A SEMIEMPIRICAL DETERMINATION OF ALPHA PARTICLE ENERGIES AND HALF-LIVES IN THE HEAVY ELEMENT REGION

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PREFACE

In his search for new nuclides showing alpha decay, the experimentalist first looks at semiempirical predictions of alpha decay energies and half-lives to find likely places where his labors may bear fruit. This was my main objective in this thesis, to aid the experimentalist.

A secondary aim was to provide data for the possible improvement of the mass formula used.

I wish to acknowledge my indebtedness to my adviser, Dr. H. Kuemmel for his invaluable suggestions and assistance throughout the completion of this thesis; and to Dr. M. Nurmia who suggested this topic for study. Also I wish to acknowledge the aid which I received from Mr. Gene Pulley, Mr. Edgar Butler, and the rest of the staff at the computer center of Oklahoma State University; and to my wife who typed the manuscript.

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CHAPTER I

INTRODUCTION

For many years numerous formulas have been obtained which give the binding energy as a function of the number of protons and neutrons in the nucleus.[1,2,3] Such relations are called mass formulas. These mass formulas usually have several terms, the form of which are known but whose relative importance is not known quantitatively. By multiplying each term by a parameter determined by empirical methods, results are obtained which agree fairly well with experiment. The experimental trend of the binding energy per nucleon may be seen in Fig. 1.

By knowing the binding energy of the parent and daughter in an alpha decay, the Q-value for the reaction may be obtained in the following way:

$$E_p = E_d + E_H + Q_\alpha$$

where E_p, E_d, and E_H are the total rest energies of the parent, daughter and alpha particle. But:

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[1]A. G. W. Cameron, "A Revised Semiempirical Atomic Mass Formula," Canadian Journal of Physics, 35 (1957), pp. 1021-1032.

[2]E. Feenberg, "Semi-Empirical Theory of the Nuclear Energy Surface," Reviews of Modern Physics, 19 (1947), pp. 239-258.

[3]A. E. S. Green and D. F. Edwards, "Dicsontinuities in the Nuclear Mass Surface," Physical Review, 91 (1953), pp. 46-63.

$$E_p = (ZM_Z + NM_N)c^2 - B_p$$

$$E_d = \left[(Z-2)M_Z + (N-2)M_N\right] c^2 - B_d$$

$$E_H = (2M_Z + 2M_N)c^2 - B_\alpha$$

where M_Z is the mass of a proton

M_N is the mass of a neutron

Z is the number of the protons in the parent

N is the number of neutrons in the parent

B_p, B_d, and B_α are the binding energies of the parent, daughter and alpha particle.

Substituting these values into the equation above and solving for Q it is found that:

(1) $$Q_\alpha = B_d - B_p + B_\alpha.$$

From this the alpha particle energy, E_α, can be obtained by using consevation of momentum and energy. Let T_d be the kinetic energy of the daughter after decay, V, be the velocity of the daughter, M_d, be the mass of the daughter, v, be the velocity of the alpha particle, and m_α, its mass.

$$Q = E_\alpha + T_d$$

$$m_\alpha v = M_d V$$

$$T_d = \tfrac{1}{2} M_d V^2$$

$$E_\alpha = \tfrac{1}{2} m_\alpha V^2$$

Eliminating T_d, V, and v from these equations:

$$Q = E_\alpha \left[1 + \frac{m_\alpha}{M_d}\right] \doteq E_\alpha \left[1 + \frac{4}{A-4}\right]$$

where A is the atomic mass number of the parent nucleus solving this for E_α:

(2)

$$E_\alpha \doteq Q\left[1 - \frac{4}{A}\right].$$

In other words the daughter nucleus carries away $\frac{4}{A}$ of the total amount of kinetic energy available. Using relations (1) and (2) above, an equation is obtained giving the alpha particle energy as a function of the number of protons and neutrons in the parent.

In order to determine the parameters in the mass formula, the method of least squares can be applied using all of the experimental values of alpha particle energies available.

Now that a reasonably accurate formula has been obtained, two things may be accomplished. First, an experimental error can be seen by comparing the experimental value with that predicted by the formula. Second, by knowing the predicted alpha particle energy, the experimentalist will know what methods to use in looking for this energy. There are several people interested in this type of prediction in their work far away from the valley of stable nuclei.[4,5] They have found unusually short life times in this region.

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[4]Antti Siivola, "On the Alpha Activity of Neurton Deficient Europium and Gadolinium Isotopes, "Annales Academiae Scientiarum Fennicae, VI. Physica, 109, 1962.

[5]M. Karras, G. Andersson, and M. Nurmia, "Search for Alpha Activity in Neutron Deficient Isotopes of Pb, Tl, Hg, Pt, and Te," (unpub. paper, University of Helsinki, 1961).

