A NEW PROCEDURE OF WATER FLOODING

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CHAPTER I

INTRODUCTION

The constantly increasing demand for petroleum products and the increasing difficulty of finding new oil fields has directed serious attention to the oil remaining in the ground after primary methods* of producing oil have been exhausted. This unrecovered oil constitutes a vast reserve of petroleum.

Nature usually provides sufficient energy for recovering only a small fraction of the total oil; therefore, to recover additional oil, artificial means of supplying energy to the reservoir must be provided. Energy by artificial means may be supplied to the reservoir during the time of production by the natural energies; or it may be supplied after the natural energies have been depleted.** If energy by artificial means is supplied to the reservoir after the natural energy has been depleted, the oil from the reservoir is then produced by the secondary recovery methods. The most common methods of supplying energy to the reservoir are the injection of gas, under

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*Primary methods of producing oil refers to the production of oil from the energies present, by nature, in the reservoir at the time of discovery - the energies may be in the form of (a) external driving fluid energy under hydrostatic head, (b) internal driving energy usually from gases in solution, (c) potential energy, (d) surface energy of reservoir fluids as a result of capillary forces.

**Natural energies are considered depleted when the cost of producing oil by these energies equals the money obtained from selling the oil which is produced.

pressure, into the reservoir, and the injection of water, under pressure, into the reservoir. Proposed methods of supplying artificial energy to the reservoirs include the underground burning of a portion of oil and the introducing into the reservoir of bacteria which attacks sulphur components of the oil. Production in some fields has been increased by the addition of vacuum to the producing well.

The present discussion is limited to the problem of artificial flooding of reservoirs with water. In this method of secondary recovery, water is injected into the oil sand with the intention of displacing the oil. The water injection and the depletion of the displaced oil are accomplished through networks consisting of injection and production wells. A network which is the most profitable for the field is used.

John F. Carll, (1) geologist, was the first to call attention to the possibilities of water flooding to increase the recovery of oil from oil sands. The exact time and place of this discovery is not definitely known. In 1921 the Pennsylvania Legislature passed a special act legalizing this practice when applied to Bradford and certain other specially named sands. Water flooding was greatly extended thereafter, and the methods of applying the water to the oil sands in order to replace the oil rapidly were improved.

In determining the appropriate flood networks to use in a

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(1) John F. Carll, "The Geology of Oil Regions of Warren, Venango, Clarion and Butler Counties," Second Geological Survey of Pennsylvania, Report III, 263, (1880).

water flooding operation, engineers are interested in the sweep efficiency* and conductivity** of the reservoir networks. Also, engineers are interested in the additional amount of capital return from the various networks.

Several types of flooding networks have been studied by Muskat and Wyckoff. (2) (3). Flooding networks which have been studied extensively are shown diagrammatically in Fig. 1. In Fig. 1 the injection and producing wells are denoted respectively by black and hollow circles. The distance between the wells in a line is denoted by "b" and that between injection and producing lines is denoted by "a".

In line-drive flood networks, the oil and water wells are arranged in parallel lines as shown in Fig. 1-a. As the ratio $\frac{a}{b}$ is varied from 0.5 to 4.0, the sweep efficiency is increased from 32 per cent to 89 per cent. For the staggered-line flood networks, Fig. 1-b, the sweep efficiency also varies as the ratio $\frac{a}{b}$ is changed. The sweep efficiency increases from 72 per cent for a ratio of 0.5 to a sweep efficiency of 92 per cent for a ratio of 4.0.

Five-spot flood networks are special cases of staggered-line-

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*Sweep efficiency of a network is the percentage of the total area flooded by the time water first appears in the producing wells.

**The conductivity of the flood is the steady state production capacity of the network per unit pressure differential between the input and the output well.

(2) Muskat, M. and Wyckoff, R. D., "Mechanics of Porous Flow Applied to Flooding Problems." Trans. A.I.M.E. 156, 219 (1933).

(3) Muskat, M. and Wyckoff, R. D., "Theoretical Analysis of Water Flooding Networks." Trans. A.I.M.E., 192, 62 (1934).

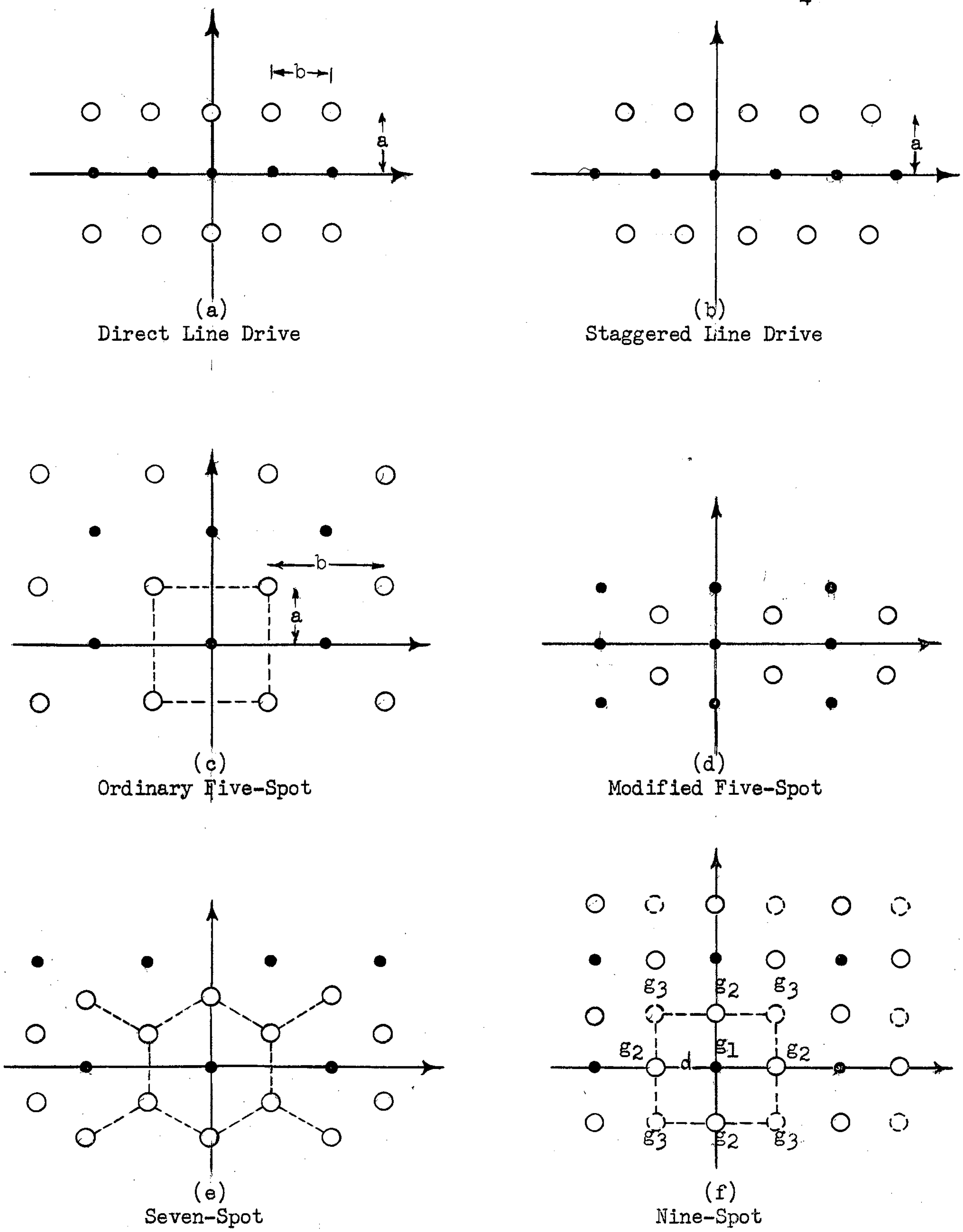

Figure 1. Schematic Representation of Flooding Networks

drive. The ordinary five-spot has an $\frac{a}{b}$ ratio of 0.5. This network has an efficiency of 72 per cent. Fig. 1-d is an example of a modified five-spot network.

The seven-spot network is given in Fig. 1-e. Its sweep efficiency is 74 per cent; however, this seven-spot network is more difficult to adjust to the boundaries of a particular field than the five-spot. This difficulty eliminates it from being used extensively. The criterion of using sweep efficiencies alone is not sufficient for determining the network to use. Since each seven-spot network element (triangle ABC) has three wells, and each five-spot network element (rectangle abcd) and line-drive network elements have two wells each, the conductivity does not offer a proper comparison between the floods, because the number of wells per acre are not the same. To compare conductivities of a five-spot flood area with the same total number of wells per acre for a seven-spot, the spacing for the seven-spot should be increased by about 7.5 per cent, because each central well in the seven-spot is associated with two peripheral wells. This makes three wells in a network element as compared to two wells in five-spot or line-drive floods. On this basis the resultant flow capacity of the seven-spot is even less than the other two networks. On such comparison it was found that the five-spot was the most favorable flood, direct-line-drive being the next, and seven-spot the last.

Upon extending this comparison to stretching and staggering the line-drive, Table I was obtained. The staggered-line-drive with $\frac{a}{b} = 1.5$, has a sweep efficiency of 80 per cent which is greater than

TABLE I

RELATIVE EFFICIENCY OF FLOOD (4)(5)

Types of Network	Relative Production Rate (Conductivities for equal well-acre densities)	Flood efficiency %
Staggered line-drive $\frac{a}{b} = 1.5$ $a = 808'$; $b = 539'$	0.383	80
Five-spot $a_5 = 660'$	0.433	72.3
Regular line-drive $\frac{a}{b} = 1.5$	.383	70.6
Seven-spot $a_7 = 709'$	.378	74

(4) Muskat, M. and Wyckoff, R. D., "Mechanics of Porous Flow Applied to Flooding Problems." Trans. A.I.M.E. 156, 219 (1933).

(5) Muskat, M. and Wyckoff, R. D., "Theoretical Analysis of Water Flooding Networks." Trans. A.I.M.E., 192, 62 (1934).

that of either the five-spot or the seven-spot. For a given well density (number of wells per acre), a 12 per cent greater pressure differential is required for the staggered-line-drive ($\frac{a}{b} = 1.5$) than for the five-spot to produce oil at equivalent rates. From the previous statement, it is obvious that this cost of producing a given quantity of oil is greater for the modified five-spot than for the five-spot.

In deciding which of the two patterns to use, engineers are interested in the net profit obtained from the oil produced. In other words, they are interested in whether the profit from the additional oil produced by the modified five-spot will more than pay for the additional cost of production.

Reviewing the nine-spot (6) pattern, Fig. 1-f, there are eight input wells surrounding each output well. There are four input wells (g_2), at a distance 'd' from the output well (g_1). Also four input wells (g_3) are located diagonally at a distance $\sqrt{2}d$ from the output well (g_1). Each input well (g_2) will contribute to the two output wells. Each input well (g_3) contributes to four output wells. Therefore, an effective number of (g_2) wells in a unit nine-spot will be two and the effective number of (g_3) wells in a unit nine-spot will be one. Hence, the magnitude of the fluid rates will be given by the equation

$$g_1 = 2g_2 + g_3$$

(6) Krutter, H. "Nine-Spot Flooding Program," Oil and Gas Journal 38 (No. 14), 50. (Aug. 17, 1939).

Comparing nine-spot with five-spot and seven-spot on a conductivity basis and considering 'N' wells in each of the flooding networks, there are N/2 output wells in the five-spot; N/3 output wells in the seven-spot, and N/4 wells in the nine-spot. Therefore, for a given pressure difference between the output and the input wells, the relative rate of production will be

$$Q_9 : Q_7 : Q_5 = 1:1.435:1.646$$

Hence the nine-spot has the lowest production rate. For the same pressure difference between the input and the output wells the nine-spot has a sweep efficiency of 52.1 per cent. The five-spot and seven-spot have a sweep efficiency of 71.8 per cent and 74.3 per cent respectively.

Work done by Muskat (7) on a nine-spot network shows that the coverage for the nine-spot will be 78.1 per cent if the rates of fluid injection into the corner and side wells are maintained in a proper ratio.

The primary criterion for successful water flooding operation is the availability of sufficient amounts of recoverable oil. Water flooding of formations with residual oil saturation below 35 per cent is a hazardous undertaking. Areal continuity and uniformity of flooded reservoirs are also prerequisites for profitable operations. On the basis of operating experiences by engineers in the industry, it is generally assumed that water flooding is most profitable if the

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(7) Muskat, M., Producers Monthly, 12, 14. (March, 1948)

