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BY THE COMMITTEE CONSISTING OF

Dr. Bryan Stanhouse, Chair

Dr. Duane Stock

Dr. William Megginson

Dr. Lubomir Litov

Dr. Scott Linn

Dr. Jaeho Kim

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Abstract

This dissertation consists of three chapters. In the first chapter, using proxies for conversion cost parameters in conjunction with a special set of default free corporate bonds, I empirically establish that the term structure of liquidity spreads was positively sloped in the financial crisis period of 2008 and negatively sloped in the subsequent post crisis period. Importantly, these results indicate the segment of the term structure that provides the largest liquidity premiums to lenders for alternative economic scenarios. At the same time, for different financial epochs, the liquidity spreads associated with different times to maturity are clear to those who issue debt.

In the second chapter, across credit ratings for alternative financial scenarios, I obtain the term structures of corporate bond liquidity premia using an enhanced version of a classic debt valuation equation. For purposes of estimation, unscented transformations (UT) accommodate the nearly two dozen nonlinear appearances of liquidity premiums in the determination of a bond's yield. A Kalman filter then extracts the otherwise latent liquidity premiums from the observed yield. I find that the term structures of liquidity premiums were upward sloping for all credit ratings and in all three economic epochs considered. In addition, the structures were much steeper during the financial crisis and steeper for lower quality bonds across all three scenarios. I also empirically establish that the magnitude of the liquidity premiums more than doubled during the recent financial crisis relative to the pre-crisis period. Furthermore, the liquidity premiums do not retreat to their pre-crisis level after the financial crisis.

Finally, in the third chapter, using a classical model of debt valuation that acknowledges liquidity premiums, I study how corporate bond market illiquidity may result in an elevated likelihood of the issuer's default. Clearly, these liquidity considerations should be included in the pricing of securities that depend on credit risk, such as credit default swaps (CDS). Empirical

analyses suggest that increases in bond illiquidity measures result in a heightened likelihood of the issuer's default and occasion a greater price of its CDS contracts. I also establish that the impact of illiquidity upon CDS premiums is more severe for less creditworthy issuers and in periods of financial stress.

Chapter 1: Estimating the Term Structure of Corporate Bond Liquidity Premiums: An Analysis of Default Free Bank Bonds

1. Introduction

The liquidity of anything that can be traded is a very important dimension of its value. Ceteris paribus, an asset that is more liquid tends to have more value than an asset that is relatively less liquid. The finance literature that describes and models liquidity has grown tremendously in recent decades. Two groundbreaking theoretical contributions have been those of Kyle (1985), who describes the basic dimensions of liquidity as trading cost, depth, and resiliency,[1] and of Acharya and Pedersen (2005), who develop a liquidity-adjusted capital asset pricing model where the asset returns depend on expected liquidity and covariance with market returns and market liquidity.[2] The liquidity of default free U.S. Treasury bonds has been analyzed by, among others, Amihud and Mendelson (1991) and, more recently, by Fontaine and Garcia (2012) and Musto, Nini and Schwarz (2018). Driessen, Nijman, and Simon (2017) analyzed differential pricing and liquidity of German short and long-term sovereign bonds. With respect to corporate bonds, Bao, Pan, and Wang (2011) examined the liquidity of corporate bonds where they use the Roll (1984) measure in preference to the bid-ask spread.[3] Lin, Wang, and Wu (2011) find that liquidity risk is an important determinant of expected corporate bond returns. Bao and Pan (2013) find that empirical volatilities of corporate bond returns are higher than those implied by the Merton (1974)

[1] Respectively, these can be represented by such measures as bid-ask spreads, the Amihud (2002) measure, and changes in dealer inventory.

[2] They find that negative shocks to liquidity reduce valuation and thus give low contemporaneous returns on equities.

[3] It is noteworthy that they find the liquidity component of the yield spread can sometimes be greater than the default spread component.

model and attribute this finding to inadequate modelling of liquidity. More recently, Bongaerts, de Jong, and Driessen (2017) analyze corporate bond liquidity as a function of both asset-specific characteristics and systematic liquidity shocks.

Investors in search and bargaining based models maximize their expected utility subject to the cost features of asset conversions. As a consequence of investor behavior, the magnitude of conversion considerations determines the shape of the term structure of liquidity spreads (TSLS). However, when financial markets transition from one economic scenario to another, a host of changes take place in these conversion costs. If the intertemporal changes are weighted by their marginal impact and then aggregated, they generate a new maturity structure of liquidity spreads. Clearly, search and bargaining models provide testable hypotheses about the shape of the TSLS in alternative economic periods. Using proxies for conversion cost parameters in conjunction with a special set of insured, default free bank bonds, we empirically establish that the term structure of liquidity spreads was positively sloped in the financial crisis of 2008 and negatively sloped in the subsequent post crisis period.[4] Importantly, these results indicate the segment of the term structure that provides the largest liquidity premiums to lenders for alternative economic scenarios. At the same time, for different financial epochs, the liquidity spreads associated with different times to maturity are clear to those who issue debt.

Of course, the attraction of using insured bank debt that participated in the guaranteed debt program was that the yield spread is exclusively the liquidity premium. However, in order to both generalize our results and to establish the robustness of our conclusions, we extended the maturity

[4] Lewis, Longstaff and Petrasek (2017) also use this data base of insured bonds to analyze different important questions than we do. In clear contrast to our work, their basic question is framed in the context of mispricing (divergence from fundamental value) of insured bonds where, for example, funding costs and slow moving capital are given as reasons for mispricing. Explaining the shape of the term structure of liquidity premiums is not mentioned in their study.

structure of our bank bonds. Extending debt to include securities which have a time to maturity greater than four years involves yield spreads that contain both a liquidity premium and a default premium. The Kalman filter routinely estimates state variables and is capable of extracting a liquidity premium from a confounded yield spread. However, our empirical analysis is now plagued with both unobserved state variables (the liquidity premium and the default premium) as well as the unknown parameters on the right hand side variables. Consequently, the conventional Kalman, which presumes knowledge of the model's parameters, cannot be directly applied to this empirical challenge. We employ a dual estimation technique to the problem of determining both the state of the dynamics and the model which gives rise to the dynamics.[5]

Section 2 of this paper contains our hypotheses. Then, sections 3 and 4 describe our data and methodology. The results of testing the hypotheses are given in section 5. Section 6 summarizes our major finding and concludes our research.

2. Hypothesis

In Duffie, Garleanu and Pedersen (2005, 2007) (DGP) "search and bargaining" based models, investors maximize the expected value of their intertemporal utility functions. DGP market equilibrium conditions produce steady state results that parsimoniously incorporate the

[5] We note that Longstaff (2004) documents regression results for the liquidity premiums of Refcorp agency bonds. On the other hand, we examine liquidity premia of corporate bonds. In contrast to Longstaff (2004) data, our insured bond premiums seem to respond positively to increases in the slope of the U.S. Treasuries and are very sensitive to changes in financial anxiety. Additionally, the differences in the specifications in Longstaff (2004) and in our study suggest that the liquidity premiums for these two types of bonds are determined by different economic variables and that the premiums themselves are different. These results highlight the distinction in the papers and ensure the independent importance of each piece of research. We also note that Schwarz (2019) examines the relative contributions of default and liquidity risks in the determination of bond spreads of European countries. In contrast to our analysis, Schwarz (2019) examines the implications of the liquidity spreads in other markets of various different countries, while we focus on the determinants of the liquidity spreads across maturities in only the U.S.

salient economic considerations of asset conversions. Significantly, the tractable pricing equations occasion yields to maturity which are amenable to a comparative static analysis of the term structure of liquidity premiums. DGP (2005, 2007) models include at least three parameters whose variations could significantly change the shape of the term structure of liquidity premiums: 1) the investor's search intensity ρ^i, 2) the holding costs δ, and 3) the joint likelihood of λ and π which characterize the probability of an adverse liquidity shock.[6,7] Plots of intertemporal comparative static behavior of these individual parameters are illustrated in Panels A, B, C, and D of Figure 1. A detailed interpretation of these figures is provided in Appendix A.

When financial markets transition from one economic scenario to another, a host of parametric changes take place, not just a solitary change in a single asset conversion cost characteristic. If the intertemporal changes in these cost considerations are weighted by their marginal impact and then summarized, they generate a new maturity structure for liquidity premiums. In order to illustrate the impact of a transition from one economic scenario to another, consider a period of stress and then a post stress period. For instance, and loosely consistent with some empirical results, we double the magnitude of δ and λ from their baseline settings, halve the value of our search parameter, and set $\pi = 1$ to generate a representation of TSLS that could

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[6] For each investor, the probability of finding a dealer follows a Poisson distribution with intensity ρ^i, where $i \in [1,2,\dots,m]$ is an index for different search intensities with $\rho^i > \rho^j$ if $i > j$. For each investor type, a shock arrives according to a Poisson process with intensity λ. The probability that the investor will become type "high", without holding cost, is 1- π and the probability he will become a "low" type, with holding cost, is π.

[7] In DGP based models the liquidity premium is defined as the yield of a bond under average search intensities (average ρ^i) minus the yield of a bond under no search frictions ($\rho^i \to \infty$). The yield to maturity, y, is the rate that makes the expected value of the discounted future cash flows equal to the price of the bond, P, for a random time to maturity. P is the average of steady state bid and ask price occasioned by the DGP model. This approach to the determination of the liquidity premium is identical to that employed by Feldhütter (2012), He and Milbradt (2014) and Chen, Cui, He, Milbradt (2018).

possibly exist during a period of stress. In contrast, we halve δ and λ, double our search parameter ρ^i and reduce π to 0.5 to occasion a post stress period.[8] The consequences of our two sets of parameters are depicted in Figures 2A and 2B. The positively sloped TSLS is an artifact of the parameter settings consistent with a period of financial stress. The negatively sloped TSLS is associated with parameters meant to characterize a post stress scenario. The contrasting schedules simply demonstrate what is possible. However, DGP based models consider the major economic features of asset conversions and, while the magnitude of the parameters is merely suggestive, the direction of their changes occasion shapes of the TSLS that are a consequence of expected utility maximizing behavior of investors. Given these two diagrams, DGP based models provide testable hypotheses about the shape of the term structure of liquidity spreads in alternative economic periods. Furthermore, we believe that proxies for DGP asset conversion parameters exist so that we will be able to empirically investigate the following hypothesis:

The term structure of liquidity premiums was upward sloping in the financial stress period of 2008 and downward sloping in a subsequent post stress scenario.

[8] Adrian, Fleming, Shachar, and Vogt (2017) find that between 2008 and 2009, the average bid-ask spread for U.S. corporate bonds widened to about 2.3% of par and then declined to pre-crisis levels of around 1%. Anderson and Stulz (2017) and Adrian, Fleming, Shachar, and Vogt (2017) also document a steep decline in the price impact measure during and after the financial crisis, from 1% to 0.4% of par value per million. Dick-Nielsen and Rossi (2018) use exclusions from a corporate bond index to show that the cost of immediacy doubled since before the 2008 crisis. Lastly, Bessembinder, Jacobsen, Maxwell, and Venkataraman (2018) document that trade execution costs nearly doubled during the financial crisis compared to the subsequent years. These three citations point to dramatic changes in the cost of asset conversions, per se, as we transition from a stress to a post stress period. In addition, we believe these references provide a loose justification for the dramatic changes we assume in the holding costs δ for Figures 2A and 2B. Furthermore, the fact that VIX nearly doubled during the stress period casually supports the changes in λ that we use to construct the representations 2A and 2B.

3. Data Description

To perform our analysis, we gathered data for four different types of bonds. The first type of bond is insured corporate bonds which were issued by numerous banks in 2008 and 2009. These bonds were insured by the U.S. Treasury and were backed by the full faith and credit of the U.S. Treasury; thus these corporate bonds had no default risk. These bonds were part of the debt guarantee program meant to stabilize the banking system. The debt guarantee limit was restricted to 125% of the face value of senior unsecured debt that was outstanding as of September 30, 2008. The last day to issue debt under the program was October 31, 2009. Any insured bonds included in the debt guarantee program matured on or before December 31, 2012, by which time all debt insurance disappeared. Bond price data for these insured corporate bonds were gathered from the Trade Reporting and Compliance Engine (TRACE), which provides comprehensive coverage of bond trades. For these insured bonds, we also gathered information such as bond maturity, issuance dates, coupon rates and other issuance characteristics from Mergent FISD.

The second type of bond is U.S. Treasury bonds which are commonly assumed free of default risk. To compute the spread of our insured corporate bonds over equal maturity Treasury bonds, we use the H-15, which publishes data on U.S. Treasury yields for fixed maturities on a daily basis. We use GOVPX to compute bid–ask spreads of Treasury bonds. Liquidity spreads are the difference in yield between insured corporate bonds and U.S. Treasury bonds of equal maturity.

The third type of bond is all U.S. corporate bonds other than the insured bonds. These other corporate bonds were used to represent market wide liquidity of the corporate bond market. TRACE is used to gather prices and yields for these bonds. To remove errors, cancellations, corrections, and reversals in the TRACE data, we filter the intraday transaction data according to the procedure described in Dick-Nielsen (2009 and 2014). Finally, to eliminate extreme outliers, the data is further treated with the median filter described in Edwards, Harris, and Piwowar (2007).

The fourth type of bond is all AAA and AA U.S bank bonds (SIC codes between 6000 to 6200) that traded between 2008 and 2012. We use these bonds in two additional tests where we attempt to generalize our results to uninsured bank bonds with a maximum maturity of up to 10 years. We use bank bonds of high credit rating because they are more similar in nature to our insured sample. We limit our maturities to 10 years because few banks issue bonds with maturities between 10 and 20 years.

We merge the above bond transaction datasets with Mergent FISD, which contains at-issuance information such as offering date, maturity, and coupon, as well as the CUSIP of the issuing firm. From FISD, we calculate the time-to-maturity of the bond and its age. Using the issuer CUSIP, we merge the resulting datasets with the issuing firm's financial reports from the quarterly files in COMPUSTAT. We obtain quarterly assets, current and long-term debt, total interest and dividends paid, financial leverage, operating margin, and volatility of assets.

In addition to bond price data for the above types of bonds, we use a battery of variables to represent financial and macroeconomic conditions such as the VIX, U.S. Treasury term structure slope, interest rate swaps, and the 30-day Fed funds futures price.

4. Methodology

Liquidity Measures

In order to thoroughly examine the impact of liquidity measures, we compute four measures: bid-ask spread, the Amihud measure, the Imputed Roundtrip Trades (Roundtrip measure), and the interquartile range. The results for the impact of the liquidity measure upon liquidity spreads are very similar for all liquidity measures where we report the results using the

Amihud measure. Figure 3 shows the strong correlation among the liquidity measures over the time period of our research.[9]

To proxy for the bid-ask spread we follow Hong and Warga (2000), and Black, Stock, and Yadav (2016) and aggregate for each bond in each day, the price (p) of the "Buys" minus the price of the "Sells" weighted by the volume size of each trade (q). We then divide this by the midpoint price between the buy and the sell in each trade:

$$bid - ask = 100 \cdot \frac{\frac{\sum_1^N p \cdot q}{\sum q_b} - \frac{\sum_1^N p \cdot q}{\sum q_s}}{\frac{\sum_1^N p \cdot q}{\sum q_b} + \frac{\sum_1^N p \cdot q}{\sum q_s}}$$

We next calculate the Amihud liquidity measure for all corporate bonds following the definition in Dick-Nielsen, Feldhütter, and Lando (2012). We aggregate the absolute value of the returns per trade and we divide by the trade volume. The Amihud measure is designed to capture the depth dimension of liquidity, i.e. the price impact of transactions adjusted transaction size:

$$A = \frac{100}{N} \sum_1^N \frac{abs(r_t - r_{t-1})}{vol \,/\, 10^6}$$

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[9] As bond market liquidity analysis has progressed, there has been debate concerning the best measure(s) of bond market liquidity, how they may or may not move together, and how bond market liquidity behaves in times of stress versus post stress. Dick-Nielsen, Feldhütter, and Lando (2012) compute a liquidity measure that is a composite of different measures and, furthermore, analyze the liquidity component (versus the default component) of corporate bond yields both before and after the financial crisis. Schestag, Schuster, and Uhrig-Homburg (2016) compute and evaluate a long list of alternative liquidity measures and find that the most commonly used measures are very strongly correlated and move together over time. Other important bond market liquidity papers include Kempf, Korn, and Uhrig-Homburg (2012) who show that current trading needs of investors determine the short end of liquidity premiums. We note that Houweling, Mentink, and Vorst (2005) find limited differences in liquidity proxies for corporate bonds. With regard to total spreads and our later analysis of different periods, Van Landshoot (2008) finds that the credit cycle as measured by default probability affects U.S. corporate bond yield spreads.

where $r_t = \log(p_t)$ is the return of the bond for the transaction t, vol is the volume traded in transaction t, and N is the number of trades in one day.

Also, following Feldhütter (2012), we compute the imputed roundtrip measure which captures pre-arranged trades. Bonds sometimes trade two or three times within a short window. This can be interpreted as a dealer prearranging the transaction and collecting the spread. Similar to Dick-Nielsen, Feldhütter, and Lando (2012), we define the roundtrip measure as the spread between the maximum price and the minimum price in such a roundtrip transaction. We also only consider roundtrip transactions where exactly two or three trades occur nearly simultaneously:

$$Roundtrip = 100 \cdot \frac{p_M - p_m}{p_M}.$$

Here p_m is the lowest price in the pre-arranged deal and p_M is the highest price in the pre-arranged deal.

Finally, we calculate the daily interquartile range as another proxy for the daily bid-ask. We calculate the interquartile range as the 75th percentile of a bond-day price minus the 25th percentile divided by the median of the bond price in that day:

$$IQR = \frac{p_{0.75} - p_{0.25}}{p_{0.50}}.$$

We also include the volatility of liquidity measures, which Dick-Nielsen, Feldhütter, and Lando (2012) have shown to be able to explain part of the liquidity spread. We calculate the volatility of each measure by calculating the standard deviation over 20 day windows. We choose 20 day windows as a compromise between sample size and explanatory power of the variable. Our results are also robust to 1-month and 3-month rolling windows.

We also include the bid-ask spread for U.S. Treasury bonds from GOVPX and CRSP as a proxy for overall liquidity in the market and our results are robust to using either. Table 1 displays means and standard deviations of liquidity measures for all bonds in TRACE (Panel A) and for our sample of insured bonds (Panel B). We summarize the correlations between the four liquidity measures used in this study in Panel C of Table 1. For statistical analysis that follows, we use the Amihud measure where, again, these statistical results are very similar if the above alternative liquidity measures are used.

Period Selection

A fundamental part of our analysis is to appropriately select periods of the time period – 2008 to 2012 – for which underlying economic conditions were different. Broadly consistent with Dick-Nielsen, Feldhütter, and Lando (2012), Acharya, Amihud, and Bharath (2013), Bessembinder, Jacobsen, Maxwell, and Venkataraman (2018), and the NBER list of economic expansions and contractions, we define October 2008 to November 2009 as the stress period and November 2009 to December 2012 as the post stress period. We deviate ever so slightly from our predecessors' periods in order to acknowledge the timing of significant changes of GDP, VIX, and Fed funds rates futures prices (10% upwards or downwards) as shown in Figure 4.[10]

Table 2 Panel A summarizes the number of insured bank bonds in the sample, the number of bank issuers, the total number of trade/days in the stress and post-stress periods, and the average number of trades per insured bond. Table 2 Panel B provides averages of the liquidity premium spreads relative to the U.S. Treasury rates and the interest swap rates for quarterly increments of the time to maturity. These averages are for the period of stress (Table 3A) and post-stress (Table 3B). Figure 5 contains raw liquidity premium spreads (insured corporate bond yields less Treasury

[10] We alternatively use the NBER definition of the periods of stress and post stress, from October 2008 to June 2009 and from June 2009 to December 2012. The results using these dates are very similar to the ones we present below.

yields of equal maturity) plotted against maturity for the different periods. The plots are averages of all bond spreads with the same time to maturity. These spreads are aggregated by maturity buckets of one-week length to avoid noise and high dispersion in the data. The green triangles provide preliminary evidence of a positive slope for the early (stress) part of the data while the blue diamonds suggest a negative slope for later (post stress) observations. Appendices B and C use alternative periods where the main results are similar to Figure 5 and other results we report below.

Maturity portfolio selection and regression framework

To test whether periods of stress and post stress have a differential impact on the TSLS, we aggregate bonds into maturity buckets. We choose two buckets – short and long – in order to parsimoniously describe the slopes of the TSLS without sacrificing sample size in each of the buckets. We alternatively used three buckets—short, medium, and long—in addition to using different periods where the results are always similar. See Appendices B and C for this robustness analysis.

We then allocate our daily bond observations into the two buckets according to the following procedure. If the time to maturity is below the 50th percentile for that period, the observation is categorized as short-term, and if it is above the 50th percentile, it is categorized as long-term. For expositional convenience, we adopt the notation ST_{INS} and LT_{INS} to characterize our relatively "short-term" and "long-term" insured bank bonds, even though the maximum time to expiry of this special sample is only 4 years.

To test hypotheses about liquidity spreads, we proceed as follows. For each period and each maturity portfolio (see period and maturity portfolio selection above), we run the regression equation found below where the liquidity spread is the yield on the insured corporate bond less the yield on the equal maturity U.S. Treasury and $Liquidity\,Measure$ is the Amihud measure. Furthermore, VIX is the volatility index, Fed funds futures is the futures contract price, Treasury slope is the yield of a 6 month Treasury less the yield of a 3 month Treasury, TTM is time to maturity, market liquidity is the liquidity measure for all corporate bonds in TRACE, volatility of liquidity is the standard deviation of the liquidity measure for the bond over a 20-day rolling window, and $e_{i,t}$ represents the error term.

$$LS_{i,t} = \alpha + \beta_1 \cdot (Liquidity\ Measure)_{i,t} + \ \beta_2 \cdot (VIX)_t + \beta_3 \cdot (Fed\ Futures\ Price)_t + \ \beta_4 \cdot (Treas\ slope, 6m - 3m)_t + \beta_5 \cdot (TTM)_{i,t} + \beta_6 \cdot (Market\ Liquidity)_t + \cdot (Volatility\ of\ Liquidity)_{i,t} + e_{i,t}$$

Here, i is one of the two periods and t is one of the two maturity buckets. We follow a panel regression in this specification and we include issuer fixed effects. We also cluster the standard errors at the firm level to account for serial correlation in the covariates.

5. Results

DGP based models generate alternative shapes of TSLS that vary according to the magnitude of parameters assigned to investor search intensity ρ, holding costs δ, the hazard rate of financial shocks λ, and the likelihood of investors being financially constrained π. Parameter assignments meant to characterize stressful periods provide a positively sloped term structure of liquidity spreads. Parameter values which describe a post stress period create a negatively sloped TSLS. Figures 2A and 2B provide a testable hypothesis for our empirical investigation and DGP asset conversion cost parameters inform the selection of RHS variables in our regression analysis.

The results from regressing liquidity spreads upon important explanatory variables related to our hypothesis are detailed in Columns 1 to 4 in Table 3. The Amihud liquidity measurement (AMIHUD), the VIX, the fed funds futures price (FFFP), and the slope of the treasury term structure of interest rates (TREASLP) are all independent variables in our empirical analysis. We also include explanatory variables not directly related to our hypotheses –market-wide liquidity (AMIHUDMKT) and volatility of liquidity (AMIHUDVOL) – as control variables following other studies such as Dick-Nielsen, Feldhütter and Lando (2012). Our right hand side variables are characterized by large differences in their respective magnitudes. Consequently, in order to facilitate a comparison of their relative impact upon the liquidity spread, we have standardized (zero mean, standard deviation of one) our regressors. After this transformation of our data, a unit increase in any independent variable corresponds to a one standard deviation increment in that variate. We run separate regressions for short and long-term maturity buckets in "stress" and in "post stress" economic periods in order to estimate the determinants of the liquidity spread for alternative times to maturity in different periods.[11] One aspect of our results is that the coefficient of determination averages 51% over our period of economic stress for short (ST_{INS}) and long-term (LT_{INS}) insured bonds, while the average R^2 for the ST_{INS} and LT_{INS} liquidity spreads in our post stress period is only 15%. In addition, LT_{INS} reduced form coefficients are larger than ST_{INS} regression coefficients for six of seven variates in the period of stress, while, in distinct contrast, the inequality is reversed for five of seven independent variables during the post stress period. These four observations point to the possibility that market participants pay very close attention to bond specific liquidity measurements, macroeconomic variables, and monetary policy during periods of economic stress. Moreover, investors translate the behavior of these variables into portfolio decisions.

[11] At this point, we reiterate that our notation ST_{INS} and LT_{INS} characterizes our short-term and "long-term" insured bank bonds, even though the maximum time to expiry in our special sample is only 4 years. At the end of this section, we extend our results by investigating the liquidity premiums of longer term bonds.

In Table 3, bond specific liquidity is quantified by the Amihud measurement, AMIHUD. In our stress period, a unit increase in the standard deviation of Amihud liquidity increases the ST_{INS} liquidity spread by 4.4 basis points and the LT_{INS} spread by 15.6 basis points.[12] Both regression coefficients are statistically significant at the 1% level. Of course, Amihud liquidity is actually a measurement of illiquidity, where an increase corresponds to an increase in the absolute value of the change in the log of the security price due to the transaction at hand. The greater the AMIHUD, the greater the illiquidity; consequently, the positive effect of AMIHUD upon both ST_{INS} and LT_{INS} liquidity spreads is well anticipated. However, remarkably, the LT_{INS} regression coefficient on AMIHUD is nearly four times larger than the impact of AMIHUD upon the ST_{INS} spread. Not only does the impact of AMIHUD increase with the time to maturity, it does so dramatically. This is emphatic support for the notion that times of stress enhance the focus of investors upon meaningful economic variables. Furthermore, changes in AMIHUD disproportionately impact the LT_{INS} yield spread since the consequences of holding LT_{INS} bonds during a financial shock can be severe.

Using the regression coefficients of AMIHUD to examine the differential change in the liquidity spread as AMIHUD changes within the stress period yields $d(LS_i) = \left[\frac{\partial LS_i}{\partial AMIHUD}\right] d(AMIHUD)$. Clearly $d(LS_{LT_{INS}})$ will be greater than $d(LS_{ST_{INS}})$, with $d(AMIHUD\ ) > 0$. Consequently, we have differential changes in the liquidity spread that contribute to the determination of the positively sloped term structure of the liquidity spread depicted in Figures 5, 6, and 8.

[12] We also used numerous alternative measures of liquidity (bid-ask spread, roundtrip, and interquartile range) where, throughout our research, all show very similar results as using AMIHUD; these results are available from the authors.

In our post stress period, AMIHUD continues to have a positive impact upon the liquidity spread. Both coefficients are statistically significant with a one standard deviation increase in AMIHUD increasing the ST_{INS} liquidity spread by 22.5 basis points and, in contrast, increasing the LT_{INS} liquidity spread by only 3.2 basis points. These coefficients detail a dramatic reversal in the relative importance of AMIHUD for short-term and long-term spreads in post stress periods. Equally notable is the fact that AMIHUD's impact upon the LT_{INS} liquidity spread is one-third of the size it was during a financial crisis.[13]

VIX is a standard measure of financial stress.[14] A standard deviation increase in VIX during the stress period increases the liquidity spread on ST_{INS} bonds 12.9 basis points and 22.3 basis points on LT_{INS} bonds where both coefficients are statistically significant. The positive reduced form coefficients are intuitive since an increase in VIX proxies an elevated level of investor anxiety which generally occasions an increase in the liquidity spread. The fact that the LT_{INS} reduced form coefficient on VIX is nearly twice as large as the ST_{INS} coefficient provides further evidence that periods of economic stress motivate longer-term security holders to pay close attention to and to react to macroeconomic factors.

The average normalized value of VIX during our stress period was 1.32 while the average normalized value of VIX for 13 months before the crisis was -0.43. This change in VIX of 1.75 standard deviations combined with our regression coefficients for LT_{INS} VIX and ST_{INS} VIX yields

[13] Relatively free of the threat of a liquidity shock, clientele effects, as suggested by Gehde-Trapp, Schuster, and Uhrig-Homburg (2016), motivate some investors to hold long-term securities with little need for a liquidity premium. Meanwhile, enhanced spreads are enjoyed by ST bond holders as compensation for incurring the repeated search costs of owning short-term bonds.

[14] VIX documents market volatility and is routinely interpreted as market anxiety. It is reasonable to consider VIX as a proxy for the DGP parameter λ, the hazard rate for the time to a financial shock. An increase in λ, in part, created the positively sloped DGP term structure of the liquidity spread found in Figure 2A. The differential impact of VIX upon dLS_{LT} and dLS_{ST} had a dramatic impact on Figure 8 and mimicked the role that $d\lambda$ played in the determination of Figure 2A.

$d(LS_{LT_{INS}}) = 0.39025$ and $d(LS_{ST_{INS}}) = 0.2275$. These differential changes in the liquidity spread during a period of economic stress contribute to the determination of the positively sloped term structure of the liquidity spread depicted in Figures 5, 6, and, most prominently, 8.

In our post stress period, VIX again registers a positive and statistically significant impact upon both ST_{INS} and LT_{INS} liquidity premiums, 0.048 and 0.032 percentage changes, respectively. However, it is very important to note that the relative magnitudes of VIX upon ST_{INS} and LT_{INS} securities is reversed. Even more importantly, the regression coefficient of VIX upon the LT_{INS} liquidity spread in our post stress period is only one seventh of its size in the earlier period. While the average VIX is zero over all three periods, the VIX retreats to an average of -0.022454 standard deviations below the mean in the post stress period.

The importance of VIX is clear; a unit increase in its standard deviation can change $LS_{LT_{INS}}$ in a stress period as much as 22.3 basis points, or, in contrast, only increase the liquidity spread for ST_{INS} bonds as little as 4.8 basis points in a post stress period. Longstaff (2004) suggests that the short-term liquidity spread is less sensitive to VIX than the $LS_{LT_{INS}}$. In contrast, our regression results record a greater increase in the $LS_{ST_{INS}}$ (4.8 basis points) than in the $LS_{LT_{INS}}$ (3.2 basis points) for a standard deviation increase in VIX during a post stress period.

We used the Fed funds futures price (FFFP) to proxy the impact of Federal Reserve behavior upon the liquidity spread. The Fed fund futures is a popular 30-day contract whose price implicitly contains investor expectations about the future cost of overnight borrowing between banks. The sustained actions of the Federal Reserve in the Fed funds market in the stress period pushed the average FFFP 1.4 standard deviations above its pre-stress level found in the 13 months before the crisis of 2008. This escalated value of the Fed funds futures price failed to impact the ST_{INS} liquidity spread in a statistically meaningful way. However, this absence of a statistically significant regression coefficient is a powerful result. It suggests a strong relationship between the ST_{INS} yields on the insured bonds and Treasury bonds. Clearly if FFFP was 1.4 standard deviations above its pre-stress level, then the Treasury yield must have fallen dramatically due to Federal Reserve policy behavior. However, ST_{INS} insured bond yields matched the deterioration of government yields and left the ST_{INS} liquidity spread unaffected by FFFP in the stress period. This remarkable interdependence between the two yields establishes the existence of market forces which preserve the short-term liquidity spread in the stress period despite aggressive monetary policy. The reduction in the anticipated Fed funds futures rate did occasion a statistically significant change in the long-term liquidity spread of -7.0 basis points.

For the post stress time period, Table 3 documents a significant positive relationship between FFFP and the liquidity spread of LT_{INS} bonds. A one standard deviation increase in the Fed funds futures price increases the LT_{INS} liquidity spread by 1.7 basis points. As in the stress period, a change in FFFP occasions no statistically significant impact upon the ST_{INS} liquidity spread. Again, we argue that the insignificant regression coefficient documents the strength of market forces to preserve the existing short-term premium regardless of Federal Reserve operations.

We maintain that monetary policy will impact segments of the term structure differently and the effect(s) will change over the business cycle. Our research analyzes short-term monetary policy by the Federal Reserve, in terms of its impact upon the Fed funds futures price. Surprisingly, the FFFP has no statistically significant impact upon the short-term liquidity in either period. This result is in contrast to Longstaff's (2004) conjecture that the greater the Federal Reserve activity at the short end of the maturity spectrum, the greater the spread. Our empirical results do establish the FFFP as having a statistically meaningful impact at the longer end of the term structure of the liquidity spread and that the direction of that effect depends on business conditions.

It is often held that the slope of the Treasury term structure proxies the expected change in risk-free yields. While we have no prior notions as to the impact of the slope of Treasury term structure upon the liquidity spread, Table 3 documents the regressor's impact with reduced form coefficients of 2.7 basis points on the ST_{INS} and 3.9 basis points on the LT_{INS} liquidity spread. The estimates are statistically significant and increasing in the TTM (time to maturity). It is possible that an increase in the slope of the Treasury term structure reflects inflationary expectations or contractionary monetary policy. Either scenario, of course, engenders the kind of investor anxiety that must be offset with a higher liquidity premium.

These regression results confirm the positively sloped term structure of the liquidity spread. For the 13 months before the crisis, the standardized slope of the term structure of treasury securities averaged 0.304045 standard deviations above the zero mean garnered over the pre-stress, stress, and post stress periods. The average normalized value of TREASLP for the stress period was 0.469915 so that the differential change in TREASLP was [0.469915 - 0.304045] = 0.165870. Combining $d(TREASLP) > 0$ with estimates for the marginal impact of the treasury slope upon the liquidity spread for ST_{INS} and LT_{INS} securities occasions a "$d(LS_i)$" that is increasing in the time to maturity and contributes to the determination of the positively sloped term structure of the liquidity spread in a period of stress.

In the post stress period, our regression analysis documents no meaningful relationship between the treasury slope and short-term liquidity premiums. However, the impact of the slope of treasury term structure upon LT_{INS} liquidity spreads is positive and statistically significant. Consistent with our earlier analysis, the regressor's impact upon the spread in a period of relative calm is proportionately much less than its impact upon the LT_{INS} premium in a crisis, 2.7 basis points and 3.9 basis points, respectively. As the threat of asset conversion diminishes, LT_{INS} bond owners become less sensitive to meaningful independent variables and become marginally more passive.

Figures 8 and 9 dramatically depict the deterioration in the liquidity spread of LT_{INS} securities as we move from a period of financial stress to a period of relative economic calm. The standardized slope of the Treasury term structure fell from 0.469915 to -0.52464 standard deviations below the mean, moving from the first to the second period where $d(TREASLP) = [-0.52464 - 0.46992]$. Consequently, the liquidity spread on LT_{INS} bonds fell by $d(LS_{LT_{INS}}) =$ (0.027)(-0.994555) = -0.026853 or -2.7 basis points. Clearly the behavior of $d(LS_{LT_{INS}}) = \frac{\partial LS_{LT}}{\partial TREASLP}\, d(TREASLP)$ contributes significantly to the transition of the term structure of the liquidity spread from the configuration of Figure 8 to the negatively sloped Figure 9.

Our hypothesis maintains that the shape of the term structure of the liquidity spread changes from a positive to a negative slope reflecting changes in a host of asset cost conversion parameters. We document empirical support for this notion with Figures 8 and 9. To generate these figures, we first collected the estimated daily values of the liquidity spread according to their time to maturity. These estimated daily values, in each of the two economic scenarios, are the product of right hand side variables and their respective parameter estimates at the time of the transaction. However, a complication arises because, on average for each day of transactions, more than a dozen estimated spreads are recorded for the same time to maturity. Consequently, in order to summarize those estimated values of the liquidity spread, our representation of TSLS minimizes the sum of the squared deviations of those computed values from a mean computed value at each TTM. Clearly the Figures 8 and 9 admit to a non-linear possibility.

The positively sloped term structure of liquidity premiums in Figure 8 (stress period) is not a surprise. Three sets of the regression coefficients were statistically significant in the determination of both the ST_{INS} and LT_{INS} liquidity spreads in this period. The impact of the regressor is greater upon the LT_{INS} liquidity premiums than the short-term liquidity spread in all three sets of stress period estimates. As the economy transitioned from our pre-stress period to the stress period, all three independent variables recorded an increase in their average standardized value.[15] These changes, taken in conjunction with the difference in the size of ST_{INS} and LT_{INS} regression coefficients, created the positively sloped term structure depicted in Figure 8.

The TSLS found in Figure 8 mimics the DGP term structure of liquidity spreads in Figure 2A in an important way; it is positively sloped in a period of economic stress. The similarity of the two diagrams emanates, first, from the ability of DGP to capture the features of asset conversion costs and, then, the success of our independent variables to proxy the behavior of DGP conversion parameters and their impact in the first period.

As the economy moved from a period of stress to relative tranquility, not only did the magnitude of our right hand side variables reflect this migration but their impact upon LT_{INS} liquidity spreads retreated. At the same time, the impact of independent variables on the ST_{INS} liquidity premiums escalated. The combined behavior of the magnitude of the variates and their estimated impact produced the negatively sloped TSLS found in Figure 9.

[15] The standardization process uses observations from all three scenarios: pre-stress, stress, and post stress. The means for the aggregation of the intervals is clearly zero. For any one period, the mean is above or below zero.

The term structure of the liquidity premiums in Figure 9 reproduces the negatively sloped TSLS provided by post stress DGP parameters in Figure 2B. The ability of Figure 9 to match the theoretically predicted shape of TSLS follows from the success of our regressors to proxy the behavior of DGP asset conversion cost considerations in the second period.[16, 17]

In Columns 5 to 7 in Table 3, we also provide alternative ways to statistically test the differential impact of the liquidity measure on the liquidity spread for ST_{INS} versus LT_{INS} bonds, in the periods of crisis and post crisis. For this purpose, we define the indicator function $1_{\{\tau=long\}}$, which takes the value 1 when the bond is characterized as an LT_{INS} debenture and 0 when the bond is classified as an ST_{INS} bond. We also define the indicator function $1_{\{period=post\}}$, which takes the value 1 if the transaction occurs during our post crisis period and 0 if it occurred during our crisis period.

In Columns 5 and 6, we summarize our estimates when we simply interact the expression $1_{\{\tau=long\}}$ with the AMIHUD liquidity measure in the crisis and post crisis periods. In our crisis

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[16] We now briefly discuss coefficients for explanatory variables that may be considered control variables. With regard to the maturity (TTM in days) within buckets for the stress period, there is a small statistically significant coefficient for the shorter maturity bucket which is consistent with the notion that the term structure is positive during a stress period. The long-term TTM coefficient in the stress period is negative but very small and not significant. For the post stress period, the short bucket TTM coefficient is negative and strongly significant which is consistent with the notion of a negatively sloped term structure during a post stress period. In contrast, the second period's long-term coefficient is not significant. In summary of the above, these results support the hypothesis that the shape of the term structure changes over time dependent upon varying asset conversion cost characteristics, where their magnitudes are generated, alternatively, by periods of stress and post stress. The coefficients for Amihud market wide measures are much smaller than for the AMIHUD measure. The volatility of the Amihud measure is clearly significant in only the post stress short maturity.

[17] We test the results of Table 3 for endogeneity difficulties in a manner similar to Dick-Nielsen, Feldhütter, and Lando (2012) in Appendix D. Issuance amount and age of issuance are used as instruments where the results are that endogeneity is not a problem. In Appendix E, we provide additional robustness tests where we regress our liquidity spreads upon the difference in liquidity measures of the insured bonds and maturity matched U.S. Treasuries. We find that similar results hold for this alternative specification.

period (Column 5), a one standard deviation increase in the AMIHUD liquidity measure is accompanied by a 6.1 bp increase in the liquidity premiums for our ST_{INS} debentures. Also in the crisis period, we document that that a one standard deviation increase in AMIHUD leads to a 7.9 basis points increase in the liquidity spreads for LT_{INS} bonds, in excess of the contribution of AMIHUD on the liquidity spreads for the ST_{INS} bonds. Put differently, a one standard deviation increase in AMIHUD results in a 14.0 bp (6.1 + 7.9) increase in the liquidity premiums for LT_{INS} bonds in the crisis period. In the post crisis period (Column 6), a one standard deviation increase in the AMIHUD measure results in a 20.1 bp increase in the liquidity spread for ST_{INS} debentures. Also in the post crisis period, we report that a one standard deviation increase in the liquidity measure of an LT_{INS} bond in the post crisis period results in only a 4.1 basis points increase in its liquidity spread. The results we present in Columns 5 and 6 are consistent with the coefficient estimates in Columns 1 through 4 in Table 3. The coefficients for the additional right hand side variables remain virtually unchanged in Columns 5 and 6, relative to those reported in Columns 1 to 4, so we do not reiterate their magnitude, statistical significance, or economic interpretation beyond what is provided in Table 3.

In Column 7 we summarize our estimates when we interact the expressions $1_{\{\tau=long\}}$ and $1_{\{period=post\}}$ with the AMIHUD liquidity measure. We first note that a 1 standard deviation increase in the AMIHUD liquidity measure of an ST_{INS} bond during the crisis period results in a 6.1 bp increase in its liquidity spread. Additionally, the results in Column 7 show that the impact of AMIHUD on the liquidity spread for LT_{INS} bonds is increased by 6.9 bp relative to the impact for the ST_{INS} FIS during our first period. This result coincides with those we report in Columns 1, 2, and 5 in Table 3 for the crisis period. In Column 7, we show that as we transition from the crisis to the post crisis period, the impact of the AMIHUD measure on the liquidity spread for ST_{INS}

bonds is increased by 14.3 basis points. Finally, our estimate for the difference-in-differences coefficient is statistically significant at the 1% level and achieves a value of -23.3 bp. This implies that as our bonds transition from the crisis to the post crisis period, the impact of a one standard deviation increase in the AMIHUD measure on the liquidity premiums for the LT_{INS} bonds is decreased by 23.3 bp relative to the ST_{INS} bonds. These results are consistent with those we report in Columns 3, 4, and 6 in Table 3 for our post crisis period.

Analyses with bonds of extended maturities

The data used in our regression analysis is limited to bank bonds that were insured by the Debt Guarantee Program. Of course, the attraction of bank debt that participated in the guaranteed debt program was that the yield spread is exclusively the liquidity premium. However, in order to both generalize our results and to establish the robustness of our conclusions, we extend the maturity structure of our bank bonds. Extending debt to include securities which have a time to maturity greater than four years will involve the yield spreads that contain both a liquidity premium and a default premium.[18] The Kalman filter routinely estimates state variables and is capable of extracting a liquidity premium from a confounded yield spread. However, our empirical analysis is now plagued with both unobserved state variables (the liquidity premium and the default premium) as well as the unknown parameters on RHS variables. Consequently, the conventional

[18] We can directly test for the existence of an interaction between liquidity and default premiums (a feedback loop) in the data that includes high grade bank bonds whose time to maturity extended beyond the expiration of the Debt Guarantee Program. Estimating the parameters of a Box/Cox model reveals no "feedback loop" in this sample of high-grade extended maturity bank bonds. However, when we extend our analysis to a sample of bank bonds with S&P credit ratings of BBB, BB, B, CCC, CC, C, and D, we are able to document the presence of an interaction between the liquidity and default premiums for a sample of lower quality bank bonds. Our empirical results tend to support the conclusions of He and Milbradt (2014) and of Chen, Cui, He and Milbradt (2018) where the calibrated feedback loop is dramatically larger for low grade bonds than for high quality debentures.

