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THE UNIVERSITY OF OKLAHOMA
GRADUATE COLLEGE

THE IDENTIFICATION OF THE MARKET AREA
FOR A DISTRIBUTION CENTER

A DISSERTATION
SUBMITTED TO THE GRADUATE FACULTY
in partial fulfillment of the requirements for the
degree of
DOCTOR OF PHILOSOPHY

BY
KUNG-MO KUO
Norman, Oklahoma

1972

THE IDENTIFICATION OF THE MARKET AREA
FOR A DISTRIBUTION CENTER

APPROVED BY

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DISSERTATION COMMITTEE

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DEDICATION

To

My Father, who will never know of this study

And To

My Mother, who works for her children's education

ACKNOWLEDGMENTS

My sincere appreciation is extended to Dr. James A. Constantin, David Ross Boyd Professor of Marketing and Transportation, and Chairman of my Advisory Committee, for his guidance, inspiration, and encouragement during the writing of this dissertation and throughout the entire doctoral program at the University of Oklahoma.

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Kung-Mo Kuo
Feb. 1972
Norman, Oklahoma

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THE IDENTIFICATION OF THE MARKET AREA
FOR A DISTRIBUTION CENTER

CHAPTER I

INTRODUCTION

Since the term "physical distribution" was first used by Fred E. Clark in 1924,¹ "physical distribution" has carried both micro and macro connotations. As used here, micro-distribution is concerned with a firm while macro-distribution refers to a region.² At the micro level, one emphasizes the maximization of profit for the firm. At the macro level, the continuous steady growth of a region is stressed. Regardless of the differences in the stresses between micro and macro distribution, current

¹Bowersox, Smykay and La Londe contended that the term "physical distribution" was first used by Fred E. Clark in his Readings in Marketing (New York: The Macmillan Co., 1924), Chapter XV. See Donald J. Bowersox, et al., Physical Distribution Management, Revised Edition (New York: The Macmillan Co., 1968), p. 8.

²Mossman and Morton define macro- and micro-distribution differently. They point out that macro focuses on systems as a whole rather than on its individual components whereas micro is concerned with various forces at work in sub-segments of a given universe or macro system. In other words, they view the system vertically. See Frank H. Mossman and Newton Morton, Logistics of Distribution Systems (Boston, Mass.: Allyn and Bacon, Inc., 1965), pp. 4-18.

trends indicate an increased attention to a broader aspect of the distribution system ranging from the concept of logistics planning for economic growth to the concept of a distribution conglomerate.³ In either case, the natural outcome of this attention is an increasing interest in the study of the role of regional distribution centers, the establishment of which becomes a "wind and grass of springtime."

The identification of the market area for a distribution center is both essential and important for comprehensive and efficient distribution center planning.

Purpose, Scope, and Methodology

Whether works pertaining to market areas are theoretical or empirical, they have centered on either of the two major fronts--consumer and firm. One emphasizes the demand side, the other, the supply side.⁴ Examples of the former are those market area studies of retailers, while

³These concepts were presented to the American Marketing Association Seminars on Physical Distribution held at the University of Oklahoma, Norman, Oklahoma, April 22-24, 1970. See James A. Constantin and William R. Southard, "Marketing Logistics Planning for Economic Development" and Bernard J. La Londe, "Facilitation Agencies as an Extension of the Arm of the Firm." Both are reports presented in AMA Seminars on Physical Distribution, Norman, Okla., April, 1970.

⁴Prof. Huff contended that a marketing area, according to earlier studies, has always been either determined by consumers or suppliers. David L. Huff, "Defining and Estimating a Trade Area," Journal of Marketing, Vol. XXVIII (July, 1964), pp. 34-8.

those market area studies of plants (manufacturers), wholesalers, and distribution units illustrate the latter. Variables chosen in these studies are generally population, distance (measured in physical terms or time units), consumer propensity to search, etc., for retailers; and distance, transportation rates, base prices, etc., for non-retailer marketing units.

Literally, no single work that considers both sides of a picture (supply and demand) can be found. The reason is simple. As McCarthy has pointed out, retail location is "primarily customer-oriented," and wholesale location is "primarily cost-oriented."⁵ This does not mean that integration of supply factors and demand factors in the market area analysis is not necessary. It only means one is more important than the other in the analysis. It seems that to consider both demand and supply factors in a market area analysis would involve many variables and would add to the technical complexity of the problem which is beyond the current state of art in this branch of the academic discipline.

The theoretical foundation for market areas for plants and those of wholesalers and distribution centers do not differ basically. As a matter of fact, they are treated in the same grand theoretical framework.

⁵E. Jerome McCarthy, Basic Marketing, Revised Edition (Homewood, Ill.: Richard D. Irwin, Inc., 1964), p. 611.

Nevertheless, the treatment of the market area of a distribution center has not been popular. Much of the literature either treats the subject superficially or takes the market area as datum.⁶ The practical application of the theory to the actual definition of the market area for a distribution center leaves much to be desired.

Also, the assumptions that transportation rates are proportionate to distance which underlie literally all the hypotheses in the early works are extremely unrealistic.

The purpose of this study is to introduce an alternative method of identifying market areas which is operational and which can be applied to a distribution center with more realistic transportation cost assumptions. To show that the new method is operational, a case of the Little Rock Distribution Center will be cited.

This study is limited to a short-run, macro, and partial spatial equilibrium analysis. This can be seen from: 1) the variables chosen, 2) time considerations, and 3) the type of distribution in question.

The difference between general and partial equilibrium analysis lies not in the presence or absence of ceteris

⁶ An exception to this statement is M.I.T. Project Bosphorus in which a differential (including waiting cost) shipping cost is used to draw a map of potential market area for the Boston port area. For a more detailed explanation, see Project Bosphorus, M.I.T. Report No. 21, Boston Port Utilization Study, presented to M.I.T., 1968 (Boston, Mass.: M.I.T. Press, 1970), pp. 188-93.

paribus assumptions. It lies in the assertion, as Blaug contended with Samuelson, that "in general equilibrium, the historical discipline of theoretical economics is practically exhausted."⁷ Since this study neither exhausts the discipline of theoretical economics nor exhausts the variables related, it is considered to be a partial equilibrium. It is concerned only with a spatial equilibrium, for the existence of market boundary among competitors and identification of the boundary is its concern.

This study is a short-run and static case because the time factor (dynamic factor) is not considered. However, the incorporation of time factors in the analysis can be made from both the demand side⁸ (consumer income changes over time, etc.), or the supply side (inventory cost over time⁹). For the long-run analysis, this dynamic factor is essential. But in the short-run case, this factor is taken as datum.

The distribution center under consideration is a facilitating agency for a region. Hence, clearly the

⁷Mark Blaug, Economic Theory in Retrospect (Homewood, Ill.: Richard D. Irwin, Inc., 1968), p. 577.

⁸Dziewonski, A Polish economist, called for an introduction of the time element into the analysis using the "tensors" concept. K. Dziewonski, "A New Approach to Theory and Empirical Analysis of Location," Regional Science Association Papers, Vol. XVI (1966), pp. 17-25.

⁹James L. Haskett, "A Missing Link in Physical Distribution System Design," in D. J. Bowersox, et al., Readings in Physical Distribution Management (London: The Macmillan Company, 1969), pp. 137-43.

interest is directed to a macro level analysis.

The theoretical foundation of this study draws heavily on those theories developed by early theorists--Lösch, Hoover, Fetter, Hyson, Isard, and others, who emphasize the cost-side in their analyses. It has therefore become natural to follow their basic, if not specific, assumptions. The basic assumption underlying this study is to hold demand factors as datum. As a significant improvement in the method, a non-proportional transportation rate structure is incorporated into the theoretical structure of this study. Instead of using the classical isotimes (and hence isopane) and the abstruse calculus or rectangular coordinate methods in drawing the market boundary among competitors, a trigonometric method will be employed. There is nothing wrong theoretically with the isotimes, calculus or rectangular coordinates approaches except that when a non-proportional rate structure is incorporated, the drawing of isotimes, and hence isopane, becomes unbearably time-consuming and tedious and the translation of calculus or rectangular coordinates concepts into practical application becomes extremely abstract.

The non-proportional transportation cost assumption underlying the hypothesis is translated into an appropriate equation showing the exponential relationship between distance and rate (cost). The assumed relationship is considered to be more realistic than plausible judging from

the existing transportation rate structures of this country.

A New Break-Point equation is formulated to find a prime equilibrium point between the center in question and each competing center.¹⁰ From the information secured by the New Break-Point equation, identification of more equilibrium points along the market boundary between the territories tributary to the spatially competing centers can be found by another set of equations employing trigonometric functions. Connections of these points constitute an identifiable market boundary line. Since the regional distribution center is oligopolistic in nature because of its high spatial differentiation and because of its locational property,¹¹ it is not difficult to identify its market area.

The empirical part of this study involves two important procedures. One is to test the rate structure

¹⁰The term "break-point" coincides with that of Converse in his "New Laws of Retail Gravitation." However, the methods are entirely different both in concept and methodology. For details of Converse's method, see P. D. Converse, "New Law of Retail Gravitation," Journal of Marketing, XIII (October, 1949), pp. 379-384. In order to avoid confusion, the method developed in this paper is termed "New Break-Point" method.

¹¹The locational property of a distribution center is that a regional distribution center in its selection of location is heavily influenced by a demand factor. As a result, a regional distribution center is invariably located at the center of population gravity of a region. This point is further attested by Greenhut. See M. E. Greenhut, "When Is the Demand Factor of Location Important?," Land Economics, XL (May, 1964), pp. 175-84.

assumption which is incorporated into the theoretical structure of the study. The transportation cost equation is transformed into a natural logarithm equation for computing the coefficient of correlation. Another step involved is to convert the distance and rate units into smaller units for facilitating the application of mathematical tables. Most of the empirical data for this study is secured from transportation tariffs and the Census of Transportation.

Since the distribution center in question is a facilitating regional agency, the market area for different commodities selected for warehousing differ greatly. Selection of commodity candidates for warehousing is briefly stated in the related chapter. However, a more detailed explanation is presented in the Appendices.

The study is presented in six chapters. Chapter I introduces the nature of the study and some basic concepts related to it. Chapter II concerns itself with a review of earlier theoretical and empirical works pertaining to market areas which provides a basic theoretical foundation of this study. In Chapter III, a new method of identifying market area is introduced. Chapter IV contains a logical extension of the new method and application of the theory in identifying the market area for a distribution center--a Little Rock case. Chapter V discusses the significance of the new method with a note on desirable further

investigation in this area. Chapter VI, the final chapter, summarizes the major points made, states the conclusions reached from the study, and outlines appropriate recommendations for use of the concepts of the new method developed.

Geonomic Space, Economic Space, and Market Area

Economic activities exist both in space and time. Space and time are vitally important in economic analyses. However, analysis considering spatial factors has not been very popular. Marshall shunted the care of economic analysis onto the track where influence of time is considered to be more important than that of space.¹² It was not until the late 1940's that attention to the space element in economic analysis was strongly called for by Isard, following the Thünen-Weber tradition, even though a pioneer spatial study made by von Thünen dated far back into the early 19th century.¹³

Analyses with emphasis on the question of "where" the economic activities are located are termed spatial

¹²According to Isard, this is called the Anglo-Saxon prejudice. Walter Isard, "The General Theory of Location and Space-Economy," Quarterly Journal of Economics, Vol. LXIII (1949), pp. 476-506.

¹³Thünen's Isolated State is considered to be the pioneer study in spatial analysis. J. H. von Thünen, Isolated State, an English edition of Der isolierte staat, Translated by Carle M. Watenberg, Edited with an Introduction by Peter Hall (Oxford, N.Y.: Der Gamon Press, 1966). The original copy was published in 1826.

economics.¹⁴ Market and market area analyses involve space elements. They ask and try to answer among other things, the question of where the marketing activities are located. Hence, market or market area analysis is a part of spatial economics. This section presents some basic concepts about space and market area to prepare for the ensuing discussions.

Geonomic Space vs. Economic Space¹⁵

In the ordinary sense, one pictures space as a three-dimensional "container" containing objectives, men or things. This is called banal space or geonomic space which explains the physical relation of objects. Banal space usually obeys Euclidian rules. The banal space of a distribution center is that in which the facilities and manpower of the distribution center are located.

As opposed to this traditional notion of space,

¹⁴For a detailed presentation of a definition of Spatial Economics and a concise explanation of approaches to it, see Edgar M. Hoover and Leon N. Moses, "Spatial Economics," International Encyclopedia of the Social Sciences, Vol. XV, pp. 95-108. The theory of spatial economy may be conceived as part of modern general equilibrium theory, or the general theory of which Walras' "general theory" is a part. Hoover took the former view and Isard took the latter.

¹⁵The concept in this section is mainly adopted from Francois Perroux, "Economic Space: Theory and Application," in John Friedman and William Alonso, Regional Development and Planning (Cambridge, Mass.: M.I.T. Press, 1960), pp. 21-36. For a more detailed philosophical explanation of space, see Bus C. Van Fraassen, An Introduction to the Philosophy of Time and Space (New York: Random House, 1970), Chapter IV.

another kind of space may be pictured. This space is defined by economic relations. One calls it an economic space. According to Perroux, "these economic spaces may conveniently be reduced to three: 1) Economic space as defined by a plan, 2) economic space as a field of forces, and 3) economic space as a homogeneous aggregate."¹⁶ Economic space carries a certain level of abstraction concerning the economic relationship explained. Such an abstraction may be that of the organization or center, service quality, prices, costs, etc. Examples of economic spaces are labor markets, security markets, money markets, market areas, etc.

In reality, spaces are also defined by the social relation and political sovereignty. Hence, there are political space and social space. Although these spaces are of importance to many in one way or another, economic space is the direct concern in this study. Figure I shows the concept of space in general and the relationship of economic space in this general framework.

According to the concept of space discussed above, an economic region is a space which is defined by either an economic plan, as field of economic force, as a homogeneous aggregate, or a combination of them. From this point of view, market area is an economic space defined as a field of forces. This "field of forces," in Perroux's

¹⁶Frnacois Perroux, op. cit., p. 26.

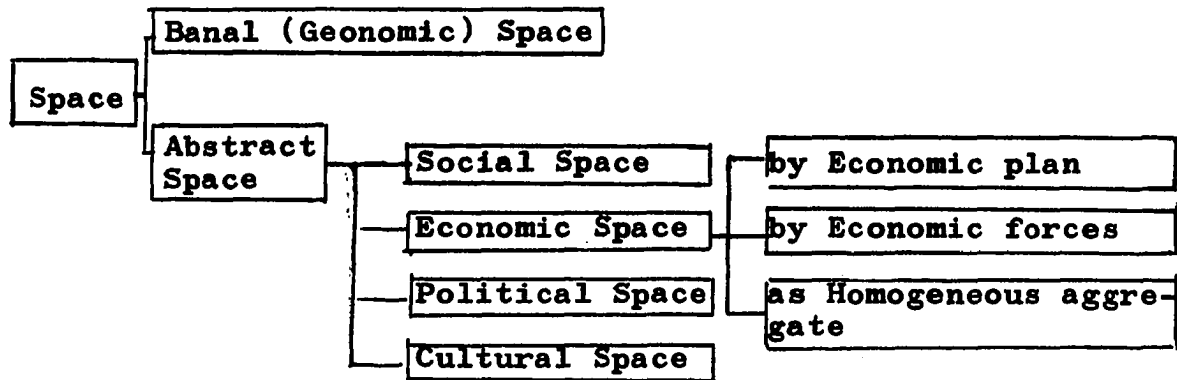


Figure 1. Space and Economic Space.

words, "as economic space consists of centers (or poles or foci) from which centrifugal forces emanate. . . ." ¹⁷ The forces may also be centrepetal. A centrepetal force forms a supply area. Figure 2 shows the market area in the conceptual framework of an economic space.

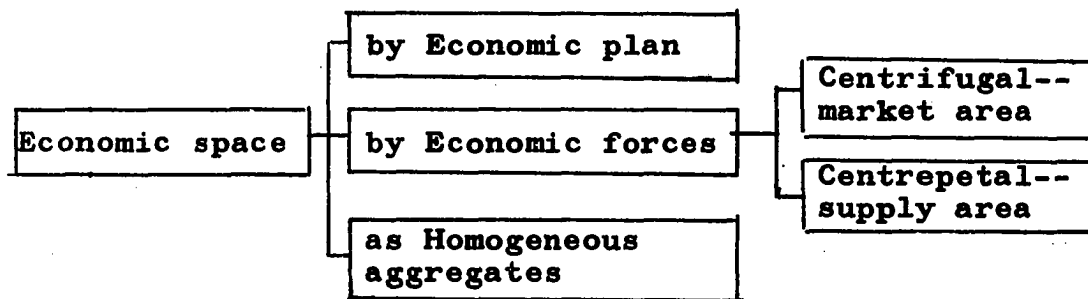


Figure 2. Economic space and market area.

The Market Area

The philosophical aspect of market area and its relation to the concept of space reviewed in the previous section gives one a clear picture of space, economic

¹⁷Ibid., p. 27.

region, and market area. It was recognized that an economic region may be viewed as an array of various kinds and sizes of markets. This seems to coincide with what Hans Weigmann expounded earlier. Nevertheless, Weigmann's approach is slightly different. His approach is called a Gestalt System which emphasizes more or less the markets of land, labor, and capital.¹⁸

As an extension to the basic concept of market area examined in the previous section, this section will deal with some more specific ideas of market and market area. After that, the definition of market area adopted in this study will be presented.

Oftentimes the terms market and market area are used interchangeably. This confusion is due to the lack of a clear understanding of the marketing process. Goods and services are originated (produced) at one place and attracted (distributed) to another place where they are

¹⁸Isard, op. cit., pp. 489-494. In Weigmann's context, the spatial markets include all commodity markets, land markets, capital markets, and labor markets. The movement of "commodity market" is traced back to labor and land through various stages of production. While this is true, one may view the commodity market as the market of the product of land, labor, and capital. Then the concept presented here differs from that of Weigmann only in emphasis. Hoover stressed that the geographic relationship between successive stages of production can be described in terms of market areas or supply areas. See E. M. Hoover, The Location of Economic Activity (New York: McGraw-Hill Book Co., Inc., 1968), p. 65. Lösch's concept of market area and development of a region attests the same idea emphasized in this paper. See August Lösch, "The Nature of Economic Regions" in John Friedmann and William Alonso, op. cit., pp. 107-115.

either consumed or used. The entire "flow" process is guided by "attractive" forces. The aggregate of these forces is, in reality, a market which consists of people who have various means of satisfying their needs and wants.¹⁹ The quantification of such a market constitutes market area. A market area can be expressed either in sales volume (abstract) or in geographical units (physical) depending upon the nature and objective of a study. The latter fits into the Committee on Definition of American Marketing Associations' definition of trade area which reads:

A district whose size is usually determined by the boundaries within which it is economical in terms of volume and cost for a marketing unit or group to sell and/or deliver a good or services.²⁰

A market area, like a market, may be identified by a generic product class, a single demographic variable, the consumer's environment, or geographical places.²¹ The

¹⁹Reed stressed the aspect of income while Levitt and Sissor emphasized the aspect of human need and want. See V. P. Reed, "Some Statistical Possibilities of Defining Market Limit," Journal of Marketing, Vol. III (1938), pp. 39-43; J. Z. Sissors, "What Is a Market," Journal of Marketing, Vol. XXX (July, 1966), pp. 17-21; and Theodore Levitt, "Marketing Myopia," Harvard Business Review, Vol. XXXVIII (July/Aug., 1960), p. 55. According to the Committee on Definition of American Marketing Association's definition, market is: 1) the aggregate of forces or conditions within which buyers and sellers make decisions that result in the transfer of goods or services, 2) the aggregate demand of the potential buyers of a commodity or service. See Committee on Definition of American Marketing Association, Marketing Definition: A Glossary of Marketing Terms (Chicago, Ill: American Marketing Association, 1960), p. 14.

²⁰C.D.A.M.A., Ibid., p. 22.

²¹Sissors, op. cit.

most commonly used way of identification is via a generic product class.

The term "market area" throughout this paper refers directly to a geographical area identified by an individual generic product class or subclass.

The Nature and Role of a Distribution Center

As the economy of this country moves into the new decade of the '70's, roughly \$475 billion worth of goods were produced to meet the needs and wants of approximately 204.8 million people who need to be fed, sheltered, transported, and entertained.²² Since the production of goods and the consumption of these goods are spatially separated, the physical transfer of goods is a must. To effect such a transfer many activities are involved, such as, warehousing, packaging, materials handling, transporting, inventory controlling, etc. These activities altogether are termed physical distribution or logistics.²³ A distribution center refers to an aggregate of fixed facilities designed to perform some, if not all, of the functions of

²²Results from the 1970 Census show that there were 204,765,770 people in this country. See "Population Activity in the United States," Population Bulletin, Vol. XXVI, No. 6 (Dec., 1970), p. 7. The seasonally adjusted annual figure for goods output was estimated to be 475.6 billion dollars. See "National Income and Product Table," Survey of Current Business, Vol. 51, No. 6 (June, 1971), p. 9.

²³For the National Council of Physical Distribution Management (NCPDM) definition, confer with Bowersox, et al., op. cit., p. 4.

physical distribution.

The Nature of the Distribution Center²⁴

As has been previously mentioned, a distribution center, whether it is a firm's distribution warehouse or a regional distribution center, is an aggregate of fixed facilities. These facilities have been broadened to include not only those of warehousing and storage (e.g., buildings, materials handling facilities, etc.), but also those of packaging and information network (e.g., intermodal containers, packaging facilities and computers). These facilities all together constitute an integrated whole to effect the "flow of goods through various steps in the production and distribution process."²⁵ This "flow through" concept encompasses the full array of functions that create not only time and place utility, but also form utility (change in package).

Originally a distribution warehouse was introduced as a storage unit. The broadening of the facility from

²⁴Distribution warehouse and distribution center have been used by many writers interchangeably. However, as the result of the broadening of the functions performed, the term distribution center seems to be more suitable than distribution warehouse.

²⁵For a more detailed explanation of the "flow" concept, see James A. Constantin, Principles of Logistics Management (New York: Appleton-Century Crofts, 1966), pp. 364-5 and Transportation and Distribution in the Arkansas River Basin Area of Oklahoma, A Report prepared for the Oklahoma Economic Development Foundation, Inc., Norman, Okla., May, 1969, pp. 24-31.

merely a storage unit to the present integrated system is the result of improvement in technological know-how and the need for lowering the ever-increasing distribution cost. Examples of this technological know-how are improvement in the information system and transportation technology.

In the future, one will see a regional distribution center controlled by a computer automated for a closer intermodal transportation coordination. Beyond this, the nation will have from 20 to 30 regional centers which are controlled by a computer system of 20 to 30 computers.²⁶

The Role of a Distribution Center

Just as the term physical distribution carries both micro and macro connotations, a distribution center may be referred to as either a micro or macro center. The differences between "micro" and "macro" centers lie in: 1) the objective of the center, and 2) the base of services, and hence the role of the center. At the micro level, the objectives are for the firm to maximize profits and "to provide the desired level of customer order delivery at lowest total cost."²⁷ Functions performed are usually

²⁶ Such a system was expounded by Professor Fitzgerald and Mr. Klawans. See Gerald A. Fitzgerald, "Why Not Public Computerized Distribution Center?," Distribution Age and Arthur H. Klawans, "The Distribution Concept of Tomorrow," both in Norman E. Daniel and J. Richard Jones, Business Logistics--Concepts and Viewpoints (Boston: Allyn and Bacon, Inc., 1969), pp. 209-216 and 217-221.

²⁷ Bowersox, et al., op. cit., p. 246. Bowersox contended that to provide the desired level of service at

limited to warehousing, storage, materials handling, and inventory control for the individual firm. It has been estimated that from one dollar sale, distribution cost amounts to 21¢; marketing cost, 27¢; manufacturing cost, 48¢; and profit, 4¢.²⁸ This means 1% saving in distribution cost will result in more than 5% increase in profit. The cost cut in distribution may be done through cost savings in either transporting, warehousing, inventory controlling, materials handling, or a combination of these.

At the macro level, the objective of a distribution center is to help provide for and service the continuous growth of the region. The center serves as a link to establish and strengthen the connections among towns and cities within a region and to improve the competitive position of the region with respect to its economic development.²⁹ The functions performed necessarily cover a full range of activities which facilitate the flow of goods and services for all industries and all consumers in the region. To be specific, these functions are: transportation

a low cost is the objective of the distribution center. While this is true from the firm's overall point of view, it may be considered as the role played by the distribution center.

²⁸B. J. La Londe, "The Decade of the Distribution Manager," Distribution Worldwide, Nov., 1970, pp. 27-32. Also Marvin Flako and James W. Taylor, "Effective Management of Total Distribution Cost," in Managing Markets for Profit (Chicago: Railway System and Management Assoc., 1965), pp. 37-78.

²⁹Constantin and Southard, op. cit., p. 32.

(inter-modal, consolidated operation, etc.), warehousing, storage, packaging, inventory management, logistics intelligence, etc. Current trends show that the scope of the regional distribution center extends to not only domestic, but also international activities. By utilizing the "inter-modal container," international shipping will be greatly facilitated by sea-rail (in many cases, air-rail) and rail-truck "container interchange."³⁰

How a distribution center can help market the economic growth of a region may be explained within a conceptual framework of the input-output method.³¹ As shown in Figure 3, the regional market in the economy may be viewed as a system of input and output with the market as a center of the system. The input of a region is the income of that region. Output is the goods and services produced in that region. According to the regional account system, annual income is equal to total goods and services produced during the accounting period (for example, period t_1).

³⁰For a case of a successful international distribution center, confer with Janet Bosworth, "Utah: An International Distribution Center," Distribution Worldwide, Vol. LXIX (Dec., 1970), pp. 19-31.

³¹The input-output method as applied in the market system was first pioneered by Elling. He perceived market system in the economy as a "complex input-output system." In this system, the input is in the form of money; output, goods and services. Market is operated with constraints on supply and demand, government regulations, and market information. See Karl A. Elling, Introduction to Modern Marketing: An Applied Approach (New York: The Macmillan Company, 1969), pp. 4-5.

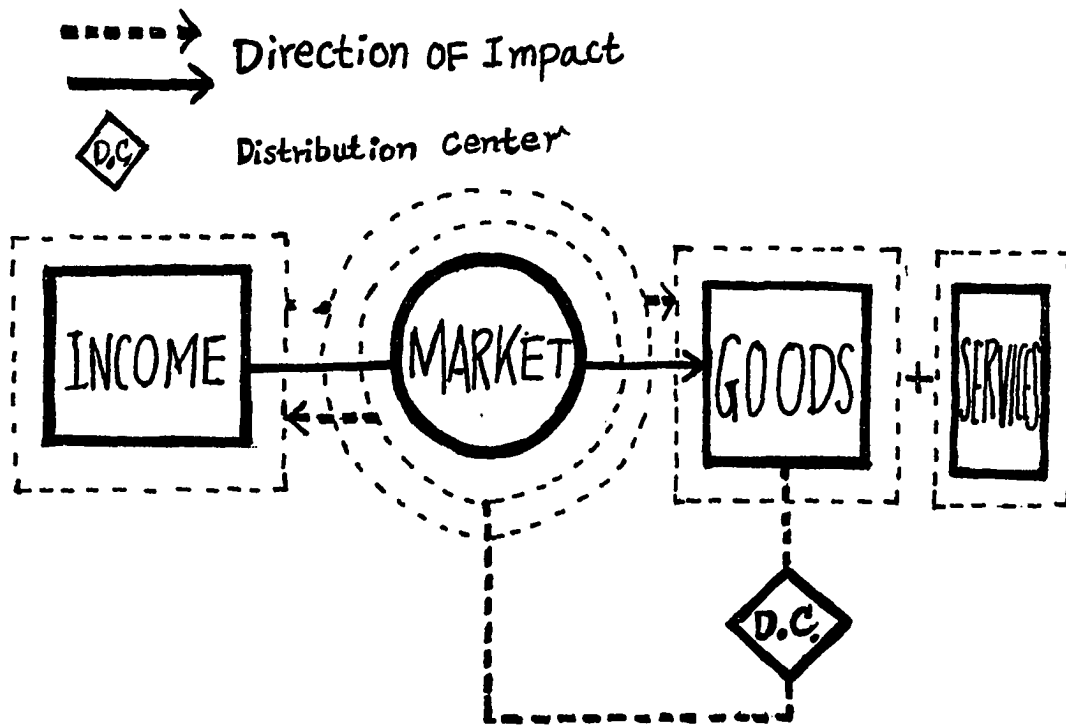


Figure 3. Distribution Center, Regional Market, and the Economic Growth of the Region.

The input and output for period t_1 is shown as the solid line. When a distribution center (shown as D.C. in Figure 3) is established for the region, the primary effect is the expansion of the market for the region due to the comparative advantage of the region resulting from the effective and efficient operation of a distribution center. Such an expansion of the market area increases the income for the region, thereby resulting in further expansion of the market. This is the acceleration effect which will finally bring an increase in output. As output increases, the

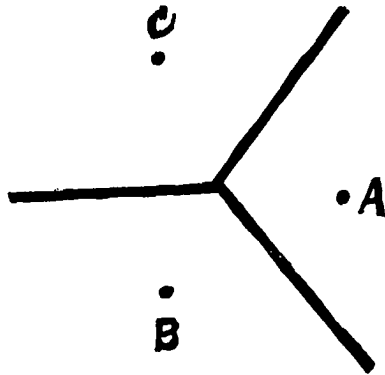
multiple effect operates; the second stage of expansion will follow. The input and output in the period t_{x+1} is shown on the dotted line which reflects the result of the extension of the market area of the region.³²

Distribution Cost and Market Area

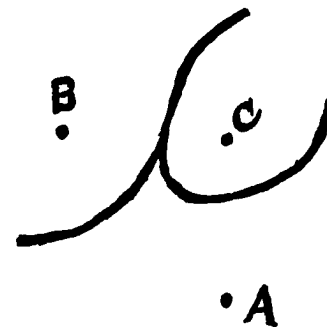
The boundary of a market area becomes identifiable when spatial equilibrium is reached. Under this condition, no rival can sell its commodities across the boundary line.³³

³²A comprehensive discussion of methods of regional analysis can be found in Walter Isard, Methods of Regional Analysis: An Introduction to Regional Science (New York: The Technology Press of M.I.T. and John Wiley and Sons, Inc., 1960).

