VOLATILITY MODELING USING HIGH FREQUENCY TRADE DATA TO IDENTIFY CRYPTOCURRENCY BUBBLES

By

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Abstract

In the light of sudden interest in Bitcoin during 2017, which saw Bitcoin growing multifold in market price, I study blockchain, Bitcoin and few other top cryptocurrencies, and examine whether Bitcoin was in a financial bubble during late 2017 and early 2018, when Bitcoin price had a sudden and dramatic run up. I conduct statistical analysis on the High Frequency Trade (HFT) data, sourced from Bloomberg and other crypto exchanges. The statistical analysis includes filtering in price data using 5%, 7% and 10% daily price jump rules (considered separately), interpolating price points between low and high prices in the time series, estimating price volatility at discrete price points, analyzing the volatility behavior and concluding whether or not price process is a strict local martingale. A bubble is confirmed if the price process is a strict local martingale, and not a true martingale. I run the test for Bitcoin, and find that Bitcoin was in intermittently in bubble during the years 2017 and 2018. I repeat the test for Ethereum, another top trading cryptocurrency, and find that Ethereum was in bubble during Nov 2017 – Feb 2018, but infrequently and for lower duration of days as compared to Bitcoin. Though Bitcoin price dramatically increased during 2017, the number of transactions and transactions volume rather fell. I develop a statistical test that can be applied on the High Frequency Trade (HFT) data of any highly traded asset to identify whether or not that asset has been in bubble during the period of consideration. I also find that Bitcoin has positive correlation with other top cryptocurrencies and almost zero correlation with S&P 500, gold, and the crude oil.

JEL Classification: C14, C58, G12, G14, G17

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CHAPTER 1

Introduction

Cryptocurrency has generated significant interest in not just investors and the financial community, but also among common public. Bitcoin, in particular, has come up strongly as an alternate source of payment. With market price of $3,955 and market capitalization of $69.65 billion (as on 25-Mar-2019), Bitcoin has also become a force to reckon with in capital markets. Bitcoin is the first ever cryptocurrency, invented in 2008 by anonymous inventor Satoshi Nakamoto, and remains the largest of all cryptocurrencies. The underlying technology of Bitcoin is blockchain: a new revolutionary technology, which has the potential to disrupt ongoing processes in many industries. Unlike traditional or 'fiat' currency, Bitcoin is neither printed nor backed by any government. New Bitcoins are awarded by the Bitcoin blockchain system to the programmers (also known as 'miners') who verify Bitcoin transactions (in form of mathematical puzzle). Bitcoin award (mining) rate is constant for every 10 minutes, and halves every four years, making the final supply of Bitcoin limited to (slightly less than) 21 million tokens. Bitcoin's value is derived from the savings in transaction that results due to using Bitcoin for payment, rather than making payment through fiat currencies (as monitoring systems are not required). Bitcoin is especially useful in making international transactions. In addition, Bitcoin derives some value from its privacy feature. Bitcoin transactions are safe and virtually impenetrable. This led to a hope that majority of financial transactions in future may be through Bitcoin, thus leading to an understanding that Bitcoin will rise in value as more Bitcoin transactions happen. This resulted in widespread speculation in Bitcoin, resulting in sudden and swift rise in its market price, especially during the latter half of 2017. Due to herd mentality, retail investors also fueled this sudden euphoria. After a steep rise in market price in December 2017, Bitcoin price again started falling in 2018. This phenomenon makes for an interesting case for study, and has some underlying questions like: Is rise in Bitcoin value based on some fundamentals or is it merely because of investors' herd mentality? What drives the fundamental value of Bitcoin? Is there a statistical test that can identify whether Bitcoin was in bubble or not? If Bitcoin was in bubble, what was the duration of the bubble? These are some of the questions I try to answer in this research.

I do a deep analysis on blockchain, Bitcoin and several other top traded cryptocurrencies. I run a statistical test on the HFT (High Frequency Trade) data of Bitcoin during 2016-18 to identify bubble in Bitcoin during this period. I repeat the procedure for Ethereum, another cryptocurrency that has the largest market cap after Bitcoin, and try to identify whether Ethereum was also in bubble during 2016-18. I consider each year during the period separately in my analysis.

1.1 Motivation

Since its introduction to the world in 2008, Bitcoin has spawned a large number of cryptocurrencies, and thus has created a legacy of its own. During this period, enormous interest has been generated in not only Bitcoin and other cryptocurrencies, but also in Blockchain – the underlying technology that has the potential to disrupt processes in many industries. Cryptocurrencies like Bitcoin are 'virtual' currencies and can be used for transactions; they are also traded on dedicated crypto exchanges, with Bitcoin leading the pack in the trading volume. The interest of investor community, both retail and institutional alike, is high in those cryptocurrencies that have suddenly risen up in market price. Bitcoin showed a remarkable run up in price since 2016, especially in the latter half of 2017. This run up in price was so sudden and swift that many investors who held Bitcoin termed it as a 'lifetime opportunity'. However, Bitcoin price fell consistently and significantly, till it fell by 75% from its all-time high (Dec-2017). When invented and released for the first time in 2009 (3-Jan-2009), Bitcoin value was merely few cents. There was a famous incidence of someone paying for a pizza using a few Bitcoins in 2009! While price of Bitcoin was a mere few cents in 2009, in 2017 it rose to few thousand dollars. From a few cents to few thousand dollars, Bitcoin came a long way in just 9 years. Majority of this run up was witnessed in 2017 itself, that too during the quarter Sep-Dec 2017. As reflected in Table 1, which shows Bitcoin price milestones, the cryptocurrency was moving ahead sluggishly till 2016, after which it accelerated suddenly. Bitcoin ran up so quickly during the fourth quarter of 2017 that it reminisced of the mad rush during the e-commerce boom of the late 1990s. Table 1 shows the Bitcoin price milestones from $1,000 to $20,000.

[Insert Table 1 here]

Figure 1 shows Bitcoin's daily price movement during 2016-18. There was a dramatic increase in Bitcoin price during the last quarter of 2017, followed by a consistent fall in 2018, especially after the first quarter of that year. Bitcoin price rise during 2017 was so significant that the Bitcoin price increased more than tenfold in the year 2017 alone.

[Insert Figure 1 here]

Along with Bitcoin, Ethereum also increased in price, almost concurrently. Figure 2 shows Ethereum's price movement during 2016-18. Its price chart is similar to Bitcoin's price chart, with dramatic rise in price by the end of 2017, and fall in the subsequent year.

[Insert Figure 2 here]

While some financial analysts were deeming it as a classic case of bubble, some were of the view that the world has suddenly discovered a new technology and mode of payment, which has strong potential to overshadow traditional mode of payment (currency and cards), and which is causing such an unheard outburst in Bitcoin price. This motivated me to conduct a study that aims to identify whether there indeed was a financial bubble in Bitcoin, and other (top) cryptocurrencies during 2017 and early 2018. I conduct a statistical analysis of price data during 2016-18 for Bitcoin and Ethereum. Along with that, I also do an in-depth analysis of blockchain, a revolutionary technology on which all cryptocurrencies are based.

1.2 Key research questions

This study focuses on the following research questions:

- Is there a statistical test to analyze and identify whether Bitcoin was in a bubble during the last quarter of 2017, when it showed dramatic increase in market price?
- Were other crypto currencies were also in bubble during this period?
- If Bitcoin was in bubble during 2017, when did it enter the bubble territory? Also, was there a bubble in Bitcoin in the preceding year (2016) and the subsequent year (2018)?
- Do the top trading cryptocurrencies move in tandem? What is the correlation between their prices?
- What is Bitcoin's correlation with various market indices and with other assets like gold, crude oil?

CHAPTER 2

Blockchain, Bitcoin and other cryptocurrencies

2.1 Blockchain

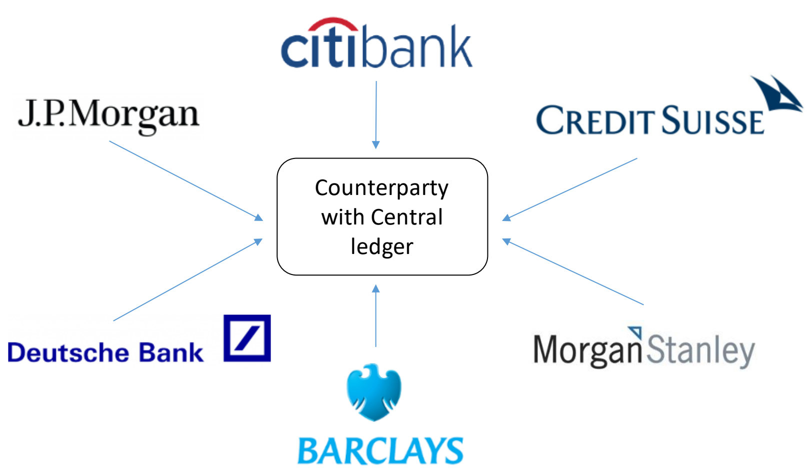
Blockchain is the underlying technology of cryptocurrency like Bitcoin. A blockchain (i.e. a 'chain of blocks') is a distributed ledger technology, and consists of database that stores transaction information in ledgers, in the form of distributed blocks. A block is a ledger, or container of data / information, which is created on regular intervals. Blocks are arranged in form of a chain, in which each successive block holds the address of the preceding block. Each new block references the preceding block, thus sequentially forming a chain of cryptographically linked transaction bundles, or blocks (Pilkington (2015), Perlman (2019)). A blockchain allows a decentralized network of economic agents to agree, at regular intervals, about the true state of shared data. This shared data can represent exchanges of currency, intellectual property, equity, information or other types of contracts and digital assets - making blockchain a general-purpose technology that can be used to trade scarce, digital property rights and create novel types of digital platforms (Cataliani and Gans (2017)). At basic level, a blockchain enables a community of users to record transactions in a shared ledger *within* that community. Each transaction is verified by the users in that community through a mechanism called 'consensus'. After verification, the transaction is published in a ledger on the blockchain platform, and cannot be changed once it is published (Perlman (2019)).

Blockchain is replicated over a peer-to-peer (P2P) network that enables multiple parties to share and modify the database in a safe and secure way even if they are unknown to each other. It is based on Distributed Ledger Technology (DLT): the blockchain consists of ledgers on distributed systems (Hileman and Rauchs

(2017)). Blockchain enables the transfer of digital files, and thus transfer of assets and other data, without relying on a Central authority. This enables network participants to independently verify the integrity of the shared database without having to rely on a trusted third party. Blockchain system is a decentralized platform on which network of computers located across the globe are linked together. These computers verify each other’s work - anyone can run programs on them - and users can pay for only what they wish to use (Pilkington (2015)). The potential Blockchain applications find use in industry processes like payments, mining, asset ownership, insurance claims, intellectual property, regular technology services, Internet of Things (IoT) integration etc. Blockchain technology has a potential to disrupt traditional industries (Hileman and Rauchs (2017)).

In traditional transactions, a neutral and trusted central agency maintains a central ledger and ensures the veracity of transactions and trust among transaction parties by tracking the movement and ownership of value. This is illustrated in the figure A, which shows traditional financial transactions process among the financial institutions.

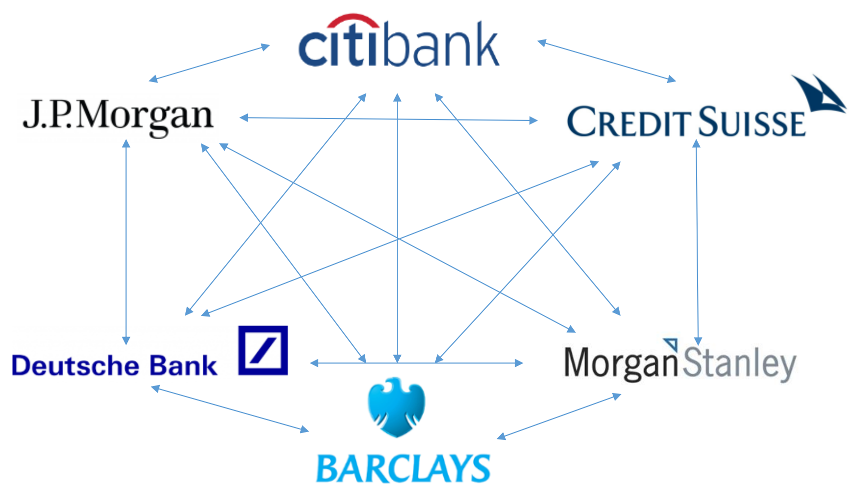
Figure A: Traditional financial transactions



[Source: Hileman and Rauchs (2017)]

Blockchain obviates the need for the central agency. The trusted third party is supplanted by the implementation of a shared public database, alteration of which requires consensus of all participants. A secure distributed ledger removes the counterparty, as the transaction record is universally visible and immutable. This is shown in the figure B, which shows financial transactions process among financial institutions on blockchain.

Figure B: Blockchain financial transactions



[Source: Hileman and Rauchs (2017)]

There are five essential components of blockchain (Hileman and Rauchs (2017), Mahmoud, Lescisin and AlTaei (2019)):

a) *Cryptography*: Use of a variety of cryptographic techniques including cryptographic hash functions, Merkle trees and public key infrastructure (private-public key pairs).

b) *P2P network*: Network for peer discovery and data sharing in a peer-to-peer fashion.

c) *Consensus mechanism*: Algorithm that determines the ordering of transactions to maintain trust among participants.

d) *Ledger*: List of transactions bundled together in cryptographically linked ‘blocks’.

e) *Validity Rules*: Common set of rules of the network that validate transactions and update ledger.

A blockchain network is defined by nodes and mesh that connects various nodes in the network. Transactions that result in transfer of assets are recorded in ledgers. The history of business transaction is visible to each node through the shared ledger technology. There is a common view of transaction history across the entire network. Blockchain uses consensus method to commit transactions to the system, implying that all parties must give consensus before a new transaction is added to the system. Each network member (on a node) has a copy of the same ledger, so asset provenance and traceability are transparent and trusted. The resulting process eliminates paper work, is quicker and efficient. The shared ledger is single, transparent and tamper-proof (Harvey (2016)).

Building blockchain involves following steps:

a) Verify each transaction, as it occurs.

b) After verification, put each transaction in a block.

c) After transactions are blocked together, connect each block to the one and after it by linking addresses (through techniques like linked lists and pointers).

d) Each block is added to the next in an irreversible chain.

2.2 Cryptocurrency

Harvey (2016) defines cryptocurrency as a digital system of non-physical tokens, which have an ascribed value. A cryptocurrency is a digital asset designed to work as a medium of exchange using cryptography to secure transactions, to control the creation of additional value units, and to verify the transfer of assets (Hardle, Harvey and Reule (2019)). Unlike a physical 'fiat currency', which is printed on paper and legally guaranteed by the government, a cryptocurrency is neither printed on pieces of paper, nor is it created by any Central Bank or government of any country. Rather, it is created by a cryptocurrency system, which works on the method of cryptography, at a rate that is defined when this system is created (Nakamoto (2008)). Many different cryptocurrencies exist, each with their own set of rules. Differences among the cryptocurrencies may involve, for example, the choice of the consensus mechanism, the latency, or the cryptographic hashing algorithms (Hardle, Harvey and Reule (2019)). The highest trading cryptocurrencies are Bitcoin, Ethereum, Bitcoin Cash and Ripple. Table 2 shows top ten cryptocurrencies by trading volume and market capitalization, and their relative proportion.

[Insert Table 2 here]

Blockchain is the underlying technology of a cryptocurrency. Hence, the key tenets of a cryptocurrency are derived from features of blockchain: cryptography, peer-to-peer information sharing, shared ledger, no central agency (rather, other users verify and authorize transactions), consensus mechanism and validation rules. Along with that, a cryptocurrency will also have digital purse where all tokens, i.e. coins will be kept. A cryptocurrency coin is not in a physical form, but rather at online, on a computer. Parham (2017) describes the process of transactions using a cryptocurrency:

"At its core, a cryptocurrency network is a distributed ledger - a type of public database that is shared, replicated, and synchronized among the members of a peer-to-peer computer network. The distributed ledger records transactions among network participants, thus keeping track of asset ownership. Every participant in the network has a unique identifier, known as an address. One person or entity may create and use many addresses. The ledger, hence, is just a list of valid transactions between addresses. The validity of the transaction is verified before it is posted, by making sure that on date ddd at time ttt, address xxx did indeed have ccc tokens of cryptocurrency, and that the (anonymous) owner of address xxx is really the one who issued the transfer order. Authenticating the transaction issuer is achieved using a cryptographic mechanism known as public-key cryptography. When an address is created for the cryptocurrency, the owner of the address also creates two keys, a public-key, which is disseminated widely, and a private-key, which is known only to the owner. When the owner issues a transaction request, they

'sign' it by attaching an encrypted version of the transaction request, using their private-key for the encryption process. An important feature of public-key cryptography is that anyone holding the public-key can decipher this signature and verify it matches the transaction it is signing. Because only the account owner has the private-key, this signature verifies they are indeed the ones who issued the transaction".

Published research in the area of Bitcoin and cryptocurrencies has focused on providing an overview of Bitcoin and its operations (Yermack (2013), Bohme, Christian, Edelman and Moore (2015)). Athey, Parashkevov, Sarukkai, and Xia (2016) combine theory and data to explain the velocity of Bitcoin and its use across countries as an investment vehicle, for gambling and illegal online markets. Because of its anonymous nature, Bitcoin has use in anonymous investments in stock markets as well. Governments across the world are bringing regulations to put control on unauthorized use of cryptocurrencies in such payment channels. Halaburda and Gandal (2014) examine competition between alternative cryptocurrencies and their differences. They find that when Bitcoin increases in value in the US dollar terms, it gains in value against other cryptocurrencies as well. Gans and Halaburda (2013) study developments in digital currency focusing on platform-sponsored currencies such as Facebook Credits. The authors find that it will not likely be profitable for such currencies to expand to become fully-convertible competitors to state-sponsored currencies. Halaburda and Sarvary (2015) point out that cryptocurrencies increase transparency by keeping the record of money transfer and payments in ledgers that are available for viewing to all market participants. Malinova and Park (2016) study the changes cryptocurrencies entail for trading behavior. The authors argue that since blockchain enhances transparency through ledgers, the implementation of blockchain technology in financial markets offers investors new options for managing the degree of transparency of their holdings and their trading intentions. Raskin and Yermack (2016) and Seretakis (2017) study the integration of cryptocurrencies with fiat-based currencies and argue that cryptocurrencies can have profound impact on the banking system and can narrow the relationship between the central banks and citizens. Rysman and Schuh (2016) study the direct use for providing citizens with central bank money and alternative payment systems. They study innovations in payments through mobile payments, faster payments and digital payments. Wright and De Filippi (2015) and Davidson, De Filippi, and Potts (2016), study implications of blockchain and cryptocurrencies for regulation and governance. Wright and De Filippi (2015) say that widespread deployment of blockchain will lead to rules administered through self-executing smart contracts and decentralized (autonomous) organizations, and will shift the balance of power away from centralized authorities in the field of communications, business, and even politics or law. This may create corporate governance issues. Davidson, De Filippi, and Potts (2016) view blockchain as new institutional technology of governance that competes with other economic institutions of capitalism, namely firms, markets, networks, and even governments. Athey, Catalini, and Tucker (2017) study the privacy trade-offs cryptocurrencies and digital wallets introduce for consumers.

2.3 Bitcoin: A Payment System

Bitcoin is a blockchain application, is independent of any central authority, and is based on an open-source peer-to-peer protocol, where all users interact and transact with each other directly. It is a digital currency and was designed as a payment system (Harvey (2016)). Bitcoin relies on cryptography to secure and validate transactions, which are pseudonymous and decentralized. In a transaction, Bitcoin can be used as a currency in exchange for goods and services. It is easily portable, divisible and irreversible. It also increases system efficiency by obviating the use of various system checks at each transaction node (Hileman and Rauchs (2017)). Through a clever combination of cryptography and game theory, the Bitcoin blockchain (a distributed, public transaction ledger) can be used by any participant in the network to verify and settle transactions. The Bitcoin protocol was first described by Satoshi Nakamoto (a pseudonym) in 2008. Nakamoto (2008) proposed Bitcoin to address an economic problem inherent in electronic commerce: the frictions and the high transaction costs of trading over the internet, particularly relevant for small-value transactions. While the key innovation in Nakamoto's paper is cryptography and computer science, it is 'economics' and the 'theory of money' that the paper impacts heavily. The concept of Bitcoin further fueled a debate on usage of cryptocurrencies, and increased awareness on it. Cryptocurrencies like Bitcoin can be used in not only payment for the financial transactions, but also they are traded like stocks on dedicated cryptocurrency exchanges. Payments are made through tokens, which are essentially digital assets that are tradable and fungible (Perlman (2019). These tokens can be transferred on across the network and can be traded on cryptocurrency exchanges, on which the investor can buy and sell cryptocurrencies like Bitcoin in fractions. In contrast, the common stocks that can be traded only in whole numbers in a stock exchange.

Each Bitcoin is divided into 100 million smaller units called *satoshis*. Bitcoin transactions are verified by programmers called ‘miners’, who are rewarded for their efforts in form of new Bitcoins, when a transaction is successfully verified and authorized by them. This process of transaction verification by other programmers, who can also be the users on the Bitcoin system, is an innovation, since there is no trusted central authority to verify and authorize the transactions. Nakamoto (2008) defines ‘mining’ as the process of transaction verification and getting awarded in lieu of that. It involves finding the next valid block to be placed on the blockchain. The objective is to competitively solve computationally difficult problems in order to limit the rate at which new blocks are created. Nodes on a blockchain network are incentivized to participate in mining as if they are the first to obtain a valid block, the distributed network rewards them with an award of cryptocurrency. This award may be in the form of new tokens, also known as a block reward or it may be collected from transaction fees. Most cryptocurrencies are designed so that all coins in circulation are generated from block rewards and once all coins have been generated, miners are then incentivized through transaction fee rewards (Mahmoud, Lescisin and AlTaei (2019)).

In case of Bitcoin, the block reward, which is paid using freshly-mined Bitcoin, is halved every 210,000 blocks – roughly every 4 years. At the inception of Bitcoin (BTC), the initial award rate was 50 BTC every 10 minutes. This has come down to 12.5 BTC during 2016-20. Therefore, the reward rate progresses in a diminishing Geometric Progression in a block of 4 years. By 2032, over 99% of all Bitcoins will have been mined. By 2048, the total reward paid to miners for mining a Bitcoin block will drop to 0.05 BTC. This is down from the initial reward of 50 BTC upon Bitcoin’s inception. Table 3 shows Bitcoin blocks and year wise number of Bitcoins issued.

[Insert Table 3 here]

2.4 Early history of Bitcoin

In its early years, Bitcoin was known to a relatively narrow community of cryptography enthusiasts. The first time the currency made it into the mainstream media was probably in June 2011, when WikiLeaks started accepting donations in Bitcoin from its supporters (Halaburda and Sarvary (2015)). WikiLeaks started accepting donations in Bitcoin, while highlighting the flexibility of the currency, its anonymity and independence from traditional financial providers. By 2013, Bitcoin started appearing to be an increasing speculative investment opportunity (Parham (2017)). Its price (i.e. exchange rate to the US dollar) increased from under $15 in Jan-2013 to over $1,200 in Dec-2013. During this time, Bitcoin also started gaining foothold in electronic commerce, when the Chinese search engine *Baidu* (world's 5th most visited site at the time) started accepting Bitcoin for payments. However, restrictions were put by the US government on digital currencies when it was revealed that Bitcoin was being used for payments in the illicit activities like drug trade by illegal websites like *Silk Road*. FBI raided the offices of this website and seized over 26,000 Bitcoins from there. Subsequently, the Chinese website *Baidu* also stopped accepting Bitcoins. In 2011, Japan-based Mt. Gox, then the largest Bitcoin exchange, experienced a security breach in which 850,000 Bitcoins worth approximately $450 million were stolen. As digital signature of a Bitcoin purse is nearly impossible to crack using brute force method, it happened only because the digital signature, or the password was known to someone who was involved in the incidence (Yermack (2013)).

Bitcoin started gaining popularity as it was touted as an instantaneous and anonymous way to make transactions, defying national boundaries, with no central bank and country as authority. Because of its anonymous nature, Bitcoins have been used in past in the criminal money laundering and tax evasion schemes (Nabilou (2019)). Chohan (2019) analyzes the growth of gambling activities on blockchain and finds that the risk of illicit activities on the Bitcoin blockchain far outweighs the recreational aspect of gambling. During its initial days, Bitcoin was increasingly perceived as a quicker and cheaper alternative to existing money, to be used in peer-to-peer transactions, international transfers, etc. Its anonymous nature and ease of online transfer made Bitcoin a prominent method of illegal money transfer. Cryprocurrencies remain one of the largest unregulated markets in the world. Foley, Karlsen and Putnins (2019) find that approximately one-quarter of Bitcoin users are involved in illegal activity. The authors estimate that in 2018, around $76 billion of illegal activity per year involved Bitcoin (equivalent to approximately 46% of bitcoin transactions). This figure is close to the scale of the U.S. and European markets for illegal drugs. However, the authors also mention that the illegal share of Bitcoin activity declines with mainstream interest in Bitcoin and with the emergence of more opaque cryptocurrencies. In recent times, some research studies have proposed increased regulations on Bitcoin and other cryptocurrencies. Since cryptocurrencies are decentralized, Nabilou (2019) argues that instead of regulating the technology or the cryptocurrencies at the code or protocol layer, the regulation should target the applications and use-cases of cryptocurrencies.

Bitcoin transactions are now not completely anonymous, and are increasingly regulated (Parham (2017)). In addition, the underlying blockchain also has complete record of the path of all the addresses Bitcoin was sent to. The record of all prior transactions is stored in the ledger, which is transparent to all Bitcoin users. Also, Bitcoin transactions are not free, as Bitcoin miners, who verify and authorize a transaction, are paid a fee for their services. This fee has remained small in past years, in tune of ~0.0001 BTC per transaction (Parham (2017)). Therefore, the main reward of Bitcoin mining remains newly issued Bitcoins. However, as supply of new Bitcoins will diminish with time, the future transaction fees will be determined by the competitive forces of supply and demand: the supply of the computing power on the side of miners, and the demand for transaction verification on the side of Bitcoin buyers and sellers (Yermack (2013)).

2.5 Bitcoin architecture

Bitcoin architecture is built on cryptography, and is underpinned by the cryptographic hash functions and digital signatures to operate the digital purses.

2.5.1 Hash Functions

A hash function is computation that transforms input data of any size, to output data of a fixed size. The input message can be any sort of data (text, character strings, binary etc.), of any length. A hash is the output of a hash function and the hash rate is the speed at which a compute is completing an operation in the Bitcoin code (Nakamoto (2008)). Bitcoin uses the *SHA-256 hash algorithm* to generate verifiably 'random' numbers in a way that requires a predictable amount of CPU effort. Bitcoin mining network's processing power is measured in hash rate. A higher hash rate is better when mining as it increases the chances of finding the next block and receiving the reward. In order for miners to confirm transactions and secure the blockchain, the hardware they use must perform intensive computational operations, which is output in hashes per second. Figure 2 shows the increasing trend of hash rate in last 2 years, given the increase in mining activity and Bitcoin award per block remaining same.

[Insert Figure 3 here]

2.5.2 Cryptographic Hash function

The cryptographic power of complex hash functions is that given a single output, it is hard to determine the input. A hash function is any computation that transforms input data of any size to output data of a fixed size. The Bitcoin blockchain encryption is virtually impenetrable and extremely secure[1]. Even if some

[1] A Cryptographic Hash Function produces output of similar length, irrespective of the input length. The input may be a word, a character string, a lengthy text, or a mix of numbers and characters. Irrespective of the input length, the output hash will be of similar size. Cryptographic hash function has a unique algorithm, which compresses the input string(s) and produces output of uniform length, blocks and size. It is nearly impossible to invert a hash (output) to its actual input. Also, no two different inputs produce the same hash (output) – a process known as 'collision'. The cryptographic hash functions are extremely safe and secure to use, and are nearly impossible to be hacked.

hacking entity is able to commandeer the processing power of the entire Bitcoin network, approximately 1.4 billion Gigahash per second, a brute force approach to finding a collision in the NSA developed SHA256 would take 1.33 x 10^{51} years. To put that in perspective, our universe is approximately 13.7 billion years old, so it would take 9,672,989,162 trillion times the life of our universe to find a collision: an impossibility! (Hileman and Rauchs (2017))

2.5.3 Digital Signatures

Signatures are generated from a hash of data to be signed, and are a private key. Digital signatures are a derivation of public-key cryptography that uses a pair of keys to ensure the integrity and provenance of messages. A message is bundled with a ‘private key’ known only to the sender; anyone with access to the sender's paired ‘public key’ may then authenticate the message (Pilkington (2015)). There are risks associated with using digital purse. A Bitcoin holder can lose Bitcoins if the private key, i.e. the digital signature to the digital purse is lost. This is akin to losing password to a mailbox or a website. However, the access to digital purse and Bitcoins can be regained after a system identification process. There is also a slight chance of digital purse being hacked. Though it would be highly improbable using the brute force method, it is still doable if the hacker knows the private key.

2.6 Bitcoin valuation

As Bitcoin gains acceptance in international payment community, it has potential to expand in number of transactions. Also, the total number of Bitcoins are finite and limited in number (unlike fiat currencies, which can be printed on requirement by a central government). In addition, Bitcoin is based on the properties of Mathematics, rather than relying on physical properties (like gold and silver) or trust in central authorities, whom the fiat currencies rely on. There are no underlying cash flows in case of a cryptocurrency (as cryptocurrency is a substitute of cash). Therefore, the Discounted Cash Flow method cannot be applied for valuation of cryptocurrency. Rather, to value a cryptocurrency, the savings resulting from the use of cryptocurrency have to be considered and analyzed under risk-neutral measure. Bitcoin transactions do not require deployment of monitoring applications, and therefore lead to dollar savings. More are the number of transactions using Bitcoin rather than credit/ debit/ cash cards or paper checks, more will be savings on transactions. Bitcoin also has value due to ‘convenience yield’, since Bitcoin has much wider use as mode of payment, due to its anonymous and decentralized nature. Like other currencies and commodities, the value of Bitcoin fluctuates wildly based on supply and demand and its perceived value. To give the current value of Bitcoin some context, according to the World Gold Council, there are 187,200 tons of gold that have been mined throughout history, with around two-thirds of that being mined since 1950. Based on a current gold price of $1300/oz. (30-April-2019), the global stock of gold is valued at $7,875 billion. In comparison, at the Bitcoin price of $5,267 (30-April-2019), and the 17.6 million Bitcoin in circulation, Bitcoin’s total value, i.e. market capitalization, is approximately $92.7 billion. At its peak price of $20,000, Bitcoin had a total market capitalization of approximately $332 billion.

Harvey (2016) and Hileman and Rauchs (2017) suggest the following valuation methods to value Bitcoins:

1) First approach is to estimate the Bitcoin transactions as a percentage of total number of transactions across the globe. Multiply that with the total GDP of the world, and divide by number of Bitcoins in circulation. However, estimating total number of Bitcoins in circulation that are used in making transactions is often a challenge.
2) A second approach is to measure savings through Bitcoin transactions, and value Bitcoin accordingly. An estimate suggests that Bitcoin transaction results in savings of ~ $3 over traditional transaction.
3) Another approach could be to ascertain the cost of mining for a particular computing power. The price of Bitcoin should be at least equal to the cost of mining Bitcoin, else there is no use of mining. A general rule will be to pursue Bitcoin mining if the cost of mining is much lesser than the Bitcoin’s ongoing market price.

This difficulty in Bitcoin valuation also leads to high volatility in Bitcoin price. The key question regarding Bitcoin volatility is whether high volatility is due to lack of collateral, or due to lack of liquidity. Because of such large volatility, it is difficult to use Bitcoin as store of value, since it carries short-term risk. Bitcoin's daily exchange rates exhibit virtually zero correlation with widely used currencies and with gold, making Bitcoin not very useful for risk management and difficult for its owners to hedge. However, Bitcoin has liquidity since it is readily convertible into US dollars on Bitcoin exchanges. Also, among the top cryptocurrencies, Bitcoin is least volatile. Table 4 shows the daily and annualized volatility of Bitcoin and other top cryptocurrencies during 2016-18. As shown in this table, Bitcoin's price volatility nearly doubled in 2017, compared to 2016. In 2018, Bitcoin remained volatile and its general price trend was downward, with volatility during the year just a bit lower as compared to that during 2017. I also find that price volatility of Ethereum, Ripple and Bitcoin is higher significantly as compared to Bitcoin.

[Insert Table 4 here]

The market capitalization of Bitcoin is defined as number of Bitcoin tokens in circulation multiplied with the unit price of Bitcoin. Figure 4 shows the Bitcoin market capitalization trend during 2016-18. I find a sudden spike in Bitcoin's market capitalization during the last quarter of 2017, after which there was a gradual fall. Similar trend in Bitcoin's market price and market capitalization (which is the product of market price and number of bitcoin's outstanding) can be attributed to the fact that the rate of new Bitcoins released in the system through mining, is constant in a block of 4 years. New coin release rate for Bitcoin has been constant since 2016 and shall remain so till 2020, after which this will again halve from the 2016-20 levels.