Kalman, which presumes knowledge of the model's parameters, cannot be directly applied to this empirical challenge. We employ a dual estimation technique to the problem of determining both the state of the dynamics and the model which gives rise to the dynamics.[19]

We utilize the expectation maximization (EM) approach, in the context of a Kalman filter, to the problem of estimating both the liquidity spread itself and the parameters that give rise to this premium. Heuristically, the KF/EM dual estimation method works by alternating between using the model to estimate the signal, and using the signal to estimate the model. In particular, we begin with some guesstimates of parameter values to generate an initial set of liquidity premiums over the sample period. Then with a collection of the expected values the liquidity premiums over the entire period a likelihood function is specified and we derive a set of maximum likelihood estimates of the model's parameters. This process iterates back and forth, minimizing the variance of the estimation error and of the prediction error until a linear combination of the estimated variances converges.[20]

The model is given by the following set of equations:

<u>Measurement equation:</u> $s_{i,t} = m_1 \cdot (liq)_{i,t} + m_2 \cdot (def)_{i,t} + \epsilon_{M,i,t}$

<u>Transition equation:</u> $\begin{bmatrix} (liq)_{i,t} \\ (def)_{i,t} \end{bmatrix} = \underline{\alpha_i} + \underline{\underline{\phi_i}} \cdot \begin{bmatrix} (liq)_{i,t-1} \\ (def)_{i,t-1} \end{bmatrix} + \underline{\underline{\beta_i}} \cdot \underline{X_{i,t}} + \underline{\epsilon_{T,i,t}}.$

Above, $s_{i,t}$ is the corporate spread and $(liq)_{i,t}$ and $(def)_{i,t}$ are the liquidity and default spreads of bond i at time t. The error term in the measurement equation, $\epsilon_{M,i,t}$, follows a normal distribution with mean zero and standard deviation σ_M. In the transition equation, $\underline{\alpha'_i} =$

[19] References for the Kalman filter/expectation maximization approach to dual estimation include Dempster, Laird, and Rubin (1977), Hamilton (1994), Harvey (1991) McLachlan (1996), and Maybeck (1979).
[20] Details of the EM algorithm that we programmed will be provided upon request.

$[\alpha_{i,1} \quad \alpha_{i,2}]$ is the vector of intercepts for each transition equation, $\underline{\underline{\phi_i}}$ is the matrix of autoregressive coefficients, $\underline{\underline{\beta_i}}$ is the matrix of control coefficients, and $\underline{X_{i,t}}$ is the vector of liquidity and default controls that includes Amihud liquidity measure, time to maturity, age, liquidity volatility, firm leverage, firm profitability, coupon amount, VIX, U.S. Treasury slope, and market liquidity. The error term in the measurement equation is a vector $\underline{\epsilon'_{T,i,t}} = [\epsilon_{1,T,i,t} \quad \epsilon_{2,T,i,t}]$, where each component follows a normal distribution with mean zero and standard deviation σ_j, with $j = 1, 2$. We estimate the parameters $m_1, m_2, \sigma_M, \underline{\alpha_i}, \underline{\underline{\phi_i}}, \underline{\underline{\beta_i}}, \sigma_1, \sigma_2$ individually for each bond in the stress and post stress periods.[21] The convergence of our estimation algorithm guarantees that the extracted signals optimally reflect daily unobserved liquidity and default premiums and that the parameters truly summarize the impact of our explanatory variables upon the liquidity spreads.

Columns 1 and 2 in Table 4 below show our parameter estimates in the stress and post stress periods when we use all available observations of the new sample. Upon examining these coefficients, it becomes apparent that several of the prominent determinants of liquidity premiums in the insured sample (Table 3) share the sign and a similar magnitude in this new extended sample. As in Table 3, the coefficient of AMIHUD is positive and statistically significant in both economic periods, highlighting that the liquidity measure contributes greatly to the determination of the liquidity spreads. However, the magnitude of the coefficient of AMIHUD is much greater in the stress period (11.5 basis points) relative to the post stress period (0.17 basis points) indicating that investors pay increased attention to liquidity measures in a period of heightened economic anxiety.

[21] We use a global optimization algorithm, particle swarm optimization, to guarantee that are estimates are global minimizers. See Kennedy and Eberhart (1995) for a good reference.

These results parallel our findings for the impact of AMIHUD upon the liquidity spread for the insured sample.

Columns 1 and 2 also show that the coefficient of TTM is statistically significant at the 1% level in the stress and post stress periods. During the stress period in Column 1, the coefficient of TTM is positive and statistically different from zero, supporting an upward sloping relationship between the liquidity premiums and the time to expiry (0.067 basis points per standard deviation increase in TTM). However, in the subsequent post stress period, the relationship between liquidity spreads and time to maturity reverses, suggesting a negative dependency between liquidity spreads and maturity (-0.16 basis points per standard deviation increase in TTM). Clearly, these coefficient estimates provide unqualified support for the upward and downward sloping nature of the relationship between liquidity spreads and time to expiry in the stress and post stress periods and are consistent with the results we document in Table 3 for the insured sample.

The impact of the remaining control variables upon liquidity spreads for this extended sample is well received. For example, the coefficient of $Leverage$ is always statistically significant in both periods, where the magnitude is larger in the crisis period than in the post crisis period. Nonpayment is an important predictor of default premiums and its importance is larger during crises. Our proxy for financial stress, VIX, is positive and statistically significant in the stress period, much like its impact for our sample of insured bank bonds.

We document the TSLS for the extended sample by averaging the liquidity premiums in monthly maturity buckets and plotting the outcome against time to maturity. The result for both economic periods is presented in Figure 10. For this sample of high grade bonds with extended maturities, the Kalman filter/EM is able to generate an upward sloping TSLS in the stress period and a downward sloping TSLS in the subsequent post stress period. This result supports our findings about the shapes of the TSLS for the insured sample in Figures 8 and 9.

We test the out-of-sample validity of our KF/EM estimates in the following fashion. We estimate the model's parameters as described above, but using a sample that contains all available uninsured bonds (with maturities up to 10 years) plus a percentage "ψ" of randomly selected insured bonds (with maturities less than 4 years). We provide two sets of coefficient estimates where ψ, the percentage of insured bonds used, equals 50% and, alternatively, 75%. Clearly, these estimates will reflect the marginal contributions of our RHS regressors in explaining liquidity and default premiums in a sample with up to 10 years to maturity and with and without an "insured" distinction. We then use the parameter estimates, along with the Kalman filter, to obtain the liquidity premiums of the remaining excluded insured bonds.

In Columns 3 and 4 of Table 4, we provide stress and post stress KF/EM estimates for the sample when $\psi = 50\%$. This additional percentage increases the number of bonds in the estimation sample from 179 to 212 in the crisis period and from 343 to 374 in the post crisis period. These coefficients closely resemble the ones in Columns 1 and 2. Most prominently, the coefficient of AMIHUD is positive and statistically significant in both economic periods, intuitively the magnitude is twice as large in the stress period (2.32 bp) relative to the post stress period (0.93 bp). Also, the coefficient of TTM is positive in the stress period (21 bp) and negative in the post stress period (-2.9 bp). The signs of the coefficient for the time-to-expiry validates our findings about the slope of the term structure of liquidity premiums in the stress and post stress periods for the insured bonds, as shown in Figures Figures 8 and 9.

We obtain out-of-sample estimates of liquidity premiums when we feed the remaining 50% of the insured bonds into a Kalman filter that incorporates the coefficients in Columns 3 and 4. If we average the liquidity premiums in monthly maturity buckets and plot the outcome against time to maturity, we obtain out-of-sample term structures of liquidity premiums in the period of stress and post stress as shown in Figure 11. Clearly, our methodology is able to generate an upward sloping TSLS in the stress period and a downward sloping TSLS in the subsequent post stress period. This result provides additional support for the shapes we generate for the insured sample in Figures 8 and 9. Clearly, our out-of-sample testing results are reassuring.

Columns 5 and 6 replicate the analysis in Columns 3 and 4, but randomly including a fraction $\psi = 75\%$ of the insured bonds. This additional percentage increases the number of bonds to 228 in the crisis period and to 390 in the post crisis period. The most important right-hand side variables in our analysis, AMIHUD and TTM, remain intact in these two Columns, compared to the results in Columns 1 to 4. We filter the liquidity premiums for the remaining 25% of the insured sample when we feed the out-of-sample data to a Kalman filter with parameters given in Columns 5 and 6. If we average the liquidity premiums in monthly maturity buckets and plot the outcome against time to maturity, we obtain out-of-sample term structures of liquidity premiums in the period of stress and post stress that confirm our predictions in Figures 8 and 9. This result is depicted in Figure 12.

Next, we provide an alternative approach for decomposing the credit spreads of our high grade bank bonds into their liquidity and default premiums. More specifically, we project the credit spreads on a variable that proxies for the liquidity risk and on variables that proxy for default risk. This method is analogous to the one presented in Dick-Nielsen, Feldhütter and Lando (2012). We estimate the OLS coefficients of the following regression for our periods of crisis and post crisis:

$$s_{i,t} = \beta_0 + \beta_1 \cdot (LM)_{i,t} + \gamma_{i,t} \cdot \underline{X_{i,t}} + \epsilon_{it}.$$

In the expressions above, i and t are indices for the bond and for the day. $s_{i,t}$ is the corporate spread, $(LM)_{i,t}$ is the AMIHUD liquidity measure, and $\underline{X_{i,t}}$ is a vector of controls that includes the same variables as in our KF/EM methodology: firm leverage, firm profitability, coupon amount, VIX, U.S. Treasury slope, time to maturity, age, liquidity volatility, and market liquidity. Then, for each bond and day, we define the "liquidity score" as the product $\beta_1 \cdot (LM)_{i,t}$, the component of the credit spread along the liquidity risk axis. This is as done in Dick-Nielsen, Feldhütter, and Lando (2012).

Columns (1) and (2) in Table 5 below summarize our coefficient estimations for the periods of crisis and post crisis. Our results closely parallel those from our insured bonds in Table 3 and those from our KF/EM dual estimation in Table 4. For example, β_1 is positive and statistically significant at the 1% in the crisis and post crisis periods. This confirms that the AMIHUD is an important determinant of the credit spreads. Yet, the magnitude of β_1 is much greater in the stress period (21.7 basis points) relative to the post stress period (5.2 basis points) indicating that illiquidity matters more during times of economic stress.

We also report in Columns (1) and (2) that the coefficient of TTM achieves a value of 50.1 basis points in the crisis period and a value of -20.7 basis points in the post crisis period. Both coefficients are statistically significant at the 1% level. These results suggest that the liquidity premiums increase with maturity during our stress period, but that they decrease with maturity in the post stress period. These regression estimates confirm our prior findings with a sample of extended maturity high grade bank bonds.

We document the TSLS obtained from this approach by averaging the "liquidity scores" in monthly maturity buckets and by plotting the outcome against the time to maturity. The resulting plots are presented in Figure 13. We find that our linear projection methodology is able to generate an upward sloping TSLS in the stress period (Panel A) and a downward sloping TSLS in the subsequent post stress period (Panel B). This result is consistent with the shapes of the TSLS generated by our KF/EM dual estimation in Figures 10, 11, and 12.

6. Conclusion

DGP (2005, 2007) based models of liquidity generate shapes of TSLS that vary according to the magnitude of parameters assigned to investor search intensity, investor holding cost, the hazard rate of financial shocks and the likelihood of investors being financially constrained. Parameter values meant to characterize stressful periods provide a positively sloped TSLS. DGP parameter magnitudes, which describe a post stress period, create a negatively sloped term structure of liquidity spreads. In this paper, with a special set of default free bonds, we used asset conversion cost parameters to inform the selection of the explanatory variables for our empirical analysis. The magnitude of our regressors in each period, as well as their relative impact upon ST and LT liquidity premiums, created term structures that support our research hypothesis: The term structure of liquidity premiums was upward sloping in the financial stress period of 2008 and downward sloping in the subsequent post stress scenario. Importantly, these results demonstrate that the TSLS can be used to inform lenders which segment of the term structure provides the largest liquidity premiums in alternative economic scenarios. At the same time, for different financial epochs, the TSLS can apprise those who issue debt of the liquidity spreads associated with different times to maturity.

Chapter 2: Nonlinear Structural Estimation of Corporate Bond Liquidity

1. Introduction

A number of research papers have shown that bond liquidity is an important factor that helps explain bond yield spreads and bond returns. Chen, Lesmond, and Wei (2007) use a large sample of U.S. corporate bonds of different investment quality and find that declines in liquidity measures lead higher yield spreads. Also, Lin, Wang, and Wu (2011) find that return on bonds with high sensitivities to aggregate liquidity exceeds that for bonds with low sensitivities to aggregate liquidity and conclude that liquidity risk is an important determinant of expected corporate bond returns. Several papers establish a relationship between bond liquidity and the business cycle. For example, liquidity premiums are a particularly large component of corporate bond yield spreads in times of financial crisis as documented by, among others, Dick-Nielsen, Fedhütter, and Lando (2012), and Friewald, Jankowitsch, and Subrahmanyam (2012).

Several studies attempt to isolate the bond premiums due to illiquidity by subtracting the yields of bonds that differ only on their liquidity dimension. For example, Amihud and Mendelson (1991) analyzed the liquidity of U.S. Treasury notes versus that of Treasury bills of equal maturity. Another set of studies assumes that if a bond's default risk is insignificant, then its spread relative to the risk-free rate may only be attributed to its degree of illiquidity. For example, Longstaff (2004) examined the spreads of Refcorp U.S. agency bonds which were backed by U.S. Treasuries and thus were free of default risk. Black, Stock, and Yadav (2016) use the spreads of a sample of default-free corporate bonds issued under the umbrella of the Debt Guarantee Program to analyze corporate bond liquidity.

Separately, many papers have attempted to estimate liquidity spreads from corporate bond trading data. Dick-Nielsen, Feldhütter, and Lando (2012) use a pooled version of several liquidity measures to linearly estimate corporate bond liquidity premia. Friewald, Jankowitsch, and Subrahmanyam (2012) use the Amihud (2002) measure, price dispersion, and the Roll (1984) measure to estimate the impact of liquidity measures on yield spreads.[22]

Although the above papers concerning corporate bond liquidity provide significant insights into liquidity premia, it is also important to note that liquidity and default interact in a nontrivial way. One group of theoretical studies in finance attempts to model how complex interactions between the default and the liquidity components may determine a bond's yield. For example, He and Xiong (2012) develop a model where a decline in aggregate liquidity evokes an increase in both liquidity yield premia and default yield premia.[23] These authors call this the "feedback effect". He and Milbradt (2014) and Chen, Cui, He, and Milbradt (2018) model the intricate relationship between endogenous liquidity and default for corporate bonds in terms of potential rollover losses for equity holders.[24]

Even though there has been a large number of publications concerning corporate bond liquidity in recent years, there has been very little analysis of the fundamental shapes of the term

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[22] Even the determination of the best measures for bond liquidity is a subject of considerable debate in the finance literature. For example, Schestag, Schuster, and Uhrig-Homburg (2016) conduct a thorough analysis of how various corporate bond liquidity measures compare to each other. Reichenbacher and Schuster (2019) develop revised liquidity measures to eliminate idiosyncratic dependence on trade size and find that these new measures explain a large part of yield spread variations.

[23] Simply put, a decline in aggregate liquidity causes an increase in required yield that makes debt more expensive and increases the likelihood the firm will cross a default threshold; of course, bond holders will demand an enhanced yield for disadvantageous changes in both liquidity and default risk.

[24] Bongaerts, de Jong, and Driessen (2017) jointly analyze the effects of bond specific liquidity, bond market liquidity, and equity market liquidity, where their focus is, in contrast to ours, upon expected returns and do not address term structures of liquidity premia.

structure of liquidity premiums. Clearly, the difficulties posed by segregating the liquidity premiums from other components from a bond's yield have impeded investigating the relationship between liquidity spreads and time to maturity. One notable exception is the work of Dick-Nielsen, Feldhütter, and Lando (2012), who present averages of the magnitude of the liquidity spread for 0-2y, 2-5y, and 5-30y maturity buckets. Their summary statistics hint at a positively sloped term structure between January 2005 and June 2009, while the theory suggested by Amihud and Mendelson (1991), Ericsson and Renault (2006) and Feldhütter (2012) prescribes a negative slope over the same period. They do not, for example, analyze how the term structure may depend upon the degree of stress in the financial system. Relatedly, there is very little analysis of how the liquidity term structure varies with credit rating. That is, for example, how does the term structure of liquidity premia of, say, a AA bond compare to that of a B bond?

The purpose of this research is to extract the liquidity premiums of corporate bonds of different credit rating in alternative economic scenarios and to document the implicit relationship between liquidity premiums and time to maturity. To guide the extraction of these premiums, we create an enhanced version of Leland and Toft's (1996) classic debt valuation equation.[25] Our model assumes the value of the firm follows a stochastic differential equation dependent upon such considerations as the expected return on the firm, the volatility of the individual firm value, and allows for bankruptcy of the firm before the bond matures.[26] Importantly, we have the liquidity premium γ_t enter the evolution of value equation as a component of the price of risk. The premium compensates the investor for holding a bond that is less than perfectly liquid.

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[25] This model is also in the spirit of He and Xiong (2012).

[26] Merton (1974) is an alternative classic model that has been criticized because default occurs only at maturity. However, we note that Feldhütter and Schaefer (2018) find that both Black and Cox (1976) and Merton-type models perform quite well when using an improved, long term history of default rates to test its accuracy.

Documenting the behavior of the liquidity premium in the context of an equilibrium debt pricing equation occasions formidable econometric challenges. Unscented transformations (UT) are used to accommodate the nearly two dozen nonlinear appearances of liquidity premiums in the determination of a bond's yield.[27] The transformed variates are then substituted into a Kalman filter to confront the problem of both documenting the state variable, extracting γ_t, and then, subsequently, estimating the model's parameters. This iterative two step procedure is provided by the Expectation/Maximization algorithm (EM).

Using corporate bond transaction data from Enhanced TRACE between January 2004 and November 2019, we find that the term structures of liquidity premiums were upward sloping for all credit ratings during the pre-crisis, crisis, and post-crisis periods.[28] In addition, the structures were much steeper during the financial crisis and steeper for lower quality bonds across all three epochs. We also empirically establish that the magnitude of the liquidity premiums more than

[27] Unlike many empirical studies that assume a linear impact of liquidity premiums upon the liquidity spreads, our analysis accommodates the nonlinearity that is commonly found in theoretical term structure models such as the ones presented by Ericsson and Renault (2006) and by Feldhütter (2012).

[28] Studying the term structure of liquidity premiums during the most recent pre-crisis, crisis, and post-crisis periods is sensible given the stark regulatory and structural contrast among those three epochs. For example, Bessembinder, Jacobsen, Maxwell, and Venkataraman (2018) find that dealer capital commitment and the average bond trade size decreased since the financial crisis as corporate bond markets move from bank-affiliated dealer capital to nonbank dealers. Dick-Nielsen and Rossi (2019) observe a large reduction in dealer inventories of corporate bonds and find the related cost of trade immediacy has increased since 2008. Adrian, Boyarchenko and Shachar (2017) find that institutions that face more regulations after the crisis reduce their trading volume and are less able to intermediate customer trades. In contrast, Trebbi and Xioa (2017) do not find a negative impact of the recent post-crisis financial regulation on corporate bond liquidity. Wang and Zhong (2019) develop theory to explain interesting aspects of corporate bond market trading. For example, they discuss why there was increased difficulty in executing large trades after the 2008 crisis. Cai, Han, Li, and Li (2019) find that the price impact of herding in corporate bond trading is highly asymmetric and that sell herding is destabilizing and particularly strong for speculative-grade illiquid bonds during the financial crisis. The severe differences in bond trading behavior during the pre-crisis, crisis, and post-crisis period make studying liquidity premiums in each epoch individually, a necessity.

doubled during the recent financial crisis relative to the pre-crisis period. Furthermore, these liquidity premiums do not retreat to their pre-crisis level after the financial crisis.

The rest of the paper is structured as follows. In section 2 we provide our model. Subsequently, in section 3 we detail our methodological approach. Section 4 describes the databases we use and details empirical proxies. In section 5 we document our empirical results. Finally, section 6 presents our conclusions.

2. Theory

We presume a representative firm has productive assets whose unleveraged value $V(s)$ follows a continuous diffusion process with a constant proportional volatility σ:

$$\frac{dV(s)}{V(s)} = (\mu(V,s) - \delta)ds + \sigma dz_s \quad (1)$$

where $\mu(V,s)$ is the total expected rate of return on asset value $V(s)$, δ is the constant fraction of the asset's value paid out to security holders, and dz_s is the increment of a standard brownian motion.[29] The process continues without limit until $V(s)$ falls to V_B, the optimal value of the firm's assets at which to declare bankruptcy. Much like Leland and Toft (1996), we characterize the default triggering barrier "b" as $\ln\left(\frac{V(s)}{V_B}\right)$ when $V(s) = V_B$.

Girsanov's theorem allows us to change the diffusion process for $V(s)$ from the real probability measure to a risk-neutral characterization so that expression (1) becomes

$$dV(s) = aV(s)ds + \sigma V(s)dz_s^* \quad (2)$$

where "a" equals $\left[(r+\gamma) - \delta - \frac{1}{2}\sigma^2\right]$ and dz_s^* is the increments of brownian motion in the risk-neutral measure. The risk-free rate is r. The liquidity premium γ enters the evolution of value

[29] We will omit the time dependency for the value of the firm $V(s)$ and for the brownian motion dz_s^* whenever it is convenient and not confusing.

equation as a component of the price of risk and it compensates the investor for holding a bond that is less than perfectly liquid.[30]

We assume that the representative firm has a bond outstanding with maturity t, continuous coupon payments $c(t)$ and a principal payment of $p(t)$. Upon bankruptcy, bondholders will recover a fraction $\rho(t)$ of V_B. If we designate $f(s, V, V_B)$ as the probability density function of the time to the bankruptcy threshold, with $F(s, V, V_B)$ as its cumulative distribution, then the discounted present value of debt is:

$$d(V, V_B, t) = \int_0^t e^{-(r(t)+\gamma(t))\cdot s} \cdot c(t) \cdot \big(1 - F(s, V, V_B)\big)\, ds + e^{-(r(t)+\gamma(t))\cdot t} \cdot p(t) \cdot \big(1 - F(t, V, V_B)\big) + \int_0^t e^{-(r(t)+\gamma(t))\cdot s} \cdot \rho(t) \cdot V_B \cdot f(s, V, V_B) ds \ . \quad (3)$$

In the expression above, the first term on the right-hand side is the present value of all the coupon payments, $c(t)$, weighted by the likelihood that they are paid up to the time of maturity of the bond. The second term is the present value of the principal, $p(t)$, discounted by $r(t) + \gamma(t)$ and weighted by the probability that the firm will not default before expiration of the debt. The final term on the right-hand side is the present value of the proportion of V_B available to bondholders at the time of default, weighted by the likelihood of a default.

[30] Note that including the liquidity premium $\gamma(t)$ in such a fashion is consistent with the bond pricing approach in Longstaff, Mithal, and Neis (2005). Weiland, Laeven, and de Jong (2017) also include a liquidity parameter in a fashion similar to our own. Also, note that this treatment of the liquidity premium corresponds to He and Xiong's (2012) characterization.

Integrating (3) by parts yields:[31]

$$d(V,V_B,t) = \frac{c(t)}{r(t)+\gamma(t)} + e^{-(r(t)+\gamma(t))t} \cdot \left(p(t) - \frac{c(t)}{r(t)+\gamma(t)}\right) \cdot \left(1 - F(t)\right) +$$

$$\left(\rho(t) \cdot V_B - \frac{c(t)}{r(t)+\gamma(t)}\right) \cdot G(t) \qquad (4)$$

where:

$$F(t) = N\left(\frac{-b - a\sigma^2 t}{\sigma\sqrt{t}}\right) + \left(\frac{V}{V_B}\right)^{-2a} \cdot N\left(\frac{-b + a\sigma^2 t}{\sigma\sqrt{t}}\right) \qquad (5)$$

$$G(t) = \left(\frac{V}{V_B}\right)^{-a+z} \cdot N(q_1) + \left(\frac{V}{V_B}\right)^{-a-z} \cdot N(q_2)$$

$$q_1 = \frac{-b - z\sigma^2 t}{\sigma\sqrt{t}}; \;\; q_2 = \frac{-b + z\sigma^2 t}{\sigma\sqrt{t}}$$

$$h_1 = \frac{-b - a\sigma^2 t}{\sigma\sqrt{t}}; \;\; h_2 = \frac{-b + a\sigma^2 t}{\sigma\sqrt{t}}$$

$$a = \frac{(r+\gamma) - \delta - \left(\frac{\sigma^2}{2}\right)}{\sigma^2}; \;\; b = \ln\left(\frac{V}{V_B}\right); \;\; z = \frac{\sqrt{(a\sigma^2)^2 + 2(r+\gamma)\sigma^2}}{\sigma^2}$$

$$V_B = \frac{\left(\frac{C}{r+\gamma}\right)\left(\frac{A}{(r+\gamma)T} - B\right) - \frac{AP}{(r+\gamma)T} - \frac{\tau C x}{r+\gamma}}{1 + \alpha x - (1-\alpha)B} \qquad (6)$$

with

$$A = 2ae^{-(r+\gamma)T}N\left(a\sigma\sqrt{T}\right) - 2zN\left(z\sigma\sqrt{T}\right) - \frac{2}{\sigma\sqrt{T}}n\left(z\sigma\sqrt{T}\right) + \frac{2e^{-(r+\gamma)T}}{\sigma\sqrt{T}}n\left(a\sigma\sqrt{T}\right) + (z-a)$$

[31] In Appendix F, we provide a more detailed presentation of our revised version of Leland and Toft's (1996) classic model.

and with

$$B = -\left(2z + \frac{2}{z\sigma^2 T}\right) N\left(z\sigma\sqrt{T}\right) - \frac{2}{\sigma\sqrt{T}} n\left(z\sigma\sqrt{T}\right) + (z - a) + \frac{1}{z\sigma^2 T} .$$

In the expression above, $N(\cdot)$ denotes the cumulative distribution of the standard normal while $n(\cdot)$ is the density function of the standard normal.[32]

The value of debt given by equation (4) is subject to multiple sources of nonlinearities and plagued by both latent variables and unknown parameters. Despite these econometric challenges, embedded within the equilibrium price of a bond is an implicit relationship between the time to maturity "t" and the liquidity premium γ_t. Calibration, of both the parameters and the variables appearing in the nonlinear configuration of equation (4), will allow us to momentarily side step the statistical issues of estimating $d(V, V_B, t)$ and generate a term structure for γ_t.

The expression for the value of debt includes nine parameters which describe firm quality, the financial/economic scenario at hand, and the interdependence of the two. Manipulation of the assigned parameters and variables will explicate the relationship between the time to maturity and liquidity premia in the context of the equilibrium price of a bond. To illustrate the possibilities, we select parameters to represent alternatively, "high" and "low" quality firms in a period of calm and then in a period of economic stress. For instance, the volatility of asset value σ would clearly increase in times of financial stress and increase more so for firms of poor quality. The risk-free rate r would be greater for both types of firms during times of heightened economic anxiety. Firm payout ratios δ are higher for low grade bonds than for high quality securities regardless of the economic circumstance. Furthermore, payout ratios are larger for both low grade and high grade

[32] We provide our modification of Leland and Toft's (1996) optimal default threshold V_B in Appendix G.

bonds in times of stress relative to periods of economic calm. Default recovery ratios deteriorate in times of economic anxiety for both high and low quality bonds.

If the 9 parameters included in equation (4) are chosen to characterize low and then high grade bonds in a period of financial calm, then the relationship between the time to maturity and the liquidity premium that will leave the equilibrium value of debt unchanged is depicted in Figure 14A. If equation (4) is depicted to, alternatively, portray low quality and then high quality firms in an epoch of economic stress, then the term to maturity of γ_t that will leave the equilibrium price of a bond undisturbed is found in Figure 14B.[33] While the consequence of our two sets of parameters are only loosely based on possible empirical results, the diagrams are an artifact of the implicit relationship between the time to maturity and the liquidity premium in the context of a widely accepted characterization of the price of debt. The figures suggest that regardless of the financial epoch, the term structure of γ_t is concave for both low and high grade bonds. In addition, there is an inverse relationship between the grade of the bond and the magnitude of the premium. At every time to maturity, the lower the quality of the bond, the larger the spread in each of the two proposed financial scenarios. For both low quality securities and for the high grade bonds respectively, the spread is larger at all times to maturity during a period of stress relative to a period of calm.

Given these two sets of diagrams, it is clear that an "enhanced" LT debt pricing model provides a vehicle for investigating the term structure of liquidity spreads. Furthermore, we believe real world proxies for the 9 parameters included in equation (4) exist, that a latent γ_t can be explicated, and the fundamentally nonlinear RHS of equation (4) can be econometrically

[33] In Appendix H, we summarize our selection of these four sets of illustrative parameters that characterize high and low quality firms in a period of stress and in a period of calm.

accommodated, so that the following questions can be empirically answered. Are term structures positively or negatively sloped? Are the structures monotonic? Are the slopes of the empirical TSLS (term structure of liquidity spreads) concave or convex? Do the slopes vary according to credit rating or according to the economic epoch at hand? Does the empirical magnitude of the liquidity premiums depend upon the credit rating of the FIS (fixed income securities) or on the existing financial scenario? Does the estimated difference in the premiums, due to bond quality, depend upon the economic epoch at hand?

3. Methodology

Perhaps the greatest empirical challenge to establishing a relationship between the term to maturity of a fixed income security and its liquidity premium is documenting the otherwise unobservable γ_t. Estimating equation (4) involves yield spreads that contain both liquidity and default premia. In order to provide a sense of how we extracted the liquidity premium from the confounded yield spread, we offer the following simple illustration.

Consider the following variables:

yield for a bond with a given time to maturity = y,
risk free rate for a given time to maturity $= r$,
yield spread for a given time to maturity $= y - r = b_1\gamma + b_2\xi + v$,
liquidity premium for a given time to maturity $= \gamma$,
default premium for a given time to maturity $= \xi$
and, measurement error for a given time to maturity $= v$.

For this example, the premiums have means of zero and we assume that $b_1, b_2, C(\gamma, \xi), \sigma_\gamma^2$, and σ_ξ^2 are known parameters. The spread, of course, is only observable in aggregate ($b_1\gamma + b_2\xi + v$). In order to generate an estimate of "γ", the liquidity premium, we minimize $E[\gamma - \theta(b_1\gamma + b_2\xi + v)]^2$ with respect to θ.

Taking the derivative, setting the result equal to zero, and, finally, distributing the expectation operator yields:

$$\theta = \frac{b_1\sigma_\gamma^2 + b_2C(\gamma,\xi)}{b_1^2\sigma_\gamma^2 + b_2^2\sigma_\xi^2 + \sigma_v^2 + 2b_1b_2C(\gamma,\xi)} \quad .$$

Thus, our best estimator of γ given that we can observe only the aggregate yield spread $(y - r = b_1\gamma + b_2\xi + v)$ is:

$$E\,(\gamma|(y-r)) = \left[\frac{b_1\sigma_\gamma^2 + b_2C(\gamma,\xi)}{b_1^2\sigma_\gamma^2 + b_2^2\sigma_\xi^2 + \sigma_v^2 + 2b_1b_2C(\gamma,\xi)}\right](y-r).$$

Our decomposition, intuitively, reveals that the greater σ_γ^2 , then the larger the proportion of ϕ attributed to $E(\gamma|(y-r)\,)$ while increases in σ_ξ^2 and σ_v^2 reduce that proportion. The static specification of the γ and ξ, above, is too simplistic to be useful per se. In addition, signal extraction in our empirical analysis is not a single stage procedure. Decomposition of $(y-r)$ is nearly continuous in each of our epochs, with current estimates of the premiums being updated with new signals constantly; for example, there are 193,970 observations in the post stress period alone.

The Kalman filter, detailed in Appendix I, initially seems like it is an ideal vehicle for our empirical aspirations. The transition equations of the Kalman clearly admit to the premiums being characterized as autoregressive.[34] In addition, the Kalman gain K_t, is capable of performing the necessary decomposition analysis (detailed above) if M is appropriately specified. In fact, K_t collapses to θ if the assumptions associated with our simple example are fully employed.

Transition equation:

[34] Preliminary statistical testing suggests the sufficiency of this parsimonious specification of the lag structure. Prior studies of corporate bond liquidity measures, such as Lin, Wang, Wu (2011), have modeled liquidity measures such as the Amihud and Pastor-Stambough measures using a first order autoregressive process. While liquidity measures and liquidity premiums are not equivalent, the work of Lin, Wang, Wu (2011) suggests that liquidity measures in general may be expressed as an AR(1) process.

Our transition equation has two latent variables, the liquidity premium $\gamma_{i,s}$ and the default premium $\xi_{i,s}$, and is characterized by:

$$\underline{\text{Transition equation:}} \quad x_{i,s} = \alpha_i + F_i \cdot x_{i,s-1} + w_{i,s}\ .$$

The variate $x'_{i,s}$ is the vector $[\gamma_{i,s}\ ,\ \ \xi_{i,s}]$, where $\gamma_{i,s}$ and $\xi_{i,s}$ are as previously defined. In the transition equation, $\alpha_i = [\alpha_{i,1} \quad \alpha_{i,2}]$ is a vector containing intercepts for each transition equation and F_i is a conformable matrix of autoregressive coefficients. The error term in the transition equation is a vector $w_{i,s} = [w_{1,i,s} \quad w_{2,i,s}]$, where each component has a mean of zero and standard deviation σ_j, for $j = 1, 2$. We estimate the parameters $\alpha_i, F_i, \sigma_1, \sigma_2$ individually for each bond in the pre-crisis, crisis, and post-crisis periods.

Measurement equations:

We utilize two measurement equations. The first one is inspired by the work of He and Milbradt (2014) and Chen, Cui, He, and Milbradt (2018) and explicitly includes a nonlinear feedback loop premium. Our second measurement equation is an artifact of equating the RHS of the value of riskless debt that is not traded, to the RHS of $d(V, V_B, t)$ which is subject to transaction costs and the possibility of default.

The interaction between liquidity and default premiums has gained some traction in the finance literature.[35] According to this research, the yield spreads of corporate bonds may be considered the sum of a liquidity premium, a default premium, a liquidity-driven-default portion, and a default driven liquidity portion.[36] Thus, to properly extract the liquidity premium signal, we must acknowledge an interaction between liquidity and default premiums.

[35] However, the contributions in this area have been almost entirely theoretical, where some of the most prominent papers are He and Xiong (2012), He and Milbradt (2014), and Chen, Cui, He, and Milbradt (2018). We are not aware of any academic research that empirically documents the importance of the feedback loop in the determination of bond credit spreads.

[36] The last two terms are collectively known as the feedback loop between liquidity and default.

Our first measurement equation is:

$$\left(\frac{\psi_{i,s}{}^{\lambda_\psi}-1}{\lambda_\psi}\right)=\alpha_{BC}+\beta_\gamma\cdot\left(\frac{\gamma_{i,s}{}^{\lambda_\gamma}-1}{\lambda_\gamma}\right)+\beta_\xi\cdot\left(\frac{\xi_{i,s}{}^{\lambda_\xi}-1}{\lambda_\xi}\right)+\beta_{\gamma,\xi}\left(\frac{\gamma_{i,s}{}^{\lambda_\gamma}-1}{\lambda_\gamma}\right)\left(\frac{\xi_{i,s}{}^{\lambda_\xi}-1}{\lambda_\xi}\right)$$
$$+\epsilon_{i,s}$$

In the equation above, i and s are indices for the individual bond and time. In addition, $\psi_{i,s}$ is the credit spread, $\gamma_{i,s}$ is the liquidity premium, and $\xi_{i,s}$ is the default premium. The terms β_γ and β_ξ measure the impact of both the liquidity premium $\gamma_{i,s}$ and the default premium $\xi_{i,s}$ in the formation of the credit spread $\psi_{i,s}$. The term $\beta_{\gamma,\xi}$ records the impact of the feedback loop in determining $\psi_{i,s}$.

Of course, the expression above corresponds to the Box/Cox transformation. The strength of this non-linear specification is that estimated values of λ_γ, λ_ξ and λ_ψ can provide different functional forms for the variates and for the induced relationship between liquidity and default premiums. The highly contrived transformations of the variates

$$\frac{\gamma_{i,s}{}^{\lambda_\gamma}-1}{\lambda_\gamma},\ \frac{\xi_{i,s}{}^{\lambda_\xi}-1}{\lambda_\xi},\text{ and }\frac{\psi_{i,s}{}^{\lambda_\psi}-1}{\lambda_\psi}$$

allow special cases for λ_i (e.g. $\lambda_i = 0$ or 1 for $i=\gamma,\xi,\psi$) to still make arithmetic and economic sense. Additionally, the expression above acknowledges that liquidity, default, and feedback loop premiums are additive as posited in He and Milbradt (2014), and Chen, Cui, He, and Milbradt (2018) and allows for a signal decomposition into these three fundamental sources of premiums.

A fixed income security's yield "y" is computed as the return to a bond conditional on it being held to maturity t without default:[37]

[37] This is a well-known textbook relationship.

$$d = \frac{c}{y}(1 - e^{-yt}) + pe^{-yt}. \quad (7)$$

In particular, given the bond price, the coupon payment, the principal, and the time to maturity, then the yield "y" can be calculated. Meanwhile, recall our debt valuation equation,

$$d(V, V_B, t) = \frac{c}{r+\gamma_t} + e^{(r+\gamma_t)t}\left(p - \frac{c}{r+\gamma_t}\right)\left(1 - F(t)\right) + \left(\rho V_B - \frac{c}{r+\gamma_t}\right)G(t), \quad (8)$$

an "informed" solution for "y" could be obtained if the RHS of (8) were equated to the RHS of (7) as in the expression below

$$\frac{c}{r+\gamma_t} + e^{(r+\gamma_t)t}\left(p - \frac{c}{r+\gamma_t}\right)\left(1 - F(t)\right) + \left(\rho V_B - \frac{c}{r+\gamma_t}\right)G(t) = \frac{c}{y}(1 - e^{-yt}) + pe^{-yt}. \, (9)$$

Since the LHS of (9) includes considerations of both trading costs and bankruptcy costs, the calculated y would include both a liquidity and a default premium. So the determination of the yield provides our model with another measurement equation and a second source of information useful in extracting the liquidity premium from the LHS of equation (9). In fact, γ_t explicitly appears in a nonlinear fashion in the debt valuation equation 4 times, there are another 6 nonlinear entries of γ_t included in $F(t)$, 10 nonlinear appearances of liquidity premiums in $G(t)$, and 4 nonlinear entries of γ_t in V_B.[38] Collectively, these terms account for 20 nonlinear appearances of the liquidity premium on the RHS of (8). We employ the "unscented transformation" algorithm (UT) to address the complications posed by these nearly two dozen sources of nonlinearities in just one measurement equation and another four sources of nonlinearities in the first measurement equation. In Appendix J, we consider a simple debt valuation problem in order to provide the reader some insight to the UT algorithm. In addition, we detail and simulate the advantage of UT over linearization as an approach to characterize nonlinear functions. Finally, in Appendix K we

[38] $F(t)$ and $G(t)$ are cumulative distributions and V_B is the endogenous bankruptcy threshold, all three of these expressions are included in the determination of $d(V, V_B, t)$.

provide a summary of just how the UT algorithm was employed in our empirical investigation of the term structure of liquidity premiums.

Once we have transformed our transition and our measurement equations by the UT algorithm, we are still confronted with a dual estimation problem. Both state variables are unobservable and the parameters which characterize the interaction between the state variables need to be identified. We use expectation maximization (EM) in the context of our unscented Kalman filter as our approach to this dual estimation problem. Introduced by Dempster, Laird, and Rubin (1977), the EM algorithm has proven to be a useful method of analysis in econometric and statistical models.[39] Heuristically, this dual estimation method works by alternating between using the model to estimate the state variables, and then using the state variables to estimate the model. In particular, we begin with preliminary values of the vector of unknown parameters in the measurement and transition equations to generate an initial set of estimates of our respective premiums over the sample periods. With a collection of the expected value of the state variables over the entire epoch, a likelihood function is specified and we derive a set of maximum likelihood estimates of the parameters. This process iterates back and forth, minimizing the variance of the estimation error and then the variance of the prediction error until a weighted linear combination of the estimated variances converges.[40]

4. Data Description

We use sum of the total book value of short (DLCQ) and long-term debt (DLTTQ) to proxy for the total principal P. We obtain these two items with quarterly frequency from the COMPUSTAT files. We utilize total debt instead of the face value of the individual bonds because

[39] For some general background, the reader is referred to McLachlan (1996), Harvey (1991), and Hamilton (1995).

[40] Details of the EM algorithm that we programmed will be provided upon request.

theoretical models assume that firms only issue debt in the form of bonds. Of course, failing to account for all debt outstanding would lead to a lower leverage and lower yields. Following a similar procedure as Eom, Helwege, and Huang (2004) and Feldhütter and Schaefer (2018), we then calculate the value of the firm V as the sum of P and the equity value of the firm, which is the number of shares outstanding (SHROUT) times the price of the firm's stock (PRC). Stock price data is obtained with a daily frequency from the CRSP records.

The variable δ captures the total payouts of the firm in the form of interest payments to debt holders, dividends to equity holders, and net stock repurchases. The data is obtained from the quarterly COMPUSTAT files as follows: interest payments are the previous fourth quarter's total interests paid (INTPNY), dividends to equity are the total annual dividends (DVY), and net stock repurchases are the previous year's net purchase or common and preferred stock (PRSTKCY). The payout ratio is then computed as the ratio of total payout divided by total assets.

The coupon c is obtained explicitly from Mergent FISD while the time to maturity is calculated daily as the number of days remaining till the expiry of the bond (maturity date minus current date). The daily yield of the bond is also directly obtained from TRACE and the spread s is defined as the daily yield minus the risk-free rate of a maturity matching U.S. Treasury bond, interpolated as explained above.

