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RESTORATION: A GAME THEORY APPROACH

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Contents

List of Figures

List of Tables

Abstract

Critical infrastructures are governed by several sectors working together to maintain social, economic and environmental well-being. These infrastructures are interdependent and rely on a complex schedule of repair after a disruptive event. Decision makers seek to restore the infrastructure networks as quickly as possible while balancing time and resource constraints. Although many models focus on a centralized view for networks, rarely is there only one decision maker for the infrastructure networks, making decentralization a more realistic view. In decentralized decision-making paradigm, individual decision makers need to decide how to prioritize areas of the network and eventually improve the aggregated infrastructure systems resilience. Existing literature advocates cooperative management strategies to enhance infrastructure systems resilience. However, there is a dearth of quantitative studies analyzing resource allocation decisions considering both decentralized and cooperative aspects. In light of cooperative game theory, interdependent infrastructure systems can be modeled as coalitions of service providers pooling their resources to meet the global performance. This work relies on coalitional game theory to address decentralized resource allocation for interdependent water distribution and road networks. Coalitional game theory addresses the fair allocation of resources for nodes that surround an important area to the infrastructure and the need to decentralize the overall interdependent network. In particular, combining coalitional game theory with weighted graphs creates an order of repair for each node in the coalitions. Subsequently, the decision makers can pass information on to the master problem, reducing the complexity of the resource allocation problem for the interdependent networks. The proposed approach is applied to water distribution and transportation networks in the City of Tampa, FL. We compare the decentralized solutions to centralized solutions in different scenarios to demonstrate the feasibility of our approach for the city-scale networks.

Keywords— Game theory, Interdependent infrastructures, Graph weighting, Network optimization, Resource allocation

1 Introduction

Cities are comprised of many subsystems that focus on one critical element, known as infrastructures. Department of Homeland Security has identified sixteen critical infrastructure sectors necessary for the functionality of the city (DHS, 2003). After a disruptive event such as a natural disaster, these infrastructures must be repaired in a timely manner to restore the city and maintain social and economic well-being. Resources such as work crews, money, and supplies must be used to repair the city. Decision makers for the city need to decide where resources should be allocated to minimize the total time it takes for the infrastructures to be repaired. Traditional resource allocation models use the centralization approach, which assumes that all geographical areas and infrastructure types share one decision maker, one set of resources, and one aggregated goal (Sharkey et al., 2015). However, infrastructures are governed by several sectors and are interdependent, such as the physical interdependency of the water and transportation infrastructures. Many pipelines run underneath roads; if a pipeline breaks, the road must be damaged in order to repair the pipeline. This leads to competing interests between the transportation infrastructure and the water infrastructure, along with separate goals for different physical locations. Hence, it is imperative to understand the collective behavior of decision makers for managing infrastructure systems.

The complexity of centralized approaches and the need for decentralized solutions have sparked a huge growth in the decision science literature that aims to find low complexity and distributed algorithms (Arnold and Schwalbe, 2002; Mohebbi and Li, 2015). In decentralized decision-making paradigm, individual decision makers need to decide how to prioritize areas of the infrastructure networks and eventually improve the aggregated infrastructure systems resilience. There are several studies in the literature advocating cooperative management strategies to enhance the resilience of infrastructure systems facing of disruptive events (see Bel et al. (2013); Hophmayer-Tokich and Kliot (2008); Whittington et al. (2005)). Nonetheless, there is a paucity of quantitative studies modeling the co-existence of cooperation and decentralization for infrastructure networks restoration. Hence, in this work, we propose a decentralized resource allocation framework for restoring interdependent infrastructure networks based on cooperative game theory.

In light of cooperative game theory, interdependent infrastructure systems can be modeled as coalitions of service providers pooling their resources to restore network components and meet the global performance. A coalition is a group of connected actors / network components which fully cooperate with one another in the coalition. In addition, the coalitions are mutually disjoint; hence, no one actor is in two coalitions. All resources that belong to each actor now belong to the group, and the actions of the actors in each coalition are decided based on the cooperative game solutions. In this work, the actors are the nodes of the various infrastructure networks. Important nodes for each infrastructure, or key nodes, are identified resulting in forming coalitions that are focused on important aspects of the network. Due to the interdependent nature of infrastructure networks, the coalitions sometimes have nodes from multiple infrastructures. Figure 1 demonstrates a sample coalition for interdependent water and transportation networks, with the key node being a water valve. The coalition is comprised of the two water nodes that are directly connected to the valve and the co-located transportation node.

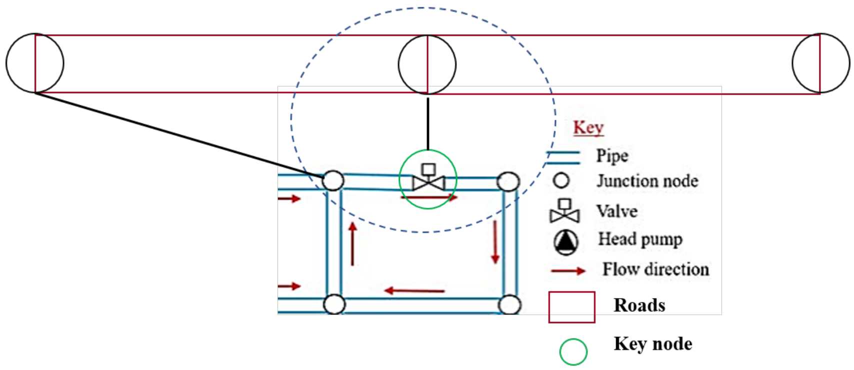

Figure 1: Sample Coalition for co-located water and transportation networks

After the coalitions are formed, each node's individual contribution to the interdependent networks can be calculated based on the cooperative game solutions. Fairness criteria for cooperative games can be determined using the concepts of the core, and the Shapley value. The core and Shapley value are the main solution concepts in cooperative games that are analogous to the Nash Equilibrium in non-cooperative games (Myerson, 1991). The core can be empty or quite large making the selection of a suitable core allocation difficult. This drawback motivated the search for a unique payoff vector known as the Shapley value. Hence, we use Shapley value to calculate the individual contribution of every node to the entire networks.