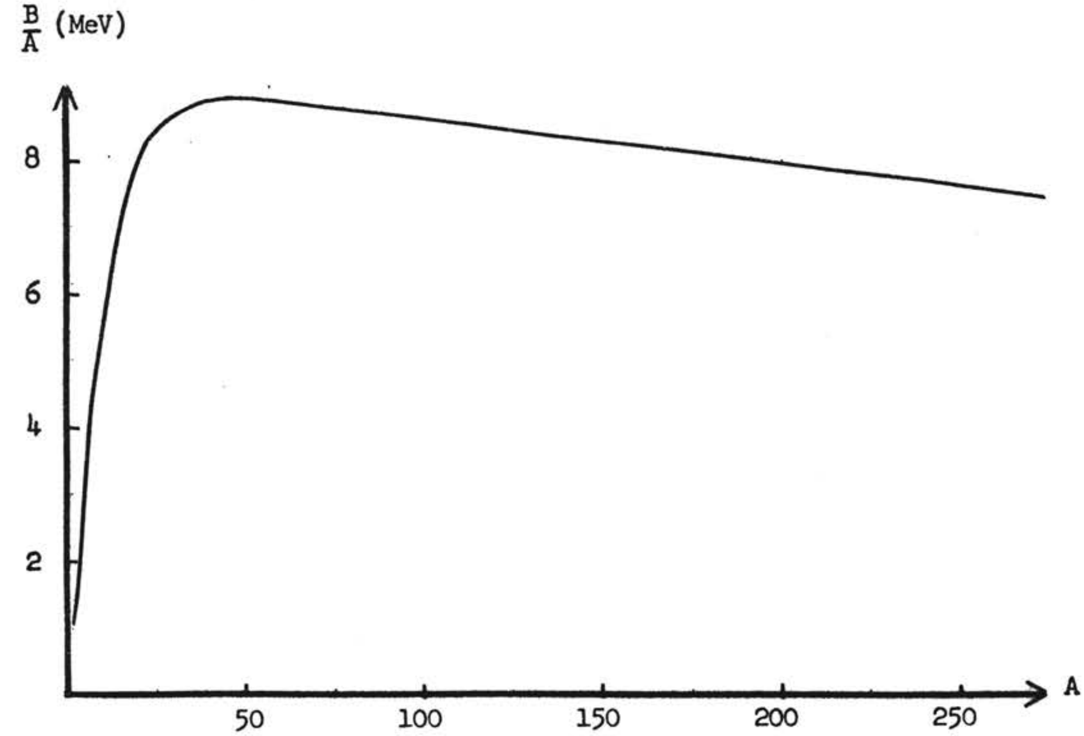

Fig.1. Binding energy per particle for the nuclides

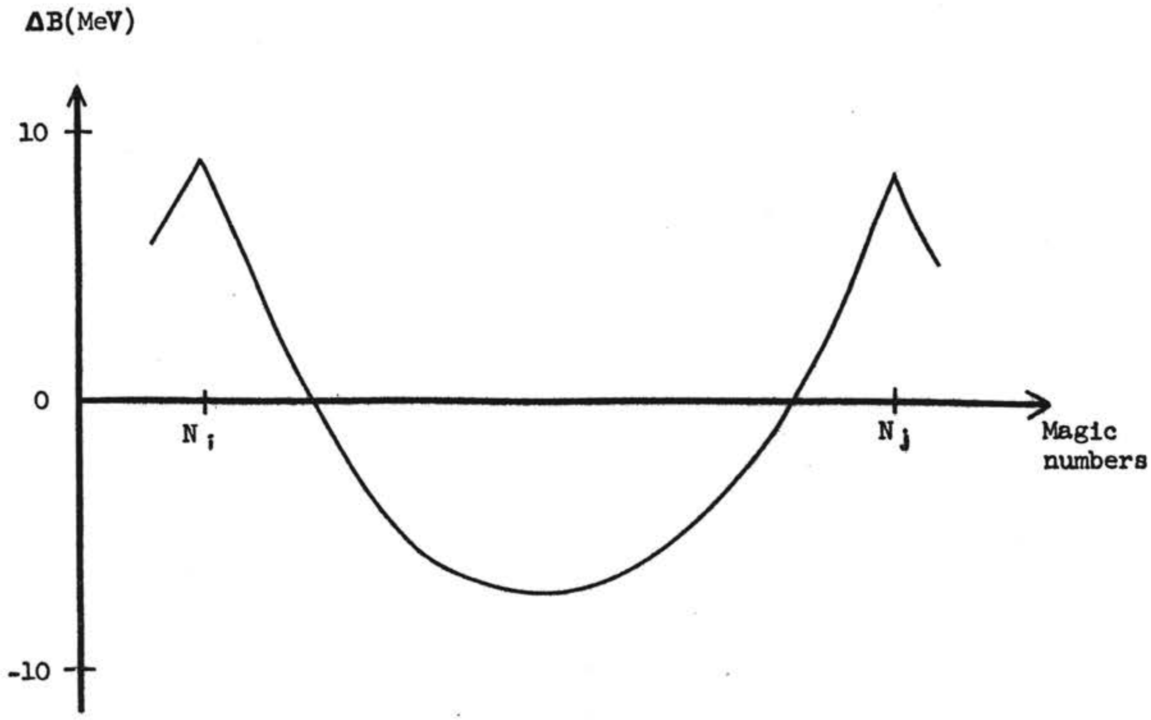

Fig.2. Deviation of experimental binding energy from B_{sw} close to magic numbers.

CHAPTER II

THE MASS FORMULA

One of the earliest mass formulas was the Bethe-Weizsacker formula which is based on the liquid drop model:

$$(3) \qquad B_{BW}(N,Z) = aA - bA^{2/3} - c\,\frac{Z(Z-1)}{A^{1/3}} - d\,\frac{(N-Z)^2}{A} + e\,\frac{\delta_{N,Z}}{A^{1/2}}$$

where B_{BW} is the binding energy, N is the number of neutrons, Z is the number of protons, A is the mass number, $\delta_{N,Z}$ is 2, 1, or 0 depending on whether the nuclide is even-even, even-odd or odd-even, or odd-odd, and the lower case letters are the parameters to be determined by experiment.[1] The first term is proportional to A because the liquid drop model assumes that the "number of bindings" is proportional to the volume. But this is an over estimate by an amount proportional to the surface, thus the second term. The third term is due to the Coulomb repulsive force. If there were no more terms, there would be a tendency for the stable nuclides to have a low proton number. The fourth term accounts for this. The fifth term accounts for the fact that odd-odd nuclei are least stable while even-even nuclei are the most stable. This formula will give quite accurate results for many nuclei but fails when the proton or neutron number is near a magic number. This is due to the

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[1] H. A. Bethe and R. F. Bacher, "Nuclear Physics," Reviews of Modern Physics, 8 (1936), pp. 165-167.

fact that if the number of protons or neutrons are at a magic number, they form a closed shell thus making the binding energy higher than would be accounted for by the Bethe-Weizsäcker formula. See Fig. 2.