The volatility of the firm's assets is calculated in a similar fashion as in Schaefer and Strebulaev (2008) and Feldhütter and Schaefer (2018). First, we compute the volatility of equity, σ_{equity}, as standard deviation of daily returns (RET from CRSP) using a rolling 1-year period and we multiply the result by $\sqrt{255}$ to annualize the standard deviation. Since the value of the firm is composed of both debt and equity, the volatility of assets is obtained from aggregating these two components as follows:

$$\sigma^2 = (1 - lev) \cdot \sigma_{equity}^2 + lev \cdot \sigma_{debt}^2 + 2 \cdot lev \cdot (1 - lev) \cdot \sigma_{equity,debt}$$

where σ_{equity}^2 and σ_{debt}^2 are the variance of equity and debt, respectively, and $\sigma_{equity,debt}$ is their covariance. Schaefer and Strebulaev (2008) find that the volatility of assets can be approximated by assuming that the volatility of debt is zero and then multiplying the result by a correction factor. Thus, we estimate the total volatility of assets as $\sigma^2 = (1 - lev)\sigma_{equity}^2$ and multiply this by 1 if $lev < 0.25$, 1.05 if $0.25 < lev < 0.35$, 1.10 if $0.35 < lev < 0.45$, 1.20 if $0.45 < lev < 0.55$, 1.40 if $0.55 < lev < 0.75$, and 1.80 if $lev > 0.75$.

We proxy for the risk-free rate by using the daily yield of a U.S. Treasury bond with the same maturity as the corporate bond. The data on the U.S. Treasury bonds is from the H-15 website maintained by Federal Reserve of St. Louis. For virtually all bonds, there are no available U.S. Treasury with the exact same time to expiry as our corporate bonds. Thus, we calculate the risk-free rate for each bond on each trading day as the interpolated yield using the Treasury term structure prevailing on that day. A statistical summary of all our variables can be found in Table 7.

5. Results

Pre-crisis Period

We consolidate bonds into three major credit rating categories: high for bonds with S&P credit ratings AAA and AA, medium for bonds with credit ratings A and BBB, and low for speculative grade bonds. Estimates are aggregated in this fashion because sometimes there are very few observations per credit rating where this is most common for the AAA bonds but also occurs for other categories. One may consider levels of liquidity premia and the shapes of the TSLP during the pre-crisis period as our "baseline" results because this period was less affected by financial stress, the regulations that followed the financial stress, and by the Volcker rule that came to full effect on June 2015. This baseline period is free of dramatic policy initiatives by the Federal Reserve.

Table 8 includes average liquidity premia from our unscented Kalman filter estimation for each credit rating category during the pre-crisis period in the first column . The average level of liquidity premiums (LP) increases with lower credit quality. For the highest grade bonds the average liquidity premium is 0.425% during the pre-crisis period. For medium quality bonds, the average liquidity premium is 0.534% while it is 0.678% for the speculative grade bonds. The next column reports liquidity premia for the crisis period where, as expected, the increase from the pre-crisis column is quite dramatic for all credit qualities. For example, the average high grade liquidity premium increases from 0.425% to 1.009%. The last column represents average post-crisis liquidity premia which are much lower than the crisis but still greater than the pre-crisis situation.

Given these averages of term premia across all maturities for different credit ratings in the three periods, we next illustrate our estimated families of term structures for 'high' quality, 'medium' quality, and 'low' quality bonds during the pre-crisis period in Figure 15. The general pattern across all credit ratings in this period is that the relationship between liquidity premiums and maturity is positive where the curves are upward sloping.

For high grade bonds, the model implies that shortest maturity bonds have a liquidity premium of about 0.38% while the longest term bonds have a liquidity premium of about 0.62%. For medium term bonds, the impact of increasing maturity tends to be stronger. That is, although the level of liquidity premium for the shortest maturity tends to be lower than for high grade at about 0.18%, the liquidity premium rises with maturity to be approximately the same for the longest maturity. For low grade bonds, the liquidity premium begins at approximately 0.20%, rises slowly until about 10 years maturity, and then rises very strongly to be over 1.0% at 20 years to maturity. In summary, the term structure tends to be concave for the highest quality bonds, approximately linear for intermediate bonds, and strongly convex for lower quality. The difference between the shortest and longest maturity clearly increases with every reduction in the credit rating where the difference due to maturity is clearly least for the highest credit quality and greatest for the lowest credit quality.

Separate from the above slopes of TSLP's of differing credit quality, we note that the general level of TSLP is higher for lower credit grades. Greater liquidity premia for lower grades is well documented in Dick-Nielsen, Feldhütter and Lando (2012) and, also, Friewald, Jankowitsch and Subrahmanyam (2012). More recently Chen, Cui, He, and Milbradt (2018) develop simulations where liquidity premia for lower grade bonds often tend to be higher for lower grade bonds. In summary, the results in Figure 15 strongly suggest that liquidity premia are unequivocally greater for longer maturity bonds, irrespective of the credit quality of the issue.

We provide statistical support for our representations of the TSLP in Table 9. We regress the liquidity premiums on time to maturity as prescribed by the equation:

$$LP = \alpha_0 + \beta_1 \cdot TTM + \beta_2 \cdot TTM \cdot 1_{\{TTM>\eta\}} + \beta_3 \cdot 1_{\{TTM>\eta\}}.$$

In the equation above, LP is the liquidity premium and TTM is the time remaining until maturity. The expression $1_{\{TTM>\eta\}}$ is an indicator function that takes the value 1 when the TTM for that observation is greater than a predetermined value "η" and takes the value zero when TTM is less than η. For each rating class, we document regressions where η takes the value of the median TTM (0.5) in each period.

In this analysis, we consider a bond short term when $TTM < \eta$. For short term bonds, the expression $\beta_2 \cdot TTM \cdot 1_{\{TTM>\eta\}}$ in the equation above is zero because $TTM < \eta$. Thus, β_1 is our estimate for the slope of the short- term bonds. Next, we consider a bond to be long term when $TTM > \eta$. For long-term bond, the expression $\beta_1 \cdot TTM + \beta_2 \cdot TTM \cdot 1_{\{TTM>\eta\}}$,of course, becomes $(\beta_1 + \beta_2) \cdot TTM$. Thus, the slope for long-term bonds is given by the sum of the coefficients β_1 and β_2.

In Columns 1, 2, and 3 of Table 9, we document our findings in the pre-crisis period for the high, medium, and low credit classes, where all the coefficients are statistically significant at 1%. In Column 1, we report the slope of the term structure of the short-term high quality bonds (β_1) as about 0.023 % per year, while the slope for the long-term bonds ($\beta_1 + \beta_2$) is 0.014 % per year (0.023 – 0.009). In Column 2, we show that for the medium quality bonds the slope for the short-term (β_1) is 0.027 % per year which is unchanged for the long- term bonds because β_2 is estimated to be zero. Finally, in Column 3, we document that the slope for the short-term low grade (β_1) is 0.026 % per year while the slope for the long-term is greater at 0.032% per year (0.026 +0.006). In all three specifications for the pre-stress period, the slopes β_1 and $\beta_1 + \beta_2$ are positive for both short and long-term bonds. However, the slope coefficient is statistically larger for the short- term relative to long term for the high grade bonds. This supports the concave TSLP that we find for this credit rating. In contrast, the slope of the long-term low grade bonds is greater than the slope of the short-term low grade bonds due to a positive β_2 coefficient of 0.006. This last result suggests a convex shape consistent with the shape depicted in Figure 15.

Crisis Period

The period of crisis starts in April 2007 and ends in June 2009. This is as defined by the NBER and as done in Bessembinder, Jacobsen, Maxwell, and Venkataraman (2018). We again refer to Table 8 where, as mentioned above, the crisis period shows a very dramatic increase in average liquidity premia for all credit qualities as compared to the pre-crisis period. Figure 16 displays the term structure of liquidity premia for the crisis period of different credit qualities where the term structure is concave and positive for all. The dramatically high levels of liquidity premiums in the crisis period are consistent with Dick-Nielsen, Feldhütter, and Lando (2012) and Friewald, Jankowitsch and Subrahmanyam (2012). We note that the term structure is mildly upward sloping for high grade bonds but much more steeply sloped for medium grade bonds. The slope for low grade bonds is greater than for high grade but less than for medium grade. Recent work by Goldstein and Hotchkiss (2019) shows that dealers have a higher propensity to offset trades the same day for speculative grade bonds which leads to a higher turnover where this might explain why the liquidity premiums are slightly lower for the bonds of the lowest quality.

As done for the pre-crisis period, we support our analysis of the TSLP during a period of crisis with the same regression specification. In Columns 1, 2, and 3 of Table 10, we document our findings in the period of crisis for the high, medium, and low credit classes, where all the coefficients are statistically significant at the 1%. In Column 1, we report the slope of the term structure of the short-term high quality bonds (β_1) as about 0.003 % per year, while the slope for the long-term bonds ($\beta_1 + \beta_2$) is about 0.002 % per year (0.003 -0.001). In Column 2, we show that for the medium quality bonds the slopes for the short-term (β_1) and long-term bonds ($\beta_1 + \beta_2$) are 0.089 % per year and 0.066 % per year, respectively. Finally, in Column 3, we document that the slope for the short-term speculative grade (β_1) is 0.042 % per year while the slope for the long-term is 0.030% per year. In all three specifications for the crisis period, the slope coefficient is positive for both short and long-term bonds. However, the coefficient is larger for the short-term bonds relative to the longer term bonds given the negative β_2 values. This suggests that larger increases in the liquidity premia occur for bonds that are close to maturity, but that these increases decline as the maturity horizon of the bond increases. This evidence clearly supports the appearance of a concave term structure of liquidity premiums in the crisis period.

For the high grade bonds, liquidity premiums are a much greater fraction of the total credit spread (default plus liquidity) during a period of crisis than in any of the other epochs. For the short-term, high quality bonds, liquidity premia represent 33% of the total credit spread, while for the long-term high quality bonds in the same economic epoch, liquidity premia constitute 48% of their total credit spread. These magnitudes are large when compared to the ratio of liquidity premia to total credit spread for the high quality bonds in the other two (non-crisis) periods. For example, for the high quality bonds in the pre-crisis period, the percentage of the credit spread due to the liquidity premium is only 6% for the short-term bonds and only 7% for the long-term bonds. During the post-crisis period, the liquidity premium is 15% of total credit spread for short-term bonds and 33% for the long-term bonds.

Post-crisis Period

The last column of Table 8 reports the average liquidity premiums during the post-crisis period for high, medium, and low grade corporate bonds where, again, these are estimated by the UKF method. For each grade, the liquidity premium is higher than the pre-crisis number but lower than the crisis. These results confirm that illiquidity in the bond market has not returned to its pre-crisis levels. This evidence supports the claims in Anderson and Stulz (2017), Adrian, Fleming, Shachar, and Vogt (2017) and Bessembinder, Jacobsen, Maxwell, and Venkataraman (2018). As before, the highest grade bonds have the lowest liquidity premium. The liquidity premium of the high quality bonds (0.722%) is about one fifth lower than the liquidity premium of the medium grade bonds (0.916%) in the post-crisis. It is curious that the liquidity premia do not decline as bonds move from the medium to the speculative grade ratings; that is, the liquidity premiums of the low quality bonds are about 1% lower than the medium grade bonds. The difference is small and is consistent with Goldstein and Hotchkiss (2019) who show that dealers have a higher propensity to offset trades in the same day for speculative grade bonds where this leads to a higher turnover where this might explain why the liquidity premiums are slightly lower for the bonds of the lowest quality.

Figure 17 displays the term structure of liquidity premia for the post-crisis period for different credit qualities where the term structure is convex and positive for all credit qualities. The high grade short-term bonds have a liquidity premium of about 0.45% while high grade long-term bonds have a liquidity premium of about 1.18%. For medium grade bonds, the liquidity premium for the shortest maturity is about 0.28% and 1.5% for the longest. Finally, the average liquidity premium of the shortest maturity low grade bond is 0.60% and 1.1% for the longest.

In Table 11 we support our depictions of the TSLP during the post-crisis period with the same regression specification used before. Column 1 includes the slope of the term structure of the short-term high quality bonds (β_1) as about 0.013 % per year, while the slope for the long-term bonds ($\beta_1 + \beta_2$) is about 0.023 % per year (0.013+0.010). In Column 2, we show that for the medium quality bonds the slope for the short-term (β_1) is 0.041 % per year and is 0.055 % per year for the long term bonds. Finally, in Column 3, we document that the slope for the short-term speculative grade (β_1) is 0.012 % per year while the slope for the long-term is 0.017% per year. It is important to contrast the behavior of β_2 in the post-crisis period to the other periods. That is β_2 is positive in post-crisis estimations for all credit qualities wherein it was typically negative for the earlier periods. This helps explain the convexity of the TSLP that we find for all credit ratings in the post-stress period.

In summary, our results show how liquidity premia behave with respect to bond maturity where we have taken special care to utilize econometric methods that fit the nonlinear character of liquidity premia found by prior researchers. Our estimations show that the character of the term structure liquidity premia changed significantly as we progressed from pre-crisis, to crisis, to post-crisis periods. In summary, the first two periods had positively sloped, concave term structures wherein the crisis period had dramatically higher liquidity premia than pre-crisis. The post-crisis term structure was quite different in that the positively sloped term structure of liquidity premia was convex instead of concave. Also, we note that the level of liquidity premia in post-crisis was lower than the crisis period but higher than pre-crisis period.

6. Conclusion

In this paper, we obtained the term structures of corporate bond liquidity premia for fixed income securities of different credit ratings in alternative economic circumstances. For purposes of estimation, unscented transformations (UT) accommodated the nearly two dozen nonlinear appearances of liquidity premiums in the determination of a bond's yield. Subsequently, a Kalman filter was used to extract an otherwise latent liquidity premium from the bond's observed yield. We found that the term structures of liquidity premiums were upward sloping for all credit ratings and in all three economic epochs considered. In addition, the structures were much steeper during the financial crisis and steeper for lower quality bonds across all three financial scenarios. Empirically, we also established that the magnitude of the liquidity premiums more than doubled during the recent financial crisis relative to the pre-crisis period. Furthermore, the liquidity premiums did not retreat to their pre-crisis level after the financial crisis.

Chapter 3: Does Corporate Bond Illiquidity Spill Over to CDS Premiums?

1. Introduction

Credit default swaps (CDSs) on corporate entities are insurance type financial instruments whereby the buyer of the protection receives a lump sum payment from the seller in the event that the firm referred to in the contract defaults on its debt obligations. Since the insurance payment only occurs if the bankruptcy provisions for the referenced firm are triggered, the key input in pricing a CDS is the likelihood that this firm defaults within a certain time horizon. Independently, such firms might also raise capital in the bond market to finance their projects. It is known that the yields on these bonds are subject to bond market illiquidity considerations which can be so severe, that they could move the firm closer to a bankruptcy event. Clearly, bond market illiquidity frictions could be priced in CDS spreads.

The purpose of this research is to document the relationship between bond market liquidity premiums and the price of credit default swaps. To guide the intuition and to motivate my empirical investigation, I use an enhanced version of Leland and Toft's (1996) (LT) classical debt valuation equation. The model assumes that the firm's probability of default depends upon such considerations as the expected return of the firm's assets, the volatility of the value of individual firm's assets, and fraction of the value of the firm's assets that are paid out over time. Importantly, the liquidity premium "γ" enters the evolution of value equation as a component of the price of risk. The premium compensates the investor for holding a bond that is less than perfectly liquid. The fair market price of a credit default swap is obtained by setting the present value of the premium payments, weighted by the probability of default, equal to the discounted expected value of the protection payment.

Using daily single-name CDS prices from the CMA database between January 2007 and November 2019 and corporate bond transaction data from Enhanced TRACE, I find results that support the predictions of my enhanced CDS pricing model. My first finding is that, after controlling for credit risk determinants, increases in the illiquidity measures in corporate bonds lead to decreases in the Bharath and Shumway (2008) distance to default measure.[41] The decreases, in the distance to default, are statistically significant. In line with the prior research in credit spreads, my results show that the magnitude of the impact of the illiquidity measures on the distance to default is between 1/10 and 1/3 of the magnitude of variables commonly included in structural models of default. Finding that exogenous shocks to bond illiquidity lead to reductions in the distance to default is an interesting result in its own right since it constitutes a first empirical test that bond market illiquidity can lead to early default, as theoretically hypothesized by He and Xiong (2012), He and Milbradt (2014), and Chen, Cui, He and Milbradt (2018).

My second finding is that, after controlling for the credit risk determinants suggested by the model and by time-to-maturity, the CDS premiums increase with increases in the illiquidity of the corporate bond market. This increase in the premiums is both statistically significant and economically meaningful even when controlling for the bid ask spread of the CDS contract. In a subsample of single-name 5-year CDS premiums, a one standard deviation increase in the illiquidity measures of the bond market leads to between a 1 bp and a 7 bp increase in the CDS spreads. Increases in bond market illiquidity, as quantified by other common measures, lead to similar increases in the CDS spreads for the 5-year tenor. This bond illiquidity premium is also present in all the other CDS tenors available in the data. All specifications seem to indicate that

[41] Simultaneity between the dependent and the main independent variable may bias the measure of the effect of debt market illiquidity on the distance to default. I address this issue by using 12 month lags of the illiquidity measure as instruments for current illiquidity.

CDS spreads reflect a sizable premium due to the illiquidity in the corporate bond market. Finally, subsample analyses suggest that the impact of bond market liquidity of CDS premiums is greater when liquidity is scarce, that is, during the financial crises and for lower quality bonds.

Whether participants in one financial market include information pertaining to a distinct market is an important question that dates back to, for example, Shiller and Beltratti (1992) and Bekaert, Engstrom and Grenadier (2010). Both papers investigate the correlation of returns between stocks and bonds. Clearly, a high correlation between the returns of different assets suggests that investors use similar fundamental information to price them. Analogously, in the market microstructure literature, Hasbrouck and Seppi (2001), Huberman and Halka (2001), and Chordia, Roll and Subrahmanyam (2000) are the first empirical works to study how liquidity measures for different stocks in NYSE might be related. They conclude that liquidity measures for different stocks are closely linked since they are determined by similar fundamental variables. Their findings suggest that either participants in the market for one stock are aware of the liquidity considerations in the market for other stocks, or that investors are simultaneously involved in more than one market, leading to similar liquidity supply and demand schedules.

Establishing the prices for corporate bonds and for credit default swaps requires similar information. Importantly, the yields on bonds are determined by the creditworthiness of the issuer, while the payment or nonpayment of the CDS contract is also dictated by the probability of default of the underlying entity. Thus, clearly, factors that might change the likelihood of default will impact the prices of both securities. There is some academic evidence which favors the presumption that these two markets are integrated. For example, Subrahmanyam, Tang and Wang (2014) find that the introduction of a CDS contract on an entity leads to an increase in the probability of default of the referenced firm. These authors explain that creditors are more willing to lend more money to the firm once their claim is insured. This, in turn, increases the firm's leverage and attracts more creditors which complicates the debt restructuring process. Thus, the likelihood of a default triggering event is elevated. In a theoretical contribution, Sambalaibat (2013) simultaneously models the markets for bonds and for credit default swaps and shows that the introduction of CDSs increases the liquidity of the underlying bonds. According to Sambalaibat (2013), the availability of a credit default swap contract attracts new investors to both markets, which enhances their respective liquidities.

Relatedly, companies may raise external capital by offerings bonds. These issuances frequently occur in over-the-counter markets which are subject to illiquidity frictions.[42] Bond issuing firms must offer a greater yield on their debt to compensate buyers for this heightened liquidity risk. Evidence of this phenomenon is presented in, for example, Goldstein, Hotchkiss and Pedersen (2019), who find a negative correspondence between bond liquidity and yields at issuance. Then, the market value of the firm's total debt, which is the aggregated value of its individual outstanding bonds, will also be partly determined by the market's liquidity frictions. The total value of the firm as well as the total value of equity does not elude these implications. Furthermore, structural models of default, such as Merton (1974), Black and Cox (1976), Leland (1994), and Leland and Toft (1996), presume that the owners of the firms select a default barrier such that, in case this barrier is crossed, it is optimal for the equity holders to shut down operations. This endogenous default threshold must then embed the same illiquidity issues that affect the market price of the bond, the value of total debt, the value of total equity, and the value of the firm. This construct points to a clear link between liquidity frictions in the secondary market for bonds

[42] See, for example, the work in Duffie, Garleanu, and Pedersen (2005, 2007) who modeled illiquidity in OTC markets in terms of "search and bargaining" considerations.

and the firm's probability of default. Under these conditions, financial instruments whose prices depend on the likelihood of bankruptcy, such as credit default swaps, might also be affected by illiquidity.

In recent decades, credit default swaps have become one of the most frequently traded financial instruments with the purpose of transferring credit risk among counterparties. Academic research has recognized this trend.[43] This paper sheds light on the literature studying the interaction between corporate bond and CDS markets (Nashikkar, Subrahmanyam and Mahanti (2011), Subrahmanyam, Tang and Wang (2014), Sambalaibat (2013), and Lee, Naranjo and Velioglu (2018)) by showing that CDS spreads reflect a premium solely attributable to liquidity frictions in the corporate bond market. The recent literature on this topic finds support for an illiquidity spillover effect from CDSs to corporate bonds. This paper describes an economic channel through which corporate bond illiquidity may spill over to credit default swap prices.

The remainder of the paper is organized as follows. In section II, I formulate the model. I describe the data in section III. In section IV, I summarize the empirical results. In section V, I summarize my conclusions.

2. Model

We presume a representative firm has productive assets whose unleveraged value $V(s)$ follows a continuous diffusion process with a constant proportional volatility σ:

$$dV(s) = (\mu(V,s) - \delta) \cdot V(s)ds + \sigma V(s)dz_s\,, \quad (1)$$

where $\mu(V,s)$ is the total expected rate of return on asset value $V(s)$, δ is the constant fraction of the asset's value paid out to security holders, and dz_s is the increment of a standard brownian

[43] See for example the work in Lando (1998) and Longstaff, Mithal and Neis (2005).

motion.[44] The process continues without limit until $V(s)$ falls to V_B, the optimal value of the firm's assets at which to declare bankruptcy. Much like Leland and Toft (1996), I characterize the default triggering barrier "b" as ln $\left(\frac{V(s)}{V_B}\right)$, when $V(s) = V_B$.

Girsanov's theorem allows us to change the diffusion process for $V(s)$ from the real probability measure to a risk-neutral characterization so that expression (1) becomes:

$$dV(s) = \left((r+\gamma) - \delta - \frac{1}{2}\sigma^2\right)V(s)ds + \sigma V(s)dz_s^* \quad (2),$$

where r is the risk-free rate and dz_s^* is the increment of a brownian motion in the risk-neutral measure. The liquidity premium "γ" enters the evolution of value equation as a component of the price of risk and it compensates the investor for holding a bond that is less than perfectly liquid.[45]

Credit default swap contracts compensate the buyers in case the referenced entity defaults on its obligations. The fair price for this contract may be found by setting the discounted expected value of the flow of premium payments "S" equal to the discounted expected value of the lump sum protection payment "R". The premium payments are assumed to occur continuously until either the expiry of the CDS or until the default of the underlying company. Similarly, the protection payment will only occur if the underlying entity fails in its commitments.

Consider the discounted expected value of the flow of premium payments $S(t)$

$$\int_0^t e^{-(r+\gamma)\cdot s} \cdot S(t) \cdot [1 - F(s, V, V_B)]\, ds, \quad \text{(i)}$$

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[44] We will omit the time dependency for the value of the firm $V(t)$ and for the brownian motion dz_s whenever it is convenient and not confusing.

[45] Note that including the liquidity premium γ in such a fashion is consistent with the bond pricing approach in Longstaff, Mithal and Neis (2005). Also, note that this treatment of the liquidity premium corresponds to He and Xiong's (2012) characterization.

which will be paid at "s" with probability of $1 - F(s)$, where $F(s)$ is the cumulative distribution function of the first passage time to default V_B. The CDS expires at t.

Rewriting (i) yields

$$S(t)\left[\int_0^t e^{-(r+\gamma)\cdot s} ds - \int_0^t e^{-(r+\gamma)\cdot s} \cdot F(s)\, ds\right] \qquad \text{(ii)}$$

and

$$\int_0^t e^{-(r+\gamma)\cdot s} ds = \frac{-1}{(r+\gamma)}\, e^{-(r+\gamma)\cdot s}\Big|_0^t$$

which becomes

$$\int_0^t e^{-(r+\gamma)\cdot s} ds = \left(\frac{1}{r+\gamma}\right)\left[1 - e^{-(r+\gamma)\cdot t}\right].$$

Rewriting the second term in (ii)

$$\int_0^t e^{-(r+\gamma)\cdot s} \cdot F(s)\, ds$$

and invoking integration by parts

$$\int_B^A u\, dv \ = uv|_B^A - \int_B^A v\, du$$

with

$$u \ = F(s)$$

$$dv = e^{-(r+\gamma)\cdot s} ds$$

so that $du = f(s)ds$ and $v = \frac{-1}{(r+\gamma)} e^{-(r+\gamma)\cdot s}$. Consequently, $\int_0^t e^{-(r+\gamma)\cdot s} \cdot F(s)\, ds$ becomes

$$\left\{\left[F(s)\frac{-1}{r+\gamma}e^{-(r+\gamma)s}\Big|_0^t\right] + \int_0^t \frac{1}{r+\gamma}e^{-(r+\gamma)s}f(s)\,ds\right\}$$

and evaluating the limits yields

$$F(t)\left(\frac{-1}{r+\gamma}e^{-(r+\gamma)t}\right) + \frac{1}{(r+\gamma)}\int_0^t e^{-(r+\gamma)s}f(s)\,ds.$$

Gathering all three terms and the expected value of the discounted premium becomes

$$S(t)\left(\frac{1}{r+\gamma}\right)\left[\left(1-e^{-(r+\gamma)\cdot t}\right) + F(t)\cdot e^{-(r+\gamma)t} - G(t)\right]$$

with

$$G(t) = \int_0^t e^{-(r+\gamma)s}f(s)\,ds.$$

The discounted expected value of the lump sum protection payment R would be written as

$$R\int_0^t e^{-(r+\gamma)s}f(s)\,ds$$

which, using the notation above, equals $R \cdot G(t)$.

Finally, equating the expected value of the discounted flow of premiums to the discounted expected value of the protection payment and then solving for $S(t)$ yields

$$S(t) = \frac{R\cdot G(t)}{\left(\frac{1}{r+\gamma}\right)\left[\left(1-e^{-(r+\gamma)\cdot t}\right) + F(t)\cdot e^{-(r+\gamma)t} - G(t)\right]}. \quad \text{(iii)}$$

The expressions for $F(t)$, $G(t)$, and V_B are provided here and derived in Appendices L and G.

$$F(t) = N\left(\frac{-b-a\sigma^2 t}{\sigma\sqrt{t}}\right) + \left(\frac{V}{V_B}\right)^{-2a}\cdot N\left(\frac{-b+a\sigma^2 t}{\sigma\sqrt{t}}\right)$$

$$G(t) = \left(\frac{V}{V_B}\right)^{-a+z} \cdot N(q_1) + \left(\frac{V}{V_B}\right)^{-a-z} \cdot N(q_2)$$

$$q_1 = \frac{-b - z\sigma^2 t}{\sigma\sqrt{t}}; \;\; q_2 = \frac{-b + z\sigma^2 t}{\sigma\sqrt{t}}$$

$$h_1 = \frac{-b - a\sigma^2 t}{\sigma\sqrt{t}}; \;\; h_2 = \frac{-b + a\sigma^2 t}{\sigma\sqrt{t}}$$

$$a = \frac{(r+\gamma) - \delta - \left(\frac{\sigma^2}{2}\right)}{\sigma^2}; \;\; b = \ln\left(\frac{V}{V_B}\right); \;\; z = \frac{\sqrt{(a\sigma^2)^2 + 2(r+\gamma)\sigma^2}}{\sigma^2}$$

$$V_B = \frac{\left(\frac{C}{r+\gamma}\right)\left(\frac{A}{(r+\gamma)T} - B\right) - \frac{AP}{(r+\gamma)T} - \frac{\tau C x}{r+\gamma}}{1 + \alpha x - (1-\alpha)B}$$

with

$$A = 2ae^{-(r+\gamma)T}N\big(a\sigma\sqrt{T}\big) - 2zN\big(z\sigma\sqrt{T}\big) - \frac{2}{\sigma\sqrt{T}}n\big(z\sigma\sqrt{T}\big) + \frac{2e^{-(r+\gamma)T}}{\sigma\sqrt{T}}n\big(a\sigma\sqrt{T}\big) + (z-a)$$

and with

$$B = -\left(2z + \frac{2}{z\sigma^2 T}\right)N\big(z\sigma\sqrt{T}\big) - \frac{2}{\sigma\sqrt{T}}n\big(z\sigma\sqrt{T}\big) + (z-a) + \frac{1}{z\sigma^2 T}\ .$$

Above, $N(\cdot)$ denotes the cumulative distribution of the standard normal while $n(\cdot)$ is the density function of the standard normal.

Equation (iii) formally acknowledges the impact of liquidity premiums upon the price of a CDS. In particular, γ_t is embedded in $F(t)$ 106 times, liquidity premiums make exactly 150 appearances in $G(t)$ and γ_t itself is found on the RHS of (iii) 3 times. In aggregate, there are 409 ($G(t)$ is both in the numerator and denominator of (iii)) instances of the liquidity premiums in the determination of the price of a credit default swap. Though the magnitude of γ_t will be in basis points, the number of occurrences of the premium demand the recognition of γ_t in any empirical investigation of the determinants of CDS.

Our enhanced CDS pricing equation provides a basis for investigating an empirical relationship between a liquidity premium and the price of credit default swap. However, the multitude of the appearances of γ_t on the RHS of (iii) and the confounded nature of their entry frustrates both calculus and casual intuition.[46] Consequently, in order to establish an "a priori" notion about the comparative static relationship between liquidity premiums and CDS prices, I used some illustrative parameters to simulate a relationship between the variates.[47] For example, I set the initial unlevered value of the assets V_0 and the corporate tax rate equal to \$100 and 0.35, respectively. I also fix the risk-free rate r and the volatility of assets σ equal to 0.05 and 0.20, respectively. Finally, I set the payout ratio δ to 0.07 and the fraction of face value paid by the insurance in default, R, equal to 0.60. Plotting the price of a CDS contract for different values of the liquidity premium results in the relationship illustrated in Figure 18. Clearly, increases in the liquidity premia result in increases in the price of CDS contracts and, more importantly, they provide a rationale to empirically investigate the relationship between the two variates.

[46] Liquidity premiums γ_t are part of the value evolution equation

$$dV(s) = \left(r + \gamma - \delta - \frac{1}{2}\sigma^2\right)V(s)ds + \sigma V(s)dz_s^*$$

consequently, increases in $\gamma(t)$ as part of $dV(s)$ would reduce the likelihood of firm default. In addition, γ_t impacts the RHS of (iii) as a discount factor, reducing the expected present value of the premium payments over time and also reducing the expected present value of the lump sum payment R.

[47] The magnitude of the parameters chosen is based on those used in Leland and Toft (1996) and He and Xiong (2012).

3. Data

The data for this paper comes from four different sources. The first source is the CMA data, owned by Intercontinental Exchange, which contains detailed end-of-day market information for over 2000 single-name credit default swaps. This database contains daily transaction information such as bid and offer par spreads, bid and offer quote spreads, bid and offer upfront fees, and mid daily high and low spreads. The CMA data also contains the restructuring clause applicable to the CDS, the coupon, the recovery rates, and the cumulative probabilities of default. This rich dataset allows for a detailed exploration of liquidity implications in credit default swaps. For example, I am able to calculate bid-ask spreads using par spreads, quoted spreads, and upfront fees. From the CMA data set, I gather all single-name contracts that are active, trade in the United States, and are denominated in U.S. dollars. I remove contracts written on sovereign entities. The time series for all variables is available from January 2006 to June 2019. The data for prices and bid-ask spreads is available for the ten tenors: 6 months and 1, 2, 3, 4, 5, 7, 10, 20, and 30 years. In total, I am able to obtain 32,026,253 observations for 1,336 single-entity contracts. Panel A of Table 12 summarizes some of the main variables in the CDS data set by sector and by time period.

The second source is TRACE which contains all intraday trading data for U.S. corporate bonds. From TRACE, I collect transaction yields, buy and sell prices, and volumes. I only collect bonds for firms that also appear on the CMA data set. Since the first observation in the CMA data occurs in 2006, I only gather bond transactions from 2006 to 2019. Using TRACE, I calculate daily bid-ask spreads, the Amihud (2002) measure, the interquartile range, and the roundtrip measure, as detailed in Dick-Nielsen, Feldhütter, and Lando (2012). Panel B of Table 12 presents averages for the most prominent variables in this data set.

The third source is Mergent FISD which contains at-issuance information on the corporate bonds. I collect the issuance and maturity date, coupon rate, and exclude any bonds with call provisions.

The fourth source is the merged files from CRSP/COMPUSTAT, from which I collect the total book value of short (DLCQ) and long-term (DLTTQ) debt to proxy for the face value of the outstanding bond. I calculate the value of the firm as the sum of the face value of the bond and the equity value of the firm, which is the number of shares outstanding (SHROUT) times the price of the firm's stock (PRC). The variable δ captures the total payouts of the firm in the form of interest payments to debt holders, dividends to equity holders, and net stock repurchases. The data is obtained from the quarterly COMPUSTAT files as follows: interest payments are the previous fourth quarter's total interests paid (INTPNY), dividends to equity are the total annual dividends (DVY), and net stock repurchases are the previous year's net purchase of common and preferred stock (PRSTKCY). The payout ratio is then computed as the ratio of total payout divided by total assets.

The LT model requires precise calculation of the volatility of the firm's assets σ. Yet, this variable knows no direct empirical counterpart. We utilize the procedure put forth in Schaefer and Strebulaev (2008) and in Feldhütter and Schaefer (2017) to develop a proxy for σ using the volatility of the firm's stock returns σ_e, the volatility of the firm's debt σ_d, the correlation between σ_e and σ_d, $\sigma_{e,d}$, and the firm's leverage lev. Following Schaefer and Strebulaev (2008), we compute the σ_e as standard deviation of daily returns (RET from CRSP) using a rolling 1-year period and we then multiply the result by $\sqrt{255}$. This last step converts the result into an annual standard deviation. Since the value of the firm's assets is no more than a weighted average between the values of debt and equity using lev as the weight, the volatility of assets may be obtained by invoking the equation for the variance of a linear combination of two random variables:

$$\sigma^2 = (1 - lev) \cdot \sigma_e^2 + lev \cdot \sigma_d^2 + 2 \cdot lev \cdot (1 - lev) \cdot \sigma_{e,d}.$$

Schaefer and Strebulaev (2008) find that this relationship can be simplified further by assuming that $\sigma_d = 0$, resulting in $\sigma^2 = (1 - \text{lev})\sigma_{\text{equity}}^2$. Finally, Schaefer and Strebulaev (2008) prescribe that the result must be multiplied by a correcting factor which takes the following values depending on the firm's leverage: 1 if $lev < 0.25$, 1.05 if $0.25 < lev < 0.35$, 1.10 if $0.35 < lev < 0.45$, 1.20 if $0.45 < lev < 0.55$, 1.40 if $0.55 < lev < 0.75$, and 1.80 if $lev > 0.75$. We mimic this procedure when calculating σ.

4. Results

Implications of changes in bond market liquidity on the likelihood of default

The calibrated results depicted in Figure 18 suggest that bond market illiquidity contributes to the determination of credit default swap prices. In the context of Leland and Toft's liquidity enhanced model, increases in the liquidity premiums engender a multitude of firm variable changes which, together, contribute to a greater likelihood of default. Thus, to establish a credible empirical channel through which liquidity premiums help determine CDS prices, I begin by studying the relationship between bond liquidity and the likelihood of default.

I use the "distance-to-default" measure proposed by Bharath and Shumway (2008) as an empirical counterpart of the likelihood of default. Their measurement is virtually identical to Merton's (1974) probability that the value of the firm is greater than the face value of the firm's outstanding debt at its time of maturity.[48] To characterize bond market liquidity, I use the TRACE

[48] Merton's (1974) model is an application of Black and Scholes' (1976) option pricing model to the value of the firm. In Merton's work, a firm's value of equity, E, can be conceptualized as a European call option written on the value of the firm, V, with a strike price equal to the face

database to calculate the bond's bid-ask spread, the Amihud (2002) illiquidity measure, and the Roll (1984) liquidity measure.[49] All these measures are aggregated at a firm-day level. Table 13 shows the results of the following regression:

$$DistanceToDefault_{(i,t)} = \alpha_0 + \alpha_1 \cdot liquidity_{(i,t)} + controls_{(i,t)} + \xi_i + \omega_t + \epsilon_{(i,t)}.$$

In the equation above, i and t are indices for the firm and for the day of the observation. In addition, $DistanceToDefault_{(i,t)}$ is Bharath and Shumway's (2008) measure, $liquidity_{(i,t)}$ is one of the three liquidity measures (bid-ask spread, the Amihud (2002) measure, or the Roll (1984)

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value of debt, B. The firm's debt is assumed to mature at time t. Substituting the value of equity, the value of debt, and the value of the firm into Black and Scholes (1976) option pricing model yields

$$E = V \cdot N(d_1) - e^{-rt} \cdot B \cdot N(d_2),$$
$$D = V - E, \text{ with}$$
$$d_1 = \frac{\ln\left(\frac{V}{B}\right) + \left(r + \frac{1}{2}\sigma^2\right)t}{\sigma\sqrt{t}}, \quad \text{and} \quad d_2 = \frac{\ln\left(\frac{V}{B}\right) + \left(r - \frac{1}{2}\sigma^2\right)t}{\sigma\sqrt{t}}.$$

This is exactly Merton's model. The term $N(d_2)$ is the risk-neutral probability that the option will be in the money at expiration (see for example Cox and Rubinstein (1985) page 205). In the context of Merton's equity valuation, this implies that $N(d_2)$ is the probability that the value of the firm is greater than the face value of debt at time t. Clearly, as a "z-score", the larger the value of d_2, the greater the probability that $V > B$ at maturity. This is precisely the reason why Bharath and Shumway (2008) denominate d_2 the "distance-to-default". Finally, note that $N(d_2)$ is only allowed to take values in the range [0,1], while d_2 is conceptually allowed to take any value in the real line. Thus, choosing d_2 over $N(d_2)$ as a dependent variable justifies the use of linear regressions as an adequate statistical estimation method.

[49] I compute a bond's bid-ask spread within a day by subtracting the daily average of the bonds "buy" price from the daily average of the bonds "sell" price. The weights used for the averages are the dollar volumes in each transactions. Thus, clearly the daily bid-ask spread is in units of dollars ($). Next, we compute the daily Amihud (2002) measure by averaging the bond's daily absolute returns, and then by dividing the result by the total bond's daily volume, expressed in millions of dollars. Since the numerator of Amihud's measure is expressed as percentage and the denominator is expressed in dollars, the units of this liquidity measure are $\frac{1}{dollar} = 1/\$$. Lastly, Roll's (1984) measure is computed by obtaining the daily covariance between consecutive bond returns, taking the squared root of the result, and then multiplying by two. Thus, Roll's measure is expressed as a percentage.

measure), and $controls_{(i,t)}$ is a vector of the covariates: leverage, profitability, payout ratio, volatility of assets ("Volty"), time-to-maturity ("TTM"), the VIX, and the slope of the U.S. Treasury bonds ("Trea. Slope"). These control variables are typically included in regressions where the credit spread is the dependent variable.[50] Finally, ξ_i symbolizes single entity fixed effects, ω_t represents day fixed-effects, and $\epsilon_{(i,t)}$ are random errors.[51]

In Column 1 of Table 13, the coefficient of "Bid Ask" is negative and statistically significant at the 1% level. This first result suggests that a one dollar widening of the bid-ask spread, an increase in bond illiquidity, results in a 0.079 standard deviation decrease in the distance to default. In Column 2 of Table 13, the coefficient of the Amihud (2002) illiquidity measure is also negative and statistically significant at the 1% significance level. The coefficient in that column asserts that a 1-unit increase in the per-dollar change in price (an increase in illiquidity) results in a 0.018 standard deviation reduction in the distance to default. Finally, Column 3 in Table 13 shows that the impact of the Roll (1984) measure is positive and statistically significant at the 1%. Thus, an increase in the one period lagged correlation between returns, a manifestation of greater liquidity, results in a 0.008 standard deviation increase in the distance to default. In summary, Table 13 suggests that increases in illiquidity may increase the possibility of a bankruptcy event. In Table 13, the magnitude of the coefficient for "Bid Ask", "Amihud", and "Roll" are between 1/10 and 1/3 of the magnitude of the coefficients for "Leverage", "Payout Ratio", and "Volty". Thus, clearly, bond liquidity measures stand out as important determinants of the "distance to default", even when compared with prominent determinants of default, provided by the classic LT framework. Furthermore, these results are in line with, for example, Dick-

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[50] See, for example, Dick-Nielsen, Feldhütter, and Lando (2012).

[51] The use of time and firm fixed effects is partially inspired by the analyses in Subrahmanyam, Tang and Wang (2014).

Nielsen, Feldhütter and Lando (2012), who find that a similar proportion of the credit spread is explained by illiquidity.

Proxies for bankruptcy, which are common in structural models of default are statistically significant in Columns 1 to 3 of Table 13. For example, in all three specifications, a 1 percent increase in the firm's ratio of total debt to total assets results approximately in a 0.002 standard deviation decrease in the distance to default. All coefficients for leverage are significant at the 1%. This result certainly implies that default becomes more likely when firms assume more debt as a fraction of their value. Also, as expected, the volatility of the firm's assets, "Volty", is statistically significant at the 1% level and negatively correlated with the firm's distance to default. Additional variables such as the firm's profitability (a percentage) and the total payout ratio seem to increase the distance-to-default, although the magnitude of these coefficients is not statistically different from zero.