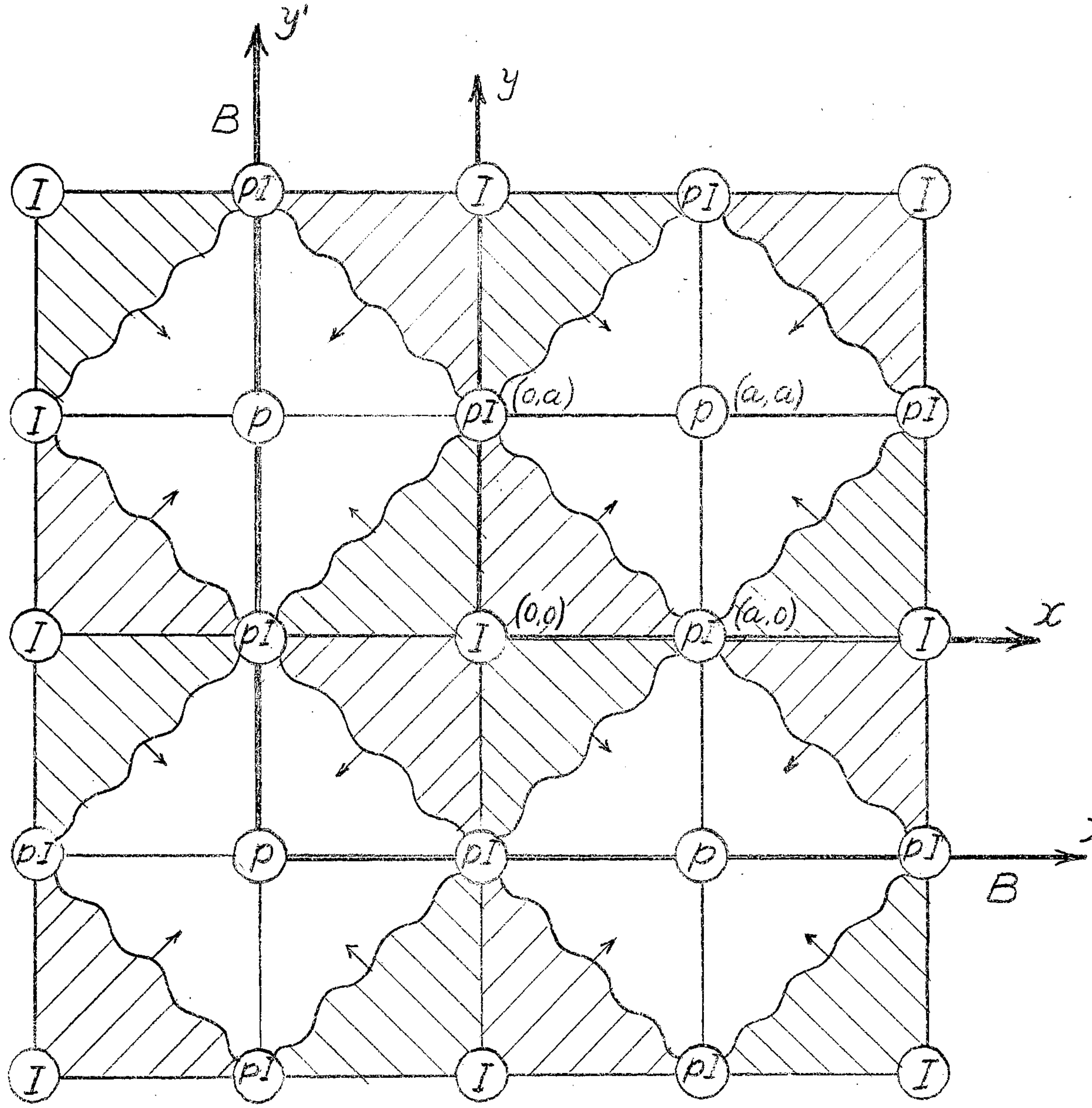

FIG. 2

Schematic Representation of New Water Flooding Network

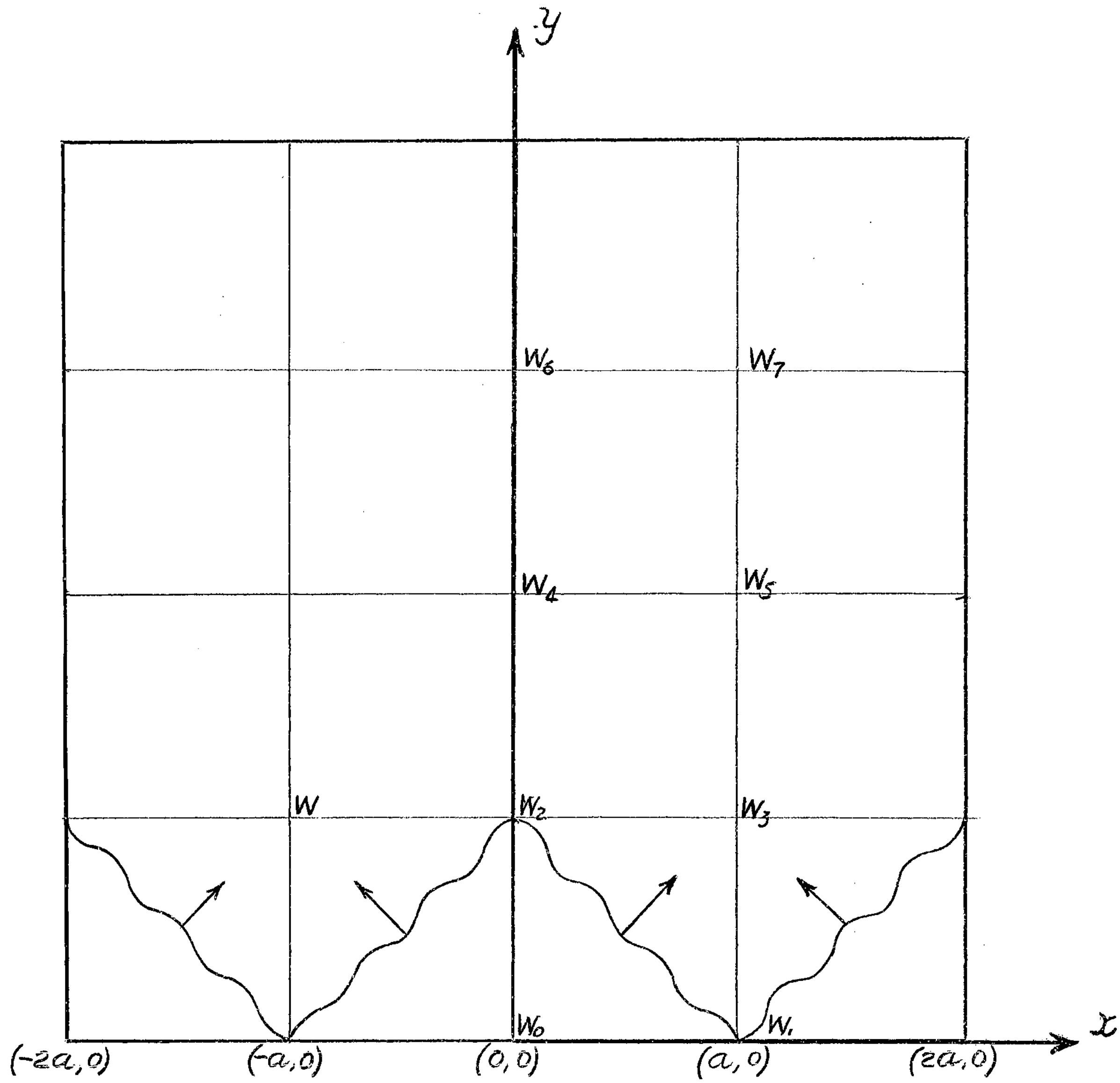

FIG. 3

Schematic Representation of New Water Flooding Network - Second Stage reservoir oil viscosity does not exceed 20 to 25 centipoises. Another criterion for successful water flooding is, of course, the comparative value of additional oil produced and the cost of production. No simple rules can be given to prescribe the amount of additional recovery required for a payout and profit. The gross return will depend upon the unit selling price of the oil and its volume. Included in the operating costs are the expenses of drilling new wells and reworking old wells. The cost of treating and pumping flood water and the availability of water should be given due consideration. Another factor for consideration is the water-oil ratio.

The question of absolute well spacing is a matter of income versus expenditure. It has been generally established that well spacing has no effect upon efficiency and only a minor effect upon conductivity. Therefore, the values of high production rate with close spacing and large investments must be balanced against lower production rate with wider spacing and proportionately lower investments. Evidently, no single formula will evaluate these factors, except as it is merely a symbolic summation of all these items which comprise the operating cost.

CHAPTER II

PROPOSED PROCEDURE FOR WATER FLOODING

The procedure can best be illustrated with the help of Fig. 2. A square field with 25 wells is imagined. There are five rows and each row has five wells. The wells are shown in circles and are labeled as P, I, and PI. There is no staggering of the wells. To start, the water is injected into the wells labeled I, while the wells labeled P and PI are the producers. When water begins to flow into wells labeled PI, these wells then become injection wells and the wells labeled I are closed. At this stage, the shape of the flood front is believed represented approximately by the wavy lines in Fig. 3.

The procedure applied to a line-drive from a side of a field can be understood more fully by referring to Fig. 3. Figure 3 is a rectangular coordinate representation of the wells in an oil field. The wells are equally spaced "a" units both in the x and y directions. Initially, water is injected into alternate wells as (-2a,o), (o,o), (2a,o). All other wells remain as producers until water breaks through into wells (-2a,a), (o,a), (2a,a), (-a,o) and (a,o). Then wells (-2a,a), (2a,a), (-a,o), (o,a) and (a,o) become additional injection wells. At this time, it is believed that the water fronts have approached the positions as represented by the wavy lines. Next, the water breaks through into wells (a,a) and (-a,a) after which the procedure is continued across the field. With this procedure there is leakage in the direction of the flood front past the producing wells.

CHAPTER III

FLOW NET DETERMINATION BY MEANS OF THE RESERVOIR ANALYZER

The flow of fluids through porous media is known to follow Darcy's Law which states that the velocity of flow is proportional to pressure gradient. Given this basic law, it is possible to obtain solutions to many problems of viscous flow of liquids by usual methods of potential theory. It is useful, therefore, to note that Darcy's Law is precisely equivalent to the law of electrical conduction. Pressure distribution in steady state flow of liquid in porous medium is equivalent to potential distribution in flow of electrical current in an electrical conducting medium. This significance is due to the fact that electrically the porous medium corresponds to a conducting medium of specific conductance. In the present analysis, fluids were assumed to be of the same density. The electrical analogy also implies that wells penetrating a fluid-bearing sand correspond to electrodes penetrating an electrical-conducting medium and that impermeability of sand facies corresponds to electrical insulation.

The analogy between the flow of an incompressible fluid through a porous medium and current through an electrical conducting medium can be understood best by inspecting certain relations. Table II lists several relations between fluid flow in a porous medium and electrical current flow in a conducting medium. Therefore, the analogy of the above considerations imply that a reservoir can be simulated

TABLE II

Hydrodynamics of Steady State Flow Through Porous Media (Incompressible Fluid)	Current Conduction
Darcy's Law	Ohm' Law
Pressure: P	Voltage: V
Pressure gradient: ΛP	Potential gradient: ΛV
$\frac{\text{Permeability}}{\text{Viscosity}}$: $\frac{k}{\mu}$	Specific conductivity: σ
Velocity vector: $\bar{v}$	Current vector: $\bar{I}$
$\bar{v} = \frac{K}{\mu} \Lambda P$	$\bar{I} = -\sigma \Lambda V$
Equi-pressure surface:	Equipotential surface:
P = constant	V = constant

by an electrical medium of similar geometry, with distributed resistances related to the reservoir rocks. The pressure history and distributions in a reservoir are identical with voltage history and distributions of the electrical analogue. Also, the current densities are identical with the fluid flux in the reservoir. A reservoir can be visualized as an interconnected network of discrete rock units with each of which definite values of pertinent physical constants can be associated. Now it is possible to represent a reservoir by an electrical medium consisting of an electrical resistant network; hence, the correspondence between the electrical flow and fluid flow system is established.

As suggested in Chapter II, the strip of field under consideration is one bounded by the lines $x = o$, $y = o$, $x = a$, and $y = a$. Taking the previously mentioned facts into consideration to represent this strip, an electrical analogue, Fig. 4, was constructed. The analogue was a resistance network. The analogue represents the xy plane of the function $p(x,y)$ of two variables. The xy plane was divided into a network or lattice squares of sides 0.05a by plotting the two families of lines

(a) $x = 0.05ma$ $m = 1,2,3 - - - 21.$

(b) $y = 0.05na$ $n = 1,2,3 - - - 21.$

Each side of the square contained a megohm resistor. The points of intersection of the resistors are called lattice points. Each lattice point is represented by the rectangular coordinate system. Thus, the point of intersection of the line $x = 0.05a$ with that of

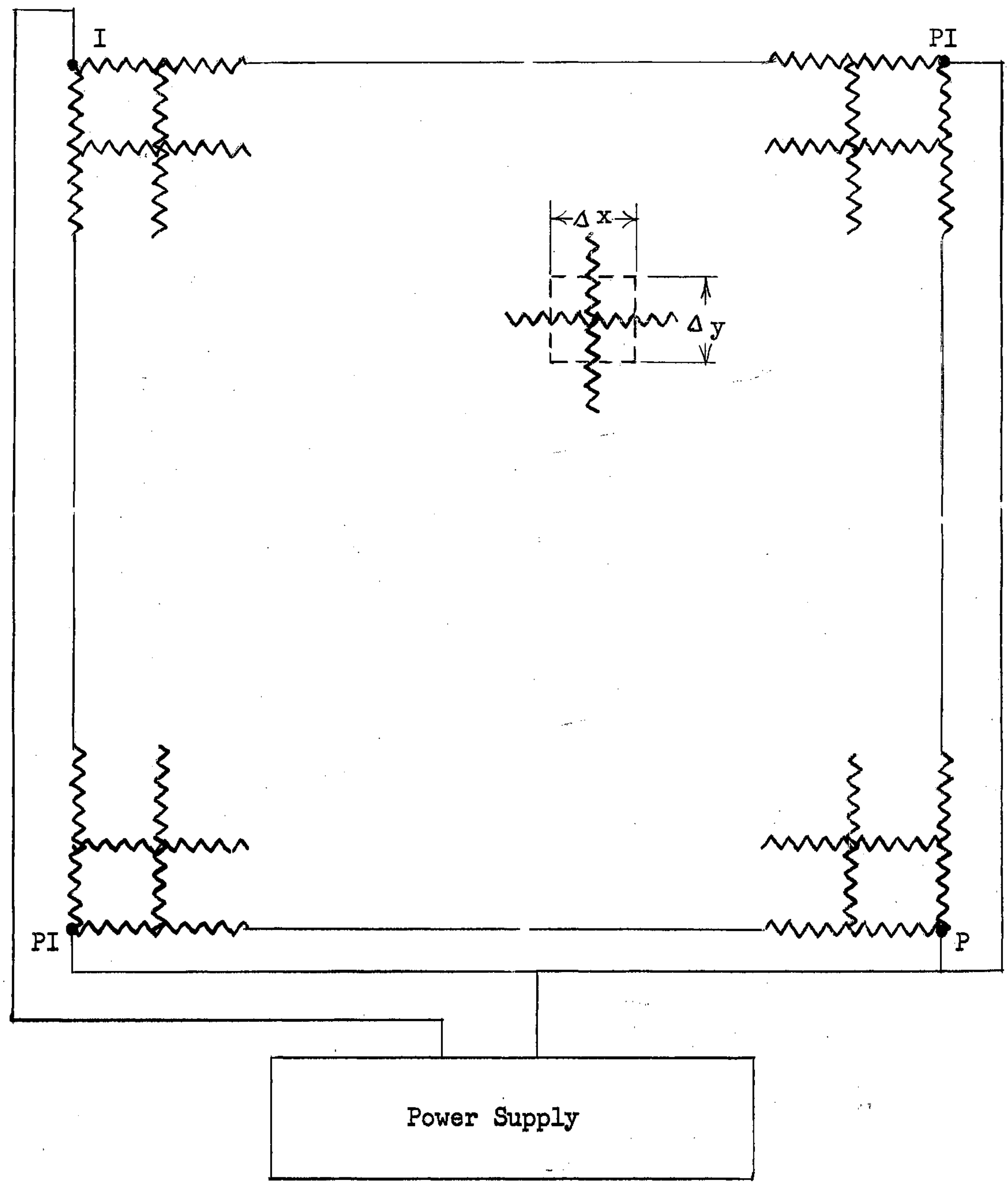

Figure 4. Circuit Diagram and Schematic Drawing of the Electric Analogue

y = 0.05a is (0.05a,0.05a) and so on.

Thus, it can be very well imagined that the whole analyzer is the summation of the small square units of sides Δx and Δy - such a unit is marked on Fig. 4 with dotted lines.

The reservoir analyzer was used to determine the potential distribution within one of the symmetrical blocks of the flood pattern for the proposed procedure of water flooding. During the initial stage one corner of the analyzer was used for the current input terminal and the other corners of the analyzer were used as current output terminals. During the final stage three corners of the analyzer were used as current input terminals and the remaining corner was used as the current output terminal. The current input terminals were analogous to producing wells.

After the potential distribution had been determined from the analyzer the equipotential lines were drawn in and the streamlines were plotted orthogonally to the equipotential lines.

The position of the advancing flood front was determined by an equation given by Calhoun. (8). The equation is

$$\Delta t_1 = \phi \frac{u}{k} \frac{(\Delta S)^2}{(\Delta P)}$$

where t = the time required for a fluid particle to move from a higher equipotential line to a lower one

ϕ = porosity of the reservoir formation

k = permeability of the reservoir formation

u = viscosity of the reservoir fluid

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(8) Calhoun, John C. Jr., Fundamentals of Reservoir Engineering, Univ. of Oklahoma Press, Norman, Oklahoma. (1953).

ΔS = distance between equipotential lines as measured along a streamline

ΔP = potential difference between two successive equipotential lines.

In order to simplify the problem as much as possible the value of terms $\phi \frac{u}{k}$ was assumed to be unity.

CHAPTER IV

DEVELOPMENT OF FLOW EQUATIONS

In this treatment and throughout the analysis of the problem, it was assumed that the sands were homogeneous and uniform in thickness; also, it was assumed the wells completely penetrated the formation. With these assumptions, the flow system was considered as two-dimensional. In a two-dimensional system, the velocity vector ($\overline{v}$) varies with only two of the rectangular co-ordinates. Physically, of course, all fluid systems necessarily extend in three dimensions. The significance of the two-dimensional system is that all the features of motion can be considered in a single plane --- the motion is identically the same in parallel planes. In this mode of attack, only horizontal two-dimensional motion is considered in the xy plane, and the velocity vector ($\overline{v}$) is independent of the vertical co-ordinate. Furthermore, it also follows that, even though gravity is acting on each fluid element, the fluid either moves bodily in a vertical direction or it has no vertical velocity. It is clear, therefore, that gravity was of no significance in this type of motion and one can use, quite rigorously, the pressure "p" as equivalent of the velocity potential.

From the previous considerations, it follows that the fundamental equation which governs horizontal two-dimensional fluid motion in steady state is:

$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} = 0 \qquad (1)$$

$$\bar{V}_x = -\frac{K}{\mu} \cdot \frac{\partial p}{\partial x} \quad ; \quad \bar{V}_y = -\frac{K}{\mu} \cdot \frac{\partial p}{\partial y} \tag{2}$$

The Laplace equation is solved by choosing a potential function which is a combination of individual functions - - the individual functions satisfy the boundary conditions. A general solution of Eq. (2) for the square (o,o), (o,a), (a,o) and (a,a) as indicated by the xy co-ordinates in Fig. 2 is

$$\phi(x,y) = \phi(x,y) + \sum_{1}^{n} \phi_m(x,y) \tag{3}$$

if $\phi(x,y)$ is any harmonic function which approximately satisfies the boundary conditions on all sides of the square, and if $\phi_m(x,y)$ is a sequence of harmonic functions which vanish on three sides of the same square.

For the first stage of the new water flooding procedure, Eq. (3) can be written as

$$\phi(x,y) = \phi_1, + \phi_2 + \phi_3 + \phi_4 + \phi_5 \; . \tag{4}$$

The function,

$$\phi_1 = \frac{P_o \; (a-x) \; (a-y)}{a^2} \tag{5}$$

is a harmonic function and is linear on the lines $x = o$ and $y = o$. It is zero on the lines $x = a$ and $y = a$ and takes on the value of the pressure at the four wells. The pressure at the injection well is P_o and the pressure at the producing wells is zero.

The function,

$$\phi_2 = \sum A_n \sinh \; n\pi(1 - \frac{y}{a}) \sin \frac{n\pi x}{a} , \qquad (6)$$

is harmonic and vanishes on the line $y = a$, $x = a$, and $x = o$.

The function,

$$\phi_3 = \sum B_n \sinh \frac{n\pi y}{a} \sin \frac{n\pi x}{a} , \qquad (7)$$

is harmonic and vanishes on the lines $y = o$, $x = o$, and $x = a$.

