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# THE IMPACT OF CREATIVITY-FOSTERING MATHEMATICS INSTRUCTION ON STUDENT SELF-EFFICACY AND MOTIVATION

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THE IMPACT OF CREATIVITY-FOSTERING MATHEMATICS INSTRUCTION
ON STUDENT SELF-EFFICACY AND MOTIVATION

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BY THE COMMITTEE CONSISTING OF

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# Abstract

Mathematics educators and researchers have described creativity as an essential part of mathematics, yet little research has been done to study the effects of fostering creativity in the tertiary mathematics classroom. This dissertation explores the impact of creativity-fostering instruction on student self-efficacy and motivation for mathematics in three parts. The first part presents a framework for characterizing creativity-fostering mathematics instruction (CFMI) which is used to study evidence of CFMI in association with qualitative changes in student self-efficacy for proving. The theoretical development and initial testing of a new instrument for measuring self-efficacy for proving is also outlined. The second part explores student problem posing as one particular instructional tool for fostering mathematical creativity. Through an illustrative case study of three students' experience problem posing, this study demonstrates the impact problem posing had on these students' motivation toward mathematics. The final part describes a large-scale quantitative study of CFMI, student self-efficacy, and student motivation. For this, two new instruments were developed for measuring CFMI and creative self-efficacy for mathematics. Limitations in the data collected constrained the study of the impact of CFMI on pre- to post-semester *change* in student self-efficacy and self-motivation. However, these methods demonstrate validity and reliability of the instrument used and provide a model for future study of the impact of CFMI on student self-efficacy and self-motivation.

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# Abbreviations

| Abbreviation | Term |
|---|---|
| 5PS | Five principles survey |
| AMTM | Academic motivation toward mathematics |
| CFA | Confirmatory factor analysis |
| CFMI | Creativity-fostering mathematics instruction |
| CSEM | Creative self-efficacy for mathematics |
| EFA | Exploratory factor analysis |
| PCA | Principle component analysis |
| PLOC | Perceived locus of causality |
| SDT | Self-determination theory |
| SEM | Structural equation modeling |
| SEP | Self-efficacy for proving |
| SLT | Social learning theory |

# Chapter 1

# Introduction

## 1.1 Motivation for this study

Students entering university often view mathematics as a series of formulas and processes to memorize. Even when instructors *describe* creative problem solving and creative discovery as a part of mathematics, students rarely *experience* mathematics as a creative discipline (Shoenfeld, 1989). Yet, to professional mathematicians, those researching and applying mathematics, creativity is considered one of the most important aspects of their work (Borwein, Liljedahl, & Zhai; 2014). This contrast illustrates a fundamental challenge to the instruction of a subject in which the discovery and formalization of ideas has occurred over centuries. However, the relative discrepancy between the way a subject is taught and the way it is experienced by professionals may be particularly problematic within mathematics.

More than any other academic subject, mathematics is known for its anxiety-inducing effects in large proportions of the population. Mathematics anxiety — or feeling of tension, apprehension, or fear that interfere with math performance (Ashcraft, 1988) — impacts as much as 25% of four-year college students and 80% of community college students (Beilock & Willingham, 2014). While researchers frequently approach mathematics anxiety from an emotional perspective studying negative beliefs and feelings regarding mathematics (Hembree, 1990), a cognitive perspective can offer further understanding of the sources of mathematics anxiety. Wilenski (1993) defined the term *epistemological anxiety* to refer to "a feeling, often in the background, that one does not comprehend the meanings, purposes, source or legitimacy of the mathematical objects one is manipulating and using" (1993, p. 172). Epistemological anxiety may stem from a lack of opportunities for students to deeply and creatively engage mathematics. Without such opportunities, students can fail to gain a deep understanding of the relationship between mathematical ideas. To relieve the frustration of not being able to gain this kind of relational understanding, students often "switch to instrumental understanding of being able to perform the requisite procedures…but [with] a sense of underlying doubt" (Tall, 2013, p. 127).

Approaches for managing mathematics anxiety have included reducing exposure to negative math attitudes, expressive writing, reappraisal and re-framing (Maloney, 2012; Beilock, 2015). While each of these techniques have been shown to reduce mathematics anxiety to some degree, these approaches are largely deficit oriented (Adiredja & Zandieh, 2020). They focus on trying to avoid, reduce, redirect, or suppress the power of negative thoughts, which may not be as effective as developing patterns of positive thoughts that preempt negative thinking. From the perspective of cognitive psychology, Bandura (1997) explained:

> Human thought can be positively proactive as well as avoidantly reactive. Thought control from unwanted trains of thoughts involves self-attraction to desired thoughts rather than solely self-distraction from unwanted thoughts. The same is true for diversionary activities. There is a marked difference between keeping busy to avoid thinking about unpleasant matters and engrossment in activities for the enjoyment they provide. In forming associations between different types of thoughts, the principles of associative learning suggest that forward cueing of positive diversions by unwanted thoughts should be more reliable than backward cueing of unwanted thoughts by positive diversions. (p. 147)

In other words, motivating and developing patterns of desired thinking in contexts that tend to trigger anxiety may more effectively reduce mathematics anxiety than simply treating or working with thoughts caused by anxiety. Although it may be helpful, even necessary, for instructors and students to recognize anxiety and its symptoms, we should also consider how to change or eliminate the source of the anxiety, e.i. the beliefs and habits underlying it. According to Bandura, "The most powerful way of eliminating intrusive ideation is by gaining mastery over threats and stressors that repeatedly trigger the perturbing trains of thought" (p. 147). Thus, in a very basic sense, mathematics students (or anyone using mathematics) must gain mastery for generating new understanding and approaches when facing an unsolved problem.

What then *positively* characterizes the knowledge, skills, habits, and beliefs that allow students to engage a new mathematical idea or problem, without fear, but with confidence in their ability to creatively navigate the unknown? To answer this, we must address underlying assumptions about the way in which new ideas are developed or learned. For this dissertation, I take a primilary Vygotskian view of learning, considering the development of habits and skills for making meaning as the result of a process of *socialization*, i.e. the long-term process by which personal habits and traits are shaped through participation in social interactions with particular demand and reward characteristics (Vygotsky, 1978). Resnick, in a discussion of the nature of mathematics, described the rationale behind this view:

> [T]he reconceptualization of thinking and learning that is emerging from the body of recent work on the nature of cognition suggests that becoming

> a good mathematical problem solver—becoming a good thinker in any domain—may be as much a matter of acquiring the habits and dispositions of interpretation and sense-making as acquiring any particular set of skills, strategies, or knowledge. If this is so, we may do well to conceive of mathematics education less as an instructional process (in the traditional sense of teaching specific, well-defined skills of items of knowledge), than as a socialization process. If we want students to treat mathematics as an ill-structured discipline—making sense of it, arguing about it, and creating it, rather than merely doing it according to prescribed rules—we will have to socialize them as much as to instruct them. (p. 58)

This perspective explicates the rationale and motivation underlying my work on this dissertation: (1) that notion that mathematics education can involve the socialization of students into an experience of mathematics as an ill-structured, **creative discipline**, and (2) that there is still relatively little understood about the student outcomes of teaching mathematics as such. Motivated by my own experience teaching mathematics, I have chosen to focus on two important and potentially necessary conditions for students to experience mathematics creatively and without anxiety:

1. Students must see themselves as capable of arriving at new solutions or insights of their own.
2. Students must possess a drive to work on a mathematics problem over a period of time, despite difficulty or uncertainty.

These two conditions can be described or summarized by the terms *self-efficacy* and *self-motivation*, respectively. Research indicates that both self-efficacy and self-motivation strongly *mediate* the ways in which students engage and experience mathematics. Self-efficacy — or one's belief in their capabilities to produce given outcomes (Bandura, 1997) — influences,

> the courses of action people choose to pursue, how much effort they put forth in given endeavors, how long they persevere in face of obstacles and failures, their resilience to adversity, whether their thought patterns are self-hindering of self-aiding, how much stress and depression they experience in coping with taxing environmental demands, and level of accomplishment they realize. (p. 3)

For this reason, low self-efficacy may serve as an underlying driver of mathematics anxiety. Conversely, at least some level of self-efficacy is necessary to engage mathematics creatively. Otherwise, "if people believe they have no power to produce results, they will not attempt to make things happen" (p. 3).

Self-motivation, though closely related to self-efficacy, involves acting in ways that are self-initiated through ownership or identification with values or reasons for acting (Ryan & Deci, 2000). Studies demonstrate an association between self-motivation and greater enjoyment, positive coping styles, expending greater effort, as well as other indicators of psychological well-being (Ryan & Deci, 2000). Moreover, longitudinal studies suggest even those with high self-efficacy do not experience these benefits *unless* they pursue and attain goals for which they are self-motivated (Sheldon & Kasser, 1998). Thus, the importance of student self-motivation and self-efficacy for mathematics is manifest through their impact on how students engages and experiences mathematics, especially in creating a sense of internal reward and satisfaction of their efforts.

Not surprisingly, studies of student self-efficacy and self-motivation for mathematics (Pajares & Kranzler, 1995; Ryan & Deci, 2000) demonstrate both constructs are negatively correlated with mathematics anxiety. Yet, in all the effort put into characterizing and identifying the problem of mathematics anxiety (which is important, necessary work), relatively little research clearly characterizes the environments by which students develop self-efficacy and self-motivation in upper-level mathematics courses. This dissertation contributes to research on mathematics education by specifically examining existing instructional practices designed to foster or enable the creative engagement of mathematics students, i.e. creativity-fostering mathematics instruction (CFMI). Then, by studying the relationship between CFMI on student self-efficacy and self-motivation, I explore the central question:

> What is the impact of creativity-fostering instruction on student self-efficacy and self-motivation toward mathematics?

Summarized below is an outline of the three parts of this dissertation that investigate this question.

## 1.2 Outline of dissertation

### 1.2.1 Part I

As an initial study of the impact of creativity-oriented instruction on student self-efficacy, Chapter 2 presents a framework for characterizing and measuring aspects of *creativity-fostering mathematics instruction* (CFMI) using Sriraman's (2005) five principles for fostering mathematical creativity. Through classroom observation, online surveys, and interviews with four students in an introduction to proofs course, I studied evidence of CFMI and its effects on student self-efficacy. Using a survey instrument designed to measure student self-efficacy for proving, the Self-efficacy for Proving Scale, changes of student self-efficacy were triangulated with qualitative coding of sources of self-efficacy described by students. This illustrated associations between four principles of CFMI and changes in student self-efficacy for proving, along with two instances where the combined use of principles may have provided students greater opportunities for building self-efficacy for proving. These results contribute to the mathematical creativity literature the idea that creativity-fostering mathematics instruction can have direct impact on the ways students gain self-efficacy toward mathematics, providing both theoretical and practical characterizations of this link.

Parts of Chapter 2 have been published in the Journal of Mathematical Behavior (Regier & Savic, 2019).

### 1.2.2 Part II

Chapter 3 focuses on *problem posing,* the activity of students authoring their own mathematical questions, as one particular tool for CFMI described in the literature (e.g. Silver, 1997). As an illustrative case-study, this project utilized self-determination theory (Deci & Ryan, 2000) to explore the motivational development of three students who participated in problem posing in the same course studied in Chapter 3. The experience of these students illustrated several ways in which problem posing can create new, more integrated habits of motivational regulation. However, the differences of these cases highlights a need for more explicit discussion of the relationship between problem posing and other educational goals. To the literature on problem posing, these findings contribute illustrations of how problem posing *can* impact the long-term motivational orientation of students, as well as a contextualized explanation of the conditions necessary for problem posing to foster the integration of motivational regulation in mathematics.

### 1.2.3 Part III

Chapter 4 presents a large-scale ($n \approx 250$) quantitative study of student perception of CFMI, self-efficacy, and motivation of students enrolled in 12 upper-level undergraduate mathematics courses. A pre/post semester correlational research design was developed utilizing: (1) a new instrument for measuring student perception of CFMI, (2) a new instrument for measuring general creative self-efficacy for mathematics, (3) the Self-efficacy for Proving Scale developed in Part I, and (4) an adaptation of the Academic Motivation Toward Mathematics Scale. The available data did not show a statistically significant relationship between CFMI and post-semester creative self-efficacy or self-efficacy for proving, though these tests were not able to control for prior student self-efficacy. Creative self-efficacy for mathematics mediated the effect of social aspects of CFMI on self-efficacy for proving, and self-efficacy for proving mediated the effect of individual aspects of CFMI on creative self-efficacy for mathematics. The models studied highlight the varying roles social and individual aspects of instruction may play on the development of student self-efficacy for proving, creative self-efficacy, and creative mathematical identity. Implications for educators and researchers are discussed at the end of Chapter 4, along with directions for future research. In particular, part III contributes to the research of undergraduate mathematics education new methods for efficiently studying the impact of CFMI on the development of student self-efficacy. Chapter 5 provides a brief summary of the main results of this dissertation and discussion of their significance for mathematics education.

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# Chapter 2

# Part I: How teaching to foster mathematical creativity may impact student self-efficacy for proving

## 2.1 Introduction

For over a century mathematics and mathematics education researchers have endeavored to better understand the role of creativity in mathematical thinking and problem solving (Haavold, 2016; Mann, 2006; Poincaré, 1946). Since mathematics education and research are both ripe with the potential for creativity, a research stream has developed offering various ways to foster creativity in mathematics (Zazkis & Holton, 2009; Leikin, 2007, 2014; Watson & Mason, 2005; Shriki, 2008; Sriraman, 2005; Savic, Karakok, Tang, El Turkey, & Naccarato, 2017; Zaslavsky, 1995). At the same time, there is little research studying the impact of fostering mathematical creativity on individual cognitive constructs such as self-efficacy (Mathisen & Bronnick, 2009).

This chapter focuses on studying the impact of fostering creativity on student self-efficacy in mathematics due to the powerful role self-efficacy plays in mediating student achievement (Pajares & Kranzler, 1995; Randhawa, Beamer, & Lundberg, 1993). In a variety of contexts, people with high self-efficacy have been shown to experience increased motivation, engagement, and resilience to adversity (Bandura, 1997); demonstrate increased use of strategic thinking; manage their time better; are more persistent; and are less likely to reject correct solutions (Bouffard-Bouchard, 1990; Bouffard-Bouchard, Parent, & Larivee, 1991). Furthermore, self-efficacy has been identified as a better predictor of mathematical performance than prior ability or experience within mathematics (Pajares & Kranzler, 1995; Siegel, Galassi, & Ware, 1985).

Tan et al. (2011) demonstrated several connections between secondary students' experience in the classroom and domain-general creative self-efficacy but called for more research designs to explore how various classroom factors impact self-efficacy. We add to this literature by studying self-efficacy for mathematical proving, a little-studied construct in mathematics education research (Iannone & Inglis, 2010). This serves to highlight the ways in which fostering creativity in tertiary mathematics education may influence student self-efficacy for mathematics.

The purpose of this article is two-fold. Firstly, we introduce a theoretically-based methodology for both quantitatively and qualitatively studying the effect of creativity-fostering instruction on student self-efficacy for proving. Secondly, we study how the sources of self-efficacy afforded by the use of creativity-fostering instruction can serve to build student self-efficacy for proving. In viewing student self-efficacy for mathematical proving as a domain-specific creative trait (Baer, 1998), this research also sheds light on what classroom environments may best foster creative capacities of mathematics students at the tertiary level.

## 2.2 Theoretical background

### 2.2.1 Mathematical creativity in the classroom

From Leikin et al.'s (2009) and Silver's (1997) use of Torrance's (1966) constructs of creativity, to use of Wallas' four stages of creativity (1926) in pure mathematics (Hadamard, 1945), mathematical creativity has often been measured or described in relation to the individual (either the student or mathematician). At the same time, there is considerable variation in the ways individual mathematical creativity has been defined (Mann, 2006). While recognizing the multi-faceted nature of creativity (Moore-Russo & Demler, 2018), for the purpose of this study, we define mathematical creativity as one's process of offering new solutions or insights that are new or unexpected for the student with respect to their mathematical background. This definition is based on Savic, El Turkey et al. (2017), influenced by the perspectives of Liljedahl and Sriraman (2006). One can further categorize this definition as relative to the individual (Beghetto & Kaufman, 2007), process-oriented (Pelczer & Gamboa Rodriguez, 2011), and domain-specific (Baer, 1998) mathematical creativity. While this definition is appropriate for studying actions for fostering students' mathematical creativity, it is methodologically difficult to measure the extent to which a student's actions or behavior are creative, i.e. new or unexpected to the creator. Thus, in this study, rather than measure what is creative, we leave that judgment to the student, looking instead at their self-efficacy for proving (Section 2.2.2).

To better understand the role of mathematical creativity in the tertiary classroom, we were interested in studying observable actions 1 instructors use to foster creativity in the mathematics classroom. Zazkis and Holton (2009) provided an overview of a range of ways instructors can foster creativity at tertiary mathematics, citing Zaslavsky's (1995) work on open-ended problems, Shriki's (2008) work on students creating new definitions, Leikin's (2007) multiple-solution tasks, and Watson and Mason's (2005) learner-generated examples. Zazkis and Holton (2009) further detailed their own tasks and classroom actions, including starting a graph theory course with an open-ended question about edges and vertices, or understanding by asking students to calculate the ratio of perimeter to the "segment connecting the 'centre' to the 'corner' " (p. 361) of squares and triangles. More recently, Leikin (2014) studied how multiple-solution tasks were used in a university course for prospective teachers, observing discussions in class that focused on the multiple geometric proofs constructed by the students and commenting on the misnomer of tasks requiring a single solution.

In researching mathematical creativity in the K-12 classroom, Sriraman (2005) conjectured five principles that can be "applied in the everyday classroom to maximize the potential for creativity in the classroom" (p. 26). These principles are derived from the mathematical creativity literature along with mathematicians' experiences of creating and publishing their results. While these principles are by no means exhaustive of the ways in which an instructor can foster mathematical creativity in the classroom, they provide a general framework for studying a range of instructor actions that can be viewed as creativity-fostering. For each of the five principles, described below, Sriraman offered the general role of each principle in relation to students, along with several explicit examples of applications of each principle. To reflect this projects' grouping of the principles, Sriraman's (2005) original ordering of the five principles has been changed below.

#### 2.2.1.1 Aesthetic Principle

The aesthetic principle involves conveying a sense of appreciation of the beauty of mathematics, especially in the discovery of new ideas and in connecting disparate ideas in mathematics. This may also include "real-world problem selection and the careful 'staging' of the discovery moment by the teacher" (p. 28). Reference to the beauty, ellegance, or efficiency of a solution to a problem may be examples of ways in which the aesthetic principle is enacted.

#### 2.2.1.2 Free Market Principle

The free market principle derives from the idea the professional mathematicians often take risks exploring unknown paths or when presenting their ideas to the scrutiny of experts. Likewise, it is important for students to experience engaging in this process. Thus, teachers should provide an environment where it is safe to develop their own ideas and present them to others.

#### 2.2.1.3 Scholarly Principle

The Scholarly Principle views creativity as contributing to or challenging known paradigms and extending the existing body of knowledge. This can involve engaging students in debate with one another's ideas and challenging one another's approaches to problems. According to Sriraman, "teachers should embrace the idea of creative deviance as contributing to the body of mathematical knowledge, and they should be flexible and open to alternative student approaches to problems" (p. 28). The scholarly principle also may be enacted through encouraging generalization, providing problem-posing opportunities to build of other's ideas, or promoting understanding and discussion of problem design.

#### 2.2.1.4 Gestalt Principle

The Gestalt Principle is concerned with providing students with enough freedom of time and space to work on problems for extended periods. Sriraman states the students should be given "suitably challenging problems over a protracted time period, thereby creating the opportunities for the discovery of an insight and to experience the euphoria of the 'Aha! moment" (p. 26).

This principle derives its name from Gestalt psychology's characterization of creativity as a four-stage process (Wallas, 1926) involving:

- preparation: working on a problem for considerable amounts;
- incubation: putting the problem aside, allowing for the use of the unconscious drive to create;
- illumination: experience of insight or 'Aha' moment; and
- verification: reflection and determination whether this solution is correct.

Although often ignored in the classroom (Sriraman, 2005), this model explains well mathematicians' own descriptions of their creative process (e.g. Hadamard, 1945; Poincaré, 1946).

#### 2.2.1.5 Uncertainty Principle

Since mathematics at the professional and practical levels is full of uncertainty and ambiguity, students should be exposed to the uncertainty and difficulty of creating mathematics. According to Sriraman, this requires instructors to "provide affective support to students who experience frustration over being unable to solve a difficult problem" (p. 28). Additionally, students should be exposed to the history of ideas or problems from mathematics and science that took centuries to develop or solve. The uncertainty principles also can involve student problem-posing and exploration of their own approaches to problems.

### 2.2.2 Self-efficacy for proving

Bandura (1997) defined perceived self-efficacy as "beliefs in one's capabilities to organize and execute the courses of action required to produce given attainments" (pg. 3) and is a central construct of social cognitive theory. According to social cognitive theory, self-efficacy is domain-specific: one's self-efficacy will vary depending both the task in question and context one is working in (Bandura, 1997). Therefore, we define self-efficacy for proving as one's beliefs in their own capabilities to organize and execute the actions required to produce justifiable mathematical proofs.

In this dissertation, we view proof at the university introductory-level as a logical justification of a mathematical statement (Weber, 2005). However, we consider this definition within a broader view of proof as "as a series of ideas and insights rather than [just] a sequence of formal steps" (Hanna & Villiers, 2012). Therefore, proving refers to the process of justifying a mathematical statement or conjecture (Hsieh, Horng, & Shy, 2012). Previous research on proving at the tertiary level has focused on students' cognition, including logical skills necessary for proving (Selden & Selden, 1995), types of reasoning and problem-solving processes used by students (Weber, 2005), and proof schemes (Harel & Sowder, 1998) or "arguments [one] uses to convince [themselves] and others of the truth or falseness of a mathematical statement" (Housman & Porter, 2003, p. 140).

There is less research studying the impact of affective aspects of students' experience, such as self-efficacy, in proving (Iannone & Inglis, 2010). Selden, McKee, and Selden (2010) studied affect in relation to the behavioral schemas used by students in proving. Iannone and Inglis (2010) studied university students' self-efficacy for proving and their proving abilities, finding a positive correlation. However, their study does not explain any cause for such a correlation. Selden and Selden (2010) identified the importance of self-efficacy in supporting students' three useful actions in proving: exploring, reworking an argument in the case of a suspected error or wrong, and validating a completed proof. They hypothesized that few students attempt these actions due to a lack of self-efficacy.

Furthermore, Selden and Selden (2013) hypothesized that self-efficacy is an important part of "much of creative cognition in general" (p. 4). In fact, one early model of creativity, the componential model of creativity (Amabile, 1983), described both domain-relevant skills (including knowledge about the domain) as well as creativity-relevant skills (i.e. exploring new cognitive pathways, suspending judgment) as components contributing to creative performance. The componential model of creativity has been widely used to study domain-general creative self-efficacy (Tan et al., 2007, 2011; Sangsuk & Siriparp, 2015). However, according to Beghetto and Karwowski (2017), there is a need for more robust, domain-specific measurements of creative self-efficacy. Thus, we add to the literature by studying self-efficacy for proving as a domain- and task-specific measurement of one's self-efficacy for a potentially creative task, mathematical proving.

In this study, we are interested in better understanding how students can gain self-efficacy for proving through classroom experience. Bandura (1997) outlined four primary sources of self-efficacy information: enactive experiences, vicarious role-modeling, verbal persuasions, and physiological reactions, described below.

#### 2.2.2.1 Enactive experiences

Enactive experiences refer to one's own successes in accomplishing a given task. An example of an enactive experience could be one's experience of solving a difficult proof and successfully explaining it to someone else. According to Bandura (p. 8), enactive experiences are often the most powerful source of self-efficacy since one's own experiences provide a reliable indication of future ability. It is of note that these sources do not directly influence self-efficacy; it is through cognitive processing and reflective thought that these sources of information are "selected, weighted, and integrated into students' self-efficacy judgements" (p. 79). However, any change in one's self-efficacy operates through one or more of these four sources.

#### 2.2.2.2 Vicarious role-modeling

Vicarious role-modeling involves observation of someone else's competencies through which, by self-comparison, the observer bases judgments of their own ability. Observing someone else present their own proof could provide some indication of the observer's own proving ability, to the degree that the observer identifies, or sees themselves similar to, the presenter. The observer could gain self-efficacy from that observation, and perhaps be more confident approaching a similar or future problem.

#### 2.2.2.3 Verbal persuasion

Verbal persuasion involves direct verbal appraisal of one's ability by someone else. Telling a student, "I believe you have the resources to prove this" can serve as some indication of ability but depends on the credibility of the persuader and the degree to which such "positive appraisal is within realistic bounds" (p. 101). Verbal persuasion is usually considered less reliable than previous two sources, since it conveys beliefs that are described rather than observed.

#### 2.2.2.4 Physiological reactions

Physiological reactions can include feelings of strength and stamina, or physical or emotional stress or fatigue. Feeling well rested or comfortable in the classroom are indicators of ability, while feeling of stress or fatigue are "signs of vulnerability to dysfunction" (p. 106).

### 2.2.3 Research question

Drawing on the above characterization of the five principles for fostering creativity and the four sources of self-efficacy, our two research questions are:

1. What methodologies are appropriate for measuring creativity-fostering instruction in the classroom and changes in student self-efficacy for proving, specifically in an introduction-to-proofs course?
2. How may teaching actions for fostering creativity (categorized by Sriraman's five principles) impact how students gain or lose self-efficacy for proving in an introduction-to-proofs course?

## 2.3 Methods

The data for this study was collected in a Discrete Mathematic course at a research-intensive university in the central USA. This course utilized inquiry-based learning, defined by the Academy of Inquiry-Based Learning as: "a form of active learning in which students are given a carefully scaffolded sequence of mathematical tasks and are asked to solve and make sense of them, working individually or in groups" (Academy of Inquiry-Based Learning, n.d.). In this course, online surveys (2.3.1), classroom observations (Section 2.3.2), and student interviews (2.3.3) were all collected and compared to explore the relationship between presence of the Five principles in the classroom and changes in student self-efficacy. This served to: (1) corroborate evidence of the use of the five principles in the classroom and (2) document changes in students' self-efficacy. We piloted these methods in an 8-week summer session. After analyzing pilot data and making changes, primary data was collected in the fall semester of 2017, taught by Dr. F. Each class was videotaped, 23 students took a beginning-of-semester online survey, 21 students took an end-of-semester online survey, and 4 students were interviewed.

### 2.3.1 Online surveys

Two online survey instruments were designed to measure student experience of the five principles and their self-efficacy for proving. This survey, named the Five Principles Survey (5PS), consisted of ten questions, two per principle, collecting students' experience of each principle. The 5PS can be found in Appendix A.1. The questions were randomized for each student. The 5PS was given at the end of the semester (Survey 3), rating their experience in Discrete Mathematics that semester The Self-efficacy for Proving Scale (SEP Scale) consisted of three specific proving statements in which students were asked to rate their confidence of five subtasks related to proving each statement. The researchers closely followed Bandura's recommendations for constructing self-efficacy scales (2006), as well as Beghetto and

Karwowski's (2017) suggestions for using and refining domain-specific measurements of creative self-efficacy. This is distinct from the common practice of using general statements of mathematical ability or experience to measure "self-efficacy" (Iannone & Inglis, 2010).

Table 2.1: Selden and Selden's (2013) continuum of problem difficulty

| Type | Description | Example |
|---|---|---|
| 1. Very-routine | Can easily be proven from previous results | Prove or disprove: If $n$ is an odd integer, then $n^4 - n$ is even. |
| 2. Moderately-routine | Requires formulating and proving a lemma (or trick) that is relatively easy to notice, formulate, and prove | Prove or disprove: The inequality $2^x \geq x + 1$ is true for every positive real number x. |
| 3. Non-routine | Requires formulating and proving a lemma (or trick) that is hard to notice, formulate, and prove | Prove or disprove: There does not exist a real number $x$ for which $x^4 < x < x^2$. |

The three proving subtasks served to orient students to the domain of interest (mathematical proving at the undergraduate level), each designed to be accessible to students with no prior experience with formal proof. Hammack's (2013) proof textbook served as the main source for these problems. To gauge gradation of challenge of the tasks, both researchers characterized each statement on the continuum of problem difficulty offered by Selden and Selden (2013, pp. 303–305), shown in Table 2.1. For each of the three surveys, the first author selected one proving statement of each difficulty type, ensuring that these statements were not included in class or in any assigned homework, quiz, or exam.

For each proving statement, to obtain a measure of student's abilities related to the process of proving and to provide context for students potentially unfamiliar with formal proving, students were asked rate their confidence in their ability (on a scale of 0% confidence to 100%) to do the following five subtasks related to proving:

- Understand and informally explain why a statement is true or false.
- Explore new ideas to come up with ways to start your proof.
- Use various representations (numbers, pictures, tables, words) to structure your thinking
- Formally write out and justify each step of your proof.
- Examine your proof for accuracy and identify any missing steps.

These subtasks were derived from Hsieh et al.'s (2012) Exploration-Proving spectrum (EP-spectrum) for proving, which centers on the concept of justification and, at the same time, considers proof in the classroom more broadly "as the product of a spectrum of activities starting with exploration, and progressing to the stages of conjecturing, informal explanation, and justification" (pg. 288). This both aligned with our perspective of proving (Section 2.2.2) and provided a more detailed characterization of the courses of actions required to produce justifiable proofs.

Three versions of this survey, each with distinct proving statements, were given, at the beginning (Survey 1), middle (Survey 2), and end (Survey 3) of the semester. All nine proving statements are provided in Table 2.5. All three online surveys are provided in Appendices A.1-A.3.

### 2.3.2 Classroom observations

One class session from the fall semester was randomly chosen from the beginning (first five weeks), middle (weeks 6–10), and end (weeks 11–15) of the fall semester. Each session was viewed by both researchers and coded for both explicit and implicit evidence of instructor use of each of the five principles of creativity. We defined explicit evidence of the principles to be overt verbal instruction aligned with one or more principle, and we defined implicit evidence to be situations or interactions from which the instructor's influence for one or more principles could be inferred. Since we chose the theoretical framing of the five principles prior to coding, this is similar to the provisional coding described by Saldaña (2009) in his fundamental qualitative research manual. Differences in codes were discussed until arriving at an agreement for each coded action. The researchers chose not to calculate Cohen's Kappa for interrater reliability and instead "rel[ied] on intensive group discussion and simple group 'consensus' as an agreement goal" (Harry, Sturges, & Klingner, 2005, p. 6; as cited by Saldaña, 2009, p. 28). This data collection method served mainly to provide direct evidence of which principles, if any, were used in the classroom.

### 2.3.3 Student interviews

Four students (with pseudonyms Fannie, Fred, Frank, and Francisca) from the Fall semester (taught by Dr. F) participated in a post-semester interview. Each student was asked questions about their classroom experience, their relative confidence in proving now, and how they gained confidence in their class. A full list of the questions is provided in Appendix A.4. Additionally, the students from the fall semester were given 30 min at the beginning of the interview to prove the same three statements used in the end-ofsemester self-efficacy survey. Then they were asked to describe their proving process and indicate whether they had previously proved seen a proof of these statements. This was to further validate the self-efficacy survey.

Each interview was transcribed, removing words such as "like" and "um" and "and stuff" for readability. Each interview was coded, once for explicit or implicit evidence of the instructor's use of Sriraman's (2005) five principles for maximizing creativity, and again for evidence of Bandura's (1997) four sources of self-efficacy. For coding the student interviews, we defined explicit evidence of the principles to be as overt teacher actions aligned with one or more principle described by student. This frequently included direct mention of the instructor. We defined implicit evidence to be situations cited by the students from which instructor influence for one or more principles could be inferred. Sources of self-efficacy were coded only when a potential source (enactive experiences, vicarious role-modeling, verbal persuasion, physiological reaction) was described in relation to a change in student confidence in proving. Codes were compared, and any discrepancies were discussed until a consensus was reached. Intersections between the five principles and self-efficacy codes were then analyzed. In our analysis, we used the general term association to discuss such intersections between one or more of the five principles and source of self-efficacy. This follows the general definition of an association as "a connection between ideas or things," distinct from the statistical use of the term. However, we note that in our analysis an association was drawn only when the language used by the student describes causality between the coded constructs.

To illustrate our coding, consider the following student response to the question, "What do you think contributed to your gaining confidence in proving?":

> Fred: When we would do the peer discussions in class, I would see how somebody else did it, and then I would be like "okay that makes a lot of sense, like how you, kind of played around with, and how you got to where you went." And usually after class, I would have a break for six hours in between my next class. So, I would, a lot of times, go back and I can redo the whole, like two homework problems.

This was coded for explicit use of the scholarly principle due to the instructor's use of peer discussion in class, specifically where students (in this case, Fred) built off and evaluated one another's ideas (in this case, "somebody else"). This response was also coded for gaining self-efficacy via vicarious role-modeling, since the Fred attributed working with his peers and observing their proving process as contributing to his gaining confidence for proving. Thus, an association between the scholarly principles and gaining self-efficacy via vicarious role-modeling could be inferred from this response.

## 2.4 Results

To begin to answer our research questions, we first describe the evidence of instructor enactment of the five principles from online surveys, classroom observations, and student interviews (Section 2.4.1). Then, we discuss the evidence from online surveys and student interviews of a pre/post-semester change in students' self-efficacy (2.4.2). Finally, having observed consistent the enactment of the five principles in class as well as a change in student self-efficacy, we present evidence from the student interviews of possible associations between the five principles and the ways students gained (or lost) self-efficacy for proving (2.4.3).

### 2.4.1 Evidence of instructor enactment of the five principles

Evidence of instructor use of all five principles were found in all three data groups. While the online surveys show common use of the five principles, we further investigated the classroom observations and student interviews in order to demonstrate concrete actions of teaching aligned with Sriraman's five principles.

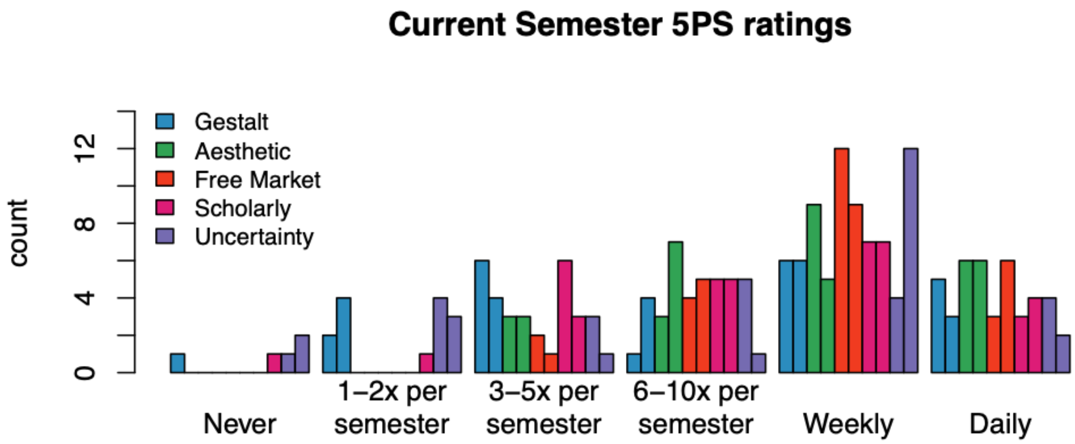


Figure 2.1: Student end-of-semester Five Principle Survey ratings

#### 2.4.1.1 Online surveys

Figure 2.1 shows a histogram of students' responses from the Five Principle Survey (5PS) given at the end of the semester. The median response for question 2, "How often did you experience the joy of arriving at a solution after working on a problem or proof for several days?" (Gestalt) was "6–10 times per semester." Question 7 (scholarly) and question 9 (uncertainty) had the same median responses. The remaining questions had a median response of "weekly."

This gave evidence that 79–100% of student responses described experiencing a given principle at least 3 times during the semester; 63–95% of student responses described experiencing a given principle at least 6 during the semester; and 42–74% of student responses described experiencing a given principle weekly or daily (see Table 2.2).

#### 2.4.1.2 Classroom observations

In the first of the three class periods that were coded, all five principles were observed. At the beginning of class, Dr. F and the teaching assistant (TA) discussed how they were giving students an opportunity to redo some of the homework problems. The TA had given feedback to some of these problems which had been turned in online through a learning management system by providing open-ended responses to students'

Table 2.2: Cumulative frequency of student rating for 5PS.

| | Gestalt | | Aesthetic | | Free Market | | Scholarly | | Uncertainty | |
|---|---|---|---|---|---|---|---|---|---|---|
| Freq of student experience | Q1 | Q2 | Q3 | Q4 | Q5 | Q6 | Q7 | Q8 | Q9 | Q10 |
| at least once | 0.95 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 0.95 | 0.89 |
| at least 3-5 times/sem | 0.89 | 0.79 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 0.95 | 0.79 | 0.79 |
| at least 6-10 times/sem | 0.63 | 0.63 | 0.84 | 0.84 | 0.89 | 0.95 | 0.68 | 0.79 | 0.68 | 0.74 |
| at least weekly | 0.58 | 0.42 | 0.68 | 0.53 | 0.74 | 0.68 | 0.47 | 0.53 | 0.42 | 0.68 |
| daily | 0.26 | 0.16 | 0.26 | 0.26 | 0.16 | 0.26 | 0.16 | 0.16 | 0.21 | 0.11 |

questions about their work, where they got stuck, etc. Several of these problems had been discussed the prior class period. Dr. F said that "we are giving you all a chance to redo some things, mainly because it is for you, not for us." This was coded for the explicit use of the Gestalt Principle for allowing students freedom of time and movement to foster "aha" experiences.

After this, Dr. F began responding to a student question about "if-then" statements, asking, " 'If x is an element of A, then x is an element of B.' Which [set theoretic] statement is that?" Students variously responded, "A union B," "A intersect B," and "A subset B." Without responding to the accuracy of any student responses, Dr. F said, "go to your notes." As students began agreeing "It's 'A subset B,' " Dr. F asked, "what is another truthful statement about this?" One student responded, "A equals B." Several students disagreed, which point Dr. F said, "Hold on, hold on, hold on. Why do you say, 'A equals B'?" This student commented, "Because if A is bigger than B, it's not contained in B. But if it's the same size as B, then they're equal," to which Dr. F responded, "In fact, 'A equals B' can be one case. There [are] many cases, and one of them is 'A equals B.' What about some others with elements?"

The above interaction was coded for implicit uses of the uncertainty, scholarly, and free market principles. The statement, "Go to your notes," was coded for use the uncertainty principle since the instructor, in not responding the accuracy of students' response, implicitly used students not knowing as an opportunity for them to be comfortable in finding their own answers to uncertainty. The instructor allowing students to debate with one-another in this interaction was coded for use of the scholarly, since this allowance helped engage students in challenging the validity of their own responses. Finally, we coded the engagement of a solution that was not fully correct ("In fact, 'A equals B' can be one case") for implicit use of the free market principle since engaging a potentially wrong solution and using it to explain how their thoughts fit into the bigger picture may have served to foster future risk taking more than focusing on the correctness of the students' response.

