

EFFECT OF VARIATION OF PARAMETERS, ON  
TEMPERATURES IN HYDRAULIC SYSTEMS

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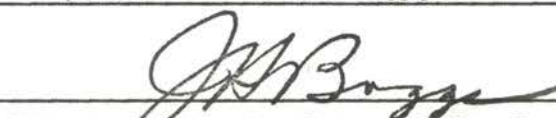
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## PREFACE

Seldom have I had the opportunity to take part in an endeavor that I have enjoyed more than the writing of this thesis. This work represents a study of the trends in hydraulic system temperatures produced by variation of system parameters.

May I express my sincere appreciation and gratitude to Dr. J. D. Parker for his valuable suggestions and criticisms of the work during its preparation, and for his time as he was never too busy for the many necessary consultations. I also wish to express my appreciation to Dr. D. R. Haworth for his timely assistance, much needed comments, and inspiration as a teacher.

Finally, I will always be grateful to my wife for her efforts in typing the final copy and for her encouragement and untiring labors which have contributed so much.

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## CHAPTER I

### INTRODUCTION

Man has long strived to utilize fluid in power systems. There is evidence that the first water wheels were built between 200 and 100 B. C. The conversion of energy stored in fossil fuels to mechanical power, using a fluid power link, was made possible by the invention of the steam engine. Along with the invention of economically useful prime movers, such as Watt's steam engine, and with the development of the factory system, there arose a need for a method of transmitting power from the point of generation to a distant power-using machine. This transmission could be accomplished mechanically, but mechanical transmission was often expensive and inconvenient. One solution to this problem was the transmission of power via a fluid medium under pressure.

Many problems have arisen in generating, transmitting, and controlling power with this type of system. One of the more recent problems, which has been brought about primarily by the increasing demands put on hydraulic systems by the introduction of faster and more complex aircraft and the development of missiles and space vehicles, is the overheating of the fluid power system.

Excessive fluid temperatures in a hydraulic system can critically affect system reliability, pump cavitation, fluid contamination and deterioration, and reservoir pressurization requirements(1).

Therefore, prior to the construction of a fluid power system, it is important for the designer to be able to calculate the temperature distribution throughout the systems so that he can determine if the heat dissipation will be adequate or, if necessary, to be able to determine the amount of heat removal required by a heat exchanger.

The prime objectives of this thesis are to show what effect variations of specific heat, pump efficiency, and over-all heat transfer coefficient have on the local temperatures of a system; to aid the designer in predicting the system temperatures; and to help him to understand better the thermal aspects of the hydraulic systems which he must design.

Since it would not be feasible to include in this thesis calculations for every possible situation, a series of examples of various simple systems will be taken to show the trends in system temperatures which are produced by variations of the parameters. All the calculations for the examples will be performed by the use of an IBM 1620 Digital Computer. Examples of the basic computer programs to be used are given in the Appendix.

The greater portion of this analysis is devoted to showing the trends in system temperatures which are produced by varying the value of specific heat over the range from 0.30

to 0.70 Btu/lb<sub>m</sub>F (this range includes most of the popular brands of hydraulic fluid presently being used in industry), while holding the other system parameters at a constant value. This effect is shown for systems in a cold environment, a hot environment, and with and without a heat exchanger. The effect of varying the specific heat on the over-all heat transfer coefficient is also shown.

In the remainder of the thesis additional examples will be used to show how the system temperatures are affected by variations in pump efficiency and the over-all heat transfer coefficient.

## CHAPTER II

### PREVIOUS INVESTIGATIONS

Prior to the recent increase in demands upon hydraulic systems, there was little concern with the effect of variations in the parameters upon the system temperatures. With the advent of high speed aircraft and space vehicles, and with emphasis on more compact hydraulic systems, the temperatures of the hydraulic systems have been affected in such a way that a new area of investigation is demanding the attention of present-day designers.

Very little information pertaining to the thesis topic is available as a resource for today's designers. As a result they must rely a great deal on the art and "know-how" that has been built around the use of petroleum oil in hydraulic systems; hence, there has been a tendency to ignore fundamental effects due to variations in fluid properties and system parameters.

Perhaps the reason for the lack of available information is that prior to the introduction of the electronic computer, this type of analysis would have required a prohibitive number of man-hours. In the Appendix a comparison will be shown of the time required for hand calculations and

the time required for an electronic computer for this type of analysis.

Fitch (2) asserts that a fluid with a high specific heat will require more thermal energy to increase the temperature of a fluid a given amount than a fluid with a low specific heat, and that a high value is important for maintaining a proper temperature level in a hydraulic system.

In the handbook of the Oronite Chemical Company (3), it is stated that fluids with a high specific heat permits the use of a smaller heat exchanger. The following analysis shows that fluids with a high specific heat may permit the use of a heat exchanger with a smaller Btu removal capacity, or it may require a larger one depending on the conditions imposed upon the system.

A list of characteristics of hydraulic fluids which are recommended as important factors to be considered when selecting a fluid for a particular system is given by Blackburn, Reethof, and Shearer (4). This list was compiled by using operating experience. One item in the list which is of interest to this analysis is that a high specific heat is an important factor in the selection of the fluid.

One other statement of interest is given by Parker and McQuiston (5). They mention the importance of selecting pumps and motors having a high efficiency.

## CHAPTER III

### THEORETICAL CONSIDERATIONS

The following derivations and discussions include the basic equations to be used throughout the remainder of this thesis to calculate the local temperatures of the hydraulic fluid at different positions in the system.

#### A. Heat Flow from Tubing

If the tubing is considered as a thermodynamic system with steady-flow conditions, with no work done on or by the system, and with no throttling, an energy balance would be stated as

heat gain by cold fluid = heat lost by hot fluid.

For this part of the analysis the fluid flowing within the tubing will be considered as the hot fluid ( $t_f$ ), and the fluid surrounding the tube (or the environment) will be considered as the cold fluid ( $t_e$ ). If the entire mass of each fluid remains at a constant temperature ( $t_f$  and  $t_e$ ), the heat flow from a hot fluid in the tubing to a cold environment can be expressed as

$$Q = UA(t_f - t_e), \quad (3-1)$$

where

$U$  = over-all coefficient of heat transfer

$A$  = surface area of the tubing.

In many instances, however, the fluid flowing through the tube may undergo a change of temperature between the entrance and exit. In addition, the environmental temperature may not be the same at all locations along the tube. As the temperature of the fluids changes, the temperature difference between the two fluids changes. Thus, there are variations in temperature differences between the hot and cold fluids at different sections of the tube.

Since the temperature difference  $(\Delta t)$  is the potential or driving force in heat transfer, it is necessary to determine the mean temperature difference  $(\Delta t)_m$  between the two fluids. One method would be to compute the arithmetic mean temperature difference denoted by

$$(\Delta t)_{am} = \frac{(\Delta t)_i + (\Delta t)_o}{2}, \quad (3-2)$$

where

$$(\Delta t)_i = t_{f,i} - t_{e,i}$$

$$(\Delta t)_o = t_{f,o} - t_{e,o}$$

$t_{f,i}$  = temperature of the fluid entering the tube

$t_{f,o}$  = temperature of the fluid leaving the tube

$t_e$  = temperature of the environment or the fluid surrounding the tube.

This result would not be a correct representation of  $(\Delta t)_m$  as is indicated by the dotted lines in Figure 1. The correct mean ordinate between the temperature curves for Figure 1 may be found by integrating to determine the area of the shaded portion and dividing that area by the horizontal length. The result is called the logarithmic mean temperature difference,

$$\Delta t_m = \frac{\Delta t_i - \Delta t_o}{\ln \frac{\Delta t_i}{\Delta t_o}} \quad (3-3)$$

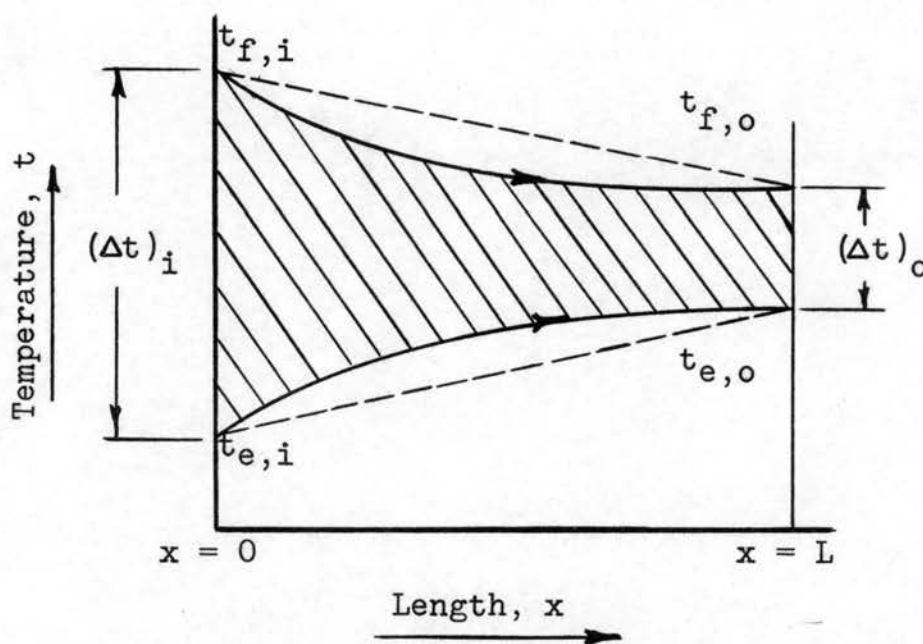


Figure 1. Temperature Distribution vs Tube Length for Parallel Flow

Since this equation is much more difficult to work with than the arithmetic mean temperature difference (Equation 3-2), it is worthwhile to show what error would be involved if the arithmetic mean temperature difference were used in place of

the logarithmic mean temperature difference.

By rearranging

$$\ln \frac{\Delta t_i}{\Delta t_o} = \ln \frac{1 + \frac{\Delta t_i - \Delta t_o}{\Delta t_i + \Delta t_o}}{1 - \frac{\Delta t_i - \Delta t_o}{\Delta t_i + \Delta t_o}},$$

which is of the general form

$$f(x) = \ln \left( \frac{1+x}{1-x} \right),$$

and when expanded to an infinite series by using Maclaurin's formula, gives

$$f(x) = 2 \left[ x + \frac{x^3}{3} + \frac{x^5}{5} + \frac{x^7}{7} + \dots \right];$$

hence,

$$\ln \frac{\Delta t_i}{\Delta t_o} = 2 \left( \frac{\Delta t_i - \Delta t_o}{\Delta t_i + \Delta t_o} \right) \left[ 1 + \frac{1}{3} \left( \frac{\Delta t_i - \Delta t_o}{\Delta t_i + \Delta t_o} \right)^2 + \frac{1}{5} \left( \frac{\Delta t_i - \Delta t_o}{\Delta t_i + \Delta t_o} \right)^4 + \dots \right].$$

Substituting this into Equation 3-3 yields

$$\Delta t_m = \frac{\frac{\Delta t_i + \Delta t_o}{2}}{1 + \frac{1}{3} \left( \frac{\Delta t_i - \Delta t_o}{\Delta t_i + \Delta t_o} \right)^2 + \frac{1}{5} \left( \frac{\Delta t_i - \Delta t_o}{\Delta t_i + \Delta t_o} \right)^4 + \dots} \quad (3-4)$$

Note that the numerator of Equation 3-4 is equal to the arithmetic mean temperature difference,

$$\Delta t_m = \frac{(\Delta t)_{am}}{1 + \frac{1}{3} \left( \frac{\Delta t_i - \Delta t_o}{\Delta t_i + \Delta t_o} \right)^2 + \frac{1}{5} \left( \frac{\Delta t_i - \Delta t_o}{\Delta t_i + \Delta t_o} \right)^4 + \dots} \quad (3-5)$$

Equation 3-5 shows the relationship between the logarithmic and arithmetic mean. If we let the quantity

$$(\Delta t_i - \Delta t_o) / \Delta t_i + \Delta t_o$$

approach zero, then the two means will approach each other.

If the ratio of the inlet temperature difference to the outlet temperature difference equals two,

$$\frac{\Delta t_i}{\Delta t_o} = 2$$

then the quantity

$$(\Delta t_i - \Delta t_o) / \Delta t_i + \Delta t_o = \frac{1}{3}$$

and the denominator of Equation 3-5 becomes 1.04; hence,

$$\Delta t_m = \frac{(\Delta t)_{am}}{1.04}.$$

The percent error which occurs by using  $(\Delta t)_{am}$  for  $(\Delta t)_m$  is then

$$\frac{(\Delta t)_{am} - (\Delta t)_m}{(\Delta t)_m} \times 100 = \left( \frac{1.04 - 1}{1} \right) 100 = 4 \text{ per cent.}$$

Therefore, if  $\Delta t_i / \Delta t_o \leq 2$ , then the arithmetic mean will differ from the logarithmic mean by less than 4 per cent.

Assuming that this accuracy is sufficient for the following analysis, the arithmetic mean temperature difference will be used in place of the logarithmic mean temperature difference to facilitate the development of an equation for the temperature drop of the fluid flowing in a tube due to heat loss to the surrounding environment.

Substituting  $(\Delta t)_{am}$  into the equation

$$Q = UA(\Delta t)_m$$

for the logarithmic mean temperature difference  $(\Delta t)_m$  gives

$$Q = UA \left( \frac{\Delta t_i + \Delta t_o}{2} \right)$$

or

$$Q = UA \left( \frac{t_i - t_e + t_o - t_e}{2} \right),$$

thus,

$$Q = \frac{UA}{2} (-2t_e + t_i + t_o). \quad (3-6)$$

Also, the heat given up by the fluid is

$$Q = \dot{m} C_p (t_i - t_o). \quad (3-7)$$

Equating Equations 3-6 and 3-7 gives

$$\dot{m} C_p (t_i - t_o) = \frac{UA}{2} (-2t_e + t_i + t_o).$$

Rearranging this equation gives a general equation for a non-adiabatic, steady-flow length of tubing, in which there is no work done on or by the system, and no throttling:

$$t_o = \frac{2t_e AU + (2\dot{m}C_p - AU)t_i}{2\dot{m}C_p + AU}. \quad (3-8)$$

It is usually more convenient to use the volume flow rate (gal/min) than the mass flow rate; by substituting  $\dot{m} = (G)(S)8.33$  into Equation 3-8, a more useful form of this equation will be obtained:

$$t_o = \frac{2t_e(A)U + [16.66(G)(S)(C_p) - (A)U] t_i}{16.66(G)(S)(C_p) + (A)U}. \quad (3-9)$$

In this equation,

$t_o$  = temperature of the fluid leaving the tube,  
degrees F

$t_i$  = temperature of the fluid entering the tube,  
degrees F

A = surface area of the tube,  $\text{ft}^2$

U = over-all coefficient of heat transfer,  
Btu/ $\text{ft}^2$  min F

G = volume flow rate, gpm

S = specific gravity

$C_p$  = specific heat, Btu/lb<sub>m</sub> F.