In order to correct the Bethe-Weizsäcker formula, the shell model must be considered.[2] Let the n'th level above the lowest level in the i'th shell be denoted by E_i. The expansion for E_i in terms of n would be:

$$E_i(n) = A_i + B_i(n-1) + C_i(n-1)^2 + \dots$$

where A_i is the bottom of the i'th shell, B_i is a linear approximation to the level distance, and C_i is a correction to this. If N' is the number of neutrons in this shell and N" is the number of holes (unoccupied levels), assuming $A_i \gg B_i \gg C_i$, the contribution to the binding energy is given by:

$$B'(N') = \sum_{n=1}^{N'} E_i(n) \doteq fN' - \tfrac{1}{2}(h + iN')N'N'' \tag{4}$$

where f is the "center of the shell" and h is approximately the level distance. In terms of the old constants:

$$f \doteq A_i + \tfrac{1}{2} B_i N_i$$

where N_i is the number of levels in the i'th shell, $B_i \doteq h$ and i is proportional to C_i. The form of equation (4) will hold true also for protons letting say Z' be the number of protons in the shell and Z" be the number of holes in it. Also there may be a term which accounts for possible coupling between neutrons and protons. Such a term should

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[2] H. Kuemmel et al., "A New Nuclidic Mass Law," (unpub., Second International Conference on Nuclidic Masses, 1964).

show less coupling for empty and filled shells than for say a shell that was half filled. This seems reasonable because closed shells are "inert", and thus have no interactions with other nucleons. Also second order perturbation theory predicts that the coupling should be proportional to the number of particles and holes of both kinds of particles in each shell. Thus there should be a term of the form

$$l\, N'\, N''\, Z'\, Z''.$$

Putting these terms together the mass formula which was used in this thesis is obtained:

$$(5)\qquad B(N,Z) = aA - bA^{2/3} - c\,\frac{Z(Z-1)}{A^{1/3}} - d\,\frac{(N-Z)^2}{A} + e\,\frac{\delta_{N,Z}}{A^{1/2}} + fN' + gZ' - \tfrac{1}{2}(h + iN')\,N'N'' - \tfrac{1}{2}(j + kZ')\,Z'Z'' + lN'N''Z'Z''$$

By using this corrected formula along with (1) a fairly accurate formula for Q is obtained:

$$(6)\qquad Q_\alpha = B_\alpha + a\left[A^{2/3} - (A-4)^{2/3}\right] + b\left[\frac{Z(Z-1)}{A^{1/3}} - \frac{(Z-2)(Z-3)}{(A-4)^{1/3}}\right] + c(N-Z)^2\left[\frac{1}{A} - \frac{1}{A-4}\right] - 2d + \tfrac{1}{2}e\left[N'N'' - (N'-2)(N''+2)\right] + \tfrac{1}{2}f\left[N'^2N'' - (N'-2)^2(N''+2)\right] + \tfrac{1}{2}g\left[Z'Z'' - (Z'-2)(Z''+2)\right] + \tfrac{1}{2}h\left[Z'^2Z'' - (Z'-2)^2(Z''\ 2)\right] + i\left[(N'-2)(N''+2)(Z'+2)(Z''+2) - N'N''Z'Z''\right]$$

where B^α is the binding energy of the alpha particle. The pairing energy has been omitted because its contribution to Q_α is negligable ($\doteq 6$ kev) due to the weak A dependence. The first term reduces to a constant. Also the first two terms of the correction terms reduce to constants. In order to determine the parameters the $82 < Z < 126$, $126 < N < 184$ region was chosen since in these shells α - decay is most prevalent. Thus not only will more accurate values for the parameters be obtained but also the formula will be of more use to the experimenter since this is the region where α- decay is most likely to be found. In this shell

$$N' = N - 126$$

$$N'' = 184 - N$$

$$Z' = Z - 82$$

$$Z'' = 126 - Z$$

After E_α has been determined the half-life can be calculated by the use of a formula derived by Bethe[3] and modified by Nurmia.[4] In the derivation of this formula a potential is assumed such as the one shown in Fig. 3. The radial wave function:

$$\Psi_r = \psi \exp(-iQ_\alpha t)$$

is assumed. Letting $\varphi = r\psi$ the equation,

$$\frac{d^2\varphi}{dr^2} + \frac{2M}{\hbar^2}\left(Q - \frac{L(L+1)\hbar^2}{2Mr^2} - V(r)\right)\varphi = 0,$$

must be satisfied. M is the reduced mass of the alpha particle. It is assumed that the alpha particle does not undergo a spin change, thus $L = 0$. The equation is solved in all three regions, the WKB approximation method being applied in regions (ii) and (iii).

For region (i): $\varphi(r) = A_1 \sin k_1(r)$

where $k_i = \left[2M(Q_\alpha - U)\right]^{1/2}/\hbar.$

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[3]E. Segrè et al, Experimental Nuclear Physics III, (New York, 1959), pp. 76-81.

[4]R. Taagepera and M. Nurmia, "On the Relations between Half-Life and Energy Release in Alpha Decay," Annales Academiae Scientiarum Fennicae, VII Physica, 76, 1961.