While the preliminary results present in Table 13 support the importance of bond illiquidity in reducing the distance-to-default, the empirical design is subject to endogeneity concerns. More specifically, the inferences in Table 13 may be affected by reverse causality, where a decrease in the Bharath and Shumway (2008) distance-to-default might lead to an increase in illiquidity and not the other way around. To address the endogeneity concerns, I implement an instrumental variable approach. I use 12-month lagged liquidity measures as an instrument for market illiquidity.[52] This has two main advantages. First, it is unlikely that a change in the distance to default at time t is able to explain changes in market liquidity at time $t - k$ for any positive integer k. If this were true, it would suggest that distance-to-default calculations incorporate market

[52] Using 12 month lagged liquidity measures is consistent with the work in Korajczyka and Sadka (2009).

information quicker than liquidity calculations. This is clearly not true. Second, prior research work in bond market, such as Lin, Wang, and Wu (2011), have proposed that bond liquidity may be predicted with time series analysis. Thus, for each point in time, I compute the predicted average of the monthly liquidity measure using 12-month lagged values of liquidity for all the bonds of the firm that traded on that month, weighted by the trading volume of each bond. The panel data ranges from 2008 to 2018 and the regressions are for firm-months, where standard errors are clustered also at the same level. The results from the second stage of the instrumental variable approach are presented in Columns 1 to 3 of Table 14, where the dependent variable is the monthly distance-to-default of the referenced firm.

In Column 1 of Table 14, I present the impact of the predicted "Bid-ask" on the distance to default. The coefficient of -0.032 is significant at the one percent level. The sign and magnitude of the regression coefficient indicate that a dollar widening of the predicted bid-ask shortens the distance to bankruptcy. This result corroborates the preliminary results shown in Column 1 of Table 13. In Column 2 of Table 14, I show that the coefficient estimate for the impact of the Dick-Nielsen's "lambda" illiquidity measure on the distance to default is -0.044. This coefficient supports the notion that increases in illiquidity reduce the distance to default. Finally, Column 3 in Table 14 presents a negative and statistically significant coefficient at the 1% level, of the impact of the predicted Amihud illiquidity measure on the distance to default. Certainly, increases in the per-dollar change in returns result in an increase the probability of a bankruptcy event. Note that the sign and the statistical significance of the coefficients for the liquidity measure remain unchanged from the ones in Table 13. This is by itself a powerful result because it indicates that reversed causality between the two variables is not an issue. Additionally, the results in Tables 2 and 3 suggest that decreases in bond market liquidity shorten the distance to default, regardless of the liquidity measure used as a right hand side regressor.

In summary, the preliminary results in Tables 2 and the instrumental variable results in Table 14 document that reductions in bond market liquidity can reflect an increase in the likelihood of a default event. Next, I empirically study the implications of changes in the bond market liquidity in the pricing of CDS contracts themselves.

Implications of changes in bond market liquidity on the prices of credit default swaps

The coefficient estimates in Tables 2 and 3 in the preceding section suggest that adverse changes in market liquidity can reduce the firm's distance to a default triggering event. This result provides some credibility for the notion that increases in γ_t can bring the company closer to bankruptcy. Additionally, the calibrated results in Figure 18 propose that increases in liquidity premiums widen the spreads of credit default swaps. To test this claim, I project the daily CDS spreads onto measures of bond market illiquidity, where the daily CDS spreads (expressed as the CDS premium payment relative to the face value of the contract) are obtained from the CMA dataset and the daily bond liquidity measures are the same as above.[53] I employ the following specification:

[53]In order to explain the dependent variables that I feature in this study, it is appropriate to discuss how market participants commonly quote credit default swap prices. CDS premiums, "S" in the model section, are quoted as an annual percentage of the face value. For example, a CDS contract with a face value of \$5 million and a 100 basis points (1%) premium implies a total annual payment of $\$5\,million \times 1\% = \$50{,}000$. Thus, CDS premiums are in fact quoted as $\frac{S}{Face\ Value}$ and expressed as basis points. This is referred to as the "par spread". One appropriate reading would be Chapter 67 of Fabozzi's "The Handbook of Fixed Income Securities", eighth edition. However, ISDA's (International Swaps and Derivatives Association Inc.) 2009 "Big Bang Protocol" standardized premiums of CDSs trading in the U.S. to either 100bp or 500bp. More details on the 2009 "Big Bang Protocol" may be found in Chapter 66 of Fabozzi's book. Of course, fixing the value of $\frac{S}{Face\ Value}$ implies that the discounted expected value of the premium payments will NOT

$$CDSspread_{(i,t)} = \alpha_0 + \alpha_1 \cdot BONDliq_{(i,t)} + \alpha_2 \cdot CDSliq_{(i,t)} + controls_{(i,t)} + \xi_i + \omega_t + \epsilon_{(i,t)}.$$

Here, $CDSspread_{(i,t)}$ is the daily CDS spread for each referenced entity, $CDSliq_{(i,t)}$ is the bid-ask spread of the CDS contract, and $BONDliq_{(i,t)}$ is an aggregate of the bond market liquidity measures of all outstanding bonds issued by the referenced entity. Typically, firms have more than one outstanding bond and they may differ by maturity. One strand of the corporate bond literature documents that liquidity premiums might be substantially different for issues in alternative segments of the term structure, even for bonds in the same credit rating category. In general, term structures of liquidity premiums are found to be increasing and concave for investment grade bonds. With this in mind, I aggregate the liquidity measures for bonds of the same firm, but different maturities in two ways: first, as the simple average of their respective liquidities and second, as the maturity-weighted average of the liquidities. In the equation above, $controls_{(i,t)}$ are covariates that explain default, ξ_i are single entity fixed effects, ω_t are day fixed-effects, and $\epsilon_{(i,t)}$ are the random error terms.

Columns 1 through 4 in Table 15 summarize the relative contributions of the bond's bid ask spread and the bond's Amihud (2002) measure in the determination of the CDS par spread, while columns 5 through 8 in Table 15 summarize the contributions of the same liquidity measures in the determination of the CDS quoted spread. The impact of all four liquidity measures on both

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equal the discounted expected value of the insurance payment in the event of the firm's default. Thus, an "upfront fee" must be paid be either the buyer or the seller of the insurance to guarantee that the net expected present value of the contract equals zero. Yet, as a market observation, oftentimes the "upfront fee" might not agree with the predictions of any CDS pricing model. In this scenario, markets participants substitute the CDS's observed "upfront fee" into a CDS pricing model and obtain the premium such that the model implied upfront fee exactly equals the observed upfront value. Such a premium is termed the "quoted spread". Again, we refer the reader to Chapter 67 of Frank Fabozzi's "The Handbook of Fixed Income Securities" eighth edition for a detailed discussion. In the results section, we alternatively use the "par spread" and the "quoted spread" as left hand side variables in the regression analyses.

the par and the quoted CDS spreads is positive and statistically significant at the 1% level for the 5-year tenor. Additionally, a one-unit (dollar for the bid-ask spread, and 1/dollar for the Amihud measure as detailed in footnote 48) increase in the liquidity measures contribute to a widening of the CDS spread between 1.3 and 2.7 basis points. Clearly, increases in bond illiquidity are positively correlated with increases in the spread of these contracts, even after accounting for the liquidity of the CDS contract itself, and after including relevant determinants of default. The signs and magnitudes of the remaining explanatory variables are consistent with conventional economic reasoning.

In Columns 1 to 8 in Table 16, I implement a similar regression analysis for each of the eight CDS tenors: 1 year, 2 years, 3 years, 4 years, 5 years, 8 years, 10 years and 30 years. For brevity, I only employ Amihud's measure as a proxy for bond market liquidity. For all tenors, the coefficient of Amihud's measure is positive and statistically significant at the 1% level. Further, the magnitude of the coefficients averages 1.1 basis points per unit increase in the bond market illiquidity. Thus, bond liquidity remains an important determinant of CDS prices even after simultaneously including the CDS liquidity as a regressor. Additionally, in each Column of Table 16, the magnitudes of the coefficients of "Amihud" are between 1/4 to 1/3 of the magnitude of the coefficient of "CDS Bid Ask". This last statement is noteworthy because it emphasizes the notion that bond illiquidity may increase the likelihood of default to such an extent that the bond illiquidity is reflected in the prices of CDSs. Furthermore, the magnitude of this impact is non-trivial. The coefficient of a bond's Amihud measure is comparable in magnitude to the coefficient of the CDS bid ask spread. This further stresses the notion that bond illiquidity, through the channel of a shorter distance to default, is a crucial determinant of CDS spreads.

Subsample analyses

I detail results when I classify the observations into the 2008 financial period and the subsequent non-crisis period. Prior research in corporate bond liquidity has found that the percentage of the corporate spread explained by liquidity increases during periods of financial instability (see for example Friewald, Jankowitsch, and Subrahmanyam (2014) and Dick-Nielsen, Feldhütter, and Lando (2012)). Thus, the contribution of bond liquidity in the determination of CDS spreads should be more pronounced during the 2008 financial crisis. I define the financial crisis from December 2007 to June 2009, which is consistent with the NBER definition and with Bessembinder, Jacobsen, Maxwell, and Venkarataraman (2018).

In Column 1 of Table 17, I summarize the coefficients of a regression of CDS prices on bond liquidity and on additional controls during the 2008 financial crisis. In Column 2 of the same table, I present a similar set of regression coefficients during the subsequent non-crisis period.

The coefficient of the Amihud measure is positive and statistically significant during the 2008 crisis period and during the subsequent non-crisis epoch. Yet, the magnitude of the contribution of the Amihud measure in the determination of the CDS spreads is almost 70% greater during the 2008 crisis period than in the non-crisis period. During the non-crisis period, a $1/\$$ increase in Amihud measure leads to a 3.2 basis points increase in the CDS spreads. However, in the 2008 crisis period, a $1/\$$ increase in the price impact measure leads to a 5.5 basis points increase in the price of the CDS contracts. In Column 3 of Table 17, I present coefficient estimates for the following regression:

$$
\begin{aligned}
CDSspread_{(i,t)} \\
&= \alpha_0 + \alpha_1 \cdot BONDliq_{(i,t)} + \alpha_2 \cdot CDSliq_{(i,t)} + \alpha_3 \cdot 1_{\tau=stress} \cdot BONDliq_{(i,t)} \\
&+ controls_{(i,t)} + \xi_i + \omega_t + \epsilon_{(i,t)}.
\end{aligned}
$$

As in the preceding analysis, i and t are indices for the bond and for the day of the observation. $CDSspread_{(i,t)}$ is the price of the CDS contract, $CDSliq_{(i,t)}$ is the bid-ask spread of the credit default swaps, $BONDliq_{(i,t)}$ is the bond Amihud measure, and $controls_{(i,t)}$ is a vector that includes common determinants of default. Additionally, the expression "$1_{\tau=stress}$" is an indicator function that takes the value 1 when the observation takes place during the 2008 stress period and 0 when the observation takes place in the subsequent non-stress period. Thus, the expression $1_{\tau=stress} \cdot BONDliq_{(i,t)}$ equals zero for observations in the non-crisis period because $1_{\tau=stress}$ is zero. During the 2008 crisis period, the expression $\alpha_1 \cdot BONDliq_{(i,t)} + \alpha_3 \cdot 1_{\tau=stress} \cdot BONDliq_{(i,t)}$ reduces to $(\alpha_1 + \alpha_3) \cdot BONDliq_{(i,t)}$ because $1_{\tau=stress}$ is one. This description implies that the coefficient α_1 provides an estimate of the impact of $BONDliq_{(i,t)}$ upon $CDSspread_{(i,t)}$ during a time of financial tranquility. Analogously, the sum of the coefficients $\alpha_1 + \alpha_3$ summarizes the impact of the bond market liquidity upon CDS spreads during the 2008 financial crisis. In addition, statistical significance of the coefficient α_3 implies that the impact of $BONDliq_{(i,t)}$ on $CDSspread_{(i,t)}$ is statistically different between the 2008 financial crisis and the subsequent non-crisis period.

In Column 3 in Table 17, the coefficient α_1 of the Amihud measure is positive and is statistically significant with a "p-value" of 0.001, implying that a one unit increase in the liquidity measure is accompanied by a 2.8 bp increase in the spread of credit default swaps during the non-crisis period. Furthermore, Column 3 also documents that α_3 is positive and statistically significant with a p value of 0.001. This result suggests that a similar increase in the Amihud liquidity measure leads to a 2.4 basis points increase in the credit spread of credit default swaps during the 2008 crisis period, in excess of the contribution of $BONDliq_{(i,t)}$ on $CDSspread_{(i,t)}$ during the non-crisis period. Put differently, a one unit increase in $BONDliq_{(i,t)}$ results in a 5.2 bp increase in $CDSspread_{(i,t)}$ during the crisis period. This result is consistent with the ones in Columns 1 and 2 in Table 17.

Next, I study the impact of corporate bond liquidity upon the CDS spreads when the data is sorted into "distressed" and "non-distressed" firms. I define a "distressed" firms as one whose cumulative probability of default is above the sample median, whereas a "non-distressed" firm is one whose cumulative probability of default is below the sample median.

Column 1 of Table 18 summarizes the coefficients of a regression of CDS spreads on bond liquidity, CDS liquidity, and on a set of default controls. Only observations that belong to "distressed" firms are used to obtain the coefficients in Column 1. Column 2 in Table 18 displays the coefficients of a similar regression, where only non-distressed firms are selected. The coefficient of the Amihud measure is positive and statistically significant in both Columns 1 and 2. Yet, the magnitude of the contribution of the bond liquidity measure in the determination of the CDS spreads is more than twice for the distressed firms than for the non-distressed companies. For the non-distressed firms, a one unit increase in Amihud leads to a 1.549 basis points increase in the CDS spreads. However, for the distressed firms, a similar increase in the price impact measure leads to a 3.9 basis points increase in the price of the CDS contracts. In Column 3 of Table 18, I summarize results for the following linear specification:

$$
\begin{aligned}
CDSspread_{(i,t)} \\
&= \alpha_0 + \alpha_1 \cdot BONDliq_{(i,t)} + \alpha_2 \cdot CDSliq_{(i,t)} + \alpha_3 \cdot 1_{type=distress} \cdot BONDliq_{(i,t)} \\
&+ controls_{(i,t)} + \xi_i + \omega_t + \epsilon_{(i,t)}.
\end{aligned}
$$

In this equation, $CDSspread_{(i,t)}$, $BONDliq_{(i,t)}$, $CDSliq_{(i,t)}$, and $controls_{(i,t)}$ have the same meaning as in Table 17. However, the expression $1_{type=distress}$ is now an indicator function that takes the value 1 when the observation belongs to a distressed company, and takes the value of 0 when the observation belongs to a non-distressed firm. Thus, clearly the expression $1_{type=distress} \cdot BONDliq_{(i,t)}$ equals zero for observations of non-distressed firms, because $1_{type=distress}$ is zero. For the distressed bonds, the expression $\alpha_1 \cdot BONDliq_{(i,t)} + \alpha_3 \cdot 1_{type=distress} BONDliq_{(i,t)}$ reduces to $(\alpha_1 + \alpha_3) \cdot BONDliq_{(i,t)}$ because $1_{type=distress}$ is one. This description implies that the coefficient α_1 provides an estimate of the impact of $BONDliq_{(i,t)}$ upon $CDSspread_{(i,t)}$, for the non-distressed companies. Analogously, the sum of the coefficients $\alpha_1 + \alpha_3$ summarizes the impact of liquidity upon the CDS prices for the distressed firms.

In Column 3 in Table 18, the coefficient α_1 of the Amihud measure is positive and is statistically significant with a p-value of 0.001. Thus, that a one unit increase in the liquidity measure is accompanied by a 1.16 bp increase in the spread of credit default swaps for the non-distressed firms. Yet, Column 3 also documents that α_3 is positive and statistically significant with a p-value of 0.001. This result suggests that a one unit increase in the Amihud liquidity measure leads to a 2.052 basis points increase in the price for credit default swaps of the distressed firms, in excess of the contribution of $BONDliq_{(i,t)}$ on $CDSspread_{(i,t)}$ for the non-distressed firms. This finding confirms the notion that the impact of the bond liquidity measures in the CDS spreads is larger for companies in financial distress.

5. Conclusions

Credit default swaps (CDSs) on corporate entities are financial instruments whereby the buyer of the protection receives a lump sum payment from the seller in the event that the firm referred to in the contract defaults on its debt obligations. Since the insurance payment only occurs if the bankruptcy provisions for the referenced firm are triggered, the key input in pricing a CDS is the likelihood that this firm defaults within a certain time horizon. Independently, such firms might also raise capital in the bond market to finance their projects. It is known that the yields on these bonds are subject to illiquidity considerations which can be so severe, that they could move the firm closer to a bankruptcy triggering event. Thus, if the bonds of the referenced firm are sufficiently illiquid, they may increase the firm's likelihood of default. Clearly, bond market illiquidity frictions must be priced in CDS spreads. Using a classical model of debt valuation enhanced with a consideration of liquidity premiums, I study how corporate bond market illiquidity may result in an elevated likelihood of the issuer's default. I use daily single-name CDS prices from the CMA data set and daily corporate bond yields from the TRACE data files. Both sources contain information from 2007 to 2019. I find that increases in bond illiquidity measures result in a heightened likelihood of the bond issuer's default. Additionally, I find that greater bond illiquidity occasions higher prices for the credit default swaps written on the bond issuer. I also empirically establish that the illiquidity implications on CDS premiums are more severe for less creditworthy issuers and in periods of financial stress.

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Appendices

Appendix A: Comparative Statics Analysis of Duffie, Garleanu, and Pedersen (2005)

In panel 1A of Figure 1, we illustrate how different term structures of liquidity premiums arise by changing ρ^i. If ρ^i is low then suitable dealers are hard to find and liquidity shocked investors would be forced to suffer the instantaneous funding cost δ. Under these conditions, investors can only wait for their position to mature. Consequently, the partial expected value of the funding cost increases with the bond horizon, leading to a larger liquidity premium for long-term bondholders and to a concave term structure for liquidity premiums. In contrast, as ρ^i increases, the average time until the arrival of the next dealer decreases. However, this waiting time could still be greater than the maturity for some short-term bonds. Thus, short-term bondholders are more likely to suffer the consequences of a liquidity shock. This scenario implies a downward sloping term structure of liquidity premiums under high values of ρ^i. (DGP and Feldhütter (2012) presume that the number of completed financial transactions in the interval t is given by a Poisson distribution with parameter $(\rho^i \cdot t)$, where ρ^i is the search intensity of traders in the ith level of investor sophistication. In DGP, ρ^i was set equal to 26 implying the expected value of the time to a completed transaction is 2 weeks. Feldhütter invokes a search intensity of 372 for his most sophisticated investor and 147 for an average search intensity trader. These two settings imply expected times to the completion of a financial transaction of 5.5 hours and 14.15 hours, respectively. In Figure 1A we choose to set ρ^i equal to 1000 and to 100 implying 2 hours and 2.5 days, respectively, to an asset conversion. The wide range in our settings of ρ^i was for illustrative purposes.) Figure 1B depicts the relationship between term to maturity and liquidity spread by changing the magnitude of the holding cost for an investor. Different values of δ are able to generate both the convex (low δ) and the concave (high δ) term structures of liquidity premiums.

The probability of an adverse liquidity shock is a circumstance in which the likelihood of a shock is high (large λ) and the probability of the investor being severely financially constrained is high (large π). In this scenario, long-term investors will likely incur higher expected cumulative holding costs. This parameterization leads to a scenario where an upward sloping term structure of liquidity premiums prevails, as illustrated in Figures 1C and 1D. (The time to a financial shock is given by the exponential distribution with $f(t) = \lambda \cdot e^{-\lambda \cdot t}$ where λ is the hazard rate. Feldhütter sets $\lambda = 3.6$ which implies that the expected time to a shock is $\frac{1\,year}{3.6} = 3\frac{1}{3}$ months. He and Milbradt (2014) set $\lambda = \frac{7}{10}$ so that expected time to a financial shock equals 1.43 years. In Figure 1C, we chose $\lambda = 1$ and $\lambda = 6$ which implies that the expected time to a financial shock is as remote as 1 year and as imminent as 2 months.) In contrast, when the likelihood of a shock itself is diminished and, at the same time, is low, long-term liquidity shocked investors enjoy a higher probability of switching types. This implies that the cumulative expected holding costs must be smaller for long-term bondholders compared to short-term bondholders. In this case, liquidity premiums must be smaller for longer maturities. This situation is presented in Figures 1C and 1D which depict a convex term structure of liquidity premiums. (The likelihood that an investor is financially constrained at the time of the economic shock, π, was set equal to 9% by both DGP and Feldhütter (2012) as a baseline representation. For illustrative purposes, we initially assigned π a value of 80%, to reflect a great deal of financial exposure for the investor to a financial shock, and then reduced that exposure to just 20% in Figure 1D.)

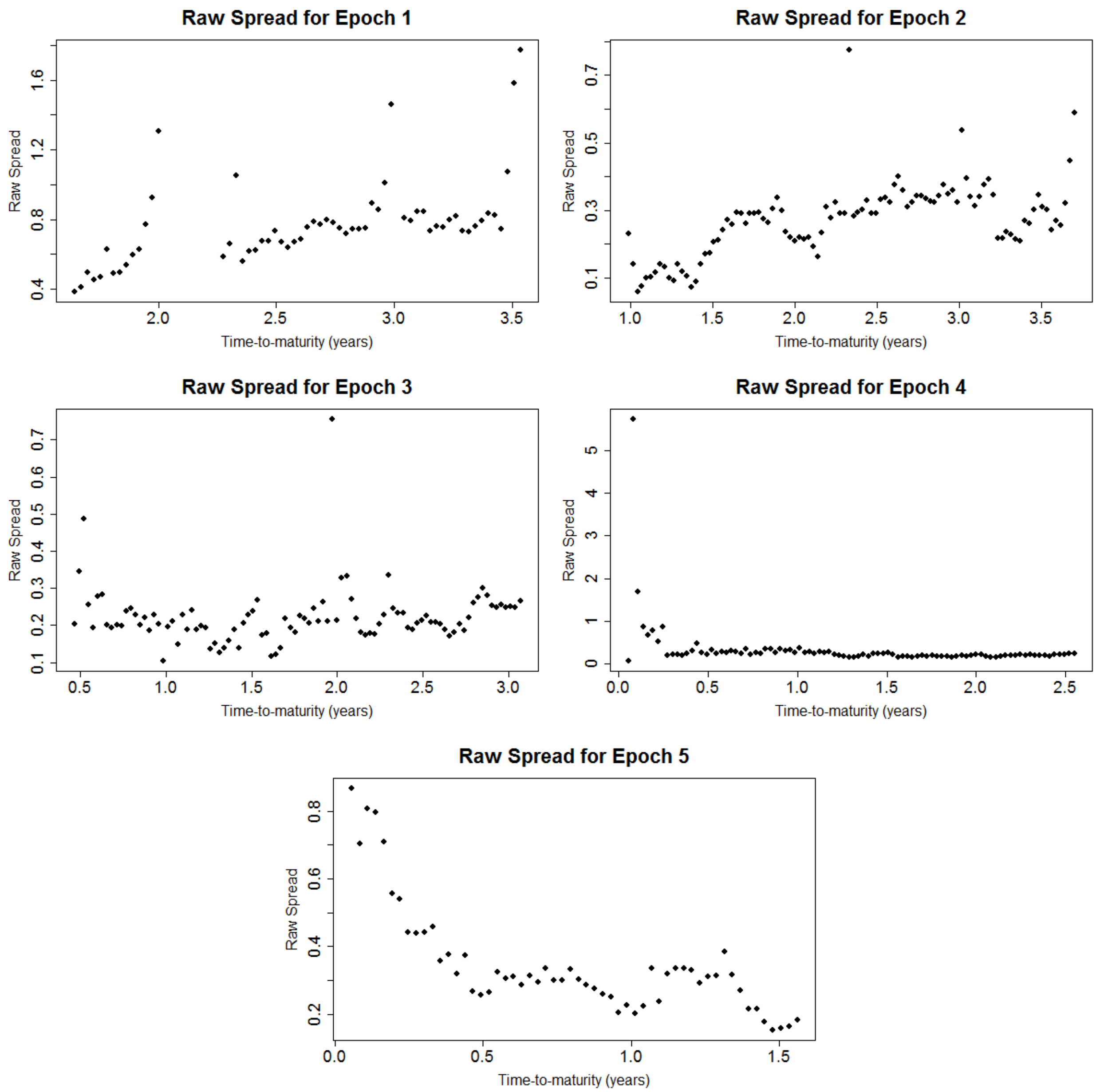
Appendix B: Liquidity spread under alternative periods

To alleviate concerns that our results are dependent upon the particular partitions of the data into our periods of “stress” and “post stress”, we provide plots and regression tables using alternative methodologies for finding date intervals. That is, we now select five new periods based on significant changes of GDP and Fed Funds futures. By identifying significant changes in the price of the Fed Funds futures (10% upwards or downwards), we believe we can identify periods of relative economic stress and, also, differing investor appetites for alternative segments of the maturity spectrum. We define five periods as follows: 1) October 1, 2008 to April 1, 2009, 2) April 1, 2009 to November 30, 2009, 3) November 30, 2009 to July 4, 2010, 4) 2010 to July 1, 2011, and 5) July 1, 2011 to December 31, 2012. Following the National Bureau of Economic Research definition of economic recession, the first two periods coincide with “stress”, while the last three periods coincide with periods of “post stress” (relative calm).

In Figure B1, we present the raw liquidity spreads for each of the five periods defined above. Periods 1 and 2, which comprise our period of stress, are clearly upward sloping, while periods 4 and 5 are downward sloping. Period 3 may be viewed as a period of transition. These results support our prior notion that periods of stress are indicative of a positive TSLS while periods of post stress are indicative of a negative TSLS.

Figure B1: Term structure of liquidity spread for each of five alternative periods

The below graphs are raw spreads of the insured corporate bonds for each of the five alternative periods. The plots are averages of all bond spreads with the same time to maturity for the period. These spreads are aggregated by weekly buckets to avoid noise and high dispersion in the data.



Appendix C: Regressions of liquidity spread using alternative periods and three maturity buckets

Similar to Appendix B, we split the data into five periods in order to demonstrate robustness to alternative stress and post stress periods. The five intervals are defined as follows: 1) October 1, 2008 to April 1, 2009, 2) April 1, 2009 to November 30, 2009, 3) November 30, 2009 to July 4, 2010, 4) July 2010 to July 1, 2011, and 5) July 1, 2011 to December 31, 2012. Furthermore, we include a medium maturity bucket in addition to short and long maturity buckets. The three buckets are defined according to whether the time remaining to maturity is below the 25^{th} percentile, between the 25^{th} and 75^{th} percentile, or above the 75^{th} percentile of times to maturity (TTM) in that period. We then perform regressions where the dependent variable is the yield of the insured corporate bond minus the yield of a treasury bond, matched by maturity. As in prior results, each right hand side variable is normalized by its mean and standard deviation over the whole sample period (October 2008 to December 2012). As before, each specification regresses the spread of the insured corporate bonds upon the liquidity measure, its volatility, the VIX, time-to-maturity of the insured corporate bond, the 6-month minus the 3-month treasury slope, a measure of market-wide liquidity, and the price of the futures contracts. The regressions include firm-fixed effects, and the standard errors of the coefficients are in parenthesis. The results in general support the notion that in periods of stress, the impact of the liquidity measure (AMIHUD) is higher for long-term bonds, while in post stress periods, the impact is higher for short-term bonds. The sign of TTM is typically positive in a stress period but typically negative post stress.

Table C1: Regression for Period 1 (October 1, 2008 to April 1, 2009)

	Regression of Liquidity Spread for Period 1		
	Insured Spread minus Treasury Yield		
	Shorter	Medium	Longer
	(1)	(2)	(3)
AMIHUD	0.050***	0.054***	0.060***
	(0.007)	(0.008)	(0.010)
AMIHUDVOL	-0.004	-0.006	0.014
	(0.008)	(0.009)	(0.011)
TTM	0.120***	0.073***	0.113***
	(0.008)	(0.009)	(0.011)
VIX	0.099***	0.110***	0.056***
	(0.007)	(0.008)	(0.011)
TREASLP	0.044***	0.042***	0.052***
	(0.007)	(0.008)	(0.011)
AMIHUDMKT	0.010	0.007	-0.014
	(0.007)	(0.007)	(0.010)
FFFP	-0.063***	-0.078***	-0.061***
	(0.007)	(0.008)	(0.011)
Constant	1.173***	1.215***	0.912***
	(0.161)	(0.145)	(0.022)
Observations	1,360	927	483
R^2	0.461	0.498	0.467
Adjusted R^2	0.449	0.483	0.449
Residual Std. Error	0.226 (df = 1331)	0.204 (df = 900)	0.197 (df = 466)
F Statistic	40.583*** (df = 28; 1331)	34.321*** (df = 26; 900)	25.506*** (df = 16; 466)
Note:			*p<0.1; **p<0.05; ***p<0.01

Table C2: Regression for Period 2 (April 1, 2009 to November 30, 2009)

	Regression of Liquidity Spread for Period 2		
	Insured Spread minus Treasury Yield		
	Shorter	Medium	Longer
	(1)	(2)	(3)
AMIHUD	0.049***	0.063***	0.249***
	(0.003)	(0.014)	(0.014)
AMIHUDVOL	-0.007*	-0.009	-0.004
	(0.003)	(0.016)	(0.013)
TTM	0.053***	0.025	-0.004
	(0.006)	(0.017)	(0.015)
VIX	0.033***	0.043**	0.056***
	(0.005)	(0.020)	(0.020)
TREASLP	0.027***	0.010	0.045**
	(0.004)	(0.018)	(0.019)
AMIHUDMKT	-0.004	-0.008	-0.026*
	(0.004)	(0.018)	(0.015)
FFFP	0.018***	-0.014	-0.030**
	(0.005)	(0.019)	(0.015)
Constant	0.165***	0.287***	0.342***
	(0.012)	(0.085)	(0.036)
Observations	1,168	2,591	1,609
R^2	0.523	0.035	0.255
Adjusted R^2	0.516	0.024	0.243
Residual Std. Error	0.101 (df = 1150)	0.646 (df = 2561)	0.455 (df = 1582)
F Statistic	74.310*** (df = 17; 1150)	3.165*** (df = 29; 2561)	20.860*** (df = 26; 1582)
Note:			*p<0.1; **p<0.05; ***p<0.01

Table C3: Regression for Period 3 (November 30, 2009 to July 4, 2010)

	Regression of Liquidity Spread for Period 3		
	Insured Spread minus Treasury Yield		
	Shorter	Medium	Longer
	(1)	(2)	(3)
AMIHUD	0.124***	0.041*	0.022***
	(0.008)	(0.023)	(0.002)
AMIHUDVOL	-0.001	-0.001	-0.007***
	(0.007)	(0.027)	(0.003)
TTM	-0.032***	0.006	0.016***
	(0.009)	(0.031)	(0.003)
VIX	0.047***	0.071**	0.035***
	(0.010)	(0.031)	(0.002)
TREASLP	-0.001	0.066***	0.002
	(0.007)	(0.023)	(0.002)
AMIHUDMKT	-0.006	-0.006	-0.001
	(0.009)	(0.030)	(0.003)
FFFP	0.0005	0.036	0.030***
	(0.008)	(0.028)	(0.003)
Constant	0.153***	0.122	0.214***
	(0.031)	(0.221)	(0.010)
Observations	743	2,056	1,201
R^2	0.361	0.030	0.406
Adjusted R^2	0.345	0.016	0.394
Residual Std. Error	0.164 (df = 723)	0.952 (df = 2026)	0.066 (df = 1177)
F Statistic	21.540*** (df = 19; 723)	2.139*** (df = 29; 2026)	34.920*** (df = 23; 1177)
Note:			*p<0.1; **p<0.05; ***p<0.01

Table C4: Regression for Period 4 (July 4, 2010 to July 1, 2011)

	Regression of Liquidity Spread for Period 4		
	Insured Spread minus Treasury Yield		
	Shorter	Medium	Longer
	(1)	(2)	(3)
AMIHUD	0.345***	0.142***	0.049***
	(0.028)	(0.004)	(0.002)
AMIHUDVOL	0.187***	0.006	-0.003
	(0.037)	(0.005)	(0.002)
TTM	-0.333***	-0.036***	-0.0003
	(0.037)	(0.004)	(0.003)
VIX	0.001	-0.024***	0.007**
	(0.044)	(0.005)	(0.003)
TREASLP	0.046	0.011***	0.001
	(0.030)	(0.004)	(0.002)
AMIHUDMKT	-0.027	0.003	0.005*
	(0.046)	(0.005)	(0.003)
FFFP	-0.026	-0.039***	-0.020***
	(0.046)	(0.005)	(0.003)
Constant	0.613***	0.129***	0.181***
	(0.184)	(0.032)	(0.007)
Observations	1,146	3,529	2,069
R^2	0.283	0.557	0.416
Adjusted R^2	0.267	0.554	0.410
Residual Std. Error	0.981 (df = 1120)	0.214 (df = 3497)	0.079 (df = 2044)
F Statistic	17.688*** (df = 25; 1120)	142.084*** (df = 31; 3497)	60.768*** (df = 24; 2044)
Note:			*p<0.1; **p<0.05; ***p<0.01

Table C5: Regression for Period 5 (July 1, 2011 – Dec 31, 2012)

	Regression of Liquidity Spread for Period 5		
	Insured Spread minus Treasury Yield		
	Shorter (1)	Medium (2)	Longer (3)
AMIHUD	0.505^{***}	0.191^{***}	0.094^{***}
	(0.037)	(0.008)	(0.005)
AMIHUDVOL	0.246^{***}	0.015	-0.020^{***}
	(0.055)	(0.009)	(0.005)
TTM	-0.091	-0.053^{***}	-0.002
	(0.059)	(0.009)	(0.006)
VIX	0.140	0.030^{**}	0.032^{***}
	(0.094)	(0.015)	(0.009)
TREASLP	0.095^{*}	-0.002	0.004
	(0.057)	(0.010)	(0.007)
AMIHUDMKT	0.090	0.033^{**}	0.011
	(0.086)	(0.014)	(0.008)
FFFP	-0.124	0.018	0.025^{***}
	(0.085)	(0.013)	(0.007)
Constant	0.222	0.144^{**}	0.261^{***}
	(0.611)	(0.073)	(0.018)
Observations	655	2,401	1,537
R^2	0.376	0.319	0.294
Adjusted R^2	0.348	0.310	0.283
Residual Std. Error	1.025 (df = 625)	0.382 (df = 2369)	0.181 (df = 1513)
F Statistic	13.013^{***} (df = 29; 625)	35.835^{***} (df = 31; 2369)	27.411^{***} (df = 23; 1513)
Note:			*p<0.1; **p<0.05; ***p<0.01

Appendix D: Potential Endogeneity of the Liquidity Measures

One potential shortcoming in the regressions that we present is that the liquidity measure might be correlated with the error term in the spread specification. To test for endogeneity bias, we perform a Durbin-Wu-Hausman test as specified in Greene (2003) and report it below in Tables D1 and D2. The logic of the Durbin-Wu-Hausman test is as follows: under the null hypothesis, both the OLS and the 2SLS estimators are consistent. Under the alternative hypothesis, only the 2SLS estimator is consistent. Thus, if we fail to reject the null hypothesis, we can safely conclude that the both estimators are consistent and there is no endogeneity bias in our regressions.

Similar to Dick-Nielsen, Feldhütter, and Lando (2012), we use the issuance amount and age of the insured bonds as instruments for liquidity. We first present the R^2 of the first stage regression, and we then present the Chi-square values of the Durbin-Wu-Hausman test. We perform regressions by period – stress and post stress – and by maturity bucket –short, medium, long.

Table D1: Issuance amount as an instrument for liquidity

Panel a: R^2 of first-stage regressions

R-squares of the 1st stage regression	**Shorter**	**Medium**	**Longer**
Stress	0.2532	0.3402	0.3928
Post Stress	0.2841	0.3518	0.2901

Panel b: X^2 values and 5% significance levels for Durbin-Wu-Hausman test

Chi-square of Durbin-Wu-Hausman test	**Short**	**Medium**	**Long**
Stress	0.5833 (33.92)	0.006012 (48.60)	0.1422 (48.60)
Post Stress	1.394 (49.80)	5.036 (50.99)	1.451 (48.60)

In panel a, we present the R^2 of the first-stage regressions – liquidity measure on issuance amount and other explanatory variables. The values for the coefficient of determination are high and we conclude that the issuance amount is a strong instrument for the liquidity measures. In panel b we present the Chi-square values for the Durbin-Wu-Hausman tests along with the 5% significance level. We can see that in all the periods and maturity buckets, the test fails to reject consistency of both estimators. The results for using only two maturity buckets are very similar.

Table D2 repeats the analysis but uses age as the instrument. Similar results hold. Thus, we can conclude that endogeneity is not a concern in our models. Again, the results for using only two maturity buckets are very similar.

Table D2: Using age of the bond as an instrument for liquidity

Panel a: R^2 of first-stage regressions

R-squares of the 1st stage regression	**Shorter**	**Medium**	**Longer**
Stress	0.60	0.10	0.34
Post Stress	0.14	0.25	0.01

Panel b: X^2 values and 5% significance levels for Durbin-Wu-Hausman test

Chi-square of Durbin-Wu-Hausman test	**Short**	**Medium**	**Long**
Stress	0.2010 (32.67)	0.7564 (47.40)	5.6278 (48.60)
Post Stress	0.02549 (48.60)	0.3259 (49.80)	0.3107 (47.40)

Appendix E: Regressions with difference in liquidity measure

We perform similar regressions as in Table 3, but, in contrast, use the difference in the liquidity measure of the corporate bonds and the U.S. Treasuries as our measure of liquidity. The regressions are for the periods of stress – October 2008 and November 2009 – and post stress – December 2009 to December 2012. We further classify the daily bond transactions into short-term and long-term depending on whether the time to maturity is above or below the median in that period. The dependent variable is the yield of the insured bond minus the yield of a treasury bond, matched by maturity. Each right hand side variable is normalized by its mean and standard deviation over the respective period, stress or post stress. BIDASK- BIDASKMKT is our liquidity measure, FFFP is the price of the price of the Fed funds 30-day futures, TREASLP is the slope of the 6-months Treasury bonds relative to the 3-months, TTM is the time-to-maturity of the insured bond, BIDASKMKT is the aggregate bid-ask measure for all outstanding corporate bonds, and BIDASKVOL is the volatility of the Bid-Ask liquidity measure for the insured bonds. The regressions include firm-fixed effects, and the standard errors of the coefficients are in parenthesis. The results in general support the notion that in periods of stress, the impact of the liquidity measure is higher for long-term bonds, while in post stress periods, the impact is higher for short-term bonds.

Regression Analysis of the Liquidity Spread (LS)

	Stress ST_{INS} (1)	Stress LT_{INS} (2)	Post Stress ST_{INS} (3)	Post Stress LT_{INS} (4)
BIDASK – BIDASK_TREA	0.019^{***}	0.083^{***}	0.076^{***}	0.004
	(0.003)	(0.006)	(0.013)	(0.007)
VIX	0.129^{***}	0.197^{***}	0.054^{*}	0.039^{***}
	(0.007)	(0.017)	(0.029)	(0.011)
FFFP	-0.001	-0.049^{***}	0.003	0.014
	(0.005)	(0.011)	(0.026)	(0.009)
TREASLP	0.029^{***}	0.027^{**}	0.011	0.034^{***}
	(0.006)	(0.013)	(0.021)	(0.011)
		Control Variables		
TTM	0.048^{***}	0.001	-0.212^{***}	0.009
	(0.004)	(0.010)	(0.021)	(0.011)
BIDASKMKT	-0.019^{***}	-0.035^{***}	0.023	-0.001
	(0.006)	(0.012)	(0.028)	(0.013)
BIDASKVOL	0.043^{***}	0.087^{***}	0.296^{***}	0.002

	(0.004)	(0.009)	(0.017)	(0.009)
Constant	0.235***	0.826***	0.211	0.130
	(0.030)	(0.289)	(0.260)	(0.184)
Observations	1,880	2,514	2,793	5,122
R^2	0.619	0.415	0.171	0.026
Adjusted R^2	0.614	0.407	0.162	0.020
Residual Std. Error	0.155 (df = 1854)	0.406 (df = 2481)	0.899 (df = 2760)	0.608 (df = 5090)
F Statistic	120.620*** (df = 25; 1854)	54.925*** (df = 32; 2481)	17.815*** (df = 32; 2760)	4.386*** (df = 31; 5090)

*p<0.1 ; **p<0.05; ***p<0.01

Appendix F: An explanation of our enhanced bond valuation equation

Following Leland/Toft model (LT) closely, we assume firm has productive assets whose unleveraged value V follows a continuous diffusion process with a constant proportional volatility σ:

$$\frac{dV}{V} = (\mu(V,s) - \delta)ds + \sigma dz$$

where $\mu(V,s)$ is the total expected rate of return on asset value V, δ is the constant fraction of value paid out to security holders, and dz is the increment of a standard brownian motion. The process continues without limit until V falls to V_B, the optimal value of the firm's assets at which to declare bankruptcy. We characterize the default triggering barrier "b" as $\ln\left(\frac{V}{V_B}\right)$ when $V = V_B$.

We have, $v = \ln\left(\frac{V}{V_B}\right)$, for $V \neq V_B$, where dv would be given by [54]

$$dv = \frac{\partial v}{\partial V} dV + \frac{1}{2}\frac{\partial^2 v}{\partial V \partial V}(dV)^2.$$

Substituting $dV = (\mu(V,s) - \delta)Vds + \sigma V dz$ into the expression for dv yields

$$dv = \frac{1}{V}\big((\mu - \delta)Vds + \sigma V dz\big) + \frac{1}{2}\left(-\frac{1}{V^2}\right)\big((\mu - \delta)Vds + \sigma V dz\big)^2$$

$$dv = (\mu - \delta)ds + \sigma dz - (1/2)\sigma^2 ds$$

$$dv = \left(\mu - \delta - \frac{1}{2}\sigma^2\right)ds + \sigma dz$$

Employing Girsanov's theorem will allow us to put this process in a risk neutral environment, so that

$$dv = \left(\mu - \delta - \frac{1}{2}\sigma^2\right)ds + \sigma(dz^* - \phi ds)$$

---

[54] The cautious reader will recognize that $\frac{d\ \ln\left(\frac{V}{V_B}\right)}{dV} = \frac{\left({}^{1}/_{V_B}\right)}{\left({}^{V}/_{V_B}\right)} = \frac{1}{V}$ which is surprising because $\frac{d\ln(V)}{dV}$ also equals $\frac{1}{V}$.

where ϕ is the price of risk, and therefore

$$dv = \left(\mu - \delta - \frac{1}{2}\sigma^2\right) ds + \sigma(dz^* - \left(\frac{\mu - (r + \gamma)}{\sigma}\right) ds)$$

or,

$$dv = \left((r + \gamma) - \delta - \frac{1}{2}\sigma^2\right) ds + \sigma dz^*$$

or,

$dv = a\ ds + \sigma\ dz^*$ with $a = \left((r + \gamma) - \delta - \frac{1}{2}\sigma^2\right)$.

