A SIMPLE METHOD FOR ANALYZING AMPLIFIER PHASE-SHIFT USING THE NEGATIVE FEEDBACK PRINCIPLE

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A SIMPLE METHOD FOR ANALYZING AMPLIFIER PHASE-SHIFT USING THE NEGATIVE FEEDBACK PRINCIPLE

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PREFACE

In man's eternal conflict with the physical elements about him, there invariably arises the question, how much?, or how many?, or how long?, about the phenomenon of immediate interest to him. When he was building his first primitive home from virgin timber it was necessary to create a gauge to determine how many units of length were to be used at various points in the structure; he also needed to know how much timber and of what weight would be necessary to protect him from the weather and other dangers. As far back as the history of mankind is known, the act of answering these questions, namely, the art of measurement, has been of vital importance.

Now, as the modern technological society reaches new heights in the development of the physical world, the measurement technique or process is becoming a science unto itself. As such its complexities are manifold and any endeavor which will result in the simplification of one of these processes should be deemed most appropriate.

It is with that purpose in mind that this thesis is written. The technique contained herein is truly simple and possesses sufficient accuracy for a multitude of practical applications.

The basic premise is predicated upon the principle of operation of the electronic audio amplifier and uses the negative feedback principle. By applying this technique to four readily obtainable voltages at the input and output of the amplifier, the phase-shift within the amplifier is obtained. Aside from the academic interest of this development, it is believed that it will have much utility if applied to low frequency amplification devices such as the servo-mechanism.

It is submitted, therefore, that the material contained in this thesis will, on a small scale, contribute to the general science of measurement, and in addition will simplify the phase-analysis of low frequency amplification devices.

ACKNOWLEDGEMENTS

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CHAPTER I

INTRODUCTION

<u>Terminology</u>

As any true philosopher knows, the proper beginning for any logical presentation of fact is the definition of terms. While this thesis is not philosophical by nature it is certainly intended to be both logical and factual. Since there will be numerous references made to specialized concepts, the following definitions are made. They are designed to be neither rigorous nor binding, but merely to assist the reader in following the author's meaning.

System - A conglomerate of electrical, mechanical, hydraulic and/or pneumatic devices associated to perform some function.

Input - An excitation, depending upon the nature of the system, which will cause the system to operate, for example, voltage, pressure, or torque.

Output - The end product of the system.

Signal - Energy which is controlled in direction, magnitude, and form for some particular application.

Automation - A term, used by industry, which includes machine operations in which one machine controls the operation of another machine.

Frequently this takes the form of an electrical device controlling some mechanical device.

Time-phase Relationship - The time variation in magnitude or amplitude of one periodic function of energy compared to some norm. If, for example, two functions of electrical energy each have a sine wave configuration, one wave is used as the norm and the other wave is said to be in phase or out of phase by some amount of time. This means that points of equal or corresponding amplitude differ along the time axis by an amount usually expressed in degrees or radians. The relationship between time and radians or degrees is,

$$\frac{1}{\text{Frequency}} = T = 2 \text{ pi}$$

radians = 360 deg. where T is the time of one period of the periodic function.

Phase-shift Plot - A graphical presentation, on either polar or cartesian coordinates, showing phase-shift plotted against frequency.

Phase Distortion - The difference in phase between input and output signals of a system using periodic energy excitation.

Transfer Function - The ratio of output to input of any given system; this describes what happens to the input as it is transferred through the system, i.e., functionally relates output to input in phase and magnitude.

System Response - The manner in which the device operates upon the input signal to produce the output signal. If an amplifier, for example, produces a voltage gain K at some frequency f with an associated phase-shift θ, then an analysis of these data would reveal the response of the system.

βK Plot - A graphical representation obtained by connecting the ends of each βK vector for various frequencies. Since K is a function of frequency its amplitude and the amplitude of βK will vary with frequency. β can be made non-frequency sensitive, however, so the βK Plot can be a phase-shift plot, thereby achieving considerable importance.

Open Loop - The status of a feedback amplifier in which the feedback network is open; or in other words, the condition of the amplifier when no feedback signal is returned to the input.

Amplifiers in General

Any system which causes a certain signal to be increased in magnitude between input and output may be considered generically as an amplifier. The elements of an amplifier may be of many types, sizes, shapes and capacities. An ordinary block and tackle is a primitive amplifier by which man can multiply an applied force in proportion to the 'mechanical advantage' of the system. In electronic usage, this 'mechanical advantage' is analogous to the amplification factor (K) of an amplifier. Mechanical gears, hydraulic pumps, amplidyne generators, are all typical examples of the amplification principle.

Analogy Between Audio- and Servo- Amplifiers

A servo-mechanism, the heart of which is the servo-amplifier, is a device which compares two signals and applies the resultant of this comparison to an appropriate apparatus which will operate in such a manner as to reduce the resultant to a minimum value. The frequency of these signals is usually well within the audio frequency range. There is, therefore, a common element, namely frequency, between the servo- and the audio- amplifier. This suggests the possibility of analyzing one using principles derived or established for the other. That is precisely what is to be done in the material to follow.

Statement of Objective

In most automatically controlled devices it is imperative to know what the time-phase relationship is between the input and output signals. This knowledge is necessary in order to be able to answer two important questions; 1) How will the system respond, time-wise?, and 2) Will the system be stable? Since the feedback or 'sampling' circuit need not introduce phase distortion of its own, a plot of the open loop phase response of the amplifier would be sufficient to answer these questions. On the other hand, perhaps the design engineer, after evaluating the amplifier phase response, decides to alter it for one reason or another; he may then intelligently assign parameters to the feedback loop to accomplish this end. In any event, the open loop phase response is an essential factor in the design of amplification systems which incorporate automatic control features.

It is the object of this thesis to establish a technique which will provide a simple method for analyzing audio-amplifier phase-shift. This phenomenon is directly analogous to the open loop phase-shift of the servo-amplifier and could be modified to apply to that device equally well.

It will be shown that by measuring the input and output voltages as shown in Figure 1, both with and without feedback (switch closed and open respectively), the amplifier open loop phase-shift may be obtained directly.

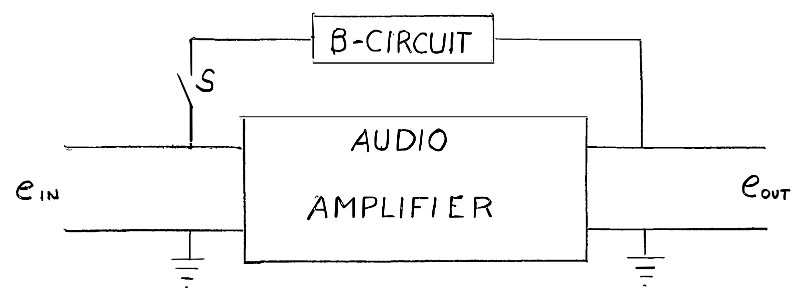

Figure 1. Block diagram of a feedback amplifier.

A phase-shift plot may be obtained either graphically or mathematically using these four measurements at suitable frequency values throughout the desired spectrum. The sampling network or β-circuit will be of an easily constructed resistive type which will introduce negligible phase-shift of its own.[1]

The ease of this technique will be apparent to those who have experienced the tedium of using a commercial phase meter over a large band of frequencies. In addition it offers the designer and technician a simple accurate tool for analyzing amplifier phase distortion. It is hoped, in general, that it will also advance amplifier measurement procedures.

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[1]Values of β are discussed in Appendix C

CHAPTER II

HISTORY OF FEEDBACK THEORY

Definition of Feedback

According to Terman (1) pages 311-313, "In the feedback amplifier, a voltage derived from the amplifier output is superimposed upon the amplifier input in such a way as to oppose the applied signal in the normal frequency range. By properly carrying out this operation, it is possible to reduce the distortion generated by the amplifier; to make the amplification substantially independent of the electrode voltages and tube constants; and to reduce greatly the phase and frequency distortion".[2]

System Stability

In order to realize the full advantages of feedback, the amplifier and its feedback network must be arranged so that oscillations do not occur. This is no problem in the so-called mid-band frequency range because there the feedback will be negative due to circuit arrangements. However, at both very low and very high frequencies, the amplifier stages produce phase-shifts that cause the phase of the

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[2]Since positive feedback in non-oscillatory amplifiers has become fact, Terman's definition should be modified to note the distinction. (2)

feedback signal to differ from the phase corresponding to negative feedback. Thus, it is possible that positive feedback could result producing oscillations and an unstable condition. Consequently, stability is a vital factor in the design and operation of feedback amplifiers.[3]

Dr. H. Nyquist and his associates in the Bell Laboratories were among the pioneers in stability research. In January 1932, Nyquist (3) published his classical paper on feedback amplifier design. His contribution enables the engineer to predict the stability of his amplifier even before its prototype is available for bench test.

Another famous paper on stabilized feedback amplifiers was published by H. S. Black (4) in 1934. His basic feedback circuit, shown in Figure 2, clearly defines the various voltages in such a network.

Black reasoned that if,

$$K' = \text{Amplification with feedback} = \frac{K}{1 - \beta K}$$

then

$$\left[\frac{\delta K'}{K'}\right]_K = \text{Ratio of change due to K} = \frac{\delta K / K}{1 - \beta K}$$

and

$$\left[\frac{\delta K'}{K'}\right]_\beta = \text{Ratio of change due to } \beta = \frac{\beta K}{1-\beta K}\left[\frac{\delta \beta}{\beta}\right]$$

So if $K\beta \gg 1$, then the K-circuit is stabilized by an amount corresponding to the reduction in amplification and the effect of introducing a gain or loss in the K-circuit is to produce no material change in the overall amplification of the system. The stability of amplification, as affected by the

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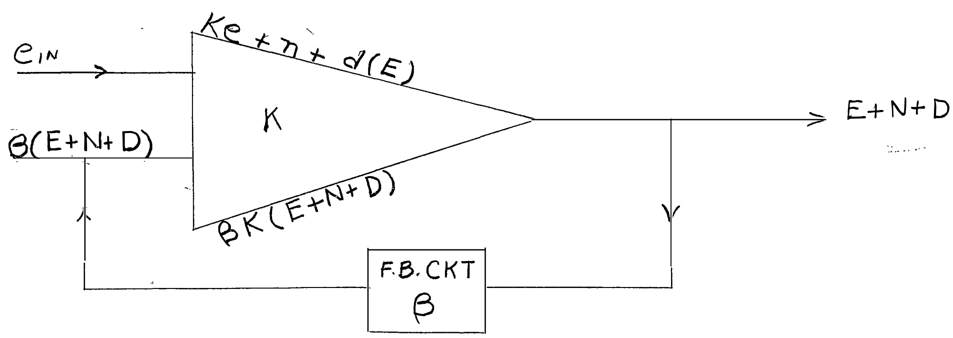
[3]Refer to Appendix A for a brief treatment of stability theory. (7, 15)

β -circuit, is neither appreciably improved nor degraded since increasing the loss in the β -circuit raises the gain by an amount almost equal to the loss. This demonstrates how the stability of amplification follows when stabilized feedback is applied to an amplifier.



Where:

e = Signal voltage

K = Amplification Factor of Amplifier

Ke = Amplifier output without feedback

n = Noise output without feedback

d(E) = Distortion without feedback

β = Propagation of feedback circuit

E = Amplifier output with feedback

N = Noise output with feedback

D = Distortion with feedback

Figure 2. Black's basic feedback amplifier defining the various voltages.

System Design

In a paper, published in 1937, Terman (5) provides a system for electronic feedback amplifier design. He demonstrates how distortion and noise are reduced by the use of feedback. Included is a simple, concise criterion for stability which will be quoted here, "When the value of A (A is Terman's symbol for amplification factor) and its conjugate are plotted as a function of frequency on rectangular co-ordinates with the real part along the x-axis and the imaginary part along the y-axis, oscillations will not occur and the system will be stable if the resulting curve does not enclose the point 1,0."

More recent information on system design was given by Mayr (6) when he concluded his work on an investigation of the necessary conditions for flat response in feedback amplifiers. It is customary to treat the performance of feedback amplifiers inside the original band-pass in terms of linearization and outside this band-pass in terms of stability. However, in his paper Mayr includes the whole spectrum from zero to infinity, in terms of frequency response. After deriving a simple general equation which gives the frequency response of the complete amplifier without feedback and of the frequency characteristic of the feedback network, a complete treatment was given for the special case of amplifiers with as many as 4 stages of R-C coupled or tuned circuit coupling, with constant feedback and equal center frequencies for all stages. He concluded with formulas which will provide characteristics and values of the component parts for preselected frequency responses.

System Analysis

Since the feedback principle, which frequently includes differential equations of the second order, offered many opportunities for elegant mathematical expression as well as engineering application, it is no wonder that mathematicians such as Bode have enjoyed a major role in the development of basic feedback theory. In the late '30s Dr. Bode, an associate of Nyquist at Bell, was developing his significant theory. He, too, was concerned with system stability and in ways and means for predicting such stability. The result of this effort was the popular Bode Diagram or asymptotic method for analyzing amplifier operation (7, 8). Refer to Figures 3a and 3b for a simple demonstration of how his system provides gain response of an amplifier. Note that the asymptotic construction is accomplished on semi-log paper. The gain could just as easily have been plotted as values of the voltage output over inout ratio provided a logrithmic axis were used.

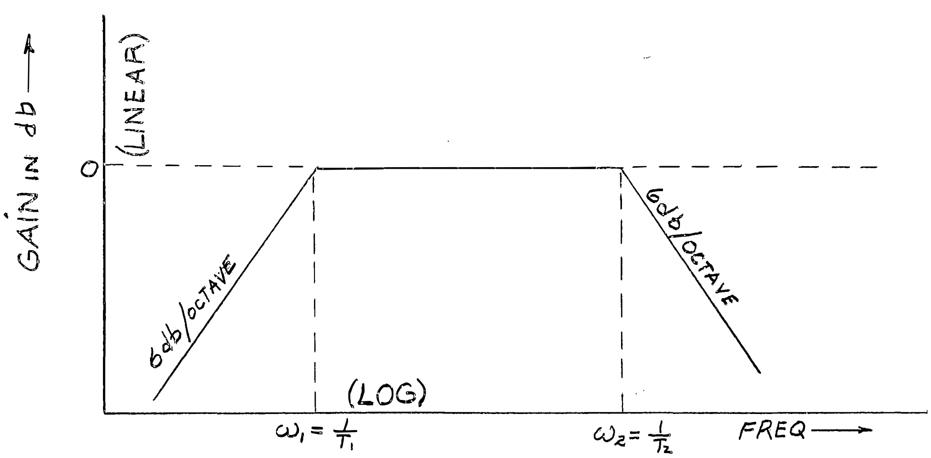


Figure 3a. The asymptotic plot of the gain response of an amplifier with a transfer function of $\frac{e_o}{e_i} = \frac{1 + jT_1\omega_1}{1 + jT_2\omega_2}$

Thus we have an asymptotic representation of the response of the device whose transfer function was given. To fill in the actual gain response it is merely necessary to place a point 3 db down from both corners of the transfer function plot and 1 db down 1 octave away from each corner and then connect these points asymptotically with the straight portions of the plot, Figure 3b.

