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THE ROLE OF TEACHER PREPARATION AND EFFICACY IN TEACHING FOR DEEPER LEARNING AND STUDENT MATHEMATICAL UNDERSTANDING IN ELEMENTARY SCHOOL: A COMPARATIVE STUDY OF UNITED STATES AND SINGAPORE

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THE ROLE OF TEACHER PREPARATION AND EFFICACY IN TEACHING FOR DEEPER LEARNING AND STUDENT MATHEMATICAL UNDERSTANDING IN ELEMENTARY SCHOOL: A COMPARATIVE STUDY OF UNITED STATES AND SINGAPORE

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Dedication

For my grandfather, who stressed the importance of receiving an education and being a life-long learner. Thank you for embedding deep within me a desire to learn which encouraged me to accomplish the goals you did not have an opportunity to pursue. I know your greatest desire for your family was for us to continue to challenge ourselves and go to college.

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Abstract

Student achievement in mathematics in the United States is lagging when compared to other subjects and other countries. This is even more apparent when considering deeper learning in mathematics, being able to apply the skills and knowledge learned to new situations. Despite changes to curriculum standards and assessments over the past decade, student achievement has made little to no improvement. The purpose of this study is to examine a country that excels at producing students who are deeper learners of mathematics, Singapore, and compare the characteristics of teachers in that country to teachers in the United States to determine what is related to the disparity between student performance in the two countries. A hierarchal linear modeling analysis was completed on data from the 2015 Trends in Mathematics and Science Study for Singapore and the United States to investigate if deeper teacher understanding of mathematics produced students who achieved at higher levels for mathematics applying and mathematics reasoning. Results suggest that teacher efficacy is crucial to student success in both countries. Teacher deeper understanding from preparedness was not necessary for students in Singapore to succeed at high levels; however, it was important for students in the United States. Teacher preparation programs in the United States would be well served to review these findings and make adjustments to their curricula to increase teacher feelings of mathematics efficacy and mathematics preparedness in order to produce teachers who are able to successfully instruct students in a way that will encourage improved performance in mathematics.

Chapter 1: Introduction

There has been a well-documented decline in the competitiveness of the United States with other countries in academic achievement, particularly in mathematics. There is a significant difference between the mathematics performance of children in the United States when compared to other countries, especially when taking the needs of 21st century skills into consideration. The performance by United States students on the mathematics portion of the Programme for International Student Assessment (PISA) when compared to students in other countries is alarming, with the United States scoring in the bottom third of participating countries. These results are not isolated to 2015; they are actually lower than US students' PISA scores from the 2012 administration (OECD, 2016). Student performance on the PISA can provide insight on their deeper learning. There are six performance levels on the PISA and attaining Levels 5 and 6 are considered "top performers" because of the complexity of the problems and the cognitive requirements to successfully complete it. In 2012, the United States only had 6.6-percent of the population score at a Level 5 in Mathematics and 2.2-percent score at a Level 6. Alternatively, Singapore was one of the top performers on the 2012 PISA, with 21.0-percent of the population scoring at a Level 5 in Mathematics and 19.0-percent score at a Level 6. Where the United States only had 8.8-percent of the population score high enough to be considered a "top performer", 40.0-percent of the Singapore population attained the necessary score as a "top performer" (Rothman, 2013).

One possible reason for this discrepancy, according the Mathematics Teaching in the 21st century study, is that teachers in the United States do not possess the same mathematical knowledge base as their corresponding colleagues in other countries

(Schmidt et al., 2007). Teachers tend to teach in the same way they were trained or how they were taught when they were students themselves (Ball, 1988). For a change to occur in the way mathematics curriculum is taught, this pattern must be broken. For students to gain deeper grasp of mathematics concepts and processes, teachers need to have a deeper understanding of the material which is being taught so this material can be taught at a deeper level.

Deeper learning is a necessary component of student learning to ensure the development of needed 21st century skills occurs. The three domains of deeper learning are cognitive, intrapersonal, and interpersonal (National Research Council, 2012). Through the cognitive domain, development of thinking, problem-solving skills, and reasoning occurs. The intrapersonal domain encompasses emotions, feelings and self-regulation while the interpersonal domain focuses on how information is expressed to others as well as how to interpret other's message and responding appropriately. These three domains are intertwined in human development and learning. To be in alignment with the characteristics of deeper learning and development of 21st century skills, mathematics instruction needs to move beyond being "a mile wide and an inch deep" for student learning to improve in this area. For student learning to improve, teachers must adapt teaching practices to align with the demands of 21st century skills. One critical component of this change in teaching practice is a shift in teacher preparation programs and professional development to emphasize the characteristics of deeper learning and providing students the 21st century skills necessary to be college and career ready.

Statement of the Problem

The National Center for Educational Statistics (NCES) administers the National Assessment of Educational Progress (NAEP) Reading and Mathematics tests to students in Grades 4, 8, and 12, typically on a two-year rotation. The goal of the Mathematics test is to measure student mathematical knowledge and the ability to use that knowledge in situations that require problem-solving. Since the same test is administered nationwide, comparisons regarding student achievement can be made among states. According to results from the NAEP test, Oklahoma continues to lag behind the national average in performance on the eighth-grade mathematics exam—only 25-percent of Oklahoma eighth graders score at or above the proficiency level on the mathematics test, compared to 33-percent nationwide. The average score for Oklahoma was not significantly different than 12 states/jurisdictions; however, the Oklahoma score was significantly lower than 34 states/jurisdictions and only significantly higher than 5 states/jurisdictions.

Currently, mathematics teachers in the United States, specifically at the middle school level, are lagging behind teachers from other nations in mathematical content knowledge and understanding (Schmidt et al., 2007). Programs for teacher preparation that generate teachers who utilize deeper learning approaches in the classroom recognize that knowledge of mathematics is a crucial component for teacher preparedness, as well as is encouraging modeling, providing formal feedback and inquiry within the classroom, and encouraging teachers to use cross-curricular instruction (Heller & Gerwin, 2016). However, novice teachers who are trained through such a program still need the support and encouragement of veteran teachers to ensure they do not revert back to older patterns of teaching when difficulties occur in the classroom or the school. Furthermore, providing professional development to veteran teachers is also crucial not only to encourage and support them in implementing this shift in their classroom practice, but also to empower them to serve novice teachers as mentors and resources for developing the skills necessary to effectively implement mathematics deeper learning and 21st century skill development in the classroom (Boaler, 2015; Dweck, 2008; Lampert, 2015; Pepin, Xu, Trouche, & Wang, 2017; Taylor, 2014).

Teachers who possess mathematics self-efficacy have confidence in their mathematical abilities and recognize that mathematics is more than memorizing rules and algorithms (Kinach, 2002; Lampert, 2015). Unfortunately, many teachers in the elementary and middle school settings do not have a high level of mathematics self-efficacy when compared to high school mathematics teachers (Midgley et al., 1989; Schmidt et al., 2007). Studies have found that the mathematics self-efficacy of the teacher has a profound effect on the mathematical comprehension of their students (Midgley, Feldlaufer, & Eccles, 1989; Newton, Evans, Leonard, & Eastburn, 2012). Furthermore, prior studies have highlighted the importance of deeper learning for student success, the need for increased mathematics professional development for mathematics teachers in the United States to remain in competition with their colleagues in other countries, and the importance of the mathematics self-efficacy on student mathematics understanding and achievement (Ball, 1988; Kinach, 2002; Lampert, 2015; Midgley et al., 1989; National Research Council, 2012; Newton et al, 2012; Schmidt et al., 2007). It is imperative for United States educators to evaluate the current capacity of mathematics teachers regarding teacher preparedness and mathematics efficacy around teaching for deeper learning. It stands to reason that only when teachers are trained to take a deeper learning approach to teaching mathematics and believe they can do it effectively can we expect deeper teaching and deeper learning to materialize in the classroom. Only then can U.S. students develop the 21st century skills they need in the today's global economy, but also experience greater mathematics success with respect to peers around the world.

Purpose

The purpose of this study is to assess the relationship between teacher preservice and in-service professional development and efficacy around deeper learning, increased teaching for deeper learning, and student understanding in the mathematics classroom. To gain a more comprehensive picture of these relationships, I will compare the two key countries who differ in their achievement of deeper learning outcomes, the US and Singapore, at the elementary school level. The following research questions will frame the dissertation study of these two countries:

1. What is the relationship between teacher preparation (both pre-service and in-service), mathematics teacher efficacy, and deeper learning in mathematics at the elementary school level?
2. What is the relationship between deeper teacher understanding in mathematics (as measured by teacher efficacy and preparation) and teaching for deeper learning practice at the elementary school level?
3. What is the relationship between teacher deeper learning practices and student deeper learning of mathematics at the elementary school level?

Chapter 2: Review of Literature

The purpose of this review of the literature is to provide a foundation for examining the role of preservice preparation and professional development on teacher confidence and beliefs and how these characteristics contribute to or hamper the development of teaching for deeper learning. The review begins by considering the role mathematics content knowledge plays in the development of teacher mathematics efficacy and beliefs. Additionally, the interaction between pre-service mathematics preparation and mathematical content knowledge, efficacy and beliefs is considered. Likewise, the role of professional development in the development of mathematical content knowledge, efficacy, and beliefs is also explored. The constructs of mathematics content knowledge, pre-service mathematics preparation, and mathematics professional development is examined through the lens of deeper learning and the resultant characteristics of the mathematics classroom where deeper learning occurs.

Mathematics Content Knowledge

Teachers have a variety of origins for the mathematics content knowledge they bring with them to the classroom. This includes the instruction received through professional development, preservice learning as part of the undergraduate program at their university, and personal experience of being a student in elementary and secondary school. It is important to recognize that preservice teachers do not enter their mathematics teaching coursework with a blank slate; they already have preconceived understandings of what and how to teach mathematics that must be taken into consideration when guiding them through their methods courses.

The goal of the methods course is to turn subject-matter knowledge into pedagogical content knowledge. Teachers tend to fall within two camps of thought: instrumentalist and relationalist. For the instrumentalist, mathematics instruction is about remembering facts, rules, and procedures to determine student achievement; understanding the "why" behind the rules is not as important as remembering the procedure to be followed. Relationalists view mathematics instruction as problem-posing, and engaging in critical and contextual thinking, and demonstrating the ability to justify and represent one's thinking as evidence of mathematical understanding (Kinach, 2002).

Ball (1988) asserted that the subject matter knowledge required for teaching is likely not the same as the knowledge required in life. Many preservice teachers' experiences with learning mathematics was focused on them reproducing what the teacher wanted with little to no emphasis placed on how they were able to arrive at the answer given. Algorithms were provided with no explanation as to the origin of the algorithm or why it worked. Subject matter content without the pedagogical component is not sufficient for producing preservice teachers who are capable of deviating from a more traditional style of mathematics teaching towards a more student-centered, constructivist teaching approach (Ball, 1988).

Mathematics content knowledge is important for not only secondary teachers, but elementary and middle school teachers as well (Schmidt et al., 2007). Foundational concepts introduced at all levels requires teachers to understand the material to successfully teach these skills to students. As students progress into high school, conceptual understanding of fundamental mathematical ideas determines future success in high school mathematics courses (Evans, 2013; Garcia, Sanchez, & Escudero, 2006;

Witterholt, Goedhart, & Suhre, 2016). In addition to the content knowledge needed, practical knowledge is also necessary. This type of knowledge includes not only the subject matter, but it also encompasses knowledge of students, student learning and understanding, purposes, curriculum, and instructional strategies.

The ability to teach well in classrooms is strongly dependent upon the mathematical content knowledge of the teacher. However, currently among elementary teachers there is a deficit in the mathematics understanding, including numeracy, quantitative literacy, computational fluency, and mathematics literacy. On average, these teachers understand approximately half of the computational processes taught in grades one through six. Additionally, introduction of pedagogical methods without an understanding of the corresponding content does not provide a significant improvement in teaching efficiency; understanding 'how' to teach the concept is not really possible without first a fundamental understanding of the concept itself (Hine, 2015).

As numerous studies have documented, many pre-service teachers have been exposed to very little mathematics in their teacher preparation program, especially at the elementary level; however, a high level of content knowledge does not necessarily guarantee either that a teacher is able to successfully convey that information to students in a way that the student can comprehend. As a result, it is important for pre-service programs to not only focus on mathematical content knowledge but also consider pedagogical content knowledge as well. As the United States Department of Education (2008) notes, "Teachers must know in detail the mathematical content they are responsible for teaching and its connections to other important mathematics, both prior and beyond to the level they are assigned to teach" (p. 37). Despite this acknowledgement, there are still a large number of pre-service elementary teachers who enter education preparation programs with a limited mathematics background on which to build their pedagogical content knowledge (Norton, 2012).

The importance of mathematics content knowledge extends beyond course instruction and includes student assessment. Teachers who have a high level of mathematics content knowledge are often more comfortable with alternative methods of student assessment than the traditional summative assessment at the end of each chapter or unit (Heritage, Kim, Vendlinski, & Herman, 2008; Randel, Apthorp, Beesley, Clark, & Wang, 2016; Webb, 2010). With formative assessment, students can get feedback at a faster rate than if only a summative assessment is used. Through formative assessment, teachers recognize student errors in thought process and correct the errors before the behavior becomes habitual. In order for this type of assessment to occur, teachers must have a solid understanding of the mathematical concepts to determine what mistakes the students are making and adjust accordingly (Randel, Apthorp, Beesley, Clark, & Wang, 2016; Webb, 2010).

Pre-Service Preparation

Pre-service preparation is a crucial component to the creation of effective novice teachers. Many prospective elementary and middle level teachers have a greater concern about teaching mathematics than for any other subject (Aljaberi & Gheith, 2018; Ball, 1988; Looney, Perry, & Steck, 2017; Schmidt et al., 2007). In order to be effective, universities must realize prospective teachers enter the program with feelings and beliefs about mathematics, primarily from past experiences as a student in a mathematics classroom. Being able to teach differently than they were taught requires preservice teachers to be equipped with not only the mathematical content knowledge necessary to be comfortable with the material but the efficacy to transfer this information in the classroom (Aljaberi & Gheith, 2018; Ball, 1988; Norton, 2018).

Teacher preparation in the United States has done little to change the methods used for teaching mathematics today when compared to previous practice (Boaler, 2015; Hiebert, Morris, & Glass, 2003). In order for teachers to effectively instruct students to mathematics proficiency, teachers must be proficient in mathematics themselves. This proficiency depends on the development of five types of mathematical competencies: conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition. When these five competencies develop in concert, the result is proficiency in mathematics. However, learning mathematical concepts alone is not enough to provide prospective teachers the foundation necessary to effectively teach mathematics. It is crucial that prospective teachers also learn how to convey mathematics knowledge to students in a way they understand through development of competencies and dispositions for teaching (Hiebert, Morris, & Glass, 2003; Kubar & Cakiroglu, 2017).

There are significant differences between the preparation of future teachers in the United States when compared to other countries such as Taiwan, South Korea, Bulgaria, Germany, and Mexico. Teacher preparation in the United States does not provide preservice teachers with the same exposure to mathematics content as the programs in other countries. On the analysis (functions) test, United States teachers scored one and one-half standard deviations below the Taiwanese future teachers and near the bottom on the algebra and functions tests when compared to the other countries studied (Schmidt et al., 2007). In Singapore, preservice teachers are subjected to a stringent screening process regarding their academic and non-academic achievements prior to admission into the teacher preparation program. Since preservice teachers enter the program with high levels of understanding of mathematical concepts, their training is more rigorous, and, for primary and middle school teachers, often in a shorter amount of time – two years in Singapore compared to four years in the United States – than what occurs in a United States teacher preparation program (Ginsburg, Leinwand, Anstrom, & Pollock, 2005). Preservice teachers in the United States who completed a secondary mathematics program had a better understanding of mathematics and outperformed their elementary and middle school program counterparts because these teachers were required to complete a greater number of advanced mathematics coursework as part of the teacher preparation program (Schmidt et al., 2007); however, the United States is still lagging behind Singapore (Ginsburg et al., 2005).

Teacher preparation in mathematics, especially for elementary preservice teachers, is vital because, many times, the primary teacher is a child's first exposure to formal numeracy concepts. Being able to help students bridge the relationship between concrete understanding and abstract thought is a crucial component of effective elementary and middle level mathematics instruction (Boran, Tarim, & Özsezer, 2018; Norton, 2018; Rieche, Leuders, & Renkl, 2019; Schmidt et al., 2007); however, many education programs do not provide preservice teachers the opportunity to develop the pedagogical understanding necessary to provide this level of instruction (Akyeampong, Lussier, Pryor, & Westbrook, 2013).

A large number of preservice teachers in the United States do not feel qualified to teach the mathematical content that will be part of their future teaching assignment

(Althauser, 2018; Ball, 1988; Hine, 2015). Elementary teachers have a greater level of mathematics anxiety than other undergraduate majors. Additionally, teachers tend to teach concepts they feel confident in, which makes teacher preparation in the field of mathematics vitally important. Many education programs do not afford preservice teachers the opportunity to become more comfortable with the mathematics. When preservice teachers are not given the opportunity to become more familiar with mathematical concepts, they are less likely to take risks in the mathematics classroom with instruction. Teacher education programs need to provide preservice teachers the guidance necessary to grow in not only mathematics content, but mathematical pedagogical understanding as well (Althauser, 2018; Hine, 2015).

Pre-service teachers enter teacher preparation programs with previous views on mathematics instruction. One of the main challenges of teacher education programs is to encourage preservice teachers to deviate from previous views and be receptive to a more student-led mathematics classroom. While many pre-service teachers have been taught mathematics in a traditional setting, the need to transition away from this type of instruction is crucial for deeper learning in mathematics. The role of the teacher preparation program is to recognize the potential biases that pre-service teachers have upon entering their preparation program and to help the pre-service teachers recognize these previously established biases. Without making teachers aware of their predispositions, there is little opportunity for change to occur once these teachers enter the mathematics classroom; the traditional approach to instruction likely to remain commonplace at the expense of more student-led, engaging forms of mathematics instruction (Kinach, 2002).