³³The equilibrium line may be linear or curvilinear, depending upon different assumptions. Hoover, in early 1940, exhibited these two different cases which are shown in the figure below.



Market areas of three producing centers with equal f.o.b. supply point and proportional transportation cost.



Market areas of three producing centers with unequal f.o.b. supply point and proportional transportation cost.

Prices for similar commodities in question at the boundary line remain the same unless there exists a strong brand loyalty or locational preference. Different commodities will have different market areas because of differences in demand conditions and cost structures. Therefore, it is utterly impossible to define a firm's or a region's aggregate market boundary. The market area that is meaningful, hence, is the market area for an individual commodity.

Although, because of the difference in primary emphasis (or orientation), the market areas for retail units have been treated as "demand determined" and those for plants, wholesalers and distribution units, "supply determined;" it is understood that, in reality, the determination of the market area depends upon both demand and supply factors which involve a complexity of variables. As a matter of fact, Greenhut contended that a cost-oriented analysis of a market area does not completely ignore the demand side of the picture for the subjective model has in essence included the demand factor via the assumption of differential delivery cost.³⁴ Likewise, a demand-oriented market area analysis does not completely negate the cost

In reality, market areas may overlap and become a zone due to: 1) the rate bracket, 2) sellers' price discrimination, and 3) imperfect interchangeability of the goods of rival production centers. For details, see Edgar M. Hoover, The Location of Economic Activity (New York: McGraw-Hill Book Company, Inc., 1948), pp. 48-60.

³⁴Greenhut, "When Is Demand Factor of Location Important?", op. cit., p. 175.

aspect. This point will be discussed later in the section in which the investigation of relationships between distribution cost and market area is made.

Market Area for Retail Unit, Plant, and Distribution Center

In the previous discussion of market area, two points have been emphasized. These are: 1) the boundary of the market area signifies the spatial equilibrium under imperfect competition; and 2) within the boundary it is economical to sell or deliver the goods. The first point applies to both retailing units and non-retailing units for the equilibrium can be viewed from either the demand side or the cost side depending upon the underlying assumptions. Market equilibrium for retail stores, under-fixed cost (hence, price) assumptions, signifies the condition in which the customer located on the boundary is indifferent about the competing suppliers in his purchasing decision. On the other hand, market equilibrium for non-retail marketing units means that on the boundary, given the demand forces, the prices for like commodities are the same regardless of the sources of supply. But the second point seems to be inadequate for retail units.³⁵ This point can be explained by the "degree of sensitivity" to cost, and hence, the difference in orientation.

³⁵Huff stresses this same point. See D. L. Huff, "Defining and Estimating a Trade Area," The Journal of Marketing, Vol. XXVIII (July, 1964), pp. 34-8.

The customers of retail units are considered to be the ultimate consumers. These customers are assumed to be willing to bear the transportation cost (cost of transporting goods bought from store to home) incurred on their shopping trip. Therefore, the retail stores are rather insensitive to the cost of delivery, given the retail price. Size and shape of the market area depend greatly upon the factors on the demand side, such as customer taste, income, propensity to travel to the stores, etc. Determination of the market area is oriented toward the customers. The market area for retailing units has thus been treated as demand determined. The market area for retailing units is simply "the area where the customer comes from."³⁶

Contrary to the retailing units, the non-retailing units are very sensitive to delivery cost mainly because the non-retailing units are positioned to bear the delivery cost. Their customers, either industrial users or other marketing agents who are strongly unwilling to pay transportation costs, are in a good position to avoid the delivery cost. Accordingly, the delivery cost places major influence on the size and shape of the market area. The market area for non-retailing units, such as distribution centers, manufacturers, and wholesalers, are said to be mainly supplier-determined.

³⁶ William Applebaun, et al., Guide to Store Location Research (Reading, Mass.: Addison-Wesley Publishing Co., 1968), p. 33.

Significance of the Distribution Cost to Market Area

Just like the influence of demand is included via differential transportation cost in non-retailing market area analysis as suggested by Greenhut,³⁷ the influence of cost is included via differential propensity to travel and/or the inverse relation between distance and market share in the analysis of retail trade area. These are reflections of the cost consumers are willing to pay (including explicit trip cost and implicit time cost incurred in the purchase of commodities). The "probability of a shopper visiting the store with respect to distance"³⁸ and the "travel time factor,"³⁹ which affects the probability of shoppers' traveling, are used to measure the cost-shopping trip relation.

The impact of distribution cost on the shape and size of the market area can be best explained through use of Hoover's price funnel and Lösch's demand cone.⁴⁰

³⁷Greenhut, "When Is Demand Factor of Location Important?," op. cit., p. 175.

³⁸L. P. Bucklin, "Trade Area Boundaries: Some Issues in Theory and Methodology," The Journal of Marketing, Vol. VIII (Feb., 1971), pp. 30-7.

³⁹Huff, op. cit.

⁴⁰Valavanis claimed that both price funnel and demand cone are Lösch's contributions. Nevertheless, it is noted Hoover expounded the concept of price funnel three years before Lösch's publication. This point can be justified by referring to E. M. Hoover, Location Theory and Shoe and Leather Industry (Cambridge, Mass.: Harvard University Press, 1937), p. 8. For Valavanis' contention, see S. Valavanis, "Lösch on Location," American Economic Review, Vol. XXXV (Sept., 1955), pp. 637-44.

Based on Launhardt-Fetter's concept of transportation cost effect, Hoover pictured the effect of transportation cost on the size of market area through his famous price funnel.⁴¹ His original concept may be extended to show the effect of other distribution costs as well as transportation cost as suggested by Alonso.⁴² As shown in Figure 4, the contour lines (or isotimes) represent the equal delivered prices while the slopes depict the transportation gradients. Since at equilibrium the delivered prices of the like commodities from sources A and B are the same, the differences in distribution cost (including transportation cost and non-transportation cost) have a preponderant effect on the size of the market area. For example, the boundary of A extends further out into B's territory ($AC > CB$) because of the lower non-transportation cost of A ($BB' > AA'$). The boundary line is shown as the heavy line in the lower part of the figure. Likewise, differences in transportation cost will also result in different market sizes and shapes via the differential slope in transportation gradients. Hoover's original version and Alonso's modification were cast on the explicit

⁴¹ Hoover credited Launhardt and Fetter for their price funnel. He also credited Palander for the development of the contour lines (isotimes). See E. M. Hoover, Location Theory and Shoe and Leather Industry, op. cit., p. 8.

⁴² Friedman and Alonso, op. cit., pp. 89-106. Alonso did not create any new concept concerning the transportation cost effect on market area. What he did was to restate Hoover's concept in a more understandable way.

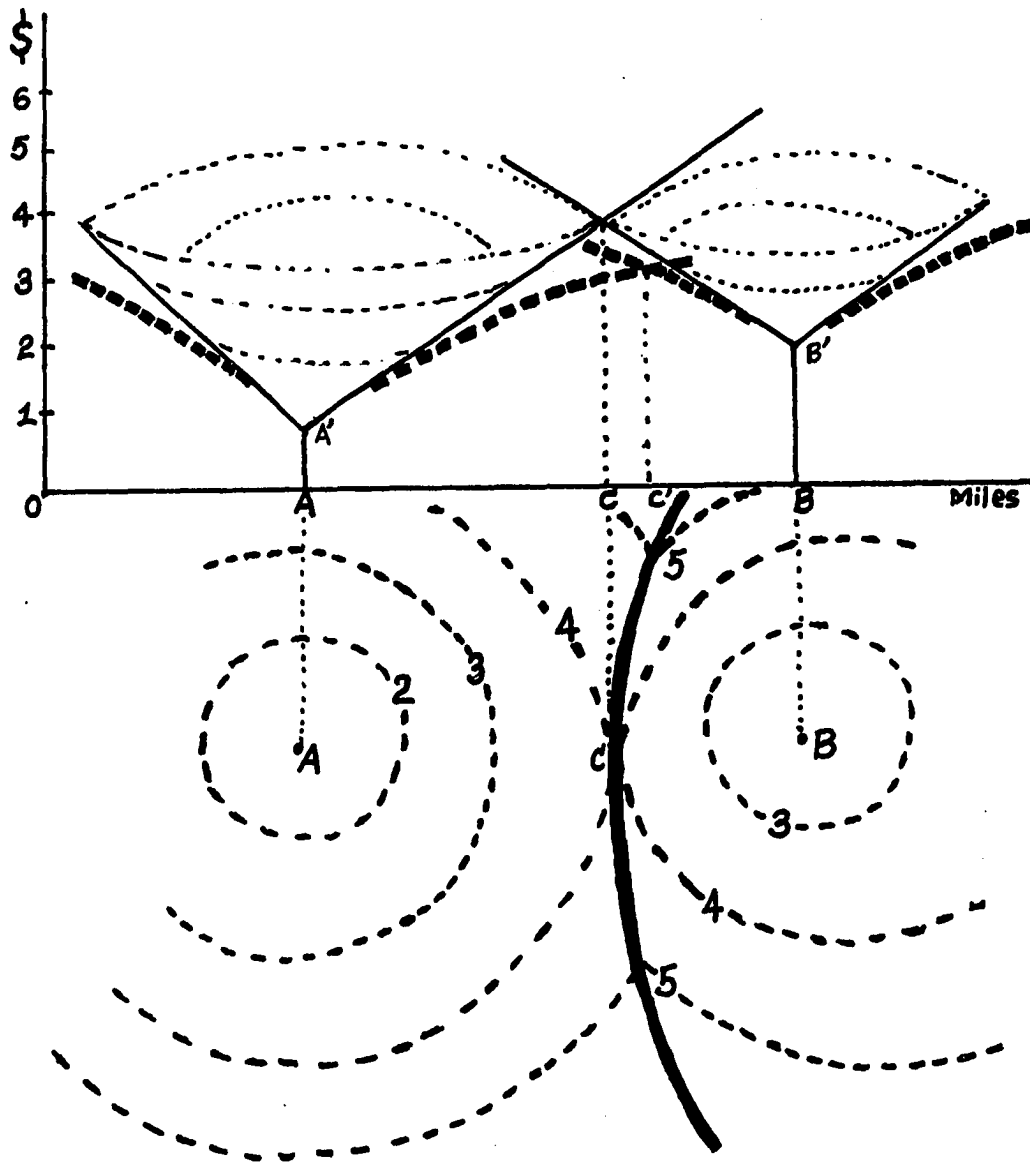


Figure 4. Distribution Cost and Market Area (1).

Note: This figure is modified from Alonso's, see footnote 42 of this chapter.

assumption of proportional transportation rate structure. One may further modify this classic example by incorporating the non-proportional transportation rate into the figure. This is shown in the heavy dotted line. The market

boundary for A extends further ($AC' > AC$).

The effect of distribution cost can also be examined from the demand side of the picture. A case of the logical extension of Lösch's demand cone concept can be made to show the impact of the distribution cost on market area.⁴³ In Figure 5A, a demand curve for a particular commodity (in Lösch's example, beer), when the space aspect of demand is introduced, is TT' . OP is the non-transportation distribution cost. PT is the transportation cost. As cost of transportation increases with distance, the price charged will be higher. Therefore, the quantity demanded will be smaller. Consequently, the demand will approach zero as the price reaches OT with transportation cost PT . PT , if transformed into distance by dividing the transportation rate into it, will become a maximum sale radius P_1T_1 in Figure 5B.⁴⁴ P_1P_1' axis shows the quantity sold which is equal to PP' in Figure 5A. The result of a larger non-transportation distribution cost, OR in Figure 5A, is a smaller market area with radius P_1T_1' shown in Figure 5B, in which case the quantity sold is P_1R_1' (RR' in Figure 5A).

⁴³For details of Lösch's demand cone, see Lösch, *op. cit.*, pp. 105-8.

⁴⁴This assumes a homogeneous area with respect to the source of raw materials and density of population. Lösch's market area argument--hexagonal market area, will be presented later in Chapter II.

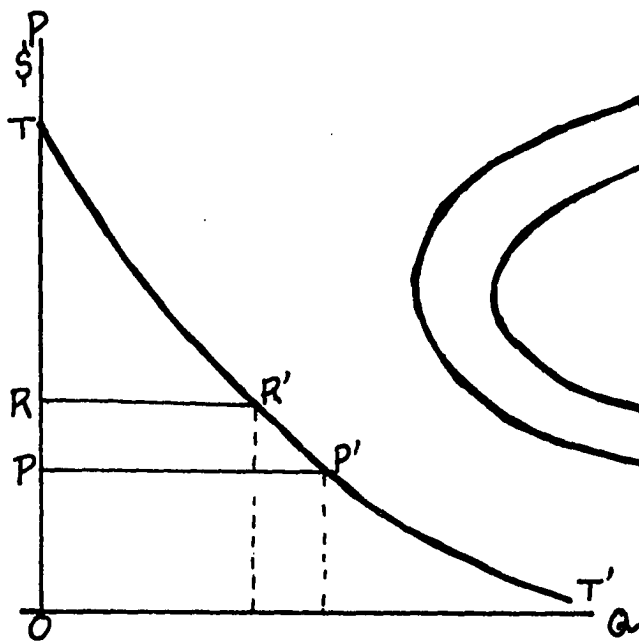


Fig. 5A

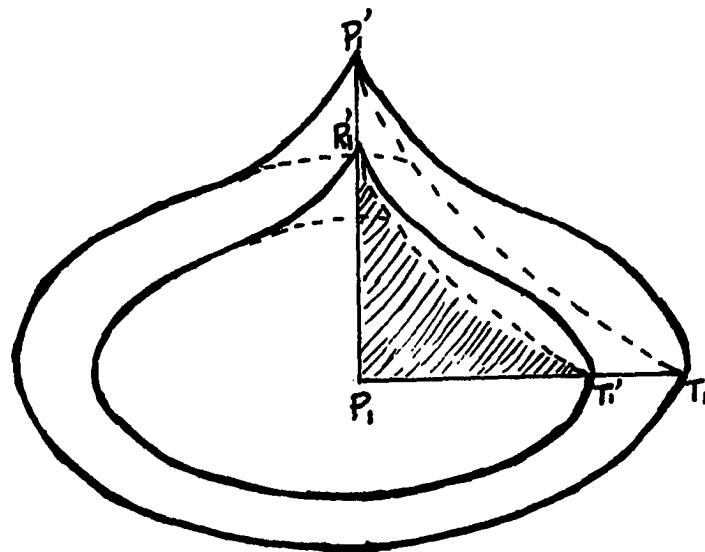


Fig. 5B

Figure 5. Distribution Cost and Market Area (II).

Note: This figure is modified from that of Lösch, see footnote 43.

CHAPTER II

EARLY THEORETICAL FORMULATIONS CONCERNING MARKET AREA

Market area analysis commenced as a segment of locational analysis. Through a series of explorations and development in this field by early theorists, market area analysis has assumed a more important role in regional analysis. In this chapter, earlier works concerning market area analyses will be reviewed. They will be classified into demand-oriented and cost-oriented analyses.

The Theory of Market Area in the Framework of the Theory of Location

The theory of location is concerned with the spatial relations of economic activities.¹ It tries to answer, in addition to the questions of "when," "what," and "why," the question of "where." Economic activities involve men, resources, and technology. The interaction of men, resources, and technology in the processes of production, distribution, and consumption reflects the spatial relation which may be

¹For a concise definition of location theory, see International Encyclopedia of Social Science, Vol. 15, pp. 95-108.

explained by way of transportation orientation, labor orientation, scale orientation (agglomeration), or a combination of these orientations. These methods are usually called the Thünen-Weber tradition.² Later developments in the theory of location are basically built upon this tradition, of course, with some modification.³ Even Isard's substitution approach, in the final analysis, is also built on the traditional methodology, for his reformulation of the transportation orientation doctrine is but to find

²The transportation orientation concept was introduced by von Thünen in his "Production Ring"--called Thünen's Ring. The concepts of labor orientation and scale orientation were introduced by Weber later. Thünen's Ring is constructed mainly through the effect of transportation on production. As Richards pointed out, the chief contribution of Von Thünen in the theory is his discussion of the effect of transportation on production. See Hoy Richards, "Transportation Cost and Plant Location: A Review of Principal Theories," in Hale Bartlett (ed.), Readings in Physical Distribution (Danville, Ill.: The Interstate Printers and Publishers, Inc., 1966), pp. 88-99. For Thünen Ring, see Johann Heinrich Von Thünen, Isolated State, translated by C. M. Wartenberg (New York: Pergamon Press, Ltd., 1966), Part I, Section I. (Original work was first published in German in 1828.) For Weber's work, see Alfred Weber, Theory of the Location of Industries, translated with an Introduction and Notes by Carl J. Friedrich (Chicago: The University of Chicago Press, 1929). (The original work was first published in 1909.)

³For a detailed development of the theory of location, see Walter Isard, Location and Space-Economy (New York: The Technology Press of M.I.T. and John Wiley and Sons, Inc., 1956), Friedman and Alonso, op. cit., Chapter 4, G. J. Karaska, "The Partial Equilibrium Approach to Location Theory: Graphic Solution," in G. J. Karosh and David F. Bramhill, Locational Analysis for Manufacturing, A Selection of Readings (Cambridge, Mass.: M.I.T. Press, 1969), pp. 22-41, and M. L. Greenhut, Plant Location, In Theory and Practice, The Economics of Space (Chapel Hill, N.C.: University of North Carolina Press, 1956), Part I, pp. 3-100.

the transportation optimal point by locating the correct substitution point between pairs of transportation inputs. These transportation inputs are treated like capital inputs and are regarded as an indication of roundaboutness in the production process conceptualized from that of Hayek.⁴

The location theory was originally developed from a study of the best location of plants and industries. Now it is concerned with a broader spectrum of analyses of the spatial relation of economic activities. In the selection of a best location, whether it is from the point of view of a firm, a project, a regional distribution center, or an industrial complex, the repercussions from the location of competing units must be considered because the interdependence of the location of all economic activities exists. The spatial relation of economic activities hence can be investigated from how an economic unit survives under spatial competition given the various orientations discussed above. The wisdom of a site selection, in the final analysis, is manifested in the size of the market of the plant, distribution center, or other economic unit.⁵ Therefore,

⁴Isard's transportation orientation doctrine can be found in Isard, op. cit., Chapters 4 and 5. For Hyake's "roundaboutness of capital" concept in relation to Isard's spatial analysis with transportation orientation, see F. Von Hayek, Pure Theory of Capital (London: Macmillan Company, 1941), p. 60.

⁵Greenhut is the major exponent of a similar concept. In Greenhut's words, "The size of a firm's market area suggests the wisdom with which its plant location has

in reality, one is mostly interested in how large his market area is. Hence, locational relation can also be represented by market area relation. Of course, the logical extension of this concept is regional economics.⁶ With this frame of reference in mind, the theory of market area in the framework of the theory of location can be expressed in Figure 6.⁷

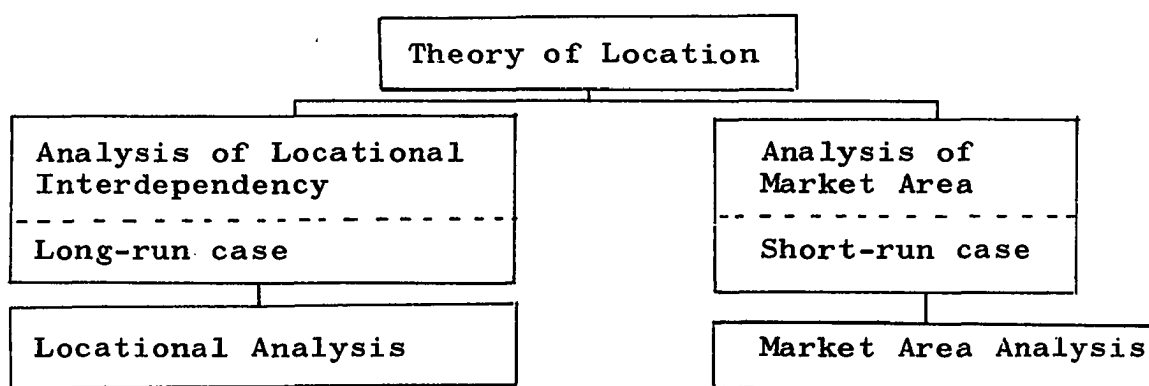


Figure 6. Market Areas Analysis and Theory of Location.

been selected." M. L. Greenhut, "The Size and Shape of the Market Area of a Firm," Southern Economic Journal, Vol. XIX (July, 1952), pp. 37-50.

⁶ A more elaborate discussion is found in August Lösch's "The Nature of Economic Region," op. cit.

⁷ The figure is constructed from Greenhut's concept with some modifications. For Greenhut's concept, see Greenhut, "The Size and Shape of the Market Area," op. cit. Greenhut accredited Fetter as the one who stimulated interest in market area analysis, and Hotelling as the one who pioneered the analysis of spatial interdependence of location. Greenhut, Ibid. Fetter's work will be discussed later in this chapter. For Hotelling's seminal work, see H. Hotelling, "Optimal Location in Spatial Competition," Journal of Political Economy, Vol. XL (1941), pp. 423-439.

The analysis of locational interdependence hypothesizes that the location is moveable in the sense that the location is adjustable over time. It is tailored for long-run analysis. In the market area analysis, one assumes a fixed location, thus it is a short-run phenomenon. The information gained from the market area analysis is useful for the long-run equilibrium analysis. In the analysis of the spatial relation of economic activities, whether it is long-run or short-run, the spatial factor is considered in two ways. These are: 1) transportation cost--cost of moving goods, the economic proximity; and 2) the effect of economic activities on others carried out in adjacent locations, neighborhood effect.⁸ Transportation cost is the price paid for conquering the resistance of space. The neighborhood effect may reduce, offset, or strengthen the transportation effect, depending upon the nature of the economic activity. It is understood that the theory of market area is only a part of spatial economics. It is that part of economic analysis which deals with partial spatial equilibrium analysis with a fixed time factor.⁹

⁸Martin Beckman, Location Theory (New York: Random House, 1968), p. 3.

⁹For a brief discussion of general spatial equilibrium analysis, see Hoover and Moses, "Spatial Economics," op. cit.

Demand-Oriented Market Area Analysis

Customers of a retailing unit may be: 1) casual customers who are occasional travelers, 2) once-for-all customers--the passing-by travelers, 3) loyal customers whether they live within the area or are from some other area, or 4) passive customers who buy only in response to specific sale promotions. It seems impossible to delineate the market boundary for a retailing unit because of the versatility of the nature of the shoppers in that unit. Even so, there are discernible patterns in the selling area of the individual retailing unit.¹⁰ The reason is rather straight-forward. Firstly, human wants and behavior have certain patterns which are the result of psychological, cultural, social, and economical backgrounds. Secondly, the response to various sales promotions has predictable patterns.¹¹

Empirically, the trade area for retailers can be delineated by analyses of customers' records (such as credit records), customer survey, customer spotting,¹² customer traffic flow analysis (Origin-Destination study),

¹⁰It is not economical to identify the market area for each individual commodity, for in each retail store there are multitudinous merchandise items.

¹¹This line of argument can be found in N. H. Schubert, "How Big Is a Market Area?," The Journal of Marketing, Vol. III (July, 1938), pp. 34-38.

¹²For details of customer spotting, see Applebaum et al., op. cit., section 9, pp. 206-231.

and customer cash flow analysis.¹³ Theoretically, a trade area may be defined by retail gravitation and the sale probabilistic methods which are discussed in more detail in the next subsections.

Retail Gravitation and Market Area

Employment of the Law of Gravitation in the analysis of retail trade area was first made by Reilly.¹⁴ As a result of his national study on city retail sales, Reilly found that the retail sale flow follows two rules:¹⁵

1) outside trade increases at about the same rate as the population of a city increases, and 2) retail trade decreases approximately in proportion to the square of the distance. If an intermediate city lies between retail center cities a and b, the share of trade that a and b can attract from the intermediate city can be shown by the following expression:¹⁶

¹³This type of study has been made by Edna Douglas, "Measuring the General Retail Trade Area--A Case Study," The Journal of Marketing, Vol. XIII (April, 1949), pp. 481-497.

¹⁴According to Huff, Reilly was the pioneer in this field. See Huff, op. cit.

¹⁵William J. Reilly, The Law of Retail Gravitation (New York: William J. Reilly, 1931), pp. 7-8.

¹⁶Ibid., p. 70. The Law of Retail Gravitation is stated as:

Two cities attract retail trade from any intermediate city or town in the vicinity of the breaking point, approximately in direct proportion to the population of the two cities and in inverse proportion to the

$$\frac{B_a}{B_b} = \left(\frac{P_a}{P_b} \right) \left(\frac{D_b}{D_a} \right)^2 \quad (1)$$

where:

B_a = share of trade from the intermediate city attracted by city a.

B_b = share of trade from the intermediate city attracted by city b.

P_a = population of city a.

P_b = population of city b.

D_a = distance from the intermediate city to city a.

D_b = distance from the intermediate city to city b.

The population of cities a and b are considered to be the forces of gravitation attracting trade to the city from

square of the distance from those two cities to the intermediate town.

See Ibid., p. 9. The distance and population exponent was determined by the formula:

$$\begin{aligned} \left(\frac{D_b}{D_a} \right)^n &= \frac{B_a}{B_b} \cdot \frac{P_a}{P_b} \\ n \log \frac{D_b}{D_a} &= \text{Log} \left(\frac{B_a}{B_b} \cdot \frac{P_a}{P_b} \right) \\ n &= \frac{\log \left(\frac{B_a}{B_b} \cdot \frac{P_a}{P_b} \right)}{\log \frac{D_b}{D_a}} \end{aligned}$$

See Ibid., pp. 70-72. For more exploration on this field of study, such as those of George Kingsley Zipf (index of interactance) and John O Stewart (index of influence); see W. Wantz, "The Topology of a Socio-Economic Terrain and Spatial Flow," Regional Science Association Paper, Vol. XVII (1966), pp. 47-61.

the intermediate city. Since it follows the Law of Universal Gravitation, Reilly called it the Law of Retail Gravitation.

By assuming an evenly distributed population in the intermediate city, Converse exhibited a convenient way of finding the breaking point for cities a and b as shown in equation (2) below:¹⁷

$$D_b = \frac{D_{ab}}{1 + \sqrt{\frac{P_a}{P_b}}} \quad (2)$$

where

D_{ab} = distance between a and b.

P_a , P_b , D_b = same as those of equation 1.

This formula is derived directly from Reilly's formula Equation (1). For the derivation of equation (2), see Appendix A.

The Law of Retail Gravitation assumes away the retail business in the intermediate town. This is very unrealistic, for the intermediate town, in fact, retains some business of its own. To consider this, Converse formulated a method to predict the proportion of retail sales a town or city will retain and the proportion it will lose. By fitting the empirical data secured from surveying somewhat

¹⁷P. D. Converse, "New Laws of Retail Gravitation," The Journal of Marketing, Vol. VIII (October, 1935), pp. 345-81. The New Law was originally formulated to apply to fashion goods or shopping goods (apparel and household furniture and furnishings).

more than 100 towns into the original formula for the Law of Retail Gravitation, Converse developed what he called the "inertia factor." The inertia factor, x in equation (3),¹⁸ is close to 4.0. (The average value for the inertia factor is 4.2 while the median value is 3.95.)¹⁹

$$\frac{B_a}{B_b} = \left(\frac{P_a}{H_b} \right) \cdot \left(\frac{x}{d} \right)^2 \quad (3)$$

where

x = inertia factor.

d = distance to the outside town.

H_b = home town population.

P_a , B_a , B_b = the same as those in equation (1).

Substituting 4 for x in equation (3), it becomes

$$\frac{B_a}{B_b} = \left(\frac{P_a}{H_b} \right) \cdot \left(\frac{4}{d} \right)^2 \quad (4)$$

Equation (4) represents what Converse called the New Law of Retail Gravitation which is stated as:

A trading center and a town in or near its trade area divide the trade of the town approximately in direct proportion to the population of the two towns and inversely as the squares of the distance factors using 4 as the distance factor of the home town.²⁰

¹⁸Ibid.

¹⁹Ibid.

²⁰Ibid. For more studies by Converse, see P. D. Converse, A Study of Retail Trade Area in East Central Illinois, University of Illinois, Bureau of Business Research, Business Study No. 2, Urbana, Ill., 1943.

Following the Reilly-Converse approach, one can delineate the trading area for the "home town" by connecting the breaking point between the home town and the surrounding outside cities. However, the method does not provide:

1) a graduated estimation above or below the break points; 2) an estimation of the unaccountable area created by the geographical spreading of the area in question; 3) an estimation of the overlapping of boundaries; and 4) differences in the market area for different commodities.²¹ Consequently, the method can only be used for a gross approximation for preliminary screening in market area analysis.

Probability of Consumer's Shopping Travel vs. Market Area

The problems of overlapping, unaccounted area, lack of graduated estimation of demand, and general (or aggregate) market area hypotheses, discussed in the previous section as the weakness of the Reilly-Converse approach, can be improved by assessing the probability of consumers' shopping travel to a retail center for each commodity in question. The market area for a specific product can be represented by a "demand surface" which consists of demand gradients reflecting the probability of consumers' shopping travel from their origins to the retail center in question.²²

²¹Some of these points were mentioned by Huff.
See Huff, op. cit.

²²Ibid.