This risk neutral brownian process can be written as:

$$dv_u = a\ du + \sigma\ dz_u^*.$$

Integrating,

$$\int_0^s dv_u = a\int_0^s du + \sigma\int_0^s dz_u^*$$

yields,

$$v_s - v_0 = a(s - 0) + \sigma\int_0^s dz_u^*$$

or,

$$E[v_s] = v_0 + a \cdot s$$

and,

$$Var(v_s) = \sigma^2 Var\left(\int_0^s dz_u^*\right)$$

or, finally

$$Var(v_s) = \sigma^2 E\left(\int_0^s dz_u^*\right)^2.$$

Furthermore, Ito's isometry allows us to write,

$E\left(\left(\int_0^s dz_u^*\right)\left(\int_0^s dz_u^*\right)\right)$ as $\int_0^s du$.

Gathering terms,

$$Var(v_s) = \sigma^2\int_0^s du$$

or

$$Var(v_s) = \sigma^2 s$$

so that v_s is normally distributed with a mean of $v_0 + as$ and a variance of $\sigma^2 s$. With these characteristics, the pdf for v must be

$$p(v_0, v; s) = \frac{1}{\sigma\sqrt{2\pi s}} e^{\frac{-(v-v_0-as)^2}{2\sigma^2 s}}$$

The variate v_s is characterized by $v(s_b)$ when v_s equals b. The likelihood $p(v_0, v; s)$ is the probability that $v_s = v$ and that the process has not reached the barrier in the interval $(0, s)$. Consequently, the cumulative probability $P\big(v(s) < b$ for $s < (s_b | v(0) = v_0)\big)$ equals $\int_{-\infty}^{b} p(v_0, v; s)dv = P(v_0, b; s)$. $P(v_0, b; s)$ equals the probability that "b" has not been reached by s which could be written as $P(v_0, b; s) =$ prob$(s_b \geq s)$ or as $P(v_0, b; s) = 1 - F(s; v_0, b)$, where $F(s; v_0, b)$ is the cumulative probability of the time to the barrier. Furthermore,

$$\frac{dP}{ds}(v_0, b; s) = -F'(s; v_0, b)$$

or

$$-\frac{dP}{ds}(v_0, b; s) = f(s; v_0, b)$$

where $f(s; v_0, b)$ is the marginal probability of the time to bankruptcy, which, upon differentiation by s, could be written as

$$f(s; v_0, b) = \frac{b-v_0}{\sigma\sqrt{2\pi s^3}} e^{\frac{-(b-v_0-as)^2}{2\sigma^2 s}}.$$

This probability density function integrates to one and the time to bankruptcy has, intuitively, a mean of $\left(\frac{b-v_0}{a}\right)$.

In our model, b is not just a threshold, but an absorbing barrier. Consequently $p(v_0, v; s)$ has to be rewritten as a linear combination $p(v_0, v; s) = p_1(v_0, v; s) + Ap_1(v_0, v; s)$ to ensure that $p(v_0, b; s)$, the probability that $v = b$, equals zero for every possible s_b. The method of images yields

$$p(0, v; s) = \frac{1}{\sigma\sqrt{2\pi s}}\left[e^{\frac{-(v-as)^2}{2\sigma^2 s}} - e^{\left(\frac{2ab}{\sigma^2} + \frac{-(v-2b-as)^2}{2\sigma^2 s}\right)}\right]$$

where $A = -e^{\frac{2ab}{\sigma^2}}$.

This solution can be easily verified by setting v equal to b where, consequently, the expression for $p(0, v; s)$ quickly reduces to 0.

Using the expression for $p(0, v; s)$ after the adjustment for the boundary condition yields the following expression for the cumulative of v:

$$P(0, b; s) = \int_{-\infty}^{b} \frac{1}{\sigma\sqrt{2\pi s}} \left[e^{\frac{-(v-as)^2}{2\sigma^2 s}} - e^{\left(\frac{2ab}{\sigma^2} + \frac{-(v-2b-as)^2}{2\sigma^2 s}\right)} \right] dv$$

If $P(v_0, b; s)$ is standardized, we have

$$P(0, b; s) = N\left(\frac{b - as}{\sigma\sqrt{s}}\right) - e^{\frac{2ab}{\sigma^2}} N\left(\frac{-b - as}{\sigma\sqrt{s}}\right)$$

where $N(\cdot)$ is the cumulative distribution for the standardized normal variate. If the expression is rewritten so that the absorbing barrier is at zero, the cumulative normal becomes

$$P(v_0, b; s) = N\left(\frac{b+as}{\sigma\sqrt{s}}\right) - e^{\frac{-2ab}{\sigma^2}} N\left(\frac{-b+as}{\sigma\sqrt{s}}\right) \quad \text{for some “}s\text{”.}$$

The term $e^{\frac{-2ab}{\sigma^2}} = e^{\frac{-2a}{\sigma^2} \cdot \ln\left(\frac{V}{V_B}\right)}$ by definition (i) and $e^{\frac{-2ab}{\sigma^2}} = \left(\frac{V}{V_B}\right)^{\frac{-2a}{\sigma^2}}$ (ii), because, taking the log of both the LHS and RHS terms of (i) yields $-\frac{2a}{\sigma^2} \cdot \ln\left(\frac{V}{V_B}\right) = -\frac{2a}{\sigma^2} \cdot \ln\left(\frac{V}{V_B}\right)$. Consequently, $P(v_0, b; s)$ becomes

$$P(v_0, b; s) = N\left(\frac{b+as}{\sigma\sqrt{s}}\right) - \left(\frac{V}{V_B}\right)^{\frac{-2a}{\sigma^2}} N\left(\frac{-b+as}{\sigma\sqrt{s}}\right).$$

Of course, for the standardized normal variate z, $N(A) = 1 - N(-A)$ due to symmetry, consequently, $P(v_0, b; s)$ becomes

$$P(v_0, b; s) = 1 - N\left(\frac{-b-as}{\sigma\sqrt{s}}\right) - \left(\frac{V}{V_B}\right)^{\frac{-2a}{\sigma^2}} N\left(\frac{-b+as}{\sigma\sqrt{s}}\right) \text{ or,}$$

$$P(v_0, b; s) = 1 - \left[N\left(\frac{-b-as}{\sigma\sqrt{s}}\right) + \left(\frac{V}{V_B}\right)^{\frac{-2a}{\sigma^2}} N\left(\frac{-b+as}{\sigma\sqrt{s}}\right) \right].$$

As detailed above, $P(v_0, b; s)$ can be written as $P(v_0, b; s) = 1 - F(s; v_0, b)$ where $F(s; v_0, b)$ is the cumulative probability of the time to the barrier. So if

$$P(v_0, b; s) = 1 - \left[N\left(\frac{-b - as}{\sigma\sqrt{s}}\right) + \left(\frac{V}{V_B}\right)^{\frac{-2a}{\sigma^2}} N\left(\frac{-b + as}{\sigma\sqrt{s}}\right) \right]$$

then,

$$F(s;v_0,b) = N\left(\frac{-b-as}{\sigma\sqrt{s}}\right) + \left(\frac{V}{V_B}\right)^{\frac{-2a}{\sigma^2}} N\left(\frac{-b+as}{\sigma\sqrt{s}}\right).$$

The expression immediately above is identical to the expression we have on page 37.

Recall

$$-\frac{d}{ds}P(v_0,b;s) = f(s;v_0,b)$$

so that the evaluation of the expression $\int_0^\infty e^{-(r+\gamma)s} f(s,v_0,b)\,ds$ will involve the differentiation of

$$P(v_0,b;s) = N\left(\frac{b+as}{\sigma\sqrt{s}}\right) - e^{\frac{-2ab}{\sigma^2}} N\left(\frac{-b+as}{\sigma\sqrt{s}}\right)$$

with respect to s. The partial derivative of the first term on the RHS can be written as $\frac{b}{\sigma\sqrt{2\pi s^3}} e^{\frac{-(b+as)^2}{2\sigma^2 s}}$, when zero is acknowledged as the absorbing barrier. The determination of $\int_0^\infty e^{-(r+\gamma)s} \frac{b}{\sigma\sqrt{2\pi s^3}} e^{-\frac{(b+as)^2}{2\sigma^2 s}} ds$ involves completing the square and if $\lambda = \frac{(a^2+2(r+\gamma)\sigma^2)^{\frac{1}{2}}}{\sigma^2}$ then the expression above eventually becomes

$$\int_0^\infty \frac{b}{\sigma\sqrt{2\pi s^3}} e^{-(1/2)\left(\frac{b+\lambda\sigma^2 s}{\sigma\sqrt{s}}\right)^2 - (1/2)b\left(\frac{2a}{\sigma^2}-2\lambda\right)} ds.$$

This expression can be rewritten as

$$e^{-(1/2)\ln\left(\frac{V}{V_B}\right)\left(\frac{2a}{\sigma^2}-2\lambda\right)} \int_0^\infty \frac{b}{\sigma\sqrt{2\pi s^3}} e^{-(1/2)\left(\frac{b+\lambda\sigma^2 s}{\sigma\sqrt{s}}\right)^2} ds$$

and, since for any A and B, $e^{A\,ln(B)} = B^A$, we have

$$\left(\frac{V}{V_B}\right)^{\lambda-\frac{a}{\sigma^2}} \int_0^\infty \frac{b}{\sigma\sqrt{2\pi s^3}} e^{-(1/2)\left(\frac{b+\left(a^2+2(r+\gamma)\sigma^2\right)^{\frac{1}{2}} s}{\sigma\sqrt{s}}\right)^2} ds$$

where the drift is now $(a^2+2(r+\gamma)\sigma^2)^{1/2}$.

Furthermore, if $\lambda - \frac{a}{\sigma^2}$ is rewritten as $-a + z$ and if $b + (a^2 + 2(r+\gamma)\sigma^2)^{1/2}s$ equals $b + z\sigma^2 s$ then we now have

$$\left(\frac{V}{V_B}\right)^{-a+z} \int_0^\infty \frac{b}{\sigma\sqrt{2\pi s^3}} e^{-(1/2)\left(\frac{b+z\sigma^2 s}{\sigma\sqrt{s}}\right)^2}\, ds.$$

For a particular fixed income security with a time to maturity of t, the expression for the likelihood of reaching the absorbing barrier b before the expiration of the bond would be:

$$\left(\frac{V}{V_B}\right)^{-a+z} \left[\int_0^t \frac{b}{\sigma\sqrt{2\pi s^3}} e^{-(1/2)\left(\frac{b+z\sigma^2 s}{\sigma\sqrt{s}}\right)^2}\, ds\right].$$

In terms of the cumulative normal, we have

$$\left(\frac{V}{V_B}\right)^{-a+z} \left[1 - N\left(\frac{b+z\sigma^2 t}{\sigma\sqrt{t}}\right)\right] \text{ or}$$

$$\left(\frac{V}{V_B}\right)^{-a+z} \left[N\left(\frac{-b-z\sigma^2 t}{\sigma\sqrt{t}}\right)\right]$$

because $N(-A) = 1 - N(A)$ for the standardized normal variate. This is exactly what we have as the first half of the expression for $G(t)$ in the theory section.

Differentiation of the second term on the RHS of $P(v_0, b; s) = N\left(\frac{b+as}{\sigma\sqrt{s}}\right) - e^{-\frac{2ab}{\sigma^2}} N\left(\frac{-b+as}{\sigma\sqrt{s}}\right)$ with respect to s almost immediately yields

$$-e^{\frac{-2ab}{\sigma^2}} \int_0^\infty \frac{-b}{\sigma\sqrt{2\pi s^3}} e^{\frac{-(1/2)(2(r+\gamma)s\sigma^2)}{\sigma^2}} e^{-(1/2)\left(\frac{(-b+as)}{\sigma\sqrt{s}}\right)^2} ds.$$

Completing the square yet again yields

$$-e^{\frac{-2ab}{\sigma^2}} \int_0^\infty \frac{-b}{\sigma\sqrt{2\pi s^3}} e^{-(1/2)\left(\frac{b-\lambda\sigma^2 s}{\sigma\sqrt{s}}\right)^2} e^{-(1/2)b(\frac{-2a}{\sigma^2}+2\lambda)} ds \quad \text{or}$$

$$e^{b\left(\frac{a}{\sigma^2} - \frac{2a}{\sigma^2} - \lambda\right)} \int_0^\infty \frac{b}{\sigma\sqrt{2\pi s^3}} e^{-(1/2)\left(\frac{b-\lambda\sigma^2 s}{\sigma\sqrt{s}}\right)^2} ds.$$

This expression can be written as

$$\left(\frac{V}{V_B}\right)^{-\left(\frac{a}{\sigma^2}+\lambda\right)} \int_0^\infty \frac{b}{\sigma\sqrt{2\pi s^3}} e^{-(1/2)\left(\frac{b-\lambda\sigma^2 s}{\sigma\sqrt{s}}\right)^2} ds.$$

Furthermore, if $\left(\frac{a}{\sigma^2}+\lambda\right)$ is written as $(a+z)$ and if $\frac{b-\lambda\sigma^2 s}{\sigma\sqrt{s}}$ is written as $\frac{b-z\sigma^2 s}{\sigma\sqrt{s}}$ then we have

$$\left(\frac{V}{V_B}\right)^{-a-z}\int_0^{\infty}\frac{b}{\sigma\sqrt{2\pi s^3}}e^{\frac{-1}{2}\left(\frac{b-z\sigma^2 s}{\sigma\sqrt{s}}\right)^2}ds$$

which can be written in terms of the cdf of the standardized normal variate as

$$\left(\frac{V}{V_B}\right)^{-a-z}\left[1-\int_{-\infty}^{\frac{b-z\sigma^2 s}{\sigma\sqrt{s}}}f(z)dz\right]$$

or as

$$\left(\frac{V}{V_B}\right)^{-a-z}\int_{-\infty}^{\frac{-b+z\sigma^2 s}{\sigma\sqrt{s}}}f(z)dz$$

due to symmetry $1-N(A)=\ N(-A)$. For a fixed income security with a time to maturity of t this expression can be written as

$$\left(\frac{V}{V_B}\right)^{-a-z}N\left(\frac{-b+z\sigma^2 t}{\sigma\sqrt{t}}\right).$$

The expression above corresponds to the second half of the RHS of our expression for $G(t)$ in our theory section.

Gathering these results, we have

$$\int_0^{\infty}e^{-(r+\gamma)s}f(s,v_0,b)\,ds=\left(\frac{V}{V_B}\right)^{-a+z}\left[N\left(\frac{-b-z\sigma^2 t}{\sigma\sqrt{t}}\right)\right]+\left(\frac{V}{V_B}\right)^{-a-z}\left[N\left(\frac{-b+z\sigma^2 t}{\sigma\sqrt{t}}\right)\right]$$

which is identical to what we provide in the theory section.

Appendix G: Optimal Bankruptcy Threshold V_B^*

By assuming that debt, tax benefits and bankruptcy costs are all corporate securities that depend on only the underlying firm value, closed form solutions are available for these features of optimal value of the firm. In particular, when securities have no explicit time dependence $F_t(V) = 0$, the value of a claim on a firm that continuously pays C ($C \geq 0$) satisfies the following ordinary differential equation

$$\frac{1}{2}\sigma^2 V^2 F_{VV}(V) + (r+\gamma)VF_V(V) - (r+\gamma)F(V) + C = 0.$$

So that,

$$F(V) = A_0 + A_1 V + A_2 V^{-x}$$

where $x = \frac{2(r+\gamma)}{\sigma^2}$.

Any time independent claim must have this functional form with the constants A_0, A_1, and A_2 being determined by the boundary conditions of the security at hand.

Given the argument above, debt, is given by $D(V) = A_0 + A_1 V + A_2 V^{-x}$ where characterizations of A_0, A_1, and A_2 are provided by the following eventualities. We assume that $D(V)$ promises a perpetual coupon payment of C unless the firm declares bankruptcy. Consequently, if $V \to \infty$ then $D(V)$ would be given by $D(V) = \frac{C}{(r+\gamma)}$ so that A_0 must be assigned the value of $\frac{C}{(r+\gamma)}$ and A_1 is clearly zero. The third boundary condition, A_2 is provided by the circumstance in which $V = V_B$. In this case, $D(V)$ collapses to $(1-\alpha)V_B$ (α is the proportional cost of bankruptcy) so that

$$D(V) = \frac{C}{(r+\gamma)} + A_2 V_B^{-x} = (1-\alpha)V_B \quad \text{and}$$

$$A_2 V_B^{-x} = \left((1-\alpha)V_B - \frac{C}{(r+\gamma)}\right) \quad \text{so that}$$

$$A_2 = \left((1-\alpha)V_B^{1+x} - \frac{C}{(r+\gamma)}V_B^x\right).$$

Substituting the expression above into $D(V) = \frac{C}{r+\gamma} + A_2 V^{-x}$ yields

$$D(V) = \frac{C}{(r+\gamma)} + \left((1-\alpha)V_B^{1+x} - \frac{C}{(r+\gamma)}V_B^x\right)V^{-x}.$$

Similar treatments of the tax benefit $TB(V)$ and the cost of bankruptcy $BC(V)$ will yield

$$TB(V) = \frac{\tau C}{(r+\gamma)} - \frac{\tau C}{(r+\gamma)}\left(\frac{V}{V_B}\right)^{-x}$$

and

$$BC(V) = \alpha V_B \left(\frac{V}{V_B}\right)^{-x}$$

where τ is the proportion of coupon payments that are tax deductible.

At this point, the total value of the firm can be characterized by

$$tv(V) = V + TB(V) - BC(V)$$

while the value of equity is given by

$$E(V) = tv(V) - D(V).$$

If the partial derivative of $E(V)$ is taken with respect to V and then set equal to zero, we have

$$V_B^* = \frac{(1-\tau)C}{(r+\gamma)+\left(\frac{1}{2}\right)\sigma^2}$$

when we evaluate the derivative at $V = V_B$. This result implies that any level of asset value which triggers bankruptcy will mean that the value of equity is zero.

In the analysis above, we used consols to insure time independence of the firm's debt service requirements. In LT's 1996 publication a stationary debt structure was employed to insure time homogenous debt cash flows and maintain the viability of the assumption that $F_t(V) = 0$. These circumstances insure that V_B^* is an endogenous constant which determines the optimal level of asset value at which to declare bankruptcy. Clearly, this debt structure recognizes that bonds can have a finite time to maturity.

In their 1996 JF paper, Leland/Toft consider an environment where at each moment in time, the firm has debt with a constant total principal P, paying a constant total coupon rate C. That is, the firm continuously retires outstanding debt and replaces it with new debt of equal coupon and principal. As long as the firm remains solvent, at any time "s" the total outstanding debt principal will be P and have a uniform distribution of maturities over the interval $(s, s+T)$. Bonds with a principal p pay a constant coupon rate c per year implying total coupon paid on all outstanding bonds of C per year. Total debt service payments are time independent and equal to $\left(C + \frac{P}{T}\right)$. In this paper, we employ LT's assumption of the firm having a stationary debt structure, but we acknowledge the role of a liquidity premium in the discount rate.

Given that the value of an individual bond $d(V; V_B; t)$ is provided in the text of this paper, the value of all outstanding bonds $D(V; V_B, T)$ would be:

$$\begin{aligned} D(V; V_B, T) &= \int_{t=0}^{T} d(V; V_B, t)dt \\ &= \frac{C}{(r+\gamma)} + \left(P - \frac{C}{(r+\gamma)}\right)\left(\frac{1-e^{-(r+\gamma)T}}{(r+\gamma)T} - I(T)\right) \\ &+ \left((1-\alpha)V_B - \frac{C}{(r+\gamma)}\right)J(T) \end{aligned}$$

where

$$I(T) = \frac{1}{T}\int_0^T e^{-(r+\gamma)t}F(t)dt = \frac{1}{(r+\gamma)T}\left(G(T) - e^{-(r+\gamma)T}F(T)\right)$$

$$J(T) = \frac{1}{T}\int_0^T G(t)dt = \frac{1}{z\sigma\sqrt{T}}\left(-\left(\frac{V}{V_B}\right)^{-a+z} N[q_1(T)]q_1(T) + \left(\frac{V}{V_B}\right)^{-a-z} N[q_2(T)]q_2(T)\right).$$

Using $tv(V; V_B)$ provided earlier in this appendix, the value of equity in this new context would be

$$E(V;V_B,T) = tv(V;V_B) - D(V;V_B,T).$$

Taking the derivative of $E(V;V_B,T)$ with respect to V and evaluating the first order condition at $V = V_B$ yields

$$V_B^* = \frac{\left(\frac{C}{(r+\gamma)}\right)\left(\frac{A}{(r+\gamma)T} - B\right) - \frac{AP}{(r+\gamma)T} - \frac{\tau C x}{(r+\gamma)}}{1 + \alpha x - (1-\alpha)B}$$

where

$$A = 2ae^{-(r+\gamma)T}N\left(a\sigma\sqrt{T}\right) - 2zN\left(z\sigma\sqrt{T}\right) - \frac{2}{\sigma\sqrt{T}}n\left(z\sigma\sqrt{T}\right) + \frac{2e^{(r+\gamma)T}}{\sigma\sqrt{T}}n\left(a\sigma\sqrt{T}\right) + (z-a)$$

and where

$$B = -\left(2z + \frac{2}{z\sigma^2 T}\right)N\left(z\sigma\sqrt{T}\right) - \frac{2}{\sigma\sqrt{T}}n\left(z\sigma\sqrt{T}\right) + (z-a) + \frac{1}{z\sigma^2 T}.$$

$N(\cdot)$ denotes the cumulative distribution of the standard normal while $n(\cdot)$ is the density function of the standard normal.

Appendix H: Calibration of LT Liquidity Premium Enhanced Model

Period	**"Firm Quality"**	r	**Leverage**	δ	σ	α	**Liq. Ratio**
Calm	Higher	2.5%	30%	4%	20%	0.4	5%
	Lower		40%	5%	30%	0.5	10%
Stress	Higher	7.5%	50%	6%	40%	0.6	10%
	Lower		60%	7%	50%	0.7	20%

Our enhanced version of the LT model is able to generate term structures of liquidity spreads that characterize periods of heightened financial uncertainty and periods of relative economic tranquility, and also for firms with varying probability distributions of the time-to-default. To illustrate the possibilities, we select parameters to be used in our enhanced LT model to represent "high" and "low" quality firms in a "calm" and in a "stress" economic periods.

In the construct of and Leland and Toft (1996), the value of the debt with maturity t, $d(t)$, the value of total debt D, the value of equity E, and the value of the firm v, are a function of 9 parameters that characterize the dynamics of the firm. These parameters are: 1) the risk-free rate r, 2) the firm payout ratio δ, 3) the volatility of the value of the unlevered firm σ, 4) the initial value of the unlevered firm V_0, 5) the maturity of the newly issued debt T, 6) the total coupons paid to all outstanding debt C, 7) the total principal value of outstanding debt P, 8) the fraction of asset lost in bankruptcy α, and 9) the tax rate τ. We enhance LT's model by additionally including the liquidity premium γ_t. We next describe our parameter calibrations.

For both types of firms in both types of economic scenarios, we assume that the initial unlevered value of the assets V_0 takes the value 100 and we assume that the corporate tax rate equals 35%. The parameters r, σ, δ, and α are calibrated to historical averages so as to represent calmed and stressed economic periods, as well as high and low quality firms. For example, we set $r = 2.5\%$ for high quality and low quality firm types in a period of financial tranquility, but we increase the interest rate to $r = 7.5\%$ for both types of firms during a time of heightened economic anxiety.

During the calmed period, we set the volatility of assets to $\sigma = 20\%$ for the high quality firm, but we modestly increase σ to 30% for the low quality firm in the same period. We increase the volatility of assets for the lower quality firm because, all else equal, increases in σ lead to an enhanced likelihood of early default. In the period of financial distress, we increase the volatility of assets for types of firms to 40% and 50% because during a period of stress there is a higher uncertainty about the future value of the firm's assets.

In the period of relative calm, we set the payout ratio δ to equal 4% for the higher graded firm and 5% for the lower graded firm. Again, we choose a slightly greater value of δ for the lower quality form because, all else equal, a larger δ implies a greater likelihood of earlier default. We increase the value of δ for both firms in the stress period to 6% and 7%.

In both economic epochs, we calibrate the recovery rate $1 - \alpha$ to a greater value for the high grade firm than for the low grade firm. This captures the notion that the higher quality firm might own assets with a greater salvage value than those of the lower quality firm. In the non-stress period, we set the recovery rates to 0.6 for the high quality firm and to 0.5 for the low quality firm. Yet, as we transition to the stress period, we slightly decrease the recovery rates to 0.4 and 0.3.

Firms in the revised LT model want to maintain a specific leverage ratio L. To achieve this desired leverage, the firms select a value of the total principal P so that the fraction of the resulting total debt D to the total value of the firm tv exactly equals the value L. Thus, the firms strategically choose the value of the principal P to meet their required capital structure. Also, to find the total coupon payments C, the enhanced LT model assumes that firm selects the coupon c so that the value of the newly issued debt $d(T)$ equals p. This of course implies that debt is issued at par value. These conditions from LT imply that the values of P and C are determined jointly as the solution to a dual minimization problem so that $L = \frac{D(P,C)}{tv(P,C)}$ and $\lim_{t \downarrow T} d(P, C, t) = p$. In our calibrations we select the leverage ratios 20% and 30% in the non-stress period, and 40% and 50% in the stress period because highly levered firms are assumed to be riskier.

The plots depicted in Figure 1 are the outcome of solving for the value of γ for different values of t, the time to maturity, and plotting the relationship between the two while maintaining the existing equilibrium value of debt.

Appendix I: the conventional Kalman filter algorithm

For expositional convenience, we provide a brief summary of the Kalman filter (KF) algorithm. Let the n state discrete-time linear system be given by the following equations:

$$x_{s+1} = F\, x_s + w_s,$$

$$y_s = M x_s + v_s,$$

$$w_s \sim (0, Q_s),$$

$$\text{and } v_s \sim (0, R_s).$$

Above, y_s equals ϕ the observed yield spread at time s, x_s equals $[\gamma_s \quad \xi_s]'$ the state vector at time s, w_s is a 2 x 1 vector of Gaussian noise and known covariance matrix Q_s, and v_s is a 1 x 1 measurement error with a mean of zero and known covariance matrix and R_s. The matrix F is the 2 x 2 system matrix and M is the 1x2 measurement system. Denote by $\hat{x}_s^-$ the a priori estimate of the state variable x_s and by $\hat{x}_s^+$ the a posteriori estimate of x_s. The state vector at time 0 is a random vector with known mean $E[x_0]$ and known covariance matrix P_0. The KF is initialized as follows:

$$\hat{x}_0^+ = E[x_0], \quad \text{and}$$

$$P_0^+ = E[(x_0 - \hat{x}_0^+)(x_0 - \hat{x}_0^+)^T] = P_0.$$

We first utilize the transition equation to propagate from the time step $(s-1)$ to s:

$$\hat{x}_s^- = F\hat{x}_{s-1}^+,$$

and the covariance in the a priori estimate is given by:

$$P_s^- = FP_{s-1}^+F^T + Q_s.$$

In the updating step, where we use the measurement equation to improve the estimation of the state vector $\hat{x}_s^-$. We can calculate the a posteriori update $\hat{x}_s^+$ as well as the a posteriori covariance P_s^+ by using the usual Kalman filter equations, where K_s denotes the Kalman gain,

$$\hat{x}_s^+ = \hat{x}_s^- + K_s[y_k - M\hat{x}_s^-]$$

$$K_s = P_s^- M^T (M P_s^- M^T + R_s)^{-1}, \text{ and}$$

$$P_s^+ = (I - K_s M) P_s^-.$$

Appendix J: A brief illustration of the UT algorithm and its advantages

To illustrate the superiority of the unscented transformation algorithm over linearization, we compare the approximations of both approaches to nonlinear functions. Our example is designed to capture the essence of UT in a simple context. For example, assume that the value of debt is given by:

$$\tilde{d} = \frac{c}{r+\gamma}\left(1 - e^{-(r+\gamma)t}\right) + p \cdot e^{-(r+\gamma)t}$$

where again, c is the coupon, $(r+\gamma)$ is the discount factor, and t is the time to maturity. With no loss in generality, assume that the bond pays no coupons, that the principal is 1, the risk-free rate r is constant, and the time to maturity is 1. We will characterize the liquidity premium as being given by $\gamma = \mu_\gamma + \epsilon_\gamma$ where μ_γ is the mean and ϵ_γ is a random variable with mean 0 and standard deviation σ_γ.

Our intent, in particular, is to compare the ability of the UT and linearization to approximate the first two orders of a Taylor expansion of the expected value of debt $\tilde{d}$, itself. That is, the objective is to accurately approximate:

$$E[\tilde{d}] = E[g(\gamma)] = E[e^{-(\gamma+r)}]$$

with the mean of two popular methodologies for the treatment of nonlinear functions. A second order expansion of $E[\tilde{d}]$ is:

$$E[\tilde{d}] = E\left[g(\mu_\gamma) + (\gamma - \mu_\gamma) \cdot \left.\frac{\partial g(\gamma)}{\partial \gamma}\right|_{\gamma=\mu_\gamma} + (\gamma - \mu_\gamma)^2 \cdot \frac{1}{2}\left.\frac{\partial^2 g(\gamma)}{\partial \gamma^2}\right|_{\gamma=\mu_\gamma} \right]$$

$$= \quad E\left[e^{-(\gamma+r)}\big|_{\gamma=\mu_\gamma} \right] - E\left[e^{-(\gamma+r)}\big|_{\gamma=\mu_\gamma} \cdot (\gamma - \mu_\gamma) \right]$$

$$+ \frac{1}{2}\, E\left[e^{-(\gamma+r)}\big|_{\gamma=\mu_\gamma} \cdot (\gamma - \mu_\gamma)^2 \right]$$

$$E\left[\tilde{d}\right] = e^{-(\mu_\gamma + r)} - e^{-(\mu_\gamma + r)} E\left[\epsilon_\gamma\right] + \frac{1}{2} e^{-(\mu_\gamma + r)} E\left[\epsilon_\gamma^2\right]$$

which equals

$$E\left[\tilde{d}\right] = e^{-(\mu_\gamma + r)} \cdot [1 + \tfrac{1}{2}\sigma_\gamma^2] \quad \text{since} \quad E\left[\epsilon_\gamma\right] = 0 \quad \text{and} \quad E\left[\epsilon_\gamma^2\right] = \sigma_\gamma^2.$$

We next show that we can fully recover the expected value of the second order Taylor series expansion of risky debt above using the mean of a UT algorithm to estimate $E\left[\tilde{d}\right]$. First, we prepare our "deterministic" sample by combining two sigma points (σ_γ, $-\sigma_\gamma$) with the sample mean of the discount factor $r + \bar{\gamma}$ and then we transform them with the nonlinear function $g(\cdot)$ so that we have $g(r + \bar{\gamma} + \sigma)$ and $g(r + \bar{\gamma} - \sigma)$. Finally, we take the expected value of a second order expansion of the sample mean of the UT variates.

$$E\left[\bar{d}_{UT}\right] = E\left[e^{-(\gamma + r)}\right]_{\epsilon_\gamma = 0} + \frac{1}{2(\kappa+1)}\frac{\partial g}{\partial \sigma_1} d\sigma_1 + \frac{1}{4(\kappa+1)}\frac{\partial^2 g}{\partial \sigma_1 \partial \sigma_1}(d\sigma_1)^2 + \frac{1}{2(\kappa+1)}\frac{\partial g}{\partial \sigma_2} d\sigma_2$$

$$+ \frac{1}{4(\kappa+1)}\frac{\partial^2 g}{\partial \sigma_2 \partial \sigma_2}(d\sigma_2)^2$$

$$= e^{-(r + \mu_\gamma)} + \frac{1}{2(\kappa+1)}\left(-e^{-(r+\mu_\gamma)}\right)d\sigma_1 + \frac{1}{4(\kappa+1)}\left(e^{-(r+\mu_\gamma)}\right)d\sigma_1^2$$

$$+ \frac{1}{2(\kappa+1)}\left(e^{-(r+\mu_\gamma)}\right)d\sigma_2 + \frac{1}{4(\kappa+1)}\left(e^{-(r+\mu_\gamma)}\right)d\sigma_2^2$$

which simplifies to:

$$E\left[\bar{d}_{UT}\right] = e^{-(\gamma + r)} \cdot [1 + \tfrac{1}{2}\sigma_\gamma^2], \quad \text{since} \quad \sigma_i = \left[(1+\kappa)\sigma_\gamma^2\right]^{1/2}.$$

Thus, with an unscented transformation, by estimating the mean using a deterministic sample of points around the mean of $(r + \bar{\gamma})$ we can get an estimate which, on average, identically captures the first two orders of a Taylor expansion of the mean of the nonlinear function $E\left[\tilde{d}\right]$ itself. In contrast, if an estimator of the value of debt relies on only the linearization of the expected value of $\tilde{d}$, a single observation on $\bar{\gamma}$ would yield

$$E\left[\bar{d}_{LIN}\right] = E\left[e^{-(r+\bar{\gamma})}\Big|_{\bar{\gamma}=\mu} + \frac{\partial\left[e^{-(r+\bar{\gamma})}\right]}{\partial\bar{\gamma}}\Bigg|_{\bar{\gamma}=\mu} d\bar{\gamma}\right]$$

where $d\bar{\gamma} = \bar{\gamma} - \mu$.

The mean of this estimator would be

$$E\left[\bar{d}_{LIN}\right] = e^{-(r+\mu)} - e^{-(r+\mu)}E[d\bar{\gamma}]$$

or simply,

$$E\left[\bar{d}_{LIN}\right] = e^{-(r+\mu)}.$$

Clearly, the mean of $\bar{d}_{LIN}$ fails to duplicate $E\left[\tilde{d}\right]$ and falls short of the ability of the $E\left[\bar{d}_{UT}\right]$ to characterize $E\left[\tilde{d}\right]$. However, the economic significance of $\bar{d}_{LIN}$'s bias will be considered in the next 4 pages

Generalizing our simple debt valuation equation to include coupons and alternative times to maturity would result in $\tilde{d} = \frac{c}{r+\gamma} + \left(p - \frac{c}{r+\gamma}\right) e^{-(r+\gamma)t}$. The expression above would allow us to conduct a comparative static analysis, alternatively varying p, σ_γ^2, r, t, μ, and c, to illustrate the economic importance of the UT approach to the treatment of nonlinearities.

We first calculate the expected value of a second order expansion of $\tilde{d}$, then we consider the results provided by $E\left[\bar{d}_{UT}\right]$ and $E\left[\bar{d}_{LIN}\right]$ for a set of baseline parameters given as follows: the maturity t equals 20 years, the principal p is \$500 million, the coupon payment c is \$20 million, the short-term interest rate r is 5%, the mean of the liquidity premium is $\mu = 0.03$, and the standard deviation of the liquidity premium is $\sigma = 0.02$. To study how changes in these variables individually worsen the estimate provided by the linearization method when compared to the UT, we then increase and decrease each variable independently of the rest and recompute the values of $E\left[\tilde{d}\right]$, $E\left[\bar{d}_{UT}\right]$, and $E\left[\bar{d}_{LIN}\right]$. We document the results from this analysis when we vary just one of the five variables in Panels A, B, C, D, and E for a time to maturity equaling 20 years. In Panel A we let p take the values \$400 million, \$500 million, and \$600 million while the remaining variables stay at their baseline levels. In Panel B, we evaluate r at 5%, 4%, and 6%. In Panel C, we equate μ to either 3%, 4%, and 5%. In Panel D, we set σ to 2%, 1%, and 3%. Lastly, in Panel E, we select c from among the values \$17.5 million, \$20 million, and \$22.5 million.

In Panel A, we calculate the deviation between the estimates for $E\left[\tilde{d}\right]$ provided by $E\left[\bar{d}_{UT}\right]$, and $E\left[\bar{d}_{LIN}\right]$ if we vary the principal amount. When p equals \$400 million, the UT computes a value of debt of \$290.13 million. This result agrees with the expected value of $\tilde{d}$. However, the linearization method predicts a value of debt of \$280.28 million, which is 3.40% lower than the correct estimate. If the principal is increased to \$600 million, the UT correctly calculates a value of debt of \$333.74 million, while the linearization method imputes a value of \$320.66 million. This implies that the prediction error of the linearization method increases to 3.92%.

In Panel B, we compare the estimates provided by the UT and the linearization, when we vary the interest rate. When r equals 5%, the UT correctly calculates a value of debt of \$311.94 million, while the linearization method assigns a value of debt of \$300.47 million. This is is a mispricing of about 3.68%. Yet, when increase r to 6%, the error slightly decreases to about 3.45%.

In the equations for $E\left[\tilde{d}\right]$, $E\left[\bar{d}_{UT}\right]$, and $E\left[\bar{d}_{LIN}\right]$ above, clearly both r and μ have identical implications for the value of debt. Thus, the linearization errors obtained when we vary μ in Panel C exactly mirror those displayed in Panel B.

Changes in the standard deviation of the liquidity premium also induce large differences in pricing between the UT and the linearization method. To illustrate this statement, in Panel D, we display results when $\sigma = 0.02$, $\sigma = 0.01$, and when $\sigma = 0.03$. These results suggest that $E\left[\bar{d}_{LIN}\right]$ deviates from the true expected value by a factor between 3.68% and 7.9%. Lastly, the results in Panel E also seem to indicate that when we vary the coupon payment from its baseline level, the UT offers an accurate estimation of the value of debt, while the linearization method deviates from the correct answer by a margin of error no less than 3.53%.

Consistent with our goal of providing reliable estimates for a term structure, we report the results provided by $E\left[\bar{d}_{UT}\right]$, and $E\left[\bar{d}_{LIN}\right]$ when we evaluate t at 10 years, 15 years, and 20 years to maturity. When t = 20 years and all other parameters are fixed at their baseline level, the UT computes a value of debt of \$311.94 million. However, the linearization method predicts a value of debt of \$300.47 million, which is 3.40% lower than the correct estimate. For a bond with 15 years to maturity, the value of debt calculated by the UT is \$261.21 million while the value of debt calculated by the linearization method is \$250.59 million. This is a difference of about 4.1%. Finally, for a bond with 10 years until expiry, the bias of the linearization is about 4.6%.

This scenario illustrates the advantage of the UT over the linearization in a simple bond pricing example with three sources of nonlinearity on the RHS of our expression for $\tilde{d}$. Yet, even in this simple example the biases from linearization can range between 3.40% and to as much as 7.8%. Furthermore, in the debt valuation equation that we estimate in the paper, there are 4 explicit nonlinear entries for the liquidity premium, another 6 nonlinear entries of γ_t are included in $F(t)$, 10 nonlinear appearances of γ_t in $G(t)$, and 4 nonlinear entries of the liquidity premium in V_B. $F(t)$, $G(t)$, and V_B are all terms included in the determination of $d(V, V_B, t)$. Clearly, the accumulated econometric advantage of a UT approach to nonlinearities in equation 4 would be monumental. The superiority of UT it is the basis for our reliance on the unscented Kalman filter at the exclusion of the extended Kalman filter as being the vehicle for our estimation for our enhanced debt valuation model.