This led to a conversation of contrapositive, inverse, and converse statements. At one point, Dr. F incorrectly stated, "this statement is the inverse," to which a student corrected "[it's the] converse, I just looked it up." This was coded for implicit use of the scholarly principle, since this illustrated a norm running throughout the course –

students had access to all the notes, were repeatedly encouraged to refer to the notes, and were challenging mathematical authority to construct their own understanding from the notes. Toward the end of this discussion, Dr. F explained, "I believe this statement [if, then] is why we learn math. I believe that calculus, adding fractions, boils down to being logical. I believe this is why for 3500 years we have been learning math: to be more precise when we speak and talk." This was coded for explicit use of the aesthetic principle. Dr. F was conveying the beauty, elegance, and precision of mathematical communication. In the remainder of this class period, we coded one or more instances of each principle. In the second class, six instances of use of the free market and scholarly principles were coded, several of which came about as a result of the instructor offering Fannie the opportunity to lead the class discussion. This action was coded for explicit use of free market principles for his encouraging Fannie to take this risk. Fannie's response to this was, "oh this is so exciting," to which she proceeded to lead class discussion (with minor guidance from Dr. F) the remainder of the class. Two instances of scholarly principle were coded in the interaction with Fannie and the class, as well as another instance of the free market principle, for Dr. F creating a safe environment for which students could engage in this discussion.

In the third class, two instances of explicit use the free market principle were coded in relation to a course requirement (worth 5% of their total grade) called "Productive Failure." This requirement involved students presenting an experience where they failed in their proving process and explaining how it proved productive in the end. In response to a student question, "How do I encourage my friend to present a productive failure?", Dr. F said, "I make mistakes in lecture, and am getting better at talking about my own failures. Having a difficulty and talking to someone else about it relieves the weight." The counted instances of the coding for the five principles for all three class periods are shown in Table 2.3.

Table 2.3: Frequency of coding of three randomly-selected class periods

| Principle | Beginning | Middle | End |
|---|---|---|---|
| Gestalt | 2 | 1 | 1 |
| Aesthetic | 3 | 1 | 0 |
| Free Market | 3 | 6 | 3 |
| Scholarly | 3 | 6 | 2 |
| Uncertainty | 2 | 3 | 6 |

#### 2.4.1.3 Student interviews

The analysis of the student interviews illustrated a range of cases in which the students cited use of the five principles in the classroom. Table 2.4 summarizes frequency for which each principle was coded in the interview. For example, Francisca stated that she experienced "inherent curiosity" from the course because

> [Dr. F] would be like "so why did you do that?" And at first it was just like "I don't know cuz that's what you do." And it was like "no, why did you do that." And like that constantly asking over and over – "well why are you doing the thing that did?" – is the reason why.

This was coded as an uncertainty principle, since the instructor utilized questioning rather than attending to correctness in the classroom. More examples of Dr. F's utilization of the five principles through the student interviews are provided in Section 2.4.3.

Table 2.4: Frequency of coding of the five principles for each interview

| Principle | Fred | Fannie | Frank | Francisca |
|---|---|---|---|---|
| Gestalt | 5 | 3 | 1 | 4 |
| Uncertainty | 4 | 1 | 4 | 5 |
| Aesthetic | 0 | 0 | 1 | 0 |
| Scholarly | 4 | 2 | 1 | 5 |
| Free Market | 1 | 2 | 1 | 1 |

### 2.4.2 Evidence of change in student self-efficacy

In this subsection, we provide the evidence of changes in student self-efficacy for proving in the online surveys and student interviews. Comparison of the beginning-of-semester and end-of-semester online surveys provide evidence that 16 out of 19 students gained self-efficacy for proving during the course. From student interviews, the results of the task-based component and their corresponding self-efficacy scores were compared. Finally, we review the evidence from the interviews of the sources of self-efficacy that were experienced by these students. Here, a potential source of self-efficacy was identified when a student acknowledged a change in their own self-efficacy in relation to one or more of the four sources of self-efficacy.

#### 2.4.2.1 Online surveys

For comparison of beginning, middle, and end-of-semester the SEP Scale ratings, each students' self-efficacy rating for a given proving statement was calculated for as the mean of the five subtask ratings for that statement. The mean and standard deviation of these ratings are shown Table 2.5. Then, each student's self-efficacy rating was calculated as the mean of the three proving statement ratings. As an example, on Survey 3, Fannie rated her self-efficacy for five subtasks (see Section 2.2.2) on the first proof statement were 100, 90, 80, 90, and 90, respectively, giving an overall self-efficacy rating of for the first proof statement of 90. Her self-efficacy for the remaining two proof statements were 82 and 90, giving an overall SEP Scale rating of 88.

Table 2.5: Mean and standard deviation of self-efficacy scores for each proof statement used

| Survey | Level | Statement | Mean | SD |
|---|---|---|---|---|
| 1 (n=23) | 1 | If $n$ is an odd integer, then $n^2 + 1$ is even | 64.09 | 15.64 |
| | 2 | If $a$, $b$, and $c$ are positive integers, and $ab$, $bc$, and $ac$, all have the same parity (are all even or all odd), then $a$, $b$, and $c$ all have the same parity. | 52.09 | 24.10 |
| | 3 | If $a$ and $b$ are integers, then $a^2 - 4b \neq 2$. | 57.57 | 21.88 |
| 2 (n=24) | 1 | If $n$ is an odd integer, then $n^2 - n$ is even. | 76.00 | 12.50 |
| | 2 | The inequality $2^x \geq x + 1$ is true for all positive real numbers $x$. | 74.17 | 14.36 |
| | 3 | There does not exist a real number $x$ for which $x^4 < x < x^2$. | 73.33 | 14.90 |
| 3 (n=21) | 1 | If $x, y \in R$, then $\vert x + y\vert \geq \vert x\vert + \vert y\vert$. | 84.67 | 10.13 |
| | 2 | If $n$ is an integer, then $1 + (-1)^n(2n - 1)$ is a multiple of 4. | 81.43 | 13.22 |
| | 3 | Every odd integer is the difference of two squares. | 77.81 | 16.50 |

The changes in student experience of the five principles and their self-efficacy can be seen in Figure 2.2, which plots the students end-of-semester vs. beginning-of-semester SEP Scale scores. The data from Fannie, Fred, and Frank are highlighted in square, triangle, and diamond respectively. We did not get a response from for Francisca on the final survey. The remaining un-interviewed students (16 out of $n = 19$) are shown in grey. As an example of one data points, we describe Fannie's data (the square data point). Her beginning-of-semester self-efficacy score was 42, plotted on the horizontal axis, and her end-of-semester self-efficacy scores was 88. The change in her self-efficacy rating, 46, is represented by the vertical distance from the square to the line $y = x$.

Several tests of the statistical significance of the changes in self-efficacy were conducted. Firstly, a one-tailed paired Wilcoxon signed ranks test was conducted ($p = 1.907 \times 10^{-5}$) giving evidence that the frequency distribution of end-of-semester SEP Scale scores are shifted to the right of the distribution of the beginning-of-semester scores. Next viewing the SEP Scale scores as a continuous variable (as the mean of 15, 11-point scales), we verified that the beginning- and end-of semester scores did not violate assumptions of normality. Then a paired two sample t-test was conducted showing a significant ($p = 3.626 \times 10^{-5}$) change in student self-efficacy ratings, with a 95% confidence interval for change in SEP Scale ratings of (14.54, 32.83).

#### 2.4.2.2 Student interviews student

Three of the four students interviewed took Survey 3 prior to their interview. Thus, we can also use the task-based portion of the student interviews to analyze the degree to

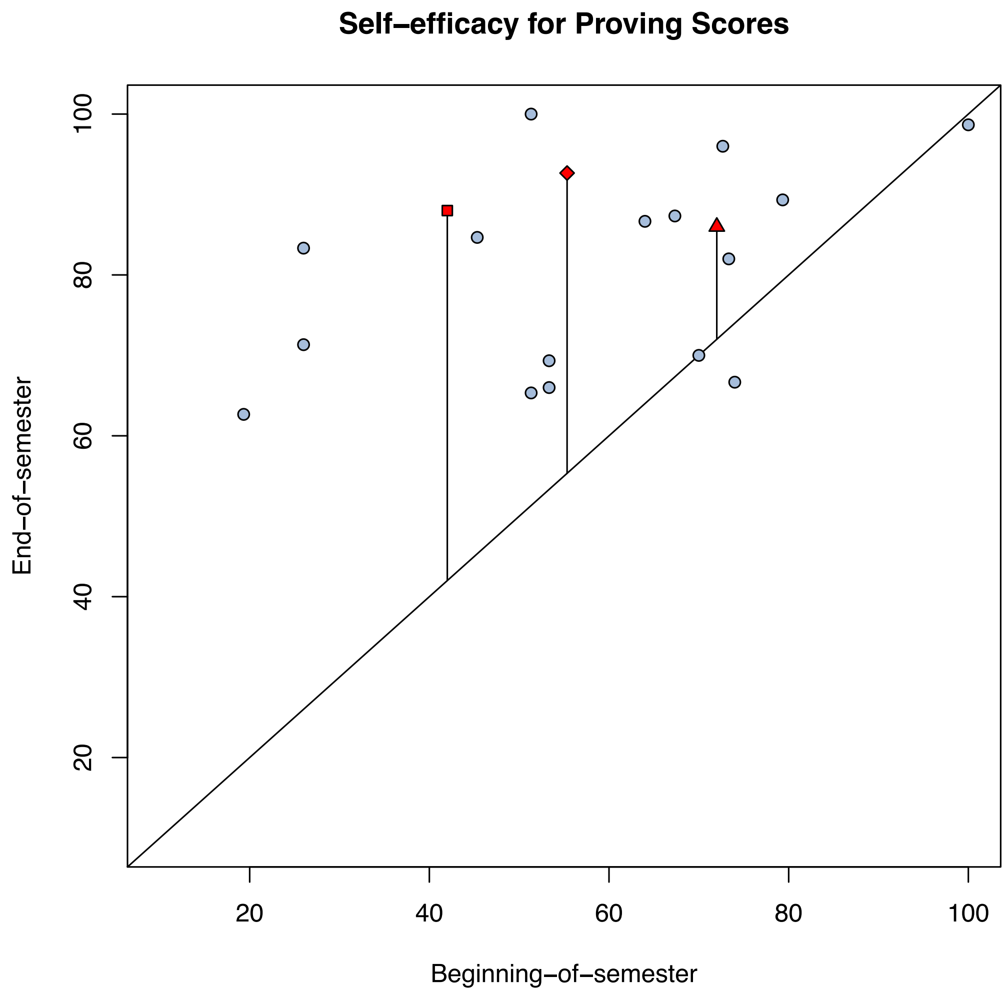


Figure 2.2: Student end-of-semester vs. beginning-of-semester SEP Scale scores

which these students' self-efficacy ratings (for the three proving statements on survey 3) reflected their proving abilities demonstrated on the actual tasks. Fannie rated her self-efficacy for proving the three statements as 90, 82, and 92 respectively, and Frank rated his self-efficacy for proving the three statements as 94, 94, and 90 respectively. Both Fannie and Frank proceeded to successfully prove all three statements. Fred rated his self-efficacy for proving the three statements as 84, 88, and 86 respectively, and proceeded to successfully solve the first statement. Fred did not finish proving the remaining statement, making several proof attempts, including a proof by induction on second, and an attempt at a proof by contradiction on the third. Frank was the only student who described having seen any of these proofs before (the first statement, the triangle inequality). The coding of the student interviews provided evidence that the four students interviewed experienced changes in self-efficacy via enactive experiences and vicarious role-modeling. Only one instance of verbal persuasion serving as a source of self-efficacy was coded, and no cases of physiological reactions serving as a source of self-efficacy were coded. The coding for each sources of self-efficacy are shown in Table 2.6. The frequency of coding for positive sources of self-efficacy are listed in each cell, along with the frequency of coding for negative sources of self-efficacy in parenthesis.

Below are examples of coding for both positive and negative sources of self-efficacy. When asked what helped her gain confidence for proving, Fannie responded:

> I think that having us do it on our own pretty much all the time was key. Because I don't think I would get the same, I don't think I would have had the same understanding of it if it was just presented to me rather than me figuring it out for myself, if that makes sense.

This was coded for evidence of enactive experiences (doing it "on our own pretty much all the time") contributing to her gaining confidence. Compare this with Fred's response to the question: "where did you struggle and what prevented you from gaining confidence?"

> I'd say like the beginning a semester… It took me till the middle of September to really devote a lot of time to this class, cuz at the beginning like I didn't think it was that challenging of a class. Then stuff started to ramp up, and I was kind of falling behind for a little bit, and like I wasn't that great at proving stuff, and like I said, at the first exam, I did. I felt super unconfident about it.

This was coded for enactive experiences (not devoting enough time to the class and "falling behind") serving as a negative source of self-efficacy information. More examples of sources of self-efficacy for the participating students are presented in the next section.

Table 2.6: Frequency of coding the sources of self-efficacy

| Source of Self-efficacy | Fred | Fannie | Frank | Francisca |
|---|---|---|---|---|
| Enactive Experiences | 3 (2) | 6 | 3 | 6 (2) |
| Vicarious Role-Modeling | 3 (1) | 4 | 2 | 2 |
| Verbal Persuasion | 0 | 0 | 0 | 1 |
| Physiological Reactions | 0 | 0 | 0 | 0 |

### 2.4.3 Evidence of associations between five principles and sources of self-efficacy in student self-efficacy

We did not have a large enough sample size (N=19) of students who took both pre- and post-semester surveys to discuss statistical correlations between use of the principles and increased student self-efficacy for proving; however, we studied the relationship between instructor use of the five principles and student-identified changes in self-efficacy by studying the intersections of the coding described in Sections 2.4.1.3 and 2.4.2.2. These comparisons are summarized in Table 2.7; each cell lists the students for which coding for instructor use a principle intersected with coding for source of self-efficacy (called an association), along with the frequency of that association in parenthesis. No coding for negative sources of self-efficacy intersected with instructor use of the five principles. The following subsections (2.4.3.1-2.4.3.4) include quotes from the four students for each of the above associations. Section 2.4.3.5 illustrates cases where students described themselves serving as potential role models for others. Throughout these sections, interview quotes coded for the **five principles** are in **bold**, and *sources of self-efficacy* are *italic.*

#### 2.4.3.1 Gestalt principle and enactive attainments

Three of the four students from the Fall semester described ways in which use of the Gestalt principle provided a positive source of self-efficacy through enactive attainments. For example, consider Fred's experience proving in the course:

> *Fred*: It's a roller coaster of a class. You reach points where you're so frustrated that you can't solve stuff, and then the satisfaction when you actually. . . You figure out how to do a **proof that you've been working on for a while**. There's really no more empowering feeling in the world. *You feel like you can do anything!* But yeah, it's the trials and tribulations. You'll struggle and then it's *figuring out how to use that struggle to achieve something,* in the future, using what you know doesn't work and like, "all right, this doesn't work. Let's try to think of something new that might work better."

This description shows evidence of "aha" experiences – the satisfaction of "figuring out

Table 2.7: Summary of associations coded between for five principle and sources of self-efficacy

| | Four sources of Self-efficacy | | | |
|---|---|---|---|---|
| Five Principles | Enactive Attainments | Vicarious Role-modeling | Verbal Persuasion | Physiological Reactions |
| Gestalt | Fred(1), Fannie(2), Francisca(2) | | | |
| Uncertainty | Fred(1), Fannie(1), Frank(1), Francisca(1) | | | |
| Aesthetic | | | | |
| Scholarly | | Fred(3), Fannie(2), Frank(1) , Francisca (1) | | |
| Free Market | | Fannie(2) | | |

how to do a proof that you've been working on for a while" – which we coded for implicit use of the Gestalt principle since the instructor was assigning problems that allowed or required this approach to proving. This segment was also coded for enactive experiences because Fred appeared to be empowered by his success in proving and considered using this experience in the face of future difficulties. Thus, struggling on difficult problems and eventually proving them contributed to Fred feeling that he could "do anything," serving as a strong indication of his future ability to prove.

#### 2.4.3.2 The uncertainty principle and enactive attainments

All four students from the fall semester described teacher actions for the uncertainty principle that were coded in association with increased self-efficacy via enactive attainment.

> *Interviewer*: How did the environment influence your learning to prove and you're gaining confidence in class?
> *Frank*: Confidence? [The fall semester course] helped me *see where a lot of pitfalls were,* and *be okay with that, but also learn to anticipate those.* The ability to anticipate those was something that was pretty valuable, I think.

Being helped to see to see where his mistakes in proving were, and be fine with them, was coded for implicit use of the uncertainty principle since Frank connected the classroom environment with ambiguity and uncertainty in the proving process. Additionally, his learning and being able to anticipate his mistakes were coded as enactive experiences.

Francisca described being challenged "every single step of the way, [this class] challenged your thinking and how you approached math." Then the interview shifted to how being challenged influenced self-efficacy.

> *Interviewer*: How do you think that contributed to your confidence in learning to prove?
> *Francisca*: At first, it was nerve-racking cuz I wasn't getting things right, and I wasn't understanding things. But over the semester and over time, I actually talked to a couple of people about this: it was like, *"you don't have to be right in this class, cuz no one's gonna be right." There's like no concept of being correct,* and once you take away the idea of being correct or being right, it makes your confidence level go up a lot more, cuz you're like, "I know that I did this, and this is what I accomplished, and so I should be proud of the work that I've accomplished."

Not having to be "right" and not considering what is the one "correct" way of doing things demonstrates implicit use of the uncertainty principle, helping students become comfortable with ambiguous, open ended, or ill-posed problems. Interestingly though, at least part of her realizing this came from her peers, evidence of vicarious role modeling. This quote shows how this shift in perspective helped Francisca reframe her own perspective of her accomplishments, allowing her experience served as a potential source of self-efficacy, not necessarily in the information directly, but the weight she gives to these or future accomplishments in making self-efficacy judgements.

#### 2.4.3.3 The scholarly principle and vicarious role-modeling

In the fall semester, all four students described teacher actions for the scholarly associated with positive sources of self-efficacy via vicarious role-modeling. The example given at the end of the Methods section is one such case. The following example, later in Fred's interview, was also coded for the scholarly principle and vicarious role-modeling:

> *Fred:* When we would do the **peer discussions in class**, I would see how somebody else did it, and then I would be like, "okay that makes a lot of sense, how do you played around with it, and how you got to where you went" ... I would, a lot of times, go back and ... redo the two homework problems. And *thinking of how the other person solved it, and then that really helped me foster ways of being more creative, as I've said, using other people [and] how other people's work is creative, as a stepping stone for how I could be more creative.*

The response in italics was coded as a source of self-efficacy because Fred have previously been asked, "What do you think contributed to your gaining confidence in proving?" and responded, "by becoming more creative."

#### 2.4.3.4 Free market principle and vicarious role-modeling

All four of the student interviews from the fall semester were coded for use of the free market principle. For Fannie, the free market principle was coded in association with gaining self-efficacy via vicarious role modeling.

> *Interviewer:* What in class contributed to your building confidence?
> *Fannie:* The **general environment of everyone not being afraid to fail**, and I think the productive failure thing kind of contributed to that. Just generally **understanding that my peers weren't going to judge me for doing something wrong** was really refreshing. That was nice. And definitely having that time to work with other people was really important, because everyone kind of had their own perspective or their own different take on the problem... *Someone next to you might have had like a different idea about it that's just as correct as yours.*

This was coded for implicit use of the free market principle because Fannie knew "her peers weren't going to judge her." It was also coded for the scholarly principle due to Dr. F allowing students to engage and understand one another's approaches to problems. Finally, because Fannie attributed her gaining confidence to being able to work with others without fear of judgment, this was coded for vicarious role-modeling.

Fannie also described the importance of "hearing other's thought processes" and of using one another's "individual strengths to come together to understand this problem and like make this proof" in relation to gaining self-efficacy for proving. Both these statements were both coded for the scholarly principle and gaining self-efficacy vicariously. However, immediately following this, she described implicit use of the free market principle through the way she experienced the environment of the class.

> *Fannie:* I also liked that **there wasn't any like super overpowering voices in the class**, because I think that might have just been a characteristic of the people in the class, or it might have been the environment... I'm not really sure. But I know that I get super intimidated when there's just one person that's constantly dominating the conversation and I think that would have made me much more hesitant to *speak up or present my proofs.* So that was kind of nice: really understanding from Day 1 that no one was going to the judge you for failure, that was a really important part of the class.

From the classroom observations, we saw Fannie's speaking up and presenting her own proofs in class as evidence her gaining self-efficacy from other students.

#### 2.4.3.5 The principles and vicarious influences toward others

Although we initially set out to code the interviews for sources of self-efficacy for students in the course, we noted two specific cases in which some principles may have encouraged students to provide sources of positive self-efficacy to other students outside the course.

> *Interviewer:* Is there anything else you gained from class?
> *Fannie:* I don't know. The ability to annoy my friends with math concepts. I was studying for my physics test the other day, and I went up to my friend, and was like "this is so cool" and it was one of the problems from my last test. I was like, "you've got to hear this. There's these things called trapezoid numbers, and they're so cool." And *I wrote it out on the chalk board*, and they're like, "okay." I'm like, "It's cool. Numbers are cool!" But I don't know. *I did gain a lot of confidence.* Ultimately that was the biggest thing. Because you know, at the beginning I was like "eh, I don't know." But, towards the end, I was like, *"I can prove things. I can do it!"*

Even Fannie showed gaining an appreciation of something new (trapezoid numbers), this was not coded for the use of aesthetic principle since there was no mention of her instructor. However, because she was compelled to explain it to her friends with confidence, becoming a potential source of self-efficacy to her friends, we coded this for vicarious influences toward others. Francisca had a similar experience.

> *Interviewer:* How long did you spend on homework?
> *Francisca:* So much time. I was like "oh it's a [sophomore level] course. It won't be…" Oh my God, so much homework, so much time. I would spend like hours. My roommates would come home, and I would be doing a problem, they'd go back to class and come back, and I'd still be doing the same problem. And they're like "why? We've been gone for two, three hours and you're doing the exact same thing." …They also thought everything that I talked about for the whole semester was just absolutely crazy. I would bring up all the terms that we would use in class like "productive failure" and all the other things, and they're just like "you're nuts." *I was like, "no no no."*

This showed that Francisca's experience of the Gestalt and free market principles may have contributed toward her speaking out to her roommates, even in the face of rejection of her ideas, demonstrating how the principles may have encouraged Francisca to become a potential source of vicarious role-modeling toward others.

## 2.5 Discussion

The first goal of this project was to develop methodologies for studying the relationship between teaching actions for fostering mathematical creativity and changes in student self-efficacy for proving. As we've demonstrated in Section 1, there is significant literature on fostering mathematical creativity, yet little research studying the effects of creativity-fostering on students (Tan et al., 2011). We suspect this is, in part, due to a lack of development of effective or efficient research methods for studying the impact of creativity-fostering instruction. The methodologies developed in this study are one step towards developing this kind of investigation. We also hope this work motivates and provides direction toward developing explicit characterizations of creativity-fostering instruction and their potential effects.

Our second goal was to develop a better understanding the possible effects of creativity-fostering instruction on student self-efficacy. We cannot claim that the whole class gained self-efficacy for proving, nor that the instructor's actions for fostering mathematical creativity in the classroom effected the whole class. However, we show, from the student interviews, the specific ways in which the students described gaining or losing self-efficacy in association with instructor use of the five principles.

### 2.5.1 Discussion of methodology

#### 2.5.1.1 Construction and implementation of online surveys

To investigate instructor actions for fostering mathematical creativity, we attempted to find an appropriate framework with teaching actions that can be observed and characterized. Sriraman's (2005) five principles, while not always providing specific examples of creativity-fostering teaching actions, gave us a theoretically-sound framework that was domain-specific and aligned with our definition of creativity. This framework proved applicable for our provisional qualitative coding, as well as for gathering evidence of student experience of creativity-fostering instruction via the online 5PS survey.

The 5PS provided evidence that the majority of students experienced each of the five principles at least 6–10 times per semester (see Section 2.4.1.1). However, in comparing student experience recorded in the 5PS with classroom observations (Figures 2.1 and 2.1), their self-reported experience of the principles was less frequent than what we recorded from class observations. This highlights the discrepancy between student experience and teacher-use of the principles. It is logical that students are not consciously aware of each time their instructor employs a principle in class. Neither need they be for it to impact them (Roediger, 1990). Thus, for future studies, we recommend rewriting the five principles survey in terms of instructor actions instead of student experience and administering this survey to both students and instructors.

If properly aligned with classroom observation protocols, such a survey may be a more efficient and reliable way to measure use of instructor actions in the classroom (Hayward, Weston, & Laursen, 2018).

The self-efficacy for proving survey (SEP Scale) appears to be a robust instrument for measuring self-efficacy for proving. The Cronbach's alpha reliability estimates of the SEP Scale were 0.92, 0.90, and 0.92 for Surveys 1, 2, and 3, respectively ( 0.9 is excellent), indicating that the SEP Scale is measuring one construct. For surveys 1 and 2, students rated lower mean self-efficacy for more difficult proving statements, providing evidence of discriminative validity. We also observed in the task-based interviews that higher student self-efficacy ratings corresponded with greater success in proving, an indication of predictive validity. Yet, we believe some improvements can be made to the SEP Scale; for example, the higher mean and standard deviation of ratings for proving statement 2 on survey 1 may be related to the task's word length. Thus, for future use of the SEP Scale, we recommend using proving statements that are brief, such as those used in survey 2 and 3. This will allow students with broad mathematical backgrounds to more easily and accurately understand and evaluate their abilities related to the task.

Separate from the statistical significance of the SEP Scale, we believe the construction of the SEP Scale warrants more consideration in the field of mathematical education research. The SEP Scale was created following Bandura's (2006) recommendations: "the construction of sound efficacy scales relies on a good conceptual analysis of the relevant domain of functioning. Knowledge of the activity domain specifies which aspects of personal efficacy should be measured" (p. 310). We attempted to construct our scale utilizing domain-specific literature about the proving process (e.g., Hsieh et al., 2012). Furthermore, the gradation of difficulty via Selden and Selden's (2013) continuum further allows the SEP Scale to capture a wide range of proving performances. Incorporating both research constructs provided us with a better, more precise understanding of self-efficacy than existing general self-efficacy scales in mathematics. For example, Iannone and Inglis (2010) utilized a self-efficacy for proving scale with statements such as "I am good at writing mathematical proofs." Earlier studies have shown that there are differences between algebra and analysis proofs (e.g., Dawkins & Karunakaran, 2016; Savic, 2017). Therefore, general statements about self-efficacy may not orient students to a reliable mathematical context for which they are gauging their own ability.

For future larger-scale quantitative studies, the pre- and post-SEP Scale and modified (as described above) end-of-semester student and instructor 5PS could be used to study the correlation between specific teaching actions and changes in student self-efficacy for proving. In particular, hierarchical linear modeling (Raudenbush & Bryk, 2002) could be used study relative or combined influence of specific teaching actions on changes in student self-efficacy for proving.

#### 2.5.1.2 Qualitative methodology via observations and interviews

The classroom observations provided a way to document instructor use of the five principles (see Section 2.4.1.2). We can see that, throughout the course, Dr. F utilized both explicit and implicit teaching actions that align with the five principles (see Table 2.2). The actions shown in Section 2.4.1.2 provide examples that further strengthen Sriraman's (2005) conjectured creativity-fostering in the classroom.

Some actions were also apparent due to the nature of the pedagogy. Dr. F utilized inquiry-based teaching, which may have played a significant role in how the scholarly and free-market principles were used. However, there are ways in which an instructor can use lecture-based pedagogy with a saturated emphasis on mathematical creativity. Omar et al. (2018) described one primarily lecture-based professor who used reoccurring assignments with open-ended questions and written reflection on their problem-solving process, assigning more grade-weight to the reflections. In this class, students transitioned into feeling "more like mathematicians" which may correspond to an increased sense of self-efficacy. We recommend for future studies to expand the methods used in this study for different pedagogical approaches.

One limitation of the methods used in this study is that we only considered creativity-fostering instruction within the classroom. According to Sriraman (2005), these principles "can be applied in the everyday classroom setting" (p. 26). However, we observed several cases in which factors outside of the classroom appeared to determine how students were influenced by the principles. For example, Dr. F's office hours appeared to have had an impact on Francisca's self-efficacy. When asked "Is there anything else Dr. F did that influenced your confidence for proving?" she replied:

> I think it was just how open he was. It was just easy to go in and approach, and like he was just like "if my doors open, just come in, and we'll talk about it for hours." It was just easy to make an appointment and like you could talk for an hour, you start about math and talk about something completely different. And that was really nice.

While this action was not coded for any of the five principles since it did not occur within the class, it illustrates how the instructor's office hours created a secondary environment where the principles may have been enacted one-on-one. Additionally, it demonstrates a context where verbal persuasion may have had a greater influence on building student self-efficacy due to the rapport Dr. F built with Francisca.

Francisca's interview also illustrated how working together with her peers outside of class may have influenced her self-efficacy. Francisca said being "challenged every single step of the way, challenged your thinking, and how you approached math" contributed to building her confidence. When asked "how?", she replied:

> At first, it was nerve-racking cuz I wasn't getting things right, and I wasn't understanding things. But over the semester and over time – I actually talked to a couple of people about this: it was like, "you don't have to be

> right in this class, cuz no one's gonna be right." There's like no concept of being correct, and once you take away the idea of being correct or being right, it makes your confidence level go up a lot more, cuz you're like "I know that I did this and this is what I accomplished, and so I should be proud of the work that I've accomplished [enactive attainment] rather than whether or not the work that I accomplished is 100% correct, and I'm 100% doing this by the book.

Again, this was not coded either the uncertainty or free market principles because it likely occurred outside of the class; however, this illustrates how norms related to use of these principles were likely enacted or reinforced through conversations students had outside the class. Therefore, the way students are assigned to work outside of class appears to play a role in how the principles influence students' self-efficacy. Other avenues to better understanding the role of the five principles play outside the class may include studying the type of problems assigned (uncertainty principle) and how problems are assigned, graded, or revised (Gestalt principle).

In the student interviews, there was only one instance (Francisca) of students describing gaining self-efficacy via verbal persuasion, and no instances of physiological reactions serving as a source of self-efficacy. This may be a result of the way interviews were conducted. The interview focused on students experience in class overall and did not explicitly focus on one-on-one interaction between student and instructor, nor the way students individually felt in class. We also did not have a way to more directly measure students emotional and physical reactions in class, which can significantly impact student self-efficacy (Bandura, 1997). It is possible that the scholarly principle can be related to students developing a sense of community and belonging in class, and in turn, help foster a sense of personal security or comfort in class. At the same time, engaging in class, or presenting one's ideas (such as through the "productive failure assignment") may also serve as a source of anxiety or fear, and thus, a negative source of self-efficacy. This would further highlight the importance of instructors attending to the scholarly and free market principles together when employing the scholarly principle.

Additionally, we recommend developing ways to investigate the quality of implementation the five principles. We conjecture that use of a principle does not behave like a binary variable in relation to student development, but rather, that specific factors enable or strengthen the impact each principle has on students. For example, consistent personal value placed on productive failure assignment throughout the class may have gone farther in showing students it is safe to take risks than simply saying "I want you to take risks." Fannie (in Section 2.4.3.4 stated as much in her response to gaining confidence, "The general environment of everyone not being afraid to fail, and I think the productive failure thing kind of contributed to that [gaining confidence]." We conjecture that qualitative analysis of instructor beliefs, values, or goals pertaining to the use each principle, as well as the alignment of those beliefs, values, or goals with teaching actions, may be one way to infer the quality or stability of their use.

### 2.5.2 Relationship between five principles and ways students gain self-efficacy

The methods used in this study — classroom observations, surveys, and student interviews — all provided evidence of instructor use of the principles within the classroom. The SEP Scale also measured a statistically significant change in students' self-efficacy. These results provide a context for which we can begin to answer the second research question: how does instructor use of creativity fostering teaching actions impact how students gain self-efficacy for proving in an introduction-to-proofs course? Coding the interviews provided evidence that the four students experienced changes in self-efficacy for proving occurred in relation to instructor use of the five principles in the following two ways:

1. Teaching actions coded for Gestalt and uncertainty principles were associated with increased self-efficacy via enactive experiences, and
2. Teaching actions coded for free market and scholarly principles were associated with increased self-efficacy via vicarious role-modeling.

Here, we reiterate that these results were found in only the students that were interviewed and not the class as a whole. In the following subsections, we discuss the implications of these associations (Sections 2.5.2.1 and 2.5.2.2). Then, we offer some explanation for why the influence the aesthetic principle, verbal persuasion, physiological reactions may not have reported (Section 2.5.2.3).

#### 2.5.2.1 Gestalt and uncertainty associated with enactive experiences

In our qualitative analysis, we observed that all four participants cited instructor actions associated with gaining of self-efficacy for proving. In particular, the Gestalt and uncertainty principles were associated with gaining self-efficacy via enactive experiences. Fred's experience (see Section 2.4.3.1) illustrated this well: "You reach points where you're so frustrated that you can't solve stuff, and then the satisfaction when you actually. . . you figure out how to do a proof that you've been working on for a while. There's really no more empowering feeling in the world. You feel like you can do anything!" Through Fred's experience, we saw Gestalt and uncertainty principles were intertwined: Fred was both given time from the professor to work on challenging proofs and was subsequently uncertain because Dr. F did not provide any proof to Fred. While giving Fred the answer or proof to a statement may have helped Fred's understanding in the short term, it would not have given him the opportunity to build self-efficacy via his own "Aha!" experiences (Savic, 2016).

Fostering creativity by employing the Gestalt and uncertainty principles to build student self-efficacy may help mitigate students' difficulties with proving, such as failure to explore for new ideas in proving, failure to rework an argument in the case of a suspected error, and failure to validate a completed proof (Selden & Selden, 2010).

Researchers have found students' problems in learning to prove can include an inability or "unwilling[ness] to generate and use their own examples," and "not know[ing] how to begin proofs" (Moore, 1994, pp. 251–252). In Furinghetti and Morselli (2009), the student Flower attempted the proof once and stated, " 'Help! I cannot do it, I still do not see anything. The deepest darkness' " (p. 81). This is in contrast to Fred: after gaining success in proving difficult statements, he was empowered with a sense that he could do anything, and thus is more likely to exhibit persistence needed in proving. Thus, we challenge instructors to consider the Gestalt and uncertainty principles when planning activities, posing problems, and engaging student participation in class.

#### 2.5.2.2 Free market and scholarly associated with vicarious role-modeling

The second main result of our investigation was that the free market and scholarly principles were associated with vicarious role-modeling as a source of self-efficacy in proving. Fannie described gaining confidence in class from learning and building from her peers' ideas, evidence of use of the scholarly principle. Immediately following this, Fannie described also gaining confidence as a result of "the general environment of not being afraid to fail," evidence of implicit use of the free market principle. Failure was then modeled in a way that promoted learning and intellectual growth, which can have a continuing impact on how students gain self-efficacy in future proof-based courses (Savic, Gunter, Curtis, & Paz Pirela, 2018).

Furthermore, it wasn't just the "productive failure" component of class that appears to have fostered student risk-taking; from the class observation (Section 2.4.1.2), Dr. F took an answer from a student that could have been deemed wrong and reframed it as a subset of the set of correct answers. This microcosmic action can have an effect on student risk-taking; the next person may be less afraid of contributing and building off others' ideas if they know that their contribution will be valued by the instructor. In turn, there is greater potential for students to gain self-efficacy from peers of perceived similar ability. Also, from the classroom observation, we observed Dr. F offering Fannie the opportunity to lead class discussion. This action demonstrates how the free market and scholarly principles can work in coordination. Fannie being comfortable enough in class to take the risk of leading discussion led to her being able to engage students in discussion and debate over the classroom material.

This result is particularly significant in light of some of our findings from the summer preliminary data collection. One student, Sam, described vicarious role-modeling from his peers as a negative source of self-efficacy:

> *Sam:* A lot of times [Dr. S] would introduce a new problem and tell us to work on it… There [were] times when he would engage the class like earlier on in the semester and I felt comfortable about like speaking up and answering occasionally, but a lot of the time I didn't feel comfortable around my peers to answer questions.

> *Interviewer:* Do you think your confidence of varied depending on the subject, or how did you become more confident by the end?
> Because you said [earlier that] you were confident.
> *Sam:* ...In this class setting I felt like there were people in this class that already knew, like there's like two people in particular, that would always answer all the questions and ... I just deferred the questions to them, so if the teacher posed a question to the class and they didn't answer it, then I felt it like "well, I definitely can't answer it if they can't."

We coded this for explicit use of the scholarly principle because the instructor was posing problems and giving students opportunities to contribute to and extend the classroom community's body of knowledge. However, Sam feeling like "I definitely can't answer it if they can't," was coded as a negative source of self-efficacy. We also noted that throughout Sam's interview the free market principle was not coded; he did not cite any way in which the instructor encouraged risk taking or provided an environment where the student felt safe to take risks. This negative source of self-efficacy cited by Sam corresponds with Bandura's (1997) observation that those "observing others perceived to be similarly [or more] competent fail lowers observers' judgment of their own capabilities and undermines their effort" (p. 87). Sam's response appears to be in direct contrast to Fannie who felt that her "peers weren't going to judge me for doing something wrong." These differences demonstrate that evidence of the five principles do not always provide positive sources of self-efficacy, and that instructors must be aware of this if they consider implementing the scholarly principle in their classroom.

#### 2.5.2.3 Potential role of the aesthetic principle, verbal persuasion, physiological reactions

Although both the classroom observations and online surveys demonstrated both classroom presence and student experience of the aesthetic principle, only Frank mentioned one teacher action for the aesthetic principle. Fannie was the only student interviewed who described appreciation of the beauty or elegance of mathematical ideas in relation to other students, not the instructor. In Section 2.4.3.5, Fannie stated one of the problems from her last test was "so cool" and wrote it out for her friends to look at instead of studying for her physics test. At the same time, observation of instructor use of the aesthetic principle was no less scarce then the Gestalt principle. It may be that instructor use of the aesthetic principle influences student self-efficacy indirectly by promoting interest, motivation, and engagement, which may mediate the influence of the other four principles on self-efficacy. For example, increased interest may contribute to students engaging difficult proofs, persisting in the face of uncertainty, building on one another's ideas, and taking risks.

Additionally, only instance of students gaining self-efficacy via verbal persuasion (Francisca) and no instances of students gaining self-efficacy via physiological reactions were coded in the student interviews (see Tables 2.2 and 2.3). This may be a result of the way interviews were conducted. The interview focused on students experience in class overall and did not explicitly focus on one-on one interaction between student and instructor, nor the way students individually felt in class. We also did not have a way to more directly measure students emotional and physical reactions in class, which can serve to impact student self-efficacy (Bandura, 1997). It is possible that the scholarly principle can be related to students developing a sense of community and belonging in class, and in turn a sense of security when in class. At the same time, engaging in class, or presenting one's ideas (such as through the "productive failure assignment") may also serve as a source of anxiety or fear, and thus, a negative source of self-efficacy. This would further highlight the importance of instructors attending to the scholarly and free market principles together when employing the scholarly principle.

This said, it is also likely that verbal persuasion and physiological reaction had a relatively small impact on students' self-efficacy for proving. According to Bandura (1997), "verbal persuasion alone may be limited in its power to create enduring increase in perceived efficacy" (p. 101). Additionally, physiological reactions tend to carry more generalized effects on self-efficacy and are more relevant to domains that involve greater demands on physical functioning.

### 2.5.3 Future research/further theoretical considerations

Although we initially set out to find the ways in which the five principles impact student self-efficacy, we recognized in the course of this research, several other factors that appear to influence the impact certain principles have on student self-efficacy for proving. Firstly, evidence of the association between the five principles and students serving as role-models toward others outside the classroom highlight the importance of these principles in changing students' attitudes toward mathematics. Fannie and Francisca actively engaged their peers in a way that demonstrated their care for the subject (Section 2.4.3.5), reflecting of a change in their own self-perceptions, or identities, concerning mathematics. Several studies have found connections between creative identity and strong self-efficacy for creative attainments (Jaussi, Randel, & Dionne, 2007; Tierney & Farmer, 2011). We suggest investigating how the use of the five principles in creativity-fostering impacts students' mathematical identities.