Shapley values can be calculated based on the number of key nodes in the coalition and their interaction with non-key nodes; however, this calculation does not consider the characteristics of the nodes. A significant characteristic of each node in the water or transportation infrastructures is the amount of flow that a node can handle (Gonzalez-Aranguena et al., 2014). For instance, the amount of water that a pump station pumps through it every day varies, and the importance of the pump station to the surrounding area is related to the amount of flow the pump services. A transportation intersection also can be defined by the amount of traffic that flows through the intersection every day, and the importance of the intersection to the surrounding area is dependent upon the amount of flow in the node. In order to capture this important characteristic in our model, a method of weighting each node in a coalition can be used to identify those nodes that have more flow and are therefore more important to the network. The coalition itself can then allocate resources in an order that is based on the Shapley values.

With the addition of coalitions, the resulting optimization of resource allocation problem is reduced in complexity. Grouping nodes together into cooperative coalitions allows each coalition to allocate its own resources to the nodes in a decentralized manner. The restoration plan from each coalition is then passed on to the master problem, which allocate resources to restore the whole interdependent networks. As a result, the amount of nodes that are considered in the master problem is reduced which subsequently influence the computational complexity.

Following this section, the remainder of this thesis is organized as follows: In section 2, the literature on two main streams of interdependent infrastructure modeling including game theory and optimization techniques is reviewed. Section 3 is devoted to defining the methodology and the proposed decentralized resource allocation framework. Section 4 provides the performance evaluations of the proposed framework applied to water distribution and transportation networks in the City of Tampa. Finally, section 5 will give insights and discussion about the proposed framework and will provide concluding remarks, limitations, and the future research directions.

2 Literature Review

The two key components of this study are optimization and game theory techniques. Each modeling approach has been applied to interdependent critical infrastructures in prior research, although the assumptions and applications vary between prior research and this study.

2.1 Optimization in Interdependent Infrastructures

Many resource allocation optimization problems in interdependent infrastructure networks rely on assumptions of centralized decision making (Kong et al., 2019; Rong et al., 2018; Sharkey et al., 2015; Zhang et al., 2018). This assumption presumes that one decision maker chooses the actions for the entire network and there is perfect knowledge of all resources and flow by the decision maker (Rong et al., 2018). In small scale disruptions, this decision maker can be the infrastructure managers; in large scale disruptions, this can be the local government (Zhang et al., 2018). The decision maker must understand the entire network that is under their jurisdiction, including resources, disruptions, interdependencies, and length of repair for each node. This is impractical in larger-scale networks. In such centralized optimization models, most models seek to maximize the resilience of the network. One approach to maximize resilience is to simply make the resilience metric the focus of the optimization model (Zhang et al., 2018). More complex models can be conceptualized and capture different aspects of the network, such as creating a two-stage optimization model that first restores the minimum level of service before minimizing the losses in the network (Kong et al., 2019). This complexity, although realistic, increases the amount of knowledge that a decision maker must know in order to make a decision.

Some studies in the literature have addressed the decentralized network optimization problem for infrastructures (e.g. see He et al. (2017); Talebiyan and Duenas-Osorio (2020)). Nonetheless, the resulting models are mainly applicable to small or county-level networks. This is due to the assumption that all actors are individually making decisions (Talebiyan and Duenas-Osorio, 2020). In addition, the interdependent nature of the infrastructures must be considered to accurately capture the true complexity of the network. Previous optimization models that account for decentralization do not always account for the interdependent nature of the infrastructures (He et al., 2017). This leaves a gap for models that include both decentralization and practical computational times for large-scale networks. To tackle this issue, we propose a decentralized optimization model for city-scale interdependent infrastructure networks. Optimizing a decentralized network allows decision makers to run the model without exceeding time constraints, thus allowing the model to be more broadly applicable.

2.2 Cooperative Game Theory in Interdependent Infrastructures

Cooperative game theory has been applied to many fields, such as economics, government policy, genetics, and healthcare systems (Choi et al., 2020; Mohebbi et al., 2020; Moretti et al., 2010). The main feature of such games is that decision makers are looking to optimize a common goal such that decisions / actions do not degrade their individual goal and performance. In order to optimize a common goal in networks, particularly one related to restoration, the characteristics of components and nodes must be understood. One key characteristic of a node can be the importance of the node in relation to the interdependent network, referred to as the Shapley value of the node. Shapley values are central to many cooperative games in a variety of scenarios (Borm et al., 2001) which provide a unique solution and allow the decision makers to make informed choices.

Decentralization of the network allows for the different sectors to cooperate while retaining their own decision making power and resources, in addition to reducing the computational complexity of the resource allocation problem (Ellinas et al., 2015). In order to decentralize the network, a way of dividing the graph must be used. One such division is based on node location. This method uses previously identified important nodes, known as key nodes, to create coalitions (Moretti et al., 2010). Thus, as the coalitions are restored, the repairs will be focused on these key nodes. There are many ways to identify important nodes in interdependent networks. However, a smaller set of key nodes can be identified by the characteristics of the physical nodes. In the water infrastructure, these key nodes are easily identified as nodes with different characteristics than others, such as reservoirs, pumps, valves, and tanks. The other nodes in a water network are simply demand nodes or pipeline junctions, neither of which control flow throughout the network. In the transportation infrastructure, these key nodes might be difficult to identify due to the lack of distinct characteristics for each node. Some work has been done in identifying key nodes such as bridges, highway intersections, and other important intersections (Vidrikova et al., 2011), but in data sets these nodes may not be clearly identified and other methods can be used as a proxy for the transportation infrastructure.