Panel A: Varying the Principal *P*

	$E(\tilde{d})$	$E(\bar{d}_{UT})$	$E(\bar{d}_{LIN})$
$P = \$400M$ $r = 0.05$ $\mu = 0.03$ $\sigma = 0.02$ $c = \$20M$	290.13	290.13	280.28
$P = \$500M$ $r = 0.05$ $\mu = 0.03$ $\sigma = 0.02$ $c = \$20M$	311.94	311.94	300.47
$P = \$600M$ $r = 0.05$ $\mu = 0.03$ $\sigma = 0.02$ $c = \$20M$	333.74	333.74	320.66

Panel B: Varying the Interest Rate $\boldsymbol{R}$

	$E(\tilde{d})$	$E(\bar{d}_{UT})$	$E(\bar{d}_{LIN})$
$P = \$500M$ $r = 0.05$ $\mu = 0.03$ $\sigma = 0.02$ $c = \$20M$	311.94	311.94	300.47
$P = \$500M$ $r = 0.04$ $\mu = 0.03$ $\sigma = 0.02$ $c = \$20M$	352.30	352.30	338.56
$P = \$500M$ $r = 0.06$ $\mu = 0.03$ $\sigma = 0.02$ $c = \$20M$	277.71	277.71	268.14

Panel C: Varying the Mean of the Liquidity Premium $\boldsymbol{\mu}$

	$E(\tilde{d})$	$E(\bar{d}_{UT})$	$E(\bar{d}_{LIN})$
$P = \$500M$ $r = 0.05$ $\mu = 0.03$ $\sigma = 0.02$ $c = \$20M$	311.94	311.94	300.47
$P = \$500M$ $r = 0.05$ $\mu = 0.02$ $\sigma = 0.02$ $c = \$20M$	352.30	352.30	338.56
$P = \$500M$ $r = 0.05$ $\mu = 0.04$ $\sigma = 0.02$ $c = \$20M$	277.71	277.71	268.14

Panel D: Varying the Standard Deviation of the Liquidity Premium σ

	$E(\widetilde{d})$	$E(\bar{d}_{UT})$	$E(\bar{d}_{LIN})$
$P = \$500M$ $r = 0.05$ $\mu = 0.03$ $\sigma = 0.02$ $c = \$20M$	311.94	311.94	300.47
$P = \$500M$ $r = 0.05$ $\mu = 0.03$ $\sigma = 0.01$ $c = \$20M$	352.30	352.30	338.56
$P = \$500M$ $R = 0.05$ $\mu = 0.03$ $\sigma = 0.03$ $c = \$20M$	326.26	326.26	300.47

Panel E: Varying the Coupon Payment c

	$E(\widetilde{d})$	$E(\bar{d}_{UT})$	$E(\bar{d}_{LIN})$
$P = \$500M$ $r = 0.05$ $\mu = 0.03$ $\sigma = 0.02$ $c = \$17.5M$	352.30	352.30	338.56
$P = \$500M$ $r = 0.05$ $\mu = 0.03$ $\sigma = 0.02$ $c = \$20M$	311.94	311.94	300.47
$P = \$500M$ $r = 0.05$ $\mu = 0.03$ $\sigma = 0.02$ $c = \$22.5M$	337.30	337.30	325.41

Appendix K: the unscented Kalman filter algorithm

We provide a summary of the unscented Kalman filter (UKF) algorithm, which is mostly based on Simon (2006). Let the n state discrete-time nonlinear system be given by the following equations:

$$x_{s+1} = f(x_s) + w_s,$$

$$y_s = g(x_s) + v_s,$$

$$w_s \sim (0, Q_s),$$

$$\text{and } v_s \sim (0, R_s).$$

Above, y_s is the observed signal at time s, x_s is the state vector at time s, and w_s and v_s are white noise processes with zero mean, and have known covariance matrices Q_s and R_s. The system equation $f(\cdot)$ and the measurement equation $g(\cdot)$ are nonlinear functions. Denote by $\hat{x}_s^-$ the a priori estimate of the state variable x_s and by $\hat{x}_s^+$ the a posteriori estimate of x_s. The state vector at time 0 is a random vector with known mean $E[x_0]$ and known covariance matrix P_0. The UKF is initialized as follows:

$$\hat{x}_0^+ = E[x_0], \quad \text{and}$$

$$P_0^+ = E[(x_0 - \hat{x}_0^+)(x_0 - \hat{x}_0^+)^T] = P_0.$$

To propagate from the time step $(s-1)$ to s, we utilize the unscented transformation algorithm (UT) where we first choose the $2n$ sigma points $\hat{x}_{s-1}^{(i)}$ given by

$$\hat{x}_{s-1}^{(i)} = \hat{x}_{s-1}^- + \tilde{x}^{(i)} \text{ for } i = 1, \dots, 2n \quad \text{with}$$

$$\tilde{x}^{(i)} = \left(\sqrt{nP_{s-1}^+}\right)_i^T \text{ for } i = 1, \dots, n, \quad \text{and}$$

$$\tilde{x}^{(n+i)} = -\left(\sqrt{nP_{s-1}^+}\right)_i^T \text{ for } i = 1, \dots, n.$$

In the equations above, $\sqrt{nP_{s-1}^{+}}$ is the matrix square root of nP_{s-1}^{+} such that $\left(\sqrt{nP_{s-1}^{+}}\right)^{T}\left(\sqrt{nP_{s-1}^{+}}\right) = \sqrt{nP_{s-1}^{+}}$ and $\left(\sqrt{nP_{s-1}^{+}}\right)_i$ is the ith row of $\sqrt{nP_{s-1}^{+}}$. Next, we use the nonlinear system equation $f(\cdot)$ to predict the vectors $\hat{x}_s^{(i)}$ from the sigma points $\hat{x}_{s-1}^{(i)}$ as follows:

$$\hat{x}_s^{(i)} = f\left(\hat{x}_{s-1}^{(i)}, u_{s-1}\right).$$

Then, we combine the $\hat{x}_s^{(i)}$ vectors to obtain the time s a priori state estimation, $\hat{x}_s^{-}$:

$$\hat{x}_s^{-} = \frac{1}{2n}\sum_{i=1}^{2n} \hat{x}_s^{(i)}.$$

We also estimate the a priori error covariance of $\hat{x}_s^{-}$, P_s^{-}, in the following fashion:

$$P_s^{-} = \frac{1}{2n}\sum_{i=1}^{2n}\left(\hat{x}_s^{(i)} - \hat{x}_s^{-}\right)\left(\hat{x}_s^{(i)} - \hat{x}_s^{-}\right)^{T} + Q_{s-1}.$$

We now turn to the updating step, where we use the measurement equation to improve the estimation of the state vector $\hat{x}_s^{-}$. First, as prescribed by the UT, we choose sigma points $\hat{x}_s^{(i)}$ given by

$$\hat{x}_s^{(i)} = \hat{x}_s^{-} + \tilde{x}^{(i)} \text{ for } i = 1, \dots, 2n \quad \text{where}$$

$$\tilde{x}^{(i)} = \left(\sqrt{nP_s^{-}}\right)_i^{T} \text{ for } i = 1, \dots, n, \quad \text{and}$$

$$\tilde{x}^{(n+i)} = -\left(\sqrt{nP_s^{-}}\right)_i^{T} \text{ for } i = 1, \dots, n.$$

We then use the nonlinear measurement equation $g(\cdot)$ to transform the sigma points into $\hat{y}_s^{(i)}$ vectors

$$\hat{y}_s^{(i)} = g\left(\hat{x}_s^{(i)}, \mathrm{t_s}\right)$$

and we combine the $\hat{y}_s^{(i)}$ vectors to obtain the predicted measurement at time s:

$$\hat{y}_s = \frac{1}{2n}\sum_{i=1}^{2n} \hat{y}_s^{(i)}.$$

We next estimate the covariance of the predicted measurement using:

$$P_y = \frac{1}{2n}\sum_{i=1}^{2n}\left(\hat{y}_s^{(i)} - \hat{y}_s\right)\left(\hat{y}_s^{(i)} - \hat{y}_s\right)^T + R_s.$$

We also estimate the cross covariance between $\hat{x}_s^-$ and $\hat{y}_s$ as follows:

$$P_{xy} = \frac{1}{2n}\sum_{i=1}^{2n}\left(\hat{x}_s^{(i)} - \hat{x}_s^-\right)\left(\hat{y}_s^{(i)} - \hat{y}_s\right)^T.$$

Finally, we can calculate the a posteriori update $\hat{x}_s^+$ as well as the a posteriori covariance P_s^+ by using the usual Kalman filter equations, where K_s is the Kalman gain,

$$K_s = P_{xy}P_y^{-1},$$

$$\hat{x}_s^+ = \hat{x}_s^- + K_s(y_s - \hat{y}_s), \text{ and}$$

$$P_s^+ = P_s^- - K_s P_y K_s^T.$$

Appendix L: An explanation of $F(t)$ and $G(t)$ as they appear in the enhanced CDS equation

Following Leland/Toft model (LT) closely, we assume firm has productive assets whose unleveraged value V follows a continuous diffusion process with a constant proportional volatility σ:

$$\frac{dV}{V} = (\mu(V,s) - \delta)ds + \sigma dz$$

where $\mu(V,s)$ is the total expected rate of return on asset value V, δ is the constant fraction of value paid out to security holders, and dz is the increment of a standard brownian motion. The process continues without limit until V falls to V_B, the optimal value of the firm's assets at which to declare bankruptcy. We characterize the default triggering barrier "b" as $\ln\left(\frac{V}{V_B}\right)$ when $V = V_B$.

We have, $v = \ln\left(\frac{V}{V_B}\right)$, for $V \neq V_B$, where dv would be given by [55]

$$dv = \frac{\partial v}{\partial V} dV + \frac{1}{2}\frac{\partial^2 v}{\partial V \partial V}(dV)^2.$$

Substituting $dV = (\mu(V,s) - \delta)Vds + \sigma V dz$ into the expression for dv yields

$$dv = \frac{1}{V}\big((\mu - \delta)Vds + \sigma V dz\big) + \frac{1}{2}\left(-\frac{1}{V^2}\right)\big((\mu - \delta)Vds + \sigma V dz\big)^2$$

$$dv = (\mu - \delta)ds + \sigma dz - (1/2)\sigma^2 ds$$

$$dv = \left(\mu - \delta - \frac{1}{2}\sigma^2\right)ds + \sigma dz$$

Employing Girsanov's theorem will allow us to put this process in a risk neutral environment, so that

$$dv = \left(\mu - \delta - \frac{1}{2}\sigma^2\right)ds + \sigma(dz^* - \phi ds)$$

---

[55] The cautious reader will recognize that $\frac{d\ \ln\left(\frac{V}{V_B}\right)}{dV} = \frac{\left({}^{1}/_{V_B}\right)}{\left({}^{V}/_{V_B}\right)} = \frac{1}{V}$ which is surprising because $\frac{d\ln(V)}{dV}$ also equals $\frac{1}{V}$.

where ϕ is the price of risk, and therefore

$$dv = \left(\mu - \delta - \frac{1}{2}\sigma^2\right) ds + \sigma(dz^* - \left(\frac{\mu - (r+\gamma)}{\sigma}\right) ds)$$

or,

$$dv = \left((r+\gamma) - \delta - \frac{1}{2}\sigma^2\right) ds + \sigma dz^*$$

or,

$dv = a\ ds + \sigma\ dz^*$ with $a = \left((r+\gamma) - \delta - \frac{1}{2}\sigma^2\right)$.

This risk neutral brownian process can be written as:

$$dv_u = a\ du + \sigma\ dz_u^*.$$

Integrating,

$$\int_0^s dv_u = a \int_0^s du + \sigma \int_0^s dz_u^*$$

yields,

$$v_s - v_0 = a(s - 0) + \sigma \int_0^s dz_u^*$$

or,

$$E[v_s] = v_0 + a \cdot s$$

and,

$$Var(v_s) = \sigma^2 Var\left(\int_0^s dz_u^*\right)$$

or, finally

$$Var(v_s) = \sigma^2 E\left(\int_0^s dz_u^*\right)^2.$$

Furthermore, Ito's isometry allows us to write,

$E\left(\left(\int_0^s dz_u^*\right)\left(\int_0^s dz_u^*\right)\right)$ as $\int_0^s du$.

Gathering terms,

$$Var(v_s) = \sigma^2 \int_0^s du$$

or

$$Var(v_s) = \sigma^2 s$$

so that v_s is normally distributed with a mean of $v_0 + as$ and a variance of $\sigma^2 s$. With these characteristics, the pdf for v must be

$$p(v_0, v; s) = \frac{1}{\sigma\sqrt{2\pi s}} e^{\frac{-(v-v_0-as)^2}{2\sigma^2 s}}$$

The variate v_s is characterized by $v(s_b)$ when v_s equals b. The likelihood $p(v_0, v; s)$ is the probability that $v_s = v$ and that the process has not reached the barrier in the interval $(0, s)$. Consequently, the cumulative probability $P\big(v(s) < b$ for $s < (s_b|v(0) = v_0)\big)$ equals $\int_{-\infty}^{b} p(v_0, v; s)dv = P(v_0, b; s)$. $P(v_0, b; s)$ equals the probability that "b" has not been reached by s which could be written as $P(v_0, b; s) = \text{prob}(s_b \geq s)$ or as $P(v_0, b; s) = 1 - F(s; v_0, b)$, where $F(s; v_0, b)$ is the cumulative probability of the time to the barrier. Furthermore,

$$\frac{dP}{ds}(v_0, b; s) = -F'(s; v_0, b)$$

or

$$-\frac{dP}{ds}(v_0, b; s) = f(s; v_0, b)$$

where $f(s; v_0, b)$ is the marginal probability of the time to bankruptcy, which, upon differentiation by s, could be written as

$$f(s; v_0, b) = \frac{b-v_0}{\sigma\sqrt{2\pi s^3}} e^{\frac{-(b-v_0-as)^2}{2\sigma^2 s}}.$$

This probability density function integrates to one and the time to bankruptcy has, intuitively, a mean of $\left(\frac{b-v_0}{a}\right)$.

In our model, b is not just a threshold, but an absorbing barrier. Consequently $p(v_0, v; s)$ has to be rewritten as a linear combination $p(v_0, v; s) = p_1(v_0, v; s) + Ap_1(v_0, v; s)$ to ensure that $p(v_0, b; s)$, the probability that $v = b$, equals zero for every possible s_b. The method of images yields

$$p(0, v; s) = \frac{1}{\sigma\sqrt{2\pi s}}\left[e^{\frac{-(v-as)^2}{2\sigma^2 s}} - e^{\left(\frac{2ab}{\sigma^2} + \frac{-(v-2b-as)^2}{2\sigma^2 s}\right)}\right]$$

where $A = -e^{\frac{2ab}{\sigma^2}}$.

This solution can be easily verified by setting v equal to b where, consequently, the expression for $p(0, v; s)$ quickly reduces to 0.

Using the expression for $p(0, v; s)$ after the adjustment for the boundary condition yields the following expression for the cumulative of v:

$$P(0, b; s) = \int_{-\infty}^{b} \frac{1}{\sigma\sqrt{2\pi s}}\left[e^{\frac{-(v-as)^2}{2\sigma^2 s}} - e^{\left(\frac{2ab}{\sigma^2}+\frac{-(v-2b-as)^2}{2\sigma^2 s}\right)}\right] dv$$

If $P(v_0, b; s)$ is standardized, we have

$$P(0, b; s) = N\left(\frac{b - as}{\sigma\sqrt{s}}\right) - e^{\frac{2ab}{\sigma^2}} N\left(\frac{-b - as}{\sigma\sqrt{s}}\right)$$

where $N(\cdot)$ is the cumulative distribution for the standardized normal variate. If the expression is rewritten so that the absorbing barrier is at zero, the cumulative normal becomes

$$P(v_0, b; s) = N\left(\frac{b+as}{\sigma\sqrt{s}}\right) - e^{\frac{-2ab}{\sigma^2}} N\left(\frac{-b+as}{\sigma\sqrt{s}}\right) \quad \text{for some “}s\text{”.}$$

The term $e^{\frac{-2ab}{\sigma^2}} = e^{\frac{-2a}{\sigma^2}\cdot\ln\left(\frac{V}{V_B}\right)}$ by definition (i) and $e^{\frac{-2ab}{\sigma^2}} = \left(\frac{V}{V_B}\right)^{\frac{-2a}{\sigma^2}}$ (ii), because, taking the log of both the LHS and RHS terms of (i) yields $-\frac{2a}{\sigma^2}\cdot\ln\left(\frac{V}{V_B}\right) = -\frac{2a}{\sigma^2}\cdot\ln\left(\frac{V}{V_B}\right)$. Consequently, $P(v_0, b; s)$ becomes

$$P(v_0, b; s) = N\left(\frac{b+as}{\sigma\sqrt{s}}\right) - \left(\frac{V}{V_B}\right)^{\frac{-2a}{\sigma^2}} N\left(\frac{-b+as}{\sigma\sqrt{s}}\right).$$

Of course, for the standardized normal variate z, $N(A) = 1 - N(-A)$ due to symmetry, consequently, $P(v_0, b; s)$ becomes

$$P(v_0, b; s) = 1 - N\left(\frac{-b-as}{\sigma\sqrt{s}}\right) - \left(\frac{V}{V_B}\right)^{\frac{-2a}{\sigma^2}} N\left(\frac{-b+as}{\sigma\sqrt{s}}\right) \text{ or,}$$

$$P(v_0, b; s) = 1 - \left[N\left(\frac{-b-as}{\sigma\sqrt{s}}\right) + \left(\frac{V}{V_B}\right)^{\frac{-2a}{\sigma^2}} N\left(\frac{-b+as}{\sigma\sqrt{s}}\right)\right].$$

As detailed above, $P(v_0, b; s)$ can be written as $P(v_0, b; s) = 1 - F(s; v_0, b)$ where $F(s; v_0, b)$ is the cumulative probability of the time to the barrier. So if

$$P(v_0, b; s) = 1 - \left[N\left(\frac{-b - as}{\sigma\sqrt{s}}\right) + \left(\frac{V}{V_B}\right)^{\frac{-2a}{\sigma^2}} N\left(\frac{-b + as}{\sigma\sqrt{s}}\right)\right]$$

then,

$$F(s;v_0,b) = N\left(\frac{-b-as}{\sigma\sqrt{s}}\right) + \left(\frac{V}{V_B}\right)^{\frac{-2a}{\sigma^2}} N\left(\frac{-b+as}{\sigma\sqrt{s}}\right).$$

The expression immediately above is identical to the expression we have on pages 62 and 63.

Recall

$$-\frac{d}{ds}P(v_0,b;s) = f(s;v_0,b)$$

so that the evaluation of the expression $\int_0^\infty e^{-(r+\gamma)s} f(s,v_0,b)\,ds$ will involve the differentiation of

$$P(v_0,b;s) = N\left(\frac{b+as}{\sigma\sqrt{s}}\right) - e^{\frac{-2ab}{\sigma^2}} N\left(\frac{-b+as}{\sigma\sqrt{s}}\right)$$

with respect to s. The partial derivative of the first term on the RHS can be written as $\frac{b}{\sigma\sqrt{2\pi s^3}} e^{\frac{-(b+as)^2}{2\sigma^2 s}}$, when zero is acknowledged as the absorbing barrier. The determination of $\int_0^\infty e^{-(r+\gamma)s} \frac{b}{\sigma\sqrt{2\pi s^3}} e^{-\frac{(b+as)^2}{2\sigma^2 s}} ds$ involves completing the square and if $\lambda = \frac{(a^2+2(r+\gamma)\sigma^2)^{\frac{1}{2}}}{\sigma^2}$ then the expression above eventually becomes

$$\int_0^\infty \frac{b}{\sigma\sqrt{2\pi s^3}} e^{-(1/2)\left(\frac{b+\lambda\sigma^2 s}{\sigma\sqrt{s}}\right)^2 - (1/2)b\left(\frac{2a}{\sigma^2}-2\lambda\right)} ds.$$

This expression can be rewritten as

$$e^{-(1/2)\ln\left(\frac{V}{V_B}\right)\left(\frac{2a}{\sigma^2}-2\lambda\right)} \int_0^\infty \frac{b}{\sigma\sqrt{2\pi s^3}} e^{-(1/2)\left(\frac{b+\lambda\sigma^2 s}{\sigma\sqrt{s}}\right)^2} ds$$

and, since for any A and B, $e^{A\,ln(B)} = B^A$, we have

$$\left(\frac{V}{V_B}\right)^{\lambda-\frac{a}{\sigma^2}} \int_0^\infty \frac{b}{\sigma\sqrt{2\pi s^3}} e^{-(1/2)\left(\frac{b+\left(a^2+2(r+\gamma)\sigma^2\right)^{\frac{1}{2}} s}{\sigma\sqrt{s}}\right)^2} ds$$

where the drift is now $(a^2+2(r+\gamma)\sigma^2)^{1/2}$.

Furthermore, if $\lambda - \frac{a}{\sigma^2}$ is rewritten as $-a + z$ and if $b + (a^2 + 2(r+\gamma)\sigma^2)^{1/2}s$ equals $b + z\sigma^2 s$ then we now have

$$\left(\frac{V}{V_B}\right)^{-a+z} \int_0^{\infty} \frac{b}{\sigma\sqrt{2\pi s^3}} e^{-(1/2)\left(\frac{b+z\sigma^2 s}{\sigma\sqrt{s}}\right)^2} ds.$$

For a particular credit default security with a time to maturity of t, the expression for the likelihood of reaching the absorbing barrier b before the expiration of the CDS would be:

$$\left(\frac{V}{V_B}\right)^{-a+z} \left[\int_0^{t} \frac{b}{\sigma\sqrt{2\pi s^3}} e^{-(1/2)\left(\frac{b+z\sigma^2 s}{\sigma\sqrt{s}}\right)^2} ds\right].$$

In terms of the cumulative normal, we have

$$\left(\frac{V}{V_B}\right)^{-a+z} \left[1 - N\left(\frac{b+z\sigma^2 t}{\sigma\sqrt{t}}\right)\right] \text{ or}$$

$$\left(\frac{V}{V_B}\right)^{-a+z} \left[N\left(\frac{-b-z\sigma^2 t}{\sigma\sqrt{t}}\right)\right]$$

because $N(-A) = 1 - N(A)$ for the standardized normal variate. This is exactly what we have as the first half of the expression for $G(t)$ in the theory section of Chapter 3.

Differentiation of the second term on the RHS of $P(v_0, b; s) = N\left(\frac{b+as}{\sigma\sqrt{s}}\right) - e^{-\frac{2ab}{\sigma^2}} N\left(\frac{-b+as}{\sigma\sqrt{s}}\right)$ with respect to s almost immediately yields

$$-e^{\frac{-2ab}{\sigma^2}} \int_0^{\infty} \frac{-b}{\sigma\sqrt{2\pi s^3}} e^{\frac{-(1/2)\left(2(r+\gamma)s\sigma^2\right)}{\sigma^2}} e^{-(1/2)\left(\frac{(-b+as)}{\sigma\sqrt{s}}\right)^2} ds.$$

Completing the square yet again yields

$$-e^{\frac{-2ab}{\sigma^2}} \int_0^{\infty} \frac{-b}{\sigma\sqrt{2\pi s^3}} e^{-(1/2)\left(\frac{b-\lambda\sigma^2 s}{\sigma\sqrt{s}}\right)^2} e^{-(1/2)b\left(\frac{-2a}{\sigma^2}+2\lambda\right)} ds \quad \text{or}$$

$$e^{b\left(\frac{a}{\sigma^2} - \frac{2a}{\sigma^2} - \lambda\right)} \int_0^{\infty} \frac{b}{\sigma\sqrt{2\pi s^3}} e^{-(1/2)\left(\frac{b-\lambda\sigma^2 s}{\sigma\sqrt{s}}\right)^2} ds.$$

This expression can be written as

$$\left(\frac{V}{V_B}\right)^{-\left(\frac{a}{\sigma^2}+\lambda\right)} \int_0^{\infty} \frac{b}{\sigma\sqrt{2\pi s^3}} e^{-(1/2)\left(\frac{b-\lambda\sigma^2 s}{\sigma\sqrt{s}}\right)^2} ds.$$

Furthermore, if $\left(\frac{a}{\sigma^2}+\lambda\right)$ is written as $(a+z)$ and if $\frac{b-\lambda\sigma^2 s}{\sigma\sqrt{s}}$ is written as $\frac{b-z\sigma^2 s}{\sigma\sqrt{s}}$ then we have

$$\left(\frac{V}{V_B}\right)^{-a-z}\int_0^\infty \frac{b}{\sigma\sqrt{2\pi s^3}}e^{\frac{-1}{2}\left(\frac{b-z\sigma^2 s}{\sigma\sqrt{s}}\right)^2}ds$$

which can be written in terms of the cdf of the standardized normal variate as

$$\left(\frac{V}{V_B}\right)^{-a-z}\left[1-\int_{-\infty}^{\frac{b-z\sigma^2 s}{\sigma\sqrt{s}}} f(z)dz\right]$$

or as

$$\left(\frac{V}{V_B}\right)^{-a-z}\int_{-\infty}^{\frac{-b+z\sigma^2 s}{\sigma\sqrt{s}}} f(z)dz$$

due to symmetry $1-N(A)=\ N(-A)$. For a CDS with a time to maturity of t this expression can be written as

$$\left(\frac{V}{V_B}\right)^{-a-z}N\left(\frac{-b+z\sigma^2 t}{\sigma\sqrt{t}}\right).$$

The expression above corresponds to the second half of the RHS of our expression for $G(t)$ in our theory section.

Gathering these results, we have

$$\int_0^\infty e^{-(r+\gamma)s}f(s,v_0,b)\,ds=\left(\frac{V}{V_B}\right)^{-a+z}\left[N\left(\frac{-b-z\sigma^2 t}{\sigma\sqrt{t}}\right)\right]+\left(\frac{V}{V_B}\right)^{-a-z}\left[N\left(\frac{-b+z\sigma^2 t}{\sigma\sqrt{t}}\right)\right]$$

which is identical to what we provide in the theory section of Chapter 3.

Table 1: Liquidity Measure Statistics

Panel A reports summary statistics for the four daily liquidity measures of all corporate bonds in the period from 2008 to 2012. The bid-ask spread, Amihud measure and interquartile range are calculated from the fixed-income transaction data available in TRACE. Details about the procedure to calculate these measures are provided in the methodology section. The bid-ask spread is the weighted average difference between sells and buys, the Amihud measure is the change in price by transaction volume, the interquartile range is the 75th percentile minus the 25th percentile divided by the median, and the roundtrip measure is the difference between the maximum and the minimum price for transactions that occur at the same time and have the same volume. Panel B reports summary statistics for the sub-sample of insured corporate bonds. The same liquidity measures are shown as in panel A and the calculation procedure is equivalent. In Panel C we report the pairwise correlations of our four liquidity measures between 2008 and 2012.

Panel A: All corporate bonds in TRACE

Statistic	N	Mean	St. Dev.	Min	Pctl(25)	Median	Pctl(75)	Max
Bid-Ask	5,163,470	42.717	67.893	-96.70	3.425	21.500	56.250	342.500
Amihud	5,460,943	53.502	117.747	0.000	1.118	13.470	48.727	799.205
IQR	6,591,554	0.006	0.023	0.000	0.000	0.002	0.007	2.973
Roundtrip	4,045,509	0.487	1.243	0.000	0.000	0.099	0.607	100.000

Panel B: Insured corporate bonds

Statistic	N	Mean	St. Dev.	Min	Pctl(25)	Median	Pctl(75)	Max
Bid-Ask	31,083	4.511	16.828	-96.700	0.000	1.913	7.224	342.500
Amihud	34,005	14.883	48.033	0.000	0.126	2.019	10.723	799.205
IQR	39,413	0.001	0.003	0.000	0.0001	0.0005	0.001	0.095
Roundtrip	24,554	0.086	0.335	0.000	0.000	0.000	0.098	41.909

Panel C: Pairwise correlations between liquidity measures

	Bid Ask	Amihud	Round-trip	IQR
Bid Ask	1	0.579	0.440	0.576
Amihud	0.579	1	0.419	0.695
Round-trip	0.440	0.419	1	0.355
IQR	0.576	0.695	0.355	1

Table 2: Summary Statistics of Sample of Insured Bonds

In this Table, we report summary statistics for our sample of insured corporate bonds. These bonds were issued by numerous banks in 2008 and 2009 and were backed by the full faith and credit of the U.S. Treasury; thus these corporate bonds had no default risk. Any insured bonds included in the debt guarantee program matured on or before December 31, 2012. Panel A presents the number of bonds, number of bond-days, and average number of transactions per bond. Panels B and C present averages of the spreads of these bonds on the maturity matched U.S. Treasury and interest rate swaps in maturity buckets in the stress (Panel B) and post stress period (Panel C).

Panel A: Observations in Stress and Post Stress Periods

	Stress	**Post stress**
Number of distinct bond issuers	30	29
Number of distinct bonds	68	68
Number of bond-days	10,392	29,018
Average transaction days per bond	153	427

Panel B: Average Insured Bond Premiums in Stress Period

TTM interval (years)	**1-1.25**	**1.25-1.5**	**1.5-1.75**	**1.75-2**	**2-2.25**
Spread (swap)	0.157	0.111	0.204	0.146	0.214
Spread (Treasury)	0.113	0.114	0.267	0.404	0.260

TTM interval (years)	**2.25-2.5**	**2.5- 2.75**	**2.75-3**	**3-3.25**	**3.25- 3.5**	**3.5 3.75**
Spread (swap)	0.411	0.158	0.195	0.254	0.183	0.457
Spread (Treasury)	0.353	0.392	0.505	0.490	0.599	0.632

Panel C: Average Insured Bond Premiums in Post Stress Period

TTM interval (years)	**0-0.25**	**0.25-0.50**	**0.60-0.75**	**0.75-1**	**1-1.25**	**1.25-1.5**	**1.5-1.75**
Spread (swap)	1.348	0.614	0.426	0.319	0.295	0.180	0.127
Spread (Treasury)	0.519	0.277	0.233	0.228	0.215	0.189	0.170

TTM interval (years)	**1.75-2**	**2-2.25**	**2.25-2.5**	**2.5-2.75**	**2.75-3**	**3- 3.25**
Spread (swap)	0.195	0.118	0.113	0.041	0.042	0.374
Spread (Treasury)	0.205	0.207	0.219	0.212	0.258	0.260

Table 3: Regression Analysis of the Liquidity Spread

This Table reports regressions of insured corporate bond liquidity spreads on our hypothesized determinants. The regressions are for the periods of stress – October 2008 to November 2009 – and post stress – December 2009 to December 2012. We further classify the daily bond transactions into ST_{INS} and LT_{INS} depending on whether the time to maturity is above or below the median in the stress period (Columns 1 to 4). In Columns 5 and 6, we present aggregate regressions for each period and we include the indicator function $1_{\{\tau=long\}}$ which takes the value 1 when the bond is characterized as being an LT_{INS} debenture at the time of the transaction and 0 when the bond is classified as an ST_{INS} FIS at the time of transaction. The dependent variable is the yield of the insured bond minus the yield of a treasury bond, matched by maturity. In Column 7, we include the term $1_{\{period=post\}}$ which is an indicator function that takes the value 1 if the bond transaction occurs during our post crisis period and 0 if it occurred during our crisis period. Each right hand side variable is normalized by its mean and standard deviation over the respective period, stress or post stress. AMIHUD is our liquidity measure, VIX is the volatility index derived from the implied volatility on S&P 500 index options, FFFP is the price of the price of the Fed funds 30-day futures, TREASLP is the 6-month Treasury yield relative to the 3-month yield, TTM is the time-to-maturity of the insured corporate bond, AMIHUDMKT is the aggregate Amihud measure for all outstanding corporate bonds, and AMIHUDVOL is the volatility of the Amihud liquidity measure for the insured bonds. The regressions include firm-fixed effects, and the standard errors of the coefficients are in parenthesis.

	Regression of Liquidity Spread						
	Insured Spread						
	Stress ST_{INS}	Stress LT_{INS}	Post Stress ST_{INS}	Post Stress LT_{INS}	Stress	Post Stress	All
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
$AMIHUD$	0.044***	0.156***	0.225***	0.032***	0.061***	0.201***	0.061***
	(0.004)	(0.009)	(0.007)	(0.003)	(0.008)	(0.006)	(0.008)
$AMIHUD \cdot 1_{\{\tau=long\}}$					0.079***	-0.160***	0.069***
					(0.011)	(0.008)	(0.012)
$AMIHUD \cdot 1_{\{period=post\}}$							0.143***
							(0.009)
$AMIHUD \cdot 1_{\{\tau=long\}} \cdot 1_{\{period=post\}}$							-0.233***
							(0.014)

$1_{\{\tau=long\}}$							-0.044*** (0.017)
$1_{\{period=post\}}$					-0.032* (0.018)	0.126*** (0.017)	0.066*** (0.013)
AMIHUD VOL	-0.0003 (0.004)	0.009 (0.009)	0.048*** (0.009)	-0.005 (0.003)	0.002 (0.007)	0.018*** (0.006)	0.014*** (0.004)
TTM	0.048*** (0.004)	-0.008 (0.009)	-0.195*** (0.010)	0.0002 (0.007)	0.059*** (0.009)	-0.156*** (0.010)	-0.113*** (0.009)
VIX	0.129*** (0.006)	0.223*** (0.015)	0.048*** (0.010)	0.032*** (0.003)	0.178*** (0.011)	0.038*** (0.005)	0.103*** (0.005)
TREASLP	0.027*** (0.005)	0.039*** (0.012)	0.002 (0.008)	0.027*** (0.008)	0.026*** (0.010)	0.030*** (0.006)	0.085*** (0.006)
AMIHUD MKT	0.002 (0.006)	-0.022* (0.013)	-0.020* (0.012)	0.005 (0.004)	-0.019* (0.011)	0.023*** (0.007)	0.004 (0.010)
FFFP	0.001 (0.005)	-0.070*** (0.010)	-0.012 (0.018)	0.017*** (0.003)	-0.034*** (0.008)	0.006 (0.006)	-0.004 (0.005)
Constant	0.240*** (0.021)	1.012*** (0.273)	0.269*** (0.062)	0.253*** (0.011)	0.437*** (0.059)	0.126** (0.050)	0.256*** (0.040)
Observations	2,285	2,850	7,321	2,917	7,220	15,337	22,557
Adjusted R^2	0.596	0.414	0.240	0.313	0.237	0.147	0.167
Residual Std. Error	0.150 (df = 2258)	0.385 (df = 2815)	0.604 (df = 7286)	0.147 (df = 2889)	0.467 (df = 7181)	0.559 (df = 15301)	0.537 (df = 22516)
F Statistic	135.955*** (df = 26; 2259)	61.870*** (df = 34; 2816)	68.978*** (df = 34; 7286)	50.183*** (df = 27; 2889)	59.964*** (df = 38; 7181)	76.668*** (df = 35; 15301)	114.234*** (df = 40; 22516)
Note:							*p<0.1; **p<0.05; ***p<0.01

Table 4: Summary of Parameter Estimates for Liquidity and Default Premiums from Kalman Filter and EM Algorithm

In this Table, we summarize the Kalman filter/EM parameters we obtain when we segregate the credit spreads into liquidity and default premiums, for a sample of high grade, U.S. bank bonds with longer maturities. We use the following measurement and transition equations:

Measurement equation: $s_{i,t} = m_1 \cdot (liq)_{i,t} + m_2 \cdot (def)_{i,t} + \epsilon_{M,i,t}$

Transition equation: $\begin{bmatrix} (liq)_{i,t} \\ (def)_{i,t} \end{bmatrix} = \underline{\alpha_i} + \underline{\underline{\phi_i}} \cdot \begin{bmatrix} (liq)_{i,t-1} \\ (def)_{i,t-1} \end{bmatrix} + \underline{\underline{\beta_i}} \cdot \underline{X_{i,t}} + \underline{\epsilon_{T,i,t}}.$

In the measurement equation, $s_{i,t}$ is the corporate spread and $(liq)_{i,t}$ and $(def)_{i,t}$ are the liquidity and default spreads of bond i at time t. The error term in the measurement equation, $\epsilon_{M,i,t}$, follows a normal distribution with mean zero and standard deviation σ_M. In the transition equation, $\underline{\alpha'_i} = [\alpha_{i,1} \quad \alpha_{i,2}]$ is the vector of intercepts for each transition equation, $\underline{\underline{\phi_i}}$ is the matrix of autoregressive coefficients, $\underline{\underline{\beta_i}}$ is the matrix of control coefficients, and $\underline{X_{i,t}}$ is the vector of liquidity and default controls that includes Amihud liquidity measure, time to maturity, age, liquidity volatility, firm leverage, firm profitability, coupon amount, VIX, U.S. Treasury slope, and market liquidity. The error term in the measurement equation is a vector $\underline{\epsilon'_{T,i,t}} = [\epsilon_{1,T,i,t} \quad \epsilon_{2,T,i,t}]$, where each component follows a normal distribution with mean zero and standard deviation σ_j, with $j = 1, 2$. We provide estimations for the stress and post stress periods using three alternative samples: without insured bonds (Columns 1 and 2), random selection of $\psi = 50\%$ of the insured bonds (Columns 3 and 4), and a random selection of $\psi = 75\%$ of the insured bonds (Columns 5 and 6). The bonds used in the sample are AAA and AA U.S. bank bonds with a maximum maturity of 10 years. p-values are reported in parenthesis.

	All extended sample		$\psi = 50\%$		$\psi = 75\%$	
	(1)	(2)	(3)	(4)	(5)	(6)
	Stress	Post Stress	Stress	Post Stress	Stress	Post Stress
α_1	0.026***	-0.167***	0.229	-0.169**	0.193***	-0.0932***
	(0.0001)	(0.000)	(0.117)	(0.014)	(0.000221)	(0.00014)
ϕ_1	0.013	0.0022	0.0038***	-0.004***	0.0041***	0.00262
	(0.695)	(0.109)	(0.0051)	(0.0016)	(0.000688)	(0.659)
$AMIHUD$	0.115**	0.0017**	0.0232***	0.0093***	0.0843**	0.00125**
	(0.0215)	(0.011)	(0.0031)	(0.0043)	(0.026)	(0.0338)
TTM	0.0666***	-0.161***	0.21**	-0.029***	0.391**	-0.087***
	(0.0001)	(0.000)	(0.0321)	(0.0007)	(0.0205)	(0.00065)
Age	0.39**	-0.076***	0.257**	-0.391**	0.234***	-0.19***
	(0.0258)	(0.0016)	(0.028)	(0.0174)	(0.00105)	(0.00001)

$AMIHUDVOL$	0.0827 (0.128)	-0.003*** (0.0031)	0.0294 (0.574)	-0.2 (0.255)	0.0563 (0.445)	-0.00041** (0.0382)
VIX	0.024*** (0.0002)	-0.006*** (0.0081)	0.0077*** (0.0014)	-0.0365** (0.045)	0.135 (0.13)	-0.00274** (0.0407)
$Slope1Y10Y$	0.0466*** (0.0005)	0.0239 (0.246)	0.0464** (0.0446)	-0.004*** (0.0032)	0.0944 (0.18)	0.0316 (0.849)
AMIHUDMKT	0.0041* (0.0764)	0.0004 (0.246)	0.0001** (0.039)	0.0032 (0.123)	0.0031** (0.0853)	0.0002 (0.687)
σ_1	0.274*** (0.000)	0.26*** (0.000)	0.264*** (0.000)	0.252*** (0.000)	0.269*** (0.000)	0.254*** (0.000)
α_2	0.448 (0.455)	0.106 (0.342)	0.378 (0.702)	0.0577*** (0.000)	0.357 (0.271)	0.0763 (0.33)
ϕ_2	0.0127 (0.682)	-0.0001** (0.0105)	0.0031*** (0.0046)	-0.0062** (0.014)	0.0069*** (0.000959)	-0.0015*** (0.00611)
$Leverage$	3.41*** (0.000)	2.61*** (0.000)	3.01*** (0.000)	1.39*** (0.000)	3.25*** (0.000)	1.7*** (0.000)
$Profit$	0.626 (0.482)	-0.382*** (0.000)	0.457 (0.709)	-0.353* (0.0804)	0.785 (0.45)	0.0245 (0.178)
$Coupon$	0.368*** (0.0057)	-0.0126** (0.0156)	-0.048*** (0.000)	-0.046*** (0.0003)	0.47** (0.0276)	0.275 (0.803)
σ_2	0.282*** (0.000)	0.252*** (0.000)	0.258*** (0.000)	0.253*** (0.000)	0.27*** (0.000)	0.257*** (0.000)
Median Max Likelihood (average initial ML = -10,000)	15.12	7.31	3.16	24.36	4.06	23.64
Number of bonds	179	343	212	374	228	390

Table 5: Regression of Credit Spread on Proxies for Liquidity and Default

This Table presents regressions of the credit spread on the liquidity measure, time to maturity, and other covariates. The dependent variable – liquidity score– was obtained by regressing the corporate yields on proxies for liquidity and default as in Dick-Nielsen, Feldhütter, and Lando (2012). Column (1) presents the results for the period of stress (October 2008 to November 2009), while column (2) shows the regression for the post stress period (November 2009 to December 2012). All standard errors are clustered at the firm-year level.

	Crisis (1)	Post Crisis (2)
$AMIHUD$	0.217***	0.052***
	(0.020)	(0.004)
$Leverage$	0.167***	0.120***
	(0.022)	(0.006)
$Profit$	-0.772***	0.136***
	(0.042)	(0.006)
$Coupon$	-0.033	0.001
	(0.021)	(0.003)
$AMIHUDVOL$	-0.066***	-0.014***
	(0.021)	(0.004)
TTM	0.509***	-0.207***
	(0.023)	(0.006)
Age	0.883***	0.005
	(0.024)	(0.007)
VIX	1.250***	0.022***
	(0.025)	(0.003)
$Slope1Y10Y$	0.034	-0.046***
	(0.022)	(0.003)
Constant	1.604***	0.311***
	(0.028)	(0.005)
Issue effects	Y	Y
Observations	4,927	5,454
Adjusted R^2	0.780	0.574
Residual Std. Error	1.107 (df = 4916)	0.203 (df = 5443)
F Statistic	1,743.833*** (df = 10; 4916)	736.658*** (df = 10; 5443)
Note:		*p<0.1; **p<0.05; ***p<0.01

Table 6: Illustrative Parameters that Represent High, Medium, and Low Quality Firms in Periods of Stress and Nonstress

In the table below, we summarize the parameters we choose to represent high, medium, and low quality firms in a period of stress and of nonstress. These are the short-term interest rate r, the firm's leverage, the firm's payout rate δ, the firm's volatility of assets σ, the borrowers recovery rate during default α, and the fraction of the credit spread that is due to illiquidity. The values in the Table below are selected to distinguish firms in a period of crisis and noncrisis, and also firms of high, medium, and low credit quality within each economic period. We are able to generate illustrative plots for each credit category in each period in Figure 11 when we input the parameters below into out Leland and Toft liquidity premium inclusive model and increase the times to maturity.

Period	Credit Quality	r	Leverage	δ	σ	α	Liq. Ratio
Nonstress	High	2.5%	40%	3%	20%	0.4	5%
	Medium		45%	4%	22.5%	0.5	7.5%
	Low		50%	5%	25%	0.6	10%
Stress	High	7.5%	50%	5%	25%	0.5	10%
	Medium		55%	6%	27.5%	0.6	15%
	Low		60%	7%	30%	0.7	20%

Table 7: Descriptive Statistics

In the table below, we report summary statistics of the main variables that we use in this paper. We obtain corporate bond credit yields and credit spreads (yield minus maturity matching U.S. Treasury rate) from the Enhanced TRACE data set. The corporate bond characteristics are obtained from FISD, and firm-level data is obtained from CRSP and COMPUSTAT. We aggregate our data by calendar months. We classify the observations into three major credit rating categories: high for bonds with S&P credit ratings AAA and AA, medium for bonds with credit ratings A and BBB, and low for speculative grade bonds. We also classify our observations into three economic periods: pre-crisis (March 2004 to October 2007), crisis (November 2007 to June 2009), and post-crisis (July 2009 to November 2019). We are able to collect 235,600 bond month observations. Panels A, B, and C contain summary statistics for our main variables in each economic period.