When using this equation it should be remembered that it is an approximation since the arithmetic mean temperature difference was used in its development in place of the logarithmic mean temperature difference. Equation 3-5 gives a relationship between these two means and can be used to evaluate the magnitude of the error.

It was stated by Parker and McQuiston (5) that in order to meet the restriction imposed by the Second Law of Thermodynamics, the inequality  $\left(\frac{2\dot{m}C_p}{U} > A\right)$  must be satisfied. This restriction can be clarified in the following manner. By equating Equations 3-6 and 3-7 to give

$$\frac{2\dot{m}C_p}{UA} (t_i - t_o) = -(2t_e - t_i - t_o),$$

and by assuming  $(t_i - t_o)$  is positive and  $\left(\frac{2\dot{m}C_p}{UA} < 1\right)$ , the following inequality is formed,

$$(t_i - t_o) > -(2t_e - t_i - t_o)$$

or

$$-2t_o > -2t_e$$

$$t_o < t_e.$$

Since  $(t_i - t_o)$  was assumed to be a positive value, or in other words, the length of piping is being cooled, it would be impossible for  $(t_o < t_e)$  because the Second Law of Thermodynamics states that it is impossible to cool a fluid below the environment temperature if the only means of removing heat from the system is by way of heat transfer to the environment.

Therefore, prior to calculating the system temperatures, the ratio  $\left(\frac{2\dot{m}C_p}{U}\right)$  should be checked for each individual component of the system to insure that its value is greater than the value of surface area selected for the respective component. If this restriction is not met, the component must be divided into smaller areas which will require the calculation of more temperatures throughout the system.

#### B. Temperature Rise in an Adiabatic Pump

An equation for the temperature differential resulting from the pumping process can easily be derived by taking an energy balance on the pump with the aid of the steady flow energy equation in which the changes in potential and kinetic energy are neglected:

$$Q - wk = \text{increase in enthalpy across pump.}$$

Gibson (6) gave the following equation for the temperature

rise across an adiabatic pump:

$$t_o - t_i = \frac{(P_o - P_i)}{62.4(C_p)(S)} \left( \frac{1 - \eta}{\eta} \right)$$

This equation in a more convenient form for this analysis is

$$t_o - t_i = \frac{0.00297(P_o - P_i)}{(S)C_p} \left( \frac{1 - \eta}{\eta} \right), \quad (3-10)$$

where the following units are used:

P = psi

C<sub>p</sub> = Btu/lb<sub>m</sub> F

η = pump efficiency, per cent

t = degrees F.

It is noteworthy that the bulk temperature rise of a given fluid for this type of pumping process is directly related to the ratio of the inefficiency to the efficiency of the pump and to the pressure difference, and it is independent of the flow rate.

### C. Temperature Rise in a Throttling Device

A relation for the temperature rise due to throttling was given by Gibson (6). In his derivation the steady flow energy equation was used with the following assumptions: No heat or work cross the boundary, and the kinetic and potential energy are considered as negligible. Using these assumptions the equation for temperature rise due to throttling is

$$t_o - t_i = \frac{P_i - P_o}{62.4(S)(C_p)} ,$$

and by using the units which are consistent with this analysis, this equation becomes

$$t_o - t_i = \frac{0.00297(P_i - P_o)}{(S)(C_p)} \quad (3-11)$$

By observing this equation and considering the assumptions made in its derivation, it can be seen that the temperature rise is independent of the size of the throttling device and of the rate of flow passing through it. For a given fluid the temperature rise given by this equation is solely a function of the drop in pressure.

#### D. Heat Exchanger

The heat removed from the fluid by a heat exchanger can be expressed as

$$Q_H = \dot{m}C_p(t_i - t_o)$$

or

$$Q_H = 8.33(G)(S)(C_p)(t_i - t_o).$$

The temperature decrease of the fluid as a result of heat removal by a heat exchanger is given by

$$t_i - t_o = \frac{Q_H}{8.33(G)(S)C_p} \quad (3-12)$$

where

$t_i$  = temperature of hydraulic fluid entering heat exchanger, F

$t_o$  = temperature of hydraulic fluid leaving heat exchanger, F

$G$  = volume flow rate, gpm

$S$  = specific gravity

$C_p$  = specific heat, Btu/lb<sub>m</sub> F.

For convenience a summary of the basic equations discussed above and which will be used throughout the thesis are as follows:

Temperature Change of Fluid Flowing Through Tubing Due to Convection with No Viscous Dissipation

$$t_o = \frac{2t_e(A)U + [16.66(G)(S)C_p - (A)U] t_i}{16.66(G)(S)C_p + (A)U} \quad (3-9)$$

Temperature Rise in an Adiabatic Pump

$$t_o - t_i = \frac{0.00297(P_o - P_i)}{(S)C_p} \left( \frac{1-\eta}{\eta} \right) \quad (3-10)$$

Temperature Rise in a Throttling Device

$$t_o - t_i = \frac{0.00297(P_i - P_o)}{(S)C_p} \quad (3-11)$$

Temperature Decrease in a Heat Exchanger

$$t_i - t_o = \frac{QH}{8.33(G)(S)C_p} \quad (3-12)$$

## CHAPTER IV

### EFFECT OF VARIATION OF PARAMETERS ON FLUID TEMPERATURES

#### A. Variation of Specific Heat

In the design of hydraulic systems the designer is faced with many problems. One might be the problem of selecting the hydraulic fluid which would best suit his particular design requirements. A question which usually arises when considering this selection is: Would it be more beneficial from a heat transfer standpoint to use a fluid of high specific heat or one of low specific heat? The following analysis will attempt to answer this question.

The effect of a variation in specific heat upon the temperature increase or decrease across individual components of the system will be considered first and then its effect upon the local temperatures of a system as a whole.

1. Effects on throttling devices. Equation 3-11 shows the temperature rise in a throttling device to be inversely proportional to the specific heat. In other words, an increase in the specific heat will cause the temperature rise due to throttling to be decreased. This decrease in the temperature rise is greater at higher pressure drops than at

lower pressure drops, as can be seen in Figure 2. As an example, if the specific heat were raised from 0.40 to 0.60 Btu/lb<sub>m</sub>F with a pressure drop of 1,000 psi, the temperature rise would be decreased by 3.2 F; and with a pressure drop of 3,000 psi, the temperature rise would be decreased by 9.6 F. It should be kept in mind that an increase in the specific heat such as this might help to alleviate hot spots in a hydraulic system caused by throttling and, also, for pumping, as will be shown later.

2. Effects on pumps. By comparing Equations 3-11 and 3-10 for throttling and pumping, respectively, it can be seen that they differ only by the factor  $(1-\eta)/\eta$ , where  $\eta$  is the efficiency of the pump. If the pump efficiency is greater than 50 per cent, which is the usual case, the above factor will be less than unity. Therefore, for systems with pump efficiencies greater than 50 per cent the temperature rise due to pump inefficiency will be less than the temperature rise due to throttling. Hence, it follows that a change in specific heat will have a greater effect on the temperature rise due to throttling than on the temperature rise due to pumping. This can be seen by comparing Figures 2 and 3, where a pump efficiency of 85 per cent was used.

3. Effects on convection from tubing. To show the effect of different specific heats on the temperature change of a fluid flowing through a length of pipe having an area of 20 ft<sup>2</sup>, Equation 3-9 will be used with the following assumptions:

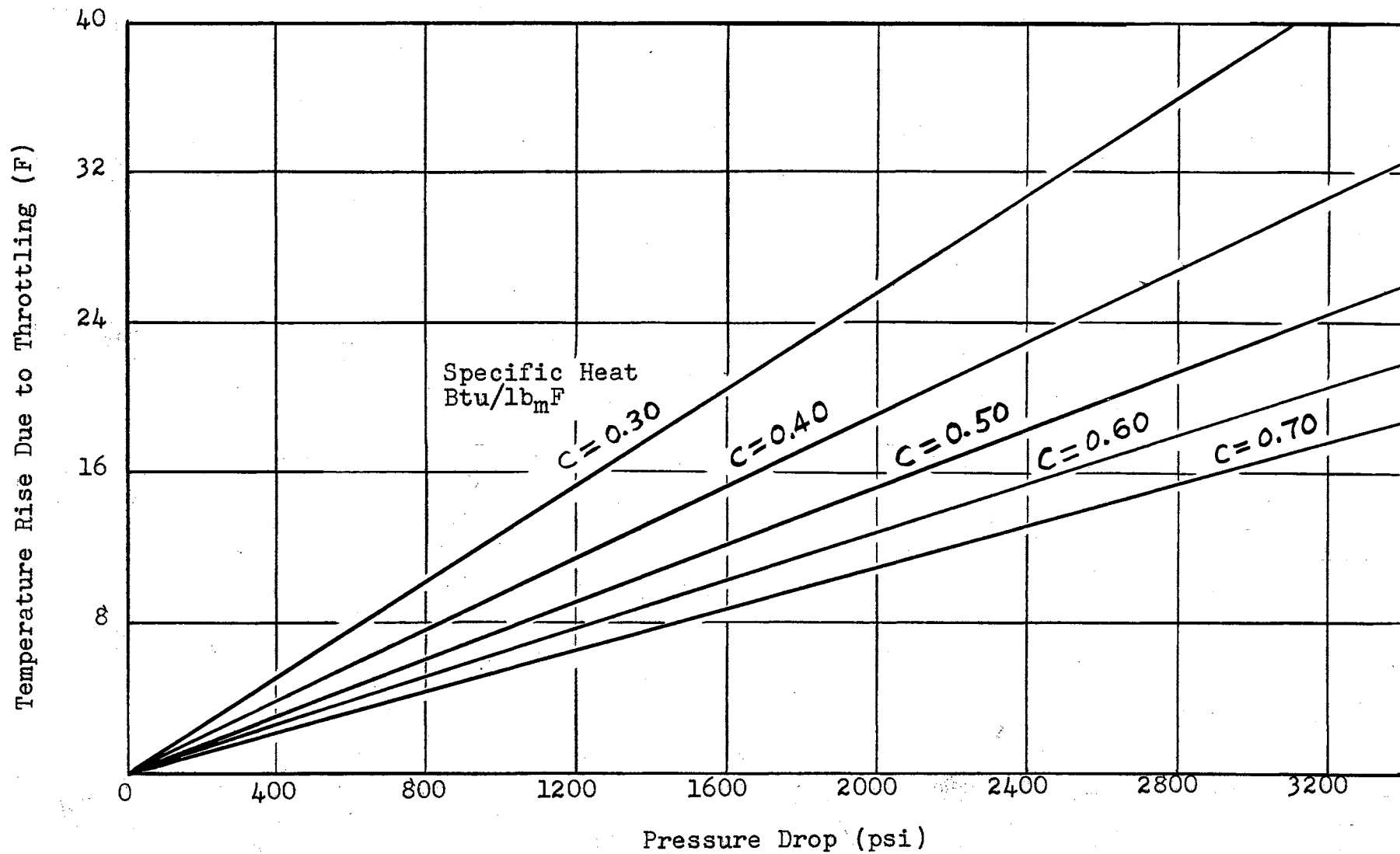


Figure 2. Temperature Rise Due to Throttling vs Pressure Drop

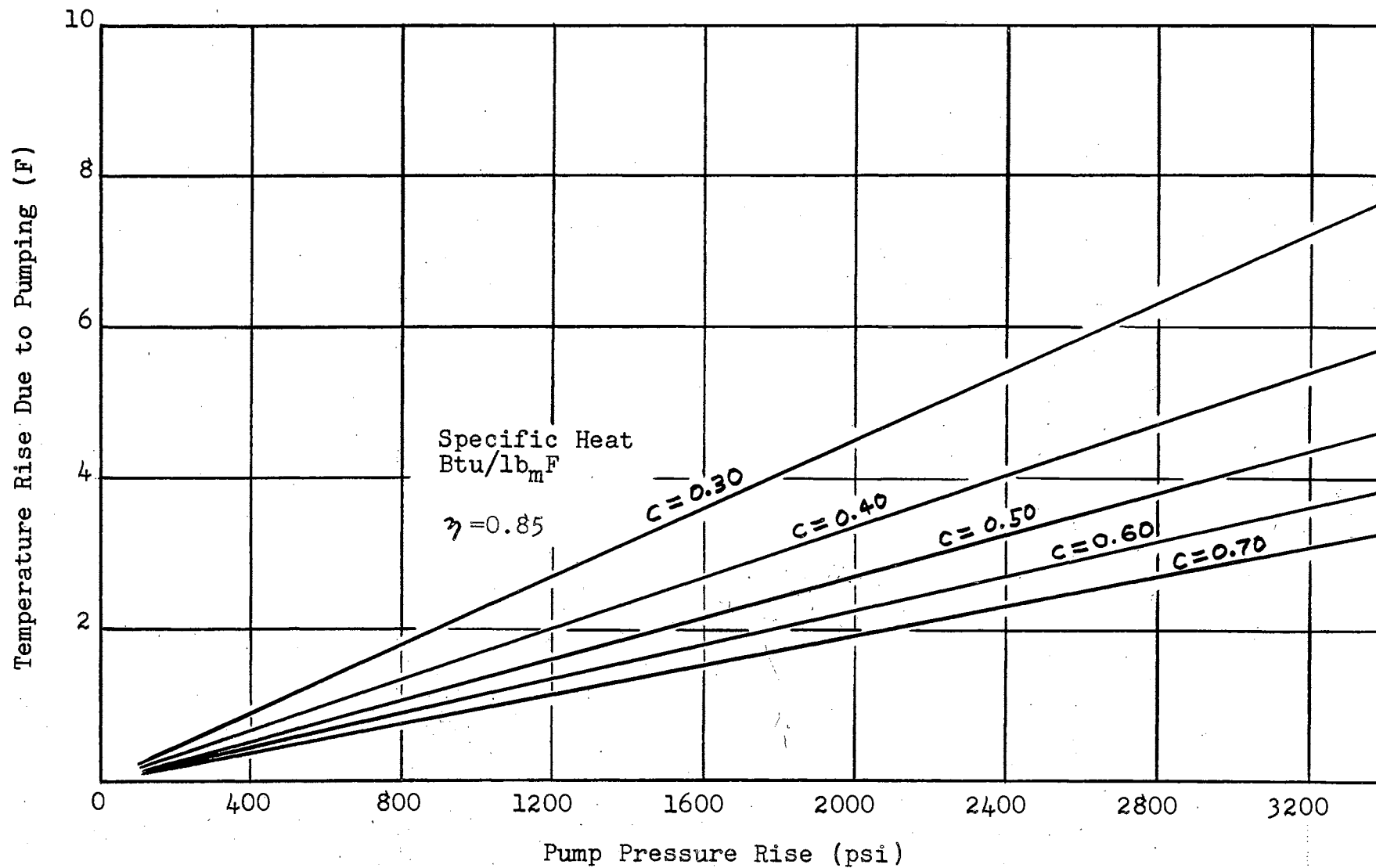


Figure 3. Temperature Rise Due to Pumping vs Pump Pressure Rise

Flow Rate (G) 1 gpm

Over-all Heat Transfer Coefficient (U) 0.025 Btu/min ft<sup>2</sup>F

Specific Gravity (S) 0.77

The results from these calculations are shown in Figure 4 for the case of a cold environment and in Figure 5 for the case of a hot environment. In either case an increase in the specific heat tends to decrease the temperature change of the fluid.

