## ESTIMATED COST FUNCTIONS FOR SELECTED

## ORLAHOMA LIVESTOCK AUCTIONS

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## CHAPTER II

INIRODUCWIION

Problem

In the past three decades, livestock auction markets in oklahoma have shown the typical growth pattern characteristic of most new industries. The first sign of new industry is its inception pexiod followet by a period of rapid growth, both in mumbers and capacity. Following this rapid expansion the teveling off period as the demand for them is fulfilled. Eventually a decime in mumbers of auctions will materialize as the low volume firms whth high unit costs fall by the wayside as competition between firms for the avoilable market tightens. Thus, the need manifests itself for incxeased research into the efficiencies of operation that may be obtaned by the existo ing livestock auction markets.

If market operators are to maintain their relative positions in the market, knowledge of increased efficiencies (both physical and eco onomic) must be considered and adopted. The froits of such suctions will increase the material welfare of mot only the market operator, but farmers, consumers, and society $\%$ well.

If one surveys the livestock auctions in oklehoms producing a marketing service, he is instantly sware of the large variation in their scale of operations. For exmmple, one would find auction volumes varying all the way from 4,000 animal units manally to those handing over 100,000 animal units. While some are operating quite efficiently,
others are operating at considerably less than the efficiency that would be possible. In most cases, the inefficiencies are due to incomplete knowledge of costs relations associated with alternative scale of plants, possible innovations, and unforseen changes in the demand for their marketing services.

Within this setting, this study deals primarily with the economic aspects of the problem. However, it is related to physical efficiencies to the degree that costs are influenced by physical relationships and comparative costs a measure of comparative physical efficiency. This provides a means of contresting one firmis costs with anothers, lending additional information that the market operator may use in making decisions as to operational changes.

This research project will not give an easy solution to the ultimate end of reducing operating costs. It is designed to present the physical environment within which auction markets operate, the theory of the firm, and Information regarding volumes and costs of marketing cattle in auction firms selected to represent a large range of operating volume. As such, its specific objectives are:
(1) To examine the general theoretical framework for cost and efficiency studies;
(2) To review alternative methodological approaches to these studies;
(3) To provide an empixical analysis of the cost relationships of certain selected livestock auctions in Oklahoma and
(4) To give an economic analysis of the results in regard to firm size and location under postulated geographical demand for services.

If the above objectives are fully realized, it should provide information which would be useful to present and potential firm operators in formulating decisions as to the scale of operations that may be most efficient for the individual conditions with which they are faced.

It is with these views in mind that this study was initiated.

CHAPTER III

## THEORETICAL FRAMEWORK

The conceptual framework for cost and efficiency firm studies can be based with some alterations, on the logical tenets of the conventional economic theory of production. In this section the theory necessary for evaluating the operations of firms and the postulation of models from which relevant economic parameters may be estimated, will be presented. Particular attention will be directed toward the scale of firm operations that would be most efficient under alternative operating conditions.

A cursory view of the procedure to be followed in the development of the theory seems warranted at this point to provide a logical frame of reference for the reader.

Any firm engaged in the process of production is faced with many complex problems. Its resources must be correctly allocated to any and all products it produces, it must produce only that amount of product which will maximize its profita and, in genexal, must strive to

Discussions of the conventional economic theory of production: Sune Carlson, A Study on the Pure Theory of Production, Iondon: PoS. King and Son, Ltd., 1939.
Paul A. Samuelson, Foundations of Economic Analysis, Cambridge:
Harvard University Press, 1947.
Kenneth E. Boulding, Economic Analysis, New Xork: Harper and Bros., 1948.

Erich Schneider, Pricing and Equilibrium, New York: The Macmillan Company, 1951.
Richard H. Leftwich, The Price System and Resource Allocation, New York: Rinehart and Company, 1955.
obtain as high a degree of efficiency in its operations as possible within its particular type of production environment.

The major portion of this chapter will deal primarily with the theory behind the production process and a presentation of the relevant criteria upon which to base the decision of how to combine resources and how much a firm should produce. In conjunction with determining the least cost combination of resources and the profit maximizing output, total cost functions will be derived. The generation of the costs in production will then be analyzed for both continuous and discrete cases. The last portion of the chapter is devoted to the examination of a few of the more important deterrents to firm operational efficiency such as, frequency of operation and seasonality of cattle marketings.

Assumptions and Definitions
Before a presentation of the relevant theory, let us define a firm and make the necessary restrictive and expository assumptions for the firm analysis that is to follow.

A firm may be defined as an institution which buys raw materials, transforms them in some manner, and then resells the new product with the purpose of making a profit from the tramsition. A plant when in operation is faced with prices for its resources plus the cost of the transformation operation. Also, there $i s$ given in the market a price for the firms finished product. At different levels of output and the accompanying amounts of inputs, the firm is thus faced with varying amounts of costs of production and the subsequent revenue from its sale. With profit maximization as one of its goals, the firm should exect that scale of plant which provides the greaicest divergence of revenue
over costs in conjunction with the demand for its product.
Along with the definition of the firm, the following relevant assumptions are made.

1. Perfect knowledge is available.
2. Production is of a timeless nature.
3. Resources will be perfectly divisable and mobile.
4. The most efficient technology will be utilized.

The Isoquant - Isocost Approach to Production Theory
This approach to the theory of production attempts to abstract from the real world of fim production only the most relevant variables with which we will be concerned. This abridgement allows us to analyze the underlying principle of production amd the consequences of alternative actions within a manageable framework. Theory also provides us with the relevant choice rules toward which the efforts are directed.

Let us illustrate with a diagram and the definitions mecessary for this approach as used in the body of the text.

1. Isoquants - These curves show the different combination of two resources ( $X$ and $Y$ ) with which a fixm can produce equal amounts of product (Z).
2. Isocosts - These curves show the different combinations of resources which the firm can purchase, given the pxice per unit of each resource, at en equal outlay of expenditures. These outlay functions denoted by $P_{1}, P_{2}, P_{3}, P_{4}$, are assumed given and fixed.
3. Isocline - This curve shows the least-cost resource combination for various outputs of product. As such, it xepresents the expansion path for the firm and is denoted 0A.
4. Ridge line - These two curves define the relevant economic sector of the isoquants within which a rational firm will operate and
are denoted $O B$. Outside of this area it would require the use of larger amounts of each factor to produce the same output and thus, by defiaftion, a larger outlay than otherwise possible.
5. Plant - A firm of fixed physical size, but free to vary the amount of product produced.


Figure 1. Hypothetical Labor-Capital Isoquant Map

Given the size plant, the prices of factors used in the production of product $Z$, the price of $Z$, the nature of the production function, it is then possible, by applying economic chotce rules, to determine the least-cost combination of resource use and the correct amount of prom duct to produce in order to maximize profits.

It is assumed that the production surface faced by a plant widl show the conventional decreasing returns in the relevant range of output as illustrated by Figure 1. As equal units of variable resources, $X$ and $Y$, are added to the production process, the output of product $Z$ will be less than twice the preceding output, i.e., $\mathrm{z}_{2}<2 \mathrm{z}_{1}, \mathrm{z}_{3}<3 \mathrm{z}_{1}$ and $\mathrm{z}_{4}<4 \mathrm{z}_{1}$ or more specifically $\left(\mathrm{z}_{3}-\mathrm{z}_{2}\right)<\left(\mathrm{z}_{2}-\mathrm{z}_{1}\right)$.

If a competitive firm producing a quantity of product $Z$, with resources $X$ and $Y$ whose prices are $P_{x}$ and $P_{y}$ respectively, had the following general production function, $Z=F(X, Y)$, the firm will minimize the cost of producing any given output at the points where $\frac{\partial Z}{\partial X} \div \frac{\partial Z}{\partial Y}=\frac{P X}{P Y}$. This condition states that the slope of the isom cost line is equal to the slope of the isoquant curve, i.e., the marginal rate of factor substitution is equal to the inverted factor price ratio. This series of tangencies to the isoquants gives the firms least-cost combinations for all outputs and a lime connecting these points of tangencies shows the expansiom path the firm should use if output is increased.

The correct amount of product to produce is based on profit maximizing criteria. Production should be expanded to the point where the cost of producing one more unit of product is just equal to its addition to total revenue when sold. Alternatively, it can be expressed in margimal terms such that the marginal physical product per dollars worth of each resource used is equal in the production of the product, and equilibrium can be expressed in equational form as follows:

$$
\frac{\frac{\partial Z}{\partial X} P_{z}}{P_{X}}=\frac{\frac{\partial Z}{\partial Y} P_{z}}{P_{y}}=1
$$

This equation may be expanded to include as many resources and products as is desirable.

Alternatively, this proposition can be stated as follows: assume that a firm accepts an order for a definite amount of product $Z$, a comstant. It is also assumed that production involves only two factors $X$ and $Y$ and that the prices of these factors, denoted by $P_{x}$ and $P_{y}$, are given to the firm and remain constant during the relevant time.

It is required to find that combination of the two factors which under these circumstances will minimize the total cost of filling the order. Total cost in this case is no longer a function of the quantity of product to be produced, but is now a function of the quantities of the two factors. Moreover, the firm can choose only among those combinations of factors that lie within its production function and as such can obtain only a relative minimization of costs, i.e., a minimum relative to the possibilities offered by the production function. Thus we have the cost function

$$
T C=X P_{x}+Y P_{y}
$$

and the production function as a side relation, subject to which total cost is to be minimized.

$$
\bar{Z}=f(X, Y),
$$

which we write,

$$
[\overline{\bar{Z}}-\mathrm{f}(\mathrm{X}, \mathrm{Y})]=0 .
$$

We now introduce this side relation into the cost function in the following manner. The expression $\bar{Z}-f\left(X_{j} Y\right)$ is equal to zero. The addition of $[\overline{\mathrm{Z}}-f(X, Y)$ to the total cost function will not change the function since it is equivalent to adding zero to the function. The lagrange multiplier, $\lambda$, is included to allow the function to be adequately constrained. Thus,

$$
T C=X P_{x}+Y P_{y}+[\overline{\bar{Z}}-f(X, Y)]
$$

This is still a function of the two variables $X$ and $Y$. $A$ necessary condition for a maximum or a minimum is that all first-order partial derivatives be zero.

$$
\begin{aligned}
& \frac{\partial X C}{\partial X}=P_{X}-\lambda \frac{\partial f(X, Y)}{\partial X}=0 . \\
& \frac{\partial T C}{\partial Y}=P_{y}-\lambda \frac{\partial f(X, Y)}{\partial Y}=0 .
\end{aligned}
$$

It remains, of course, to make sure whether our result spells an extreme value at all, and whether this extreme value is a minimum or a maximum. The sufficient condition for a minimum is that the second order partial shall be positive.

$$
\begin{aligned}
& \frac{\partial^{2} T C}{\partial X^{2}}=-\lambda \frac{\partial^{2} f(X, Y)}{\partial x^{2}}=>0 \\
& \frac{\partial^{2} T C}{\partial Y^{2}}=-\lambda \frac{\partial^{2} f(X, Y)}{\partial X^{2}}=>0
\end{aligned}
$$

Since the factor $\lambda$ is positive by assumption, these second-order partial derivatives of total cost will be positive, if $\partial^{2} f(X, Y) / \partial X^{2}$ and $\partial^{2} f(X, Y) / \partial Y^{2}$ are both negative. In order to see whether they are or not, we must ascertain their economic meaming. The first-order partials are the marginal productivities of the factors $X$ and $Y$. The secondorder partial derivatives are the first partial derivatives of these marginal productivities. The former represent the latter's rates of change with respect to the factor quantities. Therefore, if the secondorder partial derivatives are to be negative, the marginal productivities of each factor must decrease-if total product is allowed to vary, then it must increase at decreasimg rate-as further increments of the same factor are added. To express the same thing in familiar economic terminology, increasing inputs of either factor must be attended by decreasing physical returns. This is by no means always the case. But it may be averred that the condition will normally be fulfilled in the region that is relevant for the solution of our problem.

