ERROR-DETECTING RACE-FREE SEQUENTIAL SWITCHING CIRCUITS

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SWITCHING CIRCUITS

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PREFACE

From the field of switching circuits various automatic control systems have arisen, examples of which are the modern large-scale digital computers and the automatic telephone switching systems. These recent developments have done much to arouse in engineers and mathematicians alike an enthusiastic interest in regard to the unexplored possibilities inherent in the switching art. Doubtless, this presently held interest will mature in the future into valuable contributions because of the efforts of the many who will accept the challenge.

This work is tendered as a small aid for these future contributors.

I wish to express my appreciation to David L. Johnson of the Electrical Engineering staff of the Oklahoma Institute of Technology for supervising this work and acknowledge the counsel of Glenn Smith, Trade and Industrial Education, Oklahoma Agricultural and Mechanical College and Dr. Roy Deal, Mathematics Department, Oklahoma Agricultural and Mechanical College.

TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

CHAPTER I

INTRODUCTION

In one sense, the basic tool used in designing switching circuits is as old as man himself, for this tool is logic. However, it was the nineteenth century before a mathematics, Boolean algebra, was developed to represent logic.[1] In 1938 C. E. Shannon applied this mathematics to electrical switching circuits.[2]

Since 1938 much progress has been made by many engineers, progress that has developed computing machines able to think logically and remember indefinitely, capable of integrating, differentiating, and analyzing with the speed and ease of Atlas snapping a toothpick.

Regardless of the complexity or simplicity of these machines, they are still capable of making mistakes. It is with certain phases of these errors that this work is concerned.

Much of this treatise will be based upon an article by Professor D. A. Huffman recently published in the Journal of the Franklin Institute[3] because the flow table developed by Huffman was a great stride forward in the synthesis of sequential switching circuits.

Form and Symbols Used

Throughout this treatise input conditions will be designated by letters in the alphabet preceding M with the one exception of E, which will be used to designate the error circuitry. Secondary or memory

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[1]George Boole, The Laws of Thought.

[2]C. E. Shannon, "A Symbolic Analysis of Relays and Switching Circuits".

[3]D. A. Huffman, "The Synthesis of Sequential Switching Circuits."

relays will be designated by letters succeeding M, with the exception of Z, which will designate the output circuitry. Capital letters designate the relays; lower case letters designate the contacts on the relays. Normally open contacts are labeled with unprimed letters, and the normally closed contacts with the primed letters.

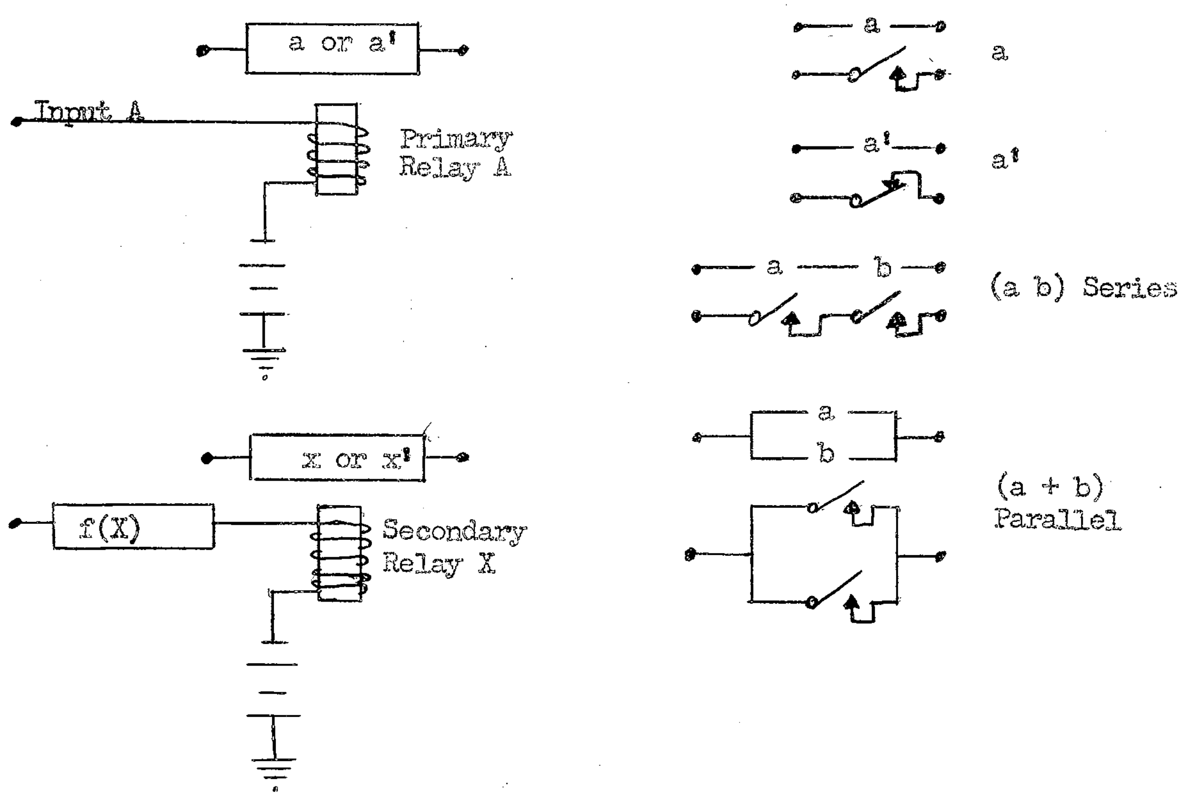

Figure 1. Graphical Symbols for Relays and Contacts.