[Insert Figure 4 here]

Bhambhwani, Delikouras and Korniotis (2019) find a significant long-run relationship between cryptocurrencies' intrinsic value and blockchain trustworthiness and transaction benefits, as well as computing power and adoption level. This hypothesis is motivated by the fact that miners expend real resources (energy) to generate the computing power required to secure and operate the blockchain. Further, an optimally performing blockchain serves as a medium for transactions and attracts users, developers, and intermediaries, thereby allowing for an increase in the network size of the cryptocurrency. The authors test this hypothesis for Bitcoin, Ethereum, Litecoin, Monero and Dash. Using dynamic least square regressions, the authors find that, on average, there is a positive and statistically significant long-term relationship among the price of each cryptocurrency with its own fundamentals (computing power and network).

2.7 Bitcoin mining and transactions

Bitcoin transactions are verified by network of computer programmers called *miners*, who are paid for their efforts in form of new Bitcoin issuance. Bitcoin creation is called ‘mining’ because it is akin to mine precious metals. Like precious metals, Bitcoin is valuable because of difficulty of creation (i.e. extraction), and scarcity. When a block is successfully mined on the Bitcoin network, there is a block reward that helps incentivize miners to secure the network (Yermack (2013), Harvey (2016)). The block reward is part of a ‘coinbase’ transaction that may also include transaction fees. The block rewards halves roughly every four years (called ‘halving’). Bitcoins have a finite supply, which makes them scarce. The total amount that will ever be issued is 21 million. *Chainalysis*, a digital forensics company, estimates that somewhere between three and four million already may have been lost (for example, through irrecoverable passwords or people who accidentally threw away Bitcoin collections back when they weren’t worth very much). Bitcoin’s value stems primarily from its scarcity and public faith in Bitcoin as a store of value, means of payment or hedge against inflation. As Bitcoin supply remains limited, price volatility is bound to increase if demand rises and more investors start trading as the cryptocurrency becomes popular due to its investment returns and the increasing awareness of blockchain applications.

The number of Bitcoins generated per block is halved every four years. The final halving will take place in the year 2140. Bitcoins are divisible to eight decimal places. During 2016-20, new Bitcoins are being created at the rate of 12.5 Bitcoins every 10 minutes. This will halve to 6.25 every ten minutes after 2020. Ultimately, total supply of Bitcoins will approach (but will never be equal to) 21 million. Figure 5 shows the Bitcoin supply trend. There is a consistent rise in the supply of Bitcoin, as mining becomes a lucrative activity due to Bitcoin price rise, and new Bitcoins consistently being mined after successful verification of transactions.

[Insert Figure 5 here]

The Bitcoin miners also have to invest in expensive computing equipment and the power necessary to find the 'hash' that wins the block. The power of the Bitcoin mining network is measured in *hashing power*, which essentially tantamount to humongous number of operations per second. The Bitcoin mining network has extremely high power capacity that can sustain as high as 9.8 x 10^{21} operations per second (Hileman and Rauchs (2017)). A transaction is when data is sent to and from one Bitcoin address (i.e. wallet) to another. The computer that implements Bitcoin protocol (i.e. Bitcoin client) saves user's Bitcoins in a file called the wallet, which the user must secure and backup. A Bitcoin wallet is a digital wallet that stores, sends, and receives Bitcoins securely. These Bitcoin wallets connect to one another over the Internet forming peer-to-peer networks, making the system a distributed one resistant to central attack. New Bitcoins are issued to competing "miners" who use their computers to generate solutions to problems that help ensure the integrity and security of the system. As the number of miners in the network changes, the problem difficulty adjusts to ensure that Bitcoins are created at a predetermined rate and not faster or slower. As problem difficulty increases and many miners try to solve concurrently, more powerful systems are required to mine Bitcoins (Hileman and Rauchs (2017)).

2.8 Economics of Bitcoin mining

Bitcoin mining is the process of using computer hardware to do mathematical calculations for the Bitcoin network in order to confirm transactions. Miners collect transaction fees for the transactions they confirm and are awarded Bitcoins for each block they verify. During 2016-20, miners are currently compensated 12.5 BTC per block. Miner's revenue streams are contingent on 1) the value of Bitcoin, and 2) the reward era's BTC pay out per block. As the average transactions per block increases, so does the average block size in MB. The proof of work difficulty is adjusted by the system such that blocks are added roughly every 10 minutes. *Difficulty* is directly related to Bitcoin mining, and how hard it is to verify blocks in the Bitcoin network. Bitcoin adjusts the mining difficulty of verifying blocks every 2016 blocks. Difficulty is automatically adjusted to keep block verification times at ten minutes. The difficulty of mining is rising as more miners join the race to mine Bitcoin. Mishra, Jacob and Radhakrishnan (2017) estimate that every 1% increase in transaction volume leads to 1.015% increase in difficulty of POW (Proof of Work) algorithm for Bitcoin mining. This is because more miners try to verify the transactions as volume increases, and thus level of difficulty increase as Bitcoin issue rate remains constant. Figure 6 shows the difficulty level in last 2 years. Like hash rate, there has been a steady and continuous rise in difficulty level, as more miners attempt Bitcoin mining using higher computing power systems (so that they can solve complex mathematical problem to verify transaction), while Bitcoin award rate remains constant per block.

[Insert Figure 6 here]

Miners verify Bitcoin transactions and are awarded in terms of Bitcoin. The direct cost of Bitcoin transaction has averaged 0.013% of daily transaction volume (Hileman and Rauchs (2017)). These are discretionary fees intended to incentivize miners to include their transaction in the next block. The total fee to miners, i.e. compensation to miners for verifying transactions and running the blockchain is actually c. 1.3% of daily transaction value. In the past, it has been much higher, peaking at 8% in 2012, and 6% in mid-2014. 1.51% of the miners' revenue is earned through transaction fees (which miners earn after successfully verifying a Bitcoin transaction).

Chinese Yuan (Renminbi) transactions have dominated Bitcoin trading volume. Figure 7 shows the trend of average daily number of transactions during the period 2016-18. Figure 8 shows the average daily Bitcoin transaction value during the same time-period. Figure 9 shows the average transaction volume during the same time-period. The trend of number of transactions and the transaction volume shows that Bitcoin transactions are not increasing with time. The transaction value has increased with Bitcoin price and fallen with the same.

[Insert Figures 7, 8 & 9 here]

Bitcoin transaction cost is due to fee paid to miners, who verify transactions and are thus awarded in form of Bitcoin (Harvey (2016)). With transaction cost still under $150 and Bitcoin price, a few thousand dollars, mining is a profitable activity. Figure 10 shows cost per Bitcoin transaction. As the graph shows, the transaction cost moves in tandem with the Bitcoin price.

[Insert Figure 10 here]

As Bitcoin popularity and acceptance as a payment mechanism grows, the number of Bitcoin wallet users are growing. Figure 11 shows the number of Bitcoin wallet users during 2016-18. There is a consistent growth in number of Bitcoin wallet users, from approximately 5.66 million in Jan-2016 to approximately 31.54 million in Dec-2018.

[Insert Figure 11 here]

With hash rate and difficulty in mining increasing exponentially, miners have to employ extremely powerful computing systems to successfully verify the transactions and thus get awarded in terms of Bitcoin. These systems are highly power intensive. Thus, profitability in Bitcoin mining is contingent upon the cost of power consumption as well. Bitcoin mining remains a profitable activity until the time mining cost (due to power consumption) remains lower than the Bitcoin market price.

Bitcoin is enabled by a network of computers running Bitcoin mining software. This software consists of a copy of all past Bitcoin transactions in the form of a blockchain, and a program that connects to peers in the network and follows a set of rules to authenticate new transactions and add blocks of these to the chain. The Bitcoin blockchain is extremely efficient in terms of data usage. Figure 12 shows the size of the Bitcoin blockchain in recent past (c.150 GB in January 2018).

[Insert Figure 12 here]

2.9 Power Consumption

Bitcoin mining is a very power intensive activity, since very high capacity computing systems are required to verify transactions. Hence, power consumption is a significant component of the Bitcoin mining cost. Miners have incentive to use higher computing power in mining, since it increases their chances of verifying transactions and thus getting reward. Higher computing power leads to higher cost of electricity, hence miners increase computing power they run until their marginal cost of electricity equals their marginal revenue. This has several implications:

■ *Energy inefficiency*: The total hash rate (the number of brute force calculations the network can achieve) is 1,630,722,753 GH per second. Given a conservative estimate of MW:GH at 0.3, the total network power draw is c.500MW. The sophisticated mining devices are energy hungry and burn 0.8 – 2.0 kWh of electricity (Mishra, Jacob and Radhakrishnan (2017)).

■ *Race to the bottom*: Electricity prices exhibit a wide regional skew, and miners based in regions with lower electricity costs have a competitive advantage, leading to a high concentration of the hash rate in lower electricity cost locations.

The more lucrative the price of Bitcoin, the more people worldwide who connect to the network to mine the digital currency and earn the rewards. The Bitcoin network regularly needs to increase the difficulty of mining to allow for more mining capacity without overloading the network. Miners must always add more power to compete with other miners for the rewards. Therefore, Bitcoin's price is directly proportional to the amount of electricity that can profitably be used in the mining process.

Statistics from *Digiconomist.com* reveals that as Bitcoin broke the $9,000 mark for the first time, the Bitcoin mining network was using more electricity in a year than the whole of Ireland. At these levels, Bitcoin uses around 300 KWh of electricity. As a comparison, one of Visa's two data centers in the US runs on about 2% of the power that Bitcoin demands. Combined, Visa's two US data centers process approximately 200m transactions per day. On the other hand, Bitcoin handles less than 350,000 per day. Such comparisons raise serious questions around Bitcoin's long-term sustainability and environmental impact. At the current growth of power consumption by the Bitcoin miners, it is surmised that if such growth rate sustains, Bitcoin mining may consume more power than many countries in the world do. China dominates the mining market, with 60-85% of all Bitcoin network processing power coming from China mining pools. Cheap electricity and labor – alongside leadership in mining hardware – are the main factors contributing to China's dominance in the mining market. At its peak, Bitcoin was consuming around 1/300 of total global power supply (*Digiconomist.com* (2018)). Table 5 shows Bitcoin's power consumption statistics. Bitcoin mining became less lucrative for Bitcoin miners in 2018 as the Bitcoin price fell consistently. This led to reduction in Bitcoin mining, with annualized mining revenues (during past 12-months) earned by Bitcoin miners also falling from $5.4 billion to $2.4 billion from May-2018 to Dec-2018, and mining cost as a percentage of mining revenue increased from 65.4% to 95.6% during the same time. Total electricity consumption due to Bitcoin mining during this period also decreased by ~31.6%, from estimated 71.12 Tera Watt-hour to 47.65 Tera Watt-hour. By December-2018, Bitcoin mining was still consuming 1/500th of global power supply.

[Insert Table 5 here]

2.10 Bitcoin and financial markets

Cryptocurrencies are traded on dedicated exchanges that function 24/7, as opposed to traditional stock exchanges that function mostly on dedicated hours on the weekdays. Some of the top exchanges that trade Bitcoin and other cryptocurrencies are *Bitfinex, Bitstamp, Coinbase, HitBTC, itBit, Kraken, Gemini* etc. Table 6 shows top Bitcoin exchanges by volume.

[Insert Table 6 here]

At the onset of the year 2017, Bitcoin essentially <u>was</u> the cryptocurrency market. Since then, over 1600 different cryptocurrencies have been launched (by 2018), thus pushing Bitcoin's share of the total cryptocurrency down to a minority share at roughly 40%, despite the historic gains of the currency in 2017. The highest trading cryptocurrencies are Bitcoin, Ethereum, Bitcoin Cash and Ripple. Bitcoin remains a dominant cryptocurrency and represents the macro landscape of the cryptocurrency market. Ethereum is the second largest traded cryptocurrency after Bitcoin. Unlike Bitcoin, whose architecture is built on Proof-of-Work algorithm, Ethereum is more robust, and is essentially a platform that can host software applications also. The name of the cryptocurrency on Ethereum platform is called *Ether*. Table 7 shows the top Ethereum exchanges by volume.

[Insert Table 7 here]

Ripple and *Bitcoin Cash* are other top traded cryptocurrencies. While *Ripple* is a separate cryptocurrency, *Bitcoin Cash* was forked out of Bitcoin on 1-Aug-2017. Since cryptocurrencies are built on a software platform, it is possible to add new features to them. This process is called *forking*. After forking, Bitcoin Cash is traded separately on exchanges. Due to their high volatility, cryptocurrencies are often compared to stocks, rather than the traditional currencies. Yermack (2013) argues that Bitcoin does not behave much like a currency according to the criteria widely used by economists. Instead Bitcoin resembles a speculative investment similar to the Internet stocks of the late 1990s.

2.11 Bitcoin - Challenges and risks

Bitcoin attempts to overcome the weaknesses of both fiat and gold-based money, by functioning as an algorithmic currency with a deterministic supply and growth rate tied to the rigor of mathematics. No government or other central authority can manipulate the supply of Bitcoins. Instead the currency is governed by cryptographic rules that are enforced by transparent computer code in a decentralized manner. Nonetheless, Bitcoin faces challenges in meeting all three criteria of a successful currency, viz. medium of exchange, unit of account and store of value (Yermack (2013). Comparing with millions of transactions every day on the Visa network, Bitcoin has a mere thousands, due to its mathematical intensity of transaction verification. Moreover, very few merchants are registered with Bitcoin exchanges like *Coinbase.* For Bitcoin to establish itself as a bona fide currency, its value will need to become more stable so that it can reliably serve as a store of value and as a unit of account in commercial markets. The high volatility of Bitcoin and other cryptocurrencies is more consistent with the behavior of a speculative investment rather than a currency.

Bitcoin’s future and legitimacy as a currency would also hinge on its integration into the web of international payments and risk management transactions. Even though it is not issued by a sovereign state, Bitcoin imparts risk to any business that accepts it for transactions, just like all other currencies. Major companies that deal in more than one currency, such as multinationals, attempt to hedge themselves against risks related to changes in those currencies’ values. Yermack (2013) states that no effective way exists to hedge Bitcoin against the value of other currencies, and the absence of any swap, forward, or other derivative markets for Bitcoin exacerbates this problem. Bitcoin transactions also are risky due to the absence of basic consumer protection, such as the provision of refunds that result from disputes between merchants and customers. While local laws may provide ground rules for resolving such disputes, because a government has no legal way to foreclose and take possession of Bitcoins, it ultimately has little ability to step in and enforce its laws. Similar problems arise in attempting to secure consumer credit denominated in Bitcoin or to pledge Bitcoins as collateral for a consumer loan. Also, due to its lack of affiliation to any sovereign agency, Bitcoin is ill-suited for use in credit markets because no government can foreclose and seize it in order to recover the loss, in the unfortunate event of a default.

Bitcoin also appears to suffer by being disconnected from the banking and payment systems of the U.S. and other countries. Most currencies are held and transferred through bank accounts, which in turn are protected by layers of regulation, deposit insurance, and international treaties. Without access to this infrastructure, Bitcoin has proven vulnerable to fraud, theft, and subversion by skilled computer hackers. However, Bitcoin bypasses the well-known flaws in standard financial security systems, which have spawned epidemics of identity theft and related problems for ordinary customers of mainstream businesses (Parham (2017)).

CHAPTER 3

Financial Bubbles

"*….What I am proposing is that we try to identify bubbles in real time, try to develop tools to address those bubbles, try to use those tools when appropriate to limit the size of those bubbles, and therefore try to limit the damage when those bubbles burst*."

- William Dudley, President of the New York Federal Reserve in 2010

3.1 Introduction

'Financial bubbles' refers to large, sustained mispricing of financial or real assets, where the mispricing may last for an extended period. Bubbles occur in periods in which the price of an asset consistently exceeds fundamentals because investors believe that they can sell the asset at an even higher price to some other investor in the future. Bubbles are caused and sustained due to investors who buy assets due to their replacement prices rather than their fundamental value. While a mere mispricing does not necessarily mean that an asset is in a bubble, a market price consistently above the fundamental price, with large volume of trading (buying and selling) in the asset, suggests a bubble. Asset valuation in bubble periods is often explosive.

Mathematically, bubble β_t at time t is defined as: $\beta_t = S_t – S_t^*$ (Protter (2013))

where, S_t : market price, and S_t^* : fundamental price of the asset

Bubble exists if $\beta_t >> 0$ for a significant period of time. A strong bubble is when market price is much greater than the fundamental price for consistent length of time. The fundamental price is expressed as a conditional expectation of future cash flows at the risk neutral measure. The risk neutral measure is unique in a complete market (where transaction costs are limited, information flow is perfect and there is a price for every asset in every possible state of the world), whereas incomplete markets have infinite number of risk neutral measures. The fundamental value of asset is derived from underlying stream of cash flows that the asset generates. If this stream is volatile, then the underlying risk of the asset increases. The swift rise and fall in the market price of Bitcoin and other cryptocurrencies indicates volatility. The question arises whether this volatility is caused due to high deviation of cryptocurrency's market price from its fundamental price. If such is the case, then this makes the case for a bubble.

3.2 History of financial bubbles

Through the history, the global economy has seen number of financial bubbles (Protter (2013)):

(a) A bubble known as Tulipmania, which occurred in Amsterdam (circa 1634 – 1637), is the first documented bubble of the modern era. As tulips became a fad, some rare tulip bulbs that were obtained through hybrid and expensive techniques led to massive speculation in the prices of bulbs, which got inflated to extremely high levels, thus creating an economy-wide bubble. When the bubble burst, the investors lost significant part of their investment.

(b) In the eighteenth century, Banque Royale (Paris, circa 1716 – 1720) financed the crown's war debts by selling off notes, giving investors the rights to the gold that was yet to be discovered in the Louisiana territories (which was part of France then). When no gold was found there, the bubble collapsed and it led to an economic catastrophe. Subsequently, the public distrust of French banks lasted for almost a century after this episode.

(c) The South Sea Company of London (circa 1711 – 1720) sold the British investors the rights to the gold of Inca and Aztec civilizations in South America, notwithstanding the fact that Spanish controlled such trade and had command of the high seas at the time. As the British public realized this, the bubble collapsed. This led to British Parliament passing the 'Bubble Act of 1720', which among other things, forbade the issuance of stock certificates.

(d) The Japanese housing bubble (circa 1970–1989), which upon collapsing led to Japan's 'lost decade' – a decade characterized with a stagnant economy and 'zombie' banks.

(e) The United States has had the most prominent financial bubbles:

- The 1816 crash due to real estate speculation.
- Construction of the Erie Canal connecting New York to Chicago through inland waterways created "irrational exuberance" (in words of Alan Greenspan), which culminated in the Crash of 1837.
- A few decades later, another wave of irrational exuberance erupted due to the construction of the railroad system within the U.S. It culminated in the Panic of 1873.
- The Wall Street panic of October 1907 saw market falling by 50%. J.P. Morgan's prestige and personality helped save the economy and the banking system. Its aftermath created the atmosphere that led to the creation and development of the Federal Reserve in 1913, via the Glass–Owen bill.
- The Great Depression of 1929 began with a great bubble built by the Florida land speculation as people were lured to buy swamp land that was touted as beautiful waterfront property. This seamlessly moved into building a massive stock market bubble, which burst and brought the Great Crash of 1929, and subsequently the economic depression.
- The rise in 'junk bond financing' by Michael Milken created a bubble in the 1980s.
- In the late 1990s, the speculation built by the commercial promise of internet (e-commerce) led to swift rise in the stock market and e-commerce stocks in particular, and created a bubble, which

when burst, led to the 'dot-com crash'. The market downturn started from March 2000, and lasted until October 2002. During this period, Nasdaq Composite index lost 78% of its value.

– The housing bubble in the US, which was tied to subprime mortgages, started in early 2000s. It led to creation of many derivative securities like ABS, CDO, CDS etc., which were created by the process of securitization, i.e. pooling of assets (mortgage loans). When the bubble burst in 2008, it created a financial tsunami, which engulfed well-respected investment banks like Lehman Brothers and Bear Stearns. The crisis had immediate cascading effect, and spread far across the world, while resulting in bankruptcy in countries like Iceland and Dubai.

3.3 Causes and stages of financial bubbles

3.3.1 Causes of financial bubbles

The *theory of rational markets* (Camerer (1989)) states that all assets should trade at their fundamental or fair value, since rational investors and traders should buy an asset if it is undervalued and sell an asset if it is overvalued. This theory eliminates the possibility of a bubble. However, this theory does not hold always, and financial bubbles do occur because of actions of irrational investors. Contrary to the common "rational expectations" framework for bubbles, economists like Hyman Minsky, Charles Kindleberger and Robert Shiller have documented that irrational behavior, ambiguous information or certain limits to arbitrage are essential drivers for bubble phenomena and financial crises (Schatz and Sornette (2019)).

Galbraith (1993) states that bubbles often come into being when there is a *sudden euphoria* because of a big change. He mentions that speculation on a grand scale occurs when there is a new (or perceived as new), technological breakthrough. It can result in over-enthusiasm and uncontrolled speculation, thus resulting in asset pricing reaching very high and unsustainable levels. This breakthrough can be in the form of new trade routes/ channels with the new world, or new infrastructure development such as the building of logistics networks like roads, railroads, canals etc. or new financing mechanisms, such as through junk bonds, or new payment channels and systems, such as e-commerce and the internet etc. Investors overreact to good news and join the bandwagon by speculating and only buying assets.

3.3.2 Stages of financial bubble

In his work on financial instability and its interaction with the economy, Hyman Minsky (1986) provided an informal characterization of bubbles and the associated bursts. Minsky identified five stages of bubbles, which are serially classified as: -

a) *Displacement*: There is a paradigm shift in the economy, due to new technology or financial innovation.

b) *Boom:* This phase is usually characterized by low volatility, credit expansion and increase in investment.

c) *Euphoria:* This stage is marked with explosive increase in asset price, extremely and unrealistically high valuations, high trading volume and high price volatility. Investors trade the overvalued asset in a frenzy.

d) *Profit taking:* At some point, sophisticated investors start reducing their positions and take their profits.

e) *Panic:* Generally, it takes a minor event to prick the bubble, and it triggers a chain of events that deflates the bubble completely and panic ensues as investors lose confidence in the asset and dump it.

Estimating the exact time when a bubble is due to collapse can be a difficult exercise and financially hazardous as well, as in words of John Maynard Keynes: “the market can remain irrational longer than you can remain solvent”. Since irrational actions of investors and traders may lead to building of a bubble, which can last longer than expected. However, the longer and bigger a bubble is, higher is its probability to burst in case of a liquidity shock to the system. Kiss, Koczy, Pinter and Sziklai (2019) prove that bubble is created by the action of risk-tolerant investors. This implies that the actions of risk-loving investors, i.e.

speculators lead to building of a bubble. If investors turn risk-averse in a short duration of time, it may lead to the bubble burst as well.

3.4 Review of financial bubbles literature

3.4.1 Causes and effects of bubbles

According to Camerer (1989), asset prices might deviate from intrinsic values based on market fundamentals, because of 'speculative bubbles' or 'fads'. Bubbles are due to market participants' actions and are behind the disruption in equilibrium prices. They are mostly due to irrational expectations of the market, but bubbles can occur even when traders act rationally and have rational expectations (e.g. Blanchard and Watson (1982), Tirole (1982, 1985)), unless a market has limitations due to finite asset life, or by the wealth or number of traders. Sometimes, prices depart from intrinsic values based on all available information because information is not perfectly aggregated by market prices (e.g. Friedman and Aoki, 1986) or because agents have different beliefs about how the economy works.

Tirole (1982, 1985) suggested that assets which might be subject to price bubbles must be durable and saleable, because an expectation of resale value is needed to generate a bubble. Scarcity or short-run inelasticity of supply of the asset is important because an asset that can be easily produced if a bubble occurs (like similar paintings by a living artist) will drive prices down and burst the bubble. Bubbles may also require an active market for assets, and a social mechanism for coordinating the common belief that a bubble exists and will continue to grow.

Blanchard and Watson (1982) pointed out that growing bubbles can have harmful real effects on the economy by drawing out inefficient supply at high prices or making asset prices poor signals (Friedman (1984)). Another argument is that bubbles harm welfare because they redistribute wealth but random redistribution is only harmful if traders are risk-averse, and risk-averse traders normally avoid a bubble since they are not sure about the long-term returns from investment in asset that is in a bubble.

Brunnermeier's (2007) says that "bubbles are typically associated with dramatic asset price increases and a subsequent collapse." This dramatic price increase is often too swift and within a short time price of an asset, or prices of group of assets, reach such dizzying heights that it is inexplicable based on fundamentals. Often, the fall in price is as swift as the rise.

In recent times, there have been some studies aiming at examining bubble like behavior in cryptocurrencies like Bitcoin. Pichl and Kaizoji (2017) conduct an empirical analysis on Bitcoin price time series in terms of standard currencies. After performing multi-scale volatility analysis from the level of tick data, through the 5-min, 1-hour and 1-day scale, the authors find that the time series of Bitcoin price in terms of other currencies like the US dollar is substantially more volatile than the currency-pair like EURUSD, with market bubbles and crashes relatively abundant. Chaim and Laurini (2018) analyze jumps in return volatility of Bitcoin and find that cryptocurrencies like Bitcoin have very high unconditional volatility and are subject to sudden, massive price swings. Chaim and Laurini (2019) analyze daily returns of Bitcoin between January 2015 and March 2018 to empirically investigate the price bubble hypothesis. Bitcoin returns exhibit characteristics similar to a bubble: the daily returns are very volatile, exhibits large kurtosis, and negative skewness (Camerer (1989)). By following previous research, the authors find that the hypothesis of Bitcoin-USD prices being a bubble, is plausible, but the evidence is inconclusive. Hafner (2018) extends traditional bubble tests to the case of time-varying volatility. The author proposes a bubble test that employs volatility model having long run deterministic component and a short run stochastic component, and finds bubble in eleven of the largest cryptocurrencies by the market capitalization, as well as in the CRIX index. However, the author finds the bubble behavior under the stochastic model to be much less pronounced than under the constant volatility. Dong et al. (2018) propose an infinite horizon model of rational asset bubbles in a dynamic new Keynesian (DNK) framework. Using this model, the authors investigate the risky and costly bubbles in cryptocurrencies in an infinite-horizon production economy with incomplete markets. In case of Bitcoin, the authors conclude the following: 1) enormous volatility, 2) price dynamics are significantly sensitive to both investor sentiment and policy stances, and 3) the market exhibits diverse cyclical features for US and China. However, the results rely heavily on the extent of the market distortion caused by the monetary policy, which as the authors say, determines the size of the Bitcoin bubbles.

3.4.2 Patterns of speculative bubbles

Hyman Minsky (1972, 1982) and Charles Kindleberger (1978, 2000) discuss three different patterns of speculative bubbles.

The first type of bubbles occur when prices rise in an accelerated manner and then crash very sharply after reaching its peak. In the end, prices drop very sharply back to a presumed fundamental level after reaching the peak, and is often characterized by panic. The general argument is that in a speculative bubble, price rises because agents (investors) expect it to keep rising and this ongoing expectation provides the increasing demand that keeps the price rising. If due to some exogenous shock the price stops rising, this breaks the expectation and the speculative demand suddenly disappears. This sends the price back to its fundamental very rapidly where there is no expectation of the price rising. In the case of the stochastically crashing *rational bubble model* of Blanchard and Watson (1982), the price rises at an accelerating rate. This occurs because the probability of a crash also rises along with the price rise and the rational agents require an ever rising risk premium to cover for this rising probability of crash. In this class of models, the price path of an asset experiencing a bubble is explosive.

The second type of bubble is when price rises and is followed by a similar decline sometime after reaching its peak. After reaching the peak, the price may last there for a while and then decline again, sometimes the decline at about the same rate as the price went up. This phenomenon is unlike the first type wherein the price declines much more rapidly than it ever rose. In this type of bubble, many agents may be quite unhappy as the price declines, but there is no general panic. Some might argue that such a pattern is not really a bubble since there is no occurrence of a dramatic crash of price. In this case, the asset price appears to move above the fundamental and then moves back down towards that fundamental. The main problem is in defining or observing such a fundamental, especially for assets like collectible items and cryptocurrencies that do not generate an income stream. Misspecified fundamentals may result in wrong valuation and asset price diverging from the fundamental.

The third type of bubble is when price rises to a peak, which is then followed by a period of gradual decline (known as the period of financial distress), to be followed by a much sharper crash at some later time. According to Kindleberger (1978, 2000), this is by far the most common type of bubble. Most of the larger and more famous historical ones conform to this bubble pattern. This includes, among others, the Mississippi bubble of 1719, the South Sea bubble of 1720 and the US stock market bubble of 1928-29. A reason of this pattern is heterogeneous behavior by agents, with some insiders getting out at the peak while others hanging on during the period of financial distress until the panic and crash.

3.4.3 Types of asset bubbles

In the arbitrage free economies that satisfy both the No-Free-Lunch-With-Vanishing-Risk (NFLVR) and complete market hypotheses (in order that both first and second fundamental theorems of asset pricing apply), and in presence of an equivalent risk-neutral measure, Jarrow, Protter and Shimbo (2007) show that bubble process can be one of three types: an asset process that is

(1) a uniformly integrable martingale, or

(2) a martingale that is not uniformly integrable martingale, or

(3) a strict local martingale that is not a martingale.

The first two kinds of bubbles exist only in infinite horizon economies. The first type of bubble represents a permanent but stochastic wedge between an asset’s fundamental value and its market price generated by a perceived residual value at time infinity. The second type of bubble is a result of trading strategies being of finite time duration, although they may be possibly unbounded. These bubbles can persist forever since they do not violate the NFLVR assumption.

The third type of bubble exists in finite horizon settings and occurs in assets with finite maturities. For this asset bubble, unprotected shorting of the asset is not feasible because, due to the admissibility condition, if the short's value gets low enough, the trading strategy must be terminated before the bubble bursts. This admissibility condition terminates the shorting and hence removes downward-selling pressure on the asset's price, thus enabling these bubbles to exist. Since shorting of asset is not permitted in this setting, there is no downward force on the asset. The asset may rise to the extent speculators in the asset want, thus creating a bubble in the process. Logically, type 3 bubbles are most relevant to actual market experiences. Determination of type 3 bubbles just tantamount to determining whether the price process under a risk neutral measure is a martingale or a strict local martingale (Jarrow, Protter and Shimbu (2007), Jarrow, Kchia and Protter (2011)). Martingale and strict local martingale processes are mutually exclusive. <u>If the asset price process is a strictly local martingale, but not a true martingale, there is a bubble</u>. However, if the asset price process is a true martingale, but not a strict local martingale, there is no bubble. Hence, A bubble is defined to be a price process which, when discounted, is a local martingale under the risk-neutral measure but not a true martingale (Cox and Hobson (2005)).

3.5 Stochastic approach for asset bubble identification

The traditional approach used to identify asset price bubbles was to estimate a model for an asset's fundamental value and then to compare it to the market price. However, the understanding that asset's fundamental value is contingent upon many parameters and thus can have multiple values, makes the evidence inconclusive (Camerer (1989)).

In the stochastic/ financial calculus literature, asset price bubbles have been characterized in frictionless, competitive, and continuous trading models using the arbitrage-free martingale pricing technology

(Loewenstein and Willard (2000), Cox and Hobson (2005), Heston, Loewenstein and Willard (2007), and Jarrow, Protter and Shimbu (2007, 2010)). Jarrow, Kchia and Protter (2011) have characterized Asset price bubbles in frictionless, competitive and continuous trading economies using the arbitrage-free martingale price approach. According to the authors, in this classical finite horizon setting, if the asset price process under a risk neutral measure is a strict local martingale (as opposed to martingale), there is a bubble.

A discrete time-dependent stochastic process is a martingale if the process path is defined by discrete points $X_{1,}$ X_2, X_3, ….., X_{n-1}, X_n , and $E(| X |) < \infty$, and $E(X_n \mid X_1, X_2, \ldots, X_{n-1}) = X_{n-1}$.

A local martingale is the localized version of a martingale. All true martingales are local martingales, but the inverse is not true. A strict local martingale is a local martingale that is not true martingale. In other words, a random process that is a local martingale but does not satisfy the martingale property (viz. $E(X_{n+1}) = X_n$) is a strict local martingale (Mijatovic and Urusov (2010)).

A local martingale can be either a true martingale or a strict local martingale. Every local martingale that is bounded from below is a supermartingale. A positive strict local martingale behaves like a supermartingale. Therefore, for a positive strict local martingale, the process is surely divergent and for every filtration, every subsequent data point is greater than the preceding data point, i.e.

$$E(X_n \mid X_1, X_2, \ldots, X_{n-1}) > X_{n-1} \qquad \text{(for every } X_i > 0\text{)}$$

Obayashi, Protter and Yang (2016) find that the price of the asset, which is exhibiting showing a bubble behavior, follows a generalized gamma distribution. Cox and Hobson (2005), Jarrow, Protter and Shimbo (2006), Jarrow, Protter and Shimbo (2010), Bayraktar, Kardaras and Xing (2011), Jarrow, Kchia and Protter (2011), Biagini, Follmer and Nedelcu (2014), Herdegen and Schweizer (2015) show that on a finite time horizon, the market price consistently exceeds the fundamental price for an extended duration of time, if and only if the market price is a strict local martingale under the selected risk-neutral measure. Hence, to determine whether a given stock has bubble pricing, it is sufficient to check whether the market price process, under a risk-neutral measure, is a strict local martingale, or alternatively, a true martingale. Asset price X_i during a bubble satisfies $X_i > 0$ and strict martingale process also mirrors a bubble behavior, of quick rise in asset price and then subsequent fall.

In this light, mathematical analysis of asset price can diagnose and detect a financial bubble in that asset. For this analysis, HFT (High Frequency Trade) data – often called tick data - is needed, for price points to be as close as possible on the timeline. HFT data can be either trade data recorded every minute, or every second, with the latter preferred if it is possible.