Solving for $P_{x}$ and $P y$ the first-order equations provide the following two equations,

$$
\begin{aligned}
& P_{X}=\lambda \frac{\partial f(X, Y)}{\partial X}, \\
& P_{y}=\lambda \frac{\partial f(X, Y)}{\partial Y} .
\end{aligned}
$$

and dividing the first by the second we obtain

$$
\frac{P_{x}}{P_{y}}=\frac{\partial f(X, Y) / \partial X}{\partial f(X, Y) / \partial Y}
$$

It may now be recalled that this is the path where the isocost is tangent to the isoquant and the $\mathrm{MRS}_{\mathrm{xy}}$ is equal to the inverse of the price ratio, and as such, is the isocline previously defined. Optimum Factor Combination ${ }^{2}$

The principles of the isoquant-1socost approach to resource utilization in the production process can be extended to plant operation, for example, to determine the optimum use of two identical machines, or in the case of auction markets, the primciple cam be applied to operations such as loading or unloading chutes.


Figure 2. Hypothetical Isoquant-Isocost Map for Production With Two Identical Machines
${ }^{2}$ Material for this section is based on the work of:
B. C. French, L. L. Samet, and R.G. Bressler, ${ }^{\text {º Economic Efficiency in }}$ Plant Operations With Special Referemce to the Marketing of Pears," Hilgardia, Volume 24, July, 1956, pp. 550-552.

As an example of this phenomena, assume that the inputs can be varied in continuous amounts (perfect divisibility). The dashed lines in Figure 2 are isoquants and the solid limes are isocosts. At low levels of output, the isocosts axe convex to the origin since average cost of one machine is less than the average cost of two machines at a less than full utilization of one machine. This convexity decreases as output is increased and becomes tangent to an isoquant at point past that of minimum average costs of one machine. As output is increased still further, the isocosts reverse themselves and become more and more concave to the oxigin.

As output is expanded from the origin long the expansion path which in this case represents equal utilizqtion of mechines, it is readily seen by inspection of Figure 2 that costs will be maximized through the range where the isocost exhibits concavity to the origin. Therefore, costs will be lower by using only one machine up to this point. Increases in output beyond this point will necessitate the use of both machines used equally to mimimize costs. A similar analysis will prevail to determime when to use more than two machines. Total Cost and the Production Function

In subsequent sectioms hypothetical total cost curves are fre quently used in a graphic analysis of firm operations. The logic by which the total cost curve is derived from a production function with given factor prices seems warranted at this poinc.

Total cost is usually thought of as function of output. However, for the purposes of this analysis, we will take total cost as given, and maximize output $Q$. The maximum output will depend on the prices of the factors of production, $X_{1}$ and $X_{2}$.

With the prices of the factors given and total cost vaxying at different levels, a series of isocosts will be obtained, each isocost corresponding to one value of total cost. The maximum output $Q$ for each total cost is determined by the point of tangency to that particular isocost (Figure 3).


Figure 3. Hypothetical Isoproduct and Isocost Map

The line ABC, (Figure 3), formed by the points of tangency is the now familiar isocline and the prices of factors are assumed to be constant throughout its length. Along this line $A B C$ and only along this line is,
(1) $\frac{\partial Q}{\partial X} \div \frac{\partial Q}{\partial Y}=\frac{P_{x}}{P_{y}}$.

From the line ABC each value of total cost is related to one value of output $Q$. Plotting down this correspondence between total cost and output, we obtain the total cost cusve (frigure 4). As long as prices remain constant, the operator will move only on the line $A B C$ (Figure 3) and $A B C$ (Figure 4) will be his cost curve.


Alternatively, this proposition can be stated as follows: Assume the production function;
(2) $Q=X X$,
and the prices of $X$ and $Y$ to be 1 and 2 respectively.
The first order partial derivatives, $\mathbb{I}_{\mathrm{o}}$., the marginal physical productivities will be
(3) $\frac{\partial Q}{\partial X}=X$
and
(4) $\frac{\partial Q}{\partial Y}=X$.

Substituting the prices of $X$ and $X$ and the values of the two firstorder partials into relstion (1) becomes
(5) $Y=\frac{X}{2}$
which is the equation for a straight lime and as such traces out the least-cost resource combimedons for various outputs of product.

The equilibrium position of the entreprenuex in this example is
described by the system;
(6) $\frac{P_{x}}{f_{x}}=\frac{P_{y}}{f_{y}}$,
(7) $Q=f(X, Y)$,
(8) $T C=X P_{x}+Y P_{y}$.
and, as such, these relations describe the behavior of the entreprenuex, where (6) is the condition for maximum output for a given cost, (7) is the technical relation given by the arts, and (8) is the definition of the total cost. In the three relations (6), (7) and (8) there are four unknows $T C, Q, X$, and $Y$ as the $p r i c e s$ of the factors are given. By expressing $Y$ in terms of $X$, equation (5), the three relations (6), (7) and (8) will involve only TC and Q and will represent the total cost function, $T C=f(Q)$.

Re-expressing the three relations (6), (7) and (8), these reliations become

$$
\left(6^{\circ}\right) \quad Y=\frac{X}{2} \quad\left(7^{0}\right) \quad Q=X Y \quad\left(8^{0}\right) \quad T C=X+2 Y
$$

Introducing ( $6^{\circ}$ ) into ( $7^{\circ}$ ) and ( $8^{\circ}$ ) we obtain

$$
Q=X \times \frac{X}{2} \quad T C=X+2 \frac{X}{2}
$$

or

$$
2 Q=x^{2} \quad T C=2 x
$$

and

$$
x=\sqrt{2 Q} \quad x=\frac{T}{2}
$$

hence

$$
T C=2 \sqrt{2 Q}
$$

This now gives the equation for determining the total cost curve when the production function and the prices of the factors are given.

## Costs of Production

In the classical treatment of the costs of production, the analw ysis of the possible production situations axe presented in terms of both the short-run and the long-run. The former exemplifying the situation where a firm is represented by a given scale of plant and, as such, has a number of fixed costs associated with it. In the latter, long-run, all costs are taken as variable with the firm able to adjust its size of plant in order to show the benefits of the economies of scale that may accrue, if any.

The following postulated cost situetions will, in general, be xem stricted to the long-run concepts:

1. Conventional theory,
2. Decreasing costs over the entire range of output,
3. Constant costs over the entrixe range of output.
4. Conventional Theory

In the long-run all costs are variable sixed costs are zero. Therefore, the TC curve starts at the origin。 Underlying each total cost curve is a physical production function, which gives rise to its general shape assuming resources are independent of resource quantities. As the resource prices, when applied to the resources used in the production of some product, form the total cost curve, it is essential that the firm use the most efficient techmiques possible to obtain the highest production relationship and consequently the lowest total cost curve. The general slope of the curve will also be affected by the price of the factors, higher prices giving xise to steeper slopes and lower prices the reverse.

In the long run, it is possible to build firms of ony given size, as all factors are variable. Supermimposing a number of differimg
size scales of plants (short-run) on the LRTC curve will show the con* trasting costs for these varying size plants (Figure 5).


Figure 5. Hypothetical Short and Long-xun Total Cost Curves (Conventiomal).

From the TC curves above, short run average cost curves and longa run average cost curves are derived which show the economies of scale that may be obtained with varying scales of plants (figure 6).


Figure 6. Hypothetical Short and Long-xun Average Cost Curves (Conventional)

As the size of plants are increased, efficiencies of scale due to division and specialization of labor, and technological factors of production occur. These economies will accrue up to a point when the optimum size plant is reached. Any increase in plant size will cause diseconomies of scale to out weigh the economies and cause their average costs to rise.

The LRAC is a line drawn tangent to the short run average cost curves and shows the least cost combination of resources and represents the planning curve for increases in scale of plants.
2. Decreasing Total Cost Over Entire Range of Output

The general shape of the total cost curve assumed under this postulated cost situation is one of increasing costs at a decreasing rate as output is increased (Figure 7). This situation may be visualized as the first sector of the conventional total cost curve just short of the inflection point.


Output of $X$

Figure 7. Hypothetical Short and Long-Run Total Cost Curves (Decreasing)

It is evident that the economies to be derived from increasing the scale of plant will be slight, except at small quantities of output, but
will continue throughout the entire range of possible outputs (Figure 8).


Figure 8. Hypothetical Short and Long-xun Average Cost Curves (Decreasing)
3. Constant Costs Over the Entire Range of Output

Firms operating undex this postulated situation would be faced with a directly proportional change in costs of operation as the prom duct output is increased, i.e., an $\alpha$ increase in resource utiliaation will bring an $\alpha$ increase in costs (Figure 9)。

With the linear total cost relation, the corresponding long-run average cost function will have a positive average cost intercept, and will be parallel to the X axis (Figure 10). In each case the short-run average cost curves will be tangent to the long-run average cost curve at their minimum cost points for all possible changes in scales of plant. It can readily be inferred then that no economies of scale will accrue to any change in scale of plant.


