AN EVALUATION OF INDIFFERENCE CURVE TECHNIQUES

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1955

Submitted to the faculty of the Graduate School of
the Oklahoma Agricultural and Mechanical College
in partial fulfillment of the requirements
for the degree of
MASTER OF SCIENCE
August, 1957

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ACKNOWLEDGMENTS

The writer wishes to express his sincere appreciation to:

Dr. Richard H. Leftwich under whose supervision this thesis was written,

Mr. Robert W. Pittman for his kind assistance in English correction, and

Mr. S. Y. Meng for his counsel in some parts of the mathematical representation.

TABLE OF CONTENTS

CHAPTER I

INTRODUCTION

It is generally accepted that indifference curve techniques play an important role in micro-economic analysis. Indifference curve techniques have been applied extensively in consumer behavior analysis, production analysis, and welfare economic analysis. The purposes of this essay are to trace the development of indifference curve techniques and to evaluate their contributions to modern micro-economic analysis.

The study is divided into four main parts: (1) historical development of indifference curve techniques, (2) consumer behavior analysis, (3) production analysis, and (4) welfare economic analysis. In Chapter II we shall present a brief survey of the development of indifference curve techniques in chronological order. In Chapter III we shall consider the application of indifference curve techniques to consumer behavior analysis. An investigation of the development and the place of indifference curve analysis in that field will be made. In Chapter IV we shall present the application of indifference curve analysis to production analysis. A summary of the development and the contributions of isoquant and iso-resource curve analysis in relation to traditional analysis will be made. In Chapter V some applications of indifference curve techniques to welfare economics will be shown. We shall study the place of indifference curve techniques in the analysis of the conditions of optimum economic welfare. The sixth and the final chapter summarizes the study.

CHAPTER II

HISTORICAL DEVELOPMENT OF INDIFFERENCE CURVE TECHNIQUES

Francis Y. Edgeworth

Francis Y. Edgeworth is generally credited with the origination of the indifference curve technique.[1] Dissatisfied with the limited single variable utility function of W. S. Jevons, he sought to generalize the utility function by making utility obtained by a consumer a function of the combination of goods he consumed rather than of any single commodity consumed. He developed a three dimensional geometric counterpart of the mathematical utility function and from this came indifference curves or contour lines of a utility surface. In applying the concept to problems of bilateral monopoly the contract curve was brought into existence. Edgeworth's indifference curves, however, were derived from a complex mathematical process and were difficult to apply. Hence, though the origination of indifference curves is attributed to him, the credit for making it useful should be shared with co-discoverers and their followers.

Rudolf Auspitz and Richard Lieben

Rudolf Auspitz and Richard Lieben were two important figures in indifference curve development. Their work, Investigation on the

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[1]Francis Y. Edgeworth, Mathematical Psychics (London, 1881).

Theory of Price, appeared in 1889.[2] They referred fully to their predecessors of the subjective value school and did not mention Edgeworth's work. However, their "satisfaction surface" and "constant-satisfaction curves" bear a very marked resemblance to Edgeworth's and many considered them as independent discoverers of the technique. Moreover, they were the first to define complementarity with any degree of precision.

Irving Fisher

Irving Fisher was the third figure in the development of indifference curve techniques. His Mathematical Investigation in the Theory of Value and Price[3] appeared three years after Auspitz and Lieben's work. He gave a more exhaustive analysis of the uses of the technique. He was influenced by Auspitz and Lieben. He only knew of Edgeworth's Mathematical Psychics after he finished his work, although their three dimensional analysis and utility surface ideas were very similar. Fisher's indifference curves were drawn in our familiar form, as different from Edgeworth's. Hence, the credit for forming a more convenient shape of the indifference curve should be attached to him.[4] He applied indifference curve analysis to most of its present uses, but his analysis was not as thorough as those at present.

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[2] R. Auspitz and R. Lieben, Untersuchungen iiber die Theorie des Precises (Leipzig, 1889).

[3] I. Fisher, Mathematical Investigation in the Theory of Value and Price (New Haven, 1925). From Transactions of the Connecticut Academy, Vol. IX, 1892.

[4] See p. 9 of this study.

Vilfredo Pareto

Vilfredo Pareto is universally known for his contribution to indifference curve analysis.[5] He was the first to give a clear exposition of indifference curves and to try to abandon the measurability of utility. He is considered as the originator of an ordinal utility system. He used the term ophelimity to replace utility, and assumed it as given instead of following Edgeworth's assumption that utility could be measured. He constructed his indifference curves from these assumed indices of ophelimity for the individual. He introduced the concept of transformation function.[6] He never did go much further in the technique of analysis than his predecessors, especially Fisher.

W. E. Johnson

Directly based on Edgeworth's work, the contributions of W. E. Johnson were made in 1913.[7] He modified Edgeworth's indifference curves and called them "iso-utility curves". He was the first to use the concept of the marginal rate of substitution, but his definition of this term was vague. He abandoned the assumption of measurability of utility.

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[5] Vilfredo Pareto, Manuale di Economia Politica (Milan, 1916).

[6] See p. 26 of this study.

[7] W. E. Johnson, "The Pure Theory of Utility Curves," Economic Journal, XXIII (1913), pp. 395-467.

Eugen Slutsky

Eugen Slutsky's important work was published in 1915.[8] His use of the concept of the marginal rate of substitution was much better than Johnson's. He introduced the concept of the compensating variation in income, so the income effect and the substitution effect of price changes could be separated. He completely freed indifference curve analysis from the assumption that utility must be measurable.

John R. Hicks and R. G. D. Allen

The work of J. R. Hicks and R. G. D. Allen was a rediscovery and further development of the indifference curve technique. Johnson's and Slutsky's works had not been widely read at this time, and Hicks and Allen popularized these earlier works. Hicks and Allen contributed their important articles in 1934 on the subject.[9] Their combined work gave an outline of modern indifference curve analysis. An ordinal utility system was developed which was based on the assumption of a given scale of preferences. Hicks and Allen pointed out that the convexity of indifference curves to the origin depends on the principle of diminishing marginal rate of substitution. They also pointed up the following problems: complementarity, the separation of income and substitution effects, the elimination of the assumption of constant marginal utility of money, the consumer's surplus analysis, and the concept

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[8] E. Slutsky, "On the Theory of the Budget of the Consumer," tr. O. Ragusa, Readings in Price Theory, Vol. VI (Chicago, 1952), pp. 27-41.

[9] J. R. Hicks and R. G. D. Allen, "A Reconsideration of the Theory of Value," Economica, I (1934), 57-76; 196-219.

of compensating and equivalent variation in income. They are credited with laying a firm foundation for modern indifference curve development.

The following chart is included to facilitate the reader's understanding of the historical development of indifference curve analysis.

Edgeworth
1881
Mathematical Psychics

1. Generalized the utility function
2. Indifference curves and the contract curve

Auspitz and Lieben
1889
Investigation on the Theory of Price

1. Precisely defined complementarity
2. Curves of constant satisfaction

Fisher
1892
Mathematical Investigation in the Theory of Value and Price

1. Completing and competing articles
2. Partial income line
3. Income and substitution effects
4. Inferior goods
5. First form of present indifference curves
6. Independently reached the generalized utility function
7. First investigation of the measurability of utility

Pareto
1906
Manual of Political Economy

1. Ophelimity as given
2. Complementarity
3. Income and substitution effects
4. Original application of indifference curves to welfare economics

Johnson
1913
"Pure Theory of Utility Curves"

1. Iso-utility curves
2. Irrelevance of measurability of utility concept
3. Vague concept of marginal rate of substitution

Slutsky
1915
"On the Theory of the Budget of the Consumer"

1. Clear concept of marginal rate of substitution
2. Concept of compensating and equivalent variation in income
3. Satisfaction as given

Hicks and Allen
1934
"A Reconsideration of the Theory of Value"

1. The construction of an ordinal utility system
2. The separation of income and substitution effects
3. Definition of complementarity improved
4. Consumer's surplus
5. Principle of diminishing marginal rate of substitution
6. Production analysis improved

CHAPTER III

THE THEORY OF CONSUMER'S BEHAVIOR

Indifference curve technique resulted from the generalization of the utility function by Edgeworth. Edgeworth visualized utility as a function of all commodities rather than a function of one commodity as Jevons had seen it. Thus:

$$(3.1) \qquad U = F(X_1, X_2, \ldots X_n)$$

where U denotes utility and $X_1, X_2, \ldots X_n$ denotes the quantities of commodities. The marginal utility of commodity X_k becomes the partial derivative of the utility function with respect to X_k, or

$$(3.2) \qquad \frac{\partial U}{\partial X_k} = F_{X_k}$$

The Formation of Indifference Curve Technique

Edgeworth

F. Y. Edgeworth, in his analysis of bilateral monopoly, introduced a rudimentary form of indifference curves and the idea of a contract curve. He began his analysis with the classical utility technique; however, when he generalized the utility function so that utility became a function of all commodities, he was able to deduce from it the idea of indifference curves.