The key characteristic in determining a bubble is the asset price volatility (which is very high in the case of bubble). The asset's price volatility is stochastic or randomly estimated by using tick price for various levels of the asset's price. Statistical estimators are applied to real-time price tick data to estimate price volatility, which is stochastic in nature. The asset's price process is defined by a standard stochastic differential equation, which is driven by a Brownian motion. The Brownian motion is based on a natural process involving the erratic, random movement of small particles suspended in gas or liquid. The Brownian motion concept is specifically used to model the martingale path, in which expected value at any point is just equal to immediate past value. The system has no memory, and past price path does not affect the expected value at any point.

Florens-Zmirou (1993) proposes a frequently used stochastic volatility estimator for discrete observations. This estimator can be employed to estimate volatility as a function of price, after which the rate of increase of the volatility function, as the asset price gets arbitrarily large, can be analyzed. Whether or not there is a bubble depends on how fast this increase occurs (its asymptotic rate of increase). There is a bubble if the rate of increase of price volatility is very high within the model's framework, and is greater than a threshold value. This methodology is in line with Jarrow, Kchia and Protter (2011) and has fairly successful rates in predicting the bubbles, with higher accuracies in cases where market volatilities can be modeled more efficiently.

Since bubbles will cause some extremely large positive price changes as they grow (especially during the last stages of bubble growth), and even larger negative price changes when they burst, the distribution of price changes will have negative skewness and large kurtosis if bubbles exist. Large kurtosis was reported by Friedman and Vandersteel (1982) and Okina (1985) for foreign exchange rates, Dusak (1973) for commodity futures prices, Fama (1976, Chapter l) for stock prices, and Blanchard and Watson (1982) for gold prices. If stock prices are rational forecasts, price changes should have a constant conditional mean (which implies independence), and prices should follow a martingale (of which a non-stationary random walk is a special kind). If prices changes are not independent, prices are driven by variation in expected returns. Speculative bubbles may arise in an economic upswing and their bursting may trigger a downswing.

3.6 The Statistical model

Let the asset price be denoted by S under risk-neutral measure Q. Since S cannot be negative, the price process is a supermartingale. No-arbitrage theory tells us that S is a local martingale under the pricing measure. The asset price S is modeled by a standard stochastic differential equation (SDE) driven by a Brownian motion W:

$$dS_t = \mu(S_t)\, dt + \sigma(S_t)\, dW_t \qquad \text{(equation 1)}$$

for all t in [0, T],

in some filtered probability space $(\Omega, \mathcal{F}, P, \mathbb{F})$, where $\mathbb{F} = (\mathcal{F}_t)_{t \geq 0}$

$\mathcal{F}$ is the filtration, which depends on the price path i.e. the path traversed by the earlier price points. The asset price S is non-negative, implying that the asset price S is a super-martingale. The asset price volatility

$\sigma(S_t)$ is stochastic since it depends on the asset price S_t, whose value is time dependent. Delbaen and Schachermayer (1998) showed that under the assumption of no arbitrage in the sense of No-Free-Lunch-with-Vanishing-Risk (NFLVR), there exists a risk neutral measure, under which the above SDE (equation 1) simplifies to:

$$s_t = s_0 + \int_0^t \sigma(s)\, dW_s \qquad \text{(equation 2)}$$

Carr, Cherny and Urusov (2007), Mijatovic and Urusov (2010) and Bernard, Cui and McLeish (2017) show that this process S, defined by equation 2, is a strict local martingale if and only if:

$$\int_{\alpha}^{\infty} \frac{x\, dx}{\sigma^2(x)} < \infty$$

for all $\alpha > 0$,

where x: price process, and

$\sigma(x)$: stochastic volatility

If the above integral is unbounded, i.e. if $I = \infty$, then the process S is a true martingale, hence there is no bubble. As shown in Jarrow, Protter and Shimbu (2007, 2010), for finite horizon economies, a bubble exists if the price process S under a risk neutral measure is a strict local martingale and not a martingale. This condition forms the basis of the bubble testing methodology (Jarrow, Kchia and Young (2011)).

Jarrow, Protter and Shimbu (2007) argue that type 3 bubbles, i.e. bubbles in finite horizon settings exist if S is a strict local martingale. Therefore, to test whether an asset is exhibiting price bubble behavior tantamount to testing only whether the above integral is finite. An intuition behind determining whether this integral should be finite (which is when the price volatility is a divergent function) or infinite (when the price volatility is a convergent function) is that price process S is always a super martingale and is a martingale if and only if the price has a constant expectation. For a strict local martingale, the expectation decreases with time.

To explain it better, consider the bubble equation:

$$\beta_t = S_t - S_t^*$$

where, S_t : market price, and S_t^* : fundamental price of the asset

The bubble, β_t is a strict local martingale by definition, since with time it grows in size, and since the fundamental price, S_t^* is a martingale (a *random walk* since it does not depend on prior price path), the market price of an asset, S_t must be a strict local martingale for bubble to exist (Jarrow, Kchia and Protter (2011), Protter (2013), Jarrow, Protter and Martin (2019)). The typical price process of a strict local martingale is to quickly rise up to highly values and then quickly decrease to small values and remain there. This is also a typical behavior of prices of assets that are undergoing speculative bubbles.

Carr, Cherny and Urusov (2007), Kotani (2006), Mijatovic and Urusov (2010) have investigated the difference between martingale and strict local martingale processes. A strict local martingale is defined by Mijatovic and Urusov (2010) as "a random process that is a local martingale but does not satisfy the martingale property". The distinction between martingale and strict local martingale is subtle, and in the case of diffusion, it tantamount to understanding the asymptotic behavior of the asset price volatility. Strict local martingale is characterized with extremely high asymptotic volatility. Hence, if the asset volatility is large enough, then it makes for the price process to be strict local martingale, and the bubble exists. The intuition is that speculative trading increases the asset volatility. The higher the speculation in an asset, the higher volatile will the market price be, and will make the case for a bubble. The difference between a martingale and strict local martingale is defined in the following mathematical theorem in Jarrow, Kchia and Protter (2011):

Theorem: If I is an integral defined as $I = \int_{\alpha}^{\infty} \frac{x\,dx}{\sigma^2(x)}$, where x is the price function, and $\sigma(x)$ is the price volatility function, α is any positive number, then price process S is a martingale if $I = \infty$ (which is when volatility remains in a range even for very large prices and a strict local martingale if $I < \infty$ (which is when the volatility becomes divergent, or takes extremely large values for large prices).

This theorem forms the basis for my bubble testing methodology. The methodology first calls for estimation of the stochastic volatility at each price point during the confidence interval, i.e. the time interval where bubble is expected and is tested for, and then estimation of the integral using discrete price points and the corresponding stochastic price volatility. To calculate the integral accurately, a continuous and uniformly integrable price function stretching from the minimum price point (in the interval) to infinity, and price volatility corresponding to those prices are needed. However, price points in tick data are discrete in nature, hence the integrand is not uniformly integrable, in strict mathematical sense. However, this integral can be approximated with a sum of large number of data points that are distributed extremely close and at uniform distance to each other. In other words, a uniform price series with very low price steps can effectively approximate the integral. To achieve this pre-condition, I generate data points by interpolating between the maximum and minimum prices in the given confidence interval. The interpolated price data points must be very close, say at a distance (step) of $0.1. The result will be a price series of the interpolated price points. I estimate the price volatility function $\sigma(x)$ corresponding to each (interpolated) price points using a stochastic volatility estimator. In theory, bubble is established if the integral, under a risk neutral measure, converges to a finite value. This would be the case if the expected asymptotic value of the stochastic volatility is very large. However, numerical integration of this definite integral, whose upper limit is infinity, is not possible. In empirical analysis, the definite integral doesn't span from α to ∞. Rather, it spans from α to β, where α is the minimum price and β the maximum price during the confidence interval, and both are finite. Numerical integration of the integrand between α and β always results in a finite value. Therefore, it is a cut-off value that decides for bubble / no bubble in the period. Hence, the decision criterion: <u>If the numerical integration results in a value that is lower than the cut-off value, there is a bubble during the period. However, if it results in a value greater than the cut-off value, then there is no bubble.</u>

3.7 Stochastic volatility estimator

I employ stochastic volatility estimator that Florens-Zmirou (1993) postulated for discrete observations. The Florens-Zmirou volatility estimator is a nonparametric estimator based on the local time of the diffusion process. For n observations, and h_n: a sequence of positive numbers converging to 0, the Florens-Zmirou stochastic variance estimator S(x) is defined as:

$$S_n(x) = \frac{\sum_{i=1}^{n} 1_{\{|S_{t_i} - x| < h_n\}} n(S_{t_{i+1}} - S_{t_i})^2}{\sum_{i=1}^{n} 1_{\{|S_{t_i} - x| < h_n\}}}. \qquad \text{(equation 3)}$$

where $h_n = n^{(-1/4)}$. For large n, $(n * h_n) \rightarrow \infty$, and $(n * h_n^4) \rightarrow 0$.

$S_n(x)$, the Florens-Zmirou stochastic variance estimator, is a consistent estimator of $\sigma^2(x)$.

The variable 'x' is the price point for which the price volatility is calculated. The term 'n' is the number of price points in the tick data series, which has trade price data for every minute (i.e. xx::yy::00). n is also equal to the number of minutes of the tick price data. Since the time series consists of tick data spaced uniformly a minute apart, the data distribution is discrete, not continuous. The term '**1**' is the function whose value is 1 if the condition $\{|S_{t(i)} - x| < h_n\}$ is satisfied. If the condition is not satisfied, the value returned by the function is 0. The term '$S_{t(i)}$' is the stochastic price at i^{th} minute in the tick data. Hence, $(S_{t(i+1)} - S_{t(i)})^2$ is the square of the difference between successive price points in the time series data. The term 'h_n' represents a sequence of positive real numbers, and is equal to $n^{(-1/m)}$. The sequence would converge to 0, as *m* increases in value. For this empirical study, I use the value: $h_n = n^{(-1/4)}$. For example, if a time series contains 10,000 data points (i.e. 10,000 minutes of tick data), the value of h_n is 10.

The required condition of **1** function in the Florens-Zmirou estimator is $|S_{t(i)} - x| < h_n$, which implies that the Florens-Zmirou estimator considers only those price points that are close enough to the price point, at which it calculates price volatility. The price points that are considered in calculation of the Florens-Zmirou estimator, lie above and below the price point at which the price volatility is calculated. Hence, the price volatility is stochastic in nature, since it is localized, i.e. depending on the location of the price point in the time series, and also is a function of time.

Jarrow, Kchia and Protter (2011) and Protter (2013) successfully use the methodology involving Florens-Zmirou stochastic volatility estimator to identify bubble in stocks. Protter (2013) conducts statistical analysis on *LinkedIn* stock, using tick data (seconds) for 5 business days (19-May-2011 to 24-May-2011) when the stock debuted on stock market, and concludes that LinkedIn stock was in bubble during that period. Protter uses the tick data consisting of price points every second, hence the price distribution is very close to being continuous. Protter (2013) also conducts this test on 6 trading days (25-Aug-2011 to 1-Sep-2011) for gold, and finds that during this period, gold was not in bubble. Jarrow, Kchia and Protter (2011) apply the same methodology on 4 e-commerce stocks, viz. *Lastminute.com*, *eToys.com*, *Infospace.com* and *Geocities.com*, to identify whether some or all of these stocks experienced Internet dot-com bubble by the turn of this century. All these stocks were often alleged to experience bubble during that era. Using this methodology, the authors find that *Lastminute.com* and *Infospace.com* stocks experienced bubble. As reflecting in their stock charts, both these stocks had a significant fall after the bubble burst. *Geocities.com* stock was not under bubble, while the statistical test on *eToys.com* remains inconclusive. The authors heavily draw results from the assessment of estimated volatility, $\sigma(x)$ vs. price (x). Since volatility is estimated as a function of price path, it can be plotted as a function of price. Volatility is very high in case of speculative bubbles, and it quickly increases with price, thus leading to the conclusion that the asymptotic value of price volatility should be extremely high. In most cases, this methodology indicates whether the stock was in a bubble or not during the time interval. In those cases where there is no clear direction of volatility as function of stock price, the test results are inconclusive.

CHAPTER 4

Hypothesis, Data and Methodology

Bitcoin price (in US dollars) rose by 63.75%, from $6,714.2 to $10,995.7 during Nov-2017, and by 75.50%, from $10,995.7 to $19,188.1 during first 19 days of Dec-2017 (Bitcoin reached all-time high on 19-Dec-2017). Such swift rise in market price of a futuristic technology like Bitcoin suggests a bubble in Bitcoin. This suspicion is further fueled by the fact that the average daily transactions of Bitcoin have not increased significantly (as suggested by Figure 9 in section 2.8), implying that Bitcoin is not as quickly embraced as a payment method as implied by its price movement. Bitcoin becomes more valuable if it is used in more payments and transactions across the globe. This is because monitoring systems are not required when cryptocurrencies like Bitcoin are used for transactions, which leads to savings. Hence, Bitcoin transaction cost is less as compared to transaction cost when fiat currencies are used for transaction. As total savings increases, Bitcoin becomes more valuable because of its limited supply. While the fiat currencies are theoretically unlimited in quantity, as they are regularly being printed by the Central Banks of their countries. On the contrary, Bitcoin's ultimate supply is limited to 21 million tokens. The economic logic suggests that Bitcoin price during 2017 end might have moved much ahead of fundamentals, thus suggesting a bubble.

4.1 Hypothesis

I propose the following two hypotheses. I test these hypotheses on the price time series of Bitcoin and Ethereum during 1-Jan-2016 to 31-Dec-2018. Based on the results of the statistical test, I decide whether to reject or not to reject a hypothesis.

H1: *Bitcoin was not in financial bubble during 2016-18.*

The average number of daily transactions in Bitcoin have not been growing significantly (and rather has decreased during certain time intervals, especially during Dec-2017 to Feb-2018 (Figure 9)), with new coins also constantly been released, this dramatic price rise of Bitcoin appears to be much ahead of fundamentals. This draws a parallel with the e-commerce bubble of the late '90s, where market was giving too much weight to future growth prospects of a futuristic technology. I propose that Bitcoin was in bubble during late 2017 and early 2018. The price chart of Bitcoin and Ethereum shows that they were moving sluggishly before 2017. Also, when the hysteria ended, Bitcoin fell for most part of the year 2018.

H2: *Ethereum was not in financial bubble during 2016-18.*

Along with Bitcoin, price of other cryptocurrencies, especially the top trading ones (refer Table 2) also increased in 2017. In my research, I analyze Ethereum price, which also significantly increased along with Bitcoin's price, during the same period. During the month of Nov-2017, Ethereum price (in US dollars) increased by 42.3%, from $303.7 to $432.2. During the first 19 days of Dec-2017, Ethereum price further increased by 85.1%, from $432.2 to $800.0. Ethereum price chart almost resembles that of Bitcoin. Hence, I propose that along with Bitcoin, Ethereum was also in financial bubble during 2016-18.

The decision to reject or not to reject each hypothesis depends on the results of the statistical test. I consider the period 1-Jan-2016 to 31-Dec-2018, viz. a period of 3 years, and conduct the statistical test for each individual year, i.e. 2016, 2017 and 2018 and report the results. I run the statistical test only on Bitcoin and Ethereum, as high frequency tick data for the period 2016-18 was available only for Bitcoin and Ethereum on the cryptocurrency exchange *Gemini* at the time of writing this dissertation. I apply the test methodology as discussed in the methodology section to identify bubble in both of these cryptocurrencies during the period 2016-18.

4.2 Data

I consider the HFT (High Frequency Trade) data of Bitcoin and Ethereum, and run my statistical model and algorithm on the same to identify bubble in the confidence intervals, i.e. the time intervals where price movement is high and chances of finding a bubble is high. The HFT data is (per minute) tick data of intraday trade. Cryptocurrencies are traded 24/7 on crypto exchanges, hence tick data for a day is for 24 hours (i.e. 1,440 minutes) starting from 00:00:00 (12 A.M.) and ending at 23:59:00 (11:59 P.M.). It is natural to view tick data as a frequently sampled collection of observations from an underlying continuous process, since data points are close and uniformly spaced. The integral for continuous process can be modeled as a sum using discrete observations, if a minute tick data is used. I source tick data from a dedicated cryptocurrency exchange called *Gemini*, where various cryptocurrencies are traded 24/7. *Gemini* is a fully US-regulated and licensed Bitcoin and Ethereum (Hardle, Harvey and Reule (2019)) exchange, and it met its capital requirements by placing all USD deposits at a FDIC-insured bank. I also source the tick data from Bloomberg; it provides intraday minute bid, ask and trade data, as well as volume per minute. However, Bloomberg provides tick data for only last 140 days. Therefore, I refer to the data from *Gemini* exchange for the time much into the past.

The tick data was available for only Bitcoin (BTC) and Ethereum (ETH) at the time of this dissertation. I use the HFT tick data for BTC and ETH for the period 2016-18, and conduct separate analysis in each case. I use trade data every minute (at :00 seconds) as well as the trade volume for analysis. There was a sudden and swift rise in Bitcoin price during 2017. An inspection of daily price data suggests me a bubble in Bitcoin price during 2017, especially towards the end of the year. I expect a bubble in Ethereum as well, since along with Bitcoin, other cryptocurrencies also rise. Ethereum has been among the largest traded cryptocurrencies after Bitcoin, hence its choice for research is logical and justified. The HFT data contains information of Open, High, Low and Close prices for every minute. I use Open Price every minute to construct the price time series and conduct analysis. Using Open price to construct the time series makes the time series balanced and less volatile, since High and Low price can occur at any time during the 60 seconds of a minute, but Open price is at fixed point of :00, and equidistant from each other at a fixed gap of 1 minute. To identify the periods of possible bubble, I put the 5%, 7% and 10% filter (described in *Methodology* section) on the daily Bitcoin price (Opening Price).

I get the daily price data of cryptocurrencies from the online sources: http://www.blockchain.com, http://www.investing.com and http://www.coindecko.com. I also download hourly data for further analysis, from the website: http://www.cryptodatadownload.com. For a few days in December 2017, Bitcoin prices jumped up astonishingly, and I use hourly prices in conjunction with the daily prices to analyze the same. I source the daily price data for S&P 500 and NYSE Tech index from Yahoo Finance and http://www.investing.com. I source the daily gold price data from http://goldprice.org, and daily crude oil price data from http://datamarket.com. For daily price data of crude oil, I consider the daily price of West Texas Intermediate (WTI) crude, which is warehoused in Cushing, Oklahoma. I calculate the daily return for day ‘t’ as $\log(P_t / P_{t-1})$.

4.3 Methodology

4.3.1 Methodology for statistical test

I use Jarrow, Kchia and Protter (2011) and Protter (2013) methodology for detecting price bubble in Bitcoin prices. The authors successfully use that methodology to identify bubble in few e-commerce stocks, viz. *LastMinute.com, eToys.com, Infospace.com* and *Geocities.com* during the e-commerce boom in the year 2000. I utilize the same methodology and apply it to Bitcoin and Ethereum to identify bubble in each of them. The methodology is applicable in case of cryptocurrencies, as cryptocurrencies are traded on dedicated crypto exchanges like stocks, with the difference that they can be traded in fractions also, unlike stocks that can be traded only in whole numbers. In line with Obayashi, Protter and Yang (2016), I impose 5% filter on the daily price series and filter in the periods that start with at least +5% movement (price jump) in daily price and end with at least -5% movement (price fall) in daily price. With +5% daily price movement, the cryptocurrency is supposed to enter the bubble phase, which ends with -5% daily price movement, at which the cryptocurrency is supposed to come out of the bubble phase. I classify *bubble birth*, when daily price movement is at least +5%, and *bubble death*, when daily price movement is at least -5%. During the bubble period, i.e. between *bubble birth* and *bubble death*, daily price may keep fluctuating, but the cryptocurrency may remain in the bubble. After a fall of -5% or more, the cryptocurrency is not supposed to enter bubble phase until there is a daily price rise of +5% or more. On lines of Jarrow, Kchia and Protter (2011), I conduct statistical test on this filtered price time series data to identify whether there was a bubble during this period or not. I separately impose 10% filter on the price series data and repeat the analysis on the filtered price series data. As expected, the time duration for +/- 10% price movement is longer and infrequent as compared to +/- 5% movement. Obayashi, Protter and Yang (2016) also consider 10% filtering rule, though they prefer 5% filtering rule more, since 5% rule filters out some bubble signal caused by noise. If price movement is more negative than -5%, and more positive than +5% on consecutive days, I consider both the days to be within the bubble period, and consider the bubble death only when price movement is -5% or more negative, and not met by price movement of at least +5% on the subsequent day. I follow similar logic for 10% filter. I filter in the period that begins with at least +10% change in price and ends with at least -10% fall in price. I also employ 7% filter for the analysis, as in the case of cryptocurrencies, 5% may be a lower filter due to their high daily volatility.

After identifying the period of (possible) bubble using the daily price series, I build the time series containing tick data using the start and the end date of the period. The time series starts at the time 00:00 (i.e. 12:00 AM) of the start date and ends at the time 23:59 (i.e. 11:59 PM) of the end date. This results in a dataset containing time (in minutes) and price (in US dollars) in adjacent columns. I calculate the third column in the dataset as $(\text{Price}_{(i+1)} – \text{Price}_i)^2$ for each i^{th} row. This dataset becomes an input file for the Matlab program that I use to estimate price volatility.

I estimate the price volatility at each price point of the price process S, by using the Florens-Zmirou (1993) volatility function estimator.

$$S_n(x) = \frac{\sum_{i=1}^{n} 1_{\{|S_{t_i} - x| < h_n\}} n(S_{t_{i+1}} - S_{t_i})^2}{\sum_{i=1}^{n} 1_{\{|S_{t_i} - x| < h_n\}}}. \qquad \text{(equation 3)}$$

where,

n: total number of price points, i.e. total number of minutes in the period for which bubble is suspected. This is the length of the time series.

h : $n^{(-1/4)}$

S_{ti} : Price at i^{th} row

$S_{t(i+1)}$: Price at $(i + 1)^{th}$ row

x : price point at which price volatility is estimated. Interpolations performed between the minimum and the maximum prices during the interval.

$S_n(x)$: Variance at x (price point)

Price volatility, $\sigma(x)$ is the square root of the variance at that price point (x).

In line with Jarrow, Kchia and Protter (2011), I first identify the time intervals for possible bubble behavior by imposing 5% filter on daily Bitcoin price series. I analyze high frequency minute price data for each such interval. Corresponding to the start and ending date, I filter in the HFT data time series, and construct datasets. I find the minimum and maximum prices in the time interval (S_{min} and S_{max} respectively) and define a price step to interpolate price points between them. In majority of the cases, I select the price step as $0.1; this value is low enough for price points to be close enough and resembling a continuous price distribution. Greater the difference between the minimum and maximum price points, larger will be the number of interpolated price points. I consider the price series starting from the minimum price, containing all the interpolated price points and ending with the maximum price points, and calculate the value of **1** function at each point. I multiply the value of **1** function with n and $(S_{t(i+1)} - S_{t(i)})^2$ for each actual price point in the tick data series and calculate the sum. Then I divide this sum by the sum of **1** function for all points in the tick data series. The resulting number is the stochastic variance at a particular price point (i.e. 'x'). The square root of the stochastic variance yields price volatility at that price point. Therefore, the stochastic price volatility is a function of the price path and time, i.e. location in the time series. Estimating the price volatility at each interpolated price points is akin to estimating the volatility distribution in the price interval.

I use a Matlab program that I have written for this project, to estimate the price volatility at each price point using the Florens-Zmirou volatility estimator. Not all price points in the tick price series will have non zero, finite price volatility. This is because trading may not happen in all the minutes, so the price will not change. This happens during dormant periods. The final price series produced by the Matlab program that employs Florens-Zmirou price volatility estimator contains only those price points that have non-zero, finite price volatility. The final output is price interpolation (with minimum and maximum prices in the interval at both ends), with price volatility estimate for every price point.

After estimating price volatility at each of the interpolated price points using Florens-Zmirou estimator. I analyze the volatility values during the interval and estimate the asymptotic behavior of volatility. I plot price volatility, $\sigma(x)$ as a function of price (x), and analyze the volatility trend during the interval to assess the asymptotic nature of volatility. Theoretically, there is a bubble if integral I is finite, i.e.

$$I = \int_{\alpha}^{\infty} \frac{x\,dx}{\sigma^2(x)} < \infty$$

However, in practice, this definite integral ranges not from α to ∞, but from α to β, where α: minimum price during the interval, and β: maximum price during the interval. For empirical research, I approximate the integral I by the sum: $\{(x/\sigma(x)^2)*\Delta x\}$ for all discrete (interpolated) price points, where x: price point at which price volatility is being calculated, $\sigma(x)$: price volatility and Δx: difference between adjacent price points. I calculate the metric: Price $(x_i)/\ \sigma^2(x_i))$ at each price point, and finally the following sum for each price point:

$$\text{Sum}(x) = \Sigma_i \{(\text{Price}(x)_i / \sigma^2(x)_i)*(x_{i+1} - x_i)\}$$

Sum(x) is the sum of the integrand (Price/Volatility^2) from α, i.e. the minimum price during the interval, to x, i.e. the price point. Price point x is a variable that varies from α to β during the interval. I plot Sum(x) as a function of price (x). I also plot price volatility $\sigma(x)$ vs. price (x). In case of bubble, the Sum is very small and the plot is flat/ sluggishly increasing, and hence theoretically the asymptotic value of integral I should be finite. In case of no bubble, the Sum is fast increasing and quickly reaches to very large value for a limited set of numbers, implying that the sum will be extremely large for large prices, and hence theoretically the integral I should have infinite value. Bubble is also characterized with extremely high volatility during the period. This is in line with the understanding that for a bubble, the integral I is finite, as compared to no bubble, for which integral I is infinite. However, in practice, the limits of the integral I are from α to β, not from α to ∞, thus making it a finite definite, rather than infinite definite integral. In short, the integral is $\int_{\alpha}^{\beta} \frac{x\,dx}{\sigma^2(x)}$, rather than $\int_{\alpha}^{\infty} \frac{x\,dx}{\sigma^2(x)}$, where α: smallest price during the period, and β: largest price during the period.

If the value of Sum(x) is very low, it suggests that that Sum(x) should remain finite for very large values of price (x). This implies a bubble. Price volatility is also abnormally high during the time interval in case of bubble. Since the integrand: $x/\sigma^2(x)$ has price (x) in the numerator, and the square of price volatility, $\sigma^2(x)$ in the denominator, theoretically the integration from α, i.e. the minimum price during the interval, to infinity should lead to a finite value if price volatility is abnormally high, and rises very fast, vis-à-vis price (x). This theoretically implies that the asymptotic value of price volatility should be infinite in such cases, and hence the value of integral I, whose limits are α and ∞, should be finite. On the other hand, if the plot of Sum(x) vs. price (x) indicates a steep rise, it implies that it would become very large for large values of x. This implies that the integral I would reach infinity for very large values (i.e. asymptotic value) of x. This implies that there is no bubble during the period. For no bubble, price volatility values are relatively low during the period, and often decrease as price increases. This theoretically implies that the asymptotic value of price volatility should be finite in such cases, and hence the value of integral I, whose limits are α and ∞, should be infinite.

Since in practice, the limits of definite integral I are α and β: both finite, hence Sum(x) during a price interval defined by α and β will always be finite. Therefore, it is the cut-off value of Sum(x) that would determine whether or not there is a bubble during a period. If the value of Sum(x) is lower than or equal to the cut-off value, then there should be a bubble. However, if the value of Sum(x) is higher than the cut-off value, there should be no bubble.

I conduct the analysis on Bitcoin and Ethereum using this methodology on all time intervals that I filter in using the 5%, 7% and 10% price filters on the daily price series of Bitcoin and Ethereum during the period

2016-18, and subsequently verify whether or not there was a bubble during each interval by conducting a statistical analysis on the tick price data corresponding to each interval.

4.3.2 Methodology of datasets construction

I source the daily price data from Bloomberg, http://www.investing.com and http://www.cryptodatadownload.com, and construct yearly datasets for 2016, 2017 and 2018 respectively. The data is in continuous time series format, as cryptocurrencies on dedicated crypto exchanges are traded 24/7, as compared to securities listed on regular stock markets like NYSE/ NADSDAQ etc, which trade only during regular market hours. I calculate daily return on day 'i' as natural log of the quotient (Price(i)/ Price(i-1)), and get the daily return posted by the cryptocurrency on each trading day. I filter in the confidence intervals by applying 5%, 7% and 10% filters respectively on the daily price data of the cryptocurrency. From the start and end dates of the confidence intervals, I filter in the tick data from the continuous tick data series and then construct final datasets in spreadsheets. The final datasets contain Date and Time (in the first column), Open price (x_i) for the minute (in the second column) and the square of difference in price between successive price points, i.e. $(x_{(i+1)} - x_i)^2$ (in the third column). I calculate the third column using prices in the second column. I repeat the procedure for Bitcoin and Ethereum respectively. These datasets become input files to the Matlab program, which calculates price volatility using the Florens-Zmirou stochastic volatility estimator at each interpolated price point between the minimum and maximum prices during the interval.

4.3.3. Methodology of finding the correlation between cryptocurrencies' returns

In order to calculate the Pearson's correlation coefficient between any two cryptocurrencies, I first consider the continuous time series of daily prices of the two cryptocurrencies for which I am calculating the correlation coefficient, and then calculate their daily returns using the daily Open price over the last trading day's Open price. I import the dataset containing daily returns of the cryptocurrencies in SAS, and run the *proc corr* procedure, which gives the Pearson's correlation coefficient and the p-value of the correlation.

4.3.4 Methodology of finding the correlation between cryptocurrencies and other assets

I calculate the correlation between daily returns of various cryptocurrencies and daily returns posted by the stock market (I use S&P 500 index as the market proxy), NYSE Tech Index, gold and crude oil. I first synchronize the daily time series data of various cryptocurrencies with that of S&P 500, gold and crude oil respectively. While the cryptocurrencies' data is a continuous time series, the market data series (S&P 500 and NYSE Tech index respectively) have breaks on every weekend, since they are recorded only when the stock market is open, which is generally on weekday during regular market hours. I delete the weekend price data for every weekend and non-trading day for all cryptocurrencies and then make new time series, using which I calculate daily returns. Hence, the daily return every opening day of the week (which is Monday in most cases), is closing price on that day as compared to the closing price on the closing day of the preceding week (which is Friday or the last trading day of earlier week). Similarly, I create separate time series of cryptocurrency prices, synchronizing with the days of gold and crude oil respectively. I run the OLS regression to calculate the correlation coefficient and the p-value between daily returns of cryptocurrency and other asset. I repeat the analysis for each of the years 2016, 2017 and 2018 respectively.

4.3.5 Methodology for identification of financial bubble

I employ the methodology discussed in sections 3.6, 3.7 and 5.2.1 respectively to identify financial bubble in an asset. I impose the 5%, 7% and 10% filters respectively on the daily price series of Bitcoin and Ethereum during 2016-18 (I consider each year separately) to identify the confidence intervals where likelihood of finding a bubble is higher. Thereafter, I construct time series datasets containing the HFT data, and run Matlab program on datasets to estimate price volatility using the Florens-Zmirou stochastic volatility estimator on the interpolated price points between the minimum and the maximum prices during the interval. The output of the Matlab program is price points (x), which are duly arranged in ascending order, starting from the minimum price α, corresponding price volatilities $\sigma(x)$, and Sum(x) in separate columns. I After cleaning the data and removing the outliers, which are often caused by times when no trade occurred in the asset), I calculate the average price volatility for the period, which is simply the average of all the estimated price volatilities for all the price points during the period. I plot the charts of price volatility $\sigma(x)$ vs. price (x) and the integral I vs. price (x). The value of integral I is numerically calculated by Sum(x), where:

$$\text{Sum}(x) = \Sigma_i \left((x_i / \sigma^2(x_i)) * \Delta x_i\right).$$

where,

x: price point where price volatility is estimated,

$\sigma(x)$: stochastic price volatility at price point x, and

Δx: difference between successive price points, and

counter i varies from 1 to n, i.e. the total number of data points in the tick price series for the interval.

I calculate the value of Sum(x) for all filtered intervals in case of various filter (5%, 7% and 10% respectively). I estimate the values of the cut-off (M) of Sum(x) using the historic data, i.e. the values of Sum(x) for the intervals in which bubble is confirmed. Bitcoin had a sudden and swift run up, characterized with unusually high volatility, during Dec-2017. During time intervals in Dec-2017, volatility is very high and is increasing quickly with price. In addition, Sum(x) is very less for these intervals and also, the plot of Sum(x) vs. price (x) is flat and remains at extremely low levels. Further, I critically analyze the time intervals during Nov-2017 to Jan-2018, and finalize the value of the cut-off (M) for each price filter. I determine cut-off values, i.e. $M_{5\%}$, $M_{7\%}$ and $M_{10\%}$ for time intervals filtered in using the 5%, 7% and 10% filters respectively. Bubble is confirmed in a time interval for which Sum(x) is less than or equal to the cut-off (M). On the other hand, if Sum(x) is greater than the cut-off, then there is no bubble during the interval.