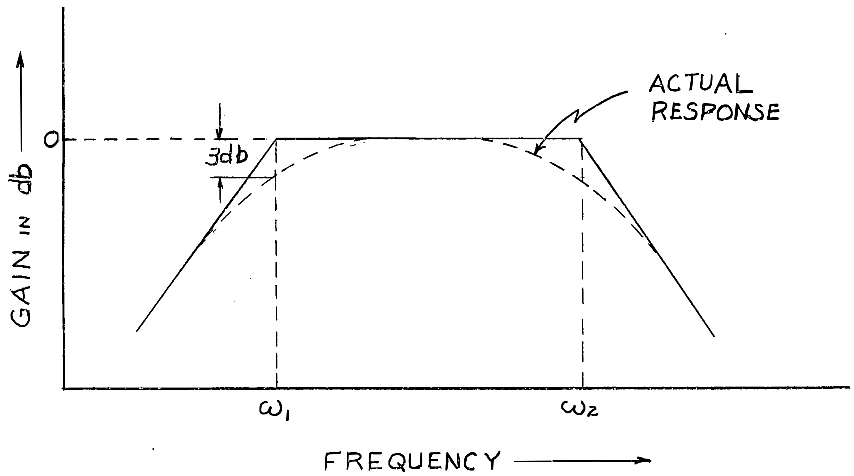


Figure 3b. The actual plot of amplifier gain obtained from the asymptotic plot.

Phase plots are obtained with equal ease. The angle at each corner of the transfer function plot is 45 degrees either leading or lagging depending upon the relative values of the Time Constants involved. In addition, at frequencies well above and below the corner frequency the angle becomes 90 degrees for that corner. Thus for our given transfer function we obtain the phase response as shown in Figure 4.

While Bode's interest was primarily of a mathematical nature, his diagrams were a boon to the simplification of of amplifier response analysis. In 1945 he published his classical treatment of passive and active network analysis and applied these principles to feedback amplifiers. This includes not only the basic network equations, but also the tools for investigating system stability, namely the poles and zeros concept.[4] While his treatment is very complete, some consider it somewhat complex from the mathematical aspect. Its true worth lies in the fact that it provides rigorous proof for the working premises of feedback theory as it is applied by the engineer.

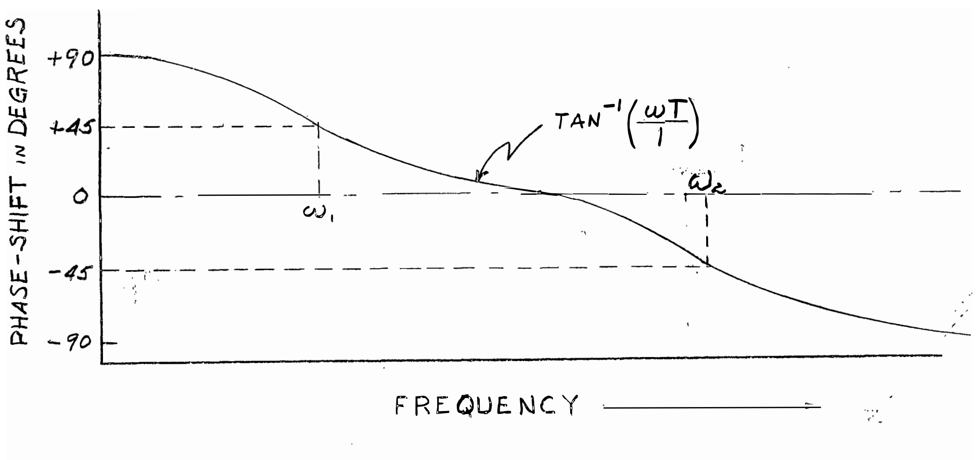


Figure 4. A phase-shift plot obtained by Bode's Method.

Positive and Negative Feedback Combined

In 1950 a very significant development was made by Miller (2) in which positive feedback was used in conjunction with negative feedback to reduce the attenuating effect of the negative return voltage. He designed a simple

[4]The zeros and poles concept is included in the stability discussion in Appendix A. (9)

two-stage audio amplifier using a combination of local positive feedback in the first stage and a moderate amount of overall negative feedback to approximate the results obtainable from conventional amplifiers using 25 db negative feedback. Figure 5 shows his basic circuit in schematic form.

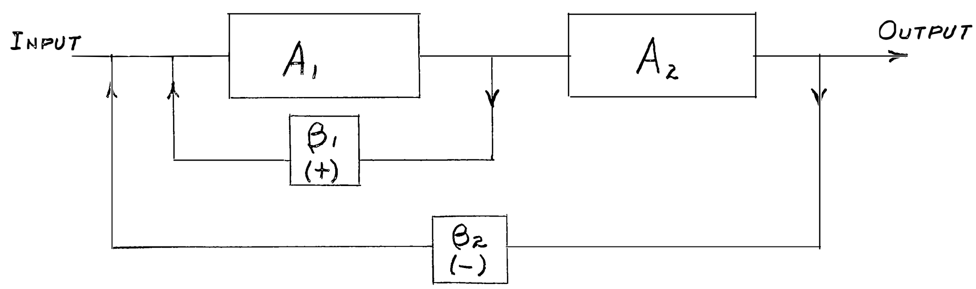


Figure 5. A block diagram of positive and negative feedback combined.

From Figure 5 it can be seen that,

$$\text{Amplification overall} = A = \frac{A_1 A_2}{1 - A_1\beta_1 - A_1 A_2 \beta_2} \cong A' = \frac{A}{1 - \beta A}$$

where $\beta A \cong A_1\beta_1 + A_1 A_2 \beta_2$

For optimum results $A_1\beta_1$ should be made equal to 1 over the useful range of frequencies.

$$\text{Then,} \qquad A = \frac{A_1 A_2}{1 - 1 - A_1 A_2 \beta_2} = -\frac{1}{\beta_2}$$

Very good results can still be obtained if $A_1\beta_1$ varies from unity by ± 20%. In order to avoid oscillations the feedback factor

$$A\beta = A_1\beta_1 + A_1 A_2 \beta_2$$

must be less than positive unity at all times. So if $A_1\beta_1 = 1$ then $A_1 A_2 \beta_2$ must be less than 0. But this cannot be done, so $A_1\beta_1$ must be other than unity at frequencies where $A_1 A_2 \beta_2$ is positive. He concludes his presentation with a discussion of his circuit for a two-stage power amplifier. The significance of his work lies in the originality of its premise. While most, if not all, previous feedback developments were devoted exclusively to the inverse type, Miller introduces a workable scheme to obtain the advantage of negative feedback without the accompanying system attenuation previously associated with this type of network.

Subminiturization

Finally, to complete this short resume of feedback theory development, a paper by Slaughter (10) is noted in which the author outlines a feedback stabilized transistor amplifier. Thus we have a relatively old principle being applied to the latest electronic amplification device giving ample evidence that the feedback principle still has much utility.

CHAPTER III

RECENT DEVELOPMENTS IN PHASE MEASURING TECHNIQUES

Electronic Phasemeters

In 1949, Florman (11) brought forth his design for an electronic phase meter. This instrument will read and record directly on calibrated scales the phase angle between two sinusoidal voltages having a voltage range of 1-30 volts over a frequency range of 100-5,000 cycles per second to a sensitivity of 0.5 degrees. The separate sine waves are converted to square waves and applied to two separate phase indicators. One indicator is ambigous about the 180 degree value and measures and records the average of the algebraic sum of the square waves in the separate channels. The other indicator is unambigous, and is operated by differentiating the square waves and applying the proper resultant pulses to fire a trigger circuit of the Eccles-Jordan type. The average plate current in the trigger tubes is directly proportional to the phase angle between the two input voltages. While this system is limited to sinusoidal inputs, the common usage of sine waves in current electronic devices makes this technique very useful.

By way of complementing Florman's method we note one proposed by Kretzmer (12) in which frequencies are extended up into the ultrasonic range. The phase difference between two periodic signals is compared in a flip-flop circuit. The average current at one of the multivibrator plates is measured and, by properly calibrating the meter, phase can be measured directly. The square wave type signal can also be handled by this method by simply differentiating and feeding into the multivibrator. The length of time the multivibrator plates spend in each state determines the average current which gives a direct phase reading.

Null Method

In 1951 a null method of measuring the gain and phase-shift of comparatively low frequency amplifiers was described (13). In this system an alternating voltage is applied to the X and Y deflection plates on a cathode ray oscilloscope. A straight line is obtained, the extremities of which are drawn on the face of the scope. Then a special circuit, Figure 6, is inserted and by realigning the trace with the dots on the tube, i.e. zero phase-shift, the actual phase-shift can be measured by the null method.

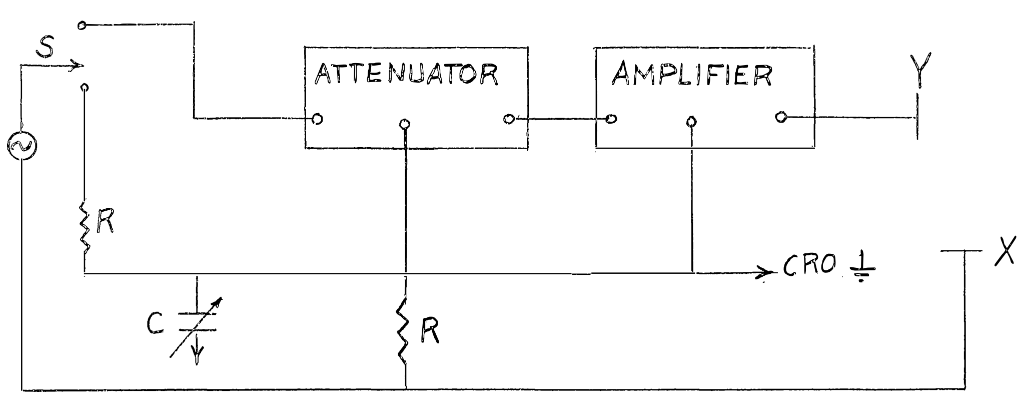


Figure 6. A schematic diagram for the null method phase measurement.

This arrangement allows for possible phase-shift in the amplifier when test frequencies are too high. The system cancels any phase-shift in the amplifier by introducing an equal and opposite phase-shift into the attenuator. In this way phase-shift as well as gain is measured.

An Indirect Method

A paper describing an indirect method for measuring phase-shift, published in 1955, will conclude this resume. (14) It states that the hunting frequency of a feedback amplifier can be changed by inserting in the loop an adjustable phase-shift network. Values of gain and phase-shift of the original loop at various hunting frequencies can then be calculated rapidly and accurately from the known parameters of the added network, a sample of which is shown in Figure 7.

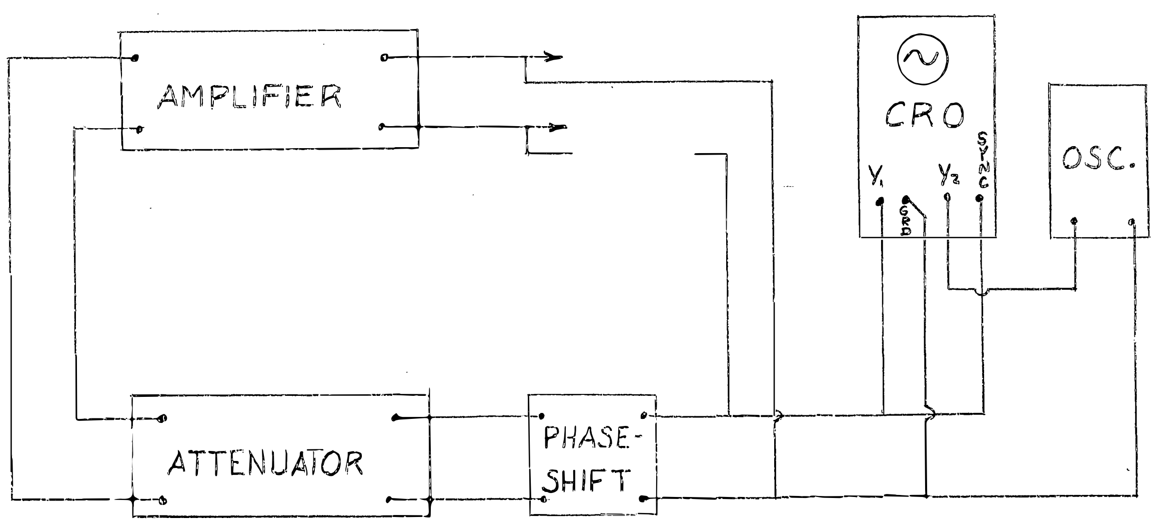


Figure 7. A block diagram of the circuit used in an indirect method of measuring phase-shift.

If a gain control is added to the negative feedback system and the loop gain is increased from zero until hunting just commences, the loop gain is then unity and the loop phase-shift 180 degrees at the hunting frequency. The addition of an extra phase-shift network at some convenient point in the loop will change the frequency of hunt. If the gain control is then readjusted to just hunt, the total loop will once more have unity gain and 180 degree phase-shift. By calculating the phase-shift of the known inserted network and subtracting from 180, the shift of the given system is obtained. The audio oscillator is used to feed a frequency signal to the dual beam cathode ray oscilloscope in order to measure the frequency of hunting.[5]

[5] An electronic switch might be used here to eliminate the need for a dual beam cathode ray oscilloscope.

CHAPTER IV

A SIMPLE PHASE MEASUREMENT TECHNIQUE

General

The forgoing resume of phase measurement methods is admittedly brief and incomplete. However, these systems are representative of those in common practice. Further investigation of available literature on the subject of audio phase measurement reveals that a truly simple system which uses only commonly available equipment is lacking. This thesis proposes a technique which uses nothing more complicated than a good quality vacuum-tube-volt-meter such as a Hewlett-Packard Model 400-C, an audio oscillator and the choice of either a short graphical construction or a brief mathematical calculation.