4. Effect on a given system. It was mentioned by Fitch (2) that a fluid with a high specific heat will require more thermal energy to increase the temperature of a fluid a given amount than a fluid with a low specific heat, and that a high value is important for maintaining a proper temperature level in a hydraulic system. The first part of this statement cannot be refuted under any circumstance; however, there is some doubt as to the validity of the second part, depending on what is considered as the system, and what the conditions are. The following analysis will help to clarify this point.

It was previously mentioned that an increase in the specific heat of a fluid would result in a decrease in the temperature drop of the fluid for the case of convection from a pipe with a cold environment, and would result in a decrease in temperature rise for the case of throttling or pumping. A system, such as shown in Figure 6, is formed by the combination of these three effects. Any variation in the specific heat of the fluid in a system such as this will have

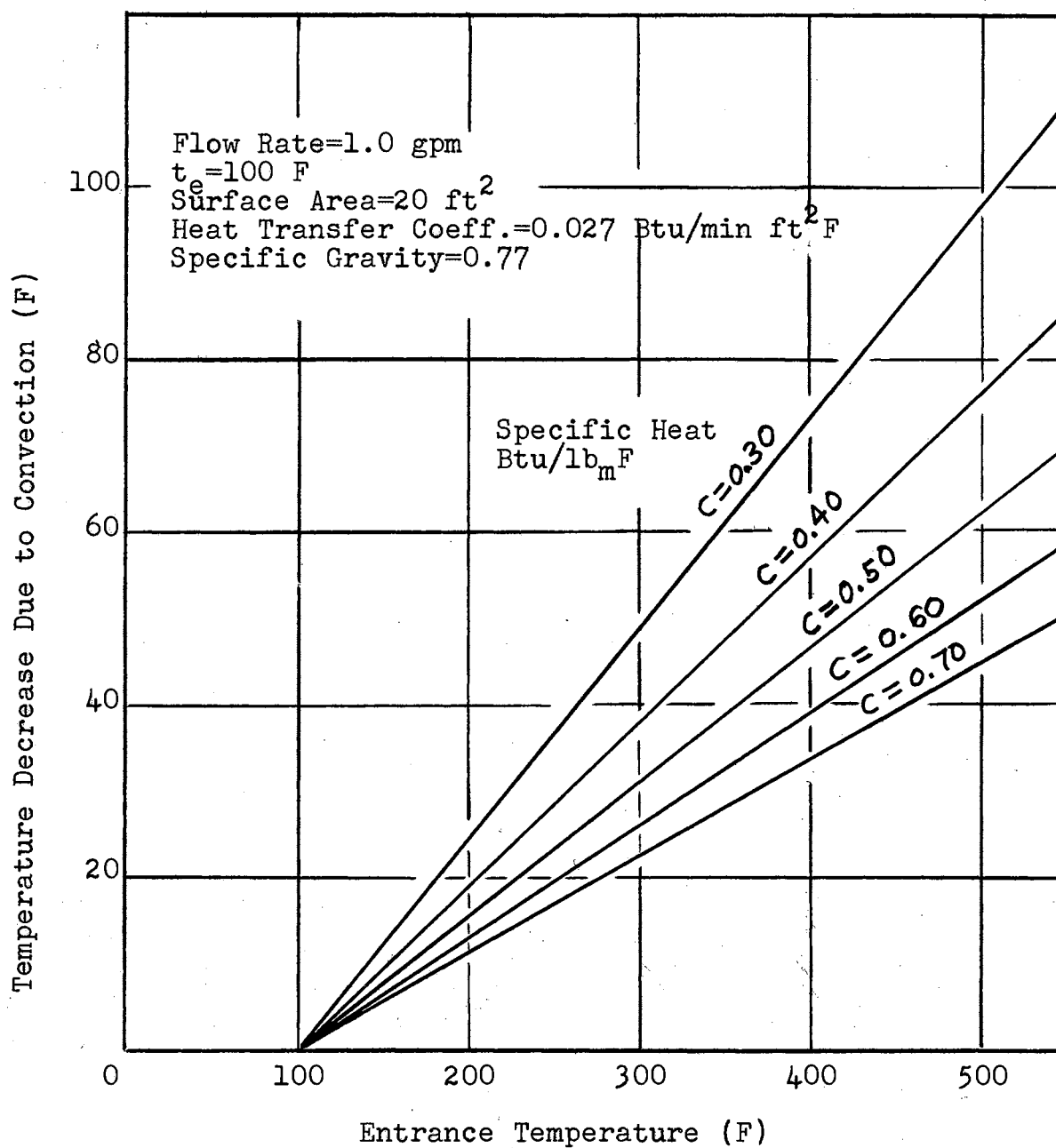


Figure 4. Temperature Decrease Due to Convection vs Entrance Temperature

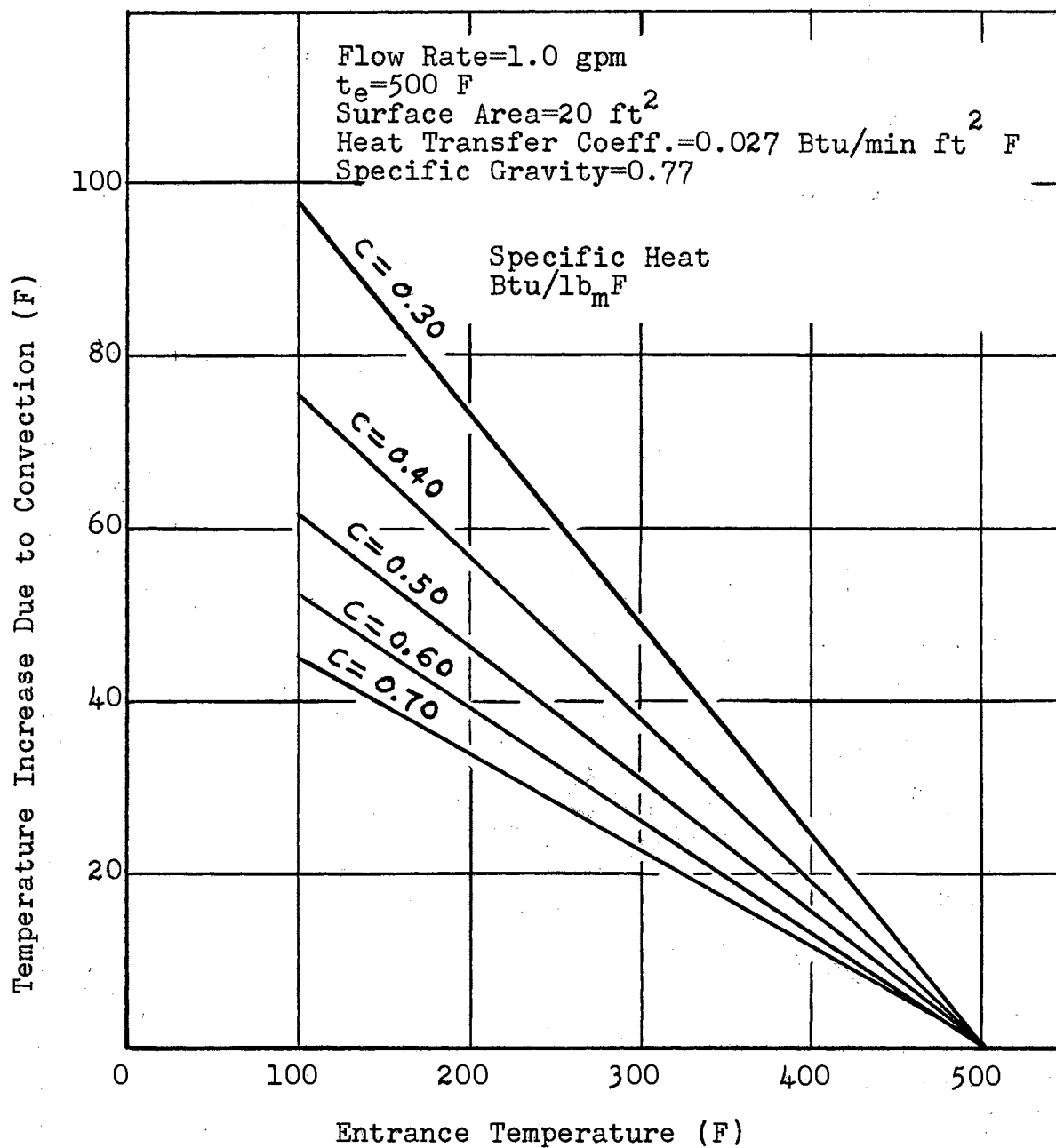


Figure 5. Temperature Increase Due to Convection  
vs Entrance Temperature

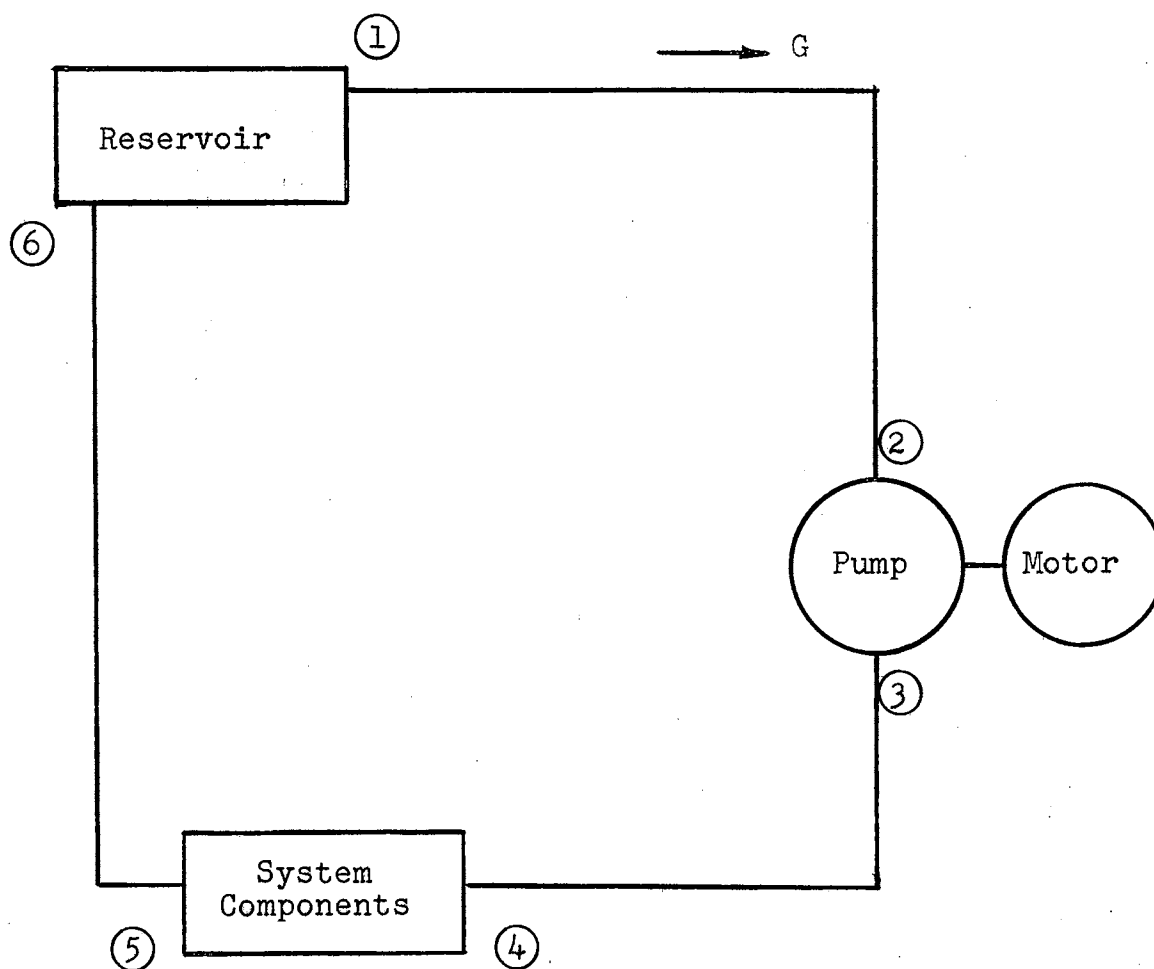


Figure 6. Hydraulic System without Heat Exchanger

little effect on the reservoir outlet temperature since there is a balancing effect between the convection portion of the system, and the pumping and throttling portion.

### Example 1

To illustrate the balancing effect mentioned above, the local temperatures for the system shown in Figure 6 will be calculated, assuming that the equilibrium temperatures of the system are greater than the environmental temperature. For this system and all other systems which follow, components such as valves, cylinders, and filters are grouped together for simplicity and referred to as system components. Since the cases to be illustrated are intended to show the trends in system temperatures produced by variations of parameters, these components are treated as throttling devices only.

Further assumptions are:

Flow (G) 5 gpm

Environmental Temperature (TE) 100 F

Pump Efficiency (ETA) 80 per cent

Over-all Heat Transfer Coefficient (U) 0.025 Btu/min ft<sup>2</sup>F

Pressure Rise Across Pump (DP) 3000 psi

Specific Gravity (S) 0.77

Surface Areas: A<sub>12</sub> = 20 ft<sup>2</sup>  
                   A<sub>34</sub> = 20  
                   A<sub>56</sub> = 20  
                   A<sub>R</sub> = 20

The quantity, A<sub>12</sub>, is the surface area of the pipe from point 1 to point 2 in the system, etc. It has been necessary to change the symbolism to be compatible with the computer.

The local temperatures for this hydraulic system and other systems which follow are calculated by the use of the equations derived in Chapter III. The procedure for carrying out these calculations will be discussed in the Appendix.

By considering the assumptions made above and by varying the specific heat (C) from 0.30 to 0.70 Btu/lb<sub>m</sub>F, the reservoir outlet temperature (T1) is found to vary from 330 to 332 F. The solution for the local temperatures for this system with the specific heat (C) varying by increments of 0.10 Btu/lb<sub>m</sub>F are listed in Table I.

TABLE I

HYDRAULIC SYSTEM WITHOUT HEAT  
EXCHANGER EXAMPLE 1

SPECIFIC HEAT (C)	LOCAL TEMPERATURES F					
Btu/lb <sub>m</sub> F	T1	T2	T3	T4	T5	T6
0.30	330	318	328	316	355	342
0.40	330	322	329	320	349	340
0.50	331	324	330	323	346	338
0.60	331	325	330	324	343	337
0.70	332	326	331	325	342	337

Probably the most significant point to note in these results is that an increase in specific heat has a leveling-out effect on the temperatures throughout the system. For example, with a specific heat of 0.30 Btu/lb<sub>m</sub>F, the maximum variation is 39 degrees as compared to 17 degrees for that by specific heat of 0.70 Btu/lb<sub>m</sub>F. One other point is that by

increasing the specific heat from 0.30 to 0.70 Btu/lb<sub>m</sub>F, the greatest increase in temperature (9 degrees) occurred at location 4, while the greatest decrease in temperature (13 degrees) occurred at location 6.