Finally, in order to accurately capture the characteristics of the network components, the networks must be weighted in some way. Weighting the network emphasizes the parts of the network that carry more importance to the city. There are multiple ways to weight a network; one approach is to weight the network by the cost of transportation (Allen and Arkolakis, 2019). However, this technique is used when plotting routes through an infrastructure. A better way to weight the infrastructures is for flow between the links in the infrastructure, which assigns weights based on existing flow (Gonzalez-Aranguena et al., 2014).

Our proposed model combines both the key-node based coalition formation and the flow-based weighting of a graph which creates a unique network that addresses both the relative importance of a node to significant areas in the interdependent infrastructures and the importance of the node characteristics. This approach is vastly different from previous approaches mentioned, due to the dual approach of weighted graphs and game theory. After the weighted coalitions are formed, the master problem of optimizing resource allocation must be addressed.

3 Analytical Modeling

The proposed decentralized resource allocation framework utilizes cooperative game theory and network optimization techniques. We first present the cooperative game formulation, solution, and then present the optimization model for interdependent infrastructure networks.

3.1 Game Formulation

3.1.1 Unanimity Game

The basic cooperative game can be defined as $< N, v >$, where N is the set of players and v is the characteristic function. In this work, N is the set of all non-key nodes and the characteristic function is a measure of the connectedness of the non-key nodes to the key nodes. Given a subset $E \subseteq N$, the characteristic function $v(E) \in \mathbb{R}$ and $v(\emptyset) = 0$. A group of nodes C can form a coalition if $C \subseteq N$.

For a unanimity game, the coalitions are represented by $u_E(C) = 1$ if $E \subseteq C$ and $u_E = 0$ if $E \nsubseteq C$, where $\emptyset \neq E \subseteq N$. Cooperative games can be written as a linear combination of unanimity games in a unique way. The coefficients of the characteristic function are $\lambda_E(v)$ for all subsets $E \in 2^N \setminus \emptyset$. Thus, the unanimity game characteristic function v is

$$\sum_{E \subseteq N, E \neq \emptyset} \lambda_E(v) u_e \tag{1}$$

We use Shapley value to solve the game. This solution can be described in several ways, and we use the following formula (Shapley, 1953):

$$\phi_i(v) = \sum_{E \subseteq N : i \in E} \frac{(|E|-1)!(|N|-|E|)!}{|N|!} [v(E) - v(E \setminus \{i\})], \quad \forall i \tag{2}$$

In our model, the Shapley value is based on the number of key nodes that are exclusively connected to the coalition (see Moretti et al. (2010)). The specific coefficients of the characteristic function must reflect this definition. To find this characteristic function, the links between key nodes and nodes must be identified as $L_E \subseteq \{\{i,k\}|i \in E, k \in K\}$ where K is the set of all key nodes. A single key node that is directly connected to a subset E can be calculated through $M_E = 1$ if $\{i,k\} \in L_E$ and equal to zero otherwise. This function needs to be summed over all of the key nodes to ensure that there are not multiple key nodes in one coalition, represented as $\lambda_E(v) = \sum_{k \in K} M_E$. The characteristic function v remains the same as Equation 1. It should be noted that coalitional structures can be determined by the vectors formed by the same function as M_E,

$$C_k(i) = 1 \text{ if } \{i,k\} \in L_E, \forall k \in K, \forall i \in E, \text{ or } C_k(i) = 0 \text{ otherwise} \tag{3}$$

3.1.2 Restricted Weighted Graphs

Having identified coalitional structures for the interdependent networks, we need to calculate the Shapley value vector for each coalition. In addition to nodes characteristics, the flow on certain links of infrastructure networks is important in calculating the Shapley value. This is due to the fact that such links might contribute more to a key node than others.

In order to allow for weighting different non-key nodes in the individual coalition, the overall network must be restricted to a graph that contains coalition C. The new set of restricted players R is simply $R \subseteq N, R \neq \emptyset$, where $C \subseteq R$. The connections in the entire graph, $I_N = \{\{i,j\}i,j \in N, i \neq j\}$, is restricted to $I_{|R}$ or the set of connections that include only those connections in R. Thus, a restricted graph is presented as $< R, I_{|R} >$. From this restricted graph, a subgraph that contains

strickly the connected components of coalition C. The set of links $\eta \subseteq I_{|R}$ where C is connected in η and the set of players $D(\eta) = \{i \in N \text{ such that } \exists j \in N \text{ where } \{i,j\} \in \eta\}$ form the new connection subgraph. Finally, the weighted graph $< N, I_w >$ is comprised of the set of players N and the weighted links $I_w = \{I, \{w_A\}_{A \in I}\}$.
However, the weighted graph must be transformed before it can be used to calculate the Shapley values of the coalition. To measure contribution, $\alpha(I_w)$, the set of proportional contribution that each coalition C contributes after the graph has been weighted, is used. For each coalition, the proportional contribution is equal to $\{\alpha_C^R(\{w_A\}_{A \in I}) \in [0,1]\}$. The exact value of α_C^R depends on the type of weight that best fits the current situation for the overall network. In our model, flow between nodes is used to weight the graph, due to the fact that the flow of water and traffic are important measures to capture in a graph. In other words, w_A is the flow on arc A and must be within $[0, \infty]$. Thus, $(\alpha)_C^R$ can be calculated as below (see Gonzalez-Aranguena et al. (2014)):

$$(\alpha)_C^R = \max_{i=1\ldots t(R)} \left\{ \frac{1}{1 + \max\limits_{L \in \eta_i^{C,R}} w_A} \right\} \text{ for } |C| \geq 2 \text{ and } = 1 \text{ if } |C| = 1 \tag{4}$$