Figure 9. Hypothetical Short and Long-run Cost Curves (Constant)


Figure 10. Hypothetical Short and Long-run Cost Curves (Constant)

## A Discontinuous Cost Function

In most production processes, there are variable factors which, by nature, are not freely divisible and must be bought or hired in large discrete units. An example of this would be a heavy piece of machinery or even a unit of labor represented by one man. For this analysis the costs of these discrete units will be represented by equipment costs. Those variable inputs which are freely divisible are designated by operating costs (Figure 11). ${ }^{3}$ As such, the assumption is made that they represent a linear cost function.


Figure 11. Hypothetical Discontinuous Cost Function

As the firm increases output of product X , it is assumed that it can produce two additional units of product per unit of time only if the firm purchases one additional piece of equipment. When the costs of these successive pieces of equipment are added to the linear
$3_{\text {This }}$ diagram is reproduced from:
H. Brems, "A Discontinuous Cost Function," American Economic Review Vol. 42, September, 1952, p. 583.
operating costs a discontinuous equipment and operating cost curve results.

The total revenue relationship is derived from a conventional linear demand function which slopes downward to the right. The point of maximum profits in conventional theory, where continuous functions are assumed, is the point where the slope of the total cost curve is equal to the slope of the total revenue curve or where MC is equal to MR. In the diagram illustrated, this would occur at about 5 units of output. However, this choice rule is not valid with the discontinuous cost function as a close examination of the diagram will reveal. Profits will be at a maximum at a point infinitesimally short of two units of output, and where $M C \neq M R$ 。

Institutional Factors as a Cause of Inefficiency in Auction Market Operations

Inherent in the operational environment within which the auction markets must function are factors which tend to put an upper limit on the degree of efficiency which any auction market may obtain. These factors are, for the most part, beyond the control of any individual market operator, but are relevant nonetheless.

Some of the major causes are postulated in the form of the following problems.

1. Frequency of operation.
2. Seasonal variability of cattle marketings.
3. Auction markets that possess some degree of monopoly control due to location.
4. Location as a factor in stabilizing the commission charges.
5. Price setting by the dominant firm.

A possible solution is presented where it seems applicable, however, in many cases the problem may not be solvable with the present state of economic theory.

## 1. Frequency of Operation

The auction markets now operating in the state of Oklahoma for the most part have only one sale day each week. This is a partial result of the small area from which they draw their cattle and an inability to obtain sufficient cattle buyers and, as a result, additional sale days may not be warranted. However, this infrequency of operation may have a marked effect on plant efficiency.

Assaming that we have a given volume of cattle to handle, the effect of building a scale of plant to accomodate them all in one day as opposed to handling them in two days may be shown graphically (Figure 12).


Figure 12. Hypothetical Total Cost Curves for Two Auctions

In this analysis, two auction sizes are built, one just large enough to handle the given volume of cattle $\mathrm{OX}_{2}$ in two days. It's fixed costs are represented by OA. The other size plant will be built
just large enough to handle the volume in one day, with its fixed costs just twice the costs of plant one. To simplify further the TVC are as sumed to be equal, (i.e., $A C=B D$ ). The small plants $T C_{s}$ is then a summation of its fixed costs $O A$ plus its variable costs AC. It is evident, therefore, that the large firm ${ }^{\circ} s C_{1}$ differs from the small plant $s$ $\mathrm{TC}_{\mathrm{s}}$ by the amount of its additional fixed costs, and subsequently a higher TC for handling the given volume in one day. If economies of scale are introduced into the analysis, the variable costs for the large firm will have a slope less than that of the small firm causing the differences in the $T C$ of the two firms to be more nearly equal at the given wolume.

## 2. Seasonal Variability of Cattle Marketings

A characteristic of the cattle production enterprise is the high degree of variability of cattle ready for market at varying times through any one year. Although the operator might correctly estimate annual volumes, variability over months leads to uncertainty as to the correct size of plant the auction market operator should build, i.e., how much flexibility should he have in his plant.


Figure 13. Hypothetical Average Cost Curves for Two Plants

If we assume that the cattle numbers vary from day to day throughout the year in a range from $X_{0}$ to $X_{4}$, the auction market operator should build a plant with a high degree of flexibility so as to keep costs minimized (Plant I, Figure 13). However, were the industry characterized by a small degree of variability in a range of $X_{2}$ to $X_{3}$, the auction market operator could build a plant with little flexibility and gear his productive processes to a small range of output, thus using the factors more efficiently, reducing his average costs (Plant II, Figure 13).

## 3. Auction Markets That Possess Some Degree of Monopoly Control Due to Location

If we assume that the auction has a given exclusive area from which the cattle are obtained, there will be no retaliation by other firms if the commission charges are changed and that entry into his exclusive territory is forbidden, the firms equilibrium position can be determined.

The demand curve faced by the auction for his services will be sloping downward to the right (Figure 14). As such it represents the average revenue that he will be able to obtain as the commission charges are altered. For example, if the commission charge is set at price at point $E$, the services demanded will be $0 X_{2}$. Each purchaser will pay a price of $X_{2} G$ or, alternatively, $O E$ and, as such, is the average price or revenue received.

The LRAC curve shows the least cost combination of resources for various outputs of services the firm can obtain. It is possible for the firm to build any size plant represented by the SRAC curves.

Assuming, as in the above diagram that the $M R$ curve cuts the LRAC curve at its minimum point, the long-run profit maximizing output will
be at an output of $\mathrm{OX}_{3}$. At any other rate of output, profits will be reduced. For example, at an output of $\mathrm{OX}_{2}$, profits will be $\mathrm{BEOX}_{2}$ which is less than for the output of $\mathrm{OX}_{3}$, which are $\mathrm{ADOX}_{3}$, This may be proven true for any other output with the situation as represented by Figure 3 . At an output of $\mathrm{OX}_{3}$, the firm will be in equilibrium both in the short and the long-run.


Figure 14. Firm Equilibrium for Hypothetical Demand Relationships and Short and Long-run Average Cost Curves

With a relaxation of some of the above mentioned assumptions, namely, nonretaliation by other firms when commission charges are altered, and that entry into the exclusive territory is blocked, a new set of factors are set into motion to cause economic profits in the long-run to be zero, i.e., total receipts being equal to the total cost that the resources could command in their next best alternative use.

The firm will still be in a market situation characterized by less than pure competition due to the advantage of location. As such, the industry will be characteristic of one within which monopolistic competition prevails. The service provided by the auction will be
differentiated in the eyes of the purchasers due to the above mentioned location advantage with respect to the purchaser of the service.

As before, the demand curve for his service faced by the auction operator will be sloping downward to the right (Figure 15). However, it will be relatively more elastic, due to the relaxation of the above mentioned assumptions.


Figure 15. Final Firm Equilibrium for Hypothetical Demand Relationship and Relevant Cost Curves

Long run equilibrium will be achieved by the firm where SRAC and LRAC are tangent to the average revenue curve, as with new firms able to enter the industry, the demand curve faced by each firm will shift downward as the new firms take up some of the available market. New firms will continue to enter until this condition exists and economic profits are reduced to zero. At this point, no new firms will enter as a less than economic profit will be realized.
4. Location as a Factor in Stabilizing the Commission Charges

If we assume that a firm has some location advantage (at a given price equal to his competitors) his services will be demanded by a
given size area. The demand curve up to his given size volume will be highly elastic because if he chooses to raise his commission price, his competitors will not necessarily follow, and the given volume he had will quickly shift to other firms.

From the point of the given volume noted above, the demand curve faced by him will be relatively more inelastic as any reduction in commission charge on his part will have the effect of enroaching upon his competitors territory. The competitor will thus, necessarily, lower his prices to retain his territory, making it more difficult for the original firm to enlarge his given area.

The demand curve faced by the firm will thus have a "kink" in it and graphically can be illustrated in the following manner (Figure 16).


Figure 16. Hypothetical Kinked Demand Relationship and Relevant Cost Curves

As the demand curve has a kink in it, $\mathrm{i}_{\mathrm{o}} \mathrm{e}$. , the elasticity changes sharply, it follows that the marginal revenue will also shift as $M R=P-\frac{P}{E}$. Thus the marginal revenue will have a discontinuous segment from a to $b$.

Cost curves $\mathrm{AC}_{1}$ and $\mathrm{MC}_{1}$ are such that the firms are making profits, and the price charged is op. Assuming that the cost curves now shift downward due to some new Innovation that reduces costs per unit of output to $\mathrm{AC}_{2}$ and $\mathrm{MC}_{2}$, the firm will not change the price charged as it will still maximize profits where $M C=M R$. likewise, if the cost curves shift upward due to an increase in resource prices, the firm will still not change the selling price op. It is thus evident that there is a wide range of different levels of cost curves under which the firm would not change its selling price. However, if the MC curves move to a point which is higher than a or lower than $b$, the price op will necessarily change if the firm is to maximize profits.
5. Price Setting by the Dominant Firm

In market situations characterized by one dominant firm and one or more smaller firms, the dominant firm sets the price and then sells the remainder after the minor firms have sold all they wish at the ruling price.

The small firms thus are in a market situation similar to one of pure competition, as they can sell all they want at the given price, Their $\mathbb{M}$ curve will coincide with the price set by the dominant firm, making the point of profit maximization by the minor firms at an output where MC is equal to $M R$ or price set by the dominant firm (Figure 17)

The supply forth coming from the minor firms may be determined by the horizontal sumation of their marginal cost curves (EMC). The market demand curve is labeled DD.

The demand curve faced by the dominant firm can now be derived. At a price set by the dominant firm at $p$, or higher, the minor firms will fill the market, so the dominant firms demand curve will start
at this price. If the price is now set at price $p_{2}$, the minor firms will sell a quantity of $C A$ leaving the quantity $A B$ unfilled. This quantity ( $A B$ ) has been superimposed on the $p_{2}$ line and is equivalent to the quantity $C E$, or the amount that the dominant firm will be able to sell at that price, and is its demand.


Figure 17. Hypothetical Demand Relation and Marginal Cost Curves (Dominant Firm)

At successively different prices, the above procedure was carried out and plotted giving the demand curve for the dominant firm,dd. The marginal revenue for the dominant firm is $M R_{d}$.

At a price $\mathrm{P}_{3}$, the minor firms will sell $0 \mathrm{X}_{\mathrm{s}}$, and the dominant firm $0 X_{d}$ or what is the remainder left by the minor firms $X_{r}-X_{S}$ and will be the profit maximizing output for all firms.

The presentation of those alternative postulated situations concludes the discussion of the economic theory needed for the construction of the firm cost models to be presented. Since no new additions were made to economic theory, a summary of each of the theories has been given instead of a complete general statement. The task is now one of
utilizing the theory presented as a tool in the analysis of structural economic relationships for auction firms.