More specifically,

$$\text{IF Sum } (x) \leq \text{cut-off (M), IMPLIES BUBBLE}$$

IF Sum(x) > cut-off (M), IMPLIES NO BUBBLE

The analysis of: (a) the plot of volatility $\sigma(x)$ vs. price (x), (b) the plot of integral (numerically calculated by Sum(x)) vs. price (x), and (c) comparison of final Sum(x) value for the interval with the cut-off M, provides with sufficient information to decide whether or not the asset was in bubble during the period of analysis. In case of a bubble, the average stochastic volatility, as estimated by the Florens-Zmirou estimator (equation 3), is very high, and generally rises consistently with the price. Also, the value of Sum(x) is lower than or equal to the cut-off M. This may imply the convergence of the integral $I = \int_{\alpha}^{\infty} \frac{x\,dx}{\sigma^2(x)}$. A combination of the abovementioned criteria (a), (b) and (c) respectively, indicates that the price process is a strict local martingale, and hence the asset is in bubble during the period. Very high volatility that is increasing with price is a recipe for a bubble during the period. If the volatility is exceptionally high through the period, it makes the case for a strong bubble (as happened during Dec-2017). Figure 13 (a) shows some representative volatility vs. price charts during the periods of bubble.

[Insert Figure 13 (a) here]

In case of no bubble, the average stochastic volatility, as estimated by the Florens-Zmirou estimator, is low, since the price volatility during the period is generally low. In addition, there is either no clear trend in price volatility, as the price increases, or the volatility *falls* with price. In either of these cases, Sum(x) quickly reaches to high values within the time interval, and its final value exceeds that of the cut-off M for that corresponding price filter. Also, the plot of Sum(x) vs. price (x), rises quickly to higher values. This suggests a divergence of the integral $I = \int_{\alpha}^{\infty} \frac{x\,dx}{\sigma^2(x)} < \infty$, and that the value of this definite integral is infinite. This indicates that the price process is *not* a strict local martingale, and hence the asset is *not* in bubble during the period. Figure 13 (b) shows some representative volatility vs. price charts during the periods of 'no bubble'.

[Insert Figure 13 (b) here]

I notice that bubble is present in those time intervals where not only price volatility is high, but also the absolute price levels are high. I observe that the chances of finding a bubble in an asset that moves from $1,000 to $2,000 during a time period are higher than the chance of finding bubble in an asset that moves from $100 to $200 in an equivalent time period.

Using the above discussed methodology, I repeat the analysis for Bitcoin and Ethereum for all years in the period 2016-18 using daily price filters 5%, 7% and 10% respectively, and report separate results for different years and price filters. I notice that Ethereum generally is at very low price levels as compared to Bitcoin (generally a ratio of ~ 1:20), with lower price volatility as well (often in ratio of 1:20). Lower volatility often makes application of this statistical methodology to identify bubble difficult. However, the fact that Ethereum price almost mirrors Bitcoin's price suggests a bubble in Ethereum for the intervals when bubble is identified in Bitcoin.

4.4 Correlation among cryptocurrencies

In accordance with the methodology described in Section 5.3, I calculate values of the Pearson's correlation coefficient and the corresponding p-values among Bitcoin, Ethereum, Ripple and Bitcoin Cash for the years 2016, 2017 and 2018 respectively. I use the daily Open prices and daily returns as posted by these currencies for the analysis. An inspection of the correlation coefficients and the corresponding p-values suggests that Bitcoin's correlation with other top cryptocurrencies, viz. Ethereum, Bitcoin Cash and Ripple is significant at 1% level. I show the correlation and regression matrix in Tables 14 (a – c).

[Insert Table 14 (a – c) here]

Since Ripple and Ethereum were introduced to the world in Jan-2017 and Aug-2017, hence I conduct the analysis on only Bitcoin and Ethereum for 2016. I find positive and significant correlation coefficient of 0.254, significant at 1% level. For 2017, I find positive correlation between Bitcoin-Ethereum, Bitcoin-Ripple, Ethereum-Ripple and Ethereum-Bitcoin Cash, significant at 1% level. The correlation coefficient of Bitcoin and Ethereum is higher in 2017, as compared to that in 2016. The correlation between Ethereum-Bitcoin Cash is also significant at 1% level, but correlation between Bitcoin-Bitcoin Cash and Ripple-Bitcoin Cash is not significant even at 10% level. The value of the correlation coefficient is positive and high (viz. 0.390, 0.303, 0.150 and 0.170 respectively). For 2018, I find that all the cryptocurrencies are positively correlated with each other at 1% significance level. The correlation coefficient is very high, and is greater than 0.7 in each case. This implies that the 4 top trading cryptocurrencies are tightly moving in tandem. I also see that the correlation coefficients for all cryptocurrency-pair in 2018 are also higher than they are for 2017, implying very strong correlation and price movement among these 4 cryptocurrencies.

4.5 Correlation of cryptocurrencies with other assets

In accordance with the methodology described in Section 5.2.4, I calculate values of the Pearson's correlation coefficient and the corresponding p-values of cryptocurrencies Bitcoin, Ethereum, Ripple and Bitcoin Cash with the stock market (S&P 500 and NYSE Tech indices respectively), precious metal like gold and precious commodity like crude oil, for the years 2016, 2017 and 2018. I proxy the stock market daily returns with S&P 500 index daily returns. I also include the Tech index, as cryptocurrency is a financial technology. I show the correlation and regression matrix in Tables 15 (a – c).

[Insert Table 15 (a – c) here]

For 2016, I conduct the regression/ correlation analysis for Bitcoin and Ethereum with S&P 500 and NYSE Tech indices, gold and crude oil (as Ripple and Bitcoin Cash didn't exist then). For this year, I do not find significant correlation of either Bitcoin or Ethereum with the above said assets at even 10% level. For 2017, I again do not find significant correlation of Bitcoin and Ripple with any asset at even 10% level. However, I find that Ethereum has significant correlation with S&P 500 and gold at 10% level. However, the correlation of Ethereum with NYSE Tech index and crude oil are not significant. I also find significant correlation of Bitcoin Cash with S&P 500 and crude oil at 5% significance level. While Bitcoin Cash has negative correlation with S&P 500 index, it has positive correlation with crude oil. I do not find significant correlation between Bitcoin Cash-NYSE Tech index and Bitcoin Cash-gold.

For 2018, I again do not find significant correlation between Bitcoin and other set of assets at even 10% level. I find similar result for Bitcoin Cash. However, I find correlation between Ethereum-NYSE Tech index and Ripple-S&P 500 index, significant at 10% level. I don't find significant correlation of both these cryptocurrencies with either gold or crude oil. In case of Ethereum, I didn't find significant correlation with S&P 500, only NYSE Tech index. Hence, I decide based on this information that Ethereum has either no or at best, a weak correlation with the stock market.

I find that though the 4 top traded cryptocurrencies move in tandem, which is revealed in their significant correlation with each other, they exhibit either no or random correlation with the stock market, precious metal like gold and precious commodity like the crude oil. Bitcoin doesn't show significant correlation with either the stock market, precious metal or precious commodity. This makes Bitcoin difficult to hedge. In addition, if there is any portfolio of cryptocurrencies, it will also be difficult to hedge with other assets. This indicates that in order to hedge cryptocurrencies against their adverse price move, dynamic hedging strategies, which are actively monitored and flexible enough to be changed with time, may be required. However, if the portfolio is sufficiently hedged by itself, then addition of Bitcoin to the portfolio *may* reduce the overall risk of the portfolio, due to low correlation of Bitcoin with other assets, most apparently with the portfolio of commodities (Symitsi and Chalvatzis (2019)).

CHAPTER 5

Empirical results

From the statistical analysis, I conclude that Bitcoin was in strong bubble during 2017, especially during the latter half of the year, when it was quickly rising in price. Interestingly, I find that Bitcoin was in bubble during 2018 as well, when it was falling in price. Much of it can be attributed to high volatility, which resulted in spike in the Bitcoin price, and resulted in short, intermittent bubbles. I find that Ethereum was in bubble intermittently during 2017-18, especially during the period Nov 2017 – Feb 2018. I did not find bubble in either Bitcoin or Ethereum during 2016. In addition, I find more bubbles in Bitcoin as compared to Ethereum during 2016-18. I also find that bubbles in Ethereum are of lower time duration as compared to average time duration of bubbles in Bitcoin.

5.1 Statistical test on Bitcoin

The statistical test indicates that Bitcoin was in bubble during the second half of 2017 and in 2018. The test also indicates that Bitcoin was not in bubble during most part of 2016. On lines of Obayashi, Protter and Yang (2016), I apply 5% and 10% filters on Bitcoin's daily price series to filter in the periods that may have witnessed bubble in Bitcoin. I also apply 7% filter on Bitcoin's daily prices. The resulting periods that are filtered in are the confidence intervals in which chances of finding bubble are high. For an x% price filter, the starting date of the confidence interval is when the daily Close price increased by x% or more as compared to the Close price in the previous day. The ending date of the confidence interval is when the daily Close price decreases by x% or more on that day as compared to the Close price of the previous day.

Since cryptocurrency trading is 24/7 on dedicated crypto exchanges, I suppose that trading day starts at 00:00:00 and ends at 23:59:59 on crypto exchange. Tick data has prices for every minute (xx:yy:00). Corresponding to the start and end date of the confidence interval, I pull in the tick price series and construct the datasets. I run the Matlab program on each dataset to estimate price volatility at each interpolated price point using Florens-Zmirou stochastic volatility estimator. The interpolated price points are interspersed at the price step of $0.1 between the minimum and maximum price points during the period. Only for few periods towards the end of 2017, when Bitcoin price ran up very fast, I use $1.0 as the price step. The price difference between minimum and maximum prices during those periods is very high as compared to other periods, and the computation is difficult in Matlab if lower price steps are used. After computation of price volatility, I remove the noise (outlier points), and plot the charts of volatility vs. price, and also the integral (numerically calculated by $\mathrm{Sum}(x) = \Sigma\ (x/\ \sigma^2(x))*\Delta x$) vs. price. I also calculate the average value of the price volatility during the interval. Using these three pieces of information, I infer whether or not bubble was there in a particular period. For a bubble, the average price volatility during the period must be very high, and the volatility should increase with price. Also, the integral should remain at low values during the period, and the slope of the curve also remaining low. From the historical data, I determine the Sum(x) cut-off value (M) for 5%, 7% and 10% price filters. I use the cut-off value of 4500 for $M_{5\%}$, 5000 for $M_{7\%}$ and 2500 for $M_{10\%}$ respectively to identify bubble in an interval. For bubble, $\mathrm{Sum}(x) \leq M$ corresponding to the price filter of the interval. This should translate to the stochastic condition for presence of bubble during the interval. For no bubble, $\mathrm{Sum}(x) > M$, corresponding to the price filter of the interval. Sum(x) takes lower values in case of bubble, while it quickly reaches high values in case of no bubble.

5.1.1 Test results for Bitcoin: Year 2016

The statistical test reveals that Bitcoin was not in a bubble during 2016. Price movement in Bitcoin during 2016 was not very high. Annualized volatility in Bitcoin for the year 2016 is just 21.0% (Table 4). While

5% filter fetches 5 periods during 2016, 7% filter fetches 3 periods and 10% filter fetches only 1 period during the year. I find no bubble during 7-Jan-2016 to 22-Jan-2016, a period when Bitcoin showed high price movement (+7.26% on opening date and -7.31% on the closing date) as well as high daily volatility (6.23%). I also do not find bubble during the months Feb – May 2016. The test for the period 28-May-2016 to 21-Jun-2016, when Bitcoin moved +11.65% on the opening date and -10.08% on the closing date, with daily volatility of 4.63%, also doesn't show a bubble. For the months of Jul-Aug 2016, I do not find any bubble. 10% filter fetches only 1 period during the year 2016, viz. 20-Jan-2016 to 21-Jun-2106. I do not find bubble for this period as well. The daily volatility for this period is low at 2.67%. The test also indicates no bubble for the period 3-Sep-2016 to 6-Jan-2017 (fetched by 5% filter). During this period, Bitcoin price increased from $573.93 to $1167.2 in 126 days. The daily price volatility is low at 2.42%. The period 23-Dec-2016 to 6-Jan-2017, fetched by 7% filter, also indicates no bubble. Bitcoin price increased from $858.37 to $1167.20 in 15 days of the period. The average daily volatility for the period is 5.73%. For all time intervals fetched by all daily price filters, I find low average price volatility, as estimated by Florens-Zmirou estimator, and Sum(x) values higher than the corresponding cut-offs. Therefore, I infer that Bitcoin was not in bubble during 2016.

[Insert Tables 8 (a-c) here]

5.1.2 Test results for Bitcoin: Year 2017

The statistical test confirms bubble during 2017, especially during the second half of the year. The bubble during the months of Nov-Dec 2017 is very strong, as indicated by very high volatility as estimated by the Florens-Zmirou stochastic volatility estimator and pretty low values of Sum(x). The plot of σ(x) vs. price (x) is fast increasing with price, and plot of Sum(x) vs. price (x) remains at very low levels, thus indicating that the asymptotic value of the integral should be finite. I use the value of Sum(x) in the time intervals during these months to determine the cut-off value (M) of Sum(x). The volatility of daily returns, at (annualized) 40.1%, is also high during 2017. The 5% daily price filter fetches 18 periods, 7% filter fetches 12 periods and 10% filter fetches 2 periods during the year. I create datasets using the high frequency tick data for the corresponding periods and estimate price volatility by running Matlab program that employs Florens-Zmirou stochastic volatility estimator. I judge whether bubble was present during a period if the value of Sum(x) is greater than the cut-off for that filter.

[Insert Tables 9 (a-c) here]

I find no bubble during the periods: 17-Jan-2017 to 9-Feb-2017, and 23-Feb-2017 to 8-Mar-2017 (fetched by 5% filter). I also do not find bubble in the periods: 11-Mar-2017 to 18-Mar-2017 and 21-Mar-2017 to 22-Mar-2017 (fetched by 5% filters). The longer period fetched by 7% filter, i.e. 17-Jan-2017 to 18-Mar-2017, also doesn't show a bubble. This indicates that Bitcoin was not in a bubble during the months Jan-Mar 2017. This is corroborated by the fact that rise in Bitcoin price during these months was rather modest, from the minimum of $834.04 in Jan 2017 to the maximum of $1264.90 in March 2017. Going forward, I do not find bubble during the period: 27-Mar-2017 to 27-May-2017 (fetched by both 5% and 7% filters). However, the period: 29-May-2017 to 12-Jun-2017 (fetched by both 5% and 7% filters) show a bubble.

Going forward, I get a conflicting result as bubble is absent for the periods: 17-Jun-2017 to 24-Jun-2017 and 27-Jun-2017 to 10-Jul-2017 (fetched by 5% filter), but I find bubble to present in the period: 17-Jun-2017 to 15-Jul-2017 (fetched by 7% filter). I find bubble during the periods: 17-Jul-2017 to 21-Jul-2017 (fetched by 5% filter, 13.61% daily volatility of returns), and 17-Jul-2017 to 25-Jul-2017 (fetched by 7% filter, 11.21% daily volatility of returns). The test further indicates a bubble for the period 27-Jul-2017 to 4-Sep-2017 (fetched by 5% filter), when Bitcoin increased from $2514.20 to $4980.00 in 40 days. I also find bubble in the subset period, viz. 5-Aug-2017 to 2-Sep-2017 (fetched by 7% filter). Therefore, I conclude that Bitcoin was in bubble during the month of Aug-2017. For the month of Sep-2017, I do not find bubble for period: 6-Sep-2017 to 8-Sep-2017 (fetched by 5% filter), but I find bubble during the period:

15-Sep-2017 to 21-Sep-2017 (fetched by both 5% and 7% filters respectively; Bitcoin price moved between $2960.6 and $4117.0 in 7 days). After 15-Sep-2017, I find bubble in all periods fetched by all price filters.

I find bubble during the periods 23-Sep-2017 to 24-Oct-2017 (fetched by 5% filter) and 25-Sep-2017 to 24-Oct-2017 (fetched by 7% filter). Further, I find bubble during the periods 29-Oct-2017 to 6-Nov-2017 (fetched by 5% filter), 13-Nov-2017 to 9-Dec-2017 (fetched by 5% filter) and 29-Oct-2017 to 9-Dec-2017 (fetched by 7% filter). The test also shows bubble during the periods 11-Dec-2017 to 22-Dec-2017 (fetched by 5% filter; daily volatility of returns 7.79%; Bitcoin price moved from $10,673.00 to $19,999 in just 12 days) and 11-Dec-2017 to 19-Dec-2017 (fetched by 7% filter; daily volatility of returns 6.54%; Bitcoin price moved from $15,269.00 to $19,999.00 in 9 days). This indicates a strong bubble during 11-Dec-2017 to 22-Dec-2017. Incidentally, 10% filter fetches a relatively longer period, from 17-Jul-2017 to 22-Dec-2017, during which Bitcoin price increased from $1,897.90 to $19,999.00 in 159 days, with daily volatility of 5.87%. The test confirms bubble during this long period. The test confirms bubble during 26-Dec-2017 to 28-Dec-2017 (fetched by both 5% and 7% filters), during which Bitcoin price moved from $13,532.00 to $16,500.00 in just 3 days. Finally, the test also confirms bubble during the period 31-Dec-2017 to 7-Jan-2018 (fetched by 5% filter; daily volatility of returns 6.75%) and 31-Dec-2017 to 8-Jan-2018 (fetched by 7% filter; daily volatility of returns 7.37%). During these periods, Bitcoin price moved between $12,540.00 and $17,222.00. The 10% filter fetches a period 26-Dec-2017 to 11-Jan-2018, during which Bitcoin price moved between $12,481.00 and $17,222.00 in 17 days. The test confirms bubble during this period.

The statistical analysis of Bitcoin price and price volatility indicates very strong bubble in Bitcoin for various periods spanning from mid of Sep-2017 till the end of the year 2017. Therefore, I conclude that Bitcoin was in strong bubble during 2017, and **I reject the principal hypothesis H1.**

5.1.3 Test results for Bitcoin: Year 2018

The statistical test confirms bubble during the periods of high price movement in 2018. This result is significant, as Bitcoin price generally fell for the year. If an asset falls in market price, but is still in bubble intermittently, it may imply that the market is unsure about the asset valuation, because of which the asset price may remain volatile even when falling in price. Therefore, asset still is in bubble as the market thinks that intrinsic value of that asset is much lower (as compared to earlier thought), which leads to high volatility at higher price levels, thus resulting in a bubble. I use the daily price filters of 5%, 7% and 10% respectively to capture high price movement and identify the confidence interval to find bubble in Bitcoin price. The price filters fetch sizeable number of periods: 5% filter fetches 16, 7% filter fetches 6 (5 of which fall in the period Jan-April 2018) and 10% filter fetch 3 periods (all 3 of them fall in the first half of the year) respectively. While periods captured by 5% filter are distributed evenly through the year, those captured by 7% and 10% filters mostly fall in the first half of the year. This is because Bitcoin price movement was higher during the first half of the year, as compared to the second half, and thus larger price filters capture periods during the first half but not as frequently during the second half of the year. I create datasets using the high frequency tick data for the corresponding periods and estimate price volatility by running Matlab program that employs Florens-Zmirou stochastic volatility estimator on these datasets. I identify whether bubble was present during a period if the value of Sum(x) is greater than or equal to the cut-off value M of Sum(x) corresponding to that price filter.

[Insert Tables 10 (a-c) here]

I find the price volatility during the months Jan-Mar 2018 to be very high. From the closing price of $15,005.85 on 2-Jan-2018, Bitcoin price fell quickly to $10,544.60 on 22-Jan-2018, and further fell to $6,838.82 on 5-Feb-2018, after which it started rising again, and increased to $11,595.54 on 5-Mar-2018, after which it again fell. Bitcoin price largely remained volatile and kept moving in the range of $6,000 -

$9,000 till 15-Nov-2018, when it breached the level of $6,000. Bitcoin finished the year 2018 at $3,832.92. The annualized Bitcoin price volatility in 2018 was 36.3%, compared to 40.1% in 2017 and 21.0% in 2016.

The statistical test confirms bubble in the periods: 20-Jan-2018 to 22-Jan-2018 (fetched by 5% filter; daily price volatility 11.02%; Bitcoin price moved between $10,049.00 and $12,999 in 3 days) and 20-Jan-2018 to 21-Jan-2018 (fetched by 7% filter; daily price volatility 14.46%; Bitcoin price moved between $11,112.00 and $12,999 in 2 days). Another period fetched by 5% filter, viz. 24-Jan-2018 to 30-Jan-2018 (daily price volatility 5.31%; Bitcoin price moved between $9,762.50 and $11,837.00 in 7 days) also shows bubble. A longer period: 20-Jan-2018 to 1-Feb-2018 (fetched by 10% filter; daily price volatility 6.53%; Bitcoin price moved between $8,428.30 and $12,999 in 13 days), also confirms a bubble. This suggests that Bitcoin was in bubble starting 20-Jan-2018, for the remaining part of the month Jan-2018. The periods in Feb-2018: 6-Feb-2018 to 22-Feb-2018 (fetched by 5% filter; daily price volatility 6.61%; Bitcoin price moved between $5,931.90 and $11,750.00 in 17 days) and 6-Feb-2018 to 21-Feb-2018 (fetched by 7% filter; daily price volatility 6.47%; Bitcoin price moved between $5,931.90 and $11,750.00 in 16 days) confirm bubble. The statistical test on the period: 26-Feb-2018 to 8-Mar-2018 (fetched by 5% filter; daily price volatility 5.08%; Bitcoin price moved between $9,074.90 and $11,650.00 in 11 days) and 26-Feb-2018 to 7-Mar-2018 (fetched by 7% filter; daily price volatility 4.94%; Bitcoin price moved between $9,399.00 and $11,650.00 in 10 days) also shows bubble. Next, the period: 11-Mar-2018 to 14-Mar-2018 (fetched by 5% and 7% filters; daily price volatility 8.04%; Bitcoin price moved between $7,937.00 and $9,887.50 in 4 days) clearly indicates a bubble. Finally, a longer period: 6-Feb-2018 to 14-Mar-2018 (fetched by 10% filter; price volatility 5.99%; Bitcoin price moved between $5,931.90 and $11,650.00 in 37 days) also confirms a bubble. 5% filter fetches next period: 19-Mar-2018 to 29-Mar-2018 (daily price volatility 4.43%; Bitcoin price moved between $6,912.10 and $9,175.80 in 11 days), during which bubble is confirmed. For the month of April-2018, 5% filter fetches a short period: 3-Apr-2018 to 4-Apr-2018 (daily price volatility 9.45%; Bitcoin price moved between $7,016.90 and $7,500.00 in 2 days), which shows a bubble. Another period in April-2018: 12-Apr-2018 to 25-Apr-2018 (fetched by 5% and 7% filters;

5.39% daily price volatility; Bitcoin price moved between \$6,767.60 and \$9,762.70 in 14 days) also confirms a bubble. A longer period: 12-Apr-2018 to 22-Jun-2018 (fetched by 10% filter; daily price volatility 3.91%; Bitcoin price movement between \$5,940.10 and \$9,940.00 in 72 days) also confirms a bubble. This shows that Bitcoin was in bubble during Jan-Apr 2018.

Statistical test on next period fetched by 5% filter, viz. 3-May-2018 to 11-May-2018 confirms bubble. However, I do not find bubble in the period: 29-May-2018 to 10-Jun-2018 (fetched by 5% filter). The next period fetched by 5% filter in June-2018, viz. 14-Jun-2018 to 22-Jun-2018 (daily price volatility Bitcoin price movement between \$5,940.10 and \$6,830.00 in 9 days) indicates no bubble. The next periods fetched by 5% filter: 29-Jun-2018 to 10-Jul-2018 and 16-Jul-2018 to 31-Jul-2018 also indicates no bubble. However, the period: 17-Jul-2018 to 5-Sep-2018 (fetched by 7% filter; daily price volatility 3.34%; Bitcoin price movement from \$5,900.00 and \$8,480.00 in 51 days) shows a bubble. I note the conflicting results, which *may* imply a *faint* bubble in Bitcoin during the month of Aug-2018. No period during Aug-2018 is fetched by any price filter.

The next period: 15-Oct-2018 to 14-Nov-2018 (fetched by 5% filter; daily price volatility 2.16%; Bitcoin price movement between \$5,300.00 and \$6,753.50 in 31 days), shows a bubble, as average volatility is high during the period, and Sum(x) has lower value than the cut-off $M_{5\%}$. Due to unavailability of tick data, I do not conduct statistical test for the next period fetched by 5% filter: 28-Nov-2018 to 3-Dec-2018. The tick data is not available for the duration 15-Nov-2018 to 5-Dec-2018. Finally, I conclude strong bubble during the last period fetched by 5% filter in the year: 17-Dec-2018 to 21-Dec-2018 (daily volatility 6.74%; Bitcoin price moved between \$3,183.60 and \$4,175.00 in 5 days). The average estimated volatility is high and the value of Sum(x) is much lower than the cut-off $M_{5\%}$, thus indicating a strong bubble. The statistical analysis concludes that Bitcoin was in bubble for many intervals during 2018.

5.2 Statistical test on Ethereum

I run the statistical test on Ethereum tick prices during the period 01-Jan-2016 to 31-Dec-2018. To identify asset bubble in Ethereum, I employ the same test methodology that I use in case of Bitcoin (Jarrow, Kchia and Protter (2011) and Protter (2013)). I use the filters of 5%, 7% and 10% respectively on the daily price series of Ethereum to filter in the periods of possible bubble. These are the confidence intervals in which chances of finding a bubble are high. For each such period, I pull in the tick price data for corresponding dates and construct the datasets. I run the Matlab program, which interpolates price points between the Minimum and Maximum prices during an interval, and estimates price volatility using Florens-Zmirou stochastic volatility estimator at each of the interpolated price points. I repeat the process for all the intervals, i.e. Ethereum price datasets. I numerically calculate the integral with $Sum(x) = \Sigma (x/ \sigma^2(x))*\Delta x$). I plot the charts of price volatility $\sigma(x)$ vs. price (x) and Sum(x) vs. price (x). I also calculate the average price volatility during the period. Ethereum price chart resembles that of Bitcoin. I notice that there was a sudden spike in Ethereum price during Dec-2017, like that in case of Bitcoin. I determine the cut-off values of Sum(x) using the historic data. The cut-off values $M_{5\%}$, $M_{7\%}$ and $M_{10\%}$ for 5%, 7% and 10% filters respectively are determined from the value of Sum(x) during Nov-Dec 2018, just like I do in case of Bitcoin. Bubble is confirmed in a period if Sum(x) is less than or equal to the cut-off M for that price filter. If Sum(x) is greater than the cut-off M for that price filter, there is no bubble. I also notice that Bitcoin:Ethereum price ratio is roughly 1:20, and that estimated price volatility for Ethereum is also much lower as compared to that of Bitcoin. This translates to Sum(x) values that are much lower in case of Ethereum compared to Bitcoin. I use the cut-off value of 200 for $M_{5\%}$, 225 for $M_{7\%}$ and 235 for $M_{10\%}$ to identify bubble.

5.2.1 Test results for Ethereum: Year 2016

Though Ethereum was released on July-2015, a fork was created in March-2016. As a result, I consider Ethereum trade data from March 2016 onwards. Though Ethereum exhibited volatility during March to

May 2016, tick data for the same period is not available. The tick data in case of Ethereum for the year 2016 is available only from May 10 onwards. I conduct the test only for those periods for which HFT tick data is available. I use the cut-off value of 200 for $M_{5\%}$, 225 for $M_{7\%}$ and 235 for $M_{10\%}$ respectively to identify bubble in an interval.

The statistical test on Ethereum for the year 2016 reveals that Ethereum was not in a bubble during 2016. I run the test for all datasets created with 5%, 7% and 10% filters respectively on Ethereum daily returns, and observe that Ethereum did not have bubble in any of the periods during 2016. I run the test on periods starting from 11-May-2016 onwards as the tick data was available only for this time duration. Because of unavailability of data before 10-May-2016, I cannot run tests for 6 periods fetched by 5% filter, 3 fetched by 7% filter and 4 periods fetched by 10% filter. I notice that the daily volatility during March-May 2016 is higher than the remaining part of the year. I run the tests on 8 periods fetched by 5% filter, 5 periods fetched by 7% filter and 3 periods fetched by 10% filter respectively, and do not find bubble in any of them. The lowest price of Ethereum during these periods was $5.99, while the highest price was $25.00. Low values of Florens-Zmirou estimated volatility and higher values of Sum(x) confirm no bubble for all periods. Therefore, the statistical test implies that Ethereum was not in bubble during 2016.

[Insert Tables 11 (a-c) here]

5.2.2 Test results for Ethereum: Year 2017

Like Bitcoin, 2017 was a year of huge gains for Ethereum. The cryptocurrency opened at $8.17 on 1-Jan-2017, and closed at $736.77 on 31-Dec-2017, while making a high of $871.89 on 19-Dec-2017. Hence, value of Ethereum jumped by 10,570% (from open to year-high) in 2017! This suggests a bubble in Ethereum during 2017. Statistical test confirms this and indicates bubble in Ethereum during 2017. However, I notice that bubble in Ethereum was not as frequent as in the case of Bitcoin during 2017. I find that Ethereum was in bubble intermittently during the months of May-Jun 2017, Jul-2017, Nov-Dec 2017.

Bubble was rather strong during latter half of December 2017, and it spilled over to the first half of January 2018 as well. I use the cut-off value of 200 for $M_{5\%}$, 225 for $M_{7\%}$ and 235 for $M_{10\%}$ respectively to identify bubble in an interval.

5% filter creates periods of possible bubbles that are of lower in duration and higher in frequency, while 7% filter crates lesser such periods, and 10% filter creates even less frequent periods, but longer in duration. Applying 5% filter to daily returns of 2017 daily price data results in 19 datasets, 7% filter results in 14 datasets and 10% filter results in 7 datasets. In case of datasets created using 5% filter, the statistical test reveals a bubble in 7 periods and no bubble in 12 periods. In datasets created using 7% filter, the statistical test reveals bubble in 8 periods and no bubble in 6 periods. In 7 datasets that are created using 10% filter, the statistical test reveals bubble in 6 periods and no bubble in 1 period.

I do not find bubble in first period in 2017 fetched by 5% filter: 17-Jan-2017 to 4-Mar-2017. I also do not find bubble in the periods fetched by 7% filter: 14-Feb-2017 to 8-Mar-2017, and that fetched by 5% filter: 9-Mar-2017 to 25-Mar-2017 (Ethereum climbed from the level of $15.11 to $53.00 in 17 days. Average daily price volatility 13.88%). I also do not find bubble in periods: 11-Mar-2017 to 3-Apr-2017 (7% filter; daily volatility 12.68%; Ethereum climbed from $18.51 to $54.54 in 24 days) and 29-Mar-2017 to 3-Apr-2017 (5% filter; daily volatility 4.97%). I also do not find bubble in periods: 8-Apr-2017 to 19-Apr-2017 and 26-Apr-2017 to 8-May-2017 (5% filter) and 13-Apr-2017 to 5-May-2017 (7% filter). In all the above cases, value of Sum(x) is lower than the respective cut-off for that price filter. The statistical test shows that Ethereum was not in bubble during Jan-Apr 2017.

I find bubble in the next period: 18-May-2017 to 26-May-2017 (fetched by both 5% filter and 7% filter; 11.68% daily price volatility; Ethereum climbed from $86.49 to $215.00 in just 9 days). The statistical test and analysis of price volatility and comparison of Sum(x) value with the cut-off confirms a bubble. I also find bubble in periods: 28-May-2017 to 14-Jun-2017 (fetched by both 5% and 7% filters; 8.56% daily price volatility; Ethereum price increased from $159.23 to $418.40 in just 18 days). The 10% filter criterion results in dataset from 11-Mar-2017 to 14-Jun-2017. Running the statistical test on the same also confirms a bubble. Ethereum price increased from $18.51 to $418.40 in just 96 days during this period, with average price volatility of 8.98%. Statistical test confirms bubble during the period. Therefore, I conclude that Ethereum was in bubble intermittently during May-Jun 2017.

The test, however, confirms no bubble during the period 27-June-2017 to 29-June-2017, when Ethereum price increased from $210.02 to $324.88 in 3 days, with average daily volatility of 11.44%. The test shows that price rise was not so steep as to create a bubble in Ethereum. There is no bubble during the period: 27-Jun-2017 to 7-Jul-2017 (fetched by 10% filter) as well. The test confirms no bubble during the periods: 2-Jul-2017 to 7-Jul-2017, 12-Jul-2017 to 13-Jul-2017 (fetched by both 5% and 7% filters) and 12-Jul-2017 to 15-Jul-2017 (fetched by 10% filter). However, bubble is confirmed during the period: 17-July-2017 to 25-July-2017 (fetched by 5%, 7% and 10% filters), in which Ethereum price increased from $153.00 to $262.80 in 9 days, with average daily volatility of 13.02%. The test confirms bubble during this period. The test also confirms bubble in the short period: 29-Jul-2017 to 30-Jul-2017 (5% filter). Therefore, I conclude that Ethereum again came into bubble territory in the second half of Jul-2017.

There is another period in 29-July-2017 to 2-Sep-2017 (fetched by 7% filter), in which Ethereum price increased from $177.53 to $395.42 in 36 days. The test shows bubble in this period. However, a coinciding period: 1-Aug-2017 to 2-Sep-2017 (fetched by 5% and 10% filters) gives conflicting results for 5% and 10% filters respectively, due to the cut-off values. While I conclude that there was a bubble in Ethereum during this period, using $M_{10\%}$ (cut-off for 10% filter), I conclude that there was no bubble, using $M_{5\%}$ (cut-off for 5% filter). This phenomenon shows that using different cut-off values for different price filters can result in different decisions for bubble/ no bubble. Due to the statistical results, I am not convinced that Ethereum was in bubble during Aug-2017.