Since a switch is either open or closed, it is a binary variable to which Boolean algebra may be applied.[4] A closed path will be represented by 1, an open path by 0. The plus (+) sign will indicate parallel paths; the times sign (•) will be used for series paths. Input information will be assumed to be in the form of grounding or un-

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[4]C. E. Shannon, "A Symbolic Analysis of Relays and Switching Circuits".

grounding input leads connected directly to the input relays. As seen in figure 1, primary relays are externally controlled, secondary relays are internally controlled. Simply because primary relays are externally controlled does not insure that they will always operate as prescribed.

<u>State-Type Sequence Diagram</u>

A sequential switching circuit is one in which the output is dependent on the sequence of inputs as well as the combination of inputs. The circuit must be able to remember what has happened in the past, digest this with what is happening at the present and decide what is to happen in the future.

At the present time, no practical system exists whereby we can designate sequential circuits by mathematics alone; therefore reference is made quite frequently to charts or diagrams. One such diagram is the state-type sequence diagram.[5]

Consider the following Example 1. Two wires bring in information in the following manner. Wire B is grounded with recurring pulses. If wire A is grounded at the end of a pulse on B and lasts until the end of the succeeding pulse, there is to be an output during the third pulse on B. See figure 2.

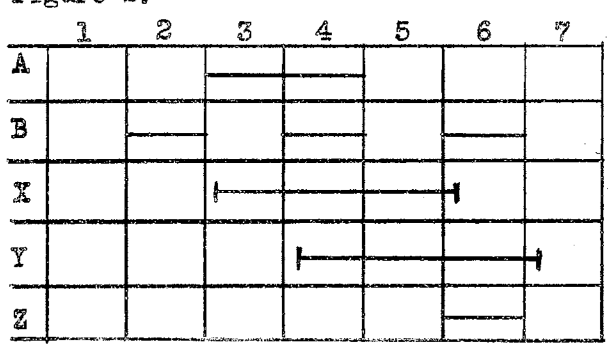

Figure 2. Sequence Diagram for Example 1.

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[5]W. Keister, A. E. Ritchie, and S. Washburn, <u>The Design of Switching Circuits</u>, p. 133.

This circuit has identical input intervals on each side of the output; therefore two secondary relays will be required.[6] These were selected as shown in figure 2. The X, Y, and Z circuits are shown in figure 3.

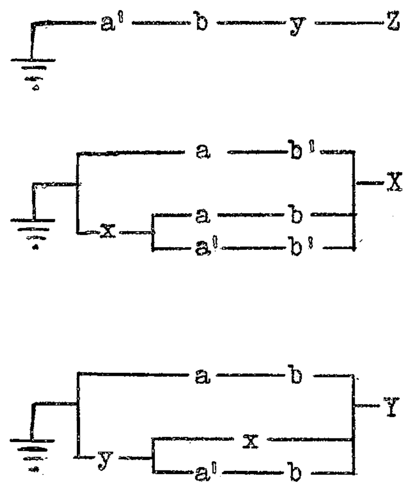

Figure 3. Circuit for Example 1.

Notice that this solution is not peculiar to this problem. For one thing, the resulting action is not dependent on the first pulse of B or on anything prior to interval three, as far as that goes. This may or may not be critical, but as the problem was originally stated it most probably would be. Also, the inputs have to come either from a human operator or from some previous switching circuit, and both of these are capable of making mistakes. The solution, not being peculiar, might work for some other sequence of A and B. If such a sequence can be found, then the solution may be of little practical value. One such sequence is shown in figure 4.

This particular sequence was chosen to illustrate two things: (1) that to find a particular peculiar solution to a sequential switch-

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[6] Ibid., p. 159.

ing circuit from a state-type diagram is very difficult and impractical, and (2) that race conditions are hazardous and sometimes lethal.

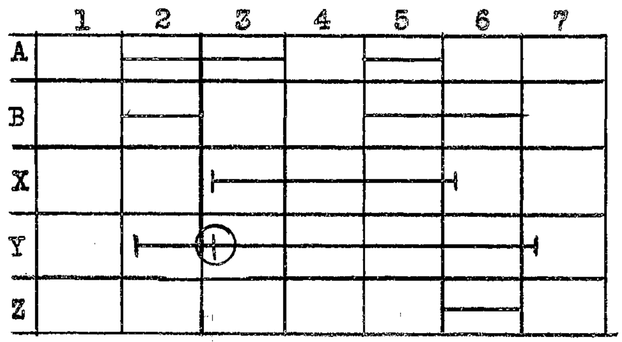

Figure 4. Alternate Sequence Diagram for Example 1.

