## FLOW CAPACITY OF A

## VISCOUS AIR FLOW METER

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## Thesis Approved:



## PREFACE

The viscous air flow meter was designed in an attempt to attain greater accuracies in the measurement of fluid flow. Since this device gives a linear relation between pressure drop versus rate of flow data, it is free from the "root mean square" errors of the conventional "kinetic type" flow measuring devices when measuring pulsating flows. It is also insensitive to installation variables since the velocities of flow are low and it may therefore be used as closely as desired to elbows, reductions, valves and gates. Since this device has been in use only a short time very little information about it is available.. The Husain Viscous Air Flow Meter was built and tested but for some unknown reason the test results were quite different from the expected results. The purpose of this paper is to show why this difference existed and to determine the flow capacity of the Husain meter.

Throughout the writing of this paper it has been presumed that the reader is femiliar with hydrodynamics and has a knowledge of conventional flow measurement by means of nozzles, orifices and venturi. Lengthy discussions of basic theory are therefore omitted. These may be found in most hydrodynamics or fluid mechanics textbooks. Only where necessary for clarity are new terms explained. The abbreviations and symbols used are all standard and are explained where necessary in the text. Sample calculations are included in the appendix.

I wish to express my thanks to Professor W.H. Easton for his invaluable aid and assistance in the writing of this paper, to Professor

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A area - flow or cross sectional, square inches or square feet
C constant coefficient
d least depth of cross section or side length of triangle, feet
F degrees Fahrenheit
k constant coefficient
L length of flow passage, feet
$P$ pressure, units as stated
Q volumetric rate of flow
r radius
t temperature Fahrenheit
V flow velocity, feet per second
W weight rate of flow, pounds per second
Y nozzle expansion factor

## Abbreviations

ASME American Society of Mechanical Engineers
cfs cubic feet per second
ft feet or foot
fps feet per second
in. inch
lb pound
rpm revolutions per minute
sq square
$\Delta$. change of value in general
$\mu$ alsolute viscosity $\mathrm{lb} \mathrm{sec} / \mathrm{ft}^{2}$
$\rho$ air density $1 b / c u f t$

## CHAPTER I

## INTRODUCTION

The accurate measurement of the amount of fluid flowing into or from a mechanical device such as an engine, turbine, pump or blower has always presented quite a problem. Ordinarily devices of the "kinetic type" such as orifices, nozzles and venturi are used because of their simplicity. Under conditions of pulsating or unsteady flow these metering devices do not provide satisfactory accuracy unless large and expensive smoothing tanks are employed since the rate of flow is roughly proportional to the square root of the measured pressure drop. Since these devices of the "kinetic type" are sensitive to both upstream and downstream effects, great care must be exercised to duplicate the conditions under which the device was calibrated in order to prevent introducing additional errors due to variations in installation. The range of flow over which any particular size orifice, nozzle or venturi gives accurate flow-metering is also quite limited.

A highly desirable metering device would be one having a pressure drop across it which is directly proportional to the flow rate, having a considerable range of flow capacity and measuring pulsating flows as accurately as steady flows. Such a device is the viscous air flow meter. This device operates in the laminar flow region rather than in the turbulent region as do meters of the orifice type.

A Viscous Air Flow Meter was designed and tested by Syed Vicar Husain but although a linear relationship was definitely found to exist between the pressure drop across the meter and the flow rate, the results lacked correlation with the theoretical calculation upon which the meter was designed. The range of flow over which linear pressure versus flow rate relationships existed was also not determined due to test equipment limitations. ${ }^{1}$

Further research and testing appeared warranted in order to find. the reason for the difference between the theoretical and the actual flow rate and to determine the flow capacity of this metering device.

With as close adherence as possible to the original test conditions, additional tests were run using a roots type blower instead of an internal combustion engine and the flow capacity of the meter was determined.

[^0]
## THEORY OF VISCOUS FLOW

The viscous element operates in the region where the flow through the element passages moves in parallel layers each with a velocity in the same direction but with a magnitude of velocity which is a function of the distance from the layer to the surfaces of the passage. In the case of circular cross section tubes these layers would be a series of hollow concentric telescopic cylinders slipping over one another, the cutermost being at rest relative to the walls of the tube, the innermost having the maximum velocity magnitude.


Figure 1. Laminar flow in circular tube

In this region of flow the effects of viscosity are so predominant that the effect of inertia may be ignored. As the rate of flow through the element increases the layers intermix, the flow becomes turbulent, and the equations for viscous flow no longer hold.