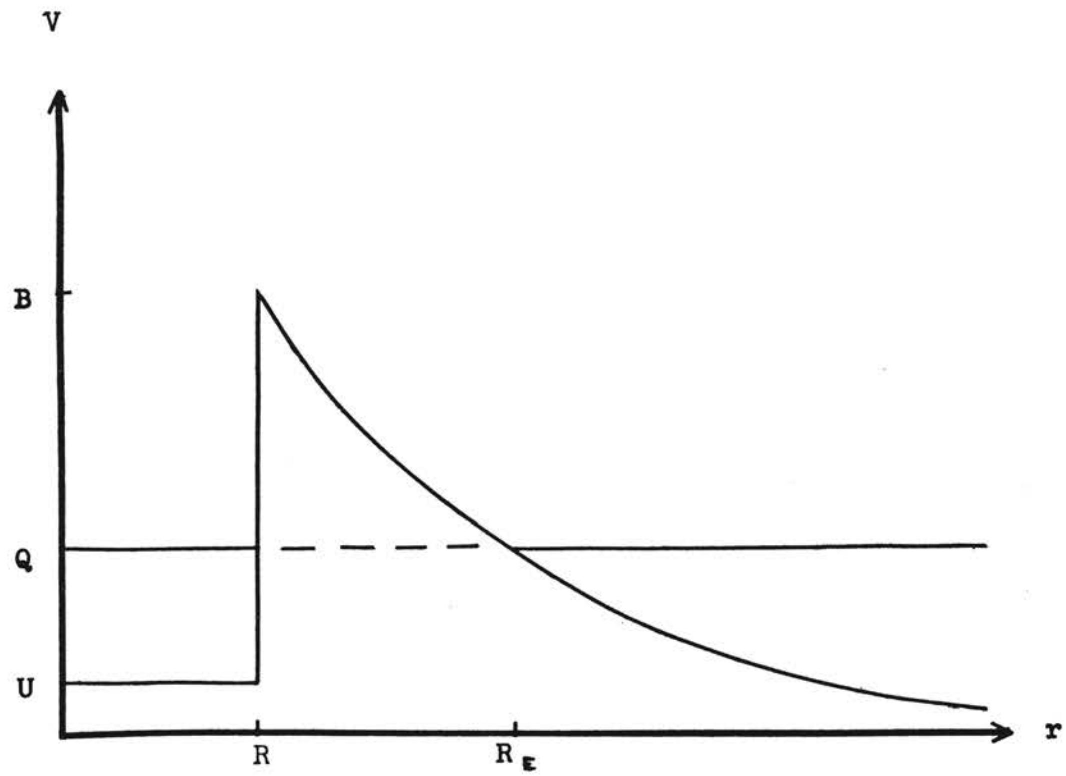

Fig.3. Nuclear Potential

For region (ii) near $r \simeq R$:

$$\varphi(r) = A_2\, k_2^{-1/2}(r) \exp\left[K_2(r)\right]$$

where

$$k_2(r) = \left[2M(V(r) - Q_\alpha)\right]^{1/2} / \hbar \text{ and}$$

$$K_2(r) = \int_r^{R_E} k_2(\rho)\, d\rho$$

For region (iii) for very large r, $\varphi(r)$ must represent a pure outgoing wave:

$$\varphi(r) = A_3 \exp(ikr)$$

where

$$k = (2MQ_\alpha)^{1/2}/\hbar.$$

By using the boundary conditions between the regions relations between the A's are found:

$$A_1 \doteq A_2 k_1^{-1} k_2^{1/2}(R) \exp\left[K_2(R)\right]$$

$$A_3 = A_2 \left[\frac{1+i}{2k}\right]^{1/2}$$

By normalizing the wave function, it is found that:

$$A_1^2 = \frac{1}{2\pi R} .$$

Since the wave function has been normalized, the decay constant is obtained directly from the solution in region (iii) for large r by

$$\lambda = \pi 4 \pi v |A_3|^2$$

where v is the relative velocity of sparation between the alpha particle and the daughter nucleus. By substituting in the values for the A's and k's, the Bethe equation is obtained:

$$(7) \quad \lambda = \frac{2^{1/2} \pi 2\, \hbar^2}{M^{3/2} R_d (B - Q)^{1/2}} \exp\left[-2 \frac{2BR_d}{\hbar v} (\alpha - \sin\alpha \cos\alpha) \right]$$

where R_d is the radius and B is the Coulomb barrier height of the daughter nucleus and α is given by $\cos^2 = EB^{-1}$. The first part of the product in the Bethe equation is the frequency that the alpha particle strikes the sides of the potential well. The exponential part gives the probability of transmission through the Coulomb barrier. To arrive at Nurmia's approximation several assumptions were made. The first part of the product in equation (7) is considered to be a constant, C_0, R_d is assumed to be equal to $r_0 A_d$ where A_d is the atomic mass number of the daughter and $r_0 = 1.5 \times 10^{-13}$ cm. With A_d approximated by 2.5 Z_d, $R_d = 1.36\, r_0 Z_d^{1/3}$. It is assumed that $B \gg Q$ making $\cos\alpha$ small, $\sin\alpha \doteq 1$ and $\alpha = \pi/2 - \cos\alpha$. Also Q may be approximated by E_α. Since $B = 2Z_d e^2 R_d$ and $\cos^2\alpha = EB^{-1}$,

$$\cos\alpha = \frac{(1.36r\ )^{1/2}}{2^{1/2}e}\,\frac{E_\alpha^{1/2}}{Z_d^{1/3}} .$$

Thus equation (7) becomes:

$$\lambda = C_0 \exp\left[-\frac{2\pi 2^{1/2} e^2 M^{1/2}}{\hbar}\,\frac{Z_d}{E_\alpha^{1/2}} + \frac{8eM^{1/2}(1.36r_0)^{1/2}}{\hbar}\,Z_d^{2/3}\right.$$

Converting this to $\log_{10}T$ where T is the half life in years:

$$(8) \qquad \log_{10}T = C_1\left(\frac{Z_d}{E_\alpha^{1/2}} - C_3 Z^{2/3}\right) - C_2$$

where $C_1 = 1.70(\text{MeV})^{1/2}$, $C_2 = 30.0$, $C_3 = 1.00(\text{MeV})^{-1/2}$ and E_α is measured in MeV. This is the equation that was used in this thesis.

CHAPTER III

DETERMINATION OF PARAMETERS

The method of least squares was used to determine the parameters for this region. To apply this method let y_i be the i'th value of $Q_\alpha - B_\alpha$, an experimental value, and let $f_i(N,Z)$ be the theoretical value of $B_d - B_p$ for the i'th parent nuclei. Therefore:

$$(9) \qquad f_i(N,Z) = \sum_{j=1}^{9} a_j x_{ij}$$

where a_j is the j'th parameter and x_{ij} is the j'th term using the N and Z values for the i'th parent nuclei. According to the method of least squares:

$$(10) \qquad \sum_{i=1}^{n} \left[y_i - f_i(N,Z) \right] x_{ik} = 0 \quad k = 1,2,\ldots,9$$

where n is the number of experimental observations. Substituting the value of $f_i(N,Z)$ from (5) into (6) and writting in matrix form:

$$X'(Y - XA) = 0$$

where Y is a column matrix whose elements are the y_i, A is a column matrix whose elements are the a_i, X is a matrix whose elements are the x_{ij}, the j'th term using the N and Z value of the i'th parent nuclei, and X' is the transpose of X. This equation represents a set of nine equations whose unknowns are the nine parameters. These equations are the conditions set on the parameters, which minimizes the sum of the squares of the residuals between the experimental values, y_i and the theoretical values, $f_i(N,Z)$. They must now be solved for the a_i:

(11) $$A = (X'X)^{-1}(X'Y)$$

In order to perform these operations a computer program was written which calculated the elements of the X and Y matrix, stored them in the proper locations, and then performed the indicated matrix operations. A difficulty arose when the computer inverted the X'X matrix. This matrix being nine by nine and the computer which was used (IBM 650) being of limited word length, the accuracy of the elements of the inverted matrix was limited. To overcome this difficulty only three parameters were calculated at a time. Older estimates of the values for the other parameters were used. In this manner the y_i became, for example:

$$y_i = Q_\alpha - B_\alpha - \sum_{j=1}^{6} a_j x_{ij}$$

while

$$f_i(N,Z) = \sum_{j=7}^{9} a_j x_{ij}.$$

It was found that the parameters were sufficiently independent of each other in the minimization of the residuals that one interation would bring the values of the parameters to well within the accuracy of the formula. (See Table II.) With the parameters thus determined the corrected mass formula can now be used to predict the alpha particle energies for this region.