One of the many hurdles that teacher preparation programs have is that some students enter the program with a negative attitude toward mathematics. In order to produce effective mathematics teachers, programs must combat this mentality because teacher attitudes toward the subject matter being taught tend to affect the willingness of the teacher to take risks in instruction. When transitioning from a traditional teaching style to one that is more student-led, a great deal of risk is taken on the part of the teacher; therefore, producing future teachers who do not have negative feelings toward mathematics is important, especially since teachers who have negative feelings toward content can unintentionally transfer those feelings to the students in the classroom (Beswick, Watson, & Brown, 2006; Looney, Perry, & Steck, 2017). Part of the responsibility of teacher preparation programs is to develop competent teachers and this cannot be accomplished without focusing on preservice teacher mathematical content knowledge as well as mathematical pedagogical knowledge. By doing so, the preservice teachers who complete such a teacher preparation program are more likely to have higher efficacy and a less negative attitude toward mathematics (Looney, Perry, & Steck, 2017).

There is potential for teacher preparation programs to influence preservice teacher mathematics efficacy. This can be accomplished through not only mathematics content courses but also strategically designed mathematics methods courses. The mathematics efficacy of a teacher has been shown to effect student achievement and student motivation; however, this cannot be achieved in a traditional preservice preparation program (Althauser, 2018; Evans, Leonard, Krier, & Ryan, 2013; Newton, Evans, Leonard, & Eastburn, 2012). In order to develop preservice teachers who have higher levels of efficacy, teacher preparation programs must make a focused effort to include curricula that will increase mathematical content knowledge as well as provide positive experiences with mathematics to help the preservice teachers feel more confident with the material. This is especially important for elementary preservice teachers who tend to be more apprehensive about mathematical content (Newton, Evans, Leonard, & Eastburn, 2012).

In many of the universities that train pre-service teachers in the United States, the education department often works independently of the mathematics department. This means that although preservice teachers are exposed to mathematics content through mathematics courses, the understanding of how to teach the content and encourage student critical thinking is not taught in concert with the material. While the National Council of Teachers of Mathematics (NCTM) advocates for student-led mathematics instruction (Ellis & Malloy, 2010), many preservice teachers who have been trained in programs where the education and mathematics departments are independent of one another do not have the mathematics efficacy necessary to implement such a teaching method in the classroom (Richardson & Liang, 2008). Since the mathematics knowledge gained by preservice teachers during their teacher preparation programs ultimately effects overall mathematics comprehension, it is vital for education programs to work in concert with the departments of the core subject areas to ensure the knowledge base of preservice teachers is developed while the pedagogical base is also being addressed (Richardson & Liang, 2008). While the United States has numerous teacher preparation programs – well over a thousand different institutions – there is only one institution for teacher preparation in Singapore, the National Institute of Education (NIE). Since the NIE focuses solely on teacher preparation, the various departments within the NIE are able to work in concert to develop the necessary skills within preservice teachers (Ginburg et al., 2005).

Teacher preparation programs that focus solely on mathematical content knowledge are not as effective as programs that include knowledge of content knowledge and teaching as well as specialized content knowledge and analytical knowledge. Preservice teachers typically receive instruction on mathematics content and mathematical pedagogy; however, these two concepts are typically delivered independently rather than concurrently. This arrangement makes it difficult for preservice teachers to develop specialized content knowledge and analytical knowledge, especially if the preservice teacher has struggled with mathematics in the past, has had a negative experience with mathematics, or a negative feeling toward mathematics. Teacher preparation programs that utilize a more holistic view of mathematics methods courses and mathematics content instruction produce preservice teachers who are more confident in mathematical content knowledge and, in turn, are more willing to use a more student-led approach to mathematics instruction in the classroom. The effects of such training remain in place for years after graduating the program, which demonstrates the effectiveness of such an approach to preservice preparation (Suppa, DiNapoli, & Mixwell, 2018).

Through the completion of successful teacher preparation programs where mathematics content knowledge is taught along with mathematics pedagogical knowledge and mathematical understanding for teaching, preservice teachers are able to increase mathematical efficacy. This is of particular importance during teacher preparation training because, once efficacy beliefs are established, they are less likely to change. Additionally, the higher level of mathematical efficacy held by a preservice teacher, the more willing they will be to engage in more student-led instruction rather than relying on traditional teaching methods, such as lecture and demonstration (Gabriele & Joram, 2007; Gonzalez & Maxwell, 2018; Swars, Daane, &Giesen, 2006). Instead, preservice teachers with higher levels of mathematical efficacy are more likely to utilize approaches that will result in student critical thinking and problem-solving as well as small-group and individualized instruction (Swars, Daane, & Giesen, 2006).

In-Service Professional Development

It is critical for teachers to maintain an active engagement in professional development once they have begun their work as classroom teachers in order to promote gains in student achievement. Mathematics knowledge is not static and requires regular maintenance and study to maintain high levels of mathematics instruction. This can be accomplished by teachers participating in professional development that focuses on mathematics content and instruction (Akiba & Liang, 2016; Louis & Marks, 1998; Wilson & Berne, 1999). Singapore teachers are encouraged to participate in 100 hours of professional development each year and given compensated leave time to accomplish this goal. Additionally, the Singapore professional development program is structured in a way where teachers are able, and encouraged, to upgrade their knowledge base on numerous topics and reach a deeper understanding of concepts. The Singapore system of professional development is highly structured and has multiple incentives to encourage teachers to participate in professional development. Conversely, professional development in the United States is less structured and, many times, not as robust in content material (Ginsburg et al., 2005). The amount of professional development required for teachers in the United States varies by state, county, and/or district. Schools and districts that promote regular participation in such professional development programs tend to have students who are more successful in mathematics because these activities improve teacher knowledge and instruction in the mathematics classroom. Even greater improvements are observed when the teachers in the mathematics department work collaboratively in professional development experiences because it allows teachers to communicate with one another and learn from the successes and failures of other teachers in the department (Akiba & Liang, 2016).

Continued support of teacher mathematical knowledge and pedagogical understanding is beneficial to ensure continued gains in student learning. Teachers who participated in mathematical professional development are exposed to strategies of utilizing mathematical concepts, such as proofs, in a way that will be of greater benefit to students, especially when these strategies encourage and support the development of critical thinking within the classroom. Additionally, through participation in professional development, teachers further develop their common content knowledge as well as their specialized content knowledge. When this occurs, teachers are then equipped to analyze student work to determine where errors have been made and what needs to be done to correct faulty understanding. By teachers being equipped to provide this type of feedback, student comprehension can improve (Lesseig, 2016).

Professional development has the potential to positively affect the mathematical knowledge of teachers. This is more likely to occur when the teacher views the professional development as relevant and/or necessary. The incorporation of real-world contexts is crucial not only for the teacher to recognize the relevance of the material but also, when the learned strategies are used in the classroom, to result in an increase student comprehension of the topic. It is particularly important for elementary and middle school teachers to be able to integrate this type of instruction into the classroom because they are typically the first exposure students have had to mathematical concepts and this experience will affect the development of the students' feelings toward mathematics (Polly, Martin, McGee, Wang, Lambert, & Pugalee, 2017).

Mathematics professional development not only provides the opportunity to increase teacher content knowledge and knowledge of teaching, but also improve teacher confidence in mathematics. This is especially important for teachers who have a high level of mathematics anxiety or low confidence in mathematics. To have a thorough understanding of conceptual numerical knowledge, teachers realize the meaning behind how numbers behave. Elementary and middle school teachers have a high need for this type of understanding because so much of elementary and middle school mathematics curriculum in the United States currently revolves around the concepts of numeracy and arithmetic (Boaler, 2015; Schmidt et al., 2007). Professional development in this area will help develop this knowledge to the level necessary to create effective learning experiences in the classroom (Saliga, Daviso, Stuart, & Pachnowski, 2015).

Teacher Efficacy

As cited in numerous studies, Bandura's definition of efficacy as "beliefs in one's capabilities to be organized and execute the courses of action required to produce given attainments" (p. 2). This has been further broken down into two expectancies: self-efficacy and outcome efficacy. Self-efficacy is content-specific and focused on the individuals perceived ability to successfully perform the necessary task at a satisfactory level of competency; alternatively, outcome efficacy is focused the individual determining if the task has been completed at a satisfactory level of competence. As an extension of Bandura's work, personal teacher efficacy has been defined as a teacher's belief in his or her skills and abilities to positively affect student achievement and is a vital component of effective instruction (Swackhamer, Koellner, Basile, & Kimbrough, 2009; Tschannen-Moran & Barr, 2004; Tschannen-Moran, Hoy, & Hoy, 1998). Teachers who have higher levels of teaching efficacy produce students who have higher levels of achievement and understanding of the content. Additionally, teachers with higher levels of efficacy are more willing to attempt a variety of instructional strategies in the classroom to aide student understanding of the material (Swackhamer, Koellner, Basile, & Kimbrough, 2009).

Mathematics teaching efficacy is a combination of two dimensions: personal mathematics teaching efficacy and mathematics teaching outcome expectancy. While teacher content knowledge is important, teacher mathematics efficacy is also important because it encompasses the teacher's belief in being able to teach effectively. Teachers with high mathematics content and low mathematics efficacy are not as effective as teachers with higher levels of mathematical efficacy (Kim & Seo, 2018; Midgley, Feldlaufer, & Eccles, 1989). Not only is teacher efficacy related to teacher behavior, but it is also related to student learning outcomes and achievement. The self-efficacy of preservice students in mathematics was a better predictor of the ability to solve mathematical problems than mathematics self-concept or prior experience with mathematics. This demonstrates the level of influence mathematics efficacy can have on both pre-service and in-service teachers. Not surprisingly, teachers' mathematics self-efficacy has a direct relationship with their mathematics teaching efficacy. Additionally, teachers with higher levels of efficacy are more willing to deviate from the traditional style of teaching and more willing to engage in more student-led strategies. This type of instruction is in alignment with NCTM's view on how to best foster student understanding of mathematical concepts and encourage students to make sense of mathematical procedures rather than simply follow the algorithm without an explanation of how the algorithm was discovered or why it works and for what type of problem it is best used. Without high levels of efficacy, teachers are more likely to rely on the traditional style of instruction through lecture, guided practice, and independent practice. Teachers with higher levels of mathematics efficacy also recognize the importance of mathematics and the various concepts presented within the classroom. These teachers are more likely to take ownership for student achievement and the role they have in the process (Beswick, Watson, & Brown, 2006; Briley, 2012; Gonzalez & Maxwell, 2018; Gulistan, Hussain, & Mushtag, 2017; Kim & Seo, 2018).

Teacher self-efficacy is a crucial component to teacher preparation since efficacy levels of the teacher are related to the instructional strategies utilized within the classroom. Those who have a high level of efficacy are more efficient in communicating mathematical concepts in the classroom and show greater classroom management skills when compared to teachers who do not have high levels of efficacy. Additionally, teacher efficacy affects not only the beliefs the teacher has in his or her ability to effectively instruct students in mathematical content, but it also affects the methods utilized for instruction. Teachers with higher levels of efficacy will be less reliant on lecture style instruction and more willing to incorporate methods and techniques that are more student-led and expand the locus of control in the classroom beyond that of the teacher. These teachers are more likely to be open-minded, innovative and remain in a consistent state of improvement when it comes to mathematics content and instructional practices (Bedir, 2015).

Preservice teachers who have low mathematics efficacy typically have higher levels of mathematics anxiety, which can then be passed on to their future students. Additionally, teachers tend to focus instruction on concepts they themselves are comfortable with, which opens up the possibility that there is a large amount of mathematical content that students are not exposed to because the teacher does not feel confident in teaching it. Pre-service teachers enter teacher preparation programs with a pre-established level of mathematics efficacy, and it is vital for teacher preparation programs to address the efficacy level of the students and work to increase mathematical efficacy. It is unlikely that preservice teachers will be forthcoming with their mathematical efficacy level, especially if it is low because they want to be viewed as capable of becoming an effective teacher; however, many pre-service elementary teachers enter teacher preparation programs already feeling like they will be unable to succeed in mathematics. Teachers who have a low level of mathematics efficacy are more likely to depend upon traditional teaching methods, such as lecture, which does not produce an environment that encourages the student-led experiences necessary for student critical thinking to develop (Althauser, 2018).

Pre-service preparation is not the only defining characteristic for teacher efficacy. One of the greatest contributors to the levels of teacher efficacy is the amount of understanding the teacher has of the mathematical content, whether this information originates from a teacher preparation program or if the teacher completed an alternative certification process. Regardless of the type of training the teacher received, the level of mathematics anxiety and teacher mathematics efficacy contributes to student understanding in the mathematics classroom. Whether it is pre-service instruction or in-service professional development, it is important for teachers to have the opportunity to become more comfortable with mathematics concepts so levels of efficacy can increase (Evans, 2013).

As previous studies have recognized, there is a relationship between teacher expectations and student achievement. It has also been found that teachers tend to take credit when students do well, but often blame external factors beyond teacher control when students perform poorly; this is even more apparent when the teacher has a low level of mathematics efficacy. Teachers with higher levels of efficacy recognize the important role they have in student comprehension, whether the students succeed or struggle, and will adjust teaching strategies and classroom procedures accordingly to enhance the student learning environment. Further, teachers with higher efficacy take "greater personal responsibility for their own actions and the performance of their students," while teachers with lower efficacy believe that student performance depends solely on the student and external factors beyond the teacher's locus of control (Guskfy, 1987).

As recognized by Guskfy, there is a relationship between teacher efficacy and student achievement. Teachers with low levels of self-efficacy more likely to rely on a lecture style classroom structure and tend to blame students for poor performance; however, teachers with high levels of efficacy are more likely to implement classroom strategies that encourage a student-led atmosphere and willing to take ownership for student achievement, or lack thereof. It is reasonable to conclude that the level of mathematics efficacy on the part of the teacher can be a viable predictor of future student achievement and research supports this conclusion. This demonstrates the importance of teacher mathematics efficacy and the need for pre-service and in-service training that supports the development of high levels of efficacy in all levels of education from elementary through high school (Gulistan, Hussain, & Mushtag, 2017).

Teacher efficacy is a fluid construct that is dependent upon the situation; teachers can have a high level of efficacy regarding one concept but have a low level of efficacy in other. There is a documented relationship between teacher efficacy and student achievement (Caprara, Barbaranelli, Steca & Malone, 2006; Kim & Seo, 2018; Klassen & Tze, 2014; Mohamadi & Asadzadeh, 2012). This relationship is even more significant when considering instructional strategies and student engagement. Teachers who believe they have the capacity to affect student learning have a higher level of efficacy than those who do not. These teachers tend to bring more enthusiasm to the classroom and are more likely to actively engage students to take ownership of their own learning experience (Kim & Seo, 2018; Tschannen-Moran & Barr, 2004).

Teachers who have a negative attitude toward mathematics can transfer that viewpoint onto their students. Additionally, negative feelings regarding mathematics can affect the ability of teachers to effectively teach the material to students. Instead of becoming more confident in mathematics, students who have been taught by a teacher with a negative opinion of mathematics are less likely to be exposed to as many mathematics concepts as they would with a teacher who holds a positive view of mathematics. These students are also less likely to enjoy or be confident in mathematics and more likely to develop anxiety toward the subject. It is important to recognize the effect teacher efficacy has on not only teacher instruction but also student understanding and achievement (Looney, Perry, & Steck, 2017).

Mathematical competence plays a significant role in the development of teacher efficacy, which research has shown to be related to a variety of student indicators, including achievement and motivation. Teacher efficacy is also associated with the willingness of a teacher to try alternative approaches to instruction and to remain persistent with students who struggle understanding mathematical content. While preservice teachers enter teacher preparation programs with a view of their mathematics efficacy, the education program can increase the mathematics efficacy of the pre-service teacher. The primary source of this growth in efficacy is in methods courses, especially if methods courses are structured in a way to enhance mathematic content knowledge and effective methods of conveying that knowledge to future students. Methods courses that continue to emphasize the use of teacher lecture rather than providing the tools necessary to implement student-led strategies in the classroom will not be as effective at improving teacher mathematics efficacy. By focusing on the methods course, teacher preparation programs are able to influence the teacher efficacy, especially for students who entered the course with the lowest levels of efficacy (Newton, Evans, Leonard, & Eastburn, 2012).

Efficacy is determined by a variety of factors, including past performance, vicarious experiences, verbal persuasion, and physiological states; however, the most effective way for individuals to develop a high level of efficacy is through mastery experiences. Self-efficacy is the belief of individuals that either “I can do” or “I cannot do”. It is important to recognize the power self-efficacy holds on teacher performance. If a teacher does not believe they have the skills necessary to be effective, the teacher will not be as effective. Typically, individuals will measure their ability to succeed upon their performance relative to others when completing the same task. In order to improve efficacy levels, pre-service and in-service teachers must be taught to measure success in terms of self-development rather than comparing to others. Additionally, providing teachers with meaningful situations in which they can experience success will help improve levels of efficacy (Peker, 2016).

Teachers with high mathematics anxiety tend to also have low levels of mathematics efficacy. These teachers are more likely to focus on basic concepts and surface level knowledge taught in a lecture style setting to the whole class. There is little to no opportunity for students to engage in collaborative activities with their peers or be allowed to engage in discovery learning. Often times these teachers, especially at the elementary school level, tend to avoid teaching mathematics due to high levels of mathematics anxiety, which is this passed on to the students in the classroom (Swars, Daane, & Giesen, 2006).