The delineation of the equilibrium market area is made by connecting the points of intersection of equal probabilistic gradients for competing centers. Each gradient represents a line of equal probability of consumers' shopping travel from their origins to the retail center. This line is usually concentric to the retail center. The probability of the consumer at origin i , travelling to retail center j , (P_{ij}), according to Huff, can be expressed as:²³

$$P_{ij} = \frac{\frac{S_j}{T_{ij}^\lambda}}{\sum_{j=1}^n \frac{S_j}{T_{ij}^\lambda}} \quad (5)$$

where

S_j = the size of a shopping center j in terms of a square foot of selling area devoted to the sale of a particular class of goods.

T_{ij} = travel time involved in getting from i to j .

λ = travel time factor reflecting the effect of travel time on different shopping trips.

²³Ibid. For further discussion of the theory, see David L. Huff, "A Probabilistic Analysis of Consumer's Behavior," in William S. Decker (ed.), Emerging Concepts in Marketing (Chicago: A.M.A., 1963), pp. 444-60. One may derive the market area in terms of the number of customers (T_j) from the result of equation (5):

$$T_j = \sum_{i=1}^n (P_{ij} \cdot C_i)$$

where C = number of customers residing in i .
See Huff, "Defining and Estimating a Market Area," op. cit.

The travel time factor λ is determined by empirical data. It varies among different products. When its value is 2, the distance-sale relation becomes a case of Reilly's contention.

The gradients for competing centers overlap. Bucklin recently developed a probabilistic method to derive a trade area "contour profile" to measure the degree of overlapping.²⁴ In Bucklin's method, a shopper's chance of visiting a particular center declines as he moves from one center to another center, which may be explained as:²⁵

$$P_a = f(D_a, T) \quad (6)$$

where

P_a = probability of a shopper visiting in relation to distance.

D_a = specific distance (from O-T).

T = total distance.

The probabilistic approaches discussed in this section implicitly assume that the shoppers who travel to the center are the customers of the center which, in many cases, may not be true, for many who travel to the center may be just window shoppers. Since breaking points between competing centers are determined by the intersection of

²⁴L. P. Bucklin, "Trade Area Boundaries: Some Issues in Theory and Methodology," The Journal of Marketing Research, Vol. VIII (Feb., 1971), pp. 30-7.

²⁵Ibid.

the gradients of equal probability for the competing centers, the equilibrium boundary exists only in relative terms--namely, the same percentage of customers who reside on the boundary is indifferent as to which center to shop.

Cost-Oriented Market Area Analysis

Since at the equilibrium boundary, the delivered prices of like commodities from different suppliers must be the same, and since the non-retail marketing units generally assume delivery services, the manufacturing costs and distribution costs become vitally important factors in determining how well the non-retail marketing units can overcome spatial resistance, given the demand condition of the commodity in question. For manufacturing plants, these costs are simply the base price and transportation cost; for wholesalers and distribution units, they are base price, transportation cost, and other distribution costs. For analytical simplicity, "other distribution costs" have been assumed to be fixed. Therefore, analysis of the size and shape of the market area for the non-retail marketing unit has been centered on the effect of the basic price and transportation cost on the market area. Early works in this area of study are numerous. In this section only the significant works are reviewed.²⁶ These are presented under

²⁶Those works that are only the modified version of the principal theories, and those works that are insignificant are not included. For example, Alonso's

the headings of Economic Law of Market Area, Isotimes Approach, Marginal Line Rule, Hexagonal Market Area, New Economic Law of Market Area, and Total Transportation Cost Approach.

The Economic Law of Market Area

A definite idea of the size and shape of the market area, under the assumptions of equal freight rate per unit of distance for all directions and fixed demand conditions, was first formulated by Fetter.²⁷ Fetter's Law of Market Areas relates the market territory to the market base price and freight rate, which is stated as:

The boundary line between the territories tributary to two geographically competing markets for like goods is a hyperbolic curve. At each point on this line the difference between freight from the two markets is just equal to the difference between the market price. Whereas on either side of this line the freight difference and price difference are equal. The relation of prices in the two markets determines the location of the boundary line: the lower the relative price the larger the tributary area.²⁸

"transportation network consideration," Dunn's "market potential concept," and "Project Bosphorus," "differential cost method," etc. are but some of many that are omitted here.

²⁷Fetter, op. cit.

²⁸Ibid. According to Greenhut, W. Launhardt, in 1885, combined an analysis of cost orientation and market area, but his reputation is lacking because of the lack of access to his writing. Later, John Bates Clark also inquired into a similar problem, especially that of the indifferent line separating the market area of rivals in his Control of Trust, First Edition (New York: Macmillan Company, 1914), Chapter IV. For further details about this line of argument, see Greenhut, "The Size and Shape of Market Areas," op. cit.

The central concept of this law is shown in equation (7) below.

$$C_A - C_B = f(\overline{AP'} - \overline{BP'}) \quad (7)$$

where

AP' , BP' = distances from centers A and B to equilibrium point P' , respectively.

C_A , C_B = market base price for center A and center B.

P' = moving point on the boundary line.

Since $(C_A - C_B)$ is constant, $f(\overline{AP'} - \overline{BP'})$ must be constant. This condition signifies that the line of moving point P' is hyperbolic. Equation (7) may be translated as shown in Figure 7.

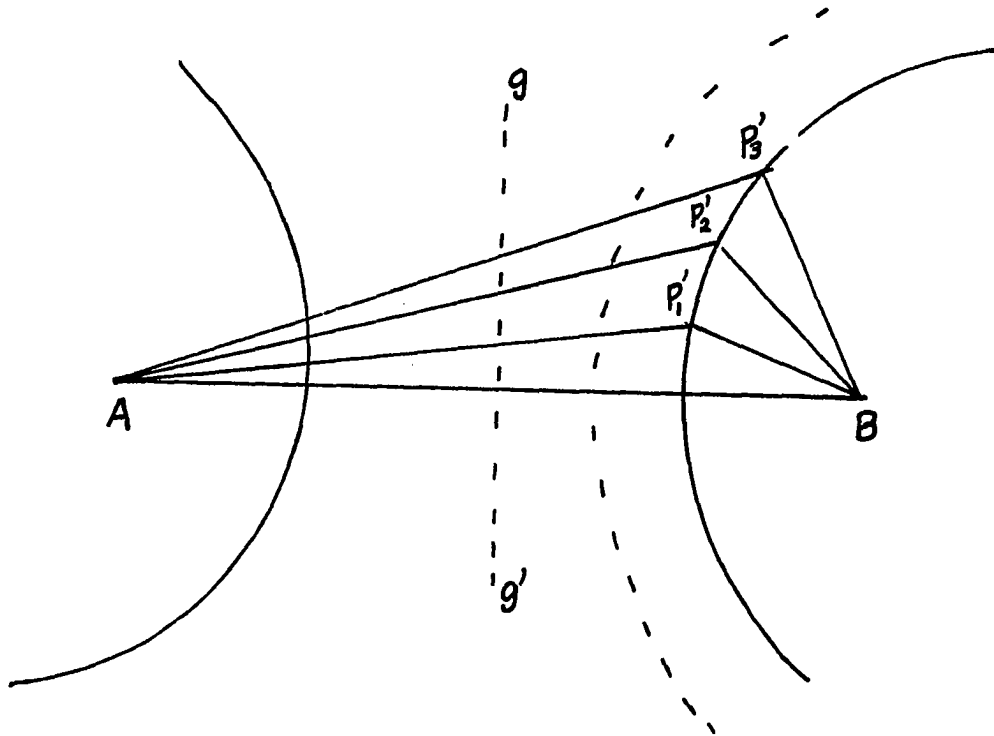


Fig. 7. Fetter's Hyperbolic Market Boundary Line Between Two Spatially Competing Suppliers.

Note: This is the graphical translation of Fetter's Law.

In the special case where $C_A = C_B$, the boundary line gg' will bisect the straight line connecting A and B.

Fetter's law uncovers the basic relation between the manufacturing cost and market area, holding the unit cost of overcoming the spatial resistance constant. Although the resultant hyperbolic boundary line is mathematically sound, the actual application of this law is limited. First, the point by point construction of the hyperbolic line involves the determination of vertices and second vertices which is necessarily a tedious pursuit.²⁹ Secondly, the equal rate per unit distance for all directions exists only in theory, not in practice.

Isotimes Approach

Using a system of contour lines which connects points of equal delivered price in the determination of the boundary line between the territories tributary to spatially competing marketing centers was first employed by Hoover.³⁰ This approach was basically built on the

²⁹The standard formula for the hyperbola is $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ (where: a is half of the transverse axis of the hyperbola, b is half of the conjugate axis of the hyperbola).

³⁰The idea of the isotime and, hence, the system of isotimes, or isopane, was first developed by Weber, the German economist, and later fully treated in the application of transportation cost by Tord Palander. See Hoover, Location Theory and the Shoe and Leather Industries, op. cit., p. 8, Footnote 3.

"Launhardt-Fetter tradition" with the same assumptions underlying the Launhardt-Fetter Method. However, Hoover's method was an improvement in that the isotimes approach has the value of practical application. As shown in Figure 8,³¹ each isotime represents the equal delivered price from the center (X or Y). These isotimes are concentric circles about each center, for the delivery cost decreases as the delivering point approaches the center.

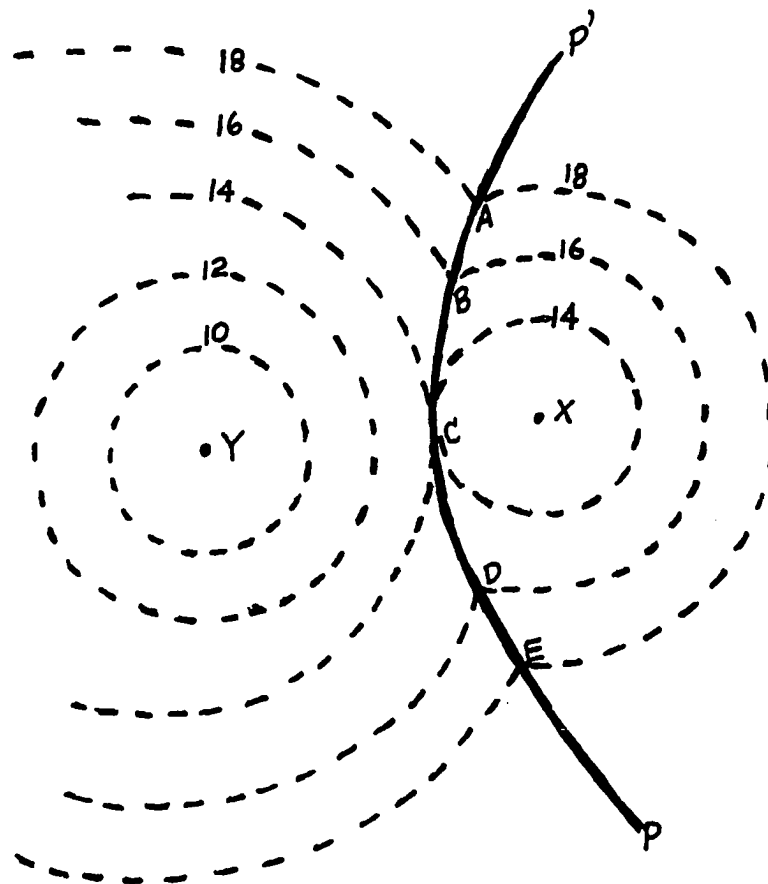


Fig. 8. Isotimes and Market Boundary.

³¹This figure is the simplified form of Hoover's. See Hoover, Ibid., p. 12, Figure 3.

The boundary line assumes a smooth hyperbolic line under the proportional freight rate assumption. For a two-competitors case, the boundary is the line connecting points A, B, C, D, and E (or PP'). With non-proportional rate structure, the boundary line will not be a smooth one.³²

Marginal Line Rule

Again following Launhardt-Fetter's concept of the market area, Hoover expounds the effect of the diminishing return upon the extent of market, citing a case of extractive industry. His line of argument may be presented with the help of Figure 9.³³ As the market area, and hence the total output, increases, the price of the goods produced at the point of production will rise. Therefore, the delivered price will then increase. Since the delivered price is the sum of the costs of delivery and manufacturing, a transportation gradient may be drawn for each expansion of production, and hence, the extension of the market area. For example, CO is the transportation gradient for market area extended from A, the center, to L; RS, A to M; and TU, A to N, etc. O, S, and U indicate the delivered

³² Hoover made a very brief comment on this point with figures, but no further discussion was made. See Hoover, Ibid., p. 13, Fig. 4.

³³ For a more detailed explanation, refer to Ibid., pp. 13-22.

price at the edge of the market area for center A. Likewise, K, I, and G indicate the delivered price at the edge of the market area for center B. The lines connecting O, S, U (for center A), and K, I, G (for center B), are referred to by Hoover as marginal lines. The intersection of these two lines, in the two competitors case, signifies the equilibrium market boundary, Z.

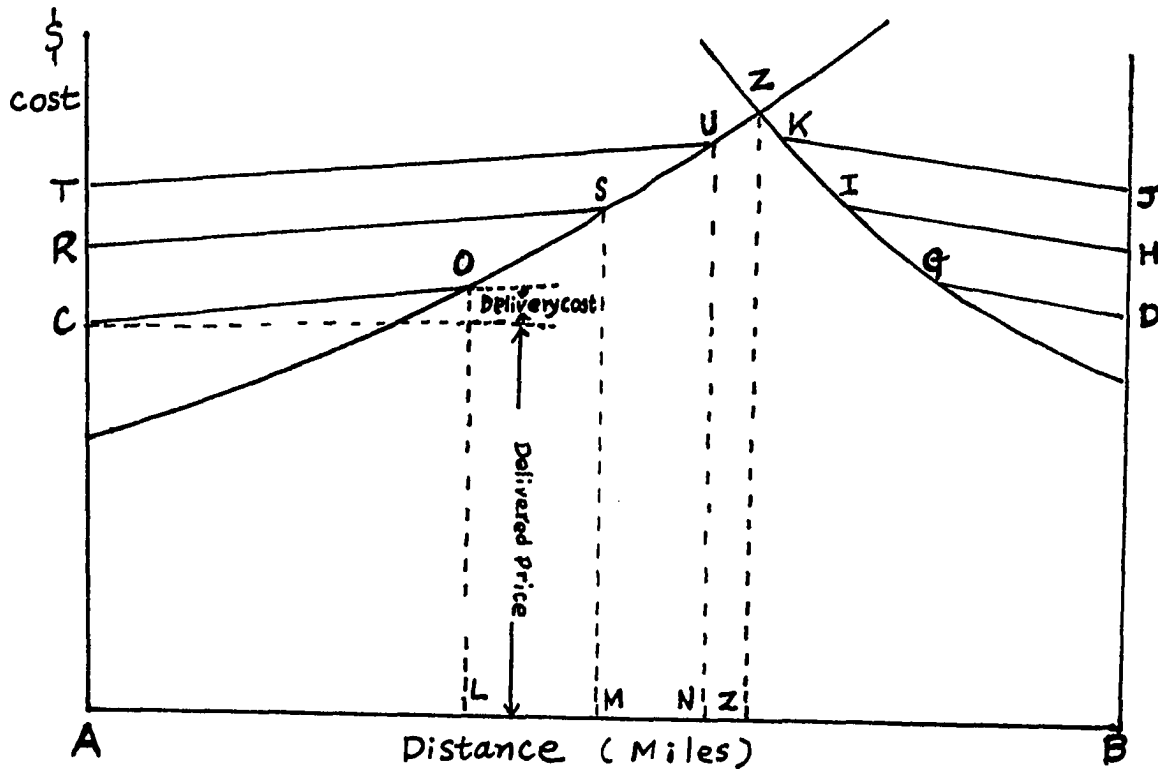


Figure 9. Marginal Line and the Extent of Market Area.

Source: E. M. Hoover, Location Theory and the Shoe and Leather Industries (Cambridge, Mass.: Harvard University Press, 1937), p. 17, Fig. 7 (some modifications made).

It is obvious that the slope of the transportation gradients and the marginal production cost are the major factors affecting the slope of the marginal line. As the

transportation rate tapers off, the transportation gradient convexes upward. Such new gradients will "retard the rise of the marginal line in its outer reaches."³⁴

The superiority of Hoover's marginal line approach is its emphasis of the marginal cost which the early theorists had long neglected.

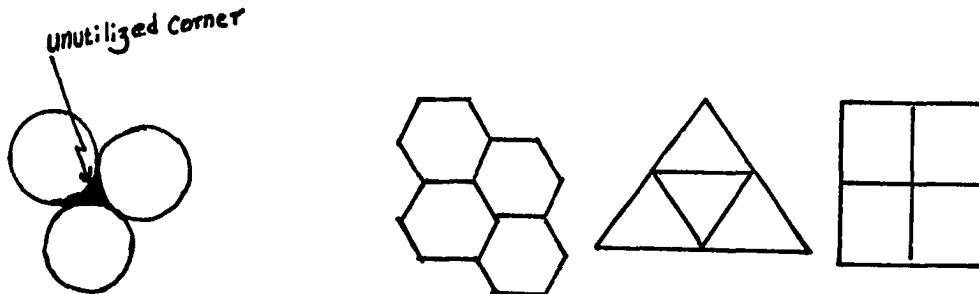
Hexagonal Market Area

Theorists up to Hoover had been more or less interested in the determination of the boundary line between markets tributary to competing suppliers. The discussion of the size and shape of the market "area" has been either alluded to or merely mentioned in passing. It was Lösch who first brought to attention such a question of market area. Assuming an equal distribution of consumers (with equal taste), adequate and equal distribution of raw materials, equal transportation rates per unit distance in all directions, and freedom of entrance and exit into business, Lösch argued that the optimal market area is shaped like a hexagon.³⁵ He contended that out of the three possible shapes, the triangle, square, and hexagon, which will eliminate the "unutilized" corner created by the circles (see Figure 10), the hexagon will have the greatest demand

³⁴Ibid., p. 21. Hoover provided only lip service to this probable situation when a non-proportional rate structure was considered.

³⁵Lösch, The Economics of Location, op. cit., pp. 105-123.

per unit of the entire area.³⁶ The equilibrium market boundary is the indifference line on which the prices of like commodities from different sources are the same.



with unutilized corners without unutilized corners

Figure 10. Market Area with Unutilized Corners and Market Area without Unutilized Corners.

It does not matter to which neighboring market customers on this line belong. The location of this boundary line may be expressed in equation (8) below:³⁷

³⁶This point was proved by Lösch using the demand cone concept. If the demand cone, as shown on page 29 of the last chapter, is cut by a plane parallel to the axis in such a way that the triangle, square, or hexagon has an equal base area, and if the proportion of the cones remaining above are compared, one will be able to find that the most area is required with a triangle and the least with a hexagon for the same value of demand. For a more rigorous proof of this statement, refer to Lösch, *Ibid.*, pp. 110-114. Lösch's hexagonal contention was refuted by Mill and others. They proved that the free entry could not result in the space filling market area. Instead, circular is the optimum shape. For a detailed discussion, see E. S. Mill, *et al.*, "A Model of Market Area with Free Entry," Journal of Political Economy, Vol. LXXII (June, 1964), pp. 278-88.

³⁷This equation is simplified from Lösch's original, rather complicated, equation. For his original equation, see Lösch, The Economics of Location, *op. cit.*, pp. 94-7.

$$A_1 + r_1 \sqrt{(x-x_1)^2 + (y-y_1)^2} = A_2 + r_2 \sqrt{(x-x_2)^2 + (y-y_2)^2} \quad (8)$$

where

A_1, A_2 = basic price for plant 1 and plant 2.

r_1, r_2 = transportation rate per unit of distance from
plant 1 and plant 2 to the equilibrium point (x, y)

x, y = locational coordinates x and y (both in terms
of distance units)

Equation (8) becomes more understandable through Figure 11.³⁸ As shown in this figure, the distance between plant 1 (x_1, y_1) and the market equilibrium point (x, y) , and the distance between plant 2 (x_2, y_2) and the market equilibrium point (x, y) can be expressed via the distances between points (x, y_1) and (x, y) and points (x_1, y_1) and (x, y_1) ; and distances between points (x, y_1) and (x, y_2) and points (x, y_2) and (x_2, y_2) , respectively, according to the Theorem of Pythagoras. They are $\sqrt{(x-x_1)^2 + (y-y_1)^2}$ and $\sqrt{(x-x_2)^2 + (y-y_2)^2}$. It is clear that equation (8) is merely an alternative way of expressing the traditional Launh-Fetter-Hoover concept.

The New Economic Law of Market Area

Fetter's hyperbolic market boundary line assumes, among other things,³⁹ the same rate per unit of distance

³⁸This figure is the geographical translation of Lösch's distance notation adopted in equation (8).

³⁹Those other things are: 1) standardized commodities, 2) complete knowledge of market, 3) given space preference, 4) given consumer's tastes and elasticity of demand for commodities.

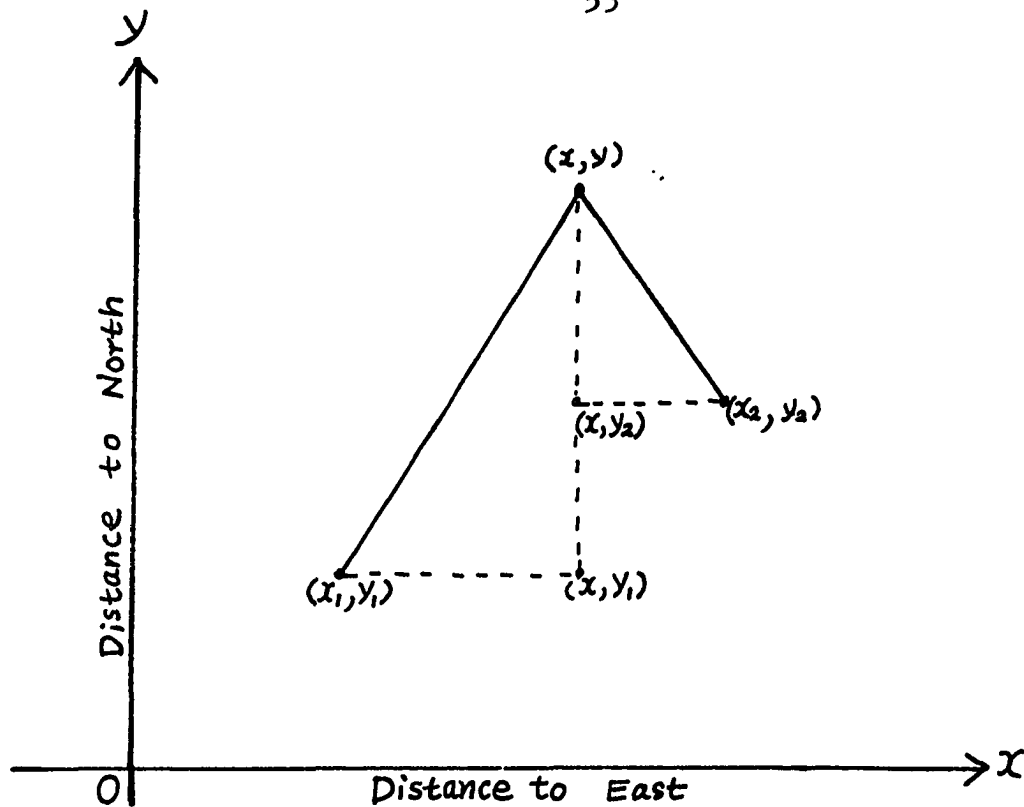


Figure 11. Market Equilibrium Point as Shown by Locational Coordinates.

for all directions. What will happen in the market area analyses if the freight rate is proportional but varies with different directions? This is the point investigated by Norwegian economists, the Hysons. The Hysons contended that the market boundary line is a hypercircle, not a hyperbola.⁴⁰ According to them, at the equilibrium state, it is equally advantageous for consumers on the boundary to buy from, for example, either market A or B. The conditions expressed by equation (9) must be met.⁴¹

⁴⁰C. P. Hyson and W. P. Hyson, "The Economic Law of Market Areas," Quarterly Journal of Economics, Vol. LXIV (1950), pp. 319-29.

⁴¹This is derived as follows:
$$\frac{P}{PA} + r \frac{\overline{PA}}{PA} = q \frac{\overline{PB}}{PB} + s \frac{\overline{PB}}{PB}$$

For further details, see Ibid.

$$\overline{PA} - \frac{s}{r}\overline{PB} = \frac{q-p}{r} \quad (9)$$

where

p, q = base prices for A and B, respectively.

r, s = freight rate from A and B, respectively.

\overline{PA} = distance from A to any point (P) on the equilibrium line.

\overline{PB} = distance from B to any point (P) on the equilibrium line.

Equation (9) defines the market boundary between A and B. Since, as shown in Figure 12, $\frac{s}{r} > 0$ and $\frac{q-p}{r} \leq 0$, equation (8) may show:⁴²

- 1) As $\frac{s}{r}$ decreases, B' market area increases, given $\frac{q-p}{r}$, resulting eventually in a circle around A.
- 2) As $\frac{s}{r}$ increases, A' market area increases, given $\frac{q-p}{r}$, resulting eventually in a circle around B.
- 3) Increases in the size of the area for B, if $\frac{q-p}{r}$ decreases (but the shape will depend on $\frac{s}{r}$).

In the special case where $s = r$, $\frac{q-p}{r} \neq 0$ (or $p \neq q$), or $s = r$, $\frac{q-p}{r} = 0$ (or $p = q$), the curve will follow Fetter's Law. Equation (9) is more general than that of Fetter's Law. The Hysons' Law may be called the "New Economic Law of Market Areas" which is stated as:

The boundary line between the territories tributary to two geographically competing markets for like goods is a hypercircle. At each point on this curve the

⁴²Ibid. For an analysis of the behavior of the hypercircles, see Appendix B.

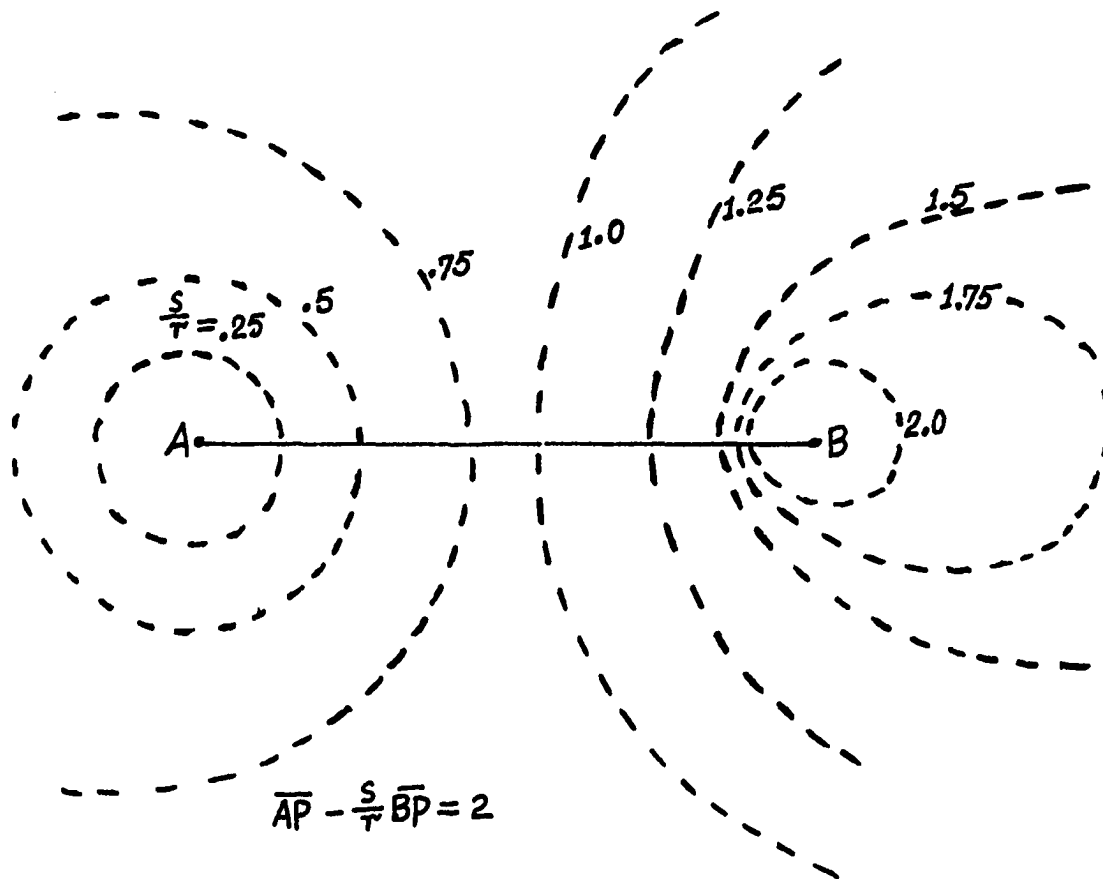


Figure 12. The Hysons' Market Boundary.

Source: C. D. Hyson and W. P. Hyson, "The Economic Law of Market Areas," Quarterly Journal of Economics, Vol. LXIV (1950), p. 322, Fig. 3.

difference between freight costs from the two markets is just equal to the difference between the market prices, whereas on either side of this line the freight differences and price differences are unequal. The ratio of the price difference to the freight rate and the ratio of the freight rate from the two markets determine the location of the boundary line; the higher the relative price, and lower the relative rate, the larger the tributary area.⁴³

It is noted that in the Hysons' hypercircle contention,

⁴³Ibid.

"the equal freight rate for all directions" assumption was, for the first time, relaxed. However, the proportional freight rate assumption still stands.

The Total Transportation Cost Approach

The cost-oriented market area analysis that embraces the total moving cost, including the moving cost of raw materials and that of finished products, was first pioneered by Isard.⁴⁴ Isard defined the condition of equilibrium boundary as the boundary where "the sum of the transportation costs on the unit production and on the raw materials required to yield the unit product is just equal to the difference between unit costs of production and the maximum price the consumer is willing to pay."⁴⁵ Symbolically, this is expressed as:⁴⁶

$$r^*s^* + \sum_{i=A}^F b_i \cdot Y_i \cdot S_i = T \quad (10)$$

where

r^* = transportation rate for a unit of product.

s^* = radius of the circle defining the boundary line.