The race condition is indicated by the circle in interval three of figure 4. Relay Y is operated in interval two by the combination ab. At the end of interval two, B releases, which would release Y, since Y is held by $(x + a'b)$. However, relay X operates in interval three by the combination ab'. Therefore, if X operates before Y is released, Y will remain operated, and an invalid output will occur.

<u>Huffman's Index</u>

Fundamentally, race conditions exist because of the finite interval between the instant a relay coil is connected to its source and the instant its contacts are operated. Digesting time might be an appropriate phrase. Since this time does exist, notice of it must be posted in the mathematics section.

It has previously been mentioned that 1 meant a closed path and 0 meant an open path. In these terms the following conditions are possible for any given relay R and its normally open contacts r.

$R = 0$
$r = 0$ relay unenergized, contacts open

$R = 1$
$r = 0$ relay energized, contacts not yet closed. (unstable)

$R = 1$
$r = 1$ relay energized, contacts closed.

$R = 0$
$r = 1$ relay unenergized, contacts not yet open. (unstable)

Notice that two of the conditions are labeled unstable. Therefore a relay has four states, two of which are stable and two of which are unstable.

Huffman has introduced a new variable[7] τ, called the transition index, which indicates the condition of a relay. If R and r are alike, the relay is stable and $\tau = 0$. If R and r are unlike, the relay is unstable and $\tau = 1$. This is equivalent to saying that

$$\tau = R + r \pmod 2$$
or
$$R = \tau + r \pmod 2$$

Addition mod 2 may be characterized by giving its addition table. That is,

$$0 + 0 = 0 \qquad \pmod 2$$

$$0 + 1 = 1 + 0 = 1 \quad \pmod 2$$

$$1 + 1 = 0 \qquad \pmod 2$$

[7] D. A. Huffman, "The Synthesis of Sequential Switching Circuits", p. 163.

CHAPTER II

TWO TECHNIQUES FOR SOLVING SEQUENTIAL SWITCHING CIRCUITS

Next, reconsider the previous problem step by step, and rearrange the state-type diagram. In this new arrangement each row will correspond to an interval in the state-type diagram. Since each interval of time in the incoming sequence is to be discreet, encircle the number designating that interval or row. The complete reasoning for this will be explained shortly. Across the top will be placed the four possible combinations of inputs to designate four columns of the resulting diagram. See figure 5.

ab: 00	10	11	01
①			
			②
	③		
		④	
⑤			
			⑥

Figure 5. Incomplete flow table for Example 1.

Each of these circled entries represents an input condition and, regardless of the complexity of the circuit, must be a stable circuit condition. This means that the transition index for each and every relay must be 0 for these combinations.

If the circuit is to be stable in each circled entry as mentioned above, a change in the input will be required in order to make any circuit changes. A change in the input corresponds to a horizontal move in the diagram of figure 5. Accordingly, the following rules may be stated.[8]

1. Each circled entry in a row of a flow table indicates a stable circuit condition, and no further changes will occur unless the input state is modified.

2. Each circled circuit condition within a row of a flow table can lead to any other circuit condition in the same row.

To move from entry 1 to entry 2 requires a horizontal movement from column 00 to column 01, and then a vertical movement from row one to row two. Since column 01 row one immediately leads to stable entry 2, it will be designated by an uncircled 2. See figure 6.

vxy \ ab	00	10	11	01
000	(1)			2
010		3		(2)
110		(3)	4	
100	5		(4)	
101	(5)			6
001	1			(6)

Figure 6. Flow Table for Example 1.

If the requirement is made that each change in input conditions has a peculiar internal or secondary relay change, each uncircled entry will be an unstable state. This means that at least one relay must

[8] *Ibid.*, p. 173.

have $\tau = 1$ for this combination of relays. It is necessary and sufficient that one and only one relay be unstable at any given time. If two relays are simultaneously unstable a race condition will result. To assure that one and only one relay be unstable at a given time requires an individual secondary relay combination for each row.

Assigning Secondary Relays.

The assigning of secondary relays requires only that they form, on an S - dimensional cube, a closed sequence line that returns to the origin with the last move. A closed sequence line of length L is any path which, starting at a given vertex of the cube, goes through L vertices of the cube subject to the conditions: (1) no vertex of the cube is entered by the line more than once, (2) the line travels only along edges of the cube, and (3) the two ends coincide. This assignment can be done in any table having an even number of rows. The number of secondary relays (S) required may be determined by the following rule.

In a table of n rows, S secondary relays will be required such that

$$2^{S-1} < n \leq 2^{S}$$

For example, if a table has seven rows, three secondary relays will be required since

$$2^{2} < 7 < 2^{3}$$

However, seven is an uneven number and a closed sequence line cannot have length seven. In this case one row would be added. Since 2^{S} will always be an even number, adding a row to an uneven-row table will not require an additional secondary relay.

The table of figure 6 has an even number of rows, and three secondary relays will be required.

$$2^2 < 6 < 2^3$$

One closed sequence line assignment is shown in figure 6. Notice that the combination 111 was avoided. After all eight possible types of sequence lines had been tried, and no one line showed an advantage, it was decided to avoid having all secondary relays on in any part of the sequence, thereby reducing power consumption. Of course, when the table has $n = 2^S$ rows this is not possible.