Summary of the Review of Literature

Teacher pre-service preparation and in-service professional development have implications for teacher content knowledge. It is from these experiences that teachers learn the material that they will ultimately be presenting to their future students. Since teacher content knowledge is tied directly to teacher efficacy and beliefs, it is crucial that teachers be given a solid foundation in their pre-service and in-service preparation upon which to build high levels of efficacy, especially in the field of mathematics since that is where many pre-service elementary teachers feel the greatest levels of anxiety and, as a result, have the lowest feelings of teaching efficacy. In order to produce students who are prepared for 21st Century careers, teacher preparation programs and professional development programs in the United States must develop teachers who have experienced deeper learning themselves and be able to transfer that experience to the classroom for their students to experience. By comparing the characteristics of teacher preparation and professional development programs completed by teachers in the United States to those completed by teachers in Singapore, where a significant portion of the student population has performed at a level consistent with deeper learners on the PISA (Rothman, 2013), it can be determined where the United States can improve teacher preparation and professional development so that United States students can experience deeper learning in the mathematics classroom. This study adds to current understandings by providing a link between teacher preparation program components and deeper learning within the classroom by studying the characteristics of classrooms where teachers have been trained in methods that encourage deeper learning and comparing them to classrooms where teachers have not been trained in this manner.

Chapter 3: Conceptual Framework

Deeper Learning

In order to be successful in the 21st Century, students must be able to problem-solve, think critically, communicate effectively, and possess levels of self-management. Deeper learning is comprised of three domains: the cognitive, interpersonal, and intrapersonal. Development of these competencies show ‘consistent, positive correlations with desirable educational, career, and health outcomes (National Research Council, 2012, p. 65). In order for students to develop these competences, it is important for students to learn in ways that support not only the retention of skills and knowledge but also the ability to apply and use that information in a variety of settings, instead of being limited to the setting in which the content was first obtained (Heller & Gerwin, 2016; National Research Council, 2012).

The demand for workers who have only basic skill sets is declining as technology advances allow for computers to replace low-skill workers. As a result, in order to be employable, students must be equipped with the ability to problem solve and think critically and be able to transfer these skills across domains. Deeper learning classrooms give students the opportunity to develop these skill sets. In order to be successful in the 21st century, students need to be able to develop transferable knowledge and skills to be able to adapt to changing situations rather than depend on repetition of a pre-established set of procedures. Students must receive the training necessary to become experts in a field where they have an interest since expert knowledge cannot be reduced to decision trees and algorithms that can be programmed into a computer. The knowledge future workers possess must be designed to be flexible and adaptive to the various situations where it is needed. Being able to think critically and problem solve are vital skills for 21st century workers. Teachers can encourage the development of this type of knowledge by focusing less on facts and procedures and more on encouraging students to recognize the characteristics of the problem at hand and use prior learning to develop a solution (Heller & Gerwin, 2016; National Research Council, 2012).

When considering cognitive development, Bloom's Taxonomy included components of knowledge, comprehension, application, analysis, synthesis, and evaluation, illustrating that learning is not limited to simply knowing the information, but learning also includes the ability to apply, analyze, and synthesize the content into other situations and uses beyond how it was originally learned (Hess, Kones, Carlock, & Walkup, 2009). Additionally, the work of Norman Webb resulted in the development of Depth of Knowledge levels, with the lowest level being recall and reproduction – characterized by rote memory or routine procedure, and the highest level being extended thinking – characterized by activities such as applying concepts to real-world situations (Hess et al., 2009). Deeper learning is characterized by the development of well-organized knowledge that can be readily accessed and applied to other situations that are relevant. Rote knowledge does not allow this type of understanding to develop. When students are taught in a method that emphasizes rote memorization, there is little opportunity for deeper learning to occur because rote memorization does not require critical thinking and does not encourage the application of knowledge outside of the confines of how the material was originally presented to the student. There are five types of knowledge: facts, concepts, procedures, strategies, and beliefs about one's own learning. In order to develop deeper learning, all five types of knowledge must be addressed simultaneously. While novice learners view these types of knowledge as being independent, expert learners recognize that all five types are interconnected and must work in concert to effectively learn the material at a deeper level (National Research Council, 2012). In the framework of the Trends in Mathematics and Science Study (TIMSS), the cognitive domain is divided into three categories: knowing, applying, and reasoning. The "knowing" component is equivalent to rote memorization and recall; however, the "applying" and "reasoning" components encompass the critical thinking and application of knowledge in a manner other than which it was expressly taught (Grønmo, Lindquist, Arora, & Mullis, 2015; Martin, Mullis, & Hooper, 2016). While the knowing component is important, applying and reasoning are also important in order for all five types of knowledge to develop concurrently.

The beliefs that students hold about learning can affect learning, performance, and motivation. Students with low mathematical beliefs rarely reflect on one's own learning or think about their own thinking, known as metacognition. Experts have strong metacognitive skills which means it is vital for students to engage in this exercise to develop these skills for future use beyond the educational setting. People who monitor their own understanding tend to have a better level of retention of material rather than people who focus on rote memory. In order to develop metacognition, learners must experience the four phases of self-regulation: forethought or planning, monitoring, regulation, and reflection. These phases can be engaged in any order but all four of them are crucial for the development of metacognition. Self-regulation is an essential component of 21st Century learning because learners must be able to set learning goals and manage the pursuit of those goals; these concepts are known in deeper learning as initiation and self-direction (National Research Council, 2012).

Teachers play a crucial role in the deeper learning of students. In order for students to experience deeper learning, teachers must present the material in a method that requires students to engage in complex thought processes rather than focusing on rote memorization of facts and ideas, this is of particular importance in mathematics (Bellanca, 2014; Boaler, 2015). For teachers to embrace this approach to instruction, mathematical content knowledge is important. Without it, teachers do not feel the high levels of efficacy necessary to implement a more student-led classroom where problem-solving and understanding the “why” behind concepts is encouraged (Ball, 1988; Hine, 2015; Norton, 2012; Randel, Apthorp, Beesley, Clark, & Wang, 2016).

To produce teachers who are able to implement deeper learning, preservice and in-service training must recognize the importance of nurturing teacher efficacy, improving mathematics content knowledge, and promoting teacher beliefs of the need for a change from previous instructional strategies to a new strategy which embraces and encourages a student-led classroom where deeper learning takes place (Althauser, 2018; Hine, 2015: Kinach, 2002; Looney, Perry, & Steck, 2017; Newton, Evans, Leonard, & Eastburn, 2012).

In Figure 1, the relationship between deeper teacher understanding, teaching for deeper learning, and student deeper learning is depicted. Student deeper learning occurs when teachers create an environment that enables students to understand the material well enough to retain material learned and apply the knowledge in a variety of situations. This environment is one where teachers understand the “why” behind mathematical concepts

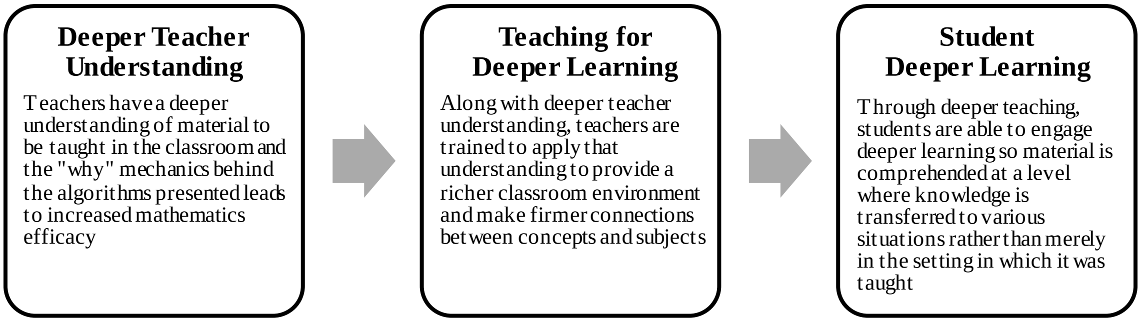

Figure 1. *Theory of Action from Deeper Teacher Understanding to Student Deeper Learning*

and can lead students to a better realization of how various concepts are related, both to other mathematical ideas as well as to situations outside the mathematics classroom. Deeper teacher understanding leads to teaching for deeper learning, which allows students deeper learning to occur; therefore, a crucial component for student deeper learning is deeper teacher understanding. Without deeper teacher understanding, it is extremely challenging to create the conditions necessary to foster student deeper learning.

Teaching for deeper learning and transfer starts with a model of student learning and guiding students on how to develop their metacognitive abilities. Additionally, teachers must develop content that enables the learner to understand the underlying principles of what is to be learned so there is an opportunity for transfer on the part of the student. This can be accomplished when learning experiences are tailored for cognitive processing. There are three types of demands on a learner's working memory: extraneous processing, which does not serve learning goals and is caused by poor instructional design; essential processing, which is necessary if a learner is to be able to mentally represent the material of the lesson and understand the material's complexity; and generative processing, which involves making sense of the material and is dependent on the learner's motivation level to dedicating themselves to make sense of the material (National Research Council, 2012).

Teacher expectations affect student performance; however, it is difficult, if not impossible for teachers to teach for deeper learning if they themselves have never developed a deeper level of understanding. Teaching for deeper learning requires teachers to expand their knowledge and competence of their field in order to promote discussion of higher-order concepts, group solving of complex problems, and public presentation of ideas and findings as well as engaging in formative assessment and revision. Though No Child Left Behind created incentive for teachers to present low-level reading and mathematics content to students, being prepared for the 21st Century requires that learners develop higher-order thinking skills. In order to do this, educators must place value on capacity rather than compliance and encourage learners to think outside of the box to solve problems presented (Heller & Gerwin, 2016; National Research Council 2012).

Traditional classrooms are structured in a way where information is presented to the students and then it is up to the student to successfully integrate this material into their previous knowledge. There is little opportunity for students to recognize how the new material relates to previous concepts or how the new information will be beneficial in their everyday lives. Without giving students the experience of knowing how information and/or skills transfer from one task to another, it is difficult for transfer to become an integral part of their learning process. To be effective, teachers need to address misunderstandings directly, treat teamwork like an outcome, use technology to support student learning, and embrace student creativity and allow it to flourish (National Research Council, 2012; Saavedra & Opfer, 2012).

In the field of mathematics, there are three major reform documents, *Curriculum and Evaluation Standards for School Mathematics* (CESSM), *Principles and Standards* *for School Mathematics* (PSSM), and *Common Core State Standards for Mathematics* (CCSSM), all of which call for mathematics learning to occur at deeper level and incorporate learning with understanding as well as the development of "usable, applicable, transferable knowledge and skills" (National Research Council, 2012, p. 119). Instructional strategies that encourage collaboration, metacognition, and motivation are critical for deeper learning to occur in the mathematics classroom; without the development of these skills students are less likely to be able to problem solve and reason. Despite the CESSM, PSSM, and CCSSM documents, mathematics instruction in the United States continues to be taught with students working in isolation, rather than collaboratively, and focusing on rote memorization and surface level knowledge, instead of developing problem-solving and critical-thinking skills.

Mathematics understanding requires the ability to reason and develop mathematically valid arguments and counterarguments regarding the material. Rather than focusing on a predetermined list of steps needed to solve routine problems, students must be taught how to think imaginatively and use the various tools and knowledge they have acquired to solve non-routine problems. When students are engaging in deeper learning, they must spend time examining the problem, considering strategies for solving the problem, reflecting on the progress made, and adjusting the approach, when necessary, to reach a solution (Boaler, 2015; National Research Council, 2012). The structure of mathematics in Singapore requires students to use more than one skill at a time to solve a problem. Additionally, there is a consistent thread of review throughout the Singapore curriculum. Rather than focusing on one concept per lesson, each lesson contains a review of previous content to help students integrate the material into working knowledge, which is crucial for deeper learning (Erbas, Alacaci, & Bulut, 2012; Ginsburg et al., 2005).

Being able to transfer learning from one subject to another is a critical component of deeper learning. Mathematics content does not exist in isolation, but is also seen in other fields, such as science. In order to develop deeper learners, teachers must embrace curriculum that is more than “a mile wide and an inch deep” and concentrate on not only the content, but also the process. Teaching for transfer requires a focused effort on the part of the teacher to help students recognize when information bridges various disciplines together (National Research Council, 2012).

When looking at the skills necessary for employees to be successful in their career, the need for basic skills is waning while the need for critical thinking and communication skills is on the rise. Saavedra and Opfer (2012) have identified seven “survival” skills: critical thinking and problem-solving, collaboration and leadership, agility and adaptability, initiative and entrepreneurialism, effective oral and written communication, accessing and analyzing information, and curiosity and imagination. These skills develop as a byproduct of effective teaching, especially deeper learning. By emphasizing the importance of higher-order thinking skills and activities that require higher-order thinking skills, students are able to learn the content at a deeper level and be able to develop these necessary skills. The purpose of education is changing and, as a result, the instructional methods and assessment strategies must also change.

Numerous studies have stressed the relationship between deeper learning and student performance (Heller & Gerwin, 2016; National Research Council et al., 2012; Taylor, 2014). Deeper learning has a well-documented positive effect on student performance, but we know less about how teachers own training in deeper learning is related to student deeper learning. Specifically, what is the relationship between teacher preparation and teacher practices? Are teachers who exhibit teaching for deeper learning characteristics within their classroom trained in a different method than those who do not exhibit these characteristics? Figure 2 illustrates the relationship posited between the different aspects of teacher preparation and practice for deeper learning.

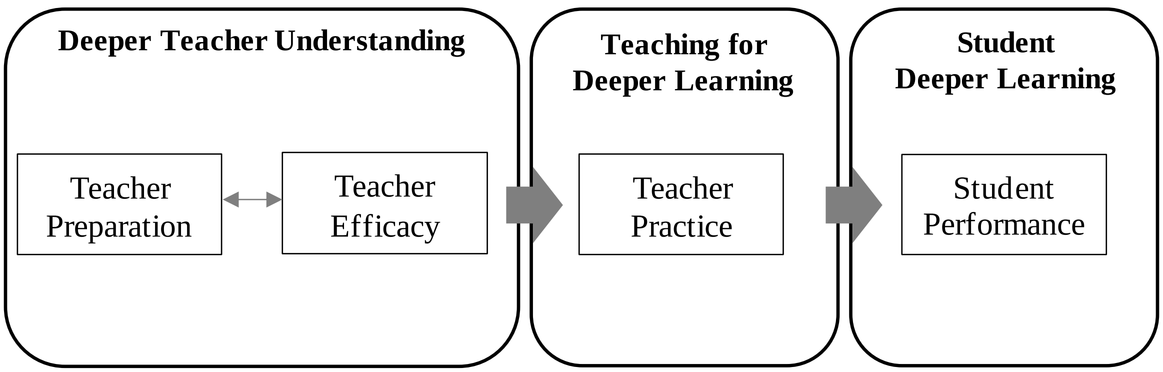

Figure 2. *Model of Posited Associations between Deeper Teacher Understanding, Teaching for Deeper Learning, and Student Deeper Learning.*

As evidenced in the review of the literature, teacher preparation has a significant effect on the development of teacher efficacy (Briley, 2012; Evans, 2013; Peker, 2016; Swackhamer, Koellner, Basile, & Kimbrough, 2009). Additionally, previous research indicates that there is a correlation between teacher efficacy and teacher beliefs (Briley, 2012; Evans, 2013; Memnun & Katranci, 2012). The content knowledge, efficacy, and beliefs of the teacher then affect teacher practice within the classroom (Beswick, Watson, & Brown, 2006; Gonzalez & Maxwell, 2018; Gulistan, Hussain, & Mushtag, 2017; Kim & Seo, 2018; Tschannen-Moran & Barr, 2004). Though previous research has studied these relationships in isolation, current research is lacking in viewing these relationships through the lens of deeper learning. This study aims to address this gap in the current body of literature by merging the framework deeper learning with that of the role of teacher preparation with teacher practice.

Current Study

The goal for this study is to investigate the relationships which exist between deeper teacher understanding and teaching for deeper learning and how those relationships influence student mathematics achievement using data from the 2015 Trends in Mathematics and Science Study (TIMSS). Deeper teacher understanding is hypothesized to be the result of teacher preparation, which influences teacher efficacy. Subsequently teaching for deeper learning, as represented by teachers reports of deeper learning teaching practices in the classroom, is hypothesized to be positively influenced through deeper teacher understanding. Below is a brief review and discussion of the measures planned to represent each of the concepts in the framework.

Deeper Teacher Understanding Components

Teacher Preparation

In order to study the effect teacher preparation has on teaching practices, specifically for teaching for deeper learning practices, it is necessary to determine what type of preparation each teacher received as well as how well prepared the teacher feels to teach mathematics concepts that are in alignment with the curriculum standards for fourth grade as well as concepts that are taught prior to and after fourth grade. To determine this information, several items from the TIMSS dataset will be included in the analysis.

Items regarding continued educational opportunities and experiences will be included in the analysis. These items will ask about participation in professional development activities, such as courses or workshops, focused on mathematics subject matter or methods. Specifically, items asking if the teacher completed professional development within the past two years in mathematics content, mathematics pedagogy/instruction, mathematics curriculum, improving students' critical thinking skills and addressing individual students' needs, as well as the total number of hours the teacher has spent in mathematics professional development in the past two years will be analyzed.

Another set of items focused on how well prepared the teacher feels to teach specific mathematics concepts will be included in the analysis. These concepts include certain skills in number sense, geometric shapes and measures, and data display.

Teacher Efficacy

The influence of teacher efficacy on student learning has been researched thoroughly. Teachers who have a higher teaching efficacy in mathematics are typically more comfortable with the activities and classroom structures that are necessary for increased student learning. Efficacy can influence how confident teachers feel to teach the material (Briley, 2012; Guskfy, 1987; Kim & Seo, 2018; Memnun & Katranci, 2012; Midgley, Feldlaufer, & Eccles, 1989; Newton, Evans, Leonard, & Eastburn, 2012).

Several items address how confident the teacher feels to inspire students to learn mathematics, show students a variety of problem-solving strategies, provide challenging tasks for the highest achieving students, and ability to adapt teaching to engage students' interests will be explored.