Other calculations were made for this system with different flow rates (G) and different values of pressure rise (DP) across the pump. These values will not be listed since the general trend was the same as found in this example. The only difference was that the greater the pressure rise (DP), the more effect a change in the specific heat had on the temperatures, and for a larger flow rate the specific heat has less effect on the system temperatures.

If the piping of the system were receiving heat from a hot environment, a change in specific heat would cause the temperatures of all three portions of the system (convection, pumping, and throttling) to change in the same sense. Therefore, it would seem logical that an increase in specific heat would cause a decrease in all the system temperatures. However, this is not the case for this system. For a system which is exchanging heat solely with a hot environment, the environment being at a higher temperature than the bulk temperature of the fluid, a heat exchanger must be provided to remove this heat plus the heat input due to the work input at the pump in order to maintain the system at the desired temperature.

An expression for the decrease in fluid temperature as it passes through a heat exchanger is given by Equation 3-12,

$$t_i - t_o = \frac{QH}{8.33(G)(S)c_p}$$

From this equation it can be seen that for an increase in specific heat, more heat must be removed by the heat exchanger to maintain a constant temperature drop ( $t_i - t_o$ ). Also, for a constant amount of heat removed from the fluid by the heat exchanger, the temperature drop ( $t_i - t_o$ ) will be decreased with an increase in specific heat.

When the effect of a change in specific heat on the system is considered, the change within the heat exchanger tends to balance with the change in the convection, throttling, and pumping portions, much the same as the convection balanced with throttling and pumping for the case of a system with a cold environment thereby resulting in a small change in the reservoir outlet temperature.

### Example 2

A system, as shown in Figure 7, exchanging heat with a hot environment, and with an increase in specific heat from 0.30 to 0.70 Btu/lb<sub>m</sub>F, will increase the reservoir outlet temperature from 188 F to 199 F. This occurs with the same assumptions that were used in Example 1, with the exception of the environmental temperature,  $T_E = 300$  F, and in addition, the heat removed by the heat exchanger was held at a constant value of 650 Btu/min.

In Table II it can be seen that for this system, varying

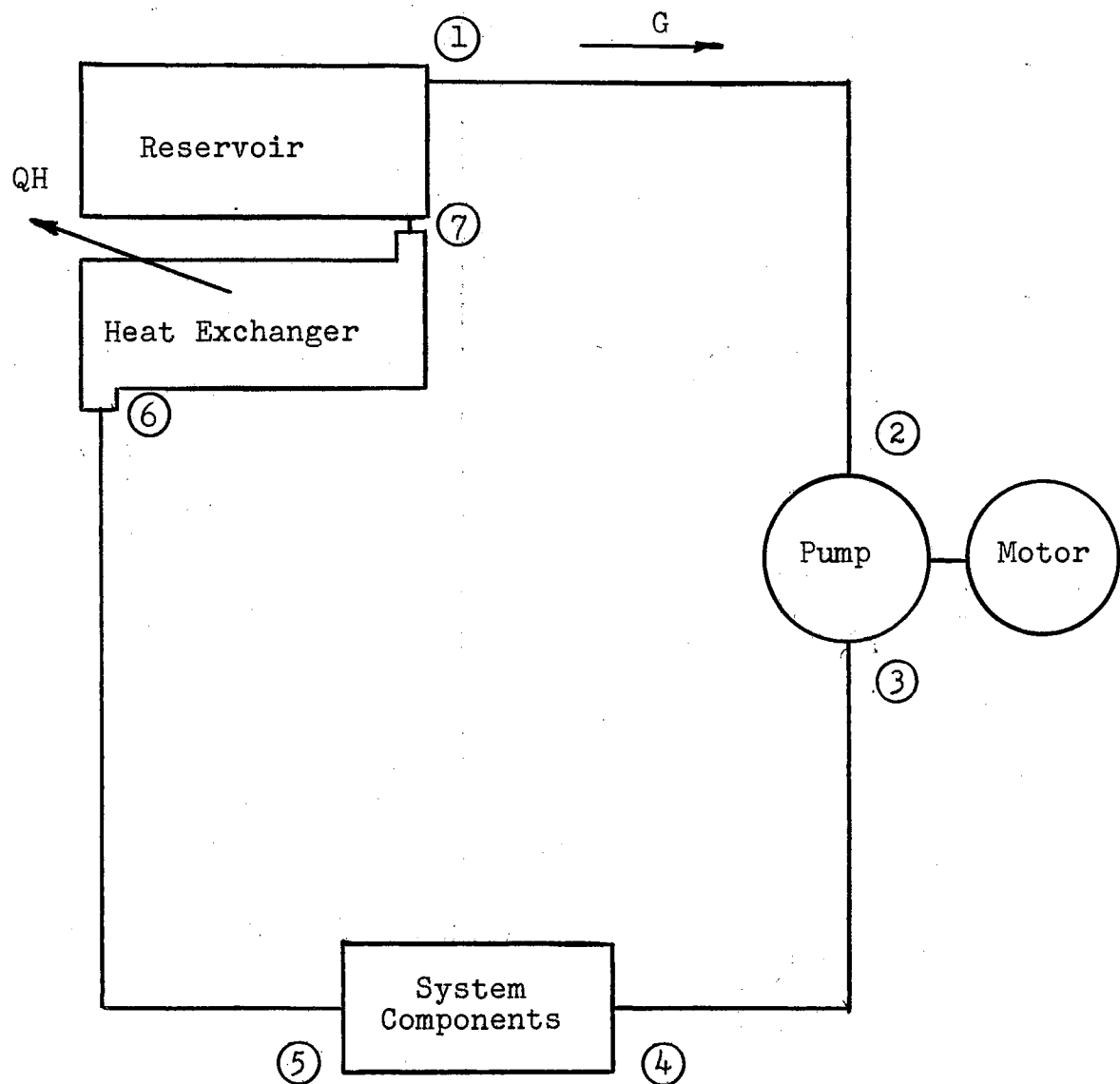


Figure 7. Hydraulic System with Heat Exchanger

the specific heat from 0.30 to 0.70 Btu/lb<sub>m</sub>F has an even greater leveling-out effect on the temperatures throughout the system than it did for the system in Example 1. Also, the highest temperature for this system was reduced by a greater amount for the same variation of specific heat than was the highest temperature of Example 1.

TABLE II

## HYDRAULIC SYSTEM WITH HEAT EXCHANGER EXAMPLE 2

SPECIFIC HEAT (C) Btu/lb <sub>m</sub> F	QH = 650 Btu/min LOCAL TEMPERATURES F						
	T1	T2	T3	T4	T5	T6	T7
0.30	188	194	203	208	247	250	182
0.40	193	197	204	208	237	239	189
0.50	196	199	205	208	231	233	193
0.60	198	201	205	208	227	229	195
0.70	199	202	206	208	224	226	197

From these observations it can be seen that the magnitude of the effect on the system temperatures by a variation of specific heat suggests the possibility of selecting a fluid with a high specific heat so the highest temperature of the system will be compatible with the recommended temperature range of the fluid.

## Example 3

To determine the size heat exchanger necessary to main-

tain the system temperatures at the desired level, it is advantageous to know what effect the specific heat of the fluid has upon the quantity of heat which must be removed by the heat exchanger. To show this, the same system will be used that was used in Example 2 (shown in Figure 7); all the conditions and assumptions will remain the same with the exception of the environmental temperature,  $T_E = 100$  F, and the flow rate,  $G = 10$  gpm. In this example, the reservoir outlet temperature ( $T_1$ ) will be held at a constant value of 300 F, and the heat removed by the heat exchanger will become the dependent variable as the specific heat is varied in the same manner as before. These results are shown in Table III.

From Table III it can be seen that the larger the specific heat of the fluid, the greater the requirement of heat removal by the heat exchanger in order to maintain the reservoir outlet temperature at 300 F.

It should be noted that by changing the specific heat over the range of values that might be found at this time (0.30 to 0.70 Btu/lb<sub>m</sub>F), the heat removal by the heat exchanger will be increased by only 2 per cent.

Since the trends shown in Examples 1, 2, and 3 are for such simple systems, one might pose the question: Would a more complex system exhibit the same trend?

Most of the more complex systems can be effectively reduced to be equivalent to either of the basic systems shown in Figure 6 or 7.

Another basic system, which is somewhat more complex

TABLE III

## HYDRAULIC SYSTEM WITH HEAT EXCHANGER EXAMPLE 3

SPECIFIC HEAT Btu/lb <sub>m</sub> F	HEAT REMOVAL (QH) Btu/min	LOCAL TEMPERATURES F						
		T1	T2	T3	T4	T5	T6	T7
0.30	512	300	295	305	299	338	332	305
0.40	516	300	296	303	299	328	324	304
0.50	518	300	297	303	300	323	319	303
0.60	520	300	297	302	300	319	316	303
0.70	522	300	298	302	300	316	314	302

than the two systems previously mentioned, is a system which furnished power for an accessory power system by using a hydraulic motor to drive an alternator.

#### Example 4

In Figure 8 the type of system mentioned above is shown with the hydraulic motor connected in parallel with the other hydraulic system components. In this example the heat removal by the heat exchanger and the local temperature will be calculated with the specific heat varying in the same manner as in previous examples, while the reservoir outlet temperature is held at a constant value of 300 F. These values are shown in Table IV. Further assumptions made are:

Flow (GA) 15 gpm

Flow (GB) 7.5 gpm

Environmental Temperature (TE) 100 F

Over-all Heat Transfer Coefficient (U) 0.025 Btu/min ft<sup>2</sup> F

Pressure Rise Across Pump (DP) 3000 psi

Specific Gravity (S) 0.77

Pump Efficiency (ETAP) 80 per cent

Hydraulic Motor Efficiency (ETAM) 85 per cent

Surface Areas: A<sub>12</sub> = 10 ft<sup>2</sup>

A<sub>34</sub> = 5

A<sub>45</sub> = 10

A<sub>67</sub> = 10

A<sub>48</sub> = 10

A<sub>910</sub> = 10

A<sub>712</sub> = 10

A<sub>1314</sub> = 5

A<sub>R</sub> = 10

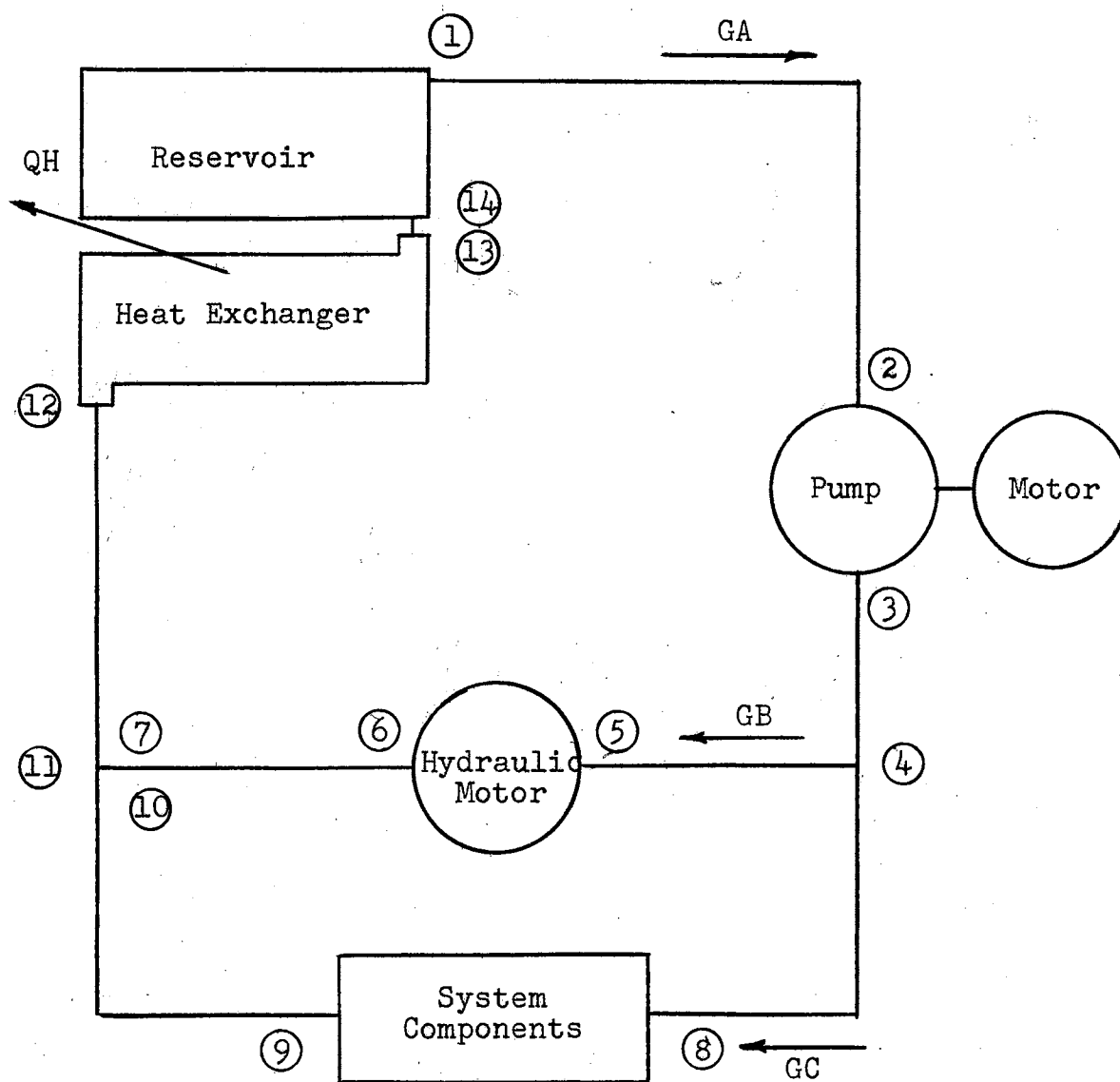


Figure 8. Hydraulic System with Heat Exchanger and Motor

TABLE IV

## HYDRAULIC SYSTEM WITH HEAT EXCHANGER AND MOTOR EXAMPLE 4

SPECIFIC HEAT Btu/lb <sub>m</sub> F	HEAT REMOVAL (QH) Btu/min	LOCAL TEMPERATURES F													
		T1	T2	T3	T4	T5	T6	T7	T8	T9	T10	T11	T12	T13	T14
0.30	500	300	298	308	307	303	309	306	303	342	338	322	320	303	302
0.40	505	300	299	306	305	303	307	304	303	332	329	316	315	302	301
0.50	508	300	299	305	304	302	306	303	302	325	323	313	312	302	301
0.60	510	300	299	304	304	302	305	303	302	321	319	311	310	301	301
0.70	512	300	299	303	303	302	304	302	302	318	316	309	309	301	301

An energy balance will now be made on the system to check the validity of this solution. The values corresponding to a specific heat of  $0.70 \text{ Btu/lb}_m\text{F}$  will be used for this check. The work input at the pump is

$$\frac{(1.482)(GA)(DP)}{(60)(ETAP)} = \frac{(1.482)(15)(3000)}{(60)(0.80)} = 1398 \text{ Btu/min.}$$

The work output at the motor is

$$\begin{aligned} \frac{(1.482)(GB)(DP)(ETAM)}{(60)} &= \frac{(1.482)(7.5)(3000)(0.85)}{(60)} \\ &= 474 \text{ Btu/min.} \end{aligned}$$

The net work input is equal to the net energy loss by heat transfer for the same increment of time

$$\begin{aligned} \text{net heat loss} &= (\text{work input at pump}) - (\text{work output at motor}) \\ &= 1398 - 474 = 924 \text{ Btu/min.} \end{aligned}$$

The net heat loss from the system is comprised of the heat removed by the heat exchanger and that which leaves by convection to the surroundings. For a specific heat of  $0.70 \text{ Btu/lb}_m\text{F}$  the heat exchanger removes  $512 \text{ Btu/min}$ , thus the heat loss by convection is

$$924 - 512 = 412 \text{ Btu/min.}$$

The approximate amount of heat loss from the system can be expressed in terms of the average surface temperature of the system, as

$$Q = UA(T_{\text{avg}} - T_E).$$

For a surface area of  $80 \text{ ft}^2$  and a  $(U)$  of  $0.025 \text{ Btu/min ft}^2\text{F}$  the average surface temperature is determined as follows:

$$T_{\text{avg}} = T_E + \frac{Q}{UA} = 100 + \frac{412}{(0.025)(80)} = 100 + 206$$

$T_{\text{avg}} = 306 \text{ F}$ , and compares well with results in Table IV.