The function,

$$\phi_4 = \sum C_n \sinh n\pi(1 - \frac{x}{a}) \sin \frac{n\pi y}{a}, \qquad (8)$$

is harmonic and vanishes on $x = a$, $y = a$, and $y = o$.

The function,

$$\phi_5 = \sum D_n \sinh \frac{n\pi x}{a} \sin \frac{n\pi y}{a} , \qquad (9)$$

is harmonic and vanishes on $x = o$, $y = o$, and $y = a$.

The terms, $A_n \sinh n\pi(1 - \frac{y}{a})$, $B_n \sinh \frac{n\pi y}{a}$, $C_n \sinh n\pi(1- \frac{x}{a})$ and $D_n \sinh \frac{n\pi x}{a}$ are respectively Fourier coefficients of functions ϕ_2, ϕ_3, ϕ_4 and ϕ_5.

$A_n \sinh \; n\pi(1 - \frac{y}{a})$ are Fourier coefficients of $P - P_o(1 - \frac{x}{a})$ on $y = o$. The value of pressure along the line $y = o$ is P.

$B_n \sinh \frac{n\pi y}{a}$ are the Fourier coefficients of P on $y = a$.

$C_n \sinh n\pi(1 - \frac{x}{a})$ are the Fourier coefficients of $P - P_o(1 - \frac{y}{a})$ on $x = o$.

$D_n \sinh \frac{n\pi x}{a}$ are the Fourier coefficients of P on $x = a$.

Substituting the values of ϕ_1, ϕ_2, ϕ_3, ϕ_4, ϕ_5 in Eq. (2) and using $P(x,y)$ for $\phi(x,y)$, the solution of the Laplace equation for obtaining pressures in the first stage of the water flooding procedure becomes

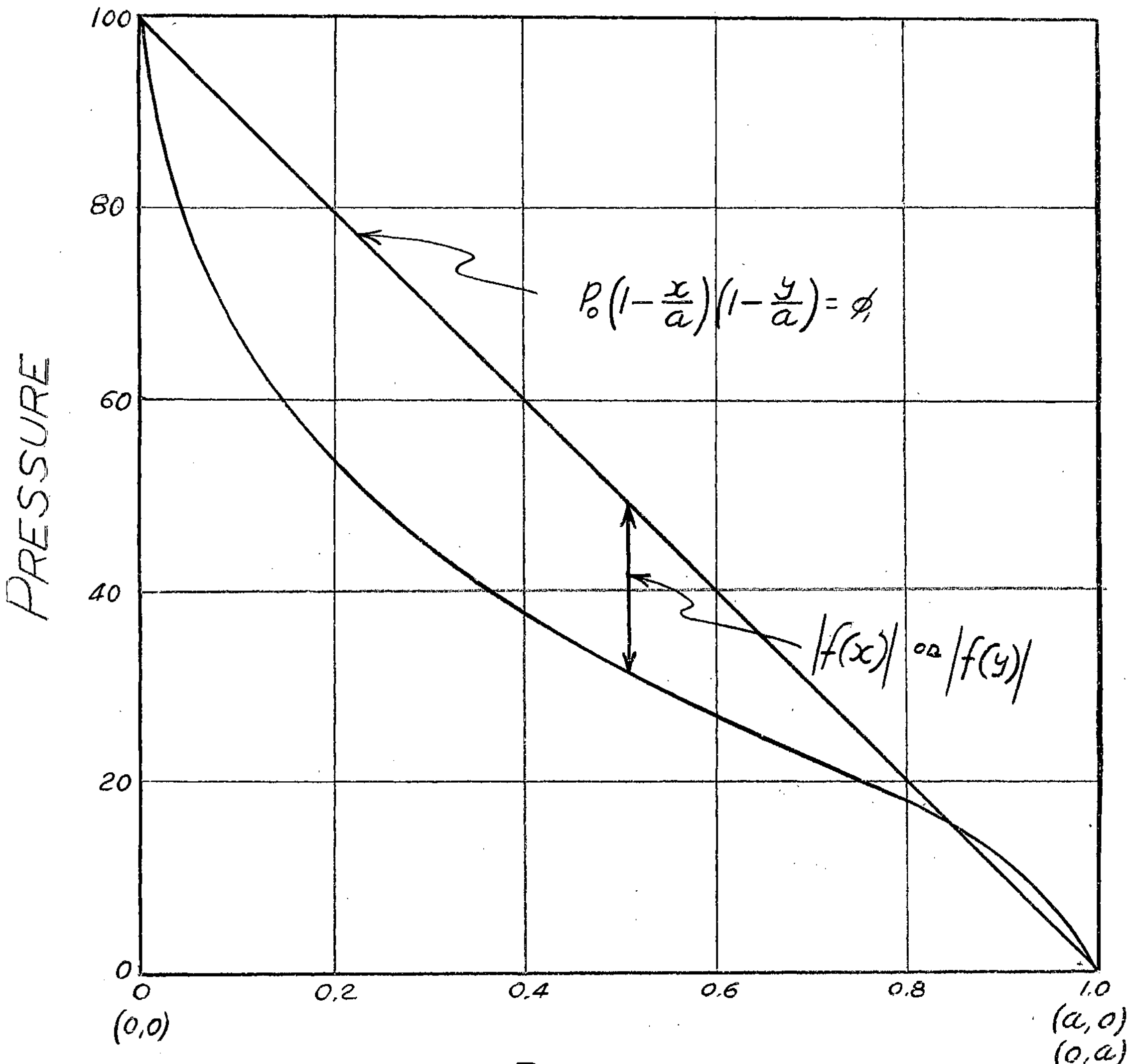
$$
\begin{aligned}
P(x,y) = {} & \frac{P_o(a-x)(a-y)}{a^2} + \sum A_n \sinh n\pi(1-\frac{y}{a})\sin\frac{n\pi x}{a} \\
& + \sum B_n \sinh\frac{n\pi y}{a}\sin\frac{n\pi x}{a} \qquad (10) \\
& + \sum C_n \sinh n\pi(1-\frac{x}{a})\sin\frac{n\pi y}{a} \\
& + \sum D_n \sinh\frac{n\pi x}{a}\sin\frac{n\pi y}{a}
\end{aligned}
$$

The pressure distributions along the boundaries for the first stage are given in Figs. 5 and 6.

In the second stage of the water flooding procedure, the producing well was taken as the origin; the two injection wells were located at (o,a) and (a,b). The (x',y') coordinate system in Fig. 2 applies to the second stage. The pressure varied along the boundaries of the square (o,o), (o,a), (a,o) (a,a) in a manner as shown in Figs. 7 and 8.

Equation (2) also holds for the solution of the Laplace equation for the second stage; however, the harmonic function ϕ, will be different for the second stage and ϕ_2, ϕ_3, ϕ_4, ϕ_5 as expressed in Eqs. (6) (7) (8) and (9) remain the same. The equation for obtaining the Fourier coefficients are different in the second stage.

For the second stage the pressure at (a,a) becomes $P = (P_o - P_c)$. The pressure at the injection well is denoted as P_o and P_c is a constant which is determined with the aid of the reservoir analyzer.