τ Table

Since, as previously mentioned, each uncircled entry is an unstable state, there must be one and only one $\tau = 1$ corresponding to each uncircled entry. The underlined entries in figure 7 correspond to the uncircled entries in figure 6. The 01,000 entry in figure 6 is an uncircled two, which indicates an unstable state that will change to circled two or 01,010.

vxy \ ab	00	10	11	01
000	000	000	000	<u>010</u>
010	000	<u>100</u>	000	000
110	000	000	<u>010</u>	000
100	<u>001</u>	000	000	000
101	000	000	000	<u>100</u>
001	<u>001</u>	000	000	000

Figure 7. τ Table for Example 1.

To change from the first row to the second row requires a change in relay X. When the combination 01,000 comes along X will be energized, making it unstable until x makes contact. When x is closed the new combination is 01,010 which is the circled entry two and the circuit is again stable.

If the solution is to be peculiar, only the uncircled entries in figure 6 should be unstable. All of the blank spaces in figure 6 are combinations that are not supposed to occur. In many instances, though, if they do occur there should be some warning. This can be accomplished by making all blank spaces stable conditions and the controlling combinations for an error relay. Thus all entries in the τ table will be zero with the exception of one $\tau = 1$ in each entry preceding each interval of the original problem.

<u>S Table</u>

The S table is to show in exactly what combinations each secondary relay is to operate. Since

$$\tau = R + r \pmod 2$$
or
$$R = \tau + r \pmod 2$$

each entry in the S table can be found by adding cyclically mod two each entry in the τ table with the secondary relay state for the same row. For example, in figure 7 the 01,000 entry is 010. Adding 000 and 010 cyclically mod two gives 010, and this then is the 01,000 entry in the S table.

After having completed the S table in the above manner, the unused combinations of secondary relays may be added as "don't care" conditions if desired. This will usually simplify the circuitry for the secondary relays. These may be used as "don't cares" only because they are mandatory in the error circuitry, as will be seen later. They are not supposed to happen, but if they do the error circuitry will take care of them.

vxy \ ab	00	10	11	01
000	000	000	000	010
010	010	110	010	010
110	110	110	100	110
100	101	100	100	100
101	101	101	101	001
001	000	001	001	001
011	DON'T CARE			
111				

Figure 8. S Table for Example 1.

These "don't care" conditions are as indicated in figure 8.

<u>Z - E Table</u>

Several times in this article reference has been made to errors and peculiar solutions. In many design problems it is essential that these be taken into account. This article is intended to show a technique of synthesis in such cases. One can not say as an absolute rule that this system will always require additional relays, although generally this is true. It will, in most instances, require additional contacts on the relays, but a problem will be presented later in this article that will require neither extra relays nor extra springs.

Since any combination of primary and secondary relays represented by the blank spaces in figure 6 are unwanted conditions, the error relay should operate when and if they occur. Thus an E is placed in each space corresponding to the blank or error conditions of figure 6. This is shown in figure 9. Also, any time the unused combinations of secondary relays occur, the error relay should operate. Thus the Z − E table will always have $n = 2^S$ rows.

vxy \ ab	00	10	11	01
000		E	E	
010	E		E	
110	E			E
100		E		E
101		E	E	
001		E	E	Z
011	E	E	E	E
111	E	E	E	E

Figure 9. Z − E Table for Example 1.

In this particular problem, an output was desired only in interval six. Therefore the Z entry will be the single combination 01,001.

If e' contacts are inserted in series with the Z network, output will be obtained only if there have been no errors.

From the S and Z − E tables to the final circuitry is but a matter of synthesizing combinational circuits. Following is one solution, not, however, the only one as anyone familar with combinational circuits will realize.