CHAPTER IV

DATA

The experimental data used in calculating the y_i is listed in Table I.[1] Some of the data in this region was not used. If all of the data that was available had been taken, the alpha particle energies which are more suitable to experimental techniques would perhaps have been given undue emphasis. There were few examples of odd-odd nuclei. However it is doubtful that this did much to invalidate the predictions because as was mentioned before the pairing energy term is negligable. Also the energies which had a sizable error ($\pm$.1 MeV) were omitted where possible. (In several areas where insufficient data was available, these more inaccurate energies were used). The alpha particle energies for bismuth ($Z = 83$) and polonium ($Z = 84$) were omitted because the daughter nuclei would lie in the next lower shell thus making equation (4) invalid.

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[1] Hans Heinrich Landolt and Richard Börnstein, "Numerical Data and Functional Relationships in Science and Technology," Group I, (Berlin, Heidelberg, Göttingen, 1961), Part 3.

CHAPTER V

RESULTS AND CONCLUSIONS

The parameters which were obtained from the data above are listed in Table II. The alpha particle energies, which were predicted from these parameters, are shown in Table III. The first column gives the name of the element, the second gives the number of protons in the parent nuclei, the third gives the number of protons, the fourth gives the predicted alpha particle energy in kev, and the fifth gives the difference between the experimental value and the predicted value in kev, the negative sign indicates that the predicted value is too high by this much. The $\log_{10}$ of the half life in years is given in the last column. The errors in $\log_{10} T$ are fairly large ($\sigma \doteq 1$), and thus $\log_{10} T$ can only be used to determine the methods to use in order to look for the alpha activities.

By studying the differences between experiment and the predictions, it can be seen that instead of being randomly distributed there are sections where the predictions are too high and sections where they are too low. For example note the elements Pu, Am, Cm, Bk, and Cf with neutron numbers 146, 147 and 148. In this section all the predictions are too low. There are several other examples of this type of systematic differences. In Fig. 4. the deviations are shown. (For the nuclides which do not have crosshatching, either the deviation was small, <100 kev,

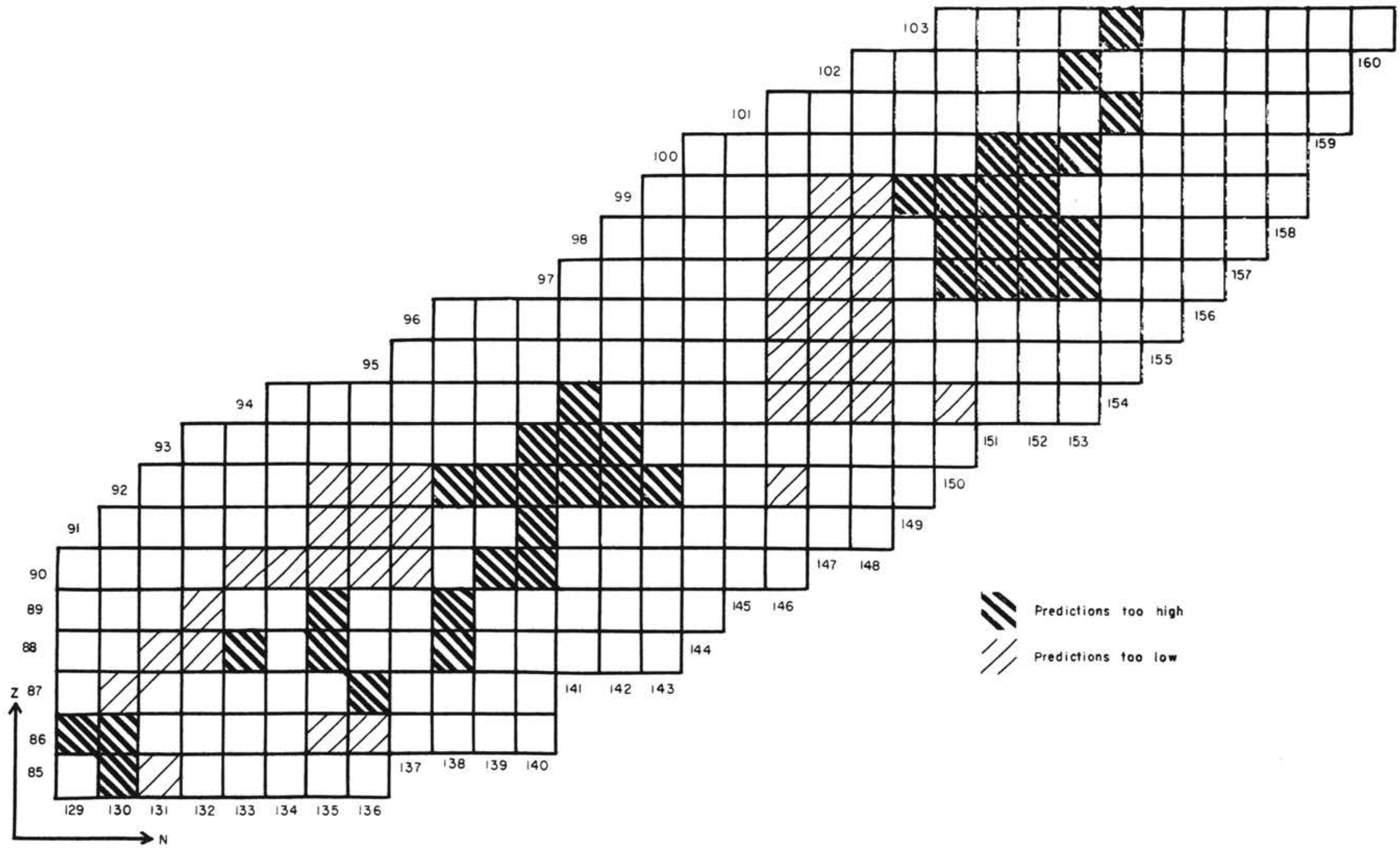

Fig. 4. Deviations of predicted alpha particle energies from experiment

or no data was available.) An attempt was made to explain this by sub-shell structure. However, in this region the nuclear deformations are quite large, and the sub-shell structure is no longer apparent.[1] According to theory, for spherical nuclei there should be a sub-shell at $Z = 92$, $N = 136$, $N = 150$.