Jean Louis Marie Poisseuille first investigated this "laminar flow" in 1842 while studying the circulation of blood in small capillaries and developed the law which bears his name. Poisseuille's law states
that the pressure drop $\Delta \mathrm{P}$ between two points a distance "L" apart in a tube of "r" radius through which a fluid of viscosity " 4 " is flowing with a mean volumetric rate of flow " $Q$ " is given by

$$
\Delta P=\frac{8 L \mu_{Q}}{\pi r^{4}}
$$

Although Poisseuille determined the equation experimentally, it has been identically developed mathematically from the general viscous flow relations equating the pressures on the ends of the layers. The general equation of flow is

$$
\frac{d P}{d X}=-M\left(\frac{\partial^{2} v}{\partial y^{2}}+\frac{\partial^{2}{ }_{V}}{\partial z^{2}}\right)
$$

where the flow is in the direction of "X" with a velocity "v ".
This general equation may also be written as:

$$
\frac{d P}{d X}=-k \frac{\mu V}{d^{Z}}
$$

where "k" is a numerical constant determined by the particular shape of passage, "d" is the least depth of section and "V" is the mean velocity of flow. Values of "k" for various cross-sections have been determined as shown in Table I.

In the design of the Viscous Air Flow Meter by Husain, triangular passages were used. The expected pressure drop for various rates of flow were calculated using Poisseuille's equation modified by the introduction of the "hydraulic mean radius" of the triangular passages. Although use of this parameter gives results which are in fairly good agreement with experimental results for turbulent flow, its use for laminar flow may give very inaccurate results. ${ }^{\text {l }}$

The theoretical pressure drops calculated using the hydraulic mean radius were found to be almost twice as great as the actual measured

[^1]pressure drops, whereas good correlation was obtained when the proper value of "k" for triangular passages was used.

## TABLE I

VALUES OF " $k$ " FOR VARIOUS CROSS-SECTIONS ${ }^{1}$

| Parallel Plates | $k=12$ |
| :--- | :--- |
| Circular Tube | $k=32$ |
| Elliptical Tube | $k=20$ for $(\mathrm{a} / \mathrm{b}=2)$ |
| Square Tube | $\mathrm{k}=28.6$ |
| Rectangular Tube | $\mathrm{k}=17.5$ for $(\mathrm{a} / \mathrm{b}=2)$ |
| Narrow isosceles triangle | $\mathrm{k}=24$ |
| Equilateral triangle of side "d" | $\mathrm{k}=80$ |

I H.F. Purday, An Introduction to the Mechanics of Viscous Flow, (Dover Publications Inc., 1949), p. 28.

CHAPTER III

STATEMENT OF PROBLEM AND ANALYSIS

The primary purpose of this investigation was to determine the flow capacity of a particular viscous air flow meter so that its range of utility might be known. A secondary objective was to discover why the actual pressure drops across the meter were far less than the calculated theoretical pressure drops.

The equation for pressure drop in a viscous flow meter is given by: $\quad \Delta \mathrm{p}=\mathrm{C}_{1} \mathrm{PQ}^{2}+\mathrm{C}_{2} \mu \mathrm{M}$ where $C_{1}$ and $C_{2}$ are constants and $e_{\text {is the fluid density. }}$

By proper design and at low rates of flow, $C_{l}$ approaches zero and the kinetic term of the equation may be ignored. At higher rates of flow the kinetic term is no longer negligible and eventually, at still higher flow rates, the flow is fully turbulent and the viscous term of the equation becomes negligible. This is the condition existant in nozzles and orifices when operating in their design region. It can be seen from the above equation that in the case of the viscous element the pressure drop is directly proportional to the volumetric rate of flow, while in the nozzle or orifice the pressure drop is proportional to the square of the volumetric rate of flow.

In order to determine the flow capacity of the Husain meter, it was necessary to either increase the rate of flow until transition to turbulent flow occured or to decrease the flow area of the meter there-
by increasing the velocity of flow.
As in the original testing of the meter, standard ASME nozzles were used for calibration.

## CHAPTER IV

## DESCRIPTION OF THE APPARATUS AND EQUIPMENT

The viscous flow meter consists of a cylindrical center section fourteen inches long containing the viscous element, the filter and the pressure pickup tubes and conical sections on either end reducing the fourteen inch inside diameter of the center section to four inches. It is sixty-three inches in overall length. A schematic view is shown in Fig. 2.

The viscous element (Fig. 3.) is made of two long strips of aluminum foil, one corrugated and one plain, wrapped around and around a central wooden core, making the element fourteen inches in diameter and 5.2 inches long. The passages are thus approximately triangular with dimensions as given in Fig. 4 .

The viscous meter was installed on the suction side of a positive displacement blower driven by a Varidrive having a 20 hp electric motor. The ASME standard nozzles were installed well upstream of the meter in accordance with ASME Power Test Codes for Flow Measurement.

Oil manometers (vertical and inclined) were used for measuring the pressure drops.

Polyethylene plastic film of 3 mil. thickness was used to reduce the flow area as shown in Fig. 5 and 6. This material was found to be pliant enough to effectively seal off the unwanted area in spite of the irregularity in the face of the viscous elements.


Figure 2. Schematic View of the Viscous Air Flow Meter


Figure 3. Schematic View of Viscous Element with Passages greatly enlarged


Figure 4. Approximate Triangular Passage


Figure 5. Viscous Element masked off for 50\% Area.


Figure 6. Viscous Element masked off for 12.5\% Area


Figure 7. Viem of Test Set Up.