#### Example 5

For many systems the designer must assure that the highest temperature within the system must not be greater than the upper temperature limit of the fluid to be used. In this example the maximum temperature of the system will be held at a constant value of  $300 \text{ F}$ , while the effect of variations in specific heat on the system temperatures and on the heat removed by the heat exchanger will be shown. The system used in Example 4 will be used in this example with the same assumptions. The computed local temperatures for this case are given in Table V.

The same system was used for Examples 4 and 5 with the same assumptions. The only difference between the two cases was that in Example 4 the reservoir outlet temperature was held at  $300 \text{ F}$ , and in Example 5 the maximum temperature of the system was held at  $300 \text{ F}$ . The results for these two examples are completely opposite, as can be seen by comparing Tables IV and V. As the specific heat was increased in Example 4 all the temperatures decreased except  $T_1$  and  $T_2$ , and the heat removed by the heat exchanger ( $Q_H$ ) increased; while, for Example 5 as the specific heat was increased all

TABLE V

## HYDRAULIC SYSTEM WITH HEAT EXCHANGER AND MOTOR EXAMPLE 5

SPECIFIC HEAT Btu/lb <sub>m</sub> F	HEAT REMOVAL (QH) Btu/min	LOCAL TEMPERATURES F													
		T1	T2	T3	T4	T5	T6	T7	T8	T9	T10	T11	T12	T13	T14
0.30	585	257	255	265	264	261	267	264	261	300	297	280	279	259	258
0.40	568	268	267	274	273	271	275	273	271	300	297	285	284	269	269
0.50	559	274	273	279	279	277	280	278	277	300	298	288	287	276	275
0.60	552	279	278	283	282	281	284	282	281	300	298	290	289	280	279
0.70	548	282	281	285	285	283	286	285	283	300	299	292	291	283	282

the temperatures increased with the exception of  $T_9$ , and the heat removed by the heat exchanger ( $Q_H$ ) decreased.

This shows that general statements cannot be made about the effects which a larger or smaller specific heat will have on the temperatures of the system or on the size of the heat exchanger. Therefore, each particular design with its individual operating requirements will require that a study be made to determine what effect a fluid with a different specific heat will have on the system. Perhaps the only general statement that can be made concerning the effects caused by variations in specific heat is that a greater value of specific heat will tend to have a leveling-out effect on the temperature throughout the system.

A variation in specific heat may have an indirect effect on the temperatures of a system by its effect upon the over-all heat transfer coefficient ( $U$ ).

Heat Transfer is concerned primarily with three basic quantities: thermal resistance, temperature difference, and rate of heat transfer, which are related by

$$Q = \frac{-\Delta t}{R_t},$$

where

$$R_t = \frac{1}{AU}.$$

Therefore, any effect that a variation in the specific heat might have upon the over-all heat transfer coefficient ( $U$ )

would likewise have some effect on the rate of heat transfer or the temperature difference. The thermal resistance ( $R_t$ ), being made up of the inside fluid film resistance, the wall resistance, and the outside fluid film resistance, can be expressed as

$$A \sum R_t = \frac{1}{U} = A_o \left( \frac{1}{A_i h_i} + \frac{r_o - r_i}{K A_m} + \frac{1}{h_o A_o} \right) \quad (4-1)$$

The relationship between the inside heat transfer coefficient ( $h_i$ ) and the specific heat for a fluid in laminar flow through a pipe is given by the following correlation (11):

$$\frac{h_i D}{K} = 1.86 (R_e)^{\frac{1}{3}} \left( \frac{D}{L} \right)^{\frac{1}{3}} \left( \frac{\mu C_p}{K} \right)^{\frac{1}{3}} \left( \frac{\mu}{\mu_s} \right)^{0.14} \quad (4-2)$$

From this it can be seen that by using a fluid of a different specific heat, the value of the inside film coefficient will change, which will in effect change the quantity of heat being transferred into or out of the system and change the system temperatures. The magnitude of this change will now be shown, using Equation 4-2 and

$$R_e = 1000$$

$$K = 0.08 \text{ Btu/hr ft sec}$$

$$\mu = 0.009 \text{ lb}_m/\text{ft sec}$$

$$D = 0.75 \text{ inch}$$

$$\mu/\mu_s = 1$$

$$L = 10 \text{ ft}$$

$$\text{For } C_p = 0.30 \text{ Btu/lb}_m \text{F}$$

$$h_i = 18 \text{ Btu/hr ft}^2 \text{F},$$

and for  $C_p = 0.70 \text{ Btu/lb}_m \text{ F}$

$$h_i = 24 \text{ Btu/hr ft}^2 \text{ F.}$$

In practical situations it would not be possible to vary ( $C_p$ ) alone, therefore, the variation in the Prandtl number and the other fluid properties would have to be considered. Since this analysis is concerned with trends, the author feels justified in varying ( $C_p$ ) alone for this idealized case.

It should be pointed out that with air surrounding the system piping, the outside film coefficient ( $h_o$ ) will be much smaller than the inside coefficient ( $h_i$ ) and will therefore be the controlling factor of the over-all heat transfer coefficient ( $U$ ). This will be illustrated by substituting the following assumed values into Equation 4-2:

$$A_o \simeq A_i \simeq A_m$$

$$r_o - r_i = 0.05 \text{ inch}$$

$$h_o = 1 \text{ Btu/hr ft}^2 \text{ F}$$

$$K = 100 \text{ Btu/hr ft F}$$

Using the value of  $h_i = 18 \text{ Btu/hr ft}^2 \text{ F}$  (the value calculated previously when  $C_p = 0.30 \text{ Btu/lb}_m \text{ F}$ ), the over-all heat transfer coefficient is

$$U = \frac{1}{\frac{1}{18} + \frac{0.05}{(100)(12)} + 1} = \frac{1}{0.055 + 0.00004 + 1}$$

$$U = 0.95 \text{ Btu/hr ft}^2 \text{ F.}$$

Substituting the value of  $h_i = 24 \text{ Btu/hr ft}^2 \text{ F}$  (the value

calculated previously when  $C_p = 0.70 \text{ Btu/lb}_m\text{F}$ ) into this same equation does not cause a significant change in the value of  $(U)$  from  $0.95 \text{ Btu/hr ft}^2\text{F}$ .

In all the equations used to calculate the local system temperatures in this analysis (Equations 3-9 through 3-12), the specific heat and specific gravity appear as a product. It is interesting to note that this product, for the most popular brands of hydraulic fluids used in industry, is a fairly constant value which ranges from about 0.370 to 0.507. These values are listed in Table VI. For this analysis, where a value of 0.77 was used for specific gravity, the range of specific heat-specific gravity product from 0.40 to 0.50 would correspond to a variation of specific heat from 0.52 to  $0.65 \text{ Btu/lb}_m\text{F}$ . The greatest change in temperature this would cause for any of the systems occurs in Example 5 where the maximum increase in temperature would be 6 degrees.

Therefore, it is reasonable to conclude that if one of the fluids listed in Table VI are selected to be used in a hydraulic system, the value of the specific heat of the fluid will have relatively little effect on the temperatures throughout the system.

This may seem like it yields the previous analysis useless; however, it is not unlikely that someday a usable fluid may be developed which has a specific heat-specific gravity product greater than 0.50 or less than 0.40.

TABLE VI

## HYDRAULIC FLUIDS

Fluid	Useful Temperature Range F	Specific Gravity	Specific Heat Btu/lb <sub>m</sub> F	Product of Specific Heat and Specific Gravity
5606	-65	.9	--	--
	160	.83	.5	.415
	275	.8	.576	.461
500A	-65	1.12	.33	.370
	100	1.05	.39	.410
	225	.99	.44	.436
7808	-50	--	--	--
	100	.915	.416	.381
	200	.876	.525	.460
8200	-65	--	--	--
	300	.845	.53	.448
	450	.78	.65	.507
8515	-65	.98	.38	.372
	300	.83	.56	.465
	450	.77	.65	--
7277B	20	.97	.41	.501
	300	.836	.545	.456
	450	--	.62	--

## B. Variation of Pump Efficiency

A pump can be described as a device for continually converting mechanical energy into the form of pressure and kinetic energy of a fluid stream. The fraction of the input energy converted to pressure and kinetic energy is a measure of the efficiency of the device, while the fraction of the input energy left over which is not successfully converted to these two forms of energy can be considered as the inefficiency of the device. The inefficiency is that fraction of the input energy which is converted to thermal energy and, in effect, increases the fluid temperature as the fluid passes through the pump.

The following analysis will show how pumps with various values of efficiency will affect the local temperatures throughout hydraulic systems.

The temperature rise of the fluid across an adiabatic pump is given by the following equation:

$$t_o - t_i = \frac{0.00297(P_o - P_i)}{(S)C_p} \left( \frac{1 - \eta}{\eta} \right). \quad (3-10)$$

It was mentioned previously that the ratio of the inefficiency  $(1 - \eta)$  to the efficiency  $(\eta)$  of the pump is a fraction which will be less than unity if  $\eta > 50$  per cent. This will nearly always be the case since the efficiency of most pumps is greater than 50 per cent. The temperature rise in a pump as a function of pressure differentials for various

pump efficiencies is shown in Figure 9. These curves are based on MIL-H-5606 fluid with a specific gravity of 0.80 and a specific heat of  $0.576 \text{ Btu/lb}_m\text{F}$  at 275 F.

To show how different values of pump efficiency affect the local temperatures throughout a system, the pump will be combined with other components to form a system, as shown in Figure 6, page 25.

The following assumptions will be used in calculating the temperatures for this system:

Environmental Temperature (TE) 100 F

Flow Rate (G) 5 gpm

Over-all Heat Transfer Coefficient (U)  $0.025 \text{ Btu/min ft}^2\text{F}$

Pressure Rise Across Pump (DP) 3000 psi

Specific Heat (C)  $0.49 \text{ Btu/lb}_m\text{F}$

Specific Gravity (S) 0.77

Surface Areas:  $A_{12} = 20 \text{ ft}^2$   
 $A_{34} = 20$   
 $A_{56} = 20$   
 $A_R = 20$

The same equations and methods will be used in this solution as were used in previous examples. The computed values of temperatures at various locations throughout the system are shown in Table VII.

These results exhibit a leveling-out trend with an increase in efficiency similar to that which an increase in specific heat did in previous examples.

To emphasize the substantial effect that variations in the pump efficiency has upon the reservoir outlet temperature

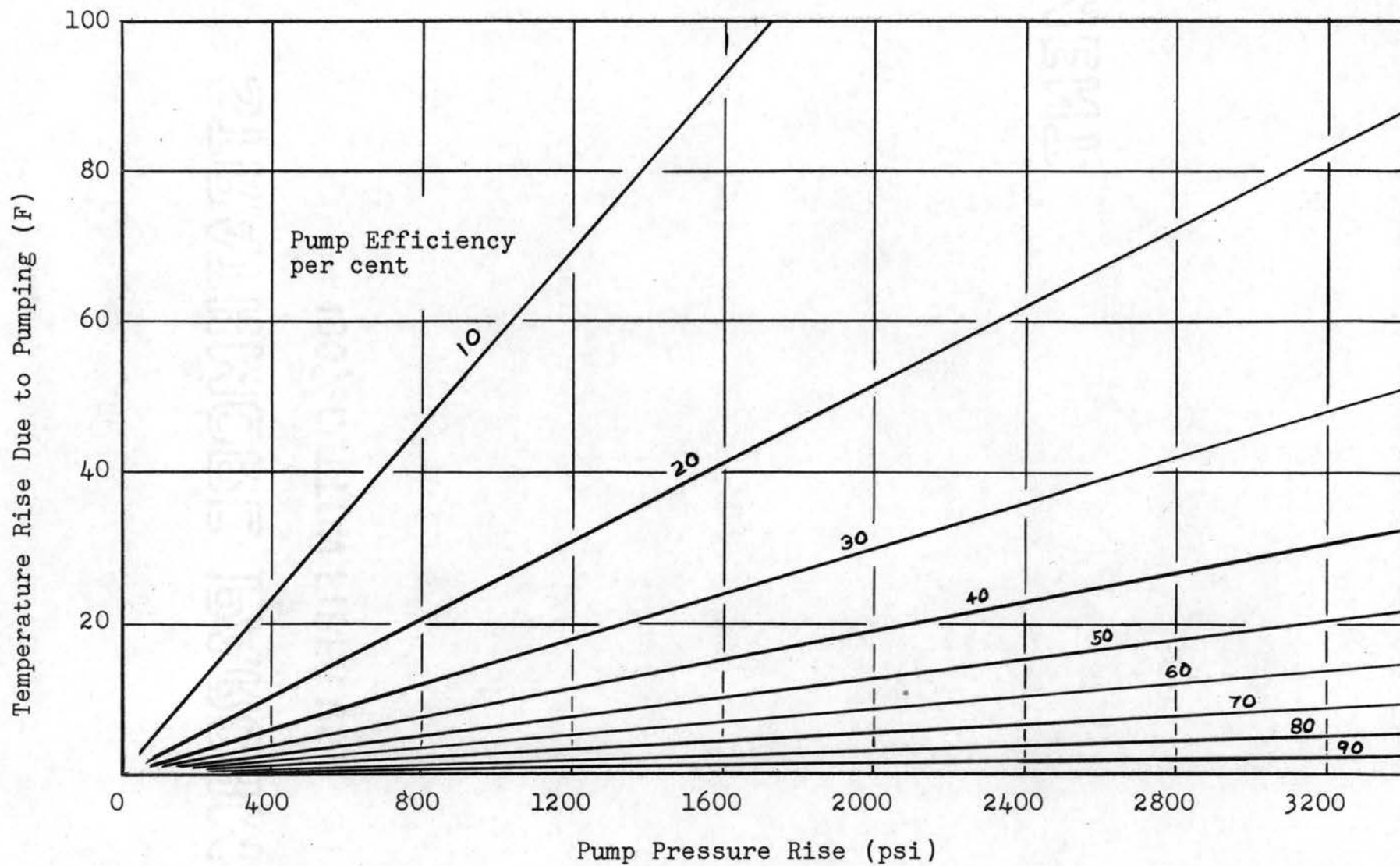


Figure 9. Temperature Rise Due to Pumping vs Pump Pressure Rise

TABLE VII

EFFECT OF VARIOUS PUMP EFFICIENCIES  
ON SYSTEM TEMPERATURES

PUMP EFFICIENCY	LOCAL TEMPERATURE F					
	T1	T2	T3	T4	T5	T6
20	1006	978	1072	1042	1056	1035
30	706	687	742	722	746	725
40	556	542	577	562	586	571
50	466	454	478	466	490	478
60	406	396	412	402	426	416
80	331	324	329	322	346	338
90	306	299	302	296	319	312
95	295	289	290	284	308	302

of this system, a graph of these two values is shown in Figure 10.