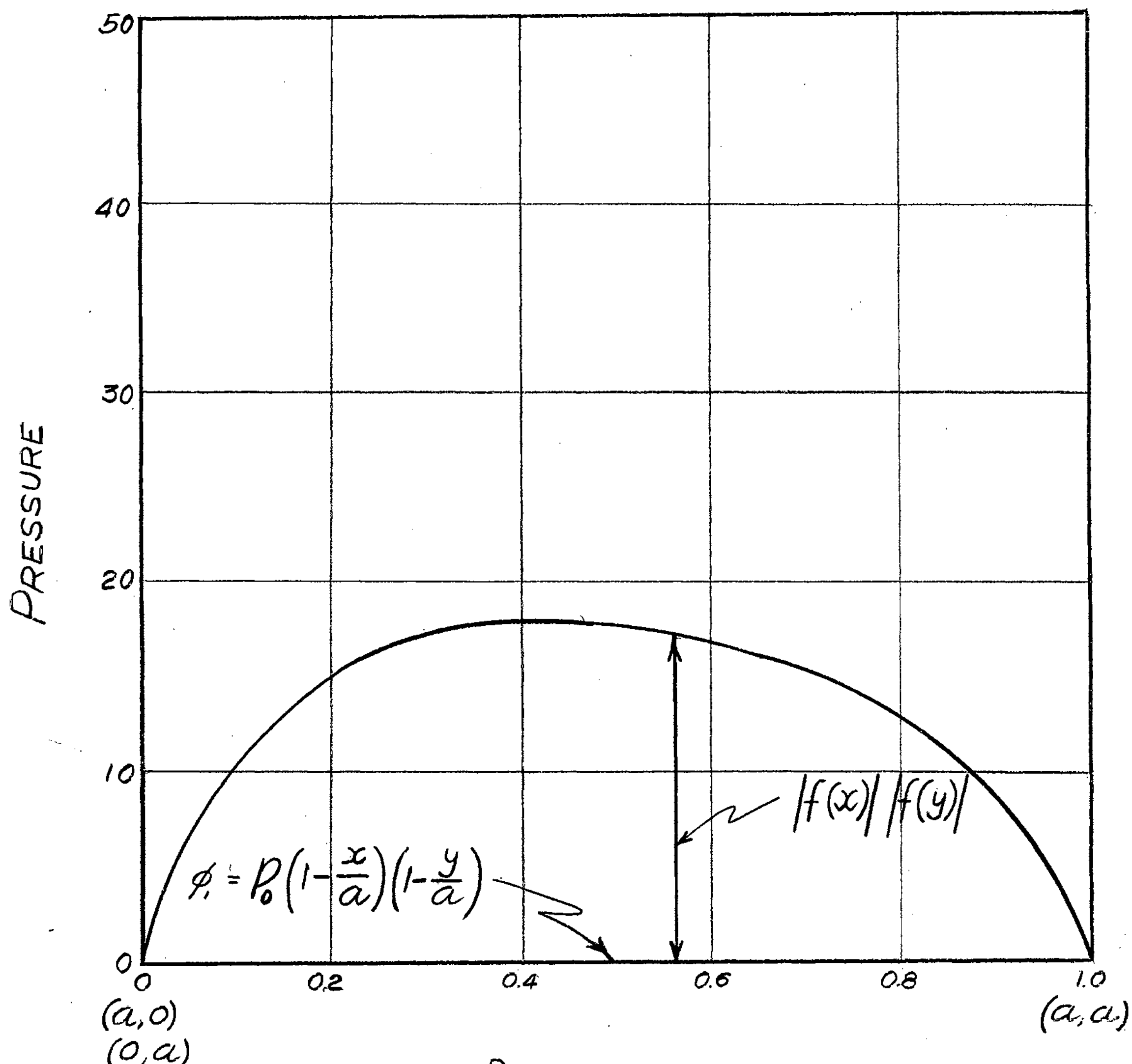


DISTANCE

PRESSURE DISTRIBUTION ALONG $(x=0)$ $(y=0)$

FIRST STAGE

Fig. 5

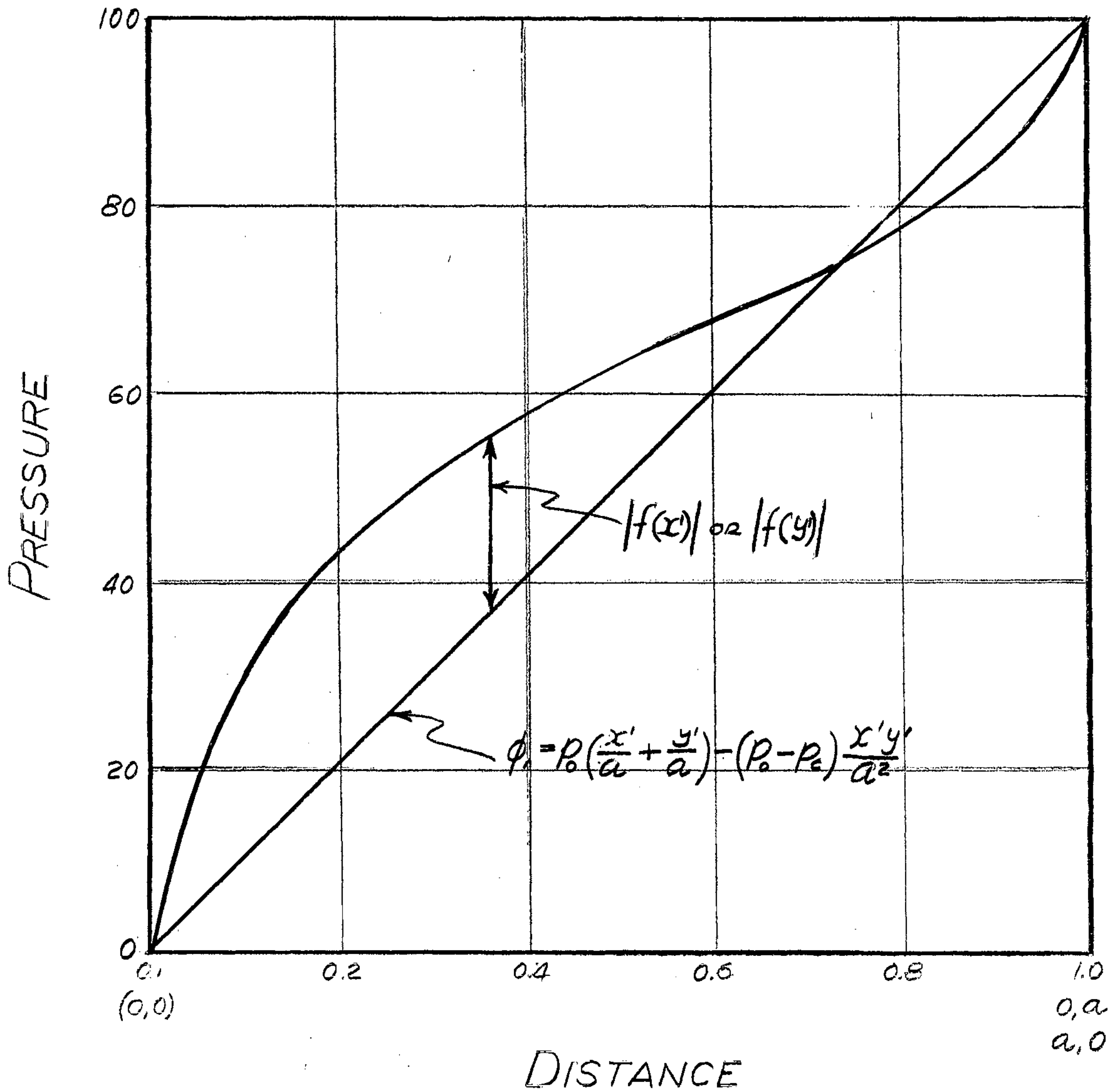


DISTANCE

PRESSURE DISTRIBUTION ALONG $(x=a)$ $(y=a)$

FIRST STAGE

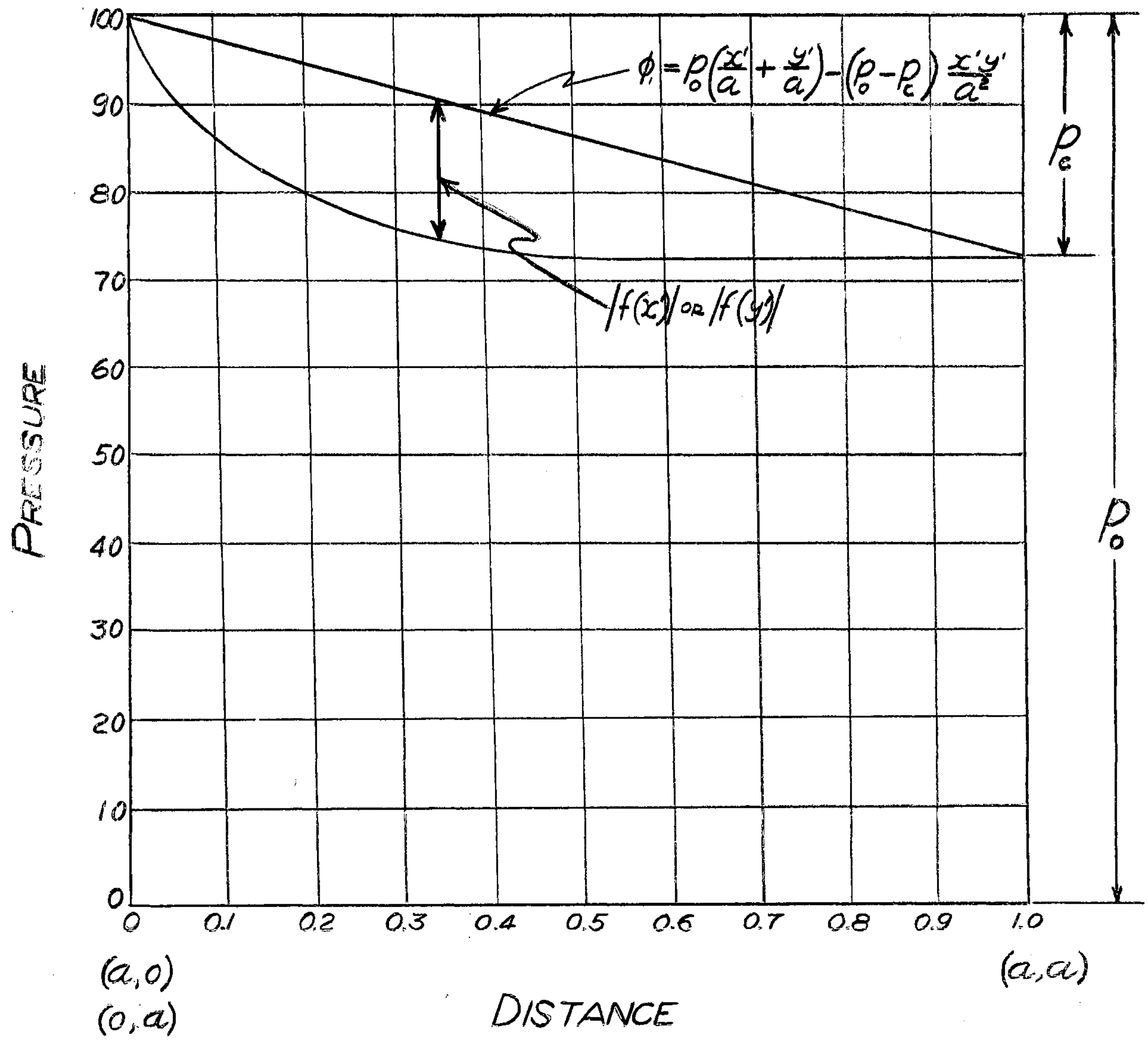
Fig. 6



PRESSURE DISTRIBUTION ALONG $(x'=0)$ $(y'=0)$

SECOND STAGE

Fig. 7



PRESSURE DISTRIBUTION ALONG $(x'=a)$ $(y'=a)$

SECOND STAGE

Fig. 8

The function ϕ_1, becomes

$$P_o(\frac{x}{a} + \frac{y}{a}) - (P_o + P_c)\frac{x'y'}{a^2} \qquad (11)$$

for the second stage. This function is harmonic and is linear on all four sides of the square (o,o), (a,a), (a,o), (o,a), and it assumes the value of the pressure at each corner.

The terms, A_n' sinh $n\pi(1 - \frac{y}{a})$, of the function ϕ_2 become the Fourier coefficients of

$$\left[P_o(\frac{x'}{a} + \frac{y'}{a}) - (P_o + P_c)\frac{x'y'}{a^2} - P\right] \text{ on } y' = 0 .$$

The terms, B_n' sinh $\frac{n\pi y'}{a}$ of the function, ϕ_3, become the Fourier coefficients of

$$\left[P_o(\frac{x'}{a} + \frac{y'}{a}) - (P_o + P_c)\frac{x'y'}{a^2} - P\right] \text{ on } y' = a .$$

The terms, C_n' sinh $n\pi(1 - \frac{x'}{a})$ of the function, ϕ_4, become the Fourier coefficients of

$$\left[P_o(\frac{x'}{a} + \frac{y'}{a}) - (P_o + P_c)\frac{x'y'}{a^2} - P\right] \text{ on } x' = 0 .$$

The terms, D_n' sinh $\frac{n\pi x'}{a}$ of the function, ϕ_5, become the Fourier coefficients of

$$\left[P_o(\frac{x'}{a} + \frac{y'}{a}) - (P_o + P_c)\frac{x'y'}{a^2} - P\right] \text{ on } x' = a .$$

Again $p(x',y') = \phi_1 + \phi_2 + \phi_3 + \phi_4 + \phi_5$ is a solution to the Laplace equation for the second stage of the water flooding procedure and can be written more specifically as

$$\begin{aligned} P(x'y') = {} & P_o(\frac{x'}{a} + \frac{y'}{a}) - (P_o + P_c)\frac{x'y'}{a^2} \\ & + \sum A_n \sinh n\pi(1 - \frac{y'}{a}) \sin n\pi\frac{x'}{a} \\ & + \sum B_n \sinh \frac{n\pi y'}{a} \sin n\pi\frac{x'}{a} \\ & + \sum C_n \sinh n\pi(1 - \frac{x'}{a}) \sin n\pi\frac{y'}{a} \\ & + \sum D_n \sinh n\pi\frac{x'}{a} \sin n\pi\frac{y'}{a} . \end{aligned} \qquad (12)$$

The Laplace partial differential equation can be replaced by a difference equation, and the difference equation can be solved by numerical methods. Since pressure distributions can be represented by a solution of the Laplace equation, the pressure at any point in a rectangular field can be determined by numerically solving the difference equation of the Laplace equation, i.e., provided the boundary conditions of the field are known.

A difference equation corresponding to a given differential equation is obtained from the corresponding difference quotients. A difference quotient (9) corresponds to the increment of a function divided by the increment of the independent variable. For example, the difference quotient for a function f(x), a single variable, is

$$\frac{f(x+h)-f(x)}{h} .$$

For a function P(x,y) of two variables the (x,y) plane was divided into a network or lattice of squares of sides "h", by drawing the two families of parallel lines

$$x = mh \qquad m = 0,1,2 \text{------}$$

$$y = nh \qquad n = 0,1,2 \text{-------}$$

as given in Fig. 9. The points of intersection of these lines are called lattice points.

The forward first difference quotient of P(x,y) with respect to x is

$$P_x = \frac{P(x+h,y)-P(x,y)}{h} . \qquad (13)$$

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(9) Scarborough, J. B., Numerical Mathematical Analysis, 2nd ed., pp 310-327, The John Hopkins Press, Baltimore. (1950).

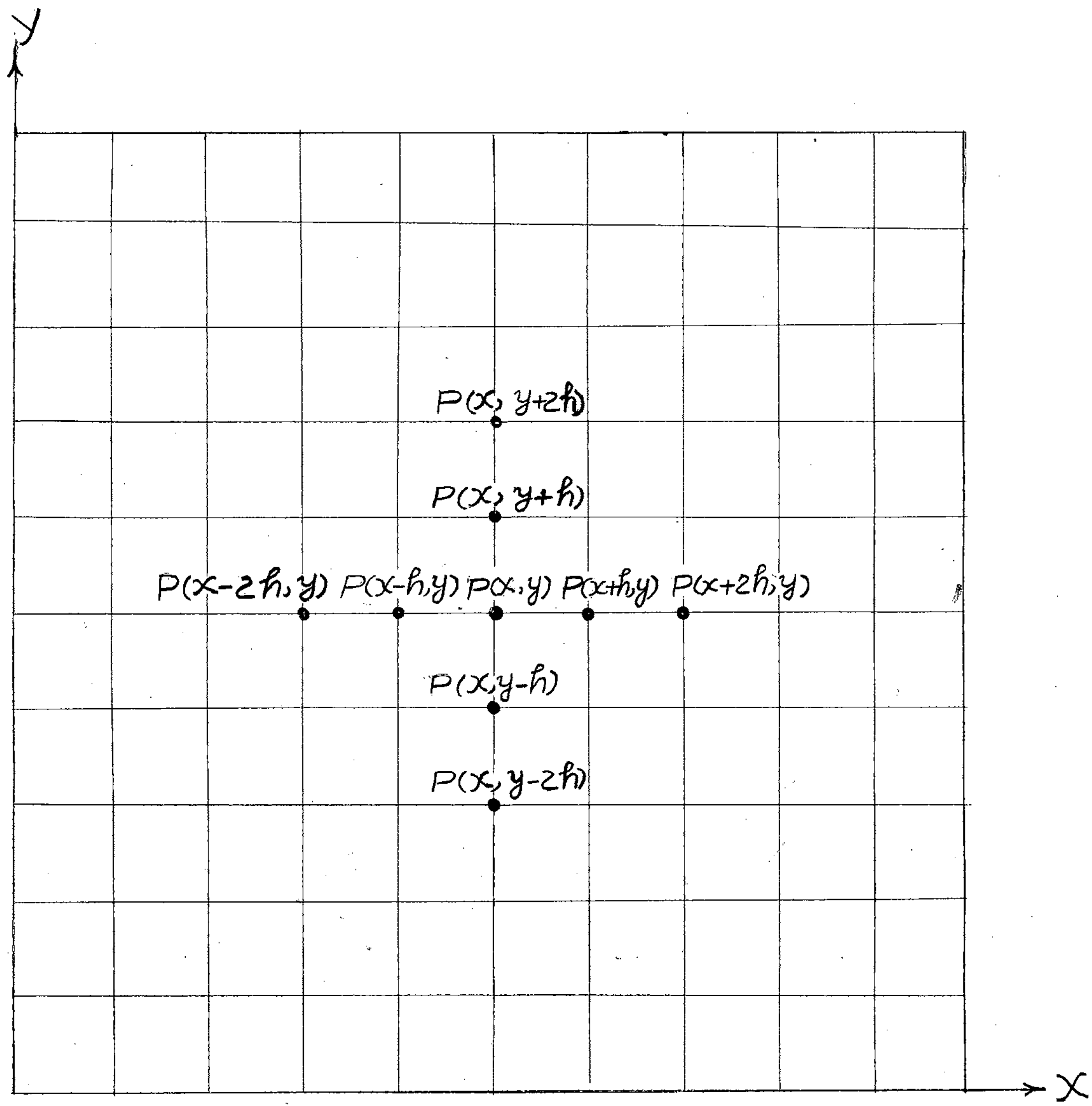

FIG. 9-a

Illustration of Divided Difference

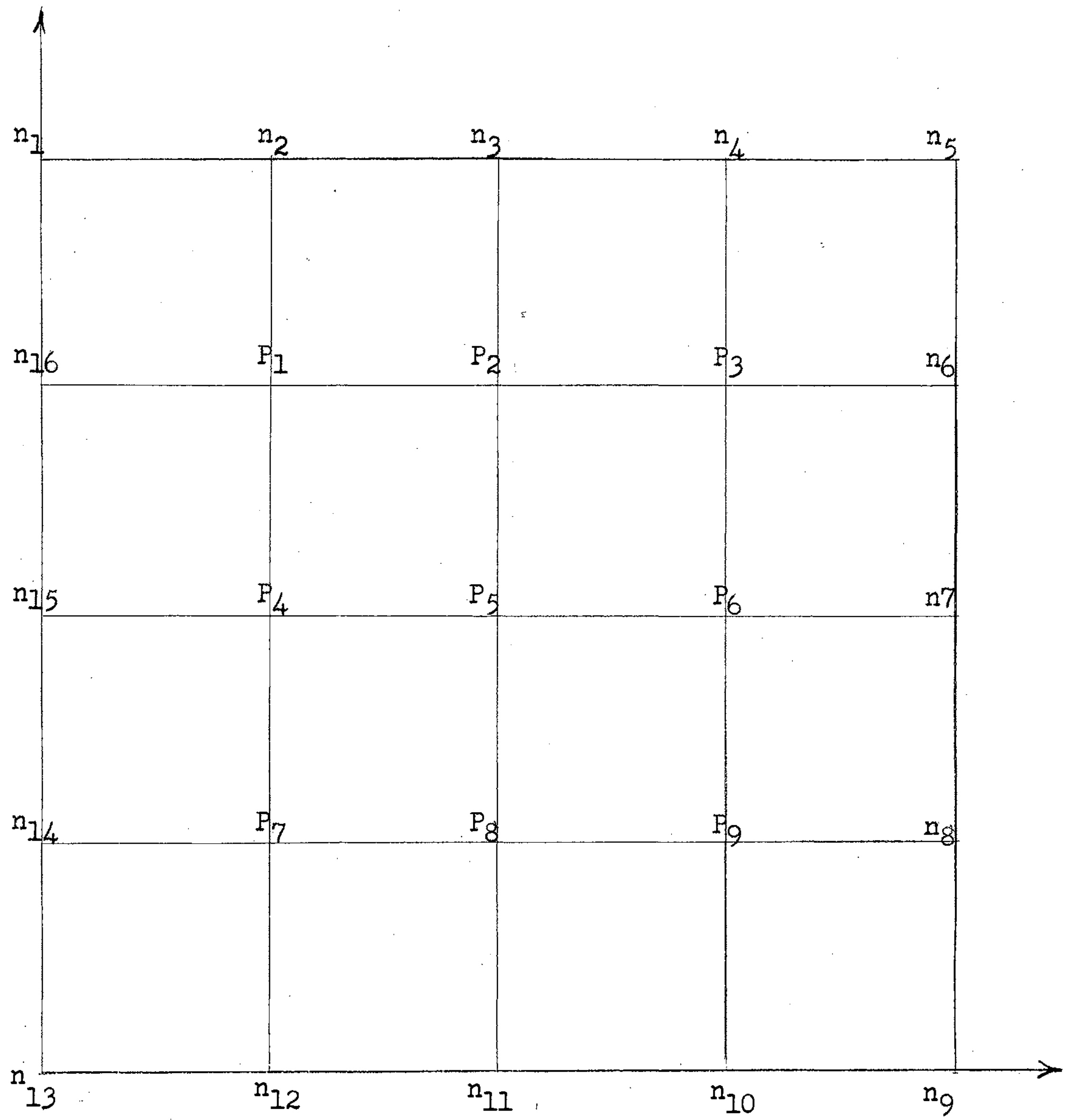

Figure 9b. Mesh Representation of Function P(x,y)

And the backward first difference quotient with respect to x is

$$P_{\bar{x}} = \frac{P(x,y) - P(x-h,y)}{h} \tag{14}$$

The second difference quotient of P(x,y) with respect to x is the difference quotients of (13) and (14) and is represented as

$$P_{\bar{x}x} = \frac{P_x - P_{\bar{x}}}{h} = \frac{\frac{P(x+h,y) - P(x,y)}{h} - \frac{P(x,y) - P(x-h,y)}{h}}{h} \tag{15}$$

$$= \frac{P(x+h,y) - 2P(x,y) + P(x-h,y)}{h^2}$$

In a similar manner the second difference quotient of P(x,y) with respect to y is determined to be as follows:

$$P_{\bar{y}y} = \frac{P_y - P_{\bar{y}}}{h} \tag{16}$$

$$= \frac{P(x,y+h) - 2P(x,y) + P(x,y-h)}{h^2} .$$

In the Laplace equation, $\frac{\partial^2 P}{\partial x^2}$ and $\frac{\partial^2 P}{\partial y^2}$ can be replaced by $P_{\bar{x}x}$ and $P_{\bar{y}y}$ respectively to give the difference equation:

$$\frac{P(x+h,y) - 2P(x,y) + P(x-h,y)}{h^2} + \frac{P(x,y+h) - 2P(x,y) + P(x,y-h)}{h^2} = 0. \tag{17}$$

Solving Eq. (17) for P(x,y), the equation

$$P(x,y) = \frac{1}{4}\left[P(x+h,y) + P(x,y+h) + P(x-h,y) + P(x,y-h)\right] \tag{18}$$

is obtained as a solution to the Laplace equation.

The inherent error in the solution of the Laplace equation with difference equations is determined by expressing the difference quotients in terms of derivatives. Taylor's formula can be used for expressing difference quotients in terms of derivatives. For a function of two variables Taylor's formula is

$$f(x+h, y+k) = \sum_{n=0}^{n=\infty} \frac{1}{n!}\left(h\frac{\partial}{\partial x} + k\frac{\partial}{\partial y}\right). \tag{19}$$

If k = 0,

$$P(x+h, y) = P(x,y) + h\frac{\partial P}{\partial x} + \frac{h^2}{2!}\frac{\partial^2 P}{\partial x^2} + \frac{h^3}{3!}\frac{\partial^3 P}{\partial x^3} + \text{----} \tag{20}$$

From Eq. (13),

$$\frac{P(x+h, y) - P(x,y)}{h} = \frac{\partial P}{\partial x} + \frac{h}{2!}\frac{\partial^2 P}{\partial x^2} + \frac{h^2}{3!}\frac{\partial^3 P}{\partial x^3} + \text{-----} \tag{21}$$

And from Eq. (14),

$$\frac{P(x,y) - P(x-h, y)}{h} = \frac{\partial P}{\partial x} - \frac{h}{2!}\frac{\partial^2 P}{\partial x^2} + \frac{h^2}{3!}\frac{\partial^3 P}{\partial x^3} + \text{-----} \tag{22}$$

Subtracting Eq. (22) from Eq. (21) and dividing by h,

$$\frac{P(x+h, y) - 2P(x,y) + P(x-h, y)}{h^2} = \frac{\partial^2 P}{\partial x^2} + \frac{2h^2}{4!}\frac{\partial^4 P}{\partial x^4} \tag{23}$$

$+$ in terms of h^4, h^6 etc.,

Which is the second-difference quotient with respect to x.