There is no bubble during the period: 5-Sep-2017 to 8-Sep-2017 (5% filter). However, the test shows bubble during the period: 15-Sep-2017 to 21-Sep-2017 (fetched by both 5% and 7% filters). 5% filter fetches a period: 23-Sep-2017 to 17-Oct-2017 (Ethereum price increased from $259.00 to $350.00 in 25 days; daily volatility of 4.00%). The statistical test shows no bubble in this period. However, 7% filter fetches a longer period, viz. 23-Sep-2017 to 10-Nov-2017. Running the statistical test on this period confirms a bubble. Thus, I again find contrasting results for periods spanning latter half of Sep-2017 and Oct-2017.

5% filter fetches a period: 3-Nov-2017 to 29-Nov-2017 (Ethereum price increased from $284.63 to $514.87 in 27 days; daily price volatility 5.05%). The statistical test confirms bubble during the period. The 7% filter creates a period, 24-Nov-2017 to 29-Nov-2017 (Ethereum price increased from $400.20 to $514.87 in 6 days; daily volatility price 8.63%). The test confirms bubble during this period. 10% filter creates a longer period: 15-Sep-2017 to 29-Nov-2017 (Ethereum price increased from $201.01 to $514.69 in a span of 76 days; daily price volatility 4.86%). The test also indicates a bubble in this period. Volatility is above the cutoff, and the Sum(x) value remains is smaller than the cutoff value. During the first half of December 2017, 5% filter fetches the period: 1-Dec-2017 to 6-Dec-2017, during which Ethereum price increased from $414.34 to $481.00 in a span of 6 days, with daily volatility of 4.97%. The test confirms that there was no bubble during this period.

However, a strong bubble comes by the end of 2017 and spills over to the next year as well. 5% filter fetches a period: 8-Dec-2017 to 15-Jan-2018, during which Ethereum price took a significant jump from $414.73 to $1420.00 in a short span of 39 days, with daily volatility of 8.12%. The statistical test confirms a strong bubble during this period. The test reveals high price volatility and lower value of Sum(x). Similarly, I find strong bubble in the period: 11-Dec-2017 to 16-Jan-2018 (7% filter), in which Ethereum price increased from $439.01 to $1420.00 in a span of 37 days, with daily volatility of 9.00%. However, 10% filter created a shorter period: 11-Dec-2017 to 22-Dec-2017, in which Ethereum price increased from $439.01 to $871.89 in a span of 12 days. I also find bubble in Ethereum during this period. A concurrent analysis of the periods 11-Dec-2017 to 16-Jan-2018 and 11-Dec-2017 to 22-Dec-2017 suggests that there might be a bubble in Ethereum during the last week of December 2017 and the first half of January 2018.

Since the statistical test strongly indicates that Ethereum was in bubble by the end of 2017, **I reject hypothesis H2 for Ethereum.**

[Insert Tables 12 (a-c) here]

5.2.3 Test results for Ethereum: Year 2018

Almost all the cryptocurrencies, including Bitcoin and Ethereum, first rise and then fell during 2018. This was like a spillover from Dec-2017. Ethereum price opened at $752.46 on 1-Jan-2018, and kept increasing till it reached all time high of $1420.00 on 13-Jan-2018. There was a 10%+ increase in daily price on the January 2nd, 7th, 9th and 12th respectively. The intraday price movement was high during the first half of January 2018 that on 13-Jan-2018, daily high was $1420.00, while the daily low was $1252.60: a difference of 13.4% from the high! The first major fall, of -19.89%, was registered on 16-Jan-2018. The second half of January remained very volatile, with daily price sometimes rising by 10% or more, and sometimes falling by 8% or more. Ethereum price further fell in Feb-2018, but still reached a month high of $972.86 after rebounding. The price started falling from Mar-2018 onwards, and ultimately reached below $100 in Dec-2018. Nevertheless, applying 5% filter to the daily price data still creates 21 datasets for the year 2018. This is because the fall in Ethereum price was not unidirectional in the year, and it kept rising intermittently and then falling. Applying 7% filter creates 14 datasets for the year 2018, while applying 10% filter creates 11 datasets for the year. I find bubble in Ethereum only intermittently during the month of Jan-2018, Feb-2018 and Mar-2018. For all the other periods created by 5%, 7% and 10% filters respectively, I do not find bubble, despite high daily volatility, in excess of 10%+ in some of the periods (3-April-2018 to 4-April-2018 and 17-Aug-2018 to 18-Aug-2018 respectively). This flows from the fact that mere high daily volatility does not imply a bubble, as volatility is created and is fueled by price movement in both the directions: up as well as down, while bubble is created only when price movement is up, and is very steep. Statistically, Sum(x) value must remain lower than the cutoff for a bubble. I use the cut-off value of 200 for $M_{5\%}$, 225 for $M_{7\%}$ and 235 for $M_{10\%}$ respectively to identify bubble in an interval.

I find bubble during the period: 2-Jan-2018 to 16-Jan-2018 (10% filter; 9.52% daily price volatility; Ethereum price moved from $750.77 to $1,420.0 in 15 days). I also find bubble during the short period: 20-Jan-2018 to 21-Jan-2018 (fetched by 5% and 7% filters; daily price volatility 13.87%; price moved from $1,010.0 to $1,164.0 in 2 days). I find bubble in a period: 24-Jan-2018 to 30-Jan-2018 (7% filter). During these 7 days, Ethereum price hovered between the minimum of $958.00 and the maximum of $1228.70, with daily volatility of 7.58%. A concurrent period is from 24-Jan-2018 to 29-Jan-2018 (5% filter). Bubble is confirmed for that period as well. I also find bubble during a longer period: 20-Jan-2018 to 2-Feb-2018 (10% filter), wherein Ethereum moved between the minimum of $740.91 and maximum of $1228.70 in a span of 14 days. Ethereum remained quite volatile during this period, and was generally going up and down: the daily volatility during this period was 7.71%. High volatility, which is increasing with price, and lower value of the integral vs. price confirms the bubble during both these periods. Further, I find bubble during the periods: 3-Feb-2018 to 18-Feb-2018 (fetched by 5% filter; daily price volatility 8.26%; Ethereum price moved between $564.56 and $994.90 in 16 days), 6-Feb-2018 to 18-Mar-2018 (fetched by 7% filter; daily price volatility 5.21%; Ethereum price moved between $564.56 and $982.84 in 31 days) and 6-Feb-2018 to 14-Mar-2018 (fetched by 10% filter; daily price volatility 5.36%; Ethereum price moved between $564.56 and $982.84 in 37 days). From the results of the statistical test, I conclude that Ethereum was in bubble during Jan-2018 to Feb-2018. For the month of Mar-2018, the intervals fetched by 7% and 10% filters show bubble till 8-Mar-2018 and 14-Mar-2018 respectively. However, another interval fetched by 5% filter, viz. 23-Feb-2018 to 8-Mar-2018 shows no bubble. Subsequent all intervals in months after Mar-2018 show no bubble. The statistical evidence indicates that Ethereum was in bubble during the months of Jan and Feb in 2018, though not in bubble during the subsequent months.

[Insert Tables 13 (a-c) here]

The statistical test proves that Bitcoin and Ethereum have both been in bubble during 2017-18, and hence I reject both the hypotheses: H1-H2 based on the statistical evidence.

CHAPTER 6

Conclusions & Remarks

The results of the statistical test indicate that Bitcoin was in bubble during the last quarter of 2017 and early 2018. This leads me to reject the principal hypothesis H1. Though the time durations vary for the datasets due to different price filters, the statistical test shows that Bitcoin was in bubble for multiple periods during the fourth quarter of 2017 and the first quarter of 2018 (starting Sep-2017). As Bitcoin's value is derived from the number of transactions, which did not increase during the time span, the fundamental/ intrinsic value of Bitcoin appears to have lagged behind the market price (which ran too fast and too much) for an extended period of time. Hence, the economic logic also suggests that Bitcoin should have been in bubble during late 2017 and early 2018. The statistical test corroborates the economic logic, and indicates that Bitcoin was in bubble during this period. The test also suggests that Bitcoin remained in bubble intermittently during 2018, even though the Bitcoin market price fell for most of the time during 2018. A reason for the same could be that market became unsure of the fundamental value of Bitcoin, and thus assumed Bitcoin had much less intrinsic price, compared to what was thought earlier. Because of this, Bitcoin price showed higher volatility at higher prices during 2018 (even though the general price trend was downward), thus showing bubble at high prices.

The statistical test shows that Ethereum was in bubble during 2017; its price movement mirrors Bitcoin's. However, I find that Ethereum was in bubble for much less duration of time as compared to Bitcoin. The test shows that Ethereum was in bubble only during months of May-2017, July-2017, Sep-2017 and during Nov-2017 to Feb-2018. Beyond Feb-2018, the statistical test doesn't convincingly indicate a bubble in

Ethereum. Since the test shows that Ethereum was intermittently in bubble during 2017 and 2018, I reject hypothesis H2 based on statistical evidence. The test also shows that neither Bitcoin, nor Ethereum was in bubble during 2016.

The regression correlation analysis during 2016-18 vividly suggests that Bitcoin and other top traded cryptocurrencies (Ethereum, Bitcoin Cash and Ripple) have strong correlation with each other at 1% significance level. I conduct analysis for each individual years, viz. 2016, 2017 and 2018 respectively. Only Bitcoin and Ethereum were trading in 2016; Ripple and Bitcoin Cash were introduced to the world in 2017. I consider the time periods for correlation-regression analysis accordingly. I find positive correlation among the cryptocurrencies, and strong correlation at 1% significance level. This shows that all top cryptocurrencies move in tandem with each other.

I conduct regression/ correlation analysis among the top 4 cryptocurrencies and other assets – stock indices (S&P 500, NYSE Tech index), gold and crude oil. An inspection of the Pearson's correlation coefficient and p-values reveals that Bitcoin doesn't have significant correlation with the stock market, gold or crude oil at even 10% significance level. For other cryptocurrencies, I do not see a significant correlation with either stock market, gold or crude oil at even 10% level during 2016. For 2017, I find Ethereum having positive correlation with S&P 500 and gold at 10% level, and Bitcoin Cash having negative correlation with S&P 500 and positive correlation with crude oil at 5% significance level. For 2018, I find Ethereum having positive correlation with NYSE Tech index and Ripple having positive correlation with S&P 500 at 10% significance level. Since these correlations are random, a lengthier period is required to confirm the relative movement of cryptocurrencies with other assets. However, correlation analysis shows that Bitcoin has virtually zero correlation (not significant even at 10% significance level) with both the stock market and gold, which makes Bitcoin difficult to hedge. However, addition of Bitcoin in a portfolio of commodities can lead to lowering of overall portfolio risk, due to low correlation of Bitcoin with commodities.

The methodology of this empirical study can be applied to any frequently traded asset whose high frequency (tick) data is available. It can be applied to identify bubble in stocks, stock indices, ETFs, commodities (precious metals, agricultural commodities etc. that generate tick data), cryptocurrencies, or even the complete stock market. An ability to confirm that the stock market is in bubble is critical, since stock market is a leading indicator of the economy, and many pension funds and banks, in which thousands and millions of investors have put in their life savings, have invested in the same. Even if a statistical test confirms that a particular asset, which has become investor's darling recently, is in a bubble, timely forewarning can be generated before the bubble bursts. The ability to tell when an asset is or is not in a bubble can also have important ramifications in the regulation of the capital reserves of banks, as well as for individual investors and retirement funds that hold assets for the long term. In case of banks, if their capital reserve holdings include large investments in asset(s) with unrealistic values due to bubbles, a shock to the bank could occur when the asset bubbles burst, potentially causing a run on the bank, as infamously happened with Lehman Brothers in 2008. If investors can spot speculative bubble in an asset, group of asset or even complete markets, they can avoid overheated asset(s) or markets and reallocate their capital. Also, if enough people realize that there is a bubble, then not many people would want to hold the assets supposedly under bubble, and maybe the asset bubbles will disappear before they get too large, without creating much damage in terms of marked-down values of the assets. This makes this research study conclusive and useful for the global financial markets.

Tables and Figures

Table 1: Bitcoin Price Milestones

This tables shows the dates on which Bitcoin price made new highs. Also mentioned are the daily high prices on the same day when Bitcoin achieved that price milestone.

Milestone	Date	Daily high	Days since last high
$1,000	11/28/2013	$1,106.54	1,230
$2,000	5/20/2017	$2,061.88	1,269
$3,000	6/11/2017	$3,025.47	22
$4,000	8/12/2017	$4,009.89	62
$5,000	9/2/2017	$5,013.91	21
$6,000	10/20/2017	$6,064.14	48
$7,000	11/2/2017	$7,355.35	13
$8,000	11/19/2017	$8,101.91	17
$9,000	11/26/2017	$9,484.91	7
$10,000	11/28/2017	$10,125.70	2
$11,000	11/29/2017	$11,517.40	1
$12,000	12/5/2017	$12,032.00	6
$13,000	12/6/2017	$14,369.10	1
$14,000	12/6/2017	$14,369.10	0
$15,000	12/7/2017	$17,899.70	1
$16,000	12/7/2017	$17,899.70	0
$17,000	12/7/2017	$17,899.70	0
$18,000	12/8/2017	$18,353.40	1
$19,000	12/16/2017	$19,716.70	8
$20,000	12/17/2017	$20,089.00	1

[Source: http://www.blockchain.com]

Table 2: Top Cryptocurrencies

This table shows top crypto currencies trading by market capitalization and their relative percentage share out of the top 10 crypto currencies.

Panel A shows top 10 crypto currencies on 31-May-2018.

Panel B shows top 10 crypto currencies on 31-Dec-2018.

Panel A

Name	Symbol	Price	Circulating supply	Market Cap ($bn)	% of top 10
Bitcoin	BTC	$ 7,563.96	17,066,712	129.09	48.39%
Ethereum	ETH	$ 582.28	99,787,268	58.10	21.78%
Ripple	XRP*	$ 0.62	39,189,968,239	24.16	9.06%
Bitcoin Cash	BCH	$ 1,000.54	17,158,250	17.17	6.44%
EOS	EOS*	$ 12.46	891,823,602	11.11	4.17%
Litecoin	LTC	$ 119.34	56,777,548	6.78	2.54%
Cardano	ADA*	$ 0.23	25,927,070,538	5.86	2.20%
Stellar	XLM*	$ 0.30	18,579,030,187	5.54	2.08%
IOTA	MIOTA*	$ 1.77	2,779,530,283	4.92	1.84%
TRON	TRX*	$ 0.06	65,748,111,645	4.02	1.51%

* : Not mineable

Panel B

Name	Symbol	Price	Circulating supply	Market Cap ($bn)	% of top 10
Bitcoin	BTC	$ 3,800.95	17,455,087	66.35	60.75%
Ripple	XRP*	$ 0.36	40,794,121,066	14.69	13.45%
Ethereum	ETH	$ 137.14	104,117,791	14.28	13.08%
Bitcoin Cash	BCH	$ 159.08	17,540,825	2.79	2.56%
EOS	EOS*	$ 2.61	906,245,118	2.37	2.17%
Stellar	XLM*	$ 0.12	19,160,776,195	2.20	2.02%
Tether	USDT*	$ 1.02	1,858,694,256	1.90	1.74%
Litecoin	LTC	$ 31.00	59,811,902	1.85	1.70%
Bitcoin SV	BSV	$ 86.51	17,539,861	1.52	1.39%
TRON	TRX*	$ 0.02	66,634,334,002	1.27	1.16%

* : Not mineable

[Source: http://coinmarketcap.com; http://cryptoreport.com]

Table 3: Bitcoin Blocks

This table illustrates the Bitcoin blocks along with the years. The first block initiated in 2009, and the last block will end somewhere around 2048. The total number of Bitcoins will get close to 21 million tokens, though not touching it completely.

Block	Reward Era	Year	Date reached	BTC / block	BTC (starting)	BTC (added)	BTC (ending)	% increase	% of BTC limit
0	1	2009	1/3/2009	50	-	2,625,000.00	2,625,000.00	infinite	12.50%
52500	1	2010	4/22/2010	50	2,625,000.00	2,625,000.00	5,250,000.00	100.00%	25.00%
105000	1	2011	1/28/2011	50	5,250,000.00	2,625,000.00	7,875,000.00	50.00%	37.50%
157500	1	2012	12/14/2011	50	7,875,000.00	2,625,000.00	10,500,000.00	33.33%	50.00%
210000	2	2013	11/28/2012	25	10,500,000.00	1,312,500.00	11,812,500.00	12.50%	56.25%
262500	2	2014	10/9/2013	25	11,812,500.00	1,312,500.00	13,125,000.00	11.11%	62.50%
315000	2	2015	8/11/2014	25	13,125,000.00	1,312,500.00	14,437,500.00	10.00%	68.75%
367500	2	2016	7/29/2015	25	14,437,500.00	1,312,500.00	15,750,000.00	9.09%	75.00%
420000	3	2016	7/9/2016	12.5	15,750,000.00	656,250.00	16,406,250.00	4.17%	78.13%
472500	3	2018	6/23/2017	12.5	16,406,250.00	656,250.00	17,062,500.00	4.00%	81.25%
525000	3	2019		12.5	17,062,500.00	656,250.00	17,718,750.00	3.85%	84.38%
577500	3	2020		12.5	17,718,750.00	656,250.00	18,375,000.00	3.70%	87.50%
630000	4	2021		6.25	18,375,000.00	328,125.00	18,703,125.00	1.79%	89.06%
682500	4	2022		6.25	18,703,125.00	328,125.00	19,031,250.00	1.75%	90.63%
735000	4	2023		6.25	19,031,250.00	328,125.00	19,359,375.00	1.72%	92.19%
787500	4	2024		6.25	19,359,375.00	328,125.00	19,687,500.00	1.69%	93.75%
	5	2025		3.125	19,687,500.00	164,062.50	19,851,562.50	0.83%	94.58%
	5	2026		3.125	19,851,562.50	164,062.50	20,015,625.00	0.83%	95.41%
	5	2027		3.125	20,015,625.00	164,062.50	20,179,687.50	0.82%	96.23%
	5	2028		3.125	20,179,687.50	164,062.50	20,343,750.00	0.81%	97.04%
	6	2029		1.5625	20,343,750.00	82,031.25	20,425,781.25	0.40%	97.45%
	6	2030		1.5625	20,425,781.25	82,031.25	20,507,812.50	0.40%	97.85%
	6	2031		1.5625	20,507,812.50	82,031.25	20,589,843.75	0.40%	98.25%
	6	2032		1.5625	20,589,843.75	82,031.25	20,671,875.00	0.40%	98.65%
	7	2033		0.78125	20,671,875.00	41,015.63	20,712,890.63	0.20%	98.84%
	7	2034		0.78125	20,712,890.63	41,015.63	20,753,906.25	0.20%	99.04%
	7	2035		0.78125	20,753,906.25	41,015.63	20,794,921.88	0.20%	99.24%
	7	2036		0.78125	20,794,921.88	41,015.63	20,835,937.50	0.20%	99.44%
	8	2037		0.390625	20,835,937.50	20,507.81	20,856,445.31	0.10%	99.54%
	8	2038		0.390625	20,856,445.31	20,507.81	20,876,953.13	0.10%	99.63%
	8	2039		0.390625	20,876,953.13	20,507.81	20,897,460.94	0.10%	99.73%
	8	2040		0.390625	20,897,460.94	20,507.81	20,917,968.75	0.10%	99.83%

[Source: http://www.blockchain.com]

Table 4: Average Volatility of Top Cryptocurrencies

This table shows the average daily & annualized volatility of Bitcoin (BTC), Ethereum (ETH), Ripple (XRP) and Bitcoin Cash (BCH) during 2016, 2017 and 2018 respectively. The volatility of daily returns is calculated using daily price data. Daily return for day 't' is calculated as $\log_e(P_t / P_{t-1})$. Volatility is calculated for those days for which daily price data is available. While Bitcoin prices are available for all days during 2016-18, Ethereum prices are available from 09-May-2016 onwards. Similarly, Ripple prices are available from 17-Jan-2017 onwards, and Bitcoin Cash prices from 14-Aug-2017 onwards. Annualized volatility is calculated from daily price volatility by multiplying daily price volatility with $(365)^{1/2}$. This is because cryptocurrencies trade daily on the cryptocurrency exchanges.

The daily prices of the cryptocurrencies are sourced from: *http://www.investing.com.*

Price volatility	BTC	ETH	XRP	BCH
2016				
daily	1.1%	2.7%	NA	NA
annual	21.0%	51.6%	NA	NA
2017				
daily	2.1%	3.1%	5.1%	5.0%
annual	40.1%	59.2%	97.4%	95.5%
2018				
daily	1.9%	2.5%	2.9%	2.9%
annual	36.3%	47.8%	55.4%	55.4%

Table 5: Bitcoin Power Consumption Statistics

This table shows Bitcoin global power consumption statistics. As it is revealed, Bitcoin mining is an extensively power consumption activity. Due to this, mining cost is around two-third of mining revenues. Panel A corresponds to 31-May-2018. On that date, Bitcoin was consuming close to 1/300th of total global power supply. Panel B corresponds to 31-Dec-2018. On that date, Bitcoin power consumption had dropped to 1/500th of total global power supply.

Panel A

Description	Value
Bitcoin's current estimated annual electricity consumption* (TWh)	71.12
Annualized global mining revenues	$5,438,032,464
Annualized estimated global mining costs	$3,556,216,066
Current cost percentage	65.40%
Country closest to Bitcoin in terms of electricity consumption	Chile
Estimated electricity used over the previous day (KWh)	194,861,154
Implied Watts per GH/s	0.2
Total Network Hashrate in PH/s (1,000,000 GH/s)	40,505.00
Electricity consumed per transaction (KWh)	1,037
Number of U.S. households that could be powered by Bitcoin	6,585,585
Number of U.S. households powered for 1 day by the electricity consumed for a single transaction	35.03
Bitcoin's electricity consumption as a percentage of the world's electricity consumption	0.32%
Annual carbon footprint (Kilo Ton of CO2)	34,851
Carbon footprint per transaction (Kg of CO2)	507.91

Panel B

Description	Value
Bitcoin's current estimated annual electricity consumption* (TWh)	47.65
Annualized global mining revenues	$2,401,444,967
Annualized estimated global mining costs	$2,298,160,525
Current cost percentage	95.60%
Country closest to Bitcoin in terms of electricity consumption	Singapore
Estimated electricity used over the previous day (KWh)	130,552,804
Implied Watts per GH/s	0.115
Total Network Hashrate in PH/s (1,000,000 GH/s)	47,120.00
Electricity consumed per transaction (KWh)	460
Number of U.S. households that could be powered by Bitcoin	4,412,201
Number of U.S. households powered for 1 day by the electricity consumed for a single transaction	15.56
Bitcoin's electricity consumption as a percentage of the world's electricity consumption	0.21%
Annual carbon footprint (Kilo Ton of CO2)	23,349
Carbon footprint per transaction (Kg of CO2)	225.64

[Source: http://www.digiconomist.net]

Table 6: Top Bitcoin Exchanges

This table shows top Bitcoin exchanges by volume, as on 31-Dec-2018. Daily (24 hours) volume is in US dollars. The table also shows number of cryptocurrency coins that are traded on the exchange.

#	Exchange Name	Daily (24-h) volume	No. of coins	Country	Established
1	Bithumb	$1,505,282,921	69	South Korea	2013
2	BitMax	$744,025,631	23	N/A	2018
3	ZB	$650,943,669	73	China	2017
4	OKEx	$639,277,148	193	Belize	2013
5	Lbank	$592,782,202	77	China	2017
6	Binance	$550,163,240	150	Malta	2017
7	Huobi	$509,398,255	179	China	2013
8	EXX	$500,941,926	88	China	2017
9	Bit-Z	$487,228,944	131	N/A	2016
10	Waves Platform	$463,876,437	40	N/A	N/A
11	CoinBene	$435,408,613	152	Singapore	2017
12	Digifinex	$426,723,551	75	Seychelles	2018
13	Idax	$356,810,772	111	Mongolia	2018
14	ZBG	$354,629,293	37	N/A	2018
15	BitOnBay	$306,994,481	21	Thailand	2018
16	Bibox	$288,773,525	80	China	2017
17	Bitmart	$239,789,143	61	Cayman Islands	2017
18	Bitforex	$227,925,198	119	Seychelles	2018
19	BW	$221,329,825	22	Australia	2018
20	Simex	$197,528,578	10	United States	2015
21	HitBTC	$196,094,060	375	Hong Kong	2013
22	Bitfinex	$180,355,796	29	British Virgin Islands	2014
23	CoinMex	$176,715,541	63	Belize	2018
24	Coneal	$164,125,737	48	Seychelles	2018
25	Coinsuper	$157,127,349	57	Hong Kong	2017

[Source: http://www.coingecko.com]

Table 7: Top Ethereum Exchanges

This table shows top Ethereum exchanges by volume, as on 31-Dec-2018.

Daily (24 hours) volume is in US dollars.

#	Exchange Name	Daily (24-h) volume
1	OKEx	$123,159,416
2	ZB	$111,402,159
3	Binance	$92,522,203
4	ZBG	$89,668,697
5	Huobi	$87,353,975
6	BW	$81,097,915
7	Bibox	$68,089,610
8	Coineal	$66,191,372
9	Bit-Z	$59,942,878
10	Bithumb	$59,171,221
11	Lbank	$56,038,577
12	Idax	$46,106,259
13	Fcoin	$44,549,755
14	Upbit	$44,437,720
15	Coinsuper	$43,349,151
16	Simex	$42,979,832
17	KKCoin	$39,329,377
18	Bitmart	$28,906,891
19	BITKER	$26,859,218
20	Digifinex	$26,107,435

[Source: http://www.coingecko.com]

Table 8a: Summary of Statistical Test for Bubble: Bitcoin (2016, 5%)

This table shows the results of the statistical test that intends to identify whether Bitcoin was in financial bubble during 2016. I consider all periods in 2016 for which Bitcoin showed a daily price movement of at least +/- 5%. A period for which Bitcoin may be in bubble, starts when daily price movement of Bitcoin, measured by the daily return for the day, is at least +5% (bubble birth), and ends when daily price movement, measured by the daily return for the day, is at least -5% (bubble death). The test gives two results: bubble / no bubble. Bubble is said to occur when the numerical integral of (price/ volatility^2) from the minimum price to the maximum price during the period, is lower than or equal to the cut-off value of 4500. If the numerical integral of (price/ volatility^2) from the minimum price to the maximum price during the period, is greater than the cut-off value of 4500, then there is no bubble during the period. The cut-off value is determined from the value of the numerical integral using the historic data.

Start date	Daily change	End date	Daily change	Days	Minutes	Min Price	Max Price	Daily volatility	Price step	N	Avg. F-Z vol.	Sum(x)	Bubble?
7-Jan-16	7.26%	22-Jan-16	-7.31%	16	23,040	$ 353.32	$ 464.00	6.23%	$ 0.1	82	77	6582	N
4-Feb-16	5.64%	27-Apr-16	-4.92%	24	120,960	$ 367.00	$ 470.69	1.70%	$ 0.1	95	145	6160	N
28-May-16	11.65%	21-Jun-16	-10.08%	25	36,000	$ 472.85	$ 788.55	4.63%	$ 0.1	194	180	6291	N
24-Jun-16	6.43%	3-Jul-16	-6.49%	14	14,400	$ 614.04	$ 702.70	4.70%	$ 0.1	82	89	6119	N
3-Sep-16	5.18%	6-Jan-17	-10.49%	126	181,440	$ 573.93	$ 1,167.20	2.42%	$ 0.1	331	384	9784	N

Table 8b: Summary of Statistical Test for Bubble: Bitcoin (2016, 7%)

This table shows the results of the statistical test that intends to identify whether Bitcoin was in financial bubble during 2016. I consider all periods in 2016 for which Bitcoin showed a daily price movement of at least +/- 7%. A period for which Bitcoin may be in bubble, starts when daily price movement of Bitcoin, measured by the daily return for the day, is at least +7% (bubble birth), and ends when daily price movement, measured by the daily return for the day, is at least -7% (bubble death). The test gives two results: bubble / no bubble. Bubble is said to occur when the numerical integral of (price/ volatility^2) from the minimum price to the maximum price during the period, is lower than or equal to the cut-off value of 5000. If the numerical integral of (price/ volatility^2) from the minimum price to the maximum price during the period, is greater than the cut-off value of 5000, then there is no bubble during the period. The cut-off value is determined from the value of the numerical integral using the historic data.

Start date	Daily change	End date	Daily change	Days	Minutes	Min Price	Max Price	Daily volatility	Price step	N	Avg. F-Z vol.	Sum(x)	Bubble?
7-Jan-16	7.26%	22-Jan-16	-7.31%	16	23,040	$ 353.32	$ 464.00	6.23%	$ 0.1	82	77	6582	N
28-May-16	11.65%	21-Jun-16	-10.08%	25	36,000	$ 472.85	$ 788.55	4.63%	$ 0.1	194	180	6291	N
23-Dec-16	6.96%	6-Jan-17	-10.49%	15	21,600	$ 858.37	$ 1,167.20	5.73%	$ 0.1	184	275	6490	N

Table 8c: Summary of Statistical Test for Bubble: Bitcoin (2016, 10%)

This table shows the results of the statistical test that intends to identify whether Bitcoin was in financial bubble during 2016. I consider all periods in 2016 for which Bitcoin showed a daily price movement of at least +/- 10%. A period for which Bitcoin may be in bubble, starts when daily price movement of Bitcoin, measured by the daily return for the day, is at least +10% (bubble birth), and ends when daily price movement, measured by the daily return for the day, is at least -10% (bubble death). The test gives two results: bubble / no bubble. Bubble is said to occur when the numerical integral of (price/ volatility^2) from the minimum price to the maximum price during the period, is lower than or equal to the cut-off value of 2500. If the numerical integral of (price/ volatility^2) from the minimum price to the maximum price during the period, is greater than the cut-off value of 2500, then there is no bubble during the period. The cut-off value is determined from the value of the numerical integral using the historic data.

Start date	Daily change	End date	Daily change	Days	Minutes	Min Price	Max Price	Daily volatility	Price step	N	Avg. F-Z vol.	Sum(x)	Bubble?
20-Jan-16	10.67%	21-Jun-16	-10.08%	154	221,760	$ 365.00	$ 788.55	2.67%	$ 0.1	306	486	4359	N

Table 9a: Summary of Statistical Test for Bubble: Bitcoin (2017, 5%)

This table shows the results of the statistical test that intends to identify whether Bitcoin was in financial bubble during 2017. I consider all periods in 2017 for which Bitcoin showed a daily price movement of at least +/- 5%. A period for which Bitcoin may be in bubble, starts when daily price movement of Bitcoin, measured by the daily return for the day, is at least +5% (bubble birth), and ends when daily price movement, measured by the daily return for the day, is at least -5% (bubble death). The test gives two results: bubble / no bubble. Bubble is said to occur when the numerical integral of (price/ volatility^2) from the minimum price to the maximum price during the period, is lower than or equal to the cut-off value of 4500. If the numerical integral of (price/ volatility^2) from the minimum price to the maximum price during the period, is greater than the cut-off value of 4500, then there is no bubble during the period. The cut-off value is determined from the value of the numerical integral using the historic data.

Start date	Daily change	End date	Daily change	Days	Minutes	Min Price	Max Price	Daily volatility	Price step	N	Avg. F-Z vol.	Sum(x)	Bubble?
17-Jan-17	8.90%	9-Feb-17	-6.09%	24	34,560	$ 834.04	$ 1,080.00	2.88%	$ 0.1	212	149	6515	N
23-Feb-17	5.83%	8-Mar-17	-6.74%	14	20,160	$ 1,108.90	$ 1,293.40	3.21%	$ 0.1	162	133	4640	N
11-Mar-17	5.32%	18-Mar-17	-9.12%	8	11,520	$ 948.60	$ 1,264.90	5.82%	$ 0.1	178	165	7464	N
21-Mar-17	7.31%	22-Mar-17	-6.81%	2	2,880	$ 990.11	$ 1,119.40	9.98%	$ 0.1	99	77	6290	N
27-Mar-17	7.61%	27-May-17	-7.29%	62	89,280	$ 960.21	$ 2,779.00	3.16%	$ 0.1	799	795	5210	N
29-May-17	7.33%	12-Jun-17	-12.41%	15	21,600	$ 2,150.00	$ 2,975.00	5.54%	$ 0.1	653	803	4120	Y
17-Jun-17	7.10%	24-Jun-17	-6.44%	8	11,520	$ 2,430.30	$ 2,785.20	4.85%	$ 0.1	241	195	5320	N
27-Jun-17	5.29%	10-Jul-17	-6.44%	14	20,160	$ 2,257.10	$ 2,637.30	3.40%	$ 0.1	293	294	4890	N
17-Jul-17	15.28%	21-Jul-17	-6.73%	5	7,200	$ 1,897.90	$ 2,950.00	13.61%	$ 0.1	458	304	4018	Y
27-Jul-17	5.51%	4-Sep-17	-8.59%	40	57,600	$ 2,514.20	$ 4,980.00	4.26%	$ 0.1	1270	792	4419	Y
6-Sep-17	4.90%	8-Sep-17	-6.67%	3	4,320	$ 4,139.20	$ 4,670.00	5.84%	$ 0.1	248	304	4982	N
15-Sep-17	14.20%	21-Sep-17	-6.97%	7	10,080	$ 2,960.60	$ 4,117.00	7.95%	$ 0.1	514	836	4434	Y
23-Sep-17	5.03%	24-Oct-17	-6.73%	32	46,080	$ 3,574.30	$ 6,178.50	3.82%	$ 0.1	975	788	2707	Y
29-Oct-17	7.61%	6-Nov-17	-5.78%	9	12,960	$ 5,699.30	$ 7,589.80	3.97%	$ 0.1	573	618	1910	Y
13-Nov-17	11.21%	9-Dec-17	-7.62%	27	38,880	$ 5,801.60	$17,685.00	6.49%	$ 0.1	2255	2840	3282	Y
11-Dec-17	11.72%	22-Dec-17	-15.94%	12	17,280	$10,673.00	$19,999.00	7.79%	$ 0.1	1233	4120	4433	Y
26-Dec-17	15.09%	28-Dec-17	-6.89%	3	4,320	$13,532.00	$16,500.00	11.53%	$ 0.1	540	1815	933	Y
31-Dec-17	11.50%	7-Jan-18	-5.62%	8	11,520	$12,540.00	$17,222.00	6.75%	$ 0.1	1003	2027	1830	Y

Table 9b: Summary of Statistical Test for Bubble: Bitcoin (2017, 7%)

This table shows the results of the statistical test that intends to identify whether Bitcoin was in financial bubble during 2017. I consider all periods in 2017 for which Bitcoin showed a daily price movement of at least +/- 7%. A period for which Bitcoin may be in bubble, starts when daily price movement of Bitcoin, measured by the daily return for the day, is at least +7% (bubble birth), and ends when daily price movement, measured by the daily return for the day, is at least -7% (bubble death). The test gives two results: bubble / no bubble. Bubble is said to occur when the numerical integral of (price/ volatility^2) from the minimum price to the maximum price during the period, is lower than or equal to the cut-off value of 5000. If the numerical integral of (price/ volatility^2) from the minimum price to the maximum price during the period, is greater than the cut-off value of 5000, then there is no bubble during the period. The cut-off value is determined from the value of the numerical integral using the historic data.