We also observed two other psychosocial mechanisms that may have influenced how the five principles may have impacted student self-efficacy for proving: the negotiation of classroom norms (Yackel, Rasmussen, & King, 2000) and integration of new mathematical standards (Bandura, 1986). We believe that classroom norms of challenging and building on one another's ideas (Section 2.4.3.3-2.4.3.4) afforded by the scholarly and free market principles may have allowed students to better internalize mathematical standards necessary to proving. If the primary external rewards (affirmation of a proposed proof, engagement of ideas by the class) are intimately related the mathematical discourse, they may provide standards for which students strive and boost competence when they are attained (Bandura, 1986). Thus, presence of classroom norms related to the scholarly and free market principles may moderate the effect of the principles on self-efficacy.

In turn, we conjecture that engagement in the Gestalt and uncertainty principles provide students with opportunities to testing and developing their own system of standards, internalizing their own sense of efficacy and motivation in proving. Both one's own experiences proving, together with comparison of those attainments within the classroom dialogue, can serve to build standards for measuring one's own proving progress and evaluating future self-efficacy. Because of this, we believe studying the interaction between classroom norms, personal standards, and student motivation may play a central role in understanding the impact fostering creativity can have on student self-efficacy for proving.

This work may also contribute to previous inconsistent findings on the influence of classroom climate on student self-efficacy. Studies of the primary (Salinas & Garr, 2010) and upper secondary school (Fast et al., 2010) have demonstrated a positive impact of learner-centered classroom climates on students' self-efficacy and achievement, while one study of tertiary students (Peters, 2013) found that learner-centered classroom climates had lower mathematics self-efficacy levels. However, in this study, we observed an inquiry-based, student-centered classroom fostering significant gains in mathematics self-efficacy. This highlights the need for understanding of the potential intersections of creativity-fostering and inquiry-based classrooms. For instance, to what degree was use of the free market and scholarly principles a natural result of the classrooms being inquiry-based? How do other inquiry-based classrooms handle the Gestalt and uncertainty principles? The tools offered in this study may advance further studies of the impact of instructor actions, and thus can lead to a better understanding of the kind of classroom environments that can best foster student development in mathematics.

## 2.6 Conclusion

In this chapter, we explored how fostering mathematical creativity in a tertiary introduction-to-proofs course may impact self-efficacy for proving. First, we developed methods grounded in previous theory of fostering mathematical creativity (Sriraman, 2005), proving (Hsieh et al., 2012; Selden & Selden, 2013), and self-efficacy (2006, Bandura, 1997) to study both creativity-fostering teaching actions and student self-efficacy for proving. Through an online student survey (5PS), classroom observations, and student interviews, we documented instructor use of teaching actions that aligned with all five principles for fostering mathematical creativity. We also developed and tested an instrument for measuring student self-efficacy for proving (the SEP Scale) which recorded a statistically-significant increase in student self-efficacy for proving from the beginning and end of semester. We provided several recommendations for future improvement and use of the methods developed in this study.

Next, we analyzed how the use of the five principles in the classroom may be related to the ways in which students gained or lost self-efficacy. Analysis of the four students interviewed provided evidence that these students gained or lost self-efficacy via enactive experiences and vicarious role-modeling in relation to instructor use of four of the five principles. In particular, the instructor fostering freedom of time and space to work on challenging problems (Gestalt principle) and exposing students to uncertainty in mathematics (uncertainty principle) appeared to support students' gaining self-efficacy via their own attainments. Additionally, the instructor allowing students to build off one another's ideas (scholarly principle) appeared to promote students gaining self-efficacy from their peers. However, without the instructor creating an environment where students could take risks (free market principle), we conjecture students would have been less likely to view their peers' participation as a positive source of self-efficacy. The four cases studied in this chapter illustrate why instructors should consider these pairs of principles in coordination (Gestalt and uncertainty; scholarly and free market). Together with research already showing the strong influence self-efficacy plays in predicting student performance (Pajares & Kranzler, 1995; Siegel et al., 1985), the results of this research can provide direction for instructors in supporting the development of students' abilities for mathematical proving.

Finally, this study may provide a new perspective for researching the effects of mathematical creativity in the classroom. It highlights the impact fostering creativity might have on constructs not frequently associated with creativity. Perhaps one goal of mathematics education, in addition to developing pedagogies that enable students to be creative, is to better understand the residual effects of students' belief in their own creative potential. We conjecture that when students feel creative, i.e. believe in their own creative potential, constructs such as self-efficacy are impacted, which can have lasting effects on students' long-term mathematical trajectory.

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# Chapter 3

# Part II: How problem posing may impact student motivation

## 3.1 Introduction

Understanding the factors that promote the development of student motivation is vital to the improvement of teaching and learning of mathematics (Pantziara & Philippou, 2013). In an meta-analysis of 65 indendent mathematica and science teaching experiments studying the impact of teaching approaches on student attitude and interest, Savelsbergh et al. (2016) concluded that of the teaching approaches considered (inquiry-based, context-based, computer-based, collaborative learning strategies, and extra-curricular activities), all demonstrated positive effects on general attitude and interest toward the subject. However, there was “little clarity about what interventions cause effects on what outcome, and under what conditions” (p. 159). The evidence of positive effects of mathematics instruction on interest was sparser, focusing primarily on classroom atmosphere and teacher behavior, with only one case studying problem posing.

While researchers have described problem posing has as a motivational tool in mathematics education (Hošpesová & Tichá, 2015; Silver, 1994), there is still little research that explains *how* problem posing impacts the motivation of students studying mathematics. This paper thus contributes to the research on motivation in mathematics in the following three ways. Firstly, by studying the types of motivational regulation experienced by students engaging in problem posing, this paper contributes to the study of problem posing two cases in which problem posing fostered the motivational development of students beyond simply connecting students to their own existing interests. Secondly, this paper contributes to the problem posing literature by providing contextualized explanation of conditions necessary for problem posing to foster the integration of motivational regulation (Ryan & Deci, 2000). Finally, through an application of self-determination theory (SDT; Ryan & Deci, 2000) to student experience problem posing, this paper offers a framework for characterizing the pedagogical purposes of problem posing in relation to the development of student motivation. Such a characterization can aid continued research and development of tools for fostering student problem posing in the classroom.

## 3.2 Literature Review

This section presents the theoretical perspectives used for studying problem posing and student motivation. Problem posing was viewed broadly drawing from a variety of literature, since an explicit characterization of problem posing in the classroom is still being developed (Cai, Hwang, Jiang, & Silber, 2000). Self-determination theory (Ryan & Deci, 2000) was used to conceptualize a range of types of student motivation to allow a fine-grained analysis of the development of motivation over time.

### 3.2.1 Problem Posing

According to Silver (1994), problem posing refers to the generation of new mathematical problems as well as the re-formulation of given problems. Researchers have variously described problem posing as a "practice deeply embedded in the activity of problem-solving" (Brown & Walter, 1983), an important part of research mathematics (Silver, 1994), a means of learning mathematics (Kilpatrick, 1987), a means for fostering mathematical creativity (Silver, 1997), an integral part of mathematical exploration (Cifarelli & Cai, 2005), a formative assessment tool for instructors (Kwek, 2015), and an aspect of general mathematics instruction (NCTM, 1991). This paper focuses on problem posing and its use as it relates to teaching and learning mathematics. For this purpose, I consider student *problem-posing* in the broad sense as "the activity of students authoring mathematical tasks" (Walkington, 2018).

There is a wide variation to the ways researchers describe problem posing in the classroom (Silver, 1994; Stoyanova & Ellerton, 1996). Brown and Walter (1983) describe the "what-if-not" strategy in which a problem is analyzed by (1) listing attributes of the problem, (2) considering what happens if an attribute is eliminated or changed, and based on this (3) posing problems. Winograd (1990) described problem posing through students writing their own story problems. Problem posing is also considered a natural part of inquiry-oriented instruction in teaching students to use self-questioning and self-regulatory techniques (Collins, 1986). At the same time, Cai, Hwang, Jiang, and Silber (2015) assert that, "few researchers have tried to describe carefully the dynamics of classroom instruction where students are engaged in problem-posing activities" (p. 22). Cai et al. continue, "because classroom instruction is generally complex, with many salient features that can be investigated, researcher will need to identify those features that are most relevant for problem posing and which may be the most influenced by introduction of problem posing" (p. 23). Since problem posing has been described as a means for improving student disposition (Silver, 1997) and motivation (Silver, 1994) toward mathematics, a better of understanding of the relationship between problem posing and student motivation may contribute toward identifying key features of problem posing in the classroom.

### 3.2.2 Motivational Regulation

A central question in teaching is how to maintain or improve intrinsic motivation for learning (Valås & Søvik 1994). Motivation in education has been researched from many perspectives. Attribution theory (Bern, 1972) describes motivation as the product of one's judgments on past performances. Expectancy-value theory (Wigfield & Eccles, 2000) modifies this perspective by characterizing achievement and choice as the product of such judgments on potential action and the value of that action. Social learning theory (SLT; Bandura, 1982) and self-determination theory (SDT; Deci & Ryan, 2000) explain motivation as mediated perceived competence, or self-efficacy. Both SLT and SDT may provide valuable perspectives to studying problem posing in describing the role of *internalization* of standards or values that regulate activity.

While SLT focuses on *intrinsic interest*, or the predisposition or tendency to engage in an activity, SDT focuses on motivation in a slightly broader sense as "energy, direction, persistence and equifinality—all aspects of activation and intention" (Deci & Ryan, 2000, p. 69). This study utilizes this later perspective, taking the view that people are motivated or "moved to act by very different types of factors, with highly varied experiences and consequences." (p. 69). Rather than simply measuring students' interest toward mathematics, I seek to understand the broad reasons why students are motivated to act in relation to mathematics.

SDT also offers two theoretical perspectives that can guide the study of student motivation toward mathematics. Firstly, SDT characterizes extrinsic and intrinsic motivation not as binary variables, but on a continuum based on by perceived locus of causality (external vs. internal). This offers finer-grain analysis than of student motivation than other motivational theories. Secondly, SDT (Deci and Ryan 2000) describes three psychological needs that foster the development of self-motivation – competence, autonomy, and relatedness. These two aspects of SDT are explained in the following subsections.

#### 3.2.2.1 SDT Continuum of Motivational Regulation

Self-determination theory (SDT) describes three intermediate forms of "extrinsic" regulation between purely external and purely internal (intrinsic) regulation—introjected, identified, and integrated. These types of regulation vary by the degree to which an external regulators or reason for acting has been *internalized* (or "taken in") and subsequently *integrated* (accepted as one's own). The more someone identifies with or takes ownership of a reason for acting, the more the perceived locus of causality (PLOC) for that action will stem from themselves. Thus, as external regulators are increasingly internalized and integrated, one experiences greater autonomy in action. In this way, SDT describes the development of self-motivation from *external regulation*, to *introjected regulation*, *identified regulation,* and finally *integrated regulation*, as described in Figure 3.1. Studies have shown that people with higher levels of integrated regulation experience greater interest and enjoyment, exert greater effort, and utilize more positive coping styles (Ryan & Connel 1989).

#### 3.2.2.2 Three psychological needs that support self-motivation

Self-determination theory also describes three innate psychological needs—competence, autonomy, and relatedness—which are the basis for self-motivation. *Competence* refers to one's sense of ability to do things successfully, which is described by SDT as facilitating intrinsic motivation. However, according to Deci and Ryan (2000), in order to develop self-motivation, "people must not only experience competence or efficacy, they must also experience their behavior as self-determined." This need to maintain a sense of self-determination, or ability to take action, is referred to as *autonomy*. This is described in attributional terms as an "internal perceived locus of causality" (deCharms, 1968). Deci and Ryan further state that "this requires either immediate contextual supports for autonomy and competence or abiding inner resources (Reeve, 1996) that are typically the result of prior developmental supports for perceived autonomy and competence" (p. 70).

The third psychological need, *relatedness* refers to the need to maintain a secure, interpersonal relational base. Deci and Ryan describe the role of this need in the development of internalization: "the primary reason people initially perform such actions is because the behaviors are prompted, modeled, or valued by significant others to whom they feel (or want to feel) attached or related. This suggests that relatedness, the need to feel belongingness and connectedness with others, is centrally important for internalization" (p. 73).

### 3.2.3 Suggested links between problem posing and the development of motivation

Existing research describes a range of potential links between problem posing and the development of motivation. A study by Kwek (2015) of cognitive factors related to problem posing suggested that, in the social setting presented in their study, affective factors such as motivation are closely related to cognitive factors, and that "student motivation, perseverance, and risk-taking are positive dispositions which students can develop which assist [students], and their teachers, to harness the benefits that problem posing can bring to the learning environment." (p. 291). However, this paper primarily

| **Regulation -** Characterization of motivation by perceived locus of causality (PLOC) | | | |
|---|---|---|---|
| **Type** | **PLOC** | **Description from Ryan & Deci, 2000** | **Description from Ryan & Connell, 1989** |
| **Non-Regulation** | None | •Behavior reflects "state of lacking the intention to act" (p. 72)<br>•Either not acting or acting without intent ("going through the motions") | (Not studied) |
| **External Regulation** | External | •Behavior "performed to satisfy an external demand or reward contingency" (p. 72).<br>•Behaviors are "interpersonally controlled" (72), i.e. controlled *by* others. | •Acting due to "external authority, fear or punishment, rule compliance" (750) |
| **Introjected Regulation** | Somewhat External | •Behavior result from "taking in a regulation but not fully accepting it as one's own." (p. 72)<br>•Behavior is externally controlled, though internally driven: "behaviors are performed to avoid guilt or anxiety or to attain ego enhancements such as pride" (p. 72)<br>•"Although internally driven, behaviors still have an external PLOC and are not really experienced as part of the self" (p. 72).<br>•Behaviors are "interpersonally controlled" (p. 72), i.e. controlled *in relation* to others. | •Acting due to a formally external regulation or value that has been "taken in" (p. 750) and is enforced through internal pressures, such as guilt, anxiety, or regulated self-esteem dynamics.<br>•Reliance on external environment is minimized, but "still retains quality of pressure and conflict, or lack of complete integration with self" (p. 750).<br>•Intermediate form or regulation between external control and regulators that have been identified/assimilated. |
| **Identified** | Somewhat Internal | •Behaviors "reflects conscious valuing of a behavioral goal or regulation, such that the action is accepted or owned as personally important" (p. 72). | •Acting for "one's own values and goals," typically expressed as, "I want…" (p. 750)<br>•Acting for reasons that are identified with self and fully assimilated. |
| **Integrated** | Internal | •Regulations which are "fully assimilated to the self, which means they have been evaluated and brought into congruence with one's other values and needs" (p. 73).<br>•Actions resulting from integrated regulation share many qualities of those resulting from intrinsic regulation, but "are still considered extrinsic because they are done to attain separable outcomes rather than for their inherent enjoyment" (p. 73). | (Not studied) |
| **Intrinsic** | Internal | •Behaviors done for their "inherent satisfactions" (p. 72). | •"Reasons for acting where the behavior is done simply for its inherent enjoyment or for fun" (p. 750). |

Figure 3.1: Types of motivational regulation characterized by SDT

is to explore the converse: how does problem posing, or cognitive factors related to problem posing, impact students' motivation for mathematics?

There is little research directly studying the effect of problem posing on cognitive and affective processes related to the development of self-motivation. The existing literature describes how problem posing can be used to link students' own interests to learning mathematics (Silver, 1994). In a series of teaching experiments Bonotto (2010) found that using problem posing along with realistic contexts familiar and meaningful to students "increased their motivation to learn even among the less able ones" (p. 27). This, however, does not describe the impact problem posing may have on the long-term processes of integration of self-regulation toward mathematics. Beyond simply motivating students in the short term, problem posing can be viewed as creating "opportunities to induce reflection as well as cognitive and metacognitive changes in students" (Bonotto, p. 27), which may be linked to changes in self-regulation.

At the same time, the mathematics education literature does highlight some indirect ways in which problem posing may enable the development of self-motivation. Hannula (2006) asserts that competence, autonomy, and relatedness can "all be met in a classroom that emphasizes *exploration, understanding and communication* instead of rules, routines and rote learning" (emphasis added, p. 176). Problem posing has been described as an integral part of mathematical *exploration* (Cifarelli and Cai, 2005), as well as supportive of conceptual *understanding* and *communication* (NCTM, 1991). This paper further explores these connections to gain further insight for how problem posing may support the development of students' sense of competence, autonomy, and relatedness?

#### 3.2.3.1 Competence

The NCTM (1991) described problem posing as is supportive of conceptual understanding, which has been correlated with self-efficacy (sense of competence) in pre-service teachers (Bleicher & Lindgren, 2005; Menon & Sadler, 2016). Silver, Ellerton, and Cai (2013) asserted that problem posing "improves students' problem-solving skills, attitudes, and confidence in mathematics, and contributes to a broader understanding of mathematical concepts and the development of mathematical thinking" (p. 2).

#### 3.2.3.2 Autonomy

Problem posing has also been described as an integral part of mathematical exploration (Cifarelli & Cai, 2005), an activity which may serve to promote student autonomy in mathematics. According to Akay and Boz (2010), "problem posing helps students to gain control from others (e.g. teachers)" (p. 61). Thus, one way in which problem posing may be supportive of integration of regulation by advancing less-controlling teaching strategies. One study by Valås & Søvik (1994) utilized SDT to study the impact of teacher controlling strategies on intrinsic motivation and found that mathematics students who considered their teachers as less controlling (more autonomy supportive) experienced greater intrinsic motivation.

Additionally, substantial research has been conducted studying the role of classroom norms in fostering student problem solving and inquiry (Rassmussen et al. 2015; Yackel et al. 2000). Yackel and Cobb (1996) described how students, in the process of negotiating socio-mathematical norms, constructed beliefs that allow them to be increasingly autonomous. It appears likely that these results may hold for students negotiating socio-mathematical norms related to student problem posing.

#### 3.2.3.3 Relatedness

Less research appears to study the impact of problem posing on student sense of relatedness. However, socio-mathematical norms related to argumentation and student problem posing are likely to impact students sense of relatedness. Bonotto's (2010) teaching experiments described the introduction of new socio-mathematical norms involved in student problem posing and whole-class discussion aimed at the "socialization of knowledge" (p. 24). This process of socialization, or the long-term process by which personal habits and traits are shaped through participation in social interactions with particular demand and reward characteristics (Vygotsky (1978), describes some of ways in which students may identify with and integrate motivational regulation for mathematics through social interaction both within and out of the classroom. Thus, a more specific question not yet studied, but beyond the scope of this paper, is, how does the negotiation of socio-mathematical norms related to student problem posing in the classroom impact students sense of competence, autonomy, and relatedness? This paper focuses on the impact of student problem posing and motivational regulation.

## 3.3 Research Questions

To gain understanding of the relationship between student problem posing and changes in students' motivational regulation toward mathematics, this research project explored the question: how does problem posing, or cognitive factors related to problem posing, impact students' motivation for mathematics?

To begin answering this question, this paper explores three related sub-questions: In the context of students' experience problem posing:

1. What types of motivational regulation characterize a student's motivation toward mathematics over time?

2. To what degree does a student attribute changes in motivational regulation to problem posing?
3. In what ways does problem posing appear to support a student's sense of competence, autonomy, relatedness?

The relationship between these questions are represented in Figure 3.2 below. Note that while is likely a reciprocal relationship between these constructs, the direction of the arrows depict the implications specifically studied in this paper.

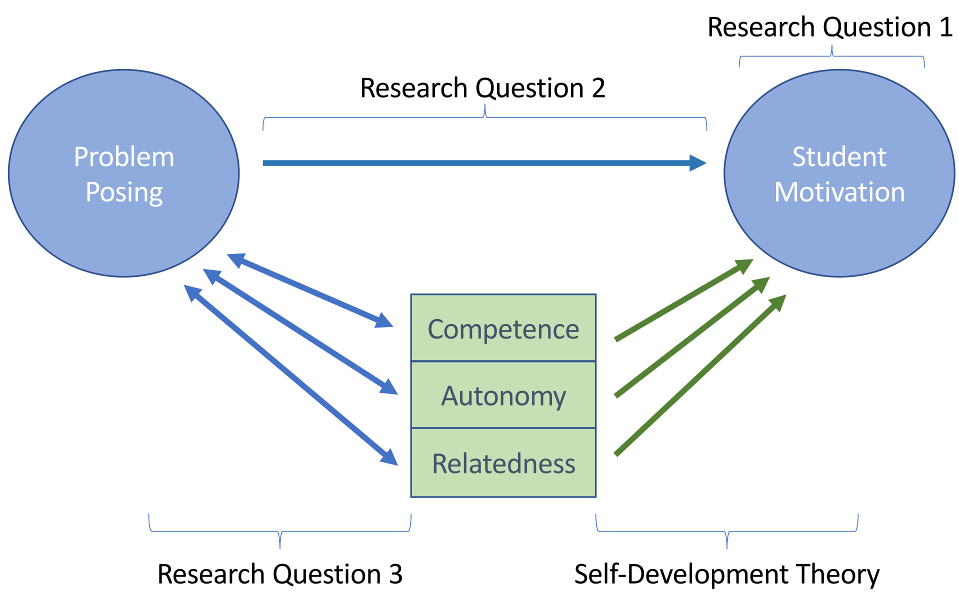



Figure 3.2: Relationship between research questions 1-3

## 3.4 Methods

This qualitative research project serves as an illustrative case study of students experience problem posing in an introduction-to-proofs course. Each class of period of this course was observed and recorded (for the project described in Chapter 2. The course, taught by Dr. F, utilized inquiry-based teaching pedagogy (Laursen et al. 2014) which involved both student problem posing in class, as well as in three types of problem posing assignments. One of these assignments, assigned twice, involved students writing "an exam like Dr. F would write." A second involved students creating their own binary operation and coming up with conjectures regarding their operation. The third, at the end of every normal assignments (assigned 2-3 times/week), students were asked to write two questions, which were periodically reviewed and discussed the following class period. Each of these assignments submitted online and collected as data. (The results section describes the extent to which theses students approached these assignments by posing their own problems.)

### 3.4.1 Participants

Two students, given pseudonyms Fred and Frank, were recruited because of their participation in interviews in December 2017. Fred had taken a Calculus II the prior semester with the same instructor. The third student, Aaron, was recruited voluntarily from among the remaining students that agreed to be contacted for follow-up research. Each student had demonstrated active participation in class. At the end of the semester, Frank, Aaron, and Fred received grades of A, B, and C, respectively, and all indicated that they had worked hard and were proud of the grade they received.

### 3.4.2 Data Collection

Interviews were conducted by the author using a semi-structured interview methodology. Questions included "Why did you take Discrete Mathematics?"; "What motivated you in class?"; "What took away motivation?" Each student's write-your-own test assignment was shown to students and used to introduce problem posing. The students interviewed were asked if and how problem posing impacted them in general, if and how it impacted their approach to Discrete Mathematics, as well as if and how it impacted their sense of competence, autonomy, and relatedness. Finally, students were asked to describe their motivation (for engaging in class, solving problems, and for posing new problems) since Discrete Mathematics. A full list of interview questions used are provided in Appendix B.

### 3.4.3 Data Analysis

Each interview was transcribed, removing phrases such as "like," "um," and "kind of" for clarity, and coded in a qualitative data analysis software package (NVivo) in three separate cycles as described below.

#### 3.4.3.1 Regulatory Style

The first cycle involved reading and coding each interview transcript for any evidence of motivational regulation as expressed in the student views concerning their engagement toward mathematics in Discrete Mathematics and in subsequent courses. This followed Hannula's (2006) view that motivation "is observable only as it manifests itself in affect and cognition, for example as beliefs, values and emotional reactions" (p. 165). Then, anything coded for evidence of motivational regulation was then sub-coded into the six types of motivational regulation detailed in Figure 3.1. This utilized the student's self-described locus of causality (PLOC) for acting and followed Ryan & Connel's (1989) research perspective that "the status of variables as real causes or motives is not directly relevant," rather the "focus is more on how persons understand and describe their own purposes for acting and the relation of such purposes to a continuum of autonomy" (emphasis added, p. 750). Because the sub-codes used for this cycle were previously defined by SDT, this coding style can be characterized as provisional coding (see Saldaña, 2013, p. 144).

#### 3.4.3.2 Problem Posing

The second cycle involved applying content-based codes for anything related to problem posing, that is, students authoring their own mathematical questions. These codes primarily fell into the following four categories:

- student's own problem posing (in or out of class)
- peers' problem posing (in or out of class)
- instructor (or teaching assistant) support for student problem posing
- student support for others' problem posing

This coding method can be described as structural coding (Saldaña, 2013, p. 84) since these codes served to categorize experiences involving to problem posing for analysis across other codes.

#### 3.4.3.3 Support for integration of motivation

The third cycle of coding involved coding each transcript for student experience of, or instructor support for, any of the three phycological needs—competence, autonomy, and relatedness—that serve as the basis for self-motivation (see Section 3.2.2.1). This process can be described as provisional coding (Saldaña, 2013), again, since these codes arose SDT's conceptual framework.

Figure 3.3, below, gives an illustration of the coding process in NVivo. These codes were next analyzed and described as summarized in the following section.

## 3.5 Results

In this section, I explore the above research questions (Section 3.3) through three respective steps of analysis:

1. First, I summarize each students' **motivation development** toward mathematics during and after their participation Discrete Mathematics based off the interview quotes coded for "regulatory style." In this, I focused primarily on motivation and motivational regulation for learning and doing mathematics, which may be conveyed through students' perspectives on engaging in class, problem solving, or problem posing. However, in the case of one student (Aaron), this also included motivational regulation for teaching mathematics.

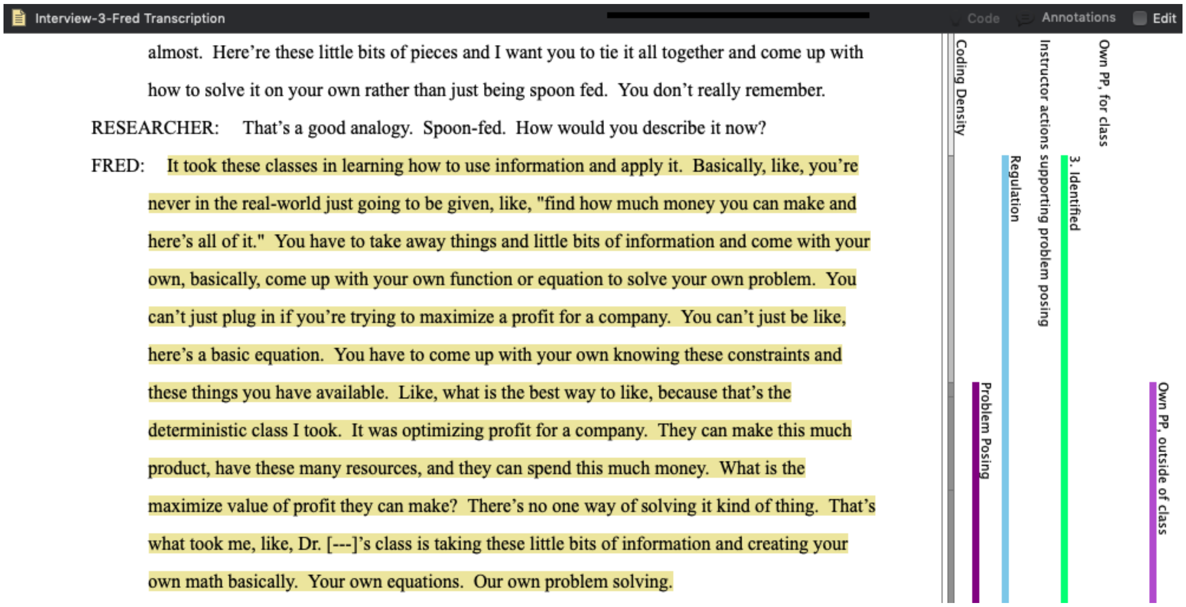



Figure 3.3: Example of coding in NVivo software

2. Second, I describe the degree to which each student **attributed** their level of (or a change in) regulatory style to problem posing. This arose from studying the intersection of coding for "problem posing" and "regulatory style."
3. Third, I discuss the degree to which problem posing appeared to have **supported the integration** of motivational regulation. This draws on analysis of the intersection of the "problem posing" and "support for self-motivation" codes.

The following three subsections (3.5.1, 3.5.2, 3.5.3) present the results pertaining these three steps of analysis respectively. For space considerations, I focus primarily on the analysis of Fred's interview, providing minimal explanation of the analysis for the Frank and Aaron. To help depict coding, words or phrases that were used directly in identifying **types of motivational regulation** are provided in **bold**. A summary of the results for each of the three students are presented at the end of this section, in Table 3.1.

### 3.5.1 Motivational Development

#### 3.5.1.1 Fred

Of the three students interviewed Fred described experiencing the clearest development from external to internal regulation for doing mathematics in Discrete Mathematics. When asked how motivated he was in Discrete Mathematics, Fred's experience illustrated a change of motivation from largely external to integrated regulation:

*Fred:* At the beginning **I wasn't really sure what was going on, so I**

> **wasn't trying to drive myself that hard** until I started to pick up on, honestly, how cool that math itself was. **It started to make me want to learn it more and apply myself more to it. So, towards the end, if I had ten minutes I would sit down and try to find a problem I hadn't done and want to mess around with it and see what I could figure out.** So, as the semester went on, I got better at it and understood the material more. I became more intrinsically motivated with it and wanted to do more of it.

Fred saying. "I wasn't really sure what was going on, so I wasn't trying to drive myself that hard" was coded for **external regulation.** Although this could be interpreted as non-regulation, from the context of although partial engagement in the class from the beginning, from the class observation and homework, it was apparent that Fred was responding to external demands of doing the assignments and coming to class. Fred's picking up on "how cool that math itself was," indicates a process of identification and integration, to the extent that his regulation for doing mathematics appears to come mostly from himself. This behavior of trying to "to find a problem I hadn't done" and wanting "mess around with it and see what I could figure out" was coded for **integrated regulation**, since it conveys a primarily internal PLOC.

Fred contrasted this with his previous experience in primary and secondary mathematics:

> *Fred:* I couldn't tell you how to do some of this stuff because you just **memorize it long enough to be able to write it on paper** and then you're just kind of like, "oh, screw it. I don't really need that anymore."

The previous behavior of "just memorize it long enough to be able to write it on paper" describes **external regulation**, something done only to satisfy an external demand.

Fred described developing, since Discrete Mathematics, the habit of asking questions in class because of "a drive to actually want to know the material." When asked, "Did you do that before," Fred responded:

> *Fred:* No, never. I was always the kid who **would just sit there and be too timid to, like, timid and cool.** You know what I mean? You don't want to be the person that asks questions and it just annoys everyone. That's something I learned in Dr. F's class. To **learn the material, you need to ask questions**.

Here we see a change from being controlled by what people thought of him (**introjected regulation**) to being controlled by the need or desire to learn the material (**identified or integrated regulation**). When asked about his motivation for solving problems since Discrete Mathematics, Fred described an almost entirely internal PLOC.

> *Fred:* I mean, again, the ability to solve problems and **the want to do math**. The want to find a flow [for a] solution to a problem. Not the solution but **the process** of how you got to an answer. It's interesting to

> me having an entire page for one solution. All the little steps in that one solution. **It's fun**.

The "want to do math," Fred's focus on the process and finding "a flow" to a problem, and it being "fun" all provide evidence of regulation by an inherent satisfaction derived from doing math (**intrinsic regulation**). Altogether, Fred's interview provided consistent evidence of a change from external to internal regulation toward mathematics.

#### 3.5.1.2 Aaron

Aaron described a similar transition from external to internal regulation for doing mathematics, as well as for teaching mathematics. He described six experiences that were coded for external and introjected regulation. For example said:

> *Aaron:* If I chose to sit back for a week and say I'm not going to engage with this as much. I could do that, but I could learn from everybody else. So, I would do, I remember **I would have a busy week and I would just kind of put minimum effort into the homework assignment and I would come to class, and I had that choice almost.**

This was coded for **introjected regulation** because his behavior was internally driven (in going to class doing minimum effort on homework) but was perceived as having an external PLOC in that his behavior is due to external factors (having a busy week).

In contrast, Aaron described 14 cases that were coded for identified, integrated, or intrinsic regulation during and since his experience in Discrete Mathematics. For example, Aaron described a time when he was "at the [gym] playing basketball and I pulled out my phone and **I started writing up a proof on it. It hit me just then. I was like, hmm, oh. I just had to start writing notes and stuff on it.**" This was coded for **identified regulation**, since solving the proof was important enough to him that he stopped playing basketball to start writing down his ideas for a proof.

Aaron also described internal regulation for teaching mathematics. The semester of the interview was conducted, Aaron had been student teaching at a local secondary school and was in the process of choosing one of two jobs offered to him teaching secondary mathematics:

> *Aaron:* I'm really excited about it and that's why **I'm looking for freedom in what I can do in the classroom.** So, like [secondary school 1], I've heard from other people that it's very strict in what you can do and what you can't do in the classroom whereas [secondary school 2] would let me do all kinds of different things. So, it's obvious **I still have to figure out what things are meaningful:** how can I structure learning in a way

> that's, one, time efficient because you have to learn so much stuff for the test, and [two], that's meaningful and gives students time to ask question.

Aaron's looking for "freedom…in the classroom" and realizing his need to "figure out what things are meaningful" was coded for **integrated regulation** since this conveys a desire to teach in a way that are in congruence with his self.

#### 3.5.1.3 Frank

Frank described little change in motivational development, already being primarily internally motivated from the start of college. Since Discrete Mathematics, Frank described being able to "read probably six pages [of an advance mathematics textbook] at a time and then it'd start a whole new concept of whole new proofs and I don't have the mental energy to select through this." When asked "what gave you the motivation to exert that energy?", Frank responded, "Coffee [and] the **material being interesting, and it making more sense than the professor**." This was coded for **integrated regulation** since this the material being interesting and "making more sense than the professor" are primarily internal reason for acting.

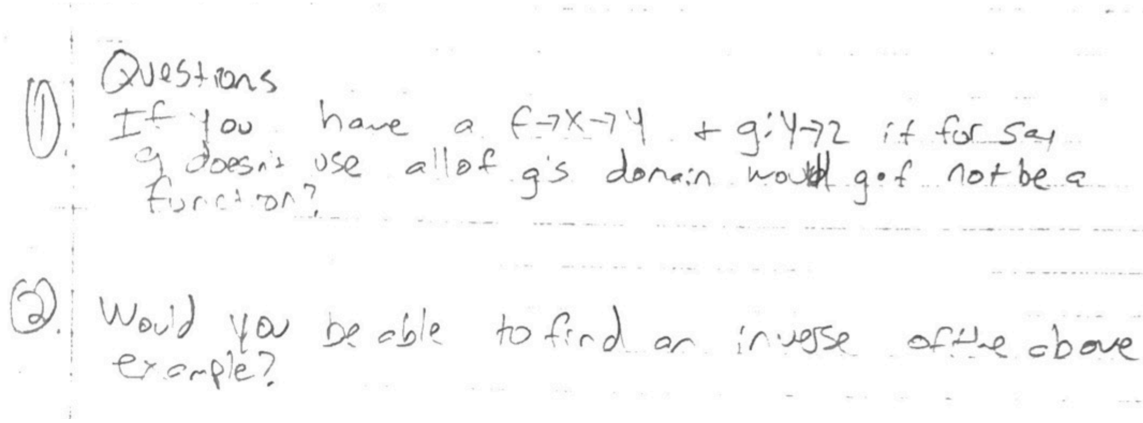
### 3.5.2 Attribution of change in Regulation to Problem Posing

#### 3.5.2.1 Fred

Overall, Fred described his experience problem posing in a way that indicates that [his own] problem posing supported integration of regulation toward mathematics. When I brought up a potential problem posing opportunity from Discrete Mathematics, saying, "I'm not sure if you remember it, Dr. F asked you before tests to write a test. Your own test," Fred responded at length:

> *Fred:* I remember that. So, I was actually, I would say, I was probably one of the better [students] at that…I remember the kid, I think his name was [—], and he sat to my left. He, like, grabbed questions from the homework and [said] "this is like what he's going to ask." I was like, "uh, he's going to give you a question you've never seen and have no idea. He just wants you to see what you can do with it." He was like, "oh, really?" I was like, "yeah." I remember my very first Calc II exam I had with Dr. F. I got a 40% on it because I just had no idea what to expect. From that point forward, I just knew for future exams…how he would phrase the material. Like, **he wanted to see your thought process and how you do math and how you go about solving math.** So, that kind of helped give me a slight advantage.

Fred understanding that Dr. F was "going to give you a question you've never seen" is evidence that Fred approached this assignment by posing his own problems. In fact, most of the problems he wrote for this assignment were novel, and some were open ended or ill-defined. As examples, two out of the six questions Fred wrote for this assignment are shown in Figure 3.4.


Questions

(1). If you have a $f \to X \to Y$ + $g: Y \to Z$ if for say g doesn't use all of g's domain would $g \circ f$ not be a function?

(2). Would you be able to find an inverse of the above example?

Figure 3.4: Example problems from Fred's write-your-own-exam assignment

At a first glance, this problem seems ill-defined, but if you allow to be a relation (and not a function), may still be a function (if is one-to-one onto the domain of ) and may have an inverse (if is one-to-one on the range of and onto on Z). When asked, "how did that assignment impact you?" Fred responded:

> *Fred:* It's still, like, he still threw some curveballs [on the exams]. I mean, you just kind of have to brace for it. Like, I knew what to expect, but you just don't know to what extent. So, I knew **not to go in and be overwhelmed during the exam.** Like if I don't understand something, because it's just Dr. F wanting you to try, not to be afraid, not [to have] a panic attack **basically because you don't know how to do a problem.**

This explanation conveys Fred's prior sense of anxiety toward the exam, evidence of **introjected regulation**, as well as a shift toward more internal regulation in choosing not to be "overwhelmed" and afraid." Fred he consciously valued Dr. F's desire for student explanation and demonstration of understanding (**identified regulation**).

Later, Fred described his own transition to more autonomous thinking in college, starting in Calculus I, but that "it really wasn't until Dr. F's Calc II where it's this whole new way of thinking about math in education and school. It's kind of abstract. Here're these little bits of pieces and I want you to tie it all together and come up with how to solve it on your own rather than just being spoon fed." I then asked, "how would you describe [your experience learning] now?"

> *Fred:* It took these classes in learning how to use information and apply it. Basically, like, you're never in the real-world just going to be given, like, "find how much money you can make and here's all of it." You kind of have to take away things and little bits of information and come with your own...function or equation to **solve your own problem**. You can't

> just plug in if you're trying to maximize a profit for a company. You can't just be like, here's a basic equation. [If you are] optimizing profit for a company…There's no one way of solving that kind of thing. That's what took me until Dr. F's class [to learn] is taking these little bits of information and **creating your own math basically. Your own equations. Our own problem solving**.

Fred adopting this view of mathematics as creating and solving his *own* problems, which Fred ascribed to Dr. F's class, is evidence of integration of regulation toward mathematics. From Fred's interview, it appears problem posing played some role in this change of regulation toward mathematics.

#### 3.5.2.2 Aaron

Aaron connected his change in motivation to the problem posing in several ways. When asked about "what did you take from Discrete Math?" Aaron mentioned exploration several times, saying things such as, "I like how he gave us like things to explore before we ever covered content in math." I used this to introduce problem posing, to which Aaron commented on.