In a similar manner the second-difference quotient with respect to y becomes

$$\frac{P(x, y+k) - 2P(x,y) + P(x, y-k)}{k^2} = \frac{\partial^2 P}{\partial y^2} + \frac{2k^2}{4!}\frac{\partial^4 P}{\partial y^4} \tag{24}$$

$+$ in terms of k^4, k^6 etc.

Upon adding Eqs. (23) and (24) and setting k = h the equation

$$P_{xx} + P_{yy} = \frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial y^2} + \frac{2h^2}{4!}\frac{\partial^4 P}{\partial x^4} + \frac{2h^2}{4!}\frac{\partial^4 P}{\partial y^4} \qquad (25)$$

$+$ in terms in h^4, h^6 etc.

The sum of all the terms in h^2, h^4, h^6, etc., represents the inherent error in the solution by difference equations. This error is negligible if h is very small.

CHAPTER V

CALCULATION OF PRESSURE DISTRIBUTIONS

For the first step in calculating pressure distributions, boundary conditions along $y = o$, $x = o$, $y = a$, and $x = a$ were determined with the reservoir analyzer. A plot of the boundary conditions for the first stage is given in Figs. 5 and 6 and a plot of the boundary values for the second stage is given in Figs. 7 and 8. Also in Figs. 5 and 6 the function $\frac{P_o(a-x)\ (a-y)}{a^2}$ is plotted and in Figs. 7 and 8 the function $P_o(\frac{x}{a} + \frac{y}{a}) - (P_o + P_c)\ (\frac{xy}{a^2})$ is plotted.

As previously developed

$$
\begin{aligned}
P(x,y) = \frac{P_o(a-x)(a-y)}{a^2} &+ \sum A_n \sinh n\pi(1-\frac{y}{a}) \sin \frac{n\pi x}{a} \\
&+ \sum B_n \sinh \frac{n\pi y}{a} \sin \frac{n\pi x}{a} \\
&+ \sum C_n \sinh n\pi(1-\frac{x}{a}) \sin \frac{n\pi y}{a} \qquad (26) \\
&+ \sum D_n \sinh \frac{n\pi x}{a} \sin \frac{n\pi y}{a}
\end{aligned}
$$

is a solution of the Laplace equation for determining the pressure distributions.

The next step in calculating the pressure distributions was the determinations of Fourier coefficients. The Fourier coefficients were determined by using the formula

$$\bar{A}_n = \frac{2}{a} \int_0^a f(x) \sin \frac{n\pi x}{a} dx \quad . \qquad (27)$$

For the second term in Eq. (26), the Fourier coefficients are

$$\bar{A}_n = A_n \sinh n\pi (1 - \frac{y}{a}) \quad . \qquad (28)$$

The value of the integral in Eq. (27) was determined by means of Simpson's rule which is a numerical method of integrating when values of f(x) are known for various values of x. Simpson's rule for numerical integration is

$$\int_{x_0}^{x+mh} f(x)\, dx = \frac{h}{3}\left[P_0 + 4P_1 + 2P_2 + 4P_3 + \text{-----} + P_m\right] \qquad (29)$$

if m is even and h is an increment for dividing the x axis to find corresponding values of (x). For calculating the Fourier coefficients the x-axis was divided into 20 equal segments, each of which was 0.05a. Corresponding to each value of x the value of f(x) was taken as the differences between the experimental curve and the curve

$$\frac{P_0(a-x)(a-y)}{a^2} \quad .$$

Combining Eqs. (27) and (29) the equation for calculating Fourier coefficients becomes

$$A_n = \frac{2}{a} \cdot \frac{h}{3} \left[P_0 \sin \frac{n\pi o}{a} + 4P_1 \sin n\pi \frac{0.05a}{a} + \text{--------} P_{20} \sin \frac{n\pi a}{a}\right] \quad . \qquad (30)$$

P_0, P_1 -----P_{20} are values of f(x) taken at the chosen values of x.

In a similar manner, the Fourier coefficients, $B_n \sinh \frac{n\pi y}{a}$, values of f(x) were determined from the differences between the experimental curve and the curve $\frac{P_o\,(a - x)(\,a - y)}{a^2}$ on the axis $y = a$. The value $\frac{P_o(a - x)(a - y)}{a^2}$ is zero along the line $y = a$.

The potentials are symmetrical about the line $y = x$; therefore the Fourier coefficients determined for $A_n \sinh n\pi(1- \frac{y}{a})$ are identical to those of $C_n \sinh n\pi(1 - \frac{x}{a})$ and the constants determined for $B_n \sinh n\pi \frac{y}{a}$ are identical to those determined for $D_n \sinh n\pi \frac{y}{a}$.

From experience, it was found that six Fourier constants for each function was sufficient for attaining calculated accuracies equivalent to accuracies with which the boundary conditions were experimentally measured.

For the second stage equation,

$$\begin{aligned} P(x'y') = {} & P_o(\frac{x'}{a} + \frac{y'}{a})-(P_o + P_c)\frac{x'y'}{a^2} \\ & + \sum A_n \sinh n\pi(1- \frac{y'}{a}) \sin \frac{n\pi x'}{a} \\ & + \sum B_n \sinh \frac{n\pi y'}{a} \sin \frac{n\pi x'}{a} \\ & + \sum C_n \sinh n\pi(1- \frac{x'}{a}) \sin \frac{n\pi y'}{a} \\ & + \sum D_n \sinh \frac{n\pi x'}{a} \sin \frac{n\pi y'}{a} \end{aligned} \tag{31}$$

the values of f(x) used in determining the Fourier constants were taken as the differences between the experimentally determined curve for pressure and the curve of $P_o(\frac{x}{a} + \frac{y}{a})-(P_o + P_c)(\frac{xy}{a^2})$. These curves were plotted along the lines $x = a$, $y = a$, $y = o$, and $x = o$.

Following the calculations of the Fourier constants, these constants were substituted into Eqs. (26) and (31); then pressures were determined for various coordinate points. The coordinate points were taken for 0.05a increments of x and y.

Sample calculations are given in the appendix.

From the calculated values, of P(x,y) equipotential lines were determined. The flow nets containing these potential lines are given in Figs. 10, 11, 12, 13, 14, 15, and 16.

To calculate numerically the pressure distributions, the iteration procedure was used. In this procedure, the area under consideration was covered with a network of 16 squares with sides of h(0.25a) units in length. The boundary conditions were determined experimentally and denoted in Fig. 9-b by n_1, n_2 etc. The unknown lattice points [interior (x,y) coordinate points] are denoted by P_1, P_2 etc.

To start the iteration process Eq. (18) was used and the central lattice point was approximated from the values of the midpoints of the horizontal and vertical boundaries -- the formula according to Fig. 9-b is

$$P_5 = \tfrac{1}{4}(n_3 + n_7 + n_{11} + n_{15}). \qquad (32)$$

Following the calculation of the central point P_5, approximate values for the points P_1, P_3, P_7, P_9 were calculated from the diagonal values of which the points are the center of a square. For instance

$$P_1 = \tfrac{1}{4}(n_1 + n_3 + P_5 + n_{15}). \qquad (33)$$

Next, a second set of values were calculated for each interior lattice point, by taking the means of the surrounding vertical and horizontal points. The second set of values for these interior points were determined in the order as the points are denoted by subscripts.

After the second set of values for the interior lattice points was determined a third set of values for these points was determined in a similar manner. This procedure was continued until additional values for each lattice point became approximately constant.

Finally the area under consideration was further divided into 64 equal squares with a new value of h one-half that for the 16 squares. The determined values of the lattice points for the 16 squares were used as diagonal points from which the first approximation of the new points were determined. After these first approximate values were determined, the procedure was continued as for the 16 square network until additional values for each lattice point became approximately constant.

DISCUSSION OF RESULTS

The flow nets in Figs. 10, 11, 12 were drawn from data obtained with the reservoir analyzer. The wavy lines in the figures are calculated flood fronts. Figure 10 gives the flood front patterns until breakthrough in wells PI. Figure 11 traces, for the final stage, the flood front pattern until breakthrough into wells P. Wells PI, and I are the injection wells for this stage. The overall sweep efficiency for this procedure was found to be 87 percent. Figure 11 also gives a flood front pattern for the final stage; however, for this stage wells labeled PI are the only injection wells.

The equipotential lines calculated by Eq. (10) are given in Fig. 13 and the equipotential lines calculated by Eq. (12) are given in Fig. 14. The equipotential lines drawn from potentials calculated by difference equations from measured boundary conditions are given in Figs. 15 and 16. The stream lines in Figs. 12, 13, 14, and 15 are lines which were drawn perpendicular to the appropriate equipotential lines.

Measured areas taken from Fig. 12 indicated that the efficiency of the new flood pattern is approximately 90 percent. This is greater than the efficiency of the ordinary five-spot pattern (10) (11) which is 72.3 percent.

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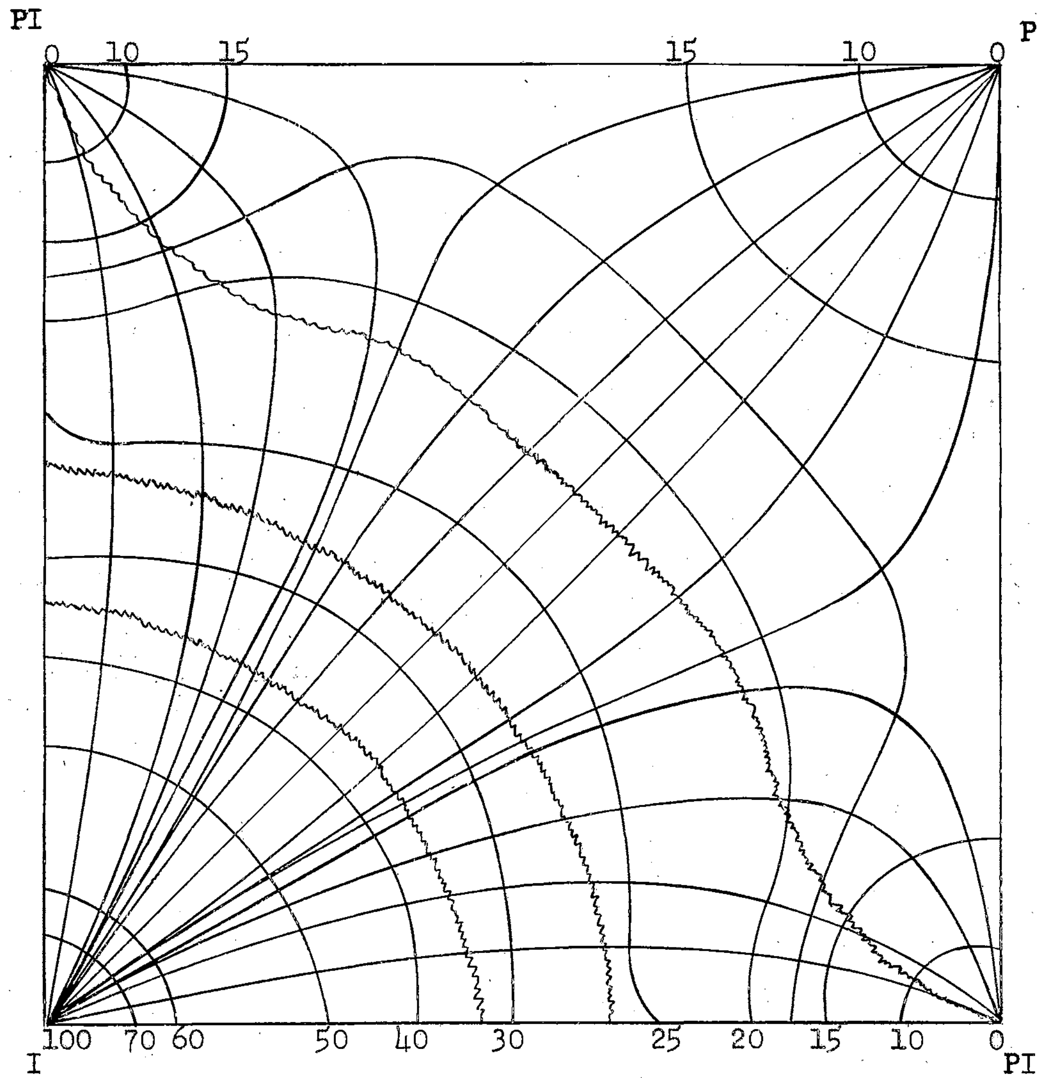
(10) Muskat,M. and Wyckoff, R. D., "Mechanics of Porous Flow Applied to Flooding Problems." Trans. A.I.M.E. 156, 219 (1933).

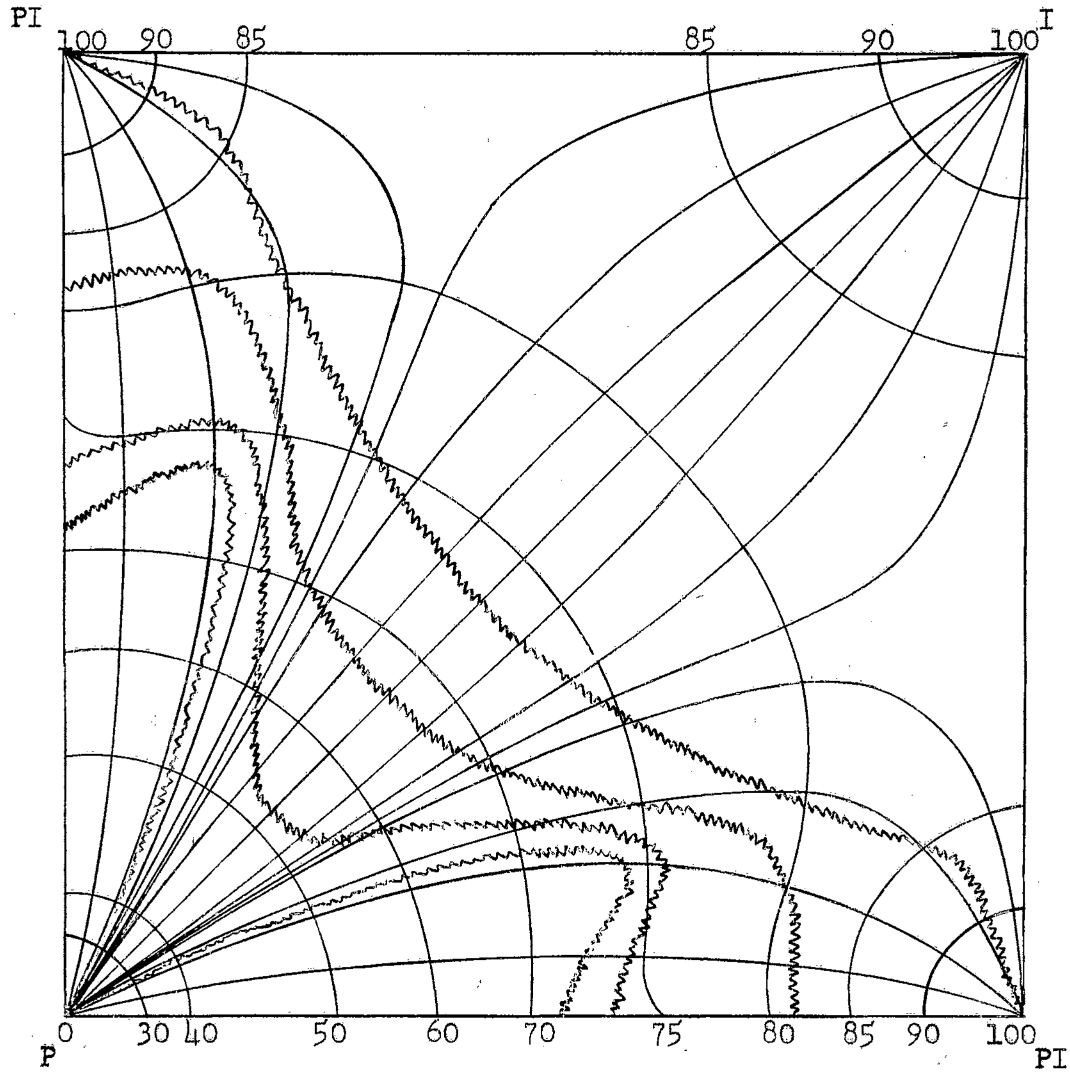
(11) Muskat, M. and Wyckoff, R. D., "Theoretical Analysis of Water Floowing Networks." Trans. A.I.M.E., 192, 62 (1934).