## CHAPTER V

PROCEDURE

After installing the meter as shown in Figure 7., the barometric pressure and room temperatures were read and recorded, the manometer scales were adjusted to zero and the Varidrive started. The speed of the "Varidrive" was varied in increments and the pressure drops across the meter and nozzle were recorded for each speed. A waste gate installed on the suction side of the blower served to reduce the flow through the meter when the blower was operating at minimum speed.

Upon plotting the results of the runs using all of the flow area of the meter, it was discovered that the capacity of the blower was insufficient to produce the quantity of flow necessary to produce fully turbulent flow in the meter. It was therefore necessary to reduce the flow area of the meter in order to increase the velocity of the flow. This was accomplished by masking off part of the face of the viscous element with polyethylene plastic film. Runs were made using one-half, one-eighth and finally one-sixteenth of the original area of the meter before complete transition to turbulent flow was attained.

## CHAPTER VI

## DATA AND RESULTS

The observed data are shown in Tables II and III. The calculated data are shown in TABLES IV through VII. The results are shown on Figures 8 through ll. Figures 8 and 9 show the rates of flow versus pressure drops across the meter at standard conditions. A comparision between the theoretical pressure drops and the actual pressure drops is also shown for the run using the full flow area of the meter. Figures 10 and 11 are plots of the pressure drops across the meter versus the pressure drops across the nozzle plotted on logarithmic paper.

## OBSERVED DATA

| Date: December 19, 1955 <br> Barometric Pressure: 29.54 in. Hg <br> Room Temperature: $70^{\circ} \mathrm{F}$ <br> Specific Gravity manometer oil: . 85 <br> ASME nozzle size: 2 in. <br> Flow meter area: 123 sq in. (100\%) |  |  | Date: December 27, 1955 <br> Barometric Pressure: 29.16 in. Hg <br> Room Temperature: $72^{\circ} \mathrm{F}$ <br> Specific Gravity manometer oil: 85 <br> ASME nozzle sine: 2 in. <br> Flow meter area: 61.5 sq in. (50\%) |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Varịrive Speed $\frac{\mathrm{rpm}}{100}$ | $\begin{gathered} \Delta P_{N} \\ (\text { nozzle }) \\ \text { in. oil } \end{gathered}$ | $\begin{aligned} & \Delta P_{M} \\ & (\text { meter }) \\ & \text { in. } \mathrm{H}_{2} \mathrm{O} \end{aligned}$ | Varidrive Speed $\frac{\mathrm{r} p \mathrm{~m}}{100}$ | $\begin{gathered} \Delta \mathrm{P}_{\mathbb{H}} \\ \text { (nozzle) } \\ \text { in. oil } \end{gathered}$ | $\underset{(\text { meter }}{\Delta \mathrm{P}_{\mathrm{M}}}$ <br> in. H |
| 5 | 2.82 | 0.190 | 4 | 1.65 | 0.280 |
| 6 | 3.88 | 0.221 | 5 | 2.55 | 0.351 |
| 7 | 5.15 | 0.259 | 6 | 3.60 | 0.425 |
| 8 | 6.50 | 0.292 | 7 | 4.98 | 0.505 |
| 9 | 8.20 | 0.330 | 8 | 6.25 | 0.577 |
| 10 | 9.90 | 0.365 | 9 | 7.80 | 0.651 |
| 11 | 11.75 | 0.400 | 10 | 9.40 | 0.723 |
| 12 | 13.75 | 0.431 | 11 | 11.12 | 0.795 |
| 13 | 15.95 | 0.466 | 12 | 13.05 | 0.870 |
| 14 | 18.36 | 0.500 | 13 | 14.85 | 0.931 |
| 15 | 20.85 | 0.532 | 14 | 16.87 | 1.001 |
| 16 | 23.42 | 0.567 | 15 | 18.50 | 1.08 |
| 17 | 26.00 | 0.600 | 16 | 20.07 | 1.15 |
| 18 | 28.70 | 0.631 | 17 | 21.45 | 1.19 |
| 19 | 31.20 | 0.660 |  |  |  |
| 20 | 33.45 | 0.685 |  |  |  |
| 21 | 35.10 | 0.706 |  |  |  |
| 22 | 35.80 | 0.710 |  |  |  |