> *Aaron:* That's one thing that I struggled with in that class because I hadn't done that yet, but I feel I've kind of gotten better at it because [every] homework he had us ask two questions. It was always do this problem and ask two questions. A lot of times I was like, what the heck am I supposed to ask? **What I found myself doing a lot of the times is I would ask a question and then I would answer it myself.** I'd be like, I can't put that question because I just answered it. So, it was actually like **I was learning by myself and learning [from] what I thought of rather than learning some kind of prescribed material.** I was thinking creatively and asking my own questions and answering them.

Aaron learning by himself from what he "thought of rather than learning some kind of prescribed material" was coded for **integrated regulation**, since his learning was described as coming from himself.

In his approach to the write-your-own test assignment, Aaron describe his original view of the goal of a test being to address or rehash a concept "you've already been taught," which was "wrong" in accordance with Dr. F's "style" of teaching and giving exams. Although he didn't approach this assignment by problem posing by definition, he demonstrated awareness of this difference (**identified regulation**) in saying "the way that his class ended up, by my read on it, **it wasn't ever a learning goal that we had already addressed in class [that was] on the tests…It was something beyond what we'd learned**."

Following this I asked Aaron to describe his motivation for the class in general:

> *Aaron:* Starting that first day and kind of establishing that relationship where, yeah, he cares for us, and he understands, **and he wants us to learn**. That helped my motivation. I definitely say that feeling of relatedness between me and [Dr. F] and the other people in the class, that helped [my] motivation.

Later, Aaron elaborated on the importance of being able to "speak our feelings or whatever and not disconnecting our classroom so much from the real world," in relation to Dr. F's valuing student contributions:

> *Aaron:* I think that was really meaningful and just seeing his kind of energy and kind of the way that he was always so engaged and loving to engage everybody in the class. He **really valued everybody's input**. That was really meaningful. The way that he kind of would, like, no matter what you said or what somebody else said, if it was wrong or if it was right or accurate or inaccurate or whatever, he always seemed to really genuinely **value that input or that question or that statement or whatever that was**.

This kind of support appears to have helped him develop self-regulation toward learning, as described by the following quotes.

> *Aaron:* I think that originally, whenever I was tasked with these problem posing assignments, like, "do this but you haven't done it before, and we haven't covered this in class, but do it." I was like, "that's a little bit scary," but going on, in **seeing how other people addressed it, [this] helped me feel like a mathematician. I was competent in mathematics. I would go home and tackle something new and then I'd come back to class and share what I learned.** So, it was like I was **doing something rather than just catching up to the teacher**. It was like I was **contributing to the whole class' learning rather than just the teacher telling us this stuff and then we're all constantly trying to catch up and understand it as well** as they do essentially

Aaron ascribed his experience "seeing how other people addressed" problem posing as contributing to his experience of being able to "go home and tackle something new and then I'd come back to class and share what I learned" (**identified regulation**). We see Aaron taking more ownership of his own learning than before through his "contributing to the whole class' learning" rather than "rather than the teacher just telling us stuff."

Overall, it is not clear from his interview whether Aaron attributed these changes in regulation to his *actual experience* posing problems, or to some other influence due to Dr. F such as his support for problem posing or his views about learning in general. Either way, habits of thinking involving problem posing appear to have become integrated into his regulation for future learning and problem solving. In response to my final question, "What do you do when you don't know what to do?" Aaron said:

> *Aaron:* **You ask questions.** You know? Yeah. When I don't know what to do, I try to break it into pieces that I do understand and **think about what I do understand and how I can tackle the problem if I understand this bit of it or this bit of it**. Like, well, I know all the pieces of it. I can do this!

#### 3.5.2.3 Frank

In the context of this interview, Frank did not attribute a change in motivation to his experience problem posing. When asked how the problem posing assignments impacted him, Frank responded:

> *Frank:* At the time I don't think a ton and I'm not sure how many long-term effects it had but I definitely think that **there's a value to trying to do that because something that I wish I would do for my classes, which I don't**, which I guess makes me question the value of it, because I think I should be doing this, but I don't.

This was coded for **introjected regulation** since it implies that he did not fully accept the practice of problem posing as his own.

This is initially surprising. Due to his previously cited highly internal regulation for doing mathematics, it seems more likely that Frank would attribute his performance on assignments or exams in this class to internal causes. However, Franks described low motivation for the problem posing assignments partly due to not perceiving them as challenging:

> *Frank:* It was so open-ended, unfortunately the downside of [problem-posing] assignments that are this open-ended and this free-form is that, really, you don't have to push yourself more than you want to.

Frank said he did not view his own problem posing as "legitimate" or "super useful" in comparison when "you're getting stuff from the professor.

### 3.5.3 Support for Integration offered by Problem Posing

#### 3.5.3.1 Fred

When asked, "how did problem posing impact your sense of confidence in this class?" Fred described gaining a sense of confidence in class and for the exams in particular:

> *Fred:* Just knowing how Dr. F asks questions, it's not to be, when you see something [you don't know], like "oh my God," in a panic. It's just like, "take a minute." Read thoroughly through **asking what the question is**

> **asking**. I'd say it helps…it's bracing or helping yourself **ease the anxiety of seeing something you don't know how to do on an exam**.

Although Fred's experience conveys a sense **introjected regulation** toward the exam in considering "how Dr. F asks questions," Fred describes a degree of self-regulation. Rather than panicking, Fred describes using questions to help "ease the anxiety of seeing something you don't know how to do." Fred then described a situation in a class since where he did panic on an early exam, then used a similar strategy of writing his own exam to prepare for the final.

> *Fred:* Yeah, for the final exam [for Deterministic Systems Models] I definitely went back and tried to create problems and think of problems that could have been on our final exam for that class. I needed it. That [first] exam kind of put me in a bad spot with my grade and I knew I had to do well in the final. So, I went through it and tried to make problems for the final. **It helped me on the final exam**.

Thus, if not problem posing in general, the use problem posing as an exam preparation strategy of appears to have given Fred a greater confidence for the exams.

When asked, "How did this assignment and problem-posing in general influence your sense of freedom towards the class and toward mathematics?" Fred described thinking that reflected high degree of self-regulation.

> *Fred:* Create a problem and think of a problem that's just out there. It's whatever. It might not even make sense when you write it down but at least you wrote it down. You were insecure about, "well, is this right?" You write a problem down and if it works you feel satisfaction and if it doesn't, you're like, "alright, well. Try again and find something that does."

Fred continued to describe Dr. F's support for failure.

> *Fred:* I mean, not to use all of Dr. F's terminology but it's okay to fail. Like if the problem doesn't make sense, so be it. It's not like it's the end of the world. Just create a new one. You have a sense of curiosity or satisfaction…of exploring with problems.

Fred's exploration of a problem, even if the problem he posed did not "make sense," gave him a "sense of curiosity or satisfaction." Contrast this with his previous experience with mathematics.

> *Fred:* So, elementary school and middle school I never really liked math, to be honest. It was basically what you'd expect from elementary school mathematics. Like, y=mx+b. Just plugging numbers basically and memorizing formulas and plugging in numbers. There was no fun in it ever. I was good at it, but high school was the same way. Here's the equations. Plug numbers into it and get the answer and if it's right it's right. If it's wrong, it's wrong. I went into college thinking that's how math was and

> especially when I took Dr. F's Calc II class and this class, I learned **that's not what math is at all**. That's like taking bits and pieces of things you don't really know and **coming to your own setup, equation, and solution on our own**...You had to come up with the flow and steps on your own. That's something it took me to get to college to learn how to do or that it even existed.

This illustrates a significant change in Fred's sense of autonomy toward mathematics, which Fred attributed to (or described in close relation to) his experience problem posing.

When asked "how did problem posing impact your sense of being related to the students around you, if at all?" Fred described his experience taking the risk in class of taking ownership of his ideas and contributing them to the class.

> *Fred:* I mean, it's always natural to feel a little insecure especially if you pose a problem that you don't know even makes sense and then going up there. So, you kind of have to have self-confidence, I'd say. Internal self-confidence. **I have no idea if this is right, but I made it and we can see it if is**. I'd say it kind of definitely helps with, you kind of have to be willing to look like a fool. Not like a fool but just, you know what I mean?
> *Researcher:* Willing to not know?
> *Fred:* Yeah. "I don't really know but here's this." So, I'd say self-confidence.
> *Researcher:* Wow. That's interesting. So, you say that helped you?
> *Fred:* To grow, definitely. I remember, in the beginning I was like, okay, I keep going back to Calc II but the first exam I had, he would do the same thing in a hundred-person lecture. He's having somebody come up to the front of the class. It takes some guts.
> *Researcher:* That's intimidating.
> *Fred:* I was so petrified. Not petrified, but I was just like, I'm not....
> *Researcher:* The first one?
> *Fred:* Yeah. It was just intimidating. He wants you to feel that so that way you feel the satisfaction when you, like, he wouldn't ever say that, but I think that's what he wants you to feel the satisfaction under the pressure and insecurity of being in front of all those people.

Fred continued to describe how supportive Dr. F was of student contributions:

*Fred:* He picks the one thing and tries to help that person. If they have one part right, like, he'll take that. He won't tell you, but he'll make a notion to if its correct. This is where you kind of need to spin off. Everything after this isn't right but you're right here so let's regroup from where you are right.

This support appears to have enabled Fred to hear problems other students were posing, and through this identify with them and develop vicariously a confidence for sharing his own problems.

#### 3.5.3.2 Aaron

Aaron described how problem posing gave him the sense that he could ask questions that were "very meaningful," and developed the sense "that I could kind of contribute or ask the same questions that all these geniuses before me have asked too." He said this "really helped me with my competence and feeling like a mathematician."

When asked. "How did problem-posing impact your sense of freedom and how you approach mathematics?" Aaron said, "I think that it was really important for me to think about what I want to think about and choose where my learning went." Aaron continued to describe how "if I chose to sit back for a week and say, 'I'm not going to engage with this as much,' I could do that" Aaron continued describing problem posing as "a criticism" of the class:

> *Aaron:* Because I remember sometimes in that class, and this would be a criticism of it I guess, but sometimes in that class I would feel like some people were kind of running away with their interests and their desires and stuff like that and kind of, it was a section of the class that was really engaged and really interested. In the question that this one person brought up and I felt like I was the only one that wasn't kind of there yet and that was probably a result of me not fully engaging with the homework at times or with the problem posing. That was just a few times in the class where I didn't do the problem posing or the homework or anything like that. I would feel like I was kind of not involved in this dialogue. It was like the real mathematicians and me sitting there like, I can't do that.... *Researcher:* Because "I didn't do the prep. I haven't thought about it yet."
> *Aaron:* M-hmm. But, I mean, that wouldn't be a criticism of the class. It'd be a criticism of me in a way. So, yeah.

This criticism of the class, being framed as criticism of himself, may have actually supported integration of regulation. He could no longer rely on outward regulation to learn but was compelled to adopt or identify with the ways Dr. F promoted learning. In fact, at end of his interview, Aaron, described an experience student teaching in which he began training his students in Geometry to pose their own problems.

Problem posing also appears to have fostered Aaron's sense of relatedness, as he described, by valuing and supporting the contributions of others in class, engaging "dialogue," and developing his view of mathematics as an ongoing collaborative human experience. Thus, through the support for problem posing offered by Dr. F as well as his peers' problem posing, Aaron appears to have developed a greater sense of relatedness in class.

#### 3.5.3.3 Frank

To Frank, the problem posing assigments, as implemented in this classroom were not supportive of his sense of competence, autonomy, or relatedness. Frank said that he did not think problem posing impacted his sense of competence, describing how the problems posed in class were "not going to imply or anything or be super useful." He did not view his problem posing in this class legitimate or worthy of any extra effort. At the same time, he did see himself in his career getting to the point of "creating things that are new and useful." Frank said, "I definitely feel confident that eventually I could architect some library or toolkit that could be very useful in different ways and I do see myself being able to build things that can be built upon." Frank described problem posing as impacting his sense of autonomy only in a superficial way, saying "it gave me a sense of, 'Oh, I can do whatever I want with this assignment,' which is usually enjoyable for sure," and described problem posing as not impacting his sense of relatedness.

## 3.6 Discussion

These findings contribute to the problem posing literature the idea that problem posing *can* have an impact on the long-term motivational orientation toward mathemathematics, as well as an explanation of some of the conditions necessary for problem posing to foster the integration of motivational regulation. For Fred, problem posing appears to have given him to opportunities to develop internal regulation for doing mathematics by giving Fred an effective mathematical approach to managing his own fear or anxiety of "seeing something you don't know how to do on an exam" (introjected source of regulation). It appears that the kind of fear or anxiety described by Fred, if allowed to persist as a primary source of regulation, obstructs the development of self-regulation toward mathematics.

From the perspective of social learning theory "it is mainly perceived inefficacy in coping with potentially aversive events that make them fearsome" (Bandura 1982, p. 136). Fred's experience illustrates how, in giving students a positive sense of self-efficacy for approaching "something you don't know how to do," problem posing *can* serve to remove external or introjected fear-based regulation, thus allowing more self-directed, autonomous forms of regulation to develop in the future. Such an interpretation offers a consistent theoretical explanation of Fred's change in regulation toward mathematics that both confirms and explains Brown & Walter's (1983) hunch that, via problem posing, "we may very well have the beginnings of a mechanism for confronting the rather widespread feelings of mathematical anxiety."

Also, Dr. F's support of problem posing (by having students present their problems and valuing student contributions) appeared to have increased Fred's sense of relatedness through other students' problem posing, which in turn contributed to his willingness

Table 3.1: Summary of results

| Research Question | Fred | Aaron | Frank |
|---|---|---|---|
| 1. What types of regulation characterize a student's motivation for mathematics over time? | Described consistent development from external to internal (identified and integrated) regulation for doing mathematics. | Described development from external to internal (identified and integrated) regulation for doing and teaching mathematics. | Described little change in regulation, appearing to be largely internally motivated from the start of college. |
| 2. To what degree did a student attribute changes in regulation to problem posing? | Attributed this change in regulation, in part, to his own experience problem posing. | Connected problem posing with this change in regulation and with taking ownership of his own learning. | Did not attribute a change in regulation to problem posing. |
| 3. In what ways does problem posing appear to support a student's sense of competence, autonomy, relatedness? | Described increased confidence for exams due to his own problem posing, an increased sense of autonomy in relation to his own problem posing and the instructor support for problem posing, and an increased sense of relatedness from his own and peers' problem posing. | Described gaining a sense of competence and being able to "choose where his learning went" from both his own problem posing and from the way class was set up in support of problem posing, and gaining a sense of relatedness from the problem posing of his peers. | Described problem posing as having little impact on his sense of competence, autonomy, and relatedness in class. |

to perform actions (pose his own problems) that were "prompted, modeled, or valued by significant others to whom they feel (or want to feel) attached or related" (Deci & Ryan, 2000, p. 73). The carrying out of those actions (i.e. problem posing) then provided more opportunities for integration of regulation related to those actions. One example of such regulation came from Fred describing problem posing as part of the larger process of his "learning" that mathematics is not replicated but created. Rather than "just plugging numbers basically and memorizing formulas and plugging in numbers" as Fred described his experience in primary and secondary school, Fred came to see mathematics as "coming to your own setup, equation, and solution on our own."

Aaron's experience described how habits of thinking involving problem posing appear to have helped foster the integration of integrated regulation toward mathematics. Aaron described problem posing in terms of by re-formulating a given problem: "I try to break it into pieces I do understand and think about what I do understand and how I can tackle the problem if I understand this bit of it or this bit of it." This perspective conveys how problem posing enabled Aaron to develop a more internal orientation (PLOC) toward mathematics.

Aaron gaining a sense of competence from problem posing, in helping him "feel like a mathematician" confirms Kwek's (2015) suggestion that "posing problems collaboratively can possibly increase one's confidence in learning and applying mathematical knowledge" (p. 289). This also helps explain how, according to SDT, further describes how problem posing contributed to his developing self-motivation for mathematics.

It is interesting that several parts of Aaron's interview attributed his change in motivation to peers' problem posing above his own problem posing experience. This demonstrates how the development of self-regulation may be impacted differently depending on the social context in which they experience or engage in problem posing. Because of Aaron's prior interest in education, it is not clear if Aaron's change in motivational regulation is explained by: (1) his actual experience asking questions, or (2) his exposure to new ideas about learning such as "the way that I understand [something new] is if I ask those questions for myself." In the first case, Aaron's own problem posing may have directly impacted his self-regulation toward mathematics as described above. In the later, class discussion of the value or use of problem posing may have served to change students views and dispositions toward learning mathematics, which may foster integration of self-regulation toward mathematics. In practice it is likely that both class discussion of problem posing and student problem posing itself operate in a coordinated way to influence students' regulation toward mathematics.

In contrast to Aaron and Fred, Frank demonstrated little change in motivational regulation from Discrete Mathematics, and Frank did not attribute any change in motivation to problem posing. In fact, he described lower motivation for the problem posing assignments than for "very challenging math problems." While he recognized the value of problem posing, he did not perceive problem posing as challenging enough to merit his effort. He said that, "unfortunately the downside of problem posing assignments that are this open-ended and this free-form is that, really, you don't have to push yourself more than you want to."

SDT explains Frank's behavior by describing how, without a sufficient level of perceived challenge, the problem posing tasks used failed to meet Frank's need for developing a greater sense of competence toward mathematics. In fact, Ryan and Connel (1989) showed that identified and intrinsic regulation, as defined by SDT, correlate with an intrinsic motivation component of a binary motivational model (Hartel, 1981) called mastery motivation. This kind of motivation, characterized by "striving for strivings toward mastery and competence are universally evident" (pp. 300-301), was not apparent in Frank's experience of problem posing. Why?

SLT provides explanation of Frank's experience in describing how "self-development is aided by a strong sense of self-efficacy to withstand failures, tempered with some uncertainty (construed in terms of the challenges of the task rather than fundamental doubts about one's capabilities) to spur preparatory acquisition of knowledge and skills" (Bandura, 1986, p. 394). Accordingly, Frank appeared to not judge the uncertainty of the problem posing tasks *as useful* to the acquisition of knowledge and skills. In spite of being one of the most mathematically advanced students in the class (from an assessment standpoint), Frank said "I haven't developed the mathematical confidence to feel like my problems are valuable." In contrast to Fred and Aaron, Frank did not see or identify himself as posing "valuable" mathematical questions. Why was Frank hesitant to see the value of his own problem posing in the context of his class' learning, problem solving, and proof writing?

One explanation for this has been offered by Silver (1994) who described that, for students already successful under more directed instruction, "there may be little desire or motivation to alter the existing power relations in the classroom, or to alter the hierarchical assumptions underlying current conceptions of mathematical performance" (Silver, 1994, p. 25). Thus, for students like Frank, explicit discussion of the fundamental assumptions of how mathematical knowledge and skills are (or can be) obtained may be needed in order to promote views of problem posing as worthy of effort. To benefit the most students possible, problem posing may need to be presented in a way that makes explicit the purposes of problem posing in relation to other educational goals. This also elevates the importance of framing problem posing and uncertainty as useful to the development of mathematical knowledge and skills.

A limitation of this research is that the results are based on students' self-reported reasons for acting, and thus, there may be bias in the explanation of their actions. However, the results for each student do appear internally consistent, and as the interviews were voluntary and over a year after their participation in Discrete Mathematics, there was little external reason to give biased responses. Additionally, the lack of an available characterizations of instructor actions used to foster or promote student problem posing highlights a need for future research developing an explicit characterization of the key features of problem-posing instruction (Cai et al., 2015) as well as the ways to measure teaching characterized by those features.

## 3.7 Conclusion

This study provides both theoretical and practical coordination between problem posing and student motivation in three ways. First, this study confirms the use of problem posing as a motivational tool beyond simply connecting students to their own existing interests (Silver, 1994). Aaron and Fred's experience provide evidence that through their own experience posing their own problems, they gained increased internal motivation toward mathematics (RQ1). Problem posing has the potential to create new internal and integrated habits of motivational regulation.

Second, this study provides explanation of *how* problem posing can impact student motivation. The analysis of Aaron and Fred's experience of problem posing illustrated how problem posing created opportunities for experiencing integration of motivational regulation by providing students with a sense of competence, autonomy, and social relatedness toward mathematics (RQ3). *At the same time,* Frank's experience provided an important illustration of how, without a sufficient sense of challenge or purpose for problem posing activities, students *may not* benefit in gaining short or long-term motivational development from problem posing. Thus, the differences in the ways in which problem posing impacted the motivation of the three students studied reinforces the need for an explicit characterization of the pedagogical purposes of problem posing in the classroom.

Finally, this research provide a framework for characterizing the pedagogical purposes of problem posing in relation to student motivational development. This framework arises directly from the application of SDT to student experience of problem posing; that is, instructor implementation of use of student problem posing should foster the following three conditions.

1. **Competence:** Problem posing should illicit a sufficient sense of challenge so that students (and instructors) perceive problems as useful to generating new mathematical competencies.
2. **Autonomy:** Problem posing should allow for student autonomy in a way that: (a) students develop a sense that *they* are a source of inquiry, and (b) positions students as sources for generating new mathematical understanding.
3. **Relatedness:** Problem posing should foster a sense of relatedness among students, so that students are exposed to their peers' thinking in a way that: (a) gives authorship of student thinking, and (b) encourages students building on one another's ideas. Productive mathematical beliefs (Schoenfeld, 1985) should be given space in the classroom to be socially transferred and reinforced, since such beliefs ultimately promote the identification and integration of the mathematical regulators described in 1. and 2. above.

In considering and applying this framework to the design of problem posing activities or assignments, instructors should recognize it is not likely that any given problem posing activity or assignment satisfy all three conditions, but that over time, problem posing activities should encompass all three to maximize potential for motivational integration. Futhermore, these three purposes can be seen as complementary:

- Firstly, some students may not perceive their ideas as contributing to the generation of knowledge, but by all students being given opportunity to pose their own questions (autonomy), instructors can highlight or develop important contributions of students' individual problems posed (relatedness). Then, through vicarious or social comparison, students are exposed to the notion they too can generate and offer useful mathematical problems.
- Secondly, due to their previous mathematical experience, a student may not respond well to the open-ended nature of problem posing tasks. In this case, reinforcing the purpose of problem posing in generating better understanding

(competence) and in highlighting contributions of previous students in prior or current classes (autonomy), students are given explicit external (but internalizable) motivation to engage in something new or unfamiliar like problem posing.

- Finally, some students may be reluctant to share their ideas in class (relatedness); but though seeing (envisioning) their ideas as contributing to the generation of classroom knowledge (competence), students are in turn individually and socially motivated to share.

This characterization represents a multifaceted approach to developing positive dispositions and identities toward mathematics. This study and future research can serve to help mathematics educators and students alike better understand how problem posing impacts motivational regulation, fostering more experiences like Aaron's, Fred's, and my own. Through this research, *I* gained greater internal motivation in seeing that as we (as students, instructors, and researchers) pose problems that are increasingly internally obtained, we gain greater access to resources within, in particular, the motivation and energy to pursue something new, difficult, and unknown.

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# Chapter 4

# Part III: A quantitative study of the impact of creativity-fostering instruction on student self-efficacy and motivation for mathematics

## 4.1 Introduction

There is a need to better understand the role creativity can play in facilitating cognitive and affective aspects of students' mathematical development. As an initial study of the impact of creativity in the classroom on students, Chapter 2 described a range ways in which creativity-fostering mathematics instruction (CFMI) was enacted in an undergraduate classroom, along with an illustration of the mechanisms by which CFMI appeared to support students' development of self-efficacy. Then, in Chapter 3, I studied one particular tool for fostering mathematical creativity, student problem-posing. By studying the potential impact of student problem posing on self-motivation for mathematics, I illustrated several ways in which problem posing appeared to have impacted individual students' motivational orientation toward mathematics. Yet, to get a clearer picture of the impact CFMI has on student self-efficacy and motivation, there is a need to quantitatively measure and study these relationships on a larger scale.

In the study of creativity-fostering instruction in general, measures of creativity-fostering or creativity-facilitating instruction have been offered. Soh's (2015) Creativity-Facilitating Teaching Index is one example. However, in reporting on the use of this instrument, Forrester and Hui (2007) concluded that "for a more reliable indicator of a creative environment we may need instruments that recognize both the complexity and flexibility of the set learning tasks(s) and the specificity of various knowledge domains." Thus, there is a need for more context-specific measures of creativity-fostering instruction within mathematics.

This paper outlines the development and use a student survey instrument designed to measuring mathematics-specific creativity-fostering instruction across upper-level undergraduate courses, the CFMI Scale. The CFMI Scale was administered in 12 upper-level mathematics courese, along with the following instruments for measuring student self-efficacy and motivation. To measure student self-efficacy, this study developed a new instrument, the Creative Self-efficacy for Mathematics (CSEM) Scale, which was implemented along with the Self-efficacy for Proving (SEP) Scale developed in Chapter 2. To study student motivation, the Academic Motivation Toward Mathematics (AMTM) Scale (Lim & Chapman, 2015) was adapted to the tertiary classroom.

As the first large-scale study of CFMI, this project contributes the research of creativity in the mathematics classroom in the following four ways. Firstly, it develops and validates a tool for measuring CFMI, the CFMI Scale, demonstrating that is a statistically reliable measurement of CFMI in the upper-level undergraduate mathematics classroom. Second, it provides initial validation of the SEP and CSEM Scales, providing direction for future use and development of these instruments. Third, this study presents robust methods for large-scale study of the impact of instruction on changes in student self-efficacy and motivation. Finally, it provides an initial quantitative exploration of the relationship between CFMI and student self-efficacy in upper-level undergraduate mathematics, describing the differentiated effects of social and individual aspects of CFMI on student task-specific and context general creative self-efficacy.

## 4.2 Literature Review

This section presents the theoretical and practical frameworks used in this study for defining and measuring creativity-fostering mathematics instruction (CFMI) and student self-efficacy. Student motivation was conceptualized using self-determination theory (Ryan & Deci) which was explained in Chapter 3.

### 4.2.1 Creativity-fostering Mathematics Instruction

There is significant effort in education research contributing to the characterization and measurement of subject-general creativity-fostering instruction. Cropley (1995) introduced nine key behaviors of teachers for facilitating creativity in their students, summarized in Table 4.1. Cropley's nine principles are derived from the "press" (Rhodes, 1961) perspective of creativity which focuses on the role of the environment in contributing to creative behavior. Soh (2000) operationalized these nine behaviors into a nine-subscales instrument for measuring instructor perspectives or beliefs regarding their instruction, called the *Creativity-Facilitating Teaching Index* (CFTIndex). Cropley (2018) further clarified his views on these principles, reinforcing their purpose as a formative tool for instructors, as well clarifying their role in researching creativity. For example, Cropley described a need to provide clearer definitions of these subscales, increased homogeneity *within* subscales, and increased distintion *between* subscales. Cropley also reinforced that what may be conducive at one stage of the creative process may not be conducive to creativity at other stages; while evaluation is important at the latter "analytic" stage of creativity, evaluation may inhibit the early "generative" stage.

Table 4.1: Cropley's (2018) nine principles for creativity-facilitating instruction

| | Cropley Principle |
|---|---|
| 1 | Motivate students to master factual knowledge, so that they have a solid base for divergent thinking. |
| 2 | Offer students opportunities to work with a wide variety of materials and under many different conditions. |
| 3 | Encourage students to learn independently. |
| 4 | Encourage flexible thinking in students. |
| 5 | Help students to learn to cope with frustration and failure, so that they have the courage to try the new. |
| 6 | Take students' suggestions and questions seriously. |
| 7 | Promote self-evaluation in students. |
| 8 | Have a cooperative, socially integrative style of teaching. |
| 9 | Delay judging students' ideas until they have been thoroughly worked out and clearly formulated. |

Following the CFTIndex's use across 9 countries across grades K-16, Soh (2015) summarized this work showing some evidence of internal consistency and concurrent validity. However, these studies focus primarily on the teacher as the unit of analysis, whereby groups of instructors' responses were aggregated and compared via subscale means. Only one study by Belio and Urtuzuastegul (2003) compared instructor scores with student scores of the CFTIndex finding that instructors rated CFTIndex higher than students for all of the 9 subscales. In Forrester and Hui's (2007) study, instructor scores for the CFTIndex subscales were used in multiple regression to predict student verbal creativity. Only the Motivation and Evaluation subscales were found as significant predictors of student creativity. Therefore, it is unclear whether instruments like the CFTIndex can reliably assess instruction in relation to its impact on students.

In a study of the impact of explicitly valuing creativity in Calculus I classrooms (Regier, Savic, & El Turkey, submitted), the CFT Index was adapted as a student instrument for measuring student perception of instruction in Calculus I classrooms. In this case, the CFTIndex demonstrated low reliability and internal inconsistencies. Furthermore, student responses were highly skewed, and factor analysis indicated that students' perceptions of creativity-fostering did not clearly distinguish CFTIndex subscales. Hayward, Weston, and Laursen (2018) demonstrated that when student survey instruments are designed to measure *what happens in the classroom,* such measurements can be an accurate indication of instruction. Therefore, this project focused on developing a student survey instrument for measuring creativity-facilitating or creativity-fostering instruction at smaller scales, i.e. at the level of individual classrooms.

For this study, the framework offered by Sriraman (2005) for supporting creativity in mathematics provided an initial characterization of creativity-fostering teaching actions. (Please refer to Chapter 2 for an outline of each principle.) In this study, we also took into consideration possible intersections between these principles, as seen in Chapter 2. The free market and scholarly principles both involve presenting one's own ideas and engaging in others', and thus can be seen as involving a social dynamic or interaction among students. The Gestalt and uncertainty principles both involve support for individual engagement in doing mathematics, in the face of challenge and uncertainty. Furthermore, fostering student problem-posing may be examples of the use of both the scholarly and uncertainty principles.

### 4.2.2 Self-efficacy

This project studies student self-efficacy from two perspectives. Firstly, we consider *self-efficacy for proving (SEP),* as defined in Chapter 2, as "one's beliefs in their own capabilities to organize and execute the actions required to produce justifiable mathematical proofs." In the important case when proving tasks are novel, i.e., students *do not already* know how to prove a given task, students belief and knowledge regarding their creative abilities are also important factors. Thus, of interest is a second kind of more general creative self-efficacy.

According to Bandura (1997), self-efficacy should be viewed as both task- and context-specific. More recently, in synthesizing the research of creative self-efficacy, Beghetto and Karwowski (2017) affirmed Bandura's view, emphasizing that "a key requirement for measuring self-efficacy beliefs is that they are tailored to elicit a person's confidence in performing specific features of a task."
Beghetto and Karwowski emphasized that measures of [creative] self-efficacy should be highly specific and focused on one's ability to perform a future impending task, but are also dynamic, i.e. sensitive to context, psychological and physiological state.

To distinguish between this perspective from the commonly used approach of studying domain and/or *context-general* creative self-efficacy (Tierney & Farmer, 2002), Beghetto and Kauffman (2013) introduced the concept of *creative metacognition* to refer to the combination of beliefs of one's own creative abilities as well as a knowledge of when, where, and why to be creative. This definition derives primarily from the first of two commonly recognized components of metacognition: *metacognitive knowledge* (Flavell, 1979). The second component, *metacognitive control and regulation* (Pintrich, Wolters, and Baxter, 2000), is not directly apparent in this definition, and though important part of metacognition, is beyond the scope of this study.

According to Beghetto and Karwowski (2017), creative metacognition, while related to creative self-efficacy, is less concerned with confidence of a particular task, but with accuracy of assessment of one's creative abilities as well as regulatory *beliefs* [cognition] related to creative performances. Creative metagognitive beliefs are concerned one's ability to make accurate self and situational judgments such as "whether creative performance is warranted and feasible in light of one's self-assessed strengths and features of the current situation" (p. 7). They are presently focused, moderately specific, and moderately stable over time. Since this perspective is focused on cognitive aspects of metacognition, and is consistent with previous conception of *metacognitive knolwedge* (Flavell, 1979), the term *creative metacognitive knowledge* will be used to clarify Beghetto and Kauffman's (2013) use of the term *creative metacognition.*

A third related construct, *creative self-belief*, was presented by Beghetto and Karwowski to describe even more general, holistic cognitive and affective judgments of one's creative abilities across a domain. Creative self-beliefs include both domain-general beliefs ("I am creative") and domain-specific beliefs ("I am creative mathematical problem solver"). Creative self-beliefs involve of combination affective ("I like creative problem solving") and cognitive ("I am good at creative problem solving") judgments regarding one's creative identity. Together, creative self-efficacy, creative metacognitive knowledge, and creative self-belief can be seen as important components of one's creative identity (Karwowski & Barbot, 2016).

This paper explores the relationship between these constructs specifically as regards to students' creative identity within mathematics. In particular, I set out to measure students' task-specific *self-efficacy for proving (SEP)* as a lens for study of creative self-efficacy. To the degree to which a prospective proving task is new to a student, the corresponding evaluation of one's SEP can be viewed as in measurement of creative self-efficacy.

Additionally, a measurement of *creative self-efficacy for mathematics (CSEM)* was developed in order to study students creative metacognition knowledge. As a context-general measurement of creative metacognitive knowledge, CSEM should be viewed as more static and stable across contexts than SEP (Pintrich, Wolters, and Baxter, 2000). Due to both the individual (enactive) and social (vicarious) factors described in Chapter 2, both SEP and CSEM may be significantly impacted by students experience in the classroom and serve as the focus of this study.

## 4.3 Research Questions and Hypotheses

In order to measure the impact of CFMI on student self-efficacy and motivation, this project firstly sought to develop, test, and validate instruments for measuring CFMI, self-efficacy, and motivation in tertiary mathematics courses. Then, with these instruments, the central question of this thesis is explored: how does teaching to foster mathematical creativity impact student self-efficacy and motivation? More precisely, this question is studied through the following research questions:

- RQ1: What is the impact of CFMI on pre- to post-semester changes in student self-efficacy for proving and creative self-efficacy for mathematics?
- RQ2: What is the impact of CFMI on pre- to post-semester changes in student motivational regulation (as described by self-determination theory (Ryan & Deci, 2000b))?

Based on the literature and the findings in Chapter 2, it is hypothesized that CFMI scales will be correlated with pre-post semester change in SEPS (RQ1) in the following ways. First, social learning theory describes that enactive experiences bear the strongest influence on self-efficacy. Thus, opportunities for working on challenging problems (Gestalt principle) and being exposed to uncertainty in mathematics (uncertainty principle) will be correlated with increased SEPS. However, the extent to which this opportunity effectively led to their own accomplishments is dependent on how students engaged those opportunities. Thus, the effect of the CFMI Gestalt subscale on SEPS will be mediated by internal regulation toward mathematics. Second, social learning theory describes that vicarious influences will have the next strongest influence on self-efficacy. Thus, opportunities for learning from one another (free market and scholarly principles) will be correlated with students gaining SEPS vicariously. However, this effect may be stronger for students with lower self-motivation. Thus, the effect of the CFMI Free Market/Scholarly subscales on SEPS will be mediated by external regulation toward mathematics.

In considering the second research question, Ryan and Deci (2000a) predicted that creativity is the result of intrinsic motivation. Thus, it is hypothesized that CFMI will be correlated with pre- to post-semester measurements of internal regulation (identified, integrated, and intrinsic regulation) toward mathematics, with post-semester measurements correlating more strongly, due to proximity of time in responses. Additionally, it is conjectured that CFMI scales will be correlated, in varying strengths, with increases in internal regulation toward mathematics. Fostering mathematical creativity will create environment in which students are more likely have opportunities to developing competence (Gestalt and uncertainty principles), acting autonomously (free market and uncertainty principles), and developing a sense of relatednedness(free market and scholarly principles), thus facilitating integration of regulation (increase in internal regulation), as described by self-determination theory (see Chapter 3).

## 4.4 Methods

This quantitative research study utilized a correlational research design. Four primary survey instruments were used: the Creativity-Fostering Mathematics Instruction (CFMI) Scale, the Self-efficacy for Proving Scale (SEP) Scale, the Creative Self-efficacy for Mathematics Scale (CSEM) Scale, and the Academic Motivation Toward Mathematics (AMTM) Scale.

## 4.5 Participants

Students in 12 upper-level mathematics courses at two large universities located in midwestern United States were asked to participate in online pre- and post-semester surveys. The number of students enrolled in each course, as well as the number of students that participated are shown in Table 4.2.

Table 4.2: Course enrollment and survey participation

| | | | Survey Participation | |
|---|---|---|---|---|
| | Course Name | Enrollment | Pre-survey | Post-survey |
| 1 | Discrete Math Structures | 32 | 28 | 25 |
| 2 | Linear Algebra I - section A | 77 | 42 | 35 |
| 3 | Linear Algebra I - section B | 33 | 26 | 20 |
| 4 | Intro to Abstract Algebra | 10 | 10 | 8 |
| 5 | Intro to Number Theory | 36 | 23 | 19 |
| 6 | Abstract Linear Algebra - A | 18 | 13 | 15 |
| 7 | Abstract Linear Algebra - B | 27 | 22 | 20 |
| 8 | Intro to Analysis I | 29 | 11 | 20 |
| 9 | Graph Theory I | 25 | 33 | 23 |
| 10 | Applied Statistical Methods - A | 33 | 30 | 25 |
| 11 | Applied Statistical Methods - B | 29 | 30 | 19 |
| 12 | Applied Statistical Methods - C | 31 | 25 | 27 |
| | Total | 380 | 293 | 256 |

Additionally, three students enrolled in an Honors Calculus I were recruited for response process interviews online (via Facetime) in the middle of the semester. Students from this course were chosen to ensure the questionnaire was accessible to any college students, especially students with little or no experience with proving. These particular students were individually selected by the instructor based on their engagement in class and perceived willingness to participate.

### 4.5.1 Data Collection

Pre-semester surveys were administered at in the first two weeks of the semester and included the SEP and AMTM Scales. Post-semester surveys were administered in the last week of class and included the CFMI, SEP, CSEM, and AMTM Scales. Additionally, students were asked three additional questions regarding their views of mathematical creativity, general perceptions of their own creativity, general perceptions of instructors' creativity-fostering instruction (see next section).

To anonymously link students' pre- and post-semester OU Institutional Review Board office recommended the use of five questions code, shown below

1. Your shoe size (ex: size 9 = 09)
2. First two letters of your favorite color (ex: Green = GR)

3. How many sisters do you have? (ex: 2 sisters = 02)
4. How many brothers do you have? (ex: no brothers = 00)
5. First letter of the city where you were born? (ex: Boston = B)

Students were given the example, "For the above example information, students would write '09GR0200B.' " This prompt was given at the end of both pre- and post-semester surveys, as well as one question regarding students' gender identity.

To administer both pre- and post-semester surveys, the author attended each course session in the first or last 15 minutes of class to explain the purpose of the research (to study the impact of creativity in instruction on motivation and confidence), the structure of the questionnaire, and that participation is entirely voluntary and anonymous, but that students' responses have the potential to contribute to research and improvements in mathematics education. Copies of the both pre- and post-semester surveys are provided in Appendices C.1 and C.2.

For the response process interviews, three students were asked to fill out online an initial version of the post-semester survey. Immediately following, they were asked to "please read each item aloud and explain why you chose the answer." If a student's reasoning did not match their answer choice, probing questions were asked in order to clarify their interpretation of the item and how it matched their answer choice and reasoning. Analysis of these interviews was used to make minor changes to the wording of the CFMI Scale, as well as to eliminate unnecessary items from the SEP and AMTM Scales. The response process interview protocol is provided in Appendix C.3.