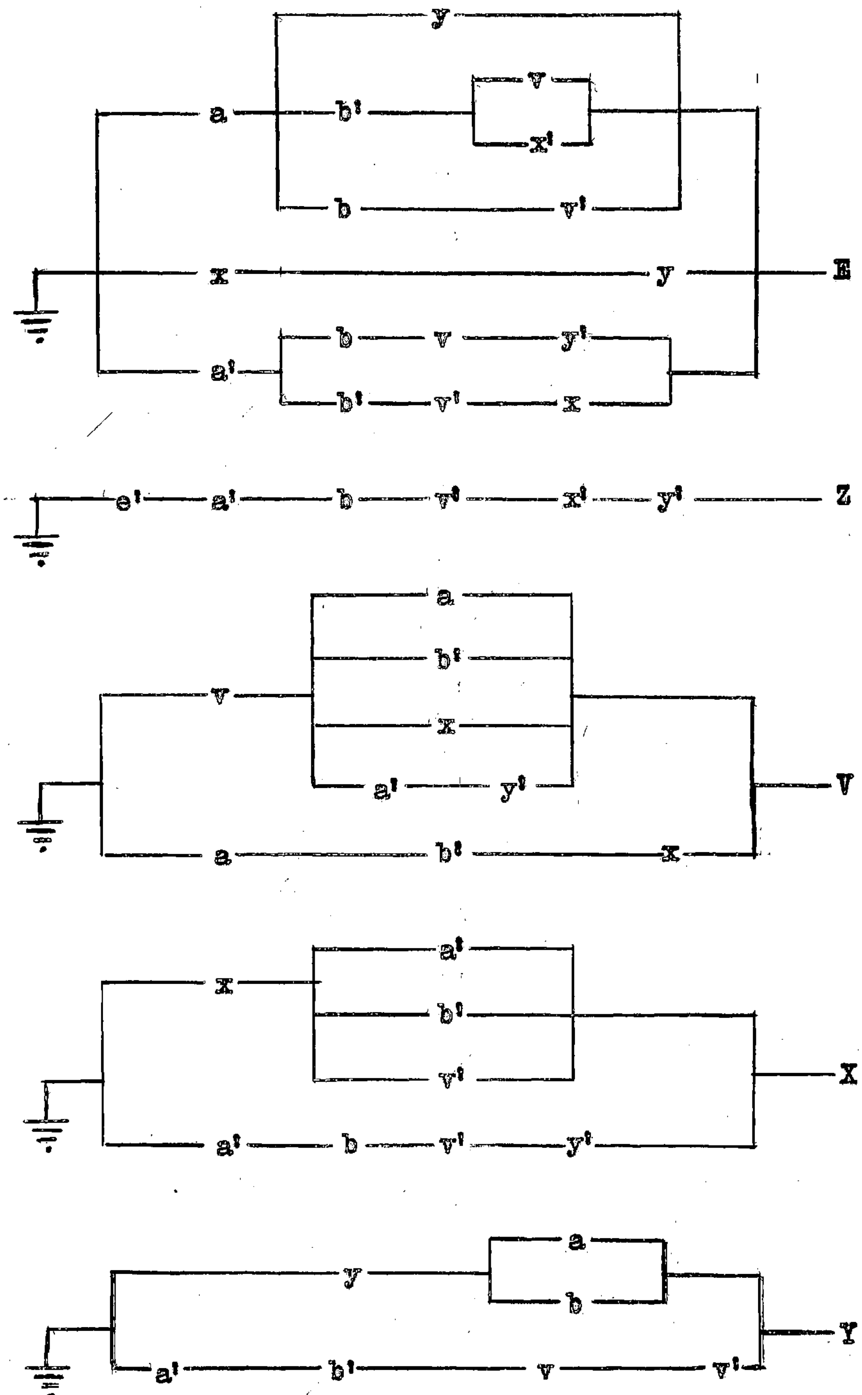

Figure 10. Alternate Circuit for Example 1.

<u>Alternative to Huffman's $2S_0 + 1$</u>

Although many problems in sequential switching circuits are so arranged that the presence of an error is not critical, one should not abandon the foregoing procedure and start merging rows.

Huffman states that any flow table with n rows can be designed which uses exactly $2S_0 + 1$ secondary relays, where S_0 is the least integer that satisfies the condition

$$n = 2^S.$$

In other words, after possible merging, a condensed flow table with n rows may not satisfy the requirements necessary to make each k-set[9] connected with $\log_2 n$ secondary relays. If this is so, it can still be synthesized with $2S_0 + 1$ secondary relays. By the procedure of this article any problem may be solved with S + 1 secondary relays, where S is such that

$$2^{S-1} < n \leq 2^S$$

in the primitive flow table. Certainly, if errors are not critical, whichever system yields the minimum number of relays and contacts should be used. Caution must be observed in minimizing relays though, since it is possible that in so doing a much greater number of springs may be required.

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[9] *Ibid.*, p. 275.

CHAPTER III

COMPARISON OF ERROR-DETECTING AND HUFFMAN TECHNIQUES

Methods Comparison

Huffman describes and neatly solves a problem of remembering.[10] For comparison of methods, the problem will be presented, solved by this author's procedure and compared with Huffman's.

The circuit of Example 2 to be synthesized has two output leads, each somewhat under the direct control of its respective input. Starting from condition one (for which neither of the inputs and neither of the outputs are grounded) grounding of the A input grounds the Z_1 output; grounding of the B input grounds the Z_2 output. But simultaneous grounding of both A and B results in no ground at the output. In the latter case, no possibility of an output ground exists until the circuit is returned to condition one by the ungrounding of both input leads.

In case a ground on the Z_1 output lead was originally obtained it will remain until such time as it is removed by the appearance of a ground signal on the B input only. Similarly, in case a ground on the Z_2 output lead was originally obtained, it will remain until such time as it is removed by an appearance of a ground on the A input only.

Since this is a selecting sequential problem, i.e., one which gives an output depending upon which sequence the input follows, a secondary relay is added across the top of the flow table. The addition

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[10] *Ibid.*, p. 280.

of a secondary relay on top gives a possible horizontal movement from an unstable state.

ab x / y	000	100	101	111	110	010	011	001	
0	(1)	2_V	9	(8)	E	6_H	(6)	(7)	FLOW
1	(3)	(2)	(9)	(10)	(4)	5_H	(5)	E	TABLE
0	00	01	01	00	00	10	00	00	τ
1	00	00	00	00	00	10	00	00	
0	00	01	11	10	00	10	10	10	S
1	01	01	11	11	01	11	11	11	
0	0	Z_1	0	Z_2	E	Z_2	Z_2	Z_2	Z – E
1	Z_1	Z_1	0	0	Z_1	0	0	E	

Figure 11. Tables for Example 2.