Gonzalez-Aranguena et al. (2014) demonstrated that the α-weighted restricted graph is decomposable on unanimity games. Hence, the modified characteristic function for the restricted weighted graph is

$$v^{I_w,\alpha}(R) = \sum_{\emptyset \neq C \subseteq R} \lambda_C(v) \alpha_C^R(\{w_A\}) \tag{5}$$

This characteristic function measures the level of maximum flow that can pass between each group of connected nodes, or route to the key node, in the coalition. According to Equation 5, the computation of the Shapley value for the corresponding game is straightforward as below.

$$\phi_i = \sum_{i \in C \subseteq N} \frac{\lambda_C(v)}{|C|}, \forall i \tag{6}$$

Simply put, this is the value that each node contributes, measured by the amount of flow that goes through the node and accounting the number of routes to the key node that the node is a part of, resulting in a vector for each coalition. It should be noted that this value will not add to one similar to other unanimity games, due to the restricted nature of the graph. Each coalition can find the Shapley value vector for itself and determine the allocation of resources and order of restoration based on this value. The allocation of resources will be fair, due to the fairness property and axioms of Shapley values. These axioms are intrinsic to the Shapley value and ensure the fairness of the result. The efficiency axiom represents the group rationality, the symmetry axiom ensures that if two players have equal contribution, they will have equal payoffs, the dummy axiom awards no payoff to players that do not add anything to the coalition, and the additivity axiom forces the combined Shapley value for two coalitions to be the same regardless of the order that they are added.

3.1.3 Proposed Algorithm

In summary, the general flow of the problem starts with the formation of the coalitions. The key nodes were previously identified; hence, the coalitions are formed around them using the direct links in the physical network and the interdependency links. Characteristic function can be calculated using weighted links for each coalition. Afterwards, the Shapley value vector can be calculated and used to create a unique order of restoration for each coalition. The coalitions can then send the order of repair to the network-wide optimization problem (i.e. master problem). The procedure is outlined below.

3.2 Optimization Model

After the coalitions are identified and the Shapley value vectors are calculated, the results can be used to optimize the resource allocation in a decentralized manner. The optimization of the resource allocation is best described as two problems: one sub-problem for coalitions and one master problem that accounts for the entire network.

3.2.1 Restoration of Coalitions

Within each coalition, the Shapley value vector can be utilized to finalize the order of restoration and the amount of resources to use. The order of repair is strictly based on the Shapley value: the damaged node with the largest Shapley value will

Procedure: Coalitional Game

---

for *all coalitions* **do**

Calculate the flow equation for each link;

Compute the characteristic function ;

Use the unanimity game formulation to calculate the Shapley value vector for the coalition;

Formulate an order of repair for disrupted nodes based on the Shapley value vector;

Send order of repair to 'Master Problem' Procedure;

end

Algorithm 1: The proposed algorithm for fair allocation of resources within coalitions be repaired first, then the second largest damaged node, and so forth. However, if the key node itself is damaged, the key node automatically is the first to receive repair resources. If the coalition has resources available to use, such as local funds, the available resources will be applied in that order.

Although the Shapley values are calculated for nodes, disruptions most often occur in the arcs of the network. To model disruption at the node level, the assumption can be made that a node is disrupted when a connected arc is disrupted. When a node receives resources from the coalition, all arcs connected to the node are restored during the time frame. If the coalition does not have enough resources or has no available resources, the order of repair will be sent to the master problem, the network-wide optimization model. Once the resource is assigned to the coalition, the nodes will be restored, still following the predetermined order of repair. If there is not enough resources to repair all nodes, the sub-problem will send the level of disrupted flow remaining back to the master problem. This cycle repeats until the coalition is fully restored. The proposed sub-problem for restoring coalitions is outlined below.

3.2.2 Interdependent Networks Optimization

The master problem received the order of repair from the coalitions and integrates it into the overall optimization model. The following network optimization problem is formulated which is mainly based off the work presented by Sharkey et al. (2015).

Sub-problem: Coalition Repair

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for *each coalition* C_k **do**

  Order list of disrupted nodes by decreasing Shapley value;

  Send order to 'Master Problem';

  Receive resources if needed from network;

  if *received resource is equal to cost of repairs* **then**

    Repair all nodes;

  end

  Repair list in order;

  Send total remaining disrupted flow to 'Master Problem';

end

Algorithm 2: The proposed algorithm for restoring coalitions

- M: Set of all infrastructures
- N^m: Set of all nodes in infrastructure m
- S^m: Set of all supply nodes in infrastructure m
- T^m: Set of all transshipment nodes in infrastructure m
- $\bar{E}^m$: Set of all arcs that can be installed in the network in infrastructure m
- E^m: Set of all initially available arcs in infrastructure m
- C: The set of all coalitions, with 0 being the centralized coalition set
- A: The set of all arcs that connect to key nodes and are therefore a part of a coalition
- Q: Set of all nodes not in a coalition in the network
- D^m: Set of all demand nodes in infrastructure m
- s_i^m: The amount of supply available at node $i \in S^m$ in infrastructure m
- d_i^m: The amount of demand at node $i \in D^m$ in infrastructure m.
- w_i^m: The weight associated with meeting one unit of demand at node $i \in D^m$ in infrastructure m.
- u_i^m: The capacity of node i in infrastructure m.
- u_{ij}^m: The capacity of arc (i, j) in infrastructure m.