#### 4.5.1.1 Creativity-Fostering Mathematics Instruction (CFMI) Scale

To ensure the content validity of a measure for CFMI, the CFMI Scale was developed using a process of deductive scale development (Hinkin, 1989) and revised after conducting response process validity interviews as explained below. The CFMI Scale was constructed primarily using Cilli-Turner, Savic, El Turkey, and Karakok's (2019) investigation of teaching actions that foster mathematical creativity. Cilli-Turner et al. analyzed interviews of 14 students experience of a Calculus I course designed to explicitly value mathematical creativity. By coding of teacher actions that contributed to students' creativity along with Sriraman's five principles, 20 explicit teaching actions were presented.

Based on these 20 actions, an initial list of student survey items was developed with the form "My instructor…" Then, three researchers (Savic, El Turkey, and Regier) worked together to align these items with both the theoretical framework of Sriraman (2009) and Savic et al. (2019). Based on the strong connection between teaching actions for the free market and scholarly principles described in Chapter 2 (Regier & Savic, 2019) and in Cilli-Turner et al. (2019), the items for these principles were grouped, given the label *FMS*. Similarly, due to the connections between the Gestalt and uncertainty principles described in Chapter 1, it was often difficult to distinguish items related to these two principles. However, these principles were labeled separately (*Gest* and *Unc* respectively), allowing for the possibility of separate and combined analysis of these principles.

Following response process validity interviews with three students, the wording of ten items was revised and one item was added, *Unc_5* to differentiate encouraging questions in one's individual problem-solving process from posing problems in class (*FMS_4*), giving a total of 22 items.

Because each of the initial CFMI Scale items were derived directly from both theoretical and practical foundation of Sriraman (2009) and Savic et al. (2019), this process of deductive scale development (Hinkin, 1989) ensured content validity. Thus, these items capture *significant aspects* of CFMI, but are by no means comprehensive. In fact, creativity may often be best supported by the convergence of many diverse unpredictable factors. The CFMI Scale simply incorporates a range of specific, identifiable aspects of instruction that the literature cites as particularly supportive of creative student behavior in mathematics.

The 22 items of the CFMI Scale, shown in Table 4.3, were administered with the prompt, "Please rate your level of agreement with each of the following statements regarding your instruction in the above math class this semester," using a 6-point Likert scale (1-Strongly disagree to 6-Strongly agree). Following the CFMI Scale, students were asked regarding about their views of mathematical creativity, general perceptions of their own creativity, general perceptions of instructors' creativity-fostering instruction, via the questions below:

1. Question C1: "What does it mean to be mathematical creativity?" (check all that apply)
    - To engage a process of offering mathematical insights that are new to me
    - To produce mathematical insights that are new to me
    - Other:

2. Question C2: "By the above definition, were you mathematically creative in this course?" (Yes or No)
3. Question C3: "By the above definition, did your instructor foster mathematical creativity in this course?" (Yes or No).

Table 4.3: CFMI Survey

| Name | Order | Item |
|---|---|---|
| Aes_1 | 5 | My instructor pointed out the beauty of certain solutions/approaches. |
| Aes_2 | 9 | My instructor pointed out connections between seemingly different mathematical ideas in class. |
| Aes_3* | 14 | My instructor pointed out only standard approaches to problems. |
| Aes_4 | 19 | My instructor pointed out simple solutions to complex problems. |
| FMS_1 | 1 | My instructor encouraged us to present our solutions/approaches. |
| FMS_2 | 4 | My instructor valued our ideas in class. |
| FMS_3* | 7 | My instructor did not encourage different approaches in class. |
| FMS_4 | 10 | My instructor encouraged us to pose our own mathematical problems to the class. |
| FMS_5 | 13 | My instructor encouraged us to debate and discuss with one another. |
| FMS_6 | 17 | My instructor recognized when a student builds on the work of another student. |
| FMS_7 | 22 | My instructor encouraged us to ask mathematical questions in class. |
| Gest_1 | 2 | My instructor assigned challenging problems and tasks. |
| Gest_2 | 6 | My instructor allowed us to approach a problem in a way that was different from theirs. |
| Gest_3* | 11 | My instructor did not allow us to revise homework problems. |
| Gest_4 | 16 | My instructor discussed how solving problems often requires a lot of time. |
| Gest_5 | 21 | My instructor allowed for freedom of time to work through problems. |
| Gest_6 | 20 | My instructor encouraged us to generalize what we learned from one problem to others. |
| Unc_1 | 3 | My instructor provided support when we were frustrated. |
| Unc_2 | 8 | My instructor discussed how it is OK to be confused while doing mathematics. |
| Unc_3* | 15 | My instructor did not emphasize the importance of asking questions in our problem solving process. |
| Unc_4 | 18 | My instructor described that doing mathematics can be challenging at times. |
| Unc_5 | 12 | My instructor encouraged us to persevere in doing mathematics. |

* These items are reverse coded.

Exploratory factor analysis and confirmatory factor analysis was used to determine the final factor structure of the CFMI Scale. Then, regression was used to study the extent to which the CFMI subscales predict the students' responses to questions C1-3. Question C3 was considered for validating and clarifying the extent to which each subscale of the CFMI Scale measured CFMI.

#### 4.5.1.2 Self-efficacy for Proving (SEP) Scale

To provide better precision for evaluation of student self-efficacy for proving, the SEPS statements piloted in Chapter 2 were re-evaluated and selected with the goal of providing tasks for which they can quickly and easily understand the statement provided. The following statement were given on the pre-semester survey.

- Statement 1 (Routine): If $n$ is odd, then $n^2 + 1$ is even.
- Statement 2 (Moderately-routine): The inequality $2^x \geq x + 1$ is true for every positive real number x.
- Statement 3 (Non-routine): If $a$ and $b$ are integers, then $a^2 - 4b \neq 2$.

Due to high positive skew of student self-efficacy ratings to Statement 1 on beginning-of-semester ratings (see Section 4.6.2.2), a routine proving state was omitted from the post-semester survey. Below are the statements given on the post-semester survey.

- Statement 4 (Moderately-routine): If $n$ is an integer, then $1 + (-1)^n(2n - 1)$ is a multiple of 4.
- Statement 5 (Non-routine): Every odd integer is the difference of two squares.

Each of these statements constitute one measurement, or one subscale of the SEP Scale, and are termed *SEP Subscales 1-5* (SEPS1-5) in the following analysis. One implicit goal in using the SEP Scale is to measure students' confidence for proving statements for which they do not already know how to prove the statement being asked. Thus, following statements 4 and 5, students were asked, "Have you previously seen or proved the above statement?" with the following response options.

1. No, I have not previously seen or proved this statement.
2. Yes, I have previously seen this statement, but haven't proved it.
3. Yes, I have proved this statement in the past
4. I may have seen something similar, but I cannot remember.

#### 4.5.1.3 Creative Self-efficacy for Mathematics (CSEM) Scale

The Creative Self-efficacy for Mathematics (CSEM) Scale consisted of 4 items measuring self-efficacy related to originality, fluency, flexibility (Torrance, 1974), collaboration in developing mathematical ideas (Tierney & Famer, 2002), along with one item pertaining to self-evaluation of creative problem-solving abilities. These items, along with their theoretical underpinnings are described in Table 4.4 below. For each of the items listed, students were asked, “Please rate how confident you are that you can do the following as of now.”

Table 4.4: CSEM items and theoretical underpinning

| Order | Item | Aspect | Definition |
|---|---|---|---|
| 1 | generate original math ideas | novelty | ability to generate novel or original ideas (Torrance, 1974) |
| 2 | solve a math problem in multiple ways | fluency | ability to generate large number of ideas (Torrance, 1974) |
| 3 | give multiple solutions to a math problem | flexibility | ability to generate diverse categories of ideas (Torrance, 1974) |
| 4 | build on the mathematical ideas of others | collaboration | ability to collaborate with others (Tierney & Famer, 2002) |
| 5 | be creative in solving math problems | general | ability to be creative according to students’ determination |

Both the SEP and CSEM Scales were constructed following Bandura’s (2006) guide for constructing, scales using 11-point 0-100% confidence ratings. After the responses process interviews, minor changes were made to *CSEM_2* and *CSEM_3*. As with the the CFMI Scale, both EFA and CFA was used to determine the factor structure of the SEP and CSEM Scales.

#### 4.5.1.4 Academic Motivation Toward Mathematics (AMTM) Scale

For measuring students’ motivational orientation, Lim and Chapman’s (2015), Academic Motivation Toward Mathematics (AMTM) Scale was adapted for this study. The AMTM Scale was designed for to measure the types of self-regulation described by self-determination theory (Ryan & Deci, 2000b) and was validated on pre-tertiary (grade 11 and 12) students. The wording of three items (*EMID4*, *IMTK2*, *IMTS2*) from the original AMTM Scale was slightly modified to make the content applicable to (upper-level) college mathematics. For example, *EMID4* was changed from “Because what I learn in mathematics now will be useful for the course of my choice in university” to “Because what I learn in mathematics now will be useful in my future studies.” *IMTK2*, *IMTS2* were slightly changed to reflect the perspective that mathematics at the tertiary level is not limited to observing and learning what expert mathematicians have come up with; rather, students can experience the joy of creating and understanding mathematics themselves. The full version of the AMTM Scale used in this study is provided in Table 4.5.

Table 4.5: Adaptation of Lim and Chapman's (2015) AMTM Scale

| Name | Item |
|---|---|
| AMOT1 | Honestly, I don't know; I feel that it is a waste of time studying mathematics. |
| AMOT2 | I can't see why I study mathematics and frankly, I couldn't care less. |
| AMOT3 | I don't know; I can't understand what I am doing in mathematics. |
| AMOT4 | I am not sure; I don't see how mathematics is of value to me. |
| EMER1 | Because without a good grade in mathematics, I will not be able to find a high-paying job later on. |
| EMER2 | In order to obtain a more prestigious job later on. |
| EMER3 | Because I want to have "the good life" later on. |
| EMER4 | In order to have a better salary later on. |
| EMIN1 | Because of the fact that when I do well in mathematics, I feel important. |
| EMIN2 | Because I want to show to others (e.g., teachers, family, friends) that I can do mathematics. |
| EMIN3 | To show myself that I am an intelligent person. |
| EMIN4 | Because I want to show myself that I can do well in mathematics. |
| EMID1 | Because I think that mathematics will help me better prepare for my future career. |
| EMID2 | Because studying mathematics will be useful for me in the future. |
| EMID3 | Because I believe that mathematics will improve my work competence. |
| EMID4 | Because what I learn in mathematics now will be useful in my future studies. |
| IMTA4 | Because I want to feel the personal satisfaction of understanding mathematics. |
| IMTK2 | For the pleasure I experience when I discover new things in mathematics that I have never seen before. |
| IMTK3 | For the pleasure that I experience in broadening my knowledge about mathematics. |
| IMTS2 | For the pleasure of being able to experience "light bulb" moments understanding something new. |
| IMTS3 | For the pleasure that I experience when I feel completely absorbed by what myself or others come up with. |

AMOT = Amotivation; EMER = External regulation; EMIN = Introjected regulation; EMID = Identified regulation; IMTA, IMTK, IMTS all are part of the IMOT or intrinsic motivation subscale

### 4.5.2 Data Analysis

To address the possibility students not carefully reading or considering their responses (i.e. "clicking" through it), the following removal criteria was developed. In the cases where either of the following two criteria held for **all** scales in a students' response, that students' response was deleted prior to analysis:

- Criteria 1: all responses were the same for a scale.
- Criteria 2: at least half of responses must be missing from a scale

This resulted in 8 responses in the pre-semester survey and 13 responses in the post-semester survey being removed. Next, scale structure was assessed using principal component analysis (PCA) and items determined conceptually inconsistent by process interviews and/or principle component analysis were subsequently removed from analysis of the scales. Based on exploratory factor analysis, several models were suggested, and confirmatory factor analysis was used to assess the model fit of these models. Finally, structural equation modeling (Bollen, 1989) was used to study the relationship between CFMI, SEP, and CSEM.

For the analysis of the instruments involving Likert-Scales (CFMI and AMTM Scales), analysis was performed using polychoric correlations. Polychoric correlations assume the variables are ordered measurements of an underlying continuum (i.e. strongly agree to strongly disagree). This matches how the CFMI and AMTM Scales were presented to students online (see Figure 4.1). All analysis was conducted using R software, primarily using the 'lavaan' and 'FactoMineR' packages.

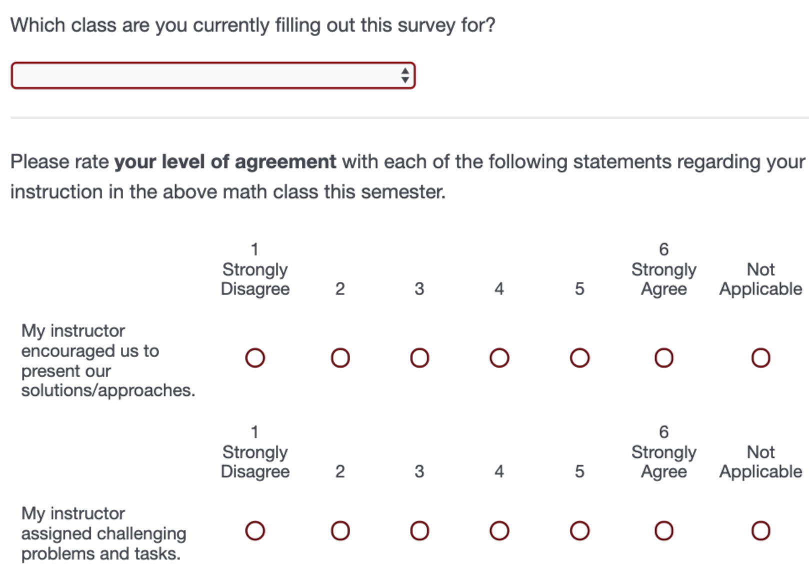



Figure 4.1: Online CFMI survey format

## 4.6 Results

This section presents the results of analysis of the CFMI, Self-efficacy, and AMTM Scales (Sections
4.6.1, 4.6.2, and 4.6.3 respectively). Since the CFMI and self-efficacy instruments used

in this study were novel, a more detailed analysis was conducted to better understand the validity of these instruments. Then, due to an inability to accurately link pre- and post-semester student responses, limited correlational analysis is presented in Section 4.6.4.

## 4.6.1 Analysis of CFMI Scale

### 4.6.1.1 Response Process Interviews

To ensure content validity of the CFMI scale, student feedback from process interviews was analyzed to modify the wording of several items. Two examples of the most significant changes (*FMS_3* and *Gest_3*) are described below. Note that the three students interviewed (Students 1, 2 and 3) are all from the same class, so inconsistent responses indicated differing interpretations of these items.

One of the items, *FMS_3,* was originally worded "My instructor penalized us for trying different approaches that didn't work." Student 2, responding "strongly disagree" said that " 'Penalized' was a weird word. It seems like only applies to test, but I read it as in class." Student 3 responded "not applicable" because they "didn't remember a time where someone didn't try something, and it didn't work; it always led to something useful." Thus, the wording of *FMS_3* was changed to "My instructor did not encourage trying different approaches."

Another item, *Gest_3* was originally worded "My instructor allowed us to turn in revisions on homework problems." Student 2, responding "Agree," said the instructor would "give us stuff to do, and we'd go over homework in class, and you could fix it. Sometimes you could go back and fix it and turn it in the next class, if you needed." However, Student 3 responded to *Gest_3* "strongly disagree," saying "when we turned in our HW problems, we could turn it in late, but we could not do revisions on then." Student 1 responded to *Gest_3* "disagree," saying "I didn't do revisions, because it didn't really matter, was more about effort and attempt, and participation." In this case, we know that the instructor allowed students to work on revisions after discussion in class before they submitted homework but did not allow revisions once homework was submitted. In this context, whether they were "allowed to turn in revisions" was interpreted differently by students. Based on this, we decided to rephrase *Gest_3* more generally as "My instructor did not allow us to revise homework problems."

### 4.6.1.2 Response Distribution of CFMI Scale

The distribution of student responses to the CFMI subscales are shown in Table 4.6. Note the high "NA" response rate to *FMS_4*, *FMS_5*, *FMS_6*. These items involve students posing problems, debating with one another, and building on one another's ideas. It was noted that these often occurred in clusters: 4 students responded "NA"

Table 4.6: CFMI Scale response frequency

| | 1 | 2 | 3 | 4 | 5 | 6 | NA |
|---|---|---|---|---|---|---|---|
| Aes_1 | 6 | 9 | 22 | 40 | 55 | 111 | 0 |
| Aes_2 | 7 | 7 | 28 | 46 | 84 | 70 | 1 |
| Aes_3 | 12 | 25 | 56 | 64 | 64 | 21 | 1 |
| Aes_4 | 9 | 12 | 24 | 54 | 66 | 77 | 1 |
| FMS_1 | 7 | 19 | 29 | 43 | 69 | 75 | 1 |
| FMS_2 | 3 | 14 | 22 | 54 | 66 | 81 | 3 |
| FMS_3 | 10 | 15 | 27 | 48 | 76 | 63 | 4 |
| FMS_4 | 22 | 54 | 56 | 45 | 29 | 24 | 13 |
| FMS_5 | 21 | 32 | 43 | 38 | 43 | 56 | 10 |
| FMS_6 | 15 | 28 | 67 | 49 | 24 | 26 | 34 |
| FMS_7 | 1 | 14 | 29 | 54 | 63 | 81 | 1 |
| Gest_1 | 1 | 3 | 4 | 22 | 70 | 143 | 0 |
| Gest_2 | 10 | 15 | 38 | 47 | 72 | 57 | 4 |
| Gest_3 | 65 | 63 | 33 | 25 | 21 | 23 | 13 |
| Gest_4 | 3 | 17 | 32 | 46 | 69 | 76 | 0 |
| Gest_5 | 25 | 24 | 50 | 52 | 50 | 41 | 1 |
| Gest_6 | 4 | 10 | 20 | 59 | 72 | 77 | 1 |
| Unc_1 | 7 | 15 | 29 | 46 | 69 | 76 | 1 |
| Unc_2 | 9 | 34 | 39 | 58 | 51 | 49 | 3 |
| Unc_3 | 8 | 11 | 30 | 41 | 82 | 70 | 1 |
| Unc_4 | 1 | 5 | 22 | 52 | 80 | 82 | 1 |
| Unc_5 | 3 | 16 | 24 | 50 | 74 | 75 | 1 |

1 = Strongly Disagree; 6 = Strongly Agree

to all *FMS_4*, *FMS_5*, and *FMS_6*, while 8 students responded “NA” to two out of these three items. It was conjectured that these students had less experience in smaller upper-level mathematics courses that could facilitate more collaborative engagement in mathematics. To test this conjecture, it was observed that out the 14 responding “NA” to two or more of the FMS items, 7 were in Applied Statistical Methods and 4 were in Linear Algebra.

The relative distribution of responses to the CFMI Subscales are shown in Figures 4.2, 4.3, and 4.4. Note the relative high frequency of positive ratings to *FMS_7* (“My instructor encouraged us to ask mathematical questions in class”) compared to *FMS_4* (“My instructor encouraged us to pose our own mathematical problems to the class”) Also note that *Gest_1* (“My instructor assigned challenging problems and tasks”) was rated very highly by students. Out of 243 total students who completed the CFMI Scale, 213 responding “5-Agree” or “6-Strongly Agree” to *Gest_1.*

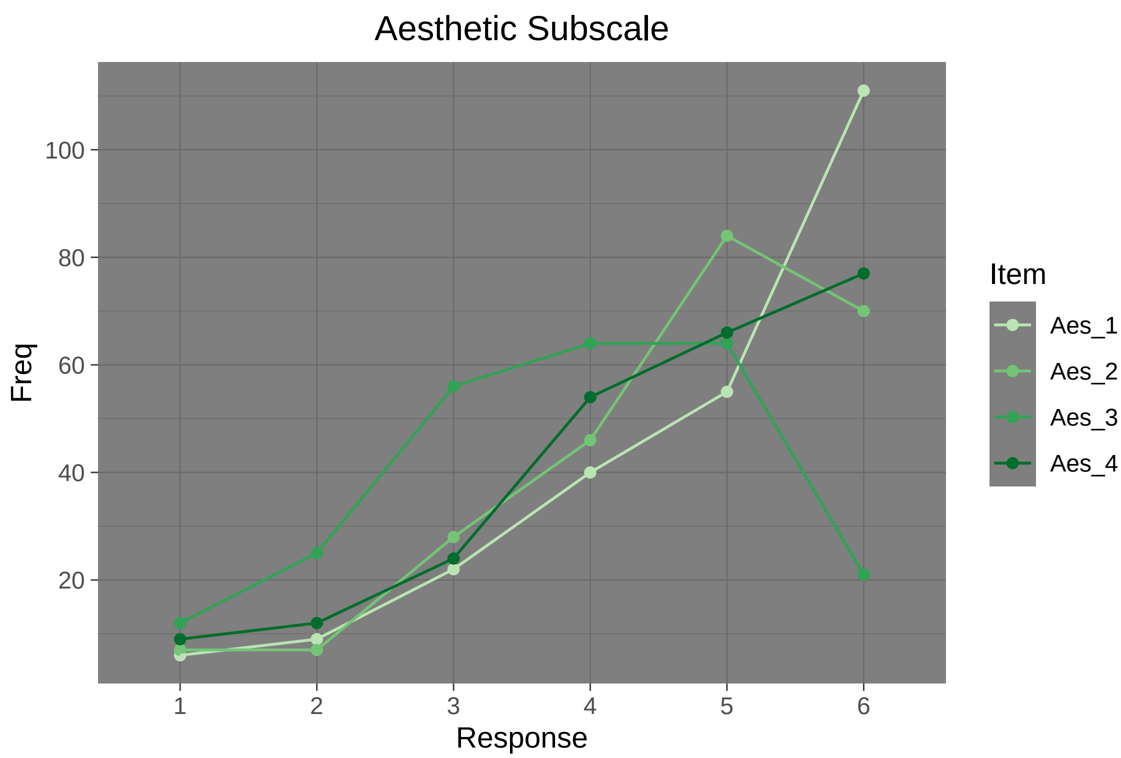



Figure 4.2: Student responses to CFMI aesthetic subscale

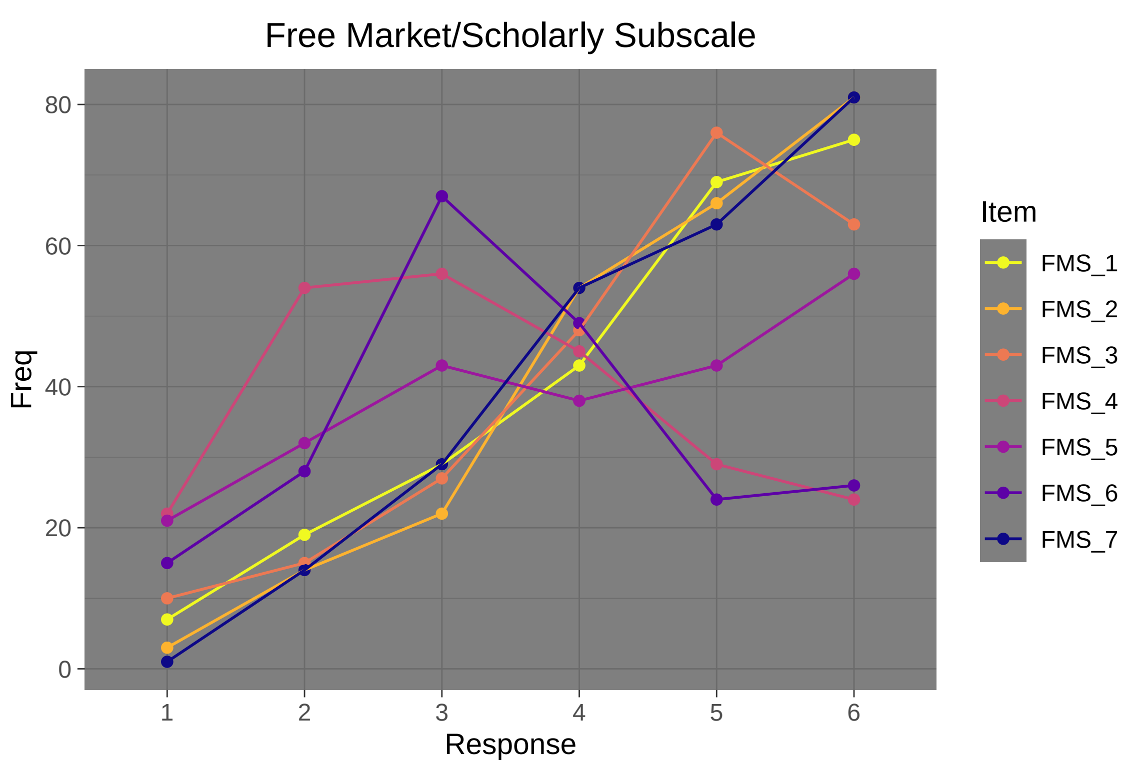



Figure 4.3: Student responses to CFMI free market/scholarly subscales

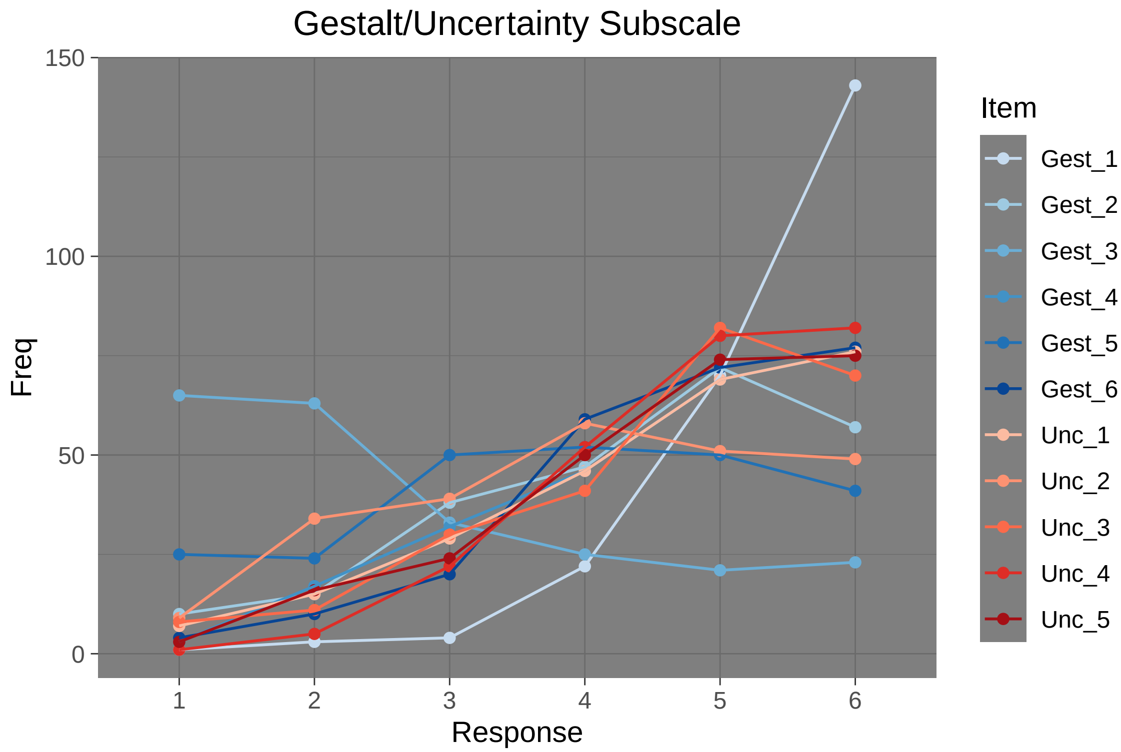



Figure 4.4: Student responses to CFMI gestalt and uncertainty subscales

#### 4.6.1.3 Exploratory Factor Analysis of CFMI Scale

To explore the factor structure of the CFMI Scale, principal component analysis was performed. Table 4.7 presents the proportion of variance explained by each factor, along with ratio of each eigenvalue to the next ($e_i/e_{i+1}$). Note that the first factor explains about 42% of the variance of the data, while the remaining factors account for a relatively small proportion of the variance. This, along with the large ratio of the largest eigenvalues over that of the second, give evidence of a single factor (Tabachnick & Fidell, 2006).

At the same time, inspection of the individual item factor loadings, shown in table 4.8, show some distinctions between the SEP and CSEM Scales. The second dimension appears to be distinguishing between somewhat between social aspects of instruction (free market/scholarly principles) and individual aspects of instruction (Gestalt/uncertainty principles).

Next, the correlation matrix of individual CFMI subscales was inspected. Two items, *Gest_1* and *Gest_3*, showed low or negative correlation with the remaining items in the Gestalt subscale. Due to this, along the previously described inconsistencies in student interpretation to *Gest_3*, these two items were removed from subsequent analysis.

The remaining subscales all demonstrated positive covariances (i.e. polychoric

Table 4.7: Variance explained by first 10 CFMI factors

| Factor (i) | Eigenvalue | % Variance Explained | Cumulative % Variance Explained | Ratio to Next Eigenvalue |
|---|---|---|---|---|
| 1 | 9.2275 | 41.94 | 41.94 | 5.3713 |
| 2 | 1.7179 | 7.81 | 49.75 | 1.2427 |
| 3 | 1.3824 | 6.28 | 56.04 | 1.1602 |
| 4 | 1.1916 | 5.42 | 61.45 | 1.1418 |
| 5 | 1.0436 | 4.74 | 66.20 | 1.0694 |
| 6 | 0.9758 | 4.44 | 70.63 | 1.2949 |
| 7 | 0.7536 | 3.43 | 74.06 | 1.1484 |
| 8 | 0.6562 | 2.98 | 77.04 | 1.0231 |
| 9 | 0.6414 | 2.92 | 79.95 | 1.1450 |
| 10 | 0.5601 | 2.55 | 82.50 | 1.0363 |

Table 4.8: Factor loadings of CFMI items

| Principle | Dim 1 | Dim 2 | Dim 3 | Dim 4 |
|---|---|---|---|---|
| Aes_1 | 0.2374 | -0.1695 | -0.1665 | -0.0627 |
| Aes_2 | 0.2343 | -0.2506 | -0.1871 | 0.1142 |
| Aes_3 | 0.1281 | 0.3087 | -0.5377 | -0.0696 |
| Aes_4 | 0.2218 | -0.1849 | -0.1803 | 0.2860 |
| FMS_1 | 0.2217 | 0.1211 | 0.1178 | 0.0309 |
| FMS_2 | 0.2489 | 0.1382 | 0.1752 | -0.1176 |
| FMS_3 | 0.1776 | 0.3184 | -0.3036 | -0.3330 |
| FMS_4 | 0.2246 | 0.0328 | 0.1304 | -0.1875 |
| FMS_5 | 0.1984 | 0.2823 | 0.3081 | -0.1753 |
| FMS_6 | 0.2057 | 0.1711 | 0.3767 | -0.0688 |
| FMS_7 | 0.2492 | 0.0178 | 0.1100 | 0.0881 |
| Gest_1 | 0.1270 | -0.3733 | -0.0758 | -0.1995 |
| Gest_2 | 0.2596 | 0.1029 | -0.0803 | -0.1744 |
| Gest_3 | 0.0555 | 0.2920 | -0.0661 | 0.6410 |
| Gest_4 | 0.1877 | -0.3699 | 0.0291 | -0.2194 |
| Gest_5 | 0.2043 | 0.0296 | 0.2147 | 0.2938 |
| Gest_6 | 0.2234 | -0.1622 | -0.1613 | 0.2426 |
| Unc_1 | 0.2507 | 0.0080 | 0.0740 | 0.0471 |
| Unc_2 | 0.2349 | 0.0364 | 0.1155 | 0.0496 |
| Unc_3 | 0.2030 | 0.2313 | -0.3082 | -0.0405 |
| Unc_4 | 0.2344 | -0.2798 | -0.0190 | 0.0242 |
| Unc_5 | 0.2421 | -0.0337 | 0.0963 | 0.1048 |

correlations). All covariances within the FMS and Unc subscales were above 0.4. For the aesthetic subscale, *Aes_3* ("My instructor pointed out only standard approaches to problems.") had a low covariance with the other three aesthetic principle items (0.272, 0.293, and 0.562 with *Aes_1, 2*, and *4*, respectively). Since this item may align more with the uncertainty principle, as a test, it was included analysis of the uncertainty scale, showing it did align closely *Unc_3* ("My instructor did not emphasize the importance of asking questions in our problem-solving process."). Thus, Aes_3 may be more appropriately considered as an aspect of *supporting uncertainty in approaching problems*. Gest_4 demonstrated low covariance with Gest_5 and Gest_6 (0.178 and 0.265 respectively).

Then, PCA was used to study individual and combined CFMI subscales separately. The percent variance and Chronbach's alpha of each subscale, along with the combined Gestalt/Uncertainty scales, and the total CFMI scale are described in Table 4.9. Values of Cronbach's alpha greater than 0.8 indicate adequate reliability for study of correlations and differences of means (Nunnally, 1978).

Table 4.9: Percent variance explained and reliability of CFMI subscales

| CFMI Subscale | % Variance Explained | Cronbach's Alpha |
|---|---|---|
| Aes | 58.69 | 0.7103 |
| FMS | 56.14 | 0.8400 |
| Gest | 51.74 | 0.6291 |
| Unc | 60.11 | 0.7916 |
| Gest/Unc | 50.09 | 0.8414 |
| FMS/Gest/Unc | 47.81 | 0.9094 |
| Full CFMI | 45.33 | 0.9117 |

#### 4.6.1.4 Confirmatory Factor Analysis of CFMI Scale

Finally, confirmatory factor analysis (CFA) was used to compare the model fit of the following potential models for measuring CFMI:

- Full 1-factor model: all four initial subscales (AES, FMS, GES, UNC) as one factor
- 3-factor model: Aes, FMS, and Gest/Unc (as one factor)
- 4-factor model: all four initial subscales as individual factors

The 4-factor model yielded a negative definite covariance matrix of the latent variables, an indication that a larger sample size is needed to test this model. Then, due to the low reliability of the aesthetic subscale, this subscale was removed from the analysis to consider the following models:

- Reduced 1-factor model: just the three subscales FMS, GES, UNC as one factor

- Reduced 2-factor model: FMS and GES/UNC (as one combined factor)
- Reduced 3-factor model: FMS, GES, UNC as individual factors.

Again, this last reduced 3-factor model yielded a negative definite covariance matrix, so was omitted from analysis. Case-wise maximum likelihood estimation of data (which imputes missing data) was not available in the 'lavaan' R package. Thus list-wise deletion of missing data was used, i.e. any subscale response with missing data was deleted from analysis. This resulted resulting in sample size of n=195 being used for all models considered. Standard (ordinary least squares) regression was used. Table 4.10 presents the model fit statistics of each of these models.

Table 4.10: CFMI model fit statistics

| Model | Latent Variables | df | Chi Sq | RMSEA (90% CI) | SRMR | CFI | TLI |
|---|---|---|---|---|---|---|---|
| 1 factor | CFMI | 170 | 342.597** | 0.072 (0.061, 0.083) | 0.075 | 0.983 | 0.981 |
| 3 factors | Aes, FMS, Gest/Unc | 167 | 310.678** | 0.067 (0.055, 0.078) | 0.070 | 0.986 | 0.984 |
| 1 factor | FMS/Gest/Unc | 104 | 240.303** | 0.082 (0.069, 0.096) | 0.070 | 0.984 | 0.981 |
| 2 factors | FMS, Ges/Unc | 103 | 231.243** | 0.08 (0.066, 0.094) | 0.068 | 0.985 | 0.982 |

Abbreviations: n = samples size, df = degrees of freedom, Chi Sq = Chi squared statistic, RMSEA = root-mean-square error of approximation, SRMR = standardized root mean, CFI = comparative fit index, TLI = Tucker-Lewis index (non-normed fit index). ** = Significant at alpha=0.001 level

All of these models demonstrate good model fit. Hooper, Coughlan, and Mullen (2008) describe the following guidelines for evaluating model fit:

- RMSEA below 0.08 indicates good model fit.
- SRMR below 0.08 indicates acceptable model fit, with well-fitting models having values less than 0.05.
- CFI values of 0.95 or greater are needed to ensure that mis-specified models are not accepted.
- TLI values of 0.95 or greater indicate good fit, though the TLI is sensitive to sample sizes and may underestimate fit for samples less than 200.

Due to good model fit and the high reliability (Cronbach's alpha) of the FMS and combined Gest/Unc factors, the more parsimonious 2-factor model was chosen for subsequent analysis. Figure 4.5 shows the standardized path coefficients (directed edges) and the covariance between factors (double-sided edge). Note the high covariance (0.93) between the FMS and Gest/Unc factors.

From this model, modification indices were considered, indicating that the inclusion of *Gest_2*, *Gest_4*, and *Unc_4* the FMS latent variable would improve model fit. Additionally, the modification indices indicated that *Gest_2/FMS_3*, *Gest_4/Unc_4*, and *FMS_5/FMS_6* may be related. Implications of this will be discussed in the discussion.