Thus, as seen in figure 11, the two input sequences may be distinguished by the relay X, one being with X, the other without.

From the S and Z – E tables the following circuit equations may be derived.

$$f\ (Y) = ab' + y$$

$$f\ (X) = a'b + x$$

$$f\ (Z_1) = x'\ (\ ay + b'y + ab'\ )$$

$$f\ (Z_2) = y'\ (\ bx + a'b + a'x\ )$$

$$f\ (E) = abx'y' + a'b'xy$$

Figure 12. Circuit Equations for Example 2.

The resulting circuits may easily be drawn, and will be left for the reader.

In general, then, selecting sequential circuits may be designed by the above procedure with the addition of secondary relays across the top of the flow table. The number of relays to be placed upon the top depends upon the number of sequences to be distinguished. One additional relay on top will distinguish between two input sequences, two will distinguish between four, three will distinguish between eight, etc.

In Huffman's solution of this problem three secondary relays were required. His flow table is as shown in figure 13.

vxy \ ab	00	01	10	11
000	(1)	2	3	10
001	(6)	(2)	9	(4)
010	(7)	8	(3)	(5)
011				
100	1	(8)	(9)	(10)
101			9	
110		8		
111				

Figure 13. Huffman's Flow table for Example 2.

There are several possibilities of errors in this flow table. If originally a ground on Z_1 was obtained by input 10, and then the input changes to 11, the output is to remain as Z_1. But if the circuitry of relay Y were to have a fault causing y to become 1, the relay conditions change to 11,011. This is a blank entry in the flow table, and can only lead to blank entries. In practice an extra relay to reset the circuit would probably be required. But this would raise to four the number of relays required, where the error-detecting method only requires three. Admittedly this is an ideal problem for the error-detecting method because fewer relays are required which is the exception and not the rule. An example will be shown later where the error-detecting method requires more equipment than Huffman's method.

The preceding problem was labeled by Huffman as a "non-ideal" situation.[11] The following problem, Example 3, was labeled as an "ideal" situation.[12]

This circuit is to have two inputs and four outputs. The input restrictions are such that only one input lead may be grounded at a time, and such that neither input lead may be grounded unless both are first ungrounded. This latter restriction prohibits the transitions in input state from 01 to 10 and vice versa. This may or may not be a strict restriction but if it is, some warning system should be provided should this condition occur.

Of the four output leads, one and only one is to be grounded at a time. With the second input ungrounded, each grounding of the first input lead is to advance the position of the output ground by one step. See figure 14.

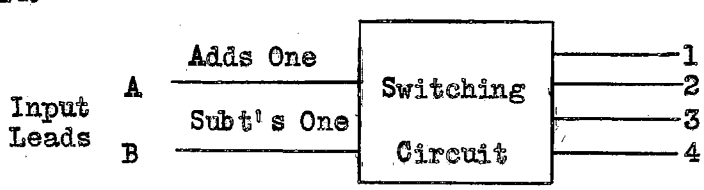

Figure 14. Circuit of Example 3.

Removal of the input ground is to have no effect upon the output. For example, if initially both input leads are ungrounded and the number 2 output lead is the one grounded, grounding of the first input

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[11]*Ibid.*, p. 280.

[12]*Ibid.*, p. 276.

lead is to remove the ground from the number 2 output lead and impress the ground instead upon the number 3 output lead. Subsequent removal of the input ground is to have no further effect on the output.

The grounding of the second input lead, on the other hand, is to make the position of the output ground retreat by one step.

Since this is a selecting sequential circuit with two possible input sequences, one secondary relay will be needed across the top of the flow table. See figure 15.

vxy \ abu	000	100	101	111	011	010	110	001
000	$(1)_1$	10_{H2}	10_{v2}	E	E	8_4	E	1_{H1}
100	1_1	E	$(10)_2$	E	E	$(5)_1$	E	2_{v2}
110	$(2)_2$	11_3	11_3	E	E	5_1	E	2_{H2}
010	2_2	E	$(11)_3$	E	E	$(6)_2$	E	3_3
011	$(3)_3$	12_4	12_4	E	E	6_2	E	3_3
111	3_3	E	$(12)_4$	E	E	$(7)_3$	E	4_4
101	$(4)_4$	9_1	9_1	E	E	7_3	E	4_4
001	4_4	E	$(9)_1$	E	E	$(8)_4$	E	1_{v1}

Figure 15. Error-Detecting Flow Table for Example 3.

If output lead number 1 is grounded at condition one, a change in input state to 10 will move to column 100 in the flow table where an unstable ten is entered which causes a move to another ten, circled or uncircled, in either the same column or the same row and adjacent to it. For the ten in column 100, the next move would be to column 101, where another unstable ten is entered. This ten then will cause a vertical move to the next row. Notice that the secondary relays are again assigned so that they would form a closed sequence line on an 3-dimensional cube.

