI/2.0
      GO TO 39906
39905 Q2R=ATAN((W12*(CDEL**2)*SIN(FIO1R(NQ2))-W22*COS(FIO1R(
     1NQ2)))/(W22*SIN(FIO1R(NQ2))+W12*(CDEL**2)*COS(FIO1R(NQ
     22))))
39906 Q2=Q2R/TROP
      GO TO 41014
39901 IF(NSLP.EQ.1) GO TO 39919
      W11=D2*COS(XO2R(NQ1))
      W21=D2*CDEL*SIN(XO2R(NQ1))
      Q11D=W21*SIN(FIO2R(NQ1))+W11*(CDEL**2)*COS(FIO2R(NQ1))
      IF(Q11D.NE.0.0) GO TO 39911
      Q1R=PI/2.0
      GO TO 39912
39911 Q1R=ATAN((W11*(CDEL**2)*SIN(FIO2R(NQ1))-W21*COS(FIO2R(
     1NQ1)))/(W21*SIN(FIO2R(NQ1))+W11*(CDEL**2)*COS(FIO2R(NQ
     21))))
39912 Q1=Q1R/TROP
      IF(NSLP.EQ.2) GO TO 41014
39919 W12=D2*COS(XO2R(NQ2))
      W22=D2*CDEL*SIN(XO2R(NQ2))
      Q22D=W22*SIN(FIO2R(NQ2))+W12*(CDEL**2)*COS(FIO2R(NQ2))
      IF(Q22D.NE.0.0) GO TO 39913
      Q2R=PI/2.0
      GO TO 39914
39913 Q2R=ATAN((W12*(CDEL**2)*SIN(FIO2R(NQ2))-W22*COS(FIO2R(
     1NQ2)))/(W22*SIN(FIO2R(NQ2))+W12*(CDEL**2)*COS(FIO2R(NQ
     22))))
39914 Q2=Q2R/TROP
41014 RETURN
      END


      SUBROUTINE XCYC(TETO,TETOR,FIO1,FIO1R,FIO2,FIO2R,XO1,X
     1O1R,XO2,XO2R,NSLP,KORCT,NQ1,NQ2,D1,D2,D3,DE,DB,DELO,CE
     2LO,Q1,Q1R,Q2,Q2R,TETO1,DTET12,QL1,QL2,XC1,XC2,YC1,YC2)
      DIMENSION TETO(30),TETOR(30),FIO1(30),FIO1R(30),FIO2(3
     10),FIO2R(30)
      DIMENSION XO1(30),XO1R(30),XO2(30),XO2R(30)
      PI=3.14159265
      TROP=PI/180.0
      SLAM=SIN(CELO*TROP)
      CLAM=COS(CELO*TROP)
      CDEL=COS(DELO*TROP)
```

```
      SDEL=SIN(DELO*TROP)
      IF(KDRCT.EQ.2) GO TO 14608
      QL1=(1.0/D2)*SQRT((D2*SDEL+D2*COS(XO1R(NQ1)))**2+(D2*C
     1DEL*SIN(XO1R(NQ1)))**2)
      QL2=(1.0/D2)*SQRT((D2*SDEL+D2*COS(XO1R(NQ2)))**2+(D2*C
     1DEL*SIN(XO1R(NQ2)))**2)
      YC1=D1*SIN(TETO1*TROP)*CLAM+D2*QL1*SIN(Q1R+ATAN(TAN(DE
     1LO*TROP)*SIN(XO1R(NQ1))))+SDEL*(DE*COS(FIO1R(NQ1))-D2*
     2SIN(FIO1R(NQ1)))
      YC2=D1*SIN((TETO1+DTET12)*TROP)*CLAM+D2*QL2*SIN(Q2R+AT
     1AN(TAN(DELO*TROP)*SIN(XO1R(NQ2))))+SDEL*(DE*COS(FIO1R(
     2NQ2))-D2*SIN(FIO1R(NQ2)))
      XC1=D1*COS(TETO1*TROP)+D2*QL1*COS(Q1R+ATAN(TAN(DELO*TR
     1OP)*SIN(XO1R(NQ1))))+(DE-D2)*SDEL*SIN(FIO1R(NQ1))
      XC2=D1*COS((TETO1+DTET12)*TROP)+D2*QL2*COS(Q2R+ATAN(TA
     1N(DELO*TROP)*SIN(XO1R(NQ2))))+(DE-D2)*SDEL*SIN(FIO1R(N
     2Q2))
      GO TO 14605
14608 QL1=(1.0/D2)*SQRT((D2*SDEL+D2*COS(XO2R(NQ1)))**2+(D2*C
     1DEL*SIN(XO2R(NQ1)))**2)
      QL2=(1.0/D2)*SQRT((D2*SDEL+D2*COS(XO2R(NQ2)))**2+(D2*C
     1DEL*SIN(XO2R(NQ2)))**2)
      YC1=D1*SIN(TETO1*TROP)*CLAM+D2*QL1*SIN(Q1R+ATAN(TAN(DE
     1LO*TROP)*SIN(XO2R(NQ1))))+SDEL*(DE*COS(FIO2R(NQ1))-D2*
     2SIN(FIO2R(NQ1)))
      YC2=D1*SIN((TETO1+DTET12)*TROP)*CLAM+D2*QL2*SIN(Q2R+AT
     1AN(TAN(DELO*TROP)*SIN(XO2R(NQ2))))+SDEL*(DE*COS(FIO2R(
     2NQ2))-D2*SIN(FIO2R(NQ2)))
      XC1=D1*COS(TETO1*TROP)+D2*QL1*COS(Q1R+ATAN(TAN(DELO*TR
     1OP)*SIN(XO2R(NQ1))))+(DE-D2)*SDEL*SIN(FIO2R(NQ1))
      XC2=D1*COS((TETO1+DTET12)*TROP)+D2*QL2*COS(Q2R+ATAN(TA
     1N(DELO*TROP)*SIN(XO2R(NQ2))))+(DE-D2)*SDEL*SIN(FIO2R(N
     2Q2))
14605 RETURN
      END