Proposed Procedure

The amplifier to be tested, as shown in Figure 8, is equipped with a resistive feedback loop and a switch to permit opening and closing this loop. The phase-shift between input and output for a particular frequency is obtained by the following steps:[6]

1. Measure the input voltage, which can be set for some convenient multiple of unity, with the

[6]For complete details on this procedure refer to Appendix D.

switch open.

2. Measure the output voltage.

3. Close the switch and repeat steps 1 and 2.

4. Divide the ratio of the output voltage to input voltage in steps 1 and 2 by the ratio of output voltage to input voltage in step 3. If the input is maintained at the same value for steps 2 and 3 this allows step 4 to be simplified and it is merely necessary to divide the output from step 2 by the output from step 3. This provides a number we shall designate A.

5. Locate the mid-band of the amplifier by varying the frequency of the oscillator and noting the band of frequencies over which the output of the amplifier is constant.

6. Measure the output of the amplifier at mid-band with the switch open. Note: It is recommended, for the sake of simplicity, that the input under all conditions of frequency and for both positions of the switch be maintained at one constant value.

7. Measure the output of the amplifier at mid-band with the switch closed.

8. Substract the ratio of input over output in step 6 from the ratio of input over output in step 7. This gives a number we shall designate B.

9. Multiply B times the ratio of voltage output over input obtained in step 6, and designate C.

10. On a piece of standard polar graph paper such as K&E Polar Coordinate #358-31, strike an arc from the point -1,0, of length A, as in Figure 9.
11. On the same piece of polar graph paper, strike an arc from the origin of length C, as in Figure 9.
12. The intersection of these two arcs establishes the angle α . This angle is the phase-shift for the amplifier without feedback for that particular frequency.

A mathematical expression, which will give the same result, is simply derived from the law of cosines and is:

$$\alpha = 180 - \cos^{-1}\left[\frac{C^2 + 1 - A^2}{2C}\right]$$

The phase response throughout the frequency spectrum is obtained for any number of desired frequencies by repeating this procedure. Steps 5 through 8 will be performed only once as the value B has been purposely made frequency insensitive and need not be changed. Once B has been obtained it will be used throughout the analysis, assuming no changes in the amplifier.

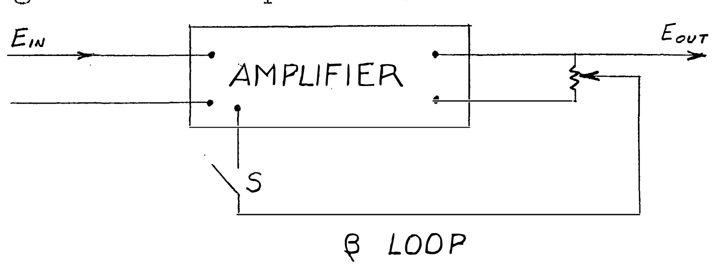
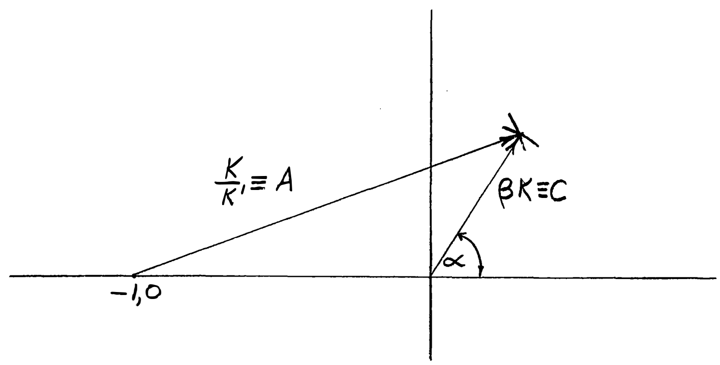


Figure 8. A block diagram of the amplifier being analyzed.



Where: K = Amplification without feedback

K' = Amplification with feedback

β = Fractional propagation of the feedback circuit

α = The desired phase-shift

Figure 9. The graphical method of obtaining the phase-shift for a particular frequency and the relationship between the numbers A and C and their physical equivalents.

CHAPTER V

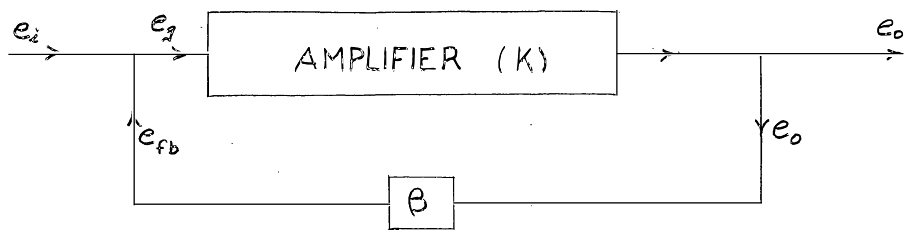
JUSTIFICATION FOR PROPOSED TECHNIQUE

Mathematical Verification

The mathematical development, which will be used to verify the proposed technique, will be essentially a vector analysis. Since a vector is a representation of a physical quantity which has both magnitude and direction, a voltage or voltage ratio with a particular phase angle attached can be so representated. Figure 10 illustrates the fundamental elements of a feedback amplifier. Each element is a physical quantity and is directed, phase-wise. Therefore, each element must be treated as a vector quantity.

The basic premise of this thesis, and the principle that is to be mathematically demonstrated, is that if an arc of vector length corresponding to the ratio K/K' is drawn from the point -1,0 in the Cartesian plane it will intersect an arc of vector length corresponding to the value βK in such a manner as to define an angle α . Furthermore, this angle is the phase-shift, for some particular frequency, of the open loop amplifier from which the values K, K', & β were obtained.

By referring to the definition of a βK-Plot and to Figure 11, it is seen that if a point on the βK-Plot can



Where:

β = Feedback Factor = $\frac{e_{fb}}{e_o}$

e_g = Actual Amplifier Input = $e_i \pm e_{fb}$

K = Amplifier Amplification without Feedback = $\frac{e_o}{e_g}$

K' = Amplifier Amplification with Feedback = $\frac{e_o}{e_i \pm e_{fb}}$

Now $e_o = K e_g$

Then $\frac{e_o}{K} = e_i + e_{fb} = e_i + \beta e_o$

Or $\frac{e_o}{e_i} = \frac{K}{1 - \beta K}$

If $\frac{e_o}{e_i} = K'$ then $\frac{K'}{K} = \frac{1}{1 - \beta K}$

Or $\frac{K}{K'} = 1 - \beta K$ {General Feedback Expression}

In the case of negative feedback, the β-factor is negative in sign, giving

$\frac{K}{K'} = 1 + \beta K$ {Negative Feedback Expression}

Figure 10. A block diagram of the basic amplifier circuit and the derivation of the general feedback expression.

be located for any one frequency, this point will yield an angle, we shall call α . Since, in this case, the quantity β was made to have no angle of its own, α is a property of K, the amplification factor of the open loop amplifier. Thus, since K is the open loop ratio of output voltage over input voltage, α is the open loop phase-shift of the amplifier.

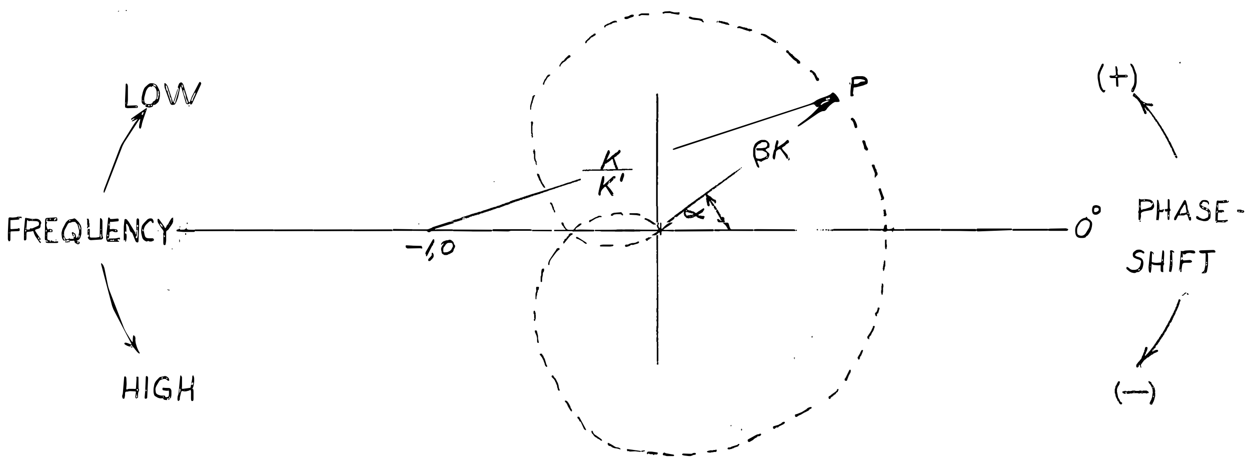


Figure 11. A typical β K-Plot showing the component vectors K/K' and β K.

We reason that, for the case of negative feedback, if;

$$\frac{K}{K'} = 1 + \beta K$$

then

$$\frac{K}{K'} - 1 = \beta K$$

These vector quantities may be manipulated in this manner because the unit vector has an associated angle of zero for all values of K,K' and β .

Now, if P is any arbitrary point on the β K-Plot, as in Figure 12a, the β K-vector, which established this point, can be resolved into its component parts bearing in mind the relationship $\beta K = K/K' - 1$. One component is -1 while the other component is K/K'. This resolution, as shown in Figure 12b, creates two vectors of lengths 1 and K/K' which originate from the point -1,0.

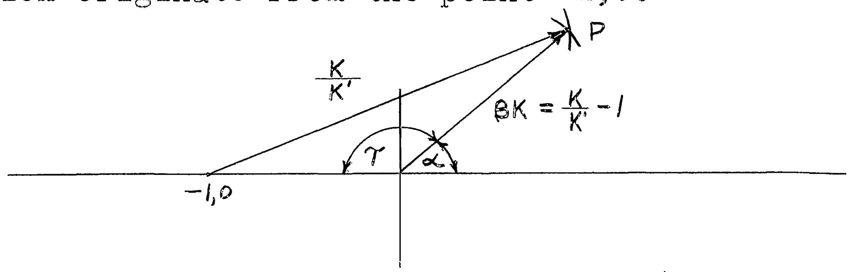


Figure 12a. The manner in which one arbitrary point P is located on the β K-Plot.

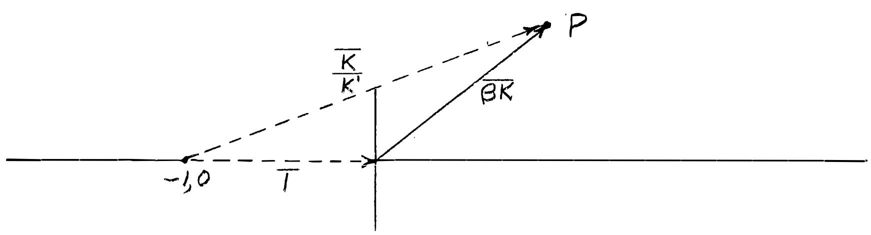


Figure 12b. The resolution of vector β K into its component parts.

Now the unit vector is necessarily fixed but the K/K' vector and the βK vector are variable and depend upon relative values of K,K' and β . However, we have shown that whatever their lengths they will always have a common point, namely point P which lies on the β K-Plot at the intersection of the two vector arcs. Therefore, for the particular arrangement of K,K' and β which corresponds to P, an angle α is located. Since K and K' are functions of frequency it is logical to conclude that the point P corresponds to some particular frequency. Thus, if a point P is found for each frequency, it will define a β K vector plot referred to the axis origin and the angle associated with each β K vector will be the open loop phase-shift of the amplifier.

The mathematical formula, which will yield the same angle as the graphical construction, is developed as follows: Refer Figure 12a;

$$\alpha = 180 - T$$

By the Law of Cosines

$$\left|\frac{K}{K'}\right|^2 = |\beta K|^2 + 1 - 2\,|\beta K| \cos T$$

From which

$$T = \cos^{-1} \frac{|\beta K|^2 + 1 - |K/K'|^2}{2\,|\beta K|}$$

$$\therefore \quad \alpha = 180 - \cos^{-1} \frac{|\beta K|^2 + 1 - |K/K'|^2}{2\,|\beta K|}$$

Therefore, the proposed technique has a foundation in mathematics. It will next be shown that it also has a foundation in physical reality, through the medium of an experimental verification.

Experimental Verification

In order to achieve an element of generality in obtaining laboratory data, parameters of the experimental audio amplifier were changed during the investigation, effectively creating two separate resistance-coupled audio amplifiers which are designated Amp-1 and Amp-2. Each amplifier was analyzed separately and has its own set of response data.[7]

The results of the experimental investigation were completely positive for both amplifiers.[8] With minor exceptions, as seen on Figures 13 and 14, the accuracy of the proposed method, hereon referred to as the graphical method, was well within the accuracy of the instruments used in measuring voltages and phase angles. Where no variance between actual and graphical values is indicated they differ by only ± 2 degrees or less. The variance seen between 1 kc and 10 kc is still within meter accuracy but is noted here to show by comparison how accurate the technique can be. The variance in the region from 20 kc to 60 kc, while not large, is probably due to a small amount of phase-shift being introduced by the feedback loop at those relatively high frequencies. The differences are not significant and do not affect the accuracy of the overall method. It is encouraging, from the servo-mechanisms aspect, to achieve such accuracy at low frequencies, for if the advantages of this technique are to be fully exploited, an application to servo problems is necessary.

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[7]For specifications of both amplifiers refer to Appendix B.

[8]For full details on the experimental procedure refer to Appendix C.