Another crucial component of efficacy is the extent to which the individual feels capable of accomplishing the specified task. In order to include this information in the analysis, items regarding how well the teacher help students appreciate the value of learning mathematics, assessing student comprehension of mathematics, improving the understanding for struggling students, making mathematics relevant to students, and development students' higher-order thinking skills will be included in the analysis.

Teacher Deeper Learning Practices

Key characteristics of deeper learning classrooms include mastery of core content, critical and creative thinking, collaboration, communication in writing and speaking, self-directed learning, and an academic mindset. For a classroom to support and promote deeper learning, these components are essential (Bellanca, 2014; National Research Council, 2012). To determine the extent to which these characteristics are present in the classroom, several items from the TIMSS dataset will be included in the analysis.

Items evaluated the mastery of core content by capturing the frequency at which teachers have students listen to an explanation of new mathematics content; listen to an explanation of how to solve problems; memorize rules, procedures, and facts; work problems with guidance, work problems together in the whole class with direct guidance, and work problems while the teacher is occupied by other tasks. Additionally, the frequency of having the students take a written test or quiz, work in mixed ability groups, work in same ability groups were considered. The frequency and length of homework were captured as well has the frequency that the teacher corrects assignments and gives feedback to the students were considered also. Items that ask about the discussion of homework in class, monitoring of homework completion, and various types of student assessment were included as well.

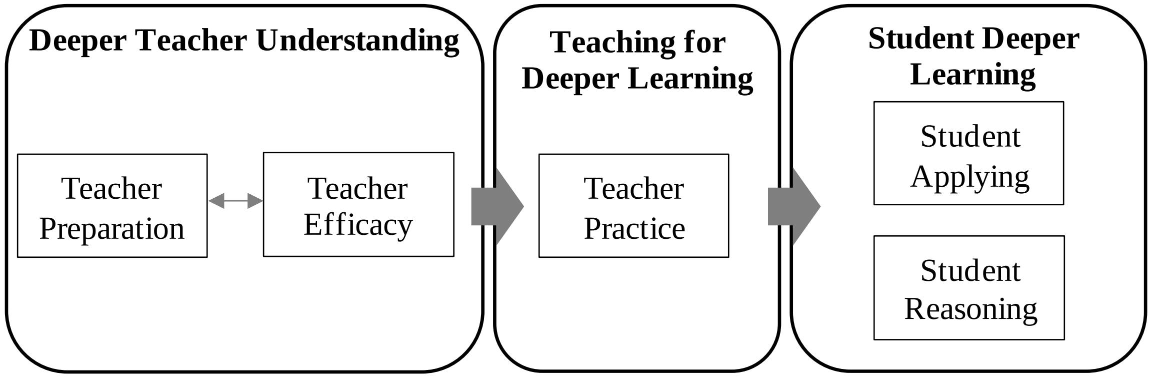

Figure 3. *Proposed Model of the Relationship between Deeper Teacher Understanding, Teaching for Deeper Learning, and Student Deeper Learning.*

Student Deeper Learning Components

The Trends in Mathematics and Science Study (TIMSS) divides the mathematics assessment items into three cognitive domains: knowing, applying, and reasoning, and reports scores on these domains for each student in grade 4 and grade 8. Since the "knowing" domain focuses on skills such as recall, recognition, classification, computation, and retrieval, this domain is not relevant to the determination of the presence of deeper learning at the student level in the classroom. Conversely, the "applying" domain focuses on skills such as determination of appropriate strategies, representation of problems in a graphical or pictorial method, and implementation of problem-solving strategies. Additionally, the "reasoning" domain focuses on skills such as analyzing, integrating, evaluating, and generalizing (Martin, Mullis, & Hooper, 2016). With this in mind, both the applying and reasoning domains encompass the characteristics of deeper learning in the classroom.

Student Applying Skills

In order to engage in deeper learning, students need to be able to not only recall mathematical content knowledge but also be able to apply that knowledge to contexts outside what the confines of the situation in which it was taught. A crucial component of the "applying" domain is the idea of problem-solving, where students are presented either a real-life situation or a hypothetical mathematics situation and then tasked with determining the best way to solve the problem, representing the problem graphically or analytically, if necessary, and then implementing the strategy to arrive at a solution. To determine the extent to which students possess the skill of applying mathematics content to problems in Singapore and the United States, the plausible values for the mathematics applying domain of the TIMSS were included in the analysis as an outcome variable.

Student Reasoning Skills

Aside from applying skills, another important component of deeper learning is the ability to engage in mathematics reasoning. This requires abilities such as logical thinking and inductive reasoning to determine the best method to use when solving a problem and the ability to justify results as being reasonable and/or adequate for the situation being considered. To determine the extent to which student possess the skill of reasoning in regard to mathematics in Singapore and the United States, the plausible values for the mathematics reasoning domain of the TIMSS were included in the analysis as an outcome variable.

Summary of the Conceptual Framework

The desired skills for teachers to use in the classroom to encourage student learning are the same skills identified by the National Research Council as being crucial for deeper learning to occur; however, it is impossible for this style of teaching, teaching for deeper learning, to develop if the teachers have not engaged in deeper learning themselves or otherwise possess a deeper understanding of what it means to teach for deeper learning. In order to produce students who are prepared for 21st Century careers, teacher preparation programs and professional development programs in the United States must develop teachers who have experienced deeper learning themselves and be able to transfer that experience to the classroom for their students to experience. By comparing the characteristics of teacher preparation and professional development programs completed by teachers in the United States to those completed by teachers in Singapore, where a significant portion of the student population has performed at a level consistent with deeper learners on the PISA (Rothman, 2013), it can be determined where the United States can improve teacher preparation and professional development so that United States students can experience deeper learning in the mathematics classroom. This study adds to current understandings by providing a link between teacher preparation program components and deeper learning within the classroom by studying the characteristics of classrooms where teachers have been trained in methods that encourage deeper learning and comparing them to classrooms where teachers have not been trained in this manner.

Chapter 4: Method

Restatement of Purpose

The purpose of this study is to assess the relationship between teacher preservice and in-service professional development and efficacy around deeper learning, increased teaching for deeper learning, and student understanding in the elementary mathematics classroom in two key countries, the United States and Singapore. There are three guiding questions in regard to these countries: (1) What is the relationship between teacher preparation (both pre-service and in-service), mathematics teacher efficacy, and deeper learning in mathematics at the elementary school level? (2) What is the relationship between deeper teacher understanding in mathematics (as measured by teacher efficacy and preparation) and teaching for deeper learning practice at the elementary school level? and (3) What is the relationship between teacher deeper learning practices and student deeper learning of mathematics at the elementary school level? The focus of this chapter will be to outline the study's method, including the data sources, population and sample characteristics, as well as the primary measures used and analyzed.

Modeling Approach and Hypotheses

This study will first test a model of the effects of teacher preparation on teacher efficacy as shown in Figure 4. The resultant relationship will be considered as Deeper Teacher Understanding. This model is composed of a hypothesis enumerated below.

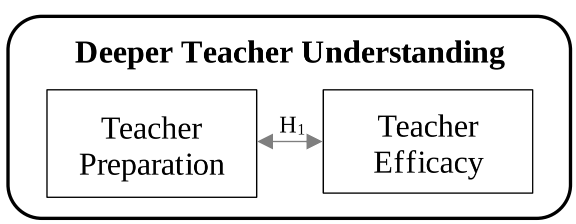

Figure 4. *Proposed Composition of Deeper Teacher Understanding as a Relationship between Teacher Preparation and Teacher Efficacy.*

Hypothesis 1 (H_1): Teacher preparation is positively related to teacher efficacy.

Once the relationship between teacher preparation and teacher efficacy has been established, the study will then test a model of the effects of deeper teacher understanding (as determined by teacher efficacy and teacher preparation) on student mathematics achievement as mediated by deeper learning practices (see Figure 5). In order to determine the relationship between deeper teacher understanding and student achievement, the following model will be investigated.

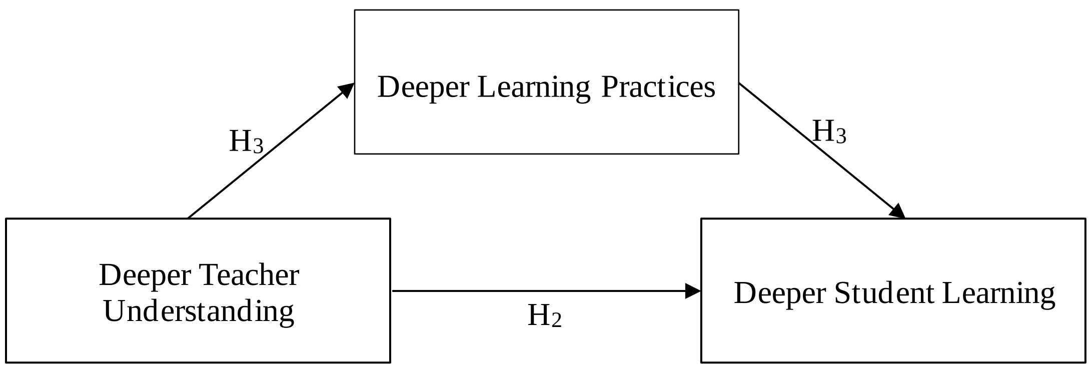

Figure 5. *Proposed Model of the Relationship between Deeper Teacher Understanding and Teaching for Deeper Learning.*

Hypothesis 2 (H_2): Deeper Teacher Understanding has a positive relationship with student mathematics applying skills (i.e., determination of appropriate strategies, representation of problems in a graphical or pictorial method, and implementation of problem-solving strategies) and mathematics reasoning skills (i.e., justification, logical thinking, and inductive reasoning).

Hypothesis 3 (H_3): The relationship between Deeper Teacher Understanding and student mathematics applying skills (i.e., determination of appropriate strategies, representation of problems in a graphical or pictorial method, and implementation of problem-solving strategies), and mathematics reasoning skills (i.e., justification, logical thinking, and inductive reasoning) is partially mediated by teacher Deeper Learning Practices.

Data Source

The data source for this study is the Trends in Internal Mathematics and Science Study (TIMSS) Database of 2015 developed by the International Association for the Evaluation of Educational Achievement (IEA). TIMSS is:

> …an international comparative study designed to measure trends in mathematics and science achievement at the fourth and eighth grades, as well as to collect information about educational contexts (such as students' schools, teachers, and homes) that may be related to student achievement (Stephens, Landeros, Perkins, & Tang, 2016, p. 1).

Data was originally collected from 57 countries within which 18,000 schools and 51,000 teachers were surveyed. The participating countries include Australia, Bulgaria, Canada, Croatia, Denmark, England, Finland, France, Germany, Hungary, Ireland, Italy, Japan, Netherlands, Poland, Portugal, Saudi Arabia, Singapore, Spain, Sweden, Turkey and the United States (Pierre, Alka, & Gabrielle, 2017). Since Singapore has been recognized for its success in mathematics deeper learning, TIMSS data from the United States and Singapore will be used for the purpose of this study (Rothman, 2013).

Sample

To obtain nationally representative samples of teachers and schools, TIMSS 2015 utilized two-stage stratified cluster sampling. In the first stage, TIMSS selected a minimum of 150 upper elementary schools per country and 150 lower secondary schools per country determined by probability-proportional-to-size (PPS) technique. Then TIMSS randomly selected students, teachers, and a school leader from each school using software from the IEA Data Processing and Research Center (DPC). The final analysis for Singapore was comprised of 6321 students nested into 315 classrooms and for the United States, there were 9121 students nested into 376 classrooms.

Measures and Instrumentation

There were three sets of questionnaires, one for students, one for teachers, and another for the school leaders. School principals, teachers, and students filled in the self-report questionnaires on paper. The questionnaire for designed for students collected information regarding students' demographics, home environment, as well as feelings regarding school in general, mathematics, and science. The questionnaire designed for teachers collected information regarding teachers' demographics, qualifications, employment characteristics, professional development participation, classroom practices, beliefs, and attitudes. The questionnaire for school principals collected information about principals' demographics, employment characteristics, school characteristics, and management and leadership. The focal study variables are all described below.

Teacher Preparation. Information regarding participant preparation was captured through items that asked about participation in various experiences such as professional development activities on teaching, and professional development in mathematics Additionally, items that ask about how well prepared the teacher feels to teach particular concepts that are characteristic of fourth grade material were also captured. For Singapore, a factor analysis with varimax rotation of the items which comprise the teacher preparation variable load on one main factor, accounting for approximately 30 percent of the variance (loadings range from 0.456 to 0.801). The reliability was calculated as $\alpha = 0.848$. For the United States, a factor analysis with varimax rotation of the items which comprise the teacher preparation variable load on one main factor, accounting for 40-percent of the variance (loadings range from 0.580 to 0.850). The reliability was calculated as $\alpha = 0.865$. Specific items included in the analysis are listed in Appendix A.

Teacher Efficacy. Information regarding teacher efficacy in the mathematics classroom was captured through items asking about the extent to which the teacher feels confident for the content, pedagogy, and classroom practice of the subject(s) taught. Information regarding beliefs in the extent the teacher can get students to believe they can do well in school work in addition to how well the teacher can craft good questions, help students think critically, and use a variety of instructional and assessment strategies also provide insight to teacher efficacy. A factor analysis with varimax rotation of the items which comprise the teacher efficacy variable load on one factor for Singapore, accounting for 62-percent of the variance (loadings range from 0.742 to 0.825). For the United States, items also load on one factor, accounting for 59-percent of the variance (loadings range from 0.694 to 0.825). The reliability was calculated as $\alpha = 0.921$ for Singapore and $\alpha = 0.913$ for the United States. Specific items included in the analysis are listed in Appendix B.

Teacher Practices. Items regarding various teaching practices were analyzed to determine the amount of deeper learning practices occurring in the classroom. These items captured information regarding the frequency of students working in together to solve problems in small groups to come up with a joint solution to a problem or task. Additionally, items captured information regarding the use of problems from everyday life or work to demonstrate why new knowledge is useful, expecting students to explain their thinking, encourage students to solve problems in more than one way, and requiring students to provide written explanations of how they solve problems. Information regarding the frequency of observation and immediate feedback occurring in the classroom was also captured as well. A factor analysis with varimax rotation of the items which comprise teacher practices load on one factor for Singapore, accounting for 52-percent of the variance (loadings range from 0.681 to 0.808). For the United States, items also load on one factor, accounting for 41-percent of the variance (loadings range from 0.446 to 0.726). The reliability was calculated as $\alpha = 0.846$ for Singapore and $\alpha = 0.743$. Specific items included in the analysis are listed in Appendix C.

Student Achievement. Academic achievement at the student level was reported for each content domain: number, geometric shapes and measures, and data display; achievement was also reported for each cognitive domain: knowing, applying, and reasoning. For the purposes of this study, achievement in the cognitive domains of applying and reasoning were captured since these domains align with deeper learning. Achievement in the cognitive domains was reported using five plausible values for each domain per student. The use of plausible values allows for the uncertainty that is common in student achievement testing. There is inherit error in analyzing student achievement since students do not consistently perform at the exact same level on a test which can affect the reliability of the results. The use of plausible values takes this into account and allows more meaningful analyses to be completed since there is not one specific score for each student, but rather a group of possible scores that the student could achieve. The use of Hierarchical Linear Modeling program allows these plausible values to be used in the analyses results in pooled estimates for the model (Raudenbush & Bryk, 2002; Tot, Koyuncu, & Gelbal, 2019). The specific items included are listed in Appendix D.

Student Control Variables. Items regarding student demographics were captured to determine the composition of the participants in the TIMSS Survey. Information regarding student gender, feelings towards mathematics, confidence in mathematics, feelings of belonging at school, and experiences with bullying at school, were captured as well as how engaging the student viewed the mathematics teacher. Specific items included in the analysis are listed in Appendix E.

Teacher Control Variables. Items regarding teacher demographics were captured to determine the composition of the participants in the TIMSS Survey. Information regarding years experience, gender, age, and highest level of education completed were captured along information about what type of preservice training the teacher received, including major area of study were included. Specific items included in the analysis are listed in Appendix F.

Table 1

Student Descriptive Statistics (Standard Deviation in parentheses) for Each Country

Characteristic	Singapore (n = 6321)	United States (n = 9121)
Sense of Belonging	9.51(1.89)	9.87(2.06)
Enjoyment of Math	9.63(1.76)	9.74(1.98)
Engaging Math teacher	9.34(1.87)	10.22(2.03)
Confidence in Math	9.12(1.79)	10.03(2.05)
Mathematics Applying	615.8(85.72)	538.1(84.02)
Mathematics Reasoning	598.4(100.8)	531.3(82.62)
Gender (percentages in parentheses)		
Female	3120 (49.4)	4669 (51.2)
Male	3201 (50.6)	4452 (48.8)
Socio-economic Status (percentages in parentheses)		
Low	749 (11.8)	1379 (15.1)
Medium	4158 (65.8)	6647 (72.9)
High	1414 (22.4)	1095 (12.0)

Descriptive Statistics

As previously mentioned, two different countries were analyzed in this study. Table 1 provides the demographic information for the student level and Table 2 provides the demographic information for the teacher level. The student level categories are fairly homogenous between the two countries as are the teacher level categories, other than level of formal education completed as well as the number of teachers who major in mathematics.

Data Analysis

The first part of this study was to determine the relationship between teacher preparedness and teacher efficacy. Items related to teacher efficacy (ATBM02A – ATBM02I) were combined through summation into a new variable, TEACHER_EFFICACY, to represent the degree of mathematical efficacy for the teacher. Items related to teacher mathematics preparation (ATBM11AA – ATBM11CB) were combined through summation into a new variable, TEACHER_PREPARED, to represent the preparedness for the teacher. Once this was completed, TEACHER_EFFICACY and TEACHER_PREPARED were analyzed to ensure the prerequisites for linear regression were met. The variable TEACHER_PREPARED had a strong, non-normal negative skew, so the variable was transformed using the formula *TEACHER_PREPAREDNESS = SQRT(k – TEACHER_PREPARED)* to normalize it, where k represents the largest value in the dataset plus one (for both Singapore and the United States, $k = 52$). With the prerequisites verified, a linear regression was completed for each country to determine the relationship between teacher preparation and teacher efficacy.