Edgeworth used two commodities in his analysis. If X_1 and X_2 represent the quantities of two commodities which an individual possesses, his utility function is

$$(3.1a) \qquad U = F(X_1, X_2).$$

Let commodities X_1, X_2 be represented by the base axes and utility be represented by the third dimension; the utility function (3.1a) can be considered as a surface which will generally have the shape of a dome; and U may be represented as any point on the length of the ordinate drawn from any point on the X_1-X_2 plane to the surface. Let the surface be cut by planes parallel to the plane of the X_1-X_2 axis, and each intersection forms a curve which may be called the indifference curve or contour line. It is the locus of points of possible combinations of X_1 and X_2 which have a given total utility for the individual. Analytically, the equation of the indifference curves system is the differential of (3.1a):[1]

$$(3.3) \qquad DU = \frac{\partial U}{\partial X_1} dX_1 + \frac{\partial U}{\partial X_2} dX_2 = 0.$$

In the case of barter of X_1 and X_2 by two individuals, the indifference curves of one individual would be:

$$\frac{\partial U}{\partial X_1} dX_1 + \frac{\partial U}{\partial X_2} dX_2 = 0.$$

The indifference curves of the other individual would be:

$$\frac{\partial \varphi}{\partial X_1} dX_1 + \frac{\partial \varphi}{\partial X_2} dX_2 = 0,$$

where $U = \varphi(X_1, X_2)$. The equilibrium rates of exchange of X_1 and X_2 for the individuals are represented by the points of tangency of the two indifference curves systems. The locus of such points can be

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[1] Edgeworth, pp. 21-22.

shown by the equation:[2]

$$(3.4) \qquad \frac{\partial U}{\partial X_1} \cdot \frac{\partial \varphi}{\partial X_2} - \frac{\partial U}{\partial X_2} \cdot \frac{\partial \varphi}{\partial X_1} = 0.$$

This indicates that at such a point of tangency the slope of the indifference curve of one individual is equal to the slope of the indifference curve of the other. Edgeworth called this locus the contract curve. These were the beginnings of indifference curve analysis, and these introductory insights were Edgeworth's contributions.

<u>Fisher</u>

Fisher was the first to put his indifference curves convex to both axes. He reached the point of generalizing the utility function eleven years after, and independent of, the work of Edgeworth. His derivation of indifference curves from the utility surface was similar to Edgeworth's, but he used the work of Auspitz and Lieben as reference.

In the analysis of consumer's behavior, Fisher introduced the price line which he called the "partial income line". The consumer will maximize his total utility at the point at which the line is tangent to an indifference curve. This condition is shown in Figure 1.[3] The line AB is the partial income line which represents maximum possible combinations of commodities X_1 and X_2 purchasable for a given amount of income. Indifference curves are 1, 2, ... 5. The equilibrium point is E. Further, if income changes, the partial income line

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[2] Ibid., p. 21.

[3] Fisher, p. 68.

will shift parallel to the previous line, and there will be a line traced out by the points of equilibrium.[4] This was later named "the income-consumption curve" by Hicks. If the price of one of the commodities being analyzed changes, the partial income line will shift clockwise or counter-clockwise and hence change the equilibrium point.

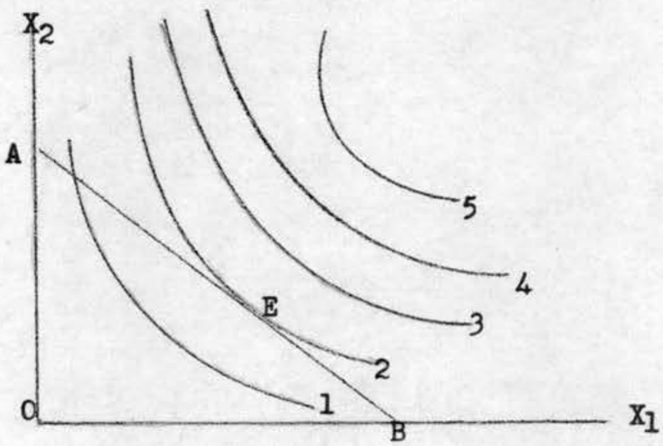

Figure 1

Fisher's analysis of inferior goods was also original. He considered two grades--A and B--of the same commodity, one of which is inferior to the other. This is shown in Figure 2.[5] If the individual is poor, he will buy the inferior one at point I near the A-axis. The rich man will select choice II, for he has a higher income.

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[4] Ibid., p. 73. Fisher only indicated that there was the line and did not make any further explanation of it.

[5] Ibid.

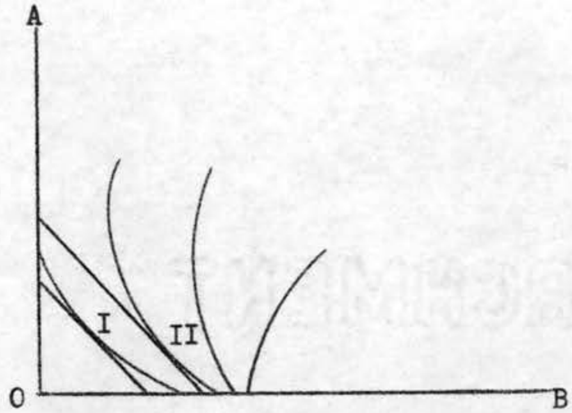

Figure 2

Pareto

Pareto attempted to construct a system of indifference curves independent of the assumption of the measurability of utility. He argued that the derivation of the indifference curves could be based on given indices of "ophelimity". Starting on this assumption, suppose a consumer is governed by his tastes alone and has 1 kg of bread and 1 kg of wine in the beginning. His scale of preference may be as follows:[6]

Bread	1.6	1.4	1.2	1.0	.8	.6
Wine	.7	.8	.9	1.0	1.4	1.8

These figures plotted on a diagram form an indifference curve. This is our familiar way of obtaining indifference curves. According to him, the consumer will be at equilibrium when the price line is tangent to an indifference curve, becuase this is the highest possible "ophelimity" the consumer's income will allow. The line which connects points of equilibrium was called the "line of exchange" by Pareto.

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[6] See P. C. Newman, A. D. Gayer, and M. H. Spencer, ed., Source Readings in Economic Thought (New York, 1954), p. 482.

Hicks

Hicks' indifference curves were also derived as the contour lines of a utility surface. He transposed the complicated mathematical processes into simple geometrical diagrams. In Figure 3,[7] OU represents the amount of utility; OX_1 and OX_2 represent amounts of two commodities; indifference curve AB is the contour line of the utility surface projected down on the X_2OX_1 plane. By eliminating the third dimension and taking these contour lines as indicators of levels of utility, he translated Marshall's utility surface into Pareto's given scale of preference. The index of utility becomes a matter of arbitrary choice, and indifference curves should be given ordinal rather than cardinal ranking.

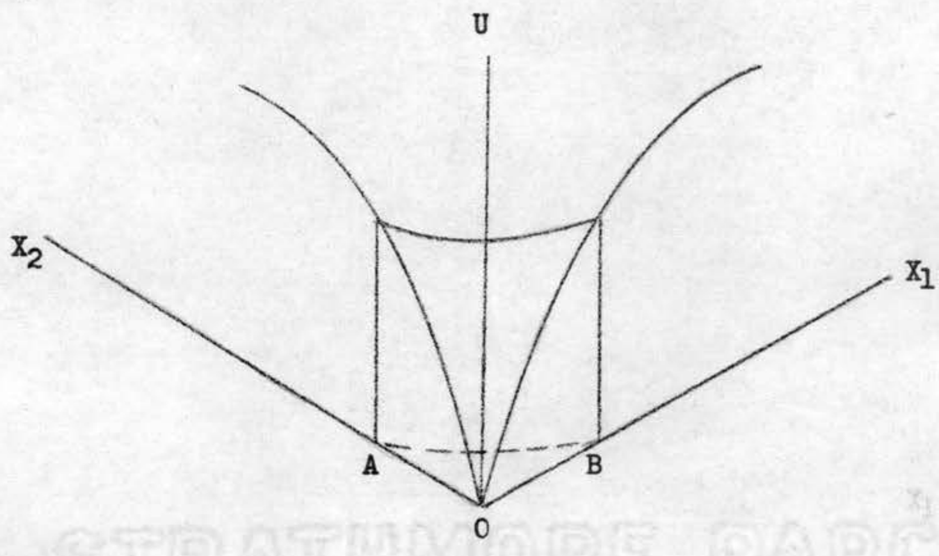

Figure 3

Using Slutsky's analysis of the nature of indifference curves, Hicks introduced the concept of the marginal rate of substitution. The marginal rate of substitution of commodity X_1 for X_2 is the

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[7] John R. Hicks, Value and Capital (2nd ed., Oxford, 1946), p. 15.

quantity of X_1 which will just compensate the consumer for the loss of a marginal unit of X_2. It is equal to the slope of the curve at any point on the curve:

$$(3.5) \qquad MRS_{x_1x_2} = \frac{dX_2}{dX_1} .$$

As indicated by Hicks, the more X_1 the consumer gets, the less X_2 he would be willing to give up for additional units of X_1. This is the principle of the diminishing marginal rate of substitution upon which the convexity of the indifference curve is based. A consumer will be at equilibrium when his price line is tangent to an indifference curve. Interpreting this in terms of marginal rate of substitution, we get:

$$(3.6) \qquad MRS_{x_1x_2} = \frac{P_{x1}}{P_{x2}} ,$$

where P_{x1} and P_{x2} denote the prices of commodities X_1 and X_2, and $\frac{P_{x1}}{P_{x2}}$ denotes the slope of the price line.