- $F(m,n) \subseteq D^m \times N^m$: The set of all parent/child node pairs in parent infrastructure m and child infrastructure n.

From these sets, the decision variables can be created. The first three decision variables are solely for the master problem which include remaining nodes that are not in coalitions. However, the other two variables gain extra indices to represent the coalitional nodes as well. For α, the decision variable that determines when the arc is repaired, an additional binary index was added to represent if either of the two nodes in the arc are in a coalition. For β, the decision variable that displays that an arc is restored, two extra indices were added. The first is the same binary index as in α, while the second represents the repair order determined by the Shapley value.

- x^m_{ijt}: The amount of flow on arc of node $(i,j) \subseteq E^m \cup \bar{E}^m$ in infrastructure m at time t.

- v^m_{it}: The amount of demand met at node $i \subseteq D^m$ in infrastructure m at time t.

- $y^{m,i}_{n,j,t}$: A binary variable for $(i,j) \subseteq F(m,n)$ representing whether sufficient demand is met at node i in infrastructure m so that node j in infrastructure n is operational at time t.

- $\alpha^m_{k,i,j,c,t}$: The binary variable which is equal to 1 if arc $(i,j) \in E^m$ in infrastructure m is completed by work crew k at time t where c is equal to 0 if the node i and j are not in a coalition and the number of the coalition otherwise.

- $\beta^m_{i,j,c,o,t}$: The binary variable which is equal to 1 if arc $(i, j) \in E^m$ in infrastructure m is available at time t where c is equal to 0 if the node i and j are not in a coalition and the coalition number otherwise and o is the order number from the game theory solution.

The objective function is to maximize the met demand.

$$Max\ Z_1 = \sum_{t=1}^{T} \sum_{m \in M} \sum_{i \in D^m} v^m_{it} \tag{7}$$

Now, the constraints can be added to the model. The first three constraints are to ensure that the flow in and out of supply nodes, transshipment nodes, and demand nodes match the needed outflow. The third constraint does have an extra value that can be utilized to ensure that the model follows the proper interdependency rules.

$$\sum_{(i,j) \in E^m \cup \bar{E}^m} x^m_{ijt} - \sum_{(i,j) \in E^m \cup \bar{E}^m} x^m_{jit} = s^m_i, t = 1, ..., T, \forall i \in S^m, \forall m \in M \tag{8}$$

$$\sum_{(i,j) \in E^m \cup \bar{E}^m} x^m_{ijt} - \sum_{(i,j) \in E^m \cup \bar{E}^m} x^m_{jit} = 0, t = 1, ..., T, \forall i \in T^m, \forall m \in M \tag{9}$$

$$\begin{aligned} \sum_{(i,j) \in E^m \cup \bar{E}^m} x^m_{ijt} - \sum_{(i,j) \in E^m \cup \bar{E}^m} x^m_{jit} = -v^m_{it} - \sum_{((i,m),(a,b),n) \in NTP} v^{mi}_{nabt}, \\ t = 1, ..., T, \forall i \in D^m, \forall m \in M \end{aligned} \tag{10}$$

The next two constraints address demand and capacity limits for the nodes and the arcs.

$$0 \leq v^m_{it} \leq d^m_i, t = 1, ..., T, \forall i \in D^m, \forall m \in M \tag{11}$$

$$0 \leq \sum_{(i,j) \in E^m \cup \bar{E}^m} x^m_{ijt} \leq u^m_i, t = 1, ..., T, \forall i \in T^m, \forall m \in M \tag{12}$$

The following constraint addresses the initially available node capacity due to the arcs that are undamaged by the disruption.

$$0 \leq x_{ijt}^{m} \leq u_{ij}^{m}, t = 1, ..., T, \forall m \in M, \forall (i,j) \in E^{m} \tag{13}$$

The next constraint links the β value to the flow. If the β value is not 1, then the arc is not available and so no flow should exist.

$$0 \leq x_{ijt}^{m} \leq u_{ij}^{m} \beta_{i,j,0,o,t}^{m}, t = 1, ..., T, \forall m \in M, \forall (i,j) \in \bar{E}^{m} \tag{14}$$

The following constraints correspond with α. The first ensures that each arc is only started to be fixed by one work crew in one time period over the entire time period. The second ties α to β, so that the arc is available after it is restored.

$$\sum_{(i,j) \in \bar{E}^{m}} \sum_{s=t}^{min\{T, t+p_{ij}^{m}-1\}} \alpha_{kij0s}^{m} \leq 1, t = 1, ..., T, \forall m \in M, \forall c \in C, k = 1, ..., K^{m} \tag{15}$$

$$\beta_{i,j,0,o,t}^{m} - \beta_{i,j,0,o,(t-1)}^{m} = \sum_{k=1}^{K^{m}} \alpha_{kij0t}^{m}, t = 2, ..., T, \forall m \in M, \forall (i,j) \in \bar{E}^{m} \tag{16}$$

Three constraints, shown below, ensure that the binary node operation variable y is linked to the demand, transshipment, and supply nodes.

$$0 \leq d_{i}^{m} - v_{it}^{m} \leq (1 - y_{n,j,t}^{m,i})(d_{i}^{m}), t = 1, ..., T, \forall (i,j) \in F(m,n), j \in N^{n}, i \in D^{m} \tag{17}$$

$$\sum_{(j,h) \in \bar{E}^{m} \cup \bar{E}^{m}} x_{jht}^{n} \leq s_{j}^{n} y_{n,j,t}^{m,i}, t = 1, ..., T, \forall (i,j) \in F(m,n), j \in S^{n}, i \in D^{m} \tag{18}$$

$$\sum_{(j,h) \in \bar{E}^{m} \cup \bar{E}^{m}} x_{jht}^{n} \leq d_{j}^{n} y_{n,j,t}^{m,i}, t = 1, ..., T, \forall (i,j) \in F(m,n), j \in D^{n}, i \in D^{m} \tag{19}$$