Table 2

Teacher Descriptive Statistics (Standard Deviations in Parentheses) for Each Country

Characteristic	Singapore (n = 315)	United States (n = 376)
Teacher Efficacy	26.09(4.71)	28.51(4.75)
Teacher Preparedness	2.70(1.27)	2.09(1.23)
Teacher Deeper Learning Practices	14.30(3.82)	18.19(2.59)
Years of experience (percentages in parentheses)		
less than 6 years	118 (37.5)	92 (24.5)
6 – 10 years	74 (23.5)	77 (20.5)
11 – 20 years	90 (28.6)	130 (34.6)
21 – 30 years	19 (6.0)	60 (16.0)
31 – 40 years	11 (3.5)	16 (4.3)
more than 40 years	3 (1.0)	1 (0.3)
Gender (percentages in parentheses)		
Female	228 (72.4)	322 (85.6)
Male	87 (27.6)	54 (14.4)
Age of Teacher (percentages in parentheses)		
Under 25	9 (2.9)	16 (4.3)
25 – 29	64 (20.3)	51 (13.6)
30 – 39	140 (44.4)	103 (27.4)
40 – 49	71 (22.5)	109 (29.0)
50 – 59	25 (7.9)	80 (21.3)
60 or older	6 (1.9)	17 (4.5)
Level of Formal Education Completed (percentages in parentheses)		
Post-secondary, non-tertiary	22 (7.0)	0 (0)
Short-cycle tertiary	41 (13.0)	0 (0)
Bachelor’s or equivalent	217 (68.9)	182 (48.4)
Master’s or equivalent	35 (11.1)	194 (51.6)
Major Area of Study – Elementary Education (percentages in parentheses)		
No	82 (26.0)	52 (13.8)
Yes	233 (74.0)	324 (86.2)
Major Area of Study – Mathematics (percentages in parentheses)		
No	117 (37.1)	346 (92.0)
Yes	198 (62.9)	30 (8.0)
Hours of Professional Development (percentages in parentheses)		
none	16 (5.1)	32 (8.5)
less than 6 hours	53 (16.8)	86 (22.9)
6 – 15 hours	119 (37.8)	112 (29.8)
16 – 35 hours	76 (24.11)	74 (19.7)
more than 35 hours	51 (16.2)	72 (19.1)

The second part of this study investigated how teacher characteristics, such as efficacy and preparation were related to student mathematics applying skills (i.e., determination of appropriate strategies, representation of problems in a graphical or pictorial method, and implementation of problem-solving strategies) and mathematics reasoning skills (i.e., justification, logical thinking, and inductive reasoning); therefore, Hierarchical Linear Modeling (HLM) was chosen for the data analysis. HLM allows for the nested nature of data and thus avoids aggregation bias which has increased its use in educational studies (Raudenbush & Bryk, 2002). As a study, TIMSS was designed as a nested sample necessitating the use of HLM. Student level data, such as achievement and demographic information, were entered at Level 1 and teacher level data, such as efficacy, professional development experience, and demographic information were entered at Level 2. Additionally, a mediation model was explored to determine what mediation, if any, occurs with deeper learning practices on the part of the teacher. Baron and Kenny (1986) developed an analytical technique to test for mediation. This technique involves three steps: first, the independent variable must have a relationship to the mediator in the first equation (path a below); second, the independent variable must be shown to have a relationship with the dependent variable in the second equation (path c below); and third, the mediator must have a relationship with the dependent variable in the third equation (path b below). The mediation effect can be determined by using the product of coefficients (path *a* x path *b*). An illustration of this as it relates to the current study is provided in Figure 6.

A series of HLM analyses were conducted to answer each research question for each country, United States and Singapore. First, a null model was run for each country to calculate ICCs and verify the existence of a relationship between a teacher and the achievement of the students taught, as determined by Student Mathematical Application scores and Student Mathematical Reasoning scores. All outcomes had over 30 percent variance at the school/classroom level.

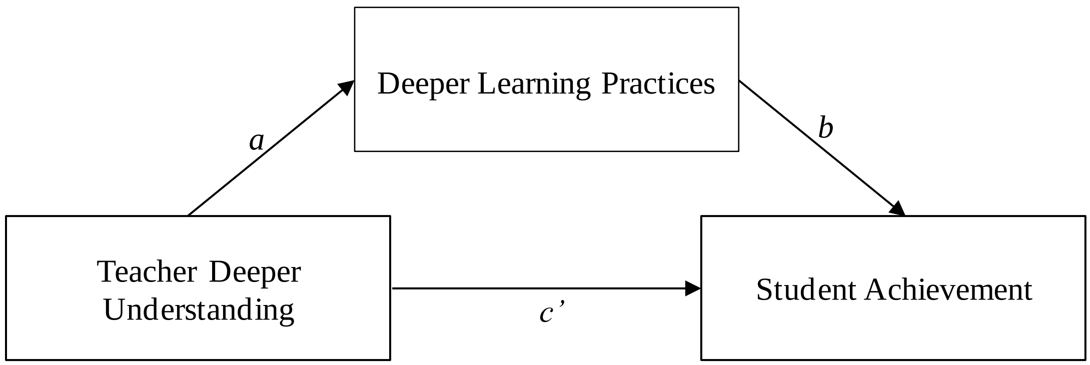

Figure 6. *Mediation Model for the Influence of Teacher Preparation and Efficacy on Student Achievement Mediated by Deeper Learning Practices*

Next, the student level data was included into the model to account for variance that may occur as a result of student characteristics. A variety of combinations were analyzed to determine which characteristics have the greatest influence on student achievement for each country. In order to get a clear picture of the effect teacher preparation/teacher efficacy has on student mathematical achievement, student level variables that had an influence on achievement were selected. For Singapore, these level-1 variables are student feelings of belonging at the school (STUDENT_BELONGING) and having an engaging mathematics teacher (STUDENT_ENGAGING_TEACHER) as well as gender (STUDENT_GENDER) and student confidence in mathematics (STUDENT_CONFIDENCE_MATH), with STUDENT_BELONGING, STUDENT_ENGAGING_TEACHER, and STUDENT_CONFIDENCE_MATH being centered around the grand mean.

The results of Model B allowed for a new model to be developed that included the teacher characteristics of preparation and (TEACHER_PREPAREDNESS) efficacy (TEACHER_EFFICACY) as well as the amount of professional development the teacher has received in the past two years (NO_PROFESSIONAL_DEVELOPMENT, LOW_PROFESSIONAL_DEVELOPMENT_HOURS, MEDIUM_PROFESSIONAL_DEVELOMENT HOURS, and HIGH_PROFESSIONAL_DEVELOPMENT_HOURS). Variables for demographic characteristics of years of experience (YEARS_TAUGHT), gender (GENDER), achievement of an advanced degree (ADVANCED_DEGREE), and specialization in mathematics (MATH_SPECIALIZATION) were also included in the analysis for Model C, with YEARS_TAUGHT, ADVANCED_DEGREE, TEACHER_PREPAREDNESS, and TEACHER_EFFICACY centered around the grand mean.

In order to determine whether teacher deeper learning practices act as a mediator on the relationship between teacher and student achievement in mathematics application and mathematics reasoning and, if so, to what extent the mediation occurred a final model, Model D, was developed. This model is identical to Model C except for the addition of the teacher deeper learning practices variable (TEACHER_DEEPER_LEARNING_PRACTICES). The final model for mathematics applying and mathematics reasoning for Singapore are listed below. Level-1 variables of socioeconomic status (STUDENT_SOCIOECONOMIC_STATUS), having an engaging mathematics teacher (STUDENT_ENGAGING_TEACHER), and student confidence in mathematics (STUDENT_CONFIDENCE_MATH) were centered around the grand mean. Likewise, level-2 variables of years experience (YEARS_TAUGHT), highest degree earned (ADVANCED_DEGREE), teacher feelings of efficacy (TEACHER_EFFICACY), teacher preparation (TEACHER_PREPARATION), and teacher deeper learning practices (TEACHER_DEEPER_LEARNING_PRACTICES) were centered around the grand mean. At Level 1, student mathematics applying achievement in Singapore was predicted to be a function of the average student achievement (β_{0j}), gender (β_{1j}), socioeconomic status (β_{2j}), belonging (β_{3j}), engaging mathematics teacher (β_{4j}), student mathematics confidence (β_{5j}), and random error (r_{ij}). The between teacher variation in student achievement was modeled as a function of the grand mean γ_{00} and the teacher characteristics in the model. A similar function was used to predict student mathematics reasoning achievement in Singapore.

Mathematics Applying

Mixed Model: $STUDENT\_MATHEMATICS\_APPLYING\_SKILLS_{ij} = \gamma_{00}$
$+ \gamma_{01}*YEARS\_TAUGHT_{j} + \gamma_{02}*TEACHER\_GENDER_{j}$
$+ \gamma_{03}*ADVANCED\_DEGREE_{j} + \gamma_{04}*MATH\_SPECIALIZATION_{j}$
$+ \gamma_{05}*TEACHER\_EFFICACY_{j} +$
$\gamma_{06}*NO\_PROFESSIONAL\_DEVELOPMENT_{j}$
$+ \gamma_{07}*LOW\_PROFESSIONAL\_DEVELOPMENT\_HOURS_{j}$
$+ \gamma_{08}*MEDIUM\_PROFESSIONAL\_DEVELOMENT\_HOURS_{j}$
$+ \gamma_{09}*HIGH\_PROFESSIONAL\_DEVELOPMENT\_HOURS_{j}$
$+ \gamma_{010}*TEACHER\_PREPAREDNESS_{j}$
$+ \gamma_{011}*TEACHER\_DEEPER\_LEARNING\_PRACTICES_{j}$
$+ \gamma_{10}*STUDENT\_GENDER_{ij}$
$+ \gamma_{20}*STUDENT\_SOCIOECONOMIC\_STATUS_{ij}$
$+ \gamma_{30}*STUDENT\_BELONGING_{ij}$
$+ \gamma_{40}*STUDENT\_ENGAGING\_TEACHER_{ij}$
$+ \gamma_{50}*STUDENT\_CONFIDENCE\_MATH_{ij}$
$+ \mu_{0j} + r_{ij}$

Mathematics Reasoning

Mixed Model: $STUDENT\_MATHEMATICS\_REASONING\_SKILLS_{ij} = \gamma_{00}$
$+ \gamma_{01}*YEARS\_TAUGHT_{j} + \gamma_{02}*GENDER_{j} + \gamma_{03}*ADVANCED\_DEGREE_{j}$
$+ \gamma_{04}*MATH\_SPECIALIZATION_{j} + \gamma_{05}*TEACHER\_EFFICACY_{j}$
$+ \gamma_{06}*NO\_PROFESSIONAL\_DEVELOPMENT\_HOURS_{j}$
$+ \gamma_{07}*LOW\_PROFESSIONAL\_DEVELOPMENT\_HOURS_{j}$

$$
\begin{aligned}
&+ \gamma_{08}*MEDIUM\_PROFESSIONAL\_DEVELOPMENT\_HOURS_j \\
&+ \gamma_{09}*HIGH\_PROFESSIONAL\_DEVELOPMENT\_HOURS_j \\
&+ \gamma_{010}*TEACHER\_PREPAREDNESS_j \\
&+ \gamma_{011}*TEACHER\_DEEPER\_LEARNING\_PRACTICES_j \\
&+ \gamma_{10}*STUDENT\_GENDER_{ij} \\
&+ \gamma_{20}*STUDENT\_SOCIOECONOMIC\_STATUS_{ij} \\
&+ \gamma_{30}*STUDENT\_BELONGING_{ij} \\
&+ \gamma_{40}*STUDENT\_ENGAGING\_TEACHER_{ij} \\
&+ \gamma_{50}*STUDENT\_CONFIDENCE\_MATH_{ij} \\
&+ \mu_{0j} + r_{ij}
\end{aligned}
$$

For the United States, the student level variables of greatest influence on were socioeconomic status (STUDENT_SOCIOECONOMIC_STATUS) and student feelings towards math (STUDENT_LIKING_MATH) as well as gender (STUDENT_GENDER) and student confidence in mathematics (STUDENT_CONFIDENCE_MATH); therefore, these variables were used in Model B for the United States instead of the same variables that had been used in the Singapore analysis with STUDENT_SOCIOECONOMIC_STATUS, STUDENT_LIKING_MATH, and STUDENT_MATH_CONFIDENCE centered around the grand mean.

The results of Model B allowed for a new model to be developed that included the teacher characteristics of preparation and (TEACHER_PREPAREDNESS) efficacy (TEACHER_EFFICACY) as well as the amount of professional development the teacher has received in the past two years (NO_PROFESSIONAL_DEVELOPMENT, LOW_PROFESSIONAL_DEVELOPMENT_HOURS, MEDIUM_PROFESSIONAL_DEVELOPMENT_HOURS, and HIGH_PROFESSIONAL_DEVELOPMENT_HOURS). Variables for demographic characteristics of years of experience (YEARS_TAUGHT), gender (TEACHER_GENDER), achievement of an advanced degree (ADVANCED_DEGREE), and specialization in mathematics (MATH_SPECIALIZATION) were also included in the analysis for Model C with YEARS_TAUGHT, ADVANCED_DEGREE, TEACHER_PREPAREDNESS, and TEACHER_EFFICACY centered around the grand mean.

In order to determine whether teacher deeper learning practices act as a mediator on the relationship between teacher and student achievement in mathematics application and mathematics reasoning and, if so, to what extent the mediation occurred a final model, Model D, was developed. This model is identical to Model C except for the addition of the teacher deeper learning practices variable (TEACHER_DEEPER_LEARNING_PRACTICES). The final model for mathematics applying and mathematics reasoning for the United States are listed below. Level-1 variables of socioeconomic status (STUDENT_SOCIOECONOMIC_STATUS), student feelings toward mathematics (STUDENT_LIKING_MATH), and student confidence in mathematics (STUDENT_CONFIDENCE_MATH) were centered around the grand mean. Likewise, level-2 variables of years experience (YEARS_TAUGHT), teacher feelings of efficacy (TEACHER_EFFICACY), teacher preparation (TEACHER_PREPAREDNESS), and teacher deeper learning practices (TEACHER_DEEPER_LEARNING_PRACTICES) were centered around the grand mean. At Level 1, student mathematics applying achievement in the United States was predicted to be a function of the average student achievement (β_{0j}), student gender (β_{1j}), student socioeconomic status (β_{2j}), student feelings toward mathematics (β_{3j}), student mathematics confidence (β_{4j}), and random error (r_{ij}). The between teacher variation in student achievement was modeled as a function of the grand mean γ_{00} and the teacher characteristics in the model. A similar function was used to predict student mathematics reasoning achievement in the United States.

Mathematics Application

Mixed Model: $STUDENT\_MATHEMATICS\_APPLYING\_SKILLS_{ij} = \gamma_{00}$
$+ \gamma_{01}*YEARS\_TAUGHT_j + \gamma_{02}*TEACHER\_GENDER_j$
$+ \gamma_{03}*ADVANCED\_DEGREE_j + \gamma_{04}*MATH\_SPECIALIZATION_j$
$+ \gamma_{05}*TEACHER\_EFFICACY_j +$
$\gamma_{06}*NO\_PROFESSIONAL\_DEVELOPMENT_j +$
$\gamma_{07}*LOW\_PROFESSIONAL\_DEVELOPMENT\_HOURS_j$
$+ \gamma_{08}*MEDIUM\_PROFESSIONAL\_DEVELOPMENT\_HOURS_j$
$+ \gamma_{09}*HIGH\_PROFESSIONAL\_DEVELOPMENT\_HOURS_j$
$+ \gamma_{010}*TEACHER\_PREPAREDNESS_j$
$+ \gamma_{011}*TEACHER\_DEEPER\_LEARNING\_PRACTICES_j$
$+ \gamma_{10}*STUDENT\_GENDER_{ij}$
$+ \gamma_{20}*STUDENT\_SOCIOECONOMIC\_STATUS_{ij}$
$+ \gamma_{30}*STUDENT\_LIKING\_MATH_{ij}$
$+ \gamma_{40}*STUDENT\_CONFIDENCE\_MATH_{ij}$
$+ \mu_{0j} + r_{ij}$

Mathematics Reasoning

Mixed Model: $STUDENT\_MATHEMATICS\_REASONING\_SKILLS_{ij} = \gamma_{00}$
$+ \gamma_{01}*YEARS\_TAUGHT_j + \gamma_{02}*TEACHER\_GENDER_j$
$+ \gamma_{03}*ADVANCED\_DEGREE_j + \gamma_{04}*MATH\_SPECIALIZATION_j$
$+ \gamma_{05}*TEACHER\_EFFICACY_j +$
$\gamma_{06}*NO\_PROFESSIONAL\_DEVELOPMENT_j +$
$\gamma_{07}*LOW\_PROFESSIONAL\_DEVELOPMENT\_HOURS_j$
$+ \gamma_{08}*MEDIUM\_PROFESSIONAL\_DEVELOPMENT\_HOURS_j$
$+ \gamma_{09}*HIGH\_PROFESSIONAL\_DEVELOPMENT\_HOURS_j$
$+ \gamma_{010}*TEACHER\_PREPAREDNESS_j$
$+ \gamma_{011}*TEACHER\_DEEPER\_LEARNING\_PRACTICES_j$
$+ \gamma_{10}*STUDENT\_GENDER_{ij}$
$+ \gamma_{20}*STUDENT\_SOCIOECONOMIC\_STATUS_{ij}$
$+ \gamma_{30}*STUDENT\_LIKING\_MATH_{ij}$
$+ \gamma_{40}*STUDENT\_CONFIDENCE\_MATH_{ij}$
$+ \mu_{0j} + r_{ij}$

Handling Missing Data

For Singapore, there were 315 complete responses out of 350 total for the teacher surveys, 10-percent missing, and 6321 complete responses out of 6347 total for the student level surveys, 0.4-percent missing. Similarly, for the United States, there were 376 complete responses out of 408 total for the teacher level surveys, 7.8-percent missing, and 9121 complete responses out of 10730 total for the student level surveys, 15-percent missing. The majority of missing teacher questionnaire cases were due to survey non-completion in which the majority of the survey was not completed. These cases had no discernable pattern and were assumed to be missing completely at random (MCAR). Thus, incomplete cases were deleted listwise from the corresponding data file prior to analysis. Missing cases at the student level were handled via the HLM program using pairwise deletion. HLM, in concert with maximum likelihood estimation is robust against bias introduced at Level 1 as a result of pairwise deletion due to random survey item non-response (Raudenbush & Bryk, 2002).