CHAPTER III

## METHODOLOGICAL APPROACHES TO COST MEASUREMENT

The problem of measuring and comparing costs may be approached in a number of alternative ways, the most efficient method depending on the specific objectives of the study and the resources available for carrying out the project.

Two of the more frequently used methods are presented in brief outline to give the relative merits of each. The latter methodological approach was employed in this study as it was felt that it more nearly fit the stated objectives of this project. Synthetic Method of Cost Analysis

This method as an approach to the derivation of cost curves of various size plants, is an outgrowth of industrial engineering. A process of production generally lends itself to being broken down into its compotient parts of operation. As a raw material enter into the production process, each process performed on the raw material may be separated into stages as it is transformed into its final form, and can be analyzed separately. This process of analyzation and summation of these individual stages has been commonly called the "building block" method.

Each stage has its own input-output function, and with suitable rates and prices applied to it, a cost curve for each stage may be derived, and an individual plant cost is the sum of the stage costs. $A$ series of alternate plant layouts or processes for the given product will give rise to a series of plant cost curves. As the plant size is
increased, a family of total cost curves will result, and the composite of these will delineate what is comonly called the long run total cost curve. The transformation of the short-run total cost curves onto a per unit basis will show overlapping average cost curves. A line tangent to the series of short run average cost curves gives rise to the traditional envelope curve or what is commonly called the economies of scale or planning curve.

At any desired rate of output on the long run average cost curve, the point of tangency of the short run average cost curve will show the optimum combination of machinery and labor used to derive that particular size plant and least-cost combination.

The Method of Cost Analysis from Accounting Records
This approach differs substantially from the previous method outlined. It has its merits in that it will give reliable estimates of the long-run planning curve and the relative efficiency between various size plants.

This method employs the use of cost accounting records of already existing firms. It is necessary to obtain reliable cost records, covering a given period of time, from firms operating at varying volumes of output.

A stratified representative sample of the varying volumes of oute put must be drawn from the industry in question so that each volume of output will be represented. The total costs of each sample firm are treated as a single observation, and a regression equation is fitted to the data providing a long-run total cost curve. Alternative Models for Analysis of Annual Cattle Volume Data

A simple regression model of total costs per animal unit handled on total animal units would probably give a reasonable good fit to the
data collected. The corresponding average cost function for this data in the form of an inverted type model such as $\bar{Y}=a+b\left(\frac{1}{X}\right)$ in which the cost function decreases rather rapidly at small output increases, and then flattens out as output is increased further would probably be more suitable.

Although this simple regression model has its uses in showing the relationship between output and costs during the period studied, it is not an appropriate estimate of the long-run average cost function.

As the size of the plant and its position on its respective cost curve are not taken into consideration in the above model, an approximate estimate of the long-run average cost function will be approached when the size of plant and plant output are perfectly correlated. A correctly estimated long-run average cost function will be estimated only when the short-run average cost curve is tangent to the long-run average cost curve. However, as short-run average cost curves are subject to change over time, care must be exercised in attempting to generalize the results over periods of time other than for the period studied.

A more reliable estimate of the long-run average cost function from empirical data, can be obtained by using a multiple regression model with a measure of capacity utilization as a second independent variable. This addition to the model takes into consideration the mainm tenance of idle plant capacity as an output that affects production costs apart from the cost of producing the output of the product. The net result will be to shift each plant along its short-run average cost function to its optimum short-run output, and the long-run average cost curve will pass through these points. The nature of the shortrun average cost curve is specified by the multiple regression model.

An illustration of the effect of the addition of an unused capacity variable is illustrated in Figure 18.


The plant size is given as $O B$ units of capacity and is currently producing $O A$ units of output at $O D$ cost per unit. The cost per unit, OD, is comprised of two segments; namely, the per unit costs incurred in producing $O A$ units of maintaining $A B$ units of idle capacity, $C D$, plus the cost per unit of output at full utilization of capacity $0 C$. Assuming that the plant curve fell on the regression line in both of the above instances, the simple regression would pass through point E, while the net regression with the addition of the capacity variable would pass through point $F$.

## CHAPTER IV

## DATA AND RESEARCH PROCEDURES

The information for the bulk of this study was obtained directly from selected livestock auction markets during the summer (1956) in the state of Oklahoma. However, additional Information was drawn from a separate livestock producer survey in order to present a more complete picture of the environment within which the livestock auctions function.

## Livestock Auction Survey

Data in the livestock auction market survey were obtained by personal interview with each auction market operator included in the sample. A detailed schedule was developed for this purpose.

The schedule was designed to provide a descriptive mosaic of the overall external and internal conditions and influences on the opeqational characteristics of the auctions. The latter section of the auction schedule provided for a detailed breakdown of the categorical expenses of maintaining and operating the physical plant; the analysis and implications of which is presented in the last chapter of this study.

Sample Design for Auction Market Survey
The study was originally designed to survey only those auctions listed under the Packers and Stockyards Act of 1921 as it was felt that the data from these auctions would, in general, be in greater detail and accuracy. However, four additional auctions, not posted under
the Act, were contacted to give a more representative picture of costs in the lower and middle volumes of cattle handled. This made a total of 27 auctions under the Act and four not posted that participated in the study.

The survey was conducted during the months of June, July and Aug. ust of 1956 during which each auction was visited. An appointment was made to see the operator prior to the visit to insure greater cooperation. The data collected on the financial operations of all firms were for the year 1955.

The size of the auctions posted range in size from 5,000 to 106,000 annual volume of all cattle handled. The volumes handled were adjusted into an animal unit measurement to place the auctions on a more homogeneous basis. The various classes of animals were broken down as follows. One horse, one head of cattle over $400 \mathrm{lbs}$. , two calves, 400 lbs. or less, two hogs or five sheep were considered one animal unit. As a result of the adjustments, the range in animal units handled changed to 4,354 to 77,572 animal units. The remaining volumes were fairly well distributed between these two limits, giving a good representation of costs for most volume levels.

Producer Survey
The data obtained in the producer survey were also obtained by personal interviews with livestock producers from a detailed schedule. In each of the surveys, information on the size of farm, type of livestock production, buying and selling practices were obtained. In addition, information regarding their individual affinities for assuming risks and general likes and dislikes of available market information at their disposal was collected. During the month of December 1955 a survey of 82 livestock producers was conducted by the Department of Agricultural

Economics at Oklahoma A. and M. College. It was designed to obtain information concerning the livestock production and marketing practices of farmers within a predominately wheat growing area of north central 0klahoma.

The sample design, in brief, consisted of six counties with a random sample of townships within these counties drawn with respect to those townships characterized as having predominately cow-calf or wheat growing enterprises, according to the county tax assessors office for cattle numbers and the Agricultural Stabilization office for wheat allotment sizes. A stratified random sample, weighted according to the numbers of each size of farm was then drawn comprising the sample of farms to be personally interviewed.

## CHAPTER V

## DESCRIPTIVE RESULTS OF THE SURVEYS

The Oklahoma State Board of Agriculture lists as of August 15, 1955 a total of one-hundred auctions in operation serving the state. These communty sales are state licensed or have state license pending. Thirty-two of these are in addition posted under the Packers and Stockyards Act, 1921.4 Under the provisions of this Act, any livestock auction that engages in inter-state commerce, or whose facilities covers an area of 20,000 square feet or more must be so posted.

These livestock auctions are located throughout the state as given in Figures 19 and 20. Those posted under the Act are predominately distributed along the perimeter of the state (Figure 20). This phenomenon probably stems from the fact that they engage in inter-state commerce.

The state has been arbitrarily divided into four regions (Figure 21). The potential amount of services demanded of the auction markets in terms of cattle and calves on farms in the respective areas has been designated, since cattle and calves make up the bulk of the animals passing through the auctions. The numbers represent the amount on farms January 1, 1954 by the United States Census of Agriculture 1954。 ${ }^{5}$
$4_{\text {United States Department of Agriculture, List of Stockyards Post- }}$ ed Under the Packers and Stockyards Act, Agricultural Marketing Service, U.S. Government Printing Office, Washington, D. C. 1954.
${ }^{5}$ Annual Report of the Oklahoma State Board of Agriculture and the Agricultural Marketing Service, USDA Cooperating, Oklahoma Agriculture, Oklahoma City, Oklahoma 1955.


Figure 19. Location of State Licensed Community Sales, With Annual Cattle Volumes Handled, Oklahoma, 1954
\%Source: State Board of Agriculture, Oklahoma City, Oklahoma.


Figure 20. Location of Auctions Posted Under the Stockyards and Packers Act 1921, with Annual Cattle Volumes Handled, Oklahoma, 1954.

Source: United States Department of Agxiculture, List of Stockyards Posted Under the Packers and Stockyards Act, Agricultural Marketing Service, U.S. Government. Printing Office, Washington, D.C. 1954.


As each region contains approximately the same number of square miles, it is evident from the numbers of cattle in each region that they are divided fairly evenly throughout the state. This in turn accounts for the even distribution of livestock auction markets throughout the state (Figures 19 and 20).

Seasonality in Livestock Receipts
Seasonal variations in livestock marketings have a profound effect on the operational efficiency of the entire marketing system, in particular the livestock auction markets. This seasonality effects adversely, the efficiency of labor and other resources used in the auction marketing process, especially during the low levels of marketings of the year.

There is also the tendency for potential auction market owners to build a scale of plant overly large to handle the estimated peak loads of marketings during the year. This creates an economic environment for the maintenance of large excess capacity facilities during the remainder of the year in which average costs will tend to be higher than they normally need to be, The foregoing facts points out that extreme caution in planning the layout of an auction market should be exercised in order to provide the needed range in cattle marketing facilities with minimization of any excess capacity.

Livestock received at the auctions sampled vary widely from month to month during the yeax. Volume during the heavy marketing season is approximately double that of the light marketing months. In addition to the monthly fluctuations, the maxketings vary also from week to week and year to year.

Reference to Figure 22 shows that total cattle receipts at the auctions studied varied considerably during the twelve month period under study. The seasonal pattern shows that during the month of


Figure 22. Seasonal Cattle Marketings at Selected Oklahoma Livestock Auctions by Regions 1955 Source: Survey data from 31 Livestock Auction Markets in Oklahoma (Summer 1956)

February, the thirty-one auctions handled less cattle than during any other month of the year. Livestock marketings during this month constituted 6.4 percent of the total marketings for the year. A two month rise in marketings followed, reaching a high point in April. A second peak in cattle marketings occurred in July when 9.4 percent of the years cattle were received. A slight drop in cattle receipts followed in August, but a third high was reached during the month of October. This highest volume month was 93 percent greater than the lowest month of February. The monthly cattle receipts at the Oklahoma City terminal market has also been included in Figure 22 as an additional comparison.