A, B, ... F = various raw materials.

b_i = constant coefficient indicating the number of units of raw material i used per unit of finished product.

⁴⁴Isard's approach is actually the synthesis of Weber and Hoover's concept. This can be seen from Isard, Location and Space-Economy, op. cit., pp. 231-2.

⁴⁵Ibid.

⁴⁶Ibid.

Y_i = transportation rate on a unit of raw material.

S_i = distance from raw material site to processing point (factory) for i .

T = difference between maximum sale price and total unit costs (excluding transport costs) which are held constant.

The equation indicating the equilibrium boundary, in the two competitors' case, may be derived as:⁴⁷

$$O_p + Y_p \cdot S_p^0 + \sum_{i=A}^F b_i \cdot Y_i \cdot S_{ip} = O_v + Y_v \cdot S_v^0 + \sum_{i=A}^F b_i \cdot Y_i \cdot S_{iv} \quad (11)$$

where

O_p, O_v = marginal cost of plant P_p and P_v .

Y_p, Y_v = transportation cost of plant P_p and P_v .

S_p^0, S_v^0 = the distances from P_p and P_v , respectively,

to any point on their common boundary line.

b_i, Y_i = the same as those for equation (10).

S_{ip}, S_{iv} = distance from the raw material site to the production point for p and v .

Isard's formula evidently still assumes the unrealistic distance proportional transportation rate and constant coefficient for raw material input. Even so, it seems that this approach is one stride toward a general spatial equilibrium. This can be seen from his inclusion of raw materials and, hence, various products.

⁴⁷Ibid., p. 237. For the derivation of this formula, see Ibid., pp. 231-7.

CHAPTER III

A NEW METHOD OF IDENTIFYING A MARKET AREA-- THE NEW BREAK-POINT APPROACH

It was noted in the previous chapters that, in analyses of market areas for retailing units it is important to know how the distance, population, and consumers' propensity to travel to a particular location affect the size and shape of the market area, assuming that the cost of a commodity is fixed. Also it was noted that, in the market area analysis for non-retailing marketing units, the emphasis is primarily on the effect of the base price and transportation cost upon the size and shape of the market area when demand is constant. The difference in such analytical emphases stems from the question of who pays the cost of overcoming the spatial resistance--the delivery cost. Distribution centers, wholesalers, and manufacturers' sales outlets bear the delivery cost, hence, cost is the primary concern in the market area analysis. Delivery cost from the retail store to the consumer's home is assumed by the ultimate consumer. Therefore, study of consumer behavior is heavily emphasized in retail market

area analyses.¹ In this chapter, an alternative method of identifying the market area for a non-retail marketing unit will be introduced. Since the new method will define the market boundary line through the approximation of the loci of break points, the method will be called the New Break-Point method.

Need for a New Approach

In the cost-oriented market area analysis, besides the ceteris paribus assumptions of various demand factors (such as tastes and preference, income, individual elasticity of demand, etc.), the following specific assumptions underlie literally all earlier theoretical constructions. These are: 1) the price of the product is basic price f.o.b. mill plus the freight charge; 2) the freight rate is proportional to distance; 3) all suppliers have equal access to consumers; and 4) transportation networks are evenly distributed. The market boundary between areas tributary to spatially competing suppliers (or centers), in the early works is identified by 1) drawing the isotimes and, hence, isopane, 2) drawing the hyperbolic line through points to points construction, 3) drawing hypercircles using the rectangular coordinates or calculus method, 4) drawing of equi-marginal lines, and 5) purely mathematical construction without considering the operational feasibility.

¹It is understood that other factors, such as floor space, number of items sold, etc., also affect the size of the market area. For more details, see Applebaum, et al.

An examination of the early works concerning market area in Chapter II raises two critical questions. They are:

1) Is the distance-proportional freight rate assumption realistic under the existing transportation rate structure of this country? If not, what are possible improvements?

2) Are the methods used for drawing the market boundary line operational in the sense that they can be employed for actual application without technical complexity? If not, what are alternative methods?

The first issue has been put aside solely for the sake of analytical simplicity. The second issue has not been answered satisfactorily because of the lack of urgency for a practical drawing of the market boundary by business organizations.

It is clear that a new method which can at least consider these two critical issues is needed. The New Break-Point method, which is introduced in this chapter, and the logical extension of the new method, which is expounded in the following chapter, are tailored to answer such a need.

The Assumptions

When the spatial equilibrium for a commodity is reached, as noted in the previous chapters, there will be an identifiable market boundary between territories tributary to competing suppliers or centers. This boundary can

be treated as consisting of loci of break-points between (or among) suppliers or centers. On these points, the selling prices of like commodities from different suppliers are the same. To the customers on the market boundary it is a matter of indifference which suppliers are used. Such an equilibrium state is based on some assumptions which may be grouped into demand related assumptions and cost related assumptions.

The fundamental demand assumption is that the consumer's demand schedule and the elasticity of demand are given. Implicit in this assumption are: 1) There is no excess demand, 2) Consumers' tastes and preferences are given, 3) Consumers' incomes are given, 4) Commodity substitution effects are given, 5) Consumers have no strong brand and institutional loyalty, and 6) There are no significant differences in the consumers' responses to sales promotion for the like commodities by competing suppliers.

The location of the break-points is greatly affected by the processing and moving costs of the competing suppliers. While the cost situation of the spatially competing suppliers may differ greatly in its complexity, for analytical convenience, some assumptions concerning the cost are necessary. These cost related assumptions are: 1) The product is sold at the price which consists of the basic price f.o.b. mill and the cost of transportation, 2) Freight cost (or rate) increases are less than proportional to

increases in distance,² and 3) The out-bound freight rate schedule for the same commodity is applicable to all directions, and such a schedule is the same for the neighboring areas.³

The second assumption will be further discussed in the next section where equations for transportation cost are developed. This second assumption is more realistic than what has been assumed for the early theoretical formulations and is considered to be an important improvement in the theoretical investigation of market area.

It is reminded that there exist implicit assumptions underlying this method. These are that pricing policy, managerial ability, organizational economy, scale economy, external economy, labor conditions, tax climates, and socio-political conditions are altogether taken as datum. That which remains to be explained consists of the critical factors which have predominant influences in the

²It is recognized that the distance rate is one of many rate systems existing in this country. Other systems are group rates, base point and related rate systems. For further detailed discussion, see D. P. Locklin, Economics of Transportation, Sixth Ed. (Homewood, Ill.: Richard D. Irwin, Inc., 1966), Chapter 9, pp. 158-1966.

³As the result of the competition between regions, or states, the rate-making authority in constructing the rate schedule tends to make it comparable to the neighboring states or region. Therefore, this assumption is quite realistic. This point is expounded by Mr. Ryan, Traffic Coordinator for the State of Oklahoma. An Interview with William Ryan, Traffic Coordinator, State of Oklahoma, Oklahoma City, Oklahoma, Sept. 20, 1971.

economic phenomena under study. Without losing the essence of the analytical framework, such an abstraction has the advantage of leading to a more comprehensive understanding of economic phenomenon.

Determination of the Distance-Moving
Cost Relation⁴

Under the prevailing rate structure, it generally holds that the rate per unit of distance decreases as the distance for which the rate applies increases. This is true for different kinds of rate schedules--class rates, commodity rates, and exception rates.⁵ In other words, the cost of moving goods increases at a decreasing rate as the distance for which the goods are moved increases. For example, Table I shows the rate schedule for moving bituminous coal from the Clinton, Richhill and Pittsburg groups to points south (Column A) and north (Column B) of the line extending from St. Louis to Kansas City. It costs 188¢ to move one hundred pounds of bituminous coal for the

⁴It is understood that the terms "moving cost" and "transportation rate" are not the same thing although the rate decision is based on the cost of moving. However, the moving cost, paid to facilitating transportation agencies are, in fact, the same as the freight rate.

⁵About 91% of interstate rail traffic is moved by commodity rates; 7%, exception rates; 1%, class rates; and 1% miscellaneous. See Constantin, Principles of Logistics Management, op. cit., p. 217. It is understood that because of the effect of competition, the cost-distance relation may be distorted and "basing points" or "basing lines" created. See Locklin, op. cit., pp. 188-189.

TABLE 1

SELECTED FREIGHT RATES FOR BITUMINOUS COAL¹
 (Between points as described in
 Notes 2 and 3)

Selected Base	Net Changes in Selected Rate (¢/cwt)	
	Column A ²	Column B ³
40	188	189
80	63	64
120	38	39
160	35	32
200	30	31
240	39	39
280	26	26
320	24	25
360	22	22
400	22	21
440	21	20
480	22	20

- Note: 1. Data selected from Tariff 2-E, page 251.
 2. From Clinton, Richhill and Pittsburg groups to points south of the line extending from St. Louis to Kansas City.
 3. From Clinton, Richhill and Pittsburg groups to points north of the line extending from St. Louis to Kansas City.

first 40 miles, 63¢ for the second 40 miles, 38¢ for the third 40 miles, and 35¢ for the fourth 40 miles, and so on. Such a distance-cost relation may be explained by equations containing either power functions, exponential functions,

or both. These equations are:

$$y = \varphi a^m \text{ with } 0 < m < 1 \quad (12)$$

$$y = h(1 - \varphi e^{-ma}) \quad (13)$$

and

$$e^y = \varphi(a+1)^m \quad (14)$$

where:

y = transportation cost

a = distance

m = parameter explaining the shape of the curve

φ = constant coefficient

h = maximum rate in the freight rate schedule.

While equation (12) is the popular formula for the non-linear relation between two variables, equations (13) and (14) are variations of other popular formulas for non-linearity, namely, $y = \varphi e^{ma}$ for equation (13) and $e^y = \varphi a^m$ for equation (14). All three equations represent the curve with positive changes and a decreasing rate of positive changes. This point is shown by the heavy line in Figure 13. This point can be seen from the fact that, for all three equations, the first derivative of y with respect to a is positive and the second derivative of y with respect to a is negative.⁶ It is noted that logically the constant φ

⁶The first and second derivatives of y with respect to a for equations (12), (13), and (14) are shown as follows:
For equation (12):

$$\frac{dy}{da} = \varphi m a^{m-1} > 0$$

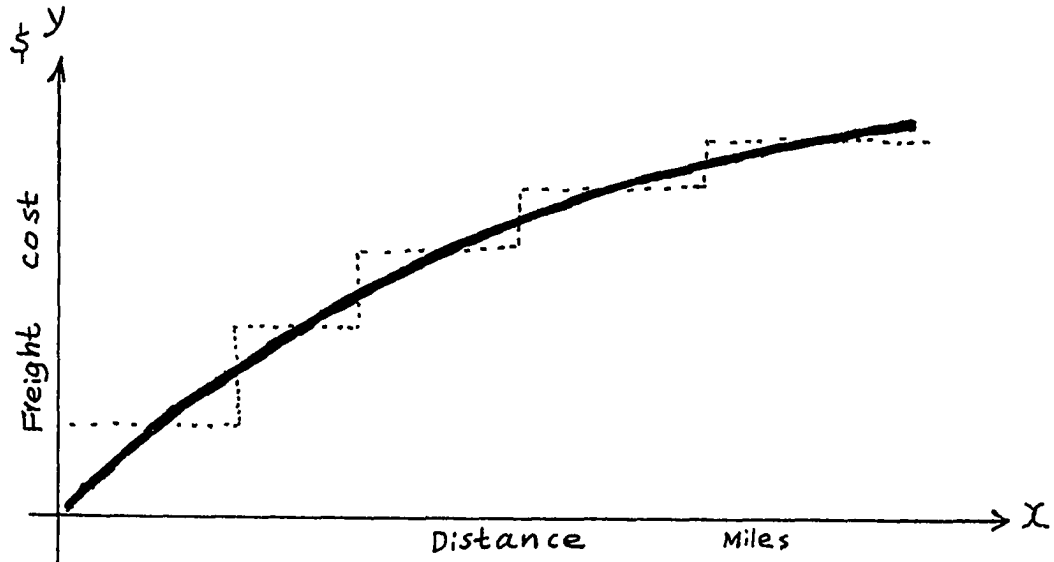


Fig. 13. General Shape of the Curve Depicting the Distance-freight Cost Relation.

$$\frac{d^2 y}{da^2} = \vartheta m(m-1)a^{m-2} < 0$$

For equation (13):

$$\frac{dy}{da} = -\vartheta h e^{-ma}(-m) > 0$$

$$\frac{d^2 y}{da^2} = \vartheta m h e^{-ma}(-m) < 0$$

For equation (14):

$$y = \log_e \vartheta + m \log_e(a+1)$$

$$\frac{dy}{da} = m \frac{1}{a+1} > 0$$

$$\frac{d^2 y}{da^2} = -m \frac{1}{(a+1)^2} < 0$$

is not needed, for, in reality, the cost of moving commodities for zero distance is zero. Even so, for more accurate representation of the current rate structure characterizing the higher starting rate in the first rate bracket of nearly all rate schedules, the constant coefficient φ is included in both equations (13) and (14). For example, in fitting the curve, the equation with φ , $e^y = \varphi(a+1)^m$, fits better than the one without φ , $e^y = (a+1)^m$. This point is illustrated in Figure 14 below.

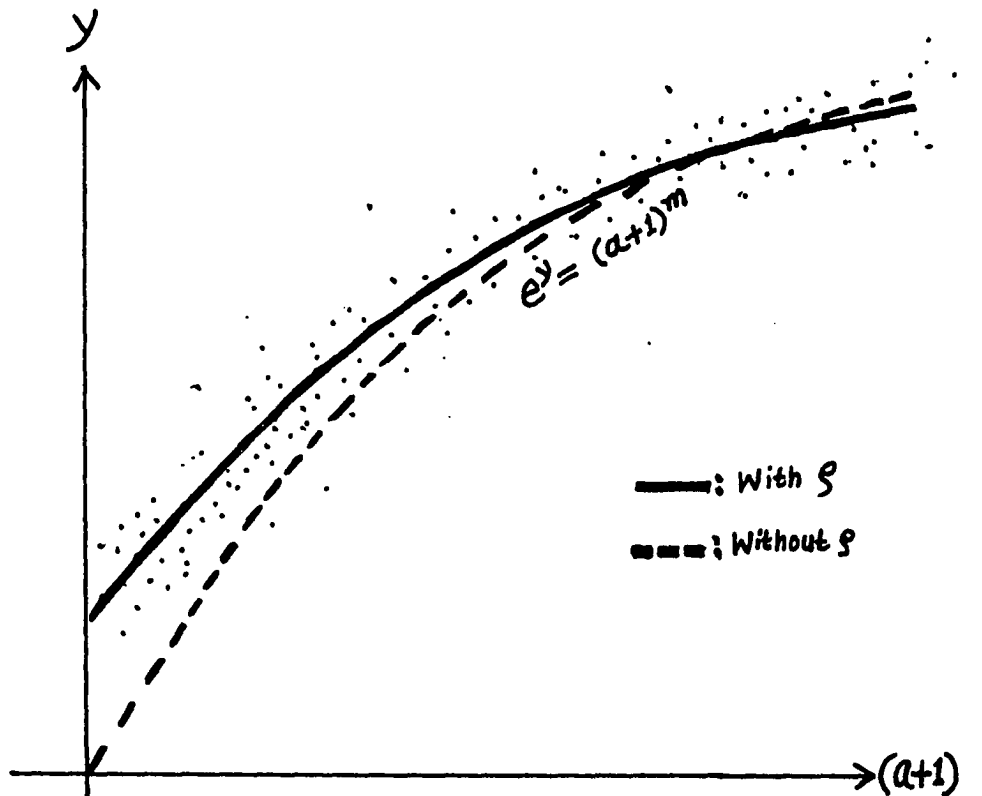


Fig. 14. Curve Fitting with and without Interception (φ).

Although all three equations can be used to explain the existing rate structure, for the break-point model to be constructed in this chapter, equations (13) and (14)

are employed. The use of equation (12) is not desirable because incorporation of such an equation into the model would introduce some difficulties, although not insoluble, in solving the break-point value in the break-point equations to be discussed in the ensuing section.⁷ As to the question of which one of the two equations selected is superior, a general rule is to use the one which fits best the individual rate schedule in question. Generally for rate schedules with a pronounced taper, the second equation is preferred. In most cases, the rate does not taper off rapidly. Therefore, the third equation may have a wider usage than the second. In building the new break-point models, the model employing equation (14), and hence (15), is identified as the first method; the model employing equation (13), the second method.

For different commodities, the rate of positive changes in moving cost differs. These differences are reflected in the parameter "m". The m value can be determined by correlation methods. To do so, one needs to effect a semilogarithmic-transformation for the distance-cost equation in question. Such a transformation is shown

⁷Using this equation will necessitate the expansion of equations with a higher degree, in order to solve a critical distance in the new break-point equation to be formulated.

below.⁸ For equation (14):

$$e^y = \varphi(a+1)^m$$

Taking the natural logarithm,

$$\ln e^y = \ln \varphi + m \ln(a+1)$$

Solving for y, gives:

$$y = \ln \varphi + m \ln(a+1) \quad (15)$$

For equation (13):

$$y = h(1 - \varphi e^{-ma})$$

Expanding the right hand side of the equation and rearranging the items, it becomes:

$$(1 - \frac{y}{h}) = \varphi e^{-ma}$$

Taking the natural logarithm, gives:

$$\ln(1 - \frac{y}{h}) = -ma + \ln \varphi \quad (16)$$

As shown in equations (15) and (16), m is the slope of the transformed curve (straight line) showing the relation of y and $\ln(a+1)$ in the case of (15), and that of $\ln(1 - \frac{y}{h})$ and a in the case of (16). Actual approximation of the m value can be made either through the use of a graph or through the use of the formula for simple correlation employing the cost (rate) and distance data from freight rate schedules.

The value of m thus secured and the value of h from various freight rate schedules will provide the essential

⁸For a detailed explanation of the double-log and semilog transformations, see Johnston, Econometric Methods (New York: McGraw-Hill Book Company, Inc., 1963), Chapters 1 and 2.

information needed for the determination of break-points to be discussed in the next section.

Determination of the Prime Break-Point

The break-point is the point at which the prices of like commodities from different suppliers are the same. The customers (ultimate consumers, retailers, wholesalers, industrial users) located on these points have no preference among the suppliers. To locate these points, the new break-point method calls for determining a break-point on the line connecting the competing suppliers before locating other points along the boundary. This point is called the Prime Break-Point. Since market price generally consists of manufacturing cost, sales cost, distribution cost, and profit, an equation can be formed to signify an equilibrium at the Prime Break-Point. As suggested in the previous section, there are two different ways of expressing the distance-cost relation. Therefore, two different equations may be constructed. The one incorporating equation (15) considered to have wider usage is called the first method and is discussed first. The one incorporating equation (13) is called the second method and is taken up after the first method.

The First Method

Following the distance-moving cost relationship represented by equation (15), the moving cost from the

competing suppliers A' and B' to the Prime Break-Point can be determined by equations (17) and (18), respectively.

(See Fig. 15 for the location of the Prime Break-Point and some symbols.)

$$y_a = \ln \varphi + m \ln(a'+1) \quad (17)$$

$$y_b = \ln \varphi + m \ln(b'+1) \quad (18)$$

where:

y_a, y_b = moving cost from A' and B' to the Prime Break-Point, respectively.

a' = distance from supplier A whose basic price is higher to the Prime Break-Point.

b' = distance from supplier B' whose basic price is lower to the Prime Break-Point.

y, m, φ = defined as before.

With equations (17) and (18) and the additional cost elements for A' and B', the Prime Break-Point can be located through the use of equation (21), which is derived from equation (19), as shown in the following paragraphs.

Since at the Prime Break-Point, as stated earlier, the price of like commodities from competing suppliers must be the same, it follows that at this point the following condition must be met:

$$A_c + A_d + A_s + A_p + Y_a = B_c + B_d + B_s + B_p + Y_b \quad (19)$$

where:

A_c, B_c = manufacturing cost for suppliers A' and B', respectively.

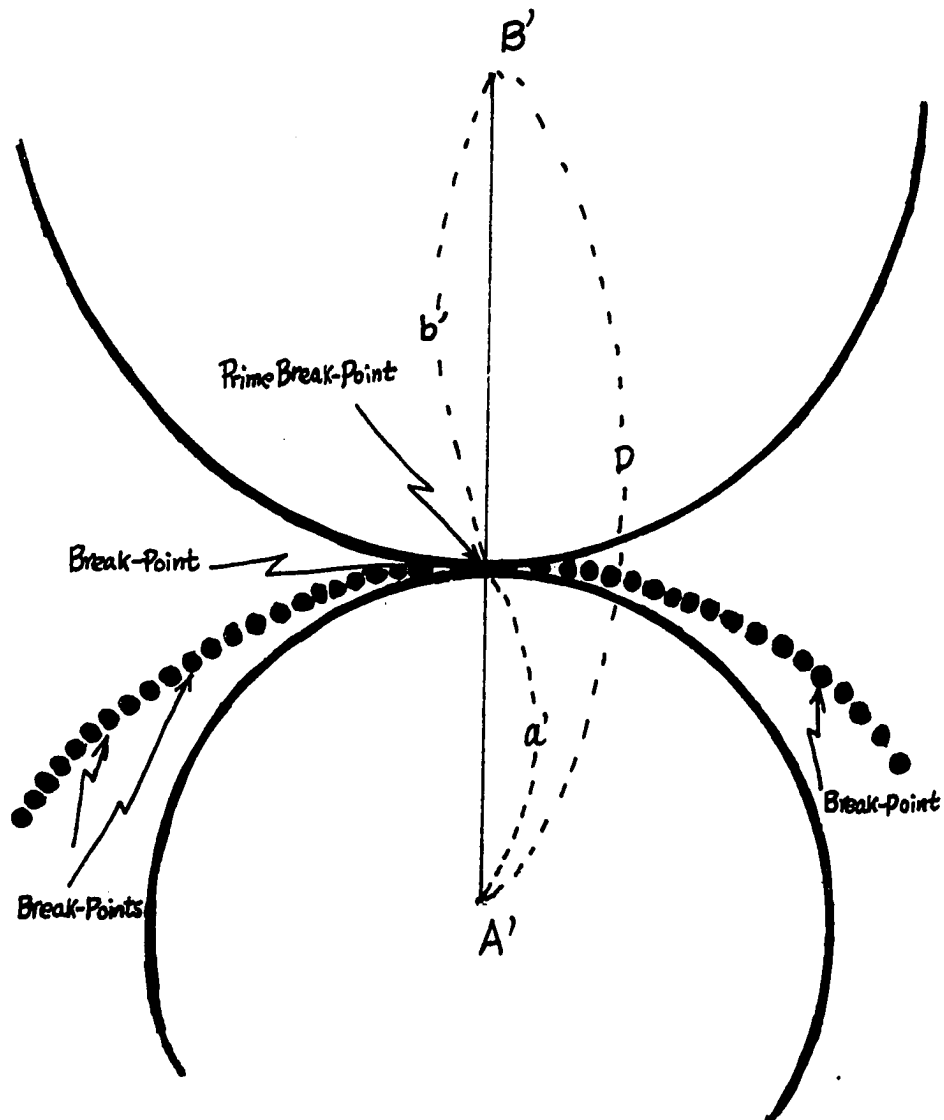


Fig. 15. The Location of the Prime Break-Point.

A_d, B_d = distribution cost excluding the cost of transportation for suppliers A' and B' , respectively.

A_s, B_s = sales cost for A' and B' , respectively.

A_p, B_p = normal profit for A' and B' , respectively.

Y_a, Y_b = moving cost from A' and B' to the Prime Break-Point, respectively.

Substituting equations (17) and (18) into equation (19):

$$A_c + A_d + A_s + A_p + m \ln(a'+1) + \ln \varphi = B_c + B_d + B_s + B_p + m \ln(b'+1) + \ln \varphi \quad (20)$$

Let $A = A_c + A_d + A_s + A_p$ and $B = B_c + B_d + B_s + B_p$,

then equation (19) becomes:

$$A + m \ln(a'+1) + \ln \varphi = B + m \ln(D-a'+1) + \ln \varphi$$

By definition $D = a'+b'$, therefore:

$$A + m \ln(a'+1) + \ln \varphi = B + m \ln(D-a'+1) + \ln \varphi$$

Dividing by m and rearranging the items, gives:

$$\ln(D-a'+1) - \ln(a'+1) = \frac{(A-B)}{m}$$

$$\ln\left(\frac{D-a'+1}{a'+1}\right) = \frac{(A-B)}{m}$$

Taking antilogarithms,

$$\frac{D-a'+1}{a'+1} = e^{(A-B)/m}$$

$$D-a'+1 = (a'+1)e^{(A-B)/m}$$

$$a'e^{(A-B)/m} + a' = D + 1 - e^{(A-B)/m}$$

$$\text{Therefore, } a' = \frac{D + 1 - e^{(A-B)/m}}{e^{(A-B)/m} + 1} \quad \text{or}$$

$$a' = \frac{D + 2}{e^{(A-B)/m} + 1} - 1 \quad (21)$$

From equation (21), if $A = B$ (which means that the costs excluding transportation cost for supplier A' and that of supplier B' are the same), the prime break-point will locate midway between A' and B'. The a value is $\frac{1}{2}D$ ($a' = \frac{D+2}{2} - 1 = \frac{D}{2}$) in this case. However, the value of a' will become greater as B becomes larger than A, and vice versa.

The Second Method

Instead of equation (15), equation (13) may be used to formulate the equations for the moving cost from competing suppliers A' and B' to the Prime Break-Point. With the same notation used in the previous section, these equations may be written as:

$$Y_a = h(1 - \rho e^{-ma'}) \quad (22)$$

$$Y_b = h(1 - \rho e^{-mb'}) \quad (23)$$

With equations (22) and (23) and the additional cost items for A' and B', an alternate break-point formula may be formulated to locate the Prime Break-Point. The result of such a formulation is shown in equation (25), which is derived from equation (24), as shown in the ensuing paragraphs.

At the Prime Break-Point, where market equilibrium is reached, the condition depicted by equation (19) in the last section must be met. Substituting equations (22) and (23) into equation (19), equation (19) becomes:

$$A_c + A_d + A_s + A_p + h(1 - \rho e^{-ma'}) = B_c + B_d + B_s + B_p + h(1 - \rho e^{-mb'}) \quad (24)$$

Substituting A for $(A_c + A_d + A_s + A_p)$, B for $(B_c + B_d + B_s + B_p)$, and D for $(a' + b')$, it becomes:

$$A + h(1 - \rho e^{-ma'}) = B + h \left[1 - \rho e^{-m(D-a')} \right]$$

Expanding the equation and rearranging the items:

$$A + h + \rho h e^{-ma'} = B + h - \rho h e^{-mD} \cdot e^{ma'}$$

$$-h e^{-ma'} \left(1 - \frac{e^{-mD}}{e^{-2ma'}} \right) = \frac{B - A}{\rho}$$

$$\text{Let } e^{-ma'} = C; \quad e^{-mD} = E$$

$$\text{then } -hC \left(1 - \frac{E}{C^2} \right) = \frac{B - A}{\rho}$$

$$-hC^2 + hE - \frac{1}{\rho} (B - A)C = 0$$

$$hC^2 + \frac{1}{\rho} (B - A)C - hE = 0$$

$$\text{therefore, } C = \frac{-\frac{1}{\rho}(B-A) \pm \sqrt{\frac{1}{\rho^2}(B-A)^2 + 4h^2E}}{2h}$$

Since $e^{-ma'} = C$, it follows:

$$-ma' = \ln C$$

$$\text{Therefore, } a' = -\frac{\ln C}{m}, \text{ or}$$

$$a' = -\frac{1}{m} \ln \left[\frac{-\frac{1}{\rho}(B-A) \pm \sqrt{\frac{1}{\rho^2}(B-A)^2 + 4h^2E}}{2h} \right] \quad (25)$$

For an a' to have meaning, the value of C must be smaller than 1. The reason is that, if C is greater than 1, the value of a' will become negative. Negative a' is

meaningless except under the circumstances when that market area of B' engulfs A'. The necessary condition of $C < 1$ can be proven by a simple example.

Pick any real number smaller than 1, e.g., 0.9, and solve for C.

$$\begin{aligned}\ln 0.9 &= 2.303 \log 0.9 = 2.303 \log 0.9 \\ &= 2.303 \log 9/10 = 2.303 (\log 9 - \log 10) < 0\end{aligned}$$

$$\text{Therefore, } a = \frac{\ln C}{-m} > 0$$

As can be seen from equations (21) and (25), as long as the values of h, m, A and B are determined, a' can be determined without difficulty. When the a' value is determined, the Prime Break-Point can be located.

The Identification of the Market Boundary Line

The Prime Break-Point, as determined by either of the two methods developed in the last section, is the market equilibrium point on the straight line connecting two geographically competing centers. (E' in Figure 16 is the Prime Break-Point.) What about the other equilibrium points which constitute part of the market boundary line? This question is considered in this section.