Chapter 5: Results

Restatement of Purpose

The purpose of this study was to determine the relationship between teacher preparation, deeper teacher understanding, and student deeper learning in the mathematics classroom. There were three guiding questions in regard to these countries: (1) What is the relationship between teacher preparation (both pre-service and in-service), mathematics teacher efficacy, and deeper learning in mathematics at the elementary school level?, (2) What is the relationship between deeper teacher understanding in mathematics (as measured by teacher efficacy and preparation) and teaching for deeper learning practice at the elementary school level, and (3) What is the relationship between teacher deeper learning practices and student deeper learning of mathematics at the elementary school level? From these questions, the following hypotheses were developed: (1) teacher preparation has a positive effect on teacher efficacy, (2) Deeper Teacher Understanding has a positive relationship with student mathematics applying skills (i.e., determination of appropriate strategies, representation of problems in a graphical or pictorial method, and implementation of problem-solving strategies) and mathematics reasoning skills (i.e., justification, logical thinking, and inductive reasoning), and (3) The relationship between Deeper Teacher Understanding and student mathematics applying skills (i.e., determination of appropriate strategies, representation of problems in a graphical or pictorial method, and implementation of problem-solving strategies), and mathematics reasoning skills (i.e., justification, logical thinking, and inductive reasoning) is partially mediated by teacher Deeper Learning Practices.

Results of Analyses

Hypothesis One

The first hypothesis focuses on the relationship between teacher preparation and teacher efficacy. This relationship was determined through linear regression using SPSS. For the United States, there was a moderate relationship between teacher preparation and teacher efficacy (r = 0.362, r^2 = 0.129) while for Singapore, there was a small to moderate relationship present between teacher preparation and teacher efficacy (r = 0.276, r^2 = 0.073). From this, it can be determined that teacher efficacy is, at least marginally, related to teacher preparation. Based on these results, Hypothesis 1 is supported.

Hypothesis Two and Hypothesis Three

The second hypothesis focuses on the relationship between deeper teacher understanding (as determined by teacher efficacy and teacher preparation) and student achievement in mathematics applying and mathematics reasoning. To test this hypothesis, the use of HLM was necessary due to the multilevel nature of the data.

The complete results of the analysis regarding the relationship between teacher deeper understanding and student mathematics applying in Singapore is summarized in Table 3. This table shows the null model, Model A, the student level variables model, Model B, the student and teacher level variables model, Model C, and the final model, Model D, with deeper learning practices added to the analysis (mediation model). Hypothesis two is represented as Model C. Throughout Models B and C, the level-1 variables of student feelings of belonging, having engaging teacher, and confidence in mathematics continue to have a small but significant effect on student achievement (*B* =

1.660, $p = 0.032$; $B = -1.952$, $p < 0.001$; and $B = 12.063$, $p < 0.001$, respectively). When the level-2 variables are introduced in Model C, teacher efficacy, teacher preparedness, and the lack of teacher professional development within the last two years have a significant effect on student achievement regarding mathematics applying ($B = 2.391$, $p = 0.002$; $B = 15.107$, $p = 0.039$; and $B = 30.950$, $p = 0.075$, respectively). The model does explain a portion of the overall variance (pseudo-$r^2 = 0.286$ at the teacher level and pseudo-$r^2 = 0.123$ at the student level).

The complete results of the analysis regarding the relationship between teacher deeper understanding and student mathematics reasoning in Singapore is summarized in Table 4. This table shows the null model, Model A, the student level variables model, Model B, the student and teacher level variables model, Model C, and the final model, Model D, with deeper learning practices added to the analysis. Hypothesis two is represented as Model C. Throughout Models B and C, the level-1 variables of student gender, having an engaging mathematics teacher, and confidence in mathematics have a significant effect on student achievement in mathematics reasoning ($B = 8.705$, $p = 0.075$; $B = -2.914$, $p = 0.022$; $B = 16.684$ and $p < 0.001$, respectively). When the level-2 variables are introduced in Model C, teacher efficacy, teacher mathematics specialization, teacher preparedness, and the lack of teacher professional development within the last two years have a significant effect on student achievement regarding mathematics reasoning ($B = 2.569$, $p = 0.002$; $B = 12.721$, $p = 0.088$; $B = 5.879$, $p = 0.028$; $B = 31.342$ and $p = 0.090$, respectively). The model does explain a portion of the overall variance (pseudo-$r^2 = 0.341$ at the teacher level and pseudo-$r^2 = 0.133$ at the student level).

The complete results of the analysis regarding the relationship between teacher deeper understanding and student mathematics applying in the United States is summarized in Table 5. This table shows the null model, Model A, the student level variables model, Model B, the student and teacher level variables model, Model C, and the final model, Model D, with deeper learning practices added to the analysis. Hypothesis two is represented as Model C. Throughout Models B and C, the level-1 variables of socioeconomic status, student feelings toward mathematics and student confidence in mathematics have a significant effect on student achievement in mathematics applying ($B = 1.512$, $p = 0.045$; $B = -2.706$, $p = 0.002$; $B = 16.695$ and $p < 0.001$, respectively). When the level-2 variables are introduced in Model C, teacher preparedness has a significant effect on student achievement regarding mathematics reasoning ($B = 23.623$, $p = 0.013$, an increase of approximately one-quarter of a standard deviation). The model does explain a portion of the overall variance (pseudo-$r^2 = 0.200$ at the teacher level and pseudo-$r^2 = 0.196$ at the student level).

The complete results of the analysis regarding the relationship between teacher deeper understanding and student mathematics reasoning in the United States is summarized in Table 6. This table shows the null model, Model A, the student level variables model, Model B, the student and teacher level variables model, Model C, and the final model, Model D, with deeper learning practices added to the analysis. Hypothesis two is represented as Model C. Throughout Models B and C, the level-1 variables of student feelings toward mathematics and student confidence in mathematics have a significant effect on student achievement in mathematics reasoning ($B = -2.990$, $p = 0.006$; and $B = 16.258$, $p < 0.001$, an increase of approximately one-fifth of a standard deviation, respectively). When the level-2 variables are introduced in Model C, teacher preparedness has a significant effect on student achievement regarding mathematics reasoning ($B = 26.021$, $p = 0.013$, an increase of approximately one-third of a standard deviation). The model does explain a portion of the overall variance (pseudo-$r^2 = 0.203$ at the teacher level and pseudo-$r^2 = 0.183$ at the student level) and is, therefore, a relevant outcome of this study. Based on these results, hypothesis two is supported.

The third hypothesis focused on the relationship between deeper teacher understanding and student deeper learning being partially mediated by teacher deeper learning practices. For this study, the technique for determining mediation developed by Baron and Kenny (1986) was used. This technique involves three steps: first, the independent variable (teacher efficacy and teacher preparedness) must relate to the mediator (deeper learning) in the first equation; second, the independent variable must be shown to relate to the dependent variable in the second equation (student mathematics applying achievement and student mathematics reasoning achievement); and third, the mediator must relate to the dependent variable in the third equation. The second of these steps was already established in the previous section.

The first step testing this hypothesis was to determine the relationship between teacher deeper understanding and deeper learning practices, which was analyzed in SPSS since it was comprised of only level-1 data (Path a). For Singapore, the relationship between teacher deeper understanding (comprised of teacher efficacy and teacher preparation) and deeper learning practices had a strong positive correlation ($r = 0.551$, $r^2 = 0.299$). The United States had a moderate positive correlation ($r = 0.329$, $r^2 = 0.103$).

The complete results of the mediation analysis regarding the relationship between teacher deeper understanding and student mathematics applying in Singapore is summarized in Table 3. This table shows the null model, Model A, the student level variables model, Model B, the student and teacher level variables model, Model C, and the final model, Model D, with deeper learning practices added to the analysis (i.e., the mediation model). Once teacher deeper learning practices are entered into the analysis, Model D, the level-1 variables of student feelings of belonging, having engaging teacher, and confidence in mathematics continue to have a small but significant effect on student achievement ($B = 1.659$, $p = 0.032$; $B = -1.951$, $p < 0.001$; and $B = 12.062$, $p < 0.001$, respectively). The level-2 variables of teacher efficacy, teacher preparedness, and the lack of teacher professional development within the last two years are still statistically significant ($B = 2.055$, $p = 0.028$; $B = 13.572$, $p = 0.035$; and $B = 31.062$, $p = 0.074$, respectively); however, teacher deeper learning practices is not statistically significant ($B = 0.705$, $p = 0.517$). It can be concluded that student achievement in mathematics applying in Singapore is not mediated by teacher deeper learning practices in the classroom. It can be seen across the models that there is still a significant amount of variance that is unexplained by these factors. The model does explain a portion of the overall variance (pseudo-$r^2 = 0.287$ at the teacher level and pseudo-$r^2 = 0.123$ at the student level).

Table 3

2-Level HLM of the Effects of Teacher Preparation and Teacher Efficacy on Student Achievement in Mathematics Applying Skills for Singapore

	Model A		Model B		Model C		Model D	
	β	*SE*	*β*	*SE*	*β*	*SE*	*β*	*SE*
Fixed Effects – Student Level								
Intercept γ_{00}	615.374	4.151***	613.169	3.959***	584.153	11.310***	584.823	11.342***
Gender, γ_{10}			4.667	2.917	3.966	2.826	3.961	2.826
Student feels belonging at school, γ_{20}			−0.005	0.537	1.660	0.657*	1.659	0.657*
Student has an engaging teacher, γ_{30}			−1.678	0.514**	−1.952	0.471***	−1.951	0.470***
Student is confident in math, γ_{40}			12.195	0.622***	12.063	0.605***	12.062	0.605***
Fixed Effects – Teacher Level								
Years of experience, γ_{01}					−0.764	0.422	−0.735	0.424~
Gender, γ_{02}					22.110	7.810**	21.629	7.813**
Graduate Degree, γ_{03}					−17.319	11.094	−16.607	11.134
Specialization in Mathematics, γ_{04}					12.232	6.992~	12.030	6.991~
Efficacy, γ_{06}					2.391	0.770**	2.055	0.930*
Preparedness, γ_{07}					15.107	2.923~	13.572	2.930*
Professional Development – None γ_{08}					30.950	17.316~	31.062	17.306~
Less than 6 hours, γ_{09}					1.807	11.919	1.412	11.926
6 – 15 hours, γ_{010}					8.400	10.236	7.955	10.252
16 – 35 hours, γ_{011}					9.345	11.028	9.374	11.020
Deeper Learning Practices, γ_{05}							0.705	1.086
Random Effects								
Level 2 variance		4699.270***		3795.355***		3354.343***		3349.825***
Level 1 variance		2878.045		2532.224		2524.307		2524.288
Level 2 r-squared				0.192		0.286		0.287
Level 1 r-squared				0.120		0.123		0.123

***$p < 0.001$, **$p < 0.01$, *$p < 0.05$, ~$p < 0.10$

Note: More than 35 hours of professional development was the comparison group. Math teacher final weights applied to the model.

The complete results of the mediation analysis regarding the relationship between teacher deeper understanding and student mathematics reasoning in Singapore is summarized in Table 4. This table shows the null model, Model A, the student level variables model, Model B, the student and teacher level variables model, Model C, and the final model, Model D, with deeper learning practices added to the analysis (i.e., the mediation model). Once teacher deeper learning practices are entered into the analysis, Model D, the level-1 variables of student gender, having an engaging mathematics teacher, and confidence in mathematics have a significant effect on student achievement in mathematics reasoning ($B = 8.699$, $p = 0.075$; $B = -2.913$, $p = 0.022$; $B = 16.682$ and $p < 0.001$, respectively). The level-2 variables of teacher efficacy, teacher mathematics specialization, teacher preparedness, and the lack of teacher professional development within the last two years are still statistically significant ($B = 2.279$, $p = 0.023$; $B = 12.545$, $p = 0.093$; $B = 4.404$, $p = 0.026$; $B = 31.434$, and $p = 0.089$, respectively). Teacher deeper learning practices is not statistically significant ($B = 0.609$, $p = 0.598$). It can be concluded that student achievement in mathematics reasoning in Singapore is not mediated by teacher deeper learning practices in the classroom. It can be seen across the models that there is still a significant amount of variance that is unexplained by these factors. The model does explain a portion of the overall variance (pseudo-$r^2 = 0.341$ at the teacher level and pseudo-$r^2 = 0.133$ at the student level).

Table 4

2-Level HLM of the Effects of Teacher Preparation and Teacher Efficacy on Student Achievement in Mathematics Reasoning Skills for Singapore

	Model A		Model B		Model C		Model D	
	β	*SE*	β	*SE*	β	*SE*	β	*SE*
Fixed Effects – Student Level								
Intercept, γ_{00}	598.924	4.433***	594.779	4.438***	564.260	11.485***	564.840	11.530***
Gender, γ_{10}			8.715	4.042~	8.705	4.040~	8.699	4.040~
Student feels belonging at school, γ_{20}			1.585	1.171	1.561	1.172	1.560	1.172
Student has an engaging math teacher, γ_{30}			−2.893	1.010*	−2.914	0.989*	−2.913	0.989**
Student is confident in math, γ_{40}			16.697	1.120***	16.684	1.146***	16.682	1.146***
Fixed Effects – Teacher Level								
Years of experience, γ_{01}					−0.882	0.448*	−0.857	0.451~
Gender, γ_{02}					24.364	8.303**	23.948	8.351**
Graduate Degree, γ_{03}					−17.712	11.737	−17.094	11.781
Specialization in Mathematics, γ_{04}					12.721	7.437~	12.545	7.438~
Efficacy, γ_{06}					2.569	0.821**	2.279	0.994*
Preparedness, γ_{07}					5.879	3.071*	4.404	3.078*
Professional Development – None, γ_{08}					31.342	18.401~	31.434	18.395~
Less than 6 hours, γ_{09}					−1.195	12.608	−1.537	12.617
6 – 15 hours, γ_{010}					7.553	10.848	7.167	10.863
16 – 35 hours, γ_{011}					9.773	11.732	9.798	11.727
Deeper Learning Practices, γ_{05}							0.609	1.153
Random Effects								
Level 2 variance		5562.652***		4142.245***		3665.041***		3661.844***
Level 1 variance		4888.703		4236.473		4236.492		4236.454
Level 2 r-squared				0.255		0.341		0.342
Level 1 r-squared				0.133		0.133		0.133

***$p < 0.001$, **$p < 0.01$, *$p < 0.05$, ~$p < 0.10$

Note: More than 35 hours of professional development was the comparison group. Math teacher final weights applied to the model.

The complete results of the mediation analysis regarding the relationship between teacher deeper understanding and student mathematics applying in the United States is summarized in Table 5. This table shows the null model, Model A, the student level variables model, Model B, the student and teacher level variables model, Model C, and the final model, Model D, with deeper learning practices added to the analysis (i.e., the mediation model). Once teacher deeper learning practices are entered into the analysis, the level-1 variables of socioeconomic status, student feelings toward mathematics and student confidence in mathematics have a significant effect on student achievement in mathematics applying (B = 1.511, p = 0.045; B = −2.710, p = 0.002; B = 16.692 and $p <$ 0.001, respectively). Level-2 variables of teacher preparedness and teacher deeper learning practices are statistically significant (B = 24.147, p = 0.020, an increase of approximately one-quarter of a standard deviation; B = 1.946 and p = 0.085, an increase of only one-fiftieth of a standard deviation, respectively). While teacher deeper learning practices is also significant, because the relationship between teacher preparedness and student achievement in mathematics applying in the United States in effect did not change, we cannot conclude that deeper learning practices is a mediator of this relationship—it is simply a covariate, and an important one at that. It can be seen across the models that there is still a significant amount of variance that is remains unexplained. The model does explain a portion of the overall variance (pseudo-$r^2 = 0.210$ at the teacher level and pseudo-$r^2 = 0.200$ at the student level.