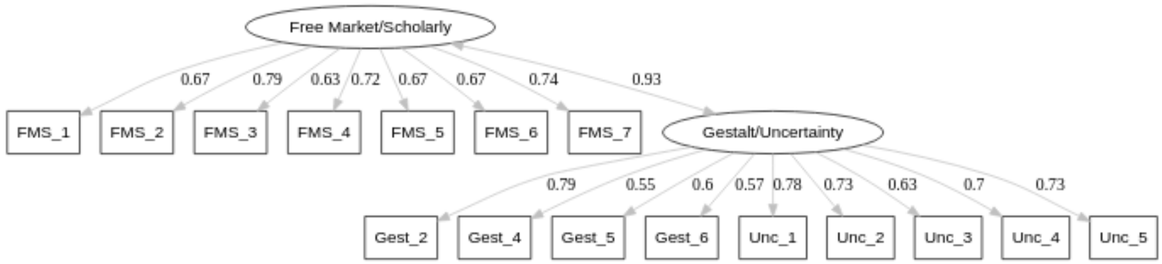



Figure 4.5: CFMI CFA model with path coefficients

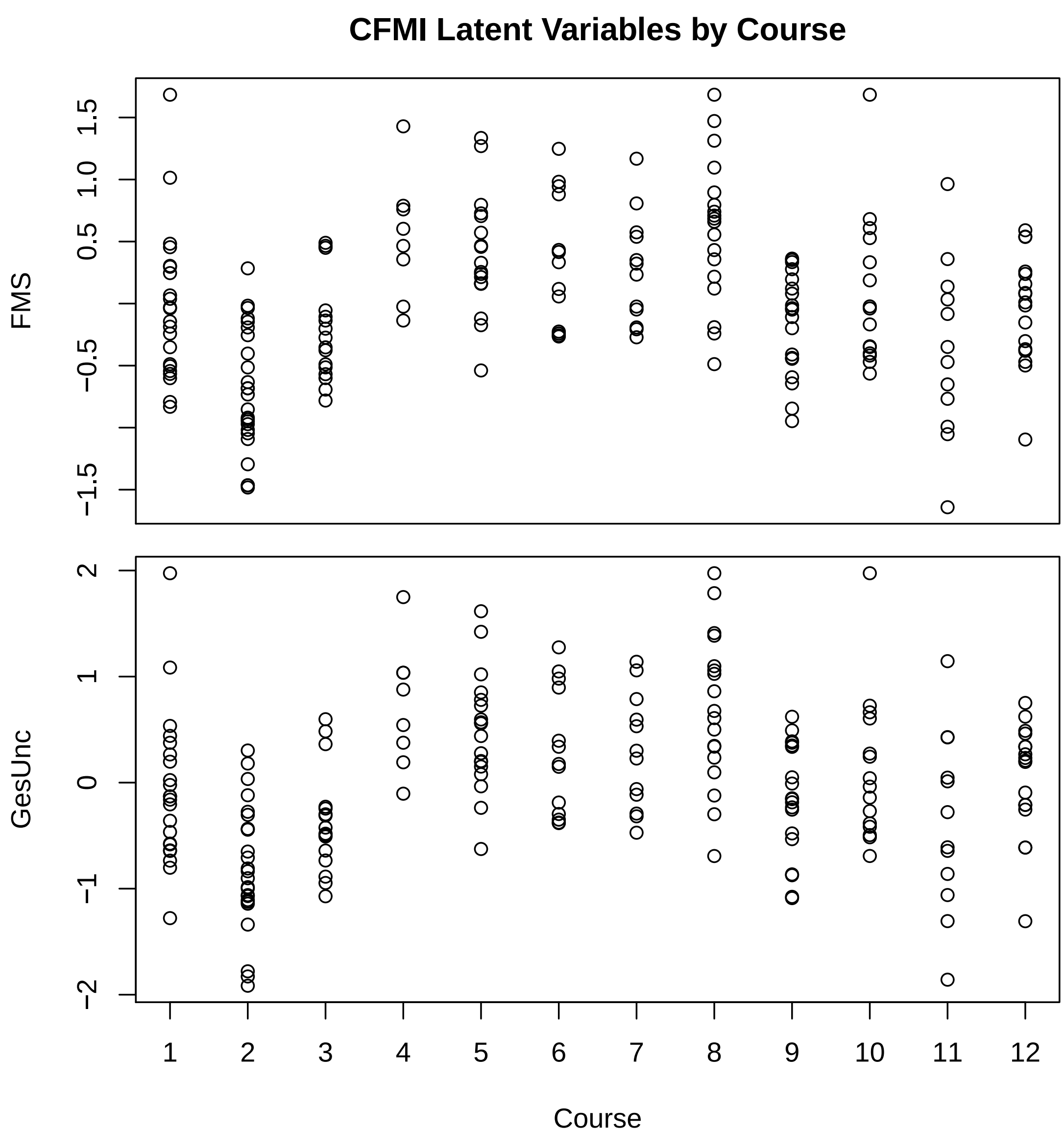



Figure 4.6: Strip chart of FMS and GesUnc latent variables by course

#### 4.6.1.5 Grouping of Data by Course

Because CFMI Scale measures potentially grouped data (by course), the hierarchical nature of the CFMI was studied in several ways. First, the stripchart of the individual factors from the final CFMI model was studied. This demonstrated moderate grouping of data (see Figure 4.6). Then intra-class correlation (ICC) was calculated following Guo (2005) by performing an unconditional ANOVA with random effects on the data with both individual ID and grouped (course) ID, that is

$$Y_{ij} = \mu + \alpha_i + \epsilon_{ij}$$

where $\alpha_i$ is the effect of course $i$, and $Y_{ij}$ is the factor score of student $j$. Form this, the ICC is found by dividing between-group variance $\sigma_\alpha$ by the total variance (sum of between-group variance and within-group variance, $\sigma_\epsilon$):

$$ICC = \frac{\sigma_\alpha}{\sigma_\alpha + \sigma_\epsilon}$$

The ICC for the FMS and GesUnc factors was 0.3341 and 0.3200 respectively. This indicates that around a third of the variance of student responses to these factors is due to group (i.e. course) characteristics, and two-thirds of the variance is due to individual characteristics. Also, these ICCs are above the suggested value of 0.25 needed for considering multilevel modeling techniques such as hierarchical linear modeling (Heinrich & Lynn, 2001). However, larger sample sizes are needed to effectively study CFMI in this way.

Next, to study the nature of the grouping in student CFMI ratings, the model predictions of the two latent variables (FMS vs. GesUnc) were plotted by course. This plot, shown in Figure 4.7, illustrates clustering of the variables by course. Students from Course 2 (large 77-student section of Linear Algebra), shown in blue, rated lower perception of instructor use of FMS and GesUnc actions. The higher-level courses, Courses 5, 6, 7, and 8 (Intro to Number Theory, Abstract Linear A and B, and Intro to Analysis, respectively) show higher ratings. Courses 3 and 9 (small section of Linear Algebra and Graph Theory I, respectively) show clusters of moderate ratings. Also note the grouping of the data along the line $y = x$. This reflects the high correlation (0.93) between the two GesUnc and FMS latent variables.

#### 4.6.1.6 Convergent Validity of CFMI

For initial study of the convergent validity of the CFMI Scale, student responses to C2 ("Were you mathematically creative in this course?") and C3 ("Did this course foster your mathematical creativity?) were considered. To ensure students brought somewhat theoretically consistent view(s) of creativity, responses to C1 ("To you, what does it mean to be mathematically creative?") were considered.

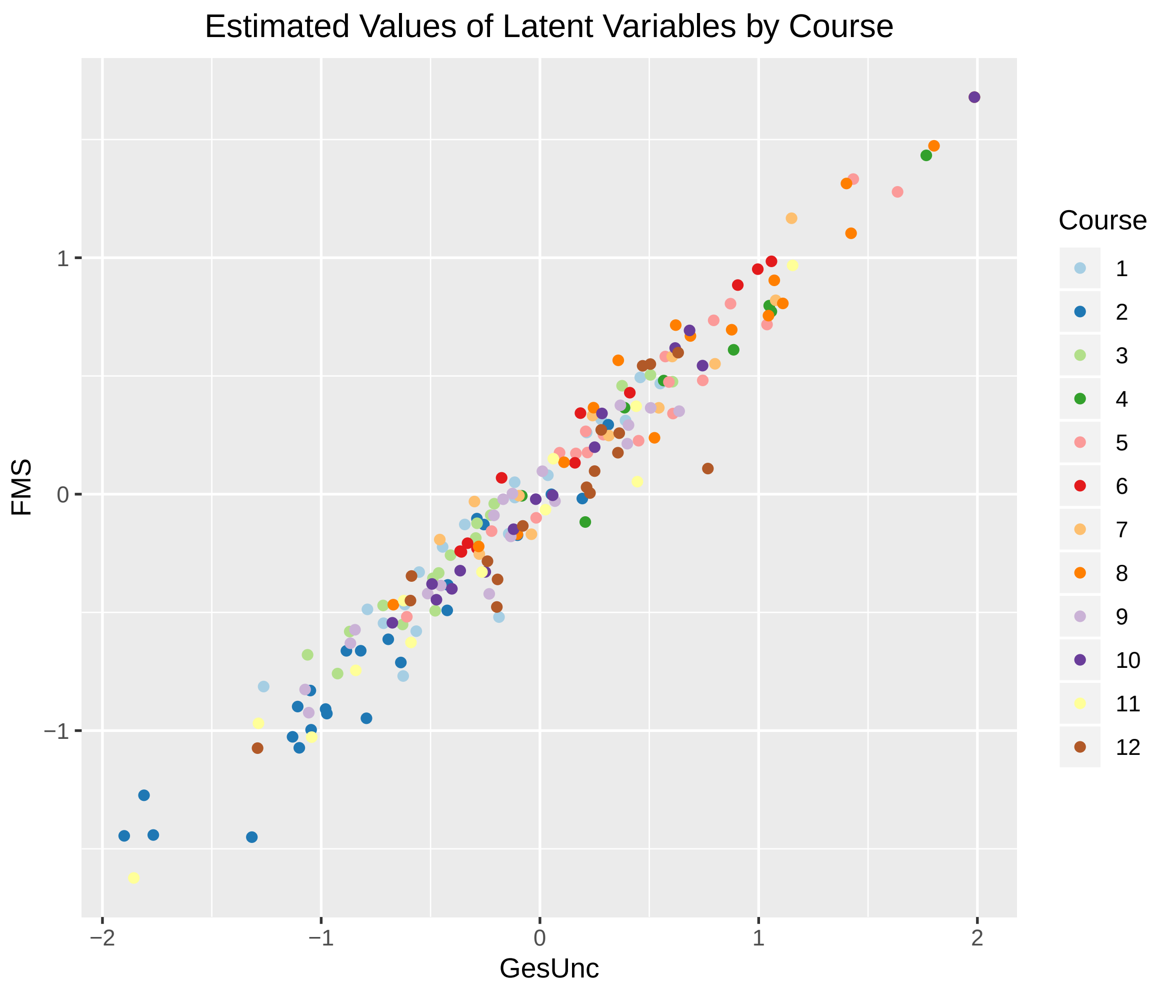



Figure 4.7: Estimated Gestalt/uncertainty and free market/scholarly latent variable estimates by course

Responses 1 correspond to a "process" definition of creativity, while response 2 corresponds to a "product" definition. Multiple choices of a definition of creativity were permitted.

Figure 4.8 presents the distribution of students' responses to question C1. Note that almost all students' views aligned the content and/or process view of creativity provided. Figure 4.9 displays the response of students to C2 and C3 by their definition of creativity (C1) and by course. This data provides a general sense of the views of students regarding mathematical creativity. Most students both described themselves as creative and said their course fostered creativity, but less so in Courses 2, 11, and 12.

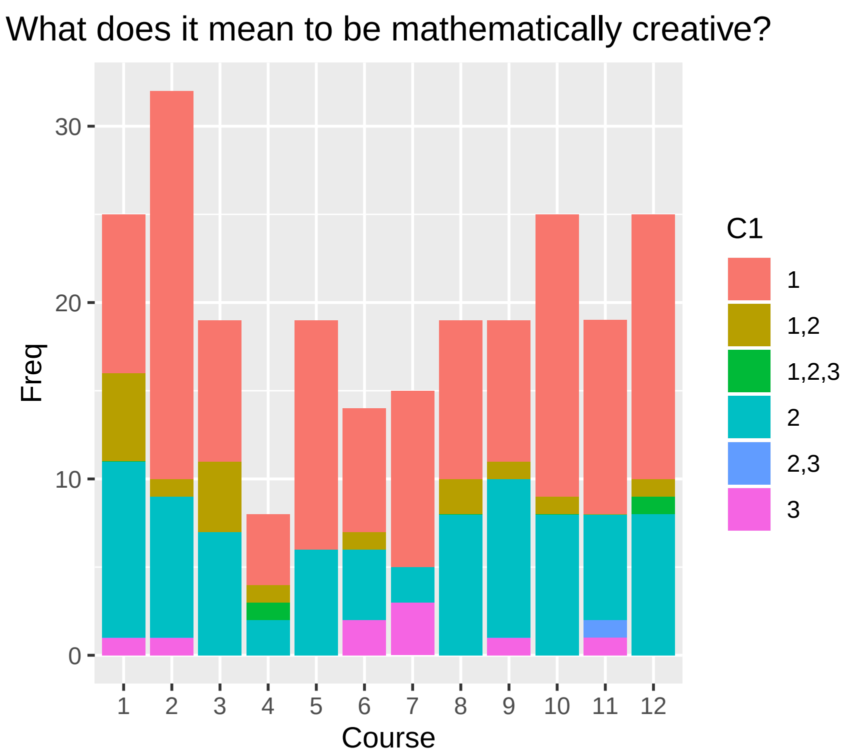



Figure 4.8: Student responses to C1 by course

With the 2-factor CFMI model described in Section 4.6.1.4, regression against student responses to C2 and C3 using SEM in the 'lavaan' package. These results are shown in Table 4.11, showing that student ratings of the FMS was a significant predictor of student perceptions of their own creativity (C2) at the $\alpha = 0.01$ level. Ratings of the FMS also were a predictor of perception of the course's creativity-fostering at the $\alpha = 0.1$ level.

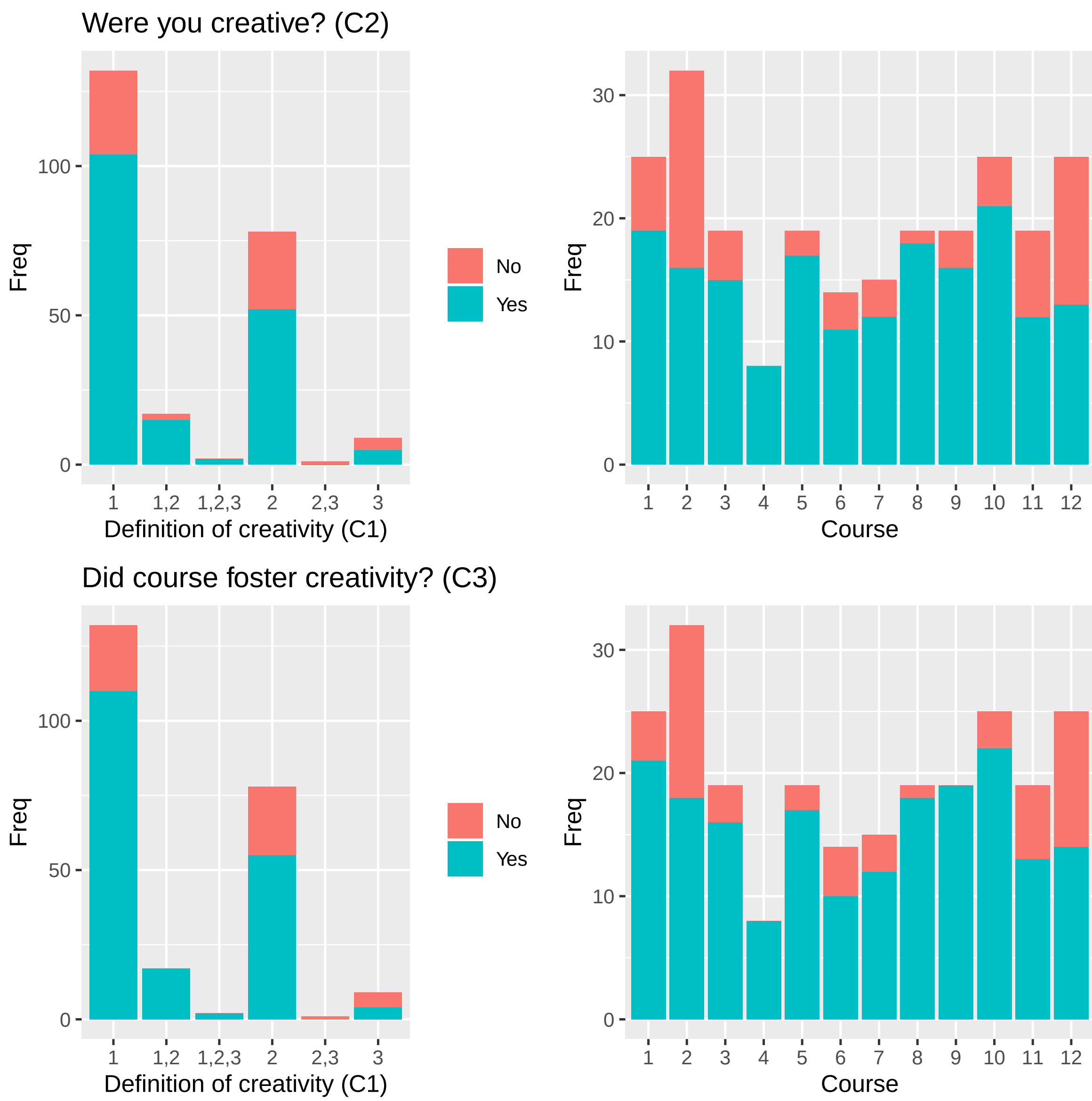



Figure 4.9: Student responses to C2 by course

Table 4.11: Regression of CFMI factors against C2 and C3

| Outcome | Predictor | Estimate | Std. Err | Z-value | P(>\|z\|) |
|---|---|---|---|---|---|
| C2 ~ | GesUnc | -0.759 | 0.513 | -1.480 | 0.139 |
| | FMS | 1.786 | 0.629 | 2.839 | 0.005 |
| C3 ~ | GesUnc | -0.152 | 0.537 | -0.282 | 0.778 |
| | FMS | 1.230 | 0.642 | 1.917 | 0.055 |

### 4.6.2 Analysis of Self-efficacy Scales

After initial study of the distribution of student responses to the self-efficacy scales used, exploratory factor analysis was used to study the relationship of the self-efficacy items. Then confirmatory factor analysis was conducted to inform model selection for future analysis.

#### 4.6.2.1 Response Process Interview

After the responses process interviews, minor changes were made to *CSEM_2* and *CSEM_3* with hopes of better distinguishing these items. These was originally worded, “solve a math problem in multiple ways,” and “generate multiple solutions,” (and revised to “solve a math problem in multiple ways” and to which one student commented, “They are the same, because the whole point of multiple ways [to solve a problem] is to find a solution.” Another student commended, “because of the way I study, [a problem] has one solution, so multiple solutions makes me more uncomfortable [uncertain in her confidence with respect to the other items].” While this response explained why she rated this item “60%”, lower than her other ratings, it does highlight that some students may have little experience approaching or solving problems from multiple perspectives.

#### 4.6.2.2 Response Distribution of Self-efficacy Scales

Of the 285 students responding to the pre-semester survey, 12 students did not respond to any items, apparently stopping the survey before continuing to the SEPS section. Table 4.12 and Figure 4.10 show the distribution of the remaining 273 student responses to the SEP Scale. In the post-semester survey, 7 students did not respond to any of the SEP or CSEM Scale items, leaving 236 student responses. Table 4.13 and Figure 4.11 show the distribution of responses to the SEP and CSEM scales.

Note that for increasingly difficult statements (from Statements 1 to 3), the distribution of responses shifts to the left, indicating that students were less confident in their abilities to prove those statements. Of some concern the high proportion of students responding “100%” to the first SEP Scale subtask, (SEPS1_1, SEPS2_1, SEPS3_1).

These questions correspond to the proving subtask, "Understand and informally explain why a statement is true or false."

Another interesting characteristic of these distributions is the proportion of students responding with "100%" confidence to *all* subtasks, demonstrated by the peak to the right. Even after removing students who indicated that had previously proven the given proof statement, this characteristic of these distributions remained the same.

On the post-semester survey, students were asked, for each proving statement: "Have you previously seen or proved the above statement?" Student responses to these questions are shown in Figure 4.14. Of concern was the possibility that those students having previously proven the statement would give skewed responses (of all 100% confidence). However, only 23.81% of total responses on the SEPS4 were "100%" and 28.33% of total responses on SEPS5 were "100%". Furthermore, the PCA plot of individual responses to SEPS4 and SEPS5 (shown in Figure 4.12) did not indicate that the responses of those having previously proven the provided statement provided significantly different self-efficacy ratings than those who hadn't. Thus, the students who had proven the given statement were included in subsequent analysis.

Table 4.12: Pre-semester SEP Scale response frequency

| | 0 | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| SEPS1_1 | 1 | 2 | 1 | 3 | 4 | 19 | 16 | 30 | 48 | 36 | 113 |
| SEPS1_2 | 5 | 9 | 14 | 26 | 17 | 36 | 32 | 36 | 41 | 20 | 37 |
| SEPS1_3 | 4 | 2 | 7 | 11 | 15 | 33 | 27 | 40 | 43 | 37 | 54 |
| SEPS1_4 | 3 | 16 | 13 | 15 | 18 | 26 | 18 | 30 | 51 | 24 | 59 |
| SEPS1_5 | 8 | 9 | 10 | 15 | 16 | 38 | 28 | 35 | 42 | 30 | 42 |
| SEPS2_1 | 1 | 3 | 10 | 5 | 16 | 21 | 16 | 35 | 42 | 47 | 77 |
| SEPS2_2 | 6 | 10 | 21 | 31 | 21 | 35 | 30 | 38 | 32 | 16 | 33 |
| SEPS2_3 | 4 | 5 | 12 | 20 | 21 | 29 | 30 | 40 | 45 | 19 | 48 |
| SEPS2_4 | 6 | 21 | 16 | 17 | 23 | 28 | 26 | 41 | 39 | 18 | 38 |
| SEPS2_5 | 5 | 11 | 26 | 11 | 23 | 38 | 28 | 39 | 31 | 34 | 27 |
| SEPS3_1 | 4 | 5 | 14 | 12 | 20 | 20 | 29 | 32 | 48 | 32 | 57 |
| SEPS3_2 | 10 | 10 | 19 | 25 | 21 | 36 | 25 | 48 | 36 | 16 | 27 |
| SEPS3_3 | 7 | 8 | 18 | 22 | 23 | 32 | 29 | 37 | 46 | 19 | 32 |
| SEPS3_4 | 10 | 18 | 21 | 23 | 26 | 30 | 21 | 37 | 33 | 23 | 31 |
| SEPS3_5 | 9 | 15 | 21 | 20 | 34 | 34 | 18 | 31 | 36 | 23 | 32 |

Table 4.13: Post-semester SEP and CSEM Scale response frequency

| | 0 | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| SEPS1_1 | 3 | 3 | 2 | 6 | 11 | 19 | 21 | 34 | 37 | 40 | 60 |
| SEPS1_2 | 7 | 7 | 9 | 15 | 18 | 26 | 33 | 33 | 35 | 23 | 30 |
| SEPS1_3 | 4 | 6 | 5 | 7 | 19 | 15 | 27 | 47 | 29 | 34 | 43 |
| SEPS1_4 | 7 | 5 | 15 | 13 | 18 | 18 | 29 | 32 | 31 | 24 | 44 |
| SEPS1_5 | 5 | 10 | 14 | 12 | 18 | 23 | 22 | 31 | 32 | 38 | 31 |
| SEPS2_1 | 2 | 5 | 4 | 14 | 16 | 19 | 28 | 24 | 48 | 29 | 47 |
| SEPS2_2 | 5 | 5 | 15 | 12 | 19 | 25 | 33 | 38 | 33 | 25 | 26 |
| SEPS2_3 | 4 | 6 | 7 | 12 | 13 | 25 | 36 | 37 | 29 | 32 | 35 |
| SEPS2_4 | 8 | 11 | 14 | 19 | 15 | 23 | 30 | 31 | 33 | 21 | 31 |
| SEPS2_5 | 7 | 12 | 12 | 20 | 17 | 19 | 23 | 40 | 37 | 21 | 28 |
| CSEM_1 | 2 | 10 | 17 | 23 | 21 | 39 | 31 | 43 | 29 | 14 | 7 |
| CSEM_2 | 2 | 3 | 3 | 11 | 9 | 24 | 35 | 43 | 37 | 31 | 38 |
| CSEM_3 | 2 | 4 | 10 | 14 | 14 | 24 | 33 | 39 | 42 | 31 | 23 |
| CSEM_4 | 2 | 3 | 7 | 18 | 12 | 28 | 19 | 31 | 49 | 37 | 30 |
| CSEM_5 | 2 | 4 | 5 | 19 | 18 | 29 | 33 | 33 | 38 | 32 | 23 |

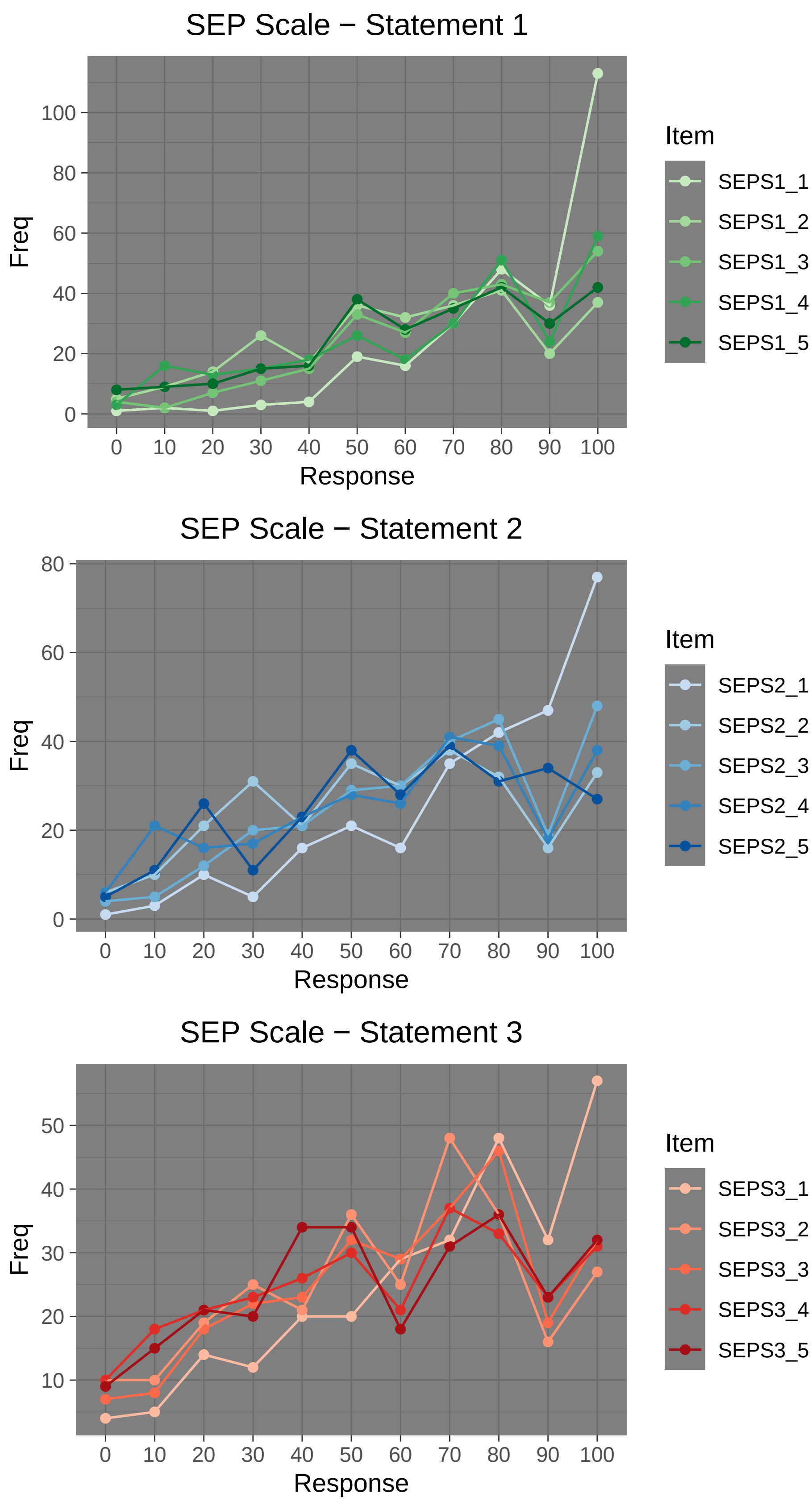



Figure 4.10: Student pre-semester responses to SEP Scale

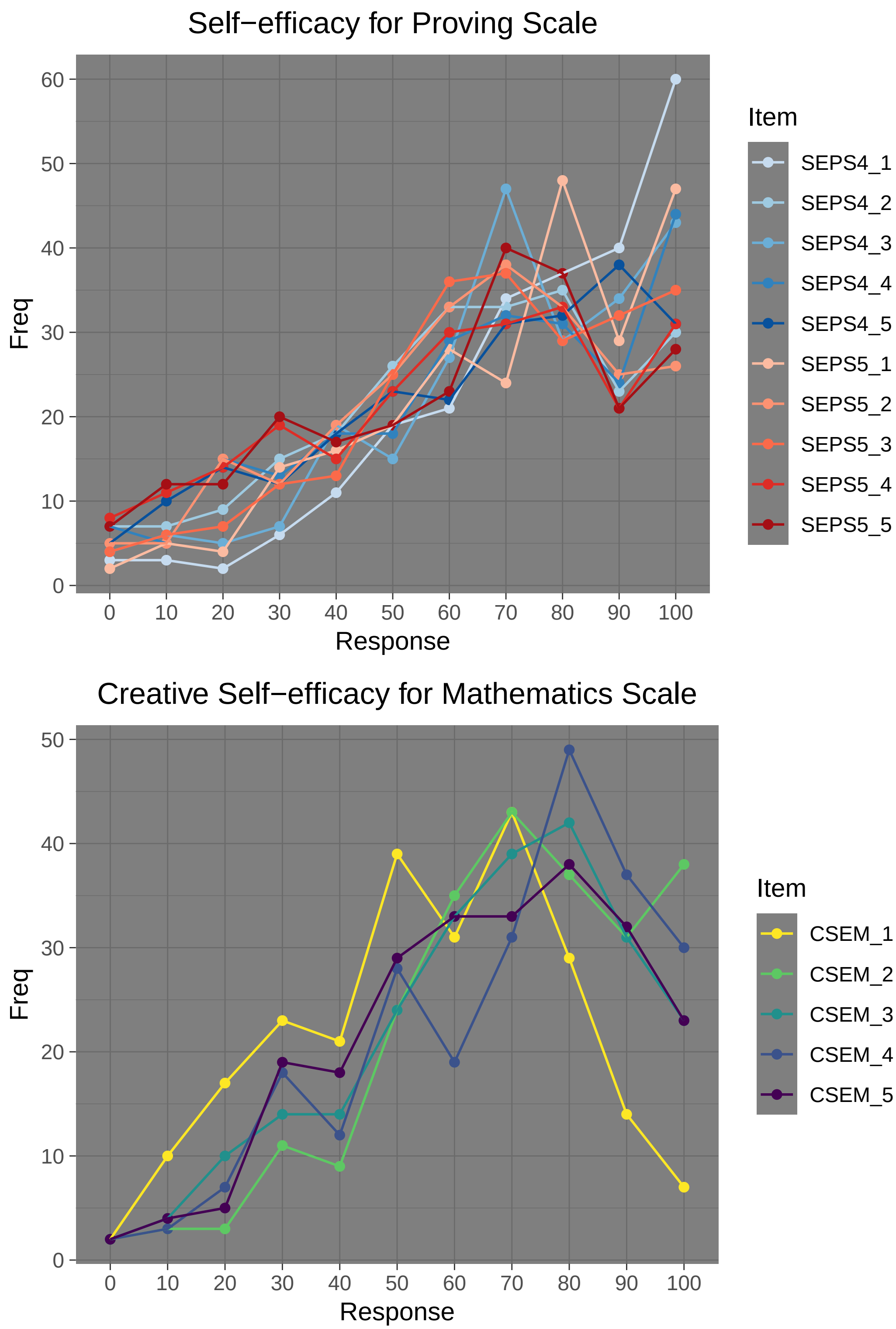
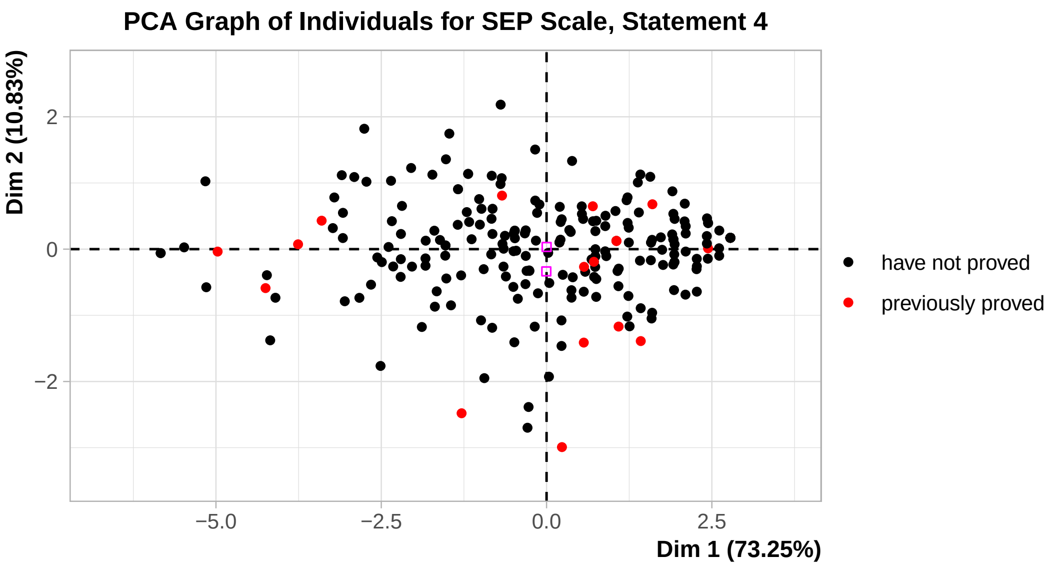



Figure 4.11: Student post-semester responses to self-efficacy scales



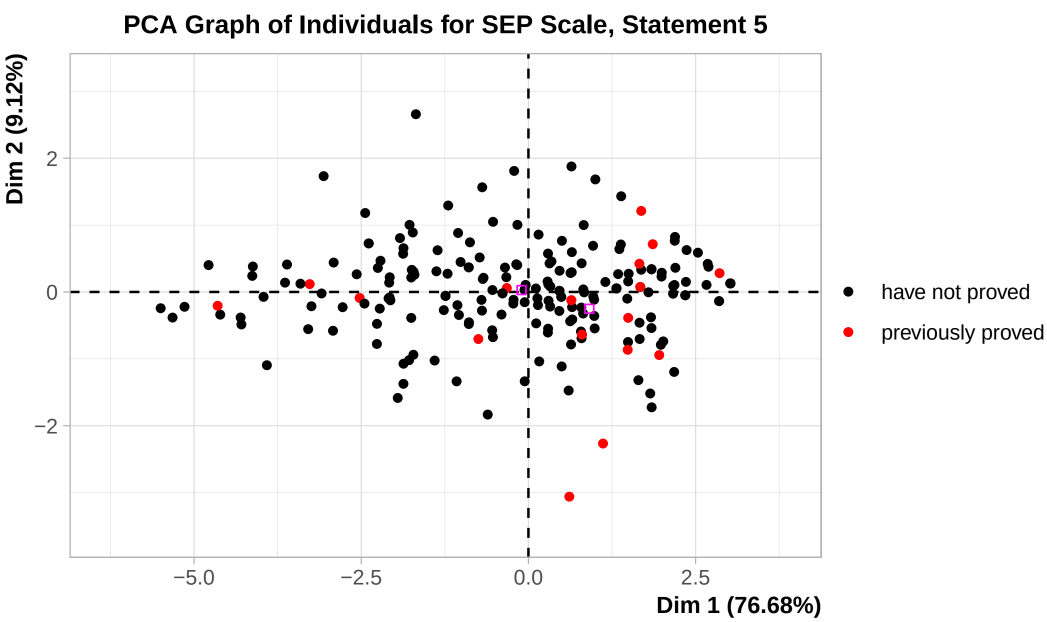




Figure 4.12: PCA graph of first two dimensions for proving statements 1 and 2 (Students who had previously proved statement are shown in red)

Table 4.14: Prior exposure to SEPS proving statements

| Have you previously seen or proved the above statement? | Statement 1 (moderately-routine) | Statement 2 (non-routine) |
|---|---|---|
| No, I have not previously seen nor proved this statement. | 112 | 156 |
| I may have seen something similar, but I cannot remember. | 75 | 54 |
| Yes, I have previously seen this statement, but haven't proved it. | 28 | 2 |
| Yes, I have proved this statement in the past | 21 | 24 |

#### 4.6.2.3 Exploratory Factor Analysis of Self-efficacy Scales

Due to the high frequency of confidence ratings of 100% for the first proving statement on the pre-survey (SEPS1), this data was omitted from analysis. The data from the four remaining SEP statements (SEPS2 and 3 on the pre-semester survey; SEPS4 and SEPS5 on the post-semester survey) were then analyzed individually and as well as together, as "stacked" responses to the same five proving subtasks. The correlation matrix of this data showed high internal correlation between the five proving sub-tasks, ranging from $\sigma_{1,2} = 0.612$ to $\sigma_{4,5} = 0.833$.

Principle component analysis demonstrated that each individual proving subtask demonstrated high internal consistency (see Table 4.15). To compare the structure of the SEP and CSEM scales, principle component analysis was conducted on the post-semester SEP subscales and CSEM Scale. Tables 4.16 and 4.17 show the relative variance explained by each factor and the factor loadings for the SEPS4, SEPS5, and CSEM Scales. While the SEP and CSEM subscales show similar loadings on the first dimension, they are clearly distinguished by the second dimension.

Table 4.15: Percent variance explained and reliabiltiy of self-efficacy subscales

| Subscale | n | % Variance Explained | Cronbach's Alpha |
|---|---|---|---|
| Pre-semester SEPS2 | 273 | 72.79 | 0.9062 |
| Pre-semester SEPS3 | 273 | 76.32 | 0.9235 |
| Post-semester SEPS2 | 236 | 73.25 | 0.9083 |
| Post-semester SEPS3 | 236 | 76.68 | 0.9234 |
| CSEM (post-semester) | 236 | 69.14 | 0.8877 |
| stacked SEPS | 1018 | 74.80 | 0.9160 |

#### 4.6.2.4 Confirmatory Factor Analysis of Self-efficacy Scales

To study model fit of the self-efficacy scales, the following self-efficacy models were considered:

Table 4.16: Relative variance explained and factors of combined self-efficacy instrument

| component | eigenvalue | % var | cumulative % var | ratio to next eigenvalue |
|---|---|---|---|---|
| comp 1 | 8.9632 | 59.75 | 59.75 | 5.6748 |
| comp 2 | 1.5795 | 10.53 | 70.28 | 2.0976 |
| comp 3 | 0.7530 | 5.02 | 75.30 | 1.1261 |
| comp 4 | 0.6687 | 4.46 | 79.76 | 1.1034 |
| comp 5 | 0.6060 | 4.04 | 83.80 | 1.2807 |
| comp 6 | 0.4732 | 3.15 | 86.96 | 1.1435 |
| comp 7 | 0.4138 | 2.76 | 89.72 | 1.0541 |
| comp 8 | 0.3926 | 2.62 | 92.33 | 1.2647 |
| comp 9 | 0.3104 | 2.07 | 94.40 | 1.1772 |
| comp 10 | 0.2637 | 1.76 | 96.16 | 1.4798 |
| comp 11 | 0.1782 | 1.19 | 97.35 | 1.2013 |
| comp 12 | 0.1483 | 0.99 | 98.34 | 1.2572 |
| comp 13 | 0.1180 | 0.79 | 99.12 | 1.3418 |
| comp 14 | 0.0879 | 0.59 | 99.71 | 2.0269 |
| comp 15 | 0.0434 | 0.29 | 100.00 | |

Table 4.17: PCA component Loadings for self-efficacy instrument

| Item | Dim.1 | Dim.2 | Dim.3 | Dim.4 | Dim.5 |
|---|---|---|---|---|---|
| SEPS4_1 | 0.7185 | -0.23 | 0.26 | -0.4000 | 0 |
| SEPS4_2 | 0.8327 | 0.00 | -0.20 | -0.1011 | 0 |
| SEPS4_3 | 0.7894 | 0.04 | -0.43 | -0.1675 | 0 |
| SEPS4_4 | 0.8471 | -0.26 | 0.21 | -0.0682 | 0 |
| SEPS4_5 | 0.8361 | -0.22 | 0.27 | -0.0495 | 0 |
| SEPS5_1 | 0.7282 | -0.40 | -0.02 | 0.0767 | 0 |
| SEPS5_2 | 0.8534 | 0.00 | -0.20 | 0.0136 | 0 |
| SEPS5_3 | 0.8326 | -0.12 | -0.44 | 0.1089 | 0 |
| SEPS5_4 | 0.7892 | -0.41 | 0.05 | 0.1991 | 0 |
| SEPS5_5 | 0.8436 | -0.33 | 0.15 | 0.1904 | 0 |
| CSEM_1 | 0.6445 | 0.44 | 0.22 | 0.3698 | 0 |
| CSEM_2 | 0.6922 | 0.47 | 0.09 | -0.2758 | 0 |
| CSEM_3 | 0.7432 | 0.45 | 0.06 | -0.2727 | 0 |
| CSEM_4 | 0.6821 | 0.42 | 0.15 | 0.1738 | 0 |
| CSEM_5 | 0.7179 | 0.40 | -0.07 | 0.2282 | 0 |

- Model 1 (one factor): all 15 items as one factor with SEP subscales items correlated
- Model 2 (two factors): *SEP Subscales 4* and *5* as one factor with correlated subscale items and CSEM as second factor
- Model 3 (three factors): *SEP Subscale 4*, *SEP Subscale 5*, and *CSEM* as separate factors with correlated SEP subscale items.
- Model 4 (three factors): Model 3 with *CSEM_2* and *CSEM_3* correlated.