In addition to the comparisons of total cattle marketings by months received at the selected auctions, a comparison of the auction receipts by months in the four designated regions was also made (Figure 23).

The regional marketings follow a very similar pattern, however, there was considerable deviations from the total auction marketings for the state, as was expected. The marketings at Oklahoma City were in one respect quite different from those obtained at the auction market interviews in that the highest peak was reached during the month of July, whereas, the highest point reached in all the regional comparisons came later in the fall during the month of October. No apparent reason or explanation was found for this occurrence.

Seasonality in Hog Marketings
An analogous group of comparisons was made for hog marketings as was previously applied to cattle.

Hogs are generally marketed in large numbers in the spring and fall months chiefly because of present farrowing practices. An inspection of Figures 24 and 25 will show that the high period in the spring was during the months of March, April and May in which approximately


Figure 23. Seasonal Catile Marketing at Selected Oklahoma Livestock Auctions for the State and the Oklahoma City Terminal Market, 1955

Source: Survey data fxom 31 livestock auction markets in oklahoma (summer 1956) Data for Oklahoma City terminal market obtained from Weekly Livestock Reports USDA and AMS cooperating, Oklahoma City, Oklahoma, 1955.


Figure 24. Seasonal Hog Marketings at Selected Oklahoma Livestock Auctions by Regions 1955
Source: Survey date from 31 livestock auction markets in Oklahoma (Summer 1956)


Figure 25. Seasonal Hog Marketings at Selected Oklahoma Livestock Auctions for State and the Oklahoma City Terminal Market, 1955

Source: Survey data fxom 31 livestock auction markets in Oklahoma (summer 1956)
Data for Oklahoma City terminal market obtained from Weekly Livestock Reports
USDA and AMS cooperating, Oklahoma City, Oklahoma, 1955.
one-third of the hogs were marketed. This phenomenon tends to offset the low marketings of cattle during the same months in which only onefifth of the marketings occur. This situation helps to use some of the available excess capacity as well as to increase the marginal productivity of resources that would otherwise be left idle if hogs were not handled.

## Livestock Consignments

The livestock producer forms the backbone of the livestock auctions in terms of supplying the livestock for auctioning purposes. The proportion of various classes of cattle consigned is shown in Table I.

TABLE I

PERCENTAGE OF CATTLE, CALVES AND HOGS CONSIGNED TO 31 LIVESTOCK AUCTIONS, BY TYPE OF SELLER, 1955

| Livestock Consigned by: | Cattle | Calves | Hogs |
| :--- | :---: | :---: | :---: |
| Livestock Producers | 82.0 | 83.0 | 95.0 |
| Dealers | 15.0 | 14.0 | 4.4 |
| Auction Personnel | 2.5 | 2.5 | 0.5 |
| Others | 0.5 | 0.5 | 0.1 |
| Totals | 100.0 | 100.0 | 100.0 |

It should be noted that in all classes of animals the livestock producer provides the major source of cattle receipts. As there is no single factor causing the producer to patronize a particular market, the auction market operator should make producers fully aware of all the services the auction market provides to maintain his good will and the continued volume consigned from the producer.

Method of Transportation and Size of Lots
The method of transportation of livestock to and from auctions was predominately by truck. Of the 31 auctions studied, only 6 had any consignments via rail transportation and they comprised less than 10 percent of their total consignments in all cases.

The average size lots brought to the market is shown in Table II. Dealers usually consign in larger lots as it is necessary for them to obtain the economies of large volumes to realize a profit in their operations. Farmers, on the other hand, frequently have a small number of cattle to sell at various times through the year, which accounts for the lower average size lots.

TABLE II
average number of head per lot consigned by producers AND DEALERS AT 31 LIVESTOCK AUCTIONS, 1955

| Type of Consignor | Cattle | Calves | Hogs |
| :--- | :---: | :---: | :---: |
| Producers | 9.9 | 10.8 | 7.8 |
| Dealers | 25.1 | 29.2 | 14.1 |

## Distances Livestock Traveled to Auctions

The livestock consigned to the various auctions predominately came from nearby farms, and as such, the auctions studied may be classified as true community sales. Approximately two-thirds of the cattle and calves came from within a radius of 24 miles of the sales and about eighty percent of the hogs were received from the same distance.

Reference to Table III shows that the number of auctions receiving any cattle over fifty miles away drops off quite rapidly. Only 81 percent of the auctions received any cattle, 74 percent any calves and 35
percent any hogs from a distance greater than fifty miles. These auctions in general, were the larger auctions and attracted sellers from a wider area as they are equipped to handle larger size consignments.

TABLE III
PERCENTAGE OF CATTLE RECEIVED AT 31 oklahoma livestock AUCTIONS BY SPECIFIED DISTANCES, 1955

| Distance Hauled in Miles | Cattle | Calves | Hogs |
| :--- | :---: | :---: | :---: |
| $0-9$ | $26^{1}$ | $25^{1}$ | $38^{3}$ |
| $10-24$ | $38^{1}$ | $38^{1}$ | $43^{4}$ |
| $25-45$ | $30^{2}$ | $30^{2}$ | $22^{6}$ |
| 50 and Over | $12^{5}$ | $12^{5}$ | $18^{7}$ |
| 133 auction markets reporting. |  |  |  |
| $3^{31}$ auction markets reporting. |  |  |  |
| $4^{29}$ auction markets reporting. |  |  |  |
| $5^{28}$ auction markets reporting. |  |  |  |
| $6_{25}^{25}$ auction markets reporting. |  |  |  |
| 711 auction markets reporting. |  |  |  |

## Selling of Livestock

Livestock producers are an important source to whom cattle are sold as well as the main source of cattle consignments. The fact that farmers along with ranchers and feeders bought approximately 30 percent of the cattle offered for sale suggested the importance of feeder and stocker cattle sold at many auctions (Table IV).

The packer and order buyer furnished the major outlet for all types of cattle, calves and hogs, the respective percentages being 50, 53, and 68.6 in that order. This is consistent with the data obtained on the percentages of cattle 45.4 , calves 51.8 and hogs 75 going for slaughter.

TABLE IV

PERCENTAGE OF THE MAJOR CLASSES OF LTVESTOCK PURCHASED BY TYPE OF BUYERS AT 31 OKLAHOMA LIVESTOCK AUCTIONS, 1955

| Type of Buyer | Cattle | Calves | Hogs |
| :--- | :---: | :---: | :---: |
| Packer and Order Buyer | 50.0 | 53.0 | 68.6 |
| Dealers | 20.4 | 19.0 | 8.4 |
| Livestock Producers | 29.5 | 28.9 | 22.9 |
| Auction Personnel | 0.1 | 0.1 | 0.1 |
| Total | 100.0 | 100.0 | 100.0 |

Of those cattle not immediately destined for slaughter, the auction market operators estimated that about 50 percent would be put on grazing, 29 percent into feed lots, and the remainder 19 percent would be used for breeding stock.

## Commissions

Auction income is derived mainly from the receiving, selling and loading out of the livestock handled. This charge is levied either on a per-head or on a percentage basis. Of those auctions interviewed, 21 of the 31 auctions based their charge on a per-head basis. The remainder, 10 , charged the fee on a percentage of the selling price. All auction markets not under the Packers and Stockyards Act may set their rates at any level they desire, however, those posted must have their rates approved.

There is some advantage in basing the charges on a per-head basis as the auction income does not fluctuate so widely as commissions based on a percentage basis, with a given change in the price of animals sold.

## Market Information

Livestock producers have a wide choice from which to obtain information regarding the current prices of cattle in all classes. Most producers do not rely on one source, but use a combination of sources from which to aid their decision pertaining to their selling and buying practices. The most popular mediums being newspapers, radio, and television in that order. Visits to auction markets as a source of price information was of somewhat lesser importance comprising approximately 25 percent of the producers answering the question. The commission firm and auction market reports were used by only 20 percent of the farmers in determining their selling and buying practices. This points up the need for auction market operators to stress this source of market information as an aid in advertising their auctions.

Adequacy of Market Information
More than four-fifths of the producers who answered this question said that the market information received was adequate. of those who expressed a negative opinion on adequacy, a variety of reasons was given. The most frequent criticism expressed was the fact that reports were received too late to be of any material benefit and a difficulty in relating them to local prices.

Desired Additional Auction Market Services
Producers, in general, were satisfied with the services provided by auction markets, however, 27 percent of those interviewed expressed a desire for some change in present practices. There was no logical grouping into which the desires could be categorized as they were so diverse. This probably stems from the fact that a number of auction
markets are located throughout the sample area and the auction market practices and services vary from auction to auction. However, one complaint that was expressed by a number of producers was concerned with the current practice of auction operators handling what was thought to be diseased animals.

Responses to Price Uncertainty
Livestock producers in producing and selling their livestock operate within an environment characterized by a high degree of price uncertainty. Their willingness to accept this uncertainty is substantiated by the very fact that cattle production practices are carried on. However, it seems logical to assume that producers would have certain limits at which they would prefer to take a guaranteed price rather than taking the chance of losing all profits.

Normally it is expected that, due to their differing innate risk assuming natures, some producers will tend to lean toward relatively more certain income situations. Also, the present liquidity of their enterprise will materially affect their decision as to the degree of risk they will subject themselves. If the possibility of an initial loss will cause undue hardships or even bankrupt their business, the producer will naturally lean toward conservation.

A set of alternative risk assuming situations was devised to obtain information from producers to determine their willingness to bear uncertainty in preference to uncertainty.

The following proposition was presented to cattle producers to obtain the desired information. "Suppose when your next lot of slaughter cattle is ready for market a buyer offers you a price that would yield you a net return of 10 dollars per-head. Would you prefer this situation to one in which there was an equal chance of a net return between

10 dollars and 20 dollars net return that would be decided by a number from 10 to 20 drawn at random?"

The producers were then offered three sets of propositions, each differing in the amount of uncertainty present. The propositions offered were as follows:

Model I - Set I
(a) Sure of 10 dollars or an equal chance for 10 to 20 dollars per-head.
(b) Sure of 12 dollars or an equal chance for 10 to 20 dollars per-head.
(c) Sure of 14 dollars or an equal chance for 10 to 20 dollars per-head.
(d) Sure of 15 dollars or an equal chance for 10 to 20 dollars per-head.
(e) Sure of 17 dollars or an equal chance for 10 to 20 dollars per-head.