Flood Front Pattern -- Initial Stage

Fig. 10

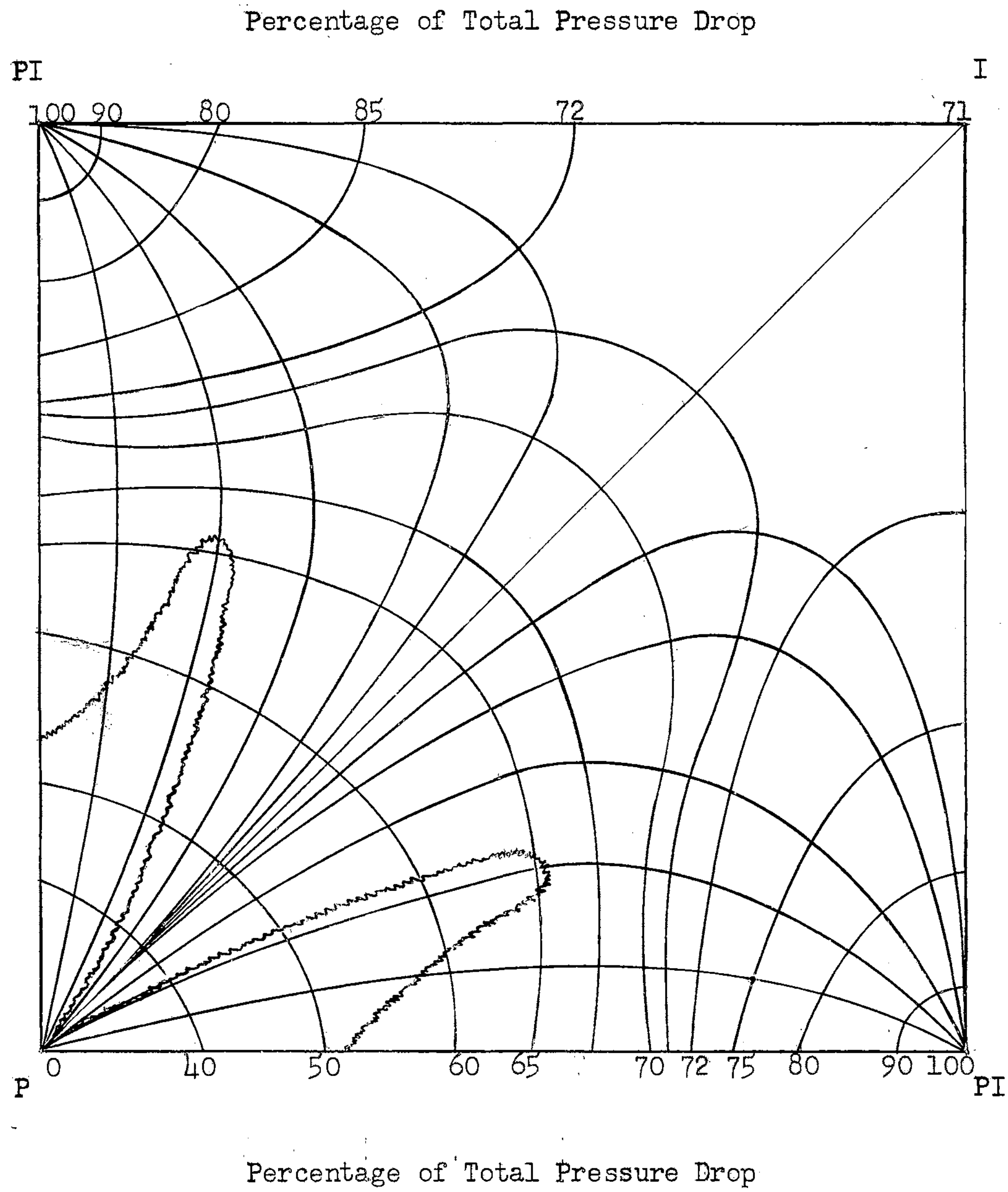
Percentage of Total Pressure Drop



Percentage of Total Pressure Drop

Flood Front Pattern - Final Stage

Fig. 11



Flood Front Pattern - Final Stage

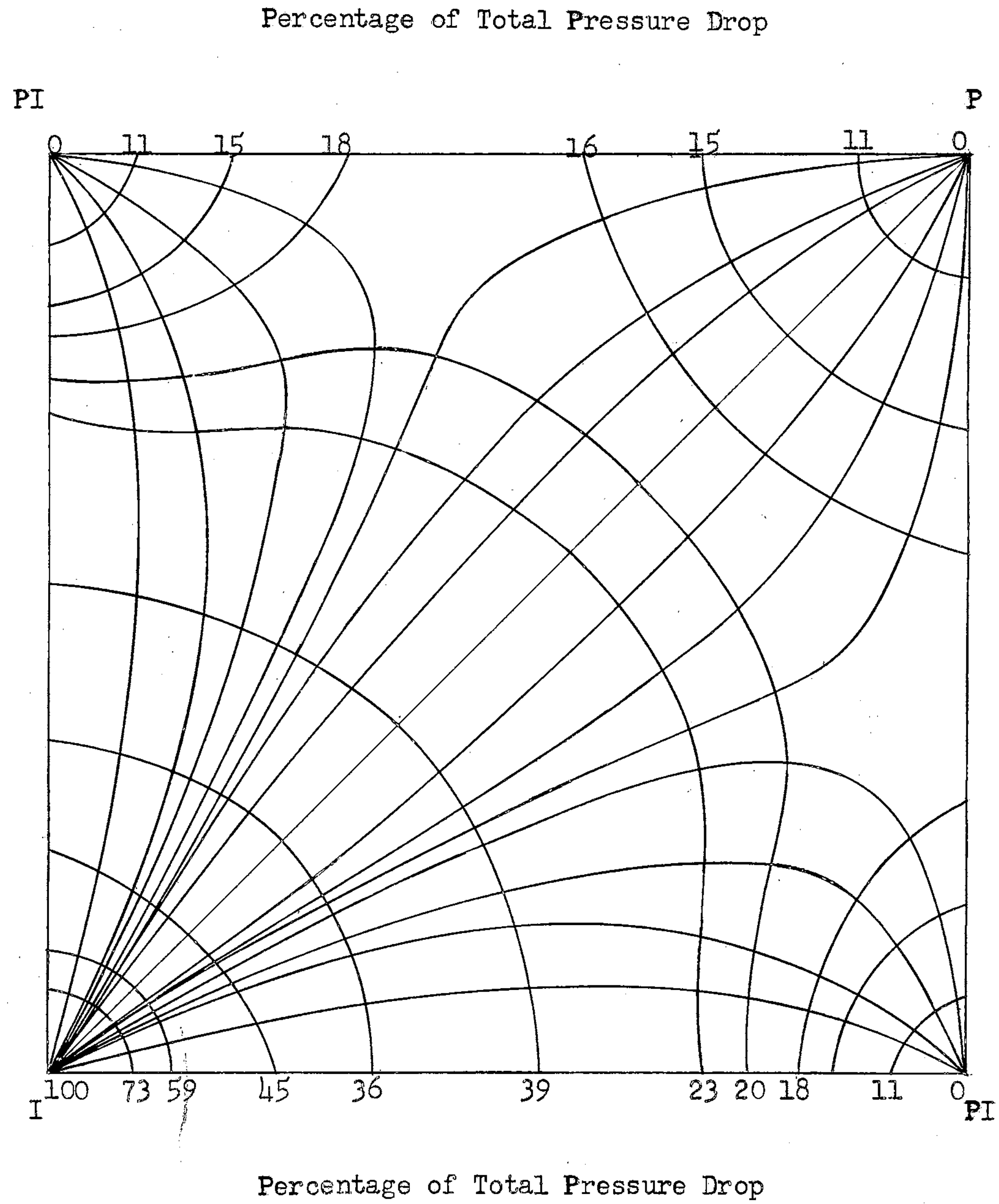
Fig. 12

The flow net diagrams obtained from numerical and classical methods of calculations are quite similar except for very minor differences in shape and position of the equipotential lines. In particular, the lines in Fig. 10 which represent 20 percent and 25 percent of the total pressure drop are concave upward near the boundary, whereas, in Fig. 13 these lines are straight. The reason for this discrepancy is that the classical method gives very accurate results near the boundary. In general, the differences can be explained in this way: the accuracy of the resulting function, ϕ, depends upon the choice of the function, ϕ_m, and the number of Fourier coefficients taken into account. The number of Fourier coefficients in this particular case is six. As a result, the calculated values at the interior points will also be approximate. More accurate results can be obtained by increasing the number of Fourier coefficients.

Comparing flow nets obtained by the numerical method with the flow net diagrams obtained from the reservoir analyzer, minute differences are found in shapes and positions of the equipotential lines. These differences can be explained as follows:

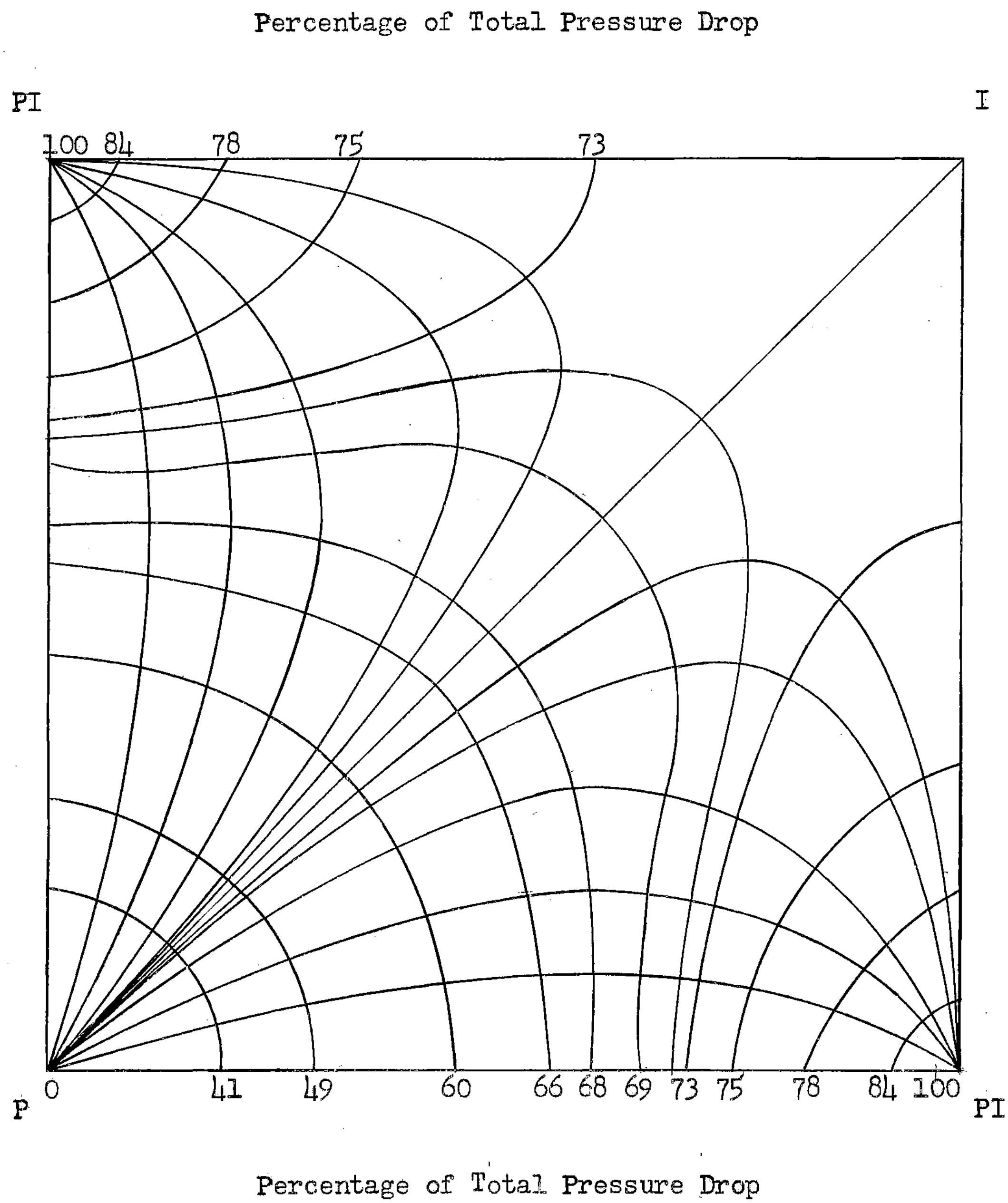
(1) An approximation is involved in the practical necessity of breaking off the averaging procedure after a finite number of steps before the limiting solution to the difference equation is actually obtained.

(2) In the above numerical method an approximation is inherent in the replacement of partial differential equations by the partial difference equations. The error introduced in this procedure will



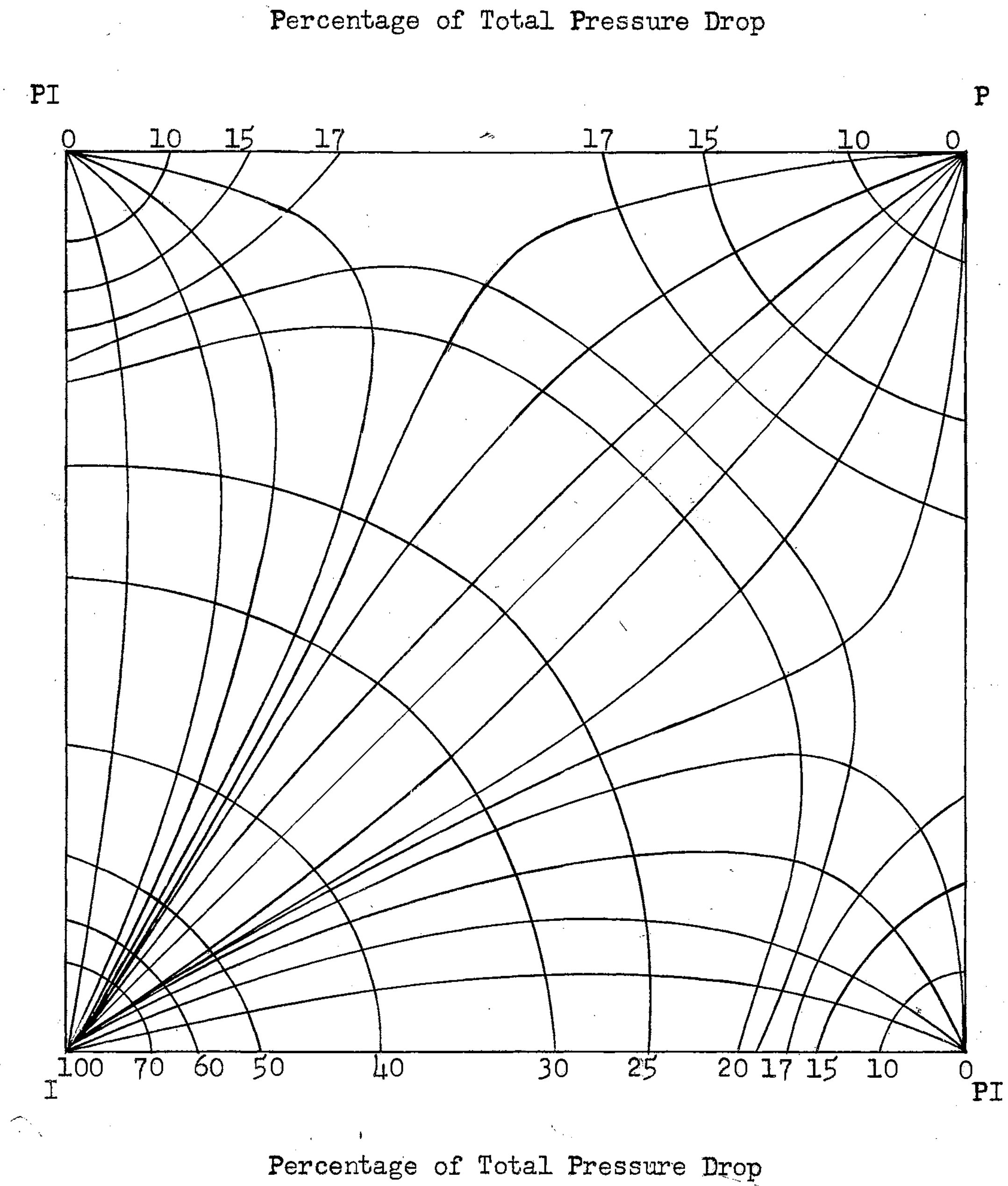
Flow Net - Initial Stage
(Classical)

Fig. 13



<u>Flow Net - Final Stage</u>
(Classical)