The next constraints were added to handle the coalitions (i.e. sub-problem). The previous constraints were only applied to those decision variables with a coalitional binary variable of 0, meaning the arcs are not connected to a node in the coalition. However, all constraints with α or β were replicated to connect the demand with β. The first additional constraint is the only constraint that directly applies only to the coalitional nodes. It ensures that the higher order nodes from any coalition get repaired before the lower order nodes. For example, all first ranked damaged nodes across all coalitions must be fixed prior to any second ranked nodes.

$$\beta^m_{i,j,c,o,t} \geq \beta^m_{k,l,c,o+1,t}, \forall (i,j),(k,l) \in A_c, \forall c \in C \tag{20}$$

$$\sum_{(i,j)\in \bar{E}^m, \notin Q} \sum_{s=t}^{min\{T, t+p^m_{ij}-1\}} \alpha^m_{kijcs} \leq 1, t = 1,...,T, \forall m \in M, \forall c \in C, k = 1,...,K^m \tag{21}$$

$$\beta^m_{i,j,c,o,t} - \beta^m_{i,j,c,o,(t-1)} = \sum_{k=1}^{K^m} \alpha^m_{kijct}, t = 2,...,T, \forall m \in M, \forall (i,j) \in \bar{E}^m \notin Q, \forall c \in C \tag{22}$$

$$0 \leq x^m_{ijt} \leq u^m_{ij} \beta^m_{i,j,c,o,t}, t = 1,...,T, \forall m \in M, \forall (i,j) \in \bar{E}^m \notin Q, \forall c \in C \tag{23}$$

After the model is run and the results are received, the master problem can send out the order of repair for the non-coalitional damaged nodes, and work crews will begin to restore the network. The proposed procedure is summarized below.

4 Validation and Performance Evaluation

This study used data from a simplified version of the water distribution and road infrastructure networks in the City of Tampa, Florida. Existing tanks, pump stations, and reservoirs from the data were used as key nodes for the water network. Valves

Procedure: Master Problem

Receive order of repairs for coalitions from the sub-problem;

Run optimization model for all nodes ;

Send resources to nodes based on results;

if *there is not enough resources to repair all disrupted nodes* **then**

  Receive disrupted flow from coalitions;

  Add penalty for unrepaired remaining flow;

  Redo optimization problem;

end

Algorithm 3: The proposed algorithm for optimization of the entire network were simulated by finding water nodes where at least four pipelines intersected. For the transportation network, there were no previously identified key nodes; thus, the betweenness centrality measure was used to simulate traffic flow, and all nodes that exceeded a certain amount of flow were considered key nodes. This assumption is based on previous work on traffic flow (Kazerani and Winter, 2009). Physical co-location was used to determine interdependencies for the water and transportation networks. The overlaid networks are illustrated in Figure 2, with the brown being the water infrastructure and the blue being the transportation network.

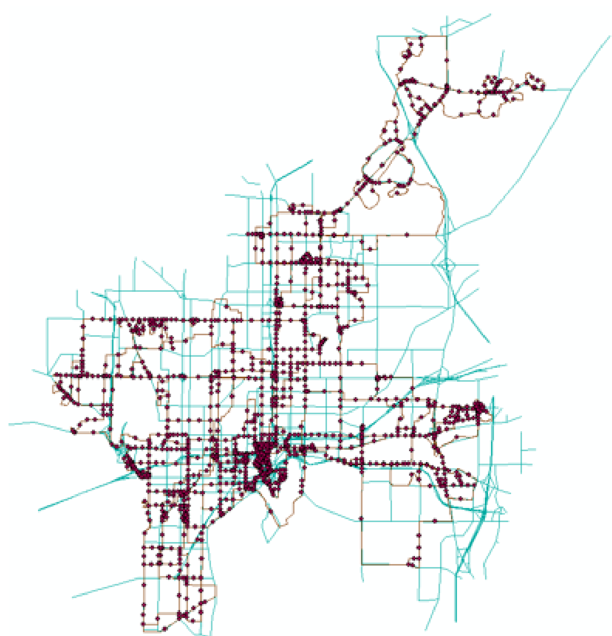

Figure 2: Interdependent water-transportation networks

In the simplified networks, there are 4312 nodes in both the transportation and water networks. There are 48 key nodes in the water network and 45 key nodes in the transportation network, based off of a betweenness threshold of greater than 0.07. This value was chosen to limit the transportation key nodes to only the highest traffic flows. Using the conservative threshold of 0.07 ensures that only the most traveled roads are chosen as key nodes. In addition, the number of key nodes in the water network is approximately the same as the number of key nodes in the transportation infrastructure. With each infrastructure accounting for roughly half of the nodes, the proportion of key nodes to non-key nodes remains roughly the same in both infrastructures. Therefore, there is a total of 93 coalitions based off of the key nodes and an example subset of these coalitions is in Table 1. The variation in the Shapley values is due to the nature of the characteristic function. Since every coalition calculates its own characteristic function based on the number of nodes in the coalition and the flow depends on the type of node, the Shapley values can vary between coalitions yet remain comparable within the coalition. Hence, although the Shapley values cannot be compared between coalitions, the coalition can still create a clear order of repair.