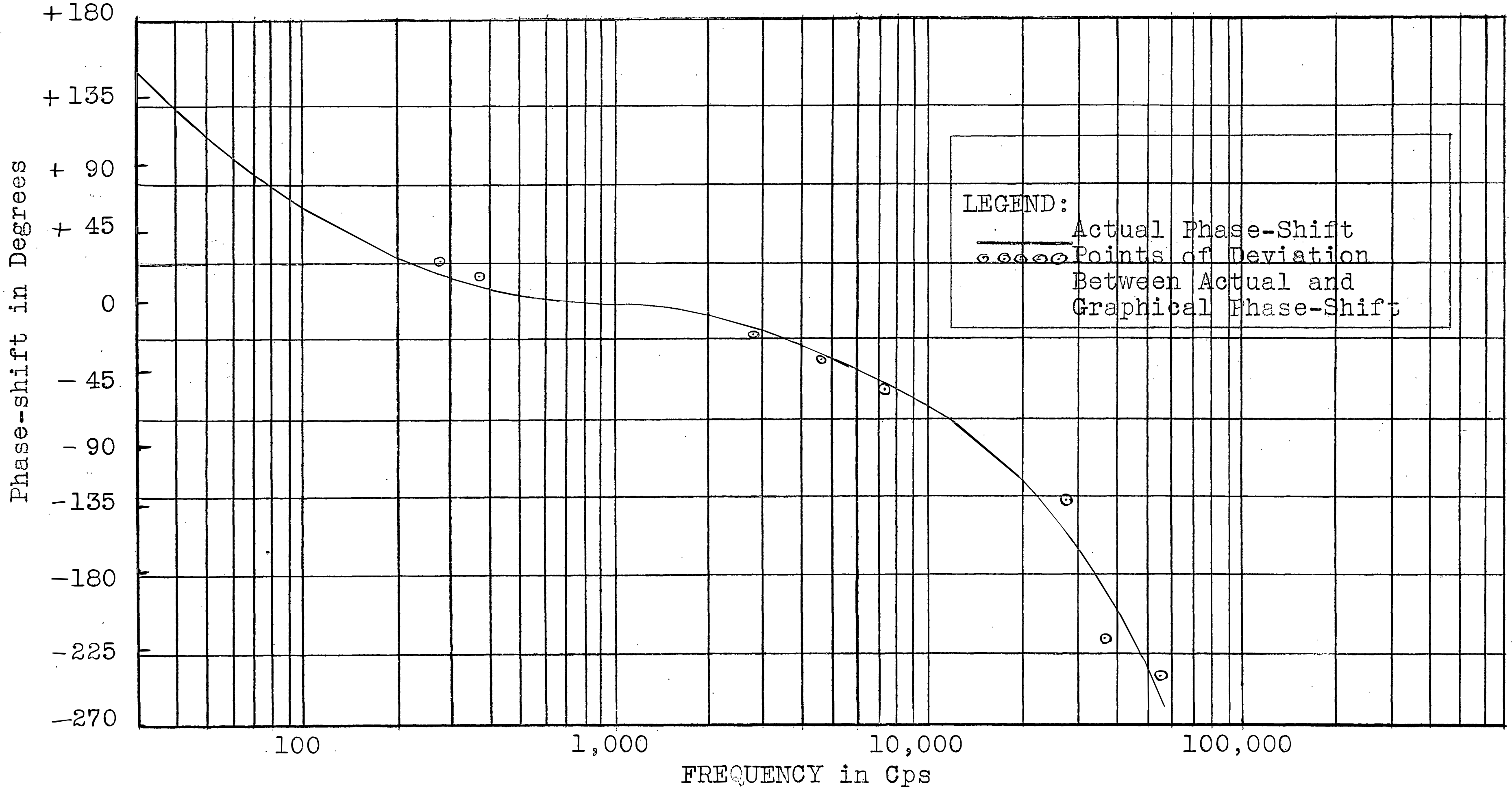

Figure 13. Comparison between the actual and the graphical phase-shift for Amp-1.

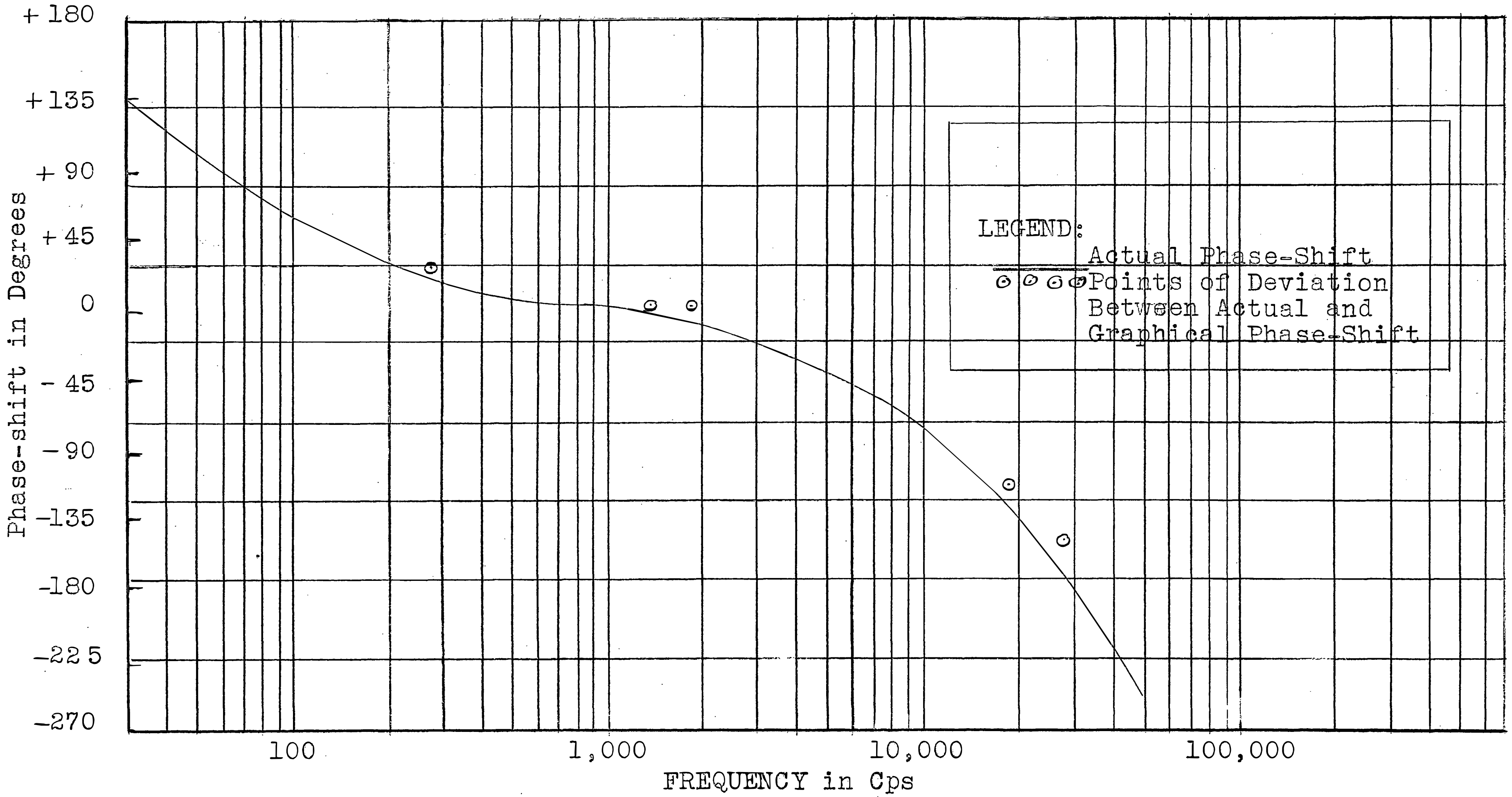

Figure 14. Comparison between the actual and the graphical phase-shift for Amp-2.

CHAPTER VI

SUMMARY AND CONCLUSIONS

Summary

The objective of this thesis was to present a simple measurement procedure which would aid the engineer in analyzing the phase-shift of a low frequency amplification system. This has been accomplished using an electronic audio amplifier, first by giving a detailed description of the method, and secondly, by justifying the method mathematically and experimentally.

A brief history of the fundamental theory involved was given along with a resume of some other methods of phase analysis. The former was intended to acquaint the reader with the basic principles of feedback amplifiers, while the latter provided a means for comparing the simple method contained herein with other methods in common use at the present time.

Qualifications and Limitations

This technique has limitations as do most measuring operations. To begin with, it is primarily intended for use in the audio frequency band (0 - 20,000 cps). At frequencies of a higher order capacitive coupling between amplifier and feedback loop is entirely possible and this would introduce error into the phase angle data. Furthermore, inductive and capacitive effects in the resistive elements of the feedback network could provide another source of error at extended frequencies.

Another limitation is inherent in the nature of the graphical construction which makes it very difficult to obtain accurate phase angles in the immediate vicinity of mid-band. This is principally due to the limitations of the VTVM used in measuring output voltages. The values of the two vectors, and thus the values of the output voltages, must be very precise in order for them to intersect properly in this region since the arcs are approaching each other tangentially. Figure 15 clearly demonstrates this situation and also shows the difficulty experienced in determining the exact point of intersection near mid-band. It is recommended, therefore, that the mathematical equation be used in conjunction with the graphical construction when arcs become overlapping. This will minimize construction errors but will not eliminate the errors completely because, as has been stated, the VTVM does not usually possess sufficient accuracy. Thus, while values of phase-shift from plus 10 degrees to minus 10 degrees will not ordinarily be obtainable with especially good accuracy, it must be remembered that the mid-band phase-shift is of very little importance. In the first place, phase-shift at mid-band is quite small in relationship to the rest of the spectrum, and secondly, stability is of no concern in that range of frequencies because the feedback will assuredly be negative by virtue of the circuit arrangement. Therefore this limitation is of very minor importance for most applications.

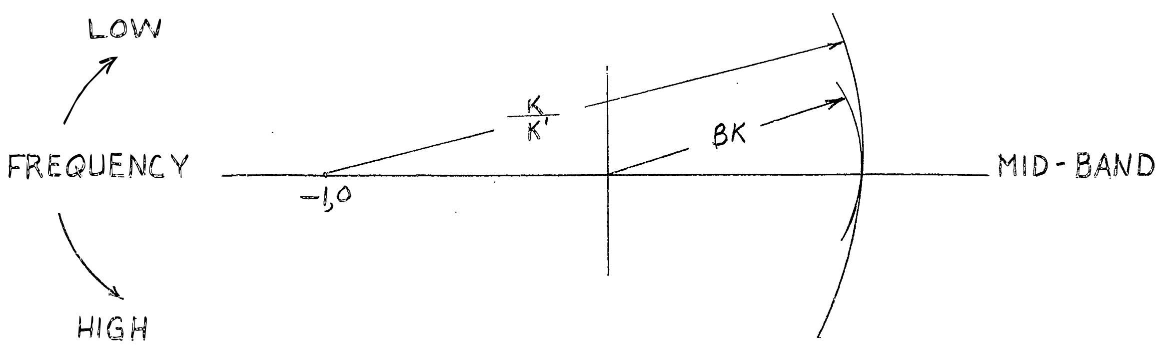

Figure 15. The intersection of the two vectors is difficult to determine in the immediate vicinity of mid-band.

Finally, there is a degree of ambiguity in the method in as much as the two vector arcs will intersect at two points for most frequencies. This is illustrated in Figure 16. Therefore, the operator must be able to interpret these points. This is not difficult, when analyzing a conventional R-C coupled audio amplifier, if it is realized that in the low half of the spectrum angles will lead and be positive and in the high frequency half the opposite is true. If the angle is determined to be either + 240 or + 120 degrees, for example, another reading for a neighboring frequency must be taken to establish the correct angle. If the frequency is increased and the new angle is seen to be either plus 100 degrees or plus 260 degrees then the original reading is plus 120 degrees and the second reading is plus 100 degrees because it is known that the phase-shift decreases as the frequency increases, up to mid-band at which point it reverses. Once this ambiguity has been resolved for any two adjacent frequencies it will cause no further question throughout the analysis.

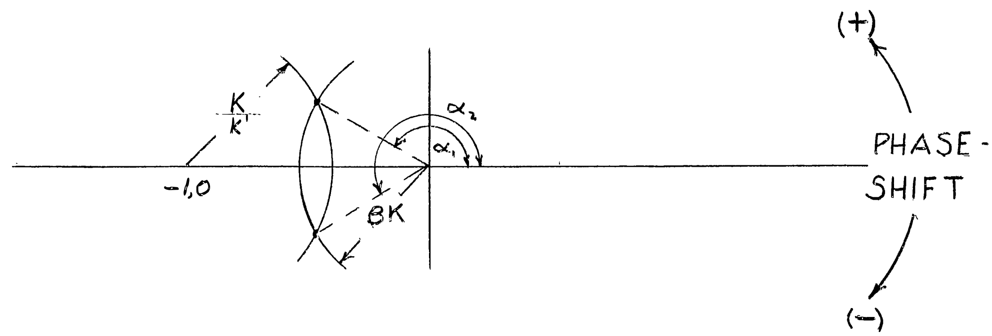

Figure 16. An ambiguity is caused by the double intersection of the two vector arcs.

Conclusions and Recommendations for Further Study

The proposed method for amplifier phase analysis has been conclusively substantiated. Laboratory comparison between values obtained from a commercial phasemeter, an inexact norm at best, and values obtained graphically are in very close agreement. Therefore, it is concluded that, subject to certain limitations, the method is as accurate as the data upon which it operated. Furthermore, it can be used to make a spot check of three of four angles each side of mid-band or it can just as easily and with little extra effort provide a complete phase response for the entire spectrum. This makes it an extremely useful device.

Since many recent developments indicate that the electromechanical analogy is applicable to servo-amplifiers to the extent that certain principles of the electronic amplifier, within limits of frequency, are directly analogous to the servo-amplifier, a simple method of determining the phase-shift of an audio amplifier could be directly applied to the evaluation of the phase relationships in the servo-mechanism. Therefore, it is recommended that the simple measuring technique, proposed herein, be considered merely as a tool to be used in future investigations concerning low frequency amplification systems. To do this it will be necessary to establish the exact nature of the analogy as it applies to audio- and servo- amplifiers. A sample of the questions related to servo-amplifiers which need to be answered are:

1. What band of frequencies are most common and over how much of this band does each system operate?
2. What type of feedback loop could be used which would be completely insensitive to frequency?
3. How and where would the feedback loop be inserted for data collection purposes?

Armed with the answers to these and other questions, this simple phase analysis method could be effectively applied, by the engineer, to both design and field problems.

In either case the operation could be performed with complete confidence, and with less equipment and time than that required by other known methods.

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APPENDIXES

APPENDIX A

STABILITY THEORY

Methods for Determining System Stability

The following methods are of classic mathematical form. Distinct from this form is the Laplace Transform approach, which will be treated later in this section. The classic methods are concerned with:

1. Determining the roots of the Characteristic Equation.
2. Determining the sign of the roots of the Characteristic Equation by Routh's Criterion.
3. Determining the complex-plane plot and applying a generalized form of Nyquist's Criterion.