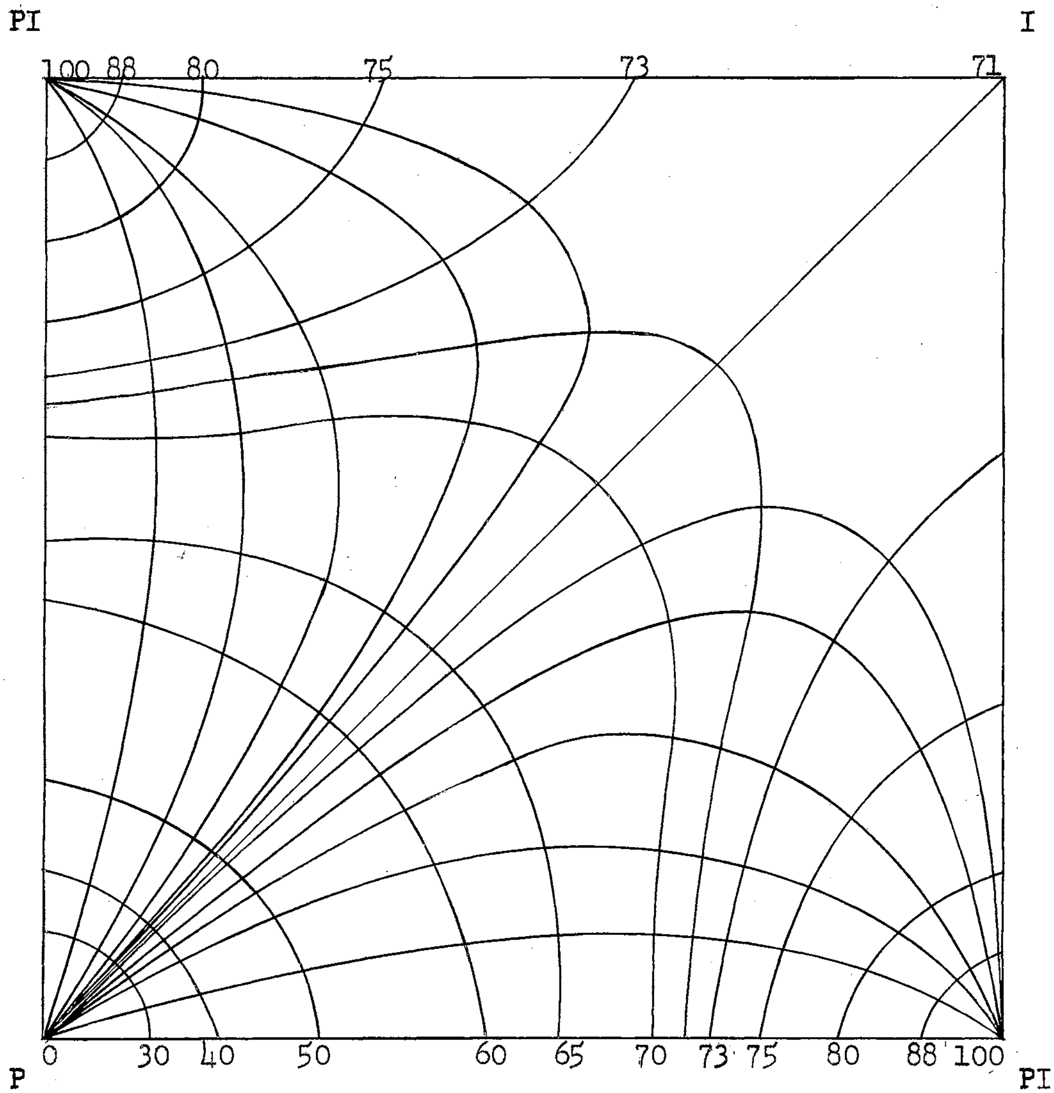
Fig. 14



Flow Net - Initial Stage
(Numerical)

Fig. 15

Percentage of Total Pressure Drop



Percentage of Total Pressure Drop

Flow Net - Final Stage
(Numerical)

Fig. 16

decrease by decreasing the size of the mesh (increasing number of internal lattice points at which the potential is computed).

The conductivities for the five-spot, first and second stage of the new procedure were determined with the aid of the reservoir analyzer. The relative flow rates of the flood patterns for a given pressure differential between input and output wells are:

$$Q_5 : Q_{1new} : Q_{2new} = 1:1.86:1.5$$

CONCLUSIONS

1. A waterflooding procedure has been presented which has a sweep efficiency of approximately 90 per cent - - a mobility ratio of 1:1 was used in determining this value. The sweep efficiency of this water flooding procedure is greater than the procedures which include either the five-spot or the seven-spot pattern.

2. A classical method of calculating potential distributions from known boundary conditions was developed and tested. Calculated results by this method corresponded very closely to experimental values obtained with the reservoir analyzer.

3. For the new flooding procedure, calculated potentials by the use of difference equations corresponded very closely to experimental values obtained with the reservoir analyzer.

4. Although an extended treatment has been given of the theoretical aspects of distributed fluid injection systems, the material thus far should be considered only as a guide for the understanding and qualitative interpretation of the geometrical features of the fluid motion in secondary recovery operations in uniform strata.

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A P P E N D I X

SAMPLE CALCULATION PRESSURE

For the calculation of Fourier's coefficients a table (Table III) consisting of seven columns was constructed. The first column was filled in with the ordinates y_0, y_1, y_2------ y_{20}. The values for $\frac{x}{a}$, $\left[P - P_o(1 - \frac{x}{a})\right]$, $\sin \frac{n\pi x}{a}$,

$\left[P - P_0(1 - \frac{x}{a})\right] \sin \frac{n\pi x}{a}$ were enlisted in second, third, fourth and fifth columns respectively. In the sixth column the number 2 was put against all even ordinates, and number 4 was put against all odd ordinates. Column seven was obtained by multiplying column five by the column seven. The summation of the seventh column (616.38) gives the value of integration involved in the Simpson's integration formula. This value was multiplied by $\frac{h}{3}$ ['h' is equal to one tenth (0.1)]. Hence, $616.38 \times \frac{0.1}{3} =$ 20.55. This is Fourier's coefficient A_1. In the same way other coefficients were calculated.

TABLE III

TABLE IV

TABLE V

TABLE VI

TABLE III

(SAMPLE CALCULATION FOR FOURIER COEFFICIENT)

(a) ordinates	(b) x/a	(c) $P-P_0(1-\frac{x}{a})$	(d) $\sin\frac{\pi x}{a}$	(e) c x d	(f)	(g) (e x f)
y_0	0.00	0	0.00	00.00	1	00.0
y_1	0.05	152	0.16	23.77	4	9.51
y_2	0.10	222	0.31	68.60	2	13.72
y_3	0.15	255	0.45	115.70	4	46.28
y_4	0.20	269	0.59	158.12	2	31.62
y_5	0.25	270	0.71	190.84	4	76.37
y_6	0.30	261	0.81	211.15	2	42.23
y_7	0.35	247	0.89	220.00	4	88.00
y_8	0.40	229	0.95	217.80	2	43.56
y_9	0.45	206	0.99	203.42	4	81.37
y_{10}	0.50	182	1.00	182.00	2	36.40
y_{11}	0.55	156	0.99	154.05	4	61.62
y_{12}	0.60	129	0.95	122.69	2	24.54
y_{13}	0.65	102	0.89	90.85	4	36.34
y_{14}	0.70	74	0.81	59.87	2	11.97
y_{15}	0.75	47	0.71	33.22	4	13.29
y_{16}	0.80	21	0.59	12.34	2	2.47
y_{17}	0.85	-2	0.45	-0.91	4	-0.36
y_{18}	0.90	-19	0.31	-5.87	2	-1.17
y_{19}	0.95	-23	0.16	-3.60	4	-1.44
y_{20}	1.00	0	0.00	0.00	1	0.00
				Total	=	616.38

TABLE IV

FOURIER COEFFICIENTS

A. First stage

$x = 0$ and $y = 0$	$x = a$, $y = a$
$A_1 = C_1 = 20.55$	$B_1 = D_1 = 20.09$
$A_2 = C_2 = 12.33$	$B_2 = D_2 = 1.24$
$A_3 = C_3 = 3.08$	$B_3 = D_3 = 2.94$
$A_4 = C_4 = 3.53$	$B_4 = D_4 = 0.24$
$A_5 = C_5 = 1.07$	$B_5 = D_5 = 1.20$
$A_6 = C_6 = 1.57$	

B. Second Stage

$x' = 0$ and $y' = 0$	$x' = a$, $y' = a$
$A_1 = C_1 = -14.27$	$B_1 = D_1 = 14.63$
$A_2 = C_2 = -12.30$	$B_2 = D_2 = 4.59$
$A_3 = C_3 = -2.06$	$B_3 = D_3 = 1.94$
$A_4 = C_4 = -3.54$	$B_4 = D_4 = 1.16$
$A_5 = C_5 = -0.54$	$B_5 = D_5 = 0.67$
$A_6 = C_6 = -1.62$	$B_6 = D_6 = 0.38$

TABLE V

PRESSURE DISTRIBUTION CALCULATIONS - INITIAL STAGE $P(x,y) = \sum_1^5 \phi$ *

(x, y)	ϕ_1	ϕ_2	ϕ_3	ϕ_4	ϕ_5	$\sum_1^5 \phi$
(0.00a, 0.00a)	100.00	0.00	0.00	0.00	0.00	100.00
(0.00a, 0.05a)	95.00	0.00	0.00	0.00	0.00	95.00
(0.05a, 0.05a)	90.25	8.54	0.04	8.54	0.04	73.26
(0.10a, 0.05a)	88.50	15.50	0.01	5.86	0.09	67.31
(0.15a, 0.05a)	80.76	19.92	0.01	4.27	0.13	56.81
(0.20a, 0.05a)	76.00	21.96	0.06	3.24	0.18	51.15
(0.25a, 0.05a)	71.25	22.35	0.04	2.49	0.29	46.89
(0.30a, 0.05a)	66.50	21.60	0.22	1.80	0.30	43.62
(0.35a, 0.05a)	61.75	20.78	0.25	1.57	0.37	40.01
(0.40a, 0.05a)	57.00	19.58	0.86	1.25	0.45	36.89
(0.45a, 0.05a)	52.25	17.91	0.27	1.01	0.56	34.14
(0.50a, 0.05a)	47.50	15.80	0.28	0.85	0.66	31.83
(0.55a, 0.05a)	42.75	13.44	0.27	0.66	0.78	29.70
(0.60a, 0.05a)	38.00	11.12	0.26	0.53	0.94	27.54
(0.65a, 0.05a)	33.25	9.04	0.25	0.45	1.12	25.12
(0.70a, 0.05a)	28.50	6.95	0.22	0.38	1.34	22.73
(0.75a, 0.05a)	23.75	4.98	0.14	0.26	1.64	20.33
(0.80a, 0.05a)	19.00	2.88	0.16	0.21	2.03	18.10
(0.85a, 0.05a)	14.25	1.02	0.12	0.15	2.59	15.80
(0.90a, 0.05a)	9.50	-0.11	0.08	0.01	3.23	12.84
(0.95a, 0.05a)	4.75	-0.67	0.04	0.05	4.26	9.71
(1.00a, 0.05a)	6.90	0.00	0.00	0.00	0.00	6.91

* ϕ's are percentage of injection pressure

(x, y)	ϕ_1	$-\phi_2$	ϕ_3	$-\phi_4$	ϕ_5	$\sum_1^5 \phi$
(0.00a, 0.10a)	85.00	0.00	0.00	0.00	0.00	85.00
(0.05a, 0.10a)	85.50	5.86	0.09	15.51	0.09	67.31
(0.10a, 0.10a)	81.00	11.14	0.17	11.14	0.17	59.06
(0.15a, 0.10a)	76.50	14.46	0.25	7.88	0.26	54.67
(0.20a, 0.10a)	72.00	16.65	0.33	6.17	0.36	49.87
(0.25a, 0.10a)	67.50	17.63	0.39	4.79	0.47	45.95
(0.30a, 0.10a)	63.00	17.79	0.45	3.77	0.59	42.48
(0.35a, 0.10a)	88.50	17.42	0.50	3.05	0.73	39.26
(0.40a, 0.10a)	54.00	16.68	0.53	2.44	0.89	36.31
(0.45a, 0.10a)	49.50	15.52	0.55	1.97	1.08	33.63
(0.50a, 0.10a)	45.00	14.02	0.57	1.61	1.29	31.22
(0.55a, 0.10a)	40.50	12.25	0.55	1.31	1.54	29.03
(0.60a, 0.10a)	36.00	10.41	0.53	1.05	1.85	26.91
(0.65a, 0.10a)	31.50	8.59	0.50	0.85	2.20	24.76
(0.70a, 0.10a)	27.00	6.77	0.45	0.68	2.43	22.43
(0.75a, 0.10a)	22.50	5.00	0.39	0.54	3.20	20.56
(0.80a, 0.10a)	18.00	3.27	0.33	0.41	3.93	18.57
(0.85a, 0.10a)	13.50	1.80	0.25	0.30	4.87	16.52
(0.90a, 0.10a)	9.00	0.79	0.17	0.19	6.16	14.34
(0.95a, 0.10a)	4.50	0.21	0.09	0.10	8.00	12.28
(1.00a, 0.10a)	11.00	0.00	0.00	0.00	0.00	11.00

(x, y)	ϕ_1	$-\phi_2$	ϕ_3	$-\phi_4$	ϕ_5	$\Sigma_1^5\phi$
(0.00a, 0.15a)	85.00	0.00	0.00	0.00	0.00	85.00
(0.05a, 0.15a)	80.75	4.28	0.13	19.92	0.13	56.81
(0.10a, 0.15a)	76.50	7.88	0.26	14.46	0.25	54.67
(0.15a, 0.15a)	72.25	10.79	0.39	10.79	0.39	51.44
(0.20a, 0.15a)	68.00	12.82	0.50	8.58	0.53	47.64
(0.25a, 0.15a)	63.75	14.02	0.60	6.74	0.70	44.29
(0.30a, 0.15a)	59.50	14.57	0.69	5.36	0.87	41.13
(0.35a, 0.15a)	55.25	14.68	0.76	4.34	1.07	38.06
(0.40a, 0.15a)	51.00	14.23	0.81	3.50	1.31	35.39
(0.45a, 0.15a)	46.75	13.46	0.84	2.84	1.57	32.86
(0.50a, 0.15a)	42.50	12.35	0.85	2.32	1.89	30.56
(0.55a, 0.15a)	38.25	10.99	0.84	1.89	2.25	28.46
(0.60a, 0.15a)	34.00	9.46	0.81	1.53	2.69	26.51
(0.65a, 0.15a)	29.75	7.84	0.76	1.24	3.11	24.52
(0.70a, 0.15a)	25.50	6.23	0.69	1.00	3.82	22.78
(0.75a, 0.15a)	21.25	4.65	0.60	0.79	4.61	21.02
(0.80a, 0.15a)	17.00	2.21	0.50	0.60	5.61	19.30
(0.85a, 0.15a)	12.75	1.99	0.39	0.44	6.89	17.60
(0.90a, 0.15a)	8.50	1.25	0.26	0.28	8.56	15.79
(0.95a, 0.15a)	4.25	0.45	0.13	0.13	10.86	14.65
(1.00a, 0.15a)	13.60	0.00	0.00	0.00	0.00	13.60

(x, y)	ϕ_1	$-\phi_2$	ϕ_3	$-\phi_4$	ϕ_5	$\Sigma_1^5\phi$
(0.00a, 0.20a)	80.00	0.00	0.00	0.00	0.00	80.00
(0.05a, 0.20a)	76.00	3.24	0.18	21.96	0.16	51.15
(0.10a, 0.20a)	72.00	6.17	0.36	1.67	0.33	49.87
(0.15a, 0.20a)	68.00	8.58	0.53	12.82	0.50	47.64
(0.20a, 0.20a)	64.00	10.35	0.69	10.35	0.69	44.67
(0.25a, 0.20a)	60.00	11.49	0.83	8.26	0.90	42.00
(0.30a, 0.20a)	56.00	12.10	0.94	6.64	1.13	39.36
(0.35a, 0.20a)	52.00	12.19	1.04	5.12	1.38	37.12
(0.40a, 0.20a)	48.00	11.92	1.11	4.40	1.69	34.48
(0.45a, 0.20a)	44.00	11.33	1.15	3.59	2.03	32.26
(0.50a, 0.20a)	40.00	10.49	1.17	2.94	2.43	30.17
(0.55a, 0.20a)	36.00	9.44	1.15	2.41	2.90	28.20
(0.60a, 0.20a)	32.00	8.28	1.11	1.96	3.45	26.32
(0.65a, 0.20a)	28.00	7.04	1.04	1.59	4.11	24.52
(0.70a, 0.20a)	24.00	5.79	0.94	1.28	4.88	22.76
(0.75a, 0.20a)	20.00	4.55	0.83	1.01	5.81	21.08
(0.80a, 0.20a)	16.00	3.39	0.69	0.77	7.03	19.56
(0.85a, 0.20a)	12.00	2.34	0.53	0.60	8.50	18.10
(0.90a, 0.20a)	8.00	1.44	0.36	0.37	10.37	16.92
(0.95a, 0.20a)	4.00	0.68	0.18	0.18	12.79	16.11
(1.00a, 0.20a)	15.40	0.00	0.00	0.00	0.00	15.40