This last model accounted for the fact that students appeared to perceive *CSEM_2* and *CSEM_3* to asking the same thing. These items were intended to measure distinct aspects of creativity, flexibility and fluency respectively; however, the response process interviews demonstrated that two out of three students were perceiving these to be the same thing. One student, when asked about their rating for *CSEM_3* ("generate multiple solutions"), responded, "same as one before [*CSEM_2*]. In calculus, every problem has one solution." Even after rewording *CSEM_3* (to "give multiple solutions to a math problem") following the responses process interviews, this pattern seemed to persist as the correlation between these two items was much higher (0.8308) than the remaining CSEM item correlations (0.6244 or lower).

The model fit statistics for these four models are shown in Table 4.18. While Model 4 demonstrated the best fit, the RMSEA (0.101) shows a moderate probability of poor model fit. To account for the students having previously proven statements, the above models were also tested using the subsample of students who hadn't seen the SEP proof statements. This gave very minor improvement in model fit indices.

In the end, Model 4 was chosen for correlational analysis as the best available model. Furthermore, the high reliability of the individual SEPS4, SEPS5, and CSEM factors gives evidence that they are each measuring one underlying construct. Figure 4.13 shows the standardized path coefficients, along with the covariances between the latent variables. All paths in the model were significant at the $\alpha = 0.001$ level. The strongest latent variable correlations, 0.87, between SEPS4 and SEPS5 give evidence of convergence validity of the self-efficacy for proving scales.

As with the CFMI model, modification indices were also considered for the final self-efficacy model. This indicated that inclusion of SEP subscale item 2 ("Explore new ideas to come up with ways to start your proof") in the CSEM latent variable would further improve model fit.

#### 4.6.2.5 Convergent Validity of Self-efficacy Scales

Questions C2 ("Were you mathematically creative in this course?") was used to study the relationship between the self-efficacy scales and creative self-belief. Regressions using the latent variables above showed that CSEM was a statistically significant predictor of student creative self-belief ($p<0.001$), but that SEP4 and SEP5 were not.

Table 4.18: Fit indices for self-efficacy models

| Model | n | Chi Sq | RMSEA (90% CI) | SRMR | CFI | TLI | AIC | BIC |
|---|---|---|---|---|---|---|---|---|
| 1 | 236 | 898.005** | 0.201 (0.190, 0.213) | 0.083 | 0.762 | 0.706 | 30312.34 | 30433.61 |
| 2 | 236 | 664.626** | 0.171 (0.159, 0.183) | 0.069 | 0.830 | 0.788 | 30081.00 | 30091.59 |
| 3 | 236 | 349.213** | 0.118 (0.105, 0.130) | 0.074 | 0.922 | 0.900 | 29769.58 | 29901.21 |
| 4 | 236 | 230.599** | 0.101 (0.088, 0.114) | 0.067 | 0.943 | 0.926 | 29697.58 | 29832.67 |

Abbreviations: n = samples size, df = degrees of freedom, Chi Sq = Chi squared statistic, RMSEA = root-mean-square error of approximation, SRMR = standardized root mean, CFI = comparative fit index, TLI = Tucker-Lewis index (non-normed fit index). ** = Significant at alpha=0.001 level

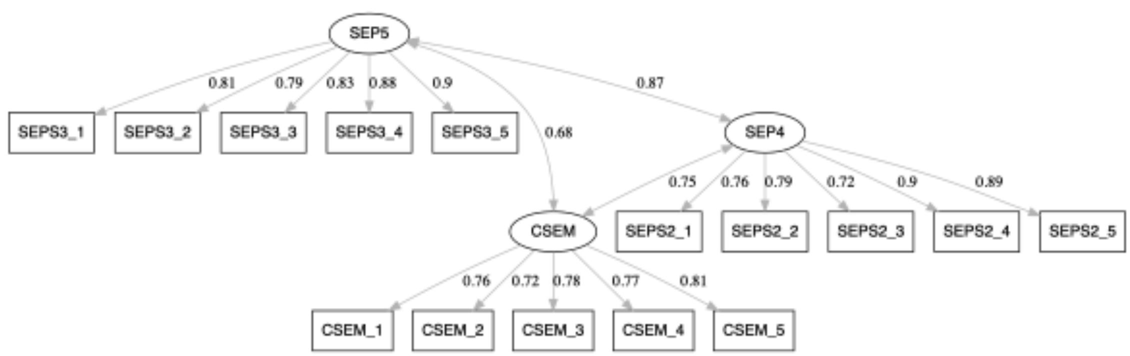



Figure 4.13: Factor structure of final self-efficacy model

### 4.6.3 Analysis of AMTM Scale

Pre-semester student responses to the AMTMS were analyzed using confirmatory factor analysis using the same 5-factor model (AMOT, EMER, EMIN, EMID, IMOT) as described by Lim and Chapman (2015). Since the AMOT factor was omitted from post-semester surveys, three-factor (EMIN, EMID, IMOT) and four-factor (EMER, EMIN, EMID, IMOT) models were considered using the combined pre- and post-semester data. The three- and four-factor models were also considered with the pre- and post-semester data separately. However, the combined four-factor model (n=416) and both three and four-factor post-semester models (n=234) yielded covariance matrices of latent variables that were not positive definite. The fit indices for the remaining models are presented in Table 4.19. These indices indicate significantly worse model fit than those described in Lim and Chapman (2015), which are reproduced in Table 4.20.

Although three questions were changed, among the subscales that were not changed (EMIN, EMER, and EMID), there were items which much lower loadings the described by Lim and Chapman (2015); there were four items with loadings between 0.49 to 0.53. These subscales also demonstrated inter-scale correlations as low as 0.171. This may be due to the differences in the samples studied. Lim and Chapman (2015) studied grade 11 and 12 students in pre-tertiary institutions in Singapore. The poor model fit found in this study may also be due to factors related to the high rate of missing data from this section of the survey such as low attention of a subset of students filling out this section of the survey. For last optional question in the post-survey ("Please let me know if you have any questions or comments about this survey, or if you would like to elaborate on any of your responses"), which one student described frustration that this survey "seemed like the same three questions reworded fifty times." Another student commented in person, that the AMTM responses scale ("Does not correspond at all", "Corresponds a little", "Corresponds moderately," etc.) confusing, though this student did not elaborate.

Table 4.19: Fit indices for AMTM Scale

| Model | n | Chi Sq | df | RMSEA (90% CI) | SRMR | CFI | TLI |
|---|---|---|---|---|---|---|---|
| Five-factor | 177 | 1400.497** | 179 | 0.197 (0.187, 0.207) | 0.176 | 0.869 | 0.846 |
| Four-factor | 182 | 1051.177** | 113 | 0.214 (0.202, 0.226) | 0.174 | 0.881 | 0.857 |
| Three-factor | 184 | 593.319** | 62 | 0.216 (0.201, 0.232) | 0.172 | 0.917 | 0.896 |
| Three-factor | 417 | 1794.999** | 62 | 0.259 (0.249, 0.270) | 0.176 | 0.737 | 0.669 |

Table 4.20: Fit indices of AMTM Scale reproduced from Lim and Chapman (2015)

| Model | n | Chi Sq | df | RMSEA (90% CI) | SRMR | CFI | TLI |
|---|---|---|---|---|---|---|---|
| Five-factor | 805 | 1674.02** | 340 | 0.070 (0.066, 0.073) | 0.052 | 0.96 | 0.95 |
| Four-factor | 805 | 2808.07** | 344 | 0.094 (0.091, 0.098) | 0.088 | 0.97 | 0.97 |

### 4.6.4 Correlational Analysis

Several complications arose from the use of the anonymous pre/post semester identification code. Firstly, for the first two classes taking the post-semester survey, Applied Statistical Methods B & C, the questions for the linking code had not been included. Thus, 36 students had no pre-semester ID code for which to link.

Of the students taking the post-survey, 188 provided codes, 173 of which were unique. Initially, it was assumed that duplicated codes described the same student. Indeed, toward the end of the process of administering surveys in class, students would comment that they had taken this survey in a previous course. These students were asked to take it again, since the CFMI Scale explores their experience in that class. However, it was found that for three responses indicating the same identification code, one of the responses was participating in a course at a different university than the other two. Thus, the sequence of questions asked **failed to create unique identifiers**!

Additionally, while administering the surveys, students indicated in person that they did not remember parts of their previous ID code. In the final optional survey question, "Please let me know if you have any questions or comments about this survey, or if you would like to elaborate on any of your responses," five students made comments like, "I can't remember if I said my shoe size was 7 or 8 last time?" or "I don't remember exactly what I put for my favorite color originally. So it might not match."

Only 71 out of the 173 unique post-semester identification codes were linked with pre-semester identification codes. Out of the 71 potential matches, 29 of these cases indicated one or more courses on pre- and post-semester surveys that were not in one-to-one correspondence. In these cases, it was not clear from the remaining data how to match those students' pre- and post-semester responses. Thus, it determined that it was not was not possible to accurately link pre- and post-semester student responses.

Thus, instead of studying the correlation between the CFMI and the *pre- to post-semester change* of student motivation and self-efficacy, analysis was limited to studing the correlation between student post-semester responses to the CFMI and self-efficacy subscales. For this, both linear regression and a mediation model were considered, described in the next subsections.

#### 4.6.4.1 SEM Regression

Using structural equation modeling (Bollen, 1989), multiple-regression was run using the final CFMI and self-efficacy models described in Sections 4.6.1.4 and 4.6.2.4, respectively. Robust estimation was used with model test statistics of $\chi^2 = 654.680$ with 293 degrees of freedom ($p < 0.001$). Below are the regression statistics of the latent variables tested.

Table 4.21: Multiple regression statistics of self-efficacy latent variables vs. CFMI

| Outcome | Predictor | Estimate | Std.Err | z-value | P(>\|z\|) | Std.lv | Std.all |
|---|---|---|---|---|---|---|---|
| SEPS4 ~ | GesUnc | 4.568 | 6.739 | 0.678 | 0.498 | 0.225 | 0.225 |
| | FMS | 6.054 | 8.407 | 0.720 | 0.471 | 0.248 | 0.248 |
| SEPS5 ~ | GesUnc | 0.279 | 8.020 | 0.035 | 0.972 | 0.012 | 0.012 |
| | FMS | 11.837 | 10.071 | 1.175 | 0.240 | 0.435 | 0.435 |
| CSEM ~ | GesUnc | 2.099 | 6.124 | 0.343 | 0.732 | 0.105 | 0.105 |
| | FMS | 11.593 | 7.725 | 1.501 | 0.133 | 0.483 | 0.483 |

Although none of the estimates are statistically significant, the strongest relationship was found to be between FMS aspects of CFMI and CSEM with a standardized estimate (correlation) of 0.464 ($p = 0.141$). This model described a strong correlation between FMS and GesUnc subscale ratings (0.938), as well as between CSEM and SEP Scale ratings (0.712), both significant at the $\alpha = 0.001$ level.

#### 4.6.4.2 Mediation

To further explore the potential relationship between the FMS and GesUnc CFMI factors and student SEP and CSEM, two mediation models were tested. The first explored the effects FMS on SEP. The second explored the effects of GesUnc (aspects of CFMI) on CSEM. These models were considered to explore the interrelated roles each aspect CFMI can play in the formation of self-efficacy, as explained by self-efficacy theory (Bandura, 1997). The broader model in which this theory is situated, triadic reciprocal causation, describes environmental factors (e.g. GesUnc and FMS aspects of CFMI) and personal determinants (e.g. SEP and CSEM) as reciprocally interacting to determine individual behavior (e.g. experiences proving). The author conjectured that the classroom mediated experiences impact CSEM in a way that changes one's experience of proving, which in turn shapes SEP. For testing these models, the SEPS4 and SEPS5 subscales were both analyzed separately, but gave very similar results. Thus, the results from using SEPS4 are used. For all tests, diagonally weighted least squares (DWLS) robust estimation was used on 188 observation. The regression estimates for the first mediation model using are shown in Table 4.22, along with the parameters defined to study the indirect and total effects of FMS (aspects of CFMI) on SEP. The Chi-squared statistic ($\chi^2 = 533.378$) on 294 degrees of freedom was significant at the $p < 0.001$ level.

Table 4.22: Regression statistics of first mediation model

| | Coefficient | Estimate | Std.Err | z-value | P(>\|z\|) | Std.lv | Std.all |
|---|---|---|---|---|---|---|---|
| Regression | | | | | | | |
| SEPS4~FMS | (a) | -4.642 | 8.722 | -0.532 | 0.595 | -0.190 | -0.190 |
| GesUnc~FMS | (b) | 1.127 | 0.082 | 13.693 | 0.000 | 0.941 | 0.941 |
| SEPS4~GesUnc | (c) | 4.344 | 6.869 | 0.632 | 0.527 | 0.213 | 0.213 |
| CSEM~FMS | (d) | 13.889 | 2.530 | 5.489 | 0.000 | 0.580 | 0.580 |
| SEPS~CSEM | (e) | 0.792 | 0.121 | 6.542 | 0.000 | 0.776 | 0.776 |
| Effect | | | | | | | |
| indirect | (b*c) | 4.898 | 7.715 | 0.635 | 0.526 | 0.200 | 0.200 |
| indirect | (d*e) | 10.996 | 2.220 | 4.952 | 0.000 | 0.450 | 0.450 |
| totaleffect | (a+b*c+d*e) | 11.252 | 2.396 | 4.697 | 0.000 | 0.460 | 0.460 |

This model described CSEM as a significant mediator of the effect of FMS (aspects of CFMI) on SEP, while FMS had no direct significant effect on SEP. Figure 4.14 illustrates the effects tested with this model, where the statistically significant effects are drawn using solid arrows and insignificant effects drawn as dashed arrows.

The second mediating model was also tested to study the effect of GesUnc (aspects of CFMI) on CSEM. The regression statistics are shown below and illustrated in Figure 4.15. This model described SEP as mediating the effects of GesUnc (aspects of CFMI) on CSEM, while individual (GesUnc) CFMI had no significant direct effects on CSEM.

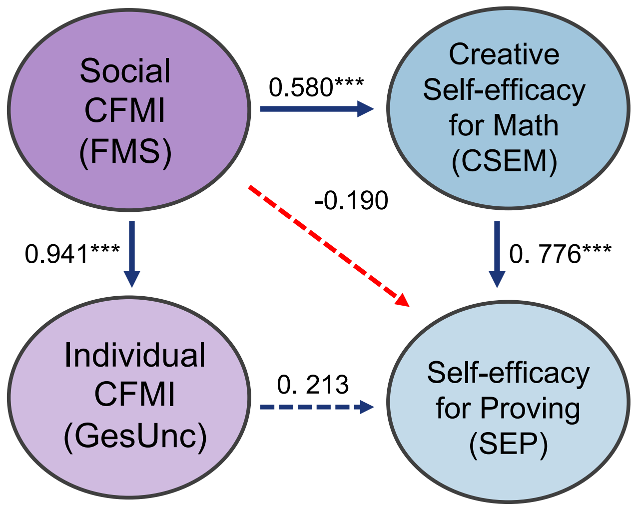



Figure 4.14: Direct and mediating effects of FMS (aspects of CFMI) on self-efficacy for proving

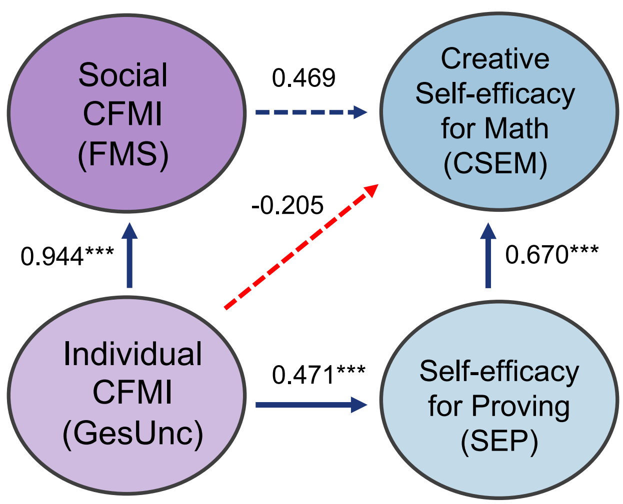



Figure 4.15: Direct and mediating effects of GesUnc (aspects of CFMI) on self-efficacy for proving

## 4.7 Discussion

Due to methodological issues, this project was not able to directly answer the research question: how does creativity-fostering instruction (CFMI) impact student self-efficacy and motivation toward mathematics? Errors in the way the anonymous code for linking pre-post semester student responses made it impossible to accurately study students' pre- to post-semester change in self-efficacy and motivation. Furthermore, the study's adaptation of the instrument for measuring motivation, the AMTM Scale (Lim & Chapman, 2015), yielded poor model fit. This demonstrates a potential need for research developing or adapting instruments for measuring motivational regulation in tertiary mathematics courses.

This said, student perception of CFMI appears to show some relationships with student end-of-semester self-efficacy, especially for the social aspects of instruction characterized by the free market and scholarly principles. More research is needed with similar populations of students utilizing improved methods for studying changes between pre- and post-semester student self-efficacy and motivation. Despite these limitations, the results of this research have important implications for educators and researchers.

### 4.7.1 Implications for Instructors

Along with the results of Chapter 2, the results of this study highlight the important role social or collaborative aspects of instruction (the free market and scholarly principles) may play in building student self-efficacy for proving. The potential effects of general creative self-efficacy for mathematics highlight the potential importance of teaching actions aligned with the free market and scholarly principles in building student self-efficacy for creative mathematical engagement. It is particularly interesting that the first mediation model **did not** show a strong direct relationship between CFMI and self-efficacy for proving. Rather, student general perceptions of their creative self-efficacy appeared to serve as a mediator between the perception of collaborate CFMI and self-efficacy for proving. Furthermore, while student perception of social and individual aspects of CFMI demonstrated differential effects on self-efficacy, social and individuial aspects of CFMI were highly correlated (0.94).

This illustrates how, while any aspect of CFMI may expose students the general idea that they can be creative, collaborative CFMI may be more effective for some students in fostering creative identities due to peer influences. Collaborative CFMI likely provides more opportunities for exposing and changing self-beliefs that limit creative engagement with mathematics, and thus may foster a deeper and more purposeful creative engagement with individual proving than is possible through individual instructor support alone. Insofar as a general sense of one's creative abilities fosters a deeper engagement and effort in proving, subsequent positive experiences proving challenging tasks then provide powerful indicators of students' ability for proving and consequently, increased SEP. A similar process may also translate process to lower level mathematics in which challenging open-ended tasks are used in association with collaborative CFMI.

Better understanding of the ways in which CFMI impacts student self-efficacy may also foster to a better understanding of how instructors develop self-efficacy for creative or innovative teaching. Although not directly apparent from this study, my own experience as an instructor illustrates the role collaborate instructor coordination and support can play in supporting self-efficacy (in a manner analagous to the role of CFMI can play in supporting student self-efficacy). In my experience, where there were community-based or community-supported efforts to change teaching practices (e.g. to utilizing group-based active learning), instructors unfamiliar to the instructional methods were first exposed to others' conviction for and successful experiences teaching a given way. Through social comparison and investment in the belief and positive experiences of other instructors, new instructors gained a general self-efficacy in their ability to use new instructional methods. This initially enabled and sustained, in the face of uncertainty, efforts to adopt new practices. Subsequent teaching experiences and reflection on those experiences highlighted both perceived successes and failures. Successes, recounted with peer support (through regular instructor meetings and one-on-one conversations) reinforced context-specific instructor self-efficacy for new practices. Ways in which instructors perceived new instructional methods to be ineffective (e.g. struggles to cover all the required material), were addressed by support from other instructors (sharing how they think about and prioritize diferent instructional goals). In this way, collaborative intructor support can provide opportunities for building and reinforcing of both context-general and context-specific instructor self-efficacy.

### 4.7.2 Implications for both Instructors and Researchers

The analysis of the student responses to the CFMI Scale can give instructors and researchers a better picture of the extent to which creativity-fostering teaching practices were commonly used or perceived as appropriate, as well as a practical understanding of how these practices appear to be related. For example, practices of encouraging students' problem-posing and building on one another's' ideas were, from students' perspectives, not a common aspect of instruction (see *FMS_4* on Figure 4.3). Over 15% of students students even described instructor practices of posing problems, debating and discussing ideas, or building on student ideas as "not applicable."

Study of the relationship between item scales also can provided interesting insight to exisiting instruction. The correlation between *Gest_4* ("My instructor discussed how solving problems often requires a lot of time.") and *Gest_5* ("My instructor allowed for freedom of time to work through problems.") was surprising low (0.178). The high proportion of student disagreeing to *Gest_5* is evidence that the instructors studied may have described how problem solving requires a lot of time, but did not as readily foster the freedom of time needed for individual problem solving or proving in the context of their class.

The CFMI item covariances, CFMI model the modification indices, and the latent variable model covariances all give evidence that the Free Market, Scholarly, Gestalt, and Uncertainty principles are very closely related. This represents one of the fundamental challenges of studying actions that can have both diverse purposes and effects. Yet, some of the connections described by this data may help inform instructors seeking to foster creativity. As indicated by CFMI model modification indices, the following Gestalt/uncertainty items appear to be particularly related to free market/scholarly principles.

- Gest_2: My instructor allowed us to approach a problem in a way that was different from theirs.
- Gest_4: My instructor discussed how solving problems often requires a lot of time.
- Unc_4: My instructor described that doing mathematics can be challenging at times.

These specific instructor actions may be related to students' perceptions of taking risks in mathematics. For example, the prospect of investing lots of time in solving or proving a problem that may not be correct or match the instructors' approach may be recognized by students as a risk. Any instructor actions that reduce this sense of risk may may improve student willingness to invest their time and energy in mathematical thinking, and subsequently increase the likelihood of students reciprocally engaging, sharing, and building on one another's ideas in class.

### 4.7.3 Implications for Researchers

Good model fit of the CFMI Scale and moderate reliability of the FMS and GesUnc variables (0.8400 and 0.8414, respectively) give evidence that the CFMI Scale is a robust instrument for measuring student perception of instruction. The high correlation (0.93) between the two GesUnc and FMS variables, as well as the significant correlation between the FMS variable and students' perceptions of creativity in the classroom and their own creativity give evidence of convergent validity. Although individual aspects of CFMI measured in this study did not correlated with student creativity, this may be due to not controlling for prior self-efficacy.

The close relationship between social and individual aspects of CFMI may be an indication that, in order for CFMI teaching actions to be effective, they must be considered and implemented holistically. The results of this study indicate that social aspects of instruction are more directly related to confidence in students' own creative abilities. It may be especially important that mathematics instruction utilize social aspects of CFMI due to fact that, in isolation, high perception of individual CFMI was correlated (though not significantly) with lower self-efficacy for proving. It may be the case that employing individual aspects of CFMI alone may have a detrimental impact on students' mathematical creativity and self-efficacy.

The relatively high frequencies of students responding "NA" to the CFMI FMS subscale raises some questions of content validity. The observed pattern of clustered "NA" responses was not observed in the other subscales, suggesting that the FMS subscale addresses teaching actions that lower-level students may not be as familiar with and/or consider as a viable part of instruction.

Chapter 2 presents evidence of content and convergent validity of the SEPS. The high reliability and factor structure of the data from this study provides evidence of internal structure validity of the SEPS. The high correlation between the SEP subscales (0.87), as well as the correlation between the CSEM and SEP subscales (0.75 and 0.68) give evidence of convergent validity of the SEP subscales. The relationship between SEPS and CSEM is also apparent from the modification indices indicating that inclusion of proving subtask 2 to the CSEM factor would improve model fit. This makes sense, as proving subtask 2 ("Explore new ideas to come up with ways to start your proof"), more than the other proving subtasks, involves exploration and potential for creative engagement. This also illustrates a practical relationship between SEP and CSEM; students' beliefs in their abilities to prove a novel task are informed by general creative metacognitive knowledge.

As indicated by the response process interviews, the high correlation between *CSEM_2* and *CSEM_3* may be due to students' experience of mathematics as primarily involving problems with only one solution. Distinctions between these items may be only appear for students with more experience or exposure to mathematical creativity. However, to better distinguish *CSEM_2* and *CSEM_3*, the wording of *CSEM_2* could be revised from "[Rate your confidence in your ability to] solve a math problem in multiple ways" to "[Rate your confidence in your ability to] approach a math problem in a variety of ways." This may better capture the definition of *fluency* as one's ability to generate large number of ideas (Torrance, 1974).

The RMSEA of the self-efficacy model was well above the recommended limit of 0.08 (Hooper et al., 2008). Thus, future modifications of the SEP Scale should be considered. One possible concern in using this SEP Scale is that students are not accurately interpreting or understanding the proving statements given. While my interaction with students indicated that they most often easily and correctly understood these statements, one student taking the post-survey commented that Statement 2 (Every odd integer was the difference of two squares) was false. After he provided a "counter example" of two even numbers, he quickly found that he had misinterpreted this statement as its converse (The difference of two squares is always odd). Especially if a student is rushing, they may misinterpret the statement provided and rate their confidence for a related, but different statement as in this case. In similar uncorrected cases, students would be rating their self-efficacy for significantly *different* constructs.

One primary concern of the correlational analysis conducted in this study is that it was not possible to control for beginning of semester student self-efficacy. Students entering the courses study were bringing over 12 years of educational experience forming their current self-efficacy. Thus, it is unlikely that any effects of CFMI on self-efficacy are observable without considering a baseline measurement of self-efficacy. Accordingly, the correlational results from this project should be viewed as tentative and serve as methodological guide for future research. The mediation models described may contribute to further study of the ways in which CFMI impacts both task-general (CSEM) and task-specific (SEP) self-efficacy. Further quantitative study of this dynamic is needed in other contexts and with more targeted pre- post-semester measurement of self-efficacy to determine if similar mediated effects are observable in other contexts, or in what ways shifts in student self-efficacy are most likely to occur.

## 4.8 Limitations and Future work

Despite challenges linking pre- and post-semester student responses, it still may be useful to use the SEP Scale to measure relative student self-efficacy for proving. Future research using the SEP or related scales should involve careful selection of proving statements that avoid terminology for which students do not have prior exposure, along with some way to verify ease and accuracy of student understanding of the statement(s) used. This could involve task-based interviews that study relative rates of mis-understanding or mis-interpretation of the statement(s) in samples from the population of interest. Additionally, the environment in which students are completing the questionnaire should be positive and used to establish appropriate motivation or incentives for students taking their time in filling out the SEP scale. Overall, students seemed interested in the survey and responded well to the encouragement "that their time in completing this survey can contribute to a better understanding of the impact of classroom environments on student confidence and motivation." Yet, to mitigate against students clicking through the survey, it may be necessary to add something like, "Please don't just click through the survey without paying attention. This will give me invalid data that will just skew any real results."

One limitation of the CFMI model is that list-wise deletion of missing data was used. This is a concern, as only 195 out of 243 responses being used, with only 9% of the data missing from the removed 48 cases. Since there is a theoretical relationship between the observed data and likelihood of missing values, this data can be considered missing at random. For example, students providing low ratings to the FMS items are more likely to describe posing problems, debating, and building on one another's ideas (FMS_4-6) as 'not applicable.' Thus, case-wise maximum likelihood estimation of the CFMI Scale data would have been more appropriate, if available.

For further validation fo the CFMI Scales, future studies should compare student responses with real-time observation protocols to study how closely does student perception of instruction compares with that of an outside observer. This, along with further study with larger sample sizes, can help determine the extent to which these effects of CFMI are due to individual differences (i.e. subjective aspects relating to perception) or group differences (i.e. objective aspects of instruction). Although the individual and group factors of instruction are likely related, a better understanding of relative effects of each on self-efficacy and motivation can guide instructor use of CFMI teaching actions.

Another limitation of this study is the poor model fit of the AMTM Scale. This study demonstrates a need for further work adapting the AMTM Scale to tertiary mathematics or developing a new scale. It was noted that the amotivation subscale may have been a source of frustration to students filling out this survey in upper-level mathematics course. Self-motivation among tertiary mathematics may operate more differently than expected in comparison to pre-tertiary students. For example, the students from this study have chosen to enroll in mathematics courses in the first place, which may not be the case in secondary school. Due to this, one approach in adapting the AMTM Scale to tertiary mathematics may include using only three subscales: introjected, identified, and intrinsic regulation. However, such a scale would be different enough from Lim and Chapman's (2015) that further research and development is needed.

Finally, it is important to note that the courses studied in this project may or may not have been representative of all upper-level courses at this institution. All instructors contacted for this project were exposed to a basic description of this project, while some engaged in longer conversations about the goals of this project in understanding the effect of instruction on student self-efficacy and motivation, and already implicitly valued this project by giving their class time for administering surveys. Thus, it may be the case that the teaching practices described in the CFMI are actually much less common in tertiary mathematics than described in this study. Yet, Tang et al. (2019) showed that CFMI can be made accessible to instructors that demonstrate an interest and openness to adopting creativity-fostering teaching practices. Due to this, continued research characterizing CFMI and its effects on students may be important means for supporting and nuturing the next generation of undergraduate mathematics students.

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# Chapter 5

# Conclusion

> The value of a psychological theory is judged not only by its explanatory and predictive power, but by its operational power to effect change...Knowing how to build a sense of efficacy and how it works provides further guidelines for structuring experiences that enable people to realize desired personal and social changes. (Bandura, 2006, p. 319)

This dissertation explored how creativity-fostering mathematics instruction (CFMI) effects change in students from three perspectives. First, in-depth study of students' experience of CFMI in relation to their proving experience provided evidence that CFMI may contribute to students gaining self-efficacy in two specific ways — individual and social. In supporting students' individual experiences of creativity, exposing students to uncertainty in mathematics (uncertainty principle) and allowing students to work on challenging problems over long periods of time (Gestalt principle) contributed to students gaining self-efficacy from their own proving experiences. In support of social experiences of creativity, allowing students to present their own ideas, debate, and build off one another's ideas (scholarly principle) enabled students to gain self-efficacy for proving vicariously, from their peers. However, these social factors were shown to also give rise to experiences that served as negative sources of self-efficacy. This finding re-enforced the important role instructors play in facilitating environments where students can safely take risks (free market principle). In Fannie's words, her instructor supported her building self-efficacy by providing a "general environment of everyone not being afraid to fail" where she understood that her "peers weren't going to judge me for doing something wrong." The impact of both individual and social aspects of CFMI highlights the importance of Sriraman's principles not only in maximizing student creativity, but moreover, in maximizing student opportunities for building self-efficacy toward mathematics.

The second perspective of this dissertation focused on one specific tool for instructors fostering mathematical creativity, problem posing, and its relation to students building self-motivation. This study illustrated how problem posing can support the integration of motivational regulation by supporting students' sense of competence in "feeling like a mathematician," allowing students the autonomy of choosing the direction and motivation for their learning, and in fostering a sense of relatedness from engaging one another's problems. While this study lacked an explicit framework or characterization of problem-posing instruction, the results suggest that the psychological needs of self-determination theory (Ryan & Deci, 2000) may serve as a useful framework for characterizing the pedagogical purposes of problem posing in the classroom. In other words, problem posing should foster students' sense of challenge in building individual confidence, promote students' sense of autonomy in learning mathematics, and facilitate students' sense of relatedness in contributing to their classroom and/or broader mathematical community.

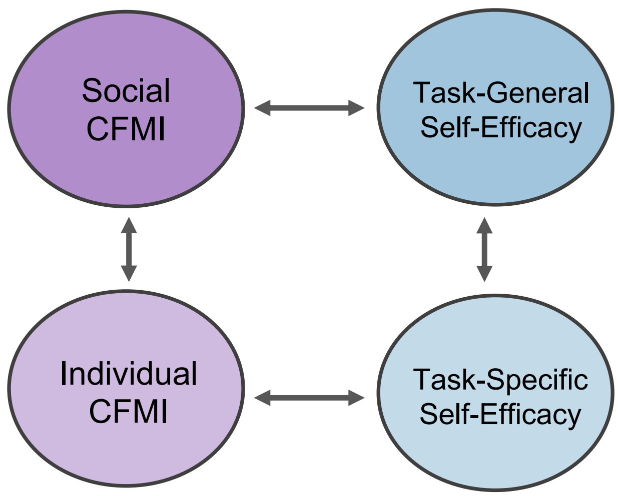


Figure 5.1: Conjectured relationship between CFMI and student self-efficacy

The third perspective considered the potential macro-scale impact of creativity-fostering instruction on student self-efficacy and motivation. While limitations to the analysis call for future work using slightly revised methods, the basic instrument used for measuring CFMI demonstrated high validity and reliability. This perspective also enabled the development and exploration of a model for the relationship between individual and social aspects of instruction on both task-general and task-specific self-efficacy for mathematics (see Figure 5.1). The data from this study provided evidence that social aspects of CFMI impacted task-specific self-efficacy for proving through promoting students' general sense of creative self-efficacy as a mediating factor. Thus, it is conjectured that social aspects of CFMI most directly impact students' general sense of creative self-efficacy. This, in turn, enables a deeper engagement with proving which ultimately serves to build task-specific self-efficacy. Furthermore, individual aspects of CFMI appeared to impact students' general sense of creative self-efficacy via students' task-specific self-efficacy for proving.

## 5.1 Future Directions

The methods used in this dissertation illustrate how the connections between CFMI, self-efficacy, and self-motivation can be studied in other undergraduate mathematics contexts as well. The conditions that helped the students studied in this dissertation succeed in building self-efficacy and motivation may similarly help students in lower-level courses. Thus, the methods developed in this dissertation should be extended to study the impact of CFMI in pre-calculus and calculus courses. Using the structure and format of the Self-efficacy for Proving Scale, an instrument can be developed for measuring self-efficacy for *problem solving* to be used in first- and second-year college classes. This dissertation also demonstrates a need to continue developing and validating both instructor and student tools for better characterizing and describing what happens in the classroom. Sections 4.7.1 and 4.7.2 provide direction for future use and modification of the CFMI, SEP, and CSEM Scales.

While I initially set out to study self-efficacy and self-motivation as positive correlates to instruction, part of the motivation for this dissertation came from a desire to better understand and address high rates of mathematics anxiety among college students. Existing studies (e.g. Pajares & Kranzler, 1995; Ryan & Deci, 2000) demonstrate that measures of mathematics self-efficacy and internal motivation are negatively correlated with mathematics anxiety. These results, together with the links between CFMI, self-efficacy, and self-motivation studied in this dissertation, position self-efficacy and self-motivation as mediators between instruction and mathematics anxiety. Hence, there is a need to study the direct effects of CFMI on mathematics anxiety. It is quite possible that certain aspects of CFMI — such as teaching mathematics as an uncertain, ill-structured discipline — may initially create more anxiety in students. Thus, the relationship between CFMI and mathematics demands further study. Future research of CFMI and anxiety should consider both direct effects and mediated effects of instruction on mathematics anxiety.

## 5.2 Closing Remarks

The explanatory tools offered in this dissertation should firstly empower mathematics instructors. The potential impact of CFMI on student self-efficacy and motivation presented outline *why* instructors should be encouraged to think and act creatively themselves and to see their classrooms as environments where the creative development of students flourishes. In such an educational context, continued research of CFMI and its informed use ultimately empowers students. Understanding the impact of CFMI can aid in the development and spread of pedagogies that most effectively provide students with opportunities for building self-efficacy and self-motivation for mathematics.

Finally, the broad implications of these ideas must not be overlooked. Using CFMI in a way that builds self-efficacy and self-motivation means students do not have to be bored or fearful of mathematics, but can enjoy mathematics. Rather than dropping out of mathematics courses, students will broaden their studies of mathematics. Instead of a short-lived mathematics career, students effectively continue to use mathematics in their future studies, their work, and their living. As a results, they become empowered as more active, persistent, and creative contributors to society.

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# Appendix A

# Supplementary resources to part I

Provided below are the surveys 1-3 and the task-based interview questions used in Part I (Chapter 2). Please do not hesitate to contact me[1] for access to the interview transcripts or student survey responses.

## A.1 Survey 1

Survey 1 from Part I is included below.

[1] paulrregier@gmail.com

## Introduction

Thank you for your participation in this survey. We appreciate your feedback!

This study takes 5-10 minutes to complete and includes two sections. You will be asked to complete each question before moving on. If at any point, you choose to end your participation in this research, you may do so; however, please email me at pregier@math.ou.edu to let me know so that I may remove you from this project.

**This research has been approved by the University of Oklahoma, Norman Campus IRB.**
**IRB Number:** 8172 **Approval Date:** 7/6/2017

*Please use the << and >> buttons on the survey to navigate forward and backwards. Previous selections may not be available when browser buttons are used for navigation.*

Click the next button to get started!

## 5 Principles Survey

Please answer the following questions based on **your experience in your previous math class.**

How often did you have the freedom (of time and space) to work on a challenging problem or proof over a period two or more days?

Never
1 to 2 times per semester
3 to 5 times per semester
6-10 times per semester
Weekly (once or more per week, on average)
Daily (once or more per class period, on average)

How often did you experience the joy of arriving at a solution after working on a problem or proof for several days?

Never

1 to 2 times per semester

3 to 5 times per semester

6-10 times per semester

Weekly (once or more per week, on average)

Daily (once or more per class period, on average)

How often did you find yourself appreciating the beauty or novelty of creating new mathematical ideas or solutions?

Never

1 to 2 times per semester

3 to 5 times per semester

6-10 times per semester

Weekly (once or more per week, on average)

Daily (once or more per class period, on average)

How often did you find yourself appreciating the beauty or novelty of creating new mathematical ideas or solutions?

Never

1 to 2 times per semester

3 to 5 times per semester

6-10 times per semester

Weekly (once or more per week, on average)

Daily (once or more per class period, on average)

How often did you feel comfortable going in a direction on a problem/proof that may or may not prove successful?

Never

1 to 2 times per semester

3 to 5 times per semester

6-10 times per semester

Weekly (once or more per week, on average)
Daily (once or more per class period, on average)

How often did you enjoy engaging new or atypical thinking in your approach to solving a problem?

Never
1 to 2 times per semester
3 to 5 times per semester
6-10 times per semester
Weekly (once or more per week, on average)
Daily (once or more per class period, on average)

How often did you engage in debate with your peers or instructor concerning how to approach a problem?

Never
1 to 2 times per semester
3 to 5 times per semester
6-10 times per semester
Weekly (once or more per week, on average)
Daily (once or more per class period, on average)

How often did you challenge the validity of your peers' or your instructor's solutions/proof?

Never
1 to 2 times per semester
3 to 5 times per semester
6-10 times per semester
Weekly (once or more per week, on average)
Daily (once or more per class period, on average)

How often did you feel comfortable considering ambiguous or ill-posed problems?

Never
1 to 2 times per semester
3 to 5 times per semester
6-10 times per semester

Weekly (once or more per week, on average)

Daily (once or more per class period, on average)

How often did you feel comfortable working on open-ended or potentially unsolvable problems?

Never

1 to 2 times per semester

3 to 5 times per semester

6-10 times per semester

Weekly (once or more per week, on average)

Daily (once or more per class period, on average)

**Block 3**

For the next three questions, please rate your confidence **in your own ability** to do the following sub-tasks related to creating a **proof,** or logical argument, of three mathematical statements.

However much you may want to prove these statements, please do not! **Just evaluate your own confidence** in your ability for each of the sub-tasks.

**Statement 1:**

If n is odd, then $n^2 + 1$ is even.