Start date	Daily change	End date	Daily change	Days	Minutes	Min Price	Max Price	Daily volatility	Price step	N	Avg. F-Z vol.	Sum(x)	Bubble?
17-Jan-17	8.90%	18-Mar-17	-9.12%	61	87,840	$ 834.04	$ 1,322.70	3.37%	$ 0.1	422	390	4891	Y
27-Mar-17	7.61%	27-May-17	-7.29%	62	89,280	$ 960.21	$ 2,779.00	3.16%	$ 0.1	799	695	5210	N
29-May-17	7.33%	12-Jun-17	-12.41%	15	21,600	$ 2,150.00	$ 2,975.00	5.54%	$ 0.1	653	803	4120	Y
17-Jun-17	7.10%	15-Jul-17	-10.27%	29	41,760	$ 1,967.30	$ 2,785.20	4.11%	$ 0.1	5681	435	3662	Y
17-Jul-17	15.28%	25-Jul-17	-7.54%	9	12,960	$ 1,897.90	$ 2,950.00	11.21%	$ 0.1	545	380	4120	Y
5-Aug-17	13.86%	2-Sep-17	-7.55%	29	41,760	$ 2,840.30	$ 4,980.00	4.28%	$ 0.1	935	733	4607	Y
15-Sep-17	14.20%	21-Sep-17	-6.97%	7	10,080	$ 2,960.60	$ 4,117.00	7.95%	$ 0.1	514	836	4534	Y
25-Sep-17	7.59%	24-Oct-17	-6.73%	30	43,200	$ 3,666.00	$ 6,178.50	3.79%	$ 0.1	1388	964	4941	Y
29-Oct-17	7.61%	9-Dec-17	-7.62%	42	60,480	$ 5,480.00	$ 17,685.00	6.18%	$ 1.0	4784	4025	1449	Y
11-Dec-17	11.72%	19-Dec-17	-8.39%	9	12,960	$ 15,269.00	$ 19,999.00	6.54%	$ 1.0	1829	2856	2245	Y
26-Dec-17	15.09%	28-Dec-17	-6.89%	3	4,320	$ 13,532.00	$ 16,500.00	11.53%	$ 1.0	540	1815	2851	Y
31-Dec-17	11.50%	8-Jan-18	-7.82%	9	12,960	$ 12,540.00	$ 17,222.00	7.37%	$ 1.0	1121	2267	3272	Y

Table 9c: Summary of Statistical Test for Bubble: Bitcoin (2017, 10%)

This table shows the results of the statistical test that intends to identify whether Bitcoin was in financial bubble during 2017. I consider all periods in 2017 for which Bitcoin showed a daily price movement of at least +/- 10%. A period for which Bitcoin may be in bubble, starts when daily price movement of Bitcoin, measured by the daily return for the day, is at least +10% (bubble birth), and ends when daily price movement, measured by the daily return for the day, is at least -10% (bubble death). The test gives two results: bubble / no bubble. Bubble is said to occur when the numerical integral of (price/ volatility^2) from the minimum price to the maximum price during the period, is lower than or equal to the cut-off value of 2500. If the numerical integral of (price/ volatility^2) from the minimum price to the maximum price during the period, is greater than the cut-off value of 2500, then there is no bubble during the period. The cut-off value is determined from the value of the numerical integral using the historic data.

Start date	Daily change	End date	Daily change	Days	Minutes	Min Price	Max Price	Daily volatility	Price step	N	Avg. F-Z vol.	Sum(x)	Bubble?
17-Jul-17	15.28%	22-Dec-17	-15.94%	159	228,960	$ 1,897.90	$ 19,999.00	5.87%	$ 10.0	447	1160	2391	Y
26-Dec-17	15.09%	11-Jan-18	-10.94%	17	24,480	$ 12,481.00	$ 17,222.00	8.25%	$ 1.0	1752	3437	1006	Y

Table 10a: Summary of Statistical Test for Bubble: Bitcoin (2018, 5%)

This table shows the results of the statistical test that intends to identify whether Bitcoin was in financial bubble during 2018. I consider all periods in 2018 for which Bitcoin showed a daily price movement of at least +/- 5%. A period for which Bitcoin may be in bubble, starts when daily price movement of Bitcoin, measured by the daily return for the day, is at least +5% (bubble birth), and ends when daily price movement, measured by the daily return for the day, is at least -5% (bubble death). The test gives two results: bubble / no bubble. Bubble is said to occur when the numerical integral of (price/ volatility^2) from the minimum price to the maximum price during the period, is lower than or equal to the cut-off value of 4500. If the numerical integral of (price/ volatility^2) from the minimum price to the maximum price during the period, is greater than the cut-off value of 4500, then there is no bubble during the period. The cut-off value is determined from the value of the numerical integral using the historic data.

Start date	Daily change	End date	Daily change	Days	Minutes	Min Price	Max Price	Daily volatility	Price step	N	Avg. F-Z vol.	Sum(x)	Bubble?
20-Jan-18	10.91%	22-Jan-18	-6.45%	3	4,320	$ 10,049.00	$ 12,999.00	11.02%	$ 0.1	915	1575	3369	Y
24-Jan-18	5.50%	30-Jan-18	-9.25%	7	10,080	$ 9,762.50	$ 11,837.00	5.31%	$ 0.1	709	1524	2816	Y
6-Feb-18	10.42%	22-Feb-18	-5.98%	17	24,480	$ 5,931.90	$ 11,750.00	6.61%	$ 0.1	1762	2192	2070	Y
26-Feb-18	7.99%	8-Mar-18	-6.09%	11	15,840	$ 9,074.90	$ 11,650.00	5.08%	$ 0.1	1074	1140	4250	Y
11-Mar-18	8.74%	14-Mar-18	-10.38%	4	5,760	$ 7,937.00	$ 9,887.50	8.04%	$ 0.1	798	1031	3953	Y
19-Mar-18	4.88%	29-Mar-18	-10.61%	11	15,840	$ 6,912.10	$ 9,175.80	4.43%	$ 0.1	905	1080	4109	Y
3-Apr-18	5.00%	4-Apr-18	-8.37%	2	1,440	$ 7,016.90	$ 7,500.00	9.45%	$ 0.1	1773	215	4457	Y
12-Apr-18	13.95%	25-Apr-18	-7.94%	14	20,160	$ 6,767.60	$ 9,762.70	5.39%	$ 0.1	9839	999	4496	Y
3-May-18	5.69%	11-May-18	-6.72%	9	12,960	$ 8,352.00	$ 9,940.00	3.56%	$ 0.1	6801	752	2121	Y
29-May-18	5.10%	10-Jun-18	-9.89%	13	18,720	$ 6,645.00	$ 7,790.20	3.48%	$ 0.1	4794	335	4829	N
14-Jun-18	5.38%	22-Jun-18	-9.99%	9	12,960	$ 5,940.10	$ 6,830.00	4.52%	$ 0.1	4506	590	8090	N
29-Jun-18	6.12%	10-Jul-18	-5.48%	12	17,280	$ 5,811.50	$ 6,810.20	2.96%	$ 0.1	5003	578	8504	N
16-Jul-18	6.02%	31-Jul-18	-5.49%	16	23,040	$ 6,335.00	$ 8,480.00	4.03%	$ 0.1	8092	865	5215	N
15-Oct-18	6.42%	14-Nov-18	-8.97%	31	44,640	$ 5,300.00	$ 6,753.50	2.16%	$ 0.1	3526	903	3903	Y
28-Nov-18	10.86%	3-Dec-18	-6.21%	6	8,640	$ 3,836.23	$ 4,472.00	6.73%				*Tick data is missing*	
17-Dec-18	9.85%	21-Dec-18	-5.66%	5	7,200	$ 3,183.60	$ 4,175.00	6.74%	$ 0.1	5118	680	2209	Y

Table 10b: Summary of Statistical Test for Bubble: Bitcoin (2018, 7%)

This table shows the results of the statistical test that intends to identify whether Bitcoin was in financial bubble during 2018. I consider all periods in 2018 for which Bitcoin showed a daily price movement of at least +/- 7%. A period for which Bitcoin may be in bubble, starts when daily price movement of Bitcoin, measured by the daily return for the day, is at least +7% (bubble birth), and ends when daily price movement, measured by the daily return for the day, is at least -7% (bubble death). The test gives two results: bubble / no bubble. Bubble is said to occur when the numerical integral of (price/ volatility^2) from the minimum price to the maximum price during the period, is lower than or equal to the cut-off value of 5000. If the numerical integral of (price/ volatility^2) from the minimum price to the maximum price during the period, is greater than the cut-off value of 5000, then there is no bubble during the period. The cut-off value is determined from the value of the numerical integral using the historic data.

Start date	Daily change	End date	Daily change	Days	Minutes	Min Price	Max Price	Daily volatility	Price step	N	Avg. F-Z vol.	Sum(x)	Bubble?
20-Jan-18	10.91%	21-Jan-18	-9.54%	2	2,880	$ 11,112.00	$ 12,999.00	14.46%	$ 0.1	4743	1210	2243	Y
6-Feb-18	10.42%	21-Feb-18	-6.80%	16	23,040	$ 5,931.90	$ 11,750.00	6.47%	$ 0.5	1230	4496	1303	Y
26-Feb-18	6.99%	7-Mar-18	-7.62%	10	14,400	$ 9,399.00	$ 11,650.00	4.94%	$ 0.1	10201	1084	4055	Y
11-Mar-18	8.74%	14-Mar-18	-10.38%	4	5,760	$ 7,937.00	$ 9,887.50	8.04%	$ 0.1	6835	1031	3953	Y
12-Apr-18	13.95%	25-Apr-18	-7.94%	14	20,160	$ 6,767.60	$ 9,762.70	5.39%	$ 0.1	9839	999	4496	Y
17-Jul-18	8.78%	5-Sep-18	-8.81%	51	69,610	$ 5,900.00	$ 8,480.00	3.34%	$ 0.5	3145	1784	2655	Y

Table 10c: Summary of Statistical Test for Bubble: Bitcoin (2018, 10%)

This table shows the results of the statistical test that intends to identify whether Bitcoin was in financial bubble during 2018. I consider all periods in 2018 for which Bitcoin showed a daily price movement of at least +/- 10%. A period for which Bitcoin may be in bubble, starts when daily price movement of Bitcoin, measured by the daily return for the day, is at least +10% (bubble birth), and ends when daily price movement, measured by the daily return for the day, is at least -10% (bubble death). The test gives two results: bubble / no bubble. Bubble is said to occur when the numerical integral of (price/ volatility^2) from the minimum price to the maximum price during the period, is lower than or equal to the cut-off value of 2500. If the numerical integral of (price/ volatility^2) from the minimum price to the maximum price during the period, is greater than the cut-off value of 2500, then there is no bubble during the period. The cut-off value is determined from the value of the numerical integral using the historic data.

Start date	Daily change	End date	Daily change	Days	Minutes	Min Price	Max Price	Daily volatility	Price step	N	Avg. F-Z vol.	Sum(x)	Bubble?
20-Jan-18	10.91%	1-Feb-18	-10.72%	13	18,720	$8,428.30	$12,999.00	6.53%	$ 0.5	787	2824	1967	Y
6-Feb-18	10.42%	14-Mar-18	-10.38%	37	53,280	$5,931.90	$11,750.00	5.99%	$ 0.5	1006	6753	133	Y
12-Apr-18	13.95%	22-Jun-18	-9.99%	72	103,680	$5,940.10	$ 9,940.00	3.91%	$ 1.0	863	2515	1792	Y

Table 11a: Summary of Statistical Test for Bubble: Ethereum (2016, 5%)

This table shows the results of the statistical test that intends to identify whether Ethereum was in financial bubble during 2016. I consider all periods in 2016 for which Ethereum showed a daily price movement of at least +/- 5%. A period for which Ethereum may be in bubble, starts when daily price movement of Ethereum, measured by the daily return for the day, is at least +5% (bubble birth), and ends when daily price movement, measured by the daily return for the day, is at least -5% (bubble death). The test gives two results: bubble / no bubble. Bubble is said to occur when the numerical integral of (price/ volatility^2) from the minimum price to the maximum price during the period, is lower than or equal to the cut-off value of 200. If the numerical integral of (price/ volatility^2) from the minimum price to the maximum price during the period, is greater than the cut-off value of 200, then there is no bubble during the period. The cut-off value is determined from the value of the numerical integral using the historic data.

Start date	Daily change	End date	Daily change	Days	Minutes	Min Price	Max Price	Daily volatility	Price step	N	Avg. F-Z vol.	Sum(x)	Bubble?
10-Mar-16	4.91%	14-Mar-16	-17.05%	5		$ 11.07	$ 15.07	12.45%				*Tick data not available*	
21-Mar-16	17.39%	25-Mar-16	-4.73%	5		$ 10.14	$ 12.47	11.43%				*Tick data not available*	
28-Mar-16	11.73%	5-Apr-16	-6.73%	9		$ 9.70	$ 12.30	5.36%				*Tick data not available*	
13-Apr-16	7.44%	18-Apr-16	-5.18%	6		$ 6.86	$ 10.90	5.85%				*Tick data not available*	
23-Apr-16	6.15%	25-Apr-16	-6.91%	3		$ 7.32	$ 8.80	6.90%				*Tick data not available*	
30-Apr-16	17.95%	3-May-16	-6.60%	4		$ 7.45	$ 10.30	11.89%				*Tick data not available*	
11-May-16	5.44%	20-May-16	-5.42%	10	14,400	$ 9.68	$ 14.77	6.20%	$ 0.1	50	58	224	N
31-May-16	10.14%	28-Jun-16	-12.46%	29	41,760	$ 9.00	$ 25.00	9.08%	$ 0.1	115	46	256	N
8-Jul-16	11.54%	2-Aug-16	-8.30%	26	37,440	$ 7.60	$ 15.72	7.16%	$ 0.1	78	49	253	N
10-Aug-16	18.80%	20-Sep-16	-4.91%	43	60,480	$ 10.50	$ 14.45	4.23%	$ 0.1	36	69	239	N
30-Oct-16	8.67%	24-Nov-16	-8.06%	26	37,440	$ 9.01	$ 11.95	2.78%	$ 0.1	26	38	304	N
30-Nov-16	5.64%	2-Dec-16	-8.98%	3	4,320	$ 7.24	$ 9.14	7.31%	$ 0.1	19	55	251	N
6-Dec-16	15.07%	23-Dec-16	-5.91%	18	25,920	$ 5.99	$ 8.75	5.11%	$ 0.1	27	48	224	N
28-Dec-16	6.46%	5-Jan-17	-7.73%	9	12,960	$ 7.22	$ 11.94	7.68%	$ 0.1	46	82	313	N

Table 11b: Summary of Statistical Test for Bubble: Ethereum (2016, 7%)

This table shows the results of the statistical test that intends to identify whether Ethereum was in financial bubble during 2016. I consider all periods in 2016 for which Ethereum showed a daily price movement of at least +/- 7%. A period for which Ethereum may be in bubble, starts when daily price movement of Ethereum, measured by the daily return for the day, is at least +7% (bubble birth), and ends when daily price movement, measured by the daily return for the day, is at least -7% (bubble death). The test gives two results: bubble / no bubble. Bubble is said to occur when the numerical integral of (price/ volatility^2) from the minimum price to the maximum price during the period, is lower than or equal to the cut-off value of 225. If the numerical integral of (price/ volatility^2) from the minimum price to the maximum price during the period, is greater than the cut-off value of 225, then there is no bubble during the period. The cut-off value is determined from the value of the numerical integral using the historic data.

Start date	Daily change	End date	Daily change	Days	Minutes	Min Price	Max Price	Daily volatility	Price step	N	Avg. F-Z vol.	Sum(x)	Bubble?
12-Mar-16	8.12%	14-Mar-16	-17.05%	3		$ 11.40	$ 15.07	17.52%				*Tick data not available*	
21-Mar-16	17.39%	24-Mar-16	-9.90%	4		$ 10.14	$ 12.47	12.58%				*Tick data not available*	
28-Mar-16	11.73%	25-Apr-16	-6.91%	29		$ 6.86	$ 12.30	5.59%				*Tick data not available*	
16-May-16	17.95%	18-Jun-16	-26.91%	34	48,960	$ 9.00	$ 25.00	8.14%	$ 0.1	116	50	369	N
19-Jun-16	8.94%	28-Jun-16	-12.46%	10	14,400	$ 10.53	$ 15.75	6.45%	$ 0.1	52	62	273	N
8-Jul-16	11.54%	2-Aug-16	-8.30%	26	37,440	$ 7.60	$ 15.72	7.16%	$ 0.1	78	49	253	N
10-Aug-16	18.80%	2-Dec-16	-8.98%	115	165,600	$ 7.24	$ 14.45	3.48%	$ 0.1	72	89	267	N
6-Dec-16	15.07%	5-Jan-17	-7.73%	31	44,640	$ 5.99	$ 11.94	5.83%	$ 0.1	58	58	294	N

Table 11c: Summary of Statistical Test for Bubble: Ethereum (2016, 10%)

This table shows the results of the statistical test that intends to identify whether Ethereum was in financial bubble during 2016. I consider all periods in 2016 for which Ethereum showed a daily price movement of at least +/- 10%. A period for which Ethereum may be in bubble, starts when daily price movement of Ethereum, measured by the daily return for the day, is at least +10% (bubble birth), and ends when daily price movement, measured by the daily return for the day, is at least -10% (bubble death). The test gives two results: bubble / no bubble. Bubble is said to occur when the numerical integral of (price/ volatility^2) from the minimum price to the maximum price during the period, is lower than or equal to the cut-off value of 235. If the numerical integral of (price/ volatility^2) from the minimum price to the maximum price during the period, is greater than the cut-off value of 235, then there is no bubble during the period. The cut-off value is determined from the value of the numerical integral using the historic data.

Start date	Daily change	End date	Daily change	Days	Minutes	Min Price	Max Price	Daily volatility	Price step	N	Avg. F-Z vol.	Sum(x)	Bubble?
13-Mar-16	16.64%	14-Mar-16	-17.05%	2		$ 11.40	$ 15.07	23.82%				*Tick data not available*	
21-Mar-16	17.39%	24-Mar-16	-9.90%	4		$ 10.14	$ 12.47	12.58%				*Tick data not available*	
28-Mar-16	11.73%	12-Apr-16	-13.25%	16		$ 7.21	$ 12.30	5.58%				*Tick data not available*	
17-Apr-16	10.77%	27-May-16	-10.90%	41		$ 7.12	$ 14.89	6.39%				*Tick data not available*	
31-May-16	10.14%	18-Jun-16	-26.91%	19	27,360	$ 5.99	$ 11.94	10.35%	$ 0.1	58	61	254	N
8-Jul-16	11.54%	24-Jul-16	-11.23%	17	24,480	$ 10.11	$ 15.72	6.94%	$ 0.1	53	60	223	N
10-Aug-16	18.80%	8-Mar-17	-12.55%	211	303,840	$ 5.99	$ 20.78	4.14%	$ 0.1	146	73	286	N

Table 12a: Summary of Statistical Test for Bubble: Ethereum (2017, 5%)

This table shows the results of the statistical test that intends to identify whether Ethereum was in financial bubble during 2017. I consider all periods in 2017 for which Ethereum showed a daily price movement of at least +/- 5%. A period for which Ethereum may be in bubble, starts when daily price movement of Ethereum, measured by the daily return for the day, is at least +5% (bubble birth), and ends when daily price movement, measured by the daily return for the day, is at least -5% (bubble death). The test gives two results: bubble / no bubble. Bubble is said to occur when the numerical integral of (price/ volatility^2) from the minimum price to the maximum price during the period, is lower than or equal to the cut-off value of 200. If the numerical integral of (price/ volatility^2) from the minimum price to the maximum price during the period, is greater than the cut-off value of 200, then there is no bubble during the period. The cut-off value is determined from the value of the numerical integral using the historic data.

Start date	Daily change	End date	Daily change	Days	Minutes	Min Price	Max Price	Daily volatility	Price step	N	Avg. F-Z vol.	Sum(x)	Bubble?
17-Jan-17	5.31%	4-Mar-17	-5.08%	47	67,680	$ 9.60	$ 10.78	3.59%	$ 0.1	111	73	387	N
9-Mar-17	7.51%	25-Mar-17	-6.04%	17	24,480	$ 15.11	$ 53.00	13.88%	$ 0.1	365	102	238	N
29-Mar-17	5.42%	3-Apr-17	-8.89%	6	8,640	$ 43.00	$ 54.54	4.97%	$ 0.1	115	120	495	N
8-Apr-17	5.35%	19-Apr-17	-5.27%	12	17,280	$ 42.42	$ 50.91	4.20%	$ 0.1	81	45	383	N
26-Apr-17	6.33%	8-May-17	-5.56%	13	18,720	$ 50.42	$ 98.79	8.59%	$ 0.1	471	29	345	N
18-May-17	9.96%	26-May-17	-9.43%	9	12,960	$ 86.49	$ 215.00	11.68%	$ 0.1	1199	252	200	Y
28-May-17	7.95%	14-Jun-17	12.22%	18	25,920	$ 159.23	$ 418.40	8.56%	$ 0.1	2138	229	127	Y
27-Jun-17	11.02%	29-Jun-17	-8.45%	3	4,320	$ 210.02	$ 324.88	11.44%	$ 0.1	1069	58	259	N
2-Jul-17	8.88%	7-Jul-17	-10.16%	6	8,640	$ 235.50	$ 293.58	6.10%	$ 0.1	560	41	325	N
12-Jul-17	18.71%	13-Jul-17	-8.45%	2	2,880	$ 182.02	$ 226.64	19.21%	$ 0.1	442	25	716	N
17-Jul-17	20.81%	25-Jul-17	-10.54%	9	12,960	$ 153.00	$ 262.80	13.02%	$ 0.1	933	171	187	Y
29-Jul-17	8.14%	30-Jul-17	-5.54%	2	2,880	$ 177.53	$ 210.00	9.67%	$ 0.1	319	36	174	Y
1-Aug-17	12.53%	2-Sep-17	-11.62%	33	47,520	$ 201.69	$ 395.42	5.03%	$ 0.1	1880	70	235	N
5-Sep-17	6.89%	8-Sep-17	-9.01%	4	5,760	$ 250.00	$ 341.24	7.55%	$ 0.1	668	66	789	N
15-Sep-17	16.26%	21-Sep-17	-9.46%	7	10,080	$ 201.01	$ 300.77	9.85%	$ 0.1	945	61	131	Y
23-Sep-17	9.04%	17-Oct-17	-5.62%	25	36,000	$ 259.00	$ 350.00	4.00%	$ 0.1	842	63	205	N
3-Nov-17	6.52%	29-Nov-17	-9.71%	27	38,880	$ 284.63	$ 514.87	5.05%	$ 0.1	1981	427	154	Y
1-Dec-17	6.58%	6-Dec-17	-8.44%	6	8,640	$ 414.34	$ 481.00	4.97%	$ 0.1	639	77	708	N
8-Dec-17	6.55%	15-Jan-18	-5.67%	39	56,160	$ 414.73	$ 1,420.00	8.12%	$ 0.1	8266	490	164	Y

Table 12b: Summary of Statistical Test for Bubble: Ethereum (2017, 7%)

This table shows the results of the statistical test that intends to identify whether Ethereum was in financial bubble during 2017. I consider all periods in 2017 for which Ethereum showed a daily price movement of at least +/- 7%. A period for which Ethereum may be in bubble, starts when daily price movement of Ethereum, measured by the daily return for the day, is at least +7% (bubble birth), and ends when daily price movement, measured by the daily return for the day, is at least -7% (bubble death). The test gives two results: bubble / no bubble. Bubble is said to occur when the numerical integral of (price/ volatility^2) from the minimum price to the maximum price during the period, is lower than or equal to the cut-off value of 225. If the numerical integral of (price/ volatility^2) from the minimum price to the maximum price during the period, is greater than the cut-off value of 225, then there is no bubble during the period. The cut-off value is determined from the value of the numerical integral using the historic data.

Start date	Daily change	End date	Daily change	Days	Minutes	Min Price	Max Price	Daily volatility	Price step	N	Avg. F-Z vol.	Sum(x)	Bubble?
14-Feb-17	14.19%	8-Mar-17	-12.55%	23	33,120	$ 11.30	$ 20.78	5.61%	$ 0.1	94	77	257	N
11-Mar-17	11.05%	3-Apr-17	-8.89%	24	34,560	$ 18.51	$ 54.54	12.68%	$ 0.1	355	107	237	N
13-Apr-17	8.11%	5-May-17	-7.76%	23	33,120	$ 46.49	$ 97.98	6.76%	$ 0.1	497	55	245	N
18-May-17	9.96%	26-May-17	-9.43%	9	12,960	$ 86.49	$ 215.00	11.68%	$ 0.1	1199	252	200	Y
28-May-17	7.95%	14-Jun-17	-12.22%	18	25,920	$ 159.23	$ 418.40	8.56%	$ 0.1	2138	229	127	Y
27-Jun-17	11.02%	29-Jun-17	-8.45%	3	4,320	$ 210.02	$ 324.88	11.44%	$ 0.1	1069	58	259	N
2-Jul-17	8.88%	7-Jul-17	-10.16%	6	8,640	$ 235.50	$ 293.58	6.10%	$ 0.1	560	41	325	N
12-Jul-17	18.71%	13-Jul-17	-8.45%	2	2,880	$ 182.02	$ 226.64	19.21%	$ 0.1	442	25	716	N
17-Jul-17	20.81%	25-Jul-17	-10.54%	9	12,960	$ 153.00	$ 262.80	13.02%	$ 0.1	933	171	187	Y
29-Jul-17	8.14%	2-Sep-17	-11.62%	36	51,840	$ 177.53	$ 395.42	5.09%	$ 0.1	2120	75	225	Y
15-Sep-17	16.26%	21-Sep-17	-9.46%	7	10,080	$ 201.01	$ 300.77	9.85%	$ 0.1	945	61	131	Y
23-Sep-17	9.04%	10-Nov-17	-7.30%	49	70,560	$ 259.00	$ 350.00	3.61%	$ 0.1	838	190	179	Y
24-Nov-17	16.12%	29-Nov-17	-9.71%	6	8,640	$ 400.20	$ 514.87	8.46%	$ 0.1	946	287	159	Y
11-Dec-17	20.04%	16-Jan-18	-19.89%	37	53,280	$ 439.01	$ 1,420.00	9.00%	$ 0.1	7902	529	173	Y

Table 12c: Summary of Statistical Test for Bubble: Ethereum (2017, 10%)

This table shows the results of the statistical test that intends to identify whether Ethereum was in financial bubble during 2017. I consider all periods in 2017 for which Ethereum showed a daily price movement of at least +/- 10%. A period for which Ethereum may be in bubble, starts when daily price movement of Ethereum, measured by the daily return for the day, is at least +10% (bubble birth), and ends when daily price movement, measured by the daily return for the day, is at least -10% (bubble death). The test gives two results: bubble / no bubble. Bubble is said to occur when the numerical integral of (price/ volatility^2) from the minimum price to the maximum price during the period, is lower than or equal to the cut-off value of 235. If the numerical integral of (price/ volatility^2) from the minimum price to the maximum price during the period, is greater than the cut-off value of 235, then there is no bubble during the period. The cut-off value is determined from the value of the numerical integral using the historic data.

Start date	Daily change	End date	Daily change	Days	Minutes	Min Price	Max Price	Daily volatility	Price step	N	Avg. F-Z vol.	Sum(x)	Bubble?
11-Mar-17	11.05%	14-Jun-17	-12.22%	96	138,240	$ 18.51	$ 418.40	8.98%	$ 0.1	3374	241	169	Y
27-Jun-17	11.02%	7-Jul-17	-10.16%	11	15,840	$ 210.02	$ 324.88	7.67%	$ 0.1	1072	86	234	Y
12-Jul-17	18.71%	15-Jul-17	-13.41%	4	5,760	$ 168.66	$ 226.64	14.20%	$ 0.1	564	45	426	N
17-Jul-17	20.81%	25-Jul-17	-10.54%	9	12,960	$ 153.00	$ 262.80	13.02%	$ 0.1	933	171	187	Y
1-Aug-17	12.53%	2-Sep-17	-11.62%	33	47,520	$ 201.69	$ 395.42	5.03%	$ 0.1	1880	70	235	Y
15-Sep-17	16.26%	29-Nov-17	-9.91%	76	109,440	$ 201.01	$ 514.69	4.86%	$ 0.1	2652	196	185	Y
11-Dec-17	20.04%	22-Dec-17	-20.18%	12	17,280	$ 439.01	$ 871.89	11.23%	$ 0.1	3284	260	158	Y

Table 13a: Summary of Statistical Test for Bubble: Ethereum (2018, 5%)

This table shows the results of the statistical test that intends to identify whether Ethereum was in financial bubble during 2018. I consider all periods in 2018 for which Ethereum showed a daily price movement of at least +/- 5%. A period for which Ethereum may be in bubble, starts when daily price movement of Ethereum, measured by the daily return for the day, is at least +5% (bubble birth), and ends when daily price movement, measured by the daily return for the day, is at least -5% (bubble death). The test gives two results: bubble / no bubble. Bubble is said to occur when the numerical integral of (price/ volatility^2) from the minimum price to the maximum price during the period, is lower than or equal to the cut-off value of 200. If the numerical integral of (price/ volatility^2) from the minimum price to the maximum price during the period, is greater than the cut-off value of 200, then there is no bubble during the period. The cut-off value is determined from the value of the numerical integral using the historic data.