Contributions of the Technique

The Construction of an Ordinal Utility System

The indifference curve technique furnishes a means of analyzing consumer behavior without assuming the measurability of utility. Edgeworth did not discover this. Fisher made the first careful examination of the possibility of discarding the assumption.[8] Pareto later introduced the concept of indices of ophelimity in order to abandon the hedonistic assumption of utility. He then reached the

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[8]Fisher, p. 89. Fisher indicated that, using indifference curve analysis for purposes of explaining consumer's reaction to price and income changes, there is no occasion to introduce total utility.

idea of a given scale of preference. However, his definition of ophelimity was really based on the measurable utility concept.[9] Hence he did not get himself free from measurable utility. As a follower of Edgeworth, Johnson believed that the analysis could be based on the ratio of marginal utilities:[10]

> The impossibility of measurement does not affect any economic problem. Neither does economics need to know the marginal (rate of) utility of a commodity. What is needed is a representation of the ratio of one marginal utility to another. In fact, this ratio is precisely represented by the slope of any point of the utility curve.

Thus the term, marginal rate of substitution, was vaguely defined. Critizing Pareto, Slutsky claimed that economic assumptions should be independent of psychological and philosophical postulates. Slutsky's definition of utility was free from the measurability assumption and he explicitly introduced the concept of the marginal rate of substitution.[11]

Hicks, completely disregarding measurable utility, constructed an ordinal utility system. It was his belief that the theory of consumer's behavior could be based on the assumption of a given scale of preference. His analysis was dependent on the concept of the marginal rate of substitution. Thus, with indifference curve analysis, he

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[9] Newman, Gayer, and Spencer, p. 482. "For an individual, ophelimity is the pleasure he derives from a certain quantity of an article when that quantity is added to a given amount (which could equal zero, let us say) of the same thing already in his possession."

[10] Johnson, p. 400.

[11] Slutsky, pp. 27-41; George J. Stigler, "The Development of Utility Theory," <u>Journal of Political Economy</u>, Vol. 58 (1950), pp. 385-386.

believed that he was "... transforming the subjective theory of value into a general logic of choice."[12]

Complementarity

Indifference curve techniques have proved to be convenient in the analysis of complementarity in some respects and have had some shortcomings in others. Some parts of the technique lay the base for the modern theory of complementarity.

Auspitz and Lieben first defined complementarity and substitutability. In the case of two commodities X_1 and X_2, whether they are completing, independent, or competing depends on whether the cross derivative of utility function is either greater than, equal to, or less than zero,[13] or

$$(3.7) \qquad \frac{\partial^2 U}{\partial X_1 \partial X_2} \gtreqless 0.$$

This is to say, if X_1 and X_2 are completing, an increase in the quantity of X_1 raises the marginal utility of X_2; if they are independent, an increase in X_1 has no influence on X_2; if they are competing, an increase in X_1 lowers the marginal utility of X_2.

Fisher adopted this definition and formulated a graphic representation of the analysis. He pointed out the extremes: perfect competing and perfect completing articles. The former were those which could be substituted in an absolute constant ratio; the latter were

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[12] Hicks and Allen, "A Reconsideration of the Theory of Value," p. 58.

[13] Henry Schultz, The Theory and Measurement of Demand (Chicago, 1938), pp. 608-609; Stigler, p. 384.

those which must be used jointly in an absolutely constant ratio. Diagrams (a), (b), (c), and (d) of Figure 4 show the analysis of competing, completing, perfect competing, and perfect completing articles.[14]

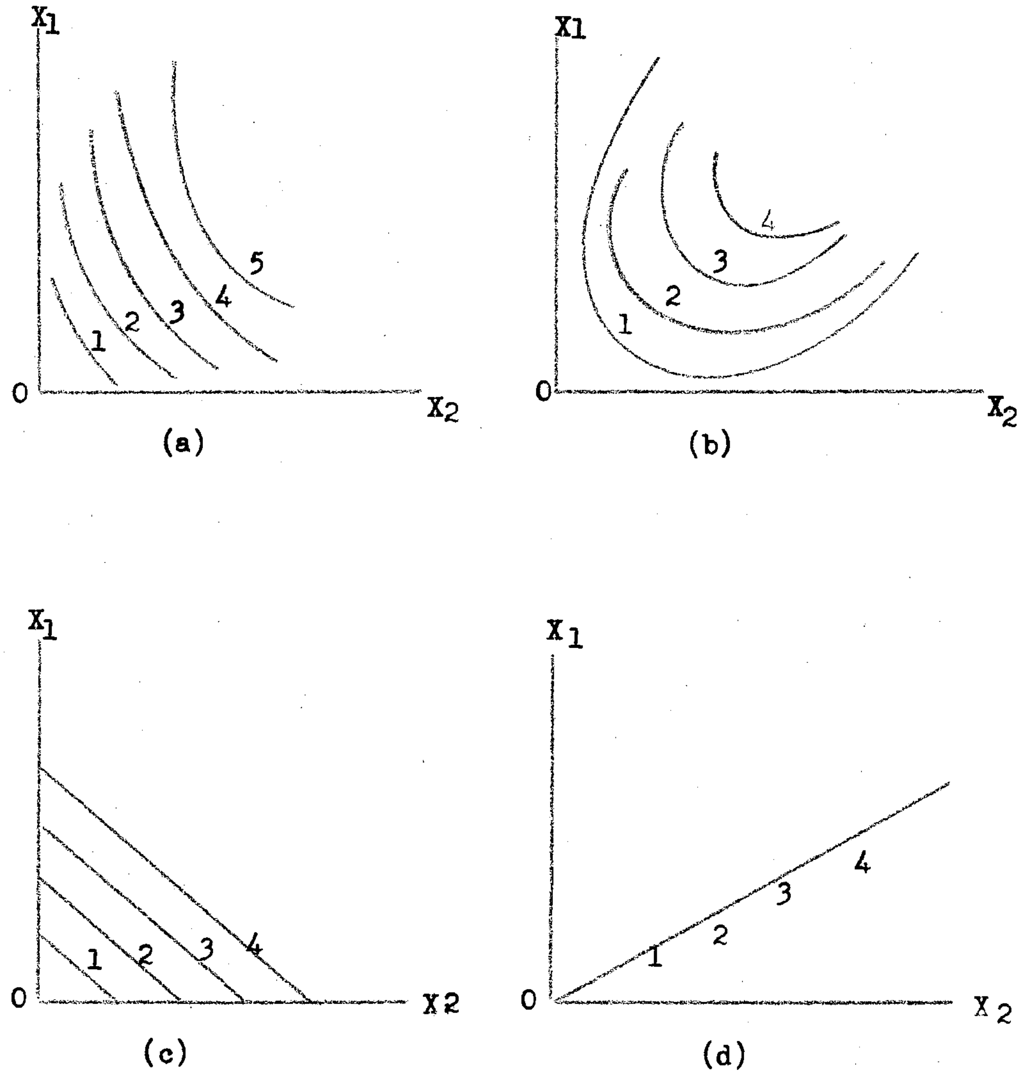

Figure 4

Pareto used his "ophelimity" concept to arrive at the same definitions of substitute goods and complementary goods. If X_1 and X_2 are

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[14] Fisher, p. 71.

complementary, an increase in X_1 raises the elementary ophelimity (or marginal utility) of X_2. If X_1 and X_2 are substitutes, an increase in X_1 lowers the elementary ophelimity of X_2. Edgeworth had shown the same results in 1897.[15]

Hicks was critical of the Edgeworth-Pareto definition. He insisted that their definition was based on the measurability of utility; consequently, he formulated a different definition:[16]

> Y is a substitute for X if the marginal rate of substitution for money is diminished when X is substituted for money in such a way as to leave the consumer no better off than before.... Y is complementary with X if the marginal rate of substitution of Y for money is increased when X is substituted for money.

This definition, though free from the measurability of utility, depends heavily on a numeraire. According to Hicks, this numeraire could be in terms of generalized purchasing power. This concept is difficult to represent graphically because the third dimension is needed to represent the numeraire. Therefore, he used verbal explanations. Using a two commodity analysis, only the substitutes relationship could be shown. The analysis of complementarity would require a third commodity outside to be substituted against as the numeraire.

The Separation of Income and Substitution Effects

Indifference curve techniques make available a convenient way of separating the income effects of a price change in one commodity from

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[15] F. Y. Edgeworth, Papers Relating to Political Economy (London, 1925), pp. 116-117. Edgeworth offered no concept of complementarity in his Mathematical Psychics. He reached his definition later than Auspitz and Lieben and Fisher. Also see Stigler, "The Development of Utility Theory," p. 384.

[16] Hicks, Value and Capital, p. 44.

the substitution effects in the consumption of that commodity. Pareto was the first to point out this possibility.[17] Slutsky added to this analysis by introducing the concept of the compensating variation in income.[18] Hicks pointed out that Marshall's analysis of demand neglected the income effect. Hicks indicated that a fall in the price of a commodity affects the demand for that commodity in two ways: (1) the price decrease will raise the individual's real income, and (2) it will change relative prices which will result in a tendency to substitute the relatively cheaper commodity for other commodities. Figure 5 shows how the income and substitution effects are separated by Hicks.[19] The consumer's original equilibrium position is point P, the point where the original price line, ML, is tangent to an indifference curve I. Suppose the price of commodity X falls, the consumer's price line will shift counter-clockwise to ML" which is tangent to a higher indifference curve II, and the new equilibrium point is point Q. Draw a line M'L' which is tangent to indifference curve II and parallel to ML. The point of tangency is P'. The movement from P to Q is equivalent to a movement from P to P' along the income-consumption curve and a movement from P' to Q along an indifference curve.