No clear evidence of a strong pairing effect can be seen by studying the deviations in detail. In some regions the alpha particle energy of the even-even nuclei are higher, while in others that of the odd-odd are higher. However there can definitly be seen some type of even-odd relationship although not the one included in the Bethe-Weizsacker formula.

Considering all the deviations available the root-mean-square error is 180 kev or an error of about 3% for an average E_α. It is doubtful that better predictions could be made with this formula by adjusting the parameters using all of the data on binding energy available such as beta decay and other reactions. By considering these other reactions, the values of the parameters may be shifted due to something inherent to the particular type of reaction that is not present in alpha decay. Perhaps for regions where sufficient data is available, more accurate results could be obtained by simply extrapolating the experimental data.[2] Say by keeping Z constant and fitting a polynomial

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[1] S. G. Nilsson and B. R. Mottelson, "The Intrinsic States of Odd-A Nuclei Having Ellipsoidal Equilibrium Shape," Mathematisk - Fysiske Skrifter Danske Videnskabernes Selskab, 1957-1961.

[2] Yamada Matumodo, "Nuclear Ground State Energies," Physical Society of Japan Journal, 16(1961) pp. 1497-1500.

in N to the known data. However, this would have two disadvantages. First, in regions where little data is available or some local effect is strong, the results may be inaccurate. Secondly, nothing could be learned about the nature of the nucleus, and the polynomials would only have practical significance.

TABLE I

EXPERIMENTAL ALPHA PARTICLE ENERGIES

Element	E (MeV)	Element	E (MeV)	Element	E (MeV)
*Lw257	8.6	*Am243	5.340	*Th227	6.036±.001
*No255	8.2	*Am241	5.5408±.0006	Th226	6.330
*No253	8.5	*Am239	5.75	Th225	6.57 ±.03
*Md255	7.34	*Am237	6.01	*Th224	7.13 ±.02
*Fm255	7.03 ±.01	Pu244	4.55	Th223	7.55 ±.10
*Fm254	7.20 ±.01	*Pu242	4.898	*Ac227	4.949±.002
*Fm253	6.94	*Pu241	4.893	*Ac225	5.8185±.0015
*Fm252	7.05 ±.02	*Pu240	5.1589±.0005	Ac224	6.17 ±.03
Fm251	6.89 ±.05	Pu239	5.147	*Ac223	6.6570±.0007
*Fm250	7.43	*Pu238	5.491±.001	*Ac222	6.96 ±.05
*Es254	6.40 ±.02	Pu237	5.65 ±.02	Ac221	7.6
*Es253	6.633±.005	*Pu236	5.763	*Ra226	4.777±.003
Es252	6.64	Pu235	5.85 ±.02	*Ra224	5.681
*Es251	6.48	Pu234	6.19	*Ra223	5.867
*Es249	6.76	*Pu233	6.30	Ra222	6.55
Es248	6.87	*Pu232	6.58	Ra221	6.71 ±.03
Es247	7.35	*Np237	4.872	*Ra220	7.43 ±.02
*Es246	7.35	*Np235	5.06 ±.02	Ra219	8.0 ±.1
*Cf252	6.112	*Np233	5.53	*Fr223	5.34 ±.08
Cf251	5.841	*Np231	6.28	*Fr221	6.332±.010
*Cf250	6.024	*U 238	4.195±.005	*Fr220	6.69 ±.03
*Cf249	6.194	*U 236	4.499±.004	Fr219	7.30 ±.02
Cf248	6.23 ±.03	U 235	4.559	*Fr218	7.85 ±.05
*Cf246	6.753	U 234	4.768	Fr217	8.3
Cf245	7.11 ±.02	*U 233	4.8157±.0005	*Rn222	5.4860±.0005
*Cf244	7.17	*U 232	5.318±.002	Rn221	6.0 ±.1
*Bk249	5.417±.015	U 231	5.45	*Rn220	6.282±.004
*Bk247	5.67	*U 230	5.884±.005	*Rn219	6.813±.002
*Bk245	6.37 ±.02	U 229	6.42 ±.02	Rn218	7.13 ±.01
*Bk244	6.67 ±.015	*U 228	6.67	Rn217	7.74 ±.03
*Bk243	6.72	U 227	6.8 ±.1	*Rn216	8.01 ±.03
*Cm248	5.054±.015	*Pa231	5.046	Rn215	8.6 ±.1
*Cm246	5.373±.010	*Pa229	5.665±.001	*At219	6.27
Cm245	5.45	*Pa228	6.1380	At218	6.63
*Cm244	5.801	*Pa227	6.526	*At217	7.051±.010
*Cm243	6.061	*Pa226	6.81 ±.05	*At216	7.79 ±.03
Cm242	6.110	*Th232	4.007±.005	At215	8.00 ±.02
Cm241	5.95 ±.02	*Th230	4.682±.010	*At214	8.78 ±.05
*Cm240	6.25	Th229	5.02	At213	9.2
Cm238	6.50	*Th228	5.421±.001		

*Nuclides used in the determination of the parameters.