## TABLE III

## OBSERVED DATA

| Date: December 29, 1955 <br> Barometric Pressure: 30.56 in. Hg <br> Room Temperature: $70^{\circ}$ <br> Specific Gravity manometer oil: . 85 <br> ASME nozzle size: 2 in. <br> Flow meter area: 14.9 sq in. $(12.5 \%)$ |  |  | Date: December 30, 1955 <br> Barometric Pressure: 29.75 in. Hg <br> Room Temperature: $70^{\circ}$ <br> Specific Gravity manometer oil: 0.85 <br> ASME nozzle size: 2 in。 <br> Flow meter area: 7.45 sq in. $(6.25 \%$ ) |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| ```Varidrive Speed rpm``` | $\begin{gathered} \Delta P_{N} \\ \left(n \circ z z l_{e}\right) \\ \text { in. oil } \end{gathered}$ | $\begin{aligned} & \Delta P_{M} \\ & (\text { meter }) \\ & \text { in. } \mathrm{H}_{2} \mathrm{O} \end{aligned}$ | Varidrive Speed $\frac{\mathrm{r} p \mathrm{~m}}{100}$ | $\begin{gathered} \Delta P_{\text {NI }} \\ (\text { nozzIe }) \\ \text { in. oil } \end{gathered}$ | $\begin{aligned} & \Delta P_{M} \\ & \text { (meter) } \\ & \text { in. } \mathrm{H}_{2} \mathrm{O} \end{aligned}$ |
| 4 | 1.58 | 1.25 | 5 | 2.35 | 3.32 |
| 5 | 2.45 | 1.56 | 6 | 3.19 | 3.92 |
| 6 | 3.58 | 1.96 | 7 | 4.32 | 4.80 |
| 7 | 4.65 | 2.30 | 8 | 5.40 | 5.60 |
| 8 | 6.05 | 2.71 | 9 | 6.70 | 6.60 |
| 9 | 7.48 | 3.10 | 10 | 8.05 | 7.56 |
| 10 | 9.00 | 3.45 | 11 | 9.50 | 8.40 |
| 11 | 10.42 | 3.80 | 12 | 11.52 | 9.70 |
| 12 | 12.35 | 4.23 | 13 | 13.30 | 10.80 |
| 13 | 14.10 | 4.60 | 14 | 15.05 | 11.75 |
| 14 | 16.00 | 5.01 | 15 | 16.60 | 12.85 |
| 15 | 17.60 | 5.30 | 16 | 18.50 | 14.05 |
| 16 | 20.95 | 5.98 | 17 | 20.05 | 15.05 |
| 17 | 23.25 | 6.40 | 18 | 21.00 | 15.60 |
| 18 | 24.80 | 6.68 |  |  |  |

TABLE IV

## CALCULATED DATA

Barometric Pressure: 14.50 psi
Room Temperature: $70^{\circ} \mathrm{F}$
Flow meter area: . 855 sq ft ( $100 \%$ area)
Nozzle discharge coefficient: 0.993
Air density: $0.0739 \mathrm{lb} / \mathrm{cu}_{\mathrm{ft}} \mathrm{ft}$
Air Viscosity: $3.79 \mathrm{X} 10^{-7} \mathrm{Ib} \mathrm{sec} / \mathrm{sq} \mathrm{ft}$

| $\begin{aligned} & \triangle \mathrm{P}_{\mathrm{N}} \\ & \mathrm{psi} \end{aligned}$ | Y* | $\stackrel{W}{1 \mathrm{~b} / \mathrm{sec}}$ | $\begin{gathered} Q \\ \text { cfs } \end{gathered}$ | $\begin{aligned} & \mathrm{V} \text { * } \\ & \text { fps } \end{aligned}$ | $\begin{aligned} & W_{S T P}^{*} \\ & 1 b^{*} / \mathrm{sec} \end{aligned}$ | $\underset{\mathrm{psi}}{\triangle \mathrm{P}_{\mathrm{Mm}^{\prime}}^{*}}$ | $\Delta \underset{\mathrm{psi}}{\mathrm{P}_{\mathrm{Mt}}}{ }^{*}$ | $\%$ diff |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| . 0861 | . 997 | . 170 | 2.30 | 2.69 | . 183 | . 00685 | . 00662 | 3.5 |
| . 157 | . 994 | . 230 | 3.12 | 3.65 | . 248 | . 00935 | . 00896 | 3.5 |
| . 250 | . 989 | . 287 | 3.89 | 4.55 | . 309 | . 0119 | . 0112 | 6.2 |
| . 352 | . 986 | . 340 | 4.61 | 5.40 | . 366 | . 0144 | . 0132 | 8.2 |
| . 487 | . 982 | . 398 | 5.40 | 6.32 | . 428 | . 0168 | . 0155 | 8.4 |
| . 636 | . 975 | . 452 | 6.13 | 7.17 | .487 | . 0192 | . 0176 | 9.1 |
| . 794 | . 969 | . 502 | 6.80 | 7.96 | . 540 | . 0217 | . 0195 | 11.1 |
| . 951 | . 964 | . 546 | 7.40 | 8.66 | . 588 | . 0238 | . 0213 | 11.7 |
| 1.07 | . 959 | . 576 | 7.80 | 9.12 | . 620 | . 0255 | . 0224 | 13.8 |
| 1.09 | . 958 | . 582 | 7.90 | 9.24 | . 626 | . 0257 | . 0227 | 13.2 |
| *Y is nozzle expansion factor |  |  |  |  |  |  |  |  |
| $V$ is flow velocity through meter |  |  |  |  |  |  |  |  |
| $W_{S T P}$ is weight rate of flow at standard temperature and pressure <br> $\mathrm{P}_{\mathrm{Mm}}$ is measured pressure drop across meter |  |  |  |  |  |  |  |  |