It can thus be concluded that it would be advantageous, from a heat transfer standpoint, to obtain a pump with the highest possible efficiency available. However, the economic aspect of the selection requires that several factors be taken into account. One such factor would be the direct relationship between the cost of the pump and the efficiency of the pump; an inefficient pump will require additional power to obtain the desired result and may also require the addition of a heat exchanger to maintain the desired temperature level.

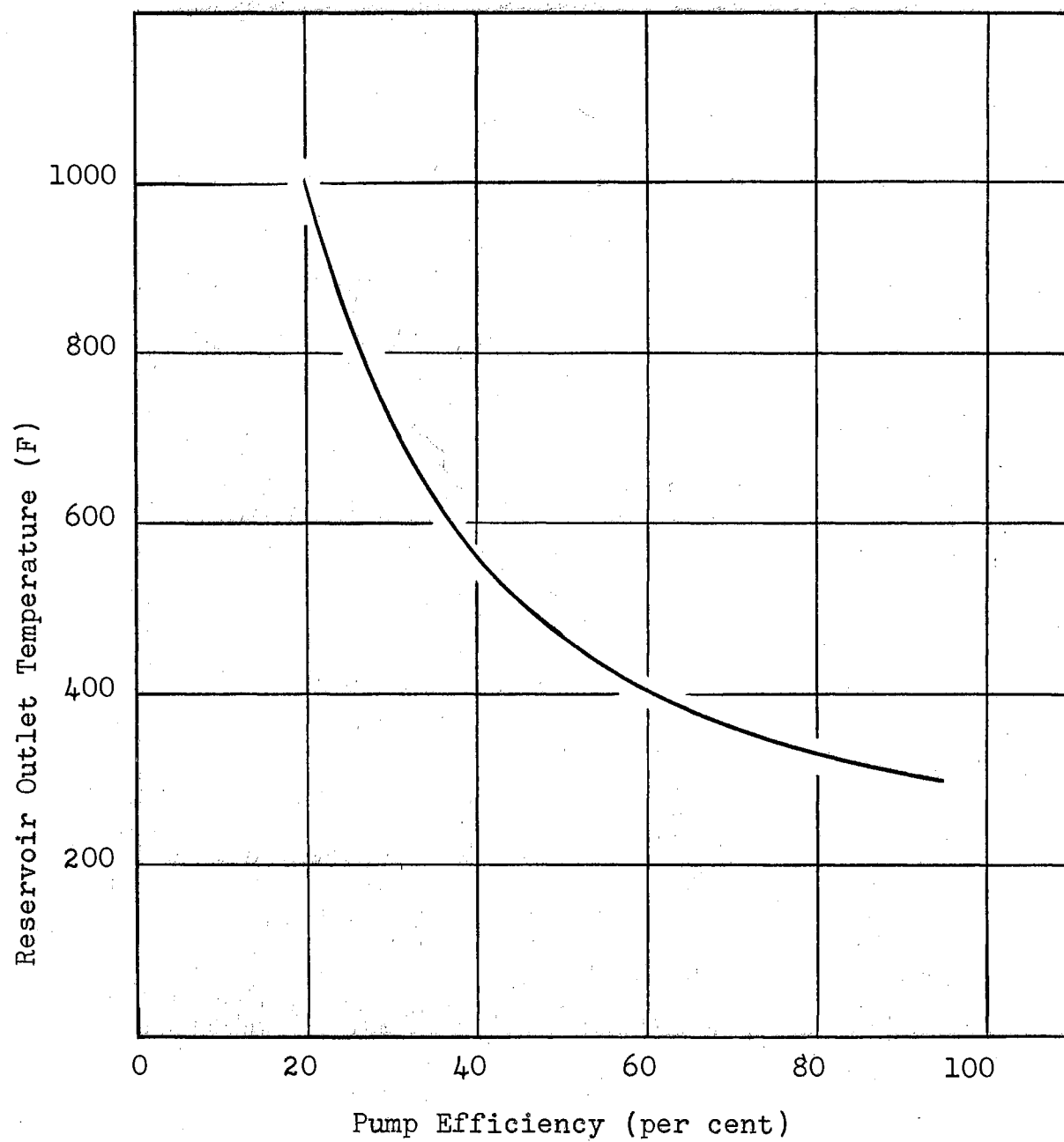


Figure 10. Reservoir Outlet Temperature  
vs Pump Efficiency

### C. Variation of Over-all Heat Transfer Coefficient

In general, heat transfer takes place from a system by conduction, radiation, or convection. For energy to be transferred by any of these modes, a temperature difference must exist since heat always flows in the direction of decreasing temperature.

The removal of heat from hydraulic systems does not involve a single mechanism of heat transfer, but a combination of all three modes. The predominant mode is generally the convection mechanism.

For practical problems the combined effect of conduction, convection, and possibly radiation can be incorporated as one over-all heat transfer coefficient.

The effect of different over-all heat transfer coefficients on the temperature decrease in the fluid, due to heat transfer from a given length of tubing, will be shown by varying the heat transfer coefficient ( $U$ ) from 0.5 to 3.0 Btu/hr ft<sup>2</sup> F in Equation 3-9,

$$t_o = \frac{2t_e(A)U + [16.66(G)(S)C_p - (A)U]t_i}{16.66(G)(S)C_p + (A)U} \quad (3-9)$$

This effect is shown in Figure 11 where it can be seen that at a flow rate of 1 gpm, the temperature decrease due to convection is largely dependent upon the value used for the heat transfer coefficient. Whereas, at a flow rate of 20 gpm the temperature decrease has very little dependency upon the

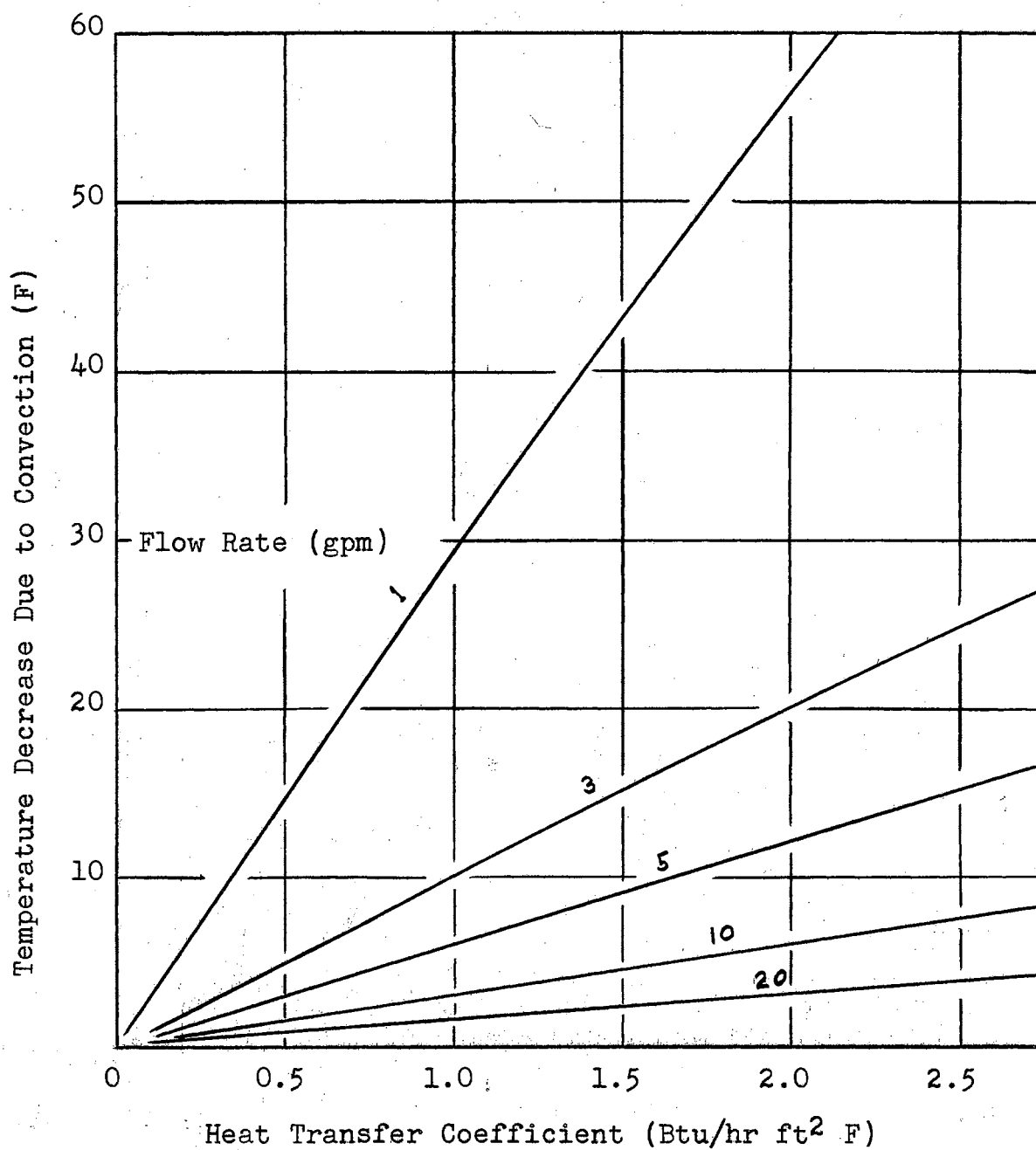


Figure 11. Temperature Decrease Due to Convection  
vs Over-all Heat Transfer Coefficient

heat transfer coefficient.

To show the effect of different over-all heat transfer coefficients on the local temperatures throughout a hydraulic system, two systems will be considered. One system will utilize a heat exchanger to maintain the equilibrium temperatures of the system at a desirable level. The other system will not use a heat exchanger.

The system with a heat exchanger, Figure 7, page 30, and with the piping exchanging heat with a cold environment, will be used with the following assumptions:

Environmental Temperature (TE) 100 F

Flow Rate (G) 5 gpm

Pressure Rise Across Pump (DP) 3000 psi

Specific Heat (C) 0.49 Btu/lb<sub>m</sub>F

Specific Gravity (S) 0.77

Pump Efficiency (ETA) 80 per cent

Heat Removed by Heat Exchanger (QH) 375 Btu/min

Surface Areas: A<sub>12</sub> = 20 ft<sup>2</sup>  
                   A<sub>34</sub> = 20  
                   A<sub>56</sub> = 20  
                   A<sub>R</sub> = 20

The solution for the local temperatures for this system with the over-all heat transfer coefficient (U) varying by increments of 0.5 Btu/hr ft<sup>2</sup>F are listed in Table VIII.

To give a graphical picture of how the variation of over-all heat transfer coefficient (U) affects the reservoir outlet temperature (T<sub>1</sub>), these values are plotted in Figure 12. The effect of variations of (U) on the system temperatures

TABLE VIII

EFFECT OF VARIATIONS OF OVER-ALL HEAT TRANSFER  
COEFFICIENT ON TEMPERATURES OF A SYSTEM  
WITH A HEAT EXCHANGER

OVER-ALL COEFFICIENT Btu/hr ft <sup>2</sup> F	LOCAL TEMPERATURE F						
	T1	T2	T3	T4	T5	T6	T7
0.5	228	230	236	234	258	256	233
1.0	160	158	164	163	187	185	161
1.5	138	136	142	141	165	163	139
2.0	126	125	131	130	153	151	127
2.5	119	118	124	123	147	144	120
3.0	115	114	120	119	142	140	116

will be opposite for the system in a hot environment than it was for the same system in a cold environment. Figure 13 shows the effect of variations of (U) on the reservoir outlet temperature for  $T_E = 300$  F and  $Q_H = 575$  Btu/min. The opposite effect mentioned above can be seen by comparing Figures 12 and 13. Since the temperatures at the remainder of the locations experience the same trend, they will not be listed for the case with the system in the hot environment.