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[17] "In passing from a certain combination of goods A, B, C, ... to another A', B', C', we may divide the operation into two: first we preserve intact the proportions of the combination and increase (or decrease) all the quantities in the same proportion; secondly, we change the proportions and so arrive definitively at the combination A', B' and c." (Pareto, p. 283.) Quoted in T. W. Hutchison, A Review of Economic Doctrines (Oxford, 1953), p. 221.

[18] Slutsky, p. 42.

[19] Hicks, Value and Capital, p. 31.

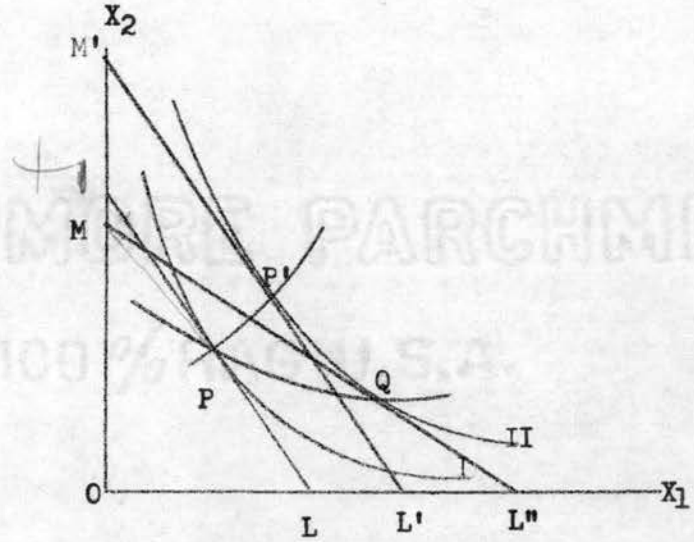
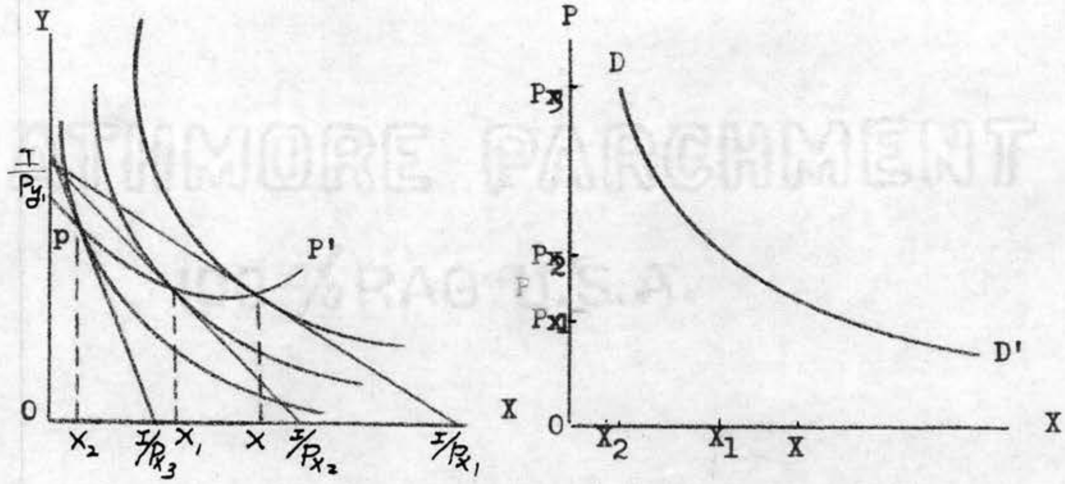


Figure 5

The movement from P to P' is the income effect and the movement from P' to Q is the substitution effect. The income effect will be of little importance when the commodity contributes only a small portion to the whole budget, and the substitution effect will be much greater than the income effect in most cases.

<u>The Derivation of the Demand Curve</u>

The next application of indifference curve techniques was the derivation of the demand curve for a commodity. The demand curve can be derived from the price-consumption curve. To analyze the demand for X, the price of Y and consumer's tastes, preferences, and income are held constant. If the price of X is varied, at prices P_{x_1}, P_{x_2}, and P_{x_3}, the corresponding quantities of X purchased are X_1, X_2, and X_3. A demand curve for X can be constructed from these data. Letting P represent prices and X the quantities, the demand curve DD' is derived. This is shown in Figure 6.



(a) Figure 6 (b)

<u>Price-Consumption Curve and Elasticity of Demand</u>

The slope of the price-consumption curve for a commodity indicates the elasticity of demand when the commodity under consideration is measured on the X-axis and generalized purchasing power is measured on the Y-axis.[20] As shown in Figure 7, AD is the price-consumption curve for X if its price is varied with a given income. Where the curve is parallel to the X-axis, the elasticity for X is unitary. This can be shown because as the price of X changes, the amount of money not spent on X remains constant; therefore, the outlay for X is unchanged. Where the curve slopes upward to the right, the demand for X is inelastic. This can be shown because as the price of X rises, the amount of money not spent on X decreases; therefore, the outlay for X increases. Where it slopes downward to the right, the demand for X is elastic. This can

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[20] Ruby T. Norris, <u>The Theory of Consumer's Demand</u> (New Haven, 1941), Chapter 2; R. H. Leftwich, <u>The Price System and Resource Allocation</u> (New York, 1955), pp. 82-84.

be shown because as the price of X rises, the amount of money not spent increases; therefore, the outlay for X decreases.

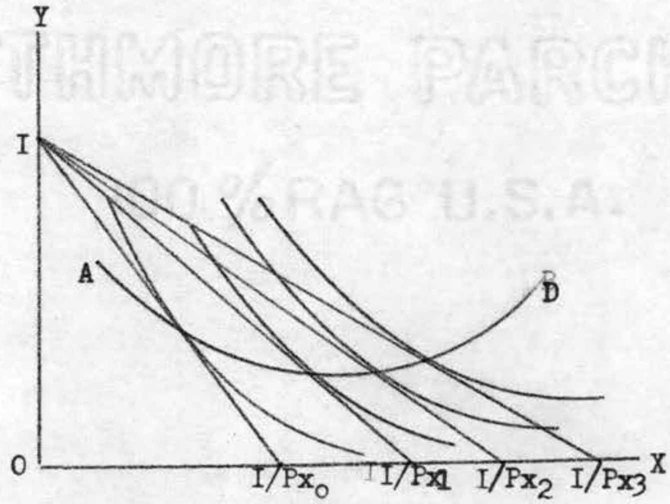

Figure 7

Income-Consumption Curve and the Engel Curve

Engel curves can be derived from the income-consumption curve. Engel curves show the changes in consumption of one commodity which result from income changes, given the commodity prices. As indicated by diagram (a) of Figure 8, at income I_1, the consumer would purchase

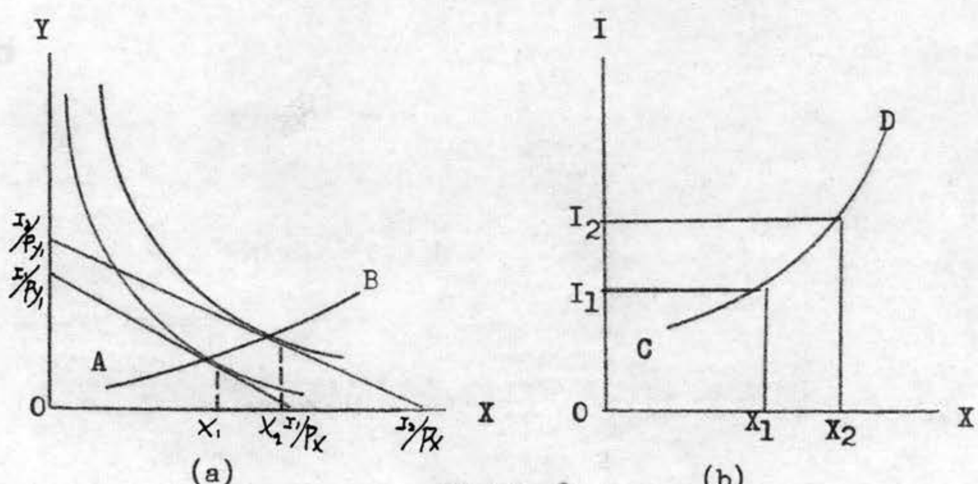

Figure 8

quantity X_1 of X; at income I_2, the consumer would purchase quantity X_2 of X. The Y-axis represents income levels in diagram (b) of Figure 8 and the X-axis represents quantities of X. The Engel curve CD may now be plotted on diagram (b) from the data obtained in diagram (a).

Evaluations

The Given Indifference Map

One merit of the indifference curve technique is the appeal of the underlying assumption, a given scale of preferences. This enables economists to discard the assumption of measurable utility and therefore frees the analysis from psychological and philosophical assumptions. Moreover, it furnishes a means of analyzing inferior goods and facilitates a more general analysis of demand theory. Within limitations, it aids in the analysis of complementarity and substitutability. It also provides a means of analyzing and separating the income and substitution effects of price changes of one commodity. It has merit because of the convenience and ease of its application to specific problems.