If circles are drawn around supply centers A' and B' with radii $\overline{A'E}$ ($=a'$, as defined before) and $\overline{B'E'}$ ($=b'$, as defined before), respectively, the areas not enclosed by the circles (the shaded areas in Figure 16) are the areas in which the other break points locate. These points can

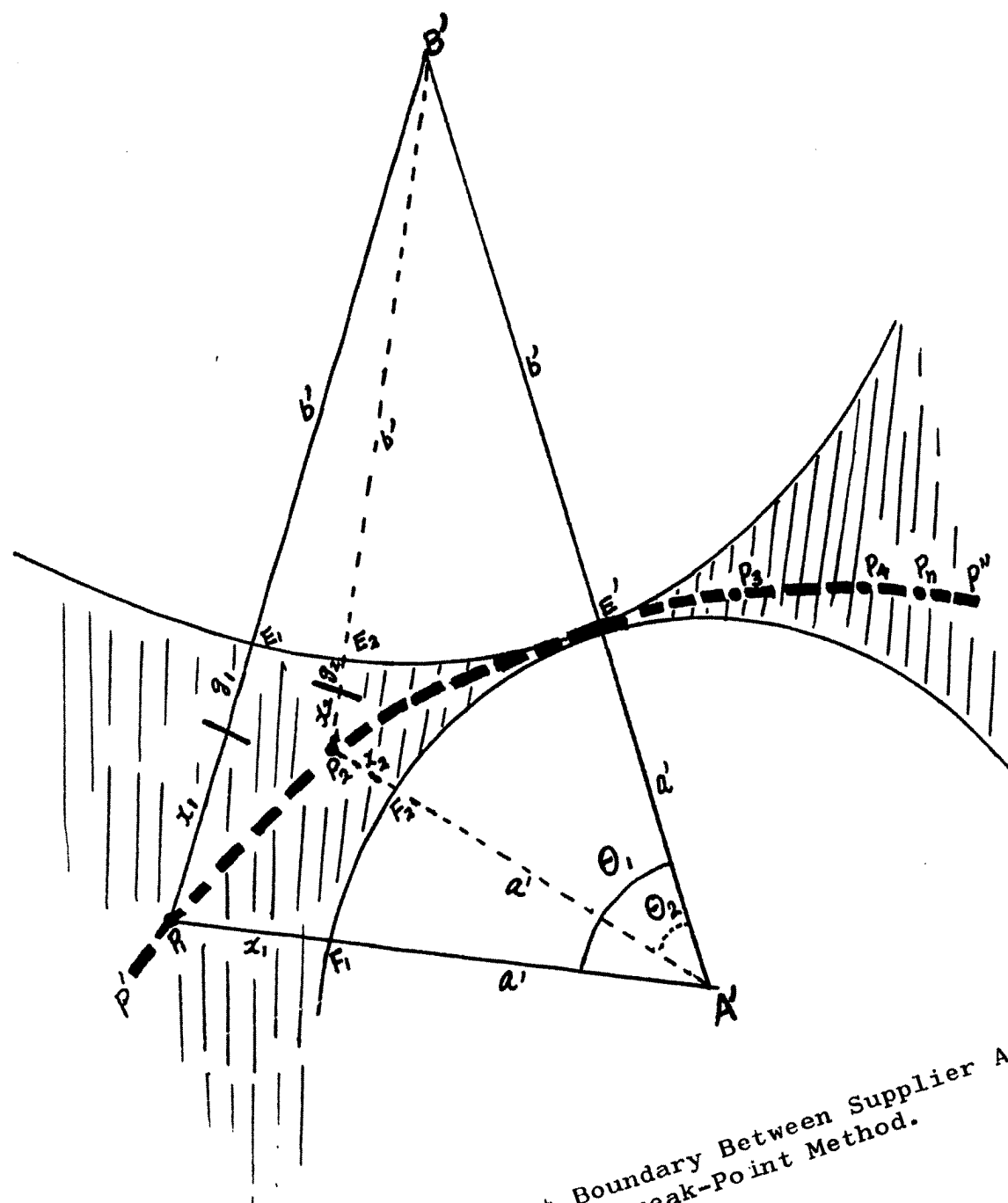


Fig. 16. The Market Boundary Between Supplier A' and B' Through the Use of New Break-Point Method.

be located through the application of the Cosine Law. The Cosine Law may be expressed as:

$$c^2 = a^2 + b^2 - 2 ab \cos \theta$$

where a, b, c = the three sides of a triangle

θ = degree of an angle facing side c .

In figure 16, $P_1, P_2, P_3, \dots, P_n$ are the moving break-points along the market's boundary line. Any of these points with supply points A' and B' constitute a triangle. For example, $P_1B'A'$ is a triangle, $P_2B'A'$ is another one, and so on. For each triangle, only one side is known, side $A'B'$. The other two sides, $A'P_1$ and $B'P_1$ for example, are unknown. This poses no problem, for these two sides are determinable via the use of distance-cost relation. The side with greater distance between the supply point and the Prime Break-Point is larger. For example, P_1B' is larger than P_1A' . As a result, the distance from the edge of the circle (E_1) to P_1 will be larger for the supplier B' , that is, $E_1P_1 > F_1P_1$. The difference between E_1P_1 and F_1P_1 may be termed as "rate-oriented differential distance," designated as g_1 . For computational convenience, the shorter of E_1P_1 and F_1P_1 , in this case F_1P_1 , is assigned x_1 . For different triangles formed with different P 's and $A'B'$, there are different x 's and corresponding g 's.

Relations between g and x are readily identifiable using the same concept underlying the break-point analysis

made at the beginning of this chapter. Since at the edges of the circles for A' and B' the total costs for the commodity delivered by them are the same, the additional costs of delivery from each edge of the circle to the equilibrium point must be the same. That is, the delivery cost for the additional distance $\overline{F_1 P_1}$ and $\overline{E_1 P_1}$ must be the same. With the same notation employed before, such a relation is shown in equations (26) and (27) using the concepts of equations (15) and (13) selected in section 3 of this chapter, respectively.

$$\begin{aligned} & m \ln(b'x+g+1) + \ln \rho - m \ln(b'+1) + \ln \rho \\ & = m \ln(a'+x+1) + \ln \rho - m \ln(a'+1) + \ln \rho \end{aligned} \quad (26)$$

$$\begin{aligned} & h \sqrt{1 - \rho e^{m(x+b'+g)}} - h(1 - \rho e^{-mb'}) \\ & = h \sqrt{1 - \rho e^{m(x+a')}} - h(1 - \rho e^{-ma'}) \end{aligned} \quad (27)$$

The relation between g and x will become clearer after solving g in equations (26) and (27).

For equation (26):

Rearranging the items in equation (26) gives:

$$\ln\left(\frac{b'+x+g+1}{b'+1}\right) = \ln\left(\frac{a'+x+1}{a'+1}\right)$$

Taking antilogarithms, rearranging the items and simplifying the equation gives:

$$g(a'+1) = b'x - a'x$$

$$\text{Therefore, } g = \frac{x(b'-a')}{a'+1}$$

Substituting $D-a'$ for b' :

$$g = \frac{x(D-2a')}{a'+1} \quad (28)$$

For equation (27):

Rearranging and simplifying the items in equation (27) gives:

$$\begin{aligned} \oint e^{-mb'} - \oint e^{-mb'} \cdot e^{-m(x+g)} &= \oint e^{-ma'} - \oint e^{-m(x+a')} \\ \left[1 - e^{-m(x+g)} \right] &= \frac{e^{-ma'}(1 - e^{-mx})}{e^{-mb'}} \end{aligned}$$

Therefore:

$$g = -\frac{1}{m} \left[\ln \left(1 - \frac{e^{-ma'}(1 - e^{-mx})}{e^{-mb'}} \right) + mx \right]$$

Substitute $(D-a')$ for b

$$g = -\frac{1}{m} \left[\ln \left(1 - \frac{e^{-ma'}(1 - e^{-mx})}{e^{-m(D-a')}} \right) + mx \right] \quad (29)$$

In equations (28) and (29), the values of m and a' can be determined via equations (15) and (21) using the first method, and equations (16) and (25) using the second method. Hence, as long as the value of x is given, the value of g can be determined. It follows that all three sides of the triangles formed with a fixed side $\overline{A'B'}$ and a moving point P ($P_1, P_2, P_3 \dots P_n$) can be determined once the x values are given. For all practical purposes, x values may be assigned although theoretically they are fixed. They are fixed in the sense that for moving point P at P_1 , the x value must be x_1 ; P_2 , x_2 ; and so forth. The logic of assigning the x value is that for any positive x value, there is a moving point P , although it may not be P_1 , but may be P_x . In Figure 16, the three sides of

triangle $A'B'P_1$ are $(a+b)$, $(b+x+g)$, and $(a+x)$. To apply the Cosine Law, the θ is assigned to the angle at the supply point nearer to the Prime Break-Point. In this case, it is the angle facing side $(b+x+g)$. From the Cosine Law, $\cos \theta$ can be explained via the three sides of the triangle. This is shown in equation (30).

For the first method incorporating equation (28), the value of θ can be determined via equation (32) which is derived from equation (30). For each x value assigned, there will be a corresponding θ . Since a' is fixed, for each $(x+a')$, there is a corresponding θ . Since $(x+a')$ is the distance from A' to the equilibrium point P , by assigning different x 's in equation (32), one will be able to locate different equilibrium points on the market boundary line. The derivation of equation (32) from equation (30) is shown below:

$$(x+g+b)^2 = (a'+x)^2 + (a'+b)^2 - 2(a'+x)(a'+b)\cos \theta \quad (30)$$

Expanding the equation, rearranging and simplifying the items gives:

$$2(b+g-a')x + g^2 + 2bg - 2a'b - 2a'^2 = -2(a'+b)(a'+x)\cos \theta$$

Solving for $\cos \theta$, gives:

$$\cos \theta = \frac{2(a'-b-g)x + 2a'(a'+b) - g(2b+g)}{2(a'+b)(a'+x)}$$

Since $b = D - a'$, $a' + b = D$, $a' - b = 2a' - D$

$$\cos \theta = \frac{2(2a' - D - g)x + 2a'D - g(2D - 2a' + g)}{2D(a' + x)} \quad (31)$$

Substituting equation (28) into equation (31) and simplifying the equation, gives:

$$\begin{aligned}
\cos \theta &= \frac{2 \left[2a' - D - \frac{x(D-2a')}{a'+1} \right] x + 2a'D - \frac{x(D-2a')}{a'+1} \left[2D-2a' + \frac{x(D-2a')}{a'+1} \right]}{2D(a'+x)} \\
&= \frac{2x(2a'-D)(a'+1)^2 - 2x^2(D-2a')(a'+1) + 2a'D(a'+1)^2}{2D(a'+x)(a'+1)^2} \\
&\quad + \frac{-2x(D-2a')(D-a')(a'+1) - x^2(D-2a')^2}{2D(a'+x)(a'+1)^2} \\
&= \frac{x(2a'-D) \left[2(a'+1)^2 + 2x(a'+1) + 2(D-a')(a'+1) + x(D-2a') \right] + 2a'D(a'+1)^2}{2D(a'+x)(a'+1)^2} \\
\therefore \cos \theta &= \frac{x(2a'-D) \{ 2(a'D) + 2a' + 2 + 2x + 2D + xD \} + 2a'D(a'+1)^2}{2D(a'+x)(a'+1)^2} \quad (32)
\end{aligned}$$

Likewise, for the second method incorporating equation (29), equation (29) may be substituted into equation (31) to eliminate the term g in equation (31). However, this is unnecessary for the formula is not simplified by doing so.

If the first method is adopted, equation (32) can be considered as the final equation for determining the equilibrium points other than the Prime Break-Point, with equation (21) as the equation for the Prime Break-Point. If the second method is adopted, equations (31) and (29) are for the break-points, and (25) for the Prime Break-Point.

Once the Prime Break-Point and the other equilibrium points are located, the market boundary line may be drawn by simply connecting these points. For a more accurate drawing of the boundary line, more equilibrium points are needed. The P'P'' line in Figure 16 is the boundary line between market areas tributary to suppliers A' and B'.

The method used for the two competitors case discussed above can be extended to a multiple competitors case. As the number of competitors increases, the drawing of the market boundary line becomes more complicated. The market area for the five competitors case is cited to illustrate the possible outcome of employing the New Break-Point method in identifying the market area of a supplier. In Figure 17, A' is the supplier whose market area is under study, and B', C', D', E' are the competitors of A'. Lines b'b'', c'c'', d'd'', and e'e'' are the market boundaries between A' and B', A' and C', A' and D', and A' and E', respectively. The shaded area $b_o c_o d_o e_o$ is the market area for A'.

The Computational Feasibility

In locating the market equilibrium points for spatially competing suppliers, equations (21), (25), (29), (31), and (32) formulated in this chapter are used. These equations involve computation of the value of the exponential function, value of the trigonometric function, value of Napierian or Natural logarithm, and the determination of the degree of an angle. Manual computation of these values is extremely tedious and time-consuming. Fortunately, mathematical tables for these values are available. For easier application of these tables, measurements of distance and units of transportation rates may be converted into smaller ones. For example, 100 miles may be converted

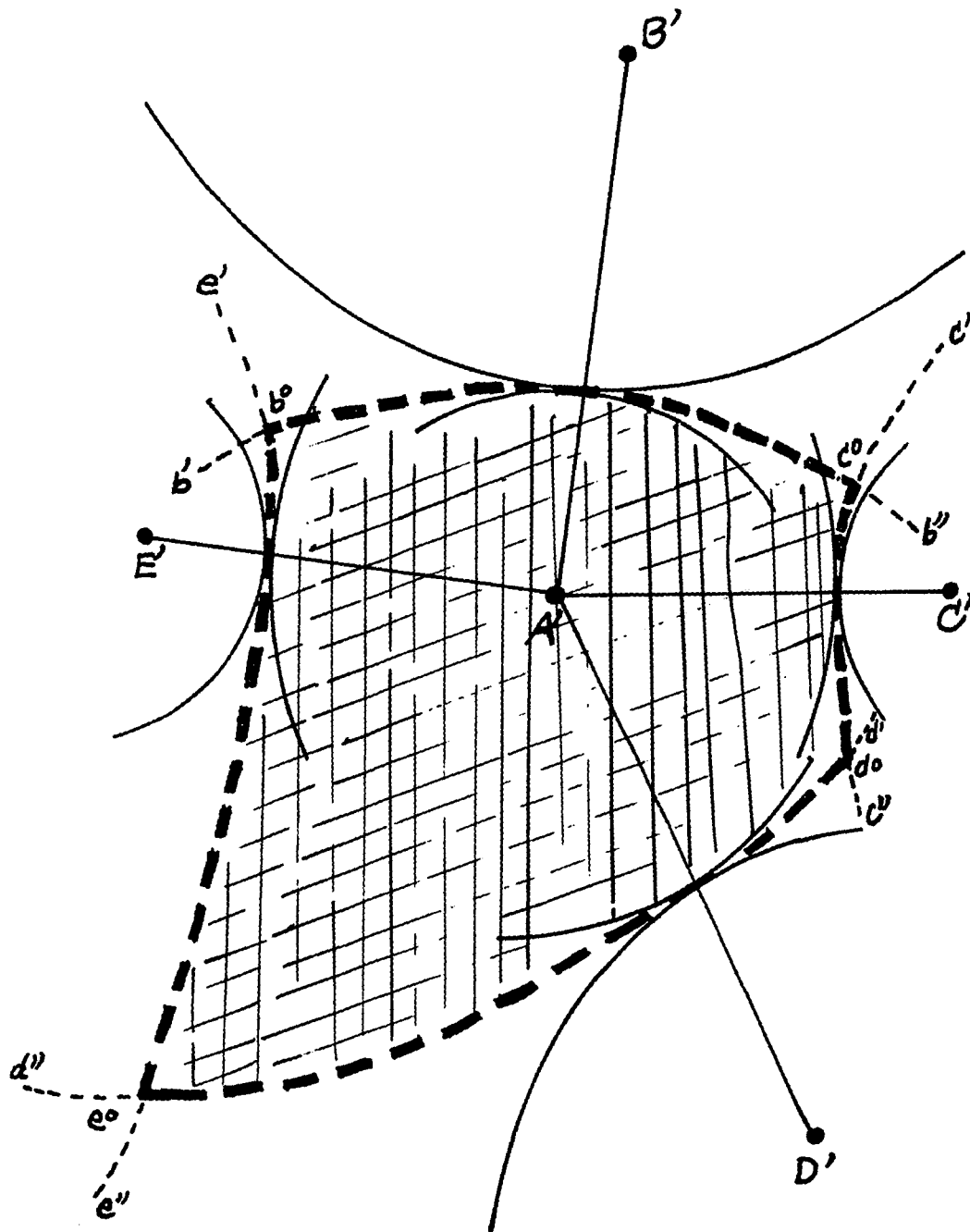


Figure 17. Probable Look of the Market Area for a Distribution Center--the Five-Competitors Case.

to 1.00 unit distance, 50 miles, 0.50 unit distance, and so on. Likewise, 1.00 may be designated for a transportation rate of 100¢ per cwt; 0.2, 20¢ per cwt, and so on.

Where conversion of the unit measurements is made, reversion is required after computation of the converted items. All these procedures are technically simple and easy to apply.

Limitations

The application of the New Break-Point method developed in this chapter is limited to:

1) business units whose delivery work is performed by facilitating distribution agencies;

2) business units which perform their own delivery services and whose delivery cost-distance relation, and hence parameter m , can be approximated with reasonable accuracy;

3) commodities for which distance rates are applicable for the outbound freight.

For some business units whose transportation cost structures show irregularity, and for some commodities for which the value of parameter m cannot be determined with reasonable accuracy, the employment of the new method is not recommended. This is because, in doing so, it will distort not only the fact, but also the logic underlying the new method.

CHAPTER IV

THE MARKET AREA OF A DISTRIBUTION CENTER

From Chapter I, it was noted that the market area of a distribution center is an economic space defined by centrifugal forces which are primarily generated from inherent characteristics of goods that can satisfy human wants. These centrifugal forces are either guided or reinforced by the cost of securing the satisfaction of human wants--namely, cost of goods to the consumers. These costs include processing costs, marketing costs and distribution costs. Analysis of a market area is treated in the same framework as that of plants and wholesalers. The method developed in the last chapter is considered to be general to all non-retailing market units. With some refinement of this general model, this chapter will specifically address itself to the market area analysis of a distribution center. In fact, it is a logical extension of Chapter III and the application of the theory thereof.

Factors Influencing Market Area of a Distribution Center

Factors affecting the spatial extent of the market area of a distribution center may be explained in three

major categories. These are: the economic forces generated from consumers, the economic forces generated from suppliers, and other forces.

The economic forces generated from consumers are those factors that influence demand for goods distributed by a distribution center. Except in the rare case in which the ultimate consumers purchase goods directly from the distribution center, the customers of a regional distribution center are generally the marketing agencies--such as retailing outlets (chain stores, variety stores, department stores, super markets, grocery stores, discount houses, mail order houses, etc.), wholesalers, agent middlemen (brokers, manufacturing agencies, sales agents) district distributors, and industrial users. Therefore, the factors affecting the demand for these institutions naturally affect the size and shape of the market area for a distribution center. These factors are ordinarily reflected in such demand characteristics of the customer as brand loyalty, purchasing patterns, income, tastes and preferences, demographic characteristics, family characteristics, etc.

The economic forces generated from suppliers are those factors that affect the economic feasibility of a distribution center to extend the market limit. To the distribution center, the immediate concern is the distribution cost. The processing cost is usually fixed by the manufacturers according to their pricing policy reflected

in the basic price. The distribution cost may be further broken down into transportation cost and non-transportation distribution cost. Non-transportation distribution cost includes inventory cost (interest, insurance, spoilage), warehouse cost (depreciation, insurance) and material handling cost, all of which may be called costs of warehousing and storage. Cost of warehousing and storage, once the warehousing capacity is determined, does not change too much. But the cost of transportation changes as the distance for which goods are moved changes. The transportation cost is considered to be the most important space-related cost factor which reflects the direct cost of overcoming the spatial resistance.

The forces affecting the market of a distribution center that cannot be clearly classified into either demand-related forces or supply-related forces are not insignificant in the market area analysis. However, their impact is usually indirect and the result of the impact oftentimes cannot be clearly identified. Some of these factors are: managerial ability, availability of labor, organizational efficiencies, geographical characteristics of the area, transportation networks, trade climates, government regulations, taxes, and other external economics.

It is understood that the three forces discussed in this section are interrelated and interact with each other in the operation of a distribution center. Market

area analysis for a distribution center, hence, includes the analysis of all these factors and their intricate phenomena. However, for analytical simplicity and for revealing the function of some important factors, many of the factors discussed in this section will be assumed away in this study. This point will become clear as the theoretical base for the analysis is taken up in the ensuing sections.

The Theoretical Base--The New Break-Point Method

Employment of the New Break-Point method in the analysis of the market area for a distribution center requires some specifications and refinements. Discussed in this section will be those specifications and refinements.

Some Specific Assumptions

Besides those assumptions underlying the general form of the New Break-Point method, including the demand factors, some supply factors, and some implicit factors, additional assumptions are specifically needed for the analysis of a distribution center.

First, a regional distribution center is considered to be oligopolistic in nature. This point can be seen from the spatial differentiation of the market area in general and the locational property of the distribution center in particular. The former was stressed by Hoover, Isard, and

Weigmann while the latter, by Greenhut.¹ Hoover stressed that the market itself is highly differentiated and the location can be considered as "a variable aspect of the production on a par with price, quality, reputation, and the like."² The fact that movement of goods always faces some spatial obstacles suggests that the markets are restricted by the spatial differentiation of the location. A traditional locational property of a distribution center is that the location is heavily determined by the demand factor. As a result, a regional distribution center is located where the population is heavily concentrated. Therefore, it is reasonable to assume that for a region there is generally one regional distribution center. The competing distribution centers are those centers of the neighboring regions. Hence, the number of competitors is not large.

Secondly, the basic prices of commodities at each competing regional distribution center are the f.o.b. production center plus the cost of transporting the commodity from the producer to each distribution center. The transportation rates for the same or like commodities are applicable to all directions for the center under study and for

¹For Hoover's contention, see Hoover, Location Theory and Shoe and Leather Industries, op. cit., p. vi. Weigmann is the then prominent German economist. For more details on his concept and Isard's, see Isard, "General Theory of Location and Space-Economy," op. cit., pp. 476-506. For Greenhut's concept, see Footnote 22.

²Hoover, Ibid.

the competing neighboring centers. This assumption is plausible for the competition among the regions tends to result in similar rate structures for the same or like commodities for different regions.

Thirdly, inbound freight is moved in the greatest feasible quantity either by rail or motor truck to save on the moving cost. The outbound freight is moved by motor truck also in greatest feasible quantity for cost saving.

Fourthly, the market area of a distribution center is limited to an area which can be reached overnight and can be served the next day. Therefore, the maximum distance from the center to the market boundary is approximately 500 miles.³ The reasonableness of this assumption is hinged on the first assumption stated earlier. If the center can serve the area covering the points which can be reached in more than two days, a country could possibly be served by a center located in the center of a country--the monopolistic situation. This situation is contrary to the first assumption stated earlier.

Lastly, non-transportation distribution costs per shipping unit (for example 100 lbs.) of like or the same

³With an average running speed of 45-50 mph, 500 miles can be reached overnight. See W. B. Saunders and Company, Economic Feasibility of Establishing a Regional Distribution Center in the Great Wilkes-Barre, Pennsylvania Area, Its Economic Impact and Physical Facility, A Report Prepared for the Economic Development Administration for the Technical Assistance Project of the U.S. Dept. of Commerce, Jan., 1966.

commodities are the same for the distribution centers among neighboring regions. This assumption is based on the assumption that, for a region, higher costs, for example, in handling and insurance, may be offset by the lower cost of land and inventory. This is a rather brave assumption. However, in order to reveal the significance of the transportation cost, both from the production center to the distribution center, and from the distribution center to the customers, this assumption is a necessary evil.

The New Break-Point Method as Applied to a Regional Distribution Center

The equations (21), (25), (29), (31), and (32) of Chapter III for the determination of various break-points can be applied to the regional distribution center with some modifications. Such modifications correspond to the specific nature of the distribution center and the additional assumptions discussed in the previous section.

As a distribution center is not a processing business unit and as by assumption the non-transportation distribution cost per transportation unit (cwt) does not change, the item A in equations (21), and (25) will have different meanings. In equations (21) and (25), the A includes processing costs, distribution costs, sales costs, and normal profits ($A = A_c + A_d + A_s + A_p$). In the context of the market area analysis for a distribution center, A will contain the basic price (A_b), and the normal profits

(A_p). Of course, by assumption, the basic price is the sum of the f.o.b. production center and the cost of transportation from the production center to the distribution center.

Since the market extent of a distribution center is conditioned, among other things discussed before, by the fact that it is desirable that delivery service be completed at latest the day immediately following the day the customer places the order in order to maintain its competitive position, the value of x to be assigned in equations (31) and (32) may not exceed $(500-a')$ miles. The market boundary line will, therefore, contain probably part of the line consisting of the moving break-points and part of the line consisting of the moving points 500 miles away from the center. In Figure 18, the heavy solid line and heavy broken line are the maximum market boundaries for the Little Rock distribution center and the St. Louis distribution center, respectively. The heavy dotted line is the break-point line between the Little Rock and St. Louis centers. The total market area in the two competitor case is the shaded area (SKOLPTA) for Little Rock, and SRNL0 for St. Louis. The case for more than two competitors would consist merely of duplicating the procedure discussed in this and the previous chapter.

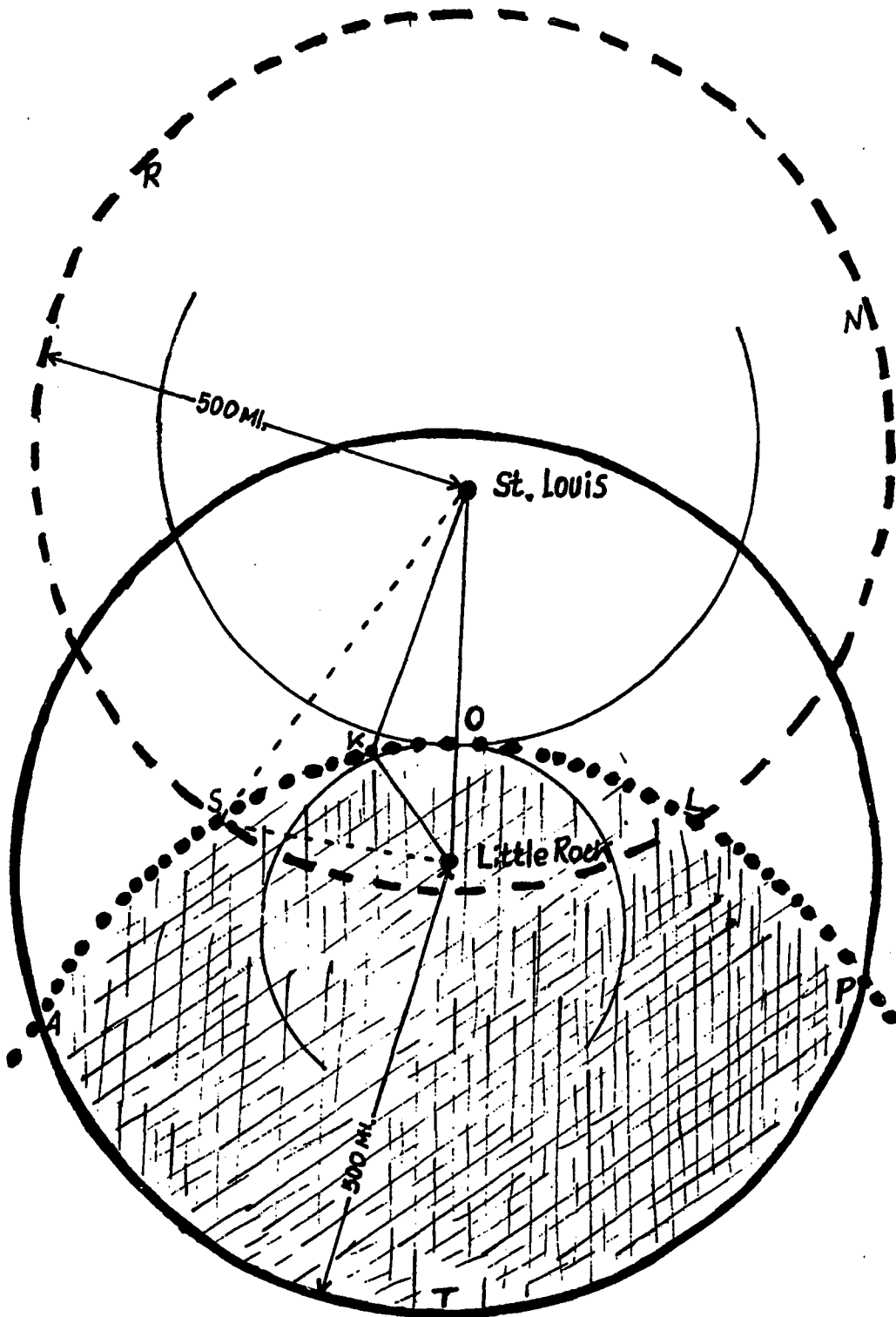


Figure 18. Probable Market Area for a Distribution Center--the Two Competitor Case.

In determining the m value from equations (15) and (16), tabulations shown in Tables 2 and 3 are desirable, for what is needed is really the slope of the straight line correlating $\ln(a+1)$ and y for equation (15) and $\ln(1 - \frac{y}{h})$ and a for equation (16).

DATA NEEDED FOR m VALUE FOR EQUATION (15)

y	a'	a'+1	ln(a'+1)
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DATA NEEDED FOR m VALUE FOR EQUATION (16)

a'	y	h	$1 - \frac{y}{h}$	$\ln(1 - \frac{y}{h})$
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For determining the value of a' , Tables 4 and 5 may be helpful.

TABLE 4

COMPUTATION OF a' VALUE FOR EQUATION (21)

between centers	(A-B)	m	(A-B)/m	$e^{(A-B)/m}$	D	a'
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TABLE 5

COMPUTATION OF a' VALUE FOR EQUATION (25)

between centers	h	h^2	(B-A)	$(B-A)^2$	hE	2h	C	lnC	a'
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Finally, in locating the break-point other than the Prime Break-Point, Table 6 is recommended for the convenience of computation.

TABLE 6

COMPUTATION OF THE BREAK-POINTS FOR EQUATIONS (31)
AND (32) (OTHER THAN PRIME BREAK-POINT)

x	a'	D	g	$\cos \theta = k$	$\cos^{-1} k = \theta$	$a' + x$
---	------	---	---	-------------------	------------------------	----------

The Empirical Aspect--The Little Rock Case

More often than not, a theory sound in logic is not sound in operation because of the difficulties encountered in its application. In this chapter, the Little Rock, Arkansas, distribution center situation will be used to show the applicability of the New Break-Point method developed in Chapter III and the preceding sections of this chapter.