The system shown in Figure 6, page 25, will be used to show how various over-all heat transfer coefficients affect the temperatures of a system without a heat exchanger. The following assumptions will be used in the calculations of the temperatures for this system:

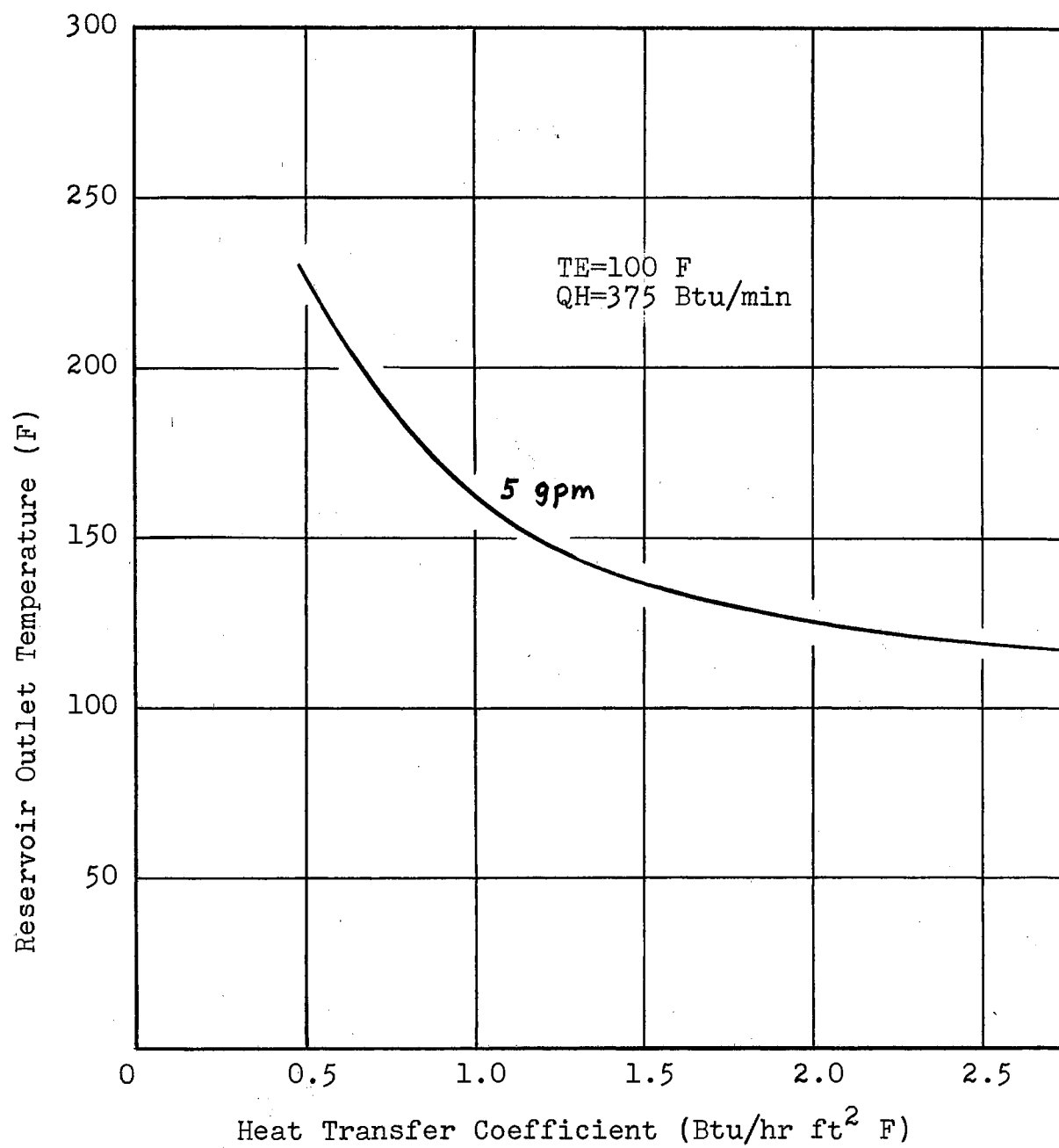


Figure 12. Reservoir Outlet Temperature vs Over-all Heat Transfer Coefficient

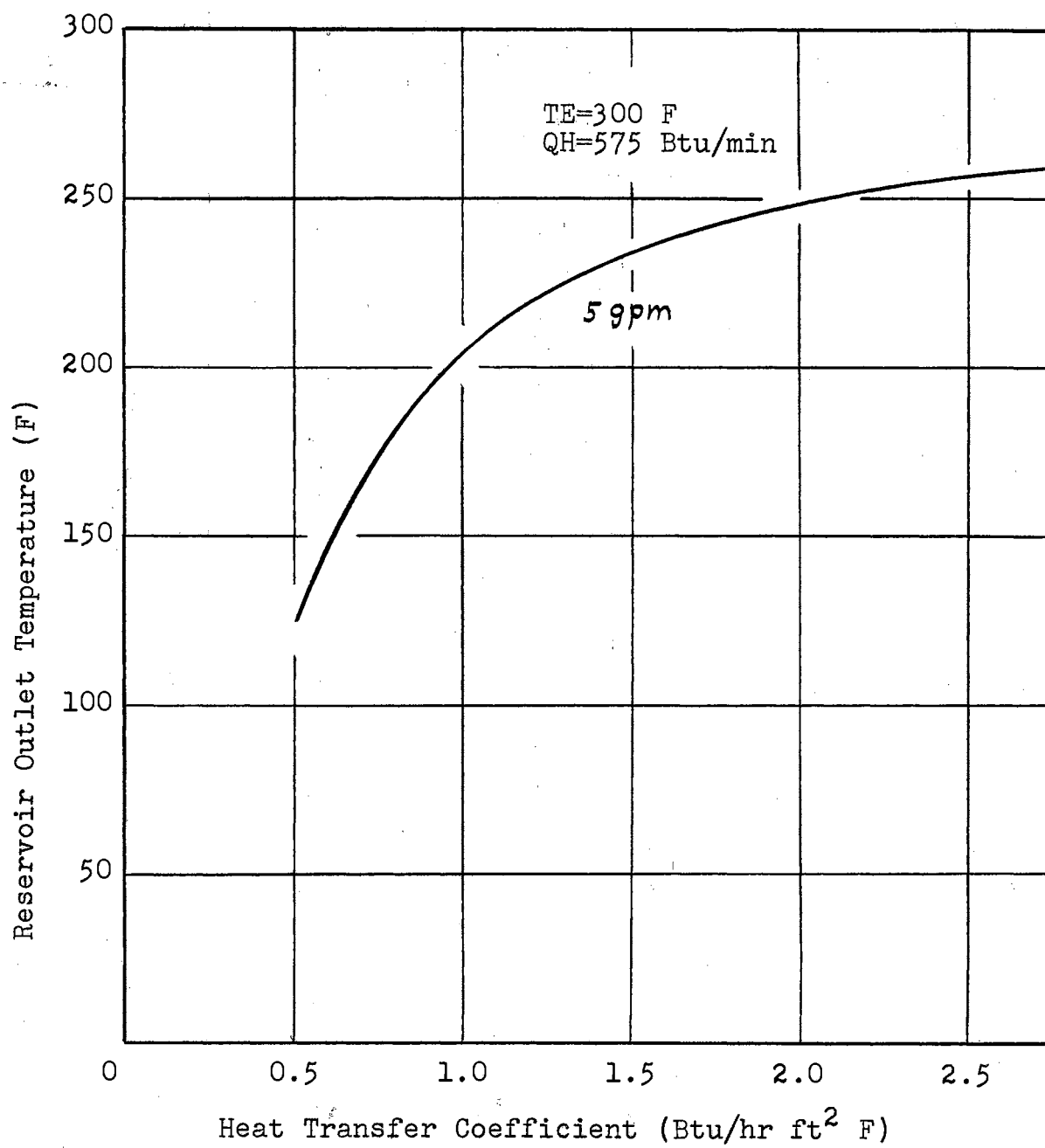


Figure 13. Reservoir Outlet Temperature vs Over-all Heat Transfer Coefficient

Environmental Temperature (TE) 100 F  
Pressure Rise Across Pump (DP) 3000 psi  
Pump Efficiency (ETA) 80 per cent  
Specific Heat (C) 0.49 Btu/lb<sub>m</sub>F  
Specific Gravity (S) 0.77  
Surface Areas: A<sub>12</sub> = 20 ft<sup>2</sup>  
                  A<sub>34</sub> = 20  
                  A<sub>56</sub> = 20  
                  A<sub>R</sub> = 20

For this solution the local temperatures throughout the system will be calculated with the over-all heat transfer coefficient (U), varying from 0.5 to 3.0 Btu/hr ft<sup>2</sup>F by increments of 0.5 Btu/hr ft<sup>2</sup>F for flow rates of 1, 5, and 10 gpm. The results of this calculation is given in Table IX. The variation of the reservoir outlet temperature (T<sub>l</sub>) with the over-all heat transfer coefficient for the three flow rates is shown in Figure 14.

By comparing the curves in Figures 12 and 14, it can be seen that the reservoir outlet temperature (T<sub>l</sub>) is much more dependent upon the heat transfer coefficient (U) for the system without a heat exchanger than for the system which has a heat exchanger. For the system without a heat exchanger, all the heat given up by the system leaves by way of convection from the tubing to the environment; while for the system which has a heat exchanger, the major portion of the heat removed from the system is removed by the heat exchanger with only a small percentage being lost by convection from the tubing to the environment. In considering all the equations used to

TABLE IX

EFFECT OF VARIATIONS OF OVER-ALL HEAT TRANSFER COEFFICIENT ON TEMPERATURES  
OF A SYSTEM WITHOUT A HEAT EXCHANGER

OVER-ALL COEFFICIENT (U) Btu/hr ft <sup>2</sup> F	FLOW RATE (G) gpm	LOCAL TEMPERATURES F					
		T1	T2	T3	T4	T5	T6
0.5	1	242	235	241	234	257	249
1.0	1	168	161	167	160	184	175
1.5	1	144	138	144	137	161	152
2.0	1	132	126	132	126	150	140
2.5	1	125	119	125	119	143	133
3.0	1	120	115	121	115	139	128
0.5	5	816	808	814	807	831	823
1.0	5	446	439	445	438	461	454
1.5	5	331	324	330	322	346	338
2.0	5	273	266	272	265	288	280
2.5	5	238	231	237	229	253	245
3.0	5	214	207	213	206	230	222
0.5	10	1533	1526	1532	1524	1548	1541
1.0	10	795	787	793	786	810	802
1.5	10	563	556	562	555	578	571
2.0	10	448	440	446	439	463	455
2.5	10	377	370	376	369	392	385
3.0	10	331	324	330	322	346	338

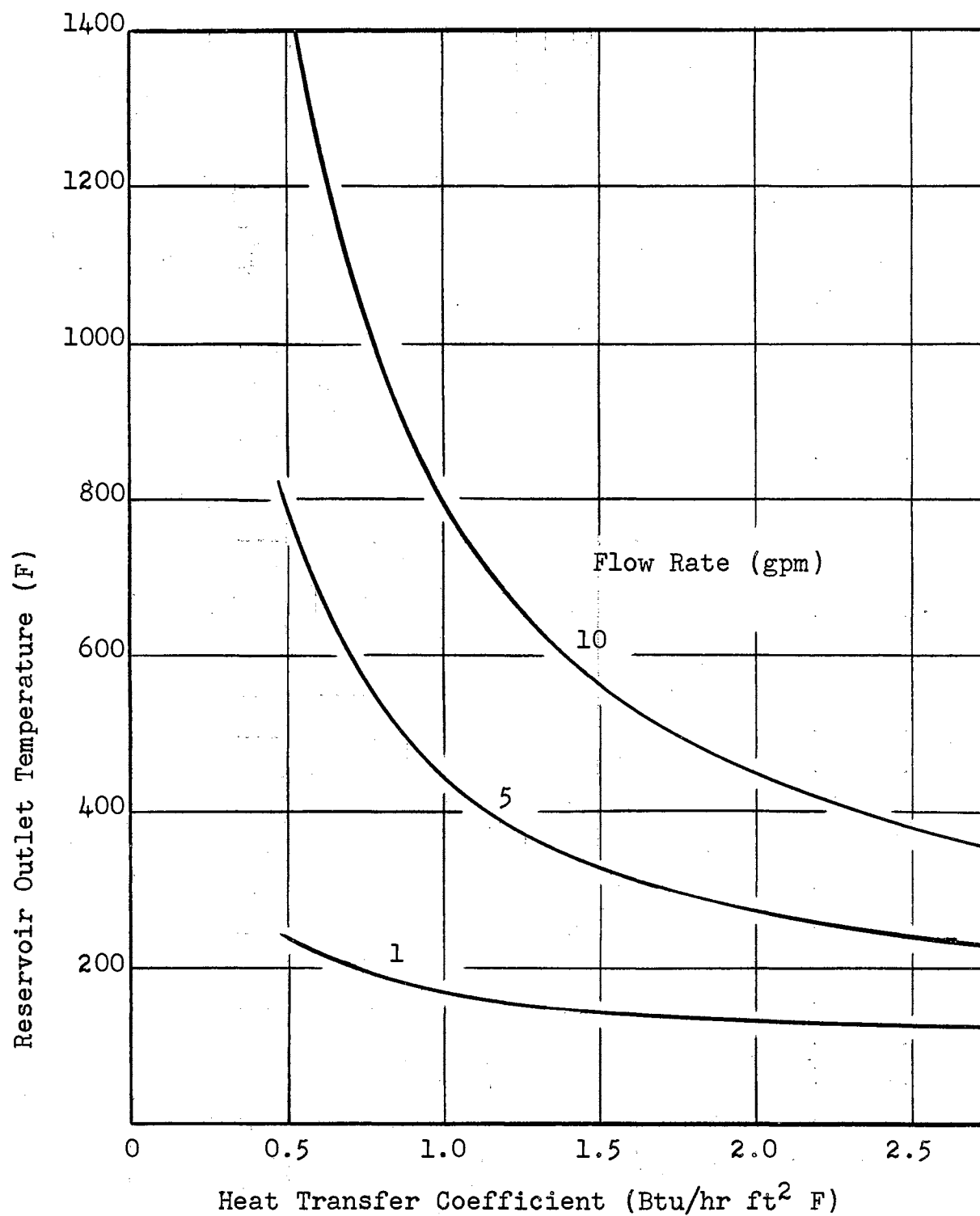


Figure 14. Reservoir Outlet Temperature vs Over-all Heat Transfer Coefficient

calculate the system temperatures, the only one containing the over-all heat transfer coefficient (U) is

$$t_o = \frac{2t_e(A) + [16.66(G)(S)C_p - (A)U] t_i}{16.66(G)(S)C_p + (A)U}, \quad (3-9)$$

which governs the temperature change in the portion of the system where the heat is given up by convection.

To show the magnitude of error in the reservoir outlet temperature involved for a certain accuracy in heat transfer coefficient, the following is offered. Assuming the over-all heat transfer coefficient was determined within 20 per cent of its true value, such as 1.5 Btu/hr ft<sup>2</sup>F, would mean that the coefficient might lie within the range of 1.2 to 1.8 Btu/hr ft<sup>2</sup>F. If the extreme value of 1.2 were used, then the error in reservoir outlet temperature (Tl) can be determined from Figure 14, the system without a heat exchanger. For flow rates of 1, 5, and 10 gpm, the errors are as follows:

Flow Rate (G), gpm	Error in Tl, degrees
1	14
5	51
10	112

For a system with a heat exchanger and using the same accuracy of over-all heat transfer coefficient as before, the error in reservoir outlet temperature can be determined from Figure 12 as 10 degrees when the flow rate (G) is 5 gpm.

In consideration of the above discussion the designer might assume that he must demonstrate more accuracy in his calculation of the heat transfer coefficient ( $U$ ) for systems which are losing their excess heat by means of convection than for systems which utilize a heat exchanger to remove this heat. In addition, the greater the flow rate ( $G$ ) required, the more accurate the designer must be in his determination of the over-all heat transfer coefficient ( $U$ ).

It should be noted that the surface area ( $A$ ) and the over-all heat transfer coefficient ( $U$ ) always appear as a product. Therefore, the value of area will have a tendency to amplify or to dampen the magnitude of the effect of variations in ( $U$ ) on the system temperatures. For example, the temperature potential of a system without a heat exchanger is affected less by a variation in ( $U$ ) if the surface area of the system is large rather than small.

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## APPENDIX

## COMPUTER TECHNIQUES

Man's productivity has been increased by the development of the computer. This increase is accomplished in several ways. (1) Through their speed computers enable man to increase his output per hour. (2) Computers are, by far, more accurate than man. (3) Computers make it possible for man to employ many mathematical methods previously considered impractical due to the time-consuming calculations involved. For example, consider attempting to solve 25 simultaneous equations by previous methods. By using the computer this problem can be solved in minutes.