      SUBROUTINE VELCTY(NTYPE,NI,TETO,CELO,DELO,D1,D2,D3,DA,
     1DB,DE,HE,FIO1,XO1,S1,VF1,VX1,VS1,AF1,AX1,AS1)
C     SUBROUTINE VELCTY IS THE SAME AS IN PROGRAM A.


C     ORDER OF DATA AND FORMAT STATEMENTS ARE AS FOLLOWS,
C     NDIM(I5)=NUMBER OF PROBLEM SETS.
C     DTET12,DFY12,CELO,DTET,DLS12(5F10.3) DTET12 IS THE
C     RANGE OF INPUT ROTATION FROM THE POSITICN FORPARAM.OF
C     CONSTRAINT Q1 TO POSITION Q2.DFY12 IS THE RANGE OF
C     OUTPUT DEFINED SIMILARLY.DLS12 IS THE RANGE OF
C     OUTPUT TRANSLATION.
C     I8,I9,IV,KD3 (4I5) I8,I9 ARE INDEXES OF DO LOOP.IF
C     I8=I9=1 THE GEOMETRIC INVERSION 1 WITH THE POSITIVE
C     SIGNED RADICAL IS OPTIMIZED.IF I8=I9=2 THE SECOND
```

```
C       INVERSION IS OPTIMIZED.
C             IV IS USED TO CALL THE CORRECT DESIRED DISPLACE-
C       MENT FUNCTIONS IN THE SUBROUTINE 'DESIRED'.
C             IF KD3=2THE INVERSION IN FIGURE 5.4 DEFINED BY
C       YO4 =YC1&0.5*(YC2-YC1)-D3*SIN(DELSLP)*CHAFFH
C       XO4 =XC1-0.5*(XC1-XC2)-D3*CHAFFY*COS(DELSLP)
C             IS OPTIMIZED.IF KD3=3 OR 4 INVER.IS DEFINED BY
C       YO4 =YC1&0.5*(YC2-YC1)&D3*SIN(DELSLP)*CHAFFY
C       XO4 =XC1-0.5*(XC1-XC2)&D3*CHAFFY*COS(DELSLP)
C             IS OPTIMIZED.
C       NTRANS(I5) IF IT IS ZERO NO APPROXIMATICN IS DONE FOR
C       TRANSLATION
C       NCLOCK(I5) IF ZERO OUTPUT ROTATES IN COUNTERCLOCKWISE
C       DIRECTION,
C       NPRINT(I5) IF ZERO  NO DISPL. NO VEL. PRINTED AFTER
C       EACH ITERATION,EXCEPT FOR THE SOLUTION REACHED.
C       NPRINT MUST NOT BE ZERO IF NFNPNT IS NOT ZERO.
C       NVLPNT(I5) IF NOT ZERO VELOCITIES AND ACCELERATIONS
C       FOR THE FINAL DIMENSIONS ARE PRINTED.
C       NITMOR (I5) IF NOT ZERO ITERATION IS DONE WITH THE
C       FIRST VALUES .THEN THE COMPUTED VALUES ARE USED AS NEW
C       DIMENSIONS.INCREMENTS ARE DIVIDED BY ACCRCY(I).
C       ITERATION IS DONE 'NITMOR' TIMES.
C       NFSYM,NSSYM,NFO,NSO,NFVO(4I5) IF NSYM=0 REFERENCE
C       CHECKS ON THE DISPLACEMENTS ARE DONE AT I=1 AND 2=NPR.
C       OTHERWISE IT IS DONE AT I=NFO AND I=NSO.