In order for any amplification system, meaning one containing active elements, to be stable it is not only necessary that it be stable in normal or steady state operation, but also that any transient disturbances decay quickly enough to permit rapid recovery by the overall output. When using the classical or Heuristic Method to solve differential equations it is necessary to select a function of current and/or voltage to represent the steady state and transient currents and voltages, for example $I_n \varepsilon^{p_n t}$, $E_n \varepsilon^{p_n t}$ or $C_k \varepsilon^{s_k t}$ where p_n and s_k represent the roots of the Characteristic Equation. For the transient terms to vanish with increasing time it is necessary that the real part of all the roots of the Characteristic Equation be negative in sign. This is the mathematical condition for stability. The magnitude of the real part of the roots of the Characteristic Equation is inversely proportional to the time required for the transient disturbance to decay to zero. Therefore the farther the roots lie to the left of the imaginary axis, the greater will be the stability as shown in Figure A-1.

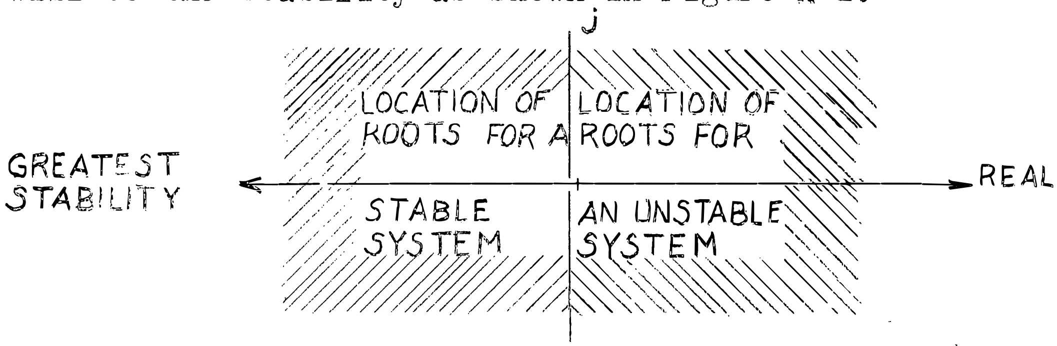


Figure A-1. Stability or instability will be greater or less depending upon the relationship between the roots of the characteristic equation and the complex plane.

The Characteristic Equation

In the network solution of differential equations, using the so-called Heuristic approach, it is required to judiciously choose values for the unknown functions such as current i(t), and voltage v(t). In as much as a preponderant number of current and voltage waveshapes can be expressed in an exponential form, let

$$i(t) = I\varepsilon^{pt}$$

then
$$\frac{di}{dt} = pI\varepsilon^{pt}$$

and $$\frac{di^2}{dt} = p^2 I \epsilon^{pt}$$

Similiarly, $$\int i(t) = \frac{I}{p} \epsilon^{pt}$$

and if the differential equation is then expressed as

$$a_0 \frac{d^n i}{dt^n} + a_1 \frac{d^{n-1} i}{dt^{n-1}} + \ldots + a_{n-1} \frac{di}{dt} + a_{n+1} \int i dt + \ldots + a_{n+q} \int^q i dt = f(t)$$

then by using the value of i(t) selected above, it can be written

$$\left(a_0 p^n + a_1 p^{n-1} + \ldots + a_{n-1} p + a_n + \frac{a_{n+1}}{p} + \ldots + \frac{a_{n+q}}{p^q}\right) I \epsilon^{pt} = f(t)$$

To reduce this equation it is set equal to zero, which is valid because the form of the transient response is independent of the applied voltage v(t). This gives,

$$\left(a_0 p^n + a_1 p^{n-1} + \ldots + a_{n-1} p + a_n + \frac{a_{n+1}}{p} + \ldots + \frac{a_{n+q}}{p^q}\right) I \epsilon^{pt} = 0$$

from which the Characteristic Equation is obtained by multiplying through by a power of p large enough to remove all p's from the denominators,

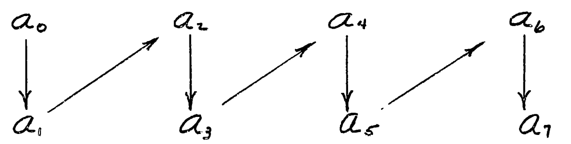
$$a_0 p^{n+q} + a_1 p^{n+q-1} + \ldots + a_{n-1} p^{1+q} + a_n p^q + a_{n+1} p^{q-1} + \ldots + a_{n+q} = 0$$

Routh's Criterion for Stability

Given a Characteristic Equation such as

$$a_0 s^n + a_1 s^{n-1} + a_2 s^{n-2} + \ldots + a_{n-1} s + a_n = 0$$

where $a_0, a_1, \ldots a_{n-1}, a_n$ are the coefficients of the powers of s, s being a transformation of p, and all coefficients of the decreasing powers of s from n to 0 are present. (If all the descending coefficients are not present, the system is unstable.) Let n equal 7, for example, and arrange the coefficients according to this array,



From this array calculate the remaining terms,

$$
\begin{array}{llll}
a_0, & a_2, & a_4, & a_6, \\
a_1, & a_3, & a_5, & a_7 \\
b_1, & b_3, & b_5, & \\
c_1, & c_3, & c_5, & \\
d_1, & d_3, & & \\
e_1, & e_3, & & \\
f_1, & & & \\
g_1, & & &
\end{array}
$$

where: $b_1 = \dfrac{a_1 a_2 - a_0 a_3}{a_1}$; $b_3 = \dfrac{a_1 a_4 - a_0 a_5}{a_1}$

$c_1 = \dfrac{b_1 a_3 - a_1 b_3}{b_1}$; $d_1 = \dfrac{c_1 b_3 - b_1 c_3}{c_1}$; etc.

Now Routh's Criterion is applied, which states that the system so represented by the Characteristic Equation is stable if all the terms in the left column of the array have the same algebraic sign. If all the signs in the left column are not the same, the number of changes of sign indicates the number of roots with positive real parts. For example, If the Characteristic Equation is,

$$s^4 + 2s^3 + 8s^2 + 4s + 3 = 0$$

then the array would be,

$$\begin{array}{lll} 1 & 8 & 3 \\ 2 & 4 & \\ 6 & 3 & \\ 3 & & \\ 3 & & \end{array}$$

and the given system is stable.

On the other hand, if the Characteristic Equation is,

$$s^4 + 2s^3 + s^2 + 4s + 2 = 0$$

then the array would be,

$$\begin{array}{lll} 1 & 1 & 2 \\ 2 & 4 & \\ -1 & 2 & \\ 8 & & \end{array}$$

and the given system is unstable. Furthermore, since the sign changes two times, there are two roots with positive real parts. The reader is referred to the literature for the Nyquist Criterion as it does not lend itself to a resume of this type (3).

Stability Analysis Using Laplace Transforms

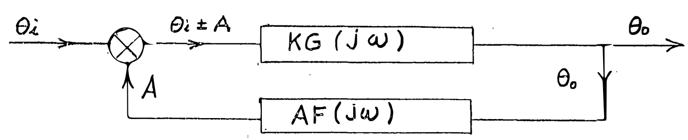
The Laplace Transform approach to stability analysis incorporates several advantages over the classical method. For instance, transient and steady state components may be handled simultaneously and thereby separate operations like obtaining the Characteristic Equation and the Particular Integral are avoided with an appreciable reduction in labor. In addition, the mathematical operations are simplified due to a characteristic of the transform which reduces the differential equation to an algebraic equation, which is easily manipulated and then retransformed to give a complete solution. Lastly, in a pure stability analysis, the Laplace method makes it possible to determine stability without retransforming thereby providing additional simplification. This last process is commonly referred to as the poles and zeros concept. Referring to Figure A-2 it will be seen that the important factor in stability considerations is the loop transfer function, which is KG(jw)AF(jw). That this is a correct interpretation may be shown from the frequency-response equation

$$\frac{\theta_o}{\theta_i}(j\omega) = \frac{KG(j\omega)}{1 + KG(j\omega)AF(j\omega)}$$

If the system is absolutely unstable and linear, then $\frac{\theta_o}{\theta_i}(jw) \longrightarrow \infty$ at some value of jw. This can happen only if the denominator of the equation becomes zero, and for this to be true KG(jw)AF(jw) must become -1.

The mathematical condition for instability is then

$$1 + KG(jw)AF(jw) = 0.$$



where: θ_i = System input expressed in radians

θ_o = System output expressed in radians

KG(jw) = The system transfer function

AF(jw) = The loop transfer function

Figure A-2. Block diagram of basic single loop feedback system.

The equation,

$$1 + KG(j\omega)AF(j\omega)$$

is actually the differential equation of the system and in general addition may be represented by the ratio of two polynomials as shown by the following transformed expression,

$$KG(s) = \frac{\theta_o(s)}{E(s)} = \frac{A(s)}{B(s)}$$

from which is obtained,

$$\frac{\theta_o(s)}{\theta_i(s)} = \frac{\frac{A(s)}{B(s)}}{1 + \frac{A(s)}{B(s)}}$$

Where the denominator is the differential equation of the system and if expanded yields,

$$\frac{B(s) + A(s)}{B(s)}$$

From which the original system equation may be generalized to give,

$$1 + KG(s) = \frac{(s-s_1)(s-s_2)(s-s_3)\ldots}{(s-s_a)(s-s_b)(s-s_c)\ldots} = 0$$

where s_1 , s_2 , s_3 , etc. are roots of the numerator, i.e., the values of s for which the numerator becomes zero; s_a , s_b , s_c , etc., are roots of denominator, and s in general is a complex variable, $s = \Delta + j\omega$. In mathematical terminology s_1 , s_2 , etc., are zeros of the equation and s_a , s_b , s_c , etc., are poles of the equation.

If it is noted that this generalized equation is actually the differential equation of the system, then it is easily seen that the only significant values are the zeros since only these can make the frequency response go to infinity. Furthermore, only those zeros which have positive real parts are of interest since only the roots which have positive real parts can cause instability in a system.

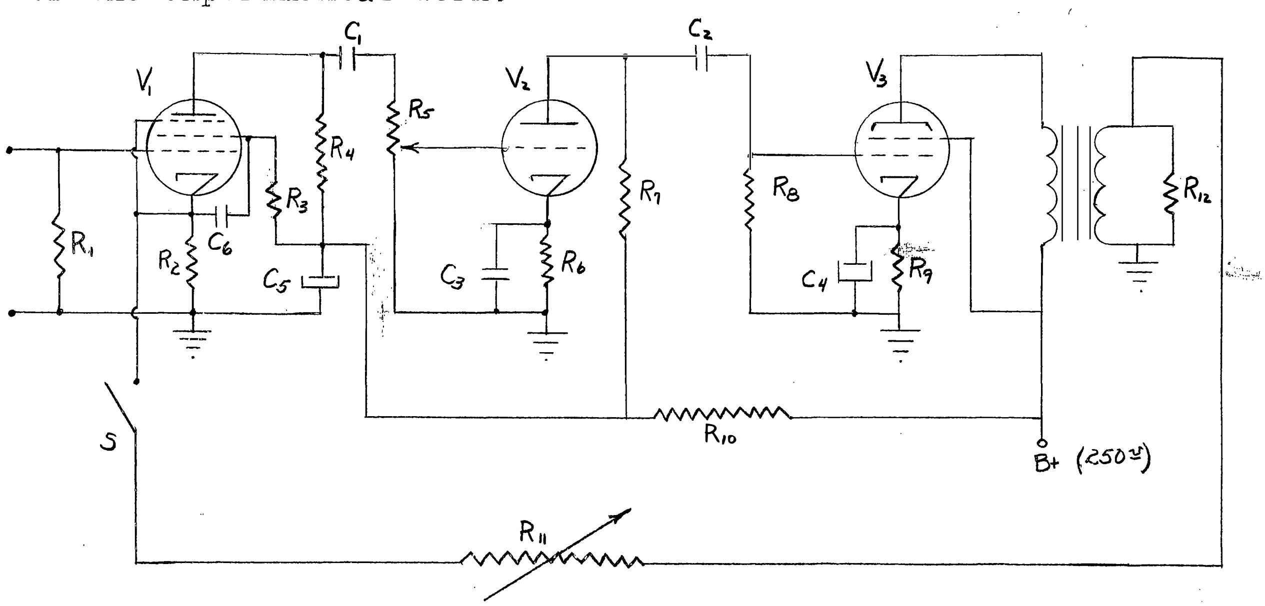
This, of course, is a generalized treatment aimed at analyzing complex feedback systems. In many specific cases it may be possible to analyze the factored Laplace form by inspection. If it can be established that there are no positive roots of s then the system will be stable.

From this brief outline, the advantages of the Laplace Transform approach to system stability analysis becomes quite apparent. And, while these methods, classical and Laplace, do not exhaust all the known processes, they represent the most commonly used techniques in current practice.

APPENDIX B

THE EXPERIMENTAL AMPLIFIER

A detailed analysis of the design of the amplifier used in obtaining experimental verification would not be relevant to the subject. However, a few points of possible interest will be discussed in an effort to present a complete picture of the experimental work.



Where:

R_1 = 500 ohms	C_1 = .02 or .1 micro-farads
R_2 = 500 ohms	C_2 = .02 micro-farads
R_3 = 270k ohms	C_3 = 25 micro-farads
R_4 = 100k ohms	C_4 = 25 micro-farads
R_5 = 500k ohms	C_5 = 8 micro-farads
R_6 = 1k ohms	C_6 = .1 micro-farads
R_7 = 50k ohms	
R_8 = 100k ohms	
R_9 = 160 ohms	
R_{10} = 15k ohms	V_1 - 6J7
R_{11} = 1.5meg ohms	V_2 - 6J5
R_{12} = 500 ohms	V_3 - 6L6

Figure B-1. A schematic of the experimental amplifier.

The amplifier, shown in Figure B-1, is a conventional three stage, R-C coupled, audio design with an untuned output stage. To this was added a simple resistive feedback loop plus a switch arranged to provide inverse feedback when desired. It is worthy to note the method used to couple the feedback signal into the first stage. Since R_2 is not by-passed it will provide a small source of degenerative feedback. However, it was determined that since this feedback would be present both with the switch S open and closed the overall results would not be materially effected. The alternative would have been to by-pass this resistor with S open and use a complex switching arrangement to develop the feedback voltage, with S closed, across some other unbypassed resistor which would then be removed from the circuit for open loop operation.