Eckert and Jones (7) give the time required to form a thousand products of two 10-digit numbers for various methods, as follows:

By hand with pencil and paper	1 week
With the aid of a key-controlled mechanical desk calculator	1 day
By an electro-mechanical calculator with automatic reading and writing	1 hour
By a small electronic calculator	1 minute
By an electronic supercalculator	1 second

Using this as a basis, and considering the IBM 1620 Digital

Computer (which was used to make the calculations for this thesis) as a small electronic calculator, the time required to do the same calculations for this thesis by hand with pencil and paper would take a person much longer.

Two basic computer programs were used for calculating the local temperatures in the foregoing analysis. The programs are written in Fortran language for the IBM 1620 computer, and are given in Tables X and XI, with respective logic flow diagrams in Figures 15 and 16. These two programs were used to calculate the local temperatures for the system shown in Figure 7, page 30. The difference between the two programs are as follows. For the program in Table X a value of  $T_1$  is selected, then the other temperatures of the system and  $Q_H$  are calculated, based upon  $T_1$  remaining at the pre-selected value. The program in Table XI utilizes an iteration procedure for the calculation of the local temperatures, using a fixed value for  $Q_H$ .

For persons unfamiliar with IBM Fortran language, an outline of the steps performed by the computer, with the aid of Figure 16, are as follows:

1. An assumed value for  $T_1$  is read into the computer on data cards along with all the system parameters.
2.  $T_2$  is calculated by using  $T_1$  and the appropriate parameters
3.  $T_3$  is then calculated by using  $T_2$ ; this routine

TABLE X  
COMPUTER PROGRAM

```

1  READ 2,A12,A34,A56,AR
2  FORMAT(F10.0,F10.0,F10.0,F10.0)
3  READ 6,S,C,U,ETA,DP,TE,T1,G
6  FORMAT(F5.0,F5.0,F5.0,F5.0,F5.0,F5.0,F5.0,F5.0)
   T2=(2.*TE*A12*U+(16.7*G*S*C-A12*U)*T1)/(16.7*G*S*C+A12*U)
   T3=.00297*DP*(1.-ETA)/(ETA*S*C)+T2
   T4=(2.*TE*A34*U+(16.7*G*S*C-A34*U)*T3)/(16.7*G*S*C+A34*U)
   T5=.00297*DP/(S*C)+T4
   T6=(2.*TE*A56*U+(16.7*G*S*C-A56*U)*T5)/(16.7*G*S*C+A56*U)
   T7=((16.7*G*S*C+AR*U)*T1-2.*TE*AR*U)/(16.7*G*S*C-AR*U)
   QH=(T6-T7)*8.35*G*S*C
20 TYPE 21,TE,G,U
21 FORMAT(4H TE=,F5.0,3H G=,F4.1,3H U=,F6.4)
25 TYPE 26,QH
26 FORMAT(4H QH=,F7.1)
30 TYPE 31,T1,T2,T3,T4
31 FORMAT(4H T1=,F7.2,4X,3HT2=,F7.2,4X,3HT3=,F7.2,4X,3HT4=,F7.2)
40 TYPE 41,T5,T6,T7
41 FORMAT(4H T5=,F7.2,4X,3HT6=,F7.2,4X,3HT7=,F7.2//)
   GO TO 5
   END

```

TABLE XI  
COMPUTER PROGRAM

```

1  READ 2,A12,A34,A56,AR
2  FORMAT(F10.0,F10.0,F10.0,F10.0)
5  READ 6,S,C,U,ETA,DP,TE,G,QH,Y,T1
6  FORMAT(F5.0,F5.0,F5.0,F5.0,F5.0,F5.0,F5.0,F5.0,F5.0,F5.0)
10 T2=(2.*TE*A12*U+(16.7*G*S*C-A12*U)*T1)/(16.7*G*S*C+A12*U)
    T3=.00297*DP*(1.-ETA)/(ETA*S*C)+T2
    T4=(2.*TE*A34*U+(16.7*G*S*C-A34*U)*T3)/(16.7*G*S*C+A34*U)
    T5=.00297*DP/(S*C)+T4
    T6=(2.*TE*A56*U+(16.7*G*S*C-A56*U)*T5)/(16.7*G*S*C+A56*U)
    T7=-QH/(8.35*G*S*C)+T6
    T1N=(2.*TE*AR*U+(16.7*G*S*C-AR*U)*T7)/(16.7*G*S*C+AR*U)
    X=ABS(T1-T1N)
    IF(Y-X)20,20,25
20  T1=T1N
    GO TO 10
25  TYPE 26,TE,G,U,C,QH
26  FORMAT(4H TE=,F5.0,3H G=,F5.1,3H U=,F6.4,3H C=,F4.2,4H QH=,F5.0)
30  TYPE 31,T1,T2,T3,T4
31  FORMAT(4H T1=,F7.2,4X,3HT2=,F7.2,4X,3HT3=,F7.2,4X,3HT4=,F7.2)
40  TYPE 41,T5,T6,T7
41  FORMAT(4H T5=,F7.2,4X,3HT6=,F7.2,4X,3HT7=,F7.2//)
    GO TO 5
    END

```

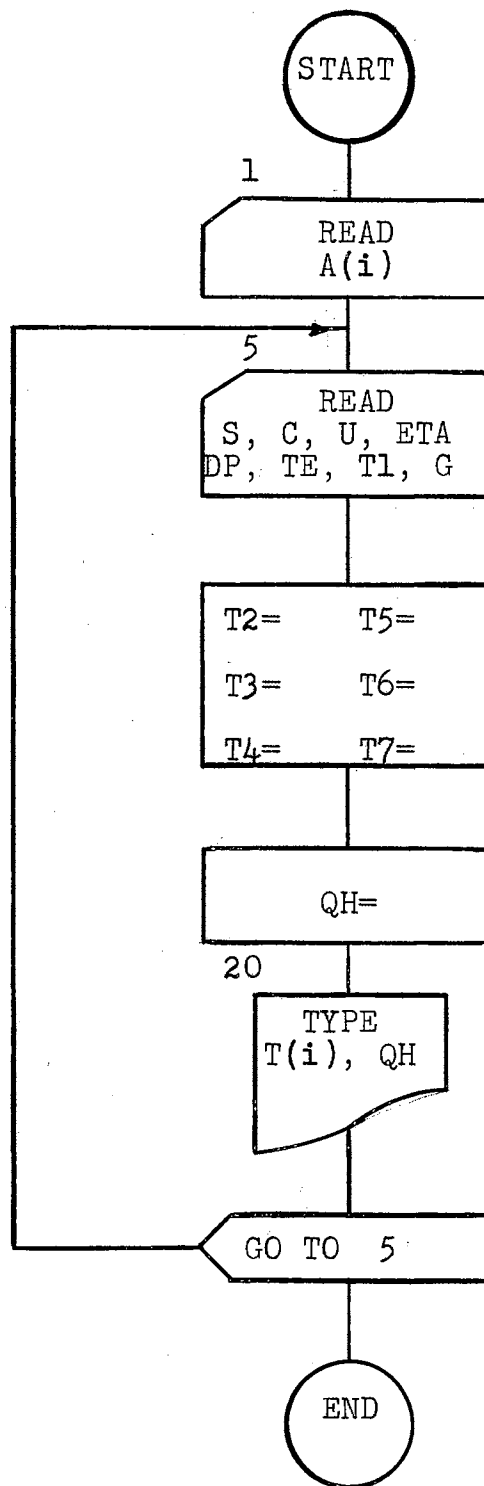


Figure 15. Computer Flow Diagram

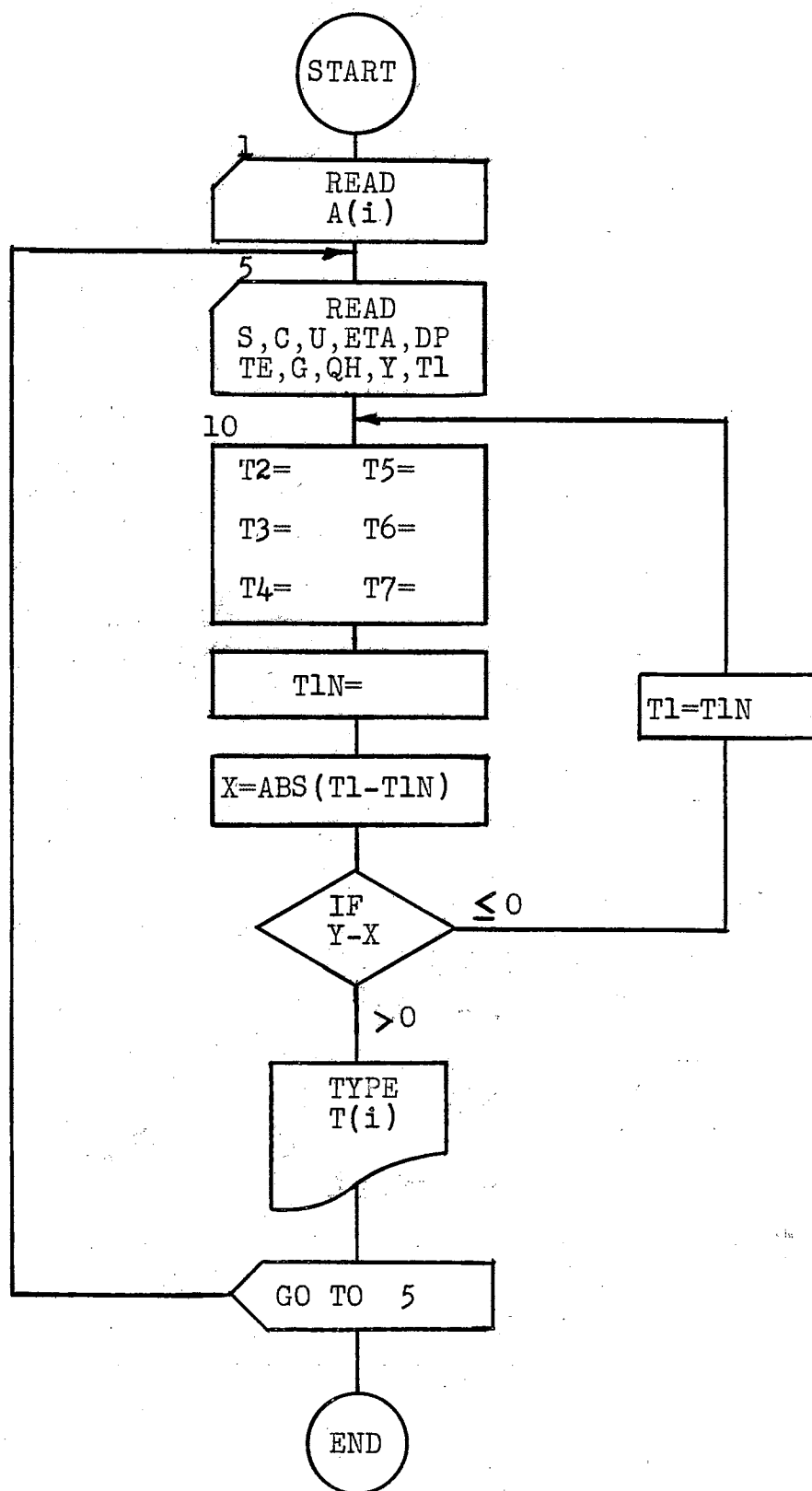


Figure 16. Computer Flow Diagram

is followed until all the temperatures are calculated.

4. By using  $T_7$  and the appropriate parameters the temperature for position 1 is calculated, and is referred to as  $T_{1N}$ .
5. The absolute value of the difference between the assumed value of  $T_1$  and  $T_{1N}$  is calculated.
6. If the absolute value of the difference is greater than or equal to an arbitrary value  $Y$  (the value of  $Y$  is read into the computer on data cards and can be varied to yield the required accuracy), then  $T_1$  is set equal to  $T_{1N}$ , and the temperatures are calculated again. This procedure is followed until the absolute value of the difference is less than  $Y$ , then the calculated value of the temperatures are typed.

The effect of the value of  $Y$  on the calculated temperatures at location number 1 is shown in Table XII for assumed values of  $T_1$  equal to 200 and 300 F.

It is noteworthy that as  $Y$  is taken smaller, more time will be required for the computer to arrive at a solution. In order to save computer time it is usually advisable to begin by using a value of, say, 0.50 for  $Y$ , which will give approximate values for the calculated temperatures. The approximate value of  $T_1$  can then be read into the computer along with a smaller value of  $Y$ , and a more accurate set of temperatures can then be calculated. This type of procedure

will usually result in a considerable saving of computer time.

TABLE XII

## CALCULATED VALUES OF T1

Y	ASSUMED VALUES		DIFFERENCE
	T1 = 200	T1 = 300	
0.50	247.60	280.24	32.64
0.10	260.60	267.11	6.51
0.05	262.18	265.47	3.29
0.01	263.51	264.17	0.66

The value to be used for Y should be determined for each particular system, depending upon the accuracy desired. A value of  $Y = 0.01$  was used for all calculations in this thesis.

It is not uncommon when designing a hydraulic system to find it necessary to insure that the maximum temperature of the system will be below a certain level. A procedure for calculating the system temperatures and QH, while holding the maximum temperature at a specified value, is as follows:

1. Use the program given in Table X for locating the maximum temperature. (For the conditions given in Example 3, the maximum is located at position 5.)
2. Rewrite the first four equations of the program in Table X, and then arrange them to form a program as given in Table XIII.

TABLE XIII  
COMPUTER PROGRAM

```

1  READ 2,A12,A34,A56,AR
2  FORMAT(F10.0,F10.0,F10.0,F10.0)
5  READ 6,S,C,U,ETA,DP,TE,T5,G
6  FORMAT(F5.0,F5.0,F5.0,F5.0,F5.0,F5.0,F5.0,F5.0)
   T4=T5-.00297*DP/(S*C)
   T3=((16.7*G*S*C+A34*U)*T4-2.*TE*A34*U)/(16.7*G*S*C-A34*U)
   T2=T3-.00297*DP*(1.-ETA)/(ETA*S*C)
   T1=((16.7*G*S*C+A12*U)*T2-2.*TE*A12*U)/(16.7*G*S*C-A12*U)
   T6=(2.*TE*A56*U+(16.7*G*S*C-A56*U)*T5)/(16.7*G*S*C+A56*U)
   T7=((16.7*G*S*C+AR*U)*T1-2.*TE*AR*U)/(16.7*G*S*C-AR*U)
   QH=(T6-T7)*8 35*G*S*C
20 TYPE 21,TE,G,U
21 FORMAT(4H TE=,F5.0,3H G=,F4.1,3H U=,F6.4)
25 TYPE 26,QH
26 FORMAT(4H QH=,F7.1)
30 TYPE 31,T1,T2,T3,T4
31 FORMAT(4H T1=,F7.2,4X,3HT2=,F7.2,4X,3HT3=,F7.2,4X,3HT4=,F7.2)
40 TYPE 41,T5,T6,T7
41 FORMAT(4H T5=,F7.2,4X,3HT6=,F7.2,4X,3HT7=,F7.2//)
   GO TO 5
END

```

3. A value for  $T_5$  can be selected for the maximum temperature level desired, and then using this as a basis, the remaining temperatures can be calculated along with the value of  $Q_H$ , which is required to maintain this temperature level.

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