Panel A: Pre-Crisis Period

High Credit Category

Statistic	N	p25	Median	p75	St. Dev.
Spread	3,142	3.873	4.803	5.295	1.162
Maturity	3,142	1.167	2.305	4.683	3.562
Leverage	3,142	0.600	0.637	0.746	0.078
Payout Ratio	3,142	0.006	0.009	0.012	0.003
Equity Volty	3,142	0.022	0.041	0.107	0.060
Coupon	3,142	4.125	5.500	7.000	1.414

Medium Credit Category

Statistic	N	p25	Median	p75	St. Dev.
Spread	894	5.868	6.065	6.387	0.504
Maturity	894	26.697	27.845	28.846	4.089
Leverage	894	0.600	0.635	0.651	0.078
Payout Ratio	894	0.006	0.011	0.012	0.003
Equity Volty	894	0.035	0.088	0.109	0.062
Coupon	894	6.125	6.125	6.125	0.593

Low Credit Category

Statistic	N	p25	Median	p75	St. Dev.
Spread	14,155	4.930	5.287	5.581	1.260
Maturity	14,155	4.107	6.414	8.496	3.400
Leverage	14,155	0.620	0.637	0.746	0.119
Payout Ratio	14,155	0.007	0.011	0.013	0.006
Equity Volty	14,155	0.022	0.048	0.109	0.066
Coupon	14,155	5.000	5.400	6.650	1.303

Panel B: Crisis Period

High Credit Category

Statistic	N	p25	Median	p75	St. Dev.
Spread	812	4.685	5.354	5.756	1.329
Maturity	812	2.062	2.873	4.094	6.618
Leverage	812	0.610	0.626	0.633	0.086
Payout Ratio	812	0.011	0.012	0.013	0.013
Equity Volty	812	0.251	0.319	0.419	0.170
Coupon	812	6.375	6.600	6.600	0.956

Medium Credit Category

Statistic	N	p25	Median	p75	St. Dev.
Spread	2,140	6.391	7.019	7.889	2.102
Maturity	2,140	17.117	25.155	28.264	9.700
Leverage	2,140	0.322	0.611	0.633	0.166
PayoutRatio	2,140	0.011	0.012	0.022	0.030
EquityVolty	2,140	0.035	0.251	0.419	0.207
Coupon	2,140	6.125	6.750	7.625	1.383

Low Credit Category

Statistic	N	p25	Median	p75	St. Dev.
Spread	20,487	5.017	6.114	8.287	1.600
Maturity	20,487	2.856	4.899	7.177	4.329
Leverage	20,487	0.456	0.611	0.773	0.206
PayoutRatio	20,487	0.010	0.012	0.013	0.034
EquityVolty	20,487	0.026	0.050	0.359	0.201
Coupon	20,487	5.000	5.500	6.500	1.439

Panel C: Post-Crisis Period

High Credit Category

Statistic	N	p25	Median	p75	St. Dev.
Spread	11,376	2.485	2.987	3.503	0.867
Maturity	11,376	4.466	6.967	9.211	7.562
Leverage	11,376	0.039	0.084	0.270	0.259
Payout Ratio	11,376	0.003	0.008	0.038	0.036
Equity Volty	11,376	0.047	0.063	0.163	0.168
Coupon	11,376	3.000	3.500	4.300	1.101

Medium Credit Category

Statistic	N	p25	Median	p75	St. Dev.
Spread	82,096	3.308	4.074	5.030	1.353
Maturity	82,096	5.197	7.682	14.090	8.667
Leverage	82,096	0.263	0.453	0.562	0.173
Payout Ratio	82,096	0.008	0.013	0.023	0.022
Equity Volty	82,096	0.027	0.054	0.134	0.191
Coupon	82,096	3.625	4.750	5.700	1.263

Low Credit Category

Statistic	N	p25	Median	p75	St. Dev.
Spread	100,498	2.004	3.456	5.669	15.010
Maturity	100,498	2.205	4.003	6.016	3.684
Leverage	100,498	0.292	0.475	0.605	0.212
Payout Ratio	100,498	0.008	0.012	0.021	0.018
Equity Volty	100,498	0.026	0.053	0.125	0.203
Coupon	100,498	3.500	5.500	6.750	2.173

Table 8: Average Liquidity Premiums (LP) by Credit Category

In this table, we report the average liquidity premiums obtained by means of our estimation procedure, aggregated by credit category and economic period. Bond observations are classified into three major credit rating categories: high for bonds with S&P credit ratings AAA and AA, medium for bonds with credit ratings A and BBB, and low for speculative grade bonds. Bond observations are additionally sorted into three economic periods: pre-crisis (March 2004 to October 2007), crisis (November 2007 to June 2009), and post-crisis (July 2009 to November 2019). The liquidity premiums are estimated for each month and for each bond using a liquidity premium inclusive model of credit spread in the context of an unscented Kalman filter and using an expectation maximization algorithm. Additional details of the model and its estimation are provided in the theory and methodology sections. In total, we are able to collect 235,600 bond month observations from March 2004 to November 2019.

	Pre-Crisis	**Crisis**	**Post-Crisis**
High Grade	0.4253%	1.0091%	0.7221%
Medium Grade	0.5346%	1.6542%	0.9168%
Low Grade	0.6784%	1.3398%	0.9072%

Table 9: Regression of Liquidity Premiums on Time to Maturity for ST and LT Bonds during Pre-Crisis Period

In the table below, we measure the slopes of the term structure of liquidity premiums for short-term and long-term bonds during the pre-crisis period. To do this, we regress the UKF extracted liquidity premiums on time to maturity (TTM) for the high (Column 1), medium (Column 2), and low (Column 3) credit categories during this period. To capture the slopes for short versus long-term bonds, we interact TTM with a dummy variable "Long", which takes the value 1 for bonds with a remaining maturity greater than the median maturity in each period, and zero otherwise.

	Liquidity Premiums		
	High Grade	Medium Grade	Low Grade
	(1)	(2)	(3)
TTM (β_1)	0.023^{***}	0.027^{***}	0.026^{***}
	(0.002)	(0.000)	(0.005)
TTM*Long (β_2)	-0.009^{***}	-0.000^{**}	0.006^{*}
	(0.001)	(0.000)	(0.003)
Long (β_3)	0.464^{***}	0.162^{***}	0.118^{***}
	(0.009)	(0.000)	(0.027)
Observations	1,498	217	11,417
R^2	0.200	0.205	0.056
Adjusted R^2	0.199	0.198	0.056
Residual Std. Error	0.211 (df = 1495)	0.210 (df = 214)	0.655 (df = 11414)
F Statistic	187.262^{***} (df = 2; 1495)	27.628^{***} (df = 2; 214)	339.911^{***} (df = 2; 11414)
Note:			$^{*}p<0.1$; $^{**}p<0.05$; $^{***}p<0.01$

Table 10: Regression of Liquidity Premiums on Time to Maturity for ST and LT Bonds during Crisis Period

In the table below, we measure the slopes of the term structure of liquidity premiums for short-term and long-term bonds during the crisis period. To do this, we regress the UKF extracted liquidity premiums on time to maturity (TTM) for the high (Column 1), medium (Column 2), and low (Column 3) credit categories during this period. To capture the slopes for short versus long-term bonds, we interact TTM with a dummy variable "Long", which takes the value 1 for bonds with a remaining maturity greater than the median maturity in each period, and zero otherwise.

	Liquidity Premiums		
	High Grade	Medium Grade	Low Grade
	(1)	(2)	(3)
TTM (β_1)	0.003^{***}	0.089^{***}	0.042^{***}
	(0.0002)	(0.004)	(0.002)
TTM*Long (β_2)	-0.001^{***}	-0.023^{***}	-0.012^{***}
	(0.0002)	(0.003)	(0.002)
Long (β_3)	1.002^{***}	1.036^{***}	1.023^{***}
	(0.001)	(0.024)	(0.013)
Observations	436	860	9,808
R^2	0.106	0.320	0.027
Adjusted R^2	0.102	0.319	0.027
Residual Std. Error	0.726 (df = 433)	0.681 (df = 857)	0.770 (df = 9805)
F Statistic	25.736^{***} (df = 2; 433)	201.895^{***} (df = 2; 857)	137.857^{***} (df = 2; 9805)
Note:			$^{*}p<0.1$; $^{**}p<0.05$; $^{***}p<0.01$

Table 11: Regression of Liquidity Premiums on Time to Maturity for ST and LT Bonds for the Post-Crisis Period

In the table below, we measure the slopes of the term structure of liquidity premiums for short-term and long-term bonds during the post-crisis period. To do this, we regress the UKF extracted liquidity premiums on time to maturity (TTM) for the high (Column 1), medium (Column 2), and low (Column 3) credit categories during this period. To capture the slopes for short versus long-term bonds, we interact TTM with a dummy variable "Long", which takes the value 1 for bonds with a remaining maturity greater than the median maturity in each period, and zero otherwise.

	Liquidity Premiums		
	High Grade	Medium Grade	Low Grade
	(1)	(2)	(3)
TTM (β_1)	0.013^{***}	0.041^{***}	0.012^{***}
	(0.004)	(0.005)	(0.004)
TTM*Long (β_2)	0.010^{***}	0.014^{***}	0.005^{*}
	(0.003)	(0.004)	(0.003)
Long (β_3)	0.342^{***}	0.153^{***}	0.478^{***}
	(0.024)	(0.030)	(0.023)
Observations	11,395	73,237	92,631
R^2	0.367	0.367	0.245
Adjusted R^2	0.367	0.367	0.245
Residual Std. Error	0.304 (df = 11392)	0.473 (df = 73234)	0.627 (df = 92628)
F Statistic	$3,300.946^{***}$ (df = 2; 11392)	$21,261.940^{***}$ (df = 2; 73234)	94.710^{***} (df = 2; 92628)
Note:			$^{*}p<0.1$; $^{**}p<0.05$; $^{***}p<0.01$

Table 12: Summary Statistics

In this table, I summarize the main variables used in this research. In Panel A, I describe the sample of credit default swap contracts which feature in the CMA data set provided by Intercontinental Exchange. The prefix *"Mid"* implies that the quantity is the midpoint between the bid and the ask prices of that variable, while the prefix *"BA"* implies the quantity is the difference between the ask and bid prices of that variable. For example, "MidParSpread" is the midpoint between the ask par spread and the bid par spread, while "BAParSpread" is the difference between the ask par spread and the bid par spread. The data set includes all single-name contracts that are active, trade in the United States, and are denominated in U.S. dollars. In Panel B, I report summary statistics for the daily liquidity measures of all corporate bonds in the period from 2006 to 2019. The bid-ask spread, Amihud measure and interquartile range are calculated from the fixed-income transaction data available in TRACE.

Panel A: Summary Statistics for Credit Default Swaps on the CMA Database

Variable	**Num. obs**	**Min**	**Q_1**	**Median**	**Mean**	**Q_3**	**Max**
MidParSpread	21,435,149	11.2	52.4	99.0	135.8	176.4	624.1
MidQuoteSpread	21,435,149	11.1	52.0	98.8	135.9	176.7	632.2
MidUpfront	21,435,149	-94.9	-2.7	-0.8	-0.8	1.8	51.0
MidPercentOfPar	21,435,149	49.0	98.2	100.8	100.8	102.7	194.9
BAParSpread	21,435,149	4.6	10.1	17.2	21.1	28.0	80.2
BAQuoteSpread	21,435,149	4.6	10.1	17.5	21.5	28.6	87.0
BAUpfront	21,435,149	0.0	0.4	0.8	1.3	1.6	27.0
BAPercentOfPar	21,435,149	0.0	0.4	0.8	1.3	1.6	27.0

Panel B: Summary Statistics for Bond Liquidity Measures

Variable	**Num. obs**	**Min**	**Q_1**	**Median**	**Mean**	**Q_3**	**Max**
Amihud measure	3,763,103	0.0	4.6	15.4	29.4	34.0	2513.1
Bond bid ask spread	1,586,512	-10.4	0.3	0.7	1.0	1.4	12.9
Amihud maturity weighted	3,763,103	0.0	4.3	15.5	32.3	37.3	2513.1
Bid ask maturity weighted	1,586,512	-10.4	0.3	0.7	1.1	1.5	12.9

Table 13: Regression of Distance to Default on Bond Liquidity Measures

In this table, I present fixed effects model regressions of the Bharath and Shumway (2008) distance to default on corporate bond liquidity measures. The dependent variable is the monthly distance to default of the reference firm. The main independent variable is the average of the daily liquidity measure for all the bonds of the firm that traded on that month, weighted by the trading volume of each bond. Columns 1 through 3 show the impact of the bid-ask spread, the Amihud (2002), and the Roll liquidity measures on the distance to default. The bond data is from TRACE FISD, and the firm level data is from CRSP COMPUSTAT. The variable definition is provided in the methodology section. The panel data ranges from 2008 to Q12019. Regressions are for firms-months and standard errors are clustered at the same level.

	Bond liquidity measures		
	Bid Ask	Amihud	Roll
	(1)	(2)	(3)
Bid Ask	−0.079***		
	(0.005)		
Amihud		−0.018***	
		(0.002)	
Roll			0.008***
			(0.002)
Leverage	−0.224***	−0.223***	−0.234***
	(0.032)	(0.032)	(0.032)
Profitability	0.161*	0.169*	0.178*
	(0.091)	(0.091)	(0.091)
Payout Ratio	0.478***	0.445***	0.445***
	(0.167)	(0.168)	(0.168)
Volty	−0.534***	−0.538***	−0.544***
	(0.012)	(0.012)	(0.012)
TTM	0.078	0.068	0.068
	(0.091)	(0.092)	(0.092)

VIX	0.002*	0.002*	0.002
	(0.001)	(0.001)	(0.001)
Treas. Slope	0.425***	0.435***	0.428***
	(0.029)	(0.029)	(0.029)
Issue effects	Y	Y	Y
Time effects	Y	Y	Y
Controls	Y	Y	Y
Observations	19,652	19,652	19,652
R2	0.136	0.128	0.125
Adjusted R^2	0.075	0.066	0.063
F Statistic (df = 8; 18349)	362.454***	336.563***	328.451***

Note: *p<0.1; **p<0.05; ***p<0.01

Table 14: Regression of Distance to Default on Predicted Bond Liquidity Measures

In this table, I present fixed effects model regressions of the Bharath and Shumway (2008) distance to default on predicted corporate bond liquidity measures. The dependent variable is the monthly distance to default of the reference firm. The main independent variable is predicted, average of the daily liquidity measure using 12-month lagged values of liquidity for all the bonds of the firm that traded on that month, weighted by the trading volume of each bond. Columns 1 through 3 show the impact of the bid-ask spread, the Amihud (2002), and the Roll liquidity measures on the distance to default. The bond data is from TRACE FISD, and the firm level data is from CRSP COMPUSTAT. The variable definition is provided in the methodology section. The panel data ranges from 2008 to Q12019. Regressions are for firms-months and standard errors are clustered at the same level.

	Bond liquidity measures		
	"Lambda"	Bid Ask	Amihud
	(1)	(2)	(3)
"Lambda" measure	−0.032***		
	(0.006)		
Bid Ask		−0.044***	
		(0.015)	
Amihud			−0.010***
			(0.006)
Leverage	−0.171***	−0.162***	−0.178***
	(0.032)	(0.031)	(0.032)
Profitability	0.136	0.111	0.142
	(0.090)	(0.089)	(0.090)
Payout Ratio	0.371**	0.352**	0.382**
	(0.167)	(0.165)	(0.167)
Volty	−0.493***	−0.485***	−0.499***
	(0.012)	(0.012)	(0.012)
TTM	0.074	0.064	0.077
	(0.091)	(0.090)	(0.091)
VIX	0.002	0.002	0.002

	(0.001)	(0.001)	(0.001)
Treas. Slope	0.424***	0.428***	0.424***
	(0.028)	(0.028)	(0.028)
Issue effects	Y	Y	Y
Time effects	Y	Y	Y
Controls	Y	Y	Y
Observations	19,652	19,652	19,652
R2	0.145	0.164	0.140
Adjusted R^2	0.084	0.105	0.079
F Statistic (df = 8; 18349)	387.773***	451.335***	374.335***
Note:			*p<0.1; **p<0.05; ***p<0.01

Table 15: Regression of CDS Spreads on Bond Liquidity Measures for 5-year Tenor

In the table below, I summarize the regression coefficients when projecting measures of CDS spreads onto bond market liquidity measures for credit default swaps of 5 year tenor. The dependent variable is either the daily mid price between the bid and the ask par spreads, $MidParSpread$ in Columns 1 to 4, mid price between the bid and the ask quoted spreads, $MidQuoteSpread$ in Columns 5 to 8. I use four different proxies for bond market liquidity measures: the bond bid-ask spread (Columns 1 and 5), the bond Amihud (2002) measure (Columns 2 and 6), a bond market maturity weighted bid-ask spread measure (Columns 3 and 7), and a bond market maturity weighted Amihud (2002) spread measure (Columns 4 and 8). The regressions also include the bid ask spread of the CDS contracts and common proxies for default that typically explain the CDS premiums. All regressions include entity fixed effects as well as year fixed effects. The panel data ranges from 2008 to Q12019. Regressions are for firms-days and standard errors are clustered at the same level.

	Regression of CDS Spread							
	MidParSpread				MidQuoteSpread			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Bond Bid Ask	2.629***				2.671***			
	(0.099)				(0.102)			
Bond Amihud		1.634***				1.638***		
		(0.060)				(0.062)		
Maturity Bid Ask			2.287***				2.315***	
			(0.098)				(0.101)	
Maturity Amihud				1.280***				1.288***
				(0.060)				(0.062)
CDS Bid Ask	34.742***	10.685***	34.762***	10.692***	34.986***	10.947***	35.007***	10.955***

	(0.197)	(0.096)	(0.197)	(0.096)	(0.202)	(0.098)	(0.202)	(0.098)
Leverage	36.063***	25.592***	36.211***	25.638***	36.721***	25.719***	36.874***	25.764***
	(0.326)	(0.190)	(0.326)	(0.190)	(0.334)	(0.195)	(0.334)	(0.195)
Recovery Rate	−10.412***	−14.430***	−10.414***	−14.442***	−10.539***	−14.594***	−10.541***	−14.606***
	(0.124)	(0.093)	(0.124)	(0.093)	(0.127)	(0.096)	(0.127)	(0.096)
Total Payout	2.714***	1.967***	2.701***	1.968***	2.643***	1.935***	2.630***	1.936***
	(0.161)	(0.138)	(0.161)	(0.138)	(0.164)	(0.142)	(0.165)	(0.142)
TTM	−4.070***	−1.623***	−4.032***	−1.580***	−4.194***	−1.723***	−4.154***	−1.680***
	(0.169)	(0.093)	(0.169)	(0.093)	(0.173)	(0.095)	(0.173)	(0.095)
Coupon	0.627	0.744	0.653	0.704	−0.629	−2.866***	−0.602	−2.906***
	(1.330)	(0.459)	(1.331)	(0.459)	(1.362)	(0.470)	(1.363)	(0.471)
VIX	9.305***	14.277***	9.354***	14.314***	9.474***	14.446***	9.524***	14.483***
	(0.087)	(0.062)	(0.087)	(0.062)	(0.089)	(0.063)	(0.089)	(0.063)
Hazard Rate	91.333***	99.777***	91.341***	99.786***	93.634***	102.708***	93.642***	102.718***
	(0.217)	(0.126)	(0.217)	(0.126)	(0.222)	(0.129)	(0.222)	(0.129)
Constant	178.940***	163.004***	179.017***	162.980***	181.260***	164.266***	181.340***	164.241***
	(1.777)	(1.024)	(1.778)	(1.024)	(1.820)	(1.049)	(1.821)	(1.049)
Entity FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Year FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Observations	142,934	304,825	142,934	304,825	142,934	304,825	142,934	304,825
Adjusted R^2	0.938	0.936	0.938	0.936	0.936	0.934	0.935	0.934

*Note: *p<0.1; ** p<0.05; ***p<0.01*

Table 16: Regression of CDS Spreads on Bond Liquidity Measures for All Tenors

In this table, I present fixed effects model regressions of corporate bond liquidity measures on CDS spreads. The dependent variable is the daily mid price between and the ask quoted spreads, $MidQuoteSpread$. The most revealing independent variable is the a bond market maturity weighted Amihud (2002) measure "Maturity Amihud". Columns 1 through 8 show the impact of the Amihud (2002) on the 1Y, 2Y, 3Y, 4Y 5Y, 10Y and 30Y tenors. The CDS spread data is from CMA, available through the Intercontinental Exchange, the bond data is from TRACE FISD, and the firm level data is from CRSP COMPUSTAT. The variable definition is provided in the methodology section. The panel data ranges from 2008 to Q12019. Regressions are for firms-days and standard errors are clustered at the same level.

	Regression of CDS Spread							
	MidQuoteSpread							
	1 yr	2 yr	3 yr	4 yr	5 yr	8 yr	10 yr	30 yr
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Maturity Amihud	0.276***	0.665***	1.038***	1.180***	1.288***	1.185***	1.065***	0.194***
	(0.038)	(0.035)	(0.052)	(0.056)	(0.062)	(0.056)	(0.050)	(0.050)
CDS Bid Ask	3.029***	2.522***	5.508***	7.417***	10.955***	2.361***	0.648***	1.507***
	(0.057)	(0.055)	(0.084)	(0.089)	(0.098)	(0.075)	(0.063)	(0.067)
Leverage	0.206*	0.174	8.558***	18.183***	25.764***	33.155***	29.321***	12.694***
	(0.112)	(0.109)	(0.167)	(0.181)	(0.195)	(0.173)	(0.155)	(0.265)
Recovery Rate	−8.332***	−10.284***	−13.218***	−14.289***	−14.606***	−16.927***	−17.827***	−8.357***
	(0.054)	(0.051)	(0.076)	(0.081)	(0.096)	(0.080)	(0.071)	(0.092)

Total Payout	2.277***	2.072***	2.384***	1.929***	1.936***	2.661***	2.444***	2.203***
	(0.073)	(0.074)	(0.115)	(0.127)	(0.142)	(0.127)	(0.114)	(0.366)
TTM	−0.111*	−1.093***	−1.709***	−1.848***	−1.680***	−0.079	−0.104	−0.769***
	(0.064)	(0.057)	(0.083)	(0.086)	(0.095)	(0.085)	(0.076)	(0.093)
Coupon	−2.372***	−4.541***	−13.883***	−14.668***	−2.906***	−7.491***	−5.298***	−13.709***
	(0.272)	(0.253)	(0.374)	(0.403)	(0.471)	(0.471)	(0.431)	(0.569)
VIX	4.574***	3.376***	9.440***	9.673***	14.483***	7.020***	5.643***	−0.262***
	(0.044)	(0.040)	(0.056)	(0.059)	(0.063)	(0.062)	(0.056)	(0.048)
Hazard Rate	66.828***	86.457***	97.504***	103.252***	102.718***	99.557***	103.841***	105.169***
	(0.056)	(0.062)	(0.105)	(0.119)	(0.129)	(0.107)	(0.096)	(0.131)
Constant	62.482***	81.697***	107.530***	136.692***	164.241***	189.458***	194.411***	185.754***
	(16.287)	(2.136)	(1.144)	(1.042)	(1.049)	(0.954)	(0.852)	(0.779)
Entity FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Year FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Observations	211,213	269,685	299,169	315,550	304,825	313,732	311,856	106,661
Adjusted R^2	0.949	0.962	0.939	0.937	0.934	0.945	0.956	0.982

*Note: *p<0.1; ** p<0.05; ***p<0.01*

Table 17: Regression of CDS Spreads on Bond Liquidity Measures During a Financial Crisis and a Financial Non-crisis Period

In the table below, I summarize the regression coefficients when projecting measures of CDS spreads onto bond market liquidity measures for credit default swaps of 5 year tenor during a financial crisis period and then, during a financial non-crisis period. The dependent variable is the daily mid price between and the ask quoted spreads, *MidQuoteSpread*. The most revealing independent variable is the a bond market maturity weighted Amihud (2002) measure "Maturity Amihud". In Column 1, I present regression coefficients for observations that belong in the 2008 financial crisis period, which started in December 2007 and ended in June 2009. In Column 2, I present regression coefficients for the subsequent non-crisis period. In Column 3 I present regression coefficients using both subsamples, and I include an indicator function $1_{\tau=stress}$which takes the value 1 if the observation belongs to the 2008 crisis period and 0 otherwise. I also include the interaction dummy $Mat.Amihud \times 1_{\tau=stress}$ which represents the marginal contribution of the bond market liquidity measure during the crisis period in the determination of the credit default spreads. All regressions include entity fixed effects as well as year fixed effects. The panel data ranges from 2008 to Q12019. Regressions are for firms-days and standard errors are clustered at the same level.

	Regression of CDS Spread		
	Insured Spread		
	Crisis	Non-crisis	Interaction
	(1)	(2)	(3)
Maturity Amihud	5.486*	3.268***	2.809***
	(0.196)	(0.054)	(0.071)
CDS Bid Ask	5.704***	6.213***	9.053***
	(0.376)	(0.088)	(0.092)
$1_{\tau=stress}$			7.391***
			(0.225)
$Mat.Amihud \times 1_{\tau=stress}$			2.375***
			(0.118)
Leverage	129.477***	18.316***	20.061***
	(7.368)	(0.180)	(0.185)

Recovery Rate	−15.168***	−14.191***	−14.756***
	(0.399)	(0.088)	(0.089)
Total Payout	147.066***	0.940***	0.598***
	(26.727)	(0.132)	(0.132)
TTM	0.328	−0.857***	−0.850***
	(0.306)	(0.086)	(0.089)
Coupon	−6.062**	−2.848***	−1.024**
	(2.863)	(0.420)	(0.440)
VIX	−7.076***	11.921***	11.970***
	(0.180)	(0.055)	(0.061)
Hazard Rate	110.156***	106.338***	104.148***
	(0.544)	(0.120)	(0.123)
Constant	199.741***	153.035***	157.125***
	(18.324)	(0.889)	(0.980)
Entity FE	Yes	Yes	Yes
Year FE	Yes	Yes	Yes
Observations	24,258	280,567	304,825
Adjusted R^2	0.965	0.952	0.942

Note: *p<0.1; **p<0.05; ***p<0.01

Table 18: Regression of CDS Spreads on Bond Liquidity Measures of High and Low Quality Firms

In the table below, I summarize the regression coefficients when projecting measures of CDS spreads onto bond market liquidity measures for credit default swaps of 5 year tenor, alternatively using either high or low quality firms. The dependent variable is the daily mid price between and the ask quoted spreads, *MidQuoteSpread*. The most revealing independent variable is the a bond market maturity weighted Amihud (2002) measure "Maturity Amihud". In Column 1, I present regression coefficients for firms whose cumulative probability of default is greater than the median. I consider these firms to be in financial "distress". In Column 2, I present regression coefficients for firms whose cumulative probability of default is below the median. I consider these firms to be "non-distressed" firms. In Column 3, I present regression coefficients using both subsamples, and I include an indicator function $1_{type=distress}$ which takes the value 1 if the firm is distressed and 0 otherwise. I also include the interaction dummy $Mat.Amihud \times 1_{type=distress}$ which represents the marginal contribution of distressed firms' bond liquidity measure in the determination of the credit default spreads. All regressions include entity fixed effects as well as year fixed effects. The panel data ranges from 2008 to Q12019. Regressions are for firms-days and standard errors are clustered at the same level.

	Regression of CDS Spread		
		Insured Spread	
	Distressed	Non-distressed	Interaction
	(1)	(2)	(3)
Maturity Amihud	3.886***	1.549***	1.160**
	(0.210)	(0.028)	(0.064)
CDS Bid Ask	7.708***	2.637***	9.415***
	(0.285)	(0.047)	(0.090)
$1_{type=distress}$			16.826***
			(0.243)
Mat.Amihud×$1_{type=distress}$			2.052***
			(0.002)
Leverage	33.693***	7.032***	19.932***
	(0.541)	(0.093)	(0.179)

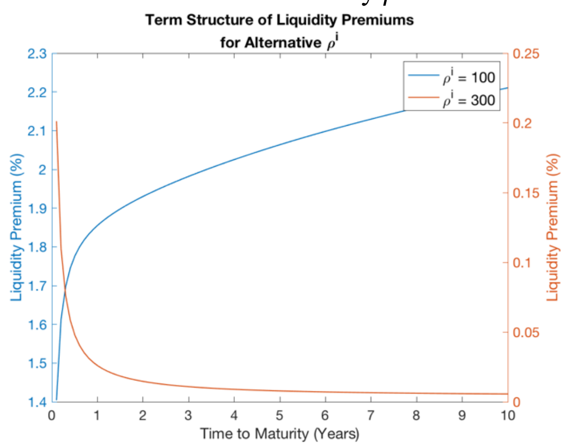
Recovery Rate	−25.020***	−5.544***	−13.781***
	(0.312)	(0.055)	(0.087)
Total Payout	0.891***	3.114***	1.923***
	(0.253)	(0.071)	(0.129)
TTM	−5.552***	−0.477***	−1.153***
	(0.453)	(0.041)	(0.087)
Coupon	−2.481*	−3.002***	1.417***
	(1.381)	(0.207)	(0.428)
VIX	23.633***	7.730***	12.524***
	(0.238)	(0.029)	(0.058)
Hazard Rate	93.052***	36.005***	86.736***
	(0.314)	(0.049)	(0.133)
Constant	367.596***	81.941***	142.253***
	(2.249)	(0.418)	(0.959)
Entity FE	Yes	Yes	Yes
Year FE	Yes	Yes	Yes
Observations	68,911	235,914	304,825
Adjusted R^2	0.818	0.917	0.945

Note: *p<0.1; **p<0.05; ***p<0.01

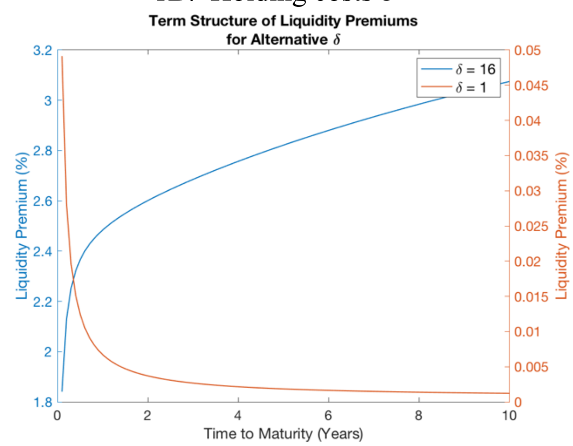
Figure 1: Comparative static analysis of DGP based term structures of liquidity premiums

These figures plot the term structure of liquidity premium for alternative values of: A) the search intensity ρ^i, B) the holding costs δ, C) the hazard rate of liquidity shocks λ, and D) the conditional probability of the liquidity shock being adverse π. In the spirit of Feldhütter (2012), the baseline parameters in the model are $\mathrm{r} = 0.1$, $\mathrm{C} = 7$, $\mathrm{F} = 100$, $\mathrm{z} = 0.9$, $\delta = 12$, $\lambda = 5$ and $\pi = 0.5$. In each of the plots below, we illustrate the impact of the variable under consideration by exclusively changing that parameter while keeping the other values fixed at their baseline specification. The liquidity premium is defined as the yield in a market where search is costly ($\rho^i =$ search intensity of the average investor) minus the yield in a market with no search costs ($\rho \to \infty$).

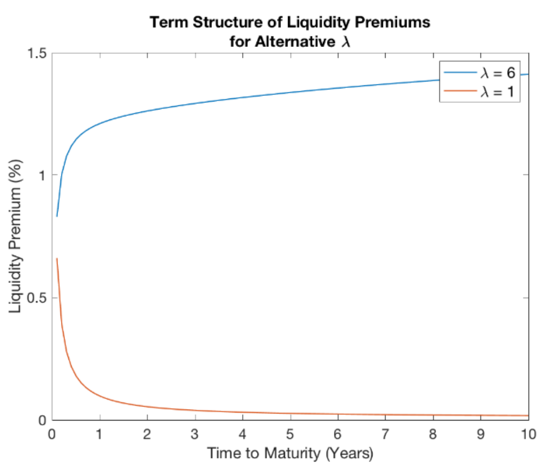
1A: Search intensity ρ^i



1B: Holding costs δ



1C: Liquidity shock hazard rate λ



1D: Probability of a binding financial constraint π

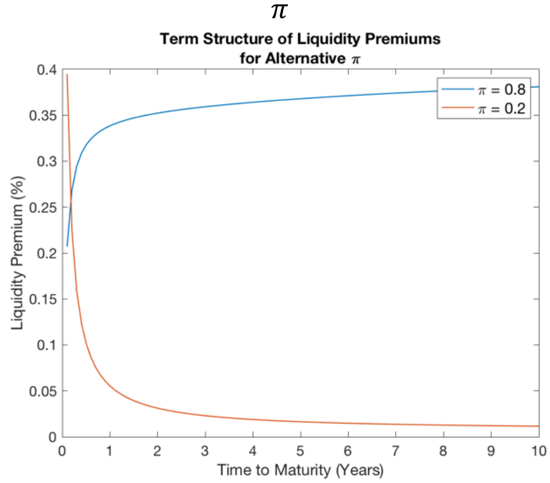
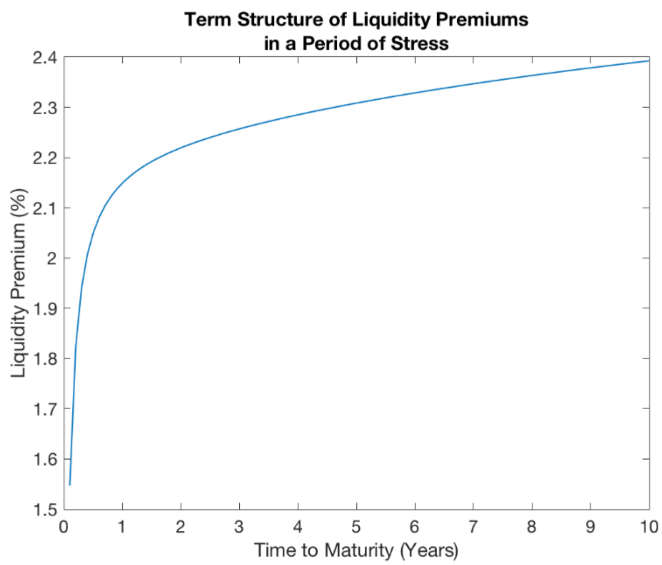


Figure 2: Illustrative DGP plots of the term structure of liquidity in periods of stress and post stress

The figures below reflect the term structure of the liquidity premium implied by the DGP model after selecting parameters for the search intensity ρ^i, the holding costs δ, the hazard rate of liquidity shocks λ, and the conditional probability of the liquidity shock being adverse π, that best represent periods of stress (Figure 2A) and post stress (Figure 2B). The motivation for our inputs in both periods is provided in the hypothesis section. Other variables of the model were chosen in the spirit of Feldhütter (2012) and are: $\mathrm{r} = 0.1$, $\mathrm{C} = 7$, $\mathrm{F} = 100$, $\mathrm{z} = 0.9$. The liquidity premium is defined as the yield in a market where search is costly (ρ^i = search intensity of the average investor) minus the yield in a market with no search costs ($\rho \to \infty$).

2A: Stress



2B: Post stress

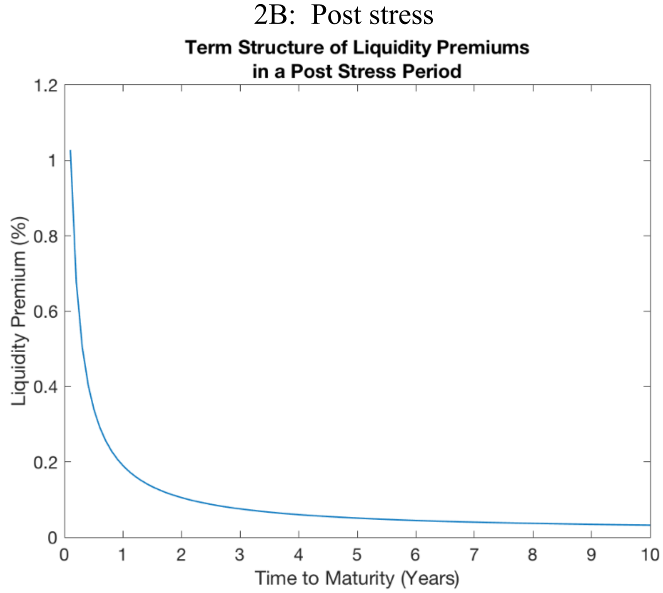


Figure 3: Time series of liquidity measures for corporate bonds

The figure below shows the time series plot from 2008 to 2012 of four liquidity measures for all corporate bonds in TRACE: The bid-ask, Amihud, Roundtrip, and interquartile range. To remove errors, cancellations, corrections, and reversals in the TRACE data, we filter the intraday transaction data according to the procedure described in Dick-Nielsen (2009). The formulas for the computation of these measures as well as the description of the data are presented in the methodology section. The liquidity measures are standardized by subtracting the mean and dividing by the standard deviation of the full window which shows that all measures behave similarly.

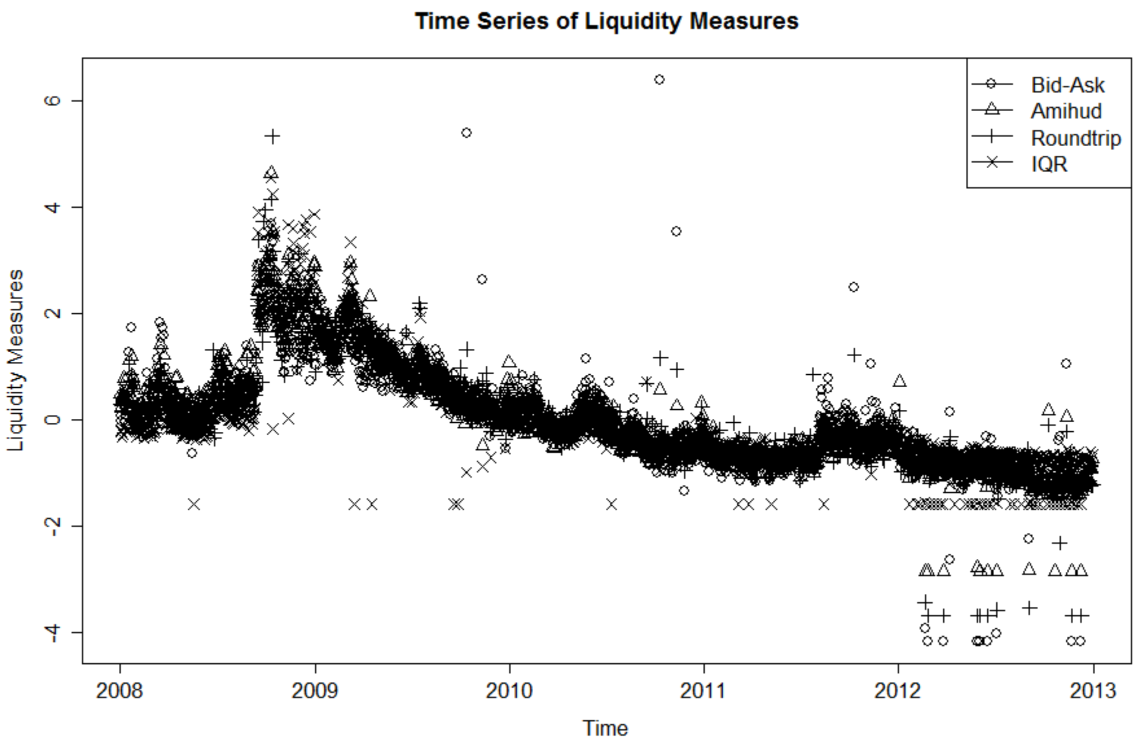


Figure 4: VIX and Fed fund futures prices

This figure plots the VIX (blue circle, axis on the left) and the price of the Fed Fund Futures (red triangle, axis on the right) for our period of interest, October 2008 to December 2012. We define the periods that we use in this study according to significant changes in the VIX and the Fed funds futures price. Periods 1 and 2 below are considered periods of "stress", while Periods 3, 4, 5, are considered " post stress" periods. The vertical lines divide the sample into the periods 1) October 1, 2008 to April 1, 2009, 2) April 1, 2009 to November 30, 2009, 3) November 30, 2009 to July 4, 2010, 4) July 4 2010 to July 1, 2011, and 5) July 1, 2011 – Dec 31, 2012.

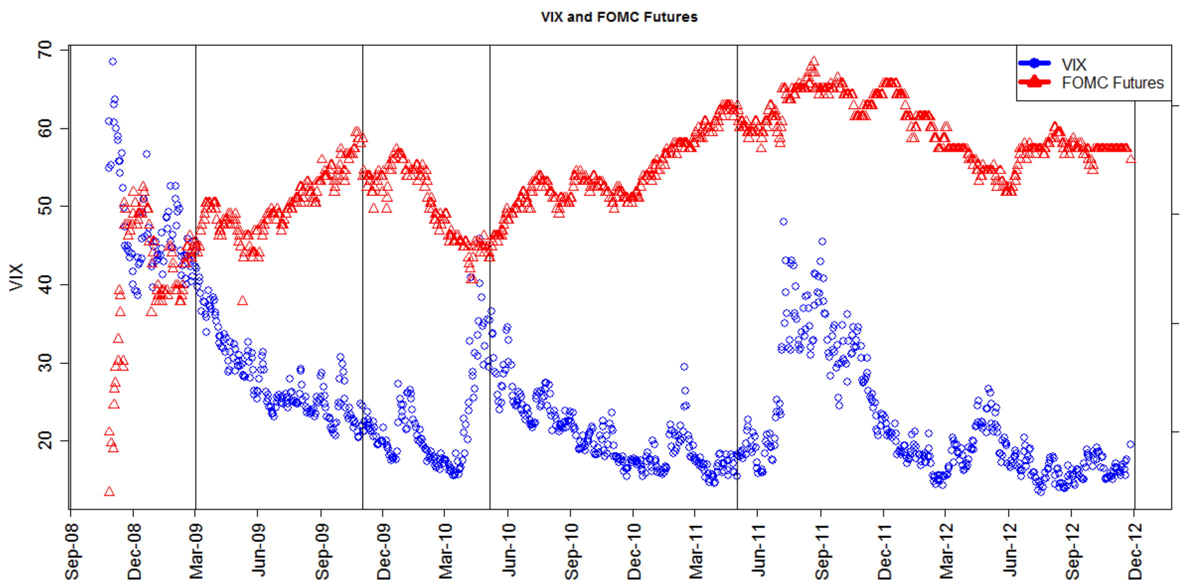


Figure 5: Term structure of raw spreads for insured corporate bonds

This figure depicts the raw spreads of the insured corporate bonds over equal maturity Treasury bonds for the two periods: "stress" and "post stress". The vertical axis is raw spread in basis points and the horizontal is time to maturity. The period of stress is from October 1, 2008 to November 30, 2009, while the post stress period goes from November 30, 2009 to Dec 31, 2012. Bond yields are obtained from TRACE and cleaned according to the Dick-Nielsen (2009) filter. Treasury yields are obtained from the H-15 website for a set of fixed maturities. The Treasury yields are interpolated to exactly match the maturity of the bonds for any given day. The plots are averages of all bond spreads with the same time to maturity. These spreads are further aggregated by weekly buckets to avoid noise and high dispersion in the data. A polynomial of best fit is also depicted in this figure.