Set II
(a) Sure of 8 dollars or an equal chance for 5 to 25 dollars per-head.
(b) Sure of 10 dollars or an equal chance for 5 to 25 dollars per-head.
(c) Sure of 12 dollars or an equal chance for 5 to 25 dollars per-head.
(d) Sure of 14 dollars or an equal chance for 5 to 25 dollars per-head.
(e) Sure of 15 dollars or an equal chance for 5 to 25 dollars per-head.
(f) Sure of 17 dollars or an equal chance for 5 to 25 dollars per-head. Set III
(a) Sure of 5 dollars or an equal chance for 0 to 30 dollars per-head.
(b) Sure of 10 dollars or an equal chance for 0 to 30 dollars per-head.
(c) Sure of 12 dollars or an equal chance for 0 to 30 do1lars per-head.
(d) Sure of 14 dollars or an equal chance for 0 to 30 dollars per-head.
(e) Sure of 15 dollars or an equal chance for 0 to 30 dollars per-head.
(f) Sure of 17 dollars or an equal chance for 0 to 30 dollars per-head.

The following alternative propositions were also asked in which the expected profit was the same in all cases, but the degree of absolute certainty varied according to the possibility of obtaining a
greater profit per-head. "Suppose when your next lot of slaughter cattle is ready for market a buyer presents these propositions to you." Model II
(a) Equal chance of a net return between 10 dollars and 20 dollars to be decided by drawing a number at random.
(b) Equal chance of a net return between 5 dollars and 25 dollars to be decided by drawing a number at random.
(c) Equal chance of a net return between 0 dollars and 30 dollars to be decided by drawing a number at random.
(d) Indifferent as to choice of above three.

In Model I, the propositions were presented to the producers in the order in each set as given. When the producer indicated that he would prefer the sure price, it was checked and then the proposition was repeated again exactly except that the next range and sure profit prices were substituted into the proposition. This procedure was again repeated for the propositions of set III.

The expected profit from each of the risk situations in models $I$ and II is the sum of the extreme values multiplied by their probabilities. This resulted in an expected payoff of 15.

When the risk situations are arrayed according to the amount of producers willing to accept uncertainty their numbers generally de* crease (Table V).

In set $I$ where the minimum profit was 10 dollars, 13 percent of the producers elected to choose a 10 dollar sure profit. When the minimum profit was changed to 5 dollars, an increase to 30 percent of the producers chose the lowest sure profit situation. It is interesting to note in set I that these 10 producers chose the poorest choice, as at this number, they had nothing to lose by taking the risk situation.
table V
MODEL I: NUMBER OF CATTLE PRODUCERS WHO CHOSE THE SURE PROFIT SITUATION RATHER THAN THE SPECIFIED RISK SITUATION RANGES, OKLAHOMA 1955*

II. Situations offering 5 dollars minimum profit

| 8 | $5-25$ | 20 | 20 | 59 |
| ---: | ---: | ---: | ---: | ---: |
| 10 | $5-25$ | 6 | 25 | 54 |
| 12 | $5-25$ | 16 | 40 | 39 |
| 14 | $5-25$ | 11 | 54 | 25 |
| 15 | $5-25$ | 13 | 71 | 8 |
| 17 | $5-25$ | 9 | 79 | 0 |

III. Situations offering 0 dollars minimum profit

| 5 | $0-30$ | 23 | 23 | 56 |
| ---: | ---: | ---: | ---: | ---: |
| 10 | $0-30$ | 6 | 29 | 50 |
| 12 | $0-30$ | 16 | 45 | 34 |
| 14 | $0-30$ | 11 | 56 | 23 |
| 15 | $0-30$ | 13 | 70 | 9 |
| 17 | $0-30$ | 9 | 79 | 0 |

[^0]To gain further insight into the producers reactions and reasons behind why he made the above choices, various questions which might have a relationship to them were asked and the results are arrayed in Table VI and VII.

TABLE VI

MODEL II: NUMBER OF CATTLE PRODUCERS WHO CHOSE THE SPECEFIED RISK CONDITIONS, OKLAHOMA 1955*

| Possible Profit from <br> Risk Situations <br> (Dollars per-head) | Added <br> Number | Cumulative <br> Number |
| :--- | :---: | :---: |
| (Range of) |  |  |
| $0-30$ | 11 | 11 |
| $5-25$ | 11 | 22 |
| $10-20$ | 48 | 70 |
| Indifferent | 7 | 77 |

* Seventy-seven producers participated in this question while five refused to answer these propositions.

In all three sets, the largest percentage of producers generally picked the highest certainty in model II and then the percentage tapered off to the proposition with the lowest certainty attached to it. This coincides with their reactions in model $I$ where the highest percentage also leaned toward certainty.

As we have said before, the degree of liquidity of the farm may play an important part in forming their choices. This fact is borne out in the question pertaining to whether or not they had a mortgage on the farm. It is quite apparent from the results that those with a mortgage generally prefer the low risk situation and those who have no mortgage are more willing to accept a high degree of uncertainty (Table VII).

TABLE VII
COMPARISON OF THE PRODUCERS RISK PREFERENCE SELECTION
WTTH CERTAIN ENVIRONMENTAL FACTORS


In complete accordance with this, though moving in the opposite direction as it would be expected to, are the percentages of producers making cash purchases for their livestock. Those paying cash are willing to accept a higher degree of uncertainty than those who use credit terms.

The above trends, though quite apparent, do not agree perfectly as postulated, however, this is probably due to the small number of producers included in the total sample, and especially in some of the items within the tables.

There appears to be no correlation between the average age of operator, or average number of acres operated with the willingness to assume risk.

CHAPTER IV

## ANALYSIS OF PLANT COSTS

The numerical data for the analysis of plant costs were obtained by: the use of accounting records in the auction market surveys. The individual cost items have been aggregated into their proper classificam tion according to conventional economic theory. As such, the appropriate breakdown includes total variable, total fixed and total costs. Each classification is treated as a separate unit to which appropriate economic interpretation and implications follow directly after the postulated model.

Hired Labor Costs

A separate analysis of hired labor costs was conducted as this segment of total plant operating costs represents a large percentage of variable costs.

In most auctions, one or more persons performed each specific job function. However, in some of the smaller auctions, one person performed more than one job. The job listings included under the hired category were bookkeeper, auctioneer, ticket writer, clerks, weigher, yard labor, both full and part-time, and the veterinarian

In some instances, it was necessary to impute a labor cost for some of the labor categories as in the situation where the bookkeeping duties were performed by the wife of the owner and, as such, was paid no specific wage. The imputed value was estimated in these cases as che average wage paid other bookkeepers for comparable size auctions,
adjusted for the number of weeks operated during the year. A similar procedure was followed in situations where one person performed more than one job function or where the owner served in one of the hired labor capacities. Except for these specific instances, all costs are actual hired labor expenses as taken from the cost accounting records of the 30 auctions listed (Table VIII). Auction numbered 31 was omitted from the analysis as it conducts more than one sale per week and, as such, it was felt that it was not homogeneous with other auctions. As only hired labor costs were obtained from auction numbered 31 , it was omitted also from all further cost analyses other than those involving hired labor costs.

In order to derive the relationship between volume handled and labor costs linear and quadratic functions were postulated. Estimation of these models resulted in the following equations:

$$
\begin{align*}
& Y=4168.23+.4646 X_{1}^{*}  \tag{4,1}\\
& (.043) \quad R^{2}=0.80 \\
& Y=3024.71+.5585 X_{1}^{*}-1.28 X_{2}  \tag{4.2}\\
& (.148) \quad(1.94) \quad R^{2}=0.81
\end{align*}
$$

Where $Y=$ total hired labor costs, $X_{1}$ the number of animal units handled and $X_{2}$ is the $X_{1}$ variable squared. In all subsequent analyses, a single asterisk will denote significance at the 5 percent probability level and the number under the coefficients in parenthesis is the standard error.

Employing a linear model yielded a statistically significant coefficient connecting $Y$ and $X_{1}$, and resulted in a significant reduction in the sum of squares of $Y$. By injecting the second variable $X_{2}$, into the relationship, a slight increase in the closeness of fit was achieved, however, the reduction in the error sum of squares for the second variable was not significant at the 5 percent probability level. A graph showing

## TABLE VIII

ANNUAL VOLUME OF ANIMAL UNITS HANDLED, ACTUAL AND IMPUTED TOTAL HIRED LABOR COSTS FOR 31 SELECTED OKLAHOMA LIVESTOCK AUCTIONS, 1955

| Auction <br> Number | Total A.U. | Hired Labor Costs |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  |  | Actual | : | Imputed |
| 1 | 4,345 | 5,528 |  | 7,140 |
| 2 | 6,824 | 4,672 |  | 5,591 |
| 3 | 7,218 | 7,727 |  | 7,548 |
| 4 | 7,892 | 9,104 |  | 8,211 |
| 5 | 9,047 | 4,850 |  | 3,417 |
| 6 | 9,240 | 10,328 |  | 8,619 |
| 7 | 9,345 | 8,872 |  | 11,220 |
| 8 | 10,548 | 10,700 |  | 11,610 |
| 9 | 12,100 | 7,956 |  | 8,211 |
| 10 | 15,949 | 8,845 |  | 9,465 |
| 11 | 16,572 | 10,236 |  | 11,913 |
| 12 | 17,703 | 13,417 |  | 11,658 |
| 13 | 17,713 | 13,772 |  | 11,985 |
| 14 | 18,614 | 10,006 |  | 11,770 |
| 15 | 19,175 | 10,244 |  | 10,452 |
| 16 | 19,212 | 13,699 |  | 14,988 |
| 17 | 20,452 | 15,126 |  | 14,106 |
| 18 | 22,946 | 16,401 |  | 17,625 |
| 19 | 23,105 | 18,384 |  | 18,360 |
| 20 | 23,721 | 17,783 |  | 15,588 |
| 21 | 25,322 | 19,480 |  | 12,769 |
| 22 | 26,333 | 22,664 |  | 17,078 |
| 23 | 26,500 | 12,980 |  | 13,515 |
| 24 | 33,857 | 25,931 |  | 20,652 |
| 25 | 34,800 | 10,684 |  | 12,049 |
| 26 | 35,460 | 26,775 |  | 18,411 |
| 27 | 38,720 | 12,435 |  | 11,976 |
| 28 | 57,129 | 34,312 |  | 25,602 |
| 29 | 59,169 | 33,020 |  | 21,632 |
| 30 | 77,572 | 37,434 |  | 25,551 |
| 31* |  |  |  |  |

* Omitted for reason given in text.
the relationship between the number of animal units handled and the cost of hired labor as explained by equation (4.1) is presented in Figure 26. This may be interpreted to read that any one animal unit reduction or increase in cattle handled will bring about a 46 cent reduction or increase in hired labor costs.