In proving/disproving **Statement 1,** what is your confidence in your ability to do each of the following?

| | Cannot do at all | | | | | Moderately certain can do | | | Highly certain can do | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | 0 | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 |
| Understand and informally explain why a statement is true or false | | | | | | | | | | | |

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | Cannot do at all | | | | Moderately certain can do | | | Highly certain can do | | |
| | 0 | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 |
| Explore new ideas to come up with ways to start your proof | | | | | | | | | | | |
| Use various representations (numbers, pictures, tables, symbols, word) to structure your thinking | | | | | | | | | | | |
| Formally write out and justify each step of your proof | | | | | | | | | | | |
| Examine your proof for accuracy and identify any missing steps | | | | | | | | | | | |

**Statement 2:**

If $a, b, c$ are positive integers, and $ab, bc, ac$ all have the same parity (are all even or all odd), then $a, b, c$ all have the same parity.

In proving or disproving **Statement 2,** what is your confidence in your ability to do each of the following?

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | Cannot do at all | | | | Moderately certain can do | | | Highly certain can do | | |
| | 0 | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 |
| Understand and informally explain why a statement is true or false | | | | | | | | | | | |
| Explore new ideas to come up with ways to start your proof | | | | | | | | | | | |

| | Cannot do at all | | | | Moderately certain can do | | | Highly certain can do | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | 0 | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 |
| Use various representations (numbers, pictures, tables, symbols, word) to structure your thinking | | | | | | | | | | | |
| Formally write out and justify each step of your proof | | | | | | | | | | | |
| Examine your proof for accuracy and identify any missing steps | | | | | | | | | | | |

**Statement 3:**

If $a$ and $b$ are integers, then $a^2 - 4b \neq 2$.

In proving/disproving **Statement 3,** what is your confidence in your ability to do each of the following?

| | Cannot do at all | | | | Moderately certain can do | | | Highly certain can do | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | 0 | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 |
| Understand and informally explain why a statement is true or false | | | | | | | | | | | |
| Explore new ideas to come up with ways to start your proof | | | | | | | | | | | |
| Use various representations (numbers, pictures, tables, symbols, word) to structure your thinking | | | | | | | | | | | |

| | Cannot do at all | | | | Moderately certain can do | | | Highly certain can do | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | 0 | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 |
| Formally write out and justify each step of your proof | | | | | | | | | | | |
| Examine your proof for accuracy and identify any missing steps | | | | | | | | | | | |

**Conclusion**

Thank you for participating in this survey!

Please let us know if you have any further questions or comments about this survey, or if you want to elaborate on any of your responses.

For correlating your responses throughout the semester, please provide your student ID number. If you prefer to remain entirely anonymous, please provide a unique number (that you can remember!) to use for future surveys.

## A.2 Survey 2

Survey 2 from Part I is included below.

**Introduction**

Thank you for your participation in this survey. We appreciate your feedback!

This study takes about 5 minutes to complete and includes one sections. You will be asked to complete each question. If at any point, you choose to end your participation in this research, you may do so; however, please email me at pregier@math.ou.edu to let me know so that I may remove you from this project.

**This research has been approved by the University of Oklahoma, Norman Campus IRB.**
**IRB Number:** 8172 **Approval Date:** 7/6/2017

*Please use the << and >> buttons on the survey to navigate forward and backwards. Previous selections may not be available when browser buttons are used for navigation.*

Click the next button to get started!

**Block 3**

For the next three questions, please rate your confidence **in your own ability** to do the following sub-tasks related to creating a **proof,** or logical argument, of three mathematical statements.

However much you may want to prove these statements, please do not! **Just evaluate your own confidence** in your ability for each of the sub-tasks.

**Statement 1:**

If $n$ is an odd integer, then $n^4 - n$ is even.

In proving/disproving **Statement 1,** what is your confidence in your ability to do each of the following?

| | Cannot do at all | | | | Moderately certain can do | | | Highly certain can do | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | 0 | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 |
| Understand and informally explain why a statement is true or false | | | | | | | | | | | |
| Explore new ideas to come up with ways to start your proof | | | | | | | | | | | |
| Use various representations (numbers, pictures, tables, symbols, word) to structure your thinking | | | | | | | | | | | |
| Formally write out and justify each step of your proof | | | | | | | | | | | |
| Examine your proof for accuracy and identify any missing steps | | | | | | | | | | | |

**Statement 2:**

The inequality $2^x \geq x + 1$ is true for every positive real number $x$.

In proving or disproving **Statement 2,** what is your confidence in your ability to do each of the following?

| | Cannot do at all | | | | Moderately certain can do | | | Highly certain can do | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | 0 | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 |
| Understand and informally explain why a statement is true or false | | | | | | | | | | | |

| | Cannot do at all | | | | Moderately certain can do | | | Highly certain can do | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | 0 | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 |
| Explore new ideas to come up with ways to start your proof | | | | | | | | | | | |
| Use various representations (numbers, pictures, tables, symbols, word) to structure your thinking | | | | | | | | | | | |
| Formally write out and justify each step of your proof | | | | | | | | | | | |
| Examine your proof for accuracy and identify any missing steps | | | | | | | | | | | |

**Statement 3:**

There does not exist a real number $x$ for which $x^4 < x < x^2$.

In proving/disproving **Statement 3,** what is your confidence in your ability to do each of the following?

| | Cannot do at all | | | | Moderately certain can do | | | Highly certain can do | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | 0 | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 |
| Understand and informally explain why a statement is true or false | | | | | | | | | | | |
| Explore new ideas to come up with ways to start your proof | | | | | | | | | | | |

| | Cannot do at all | | | | Moderately certain can do | | | Highly certain can do | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | 0 | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 |
| Use various representations (numbers, pictures, tables, symbols, word) to structure your thinking | | | | | | | | | | | |
| Formally write out and justify each step of your proof | | | | | | | | | | | |
| Examine your proof for accuracy and identify any missing steps | | | | | | | | | | | |

**Conclusion**

Thank you for participating in this survey!

Please let us know if you have any further questions or comments about this survey, or if you want to elaborate on any of your responses.

For correlating your responses throughout the semester, please provide your student ID number. If you prefer to remain entirely anonymous, please provide a unique number (that you can remember!) to use for future surveys.

## A.3 Survey 3

Survey 3 from Part I is included below.

## Introduction

Thank you for your participation in this survey. We appreciate your feedback!

This study takes 5-10 minutes to complete and includes two sections. You will be asked to complete each question before moving on. If at any point, you choose to end your participation in this research, you may do so; however, please email me at pregier@math.ou.edu to let me know so that I may remove you from this project.

**This research has been approved by the University of Oklahoma, Norman Campus IRB.**
**IRB Number:** 8172 **Approval Date:** 7/6/2017

*Please use the << and >> buttons on the survey to navigate forward and backwards. Previous selections may not be available when browser buttons are used for navigation.*

Click the next button to get started!

## 5 Principles Survey

Please answer the following questions based on **your experience this semester** in this class, Discrete Math Structures.

How often did you have the freedom (of time and space) to work on a challenging problem or proof over a period two or more days?

Never
1 to 2 times per semester
3 to 5 times per semester
6-10 times per semester
Weekly (once or more per week, on average)

Daily (once or more per class period, on average)

How often did you experience the joy of arriving at a solution after working on a problem or proof for several days?

Never
1 to 2 times per semester
3 to 5 times per semester
6-10 times per semester
Weekly (once or more per week, on average)
Daily (once or more per class period, on average)

How often did you find yourself appreciating the beauty or novelty of creating new mathematical ideas or solutions?

Never
1 to 2 times per semester
3 to 5 times per semester
6-10 times per semester
Weekly (once or more per week, on average)
Daily (once or more per class period, on average)

How often did you find yourself appreciating the beauty or novelty of creating new mathematical ideas or solutions?

Never
1 to 2 times per semester
3 to 5 times per semester
6-10 times per semester
Weekly (once or more per week, on average)
Daily (once or more per class period, on average)

How often did you feel comfortable going in a direction on a problem/proof that may or may not prove successful?

Never
1 to 2 times per semester
3 to 5 times per semester

6-10 times per semester
Weekly (once or more per week, on average)
Daily (once or more per class period, on average)

How often did you enjoy engaging new or atypical thinking in your approach to solving a problem?

Never
1 to 2 times per semester
3 to 5 times per semester
6-10 times per semester
Weekly (once or more per week, on average)
Daily (once or more per class period, on average)

How often did you engage in debate with your peers or instructor concerning how to approach a problem?

Never
1 to 2 times per semester
3 to 5 times per semester
6-10 times per semester
Weekly (once or more per week, on average)
Daily (once or more per class period, on average)

How often did you challenge the validity of your peers' or your instructor's solutions/proof?

Never
1 to 2 times per semester
3 to 5 times per semester
6-10 times per semester
Weekly (once or more per week, on average)
Daily (once or more per class period, on average)

How often did you feel comfortable considering ambiguous or ill-posed problems?

Never
1 to 2 times per semester
3 to 5 times per semester

6-10 times per semester

Weekly (once or more per week, on average)

Daily (once or more per class period, on average)

How often did you feel comfortable working on open-ended or potentially unsolvable problems?

Never

1 to 2 times per semester

3 to 5 times per semester

6-10 times per semester

Weekly (once or more per week, on average)

Daily (once or more per class period, on average)

### Block 3

For the next three questions, please rate your confidence **in your own ability** to do the following sub-tasks related to creating a **proof,** or logical argument, of three mathematical statements.

However much you may want to prove these statements, please do not! **Just evaluate your own confidence** in your ability for each of the sub-tasks.

**Statement 1:**

If $x, y$ are real numbers, then $|x + y| \leq |x| + |y|$.

In proving/disproving **Statement 1,** what is your confidence in your ability to do each of the following?

| | Cannot do at all | | | | | Moderately certain can do | | | Highly certain can do | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | 0 | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 |
| Understand and informally explain why a statement is true or false | | | | | | | | | | | |

| | Cannot do at all | | | | Moderately certain can do | | | Highly certain can do | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | 0 | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 |
| Explore new ideas to come up with ways to start your proof | | | | | | | | | | | |
| Use various representations (numbers, pictures, tables, symbols, word) to structure your thinking | | | | | | | | | | | |
| Formally write out and justify each step of your proof | | | | | | | | | | | |
| Examine your proof for accuracy and identify any missing steps | | | | | | | | | | | |

**Statement 2:**

If $n$ is an integer, then $1 + (-1)^n(2n - 1)$ is a multiple of 4.

In proving or disproving **Statement 2,** what is your confidence in your ability to do each of the following?

| | Cannot do at all | | | | Moderately certain can do | | | Highly certain can do | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | 0 | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 |
| Understand and informally explain why a statement is true or false | | | | | | | | | | | |
| Explore new ideas to come up with ways to start your proof | | | | | | | | | | | |

| | Cannot do at all | | | | Moderately certain can do | | | Highly certain can do | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | 0 | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 |
| Use various representations (numbers, pictures, tables, symbols, word) to structure your thinking | | | | | | | | | | | |
| Formally write out and justify each step of your proof | | | | | | | | | | | |
| Examine your proof for accuracy and identify any missing steps | | | | | | | | | | | |

**Statement 3:**

Every odd integer is the difference of two squares.

In proving/disproving **Statement 3,** what is your confidence in your ability to do each of the following?

| | Cannot do at all | | | | Moderately certain can do | | | Highly certain can do | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | 0 | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 |
| Understand and informally explain why a statement is true or false | | | | | | | | | | | |
| Explore new ideas to come up with ways to start your proof | | | | | | | | | | | |
| Use various representations (numbers, pictures, tables, symbols, word) to structure your thinking | | | | | | | | | | | |

| | Cannot do at all | | | | Moderately certain can do | | | | Highly certain can do | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | 0 | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 |
| Formally write out and justify each step of your proof | | | | | | | | | | | |
| Examine your proof for accuracy and identify any missing steps | | | | | | | | | | | |

### Conclusion

As a final part of research, we may, if you consent, consider using your written work from this course in our analysis. There are no risks or benefits to your consenting or not consenting. If you do not consent, you will not be penalized or lose benefits or services unrelated to the research. The data you provide will be retained in anonymous form. Only after all final grades have been submitted will Dr. Savic have access to the de-identified data.

Please check all the options that you agree to:

I agree for the researcher to use my work posted Canvas (homework/quizzes) in this study.

Yes

No

I agree for the researcher to use grades in this study.

Yes

No

I agree for the researcher to use my final exam in this study.

Yes

No

Thank you for participating in this survey!

Please let us know if you have any further questions or comments about this survey, or if you want to elaborate on any of your responses.

## A.4 Task-based interview questions

Students were given the three proving statements from the Survey 3 and given 30 minutes to attempt the proofs.

**After Task:**

1. Tell me about your proving process?
2. What tools did you use?
3. Have you seen any of these proofs before?

**Regarding Class Experience:**

4. Tell me a little about your experience in this class. Describe your experience of learning to prove mathematical statements.
5. What is your definition of mathematical creativity? Did you think you were creative in this class?
6. At the beginning of this semester, how confident were you in proving mathematical statements? What about now?
7. What do you think contributed to building your confidence?
    - How did the homework contribute to your gaining confidence in proving?
    - What is the longest you worked on a problem?
    - How did you approach open-ended problems?
    - How did the following influence your learning to prove?
        - [Dr. S]?
        - Class discussion?
        - Working in groups?
8. What would have helped you build your confidence in proving?
9. Where did you struggle the most in this class? What prevented you from gaining confidence?

**Optional:**

10. What would you have changed about this class?
11. Overall, what did you gain from this course?
12. What else would you say about your experience in this class?

# Appendix B

# Supplementary resources to part II

Provided below are interview questions used in surveys used in Part II (Chapter 4). Please contact me[1] for access to the interview transcripts in either coded or uncoded format.

**Introduction:**

1. Why did you take Discrete Math?
2. What is the most important thing you learned?
3. What is the significance of the what you learned?
4. How internally/intrinsically motivated were you for Discrete Math?
5. What motivated you? 6 What took away motivation?

[At some point, introduce discussion of problem posing, e.g. "Do you recall assignment where you were asked to 'make a test that [Dr. F] would make'? (for test 1 and test 2)." Show pdf of their work on my tablet to refresh memory.]

7. How did this assignment impact you?
8. Describe your experience working on this assignment. (Which of these questions were most interesting to you?)
9. How did asking these questions impact you?

**Competence:**

10. How did this assignment problem posing impact your sense of confidence with mathematics?

**Autonomy:**

11. How did this assignment problem posing impact your sense of freedom towards mathematics?

[1] paulrregier@gmail.com

**Integration:**

12. Did this assignment (or class in general) impacted your personal views toward mathematics?
13. Are there any particular perspectives you think you gained from this class? Since this class?
14. Do you feel you have adopted any new mathematical beliefs/values in this class? since this class?
15. How do your own personal beliefs and values relate to what you were doing in class? (Personally important to you or not?) Has this changed?

**Relatedness:**

16. How did this assignment problem posing impact your sense of being related to those in your class?
17. How did hearing other people pose problems impact you?
18. How did it impact did your motivation for posing and solving your own mathematical problems?

**General Motivation:**

19. How did this assignment impact your overall motivation? Is there any way other these assignments impacted you?
20. Can you give another example of a time where you were motivated to ask your own mathematical questions since this class?
21. Is there any way [Dr. F] impacted your motivation for posing your own mathematical questions? How?
22. How did posing your own problems impact your approach to this class?
23. Since this class, describe your motivation for
    - engaging in class
    - solving problems
    - posing new mathematical problems.

**Conclusion:**

24. What are you studying now?
25. Did your experience in this class (problem posing) impact your direction of study? How?
26. [Optional] In problem solving, "what do you do when you don't know what to do?" (Sowder, 1985)[2]
27. [Optional] In this class you asked questions on each assignment. How do you think this impact you?

[2]Threadgill-Sowder, J., Larry, S., Moyer, J. C., & Moyer, M. B. (1985). Cognitive variables and performance on mathematical story problems. *The Journal of Experimental Education, 54*(1), 56-62.

# Appendix C

# Supplementary resources to part III

Provided below is the response process interview protocol, as well as copies of the pre- and post- semester surveys used in Part III (Chapter 4). Please contact me[1] for access to the student data and/or R code used in analysis of this data.

## C.1 Pre-semester survey

The pre-semester survey from Part III is provided below.

[1]paulrregier@gmail.com

**Introduction**

**Would you like to be involved in research at the University of Oklahoma?**
I am Paul Regier, a graduate student in the OU Mathematics department, and I want to invite you to participate in a research project, entitled *Creativity, Motivation and Self-efficacy for Proving*. This research is being conducted at the University of Oklahoma. You were selected as a possible participant because of your enrollment in an upper level mathematics course. You must be at least 18 years of age to participate in this study.
**Please read this document and contact me to ask any questions that you may have BEFORE agreeing to take part in my research.**

**What is the purpose of this research?** The purpose of this research is to study the effect of various aspects of instruction on students' motivation and self-efficacy (belief in their own confidence) for mathematical proving.
**How many participants will be in this research?** About 400 people will take part in this research.
**What will I be asked to do?** If you agree to be in this research, you will be asked to complete two short questionnaires (at the beginning and end of this current semester).
**How long will this take?** The questionnaires will take less than 10 minutes (for this questionnaire) and 15 minutes (for the second questionnaire) to complete.
**What are the risks and/or benefits if I participate?** There are no risks and no benefits from being in this research.
**Will I be compensated for participating?** You not be reimbursed for your time and participation in this research.
**Who will see my information?** All data will be collected via an online survey system that has its own privacy and security policies for keeping your information confidential. There will be no In research reports, there will be no information that will make it possible to identify you. Research records will be stored securely and only approved researchers and the OU Institutional Review Board will have access to the records.
**Will my identity be anonymous?** Your participation on with this survey is fully anonymous.
**What will happen to my data in the future?** We will not share your data or use it in future research projects.

**Do I have to participate?** No. If you do not participate, you will not be penalized or lose benefits or services unrelated to the research. If you decide to participate, you don't have to answer any question and can stop participating at any time.

**Who do I contact with questions, concerns or complaints?** If you have questions, concerns or complaints about the research please contact me (Paul Regier) in person or by email (paulregier@ou.edu).

You can also contact my faculty advisor, Milos Savic at savic@ou.edu, or the University of Oklahoma – Norman Campus Institutional Review Board (OU-NC IRB) at 405-325-8110 or irb@ou.edu if you have questions about your rights as a research participant, concerns, or complaints about the research and wish to talk to someone other than the researcher(s) or if you cannot reach the researcher(s).

*Please print this document for your records. By providing information to the researcher(s), I am agreeing to participate in this research.*

**This research has been approved by the University of Oklahoma, Norman Campus IRB.**

**IRB Number:** 10965 **Approval date:** 07/18/2019

Click to write the question text

I agree to participate

I do not want to participate

**Block 3**

In the items below, select the the response that best indicates the extent to which each of the following items presently corresponds to one of the reasons why you spend time studying mathematics.

Why do you spend time studying mathematics?

| | Does not Correspond at all | Corresponds a little | Corresponds moderately | Corresponds a lot | Corresponds exactly |
|---|---|---|---|---|---|

| | Does not Correspond at all | Corresponds a little | Corresponds moderately | Corresponds a lot | Corresponds exactly |
|---|---|---|---|---|---|
| For the pleasure of being able to experience "light bulb" moments understanding something new. | O | O | O | O | O |
| I don’t know; I can’t understand what I am doing in mathematics. | O | O | O | O | O |
| Honestly, I don’t know; I feel that it is a waste of time studying mathematics. | O | O | O | O | O |
| In order to have a better salary later on. | O | O | O | O | O |
| For the pleasure that I experience when I feel completely absorbed by what myself or others come up with. | O | O | O | O | O |
| For the pleasure that I experience in broadening my knowledge about mathematics. | O | O | O | O | O |

| | Does not Correspond at all | Corresponds a little | Corresponds moderately | Corresponds a lot | Corresponds exactly |
|---|---|---|---|---|---|
| Because I want to show to others (e.g., teachers, family, friends) that I can do mathematics. | O | O | O | O | O |
| Because I want to show myself that I can do well in mathematics. | O | O | O | O | O |
| Because without a good grade in mathematics, I will not be able to find a high-paying job later on. | O | O | O | O | O |
| Because what I learn in mathematics now will be useful in my future studies. | O | O | O | O | O |

| | Does not Correspond at all | Corresponds a little | Corresponds moderately | Corresponds a lot | Corresponds exactly |
|---|---|---|---|---|---|
| Because of the fact that when I do well in mathematics, I feel important. | | | | | |
| Because I want to have “the good life” later on. | | | | | |

| | Does not Correspond at all | Corresponds a little | Corresponds moderately | Corresponds a lot | Corresponds exactly |
|---|---|---|---|---|---|
| I can’t see why I study mathematics and frankly, I couldn’t care less. | | | | | |
| Because I think that mathematics will help me better prepare for my future career. | | | | | |
| To show myself that I am an intelligent person. | | | | | |
| Because studying mathematics will be useful for me in the future. | | | | | |
| In order to obtain a more prestigious job later on. | | | | | |
| Because I believe that mathematics will improve my work competence. | | | | | |

| | Does not Correspond at all | Corresponds a little | Corresponds moderately | Corresponds a lot | Corresponds exactly |
|---|---|---|---|---|---|
| Because I want to feel the personal satisfaction of understanding mathematics. | | | | | |
| For the pleasure I experience when I discover new things in mathematics that I have never seen before. | | | | | |

| | Does not Correspond at all | Corresponds a little | Corresponds moderately | Corresponds a lot | Corresponds exactly |
|---|---|---|---|---|---|
| I am not sure; I don't see how mathematics is of value to me. | O | O | O | O | O |

**Block 3**

For the following three mathematical statements, please rate your confidence **in your own ability** to do the following sub-tasks related to creating a **proof,** or logical argument, of the truth or falsehood of each statement.

However much you may want to prove these statements, please do not! **Just evaluate your own confidence** in your ability to carry out each of sub-task.

**Statement 1:**
If n is odd, then $n^2 + 1$ is even.

In proving/disproving **Statement 1,** what is your confidence in your ability to do each of the following?

| | Cannot do at all | | | | Moderately certain can do | | | Highly certain can do | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | 0 | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 |
| Understand and informally explain why a statement is true or false | | | | | | | | | | | |
| Explore new ideas to come up with ways to start your proof | | | | | | | | | | | |
| Use various representations (numbers, pictures, tables, symbols, word) to structure your thinking | | | | | | | | | | | |

| | Cannot do at all | | | | Moderately certain can do | | | Highly certain can do | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | 0 | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 |
| Formally write out and justify each step of your proof | | | | | | | | | | | |
| Examine your proof for accuracy and identify any missing steps | | | | | | | | | | | |

**Statement 2:**

The inequality $2^x \geq x + 1$ is true for every positive real number $x$.

In proving or disproving **Statement 2,** what is your confidence in your ability to do each of the following?

| | Cannot do at all | | | | Moderately certain can do | | | Highly certain can do | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | 0 | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 |
| Understand and informally explain why a statement is true or false | | | | | | | | | | | |
| Explore new ideas to come up with ways to start your proof | | | | | | | | | | | |
| Use various representations (numbers, pictures, tables, symbols, word) to structure your thinking | | | | | | | | | | | |
| Formally write out and justify each step of your proof | | | | | | | | | | | |
| Examine your proof for accuracy and identify any missing steps | | | | | | | | | | | |

**Statement 3:**
If $a$ and $b$ are integers, then $a^2 - 4b \neq 2$.

In proving/disproving **Statement 3,** what is your confidence in your ability to do each of the following?

| | Cannot do at all | | | | Moderately certain can do | | | | Highly certain can do | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | 0 | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 |
| Understand and informally explain why a statement is true or false | | | | | | | | | | | |
| Explore new ideas to come up with ways to start your proof | | | | | | | | | | | |
| Use various representations (numbers, pictures, tables, symbols, word) to structure your thinking | | | | | | | | | | | |
| Formally write out and justify each step of your proof | | | | | | | | | | | |
| Examine your proof for accuracy and identify any missing steps | | | | | | | | | | | |

**Conclusion**

**Thank you for participating in this survey!**

You will be asked to take a similar survey **again at the end of the semester.**
For correlating your response between now and then, please provide the following information, in code, as described below:

1. Your shoe size (ex: size 9 = **09**)

2. First two letters of your favorite color (ex: Green = **GR**)
3. How many sisters do you have? (ex: 2 sisters = **02**)
4. How many brothers do you have? (ex: no brothers = **00**)
5. First letter of the city where you were born? (ex: Boston = **B**)

For the above example information, you would write "**09GR0210B**"

**Optional:** Are there any questions that did not make sense? Or any questions you have about this questionnaire?

## C.2 Post-semester survey

The pre-semester survey from Part III is provided below.

**Introduction**

Thank you for your participation in this survey. I greatly appreciate your feedback!

This study takes 10-15 minutes to complete and includes four sections. You will be asked to complete each question before moving on. If at any point, you choose to end your participation in this research, you may do so.

**What is the purpose of this research?** The purpose of this research is to study the effect of various aspects of instruction on students' motivation and self-efficacy (belief in their own confidence) for mathematical proving.
**Will my identity be anonymous?** Your participation on with this survey is fully anonymous.
**What will happen to my data in the future?** We will not share your data or use it in future research projects.
**Do I have to participate?** No. If you do not participate, you will not be penalized or lose benefits or services unrelated to the research. If you decide to participate, you don't have to answer any question and can stop participating at any time.

If you have questions or concerns about the research please contact me (Paul Regier) in person or by email (paulregier@ou.edu). You can also contact my faculty advisor, Milos Savic at savic@ou.edu, or the University of Oklahoma – Norman Campus Institutional Review Board (OU-NC IRB) at 405-325-8110 or irb@ou.edu if you have questions, concerns, or complaints.

This research has been approved by the University of Oklahoma, Norman Campus IRB.
**IRB Number:** 10965 **Approval Date:** 07/18/2019

Please use the << and >> buttons on the survey to navigate forward and backwards. Previous selections may not be available when browser buttons are used for navigation. Click the next button to get started!

**CFMI**

Which class are you currently filling out this survey for?

Please rate **your level of agreement** with each of the following statements regarding your instruction in the above math class this semester.

| | 1 Strongly Disagree | 2 | 3 | 4 | 5 | 6 Strongly Agree | Not Applicable |
|---|---|---|---|---|---|---|---|
| My instructor encouraged us to present our solutions/approaches. | ○ | ○ | ○ | ○ | ○ | ○ | ○ |
| | 1 Strongly Disagree | 2 | 3 | 4 | 5 | 6 Strongly Agree | Not Applicable |
| My instructor assigned challenging problems and tasks. | ○ | ○ | ○ | ○ | ○ | ○ | ○ |
| | 1 Strongly Disagree | 2 | 3 | 4 | 5 | 6 Strongly Agree | Not Applicable |
| My instructor provided support when we were frustrated. | ○ | ○ | ○ | ○ | ○ | ○ | ○ |
| | 1 Strongly Disagree | 2 | 3 | 4 | 5 | 6 Strongly Agree | Not Applicable |
| My instructor valued our ideas in class. | ○ | ○ | ○ | ○ | ○ | ○ | ○ |
| | 1 Strongly Disagree | 2 | 3 | 4 | 5 | 6 Strongly Agree | Not Applicable |
| My instructor pointed out the beauty of certain solutions/approaches. | ○ | ○ | ○ | ○ | ○ | ○ | ○ |
| | 1 Strongly Disagree | 2 | 3 | 4 | 5 | 6 Strongly Agree | Not Applicable |

| | 1 Strongly Disagree | 2 | 3 | 4 | 5 | 6 Strongly Agree | Not Applicable |
|---|---|---|---|---|---|---|---|
| My instructor allowed us to approach a problem in a way that was different from theirs. | ○ | ○ | ○ | ○ | ○ | ○ | ○ |
| | 1 Strongly Disagree | 2 | 3 | 4 | 5 | 6 Strongly Agree | Not Applicable |
| My instructor did not encourage different approaches in class. | ○ | ○ | ○ | ○ | ○ | ○ | ○ |
| | 1 Strongly Disagree | 2 | 3 | 4 | 5 | 6 Strongly Agree | Not Applicable |
| My instructor discussed how it is OK to be confused while doing mathematics. | ○ | ○ | ○ | ○ | ○ | ○ | ○ |
| | 1 Strongly Disagree | 2 | 3 | 4 | 5 | 6 Strongly Agree | Not Applicable |
| My instructor pointed out connections between seemingly different mathematical ideas in class. | ○ | ○ | ○ | ○ | ○ | ○ | ○ |
| | 1 Strongly Disagree | 2 | 3 | 4 | 5 | 6 Strongly Agree | Not Applicable |
| My instructor encouraged us to pose our own mathematical problems to the class. | ○ | ○ | ○ | ○ | ○ | ○ | ○ |
| | 1 Strongly Disagree | 2 | 3 | 4 | 5 | 6 Strongly Agree | Not Applicable |
| My instructor did not allow us to revise homework problems. | ○ | ○ | ○ | ○ | ○ | ○ | ○ |
| | 1 Strongly Disagree | 2 | 3 | 4 | 5 | 6 Strongly Agree | Not Applicable |

| | 1 Strongly Disagree | 2 | 3 | 4 | 5 | 6 Strongly Agree | Not Applicable |
|---|---|---|---|---|---|---|---|
| My instructor encouraged us to persevere in doing mathematics. | ○ | ○ | ○ | ○ | ○ | ○ | ○ |
| | 1 Strongly Disagree | 2 | 3 | 4 | 5 | 6 Strongly Agree | Not Applicable |
| My instructor encouraged us to debate and discuss with one another. | ○ | ○ | ○ | ○ | ○ | ○ | ○ |
| | 1 Strongly Disagree | 2 | 3 | 4 | 5 | 6 Strongly Agree | Not Applicable |
| My instructor pointed out only standard approaches to problems. | ○ | ○ | ○ | ○ | ○ | ○ | ○ |
| | 1 Strongly Disagree | 2 | 3 | 4 | 5 | 6 Strongly Agree | Not Applicable |
| My instructor did not emphasize the importance of asking questions in our problem solving process. | ○ | ○ | ○ | ○ | ○ | ○ | ○ |
| | 1 Strongly Disagree | 2 | 3 | 4 | 5 | 6 Strongly Agree | Not Applicable |
| My instructor discussed how solving problems often requires a lot of time. | ○ | ○ | ○ | ○ | ○ | ○ | ○ |
| | 1 Strongly Disagree | 2 | 3 | 4 | 5 | 6 Strongly Agree | Not Applicable |
| My instructor recognized when a student builds on the work of another student. | ○ | ○ | ○ | ○ | ○ | ○ | ○ |
| | 1 Strongly Disagree | 2 | 3 | 4 | 5 | 6 Strongly Agree | Not Applicable |

| | 1 Strongly Disagree | 2 | 3 | 4 | 5 | 6 Strongly Agree | Not Applicable |
|---|---|---|---|---|---|---|---|
| My instructor described that doing mathematics can be challenging at times. | O | O | O | O | O | O | O |
| My instructor pointed out simple solutions to complex problems. | O | O | O | O | O | O | O |
| My instructor encouraged us to generalize what we learned from one problem to others. | O | O | O | O | O | O | O |
| My instructor allowed for freedom of time to work through problems. | O | O | O | O | O | O | O |
| My instructor encouraged us to ask mathematical questions in class. | O | O | O | O | O | O | O |

## Creativity

To you, what does it mean to be mathematically creative?

To engage a process of offering mathematical insights that are new to me

To produce mathematical insights that are new to me

[ ] Other:

By your above definition, were you mathematically creative in this course?

Yes

No

By your above definition, did this course foster your mathematical creativity in this course?

Yes

No

Please rate how confident you are that you can do the following **as of now**, using the scale below.

| | 0 = Cannot do at all | | | | | | 100 = Certain can do | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | 0 | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 |
| generate original math ideas | | | | | | | | | | | |
| solve a math problem in multiple ways | | | | | | | | | | | |
| give multiple solutions to a math problem | | | | | | | | | | | |
| build on the mathematical ideas of others | | | | | | | | | | | |
| be creative in solving math problems | | | | | | | | | | | |

**SEPS**

For the next two questions, please rate your confidence **in your own ability** to do the following sub-tasks related to creating a **proof,** or logical argument, of three mathematical statements.

However much you may want to prove these statements, please do not! **Just evaluate your own confidence** in your ability for each of the sub-tasks.

**Statement 1:**
If $n$ is an integer, then $1 + (-1)^n(2n - 1)$ is a multiple of 4.

In proving or disproving **Statement 1,** what is your confidence in your ability to do each of the following?

| | Cannot do at all | | | | | Moderately certain can do | | Highly certain can do | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | 0 | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 |
| Understand and informally explain why a statement is true or false | | | | | | | | | | | |
| Explore new ideas to come up with ways to start your proof | | | | | | | | | | | |
| Use various representations (numbers, pictures, tables, symbols, word) to structure your thinking | | | | | | | | | | | |
| Formally write out and justify each step of your proof | | | | | | | | | | | |
| Examine your proof for accuracy and identify any missing steps | | | | | | | | | | | |

Have you previously seen or proved the above statement?

No, I have not previously seen or proved Statement 1.

Yes, I have previously seen Statement 1, but haven't proved it.

Yes, I have proved Statement 1 in the past.

I may have seen something similar, but I cannot remember.

**Statement 2:**

Every odd integer is the difference of two squares.

In proving/disproving **Statement 2,** what is your confidence in your ability to do each of the following?

| | Cannot do at all | | | | Moderately certain can do | | | Highly certain can do | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | 0 | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 |
| Understand and informally explain why a statement is true or false | | | | | | | | | | | |
| Explore new ideas to come up with ways to start your proof | | | | | | | | | | | |
| Use various representations (numbers, pictures, tables, symbols, word) to structure your thinking | | | | | | | | | | | |
| Formally write out and justify each step of your proof | | | | | | | | | | | |
| Examine your proof for accuracy and identify any missing steps | | | | | | | | | | | |

Have you previously seen or proved the above statement?

No, I have not previously seen or proved Statement 2.

Yes, I have previously seen Statement 2, but haven't proved it.

Yes, I have proved Statement 2 in the past.

I may have seen something similar, but I cannot remember.

**AMSTM**

This is the last section! Almost done!

For the items below, select the response that best indicates **the extent to which** each of the following items presently explains **why you spend time studying mathematics.**

Why do you spend time studying mathematics?

| | Does not Correspond at all | Corresponds a little | Corresponds moderately | Corresponds a lot | Corresponds exactly |
|---|---|---|---|---|---|
| Because I want to feel the personal satisfaction of understanding mathematics. | O | O | O | O | O |
| | Does not Correspond at all | Corresponds a little | Corresponds moderately | Corresponds a lot | Corresponds exactly |
| Because without a good grade in mathematics, I will not be able to find a high-paying job later on. | O | O | O | O | O |
| | Does not Correspond at all | Corresponds a little | Corresponds moderately | Corresponds a lot | Corresponds exactly |
| Because of the fact that when I do well in mathematics, I feel important. | O | O | O | O | O |
| | Does not Correspond at all | Corresponds a little | Corresponds moderately | Corresponds a lot | Corresponds exactly |
| Because I think that mathematics will help me better prepare for my future career. | O | O | O | O | O |
| | Does not Correspond at all | Corresponds a little | Corresponds moderately | Corresponds a lot | Corresponds exactly |
| For the pleasure I experience when I discover new things in mathematics that I have never seen before. | O | O | O | O | O |

| | Does not Correspond at all | Corresponds a little | Corresponds moderately | Corresponds a lot | Corresponds exactly |
|---|---|---|---|---|---|
| | Does not Correspond at all | Corresponds a little | Corresponds moderately | Corresponds a lot | Corresponds exactly |
| In order to obtain a more prestigious job later on. | ○ | ○ | ○ | ○ | ○ |
| | Does not Correspond at all | Corresponds a little | Corresponds moderately | Corresponds a lot | Corresponds exactly |
| Because I want to show to others (e.g., teachers, family, friends) that I can do mathematics. | ○ | ○ | ○ | ○ | ○ |
| | Does not Correspond at all | Corresponds a little | Corresponds moderately | Corresponds a lot | Corresponds exactly |
| Because studying mathematics will be useful for me in the future. | ○ | ○ | ○ | ○ | ○ |
| | Does not Correspond at all | Corresponds a little | Corresponds moderately | Corresponds a lot | Corresponds exactly |
| For the pleasure that I experience in broadening my knowledge about mathematics. | ○ | ○ | ○ | ○ | ○ |
| | Does not Correspond at all | Corresponds a little | Corresponds moderately | Corresponds a lot | Corresponds exactly |
| Because I want to have “the good life” later on. | ○ | ○ | ○ | ○ | ○ |
| | Does not Correspond at all | Corresponds a little | Corresponds moderately | Corresponds a lot | Corresponds exactly |
| To show myself that I am an intelligent person. | ○ | ○ | ○ | ○ | ○ |
| | Does not Correspond at all | Corresponds a little | Corresponds moderately | Corresponds a lot | Corresponds exactly |

| | Does not Correspond at all | Corresponds a little | Corresponds moderately | Corresponds a lot | Corresponds exactly |
|---|---|---|---|---|---|
| Because I believe that mathematics will improve my work competence. | O | O | O | O | O |

| | Does not Correspond at all | Corresponds a little | Corresponds moderately | Corresponds a lot | Corresponds exactly |
|---|---|---|---|---|---|
| For the pleasure of being able to experience "light bulb" moments in understanding something new. | O | O | O | O | O |

| | Does not Correspond at all | Corresponds a little | Corresponds moderately | Corresponds a lot | Corresponds exactly |
|---|---|---|---|---|---|
| In order to have a better salary later on. | O | O | O | O | O |

| | Does not Correspond at all | Corresponds a little | Corresponds moderately | Corresponds a lot | Corresponds exactly |
|---|---|---|---|---|---|
| Because I want to show myself that I can do well in mathematics. | O | O | O | O | O |

| | Does not Correspond at all | Corresponds a little | Corresponds moderately | Corresponds a lot | Corresponds exactly |
|---|---|---|---|---|---|
| Because what I learn in mathematics now will be useful in my future studies. | O | O | O | O | O |

| | Does not Correspond at all | Corresponds a little | Corresponds moderately | Corresponds a lot | Corresponds exactly |
|---|---|---|---|---|---|
| For the pleasure that I experience when I feel completely absorbed by what myself or others come up with. | O | O | O | O | O |

**Conclusion**

For correlating your responses between now and the beginning of semester, please provide the following information, in a code, as described below:

1. Your shoe size (ex: size 9 = **09**)
2. First two letters of your favorite color (ex: Green = **GR**)
3. How many sisters do you have? (ex: 2 sisters = **02**)
4. How many brothers do you have? (ex: no brothers = **00**)
5. First letter of the city where you were born? (ex: Boston = **B**)

For the above example information, you would write "**09GR0200B**"

What is your gender?

Female

Male

Prefer to self-describe

Prefer not to say

**Thank you for participating in this survey!**

Please let me know if you have any questions or comments about this survey, or if you would like to elaborate on any of your responses.

Powered by Qualtrics

## C.3 Response process interview protocol

1. Initial explanation of interview - “Thank you for being willing to participate. This will take about 30 minutes. Here’s how this will go…”
2. Initial questions: “What course are you taking? How’s it going, reason for taking course?”
3. Send link for survey on skype messaging: “Take time filling out. Let me know when you are done.” (I will be able to look at responses on Qualtrics.)
4. Ask students - “read each item aloud, and explain why you chose the answer.” If a student’s reasoning did not match their answer choice, probing questions were asked in order to clarify their interpretation of the item and how it matched their answer choice and reasoning. (Take notes one responses)
5. Give gift card info