TABLE V

## CALCULATED DATA

Barometric Pressure: 14.30 psi
Room Temperature: $72^{\circ} \mathrm{F}$
Flow meter area: 0.427 sq ft ( $50 \%$ Area)
Nozzle discharge coefficient: . 993
Air Density: $0.0732 \mathrm{lb} / \mathrm{cu}^{\mathrm{f}} \mathrm{t}$
Air Viscosity: $3.80 \times 10^{-7} \mathrm{lb} \mathrm{sec} / \mathrm{sq} \mathrm{ft}$

| $\begin{gathered} \Delta \mathrm{P}_{\mathrm{N}} \\ \mathrm{psìi} \end{gathered}$ | Y* | $\begin{gathered} \mathrm{W} \\ \mathrm{lb} / \mathrm{sec} \end{gathered}$ | $\stackrel{\mathrm{Q}}{\mathrm{cfs}}$ | $\begin{aligned} & V^{*} \\ & f p s \end{aligned}$ | $\begin{aligned} & W_{S T P}^{*} \\ & I \mathrm{~b} / \mathrm{P}_{\mathrm{sec}}^{*} \end{aligned}$ | $\underset{\mathrm{psi}}{\Delta \mathrm{P}_{\mathrm{Mm}^{*}}}$ | $\begin{aligned} & \Delta \mathrm{P}_{\mathrm{Mt}} \\ & \mathrm{psi} \end{aligned}$ | $\begin{gathered} \% \\ \text { diff. } \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| . 0505 | . 998 | . 130 | 1.78 | 4.17 | . 138 | . 0101 | . 0102 | -1 |
| . 0782 | . 997 | . 161 | 2.20 | 5.16 | . 171 | . 0127 | . 0127 | 0 |
| . 110 | . 995 | . 192 | 2.62 | 6.14 | . 202 | . 0153 | . 0151 | 1.3 |
| . 153 | . 994 | . 225 | 3.07 | 7.20 | . 239 | . 0182 | . 0177 | 2.8 |
| . 192 | . 993 | . 253 | 3.46 | 8.11 | . 269 | . 0208 | . 0200 | 4.0 |
| . 239 | . 991 | . 280 | 3.82 | 8.95 | . 297 | . 0235 | . 0220 | 6.8 |
| . 289 | . 989 | . 306 | 4.18 | 9.80 | . 325 | . 0260 | . 0241 | 7.9 |
| . 341 | . 987 | . 334 | 4.56 | 10.7 | . 354 | . 0286 | . 0263 | 8.75 |
| . 400 | . 984 | . 360 | 4.92 | 11.5 | . 382 | . 0312 | . 0283 | 10.2 |
| . 517 | . 979 | . 406 | 5.54 | 13.0 | . 431 | . 0362 | . 0320 | 13.1 |
| . 568 | . 977 | . 435 | 5.94 | 13.9 | . 462 | . 0389 | . 0342 | 13.7 |
| . 615 | . 975 | . 442 | 6.03 | 14.2 | . 469 | . 0415 | . 0349 | 18.9 |
| . 658 | . 974 | . 457 | 6.24 | 14.6 | . 485 | . 0429 | . 0359 | 19.5 |

*See TABLE IV

TABLE VI

## CALCIJLATED DATA

| Barometric Pressure: 15.00 psi <br> Room Temperature: $70^{\circ} \mathrm{F}$ <br> Flow meter area: . 1035 sq ft ( $12.5 \%$ Area) <br> Nozzle discharge coefficient: . 993 <br> Air density: $0.0763 \mathrm{lb} / \mathrm{cu}_{\mathrm{ft}}^{\mathrm{ft}}$ <br> Air viscosity: $3.79 \times 10^{-7} \mathrm{lb} \mathrm{sec} / \mathrm{sq} \mathrm{ft}$ |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\triangle \mathrm{P}_{\mathrm{N}}$ | Y* | $\begin{gathered} \mathrm{W} \\ \mathrm{lb} / \mathrm{sec} \end{gathered}$ | $\begin{gathered} 0 \\ c f s \end{gathered}$ | $\begin{gathered} V^{*} \\ \mathrm{fps} \end{gathered}$ | $\begin{gathered} W_{S T P} \\ \mathrm{ni} / \mathrm{sec} \end{gathered}$ | $\underset{\mathrm{psI}}{\Delta \mathrm{P}_{\mathrm{Mm}}}{ }^{*}$ | $\underset{\mathrm{psi}}{\Delta \mathrm{P}_{\mathrm{Mt}}}$ | $\begin{gathered} \text { \% } \\ \text { diff. } \end{gathered}$ |
| . 0482 | . 998 | . 129 | 1.69 | 16.3 | . 131 | . 0451 | . 0417 | 8.1 |
| . 0746 | . 997 | . 161 | 2.11 | 20.5 | . 163 | . 0563 | . 0505 | 11.5 |
| . 109 | . 995 | . 177 | 2.32 | 22.4 | . 179 | . 0708 | . 0551 | 28.5 |
| . 142 | . 994 | . 221 | 2.77 | 26.8 | . 225 | . 0830 | . 0660 | 26.0 |
| . 184 | . 993 | . 250 | 3.28 | 32.2 | . 252 | . 0978 | . 0768 | 27.6 |
| . 228 | . 992 | . 280 | 3.67 | 35.5 | . 284 | . 112 | . 0873 | 28.6 |
| . 274 | . 991 | . 308 | 4.04 | 39.0 | . 313 | . 124 | . 0960 | 29.2 |
| . 318 | . 988 | . 330 | 4.45 | 43.0 | . 335 | . 137 | . 106 | 29.6 |
| . 376 | . 985 | . 356 | 4.67 | 45.1 | ' . 361 | . 152 | . 111 | 37.0 |
| . 430 | . 983 | . 381 | 5.10 | 49.3 | . 387 | . 166 | . 121 | 37.0 |
| . 487 | . 981 | . 406 | 5.32 | 51.4 | . 412 | . 181 | . 126 | 43.5 |
| . 537 | . 979 | . 425 | 5.57 | 53.8 | . 431 | . 191 | . 132 | 44.6 |
| . 640 | . 976 | . 462 | 6.05 | 58.5 | ! 468 | . 216 | . 144 | 49.8 |
| . 710 | . 972 | . 485 | 6.36 | 61.5 | . 492 | . 231 | . 151 | 53.0 |
| . 756 | . 971 | . 499 | 6.53 | 63.1 | . 506 | . 241 | . 155 | 55.5 |
| *See TABLE IV |  |  |  |  |  |  |  |  |