The Equilibrium Condition

The equilibrium condition explained by indifference curve analysis can be stated in terms of marginal utility.[21] If indifference curve analysis is used, as listed in (3.3), the equation of the indifference

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[21] Hicks, Value and Capital, Appendix to Chapter I, pp. 305-306; Leftwich, Appendix to Chapter V, pp. 85-86.

curves system is: $\frac{\partial U}{\partial X_1}dX_1 + \frac{\partial U}{\partial X_2}dX_2 = 0,$

or in terms of (3.2):

$$(3.8) \quad F_{x_1}dX_1 + F_{x_2}dX_2 = 0.$$

The slope of the indifference curve at any point is: $\frac{dX_2}{dX_1} = -\frac{F_{x_1}}{F_{x_2}}.$

The equation of the price line is:

$$(3.9) \quad P_{x_1}X_1 + P_{x_2}X_2 = C.$$

The slope of the price line would be: $\frac{dX_2}{dX_1} = -\frac{P_{x_1}}{P_{x_2}}.$

A consumer will be in equilibrium when his marginal rate of substitution (the slope of the indifference curve) is equal to the slope of the price line:

$$\frac{F_{x_1}}{F_{x_2}} = \frac{P_{x_1}}{P_{x_2}} \qquad \left(\text{where } \frac{F_{x_1}}{F_{x_2}} = MRS_{x_1x_2}\right),$$

where F_{x_1} and F_{x_2} are the marginal utilities of X_1 and X_2 respectively. Hence we reach the conclusion of classical utility analysis that a consumer will be in equilibrium when:

$$(3.10) \quad \frac{MU_{x_1}}{P_{x_1}} = \frac{MU_{x_2}}{P_{x_2}}.$$

The Empirical Derivation of the Indifference Function

It has been suggested that the indifference function could be derived from empirical data.[22] With the development of econometrics, the statistical approach plays an increasingly important role in

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[22] L. L. Thurstone, "The Indifference Function," The Journal of Social Psychology, 1931.

economic analysis. An empirical derivation of the curve, it is argued, would give it more significance. However, since the perference and opportunity factors may influence each other, some writers have pointed out the limitations of this finding in the theory of consumer's behavior.[23] Nevertheless, by using the empirical derivation to show the variation of theory from the real world, the indifference curve technique may be better than the marginal utility approach in this respect.

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[23]W. A. Wallis and F. Friedman, "The Empirical Derivation of Indifference Function," Studies in Mathematical Economics and Econometrics (Chicago, 1942), pp. 175-189.

CHAPTER IV

THE THEORY OF PRODUCTION

This chapter is concerned with the development of indifference curve techniques in production theory. The chapter is divided into two parts: (1) development of iso-resource curves or product-product relationship analysis, and (2) development of isoquants or factor-factor relationship analysis. Following these, an evaluation will be presented.

The Development of Iso-Resource Curve Analysis

Early Contributions

Fisher was the first to apply indifference curve techniques to production theory.[1] His producer's indifference map was plotted upside down and contained inflexion points as compared to the present form. In Figure 9,[2] axes OA and OB show the quantities of commodities A and B. Each indifference curve shows the constant disutility to the producer for varying combinations of A and B produced. Price line CD shows the locus of production combinations which can be sold for a fixed amount of money. The producer will reach equilibrium at point I, which is the point of tangency between the price line and an

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[1] Fisher, p. 76.

[2] Ibid.

indifference curve. This shows the largest amount of money obtainable for given disutility. The indifference curves are assigned negative numbers for they represent constant disutilities to the producer. This was the origination of the iso-resource analysis.

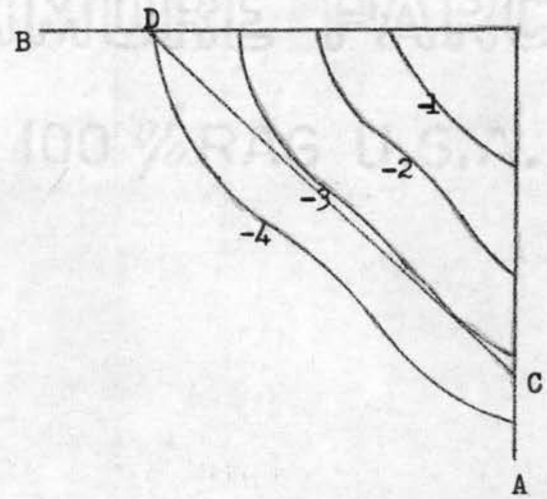

Figure 9

Pareto was the first to introduce the concept of a transformation function. According to him, production is the transformation of certain goods and services into other goods and services,[3] or

$$(4.1) \quad V = \varphi(X_1, X_2, \ldots X_n).$$

V denotes the quantity of a factor of production, and X_1, X_2, ... X_n denote the quantities of products which that factor can produce. The partial derivative of the above transformation function with respect to X_1 is called the marginal coefficient of production of V for X_1, or

$$(4.2) \quad DV = \frac{\partial \varphi}{\partial X_1}.$$

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[3] Henry Schultz, "Marginal Productivity and the General Pricing Process," Journal of Political Economy, XXXVII (1929), pp. 505-551; George J. Stigler, Production and Distribution Theories (New York, 1941), pp. 364-378.

Modern iso-resource curve analysis and technical complementarity analysis are based on the above concepts.

Iso-Resource Curve Analysis

An iso-resource curve shows the different product combinations obtainable from given quantities of factors of production. This analysis developed from Pareto's transformation function and is sometimes called product-product relationship analysis. As shown in Figure 10, the X_1-axis represents quantities of product X_1, and the X_2-axis represents quantities of product X_2; iso-resource curves 1, 2, and 3 show the different combinations obtainable from given quantities of factors V_1, V_2, and V_3.[4]

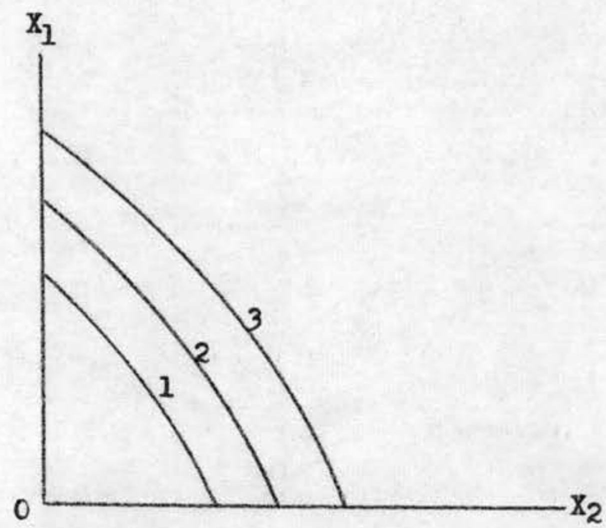

Figure 10

Each transformation curve is derived by considering given quantities of resources. The slope of the iso-resource curve is called

[4]Earl O. Heady, Economics of Agricultural Production and Resource Use (New York, 1952), pp. 203-221.

the marginal rate of transformation between two products. The marginal rate of transformation of product X_1 for X_2 is the quantity of X_1 which must be given up if an additional unit of product X_2 is to be produced. The curvature of iso-resource curves depends upon the marginal rate of transformation between products. The curve will be linear when the marginal rate of transformation is constant. This indicates that for each additional unit of one product, a constant amount of the other product must be sacrificed. The curve will be convex to the origin if the marginal rate of transformation between the two products is decreasing. This indicates that for each additional unit of one product, smaller and smaller amounts of the other product must be sacrificed. The curve will be concave to the origin when the marginal rate of transformation between the two products is increasing. This indicates that for each additional unit of one product, larger and larger amounts of the other product must be sacrificed. Owing to the operation of the law of diminishing returns, successive increases in output of one product entail increasing sacrifice of the other in most cases, thus the above analysis is a valid generalization.

Technical Complementarity

Sune Carlson introduced the concept of technical complementarity of products.[5] Carlson indicated that, when a given amount of production factor V is used in the production of products X_1 and X_2, the transformation function (4.1) is:

$$(4.1a)\ V = \varphi(X_1, X_2).$$

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[5]Sune Carlson, A Study on the Pure Theory of Production (London, 1939), p. 78.

The marginal coefficient of production for X_1 would be $\frac{\partial V}{\partial X_1}$ and for X_2 would be $\frac{\partial V}{\partial X_2}$. Carlson pointed out that if an increase in the production of one product causes the marginal coefficient of the other to increase, the products are technical complements. Examples would be gas and coke or cotton and cotton seed. If an increase in one product causes the marginal coefficient of the other to decrease, the products are technical substitutes. An example would be corn and wheat. If an increase in one product causes no change in the marginal coefficient of the other, the products are independent. If the input of V is considered disutility to a producer, the conditions of technical complementarity are a reverse of the conditions of complementarity among goods in consumer's behavior analysis. As Carlson pointed out, the technical complementarity, independence, or technical substitutability of the two products depends on whether the cross derivative of the transformation function (4.1a) is less than, equal to, or greater than zero:

$$(4.3) \quad \frac{\partial^2 V}{\partial X_1 \partial X_2} \lesseqgtr 0.$$

This form is the mathematical representation of the above analysis.