Table 5

2-Level HLM of the Effects of Teacher Preparation and Teacher Efficacy on Student Achievement in Mathematics Applying Skills for the United States

	Model A		Model B		Model C		Model D	
	β	*SE*	β	*SE*	β	*SE*	β	*SE*
Fixed Effects – Student Level								
Intercept γ_{00}	539.683	2.591***	541.280	2.673***	538.341	8.671***	540.309	8.697***
Gender, γ_{10}			−3.371	2.089	−3.355	2.088	−3.377	2.089
Student likes learning math, γ_{20}			−2.706	0.700**	−2.706	0.700**	−2.710	0.700**
Student is confident in math, γ_{30}			16.705	0.587***	16.695	0.586***	16.692	0.586***
Socioeconomic status, γ_{40}			1.510	0.711*	1.512	0.710*	1.511	0.709*
Fixed Effects – Teacher Level								
Years of experience, γ_{01}					0.447	0.287	0.518	0.288~
Gender, γ_{02}					1.467	6.903	−0.244	6.932
Graduate Degree, γ_{03}					−3.710	4.994	−4.439	4.988
Specialization in Mathematics, γ_{04}					1.257	7.345	1.631	7.308
Efficacy, γ_{06}					0.834	0.551	0.494	0.579
Preparedness, γ_{07}					23.623	2.132*	24.147	2.121*
Professional Development – None γ_{08}					8.260	9.787	7.827	9.736
Less than 6 hours, γ_{09}					8.917	7.355	9.594	7.327
6 – 15 hours, γ_{010}					2.509	6.991	1.895	6.961
16 – 35 hours, γ_{011}					−0.445	7.598	−0.912	7.563
Deeper Learning Practices, γ_{05}							1.946	0.988*
Random Effects								
Level 2 variance		2024.438***		1726.057***		1618.987***		1599.470***
Level 1 variance		4756.098		3802.557		3802.645		3802.508
Level 2 r-squared				0.147		0.200		0.210
Level 1 r-squared				0.200		0.196		0.200

***$p < 0.001$, **$p < 0.01$, *$p < 0.05$, ~$p < 0.10$

Note: More than 35 hours of professional development was the comparison group. Math teacher final weights applied to the model.

The complete results of the mediation analysis regarding the relationship between teacher deeper understanding and student mathematics reasoning in the United States is summarized in Table 6. This table shows the null model, Model A, the student level variables model, Model B, the student and teacher level variables model, Model C, and the final model, Model D, with deeper learning practices added to the analysis (i.e., the mediation model). Throughout Models B, C, and D. When the level-2 variables are introduced in Model C, teacher preparedness has a significant effect on student achievement regarding mathematics reasoning ($B = 26.021$, $p = 0.013$, an increase of approximately one-third of a standard deviation). Once teacher deeper learning practices are entered into the analysis, the level-1 variables of student feelings toward mathematics and student confidence in mathematics have a significant effect on student achievement in mathematics reasoning ($B = -2.995$, $p = 0.006$; and $B = 16.258$, $p < 0.001$, an increase of approximately one-fifth of a standard deviation, respectively). The level-2 variable of teacher preparedness still has a significant effect ($B = 26.510$, $p = 0.013$, an increase of approximately one-third of a standard deviation) and teacher deeper learning practices is statistically significant ($B = 1.892$, $p = 0.046$, an increase of one-fiftieth of a standard deviation). While teacher deeper learning practices is also significant, because the relationship between teacher preparedness and student achievement in mathematics reasoning in the United States in effect did not change, we cannot conclude that deeper learning practices is a mediator of this relationship—it is simply a covariate, and an important one at that. It can be seen across the models that there is still a significant amount of variance that is still unexplained. The model does explain a portion of the overall variance (pseudo-$r^2 = 0.212$ at the teacher level and pseudo-$r^2 = 0.183$ at the student level).

Table 6

2-Level HLM of the Effects of Teacher Preparation and Teacher Efficacy on Student Achievement in Mathematics Reasoning Skills for the United States

	Model A		Model B		Model C		Model D	
	β	*SE*	β	*SE*	β	*SE*	β	*SE*
Fixed Effects – Student Level								
Intercept γ_{00}	532.516	2.761***	532.232	2.924***	528.752	8.398***	530.659	8.418***
Gender, γ_{10}			0.285	2.211	0.300	2.212	0.276	2.212
Student likes learning math			−2.990	0.824**	−2.990	0.825**	−2.995	0.824**
Student is confident in math			16.273	0.644***	16.261	0.644***	16.258	0.644***
Socioeconomic status			1.443	0.977	1.444	0.977	1.443	0.976
Fixed Effects – Teacher Level								
Years of experience, γ_{01}					0.405	0.269	0.474	0.270
Gender, γ_{02}					1.561	6.536	−0.100	6.563
Graduate Degree, γ_{03}					−3.017	4.790	−3.727	4.783
Specialization in Mathematics, γ_{04}					1.747	6.923	2.112	6.885
Efficacy, γ_{06}					0.759	0.531	0.429	0.558
Preparedness, γ_{07}					26.021	2.044*	26.510	2.033*
Professional Development – None γ_{08}					8.415	9.352	7.999	9.304
Less than 6 hours, γ_{09}					8.967	7.050	9.629	7.022
6 – 15 hours, γ_{010}					2.186	6.693	1.597	6.667
16 – 35 hours, γ_{011}					0.154	7.269	−0.294	7.265
Deeper Learning Practices, γ_{05}							1.892	0.947*
Random Effects								
Level 2 variance		1824.652***		1551.356***		1455.046***		1436.944***
Level 1 variance		4742.281		3873.813		3873.973		3873.783
Level 2 r-squared				0.150		0.203		0.212
Level 1 r-squared				0.183		0.183		0.183

***$p < 0.001$, **$p < 0.01$, *$p < 0.05$, ~$p < 0.10$

Note: More than 35 hours of professional development was the comparison group. Math teacher final weights applied to the model.

Based on these results, hypothesis three is not supported for the United States because we cannot conclude that mediation occurs in the relationship between teacher preparedness and student achievement in mathematics applying or mathematics reasoning. Instead deeper learning practices are an important covariate of these relationships. For Singapore, deeper learning practices do not mediate the relationship between teacher deeper understanding and student mathematics achievement for either applying or reasoning. Additionally, in Singapore teacher deeper learning practices are not a covariate in the relationship between teacher deeper understanding and student achievement in mathematics applying or mathematics reasoning.

Chapter 6: Discussion. Conclusions, and Suggestions for Future Research

The purpose of this study was to determine the relationship between teacher preparation, deeper teacher understanding, teaching for deeper learning, and student understanding in the elementary mathematics classroom in two key countries, the United States and Singapore. There are three guiding questions in regard to these countries: (1) What is the relationship between teacher preparation (both pre-service and in-service), mathematics teacher efficacy, and deeper learning in mathematics at the elementary school level?, (2) What is the relationship between deeper teacher understanding in mathematics (as measured by teacher efficacy and preparation) and teaching for deeper learning practice at the elementary school level, and (3) What is the relationship between teacher deeper learning practices and student deeper learning of mathematics at the elementary school level? From these questions, the following hypotheses were developed: (1) teacher preparation has a positive effect on teacher efficacy, (2) Deeper Teacher Understanding has a positive relationship with student mathematics applying skills (i.e., determination of appropriate strategies, representation of problems in a graphical or pictorial method, and implementation of problem-solving strategies) and mathematics reasoning skills (i.e., justification, logical thinking, and inductive reasoning), and (3) The relationship between Deeper Teacher Understanding and student mathematics applying skills (i.e., determination of appropriate strategies, representation of problems in a graphical or pictorial method, and implementation of problem-solving strategies), and mathematics reasoning skills (i.e., justification, logical thinking, and inductive reasoning) is partially mediated by teacher Deeper Learning Practices. In the final chapter, the results of the study will be summarized, followed by a discussion of the results, and implications for policy and/or practice. Additionally, the limitations of this study and recommendations for future study will be addressed.

Summary of Results

The analysis results indicate that hypothesis one is supported; teacher preparation does have a positive effect on teacher efficacy, more so in the United States than Singapore. In regard to hypothesis two, a positive relationship exists between deeper teacher understanding and student achievement in mathematics applying and mathematics reasoning; therefore, hypothesis two is supported. When considering hypothesis three, the relationship between deeper teacher understanding and student achievement is not mediated by deeper teaching practices in Singapore or the United States. Deeper learning practice has a positive relationship on student achievement in mathematics applying and mathematics reasoning in the United States; however, little to no relationship exists between these two variables in Singapore. Teacher deeper learning practices are not a mediator but rather yet another predictors/covariate in the relationship between teacher preparedness and student achievement in mathematics applying and mathematics reasoning in the United States. It can be concluded that hypothesis three is not supported in Singapore or the United States.

Based on the results of these three hypotheses, following conclusions can be made regarding the three guiding questions for this study:

(1) What is the relationship between teacher preparation (both pre-service and in-service), mathematics teacher efficacy, and deeper learning in mathematics at the elementary school level? There is a moderate positive relationship between teacher preparation and mathematics teacher efficacy in the United States. There

is a weak to moderate positive relationship between teacher preparation and mathematics teacher efficacy in Singapore.

(2) What is the relationship between deeper teacher understanding in mathematics (as measured by teacher efficacy and preparation) and teaching for deeper learning practice at the elementary school level? There is a strong positive relationship between teacher deeper understanding in mathematics and deeper learning practices in Singapore; furthermore, there is a moderate positive relationship between teacher deeper understanding in mathematics and deeper learning practices in the United States.

(3) What is the relationship between teacher deeper learning practices and student deeper learning of mathematics at the elementary school level? There is a stronger relationship between teacher deeper learning practices and student deeper learning of mathematics for students in the United States than in Singapore. This relationship is statistically significant in the United States but not in Singapore.

One somewhat surprising discovery during the analysis is the variation in significant indicators of success at the student level. For Singapore, student feelings of belonging at school, presence of an engaging teacher, and student confidence in mathematics had a significant relationship with performance in regard to mathematics applying and mathematics reasoning; however, when these same factors were considered in the United States model, only the student confidence in mathematics had a significant relationship with student performance. Instead, factors of student enjoyment of math and socioeconomic status had a significant relationship with performance for students in the United States. These factors were then considered in the Singapore model but removed because the factors did not reflect a significant relationship to student performance. The goal of the models was to determine what factors contributed to student achievement and as a result, the final models for the two countries varied in student level indicators to reflect the different factors that are present at the student level and relate to student achievement in each country.

Additionally, the gender of the student was significant in Singapore but not in the United States. For mathematics reasoning skills, females significantly outperformed males in Singapore. This was not the case for mathematics applying skills in Singapore. However, it is an interesting result since the stereotypical successful math student is male. For both mathematics applying and mathematics reasoning, Singapore students who had a female teacher performed at statistically significant higher levels than students who had a male teacher. This relationship was not present in the United States data. One common thread throughout all of the models for each country is the crucial importance of student confident in mathematics. This was significant at the $p < 0.001$ level for both mathematics applying and mathematics reasoning for Singapore and the United States. Teachers not only have the important task of providing students with mathematics instruction but also engaging in activities that can help bolster student confidence in mathematics since this is so vitally important for student achievement.

Discussion

This study originated from an interest in student performing in deeper learning regarding mathematics in the United States. The primary goal was to determine what was causing the lack of student understanding of mathematics at deeper levels. In order to determine this, it became clear that a comparative study between the United States and a country that was succeeding at producing students who were exhibiting deeper learning in mathematics was necessary. After reviewing various student achievement reports, one clear choice for this comparison was Singapore. Where the United States only had 8.8-percent of the population score high enough to be considered a “top performer”, that is a student who exhibits the characteristics of deeper learning, 40.0-percent of the Singapore population attained the necessary score as a “top performer” (Rothman, 2013). This startling figure supported the selection of Singapore for the analysis and led to the question “what is Singapore doing that the United States is not?”

It is evident from the findings of this study that student achievement is a multifaceted concept with numerous variables relating to the outcome. From the review of the literature, this complexity was to be expected. For both countries, teacher preparedness was a significant predictor of student achievement in mathematics applying and mathematics reasoning; however, this was not the case for teacher efficacy, which begs the question: why? What makes Singapore and the United States so different that teacher efficacy is significant in Singapore but not the United States? A few possible rationales come to mind.

First, the United States has a plethora of institutions across the country that are tasked with training future teachers where Singapore has one single institution for this same task. In the United States, each state has several institutions of higher learning with teacher preparation programs. This allows for preservice teachers to receive a wide range of training experience, with some programs being far more successful than others in creating teachers who are confident in their mathematics abilities and can bring that confidence into the classroom. While there are accreditation organizations in place to ensure the quality of teacher preparation programs, the sheer number of institution options introduces a multitude of variables, such as curriculum components, internship experiences, and pedagogical beliefs, into training of preservice teachers. While having numerous institutions allows preservice teachers to select a program that may be in line with their interests, it also creates less standardization in the instruction and training of preservice teachers.

Additionally, where the admission requirements for the United States institutions are varied in complexity, with some institutions having fairly lax requirements compared to others, there is one set of criteria for the Singapore teacher preparation program and the requirements candidates must meet to enroll in that program are high (Ginsburg, Leinwand, Anstrom, & Pollock, 2005). This sets a very different stage for candidates in Singapore when compared to the United States. Where Singapore is able to focus on the pedagogical understanding when instructing preservice teachers, the United States must address any gaps in the preservice teacher’s knowledge of core subjects prior to or in concert with providing instruction regarding pedagogy (Ginsburg, et al., 2005). While the “knowing” domain is important, the ability to apply and reason is crucial to deeper learning (Bellanca, 2014; Boaler, 2015). Since preservice teachers in Singapore enter the teacher preparation program with a solid foundation of mathematics content knowledge, the focus of their training can be on the reasoning and applying components of mathematics. Many education programs in the United States do not afford preservice teachers the opportunity to become more comfortable with mathematics and, as a result, may be more reluctant to venture into the areas of applying and reasoning in their classroom. Teachers tend to teach concepts they feel confident in and so teacher education programs in the United States need to provide preservice teachers with the guidance necessary to grow in mathematical pedagogical understanding to help promote the inclusion of mathematics applying and mathematics reasoning in the classroom (Althauser, 2018; Hine, 2015; Looney, Perry, & Steck, 2017).

If the preservice teacher has been trained in a program where the two are taught independently, it can be difficult for the teacher to develop specialized content knowledge and analytical knowledge. This creates a self-sustaining cycle of instructional practice and that is likely to continue unchanged without a deliberate effort to address the flaws in the current system and develop a strategy for transitioning to a classroom where deeper learning can, and does, occur because teachers tend to teach the way in which they were taught (Ball, 1988). When preservice teachers are trained in a program that provides a more holistic view of mathematics, one where content and pedagogy are taught concurrently, the teacher is more willing to develop student-led approach to mathematics instruction in the classroom. The benefit of this transition is that, since teachers tend to teach the way in which they were taught, once the change occurs, it will be self-sustaining as well because the effects of training teachers to effectively implement a student-led classroom remain in place for years after graduating from their teacher preparation program (Suppa, DiNapoli, & Mixwell, 2018). When teachers encounter difficulty in the classroom and revert to their "comfort zone" approach to teaching, it will still have the characteristics of the deeper learning classroom because that is the method in which they received their instruction.

For Singapore, there were a variety of characteristics at the teacher level that made a significant difference in student achievement, such as gender, mathematics specialization, efficacy and preparedness; however, for the United States, there were two key indicators at the teacher level: teacher preparedness and deeper learning practices. When looking at the composition of teachers for each of the countries, some of the difference may be explained. In Singapore, 62.9-percent of elementary teachers had a specialization in mathematics during their teacher preparation program where, in the United States, only 8.0-percent had this specialization. With such a small percentage having this type of training, it would stand to reason that the relationship between teacher specialization in mathematics and student achievement would not be statistically significant.

From previous research, it can be seen that teacher efficacy is a key component of effective instruction (Swackhamer, Joellner, Basile, & Kimbrough, 2009; Tschannen-Moran, Hoy, & Hoy, 1998; Tschannen-Moran & Barr, 2004). Teachers who have high levels of teaching efficacy tend to produce students who have a better understanding of the content and have higher levels of achievement. This is true in the mathematics classroom as well, where teachers with higher levels of mathematics efficacy tend to be more effective than teachers with low levels of mathematics efficacy, even when both types of teachers have high levels of mathematics content knowledge (Midgley, Feldlaufer, & Eccles, 1989; Kim & Seo, 2018). When looking at the Singapore analysis results, teacher efficacy is statistically significant in both models where teacher level factors were considered; however, the statistical significance of teacher efficacy decreases from $p < 0.01$ to $p < 0.05$ when teacher deeper learning practices are added into the model, even though these practices did not have a statistically significant relationship with student achievement. Conversely, for the United States, teacher efficacy did not have statistical significance in either model, but teacher deeper learning practices were statistically significant for student achievement in both mathematics applying, that is determination of appropriate strategies, representation of problems in a graphical or pictorial method, and implementation of problem-solving strategies, and mathematics reasoning skills, that is justification, logical thinking, and inductive reasoning.

Implications for Policy and/or Practice

Student achievement depends not only upon the training the teacher received to become a teacher, but also student characteristics and teacher characteristics as well. As seen in the analysis results, student characteristics play a significant role in the performance of the student. The same can be said about teacher characteristics as well; however, these are things schools and teacher preparation institutions have little to no control of and must strive to help all students succeed. One thing that teacher preparation institutions can control is what type of experience the prospective teacher has while at the institution and what tools they are equipped with when they enter the classroom.

One important factor for student achievement in Singapore is teacher efficacy, which is in alignment with previous research regarding teacher efficacy. Teachers who have higher levels of efficacy produce students who achieve at higher levels (Caprara, Barbaranelli, Steca, & Malone, 2006;Kim & Seo, 2018; Klassen & Tze, 2014; Mohamadi & Asedzedeh, 2012). Teachers who have higher levels of mathematics efficacy produce students who perform better in regard to mathematics applying and mathematics reasoning; therefore, teacher preparation institutions would be well served to make a concerted effort to recognize perspective teachers who have low efficacy and make strides to help these teachers with improve their efficacy and confidence in mathematics.

Simply forcing these teachers to take additional mathematics classes is not enough because this may only exacerbate their feelings of insecurity on the subject if they struggle in the class, as previous research has indicated (Althauser, 2018; Looney, Perry, & Steck, 2017; Swars, Daane, & Giesen, 2006). Taking the necessary steps to cultivate mathematical understanding and help these teachers experience success in mathematics will no doubt be time consuming; however, the payout for this extra time will be the production of students who are able to understand mathematics at a deeper level and apply it to situations beyond the isolated setting in which the material was learned.