TABLE II

THE PARAMETERS

Parameter	First Iteration	Second Iteration
a	20.673	21.949
b	0.57418	0.56447
c	17.250	17.465
d	28.113	27.914
e	0.0495	0.0457
f	-0.000186	-0.000138
g	0.0311	0.0321
h	-0.00120	-0.00113
i	0.0000762	0.0000776

TABLE III

PREDICTED ALPHA DECAY ENERGIES, THE RESIDUALS, AND THE PREDICTED LOG HALF LIFE

ELEMENT	Z	N	E_α	ΔE_α	LOG T
AT	85	129	8713	70	-15.03
	85	130	8185	-200	-13.35
	85	131	7624	170	-11.74
	85	132	7136	- 86	-10.02
	85	133	6645	- 10	- 8.17
	85	134	6135	130	- 5.97
	85	135	5623		- 3.33
	85	136	5153		- 0.70
RN	86	127	9767		-17.64
	86	128	9203		-16.14
	86	129	8694	-100	-14.71
	86	130	8152	-140	-13.19
	86	131	7681	60	-11.56
	86	132	7207	- 80	- 9.83
	86	133	6714	98	- 7.98
	86	134	6218	63	- 5.77
	86	135	5763	200	- 3.62
	86	136	5320	165	- 1.03
	86	137	4889		+ 1.20
	86	138	4515		+ 3.92
	86	139	4061		+ 7.92
	86	140	3648		+11.64
FR	87	127	9658		-16.91
	87	128	9169		-15.52
	87	129	8648		-14.22
	87	130	8194	100	-12.85
	87	131	7738	110	-11.39
	87	132	7263	40	- 9.65
	87	133	6784	- 90	- 7.80
	87	134	6345	- 13	- 6.03
	87	135	5916		- 4.15
	87	136	5500	-160	- 1.88
	87	137	5139		+ 0.56
	87	138	4699		+ 3.52
	87	139	4299		+ 6.42
	87	140	3942		+ 9.23
RA	88	127	9611		-16.47
	88	128	9110		-15.22
	88	129	8675		-13.90
	88	130	8237		-12.69
	88	131	7780	200	-11.23
	88	132	7319	110	- 9.68
	88	133	6897	-190	- 7.83
	88	134	6484	70	- 6.07
	88	135	6083	-216	- 4.20
	88	136	5735	- 55	- 2.46
	88	137	5310		- 0.06

III (CONTINUED)

ELEMENT	Z	N	E_α	ΔE_α	LOG T
RA	88	138	4924	-148	+ 2.22
	88	139	4579		+ 4.68
	88	140	4213		+ 7.33
	88	141	3904		+10.20
	88	142	3576		+13.72
	88	143	3257		+17.58
AC	89	128	9124		-14.91
	89	129	8705		-13.92
	89	130	8267		-12.35
	89	131	7824		-11.07
	89	132	7419	200	- 9.51
	89	133	7023	- 60	- 8.08
	89	134	6637	19	- 6.34
	89	135	6305	-140	- 4.96
	89	136	5895	- 77	- 3.02
	89	137	5523		- 0.95
	89	138	5191	-242	+ 0.97
	89	139	4838		+ 3.33
	89	140	4541		+ 5.54
	89	141	4223		+ 7.55
	89	142	3915		+10.80
	89	143	3617		+13.95
	89	144	3314		+17.37
TH	90	129	8722		-13.42
	90	130	8299		-12.19
	90	131	7912		-11.08
	90	132	7534		- 9.73
	90	133	7164	390	- 8.10
	90	134	6847	280	- 7.03
	90	135	6453	120	- 5.23
	90	136	6096	233	- 8.10
	90	137	5778	258	- 1.80
	90	138	5439	- 18	+ 0.07
	90	139	5153	-130	+ 2.05
	90	140	4848	-166	+ 3.86
	90	141	4551		+ 6.09
	90	142	4263	257	+ 8.13
	90	143	3971		+11.04
	90	144	3717		+13.37
	90	145	3442		+16.29
	90	146	3177		+19.91
PA	91	130	8375		-12.04
	91	131	8015		-11.11
	91	132	7664		- 9.77
	91	133	7362		- 8.56
	91	134	6985		- 7.08
	91	135	6644	170	- 5.75
	91	136	6339	186	- 4.35
	91	137	6015	123	- 2.89
	91	138	5742	- 81	- 1.08

III (CONTINUED)

ELEMENT	Z	N	E_∞	ΔE_∞	LOG T
PA	91	139	5449		+ 0.26
	91	140	5164	-119	+ 2.25
	91	141	4888		+ 4.06
	91	142	4606		+ 6.30
	91	143	4362		+ 8.34
	91	144	4097		+10.50
	91	145	3841		+12.79
	91	146	3623		+15.23
	91	147	3384		+17.83
	91	148	3199		+20.13
U	92	131	8135		-11.32
	92	132	7850		- 9.99
	92	133	7490		- 8.59
	92	134	7165		- 7.33
	92	135	6876	100	- 6.24
	92	136	6566	100	- 4.87
	92	137	6307	110	- 3.68
	92	138	6027	-124	- 2.44
	92	139	5756	-310	- 0.88
	92	140	5491	-174	+ 0.74
	92	141	5221	-406	+ 2.46
	92	142	4987	-220	+ 3.97
	92	143	4733	-174	+ 5.86
	92	144	4486	13	+ 7.53
	92	145	4277		+ 9.63
	92	146	4046	149	+11.86
	92	147	3868		+13.02
	92	148	3653		+15.46
	92	149	3462		+17.61
NP	93	132	7969		-10.42
	93	133	7661		- 9.05
	93	134	7387		- 8.23
	93	135	7093		- 6.74
	93	136	6848		- 5.85
	93	137	6583		- 4.47
	93	138	6324	- 40	- 3.75
	93	139	6073		- 2.01
	93	140	5815	-290	- 0.71
	93	141	5592		+ 0.64
	93	142	5349	-290	+ 1.49
	93	143	5112		+ 3.55
	93	144	4912	- 41	+ 4.78
	93	145	4690		+ 6.71
	93	146	4519		+ 7.72
	93	147	4313		+ 9.47
	93	148	4128		+11.30
	93	149	3951		+12.83
	93	150	3780		+14.43
PU	94	134	7596		- 8.27
	94	135	7366		- 7.43

III (CONTINUED)