Start date	Daily change	End date	Daily change	Days	Minutes	Min Price	Max Price	Daily volatility	Price step	N	Avg. F-Z vol.	Sum(x)	Bubble?
20-Jan-18	10.98%	21-Jan-18	-8.64%	2	2,880	$ 1,010.00	$ 1,164.00	13.87%	$ 0.1	1383	164	185	Y
24-Jan-18	8.16%	29-Jan-18	-5.74%	6	8,640	$ 958.00	$ 1,228.70	6.66%	$ 0.1	2342	181	137	Y
3-Feb-18	5.50%	18-Feb-18	-6.02%	16	23,040	$ 564.56	$ 994.90	8.26%	$ 0.1	3660	377	155	Y
23-Feb-18	6.59%	8-Mar-18	-6.80%	14	20,160	$ 687.02	$ 894.00	3.84%	$ 0.1	1692	132	226	N
11-Mar-18	5.51%	14-Mar-18	-11.24%	4	5,760	$ 584.99	$ 739.00	6.91%	$ 0.1	1180	73	326	N
3-Apr-18	7.92%	4-Apr-18	-8.87%	2	2,880	$ 371.08	$ 418.69	11.87%	$ 0.1	455	131	252	N
12-Apr-18	14.85%	25-Apr-18	-12.18%	14	20,160	$ 413.00	$ 710.00	6.71%	$ 0.1	2462	125	230	N
28-Apr-18	6.14%	7-May-18	-4.94%	10	14,400	$ 629.67	$ 830.55	5.27%	$ 0.1	1888	134	249	N
13-May-18	6.64%	17-May-18	-5.32%	5	7,200	$ 661.32	$ 741.12	4.48%	$ 0.1	761	84	322	N
29-May-18	10.64%	10-Jun-18	-11.65%	13	18,720	$ 504.00	$ 628.00	5.16%	$ 0.1	1147	100	473	N
14-Jun-18	9.10%	15-Jun-18	-6.09%	2	2,880	$ 460.60	$ 525.19	10.74%	$ 0.1	593	46	283	N
2-Jul-18	5.11%	10-Jul-18	-8.24%	9	12,960	$ 428.32	$ 494.77	3.92%	$ 0.1	624	59	266	N
16-Jul-18	6.43%	31-Jul-18	-5.51%	16	23,040	$ 427.57	$ 515.44	3.60%	$ 0.1	802	79	256	N
17-Aug-18	10.51%	18-Aug-18	-7.26%	2	2,880	$ 283.01	$ 321.08	12.57%	$ 0.1	376	26	302	N
13-Sep-18	15.31%	17-Sep-18	-10.88%	5	7,200	$ 182.60	$ 226.86	9.62%	$ 0.1	407	74	322	N
21-Sep-18	10.47%	24-Sep-18	-6.81%	4	5,760	$ 221.12	$ 254.55	7.42%	$ 0.1	312	70	232	N
27-Sep-18	6.91%	11-Oct-18	-15.65%	15	21,600	$ 185.41	$ 238.00	4.95%	$ 0.1	410	58	279	N
15-Oct-18	8.72%	14-Nov-18	-10.92%	31	44,640	$ 165.00	$ 222.78	3.14%	$ 0.1	490	88	255	N
28-Nov-18	11.00%	3-Dec-18	-6.56%	6	8,640	$ 106.93	$ 127.85	6.85%				*Tick data is missing*	
17-Dec-18	12.45%	21-Dec-18	-5.99%	5	7,200	$ 83.65	$ 118.97	9.16%	$ 0.1	315	122	233	N
28-Dec-18	20.07%	31-Dec-18	-5.69%	4	5,760	$ 137.27	$ 146.04	11.36%	$ 0.1	235	85	244	N

Table 13b: Summary of Statistical Test for Bubble: Ethereum (2018, 7%)

This table shows the results of the statistical test that intends to identify whether Ethereum was in financial bubble during 2018. I consider all periods in 2018 for which Ethereum showed a daily price movement of at least +/- 7%. A period for which Ethereum may be in bubble, starts when daily price movement of Ethereum, measured by the daily return for the day, is at least +7% (bubble birth), and ends when daily price movement, measured by the daily return for the day, is at least -7% (bubble death). The test gives two results: bubble / no bubble. Bubble is said to occur when the numerical integral of (price/ volatility^2) from the minimum price to the maximum price during the period, is lower than or equal to the cut-off value of 225. If the numerical integral of (price/ volatility^2) from the minimum price to the maximum price during the period, is greater than the cut-off value of 225, then there is no bubble during the period. The cut-off value is determined from the value of the numerical integral using the historic data.

Start date	Daily change	End date	Daily change	Days	Minutes	Min Price	Max Price	Daily volatility	Price step	N	Avg. F-Z vol.	Sum(x)	Bubble?
20-Jan-18	10.98%	21-Jan-18	-8.64%	2	2,880	$ 1,010.00	$ 1,164.00	13.87%	$ 0.1	1383	164	185	Y
24-Jan-18	8.16%	30-Jan-18	-8.77%	7	10,080	$ 958.00	$ 1,228.70	7.58%	$ 0.1	2393	191	142	Y
6-Feb-18	12.48%	8-Mar-18	-6.80%	31	44,640	$ 564.56	$ 982.84	5.21%	$ 0.1	3387	387	195	Y
3-Apr-18	7.92%	4-Apr-18	-8.87%	2	2,880	$ 371.08	$ 418.69	11.87%	$ 0.1	455	132	252	N
12-Apr-18	14.85%	23-May-18	-10.06%	42	60,480	$ 413.00	$ 830.55	5.71%	$ 0.1	3664	112	262	N
29-May-18	10.64%	10-Jun-18	-11.65%	13	18,720	$ 504.00	$ 628.00	5.16%	$ 0.1	1147	100	473	N
14-Jun-18	9.10%	22-Jun-18	-12.06%	9	12,960	$ 450.00	$ 546.26	6.19%	$ 0.1	898	96	326	N
17-Aug-18	10.51%	18-Aug-18	-7.26%	2	2,880	$ 283.01	$ 321.08	12.57%	$ 0.1	376	26	302	N
13-Sep-18	15.31%	17-Sep-18	-10.88%	5	7,200	$ 182.60	$ 226.86	9.62%	$ 0.1	407	74	322	N
21-Sep-18	10.47%	24-Sep-18	6.81%	4	5,760	$ 221.12	$ 254.55	7.42%	$ 0.1	312	70	252	N
27-Sep-18	6.91%	11-Oct-18	-11.65%	15	21,600	$ 185.41	$ 238.00	4.95%	$ 0.1	410	58	279	N
15-Oct-18	8.72%	14-Nov-18	-10.72%	31	44,640	$ 165.00	$ 222.78	3.14%	$ 0.1	490	88	255	N
28-Nov-18	11.00%	5-Dec-18	-7.54%	8	11,520	$ 102.21	$ 127.85	6.41%				*Tick data is missing*	
17-Dec-18	12.45%	25-Dec-18	-6.85%	9	12,960	$ 83.65	$ 159.01	8.11%	$ 0.1	691	125	275	N

Table 13c: Summary of Statistical Test for Bubble: Ethereum (2018, 10%)

This table shows the results of the statistical test that intends to identify whether Ethereum was in financial bubble during 2018. I consider all periods in 2018 for which Ethereum showed a daily price movement of at least +/- 10%. A period for which Ethereum may be in bubble, starts when daily price movement of Ethereum, measured by the daily return for the day, is at least +10% (bubble birth), and ends when daily price movement, measured by the daily return for the day, is at least -10% (bubble death). The test gives two results: bubble / no bubble. Bubble is said to occur when the numerical integral of (price/ volatility^2) from the minimum price to the maximum price during the period, is lower than or equal to the cut-off value of 235. If the numerical integral of (price/ volatility^2) from the minimum price to the maximum price during the period, is greater than the cut-off value of 235, then there is no bubble during the period. The cut-off value is determined from the value of the numerical integral using the historic data.

Start date	Daily change	End date	Daily change	Days	Minutes	Min Price	Max Price	Daily volatility	Price step	N	Avg. F-Z vol.	Sum(x)	Bubble?
2-Jan-18	14.13%	16-Jan-18	-19.89%	15	21,600	$ 750.77	$ 1,420.00	9.52%	$ 0.1	5459	384	192	Y
20-Jan-18	10.98%	2-Feb-18	10.90%	14	20,160	$ 740.91	$ 1,228.70	7.71%	$ 0.1	3509	333	187	Y
6-Feb-18	12.48%	14-Mar-18	-11.24%	37	53,280	$ 564.56	$ 982.84	5.36%	$ 0.1	3836	350	192	Y
12-Apr-18	12.85%	25-Apr-18	-12.18%	14	20,160	$ 413.00	$ 710.00	6.71%	$ 0.1	2462	125	250	N
3-May-18	13.19%	23-May-18	-10.06%	21	30,240	$ 564.00	$ 830.55	5.23%	$ 0.1	2344	131	245	N
29-May-18	10.64%	10-Jun-18	-11.65%	13	18,720	$ 504.00	$ 628.00	5.16%	$ 0.1	1147	100	473	N
17-Aug-18	10.51%	5-Sep-18	-19.78%	20	28,800	$ 226.69	$ 321.08	6.23%	$ 0.1	765	73	501	N
13-Sep-18	15.31%	17-Sep-18	-10.88%	5	7,200	$ 182.60	$ 226.86	9.62%	$ 0.1	407	74	322	N
21-Sep-18	10.47%	11-Oct-18	-15.65%	31	44,640	$ 185.41	$ 254.55	5.14%	$ 0.1	598	73	259	N
28-Nov-18	11.00%	6-Dec-18	-10.36%	9	12,960	$ 91.47	$ 127.85	6.77%				*Tick data is missing*	
17-Dec-18	12.45%	27-Dec-18	-11.86%	11	15,840	$ 83.65	$ 159.01	8.90%	$ 0.1	680	117	265	N

Table 14a: Correlation Between Cryptocurrencies' Returns (2016)

This table shows Pearson's correlation coefficients and p-values between the daily returns of top cryptocurrencies in the year 2016. p-values are shown in parenthesis. Level of significance is denoted by stars (*). Correlation between daily returns of any two cryptocurrencies is calculated for the date range for which price data is available in both cryptocurrencies. Pearson's correlation coefficients and p-values are estimated using *proc corr* in *SAS*. A correlation value close to 1.0 indicates a strong positive linear relationship between the two variables. On the other hand, correlation value close to -1.0 indicates strong negative linear relationship between the two variables. The daily prices of the cryptocurrencies are sourced from: *http://www.investing.com.* Following are the dates in 2016 for which daily prices of cryptocurrencies are available:

BTC (Bitcoin): 01-Jan-2016 to 31-Dec-2016

ETH (Ethereum): 09-May-2016 to 31-Dec-2016

XRP (Ripple): NA

BCH (Bitcoin Cash): NA

	BTC	ETH	XRP	BCH
BTC	1.000			
ETH	0.254 (0.0001)***	1.000		
XRP	NA	NA	1.000	
BCH	NA	NA	NA	1.000

* p < 0.1
** p < 0.05
*** p < 0.01

Table 14b: Correlation Between Cryptocurrencies' Returns (2017)

This table shows Pearson's correlation coefficients and p-values between the daily returns of top cryptocurrencies in the year 2017. p-values are shown in parenthesis. Level of significance is denoted by stars (*). Correlation between daily returns of any two cryptocurrencies is calculated for the date range for which price data is available in both cryptocurrencies. Pearson's correlation coefficients and p-values are estimated using *proc corr* in *SAS*. A correlation value close to 1.0 indicates a strong positive linear relationship between the two variables. On the other hand, correlation value close to -1.0 indicates strong negative linear relationship between the two variables. The daily prices of the cryptocurrencies are sourced from: *http://www.investing.com.* Following are the dates in 2017 for which daily prices of cryptocurrencies are available:

BTC (Bitcoin): 01-Jan-2017 to 31-Dec-2017

ETH (Ethereum): 01-Jan-2017 to 31-Dec-2017

XRP (Ripple): 17-Jan-2017 to 31-Dec-2017

BCH (Bitcoin Cash): 14-Aug-2017 to 31-Dec-2017

(Bitcoin Cash was forked out of Bitcoin in Aug-2017)

	BTC	ETH	XRP	BCH
BTC	1.000			
ETH	0.390	1.000		
	(0.0001)***			
XRP	0.150	0.170	1.000	
	(0.005)***	(0.0015)***		
BCH	0.051	0.303	0.107	1.000
	(0.548)	(0.0003)***	(0.2113)	

* $p < 0.1$
** $p < 0.05$
*** $p < 0.01$

Table 14c: Correlation Between Cryptocurrencies' Returns (2018)

This table shows Pearson's correlation coefficients and p-values between the daily returns of top cryptocurrencies in the year 2018. p-values are shown in parenthesis. Level of significance is denoted by stars (*). Correlation between daily returns of any two cryptocurrencies is calculated for the date range for which price data is available in both cryptocurrencies. Pearson's correlation coefficients and p-values are estimated using *proc corr* in *SAS*. A correlation value close to 1.0 indicates a strong positive linear relationship between the two variables. On the other hand, correlation value close to -1.0 indicates strong negative linear relationship between the two variables. The daily prices of the cryptocurrencies are sourced from: *http://www.investing.com.* Following are the dates in 2018 for which daily prices of cryptocurrencies are available:

BTC (Bitcoin): 01-Jan-2018 to 31-Dec-2018

ETH (Ethereum): 01-Jan-2018 to 31-Dec-2018

XRP (Ripple): 01-Jan-2018 to 31-Dec-2018

BCH (Bitcoin Cash): 01-Jan-2018 to 28-Oct-2018

(In Nov-2018, Bitcoin Cash was forked into Bitcoin ABC and Bitcoin SV respectively)

	BTC	ETH	XRP	BCH
BTC	1.000			
ETH	0.820 (0.0001)***	1.000		
XRP	0.713 (0.0001)***	0.765 (0.0001)***	1.000	
BCH	0.809 (0.0001)***	0.794 (0.0001)***	0.705 (0.0001)***	1.000

* $p < 0.1$
** $p < 0.05$
*** $p < 0.01$

Table 15a: Correlation of Cryptocurrencies' Returns with Various Other Asset Classes (2016)

This table shows Pearson's correlation coefficients and p-values between the daily returns of Bitcoin and other cryptocurrencies with the daily returns posted by other assets like S&P500, Tech index, Gold and Crude Oil for the year 2016. p-values are shown in parenthesis. Tech Index is the Index of technology companies listed in NYSE. Since cryptocurrencies trade like stocks and are based on financial technology, their performance is compared with both the S&P 500 index and the Tech Index. Daily returns on the day 't' are defined as $\log_e(P_t/P_{t-1})$. While cryptocurrencies can trade of all the days of a year, other assets trade only when the stock markets/ commodity markets open. Only those days are considered for which trading occurred in both crypto exchanges and stock markets/ commodity markets. The daily prices are sourced from: *http://www.investing.com.* Pearson's correlation coefficients and p-values are calculated using *proc corr* in *SAS*. A correlation value close to 1.0 indicates a strong positive linear relationship between the two variables. On the other hand, correlation value close to -1.0 indicates strong negative linear relationship between the two variables. Following are the dates in 2016 for which daily prices of cryptocurrencies are available:

BTC (Bitcoin): 01-Jan-2016 to 31-Dec-2016

ETH (Ethereum): 09-May-2016 to 31-Dec-2016

XRP (Ripple): NA

BCH (Bitcoin Cash): NA

	S&P 500	Tech Index	Gold	Crude Oil
BTC	-0.001	0.107	0.046	0.034
	(0.990)	(0.866)	(0.429)	(0.580)
ETH	-0.078	-0.047	-0.029	-0.068
	(0.323)	(0.553)	(0.689)	(0.374)
XRP	NA	NA	NA	NA
BCH	NA	NA	NA	NA

* p < 0.1
** p < 0.05
*** p < 0.01

Table 15b: Correlation of Cryptocurrencies' Returns with Various Other Asset Classes (2017)

This table shows Pearson's correlation coefficients and p-values between the daily returns of Bitcoin and other cryptocurrencies with the daily returns posted by other assets like S&P500, Tech index, Gold and Crude Oil for the year 2017. p-values are shown in parenthesis. Tech Index is the Index of technology companies listed in NYSE. Since cryptocurrencies trade like stocks and are based on financial technology, their performance is compared with both the S&P 500 index and the Tech Index. Daily returns on the day 't' are defined as $\log_e(P_t/P_{t-1})$. While cryptocurrencies can trade of all the days of a year, other assets trade only when the stock markets/ commodity markets open. Only those days are considered for which trading occurred in both crypto exchanges and stock markets/ commodity markets. The daily prices are sourced from: *http://www.investing.com.* Pearson's correlation coefficients and p-values are calculated using *proc corr* in *SAS*. A correlation value close to 1.0 indicates a strong positive linear relationship between the two variables. On the other hand, correlation value close to -1.0 indicates strong negative linear relationship between the two variables. Following are the dates in 2017 for which daily prices of cryptocurrencies are available:

BTC (Bitcoin): 01-Jan-2017 to 31-Dec-2017

ETH (Ethereum): 01-Jan-2017 to 31-Dec-2017

XRP (Ripple): 17-Jan-2017 to 31-Dec-2017

BCH (Bitcoin Cash): 14-Aug-2017 to 31-Dec-2017

	S&P 500	Tech Index	Gold	Crude Oil
BTC	0.056	0.051	-0.015	-0.020
	(0.379)	(0.422)	(0.798)	(0.743)
ETH	0.119	0.083	0.112	0.027
	(0.059)*	(0.189)	(0.055)*	(0.666)
XRP	-0.032	-0.006	0.040	0.053
	(0.626)	(0.927)	(0.503)	(0.406)
BCH	-0.218	-0.107	0.038	0.215
	(0.033)**	(0.300)	(0.691)	(0.032)**

* $p < 0.1$
** $p < 0.05$
*** $p < 0.01$

Table 15c: Correlation of Cryptocurrencies' Returns with Various Other Asset Classes (2018)

This table shows Pearson's correlation coefficients and p-values between the daily returns of Bitcoin and other cryptocurrencies with the daily returns posted by other assets like S&P500, Tech index, Gold and Crude Oil for the year 2018. p-values are shown in parenthesis. Tech Index is the Index of technology companies listed in NYSE. Since cryptocurrencies trade like stocks and are based on financial technology, their performance is compared with both the S&P 500 index and the Tech Index. Daily returns on the day 't' are defined as $\log_e(P_t/P_{t-1})$. While cryptocurrencies can trade of all the days of a year, other assets trade only when the stock markets/ commodity markets open. Only those days are considered for which trading occurred in both crypto exchanges and stock markets/ commodity markets. The daily prices are sourced from: *http://www.investing.com.* Pearson's correlation coefficients and p-values are calculated using *proc corr* in *SAS*. A correlation value close to 1.0 indicates a strong positive linear relationship between the two variables. On the other hand, correlation value close to -1.0 indicates strong negative linear relationship between the two variables. Following are the dates in 2018 for which daily prices of cryptocurrencies are available:

BTC (Bitcoin): 01-Jan-2018 to 31-Dec-2018
ETH (Ethereum): 01-Jan-2018 to 31-Dec-2018
XRP (Ripple): 01-Jan-2018 to 31-Dec-2018
BCH (Bitcoin Cash): 01-Jan-2018 to 28-Oct-2018

	S&P 500	Tech Index	Gold	Crude Oil
BTC	0.074	0.091	-0.030	-0.005
	(0.241)	(0.153)	(0.592)	(0.938)
ETH	0.096	0.107	0.010	-0.001
	(0.129)	(0.091)*	(0.864)	(0.990)
XRP	0.110	0.101	0.011	0.010
	(0.082)*	(0.111)	(0.850)	(0.866)
BCH	0.063	0.039	-0.067	0.088
	(0.362)	(0.571)	(0.284)	(0.191)

* $p < 0.1$
** $p < 0.05$
*** $p < 0.01$

Figure 1: Daily Bitcoin Price

This figure shows daily price movement in Bitcoin for the period: 1-Jan-2016 to 31-Dec-2018. Highest Bitcoin price: $19,289.79 (closing) on 17-Dec-2017. The price peaked at $19,783.21 on same day.

Bitcoin price on 1-Jan-2016: $432.33; Bitcoin price on 30-Jun-2016: $637.96

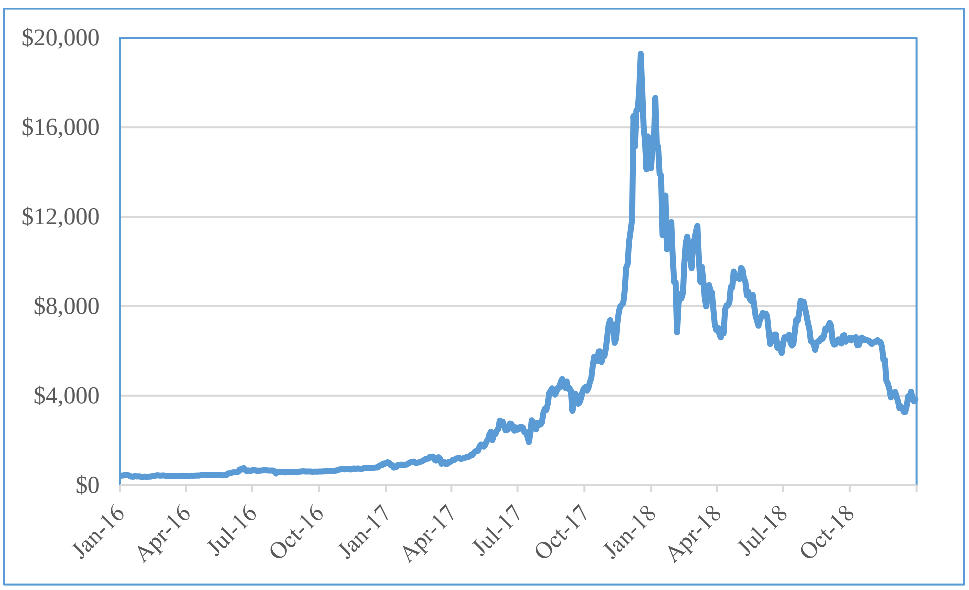
Maximum Bitcoin price (2016): $967.48; Minimum Bitcoin price (2016): $373.04

Bitcoin price on 1-Jan-2017: $997.73; Bitcoin price on 30-Jun-2017: $2,477.64

Maximum Bitcoin price (2017): $19,289.79; Minimum Bitcoin price (2017): $785.22

Bitcoin price on 1-Jan-2018: $15,005.86; Bitcoin price on 30-Jun-2018: $5,908.70

Maximum Bitcoin price (2018): $17,319.20; Minimum Bitcoin price (2018): $3,271.24



[Data source: http://www.blockchain.com]

Figure 2: Daily Ethereum Price

This figure shows daily price movement in Ethereum for the period: 27-May-2016 to 31-Dec-2018. Highest Ethereum price: $1,386.02 (closing) on 13-Jan-2018. The price peaked at $1,419.96 on same day.

Ethereum price on 27-May-2016: $11.25; Ethereum price on 30-Jun-2016: $12.49

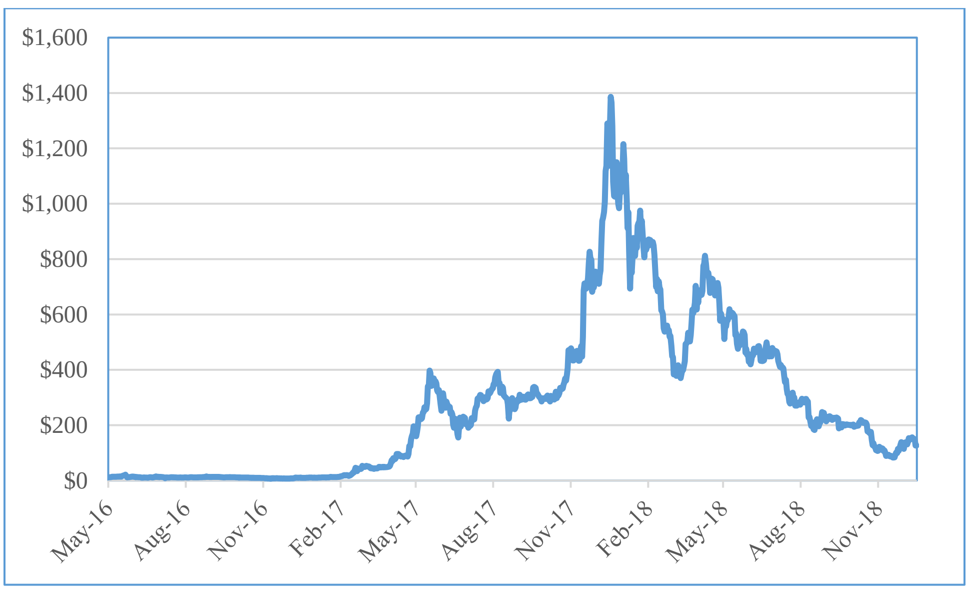
Maximum Ethereum price (2016): $21.09; Minimum Ethereum price (2016): $6.69

Ethereum price on 1-Jan-2017: $8.18; Ethereum price on 30-Jun-2017: $280.04

Maximum Ethereum price (2017): $826.65; Minimum Ethereum price (2017): $8.18

Ethereum price on 1-Jan-2018: $759.03; Ethereum price on 30-Jun-2018: $453.29

Maximum Ethereum price (2018): $1,386.02; Minimum Ethereum price (2018): $83.0



[Data source: http://www.cryptodatadownload.com]

Figure 3: Hash Rate

This figure shows monthly average hash rate at which Bitcoin network is performing. Hash rate is measured in tera hashes (trillions of hashes) per second. The period is from Jan-2016 to Dec-2018.

Monthly average hash rates:

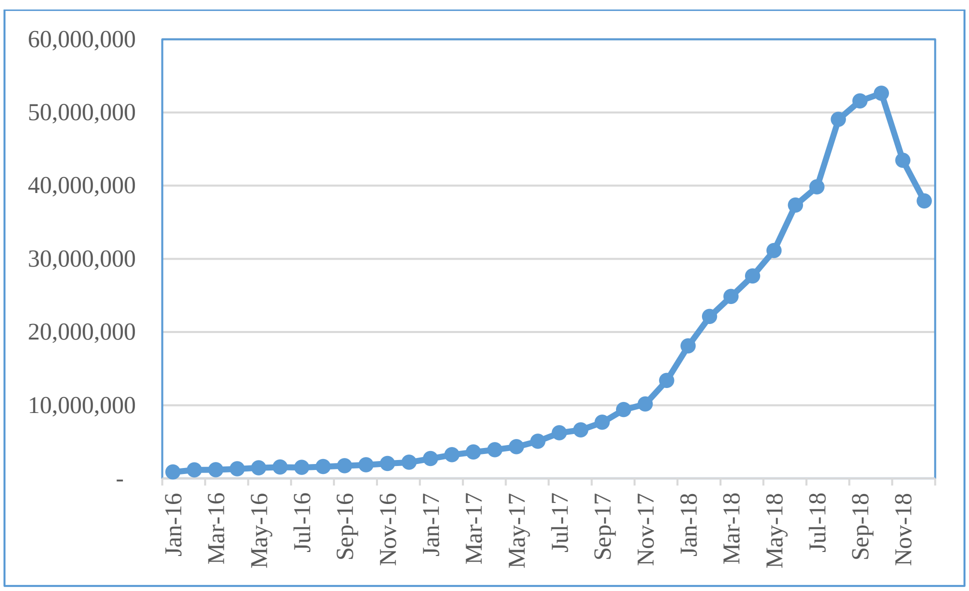
Jan-2016: 0.85 mn tera hashes/s; Jun-2016: 1.54 mn tera hashes/s;

Jan-2017: 2.76 mn tera hashes/s; Jun-2017: 5.07 mn tera hashes/s;

Nov-2017: 9.87 mn tera hashes/s; Dec-2017: 13.43 mn tera hashes/s;

Jan-2018: 18.09 mn tera hashes/s; Jun-2018: 37.33 mn tera hashes/s;

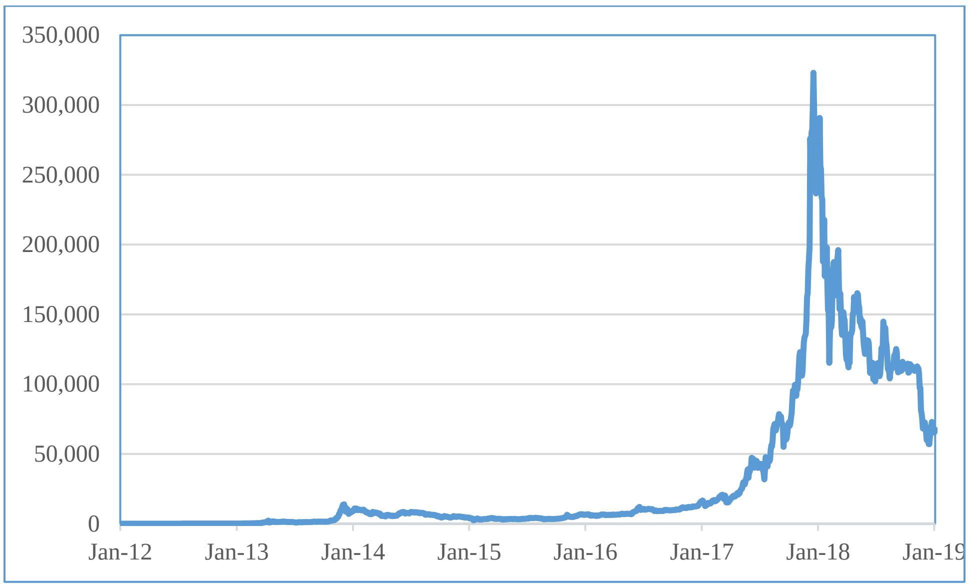
Oct-2018: 52.61 mn tera hashes/s; Dec-2018: 37.89 mn tera hashes/s



[Data Source: http://www.blockchain.com]

Figure 4: Bitcoin Market Capitalization

This figure shows Bitcoin market capitalization in million US dollars. Market capitalization of Bitcoin is measured as a product of Bitcoin unit price (in dollars) and number of Bitcoins in circulation, i.e. coins that are already mined. The market price of Bitcoin measured as daily average market price across major Bitcoin exchanges.

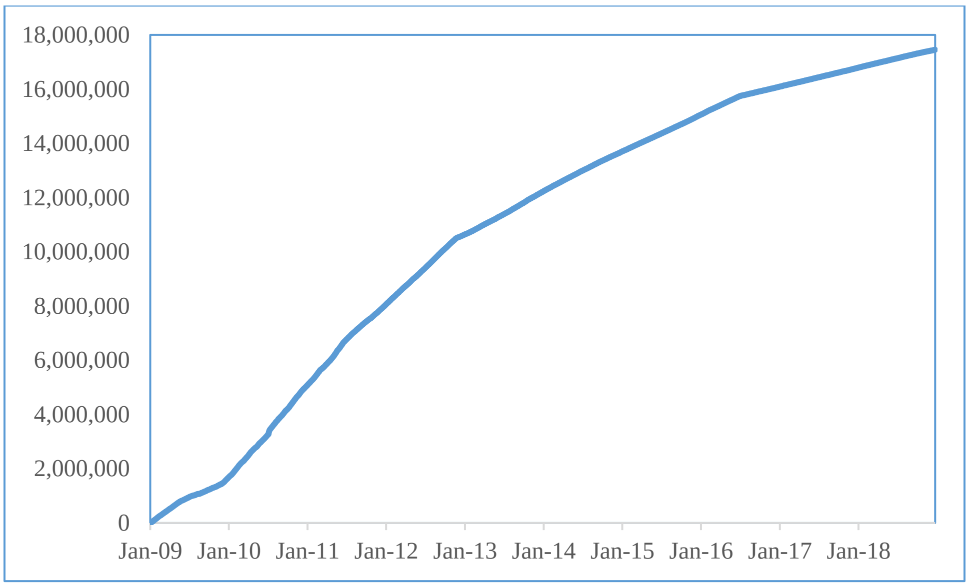


[Data source: http://www.blockchain.com]

Figure 5: Bitcoins in Circulation

This figure shows number of Bitcoins that were mined and were in circulation. Bitcoins are mined round the clock on Bitcoin exchanges. New Bitcoins are awarded in the block of 10 minutes to miners who successfully verify Bitcoin transactions.

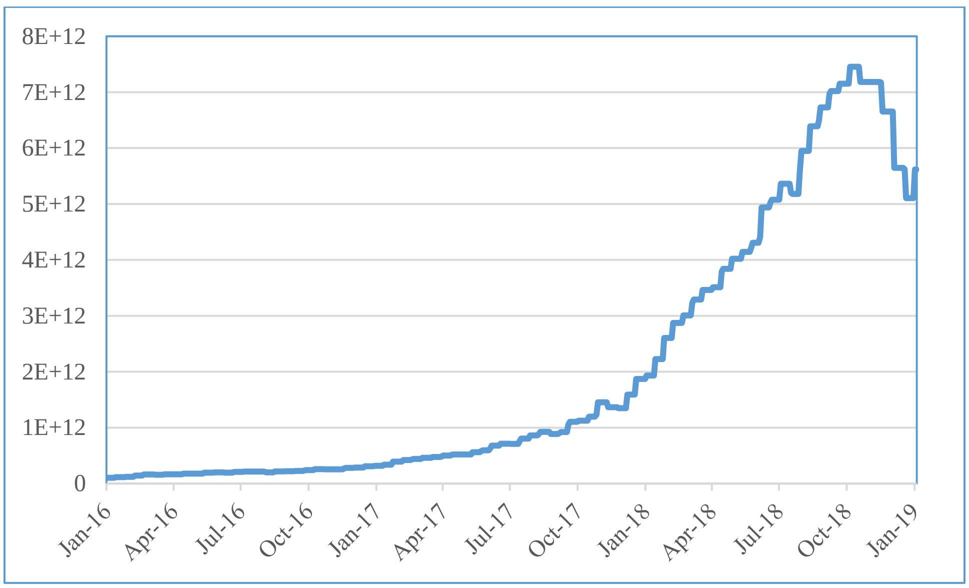
Bitcoin supply crossed 10 million tokens on 22-Sep-2012, 12 million tokens on 17-Nov-2013, 14 million tokens on 30-Mar-2015, 16 million tokens on 21-Nov-2016 and 17 million tokens on 26-Apr-2018. Final supply of Bitcoin: (nearly, but never equal to) 21 million tokens.