A change in input from 00 to 10 is to advance the output ground by one, so condition ten must be a ground on output lead number 2. The subsequent removal of the ground on input A will change the top relay state from 101 to 001, then to 000. This was not to affect the output, so entry two in column 000 will retain the ground on output lead number 2. Should the input then change to 01, the top relay state will be 010 where an unstable five is entered. This causes a change vertically to column 010 row 100.

Each change in input state to 01 is to retreat the output ground by one step so condition five must represent a ground on output lead number 1.

If the input changes from 01 to 10 or vice versa, the top relay state will change from 100 to 010, or from 101 to 011, or the reverse of these two. There are no stable entries in the 100 column so a change from 100 to 010 will not happen unless it goes through the E entry first. Likewise any one of the unwanted changes will result in passing through an E entry or stopping in one.

The τ and S tables were derived in the usual manner with the top secondary relay being listed first. Thus the entry 1 000 in the τ table means that the top or X secondary relay is in an unstable condition and a horizontal move will result. See figure 16 on page 22.

From the S and Z – E tables, figures 17 and 18 on pages 22 and 23, the final circuit may be designed by means well established.[13]

One possible solution is given by the algebraic expressions shown in figure 19 on page 23.

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[13] W. Keister, A. E. Ritchie, and S. Washburn, <u>The Design of Switching Circuits</u>, chs. 5 and 6.

vxy \ abu	000	100	101	111	011	010	110	001
000	0 000	1 000	0 100	0 000	0 000	0 001	0 000	1 000
100	0 100	0 000	0 000	0 000	0 000	0 000	0 000	0 010
110	0 000	1 000	0 100	0 000	0 000	0 010	0 000	1 000
010	0 100	0 000	0 000	0 000	0 000	0 000	0 000	0 001
011	0 000	1 000	0 100	0 000	0 000	0 001	0 000	1 000
111	0 100	0 000	0 000	0 000	0 000	0 000	0 000	0 010
101	0 000	1 000	0 100	0 000	0 000	0 010	0 000	1 000
001	0 100	0 000	0 000	0 000	0 000	0 000	0 000	0 001

Figure 16. τ Table for Example 3.

vxy \ abu	000	100	101	111	011	010	110	001
000	0 000	1 000	1 100	1 000	1 000	0 001	0 000	0 000
100	0 000	0 100	1 100	1 100	1 100	0 100	0 100	1 110
110	0 110	1 110	1 010	1 110	1 110	0 100	0 110	0 110
010	0 110	0 010	1 010	1 010	1 010	0 010	0 010	1 011
011	0 011	1 011	1 111	1 011	1 011	0 010	0 011	0 011
111	0 011	1 111	1 111	1 111	1 111	0 111	0 111	1 101
101	0 101	1 101	1 001	1 101	1 101	0 111	0 101	0 101
001	0 101	0 001	1 001	1 001	1 001	0 001	0 001	1 000

Figure 17. S Table for Example 3.

vxy \ abu	000	100	101	111	011	010	110	001
000	1	2	2	E	E	4	E	1
100	1	E	2	E	E	1	E	2
110	2	3	3	E	E	1	E	2
010	2	E	3	E	E	2	E	3
011	3	4	4	E	E	2	E	3
111	3	E	4	E	E	3	E	4
101	4	1	1	E	E	3	E	4
001	4	E	1	E	E	4	E	1

Figure 18. Z - E table for Example 3.

$$f(E) = b(a + u) + au'[v(x'y' + xy) + v'(xy' + x'y)]$$

$$f(Z_1) = b'x[ay(u + v) + a'(u'y' + uv)] + a'bu'vy'$$

$$f(Z_2) = b'y'[uv(a' + x') + a'u'x + av'x'] + a'bu'v'x$$

$$f(Z_3) = b'x[a'y(v' + u') + y'(av + uv')] + a'bu'vy$$

$$f(Z_4) = b'y[a'x'(v + u') + x(av' + uv)] + a'bu'v'x'$$

$$f(U) = u[a + b + y(x'v' + xv) + y'(v'x + vx')] + ab'[y(xv' + vx') + y'(v'x' + vx)]$$

$$f(V) = v(b + au' + a'u) + b'[y(aux + a'u'x') + y'(aux' + a'u'x)]$$

$$f(X) = x[a + a'y(b + u' + v') + a'y'(b' + u + v')] + x'a'v(bu' + b'u)$$

$$f(Y) = y[u(a + b + v + x) + u'(a + b' + v + x')] + y'v'a'(b'x + bx')$$

Figure 19. Circuit Equations for Example 3.

CHAPTER IV

REVIEW OF ERROR-DETECTING TECHNIQUE

Purpose.

For some sequential circuits the existence of races or the possibility of errors can not be tolerated. This article describes a method for synthesizing circuits in such a manner that races can not exist, and if an error is made, it is easily detected.

This method will usually entail the use of more equipment than is necessary to get just a solution. It then becomes the engineer's prerogative as to which method is the most advantageous.

Procedure.

1. From the given information, determine the number of discreet time intervals and arrange them in the proper sequential order.