ELEMENT	Z	N	E_{∞}	ΔE_{∞}	LOG T
PU	94	136	7115		- 6.35
	94	137	6871		- 5.45
	94	138	6633	- 50	- 4.29
	94	139	6388	- 90	- 3.08
	94	140	6177	10	- 2.08
	94	141	5945	-100	- 1.04
	94	142	5720	043	+ 0.29
	94	143	5529	120	+ 0.84
	94	144	5317	173	+ 2.85
	94	145	5154	- 8	+ 3.75
	94	146	4956	202	+ 4.98
	94	147	4779	114	+ 6.59
	94	148	4608	289	+ 7.59
	94	149	4444		+ 8.97
	94	150	4317	230	+10.04
	94	151	4153		+12.27
	94	152	4025		+13.05
	94	153	3874		+14.24
AM	95	137	7171		- 6.40
	95	138	6940		- 5.50
	95	139	6742		- 4.35
	95	140	6522		- 3.63
	95	141	6308		- 2.65
	95	142	6128	-120	- 1.64
	95	143	5927		- 0.60
	95	144	5773	- 20	+ 0.47
	95	145	5584		+ 1.59
	95	146	5414	126	+ 2.45
	95	147	5251		+ 3.64
	95	148	5094	250	+ 4.25
	95	149	4972		+ 5.82
	95	150	4814		+ 6.79
	95	151	4690		+ 7.79
	95	152	4544		+ 8.82
	95	153	4389		+10.25
	95	154	4285		+10.98
CM	96	138	7286		- 6.23
	96	139	7080		- 5.56
	96	140	6878		- 4.65
	96	141	6710		- 3.94
	96	142	6519	- 20	- 3.22
	96	143	6375		- 2.48
	96	144	6195	60	- 1.46
	96	145	6035	- 90	- 0.68
	96	146	5880	230	+ 0.38
	96	147	5730	331	+ 0.93
	96	148	5614	186	+ 1.78
	96	149	5462	- 10	+ 2.64
	96	150	5344	29	+ 3.53
	96	151	5203		+ 4.44

III (CONTINUED)

ELEMENT	Z	N	E_{α}	ΔE_{α}	LOG T
CM	96	152	5053	1	+ 5.37
	96	153	4952		+ 6.33
	96	154	4842		+ 6.99
	96	155	4738		+ 7.65
BK	97	141	7094		- 5.17
	97	142	6960		- 4.71
	97	143	6791		- 3.77
	97	144	6640		- 3.29
	97	145	6493		- 3.05
	97	146	6352	370	- 1.80
	97	147	6243	430	- 1.29
	97	148	6098	270	- 0.50
	97	149	5986		+ 0.29
	97	150	5851	-180	+ 0.84
	97	151	5706		+ 1.67
	97	152	5609	-192	+ 2.24
	97	153	5503		+ 2.82
	97	154	5402		+ 3.71
	97	155	5321		+ 4.01
	97	156	5244		+ 4.63
CF	98	142	7370		- 5.90
	98	143	7229		- 5.23
	98	144	7093		- 4.77
	98	145	6960		- 4.31
	98	146	6859	310	- 3.83
	98	147	6722	390	- 3.11
	98	148	6616	137	- 2.62
	98	149	6487		- 2.37
	98	150	6349	-110	- 1.36
	98	151	6256	- 62	- 0.32
	98	152	6154	-130	- 0.32
	98	153	6057	-216	+ 0.21
	98	154	5978	134	+ 0.21
	98	155	5904		+ 1.30
	98	156	5806		+ 1.58
	98	157	5727		+ 1.87
ES	99	143	7677		- 6.84
	99	144	7554		- 6.41
	99	145	7462		- 5.97
	99	146	7333		- 5.53
	99	147	7234	120	- 4.84
	99	148	7112	240	- 4.61
	99	149	6981	-110	- 3.91
	99	150	6894	-130	- 3.44
	99	151	6797		- 2.95
	99	152	6703	-220	- 2.46
	99	153	6628	20	- 2.46
	99	154	6557	76	- 1.96
	99	155	6461	- 60	- 1.45
	99	156	6383		- 0.94

III (CONTINUED)

ELEMENT	Z	N	E_{α}	ΔE_{∞}	LOG T
ES	99	157	6338		- 0.94
	99	158	6254		- 0.41
FM	100	144	8051		- 7.96
	100	145	7931		- 7.55
	100	146	7841		- 7.13
	100	147	7727		- 6.70
	100	148	7603		- 6.26
	100	149	7522		- 5.82
	100	150	7430	0	- 5.60
	100	151	7342	-450	- 5.38
	100	152	7270	-220	- 4.92
	100	153	7202	-263	- 4.46
	100	154	7110	90	- 4.23
	100	155	7034	0	- 3.76
	100	156	6990		- 3.52
	100	157	6907		- 3.28
	100	158	6828		- 3.04
MD	101	146	8330		- 8.43
	101	147	8214		- 8.02
	101	148	8140		- 8.02
	101	149	8054		- 7.60
	101	150	7972		- 7.18
	101	151	7905		- 7.18
	101	152	7841		- 6.76
	101	153	7752		- 6.32
	101	154	7680	-340	- 6.11
	101	155	7637		- 6.11
	101	156	7557		- 5.66
	101	157	7479		- 5.44
	101	158	7430		- 5.22
	101	159	7357		- 4.99
NO	102	148	8669		- 9.28
	102	149	8593		- 9.08
	102	150	8532		- 8.88
	102	151	8473	30	- 8.68
	102	152	8389		- 8.48
	102	153	8320	-100	- 8.07
	102	154	8280		- 8.07
	102	155	8202		- 7.87
	102	156	8125		- 7.87
	102	157	8078		- 7.24
	102	158	8006		- 7.03
	102	159	7963		- 7.03
LW	103	150	9097		-10.50
	103	151	9018		-10.12
	103	152	8954		- 9.93
	103	153	8917		- 9.93
	103	154	8842	-200	- 9.55
	103	155	8769		- 9.35

III (CONTINUED)

ELEMENT	Z	N	E_{α}	ΔE_{α}	LOG T
LW	103	156	8723		- 9.35
	103	157	8652		- 8.96
	103	158	8609		- 8.96
	103	159	8581		- 8.76
	103	160	8514		- 8.56

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VITA

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