The Isoquants and Isocosts Analysis

Sune Carlson was the first to introduce the typical isoquant analysis comprehensively in English.[6] Carlson's exposition began

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[6]Carlson wrote her book within the year 1935-36. She referred fully to E. Schneider, Theorie der Produktion (Wien, 1934); and Frisch's unpublished materials, "Tekniske og økonomiske produktivitetslover," (Mimeographed lectures) (University of Oslo). She gave little credit to Hicks and Allen's article. Therefore, she was the first to introduce a complete indifference curve analysis of production theory in English.

with a strictly static state. Her analysis was of two variable factors of production, V_1 and V_2, used to produce X with the method of production held constant. Thus the production function or the relationship between output and input would be:

$$(4.4)\quad X = \varphi(V_1, V_2).$$

The partial derivative of the production function with respect to factor V_1 is the marginal productivity of V_1, or

$$(4.5)\quad \frac{\partial X}{\partial V_1} = \varphi V_1$$

In Figure 11, the inputs of V_1 and V_2 are measured on the bases axes, the vertical axis represents output of X, and the production surface OACD denotes the locus of outputs of all possible factor combinations in the production of X.[7]

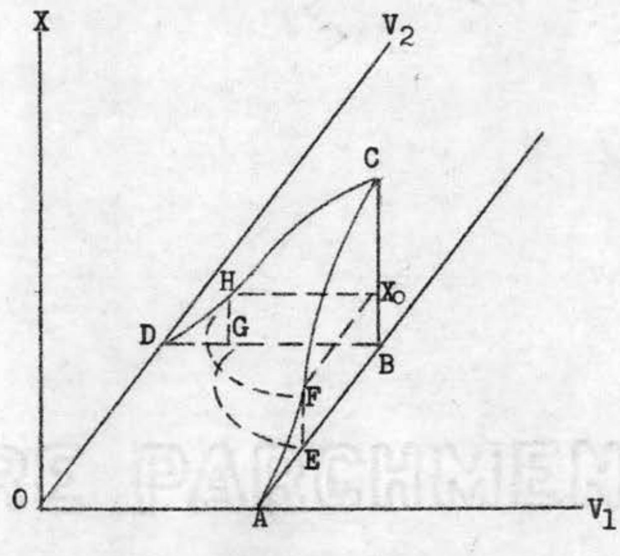

Figure 11

When the input of V_1 is held constant at OA, output will increase along AC when the input of V_2 is increased. The slope of the curve AC

[7]Carlson, p. 18.

indicates the marginal productivity of V_2. Similarly, when V_2 is held constant at CD, the marginal productivity of V_1 is indicated by the slope of the curve DC. The contour lines, which represent equal outputs of X for different combinations of factors, are called isoquants. When the production of X is a fixed amount, X_o, the production relationship is $X_o = \varphi(V_1, V_2)$. The equation of the isoquants system can be obtained by differentiating the above function:

$$(4.6)\quad 0 = \varphi_{v_1}dV_1 + \varphi_{v_2}dV_2.$$

The slope of an isoquant would be:

$$(4.6a)\quad \frac{dV_2}{dV_1} = -\frac{\varphi_{v_1}}{\varphi_{v_2}}.$$

Constant cost combinations of the factors can be represented by another system of curves--the isocost curves. An isocost curve shows the different combinations of factors which the firm can purchase with a given outlay. When the prices of the two factors are fixed, and the outlays are changed, the isocosts will be parallel lines. Therefore, the equation of the isocost system is:

$$(4.7)\quad p_1V_1 + p_2V_2 = K.$$

The prices of the factors are represented by p_1 and p_2. The slope of an isocost can be obtained by differentiating the above equation:

$$(4.7a)\quad \frac{dV_2}{dV_1} = -\frac{p_1}{p_2}.$$

The minimum cost combinations for a given amount of the product will be located by the point of tangency of an isocost and an isoquant, or

$$(4.8)\quad \frac{\varphi_{v_1}}{\varphi_{v_2}} = -\frac{p_1}{p_2}.$$

Carlson also introduced the concept of isoclines. An isocline is a line which connects the points of equal slopes on different isoquants and therefore indicates constant ratios of marginal productivities of different factor combinations for different product levels. The ridge lines or border lines are a special type of isocline. They are used to mark off the relevant sector of the isoquant system. As shown in Figure 12 (a), OA intersects each isoquant at the point where it is vertical, OB intersects each isoquant at the point where it is horizontal. The marginal productivity of factor V_2 is negative at any point which is above OA. The marginal productivity of factor V_1 is negative at any point which is to the right of OB. Hence, the most efficient combination must be somewhere between line OA and OB. The line OD which joins the points of tangency of isocosts and isoquants, the least cost combinations, is called the expansion path. Given the prices of V_1 and V_2, it is the most efficient path for the producer who wants to expand or contract his output to follow.

John M. Cassel developed the same system that Carlson developed at almost the same time.[8] He pointed out the convenience of using isoquant analysis when dealing with more than one factor in order to trace out the law of variable proportions. He also mentioned how the three stages of production in traditional analysis could be shown in a more general way by isoquant analysis.[9]

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[8] John M. Cassel, "On the Law of Variable Proportions," Explorations in Economics (New York, 1936), pp. 223-236. Reprinted in Readings in the Theory of Income Distribution (Philadelphia, 1946), pp. 103-118.

[9] See the following section.

<u>The Derivation of the Product Curve</u>

The product curve for one factor can be obtained from the isoquant system. In Figure 12 (a), factor V_2 is held constant at V, the product curve for V_1 can be obtained by successive increases in the quantity of V_1 used. The quantities of V_1 used, V_a, V_b, ... V_d, and the corresponding product levels, X_o, X_1, ... X_3, are plotted in Figure 12 (b) to obtain the total product curve for V_1.

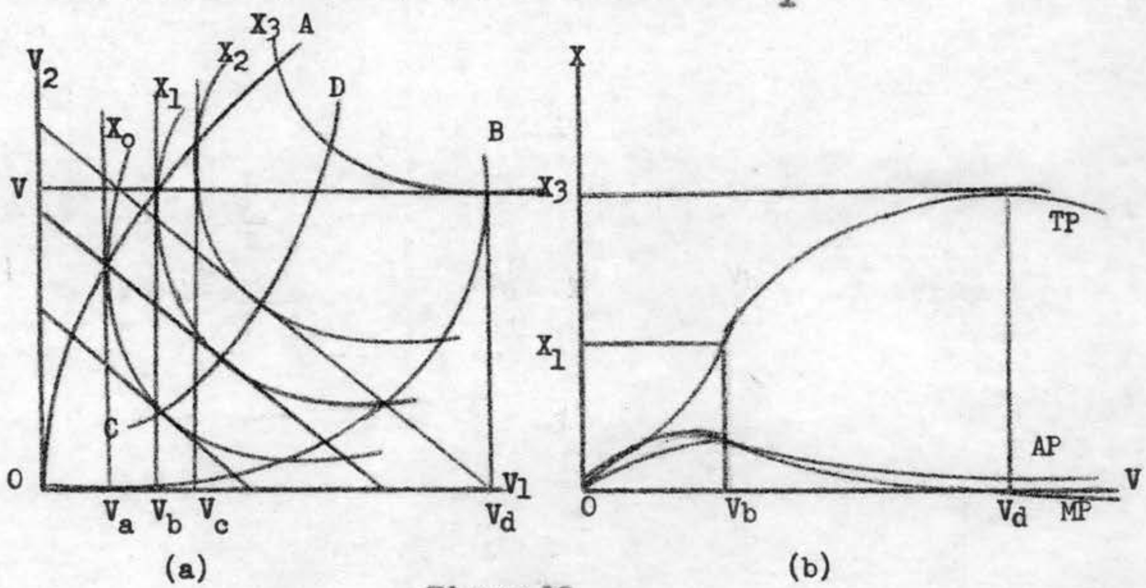

Figure 12

Cassel pointed out that in analyzing inputs of V_1, the three stages of production can be shown on the isoquant map. In Figure 12 (a), for any quantity of V_1 to the left of ridge line OA with a fixed amount of V_2, average product of V_1 is increasing when the quantity of V_1 is increased. This is the condition of the first stage of production and is shown in Figure 12 (b) by that part of the product curve up to input V_b of V_1. For any amount of V_1 within the two ridge lines OA and OB, average product and marginal product of V_1 are decreasing, but marginal product of V_1 is still positive. This is the condition of the second stage of production and is shown by the part of the product curve between inputs V_b and V_d of V_1. For any amount of V_1 to the right of OB, marginal product of V_1 is negative. This is the condition of the third stage and is shown by the part of the product curve where inputs of V_1 are larger than V_d.

Evaluation

The Technique

The application of indifference curve techniques to production theory illuminates some dark corners of the traditional theory. One of its merits is that it is more penetrating and more comprehensive. The isoquant technique shows in generalized form the law of variable proportions. It furnishes a more simple and general way of obtaining minimum cost conditions. The expansion path indicates the most efficient path the producer can take as he expands or contracts his output. The iso-resource curve technique furnishes a means of analyzing the production relationships between two products. It throws some light on the nature of technical complementarity in the production of two or more products. The technique is valuable in analyzing production behavior from different points of view--the exposition of factor-factor relationships and the product-product relationships.

Empirical Uses

Indifference curve analysis of production lends itself to empirical investigations.[10] Thus it has an advantage over traditional

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[10]For some empirical derivation examples, see Heady, pp. 142-143; pp. 155-157 ff.

production analysis. Further, the production function is more concrete than the utility function. It exists and its existence is apart from the subjective judgement of producers and consumers. Because of this characteristic, the empirical derivations of indifference curve technique of production have more significance than empirical derivations of consumer indifference curves, as we pointed out in consumer's behavior analysis.