Teacher preparedness is also a key factor to consider when looking at improving student achievement, especially in deeper learning characteristics such as mathematics applying and mathematics reasoning. In both Singapore and the United States, teachers with higher levels of preparedness produced students who perform better in mathematics applying and mathematics reasoning. It is important to note, however, that this level of preparedness needs to occur pre-service rather than in-service. As the models for both Singapore and the United States show, the lack of recent professional development experience in mathematics produced greater gains in student performance than participation in professional development produced. This, like the relationship between teacher efficacy and student learning, supports the importance of the components of teacher preparation program. Teacher preparation in the United States has done little to change the methods used for teaching mathematics today when compared to previous practice (Boaler, 2015; Hiebert, Morris, & Glass, 2003). When reviewing teacher preparation programs, institutions should bear in mind the important role of teacher efficacy and teacher preparation regarding student deeper learning and focus curriculum for preservice teachers on experiences that will promote an increase in these two key areas.

Although, the importance of effective teacher preparation programs cannot be ignored, once teachers have entered the profession, it then falls on their district to help teachers who may struggle in producing students who are deeper learners to improve their teaching practices. Simply sending teachers to additional professional development is not enough and, in some cases, as the results of this study indicate is actually counterproductive because it could lead to a decrease in student achievement regarding mathematics applying and mathematics reasoning. Professional development can positively affect the mathematical knowledge of teachers; however, this is more likely when the teacher views the professional development as relevant and/or necessary (Polly, Martin, McGee, Wang, Lambert, & Pugalee, 2017). Opportunities should not be solely focused on teacher content knowledge or knowledge of teaching, but also on improving teacher confidence in mathematics (Boaler, 2015; Schmidt, et al, 2007). By doing so, teachers can develop their knowledge to the level necessary to create effective learning experiences in the classroom (Saliga, Daviso, Stuart, & Pachnowski, 2015). Districts would be well served to consider experiences that are more personalized and meaningful to the teacher instead of simply enrolling a struggling teacher in vast amount of professional development in hopes that will produce the desired improvement in the classroom.

Limitations

When considering the implications from this study, it is important to recognize the limitations of it. The most apparent limitation that there is substantial amount of variance in student achievement that was not explained by the student level or teacher level factors analyzed in this study; however, due to the structure of the TIMSS study, these were the only levels that were able to be included in the analysis. The causes for this variance could be at the school, district, or regional (i.e., state) level. The TIMSS study is structured in a way that only a small number of classrooms per school site are selected for inclusion in the study. The climate and culture of the school has an important role on both student achievement and teacher efficacy (Chong, Klassen, Haun, Wong, & Kates, 2010). Additionally, school and district leadership play a vital role in the teacher practices and student achievement because the administration can encourage teachers to shift their classroom from teacher-led to one that is more student-led as well as provide teachers with professional development opportunities that are relevant, meaningful, and in alignment with the vision of the school. Although the importance of the school and district characteristics as well as school and district leadership cannot be ignored, the requirements for studying the school level were not met and as a result this study was unable to nest the classrooms into schools and/or districts. Without nesting at the school level, the analysis was limited to the student and classroom levels.

Another potential limitation to the study is the lack of information on where the teachers in the United States received their training. Just as the importance of the school and district characteristics and leadership cannot be ignored, the characteristics of the teacher preparation program are also a vital component to be considered. While Singapore has one institution for teacher preparation, the United States has numerous institutions in each state, making it difficult to determine if the differences in teacher efficacy and/or teacher preparation are a result of the instruction that the teacher received at their teacher preparation institution or from experiences either before or after their training. Additionally, the large number of institutions in the United States makes it difficult to determine if certain characteristics of teacher preparation programs have greater significance on teacher effectiveness.

This data is from one level of a single cycle of TIMSS, the elementary level of the 2015 cycle, which introduces another limitation to this study. The generalizability of findings is limited to the elementary level. Performing similar analyses on other cycles, such as the 2019 cycle, could provide more information about the relationship between teacher deeper understanding and student deeper learning. By incorporating more than one cycle, the longitudinal trends can be explored which would provide a richer landscape for the analysis of this relationship and any changes that occur.

Suggestions for Further Research

While this study does address a gap in the current literature regarding student achievement, deeper learning, and the importance of the teacher on deeper learning experiences, additional research is required in this area. The degree of variance that remains unexplained demonstrates the need for additional study to be completed to determine where this additional variance originates. There are several possibilities as to the origin of this unexplained variance.

One of the possibilities, and a likely source, is differences that exist at the school and/or district level. The importance of the beliefs and leadership of school and district administration cannot be ignored; however, due to the composition of the TIMSS data, this study was not able to analyze the school or district level because the minimum requirements for nesting the classrooms into schools and/or districts for analysis was not met. As a result, it was not possible to determine what school level variables, if any, related to student performance in regard to mathematics deeper learning. With this limitation in mind, the exploration of school level factors that could be related to student achievement in deeper learning areas recommended for future research.

Additional study of the inclusion of deeper learning practices in teacher preparation programs would also be beneficial. Teachers tend to teach the way they were trained (Ball, 1988). As a result, changes to teacher preparation programs may be necessary to provide preservice teachers with the training necessary to be successful at teaching for deeper learning; however, this change needs to be calculated and not simply an arbitrary action with the hope that the desired results will happen. In order to make meaningful changes to teacher preparation components, additional research is necessary on what institutions are doing now, what actions are successful, and what actions are not.

The implications for the United States are alarming and, as such, illustrate the necessity of additional research on what attributes of the current teacher preparation program construction is contributing to the success of the teachers produced by it and what attributes are in need of revision or restructuring to improve the performance of the teacher. One possible starting place would be to consider how closely the teacher preparation programs in the United States mirror that of Singapore and what characteristics of the Singapore model, if any, can be adapted and implemented into United States teacher preparation programs.

Conclusion

There are many factors that influence the performance of students including, but certainly not limited to, the students, teacher, school, and location. It is crucial that this be recognized when looking at how to improve student performance. What works for one student may not work for another. What works for one teacher may not be as successful for another teacher. The same can be said about schools, locations, states, countries, et cetera; however, recognizing the common threads that are woven throughout these differences can lead to initiatives that will produce the desired results for our students.

While many areas of the United States are experiencing a vast teacher shortage, despite the availability of institutions for teacher preparation, admission into the Singapore teacher preparation program is competitive and sought after by those hoping to become educators (Ginsburg, Leinwand, Anstrom, & Pollock, 2005). This allows Singapore to be more selective in their admission process and focus teacher preparation on areas more closely associated with pedagogy, which shortens the amount of time the preservice teacher must spend in training prior to entering the classroom (Ginburg et al., 2005).

In order to be successful in the future workforce, it is vital for students to be able to understand concepts at a deeper level because that is where problem-solving begins and the creation of a generation of people who can think beyond the confines of a multiple choice question is nurtured. There is much work to be done in the United States to break the pattern of teaching surface-level information and begin the exploration of the depths of knowledge that students in other countries are already diving into and applying to the new situations they encounter. For this to become a reality, more must change than simply the type of testing that students complete, curriculum standards, or the introduction of more technology to the classroom. Teacher preparation programs must adapt to meet the needs of the future workforce by equipping teachers with the skills necessary to lead their students into the realm of deeper learning. A change of this magnitude will not be easy nor will it happen quickly; however, it is vital to develop the United States educational program into one that routinely produces deeper learners rather than these students being a rare product of the educational system as it is today. While there are schools, such as High Tech High in San Diego, California, who are moving to this type of learning environment, it is not common-place and will not be self-sustaining without a greater number of schools, districts, and preservice programs aligning with this initiative.

As with so many initiatives, there is an unknown amount of time that will pass before the benefits of a change to deeper learning will be easily seen. Policy makers and school leaders may be reluctant to continue toward full implementation of deeper learning when immediate results are not easily seen; however, if sustained the result will be a new generation of students who are capable of a greater level of problem-solving and abstract thought than what schools are currently producing. This is the type of student who will be able to succeed in future workplace experiences. Some of these students will ultimately decide to become educators, and they will be able to pass their knowledge of problem-solving skills and deeper learning to the next generation of students, creating and infinite loop of development of strategic thinking citizens. The future of our country rests on the changes we implement with the students of today.

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Appendix A

Trends in Mathematics and Science Study (TIMSS) Items Included in Analysis

Teacher Preparation (pre and in-service)

Code		Item
ATBM09A	–	In the past two years, have you participated in professional development in any of the following? Mathematics content
ATBM09B	–	In the past two years, have you participated in professional development in any of the following? Mathematics pedagogy/instruction
ATBM09C	–	In the past two years, have you participated in professional development in any of the following? Mathematics curriculum
ATBM09E	–	In the past two years, have you participated in professional development in any of the following? Improving students' critical thinking and problem-solving skills
ATBM09F	–	In the past two years, have you participated in professional development in any of the following? Mathematics assessment
ATBM09G	–	In the past two years, have you participated in professional development in any of the following? Addressing individual students' needs
ATBM10	PD_0 PD_1 PD_2 PD_3 PD_4	In the past two years, how many hours in total have you spent in formal <in-service/professional development> for mathematics?; PD_0 = no professional development, PD_1 = less than 6 hours, PD_2 = 6 – 15 hours, PD_3 = 16 – 35 hours, PD_4 = more than 35 hours
ATBM11AA	–	How well prepared do you feel you are to teach the following mathematics topics? Number: Concepts of whole numbers, including place value and ordering

ATBM11AB	–	How well prepared do you feel you are to teach the following mathematics topics? Number: Adding, subtracting, multiplying, and/or dividing whole numbers
ATBM11AC	–	How well prepared do you feel you are to teach the following mathematics topics? Number: Concepts of multiples and factors; odd and even numbers
ATBM11AD	–	How well prepared do you feel you are to teach the following mathematics topics? Number: Concepts of fractions (fractions as parts of a whole or of a collection, or as a location on a number line)
ATBM11AE	–	How well prepared do you feel you are to teach the following mathematics topics? Number: Adding and subtracting with fractions, comparing and ordering fractions
ATBM11AF	–	How well prepared do you feel you are to teach the following mathematics topics? Number: Concepts of decimals, including place value and ordering, adding and subtracting with decimals
ATBM11AG	–	How well prepared do you feel you are to teach the following mathematics topics? Number: number sentences
ATBM11AH	–	How well prepared do you feel you are to teach the following mathematics topics? Number: Number patterns
ATBM11BA	–	How well prepared do you feel you are to teach the following mathematics topics? Geometric Shapes and Measures: Lines: Measuring, estimating length of; parallel and perpendicular lines
ATBM11BB	–	How well prepared do you feel you are to teach the following mathematics topics? Geometric Shapes and Measures: Comparing and drawing angles
ATBM11BC	–	How well prepared do you feel you are to teach the following mathematics topics? Geometric Shapes and Measures: Using

		informal coordinate systems to locate points in a plane
ATBM11BD	–	How well prepared do you feel you are to teach the following mathematics topics? Geometric Shapes and Measures: Elementary properties of common geometric shapes
ATBM11BE	–	How well prepared do you feel you are to teach the following mathematics topics? Geometric Shapes and Measures: Reflections and rotations
ATBM11BF	–	How well prepared do you feel you are to teach the following mathematics topics? Geometric Shapes and Measures: Relationships between two-dimensional and three-dimensional shapes
ATBM11BG	–	How well prepared do you feel you are to teach the following mathematics topics? Finding and estimating areas, perimeters, and volumes
ATBM11CA	–	How well prepared do you feel you are to teach the following mathematics topics? Data Display: Reading and representing data from tables, pictographs, bar graphs, or pie charts
ATBM11CB	–	How well prepared do you feel you are to teach the following mathematics topics? Data Display: Drawing conclusions from data displays
	TP	Sum of ATBM11AA, ATBM11AB, ATBM11AC, ATBM11AD, ATBM11AE, ATBM11AF, ATBM11AG, ATBM11AH, ATBM11BA, ATBM11BB, ATBM11BC, ATBM11BD, ATBM11BE, ATBM11BF, ATBM11BG, ATBM11CA, ATBM11ATBMCB
–	TPSQRT	TP transformed to normalize data for a left skew

Appendix B

Trends in Mathematics and Science Study (TIMSS) Items Included in Analysis

Items for Teacher Efficacy

TIMSS Code	Analysis Code	Item
ATBM02A	–	In teaching mathematics to this class, how would you characterize your confidence in doing the following? Inspiring students to learn mathematics
ATBM02B	–	In teaching mathematics to this class, how would you characterize your confidence in doing the following? Showing students a variety of problem-solving strategies
ATBM02C	–	In teaching mathematics to this class, how would you characterize your confidence in doing the following? Providing challenging tasks for the highest achieving students
ATBM02D	–	In teaching mathematics to this class, how would you characterize your confidence in doing the following? Adapting my teaching to engage students' interest
ATBM02E	–	In teaching mathematics to this class, how would you characterize your confidence in doing the following? Helping students appreciate the value of learning mathematics
ATBM02F	–	In teaching mathematics to this class, how would you characterize your confidence in doing the following? Assessing student comprehension of mathematics
ATBM02G	–	In teaching mathematics to this class, how would you characterize your confidence in doing the following? Improving the understanding of struggling students
ATBM02H	–	In teaching mathematics to this class, how would you characterize your confidence in doing the following? Making mathematics relevant to students
ATBM02I	–	In teaching mathematics to this class, how would you characterize your confidence

		in doing the following? Developing students' higher-order thinking skills
–	TCHEFFIC	Sum of items ATBM02A, ATBM02B, ATBM02C, ATBM02D, ATBM02E, ATBM02F, ATBM02G, ATBM02H, ATBM02I

Appendix C

Trends in Mathematics and Science Study (TIMSS) Items Included in Analysis

Teacher Deeper Learning Practices

TIMSS Code	Analysis Code	Item
ATBG14A	–	How often do you do the following in teaching this class? Relate the lesson to students' daily lives
ATBG14B	–	How often do you do the following in teaching this class? Ask students to explain their answers
ATBG14D	–	How often do you do the following in teaching this class? Ask students to complete challenging exercises that require them to go beyond the instruction
ATBG14E	–	How often do you do the following in teaching this class? Encourage classroom discussions among students
ATBG14F	–	How often do you do the following in teaching this class? Link new content to students' prior knowledge
ATBG14G	–	How often do you do the following in teaching this class? Ask students to decide their own problem-solving procedures
ATBG14H	–	How often do you do the following in teaching this class? Encourage students to express their ideas in class
–	TCHDLP	Sum of items ATBG14A, ATBG14B, ATBG14C, ATBG14D, ATBG14E, ATBG14F, ATBG14G, ATBG14H

Appendix D

Trends in Mathematics and Science Study (TIMSS) Items Included in Analysis

Student Achievement

TIMSS Code	Analysis Code	Item
ASMAPP01	ASMAPP01	1st Plausible Value for Math Applying
ASMAPP02	ASMAPP02	2nd Plausible Value for Math Applying
ASMAPP03	ASMAPP03	3rd Plausible Value for Math Applying
ASMAPP04	ASMAPP04	4th Plausible Value for Math Applying
ASMAPP05	ASMAPP05	5th Plausible Value for Math Applying
ASMREA01	ASMREA01	1st Plausible Value for Math Reasoning
ASMREA02	ASMREA02	2nd Plausible Value for Math Reasoning
ASMREA03	ASMREA03	3rd Plausible Value for Math Reasoning
ASMREA04	ASMREA04	4th Plausible Value for Math Reasoning
ASMREA05	ASMREA05	5th Plausible Value for Math Reasoning

Appendix E

Trends in Mathematics and Science Study (TIMSS) Items Included in Analysis

Student control and/or demographic information

TIMSS Code	Analysis Code	Item
ITSEX	FEMALE	Student Gender
ASBG05A	–	Do you have any of these things at your home? A computer or tablet of your own
ASBG05B	–	Do you have any of these things at your home? A computer or tablet that is shared with other people at home
ASBG05C	–	Do you have any of these things at your home? Study desk/table for your use
ASBG05D	–	Do you have any of these things at your home? Your own room
ASBG05E	–	Do you have any of these things at your home? Internet connection
ASBG05F	–	Do you have any of these things at your home? Your own mobile phone
ASBG05G	–	Do you have any of these things at your home? A gaming system
ASBG05H	–	Do you have any of these things at your home? <country-specific indicator of wealth>
ASBG05I	–	Do you have any of these things at your home? <country-specific indicator of wealth>
ASBG05J	–	Do you have any of these things at your home? <country-specific indicator of wealth>
ASBG05K	–	Do you have any of these things at your home? <country-specific indicator of wealth>
–	SES	Sum of ASBG05A, ASBG05B, ASBG05C, ASBG05D, ASBG05E, ASBG05F, ASBG05G, ASBG05H, ASBG05I, ASBG05J, ASBG05K
ASBGSSB	BELONG	Student sense of school belonging

ASBGSLM	LIKEMATH	Student enjoyment of studying math
ASBGEML	ENGMTCHR	Student has an engaging mathematics teacher
ASBGSCM	CONFIDMT	Student feels confident in mathematics

Appendix F

Trends in Mathematics and Science Study (TIMSS) Items Included in Analysis

Teacher control and/or demographical information

TIMSS Code	Analysis Code	Item
ATBG01	YRSTCH	By the end of this school year, how many years will you have been teaching altogether?
ATBG02	FEMALE	Are you female or male?
ATBG03	AGE	How old are you?
ATBG04	DEGREE	What is the highest level of formal education you have completed?
ATBG05AA	MAJELEM	During your <post-secondary> education, what was your major or main area(s) of study? Education – Primary/Elementary
ATBG05AB	MAJSEC	During your <post-secondary> education, what was your major or main area(s) of study? Education – Secondary
ATBG05AC	MAJMATH	During your <post-secondary> education, what was your major or main area(s) of study? Mathematics
ATBG05BA	SPECMATH	If your major or main area of study was education, did you have a <specialization> in any of the following? Mathematics
ATBM01	MATHTIME	In a typical week, how much time do you spend teaching mathematics to the students in this class (minutes)?