Imputed Labor Costs
During the analysis of the hired labor costs, it was noticed that there were large deviations in the amount of wages paid at various auctions. It was thought that these differing wage rates were a partial influence in magnifying the deviations around the regression line. To test this hypothesis, imputed hired labor costs were estimated for all job categories such that each auction ${ }^{0}$ s labor costs were placed on an equally weighted basis.

To test the above mentioned hypothesis regarding the differences of wage rates paid hired labor, the same form of models were fitted to the imputed hired labor values. The estimating equations and their coefficients are as follows:

$$
\begin{align*}
& \mathrm{Y}=6771.44+.2768 \mathrm{X}_{1}^{*}  \tag{4.3}\\
& (.0315) \quad \mathrm{R}^{2}=0.73 \\
& \mathrm{Y}=4629.19+.4525 \mathrm{X}_{2}^{*}-.241 \mathrm{X}_{2}  \tag{4.4}\\
& (.1030)(.1348) \quad R^{2}=0.76
\end{align*}
$$

Where $Y=$ is total inputed labor costs, $X_{1}$ the number of animal units handled and $X_{2}$ is the $X_{1}$ variable squared.

Employing a linear model yielded a statistically significant coefficient connecting $Y$ and $X_{1}$, and resulted in a significant reduction in the sum of squares of $Y$. By injecting the second variable, $X_{2}$, into the relationship, a slight increase in the closeness of fit was achieved, however, the reduction in the error sum of squares for the second


Figure 26. Estimated Total Hired Labor Costs for 30 Livestock Auctions, Oklahoma, 1955
variable was not significant at the 5 percent probability level.
A comparison of the correlation coefficients of the equations for actual and imputed hired labor costs shows that rather than increasing the amount of variation explained by the imputed costs, a reduction has occurred. This would lead us to reject our previous stated hypothesis. However, the failure to increase the closeness of the fit could possibly lie in the various marginal physical products of labor for differing hired personnel, i.e, the workers are paid different wages according to their value of marginal product. To assign each worker an equal wage rate could be distorting the value of their services in the operation of the auction market.

Total Variable Costs
Variable costs as used in this study refer to all costs associated with conducting the operation of the auction minus all costs that would be incurred if the plant were left idle, i.e., the fixed costs. No attempt was made to table all the separate variable cost items, as they are too numerous. Instead, only the major categories are listed to give a general picture of the items ancluded (Table IX). Data relating to total variable costs were obtained from 29 livestock auctions.

The major variable cost item is the hired labor costs, not including the supervisory personnel, which were not included in the total variable costs as the owner in most cases performs two and sometimes three different positions at a single auction. Thus, it was felt that it would be extremely difficult, if not impossible, to arrive at any realistic estimation of the market value for his services.

In order to derive the relationship between volume handled and total variable costs a linear function was pastulated:

TABLE IX

COMPONENIS OF TOTAL, VARIABLE AND FTXED COSTS FOR 31 SELECTED OKLAHOMA LIVESTOCK AUCTIONS, 1955

| Total Costs | Total Variable Costs | Total Fixed Costs |
| :--- | :--- | :--- |
| Hired labor | Hired labor | Rent |
| Office expenses | Office expenses | Insurance |
| Utilities | Utilities | Taxes |
| Yard and barn expense | Yard and barn expense | Interest |
| Transportation | Transportation | Depreciation |
| Advertising | Livestock losses |  |
| Livestock losses |  |  |
| Miscellaneous |  |  |
| Rent |  |  |
| Insurance |  |  |
| Interest |  |  |
| Inepreciation |  |  |

$$
\begin{equation*}
\mathrm{Y}=5485.78+.770 \mathrm{X}_{1}^{*} \tag{4.5}
\end{equation*}
$$

$$
\begin{equation*}
\mathbf{R}^{2}=0.75 \tag{.085}
\end{equation*}
$$

In equation (4.5) the total variable costs $Y$ were taken as a function of the number of animal units handled $X_{1}$ the equation resulted in a large Y -intercept which is not consistent with logic in that the total variable costs should be zero when no animal units are handled. As the regression coefficient is significant at the 5 percent probability level there is the indication that a significant reduction in the sum of squares of $Y$. A one animal unit increase in animals handled will bring about a 77 cent increase in total variable costs. A graph showing this relationship is presented in figure 27.

In order to ascertain if the function rises at an increasing or decreasing rate, the following quadratic model was fitted. The following estimating equation and regression coefficients resulted:

$$
\begin{align*}
Y=2800.30+ & 1.022 X_{1}^{*}-.004 X_{2}  \tag{4.6}\\
& (.301) \quad(.005) \quad R^{2}=0.75
\end{align*}
$$

Where $Y$ is total variable costs, $X_{1}$ the number of animal units handled and $X_{2}$ is the $X_{1}$ variable squared.

Employing the quadratic model yielded a statistically significant coefficient connecting $Y$ and $X_{1}$ and resulted in a significant total reduction in the sum of squares of $Y$. The addition of the second variable $X_{2}$ into the relationship did not change the degree of closeness over the linear model equation (4.5) and the reduction in the error sum of squares for the second variable was not significant at the 5 percent probability level. However, this addition of the second variable lowered the Y -intercept and introduces the range of decreasing costs to scale which is consistent with economic theory as indicated by the b values computed for each variable. However, only the $X_{1}$ coefficient is significant at


Figure 27. Relationship Between Total Variable Costs and the Annual Volume of Animal Units Handled by 29 Livestock Auctions, Oklahoma, 1955
the 5 percent probability level.
In order to ascertain if total available capacity affects total variable costs, a quadratic model was postulated. Estimation of these models resulted in the following equation:

$$
\begin{align*}
& \mathrm{Y}=6128.22+.782 \mathrm{X}_{1}^{*}-.029 \mathrm{X}_{2}^{*}  \tag{4.7}\\
&(.091) \quad(.101) \quad \mathrm{R}^{2}=.75
\end{align*}
$$

Where $Y$ is total variable costs, $X_{1}$ the number of animal units handled and $X_{2}$ a measure of the percentage of the total possible plant capacity utilized. ${ }^{6}$

The addition of the $X_{2}$ variable did show significance at the 5 percent probability level and does provide decreasing costs as the percentage of capacity is increased (Figure 28). In this figure the regression line denoted (b) was calculated from equation (4.7) using 50,000 animal units as the $X_{1}$ variable and fifty percent of capacity utilization for the $X_{2}$ variable. Alternatively, the use of one-hundred percent of capacity at 50,000 animal units provides a reduction in total variable costs of approximately 1,450 dollars, regression line (a).
${ }^{6}$ A separate letter of inquiry was sent to each auction market operator concerning the numbers of each type of animals that the auction could accommodate at one time in the sellers pens. These figures were then adjusted to place them in terms of animal units. For those auctions which did not answer the inquiry, an estimation of the total animal units that could be accommodated was obtained by calculating the average square feet per animal unit for those auctions of comparable size to the auctions which did not reply and dividing this figure into the total square feet of the auction with the missing data. Each auction's plant capacity for one unit of time was then multiplied by the number of week the auction held a sale in order to place the animal units handled on an annual basis. The percentage of capacity was then computed by dividing the actual animal units it had handled during the year, by the above total potential capacity.


Figure 28. Relationship Between Total Variable Costs and the Annual Volume of Animal Units Handled and Percentage of Capacity Utilized by 29 Livestock Auctions, Oklahoma, 1955

Another model fitted to the total variable cost data in the form of a quadratic function provided the following estimating equations:

$$
\begin{align*}
Y=-8587.77+.200 X_{1}^{*}+.515 X_{2}^{*}  \tag{4.8}\\
(.018) \quad(.089) \quad R^{2}=0.83
\end{align*}
$$

Where $Y$ is the total variable costs, $X_{1}$ measure of che cotal possuble capacity of the plant in animal units as explained above (footnote 6) and $X_{2}$ the percentage of capacity utilized.

This model provided a closer fit than any model postulated for total variable cost data and both regression coefficients are positive and significant at the 5 percent level. However, the estimating equation has a large negative $Y$-intercept which would provide an average variable cost curve that would be increasing throughout, well into the relevant output range which is not a logical concept.

Total Fixed Costs
Total fixed costs, as defined for the purpose of the study, include all those costs that would be incurred even if the firm ceased to operate. See Table IX for items in this category.

The depreciation and interest on the original and improvement investments on the buildings and yards was calculated by depreciating them over a twenty year period and adding three percent interest cost. For office equipment and loud speaker system, a ten year depreciation period was used.

In each case, the investments were based on the amount the owner paid originally, plus an estimate of the value of the improvements he had made.

The calculated fixed costs at most auctions were a relatively sma1l part of total costs having a mean value of approximately 4,000 dollars.

In order to derive the relationship between volume handled and total fixed costs a linear function was postulated. Estimation of thos model resulted in the following equation:

$$
\begin{align*}
Y=770.42+ & .155 X_{1}^{*}  \tag{4.9}\\
& (.018) \quad R^{2}=.73
\end{align*}
$$

Where $Y$ is total fixed costs and $X_{1}$ is the number of animal units handled。

Employing a linear model yielded a statistically significant coefficient connecting $Y$ and $X_{1}$, and resulted in a significant reduction in the total sum of squares of $Y$. A graph showing the relationship between the number of animal units handled and the total fixed costs as explained by equation (4.9) is presented in Figure 29. The regression line obtained is in accordance with economic theory. Total Costs

The summation of the total variable and total fixed costs gives rise to the sum total of all expenses incurred in the operation of the livestock auction market (Tables IX and X). The figures on unused capacity were computed by substracting the actual cattle marketings from the possible annual cattle marketings for each plant if operated at total capacity. The per animal units costs shown in Table $X$ were obw tained by dividing the total cost by the actual cattle marketed without regard to unused capacity.