Capacitors C_1 and C_2 were deliberately made too small for optimum results in an effort to lessen the bandwidth. A wide bandwidth, it was felt, would unnecessarily increase the labor of data accumulation and reduction without providing any compensating advantages. A transformer with poor response characteristics was specified for the same reason.

Three stages were used in order to assure ample gain during feedback operation. A lesser number of stages might have required higher operating conditions per stage with an undesireable increase in distortion.

The value of R_{11} was determined experimentally. Originally it was only 200k ohms but this value resulted in oscillation at low and high frequencies. Figure B-2 illustrates why the oscillations occured when β was made too large as a result of R_{11} being too small. Actually R_{11} is composed of a fixed resistor of 1 meg ohm and a 500k ohm potentiometer in series, making it possible to vary β.

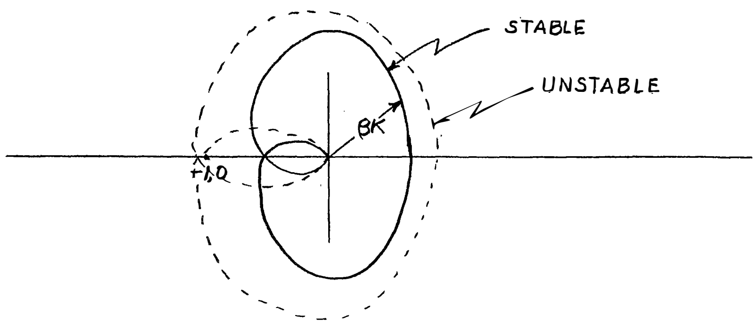


Figure B-2. A typical βK response curve showing instability as a function of β.

The measured gain response of the amplifier, shown in Figures B-3 and B-4, are fairly typical except for a peaking affect at the high end of the frequency spectrum which shows on the plot as a decrease in the slope of the curve above 10 kc. This is attributed to a resonant condition cause by stray capacitance resonating with leakage inductance in the poor quality transformer. The result of this effect is an unintentional improvement in the high frequency response. The overall result is a widening of the bandwidth. In as much as the phase-shift of the amplifier without feedback is the prime purpose of the proposed technique, the response without feedback is the characteristic under discussion. However, as feedback theory indicates, gain response is generally widened by the use of negative feedback. This is clearly shown in Figures

B-3 and B-4, and since the open loop system has a small source of negative feedback in the first stage this might account for some additional widening in the open loop response.

In order to obtain data from two amplifiers, rather than one, the circuit shown in Figure B-1 was designated Amp-1 and by changing C_1 from .02 micro-farads to .1 micro-farads and shunting the input to the last stage by a 500 micromicro-farad capacitor, another amplifier was effectively created and is designated Amp-2. That these changes affected the amplifier with a distinct gain and phase response is clearly shown on Figures B-5 and B-6. While the changes in low response expected from the increase in C_1 is not as apparent as it could be in the gain response, which is probably due to transformer cut-off, the phase-shift response is notably different. Furthermore, the effect of shunting the output of Amp-2 is quite evident in the high frequency region. The gain is seen to drop off, beginning at about 5 kc, which corresponds to the calculated situation.

In any event, the changes are sufficient to cause a difference in the data used in the graphical method and therefore constitute the element of generality sought. It is safe to conclude that any conventional R-C coupled audio amplifier would be analyzed as accurately as were Amp-1 and Amp-2.

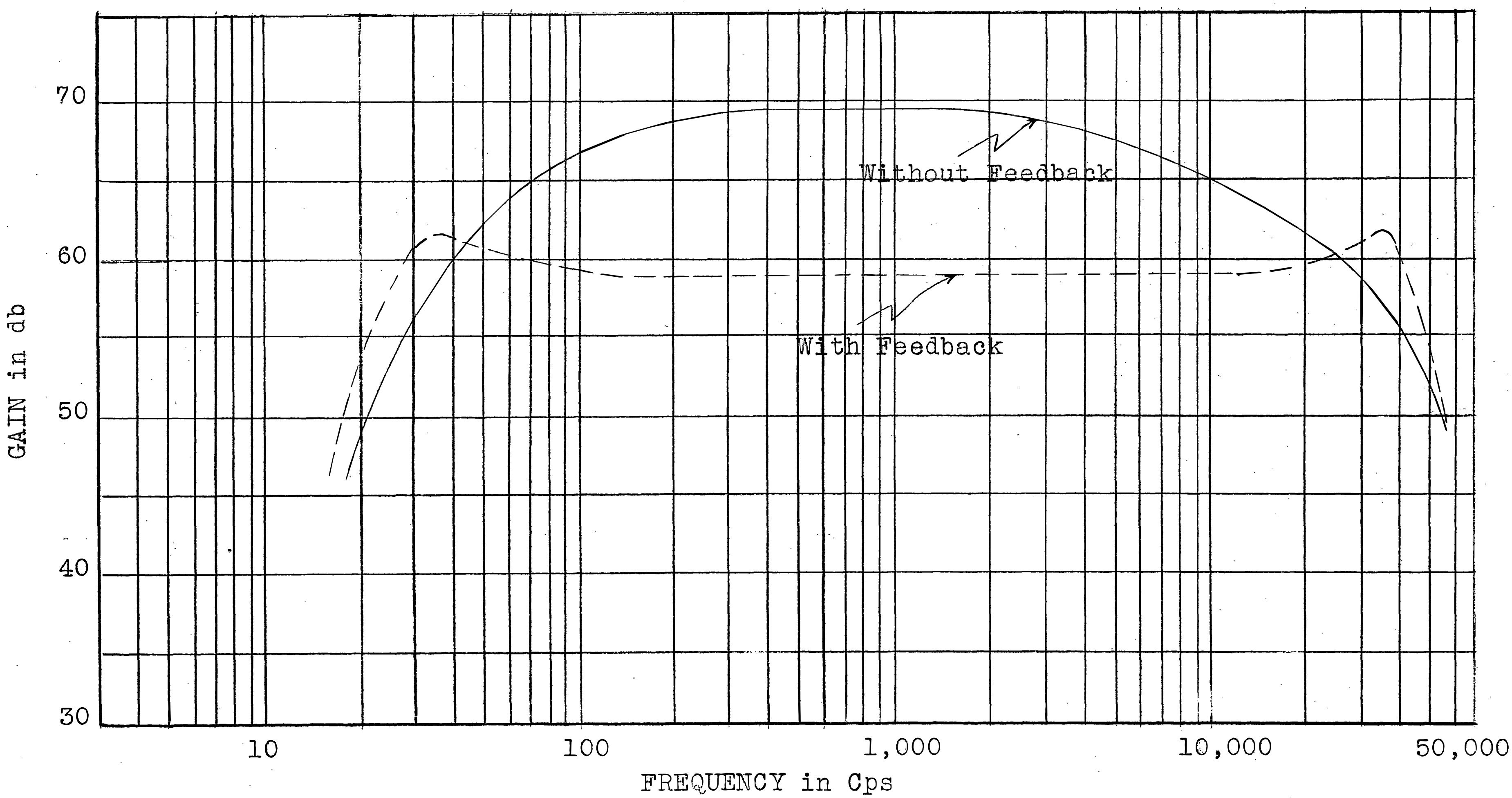


Figure B-3. Gain response, with and without feedback, for Amp-1.

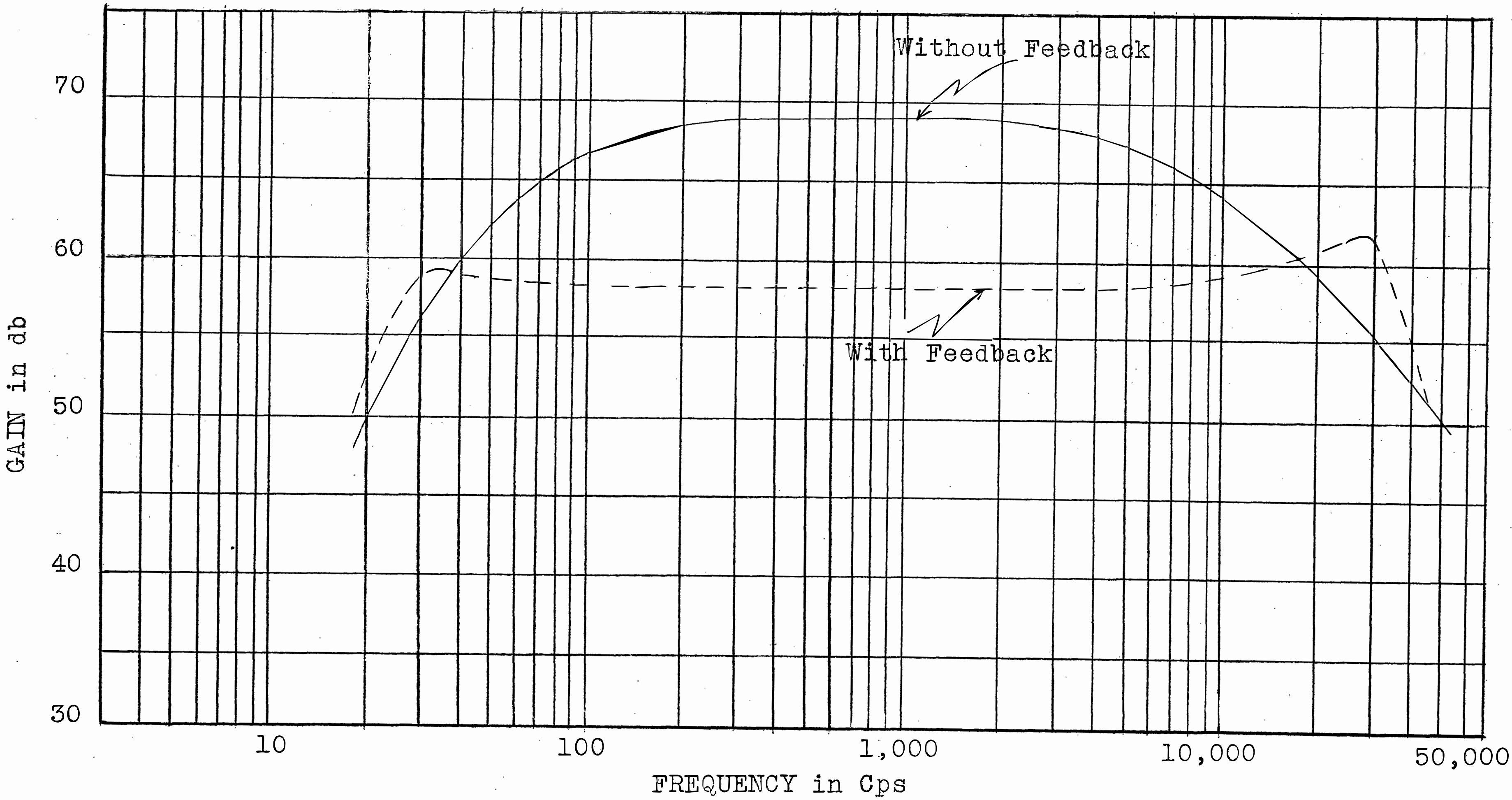


Figure B-4. Gain response, with and without feedback, for Amp-2.

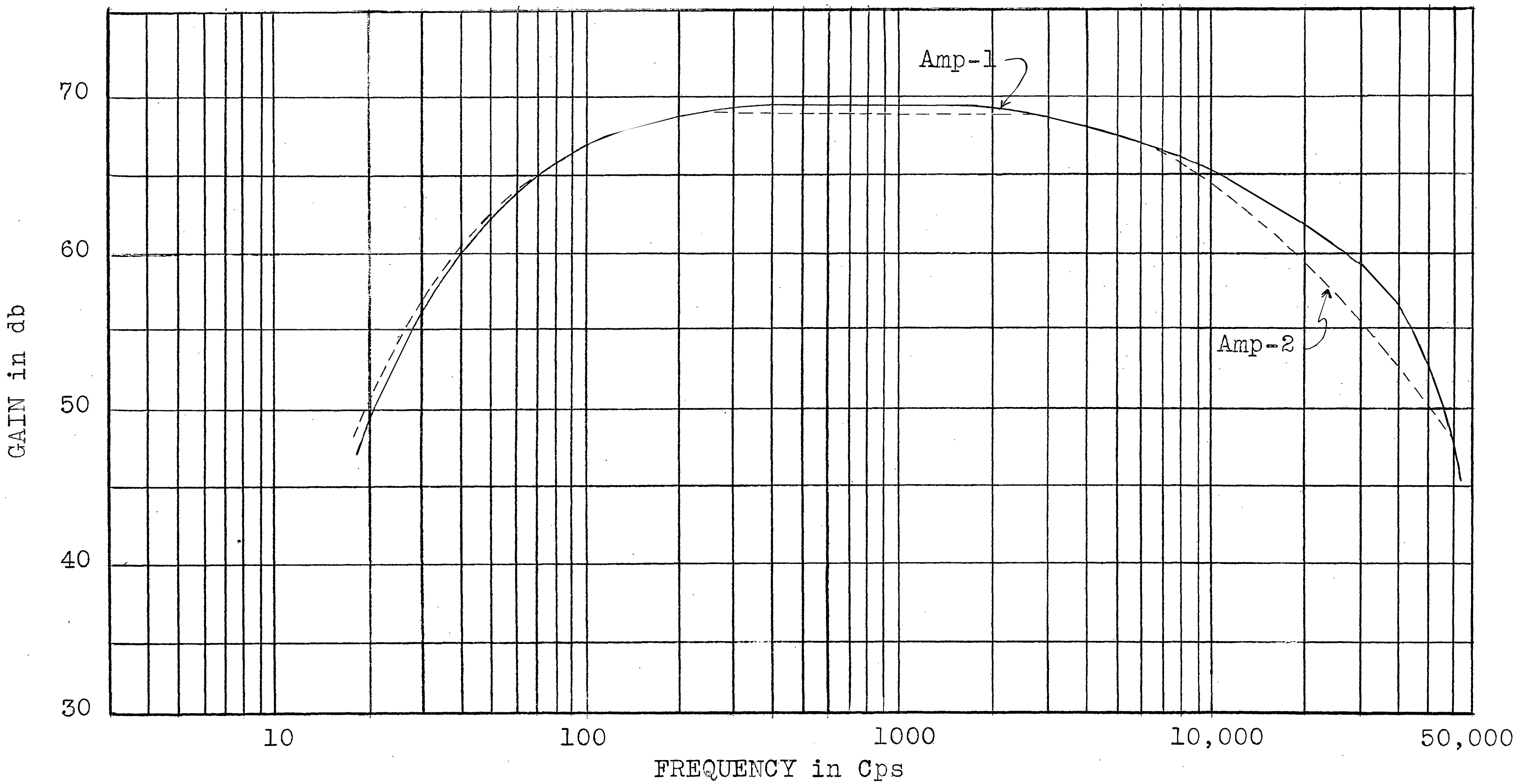


Figure B-5. Comparison between the gain response of Amp-1 and Amp-2.