[Data source: http://www.blockchain.com]

Figure 6: Difficulty of Mining

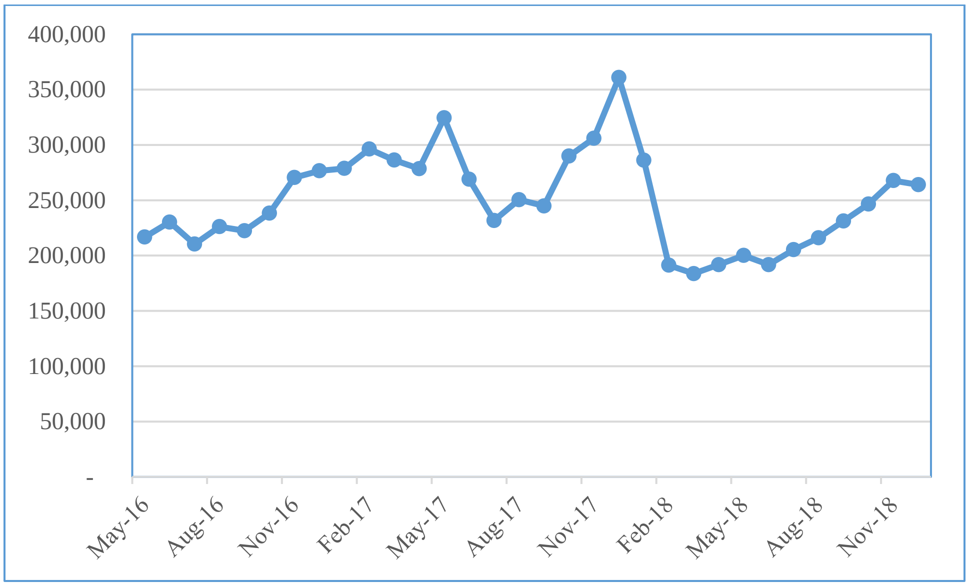
This figure shows monthly average difficulty of mining vs. hashing power employed by network of miners. The character ‘E’ denotes ’10 raised to the power of’. The period is from Jan-2017 to Dec-2018. The difficulty is rising constantly, with more miners joining the bandwagon, and trying to solve mathematical problems for transaction verification (i.e. mining) using high power systems, but total amount of coin award remaining constant per block of 10 minutes. Hence, this is a classic case of demand and supply, with supply remaining constant and demand rising quickly, thus resulting in quick rise in difficulty level of mining. Till Nov-2017, the difficulty level was rising at a constant rate. After Nov-2017, the rate of increase in the difficulty level increased, but started falling again in Dec-2018



[Data Source: http://www.blockchain.com]

Figure 7: Average Daily Bitcoin Transactions

This figure shows monthly average daily (confirmed) Bitcoin transactions across the globe. Number of confirmed Bitcoin transactions peaked to 360,989 in Dec-2017. From there, it fell to 191,293 in Feb-2018, before rising again to 263,993 in Dec-2018.



[Data Source: http://www.blockchain.com]

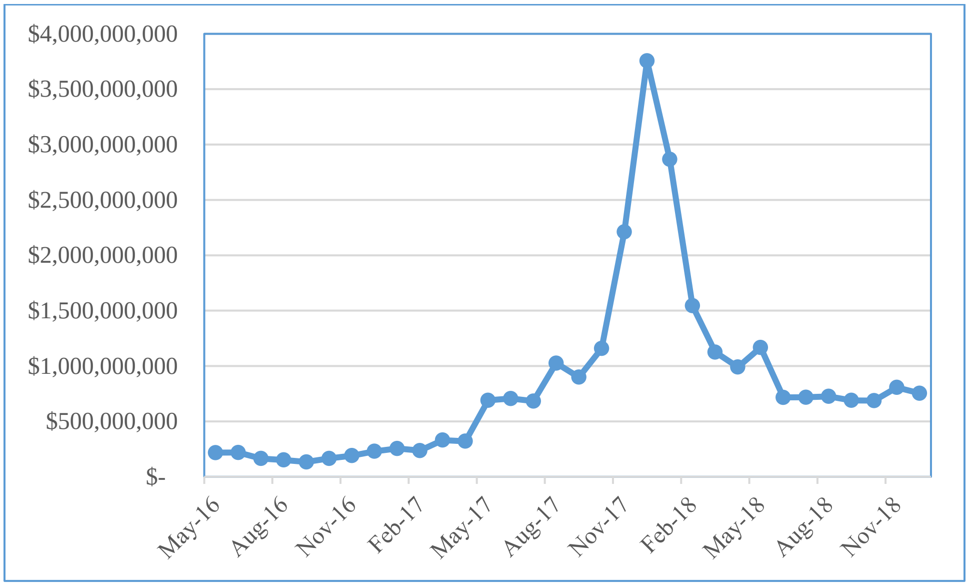
Figure 8: Average Daily Bitcoin Transactions Value

This figure shows the monthly average daily Bitcoin transactions value, in US dollars. There was a steep climb in average daily transaction value during Sep-2017 to Dec-2017, after which it started falling.

Average daily Bitcoin transaction value ($mn):

Sep-2017: $899 mn; Oct-2017: $1,158 mn; Nov-2017: $2,210 mn; Dec-2017: $3,756 mn; Jan-2018: $2,867 mn; Feb-2018: $1,544 mn; May-2018: $1,167 mn. After May-2018, the average daily transaction value has been around $750 mn.

Bitcoin transaction value is directly related to the Bitcoin price, and since Bitcoin price peaked in Q4-2017, so did the average daily transaction value.



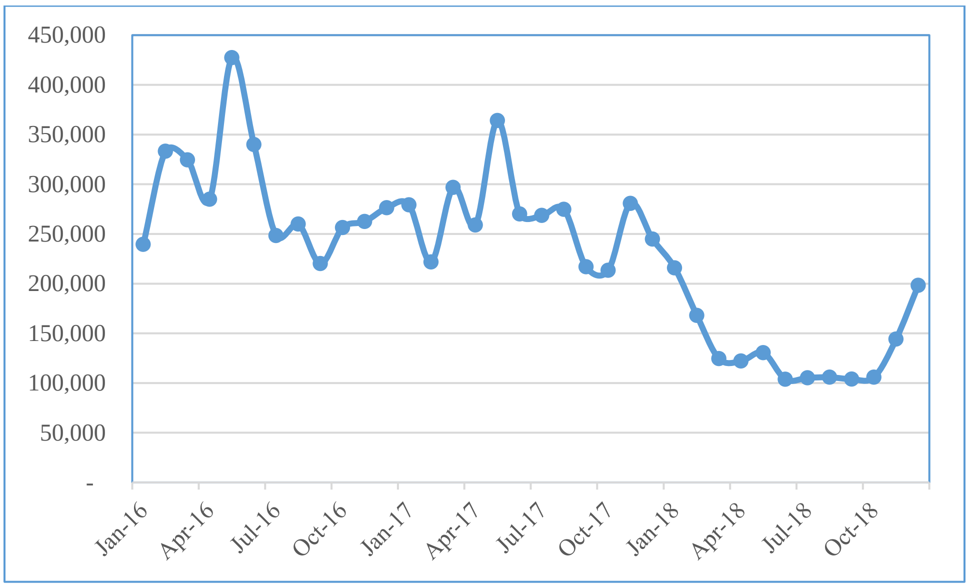
[Data Source: http://www.blockchain.com]

Figure 9: Average Daily Bitcoin Transaction Volume

This figure shows monthly average of daily Bitcoin transaction volume in terms of Bitcoins, from Jan-2016 to Dec-2018.

Average daily Bitcoin transaction volume:

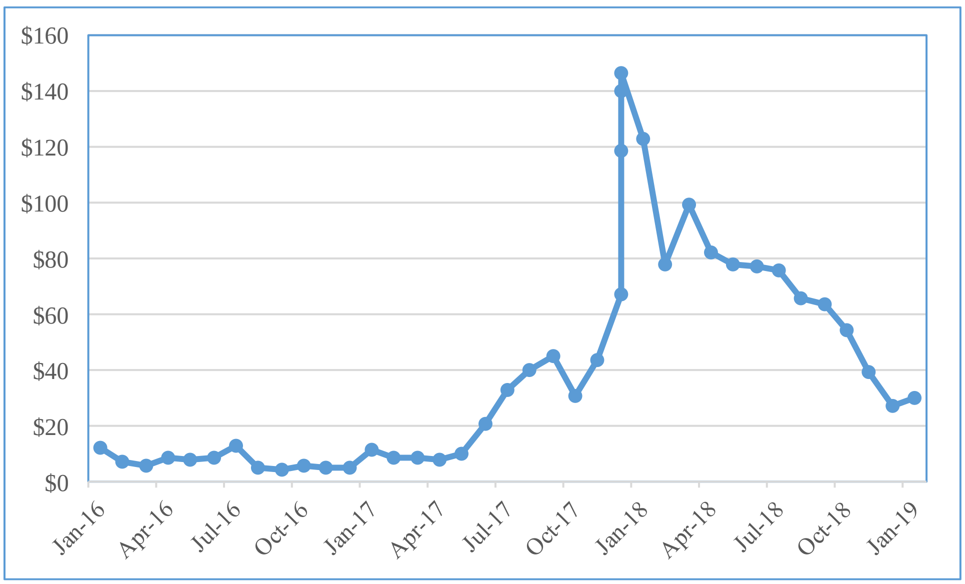
Jan-2016: 239,379 Bitcoins; Jun-2016: 339,911 Bitcoins; Jan-2017: 279,290 Bitcoins; Jun-2017: 270,006 Bitcoins; Jan-2018: 215,835 Bitcoins; Jun-2018: 103,683 Bitcoins; Dec-2018: 198,245 Bitcoins.



[Data Source: http://www.blockchain.com]

Figure 10: Cost Per Bitcoin Transaction

This figure shows the daily cost per Bitcoin transaction in US dollars during the period Jan-2016 to Dec-2018. Cost per Bitcoin transaction is calculated by dividing total miners' revenue by total number of Bitcoin transactions for the day. The transaction cost was very low in 2016; it started climbing sharply in 2014. The transaction cost peaked on 25-Dec-2017, with value $146.6 per transaction. After Bitcoin price started falling in Jan-2018, Bitcoin transaction cost also fell along with that. Transaction cost on 31-Dec-2018: $31.7 per transaction. Earlier peak was on 1-Jan-2014: $90.2 per transaction.



[Data Source: http://www.blockchain.com]

Figure 11: Average Bitcoin Wallet Users

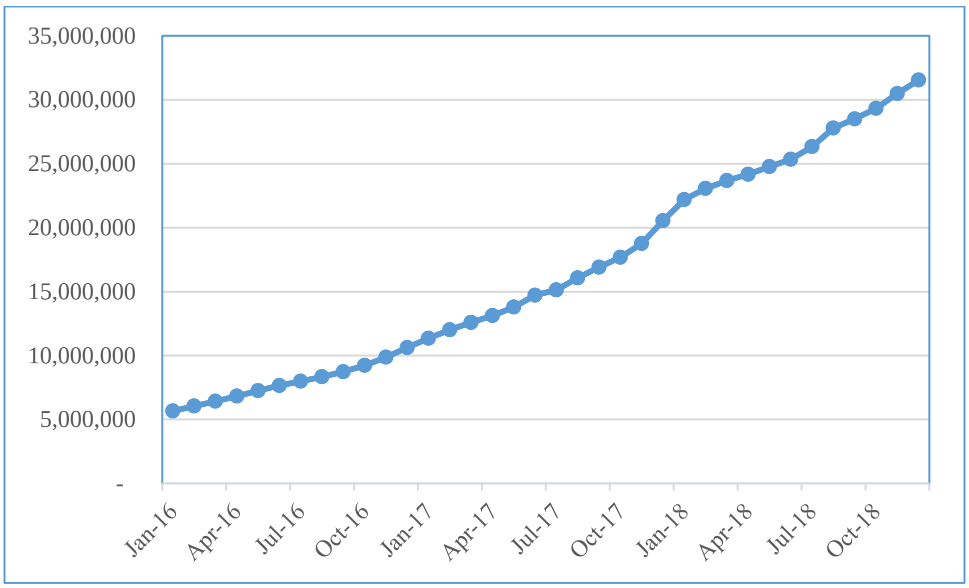
This figure shows average Bitcoin wallet users every month from Jan-2016 to Dec-2018. As the figure shows, the Bitcoin wallet users are continuously increasing.

Average number of Bitcoin wallet users:

Jan-2016: 5.66 mn; Jun-2016: 7.65 mn;

Jan-2017: 11.35 mn; Jun-2017: 14.72 mn;

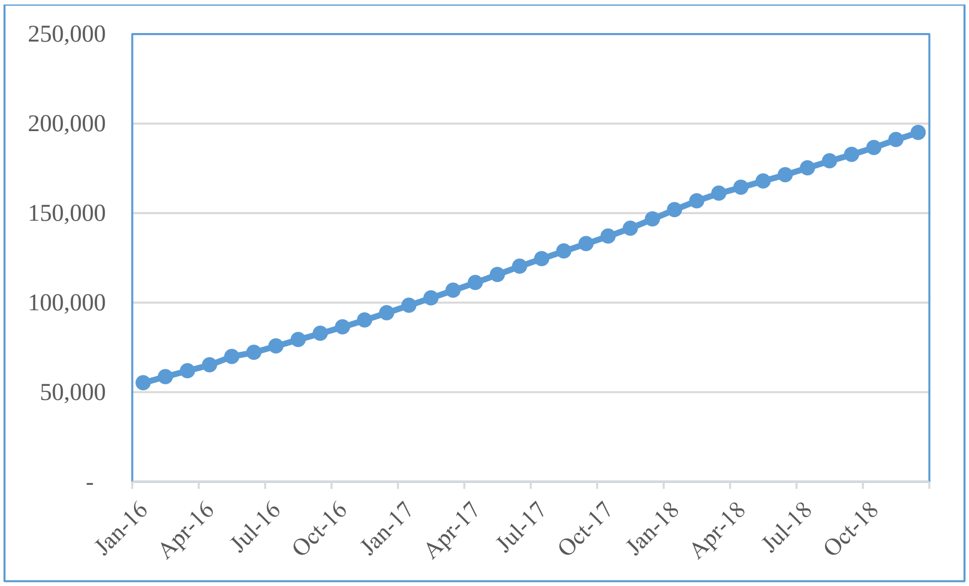
Jan-2018: 22.19 mn; Jun-218: 25.35 mn; Dec-2018: 31.54 mn



[Source: http://www.blockchain.com]

Figure 12: Average Blockchain Size

This figure shows average blockchain size in MB from Jan-2016 to Dec-2018. The total size of blockchain includes all block headers and transactions, but does not include database indexes. With more and more transactions Bitcoin transactions, average blockchain size is continuously increasing. With all transactions, the blockchain size on 31-Dec-2018 was just 196,966.7 MB (approximately 197 GB). This makes blockchain an efficient holder of transaction data.



[Data Source: http://www.blockchain.com]

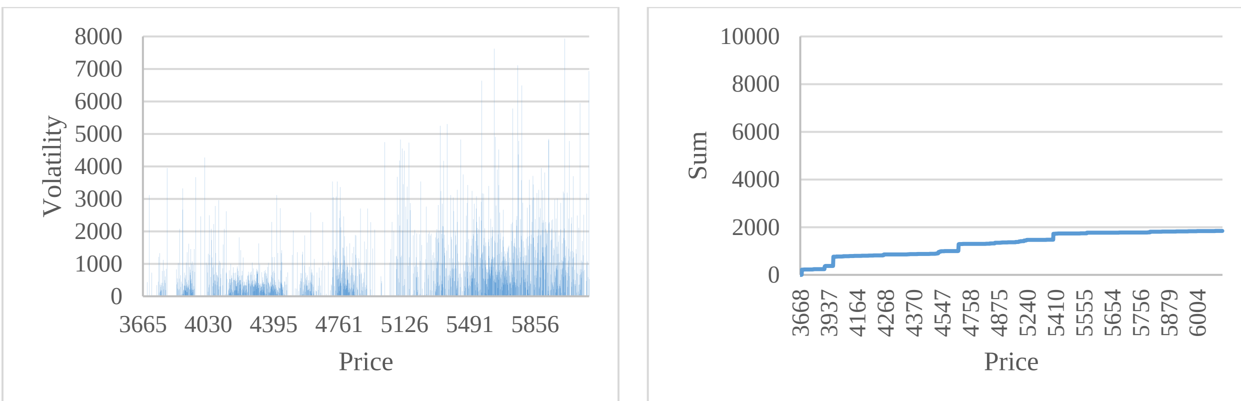
Figure 13a: Volatility vs. Price charts (bubble)

This figure shows representative price volatility vs. price charts, when the test is run on Bitcoin tick data, and is conclusive about presence of a bubble during the period for which the test is run. Price volatility is estimated by running Florens-Zmirou volatility estimator on the Bitcoin tick data. In case of bubble, price volatility is very high and generally increasing with price.

Panel A shows the chart of estimated price volatility vs. price during the period 25-Sep-2017 to 24-Oct-2017

Panel B shows the chart of estimated price volatility vs. price during the period 6-Feb-2018 to 21-Feb-2018

Panel A



Panel B

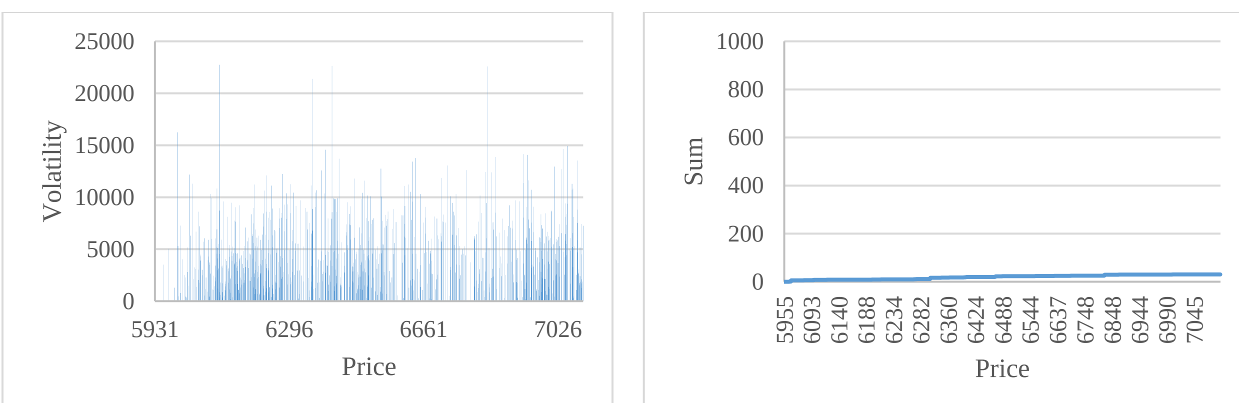


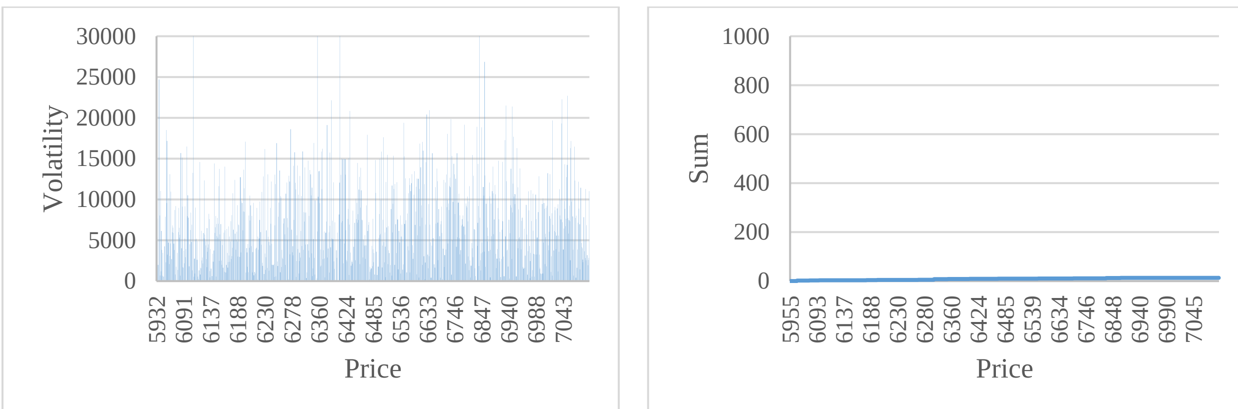
Figure 13a (continued): Volatility vs. Price charts (bubble)

This figure shows representative price volatility vs. price charts, when the test is run on Bitcoin tick data, and is conclusive about presence of a bubble during the period for which the test is run. Price volatility is estimated by running Florens-Zmirou volatility estimator on the Bitcoin tick data. In case of bubble, price volatility is very high and generally increasing with price.

Panel C shows the chart of estimated price volatility vs. price during the period 6-Feb-2018 to 14-March-2018

Panel D shows the chart of estimated price volatility vs. price during the period 29-Oct-2017 to 9-Dec-2017

Panel C



Panel D

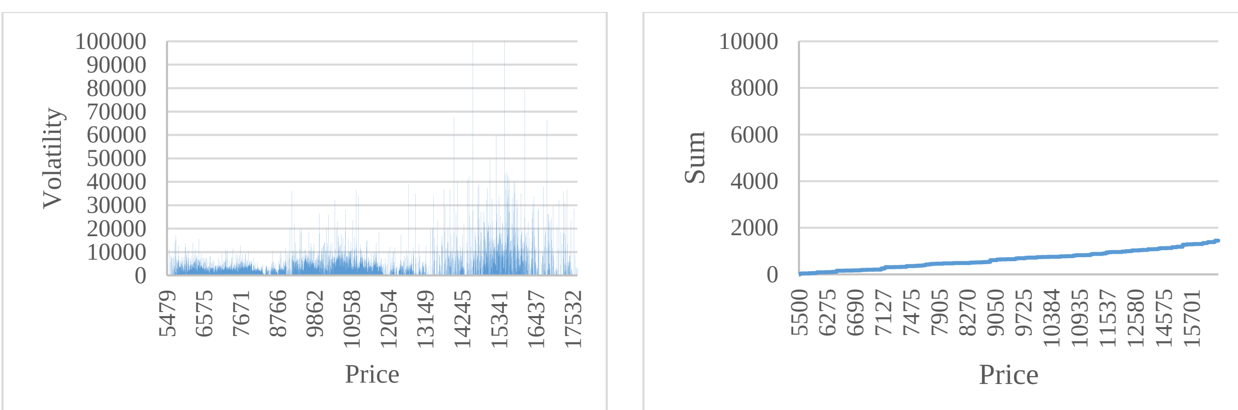


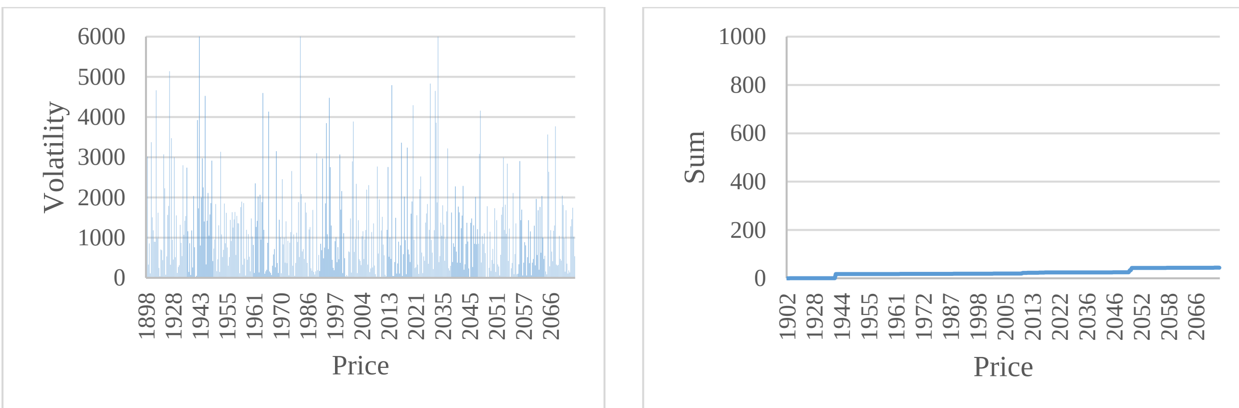
Figure 13a (continued): Volatility vs. Price charts (bubble)

This figure shows representative price volatility vs. price charts, when the test is run on Bitcoin tick data, and is conclusive about presence of a bubble during the period for which the test is run. Price volatility is estimated by running Florens-Zmirou volatility estimator on the Bitcoin tick data. In case of bubble, price volatility is very high and generally increasing with price.

Panel E shows the chart of estimated price volatility vs. price during the period 17-Jul-2017 to 22-Dec-2017

Panel F shows the chart of estimated price volatility vs. price during the period 26-Dec-2017 to 11-Jan-2018

Panel E



Panel F

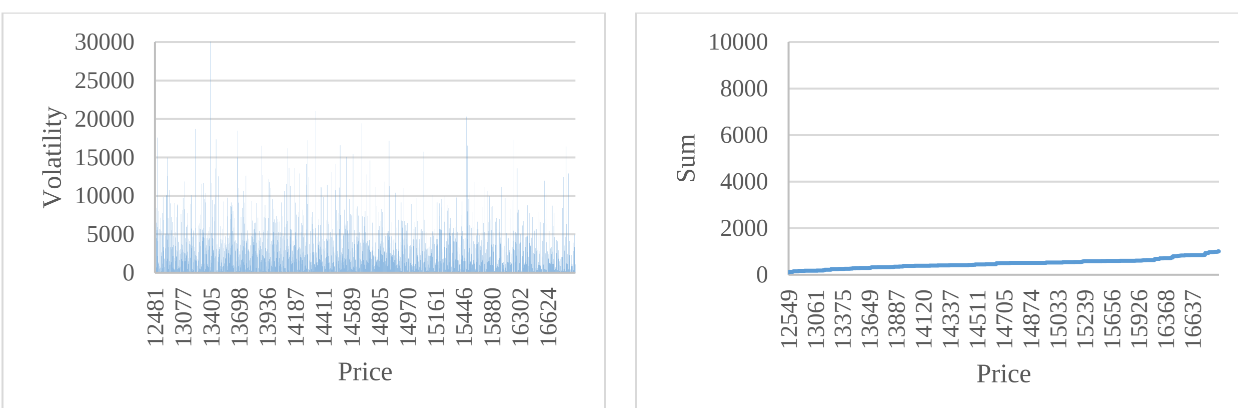


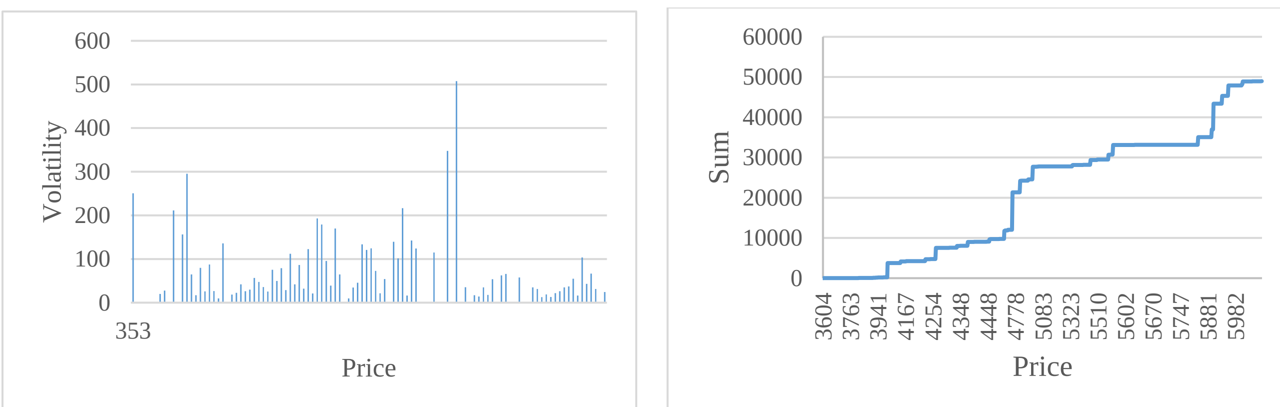
Figure 13b: Volatility vs. Price charts (no bubble)

This figure shows representative price volatility vs. price charts, when the test is run on Bitcoin tick data, and is conclusive about absence of a bubble during the period for which the test is run. Price volatility is estimated by running Florens-Zmirou volatility estimator on the Bitcoin tick data. In case of no bubble, price volatility is low and either decreasing with price, or there is no clear trend, e.g. price volatility may rise and fall frequently during the period.

Panel A shows the chart of estimated price volatility vs. price during the period 7-Jan-2016 to 22-Jan-2016

Panel B shows the chart of estimated price volatility vs. price during the period 24-Jun-2016 to 7-Jul-2016

Panel A



Panel B

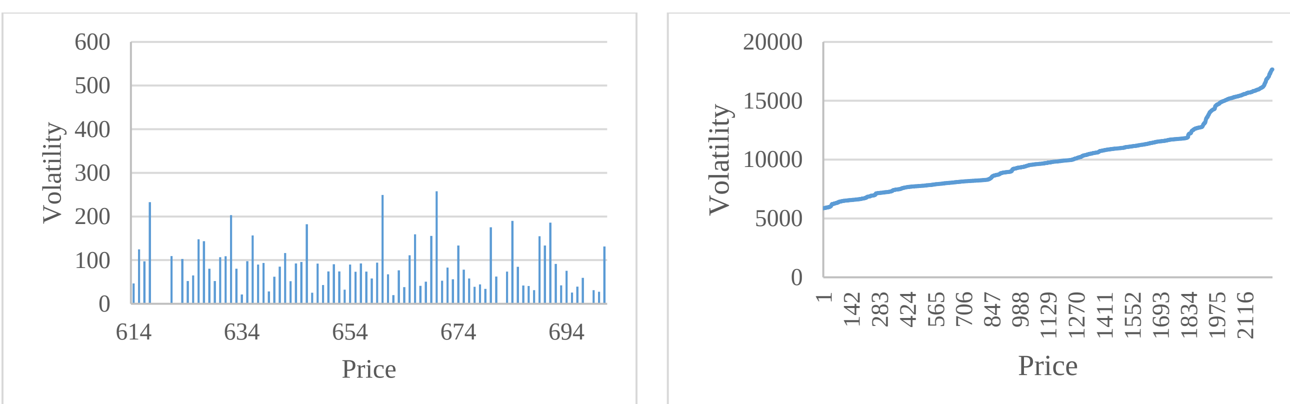


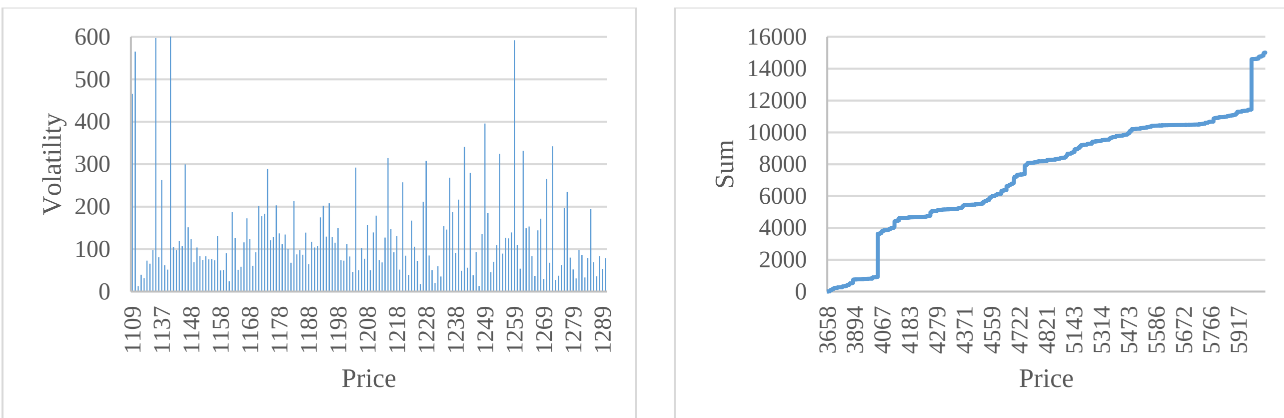
Figure 13b (continued): Volatility vs. Price charts (no bubble)

This figure shows representative price volatility vs. price charts, when the test is run on Bitcoin tick data, and is conclusive about absence of a bubble during the period for which the test is run. Price volatility is estimated by running Florens-Zmirou volatility estimator on the Bitcoin tick data. In case of no bubble, price volatility is low and either decreasing with price, or there is no clear trend, e.g. price volatility may rise and fall frequently during the period.

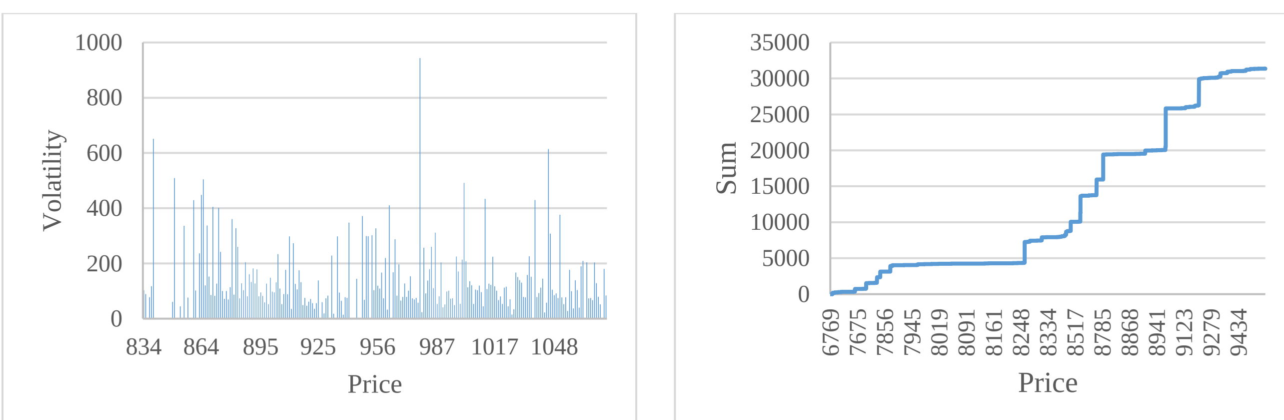
Panel C shows the chart of estimated price volatility vs. price during the period 23-Mar-2017 to 8-Mar-2017

Panel D shows the chart of estimated price volatility vs. price during the period 17-Jan-2017 to 9-Feb-2017

Panel C



Panel D



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