Selection of the Total Cost Function - The Volume Variable

When graphing the total cost of animal units handled against the total animal units handled, a slight curvature is apparent for the auction market study, i.e., as the number of animal units handled is increased, the total cost of handing then increases but at a slightm ly decreasing rate, The curvature is so slight that a linear


Figuxe 29. Relationsinip Between Total Fixed Costs and the Annual Volume of Animal Units Handled for 29 Livestock Auctions, Oklahoma, 1955

TABLE X

## ANNUAL VOLUME, TOTAL COSTS AND UNUSED CAPAGITY BY PLANTS FOR 31 SELECTED OKLAHOMA LIVESTOCK AUCTIONS, 1955


$1_{\text {Plants excluded for reasons given in text }}$.
regression of total cost on the output variable provides a good fit. The solution of the linear model provided a positive $Y$ intercept of a reasonable magnitude, the corresponding average cost function is non linear, decreasing at a diminishing rate as output increases. However, had the solution resulted in a negative $Y$ intercept, the corresponding average cost function would have increased at the smaller outputs, as output increases, and possibly will into the relevant output range. Therefore, it is necessary to have a high degree of accuracy on the observations at the extreme lower end of the output range if dependable results are to be obtained with the model.

To avoid the difficulties involved with the possibility of obtaining a negative $Y$ intercept, a total cost regression equation nonlinear in the volume variable and passing through the origin can be filled. This assumes that total cost is zero when both output and unused capacity are zero which is not illogical. However, there is some difficulty in choosing the most suitable equation. With the apparent curvalinearity noted above, an equation in which an exponent of slightly less than one is needed on the volume variable. This exponent establishes the curvature of both the total cost function and the average cost function. Its value is not determined by the least-sqquares method of fitting the regression equation but must be taken as given. For example, $Y=b_{1} X_{1}{ }^{.9}$. The precise value for the exponent can be determined by fitting a number of different equations with differing exponents on the volume variable and then using the one that fits the data the most closely.

Selection of the Total Cost Function - The Capacity Variable The selection of the total cost regression function with respect to the volume variable dictates the curvature of the long-run cost
function. ${ }^{7}$ The alternative models that may be used are presented in Chapter III under methodological approaches to cost measurement, page 31.

The form of this total cost regression function with respect to the capacity variable determines the shape of the short-run total cost functions. If it assumed that the unused plant capacity is linearly related to thetotal cost of handling animal units, the model is a simple one with respect to the idle capacity.

A family of short-run average cost curves may be dexived from the above model. Each short-run average cost curve will terminate at the point of intersection with the long-run average cost curve, and become infinitely elastic with it. Each shortorun cost curve originates at a common point at infinity on the $Y$ axis, and at the point of intersection with the long-run cost curve, each short-run curve will rep . resent its plant capacity, and then flatten out along with the long-run cost curve. Thus, although per unit costs of idle plant capacities are constant for all plant capacities, cost per animal unit of cattle handled for the given constant will decrease as output is increased.

The family of short-run average cost functions derived from the above model differ from the conventional envelope curve as portrayed in conventional theory. However, except for the fact that the curves do not show increasing costs beyond the point of intersection with the

[^1]long-run cost curve, similar and useful conclusions may be drawn from them.

Estimates from Analysis of Annual Data
The models used in the analysis of the annual data in Table $X$ were,

$$
\begin{align*}
& Y=b_{1} X_{1} \cdot 5+b_{2} X_{2}  \tag{4.13}\\
& Y=b_{1} X_{1} \cdot 7+b_{2} X_{2}  \tag{4.14}\\
& Y=b_{1} X_{1} \cdot 9+b_{2} X_{2}  \tag{4.15}\\
& Y=a+b_{1} X_{1}+b_{2} X_{2} \tag{4.16}
\end{align*}
$$

where in all cases,

$$
\begin{aligned}
& Y=\text { total annual cost of operating auctions, } \\
& X_{1}=\text { annual volume of animal units handled, } \\
& X_{2}=\text { unused animal unit capacity on an annual basis. }
\end{aligned}
$$

Data relating to the above variables are given in Table $X$. When fitted to the data in Table $X$ for the 29 auction markets retained by the method of least squares, all of the models provided approximately the same $R^{2}$, so it was decided arbitrarily to use models (4.14) and (4.16) to express the cost relations.
, Model (4.14) provided the regression equation

$$
\begin{align*}
\mathrm{Y}= & .0267 \mathrm{X}_{1} \cdot 7^{*}+.069 \mathrm{X}_{2}^{*}  \tag{4.17}\\
& (.0022) \quad(.019) \quad \mathrm{R}^{2}=0.93
\end{align*}
$$

while equation (4.16) provided the regression equation

$$
\begin{align*}
Y=3510.23+.9426 X_{1}^{*}+.049 X_{2}^{*}  \tag{4.18}\\
(.079)(.021) \quad R^{2}=0.92
\end{align*}
$$

In both estimating equations the two regression coefficients were significant at the 5 percent probability level.

The two estimating long-run average cost curves in Figures 30 and 31 were computed by setting $X_{2}=0$ in equations (4.17) and (4.18)


Figure 30. Estimated Shoxt and Long-run Average Cost Functions for 29 Selected Oklahoma Livestock Auctions, 1955 (Statistical Model 4.14)


Figure 31. Estimated Short and Longmrun Average Cost Functions for 29 Selected Oklahoma Livestock Auctions, 1955 (Statistical Model 4.16)
respectively, solving for a series of total costs associated with a given series of values for volume of animal units handled, and dividing the result in each case by the number of animal units handled. The estimated short-run cost functions for the several capacities in Figures 30 and 31 were computed from equations (4.17) and (4.18) respectively by calculating the total cost for an auction of a given size, i.e., any point on the long-run average cost curve. From this point the total cost is then computed for a given decrease in $X_{1}$ and the additional cost of the unused capacity is added to it and the total is divided by the remaining value of $X_{1}$. This procedure is followed to obtain enough points to provide a smooth continuous curve.

The nature of the short-run curves are not in the strictest sense like those of conventional economic theory as they terminate (because infinitely elastic) with the long-run average cost curve. Pure theory would dictate that a range of costs that increase at an increasing rate beyord the optimum point should be evident. However, the curves derived lead to similar conclusions to those drawn from the more usual envelope curves. For example, these curves indicated that the lowest cost for any output can be obtained in the smallest plant capable of producing that output, i.e., the short-run curves do not intersect. They also indicate that a large plant, can be operated at a lower cost at less than optimum output more efficiently than a very small plant can at optimum output. For example, a large plant operating at 70,000 animal units handled annually at optimum output can operate at 50,000 animal units annually at a lower average cost than a small plant which has its optimum output at 10,000 animal units handled annually.

The fact that the long-run average cost curve flattens out rather rapidly as output is increased probably explains part of the nature of
the auction market enviroment. There are many auctions of various sizes dispersed rather evenly through the state of oklahoma. The economies of scale derived from building large auctions to serve a large area appear to be not practical when based on the analysis of the data obtained. A smaller auction with essentially the same costs per unit of animals handled can successfully compete with the larger size auction thereby cutting down on the area served by potentially large scale auctions. This would tend to make the auction marketing business a highly competitive one as., in reality, it is.

## CHAPTER VI

## SUMMARY AND CONCLUSIONS

The central problem area of this study involved the determination of the actual cost relationships for selected Oklahoma livestock auctions. In order to realize this objective, a theoretical framework, within which the problem is contained, was formulated. Alternative methodological approaches to the estimation of cost relationships were examined. Under the restrictions of time, labor and funds available, data generated from auction market cost accounting records, with certain statistical variations, were chosen as the most appropriate method.

Given the choice of problem areas and the methodology to be applied, a schedule was developed to collect the relevant information through the medium of personal interviews with livestock auction operators. The central core of the schedule pertained to a detailed breakdown of all costs associated with the operation of a livestock auction.

Alternative economic models were postulated for the generation of the data relating to the conventional economic breakdown of total variable, total fixed and total costs. By employing appropriate statistical techniques, estimates wexe obtained for each of the postulated models and the results were subjected to statistical and economic tests. From the estimated relationships long-run average cost curves were derived showing the economies to be realized from various scale of plants. An economic analysis was made for each of the estimated relationships.

Inherent in the environment within which the auctions must function are institutional factors which tend to set limits to the degree of operational efficiency an auction market may obtain. Two of the more important institutional factors found as a cause of inefficiency were; (1) the present practice of operating the auctions with only one sale day per week, thus leaving the physical plant idle the major part of the time and (2) the high degree of seasonality of livestock marketings during any one year. This phenomenon added an additional element to inefficiencies in the sense that it increased the uncertainty of the auction market owners decision as to the correct scale of plant to build. The result of this inability to predict the number of cattle to be marketed in any one sale day led the owners, in many instances, to build a scale of plant overly large to handle their estimated volumes of cattle.

Both linear and quadratic models were postulated as an explanation of total variable costs. As the second variable in the quadratic equation did not show a statistically significant reduction in the total sum of squares of error of the dependent variable, it was concluded that the linear function on total variable costs as a function of animal units handled provided the better estimating equation.

A linear model was postulated for the explanation of total fixed costs. This model stipulated that total fixed costs were a linear function of animal units handled, and provided a statistically significant regression coefficient.

The postulated models for total costs were in the form of both linear and non-linear functions. On the basis of statistical tests, it was concluded that an equation in the form of total costs as a function of animal units handled and unused capacity provided the most relevant variables to explain the data. The long-run average cost curves were
derived from this general form of estimating equation. The general shape of which slopes sharply downward to the right at its outset and then levels as output is increased.

The nature of the short-run average cost curves were not in the strictest sense like those of conventional economic theory as they terminated with the long-run average cost curve. However, the curves did lead to similar conclusion to those drawn from the typical envelope curve. For example, the curves indicated that the lowest cost for any output can be obtained in the smallest plant capable of providing that output. They also indicated that a large plant can be operated at a lower cost, at less than optimum output, more efficiently than a very small plant at optimum output. For example, a large plant opexating at 70,000 animal units handled annually at optimum output can operate at 50,000 animal units handled annually at a lower average cost than a small plant which has its optimum output at 10,000 animal units handled annually.

The subsequent economic analysis of the long-run average cost curve led to the conclusion that few economies of scale are to be derived from increasing the size of auction markets except in the lower output ranges of from approximately zero to 40,000 animal units handled annually.

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# THESIS TITLE: ESTIMATED COST FUNCTIONS FOR SELECTED OKLAHOMA LIVESTOCK AUCTIONS 

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TYPIST: Gwen Martin


[^0]:    * Seventy-nine producers participated in this question while three refused to answer these propositions.

[^1]:    7 This approach is based largely on the work of: Richard Phillips, "Empfrical Estimates of Cost Functions for Mixed Feed Mills in the Midwest," Agricultural Economics Research Vol. VIII, No. 1, January, 1956, pp. 1-8.