For each commodity, there will be a market area. The market area for different commodities varies greatly. Therefore, for the overall planning of a regional distribution center, the selection of the commodities to be warehoused becomes one of the very important procedures. A method of selecting commodity candidates for warehousing in the distribution center is developed and presented in Appendix C. The commodity to be used for illustration in this study is the outcome of the selection employing this method. The result of the selection shows that canned fruits and vegetables are the No. 1 candidate from consumer's goods items (see Table 3 of Appendix C). Therefore, canned fruits and vegetables, to be specific, the hot-packed canned orange juice, will be cited for illustration of the application of the New Break-Point method. Except the first subsection dealing with the sources of data, the arrangement of the rest of the subsections follows closely the order of the sections in the last chapter.

Sources of Data⁴

The information needed for locating the Prime Break-Point and the various break-points is the distance between competing centers (D), the differences in the basic prices between centers (A-B), and the parameter m. The value of D can be obtained from Rand McNally's Commercial Atlas and Marketing Guide, Household Goods Carrier Bureau's Household Goods Carrier's Mileage, or freight tariff (base number).⁵ For this study the Household Goods Carrier's Mileage Guide was used.

The differences in basic prices of competing distribution centers, e.g. (A-B), as a matter of fact, are the sum of the difference in the manufacturer's list prices and the differences in cost of moving the commodity from the manufacturing point (production center) to the distribution center. Manufacturing price lists can be obtained

⁴Most of the tariff data was secured through assistance of Mr. William Ryan, to whom the author is greatly obliged. Interviews with William Ryan, Traffic Coordinator for the State of Oklahoma, Oklahoma City, Oklahoma, Sept.-Dec., 1971.

⁵It is recognized that mileages shown in Household Carrier Bureau's Household Goods Carrier's Mileage Guide and the Tariff (Base No.) are slightly different from the actual physical mileage. Nevertheless, it is very close to the physical mileage. In drawing the market boundary line, these mileages are to be converted proportionally to the actual physical mileage. For the sources of mileage guide, see: Rand McNally and Company, 1971 Commercial Atlas and Marketing Guide (New York: Rand McNally and Company, 1971), p. 24 and Household Goods Carrier's Bureau, Mileage Guide (New York: Rand McNally and Company, 1967), pp. 78, 149, 216, 217.

from individual producers, while the cost of moving can be obtained from transcontinental or interstate freight tariffs. In determining the inbound transport cost, one needs to know the major source of supply, in other words, the location of the production center. The related data for the location of the production center can be secured from the latest Census of Transportation.⁶

Parameter m can be determined from various freight tariffs. For this study, transcontinental and interstate tariffs are the major sources of inbound and outbound freight rates. For the major sources of data, see Appendix D.

Testing of the Transportation Cost Equations

The m value needed for the calculation of the Prime Break-Point and various equilibrium points on the market boundary is determined by fitting the current rate schedule into the curve represented by equations (15) and (16) which are transformed from equations (13) and (14), respectively. The m value is merely the slope of the straight line correlating y and $\ln(a+1)$ for equation (15), and $\ln(1 - \frac{y}{h})$ and a for equation (16). The validity of m value

⁶The latest Census of Transportation, published in Nov., 1970, consists of 3 volumes. The information pertaining to the commodity transportation survey is published in Vol. III. See Bureau of Census, 1967 Census of Transportation, Vol. III (U.S. Dept. of Commerce, 1970), Part I, II and III.

is hinged upon how good the freight rate schedule is represented by these equations. In determining the m value through the use of the correlation technique, one will not only obtain m value, but also some most commonly used measurements of testing the validity of the estimating equations. These are the coefficients of determination (R^2), t value, and the standard error of the estimate.

Equations (15) and (16), and hence (13) and (14), are tested by various current freight rate schedules. In order to cover a wide range of different rate schedules, twelve rate schedules are selected. These schedules include intra-state and inter-state class, commodity, rail and motor freight rates. Since canned fruits and vegetables are selected for illustration, all rate schedules used for testing are those of canned fruits and vegetables.

As mentioned in section three of Chapter III, a closer representation of the cost-distance function can be made through the introduction of the constant coefficient ϕ into the equation. In order to show this point, the equations without ϕ are also tested. Therefore, for each set of rate schedules, four equations are tested. These four equations are restated and numbered as follows:

$$\text{I) } \ln(1 - \frac{y}{h}) = \ln \phi - ma$$

$$\text{II) } \ln(1 - \frac{y}{h}) = -ma$$

$$\text{III) } y = \ln \phi + m \ln(a+1)$$

$$\text{IV) } y = m \ln(a+1)$$

A brief summary of the results of the regression analysis is shown in Table 7.

In the case of canned fruits and vegetables, as one can see from Table 7, the third equation appears to have the highest R^2 among the four equations in all sets of rate schedules put to test. The R^2 for equation III in all cases exceeds .95. While not all t values for m for the third equation are the highest of the four equations, they are statistically acceptable at the significance level of 0.001. With the determined degree of freedom for the individual set of data, the computed t value for m for the equation, in all cases, exceeds the desirable t value at the significant level of 0.001.⁷ Therefore, for the canned fruits and vegetables (to be specific, the hot-packed 46 oz. canned orange juice) cited for illustration in this section, equation (15), the third equation in testing, is used for the ensuing break-points analyses. Therefore, for the computation of the Prime Break-Point and the other break-points, equations (21) and (32) will be adopted.

Locating the Prime Break-Point

From equation (21), the distance between the center and the Prime Break-Point is determinable if the distance

⁷The degrees of freedom for this analysis is $n-1$, where n is the number of observations. The desired t value can be found from Frederick E. Croxton and Dudley Cowden, Applied General Statistics, Second Edition (Englewood Cliffs, N.J.: Prentice Hall, Inc., 1955), pp. 750-751.

TABLE 7
SUMMARY OF THE RESULTS OF REGRESSION ANALYSIS*

Rate Schedule	Equation I				Equation II			
	R ²	t	m	Std. Error	R ²	t	m	Std. Error
Oklahoma Commodity Motor (1000 lb min.)	.888	-17.3	-.411	.0237	.883	-32.3	-.385	.0119
Arkansas Class Motor (1000-2000)	.915	-20.2	-.613	.0303	.902	-36.8	-.672	.0182
Arkansas Class Motor (A.Q.)	.915	-20.2	-.624	.0309	.904	-36.8	-.681	.0184
Arkansas Commodity Motor (20,000 min.)	.890	-17.5	-.562	.0320	.890	-31.2	-.568	.0182
Arkansas Commodity Motor (68,000 min.)	.767	-11.6	-.615	.0529	.761	-18.3	-.572	.0313
Arkansas Commodity Motor (1000 min)	.913	-18.9	-.579	.0305	.912	-37.8	-.595	.0157
Interstate (SW Line Commodity Rail 60,000 min)	.897	-23.3	-.170	.0073	.882	-38.0	-.151	.0039

TABLE 7 (Continued)

Rate Schedule	Equation I				Equation II			
	R ²	t	m	Std. Error	R ²	t	m	Std. Error
Interstate SW Line Commodity Rail 48,000 min	.904	-25.3	-.166	.0065	.901	43.9	-.158	.0036
Oklahoma Commodity Motor 8000 min	.844	-16.9	-.415	.0245	.841	25.5	-.396	.0155
Interstate SW Line Commodity Motor 8000 min	.754	-15.7	-.179	.0113	.752	-25.	-.171	.0069
Interstate SW Line Commodity Motor 14000 min	.844	-20.9	-.191	.0091	.840	-32.5	-.181	.0055
Interstate SW Line Commodity Motor 30,000 min.	.892	-25.6	-.180	.0069	.889	-41.	-.173	.0042

TABLE 7 (Continued)

Rate Schedule	Equation III				Equation IV			
	R ²	t	m	Std. Error	R ²	t	m	Std. Error
Oklahoma Commodity Motor (1000 lb min.)	.972	36.6	1.038	.0283	.957	101.8	1.599	.0113
Arkansas Class Motor (1000-2000)	.993	72.0	.867	.0120	.564	34.9	1.381	.0395
Arkansas Class Motor (A.Q.)	.993	75.1	.918	.0122	.562	34.9	1.464	.0420
Arkansas Commodity Motor (20,000 min.)	.994	76.5	.450	.0058	.887	54.1	.582	.0107
Arkansas Commodity Motor (68,000 min.)	.972	38.0	.794	.0209	.917	59.3	.963	.0162
Arkansas Commodity Motor (1000 min)	.980	40.8	.893	.0218	.749	46.1	1.296	.0281
Interstate (SW Line Commodity Rail 60,000 min)	.951	34.6	.457	.0132	.905	63.9	.361	.0056

TABLE 7 (Continued)

Rate Schedule	Equation III				Equation IV			
	R ²	t	m	Std. Error	R ²	t	m	Std. Error
Interstate SW Line Commodity Rail 48,000 min	.959	40.0	.445	.0111	.950	93.4	.404	.0043
Oklahoma Commodity Motor 8000 min	.970	41.5	.847	.0204	.955	78.1	.938	.0120
Interstate SW Line Commodity Motor 8000 min	.979	16.9	1.498	.0862	.768	35.3	1.334	.0318
Interstate SW Line Commodity Motor 14000 min	.960	44.1	.783	.0177	.959	101.2	.758	.0074
Interstate SW Line Commodity Motor 30,000 min	.964	46.4	.733	.0157	.963	.713	107.2	.0066

*For the input data see Appendix E. For the sources of input data, see also Appendix D and subsection 1 of this section. OLSQ programming of IBM360 is employed for the analysis.

between the two competing centers (D), the differences between the basic prices of the competing center (A-B), and the parameter m are known. The values of D, (A-B), and m for the hot-packed 46 oz. canned orange juice, and the result of the computation of the a' value (the distance between the center and the Prime Break-Point) using equation (21) are shown in Table 8.

TABLE 8
COMPUTATION OF a' VALUE

Competing Centers	(A-B) ¹ (in \$)	m ²	$\frac{(A-B)}{m}$	$e^{(A-B)/m}$	D ³ (in 100 miles)	a', ⁴ (in 100 miles)	a', ⁵ adjusted (in 100 miles)
L.R. and Dallas	0.30	.73	0.41	1.51	3.28	1.10	1.07
L.R. and Memphis	0.41	.73	0.56	1.75	1.38	0.23	0.22
L.R. and St. Louis	0.75	.73	1.03	2.80	3.55	0.46	0.41

Note: 1. A represents the higher of the two basic prices.
 2. From the last figure in column 3 of Table 7 (p.105)
 3. "Transportation Mileage," see footnote 3.
 4. Represents the distance from the center with the higher basic price to the Prime Break-Point.
 5. Converted from "transportation mileage" to physical mileage by multiplying (Physical Mileage of D) / (Transportation Mileage of D)

The figures in the first column of Table 8 are the difference in basic prices between the competing centers. The basic prices, in turn, consist of the manufacturer's sales price and the cost of moving from the production

center to the distribution center. (By the assumption stated earlier, the non-transportation distribution costs per unit for the competing centers are the same. Therefore, this cost factor is not included here.) For the product cited, namely 46 oz. canned orange juice, there are two major production centers in this country. These are Florida and California.⁸ Therefore, in computing the moving cost, different rate structures are involved. Of course, the manufacturers' sales prices are also different at these two centers. Although there is the 100,000-pound minimum car-load least expensive commodity rate for transcontinental east-bound traffic from California, the practical application of this rate is limited to only a few larger distribution centers.⁹ For a smaller regional distribution center,

⁸U.S. Dept. of Commerce, U.S. Industrial Outlook 1970 (Washington, D.C.: Government Printing Office, n.d.), p. 85. It is understood that Texas (Rio Grande Valley) also produces orange juice. However, the volume is too small for interregional marketing.

⁹Interviews with Merl Fernberg, V.P., Collins Dietz and Morris Co.; A. E. Murray, Buyer for Scrivner Boogart; C. Pierce, Transportation Rate Clerk, Transcon Lines, Inc.; William Ryan, Traffic Coordinator for the State of Oklahoma; and Merlin Dickerson, Buyer for Safeway, OKC Regional Division. Interviews were conducted on Dec. 13 and 14, 1971, at Oklahoma City. According to them, some canned orange juice is from California, for example, the Del Monte 46 oz. can, which is the competing brand of Treesweet. These canned juices are shipped to a bigger distribution center with 100,000 lbs. minimum rate and then consigned to the OKC area or a smaller distribution center. To the consignee, the amount of the consolidation fee will literally offset the rate advantage from California.

The 100,000 lb. minimum carload commodity rate is 173¢/cwt for Little Rock, St. Louis, Dallas, and Memphis alike. See Tariff 2-J. Transcontinental East-Bound

the rates applicable are generally higher for inbound traffic from California than the inbound traffic from Florida as far as canned orange juice is concerned. This point can be seen clearly from Table 9. Therefore, the area under

TABLE 9
COMPARATIVE INBOUND FREIGHT RATES FROM FLORIDA
AND CALIFORNIA TO SELECTED POINTS*

From S. Calif. to	Rate (\$1.00)	Minimum Volume(lb)	From Central Florida to	Rate (\$1.00)	Minimum Volume(lb)
Little Rock	4.37	30,000	Little Rock	2.18	24,000
Dallas	2.90	40,000	Dallas	2.48	24,000
St. Louis	3.04	40,000	St. Louis	1.43	24,000
Memphis	4.37	30,000	Memphis	1.77	24,000

*Rate schedules selected are those providing the lowest rates applicable in each case excluding the transcontinental commodity carload rail rates. The reason for not including the transcontinental carload rates is, as stated earlier, that this rate schedule, in actual practice, is limited to a very few larger distribution centers as far as canned orange juice is concerned.

SOURCES: Tariff 20-B, Rocky Mountain Motor Traffic Tariff Bureau, Denver, Colorado, July 26, 1969, pp. 783-784; Supplement 37 to Tariff 26-B, Rocky Mountain Motor Tariff Bureau, Denver, Colorado, March 17, 1971; and Tariff 12-Y, South-North Commodity Tariff, issued by John M. Womack, Louisville, Kentucky, March 17, 1971.

study can be reasonably assumed to have been served mainly from the Florida production center. The factory price for

Commodity Tariff, Transcontinental Freight Bureau, April 1, 1971, pp. 34, 237, 171, and Supplement 63 to Tariff 2-J, Transcontinental Freight Bureau, Oct., 1971, pp. 108-9.

46 oz. canned orange juice at the production center in Florida is \$4.40 for a box of 12 cans which weighs approximately 44 pounds including the case.¹⁰ The freight rates applicable are \$2.18, \$1.43, \$1.77, and \$2.48 for Little Rock, St. Louis, Memphis and Dallas, respectively.¹¹

The m value for canned orange juice depends upon the rate schedule selected for use. For this analysis, the interstate motor freight tariff for 30,000 lbs minimum is selected.¹² The reasons for such a selection are:

- 1) The rates for 30,000 lbs. minimum is the least rate applicable for interstate movement within the area under study. The other rate schedules are substantially higher than this one.
- 2) The majority of freight movements (about 73%) under 600 miles are shipped by 30,000 lbs. or more.¹³

¹⁰The factory price is the prevailing price in Dec., 1971, at the Central Florida production point, Winter Haven. An interview with Merl Fernberg, V.P., Collins Dietz and Morris Company, OKC, Okla., Dec. 13, 1971.

¹¹Tariff 501-C, Southern Motor Carrier Rate Conference, Atlanta, Georgia, Oct. 11, 1971, p. 590; Tariff 505-C, Southern Motor Carrier Rate Conference, Atlanta, Georgia, May 2, 1971, pp. 75 and 386; and Tariff 12-Y, South-North Commodity Tariff, issued by John M. Womack, Louisville, Kentucky, March 17, 1971, Item 51850. It is noted that, except St. Louis for which the commodity rate applies, these rates are class rates.

¹²Tariff-1, Southwestern Commodity Tariff, issued by Joe E. Kinard, Dallas, Texas, Sept. 8, 1970, p. 340.

¹³Bureau of Census, 1967 Census of Transportation, op. cit., Volume III, Part 1, p. 87.

The Household Goods Carrier's Mileage Guide is used to determine the value of D .¹⁴ D value thus determined is the so-called Mover's Mileage, which is slightly different from the actual physical mileage. Therefore, for the computation of a' value, this mileage is used. However, for the drawing of the market boundary line to be discussed in the ensuing subsection, this a' value needs to be adjusted to a physical mileage. This adjustment is made simply by multiplying a' with the ratio of the physical mileage of D and the transportation mileage (or mover's mileage). See also Note 5 of Table 8.

Drawing of Market Boundary Line

Once the Prime Break-Points between the Little Rock center and other competing centers are known, the other break-points along the boundary lines between these competing centers can be located through the use of equation (32) developed in the last chapter. The result of computation using equation (32) is shown in Table 10 below.

Column V of Table 10 shows the degree of angle at the center with the higher basic price. Column VI shows the distance between the moving break-points and the center with the higher basic price. The market boundary lines between Little Rock and Dallas, Little Rock and Memphis, and Little Rock and St. Louis can be drawn from the data in columns V

¹⁴See footnote 3 of the chapter.

TABLE 10

SUMMARY OF THE COMPUTATION OF BREAK-POINTS
USING EQUATION (32)*

Compet- ing Cen- ters	I x (100 mi.)	II a' adjusted (100 mi.)	III D (100 mi.)	IV $\cos \theta$ = K	V $\theta = \cos^{-1} K$	VI a' + x adjusted (100 mi.)
Little	0.25	1.07	3.00	0.6978	45°45'	1.32
	0.50	1.07	3.00	0.4785	61°25'	1.57
Rock	0.75	1.07	3.00	0.3079	72°07'	1.82
	1.00	1.07	3.00	0.1685	80°18'	2.07
and	2.00	1.07	3.00	-0.2301	103°18'	3.07
	3.00	1.07	3.00	-0.5917	126°17'	4.07
Dallas	3.50	1.07	3.00	-0.6384	129°20'	4.57
Little	0.10	0.22	1.37	0.2571	75°06'	0.32
	0.15	0.22	1.37	0.0206	88°49'	0.37
Rock	0.20	0.22	1.37	-0.1687	99°43'	0.42
	0.25	0.22	1.37	-0.3264	109°03'	0.47
and	0.30	0.22	1.37	-0.4602	117°24'	0.57
	0.40	0.22	1.37	-0.6825	133°03'	0.62
Memphis	0.55	0.22	1.37	-0.9418	160°20'	0.77
Little	0.10	0.41	3.14	0.2869	73°25'	0.51
	0.15	0.41	3.14	0.0131	89°15'	0.56
Rock	0.20	0.41	3.14	-0.2236	102°05'	0.61
and	0.25	0.41	3.14	-0.4318	115°35'	0.66
St.	0.30	0.41	3.14	-0.6173	128°07'	0.71
	0.35	0.41	3.14	-0.7864	141°51'	0.76
Louis	0.40	0.41	3.14	-0.9373	159°37'	0.81

*Data in Column II are from Table 8. Data in Column III are the approximate physical mileage.

and VI. The resultant market area for the Little Rock Distribution Center is shown in Fig. 19. It is the shaded area encircled by a portion of the boundary line between the St. Louis center and the Little Rock center (FF'); and a portion of the boundary line between Memphis center and

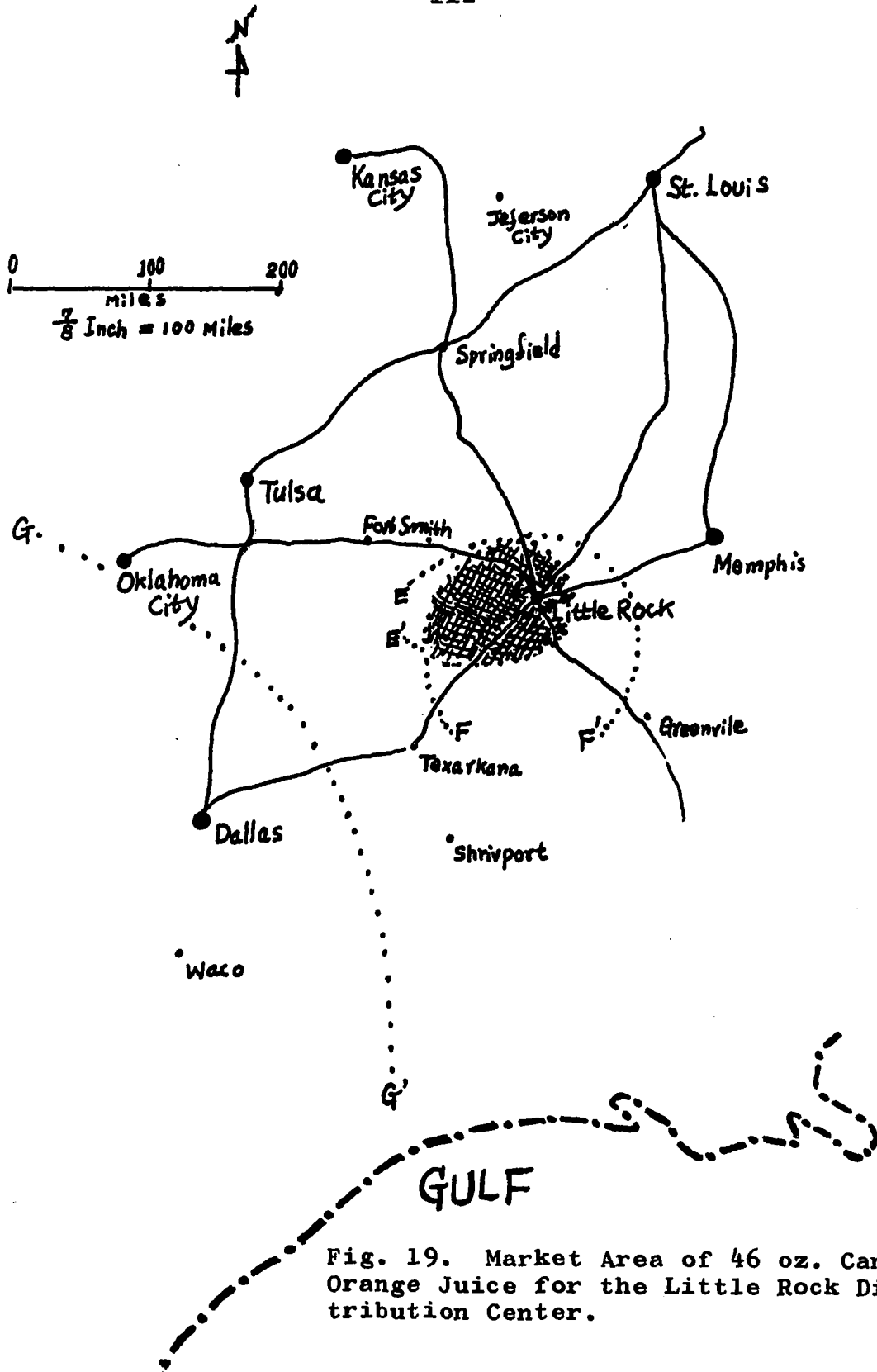


Fig. 19. Market Area of 46 oz. Canned Orange Juice for the Little Rock Distribution Center.

Little Rock center (EE'). GG', theoretically, is the boundary line between Little Rock and Dallas. In this specific case, this GG' line is meaningless because it will never contact either the EE' or FF' line, thereby leaving only the EE' and FF' lines the determining ones.

Limitations and Comments

The New Break-Point Method, when it is applied to a distribution center, obeys the limitations stated in the last chapter. It is understood that, in practice, the organizational relations and behavior, such as the business relation between an individual distribution center and its suppliers, affect greatly the inbound moving cost and the basic price f.o.b. and, hence, the total basic price for a center. This aspect of the analysis is beyond the capacity of the new method developed in this study, and will affect the fidelity of the new method.

CHAPTER V

THE SIGNIFICANCE OF THE NEW METHOD AND DESIRABLE FURTHER INVESTIGATION

The special characteristics of the New Break-Point method developed in the last two chapters are: 1) removal of the distance-proportional transportation cost assumption, and 2) employment of the Cosine Law and polar coordinates in drawing the marketing boundary line, which is both simple in concept and feasible in operation. In this chapter, the significance of these two characteristics will be discussed, followed by brief recommendations for desirable further investigation.

The Significance of the New Break-Point Method

Incorporation of a more realistic distance-non-proportional transportation cost structure into the analytical model for the market area of a distribution center reveals the fact that the market area for the center with a lower basic price tends to be underestimated under the traditional framework of analysis in which the distance-proportional transportation cost is assumed. This point can be clearly illustrated via Figure 20 for the two competitor case.

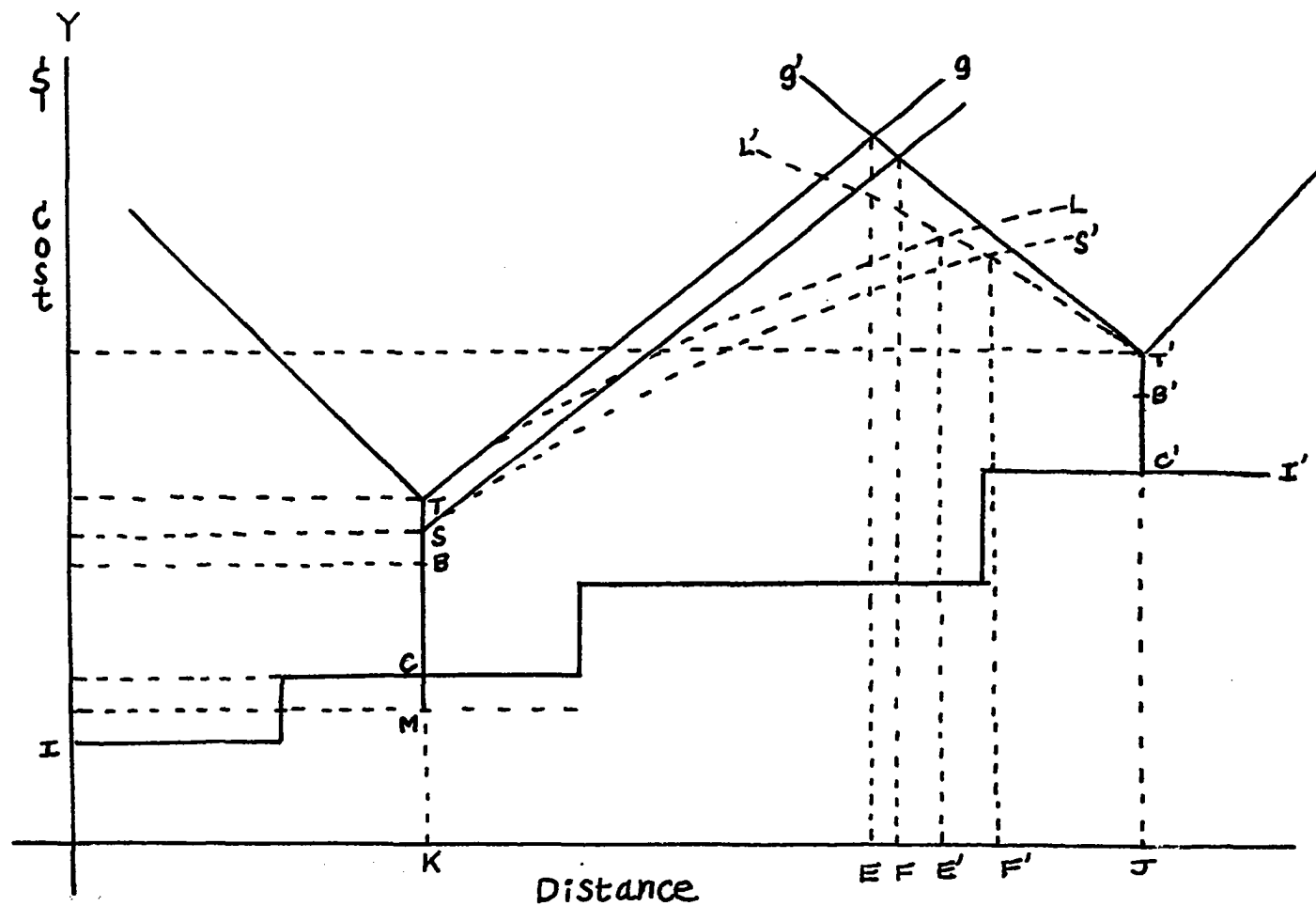


Fig. 20. Different Transportation Cost Assumptions and the Extent of Market Areas for Competing Distribution Centers.

In Figure 20, the stepwise slope II' shows the inbound transportation "cost stairs" which reflect the group rate system prevailing in the current rate structure for the inbound volume long haul. BC , $B'C'$ and TB , $T'B'$ are the f.o.b. production center and non-transportation distribution cost for the two competing distribution centers located at K and J , respectively. Tg and $T'g'$ show the transportation gradients under the traditional proportional rate assumption while TL and $T'L'$ are the transportation gradients under the non-proportional transportation cost assumption. As one can visualize from Figure 20, KE' is greater than KE . That is to say the extent of the market for the center with a lower basic price ($= KC+CB+BT < JC'+C'B'+B'T'$) is underestimated under the proportional cost assumption.

Corollary to the point discussed above is that a small reduction in the basic price, whether it is the reduction in the inbound rate or other costs, will result in a greater increase in the extent of the market for the lower basic price center. In other words, the increase in the market extent as the result of lowering the basic price is underestimated in the traditional market analysis. For example, if the inbound rate is reduced by $CM (= TS)$, the market boundary will extend from KE to KF under the proportional cost assumption; KE' to KF' , under the non-proportional cost assumption. Under a tapering rate

situation, EF must be smaller than $E'F$.

The significance of the operational aspect of the new method can be explained from two perspectives--namely, the simplicity of the concept underlying the theoretical framework and the easiness in the application of the theoretical model in practice. The former is essential to the latter.