CHAPTER V

WELFARE ECONOMICS

Welfare economics deals with the well-being of an economic society. It is concerned with the optimum conditions of economic welfare. There are five primary marginal conditions which must be met to attain maximum welfare. These are (1) the optimum allocation of products among consumers, (2) the optimum allocation of factors of production among products, (3) the optimum factor-product relationships, (4) the optimum degree of specialization, and (5) the optimum direction of production.[1] This chapter presents indifference curve techniques as used to analyze those marginal conditions of maximum welfare.

In our analysis of the marginal conditions of maximum welfare, we shall make the following assumptions:

(1) Each individual has one utility function and owns definite quantities of each product and each factor.

(2) Each firm has a given transformation function determined by the "state of the arts".

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[1]Melvin W. Reder, Studies in the Theory of Welfare Economics (New York, 1947), Chapter II.

The Optimum Allocation of Products

If fixed amounts of products X_1 and X_2 are distributed randomly among a group of consumers, and if the quantities of all other products consumed by them are given, the optimum allocation of products occurs when the distribution of those products is such that no change could be made to increase one consumer's well-being without decreasing some other consumer's well-being. Expressed in terms of indifference curve analysis, the optimum allocation of products occurs when they are so distributed that the marginal rate of substitution between any two products is equal for all consumers.

The optimum allocation of two products between two consumers can be shown by an application of indifference curve analysis--the box analysis.[2] A typical box analysis is presented in Figure 13.[3] The horizontal lines of the diagram measure the total quantity of product X_1. The vertical lines of the diagram measure the quantity of product X_2. Any point within the rectangle represents a given distribution of these two products between the two consumers, O and P. Lines 1, 2, 3, and 4 are consumer O's indifference curves. Lines a, b, c, and d are consumer P's indifference curves. Since the indifference curve system of P is drawn upside down, P's quantities of X_2 are measured on the vertical line PX_2, and P's quantities of X_1 are measured on the

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[2]A crude sort of box analysis was originated by Edgeworth (see his Mathematical Psychics, p. 29). After him, A. L. Bowley modified this device into the form usually applied today. See Bowley's The Mathematical Groundwork of Economics (London, 1924), Chapter I.

[3]Kenneth E. Boulding, "Welfare Economics," A Survey of Contemporary Economics (Chicago, 1953), B. F. Haley, ed., p. 14.

horizontal line PX_1. The locus of the points of tangency of the two sets of indifference curves, CC', is the contract curve. From any point not on this curve, say M, we can move to the contract curve at N along an indifference curve b to a higher indifference curve for O, indifference curve 3. Consumer O's well-being has been increased and P's well-being has not been decreased but has remained the same. Similarly, a movement along the indifference curve 2 for consumer O from M to W will increase P's well-being without a decrease in O's well-being. A movement from M to any point on the contract curve between MN and MW represents a gain to both parties. It is not possible to move from one point to another on the contract curve, however, without decreasing at least one consumer's well-being. Any point on the contract curve, then, represents a possible optimum allocation of products. Thus an optimum allocation of products occurs when the distribution is such that no change could be made to increase one consumer's well-being without decreasing some other consumer's well-being.

Since the contract curve is the locus of the tangency points between indifference curves of two consumers, it can also be shown mathematically. From (3.8), the slope of any curve of one consumer's indifference curve system would be: $\frac{dX_2}{dX_1} = -\frac{F_{x_1}}{F_{x_2}}$.

The slope of any curve of the other indifference curve system would be: $\frac{d'X_2}{d'X_1} = -\frac{F'_{x_1}}{F'_{x_2}}$. The equation of the contract curve then is:

$$(5.1) \quad \frac{F_{x_1}}{F_{x_2}} = \frac{F'_{x_1}}{F'_{x_2}} .$$

Since the marginal rate of substitution of X_1 for X_2 is the slope of the indifference curve, the above equality can be restated:

$$(5.2)\quad MRS_{x_1x_2} = MRS'_{x_1x_2} .$$

This is the necessary condition for an optimum allocation of products among consumers shown in the simple form.

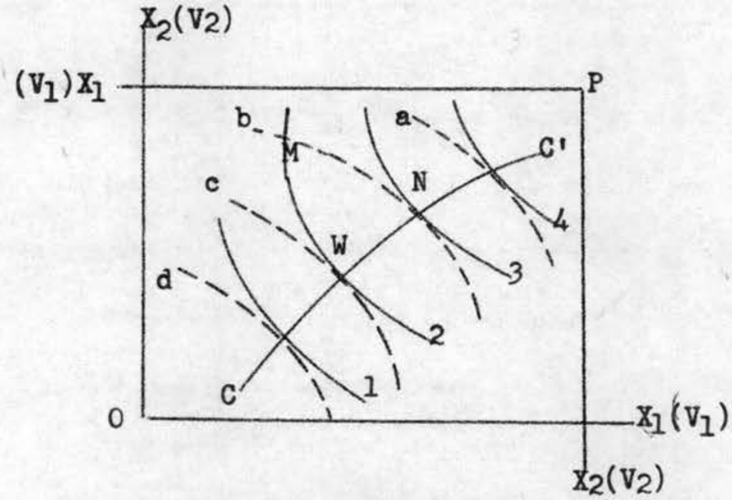

Figure 13

The Optimum Allocation of Factors

This condition refers to the optimum utilization of factors of production. If fixed amounts of production factors, V_1 and V_2, are used in the production of products and the quantities of all other factors and products are held constant, the optimum allocation of factors occurs only if the marginal rate of technical substitution between the two factors is the same for every product in production.

If two products are produced, this condition can be shown also in Figure 13. The horizontal lines of the diagram are now used to measure the quantity of V_1 and the vertical lines of the diagram are used to measure the quantity of V_2. Lines 1, 2, 3, and 4 are the isoquants for product O. Lines a, b, c, and d are the isoquants for product P. Any point within the diagram represents a given allocation of the two factors between the two products. A movement from M to any point on the contract curve between MN and MW represents an increase in the production of either O or both with no increase in the quantities of the two factors. Thus for an optimum allocation of factors to occur, the marginal rate of technical substitution between the two factors must be the same for every product which they are used to produce.

The necessary conditions for an optimum allocation of factors can also be shown mathematically. From (4.6a), the slope of an isoquant of one isoquant system can be shown as: $\frac{dV_2}{dV_1} = -\frac{\varphi v_1}{\varphi v_2}$.

The slope of an isoquant of the other isoquant system can be shown as:

$$\frac{d'V_2}{d'V_1} = -\frac{\varphi' v_1}{\varphi' v_2}.$$

The contract curve is the locus of the tangency points of the two sets of isoquants, thus

$$(5.3)\quad \frac{\varphi v_1}{\varphi v_2} = \frac{\varphi' v_1}{\varphi' v_2}.$$

Since the slope of an isoquant equals the marginal rate of technical substitution between the two factors, the above equation can be stated as:

$$(5.4)\quad MRTS_{v_1 v_2} = MRTS'_{v_1 v_2},$$

which is the necessary condition for maximum production. The sufficient condition for maximum production will be presented in the next section.

The Optimum Factor-Product Relationship

This marginal condition of maximum welfare relates to the allocation of a single factor of production between two producers. The quantities of all other products produced and factors used are assumed to be constant. The optimum factor-product relationship will be reached when the marginal productivity of factor V for product X is the same for any pair of producers using V and producing X.

This condition requires the equality of marginal productivities of the production functions for the two producers. As shown in (5.3), the necessary condition for an optimum allocation of factors is:

$$\frac{\varphi_{v_1}}{\varphi_{v_2}} = \frac{\varphi'_{v_1}}{\varphi'_{v_2}} .$$

Following this, the consistent conditions for optimum allocation of factors are:

$$(5.5) \quad \varphi_{v_1} = \varphi'_{v_1}$$

$$(5.6) \quad \varphi_{v_2} = \varphi'_{v_2}$$

Or, in terms of (4.5) we get:

$$(5.5a) \quad \frac{\partial X}{\partial V_1} = \frac{\partial' X}{\partial V_1}$$

$$(5.6a) \quad \frac{\partial X}{\partial V_2} = \frac{\partial' X}{\partial V_2}$$

which indicate that for optimum factor-product relationships the marginal productivity of factor V must be the same for any pair of producers.

The Optimum Degree of Specialization

This condition of maximum welfare specifies a necessary condition for determining the optimum output of each product by each producer. Given the quantities of the factors each producer uses and the quantities of other products he produces, the marginal rate of transformation between any two products must be the same for any two producers who produce both of them.