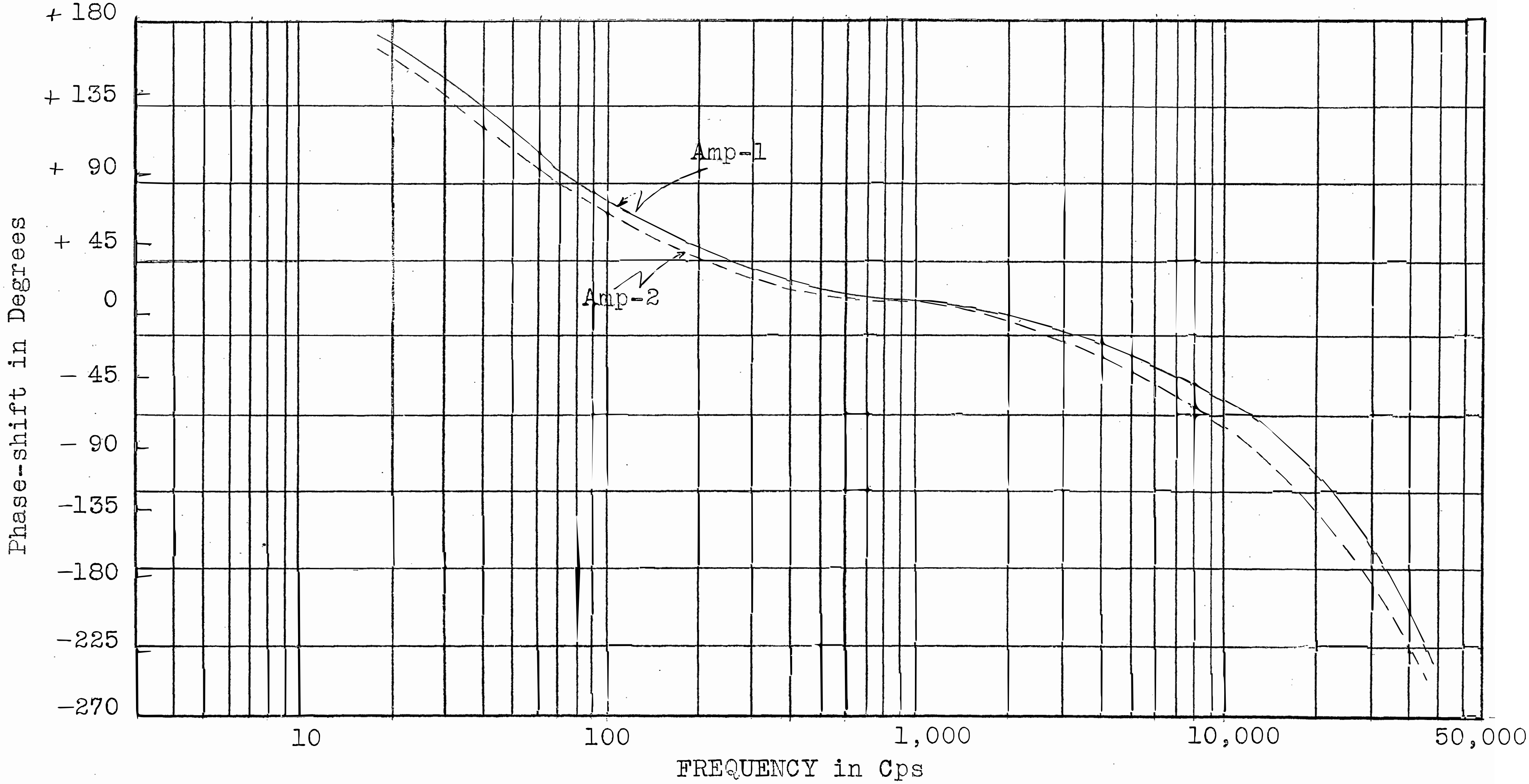


Figure B-6. Comparison between the phase response of Amp-1 and Amp-2.

APPENDIX C

LABORATORY PROCEDURE FOR EXPERIMENTAL VERIFICATION

General

Both amplifiers, Amp-1 and Amp-2 were analyzed in exactly the same manner. The same procedure is applied to each in obtaining laboratory data. This was done to insert a degree of generality in the results.

Determining Values for β

The amount of feedback returned to the input is limited by stability considerations and is a function of the β factor. Thus, if too much feedback is used, the β K-plot will encircle the point -1,0 and oscillations will result. An accurate determination of β is, therefore, very important. It was found, during experimentation, that the larger the amount of feedback, the more accurate the results. This, of course, is true only up to a certain point. Thus it was decided to vary the potentiometer in the feedback loop until oscillation occured at very high and very low frequencies and then reduce the feedback until no suggestion of oscillation was detected. The stability of the amplifier was then checked over the entire band by slowly varying the frequency and observing the output waveshape on the oscilloscope.

The setting of the gain control is also an important preliminary. It not only affects the stability but also the effectiveness of the graphical phase angle technique. If the gain is set too low the voltage measuring devices may lack sufficient accuracy to provide vector arcs which will intersect at very low frequencies. If the gain is set too high the feedback must be too small for good accuracy. These two factors, gain and feedback must be properly balanced to achieve good results. There is no clear cut way for this balance to be accomplished, however, in Appendix D the author outlines a system which worked quite well.

A feedback of approximately -10.5 db, at mid-band, was used for Amp-1, giving a value for β of .000855. Any value of feedback between 5 - 15 db should give satisfactory results. Since the gain must be kept sufficiently high in order for the vectors to intersect at very low frequencies, a gain of 69 db was used. The corresponding values for Amp-2 are only slightly different, β = .000876 or -11.3 db of feedback, with a gain of 68.8 db.

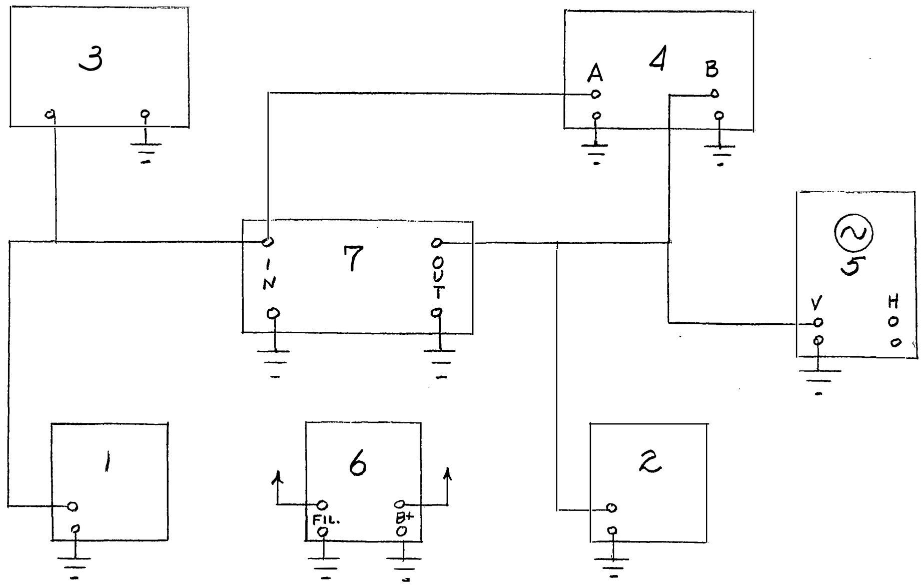
As has been shown previously, the vector arcs begin to overlap as the frequency is approaching mid-band. Consideration was given to varying β at one or more points in the band so that these arcs might intersect more precisely. However, in the interest of simplicity, this was not included in the procedure. Aside from introducing additional adjustments, varying β was not necessary because the intersections are definite enough for all frequencies except those within the zone of inaccuracy, near mid-band. This zone is created by factors beyond the control of β and so would not be improved by manipulating β.

Obtaining Data

All equipment used in the experimental investigation is shown in Figure C-1, along with a connection diagram. A preliminary set of phase and gain response data was obtained at the outset in order to arrive at some notion concerning the bandwidth of the amplifier. This revealed a 'hump' in the feedback response which is probably due to leakage inductance and stray capacitance in the output transformer.

Unit #8 was used to compare with #4 and they were found to be in agreement within 4 degrees at any frequency. After that a few phase angles were plotted on graph paper to see if the original premise had foundation in fact. While results were generally positive, they were not as accurate as it was believed possible. So a series of measurement refinements were initiated to track down the elements which were causing the discrepancies.

One of the first parameters to be suspected was the β -factor. Originally, this had been calculated from the values of the resistive voltage divider from which the feedback voltage was obtained. Due to possible interaction between circuit elements while under operating conditions, this value of β was not accurate enough. It should be



The numerals in each block identify pieces of equipment as follows:

1. Hewlett-Packard (Model 400-C), Vacuum-tube-voltmeter.
2. Hewlett-Packard (Model 400-A), Vacuum-tube-voltmeter.
3. Hewlett-Packard (Model 650-A), Test Oscillator.
4. Deltron (Model 100-A), Phasemeter.
5. General Electric (Type 2-B), Oscilloscope.
6. Regulated Power Supply.
7. Experimental Amplifier.
8. McCollum (Model 1-A), Phasemeter. *

* Unit #8 was used only to check the accuracy of unit #4.

Figure C-1. Connection diagram of equipment used in experimental investigation along with a description of each unit.

noted, at this point, that after the phase angle passes plus 90 degrees on its way to 0 degrees, the accuracy of all values becomes critical. Because the arcs of the two vectors are not only becoming quite large but are also approaching each other tangentially, a slight error in a voltage measurement could result in a phase angle error of 10 - 15 degrees. Of course, an error in the values used for the multiplier β would also produce error and β must be known exactly. This can be accomplished quite simply. From the general expression for negative feedback we see that,

$$\frac{K'}{K} = \frac{1}{1 + \beta K}$$

from which, $\frac{K}{K'} = 1 + \beta K$

At the exact center of mid-band frequencies, when phase-shift is zero, these vectors may be manipulated algebraically, since no angle is attached to them. This results in the desired expression,

$$\beta = \frac{1}{K'} - \frac{1}{K}$$

Therefore, in order to obtain a sufficiently accurate β it is merely necessary to measure the output voltages for open and closed loop operation at mid-band and calculate β directly.

The Vacuum-tube-volt-meters were then given a brief calibration as a matter of routine and were all within specified accuracies. However, since these types of meters are most accurate in the upper quarter of their full scale movement, all readings were taken in the lowest possible multiplier range. It was noted, furthermore, that when voltage readings were taken below the half-scale point, error in the graphically obtained phase angles resulted.

Finally a data run was made on each amplifier and the figures applied to the graphical phase angle technique. This data for both amplifiers is shown on Table C-1.

Use of Graphical Method

After a complete set of output voltages was secured, the vector lengths K/K' and β K were calculated. These values were then struck on polar graph paper and an angle was measured for each frequency. When the frequency approached mid-band the mathematical expression was employed. The results were excellent, giving testimony to the effectiveness of the attempted procedural refinements. (Appendix D exemplifies the entire experimental procedure.)

TABLE C-1

TABULATED RESPONSE DATA FOR AMP-1 & AMP-2

	Amp-1				Amp-2			
Frequency in Cps	Phase-shift in Degrees ACTUAL	GRAPH	Output Volts w/o FB	Output Volts w/ FB	Phase-shift in Degrees ACTUAL	GRAPH	Output Volts w/o FB	Output Volts w/ FB
20	+ 172		2.2	2.8	162		2.4	3.1
30	+ 158	163	5.4	9.5	137	137	5.6	7.85
50	+ 115	113	11.8	10.6	102	103	11.6	9.2
70	+ 90	91	16.3	9.6	82	82	16.0	8.75
100	+ 68	68	20.8	8.9	60	61	20.1	8.35
150	+ 47	47	24.0	8.5	41	43	23.3	8.2
200	+ 34	35	25.7	8.4	29	31	25.0	8.1
300	+ 22	26	27.0	8.3	18	23	26.2	8.1
400	+ 14	18	27.6	8.3	11	11	26.9	8.05
600	+ 7	18	28.0	8.3				
800	+ 3	0	28.0	8.25	1	0	27.3	8.05
1,000	+ 0	0	28.0	8.25	0	0	27.3	8.05
1,500	- 3	0	28.0	8.25	7	0	27.2	8.0
2,000	- 9	10	27.6	8.25	12	0	26.9	8.0
3,000	- 17	20	26.8	8.25	23	25	25.7	8.05
5,000	- 31	37	24.0	8.25	41	43	23.0	8.15
8,000	- 50	54	20.1	8.25	64	64	18.7	8.3
10,000	- 61	63	18.0	8.25				
15,000	- 80	80	14.5	8.4	102	100	12.1	9.1
20,000	- 100	95	12.0	8.7	129	115	10.0	9.9
30,000	- 147	126	9.9	11.5	173	150	6.7	11.5
40,000	- 197	216	7.4	12.0	212	209	4.5	6.65
50,000	- 223	219	4.8	6.7	249	240	3.35	3.9
60,000	- 253	238	2.8	3.2	*	255	1.64	1.7
70,000	*	225	1.66	1.72	*	270	.9	.9

* Note: The Phasemeter did not have sufficient gain at these high frequencies to register an angle.

APPENDIX D

SAMPLE APPLICATION OF THE NEW TECHNIQUE

General

Amplifier Amp-2 will be analyzed herein as a typical amplifier for which the phase response is desired. All pertinent values will be recorded on a master data sheet, Table D-1, and the graphical method will be demonstrated.

Preliminary Preparations

The first step would be to run a spot gain response to learn the magnitude of frequencies involved and to locate K, the open loop amplification factor at mid-band. Next a suitable feedback loop must be designed. Bearing in mind that this loop must be purely resistive, the transfer function (β) must be found. A rule of thumb which might be helpful at this early stage is that due to the geometry of the vector manipulation, β should fall between the values of 1/K and 1/K/2. Using this guide the resistive transfer function is designed. In this case K is found to be 2730 so β would fall between .000366 and .000735. Figure D-1 shows how this is arranged with a potentiometer to allow a fine adjustment.