For the drawing of the market boundary line, the methods used have been, as discussed in Chapter II, those of isopane, calculus, and rectangular coordinates. Conceptually, the calculus and rectangular coordinates approaches are quite abstract. Hence, these approaches have remained purely theoretical pursuits, for the application of these abstruse concepts in the practical drawing of the market boundary line encounters computational complexity and awkwardness in determining the moving points on either the hyperbolic line, hypercircle, or hexagonal boundary. This point may be considered as the major reason why Fetter, followed by Lösch, and even down to Isard, did not concern himself with the practical application of the theory he developed. As to the isopane approach represented in Hoover's theory, followed by Alonso, Dunn, and many others, although simple in concept, the practical application of the concept in drawing the market boundary involves tedious and time-consuming tasks. One can easily understand this point by considering the number of points of equal delivered

price needed for an isotime and the number of isotimes needed for the isopane.

As compared to the methods that have been used, the trigonometric method developed in this study, for all practical purposes, is both simple in concept and easy to apply. It is simple because only the concepts of the Cosine Law and the exponential function are involved in the entire methodological process. It is said to be easy because once the parameter m , distance D , and Prime Break-Point distance a' are known, the moving point p on the boundary line can be readily located by applying the concept of polar coordinates, as illustrated in the empirical part of this study.

Desirable Further Investigation

Information concerning the market area is essential to an effective and efficient planning and operation of a distribution center. The New Break-Point method formulated in this study furnishes the information pertaining to the geographical boundary of the market area of a distribution center. This market boundary is based on the fixed demand on which, in turn, the service capacity of a center is based. The non-transportation distribution cost, as well as the transportation cost for a center, in turn, is affected by the size or capacity of the center. Therefore, it is desirable that some demand factors, if not all, be brought into the theoretical framework of the market area analysis. In doing so, it seems highly feasible that an

optimum market boundary may be identifiable. To pursue this line of investigation, the formulation of a DQ curve (Distance-Quantity curve) seems to be constructive. The concept of the DQ curve and, hence, a possible approach to identifying the optimum market area for a distribution center, can be briefly explained via Figure 21.

In Figure 21, (a) shows the price-distance relation, the PD curve; (b), demand curve; (c), operator for facilitating the derivation of the DQ curve; (d) the DQ curve. The DQ curve is derived by tracing the functional relations from (a) to (b), (c), and (d). The DQ curve, thus derived, shows the relation between distance and quantity demanded. From this curve, one can compute the total equilibrium demand corresponding to the market area identified via the New Break-Point method. If the equilibrium demand for a commodity is larger than the "planned demand," the service capacity of the distribution center is said to be less than optimal. If it is so, an adjustment of capacity will take place. The adjustment process will continue until the gap between the equilibrium demand and the "planned demand" narrows down to a possible minimum, probably to zero. This condition, as shown in Figure 21(d), signifies the gradual merger of $D'Q'$ and $D''Q''$, for as the capacity is further and further expanded, the reduction in non-transportation distribution cost (shown as AP , AP' , or AP'') per unit service becomes smaller and smaller. The resultant

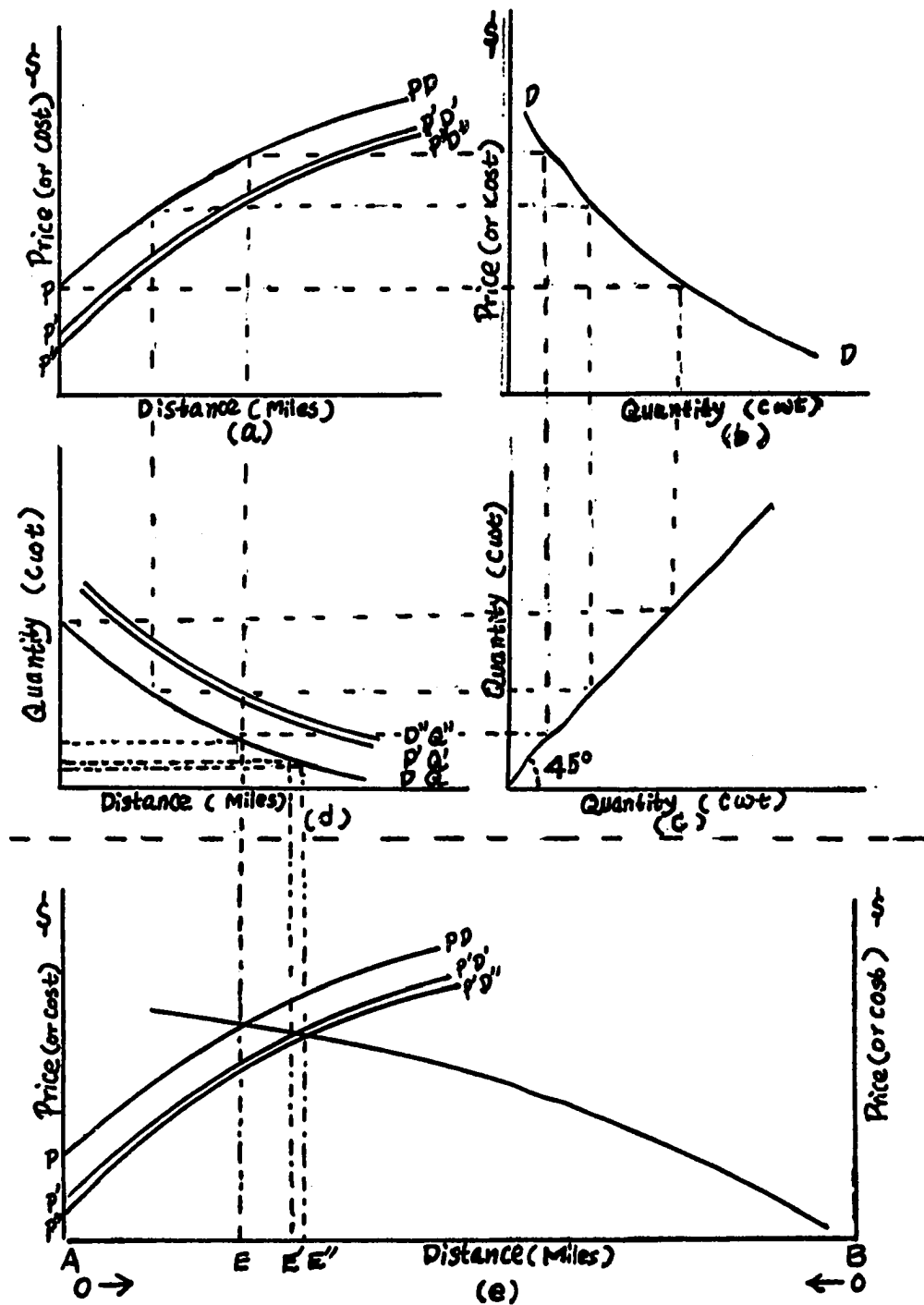


Fig. 21. The Derivation of the Negative DQ Curve.

- (a) Price-Distance Curve (b) Demand Curve (c) Operator
 (d) DQ Curve (e) Cost-Distance Relation

reduction in cost will expand the market extent of the center. For example, [in Fig. 21(e)], for center A, it expands from AE to AE' as the result of cost reduction from AP to AP'. The market boundary corresponding to the optimum capacity of the distribution center may be considered as the optimal market boundary.

The identification of the optimal market area for a distribution center hinges upon the understanding of the behavior of the non-transportation distribution cost, the characteristics of the demand function for a distribution center, and hence, the interaction of these costs and demands which are reflected in the DQ curve. Although all these terms are very popular, further investigation is much to be desired.

CHAPTER VI

SUMMARY, CONCLUSION, AND RECOMMENDATIONS

A market area is a quantified market expressed in terms of geographical units. It is usually identified by a generic product class. An analysis of a market area which is generally considered as a short-run partial spatial equilibrium analysis may be demand-oriented or cost-oriented depending upon the significance of the impact of the demand factors and supply factors on the extent of the market. An example of demand-oriented market area is that of retailers; examples of cost-oriented market area, plants, distribution centers, and other non-retailing business units.

The analyses of the market area for a distribution center and for a plant and other non-retailing business units are built on the same theoretical foundation that stresses the effect of transportation cost and basic price on the extent of a market. Early theoretical works concerning a cost-oriented market area have been constructed on, among other things, the assumptions of fixed demand and proportional moving-cost-distance relationships. Market areas are generally identified via: drawing of isotimes (hence isopane); drawing of hyperbolic market boundary

through point to point construction employing calculus or analytical geometry; drawing of marginal lines through point to point approximation of marginal delivered prices; drawing of a hypercircle via rectangular coordinates; and highly abstract formulas.

A review of early theoretical formulations uncovers two possible improvements. These are:

1) Removal of the linear transportation cost assumption, and

2) Improvement in the operational aspects of theory. Tailored to fulfill these improvements is the New Break-Point method developed in this study.

A market boundary is viewed as consisting of moving points signifying the market equilibrium between or among the spatially competing suppliers. These moving points are called Break-Points. The moving point located on the straight line connecting two competing suppliers is called the Prime Break-Point. The New Break-Point method calls for determining the locations of the Prime Break-Point and the Break-Points.

The Prime Break-Point can be located via approximation of the distance between the supplier with the higher basic price and the Prime Break-Point, incorporating distance-nonproportional transportation cost equations.

It is found that there are two desirable equations that represent the assumed distance-non-proportional transportation cost structure. These are:

$$e^y = \rho(a + 1)^m \quad \text{and}$$

$$y = h(1 - \rho e^{-ma})$$

where:

y = moving cost

a = distance

m = parameter explaining the shape of the curve

ρ = constant coefficient

h = maximum rate in the freight rate schedule

Generally, for a rate schedule with a pronounced taper, as distance increases, the second equation represents the better cost-distance relation than the first one, and vice versa.

Incorporating either the first transportation cost equation or the second transportation cost equation, the distance between the supplier with the higher basic price and the Prime Break-Point can be determined by the equations below:

$$a' = \frac{D + 2}{e^{(A-B)/m} + 1} - 1 \quad (\text{incorporating the first transportation cost equation})$$

or

$$a' = -\frac{1}{m} \ln \left[\frac{-\frac{1}{\rho}(B-A) \pm \sqrt{\frac{1}{\rho^2}(B-A)^2 + 4h^2E}}{2h} \right] \quad (\text{incorporating the second transportation cost equation})$$

where:

a' = distance between the supplier with the higher basic price and the Prime Break-Point

A, B = basic prices for suppliers A' and B'

$$E = e^{-mD}$$

D = distance between A' and B'

m, h = as defined previously

Once the value of a' is determined, the equilibrium points other than the Prime Break-Point can be located through the application of the Cosine Law and the concept of polar coordinates. As a result, the break-points may be located via use of the two formulas below--the first formula incorporating the first transportation cost equation and the second formula incorporating the second transportation cost equation.

$$\cos \theta = \frac{x(2a' - D) \left(2a'D + 2a' + 2 + 2X + 2D + XD \right) + 2a'D(a' + 1)^2}{2D(a' + X)(a' + 1)^2}$$

where:

x = any real number greater than 0

a', D = as stated before

or

$$\cos \theta = \frac{2(2a' - D - g)X + 2a'D - g(2D - 2a' + g)}{2D(a' + X)}$$

where

$$g = -\frac{1}{m} \left[\ln \left\{ 1 - \frac{e^{-ma'}(1 - e^{-mx})}{e^{-m(D - a')}}} \right\} + mx \right]$$

$a' D$ = as stated before

X = any real number greater than 0

The Prime Break-Point and break-points thus determined are known to be the market equilibrium points between the supplier whose market area is under study and the competing suppliers. By connecting these points, the market boundary lines between the supplier and the competing suppliers can be identified. The market area for the supplier under study is the area not encroached upon by its competitors.

The application of the New Break-Point method to a regional distribution center requires some specifications and refinements corresponding to the specific nature of a distribution center and additional assumptions underlying the analysis. As a distribution center is not a processing business unit and as by assumption the non-transportation distribution cost per transportation unit does not change, the symbol A in the analytical model will represent the normal profit and the basic price at the distribution center. The basic price is the sum of the f.o.b. production center and the cost of transporting the commodity from the production center. Also by the oligopolistic assumption of the nature of the distribution center and the result of the competition in services, the market area for a distribution center is limited to a maximum of 500 miles radius. Hence, the value of x to be assigned in the New Break-Point model can not exceed $(500 - a')$ miles.

The analytical model developed in this study is

applied to the case of the proposed Little Rock Regional Distribution Center. The result of this empirical aspect of the study reveals that the transportation cost assumption underlying the New Break-Point method is quite realistic and supported by the statistical results from a wide range of existing transportation rate schedules; and that the method is operational in the sense that there is no computational difficulty or abstruse concept involved in the new method. The result of the analysis of the market area for canned orange juice, selected by a method of choosing the warehouse candidates for a distribution center developed and presented in Appendix C of this study, has manifested that it is consistent with the purported significant characteristics of the new method formulated in this study.

To avoid possible distortion of the facts and the logic underlying the new method, the New Break-Point method is not recommended for the distribution center whose cost-distance relation shows irregularity, and hence the parameter m cannot be approximated with reasonable accuracy. In applying the new method, it should be understood that the organizational behavior, such as business relationships between an individual distributing center and its suppliers, affects the inbound moving cost and the f.o.b. price, thereby affecting the fidelity of the New Break-Point method.

It has been noted that in the analysis of a market area, the demand aspect of the analysis is assumed to be

fixed. With the New Break-Point method as a basic tool, it seems highly likely that some, if not all, *ceteris paribus* assumptions of demand can be relaxed by introducing a demand curve and constructing a DQ curve (Distance-Quantity curve). Better understanding of the DQ function is hinged upon the better understanding of the behavior of the demand curve and non-transportation distribution cost functions. Further investigation into these theoretical fronts leave much to be desired, although these terms are not unknown.

APPENDICES

APPENDIX A

DERIVATION OF CONVERSE'S BREAKING-POINT FORMULA

The New Law of Retail Gravitation was fundamentally based on the Reilly concepts. The formula $D_b = \frac{D_{ab}}{1 + \sqrt{\frac{P_a}{P_b}}}$, as shown in Equation (2) of Chapter II, was derived from Reilly's $\frac{B_a}{B_b} = \left(\frac{P_a}{P_b}\right)\left(\frac{D_b}{D_a}\right)^2$, the Equation (1) of Chapter II. With the same symbol assignments for the formula stated in Chapter II, the step to step derivation was shown by Professor Huff as follows:*

$$1) \frac{B_a}{B_b} = 1$$

$$2) \left(\frac{P_a}{P_b}\right)\left(\frac{D_b}{D_a}\right)^2 = 1$$

$$3) \frac{D_b}{D_a} = \sqrt{\frac{P_b}{P_a}}$$

$$4) D_a = D_{ab} - D_b$$

$$5) \frac{D_b}{D_{ab} - D_b} = \sqrt{\frac{P_b}{P_a}}$$

$$6) \frac{D_{ab}}{D_b} - 1 = \sqrt{\frac{P_a}{P_b}}$$

$$7) \frac{D_{ab}}{1 + \sqrt{\frac{P_a}{P_b}}} = D_b$$

*David L. Huff, "Defining and Estimating a Trade Area," The Journal of Marketing, Vol. 28 (July, 1964), p. 35.

APPENDIX B

BEHAVIOR OF HYPERCIRCLES*

A complete analysis of the behavior of this family of curves has been made by F. Gomes Teixeira.** The following simple discussion may be sufficient for most readers.

Consider the function:

$$F(P) = \overline{AP} - \frac{s}{r}\overline{BP} - \frac{q-p}{r}.$$

Using the familiar rectangular coordinates:

Let P be denoted by (x,y), then

$$\overline{AP} = \sqrt{(x+a)^2 + y^2}, \quad \overline{BP} = \sqrt{(x-a)^2 + y^2}.$$

When $y = 0$,

$$F(P) = F(x,0) = |(x+a)| - \frac{s}{r}|(x-a)| - \frac{q-p}{r}.$$

The function $f(x) = |x - 0|$ is continuous, and since the sum of two continuous functions is also a continuous function, $F(x,0)$ is a continuous function.

$$\begin{aligned} \text{For } x \geq a, \quad F(x,0) &= x + a - \frac{s}{r}(x-a) - \frac{q-p}{r} \\ &= x(1-\frac{s}{r}) + a(1+\frac{s}{r}) - \frac{q-p}{r} \end{aligned} \quad (1)$$

*Adopted from C. D. Hyson and W. P. Hyson, "The Economic Law of Market Area," Quarterly Journal of Economics, Vol. LXIV (1950), pp. 319-29.

**F. Gomes Teixeira, Traite des Courbes Speciales Remarkables (Coimbre, 1908), Vol. I, pp. 218-233.

$$\begin{aligned} \text{For } -a \leq x \leq a, F(x,0) &= x + a - \frac{s}{r}(a-x) - \frac{q-p}{r} \\ &= x(1+\frac{s}{r}) + a(1-\frac{s}{r}) - \frac{q-p}{r} \end{aligned} \quad (2)$$

$$\begin{aligned} \text{For } x \leq -a, F(x,0) &= -(x+a) + \frac{s}{r}(x-a) - \frac{q-p}{r} \\ &= -x(1-\frac{s}{r}) - a(1+\frac{s}{r}) - \frac{q-p}{r} \end{aligned} \quad (3)$$

Therefore, in each of the three intervals, $x > a$, $-a \leq x \leq a$, $x \leq -a$, $F(x,0)$ is linear.

Using (1), if $\frac{s}{r} > 1$, $F(x,0) \rightarrow -\infty$ as $x \rightarrow +\infty$.

If $\frac{s}{r} = 1$, $F(x,0) = 2a - \frac{q-p}{r}$,

$F(x,0) > 0$ if $2a > \frac{q-p}{r}$,

and $F(x,0) < 0$ if $2a < \frac{q-p}{r}$.

If $\frac{s}{r} < 1$, $F(x,0) \rightarrow +\infty$ as $x \rightarrow +\infty$.

Using (2), $F(+a,0) = 2a - \frac{q-p}{r}$.

If $2a > \frac{q-p}{r}$, $F(+a,0) > 0$,

if $2a < \frac{q-p}{r}$, $F(+a,0) < 0$.

$F(-a,0) = -2a\frac{s}{r} - \frac{q-p}{r}$.

If $-2a(\frac{s}{r}) > \frac{q-p}{r}$, $F(-a,0) > 0$,

if $-2a(\frac{s}{r}) < \frac{q-p}{r}$, $F(-a,0) < 0$.

Using (3), if $\frac{s}{r} > 1$, $F(x,0) \rightarrow -\infty$ as $x \rightarrow -\infty$.

If $\frac{s}{r} = 1$, $F(x,0) = -2a - \frac{q-p}{r}$,

$F(x,0) > 0$ if $\frac{q-p}{r} < -2a$,

and $F(x,0) < 0$ if $\frac{q-p}{r} > -2a$.

If $\frac{s}{r} < 1$, $F(x,0) \rightarrow +\infty$ as $x \rightarrow -\infty$.

The intersections of the hypercircles with the x -axis will occur when $F(x,0) = 0$. Suppose now that $\frac{s}{r} > 1$, and $-2a\frac{s}{r} < \frac{q-p}{r} < 2a$. It should be clear from the preceding analysis that $F(x,0)$ will vanish once in the interval $x > a$, and

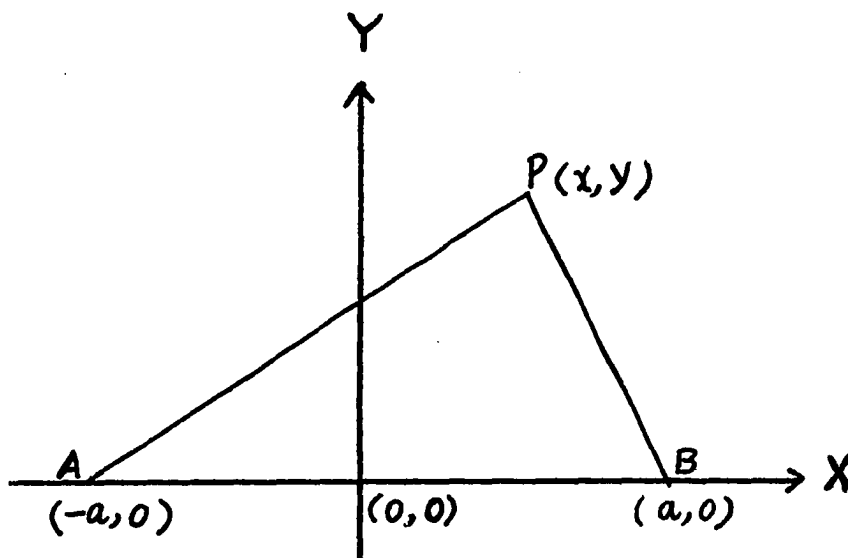


Figure 5

once in the interval $-a < x < a$. At the same time, since $F(x, 0) = |x+a| - \frac{s}{r}|x-a| - \frac{q-p}{r}$ is continuous for all values of x , and linear in each of the three intervals $x > a$, $-a < x < a$, $x < -a$; and since $F(x, 0) \rightarrow -\infty$ as $x \rightarrow +\infty$ and $F(x, 0) \rightarrow -\infty$ as $x \rightarrow -\infty$, $F(x, 0)$ can vanish at most twice. Therefore, when $\frac{s}{r} > 1$, and $-2a\frac{s}{r} < \frac{q-p}{r} < 2a$, there are two and only two intersections of the hypercircles with the x -axis.

In a similar manner the whole of Table 1 may be derived. This shows how the hypercircles will intersect the x -axis for all possible values of $\frac{q-p}{r}$, and $\frac{s}{r}$. (Bear in mind that $\overline{AB} = 2a$.) All hypercircles are symmetrical in the x -axis. Except for the case $\frac{q-p}{r} = 0$, the circle, they are symmetrical in no other line. The case $\frac{s}{r} = 1$,

the hyperbola, has but one intersection with the x-axis. Thus a general idea of the shape of any one curve may be obtained by a comparison of the values of $\frac{q-p}{r}$, \overline{AB} , and $\frac{s}{r}$.

TABLE 1

$\frac{s}{r}$	$\frac{q-p}{r}$	Intersection on X-Axis Between and	
$\frac{s}{r} > 1$	$\frac{q-p}{r} > 2a$	none	none
	$2a > \frac{q-p}{r} > -2a\frac{s}{r}$	$(a, 0)$ $(-a, 0)$	$(+\infty, 0)$ $(0, 0)$
	$-2a(\frac{s}{r}) > \frac{q-p}{r}$	$(a, 0)$ $(-\infty, 0)$	$(+\infty, 0)$ $(-a, 0)$
$\frac{s}{r} = 1$	$\frac{q-p}{r} > 2a$	none	none
	$2a > \frac{q-p}{r} > 0$	$(0, 0)$	$(a, 0)$
	$0 > \frac{q-p}{r} > -2a$	$(-a, 0)$	$(0, 0)$
	$-2a > \frac{q-p}{r}$	none	none
$\frac{s}{r} < 1$	$\frac{q-p}{r} > 2a$	$(a, 0)$ $(-a, 0)$	$(+\infty, 0)$ $(a, 0)$
	$2a > \frac{q-p}{r} > -2a\frac{s}{r}$	$(-a, 0)$ $(-\infty, 0)$	$(a, 0)$ $(-a, 0)$
	$-2a(\frac{s}{r}) > \frac{q-p}{r}$	none	none

APPENDIX C

PROCESS FOR SELECTING COMMODITY CANDIDATES FOR WAREHOUSING

The functions of a warehouse are to facilitate the efficient flow of goods in the channels and to store the goods as the condition requires in the process of flow. At a particular warehouse location, not all commodities are suitable for warehousing services. Identification of those commodities which are best suited for warehousing at a particular location becomes one of the most important steps in the planning and operation of a distribution center.

Generally, the commodities that are preferably demanded, that can bear substantial transportation cost, and that can save some transportation cost under certain transportation rate structures would be considered as good candidates for warehousing. The process of selecting such commodities that will meet these qualifications is shown in Figure 1. Three steps are involved in the process of selection. Each step is considered to be a screening process. The first screening step (Process I) is tagged as "Demand Condition;" the second step (Process II), "Distance Condition;" the third step (Process III), "Weight Condition." These steps

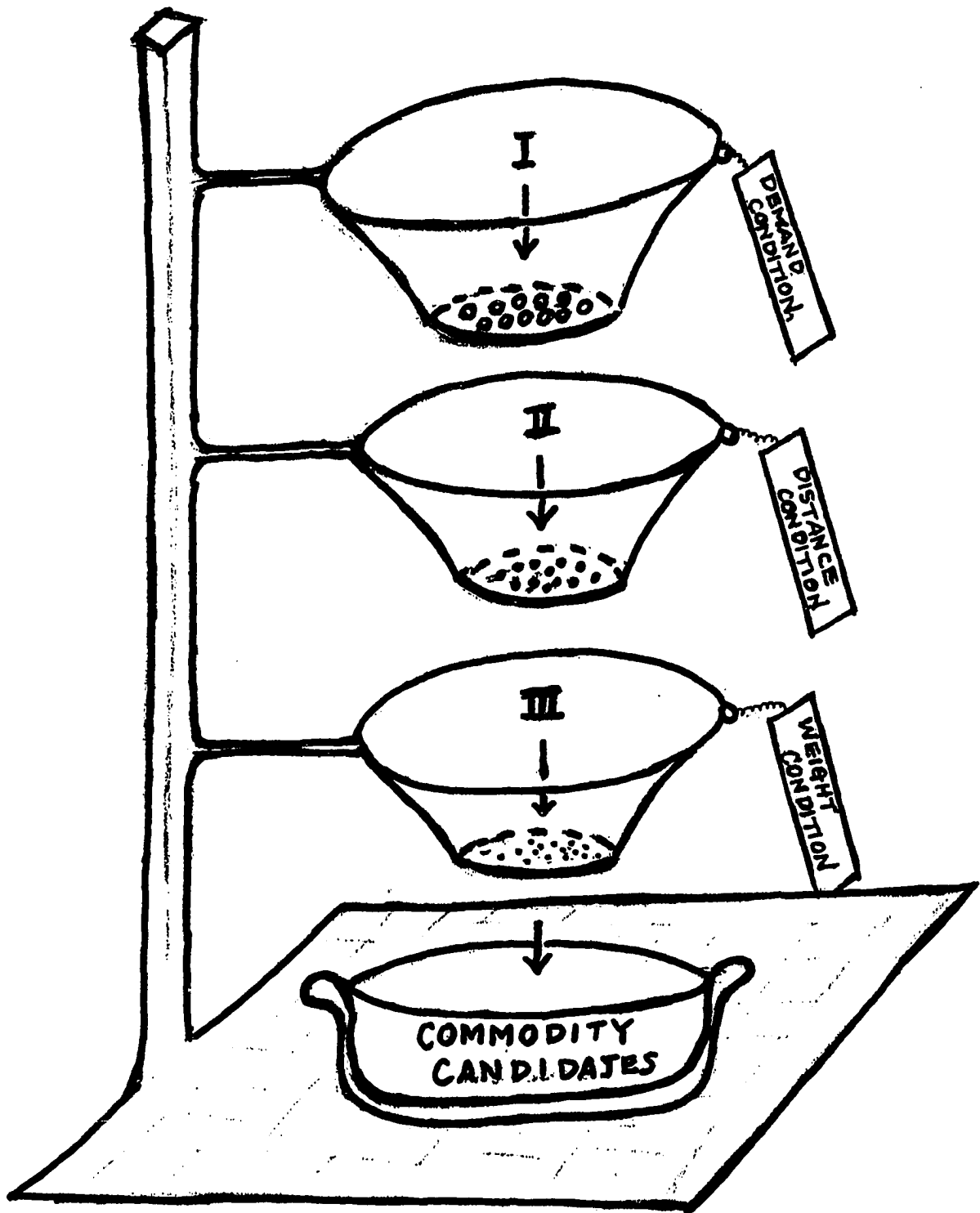


Figure 1. Process for Selecting Commodity Candidates for Warehousing.

will be discussed separately in more detail.

Process I

Commodities that are demanded in a particular region invariably are reflected in the freight movement within that region and/or between that region and other regions. For the warehousing services at a regional distribution center, an index called Commodity Preference Index (written as C-P index hereafter) can be used to detect the likely potential commodity candidates. C-P index is computed by dividing the index of excess freight received by a region into the index of purchasing power for the same region. Index of excess freight received is merely the excess of freight received over freight shipped for a region expressed in the percentage of total national freight movement for particular commodities. Data for the index of excess freight received may be secured from the Bureau of Census's Census of Transportation, Vol. III.¹ Information pertaining to the index of purchasing power may be secured from Survey of Buying Power conducted and published annually by the Sales Management, Inc., and Directory of Key Plants, also published annually by the Sales

¹The latest Census of Transportation was published in 1970 and consists of three volumes. The information pertaining to the commodity transportation survey is published in Vol. III. See Bureau of Census, 1967 Census of Transportation (Washington, D.C.: Government Printing Office, 1970), Vol. III, Parts 1, 2, and 3.

Management, Inc.² Index of purchasing power may be divided into the index of purchasing power for consumer goods and that of industrial goods. While the former is readily available from the Survey of Buying Power, the latter can be computed through the use of the Directory of Key Plants. The index of purchasing power for consumer goods shows the purchasing power in percentage of U.S. total purchasing power which is computed by weighting 2 for population, 3 for effective buying income (EBI), and 5 for the total retail sales.³ In a similar manner, one can compute the index of purchasing power for industrial goods by weighting 2 for corporate earnings, 3 for the number of employees, and 5 for the total industrial sales. The factors involved and the process of computation of the C-P index are illustrated in Figure 2.

The resultant index will manifest the degree of commodity preference for a region. The higher the index, the better the commodity is as a warehousing candidate.