Under the current framework, calculating the characteristic function for the weighted graphs is limited to direct connections. Given that the coalitions only contain nodes that are directly connected to the key node, these possible paths are limited to direct connections and the potential interdependent node. The flow along the interdependent node is assumed to be the same as the maximum amount of flow on the other

Key Node	**Nodes in Coalition**	**Infrastructure**	**Shapley Value**
J-103410	7324	Transportation	.064
	A-58369	Water	.042
	A-54531	Water	.042
	A-66147	Water	.00076
	A-25129	Water	.00076
7476	A-66724	Water	1.1839
	7493	Transportation	1.1839
	7464	Transportation	1.1837
	7472	Transportation	1.1203
	7503	Transportation	1.1203
HSP	HRR	Water	.000018
	J-87510	Water	.000018

Table 1: Subset of Coalitions in Order of Repair from Tampa, FL

links. This is due to the fact that the interdependent node is assumed to be at least as important as the most important direct connection node.

A percentage of the set of all arcs was randomly selected to be disrupted. Here, four scenarios chosen were 5% disruption, 10% disruption, 12% disruption, and 15% disruption. We also assumed that at least half of the selected disrupted arcs belonged to the identified coalitions. If the arc was connected to a key node or in a coalition, the proposed methodology was applied and the order of restoration was formed. As mentioned before, the key node is always placed in the first spot of the order of repair if it is damaged. To evaluate the performance of the proposed approach, we compute the time it takes to solve the optimization model for the centralized versus the decentralized model. Table 2 summarizes the computational time for both models. It can be observed that there is a significant difference between the centralized and proposed models in terms of computational time. In addition, when the objective values are compared (see Figure 3), the proposed method reaches a higher value for restored flow.

Additionally, the proposed method improves the met demand faster than the centralized method. In the water network, both methods start at zero demand met, but the proposed method outpaces the centralized method in rapid demand growth (see Figure 4). In the transportation infrastructure, the models also both start at the same demand. The met demand of the proposed model for transportation increases faster, but does end up slightly below the centralized model (see Figure

5). However, the overall demand of the proposed method is better.

Percent	**Method**	**Obj. Value**	**Time (Secs)**	**Optimality Gap**
5%	Centralized	3797000	2619.41	0%
	Proposed	3870000	179.23	0%
10%	Centralized	1198000	76098.09	.01%
	Proposed	1551000	231.6	.004%
12%	Centralized	1053261	28100.16	5%
	Proposed	1448000	194.28	4.76%
15%	Centralized	750100	135676.08	5%
	Proposed	1484000	220.71	4.21%