2. Form a flow table with one column for each possible combination of input leads.

If the problem is one of selecting sequences, add the needed secondary relays on top so that there will be one column for each possible combination of these secondary relays and the primary relays. The number of secondary relays needed across the top will be determined by the number of individual sequences to be chosen from. One secondary relay on top will distinguish between two sequences, two between four, etc. If S_t represents the number of secondary relays assigned to the top, the flow table will have $2^{L + S_t}$ columns, where L is the number of primary relays.

3. Assign the intervals determined in 1 to the flow table so that there is one and only one per row. Circle each entry. In the selecting

sequential flow table, there may be more than one per row if it is so arranged that a change in input will not cause a move from one circled entry to another.

4. Assign an uncircled entry to each circled entry so that they are in the same column and are in adjacent rows. This requires the uncircled entry to be in either the row immediately preceding or succeeding the circled entry's row. The first and last rows are considered adjacent.

In the selecting flow table, an uncircled entry may be in the same row as another entry of the same number either circled or uncircled. If so, each uncircled entry of a given number must be horizontally adjacent to another entry of the same number either circled or uncircled.

5. Assign secondary relays to the rows so that a different combination represents each row, and arrange these combinations in an order so that they would form a closed sequence line on an S - dimensional cube. This requires an even number of rows; therefore, one row may have to be added to the table.

6. Label each unoccupied space with an E, indicating the error circuitry. If the table has less than 2^S rows, the unused combinations of secondary relays must be included in the error circuitry, and may or may not be included as "don't care" conditions in the S circuitry.

7. Derive the τ table by making $\tau = 0$ for each relay in each entry corresponding to a circled or E entry in the flow table. Also in each entry corresponding to an uncircled entry in the flow table, one and only one relay should have $\tau = 1$.

8. By adding τ and r cyclically mod 2 to obtain R, form the

S table.

9. Form the Z - E table by entering Z in the proper space to give the required output and E in each space that should indicate an error.

10. Solve by combinational methods the problem presented by the S and Z - E tables.

CHAPTER V

SUMMARY AND CONCLUSIONS.

A sequential switching circuit is a combination of two or more switches which are either electronically, mechanically or manually controlled and operate in a certain sequence or order to produce some prescribed output at any given time. The circuit must have some method of remembering what has happened in the past. Then the past must be digested with the present, and the circuit must decide the future events.

Several methods exist for synthesizing sequential switching circuits with each method having some advantage over the others. However, two outstanding disadvantages exist in every previous known method. One is that the solution to a problem in most cases is not peculiar to that problem. That is, there exist other sequences for which the circuit would work equally well. If this is so the circuit can not usually indicate whether an error has been made. These errors could be either internal or external. Another disadvantage of existing methods is the difficulty in locating race conditions and also determining whether these races are hazardous or not. Race conditions are caused by two or more switching devices being energized at the same time. A race then occurs between switches to see which one makes or breaks contact first.

A new method is introduced in this work which guarantees the following three things.

1. The final solution will be peculiar to the problem. That is, it will work for no other sequence.

2. If an error is made, either internally or externally,

a warning will be given.

3. There are no possible races.

The method is based on a modification of the very important flow table techniques introduced by D. A. Huffman in the Journal of the Franklin Institute, March 1954. In order to eliminate races secondary relay combinations are assigned to each row of a primitive flow table such that they would form a closed sequence line on an S - dimensional cube, where S is the number of secondary relays. Error detection is achieved by entering undesirable combinations in a warning circuit.

This method also has its disadvantages. The main one is that it usually requires additional equipment. Whether its advantages outweigh its disadvantages depends on the problem restrictions and the percent of extra equipment required. Its advantages are believed to be sufficient to warrant investigating by this method any problem that arises.

BIBLIOGRAPHY

Birkoff and MacLane. A Survey of Modern Algebra. New York, The Macmillan Company, 1941.

Boole, George. The Laws of Thought. Queens College, Dublin, 1854.

Huffman, D. A. "The Synthesis of Sequential Switching Circuits." Journal of the Franklin Institute. Part 1 - Vol. 257, No. 3, March 1954, pp. 161-190; Part 2 - Vol. 257, No. 4, April 1954, pp. 275-303.

Karnaugh, M. "The Map Method for Synthesis of Combinational Logic Circuits". AIEE Technical Paper No. 53-217, April 1953.

Keister, W., A. E. Ritchie, and S. Washburn. The Design of Switching Circuits. D. Van Nostrand Company, Inc. 1951.

Montgomerie, G. A. "Sketch for an Algebra of Relay and Contactor Circuits". Jour. IEE, Vol. 95, Part III, 1948, pp. 303-312.

Shannon, C. E. "A Symbolic Analysis of Relays and Switching Circuits". Trans. AIEE, Vol. 57, 1938, pp. 713-723.

Shannon, C. E. "The Synthesis of Two-Terminal Switching Circuits". Bell System Tech. Jour., Vol. 28, 1949, pp. 59-98.

Veitch, E. E. "A Chart Method for Simplifying Truth Functions". Proc. Assn. for Computing Machinery, May 1952.

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