C       NQ1,NQ2 (2I5) INDICATE AT WHAT WALUES OF TET(NQ1) AND
C       TET(NQ2) THE PARAMETERS Q1 AND Q2 ARE INTRODUCED.
C       KR2CH,KRFCH,KRSCH (3I5) THE ONE NOT ZERO DEFINES WHICH
C       RESIDUAL SUM IS MINIMIZED.R**2,RF**2,OR RS**2
C       NPR,NPRL(2I5) NPR IS THE NUMBER OF PRECISION POINTS IN
C       THE DISPLACEMENT,NPRL IS THE NUMBER OF THE PRECISION
C       POINTS CONSIDERED IN THE MINIMIZATION PROCES.
C       R2MX,RF2MX,RS2MX,R2SEE,DIFFMX,DIFSMX(6F10.5)
C       ACCRCY(I),I=1,8 (8F9.5)
C       NTDATA(I5),IF ZERO IT COMPUTES SYO AND SF IN THE
C       SUBROUTINE 'DESIRED'.OTHERWISE INSERT DATA FOR 'TET(I)
C       SYO(I),SF(I),I=1,NPR'
C       MDELZRO(I5) IT IS NOT ZERO WHEN DELTA IS ZERO.
C       NSLP(I5) IT DEFINES WHERE THE CONSTRAINT FOR
C       INSTANTANEOUS DWELL IS.IF NSLP=1 CONSRAINT IS AT TETO2
C       Q2 IS THE SLOPE OF THE NORMAL AT TETO2.IF NSLP=2
C       CONSTRAINT IS AT TETO1 AND Q1 IS THE SLOPE OF THE
C       NORMAL AT TETO1.IF NSLP=0 CONSTRAINT IS AT BOTH TETO1
C       AND TETO2.IF NSLP,2 THERE ARE NO CONSTRAINTS.
C       D(I),I=1,8 (8F9.5) INITIAL VALUES OF DIMENSIONS.
C       D(I)=DE,D(2)=TETO1
C             D(3)=DELO,D(4)=D1,D(5)=D2,D(6)=D3,D(7)=DA,D(8)=DB
C       IF Q1 IS A VARIABLE D(6)=Q1.IF Q2 IS A VARIABLE D(7)=
C       Q2.
C       DDD(I),I=1,8 (8F9.5) INITIAL INCREMENTS IN THE UNKNOWN
C        DIMENSIONS.
```

```
C       NIT(I),I=1,8 (8I5) NUMBER OF INCREMENTS.
C       WHEN ALL NIT(L) ARE ZERO IT COMPUTES WITH THE INITIAL
C       VALUES ONCE.
C            IF NIT(K)=0 NO ITERATION IS NEEDED FOR THE KTH DI
C       MENSION.
C       THE OUTPUT IS PRINTED IN THE FOLLOWING CRDER,
C       KDRCT,IV(2I5)
C       DIMENSIONS AFTER EACH ITERATION.,THEN R**2,RF**2,RS**2
C       ,DIFFCH,AND DIFSCH
C       Q1,Q2,DELSLPO
C         TETO1P,FOPLNO. THESE ARE PRINTED FOR PLANE MECHANISM
C       SMQO(I),I=1,8 EQUATIONS OF CONDITIONS.THEY MUST VANISH
C       THE OUTPUT DISPLACEMENTS ARE PRINTED IN THE FOLLOWING
C       ORDER,
C       TETO,FIO,SYOG,SYO,S,SFG,SF,XO,DIFROT,DIFTRN.