| Barometric Pressure: 14.60 <br> Room Temperature: 700 F <br> Flow meter area: . $0517 \mathrm{sq} \mathrm{ft}(6.25 \%$ Area) <br> Nozzle discharge coefficient: . 993 <br> Air density: $.0744 \mathrm{lb} / \mathrm{cu} \mathrm{ft}$ <br> Air viscosity: $3.79 \times 10^{-7} \mathrm{Ib} \mathrm{sec} / \mathrm{sq} \mathrm{ft}$ |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\underset{\mathrm{psi}}{\Delta P_{N}}$ | Y* | $\begin{gathered} W \\ \mathrm{lb} / \mathrm{sec} \end{gathered}$ | $\begin{gathered} \text { Q } \\ \text { cfs } \end{gathered}$ | $\begin{aligned} & V^{*} \\ & \text { fps } \end{aligned}$ | $\begin{aligned} & W_{S S P}{ }^{*} \\ & 1 \mathrm{~B} / \mathrm{sec} \end{aligned}$ | $\underset{\mathrm{psi}}{\Delta \mathrm{P}_{\mathrm{Mm}}{ }^{*}}$ | $\begin{aligned} & \Delta P_{M i t} \\ & p s i \end{aligned}$ | $\begin{gathered} \% \\ \text { diff. } \end{gathered}$ |
| . 0722 | . 998 | . 156 | 2.10 | 40.7 | . 162 | . 120 | .100 | 20 |
| . 0975 | . 995 | . 180 | 2.42 | 46.8 | . 187 | . 141 | . 108 | 30.8 |
| . 132 | . 994 | . 210 | 2.82 | 54.6 | . 218 | . 173 | . 134 | 28.4 |
| . 164 | . 993 | . 234 | 3.15 | 61.0 | . 243 | . 202 | . 150 | 34.6 |
| . 205 | . 992 | . 261 | 3.51 | 68.0 | . 271 | . 238 | . 167 | 42.5 |
| . 246 | . 991 | . 286 | 3.85 | 74.5 | . 296 | . 272 | . 183 | 48.6 |
| . 291 | . 989 | . 311 | 4.18 | 81.0 | . 323 | . 303 | . 199 | 52.2 |
| . 353 | . 987 | . 342 | 4.60 | 89.0 | . 356 | . 349 | . 219 | 59.4 |
| . 408 | . 984 | . 366 | 4.92 | 95.2 | . 381 | . 390 | . 234 | 71.0 |
| . 461 | . 981 | . 389 | 5.23 | 101 | . 405 | . 424 | . 248 | 71.0 |
| . 510 | . 979 | . 440 | 5.92 | 114 | . 457 | . 464 | . 280 | 65.8 |
| . 568 | . 978 | . 467 | 6.28 | 121 | . 486 | . 507 | . 298 | 70.0 |
| . 628 | . 975 | . 484 | 6.50 | 126 | . 504 | . 563 | . 310 | 81.5 |
| *See TABLE IV |  |  |  |  |  |  |  |  |



Figure 8. Pressure Drops Across Meter Versus Rates of Flow For 100\% and 50\% Flow Area


Figure 9. Pressure Drops Across Meter Versus Rates of Flow For $12.5 \%$ and $6.25 \%$ Flow Area