Table 2: Results from Case Study

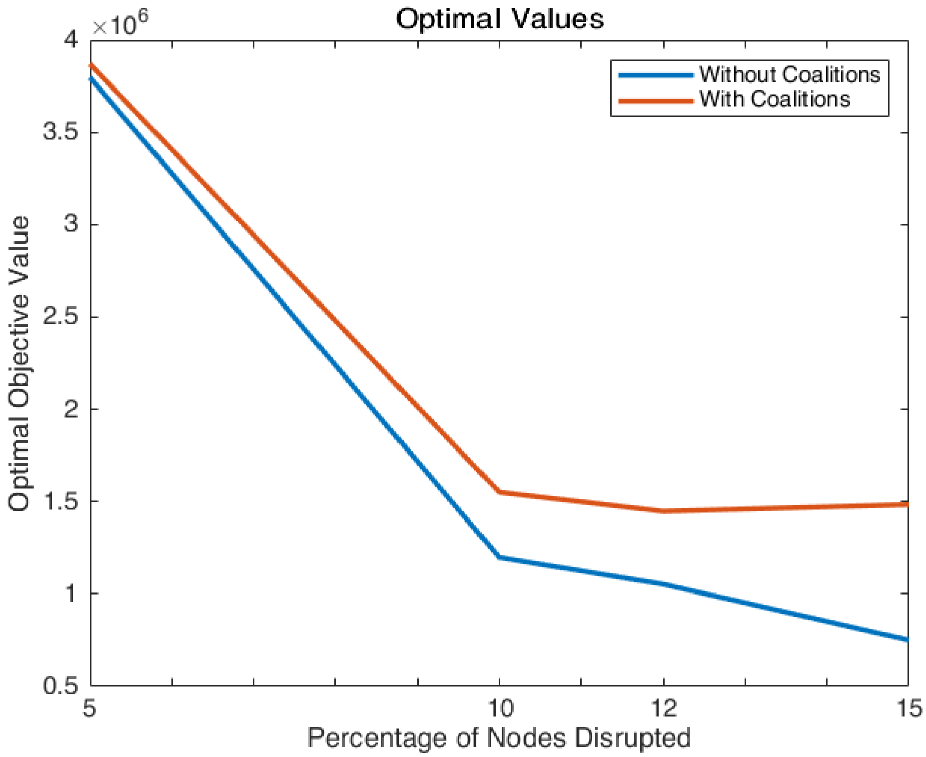

Figure 3: Optimal Values for Both Methods

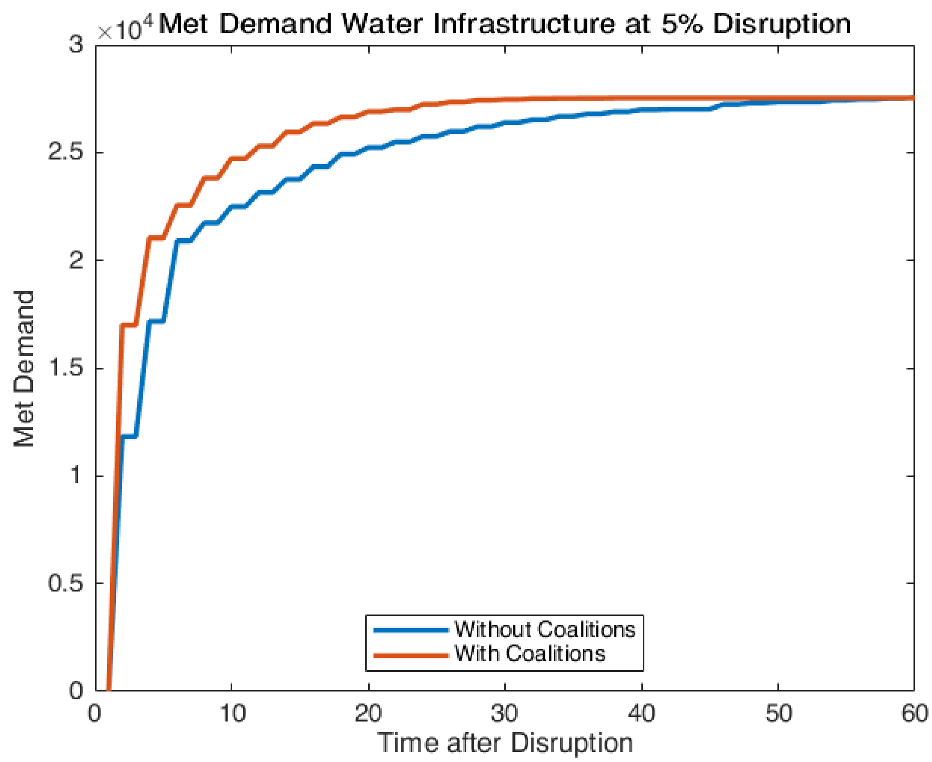

Figure 4: Met Demand in the Water Infrastructure at 5% Disruption

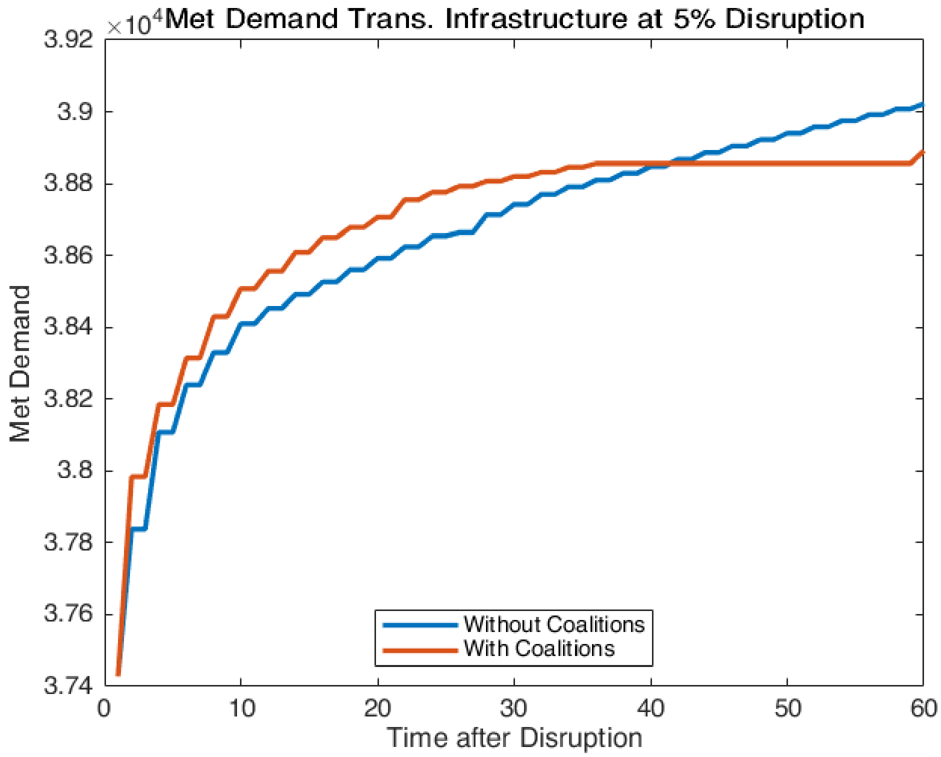

Figure 5: Met Demand in the Transportation Infrastructure at 5% Disruption

5 Conclusion

We proposed a decentralized resource allocation model for interdependent infrastructure networks restoration using cooperative game theory. We first identified coalitional structures in the interdependent networks, and then calculated the order of restoration for network components within coalitions using Shapley values. Our proposed approach combined coalitional game theory with weighted graphs to address the fair allocation of resources in a decentralized manner. The restoration plan from coalitions were then passed on to the master optimization problem, which allocate resources to restore the whole interdependent networks. We applied our framework to water distribution and road networks in the City of Tampa, FL. To evaluate the model's performance, we calculated and compared the computational time for both centralized and decentralized models where four hypothetical disruption scenarios were simulated. The results demonstrated that the decentralized model outperforms the centralized counterpart in terms of computational time and the trajectory of the system performance (met demand) over time.

In future work, more types of interdependence can be used, reflecting the complexity of the infrastructure networks. These additional interdependencies, such as functional interdependencies, will contribute to the realism of the network formulation without increasing the complexity in a significant manner. Additional infrastructures can also be incorperated, such as the power infrastructure. With larger networks, the coalitions perhaps could be expanded, either by introducing more key nodes or relaxing the direct connections definition. However, future research needs to examine if this model could be scaled up to larger networks. In addition, this research can be expanded to include resources that are controlled by the coalitions, giving more control to the decision makers of the coalitions. Depending on the infrastructure and the geographical area, future work in this direction might better capture the circumstances around the disruption. The optimization problem can be expanded to include cost to see if the proposed model could reduce the cost of restoring the network.

As large scale disruptions continue to change life in cities, decision makers need a reliable and rapid way to prioritize different areas of infrastructures and eventually enhance infrastructure systems resilience. This research can further their efforts and their resources toward providing a clear restoration plan.

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