As shown in Figure 14 (a),[4] let AB represent the transformation curve between X and Y for producer I. Suppose he produces RS of Y and OR of X. As shown in Figure 14 (b), let CD represent the transformation curve for producer II and suppose that he produces TU of Y and OT of X. Superimpose Figure 14 (a) on Figure 14 (b), rotating the axes

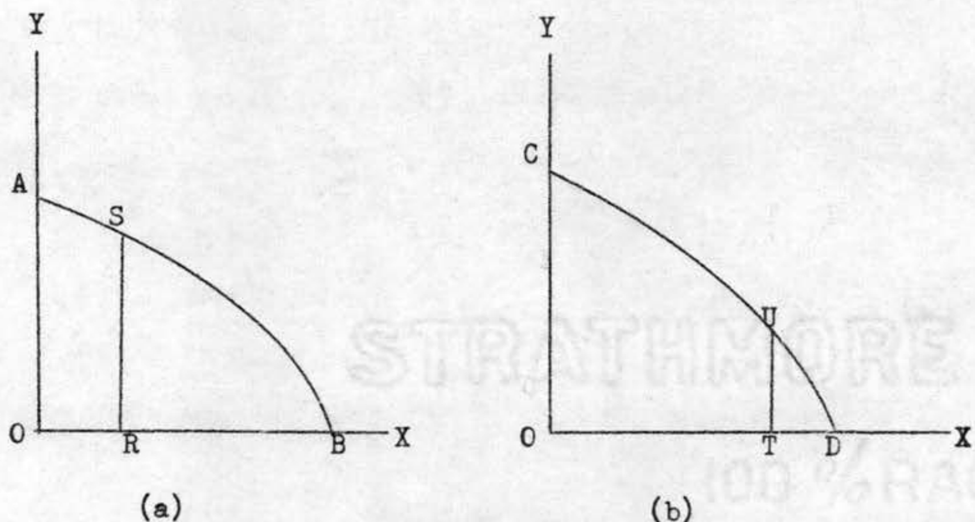

Figure 14

of Figure 14 (b) through 180 degrees and shifting the origin so that S and U coincide at point S. The result in shown in Figure 15.

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[4]See Reder, p. 23-26 for Figures 14 (a), 14 (b), and Figure 15.

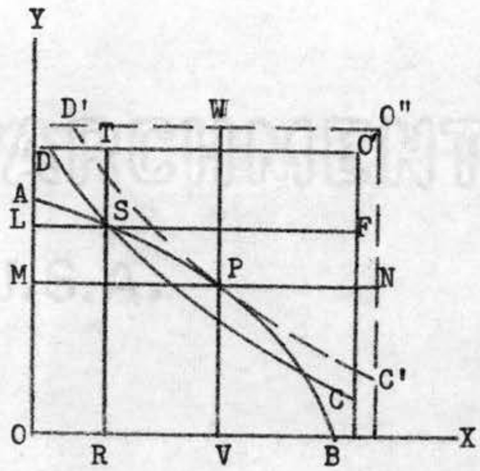

Figure 15

The combined output of Y, RT, is the summation of RS and TU. Similarly, the combined output of X is LF. Now shift the axes of producer II until CD becomes tangent to AB at P. The new position is shown by dotted lines and curve C'D'. The new combined output of Y is VW which is greater than RT. This indicates that, unless the marginal rate of transformation between the two products is the same for both producers, it will be possible to increase the combined outputs of both products without changing the outputs of factors or the outputs of other products. Therefore, for welfare to be maximized, the marginal rate of transformation between any two products must be the same for any two producers that produce them both, or

$$(5.7) \quad MRT_{x_1x_2} = MRT'_{x_1x_2}$$

where $MRT_{x_1x_2}$ denotes the marginal rate of transformation between two products X_1 and X_2.

The Optimum Direction of Production

The first of our marginal conditions of maximum welfare serves to determine the optimum allocations of products among consumers. The second, third, and fourth conditions specify the optimum conditions of production. The fifth condition relates to the technical conditions of production to conform to the structure of consumers' preferences.

Given the amounts of the factors used and the other products produced, the optimum direction of production is that the marginal rate of transformation for the community between two products must be the same as the marginal rate of substitution between them for any person.

This condition can be shown in Figure 16. Given the quantities of the factor each producer uses and the quantities of other products he produces, let curve AB represent a community transformation curve between two products X_1 and X_2. Given the amount of products, let CD stand for a community indifference curve of a society which consumes both X_1 and X_2. The community indifference curve CD intersects the transformation curve AB at point P. Under this situation, the community is able to move from P to Q which is on a higher community indifference curve. Thus the relative production of X_1 and X_2 could be altered in such a way as to increase consumers' well-being. The optimum direction of production can be reached only if the marginal rate of transformation for the community between two products is the same as the marginal rate of substitution between two products is the same for a community, or

$$(5.8) \quad CMRT_{x_1x_1} = CMRS_{x_1x_2}$$

where $CMRT_{x_1x_2}$ stands for community marginal rate of transformation and $CMRS_{x_1x_2}$ stands for the community marginal rate of substitution (or, what amounts to the same thing, the marginal rate of substitution of X_1 for X_2 for each consumer in the community).

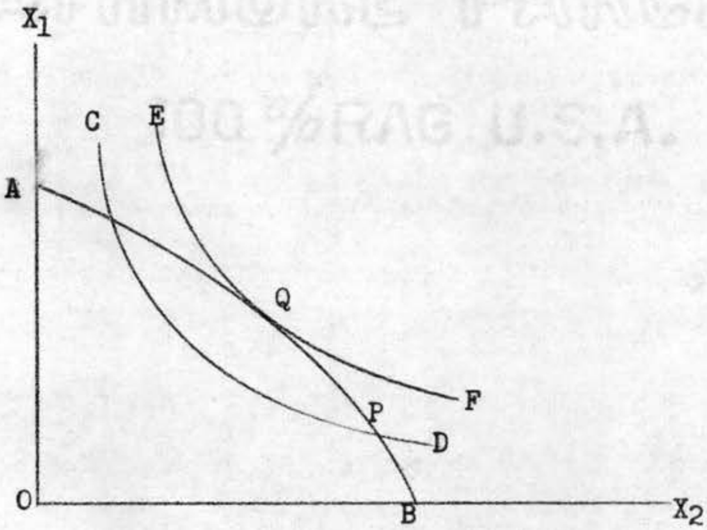

Figure 16

Evaluation

Since modern welfare economic analysis centers around the conditions of maximum welfare, indifference analysis plays an important role in that field. It is generally accepted that the application of indifference curve has special convenience in the analysis of those marginal conditions which we have discussed. This is the merit of indifference curve technique in welfare economic analysis.

The shortcomings of indifference technique in welfare economic analysis, however, should be also noted. In the analysis of optimum allocations of products among consumers, the contract curve represents a possible optimum condition. But which point is the "optimum" optimum? There should be a point on the contract curve which represents the "optimum optimum". This point cannot be shown by the box analysis. It involves interpersonal comparison of satisfaction. Further, since the optimum direction of production will occur when a community indifference curve is tangent to a community transformation curve, the consideration arises again. The establishing of a community indifference curve presupposes interpersonal comparison of satisfaction also.

The device of isoquants, iso-resources, and factor-product relationship analysis in welfare economic analysis is more concrete than indifference curve analysis for consumers. The production function and the transformation function are free from subjective personal judgements. Thus the construction of a community transformation curve is obtainable.

Nevertheless, the techniques have aided in the development of welfare economics:

> ... it is only a slight exaggeration to claim that modern welfare economics has developed largely as a result of the invention of this powerful device.[5]

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[5]Boulding, "Welfare Economics," p. 12.

CHAPTER VI

CONCLUSION

Indifference curve technique was devised by Francis Y. Edgeworth in 1881 from his attempts to generalize the utility function. After Edgeworth, the works of Auspitz and Lieben, Irving Fisher, Vilfredo Pareto, W. E. Johnson, and Eugen Slutsky gradually perfected and extended the application of the technique. However, it was not considered a particularly useful tool until it was resurrected by John R. Hicks and R. G. D. Allen in "A Reconsideration of the Theory of Value" which was published in 1934. Since that date, indifference curve techniques have been used extensively in the fields of consumer behavior analysis, production analysis, and welfare economic analysis.

Indifference curve analysis illuminates some dark corners in consumer behavior analysis. One merit of it is the appeal of the underlying assumption, a given scale of preferences. This enables economists to discard the assumption of measurable utility and therefore frees the analysis from psychological and philosophical assumptions. Moreover, it furnishes a means of analyzing inferior goods and facilitates a more general analysis of demand theory. Within limitations, it aids in the analysis of complementarity and substitutability. It also provides a means of analyzing and separating income and substitution effects of price changes of one commodity. It has some merit because indifference curves show some possibility of being derived from empirical data.

A more penetrating and comprehensive investigation of production is possible when the indifference curve technique is used. The iso-quant technique shows factor-factor relationship. It furnishes a more simple and general way of obtaining minimum cost conditions. The expansion path indicates the most efficient path the producer can take as he expands or contracts his output. The iso-resource curve technique furnishes a means of analyzing the production relationship between two products. It throws some light on the nature of technical complementarity in the production of two or more products. The technique is valuable in analyzing production behavior from different points of view--the exposition of factor-factor relationship and the product-product relationship.

The application of indifference curve analysis to welfare economics is one of the technique's broader uses. It has some convenience in analyzing primary marginal conditions of "maximum welfare". The optimum allocation of two products among two consumers can be shown by the box analysis. The contract curve shows a possible optimum condition though the "optimum optimum" cannot be shown on it. The determination of the "optimum optimum" involves interpersonal comparison of satisfaction. The optimum allocation of two factors between two producers can be shown also by the box analysis. The tangency points of the two sets of isoquants indicate the possible optimum allocations. The optimum factor-product relationship requires the equality of the marginal productivities of a production factor between producers. The optimum degree of specialization will be achieved if the marginal rate of transformation between two products for one producer is equal to that for the other. This can be shown as the tangent point of transformation curves for two producers. The optimum direction of production can be indicated by the tangency of a community transformation curve to a community indifference curve. The shortcoming of this device is that the establishing of a set of community indifference curves presupposes interpersonal comparisons of satisfaction. However, indifference curve techniques did aid in the development of welfare economics.

Since Edgeworth's Mathematical Psychics in 1881, indifference curve techniques have become a useful and powerful tool in microeconomic analysis.

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