Figure 10. Pressure Drops Across Meter Versus Pressure Drops Across Nozzle


## DISCUSSION OF RESULTS

Figure 8 shows that for rates of flow up to approximately $0.6 \mathrm{lb} / \mathrm{sec}$ when using the total flow area, the viscous air flow meter has a pressure drop across it which is directly proportional to the rate of flow. Above this critical flow rate, transition to turbulent flow can be seen to occur in that the curves become parabolic. Figures 8 and 9 show that for the runs using less than the full area, the transition occurs at correspondingly lower flow rates. The graphs on the logarithmic paper, Figures 10 and 11 , show that while the meter is operating in the viscous region it has pressure drops across it which bear a square root relationship (slope equal to $1 / 2$ ) to the pressure drops across the nozzle but on reaching the transition region the slope gradually changes from $1 / 2$ to 1 . For the run using one-sixteenth of the total area, Figure 11 shows graphically that the viscous meter at higher flow rates is operating exactly like a nozzle, in that the slope is 1.

The theoretical pressure drops calculated using the general flow equation are found to agree closely with the measured pressure drops while the flow is predominantly laminar. A value of "k" of 80 from Table I for an isosceles triangle and a side length "d" of . 00608 ft were used in the calculation. This side length was computed by taking the average of the sum of the lengths of the sides of the triangular passage used in the meter.

## CHAPTER VIII

## SUMMARY AND CONCLUSIONS

The purpose of this investigation was to determine the flow capacity of a viscous air flow meter and to seek correlation between theoretical and experimental results. The results show that for flow rates up to approximately $0.6 \mathrm{lb} / \mathrm{sec}$ or 8 cfs a linear relationship exists between pressure drop across the meter and the rate of flow. Above this flow rate the meter can still be used equally as well as a nozzle or orifice by calibrating it against a standard nozzle.

The critical velocity of flow above which laminar flow no longer exists was found to be about $9 \mathrm{ft} / \mathrm{sec}$ for this particular meter. This corresponds to a Reynolds Number of approximately 350 if the mean side length of the triangular passage is used as the length dimension or of approximately 540 if the length of the longest side is used.

The calculated theoretical pressure drops for the flows below the critical velocity were found to agree closely with the actual measured drops when the general flow equation was used. The pressure drops calculated using Poisseuille"s equation with the "hydraulic mean radius" substituted for the radius of the tube divided by two, were found to be $78 \%$ greater than those found using the general equation. This comparison is shown in Appendix $A$. For flow in the laminar or viscous region the use of the "hydraulic mean radius" parameter is thus seen to introduce large errors. The "hydraulic mean radius" is useful only for
computing pressure drops when the boundary layer is thin as in turbulent flow.

The correlation between the theoretical and the experimental results was much better then had been expected since the passages were not truly isosceles trianges, no particular dynamic design precautions were taken, no effects of relative humidity were considered, and the pressure taps were external to the flow passages.

The use of a positive displacement rotary lobe blower caused the flow to be extremely pulsating even though some smoothing was accomplished by the use of a muffler. The nozzle is particularly sensitive to pulsations. Since the velocity of flow through the viscous element was found from the volumetric rate of flow determined by the pressure drops across the nozzle, it is believed that much of the devim ation between the theoretical and measured pressure drops across the meter was introduced by the inaccuracy of the flow measurement. Further investigation might be undertaken using either a steady source of flow such as a centrifugal blower or adequate smoothing tanks in order to eliminate inaccuracies due to flow measurement by means of a nozzle. The flow meter might also be calibrated using water as the fluid. A weighing tank and stop watch could then be used to measure the flow.

Binder, R.C., Fluid Mechanics, (3rd edition, Prentice-Hall, Inc., New York, 1955).

Goldsmith, Herbert, The Linear Flow Element As A Precision Steam Flow-Meter, National Instrument Laboratories Inc., (Riverdale, Md., 1955), (mimeographed).

Husain, Syed Vicar, The Design and Calibration of a Viscous Flow Meter, (unpublished Masters Report, Oklahoma A. and M. College, May, 1955).

Lamb, Horace, Hydrodynamics, (5th edition, Cambridge University Press, 1924).

Purday, H.F., An Introduction to the Mechanics of Viscous Flow, (Dover Publications, Inc., 1949).

Rouse, Hunter and Howe, J.W., Basic Mechanics of Fluids, (John Wiley and Sons, Inc., 1953).

Von Mises, Richard, Theory of Flight, (lst edition, McGraw-Hill Book Company Inc., 1945).
"Displacement Compressors, Vacuum Pumps and Blowers", ASME Power Test Codes, (1939).
"Flow Measurements", Part 5., ASME Power Test Codes, (1949).
"Vol-O-Flow Meter for Accurate Gas Flow Measurement", National Instrument Laboratories, Inc.