For the first screening, one will select a smaller number of candidates from the top of the list arranged in descending order according to the value of the index. The

²Survey of Buying Power is published annually in June.

³According to Dr. Hong, Managing Director of Market Statistics for Sales Management, Inc., the 2, 3, and 5 weights are based on the judgment built on years of experience by the Sales Management staffs and executives. Correspondence with Dr. Alfred Hong, Managing Director, Market Statistics, Sales Management, Inc., July 15, 1971.

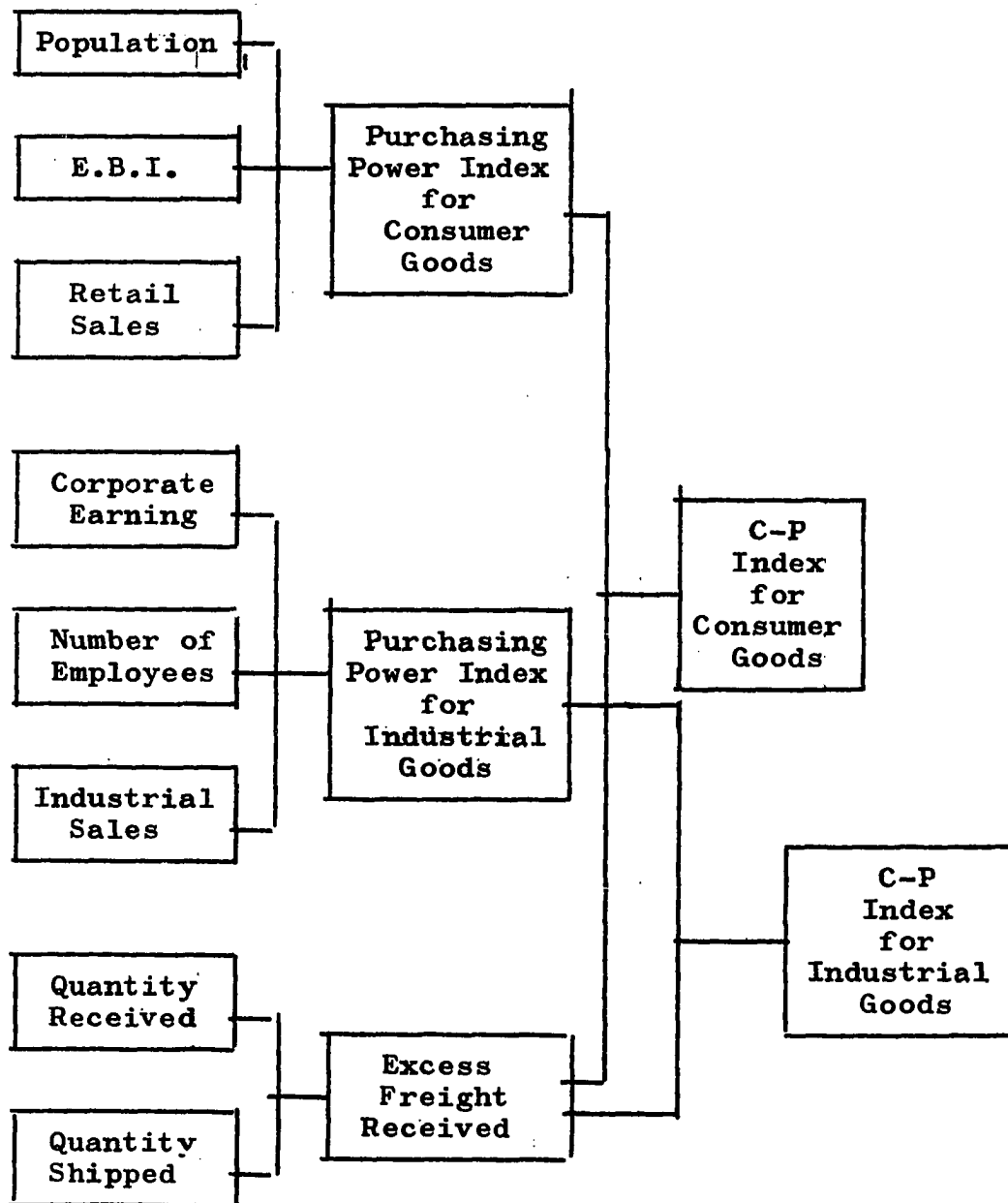


Figure 2. Factors Associated with C-P Index for a Region.

number of candidates to be selected from this first screening process depends upon the number of commodities to be warehoused. For illustrative purposes, 15 candidates are selected from this first process (shown in Table 1).

TABLE I

RESULT OF SELECTING PROCESS I*
(C-D Index)

TCC** Code	Freight Received % of Nation	Freight Shipped % of Nation	Excess Freight Received % of Nation	C-D Index
326	16.4	4.5	11.9	1.40
283	12.6	1.0	11.6	1.36
316	7.0	--	7.0	0.87
284	9.8	3.9	5.9	0.69
364	6.1	0.9	5.9	0.69
207	7.9	2.3	5.6	0.66
371	6.6	1.0	5.6	0.66
365	7.6	2.3	5.3	0.62
239	9.9	5.7	4.2	0.49
221	5.8	2.0	3.8	0.45
204	15.1	11.7	3.4	0.40
201	9.1	5.9	3.2	0.38
203	6.2	3.4	2.8	0.33
222	2.6	--	2.6	0.31
202	2.7	0.7	2.0	0.24

*The freight data are taken from Bureau of Census' 1967 Census of Transportation (Washington, D.C.: Government Printing Office, 1970), Vol. IV, Part 3. The consumer purchasing power index for C-D index is that of the corresponding year. See Sales Management, Vol. 100, p. B-7.

**For the translation of TCC code, see Table IV.

Process II

To say how well a commodity can bear the transportation cost is tantamount to saying how far the commodity can be shipped and marketed, other things being equal. This quality can be measured by a Critical Distance Share (C-D share hereafter) which is expressed in percentage of total freight movement. C-D share is the cumulative percentage of the freight moved beyond a critical distance, for example 500 miles or more. The process of computing C-D share is shown in Figure 3.

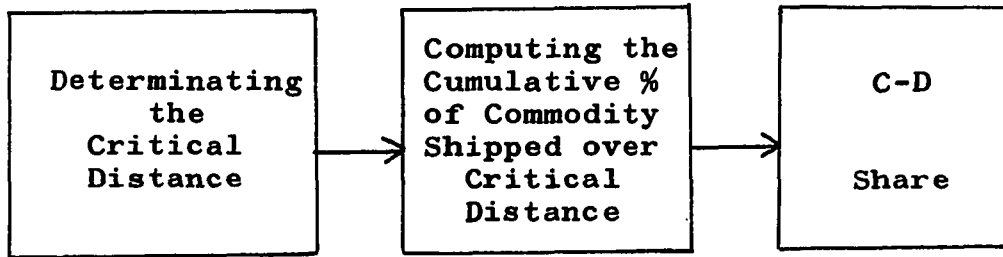


Figure 3. Process of Computing C-D Share.

The critical distance for a commodity depends mainly upon the competitive conditions. For a distribution center, the critical distance can be assumed to be 500 miles. The reason is that, for a commodity which is mainly shipped less than 500 miles, the delivery can be economically made by the manufacturers. Only those commodities which need to be transferred to areas beyond the "economic hand" of manufacturers are here considered to be the probable candidates for warehousing in a distribution center.

For Process II, C-D index for those commodities selected from the first process are computed and arranged in descending order. Selection of the commodities in this process is made from the top of the list. Again, the number of commodities to be selected depends upon the desired number of commodities to be warehoused in the last analysis. For this example, 10 commodities are selected. The result of such a selection is shown in Column A of Table 2.

TABLE 2
RESULT OF SELECTION PROCESSES II AND III*

TCC** Code	A (C-D Share) %	TCC** Code	B (F-W Share) %
316	53.3	203	80.8
365	48.4	204	78.8
364	46.7	202	77.2
207	44.7	284	60.6
203	44.6	201	60.6
326	44.6	283	59.3
221	41.1	207	51.5
239	39.7	371	43.3
222	38.0	326	32.7
201	36.3	239	23.2

*All data are secured from Bureau of Census, 1967 Census of Transportation (Washington, D.C.: Government Printing Office, 1970), Vol. III, Part 3.

**For the translation of TCC code, see Table IV.

Process III

Under certain conditions, for example, volume movement (e.g., TL, CL or Train Load), some commodities can save more transportation cost than others. This is hinged upon the demand condition, availability of transportation facilities, and the nature of the commodity. Without going into the complexity of such an analysis, one can determine the commodities which are the potential cost savers. The Freight-Weight Share (called F-W share hereafter) may be used to detect such commodities. From Census of Transportation, one can compute the cumulative percentage of freight that is moved more than TL or CL. The higher the percentage, the better the commodity is as a potential saver. For example a CL is about 40,000 lbs, and therefore, one may compute the cumulative percentage of freight movement by 40,000 lbs or more. In the example cited here, the cumulative percentages of shipment with 30,000 lbs or more for the commodities selected from the second screening process are computed. F-W share for the commodities selected from the second screening process is shown in Column B of Table 2. TCC 203 (canned and preserved fruits, vegetables, and seafoods) appears to be the No. 1 candidate; TCC 204 (grain mill products), No. 2; TCC 202 (dairy products), No. 3; etc. A summary of the results of selection processes I, II, and III are shown in Table 3.

TABLE 3
SUMMARY OF THE RESULTS OF SELECTION
PROCESSES I, II, AND III*
(For Consumer Goods.)

Process I (C-P Index)		Process II (C-D Share)		Process III (F-W Share)	
TCC	Rank	TCC	Rank	TCC	Rank
326	1	316	1	203	1
283	2	365	2	204	2
316	3	364	3	202	3
284	4	207	4	284	4
364	5	203	5	201	5
207	6	326	6		
371	7	221	7		
365	8	239	8		
239	9	222	9		
221	10	201	10		
204	11				
201	12				
203	13				
222	14				
202	15				

*Data adopted from Tables 1 and 2.

**For the translation of TCC code, see Table 4.

TABLE 4

SELECTED MAJOR TCC 3-DIGIT CLASSES*

TCC	Commodity Description
201	Meat, Poultry, and Small Game; Fresh, Chilled or Frozen
202	Dairy Products
203	Canned and Preserved Fruits, Vegetables, and Seafoods
204	Grain Mill Products
206	Sugar, Beet and Cane
207	Confectionery and Related Products
208	Beverages and Flavoring Extracts
209	Miscellaneous Food Preparations and Kindred Products
221	Cotton Broadwoven Fabrics
222	Man-Made Fiber and Silk Broadwoven Fabrics
227	Carpets, Rugs, and Mats, Textile
228	Yarn and Thread (Cotton, Wool, Silk, and Man-Made Fiber)
229	Miscellaneous Basic Textiles
231	Men's, Youths', and Boys' Clothing
233	Women's, Misses', Girls', and Infants' Clothing
239	Miscellaneous Fabricated Textile Products
242	Lumber and Dimension Stock and Miscellaneous Sawmill and Planing Mill Products
243	Millwork and Prefabricated Wood Products, Including Plywood and Veneer
249	Miscellaneous Wood Products
251	Household and Office Furniture

TABLE 4 (Continued)

TCC	Commodity Description
262	Paper (Except Building Paper)
263	Paperboard, Fiberboard, and Pulpboard (Except Insulating Board)
264	Converted Paper and Paperboard Products (Except Containers and Boxes)
265	Containers, Boxes and Related Products, Paperboard, Fiberboard, and Pulpboard
281	Industrial Chemicals
282	Plastic Materials and Plasticizers, Synthetic Resins, Rubbers, and Fibers
283	Drugs (Biological Products, Medicinal Chemicals, Botanical Products, and Pharmaceutical Preparations)
284	Soap and Detergents, Cleaning Preparations, Perfumes, Cosmetics, and Other Toilet Preparations
285	Paints, Varnishes, Lacquers, Enamels, and Allied Products
287	Agricultural Chemicals
289	Miscellaneous Chemical Products
291	Products of Petroleum Refining
295	Paving and Roofing Materials
301	Tires and Inner Tubes
306	Miscellaneous Fabricated Rubber Products
307	Miscellaneous Plastic Products
314	Footwear (Except Rubber)
316	Luggage and Handbags (All Materials), and Other Personal Leather Goods
322	Glass and Glassware, Pressed and Blown
324	Hydraulic Cement

TABLE 4 (Continued)

TCC	Commodity Description
325	Structural Clay Products
326	Pottery and Related Products
327	Concrete, Gypsum, Plaster, and Plaster Products
329	Abrasives, Asbestos, and Miscellaneous Nonmetallic Mineral Products
331	Steel Works and Rolling Mill Products
332	Iron and Steel Castings
333	Nonferrous Metals Primary Smelter Products (Slab, Ingot, Pig, etc., and Residues)
335	Nonferrous Metal Basic Shapes
336	Nonferrous and Nonferrous Base Alloy Castings
339	Miscellaneous Primary Metal Products
341	Metal Cans
342	Cutlery, Hand Tools, and General Hardware
343	Plumbing Fixtures and Heating Apparatus (Except Electric)
344	Structural and Miscellaneous Metal Products
345	Bolts, Nuts, Screws, Rivets, Washers, and Other Industrial Fasteners
346	Metal Stampings
348	Miscellaneous Fabricated Wire Products
349	Miscellaneous Fabricated Metal Products
351	Engines and Turbines
352	Farm Machinery and Equipment
353	Construction, Mining, and Materials Handling Machinery and Equipment

TABLE 4 (Continued)

TCC	Commodity Description
354	Metalworking Machinery and Equipment
355	Special Industry Machinery (Except Metalworking Machinery)
356	General Industrial Machinery and Equipment
357	Office, Computing, and Accounting Machines
358	Service Industry Machines
359	Miscellaneous Machinery and Parts (Except Electrical)
361	Electrical Transmission and Distribution Equipment
362	Electrical Industrial Apparatus
363	Household Appliances
364	Electric Lighting and Wiring Equipment
365	Radio and Television Receiving Sets (Except Communication Types), Phonographs, and Phonograph Records
366	Communication Equipment
367	Electronic Components or Accessories
369	Miscellaneous Electrical Machinery, Equipment, and Supplies
371	Motor Vehicles and Motor Vehicle Equipment
372	Aircraft and Parts
379	Miscellaneous Transportation Equipment
382	Measuring, Controlling, and Indicating Instruments
386	Photographic Equipment and Supplies

*Bureau of Census, 1967 Census of Transportation (Washington, D.C.: Government Printing Office, 1970), Vol. III, Part 3, pp. 199-200.

APPENDIX D

MAJOR SOURCES OF INFORMATION FOR THE NEW
BREAK-POINT ANALYSIS

<u>Information</u>	<u>Sources</u>
Freight Movements	The Bureau of Census. <u>1967 Census of Transportation</u> . Washington, D.C.: Government Printing Office, 1970.
Transportation Rates (costs)	Various freight tariffs issued by various rate making authorities.
Distance Mileages	Rand McNally and Company. <u>1971 Commercial Atlas and Marketing Guide</u> . New York: Rand McNally and Company, 1971. Household Goods Carriers Bureau. <u>Mileage Guide</u> . New York: Rand McNally and Company, 1967.
Commodity Price f.o.b. Supplier	Various commodity catalogs published by various producers.

APPENDIX E

INPUT DATA FOR THE TESTING OF
TRANSPORTATION COST EQUATIONS**

TABLE 1
ARKANSAS CLASS RATES FOR MOTOR FREIGHT*

Rates			Rates		
Distance	A.Q.	1000- 2000 lbs	Distance	A.Q.	1000- 2000 lbs
12.5	83.0	78.0	88.0	117.0	110.0
28.0	87.0	82.0	93.0	119.0	112.0
33.0	89.0	85.0	98.0	121.0	113.0
38.0	94.0	89.0	105.5	125.0	118.0
43.0	97.0	91.0	115.5	129.0	122.0
48.0	99.0	93.0	125.5	133.0	125.0
53.0	101.0	96.0	135.5	135.0	127.0
58.0	103.0	97.0	145.5	140.0	133.0
63.0	106.0	100.0	155.5	142.0	134.0
68.0	109.0	102.0	165.5	146.0	138.0
73.0	110.0	103.0	175.5	149.0	140.0
78.0	113.0	107.0	185.5	153.0	144.0
83.0	115.0	109.0	195.5	155.0	146.0

**All data are for canned vegetables and fruits.
The distance data are the mid-values of distance brackets.

TABLE 1 (Continued)

Rates			Rates		
Distance	A.Q.	1000- 2000 lbs	Distance	A.Q.	1000- 2000 lbs
205.5	160.0	151.0	331.5	197.0	185.0
215.5	162.0	152.0	351.5	202.0	190.0
225.5	166.0	157.0	371.5	207.0	196.0
235.5	168.0	159.0	391.5	212.0	200.0
255.5	175.0	164.0	411.5	216.0	204.0
271.5	179.0	169.0	431.5	221.0	209.0
291.5	185.0	175.0	451.5	226.0	214.0
311.5	190.0	179.0			

*Tariff 32-G, Arkansas Tariff, Southwestern Motor Freight Bureau, Inc., Dallas, Texas, Feb. 10, 1969, pp. 88, 92-3.

TABLE 2

OKLAHOMA COMMODITY RATES FOR MOTOR FREIGHT*

1,000 lbs. min.				8,000 lbs. min.			
Distance	Rates	Distance	Rates	Distance	Rates	Distance	Rates
40.0	86.0	310.0	157.0	5.0	33.0	125.0	75.0
87.5	87.0	330.0	159.0	12.5	36.0	135.0	78.0
92.5	90.0	350.0	167.0	17.5	38.0	145.0	80.0
97.5	92.0	370.0	172.0	22.5	40.0	155.0	84.0
105.0	97.0	390.0	178.0	27.5	42.0	165.0	86.0
115.0	99.0	410.0	182.0	32.5	44.0	175.0	87.0
125.0	102.0	430.0	186.0	37.5	45.0	185.0	90.0
135.0	107.0	450.0	189.0	42.5	48.0	195.0	92.0
145.0	109.0	470.0	200.0	47.5	50.0	205.0	97.0
155.0	112.0	490.0	202.0	52.5	52.0	215.0	99.0
165.0	116.0	510.0	205.0	57.5	53.0	225.0	100.0
175.0	119.0	530.0	210.0	62.5	56.0	235.0	103.0
185.0	124.0	550.0	215.0	67.5	57.0	250.0	106.0
195.0	125.0	570.0	224.0	72.5	60.0	270.0	112.0
205.0	130.0	590.0	226.0	77.5	62.0	290.0	115.0
215.0	132.0	610.0	232.0	82.5	63.0	310.0	125.0
225.0	135.0	630.0	236.0	87.5	66.0	330.0	129.0
235.0	139.0	650.0	240.0	92.5	68.0	350.0	131.0
250.0	142.0	670.0	245.0	97.5	69.0	370.0	139.0
270.0	147.0	690.0	251.0	105.0	72.0	390.0	141.0
290.0	152.0			115.0	74.0	410.0	150.0

TABLE 2 (Continued)

1,000 lbs. min.		8,000 lbs. min.	
Distance	Rates	Distance	Rates
		430.0	155.0
		450.0	159.0
		470.0	164.0
		490.0	165.0
		510.0	170.0
		530.0	174.0
		550.0	180.0
		570.0	184.0
		590.0	186.0
		610.0	190.0
		630.0	196.0
		650.0	201.0
		670.0	205.0
		690.0	207.0

*Tariff 27-E, Oklahoma Class and Commodity Tariff,
Southwestern Motor Freight Bureau, Inc., Dallas, Texas, Feb.
9, 1970, p. 120.

TABLE 3

ARKANSAS COMMODITY RATES FOR MOTOR FREIGHT*

1,000 lbs. min.		8,000 lbs. min.		20,000 lbs. min.	
Distance	Rates	Distance	Rates	Distance	Rates
25.0	90.0	10.0	41.0	25.0	10.0
57.5	92.0	22.5	42.0	26.0	22.5
62.5	94.0	27.5	44.0	27.0	27.5
67.5	96.0	32.5	47.0	28.0	32.5
72.5	97.0	37.5	48.0	30.0	37.5
77.5	100.0	42.5	51.0	32.0	42.5
82.5	103.0	47.5	53.0	34.0	47.5
87.5	105.0	52.5	55.0	36.0	55.0
92.5	106.0	57.5	57.0	38.0	65.0
97.5	107.0	62.5	59.0		
105.0	111.0	67.5	61.0	40.0	75.0
115.0	114.0	72.5	63.0	43.0	82.5
125.0	118.0	75.5	65.0	44.0	87.5
135.0	120.0	82.5	66.0	45.0	92.5
145.0	125.0	87.5	68.0	46.0	97.5
155.0	128.0	92.5	70.0	47.0	105.0
165.0	131.0	97.5	72.0	48.0	115.0
175.0	134.0	105.0	75.0	52.0	125.0
185.0	136.0	115.0	78.0	54.0	135.0
195.0	138.0	125.0	79.0	55.0	145.0
205.0	144.0	135.0	83.0	57.0	155.0
215.0	147.0	145.0	85.0	58.0	165.0

TABLE 3 (Continued)

1,000 lbs. min.		8,000 lbs. min.		20,000 lbs. min.	
Distance	Rates	Distance	Rates	Distance	Rates
225.0	150.0	155.0	88.0	60.0	175.0
235.0	153.0	165.0	90.0	62.0	185.0
250.0	158.0	175.0	91.0	63.0	195.0
270.0	163.0	185.0	95.0	64.0	205.0
290.0	167.0	195.0	96.0	65.0	215.0
310.0	174.0	205.0	102.0	66.0	225.0
330.0	178.0	215.0	105.0	68.0	235.0
350.0	184.0	225.0	106.0	72.0	250.0
370.0	188.0	235.0	109.0	73.0	270.0
390.0	194.0	250.0	111.0	76.0	290.0
410.0	199.0	270.0	119.0	78.0	310.0
430.0	205.0	290.0	122.0	81.0	330.0
450.0	208.0	310.0	132.0	83.0	350.0
470.0	212.0	330.0	135.0	85.0	370.0
490.0	218.0	350.0	140.0	87.0	390.0
		370.0	146.0	91.0	410.0
		390.0	150.0	93.0	430.0
		410.0	158.0	96.0	450.0
		430.0	164.0	99.0	470.0
		450.0	167.0	101.0	490.0
		470.0	174.0		
		490.0	175.0		

*Tariff 32-G, Arkansas Tariff, Southwestern Motor Freight Bureau, Inc., Dallas, Texas, Feb. 10, 1969, p. 113.

TABLE 4

INTERSTATE COMMODITY RATES FOR MOTOR FREIGHT*

Distance	Rates		
	8,000 lbs. min.	14,000 lbs. min.	30,000 lbs. min.
17.5	55.0	46.0	33.0
37.5	56.0	47.0	34.0
42.5	59.0	51.0	36.0
47.5	61.0	54.0	37.0
52.5	63.0	55.0	40.0
57.5	65.0	56.0	40.0
62.5	69.0	59.0	43.0
67.5	71.0	61.0	43.0
72.5	74.0	63.0	45.0
77.5	76.0	65.0	45.0
82.5	77.0	66.0	47.0
87.5	79.0	69.0	49.0
92.5	81.0	71.0	51.0
97.5	83.0	74.0	53.0
105.0	87.0	77.0	54.0
115.0	90.0	78.0	55.0
125.0	91.0	79.0	57.0
135.0	97.0	84.0	59.0
145.0	99.0	87.0	61.0
155.0	102.0	90.0	63.0
165.0	104.0	92.0	64.0
175.0	105.0	95.0	66.0

TABLE 4 (Continued).

Distance	Rates		
	8,000 lbs. min.	14,000 lbs. min.	30,000 lbs. min.
185.0	110.0	99.0	69.0
195.0	111.0	100.0	70.0
205.0	118.0	102.0	71.0
215.0	121.0	107.0	74.0
225.0	122.0	110.0	75.0
235.0	127.0	110.0	77.0
250.0	129.0	112.0	79.0
270.0	138.0	117.0	80.0
290.0	141.0	120.0	84.0
310.0	153.0	122.0	87.0
330.0	156.0	124.0	90.0
350.0	162.0	133.0	92.0
370.0	169.0	135.0	97.0
390.0	173.0	139.0	99.0
410.0	183.0	142.0	101.0
430.0	190.0	146.0	103.0
450.0	193.0	151.0	108.0
470.0	201.0	158.0	110.0
490.0	203.0	159.0	112.0
510.0	207.0	163.0	113.0
530.0	213.0	167.0	118.0
550.0	217.0	170.0	120.0
570.0	225.0	176.0	122.0

TABLE 4 (Continued)

Distance	Rates		
	8,000 lbs. min.	14,000 lbs. min.	30,000 lbs. min.
590.0	227.0	179.0	124.0
610.0	233.0	185.0	128.0
630.0	238.0	187.0	131.0
650.0	245.0	189.0	133.0
670.0	250.0	194.0	135.0
690.0	252.0	196.0	139.0
710.0	256.0	201.0	141.0
730.0	258.0	206.0	142.0
750.0	267.0	208.0	145.0
770.0	271.0	211.0	148.0
790.0	272.0	213.0	151.0
812.5	277.0	216.0	154.0
837.5	282.0	219.0	156.0
862.5	288.0	225.0	158.0
887.5	293.0	229.0	161.0
912.5	296.0	233.0	163.0
937.5	303.0	234.0	166.0
962.5	307.0	239.0	168.0
987.5	309.0	243.0	170.0
1012.5	317.0	247.0	173.0
1037.5	321.0	251.0	176.0
1062.5	326.0	253.0	180.0
1087.5	331.0	256.0	181.0

TABLE 4 (Continued)

Distance	Rates		
	8,000 lbs. min.	14,000 lbs. min.	30,000 lbs. min.
1112.5	334.0	261.0	186.0
1137.5	343.0	267.0	188.0
1162.5	345.0	276.0	189.0
1187.5	352.0	273.0	191.0
1212.5	355.0	277.0	194.0
1237.5	359.0	281.0	196.0
1262.5	364.0	285.0	200.0
1287.5	370.0	288.0	201.0
1312.5	373.0	291.0	206.0
1337.5	379.0	296.0	209.0
1362.5	382.0	300.0	211.0
1387.5	386.0	305.0	213.0
1412.5	392.0	307.0	216.0
1437.5	397.0	310.0	218.0
1462.5	402.0	312.0	221.0
1487.5	405.0	320.0	225.0

*Tariff 1, Southwestern Commodity Tariff, Southwestern Freight Bureau, Inc., Dallas, Texas, Sept. 8, 1970, pp. 339-340.

TABLE 5

INTERSTATE (SW LINE) COMMODITY RATES
FOR RAIL FREIGHT*

48,000 lbs. min.				60,000 lbs. min.			
Distance	Rates	Distance	Rates	Distance	Rates	Distance	Rates
22.5	21.5	350.0	49.5	27.5	19.5	450.0	50.5
50.0	22.5	370.0	50.5	77.5	20.5	470.0	53.5
60.0	23.5	390.0	51.5	105.0	21.5	490.0	55.5
70.0	24.5	410.0	55.5	120.0	22.5	510.0	57.5
82.5	25.5	430.0	57.5	135.0	23.5	530.0	58.5
95.0	27.5	450.0	59.5	145.0	24.5	550.0	60.5
105.0	29.5	470.0	62.5	160.0	26.5	570.0	61.5
120.0	30.5	490.0	64.5	185.0	27.5	590.0	62.5
135.0	31.5	510.0	67.0	205.0	28.5	610.0	63.5
145.0	32.5	530.0	68.0	220.0	29.5	630.0	64.5
160.0	33.5	550.0	69.0	235.0	31.5	650.0	67.0
182.0	35.5	570.0	71.0	250.0	33.5	670.0	70.0
195.0	36.5	590.0	72.0	270.0	35.5	690.0	72.0
205.0	37.5	610.0	73.0	290.0	36.5	710.0	73.0
220.0	38.5	630.0	74.0	310.0	38.5	730.0	74.0
235.0	39.5	650.0	76.0	330.0	39.5	750.0	75.0
250.0	41.5	670.0	79.0	350.0	40.5	770.0	78.0
270.0	44.5	690.0	81.0	370.0	41.5	790.0	79.0
290.0	45.5	710.0	82.0	390.0	43.5	812.5	80.0
310.0	47.5	730.0	83.0	410.0	46.5	837.5	81.0
330.0	48.5	750.0	84.0	430.0	48.5	862.5	82.0

TABLE 5 (Continued)

48,000 lbs. min.				60,000 lbs. min.			
Distance	Rates	Distance	Rates	Distance	Rates	Distance	Rates
770.0	87.0	1162.5	107.0	887.5	84.0	1237.5	102.0
790.0	88.0	1187.5	109.0	925.0	86.0	1262.5	103.0
812.5	89.0	1212.5	110.0	962.5	88.0	1287.5	104.0
837.5	90.0	1237.5	111.0	987.5	89.0	1312.5	105.0
862.5	91.0	1262.5	112.0	1012.5	90.0	1337.5	106.0
887.5	92.0	1287.5	113.0	1037.5	92.0	1362.5	108.0
925.0	95.0	1312.5	114.0	1062.5	94.0	1387.5	110.0
962.5	97.0	1337.5	115.0	1087.5	96.0	1412.5	113.0
987.5	98.0	1362.5	117.0	1125.0	97.0	1437.5	116.0
1012.5	99.0	1387.5	119.0	1162.5	98.0	1462.5	119.0
1037.5	100.0	1412.5	122.0	1187.5	100.0	1487.5	122.0
1062.5	103.0	1437.5	125.0	1212.5	101.0		
1087.5	104.0	1462.5	128.0				
1112.5	105.0	1487.5	131.0				
1137.5	106.0						

*Tariff SW 2004-I, SW Line Freight Southwestern

Freight Bureau, St. Louis, Mo., Dec. 10, 1968, pp. 195, 262-266.

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