        SUBROUTINE DSIRED(DTET,DFY12,DTET12,DLS12,NPR,KDRCT,IV
       1,TET,SYO,SF)
        DIMENSION TET(30),SYO(30),SF(30)
        PI=3.14159265
        TROP=PI/180.0
        DO 100 I=1,NPR
        IF(I.GT.1) GO TO 101
        TET(I)=0.0
        GO TO 102
101     I1=I-1
        TET(I)=TET(I1)&DTET
102     IF(KDRCT.EQ.1) GO TO 103
        IF(KDRCT.EQ.2) GO TO 104
103     IF(IV.EQ.1) GO TO 1031
        IF(IV.EQ.2) GO TO 1032
        IF(IV.EQ.3) GO TO 1033
        IF(IV.EQ.5) GO TO 1035
1031    SYO(I)=-DFY12*      ((TET(I)/DTET12)**2)
        SF(I)=DLS12*(1.0-(1.0-TET(I)/DTET12)**2)
        GO TO 10001
1032    IF(TET(I).GT.210.0) GO TO 10321
        SYO(I)=-DFY12*0.5*(1.0-COS(TET(I)*180.0*TROP/210.0))
        SF(I)=0.5*DLS12*  (1.0-COS(TET(I)*180.0*TROP/210.0))
        GO TO 10001
10321   SYO(I)=-0.5*DFY12*(1.0&COS((TET(I)-210.0)*180.0*TROP/1
       150.0))
        SF(I)=0.5*DLS12*  (1.0&COS((TET(I)-210.0)*180.0*TROP/1
       150.0))
        GO TO 10001
1033    SYO(I)=-DFY12*((TET(I)/DTET12)**2)
        SF(I)=-DLS12*TET(I)/DTET12
        GO TO 10001
 1035   IF(TET(I).GT.180.0) GO TO 10351
        SYO(I)=2.8125*((TET(I)/45.0)**3)
        GO TO 10352
```

```
10351 SYO(I)=-2.8125*(((360.0-TET(I))/45.0)**3)
10352 SF(I)=0.
      GO TO 10001
104   IF(IV.EQ.1) GO TO 1041
      IF(IV.EQ.2) GO TO 1042
      IF(IV.EQ.5)GO TO 1045
      GO TO 100
1041  SYO(I)=0.0
      SF(I)=DLS12 *(1.0-(1.0-TET(I)/(0.5*DTET12))**2)*(-1.0)
      GO TO 10001
1042  SYO(I)= DFY12*((TET(I)/DTET12)**2)
      SF(I)=-DLS12*TET(I)/DTET12
      GO TO 10001
1045  IF(TET(I).GT.180.0) GO TO 10451
      SYO(I)=2.8125*((TET(I)/45.0)**3)
      GO TO 10452
10451 SYO(I)=-2.8125*(((360.0-TET(I))/45.0)**3)
10452 SF(I)=0.
10001 IF(SYO(I).LT.0.0) SYO(I)=360.0&SYO(I)
100   CONTINUE
      RETURN
      END
```

VITA 2

Cemil Bagci

Candidate for the Degree of

Doctor of Philosophy

Thesis: THE RSRC SPACE MECHANISM--ANALYSIS BY 3x3 SCREW MATRIX, SYNTHESIS FOR SCREW GENERATION BY VARIATIONAL METHODS

Major Field: Mechanical Engineering

Biographical:

Personal Data: Born December 1, 1932, in Isparta, Turkey. Married to Sarah Alice Vanmeter, October 20, 1962. Has daughter Cemile Hacer, Sons Ismail Cemil, Attila Cemil, Ali Cemil.

Education: Graduated from Technical Institute in Isparta (Isparta Erkek Sanat Enstitüsü), Turkey, 1950; received Bachelor of Science in Technical Education from Technical Teacher's College (Erkek Teknik Örgretmen Okulu) Ankara, Turkey, 1954; received the Bachelor of Science and the Master of Science degrees in Technical Education while completing requirements for the Bachelor of Science degree in Mechanical Engineering, from Oklahoma State University in 1962 and 1963, respectively; received the Master of Science degree in Mechanical Engineering from Oklahoma State University in 1964; participated in the NSF Summer Institute on "Matrix Methods of Structural Analysis," Madison, Wisconsin, eight weeks, 1968; completed requirements for the Doctor of Philosophy degree at Oklahoma State University in August, 1969.

Professional Experience: Teaching at Technical Institutes in Çanakkale, Merzifon, Nevşehir, Turkey, 1954 to 1960; teaching and research assistant at Oklahoma State University, 1963-1967; has been teaching as assistant professor in the Department of Mechanical Engineering, Tennessee Technological University from September, 1967.