## APPENDIX A

The use of Poiseuille's equation for flow in a tube modified by the parameter "hydraulic mean radius", can be shown to give results 78\% greater than the values given by the general flow equation as follows:

As explained earlier, Poisseuille's equation is

$$
\Delta P_{P}=\frac{8 L \mu_{Q}}{\pi r^{4}}
$$

or since $Q=A V=\pi r^{2} V$ then

$$
\Delta P_{P}=\frac{8 M V L}{r^{2}}
$$

The hydraulic mean radius " $m$ " is defined as cross-sectional area wetted perimeter

It is seen that for a circular tube of radius "r",
or

$$
\begin{gathered}
m=\frac{\pi r^{2}}{2 \pi r}=\frac{r}{2} \\
r=2 \mathrm{~m}
\end{gathered}
$$

With the introduction of this parameter, Poisseuille's equation becomes

$$
\Delta P_{P}=\frac{2 \mu V L}{m^{2}}
$$

The general flow equation yields upon integration

$$
\Delta P_{G}=\frac{k \mu v L}{d^{2}}
$$

where "k" is determined by the laminar velocity distribution in a particular cross-section flow passage, and "d" is either the least depth of section (ie: the shortest side of a rectangular passage or the minor diameter of an eliptical passage) or the side length in case of an isosceles triangle。

Forming a ratio of the two equations gives:

$$
\frac{\Delta P_{P}}{\Delta P_{G}}=\frac{\frac{2 \mu V I}{m^{2}}}{\frac{k \mu V I}{d^{2}}}=\frac{2}{k} \frac{d^{2}}{m^{2}} \quad \text { and upon }
$$

substituting the values for the triangular passages used in the viscous flow meter

$$
\begin{array}{r}
\frac{\Delta P_{P}}{\Delta P_{G}}=\frac{2}{80} \frac{(.00608)^{2}}{(.00072)^{2}}=1.78 \\
\Delta P_{P}=1.78 \mathrm{X} \Delta P_{G}
\end{array}
$$

The following sample calculations are for the first line of Table V. which was the run using $50 \%$ of the total flow area of the meter. The conversion of the pressure drops in inches of oil and inches of water to pounds per square inch is not shown.

The weight rate of flow was calculated using the pressure drop across the nozzle by means of the following equation as found in the ASME Test Codes:

$$
\begin{aligned}
W=.688 \mathrm{ACY} \sqrt{\rho \triangle P_{N}} & =.688 \mathrm{X} \pi \mathrm{X} .993 \mathrm{X} .998 \sqrt{.073 \mathrm{X} .0505} \\
& =.130 \mathrm{lb} / \mathrm{sec}
\end{aligned}
$$

where $A$ is the area of the nozzle throat in square inches, $C$ is the coefficient of discharge for the nozzle, $Y$ is the expansion factor determined by the pressure ratio upstream and downstream of the nozzle, $\rho$ is the air density in $1 \mathrm{~b} / \mathrm{cu} \mathrm{ft}$ and $\Delta P_{N}$ is the pressure drop across the nozzle in psi.

The volumetric rate of flow was found by dividing the weight rate of flow by the density.

$$
Q=\frac{W}{\varrho}=\frac{.130}{.0732}=1.78 \mathrm{cfs}
$$

The velocity of flow through the meter was found by dividing the volumetric rate of flow by the area of the flow meter.

$$
V=\frac{0}{A}=\frac{1.78}{.427}=4.17 \mathrm{fps}
$$

The theoretical pressure drop across the meter was calculated by the following equation:

$$
\begin{aligned}
\Delta P_{M t}=\frac{80 \mu V I}{d^{2} \times 144} & =\frac{80 X}{}-\frac{3.80 \times 10^{-7} \times 4.17 X}{(.00608)^{2} \times 144} \times 433 \\
& =.0102 \mathrm{psi}
\end{aligned}
$$

where $\mu$ is the absolute viscosity of the air in $l \mathrm{~b}$ sec $/ \mathrm{ft}^{2}, \mathrm{~V}$ is the mean velocity of flow, in fps, $L$ is the length of the flow passage in ft, $d$ is the mean side length of the triangular passage in ft.

The weight rate of flow at standard conditions was found by multiplying the flow rate at test condition by the ratios of viscosities, pressures and temperatures as follows:

$$
W_{S T P}=W_{t} \times \frac{\mu_{t}}{\mu 60} \times \frac{14.7}{P_{t}} \times \frac{\mathrm{Tt}}{520}=1.30 \times-\frac{3.80}{3.74} \times \frac{1.47}{1.43} \times \frac{532}{520}=1.38 \mathrm{cfs}
$$

The absolute viscosity was found using the following equation where $t$ is in degrees $F$ :

$$
\begin{aligned}
\mu_{t} & =(3408+5.483 \mathrm{t}) \times 10^{-10} \\
& =(3408+5.483 \times 72) \times 10^{-10} \\
& =3.80 \times 10^{-7} 1 \mathrm{~b} \mathrm{sec} / \mathrm{sq} \mathrm{ft}
\end{aligned}
$$

The per cent differences between the actual pressure drop and the theoretical pressure drop was calculated as follows:

$$
\begin{aligned}
\% \text { diff } & =\frac{\Delta P_{\mathrm{Mm}}-\Delta P_{\mathrm{Mt}}}{\Delta P_{\mathrm{Mt}}} \times 100 \\
& =\frac{.0101-.0102 \times 100}{.0102} \times-1 \%
\end{aligned}
$$

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