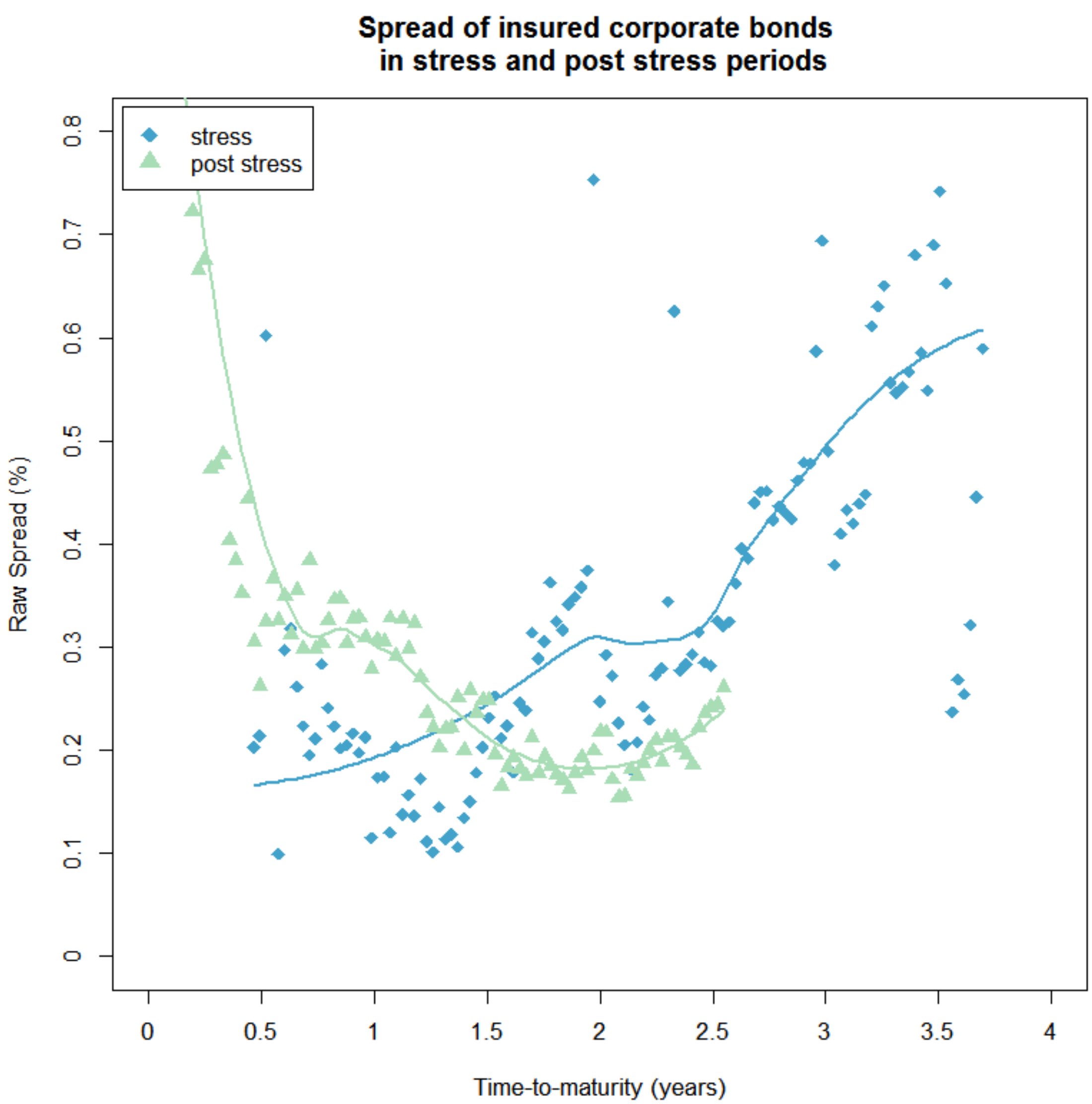


Figure 6: Term structure of liquidity spreads for insured corporate bonds for period of stress

This figure depicts the term structure of fitted spreads of the insured corporate bonds over equal maturity Treasury bonds for the period of "stress" from October 1, 2008 to November 30, 2009. The fitted spread is the predicted value of the liquidity spread (LS) from the regression of spread on the liquidity measure, the VIX, the Fed Fund futures, and other explanatory variables. Insured corporate bond yields are obtained from TRACE and cleaned according to the Dick-Nielsen (2009) filter. Treasury yields are obtained from the H-15 website. The plots are averages of all insured corporate bond spreads with the same time to maturity. These spreads are aggregated by weekly buckets because of noise and high dispersion in the data.

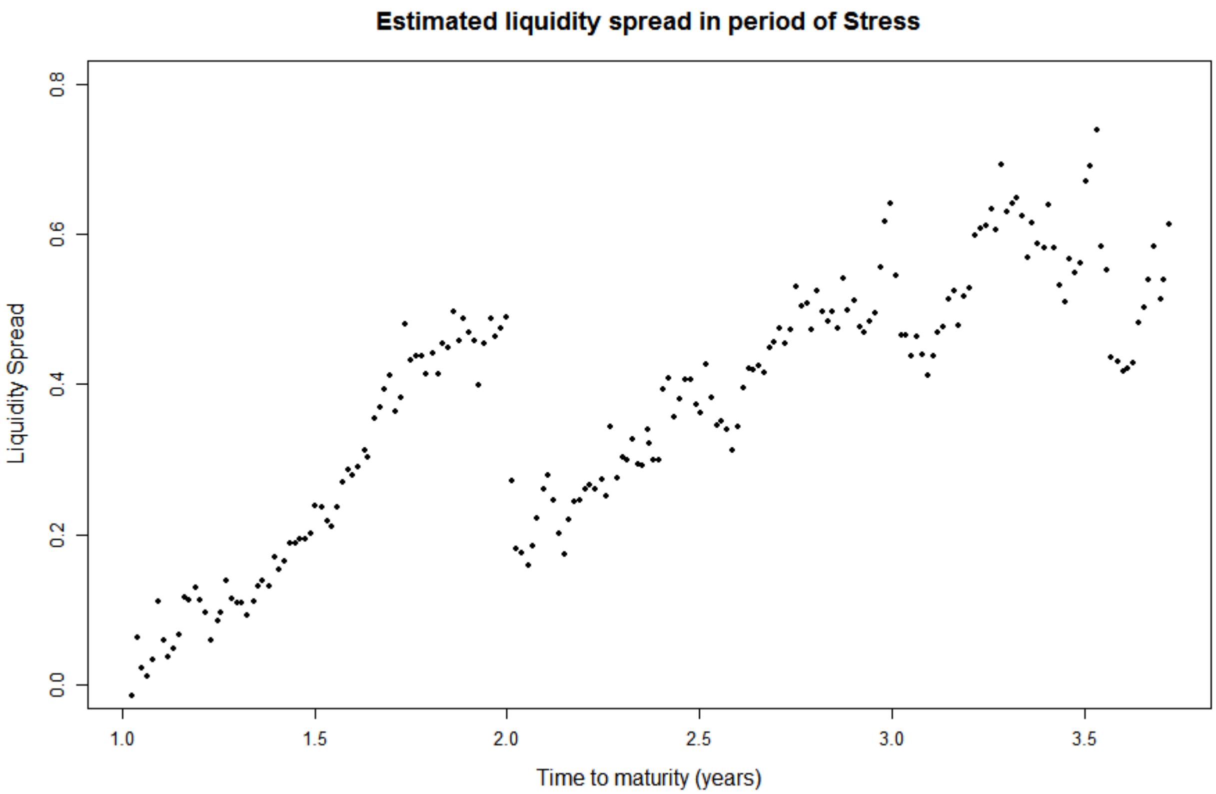


Figure 7: Term structure of liquidity spreads for insured corporate bonds for post stress period

This figure depicts the term structure of fitted spreads of the insured corporate bonds over equal maturity Treasury bonds for the post stress period, from November 30, 2009 to Dec 31, 2012. The fitted spread is the predicted value of the liquidity spread (LS) from the regression of spread on the liquidity measure, the VIX, the Fed Fund futures and other explanatory variables. Bond yields are obtained from TRACE and cleaned according to the Dick-Nielsen (2009) filter. Treasury yields are obtained from the H-15 website. The plots are averages of all bond spreads with the same time to maturity. These spreads are aggregated by weekly buckets because of noise and high dispersion in the data.

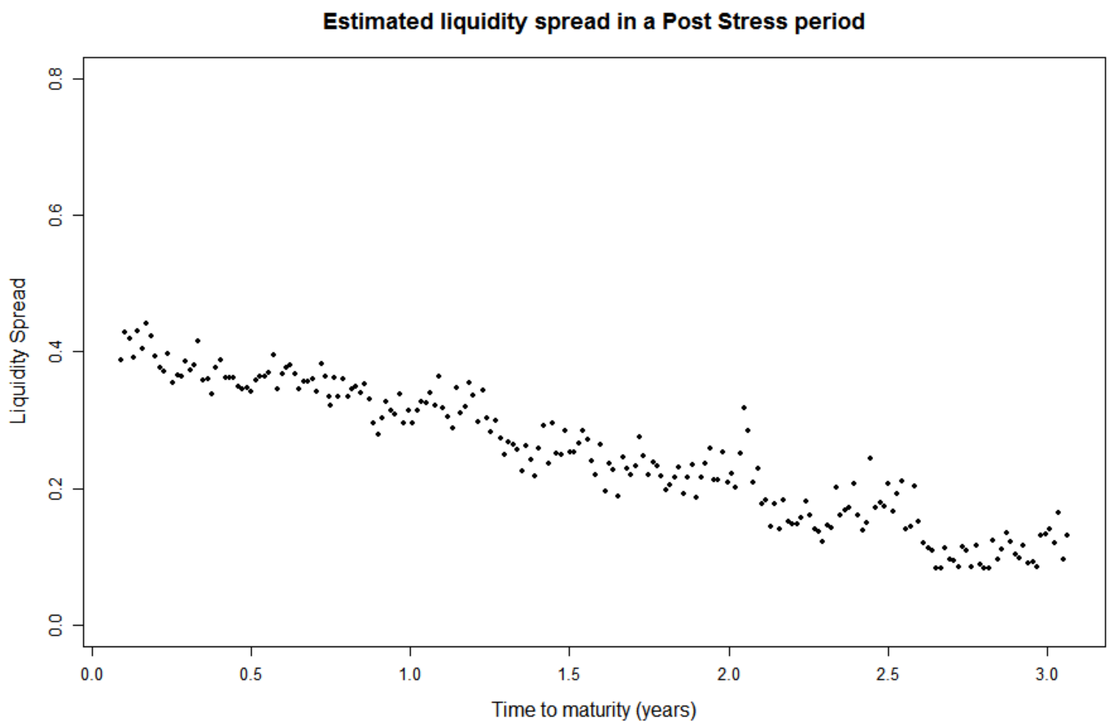


Figure 8: Term structure of insured corporate bond liquidity spreads for period of stress

To generate this figure, we first collected the estimated daily values of the liquidity spread, according to their time to maturity. These estimated daily values for the period of stress are the product of the right hand side variables and their respective parameter estimates at the time of transaction. On average, for each day of transaction, more than a dozen estimated spreads are recorded for the same time to maturity. Consequently, in order to summarize those values of relative variations in time to maturity, we minimized the sum of the squared deviations of these computed values from a mean computed value at each time to maturity (TTM). Thus, our characterization of the relationship between liquidity spread and the time to maturity is depicted in this figure. This figure depicts the term structure of spreads of the insured corporate bonds over equal maturity Treasury bonds for the period of "stress" from October 1, 2008 to November 30, 2009. Bond yields are obtained from TRACE and cleaned according to the Dick-Nielsen (2009) filter. Treasury yields are obtained from the H-15 website. These spreads are aggregated by weekly buckets because of noise and high dispersion in the data.

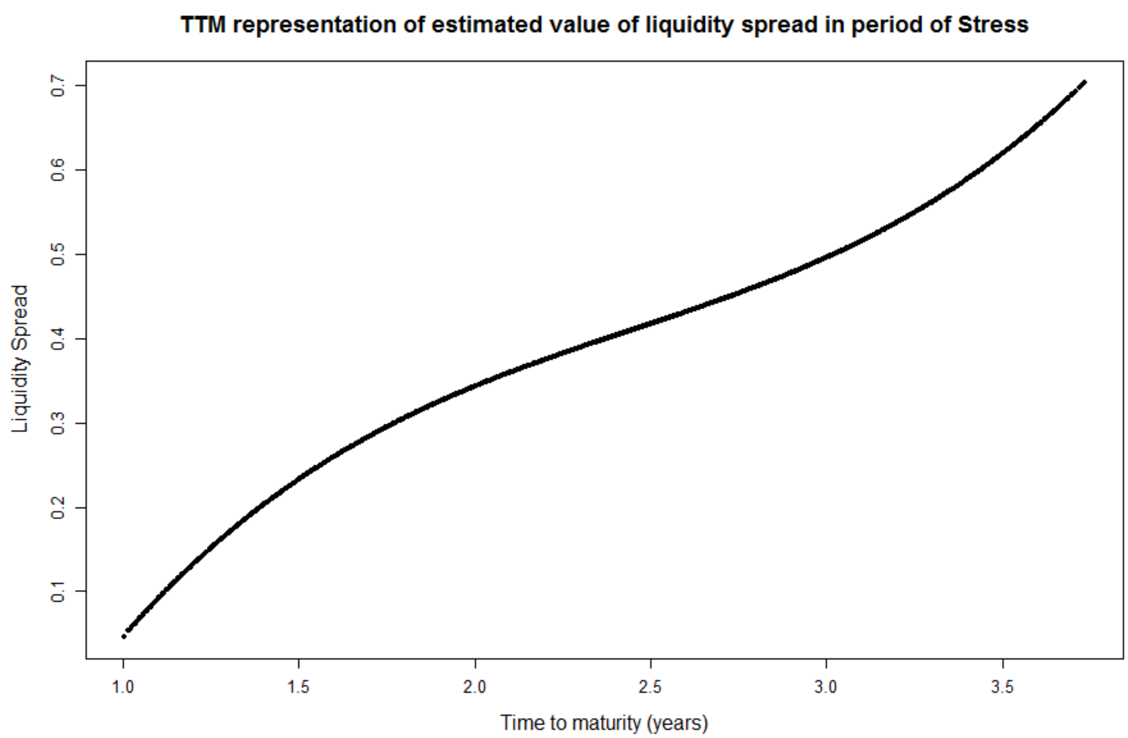


Figure 9: Term structure of insured corporate bond liquidity spreads for post stress period

To generate this figure, we first collected the estimated daily values of the liquidity spread, according to their time to maturity. These estimated daily values for the post stress period are the product of right hand side variables and their respective parameter estimates at the time of transaction. On average for each day of a transaction, more than a dozen estimated spreads are recorded for the same time to maturity. Consequently, in order to summarize those values of relative variations in time to maturity, we minimized the sum of the squared deviations of these computed values from a mean computed value at each TTM. Our characterization of the relationship between liquidity spread and the time to maturity is depicted in this figure. Bond yields are obtained from TRACE and cleaned according to the Dick-Nielsen (2009) filter. Treasury yields are obtained from the H-15 website. These spreads aggregated by weekly buckets because noise and high dispersion in the data.

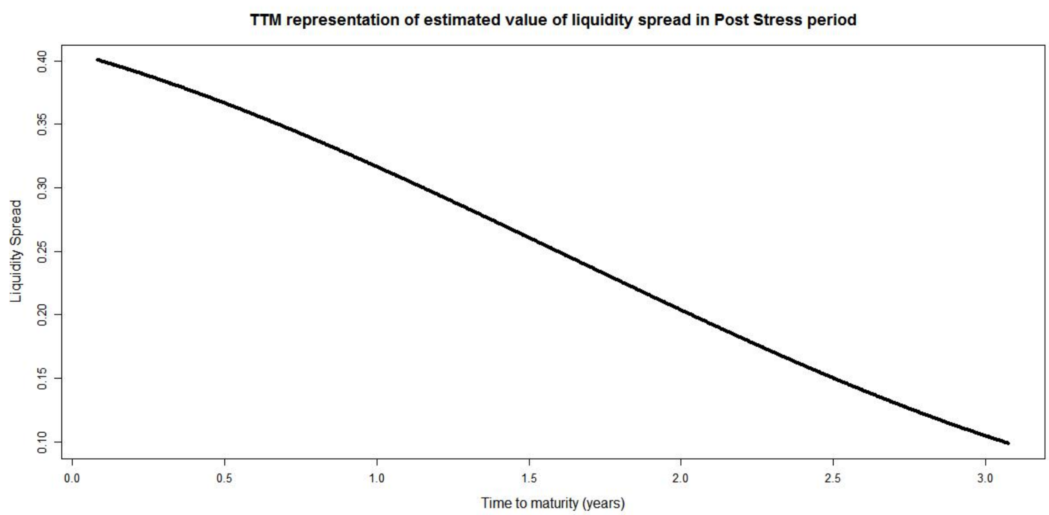
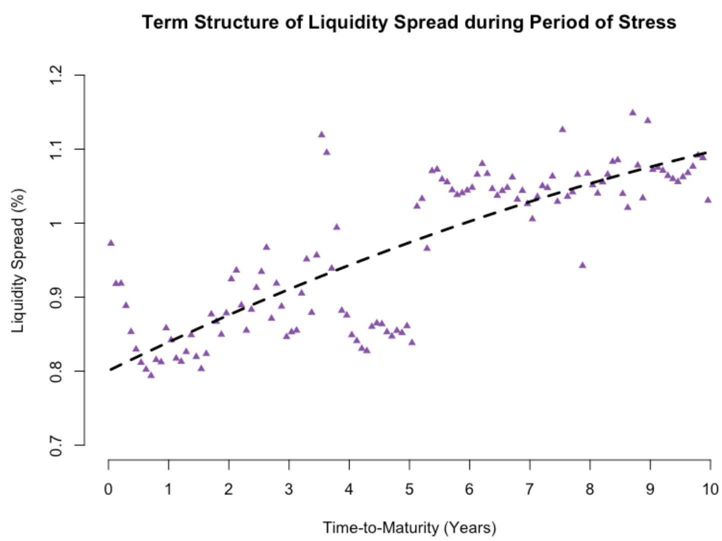


Figure 10: KF/EM implied term structures of liquidity spreads for extended sample of AAA and AA bank bonds in periods of stress and post stress

These figures depict the term structure of liquidity spreads for a sample of AAA and AA bank bonds with maturities up to 10 years. This sample excludes our insured bonds and corresponds to the parameter estimates in Columns 1 and 2 of Table 4. The liquidity spreads are extracted using a Kalman filter and the expectation maximization algorithm as presented in the methodology section. The liquidity premiums are aggregated into monthly time to maturity buckets, averaged and then plotted against TTM. The term structures of liquidity premiums are for the “stress” period (Panel A) from October 1, 2008 to November 30, 2009, and for the “post stress” period (Panel B), from November 30, 2009 to Dec 31, 2012. To better visualize the slopes, we include a second degree polynomial of best fit for each period.

Panel A: Stress Period



Panel B: Post Stress Period

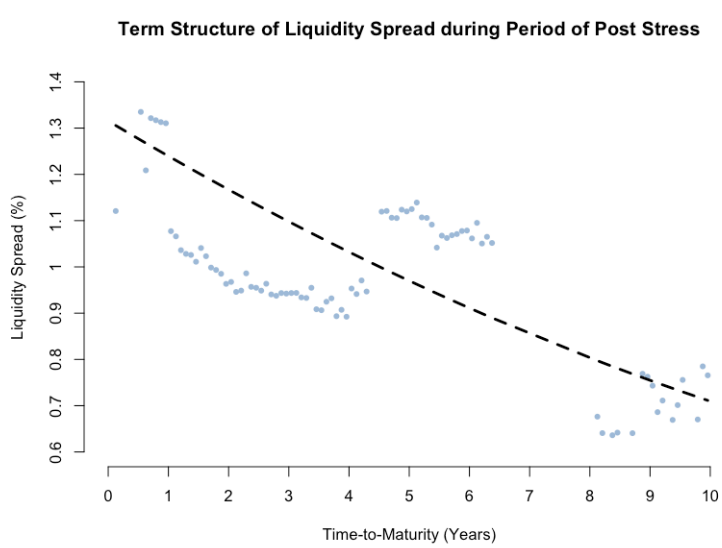


Figure 11: Kalman filter implied term structures of liquidity spreads for out of sample set of insured bonds when $\psi = 50\%$

Below, we depict out-of-sample estimation of the term structure of liquidity spreads for the insured bonds. We estimate the parameters from our KF/EM methodology using a sample that combines all uninsured bonds and 50% of our insured sample. We then estimate the liquidity premiums by feeding the sample excluded from the estimation to a Kalman filter with the previously estimated parameters in Columns 3 and 4. The term structures of liquidity premiums are for the "stress" period (Panel A), from October 1, 2008 to November 30, 2009, and for the "post stress" period (Panel B), from November 30, 2009 to Dec 31, 2012. To better visualize the slopes, we include a second degree polynomial of best fit for each period.

Panel A: Stress Period

Panel B: Post Stress Period

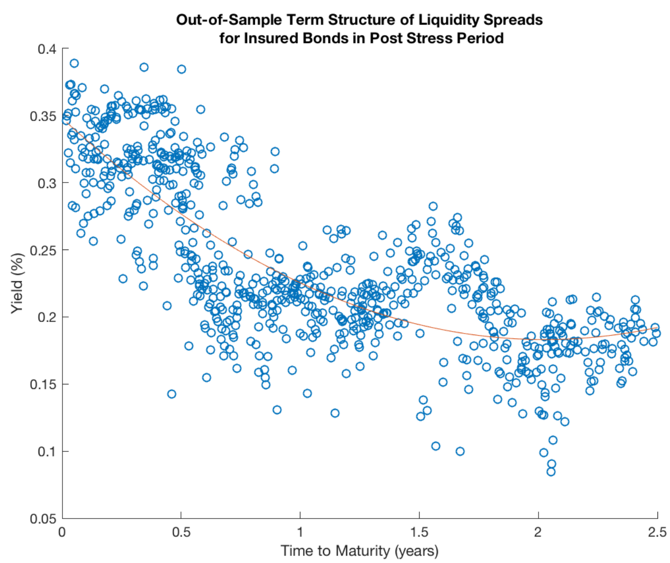
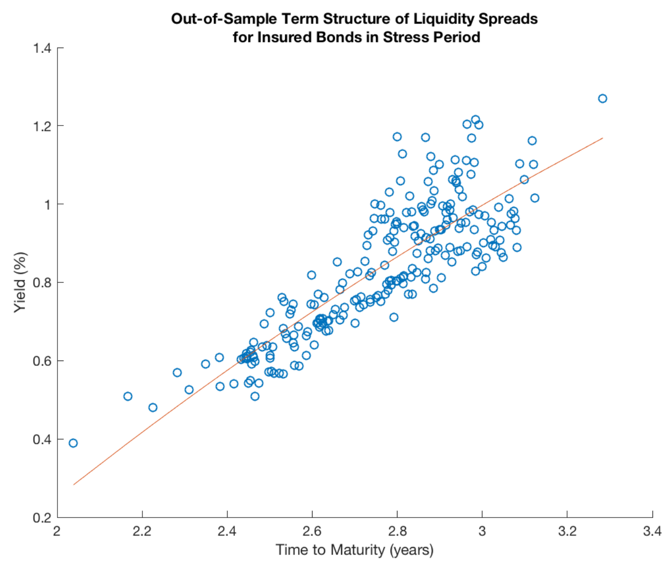


Figure 12: Kalman filter implied term structures of liquidity spreads for out of sample set of insured bonds when $\psi = 75\%$

Below, we depict out-of-sample estimation of the term structure of liquidity spreads for the insured bonds. We estimate the parameters from our KF/EM methodology using a sample that combines all uninsured bonds and 75% of our insured sample. We then estimate the liquidity premiums by feeding the sample excluded from the estimation to a Kalman filter with the previously estimated parameters in Columns 5 and 6. The term structures of liquidity premiums are for the "stress" period (Panel A), from October 1, 2008 to November 30, 2009, and for the "post stress" period (Panel B), from November 30, 2009 to Dec 31, 2012. To better visualize the slopes, we include a second degree polynomial of best fit for each period.

Panel A: Stress Period



Panel B: Post Stress Period

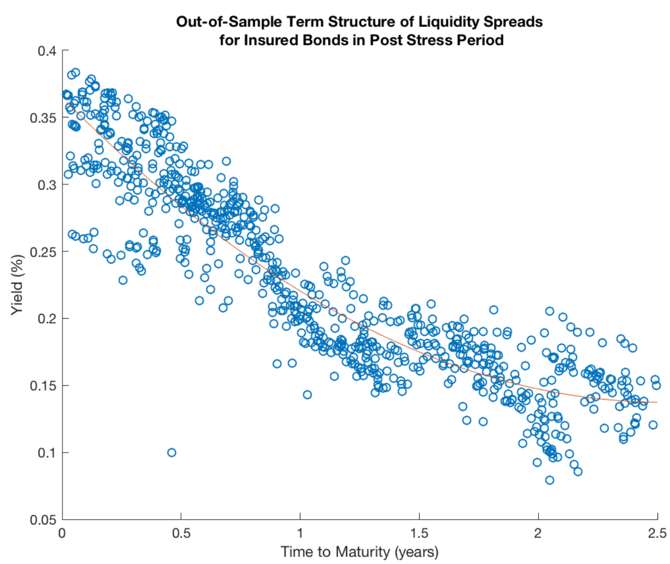
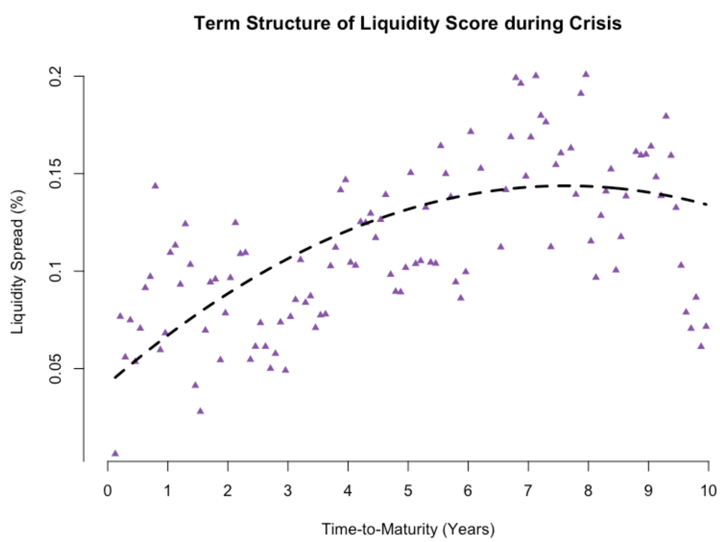


Figure 13: Term Structures of Liquidity Scores in Periods of Stress and Post Stress Implied from Projection of Credit Spread onto Liquidity and Default Proxies

These figures depict the term structure of "liquidity scores" for a sample of AAA and AA bank bonds with maturities up to 10 years. This sample excludes our insured bonds. We obtained these liquidity scores using a procedure similar to the one detailed in Dick-Nielsen, Feldhütter, and Lando (2012) and summarized in the methodology section. The liquidity scores are aggregated into monthly time to maturity buckets, averaged and then plotted against TTM. The term structures of liquidity premiums are for the crisis period (Panel A), from October 1, 2008 to November 30, 2009, and for the "post stress" period (blue, circle), from November 30, 2009 to Dec 31, 2012. To better visualize the slopes, we include a second degree polynomial of best fit for each period.

Panel A: Crisis Period



Panel B: Post Stress Period

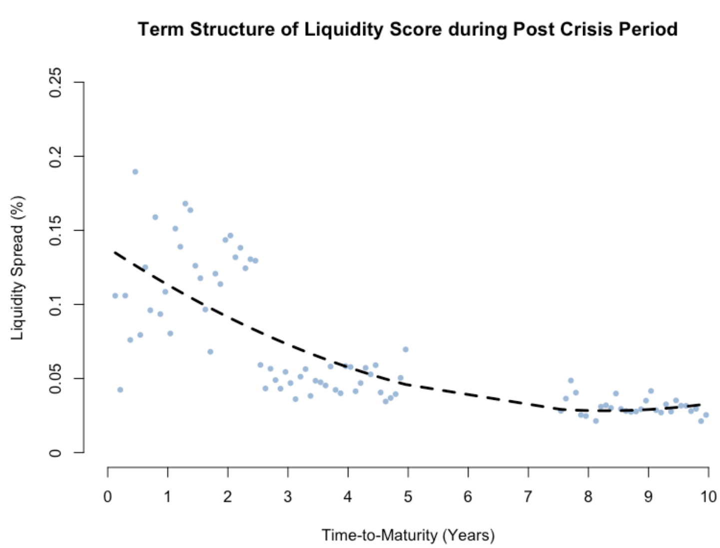
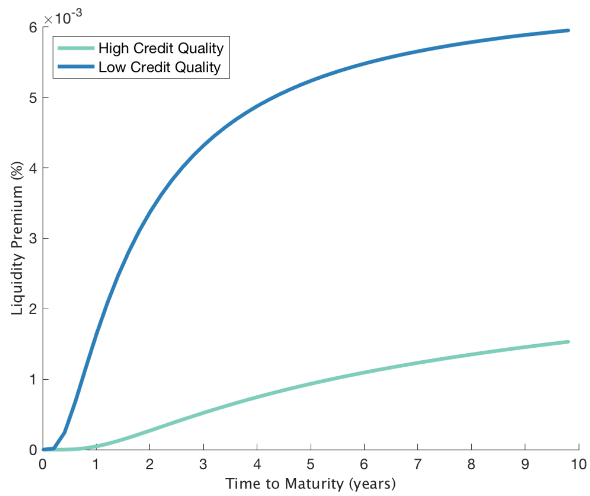


Figure 14: Illustrative plots of the term structure of liquidity premiums in periods of nonstress and stress for high and low quality bonds

The figures below reflect the term structure of the liquidity premium implied by liquidity premium inclusive model after selecting parameters for the short-term interest rate r, the firm's leverage, the firm's payout rate δ, the firm's volatility of assets σ, the fraction of asset value lost in bankruptcy α, and the fraction of the credit spread that is due to illiquidity, that best represent a period of nonstress (Figure 11A) and a period of stress (Figure 11B). In addition, in each period we present the term structures of liquidity premiums for high and low credit quality firms. The parameters used for each credit category in each firm are displayed in Table 6, while the motivation for these variables is provided in Appendix H.

Panel A: Nonstress period



Panel B: Period of Relative Stress

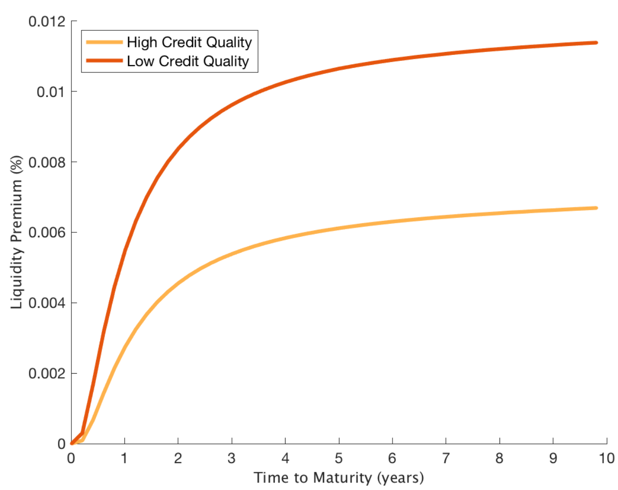


Figure 15: Estimated term structure of the liquidity premium per credit category for the pre-crisis period

The figures below summarize term structures of liquidity premiums during the pre-crisis period for the three credit rating categories: high, medium and low. The pre-crisis period starts in March 2004 and ends in October 2007. The liquidity premiums are estimated for each month and for each bond using a liquidity premium inclusive model of credit spread in the context of an unscented Kalman filter using an expectation maximization algorithm. Additional details on the model and its estimation are provided in the theory and methodology sections. The liquidity premiums are averaged by credit category and by maturity buckets. The figure below shows the result of plotting these averages versus the time to maturity for each credit category. In total, we are able to collect 235,600 bond month observations from March 2004 to November 2019, where 18,191 observations belong to the pre-crisis period.

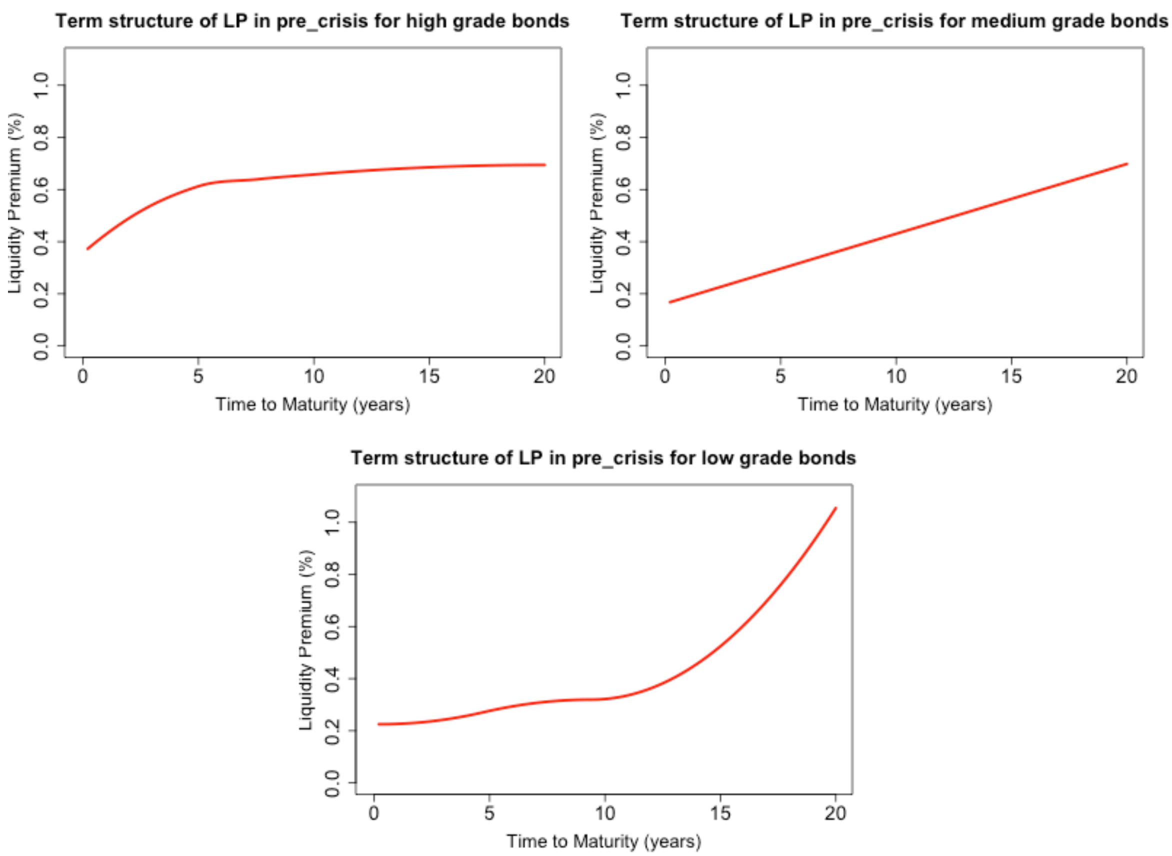


Figure 16: Estimated term structure of the liquidity premium per credit category for the crisis period

The figures below summarize term structures of liquidity premiums during the crisis period for three rating classes: high, medium, and low. The crisis period starts in November 2007 and ends in June 2009. The liquidity premiums are estimated for each month and for each bond using a liquidity premium inclusive model of credit spread in the context of an unscented Kalman filter using an expectation maximization algorithm. Additional details on the model and its estimation are provided in the theory and methodology sections. The liquidity premiums are averaged by credit category and by maturity buckets. The figure below shows the result of plotting these averages versus the time to maturity for each credit category. In total, we are able to collect 235,600 bond month observations from March 2004 to November 2019, where 23,439 observations belong to the crisis period.

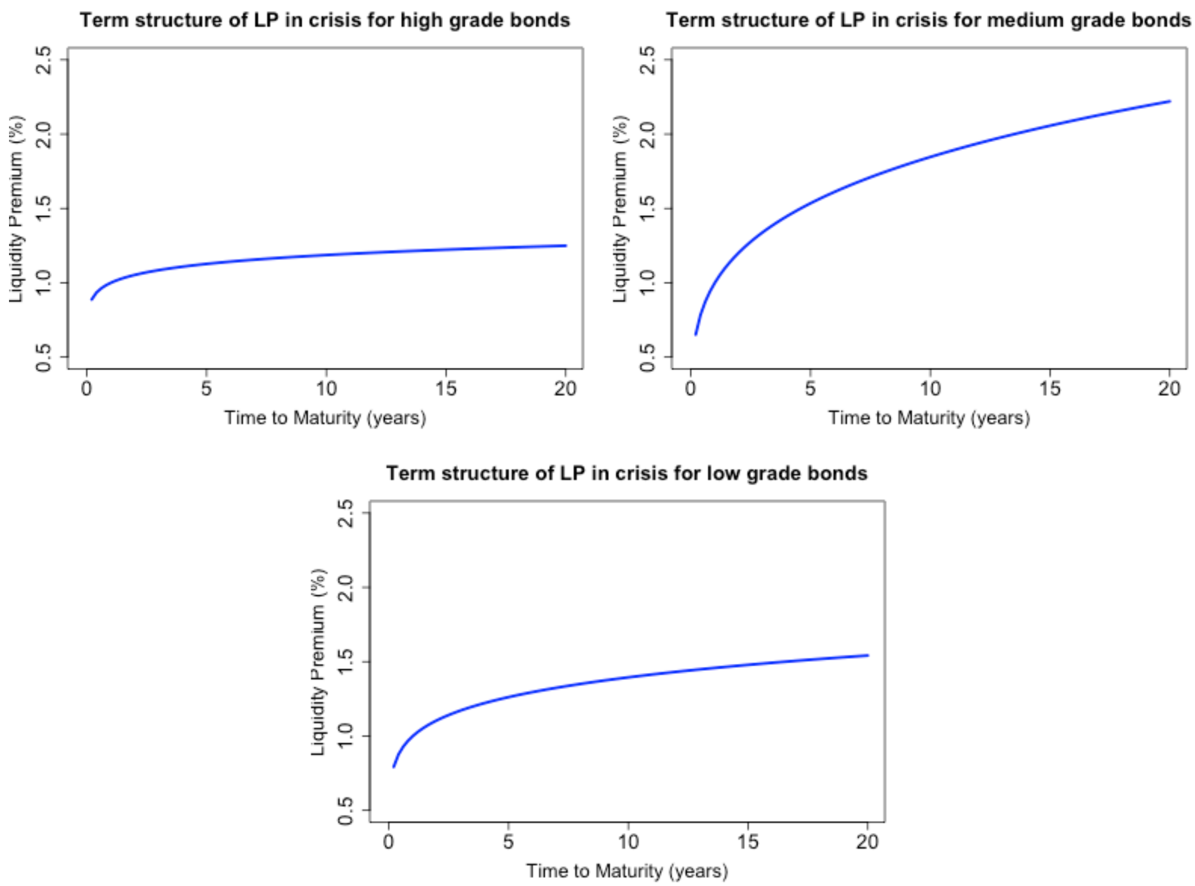


Figure 17: Estimated term structure of the liquidity premium per credit category for the post-crisis period

The figures below summarize term structures of liquidity premiums during the post-crisis period for three different corporate credit ratings: high, medium, and low. The post-crisis period starts in July 2009 and ends in November 2019. The liquidity premiums are estimated for each month and for each bond using a liquidity premium inclusive model of credit spread in the context of an unscented Kalman filter using an expectation maximization algorithm. Additional details on the model and its estimation are provided in the theory and methodology sections. The liquidity premiums are averaged by credit rating and by maturity buckets. The figure below shows the result of plotting these averages versus the time to maturity for each credit rating.

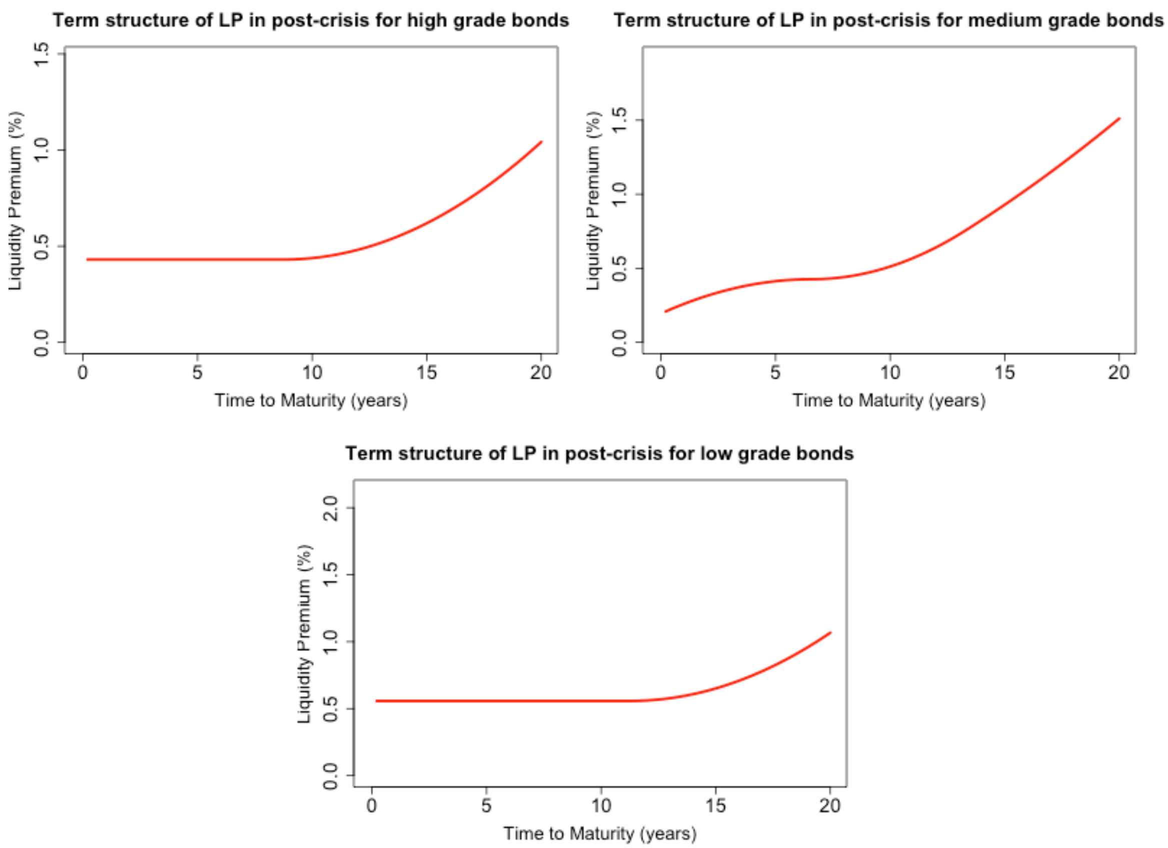


Figure 18: Illustrative plot of the relationship between liquidity premiums γ_t and the price of credit default swaps

The figure below reflects the price of credit default swaps as a function of liquidity premiums, implied by the model after selecting parameters for the short-term interest rate *r*, the firm's payout rate *δ*, the firm's volatility of assets *σ*, and the fraction of the face value of the bond paid by the insurance *R*, that best represent a typical firm in the economy. The parameters used are based on the ones in Leland and Toft (1996) and He and Xiong (2012) and are provided in the theory section.