It is assumed that the amplifier is equipped with a gain control which will be needed to obtain the proper balance between gain and feedback.

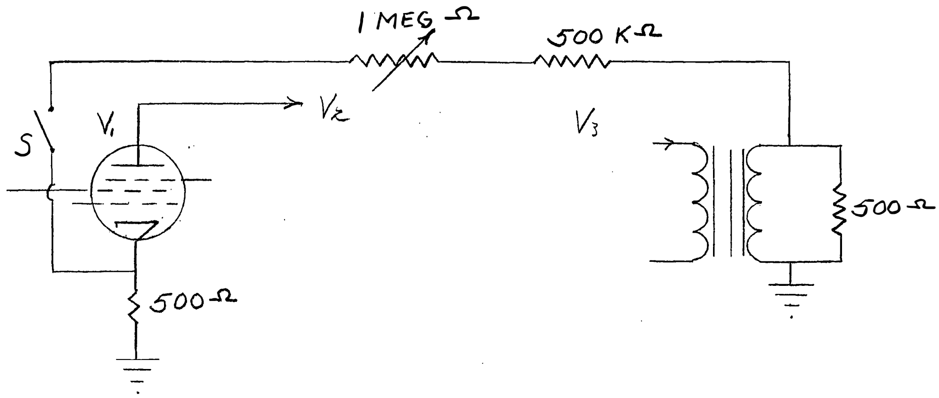


Figure D-1. Showing preliminary feedback loop from which values of β from .001 to .000333 are obtainable.

The next step is to set the gain and β controls to a more exact degree. As has been stated, the vectors may fail to intersect at very low frequencies. So the gain must be made large enough, within limits of stability, to overcome this difficulty. This is done quite simply. After the gain is set, take a voltage output reading with the feedback loop open and closed with the signal generator set at the lowest frequency of interest. From these two values calculate K/K' and β K (assuming that β has been determined from the mid-band amplification values of K and K' as outlined in Appendix C). If these two calculated figures do not add up to unity, they will not intersect and the gain must be increased slightly. It may not be possible to get intersection at the lowest frequency but it should be in the very low range. In this case it was affected at 30 cps which was satisfactory. Then the

β control is rotated until βK becomes large enough to cause oscillation at very low and very high frequencies. Then β is reduced well below the unstable value and the output waveshape is observed on the oscilloscope to make certain that no oscillation is present at any frequency in the band. β is then recalculated from the new values of K and K'; this turned out to be,

$$K = 2730$$

$$K' = 805$$

$$\beta = 1/K' - 1/K = .000876$$

Since a high value for β is desireable, this figure is very acceptable even though it is slightly higher than the rule of thumb indicated. Equipped with an accurate β and knowing the frequencies over which the phase response is desired we are ready to record data.

Obtaining Data

The Hewlett-Packard 400-C will be used to measure input voltage, which will be held constant at .01 v. The Hewlett-Packard 400-A will be used to measure output voltage. The oscilloscope will be used to insure against oscillations within the amplifier.

The signal generator is set at 20 cps and the output voltages recorded for open loop and closed loop operation. Then the frequency is increased with similar voltage readings being taken for each frequency until the entire band is covered. Since the input voltage is liable to vary, due to changing input impedances at higher frequencies, it should not be assumed constant and must be checked before recording each voltage measurement.

Next the K/K' and β K vector lengths are calculated for each frequency. This data is entered on the master data sheet Table D-1 and no further data is needed.

Using Data

A piece of K&E Polar Coordinate # 358-31 graph paper will be used for all phase angle construction. For convenience it should be attached to pieces of paper as shown in Figure D-2 to allow for arcs which are too large for the graph paper when using the full scale.

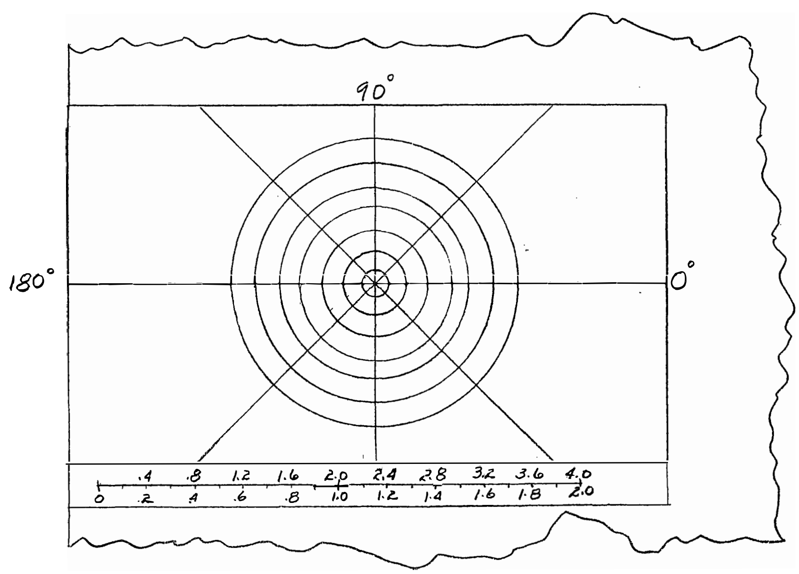


Figure D-2. Showing the arrangement of graph paper and the full and half scales which will be used.

TABLE D-1

MASTER DATA SHEET FOR AMPLIFIER AMP-2

Frequency Cps	Phase-shift Degrees Actual	Phase-shift Degrees Graph	Output Volts w/o FB	Output Volts w FB	Gain db w/o FB	Gain db w FB	K/K'	βK #
20	+ 162	*	2.4	3.1	47.6	49.8	.775	.21
30	+ 137	137	5.6	7.85	54.96	58	.714	.491
50	+ 102	103	11.6	9.2	61.3	58.28	1.26	1.016
70	+ 82	82	16.0	8.75	64	58.8	1.83	1.4
100	+ 60	61	20.1	8.35	66	58.6	2.41	1.76
150	+ 41	43	23.3	8.2	67.4	58.2	2.845	2.04
200	+ 29	31	25	8.1	68	58	3.09	2.19
300	+ 18	23	26.2	8.1	68.4	58	3.235	2.29
400	+ 11	11	26.9	8.06	68.6	58	3.34	2.355
800	+ 1	0	27.3	8.05	68.8	58	3.39	2.39
1,000	- 1	0	27.3	8.05	68.8	58	3.39	2.39
1,500	- 7	0	27.2	8.0	68.8	58	3.4	2.385
2,000	- 12	0	26.9	8.0	68.6	58	3.36	2.35
3,000	- 23	25	25.7	8.05	68.2	58	3.19	2.25
5,000	- 41	43	23	8.15	67.2	58.2	2.825	2.015
8,000	- 64	64	18.7	8.3	65.4	58.2	2.255	1.64
15,000	- 102	101	12.1	9.1	61.6	59.2	1.33	1.06
20,000	- 129	115	10.0	9.9	60	60	1.01	.876
30,000	- 173	150	6.7	11.5	56.6	61.3	.582	.587
40,000	- 212	209	4.5	6.65	53	56.6	.677	.395
50,000	- 249	240	3.35	3.9	50.6	51.8	.86	.293
60,000	- ‡‡	255	1.64	1.7	44.4	44.6	.966	.144

β = .000876

* Amplifier gain was too low to cause an intersection of vector arcs.

‡‡ Phasemeter lacked sufficient gain to yield an angle at this frequency.

The remainder of the process is simple and direct. Referring to the data sheet, for 20 cycles, we see that $K/K' + \beta K \neq$ unity so the graphical construction will not render an angle because the vectors do not intersect. However, at 30 cycles an arc of $K/K' = .714$ intersects with an arc of $\beta K = .491$ giving an angle of either + 223 or + 137 degrees. This ambiguity is easily resolved by taking the next frequency which is 50 cps and striking an arc of $K/K' = 1.26$ and an arc of $\beta K = 1.016$. These intersect and give an angle of either + 258 or + 102. We immediately log this angle as + 102 and the previous angle as + 137 because it is known that for this type of amplifier, the phase angle is positive in the low frequency range and will decrease as the frequency is increased. This ambiguity will no longer concern us. The next angle, from $K/K' = 1.83$ and $\beta K = 1.4$, is + 82 degrees and so forth until we arrive at a frequency which gives a phase angle of about + 10 degrees, at which point accuracy will be difficult until the frequency is high enough to cause the phase angle to pass - 10 degrees. Following this procedure, the remainder of the phase angles are obtained.
In Figure D-3, which illustrates this process for several angles, half scale was used exclusively due to space limitations. However, in practice a full scale would be used until the lengths of the vector arcs become too long, and then half-scale is used. In this particular example, full scale would be used until the vector length of either

K/K' or β K became longer than 2.0, which occurs in the vicinity of an angle of + 90 degrees. It is convenient to use the scale which is located on the polar graph paper and Figure D-2 illustrates how this scale can be calibrated for full and half-scale.

Table D-2 shows a few calculated angles, obtained with the mathematical equation previously derived, compared with graphical and actual values. It is interesting to note how closely they agree.

This data was originally taken to verify the new technique and phase-shift was measured with the Deltron Phasemeter to be used as a norm of the actual phase-shift. It will be noted, on the master data sheet Table D-1, how closely the ACTUAL and GRAPHICAL values compare. When the number of possible sources of error are considered, this agreement is remarkable.

TABLE D-2

Comparison Between Actual, Graphical & Calculated Phase Angles Plus a Sample Calculation

Frequency	Actual	Graphical	Calculated
30 cps	+ 137°	+ 137°	+ 138°
70	+ 82°	+ 82°	+ 82.4°
400	+ 11°	+ 11°	+ 11°
5000	- 41°	- 43°	- 43.3°
40,000	- 212°	- 209°	- 208°

Sample Calculation:

The equation for phase-shift is:

$$\alpha = 180 - \cos^{-1} \frac{\beta K^2 + 1 - \left(\frac{K}{K'}\right)^2}{2 \beta K}$$

Using values from Table D-1 for 400 cps

$$\alpha = 180 - \cos^{-1} \frac{2.355^2 + 1 - 3.34^2}{2 \times 2.355}$$

$$= 180 - \cos^{-1} - \frac{4.6}{4.71}$$

$$= 180 - (180 - 11) = +11^\circ$$

$$\therefore \quad \alpha = +11^\circ$$

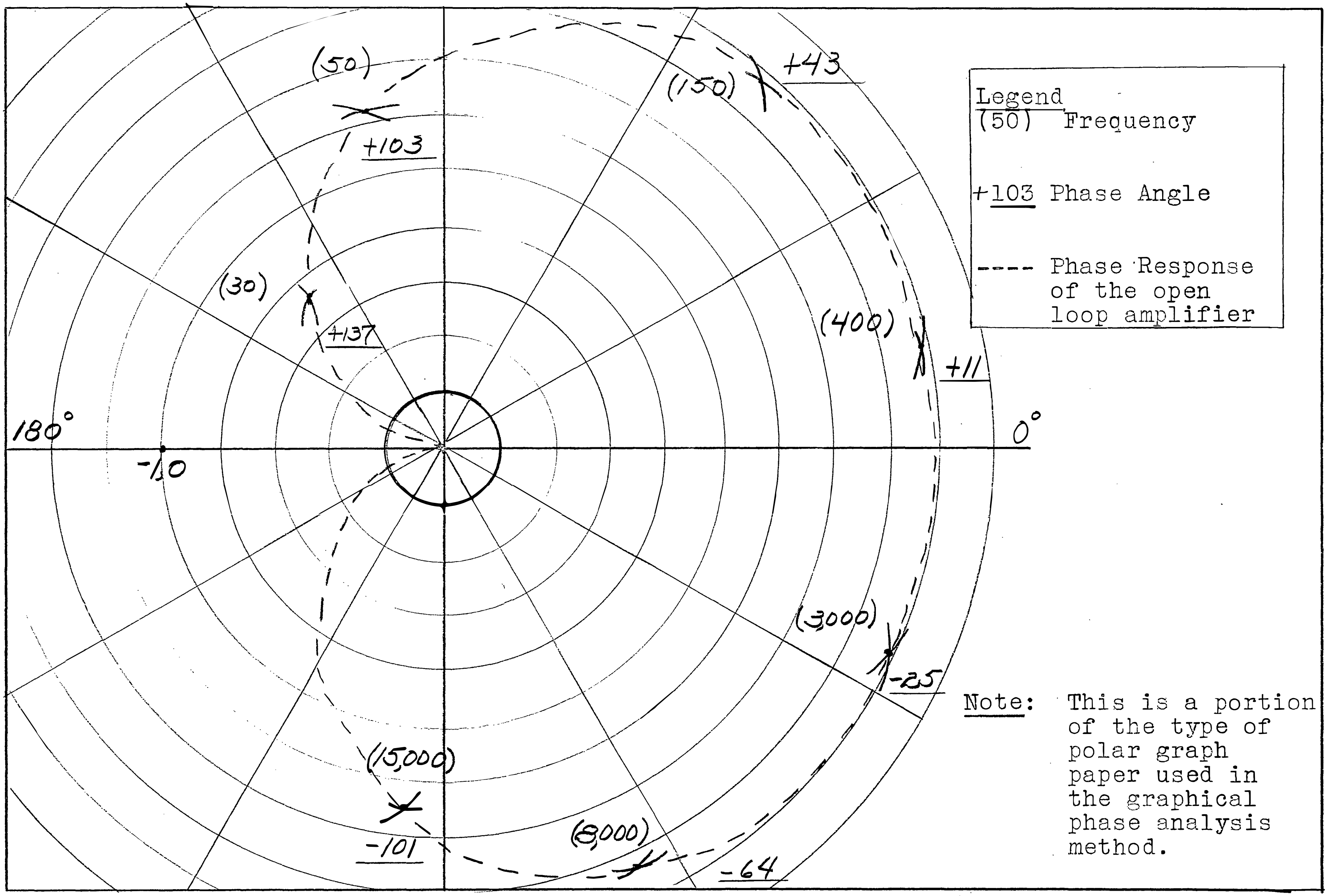


Figure D-3. A sample of the graphical construction

VITA

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Master of Science

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THESIS TITLE: A SIMPLE METHOD FOR ANALYZING AMPLIFIER PHASE-SHIFT USING THE NEGATIVE FEEDBACK PRINCIPLE

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THESIS ADVISER: Dr. H. T. Fristoe

The content and form have been checked and approved by the author and thesis adviser. The Graduate School Office assumes no responsibility for errors either in form or content. The copies are sent to the bindery just as they are approved by the author and faculty adviser.

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