(x, y)	ϕ_1	$-\phi_2$	ϕ_3	$-\phi_4$	ϕ_5	$\Sigma_1^5\phi$
(0.00a, 0.25a)	75.00	0.00	0.00	0.00	0.00	75.00
(0.05a, 0.25a)	71.25	2.49	0.29	22.36	0.20	46.90
(0.10a, 0.25a)	67.50	4.79	0.47	16.73	0.39	45.95
(0.15a, 0.25a)	63.75	6.74	0.70	14.02	0.60	44.29
(0.20a, 0.25a)	60.00	8.26	0.90	11.49	0.83	41.98
(0.25a, 0.25a)	56.25	9.31	1.08	9.31	1.08	39.79
(0.30a, 0.25a)	52.50	9.92	1.23	7.58	1.36	37.59
(0.35a, 0.25a)	48.75	10.15	1.36	6.22	1.66	35.40
(0.40a, 0.25a)	45.00	10.04	1.44	5.10	2.02	33.33
(0.45a, 0.25a)	41.25	9.65	1.50	4.18	2.43	31.34
(0.50a, 0.25a)	37.50	9.03	1.51	3.45	2.91	29.44
(0.55a, 0.25a)	33.75	8.24	1.49	2.83	3.46	27.63
(0.60a, 0.25a)	30.00	7.30	1.43	2.31	4.11	25.92
(0.65a, 0.25a)	26.25	6.30	1.34	1.89	4.88	24.28
(0.70a, 0.25a)	22.50	5.26	1.21	1.52	5.78	22.71
(0.75a, 0.25a)	18.75	4.23	1.06	1.20	6.84	21.23
(0.80a, 0.25a)	15.00	3.23	0.88	0.92	8.16	19.89
(0.85a, 0.25a)	11.25	2.31	0.68	0.67	9.74	18.70
(0.90a, 0.25a)	7.50	1.47	0.46	0.43	11.66	17.71
(0.95a, 0.25a)	3.75	0.69	0.23	0.21	13.93	16.84
(1.00a, 0.25a)	16.60	0.00	0.00	0.00	0.00	16.60

(x, y)	ϕ_1	$-\phi_2$	ϕ_3	$-\phi_4$	ϕ_5	$\Sigma_1^5\phi$
(0.00a, 0.30a)	70.00	0.00	0.00	0.00	0.00	70.00
(0.05a, 0.30a)	66.50	1.80	0.30	21.60	0.22	43.62
(0.10a, 0.30a)	63.00	3.77	0.59	17.79	1.25	42.48
(0.15a, 0.30a)	59.50	5.36	0.87	14.57	0.69	41.13
(0.20a, 0.30a)	56.00	6.64	1.13	12.08	0.94	39.36
(0.25a, 0.30a)	52.50	7.58	1.36	9.92	1.23	37.59
(0.30a, 0.30a)	49.00	8.18	1.55	8.18	1.54	35.73
(0.35a, 0.30a)	45.50	8.46	1.70	6.76	1.90	33.88
(0.40a, 0.30a)	42.00	8.47	1.81	5.59	2.30	32.06
(0.45a, 0.30a)	38.50	8.22	1.88	4.62	2.76	30.30
(0.50a, 0.30a)	35.00	7.77	1.90	3.82	3.30	28.60
(0.55a, 0.30a)	31.50	7.16	1.87	3.16	3.92	26.97
(0.60a, 0.30a)	28.00	6.42	1.79	2.59	4.65	25.43
(0.65a, 0.30a)	24.50	5.60	1.68	2.12	5.49	23.95
(0.70a, 0.30a)	21.00	4.74	1.52	1.71	6.50	32.57
(0.75a, 0.30a)	17.50	3.86	1.33	1.36	7.64	21.24
(0.80a, 0.30a)	14.00	3.00	1.10	1.01	9.03	20.12
(0.85a, 0.30a)	10.50	2.18	0.85	0.75	10.65	19.06
(0.90a, 0.30a)	7.00	1.42	0.58	0.49	12.55	18.22
(0.95a, 0.30a)	3.50	0.69	0.29	0.24	14.70	17.55
(1.00a, 0.30a)	17.40	0.00	0.00	0.00	0.00	17.40

(x, y)	ϕ_1	$-\phi_2$	ϕ_3	$-\phi_4$	ϕ_5	$\Sigma_1^5\phi$
(0.00a, 0.35a)	65.00	0.00	0.00	0.00	0.00	65.00
(0.05a, 0.35a)	61.75	1.57	0.37	20.78	0.25	40.01
(0.10a, 0.35a)	58.50	3.05	0.73	17.42	0.50	39.26
(0.15a, 0.35a)	55.25	4.34	1.07	14.68	0.76	38.06
(0.20a, 0.35a)	52.00	5.12	1.38	12.14	1.04	37.12
(0.25a, 0.35a)	48.75	6.22	1.66	10.14	1.36	35.40
(0.30a, 0.35a)	45.50	6.76	1.90	8.46	1.70	33.88
(0.35a, 0.35a)	42.55	7.04	2.09	7.04	2.09	32.34
(0.40a, 0.35a)	39.00	7.09	2.22	5.88	2.52	30.78
(0.45a, 0.35a)	35.75	6.93	2.30	4.89	3.02	29.26
(0.50a, 0.35a)	32.50	6.60	2.32	4.07	3.61	27.76
(0.55a, 0.35a)	29.25	6.23	2.29	3.78	4.28	26.20
(0.60a, 0.35a)	26.00	5.56	2.20	2.79	5.06	25.10
(0.65a, 0.35a)	22.75	4.92	2.05	2.28	5.96	23.56
(0.70a, 0.35a)	19.50	4.22	1.86	1.85	7.03	22.32
(0.75a, 0.35a)	16.25	3.50	1.62	1.46	8.22	21.12
(0.80a, 0.35a)	13.00	2.78	1.35	1.12	9.63	20.07
(0.85a, 0.35a)	9.75	2.06	1.04	0.82	11.29	19.21
(0.90a, 0.35a)	6.50	1.36	0.71	0.53	13.16	18.48
(0.95a, 0.35a)	3.25	0.67	0.36	0.26	15.12	17.79
(1.00a, 0.35a)	17.90	0.00	0.00	0.00	0.00	17.90

(x, y)	ϕ_1	$-\phi_2$	ϕ_3	$-\phi_4$	ϕ_5	$\Sigma_1^5\phi$
(0.00a, 0.40a)	60.00	0.00	0.00	0.00	0.00	60.00
(0.05a, 0.40a)	57.00	1.25	0.45	19.58	0.26	36.88
(0.10a, 0.40a)	54.00	2.43	0.89	16.68	0.53	36.31
(0.15a, 0.40a)	51.00	3.50	1.31	14.23	0.81	35.39
(0.20a, 0.40a)	48.00	4.40	1.69	11.92	1.11	34.48
(0.25a, 0.40a)	45.00	5.10	2.02	10.04	1.44	33.32
(0.30a, 0.40a)	42.00	5.59	2.30	8.47	1.81	32.06
(0.35a, 0.40a)	39.00	5.88	2.52	7.09	2.22	30.78
(0.40a, 0.40a)	36.00	5.97	2.68	5.97	2.68	29.42
(0.45a, 0.40a)	33.00	5.91	2.78	5.00	3.21	28.08
(0.50a, 0.40a)	30.00	5.66	2.80	4.19	3.83	26.78
(0.55a, 0.40a)	27.00	5.30	2.76	3.49	4.53	25.50
(0.60a, 0.40a)	24.00	4.84	2.65	2.90	5.35	24.26
(0.65a, 0.40a)	21.00	4.31	2.48	2.38	6.28	23.07
(0.70a, 0.40a)	18.00	3.72	2.25	1.43	7.39	21.99
(0.75a, 0.40a)	15.00	3.10	1.97	1.53	8.60	20.93
(0.80a, 0.40a)	12.00	2.47	1.63	1.18	10.03	20.02
(0.85a, 0.40a)	9.00	1.84	1.26	0.86	11.71	19.28
(0.90a, 0.40a)	6.00	1.22	0.86	0.56	13.59	18.67
(0.95a, 0.40a)	3.00	0.61	0.44	0.28	15.64	18.20
(1.00a, 0.40a)	18.10	0.00	0.00	0.00	0.00	18.10

(x, y)	ϕ_1	$-\phi_2$	ϕ_3	$-\phi_4$	ϕ_5	$\sum_1^5 \phi$
(0.00a, 0.45a)	55.00	0.00	0.00	0.00	0.00	55.00
(0.05a, 0.45a)	52.25	1.01	0.55	17.91	0.27	34.14
(0.10a, 0.45a)	49.50	1.97	1.08	15.52	0.55	33.63
(0.15a, 0.45a)	46.75	2.84	1.57	13.46	0.84	32.86
(0.20a, 0.45a)	44.00	3.59	2.03	11.33	1.15	32.26
(0.25a, 0.45a)	41.25	4.18	2.43	9.65	1.50	31.34
(0.30a, 0.45a)	38.50	4.62	2.76	8.22	1.88	30.30
(0.35a, 0.45a)	35.75	4.89	3.02	6.93	2.30	29.26
(0.40a, 0.45a)	33.00	5.00	3.21	5.91	2.78	28.08
(0.45a, 0.45a)	30.25	4.97	3.32	4.97	3.32	26.95
(0.50a, 0.45a)	27.50	4.81	3.35	4.18	3.95	25.81
(0.55a, 0.45a)	24.75	4.53	3.30	3.50	4.67	24.68
(0.60a, 0.45a)	22.00	4.17	3.17	2.93	5.50	23.57
(0.65a, 0.45a)	19.25	3.34	2.96	2.41	6.45	22.46
(0.70a, 0.45a)	16.50	3.26	2.69	1.96	7.58	21.55
(0.75a, 0.45a)	13.75	2.74	2.35	1.56	8.79	20.60
(0.80a, 0.45a)	11.00	2.21	1.96	1.20	10.23	19.77
(0.85a, 0.45a)	8.25	1.65	1.51	0.87	11.91	19.15
(0.90a, 0.45a)	5.50	1.10	1.03	0.56	13.79	18.66
(0.95a, 0.45a)	2.75	0.55	0.52	0.28	15.89	18.33
(1.00a, 0.45a)	18.10	0.00	0.00	0.00	0.00	18.10

(x, y)	ϕ_1	$-\phi_2$	ϕ_3	$-\phi_4$	ϕ_5	$\sum_1^5 \phi$
(0.00a, 0.50a)	50.00	0.00	0.00	0.00	0.00	50.00
(0.05a, 0.50a)	47.50	0.80	0.66	15.80	0.28	31.83
(0.10a, 0.50a)	45.00	1.61	1.29	14.02	0.56	31.22
(0.15a, 0.50a)	42.50	2.32	1.89	12.35	0.85	30.56
(0.20a, 0.50a)	40.00	2.94	2.43	10.49	1.17	30.17
(0.25a, 0.50a)	37.50	3.45	2.91	9.03	1.51	29.44
(0.30a, 0.50a)	35.00	3.82	3.30	7.77	1.90	28.60
(0.35a, 0.50a)	32.50	4.07	3.51	6.60	2.32	27.76
(0.40a, 0.50a)	30.00	4.19	3.82	5.66	2.80	26.78
(0.45a, 0.50a)	27.50	4.18	3.95	4.81	3.35	35.81
(0.50a, 0.50a)	25.00	4.07	3.98	4.07	3.98	24.82
(0.55a, 0.50a)	22.50	3.86	3.92	3.42	4.72	23.83
(0.60a, 0.50a)	20.00	3.57	3.76	2.87	5.53	22.85
(0.65a, 0.50a)	17.50	3.18	3.52	2.38	6.48	21.94
(0.70a, 0.50a)	15.00	2.82	3.20	1.94	7.60	21.04
(0.75a, 0.50a)	12.50	2.39	2.80	1.55	8.80	20.17
(0.80a, 0.50a)	10.00	1.93	2.33	1.94	10.23	19.44
(0.85a, 0.50a)	7.50	1.45	1.80	0.87	11.89	18.88
(0.90a, 0.50a)	5.00	0.97	1.23	0.57	13.75	18.44
(0.95a, 0.50a)	2.50	0.49	0.82	0.28	15.85	18.21
(1.00a, 0.50a)	17.90	0.00	0.00	0.00	0.00	17.90

(x, y)	ϕ_1	$-\phi_2$	ϕ_3	$-\phi_4$	ϕ_5	$\sum_1^5 \phi$
(0.00a, 0.55a)	45.00	0.00	0.00	0.00	0.00	45.00
(0.05a, 0.55a)	42.75	0.67	0.78	13.44	0.27	39.70
(0.10a, 0.55a)	40.50	1.30	1.54	12.25	0.55	29.03
(0.15a, 0.55a)	38.25	1.89	2.25	10.99	0.85	28.46
(0.20a, 0.55a)	36.00	2.41	2.90	9.44	1.15	28.20
(0.25a, 0.55a)	33.75	2.83	3.46	8.24	1.49	27.63
(0.30a, 0.55a)	31.50	3.16	3.92	7.16	1.87	26.97
(0.35a, 0.55a)	29.25	3.37	4.27	6.23	2.29	26.20
(0.40a, 0.55a)	27.00	3.49	4.53	5.30	2.76	25.50
(0.45a, 0.55a)	24.75	3.50	4.67	4.53	3.29	24.68
(0.50a, 0.55a)	22.50	3.42	4.70	3.86	3.92	23.83
(0.55a, 0.55a)	20.25	3.26	4.62	2.26	4.62	22.97
(0.60a, 0.55a)	18.00	3.03	4.44	2.75	5.44	22.10
(0.65a, 0.55a)	15.75	2.75	4.16	2.28	6.37	21.24
(0.70a, 0.55a)	13.50	2.42	3.78	1.87	7.46	20.45
(0.75a, 0.55a)	11.25	2.06	3.31	1.49	8.64	19.65
(0.80a, 0.55a)	9.00	1.67	2.76	1.16	10.04	18.97
(0.85a, 0.55a)	6.75	1.27	2.13	0.84	11.66	18.43
(0.90a, 0.55a)	4.50	0.85	1.46	0.55	13.47	18.02
(0.95a, 0.55a)	2.25	0.43	0.74	0.27	15.47	17.76
(1.00a, 0.55a)	17.50	0.00	0.00	0.00	0.00	17.50

(x, y)	ϕ_1	$-\phi_2$	ϕ_3	$-\phi_4$	ϕ_5	$\sum_1^5 \phi$
(0.00a, 0.60a)	40.00	0.00	0.00	0.00	0.00	40.00
(0.05a, 0.60a)	38.00	0.54	0.94	11.12	0.26	27.54
(0.10a, 0.60a)	36.00	1.05	1.85	14.01	0.53	26.91
(0.15a, 0.60a)	34.00	1.53	2.69	9.46	0.81	36.51
(0.20a, 0.60a)	32.00	1.96	3.45	8.28	1.11	26.32
(0.25a, 0.60a)	30.00	2.31	4.11	7.30	1.43	25.92
(0.30a, 0.60a)	28.00	2.59	4.65	6.42	1.79	25.43
(0.35a, 0.60a)	26.00	2.79	5.06	5.56	2.20	25.10
(0.40a, 0.60a)	24.00	2.90	5.35	4.84	2.65	24.26
(0.45a, 0.60a)	22.00	2.93	5.50	4.17	3.17	23.57
(0.50a, 0.60a)	20.00	2.87	5.53	3.57	3.76	22.85
(0.55a, 0.60a)	18.00	2.75	5.44	3.03	4.44	22.10
(0.60a, 0.60a)	16.00	2.58	5.23	2.57	5.23	21.33
(0.65a, 0.60a)	14.00	2.33	4.90	2.14	6.12	20.55
(0.70a, 0.60a)	12.00	2.06	4.46	1.76	7.17	19.82
(0.75a, 0.60a)	10.00	1.75	3.91	1.41	8.30	19.05
(0.80a, 0.60a)	8.00	1.43	3.26	1.09	9.66	18.40
(0.85a, 0.60a)	6.00	1.09	2.53	0.80	11.22	17.86
(0.90a, 0.60a)	4.00	0.72	1.73	0.53	12.94	17.42
(0.95a, 0.60a)	2.00	0.36	0.88	0.26	14.81	17.06
(1.00a, 0.60a)	16.90	0.00	0.00	0.00	0.00	16.90

(x, y)	ϕ_1	$-\phi_2$	ϕ_3	$-\phi_4$	ϕ_5	$\sum_1^5 \phi$
(0.00a, 0.65a)	35.00	0.00	0.00	0.00	0.00	35.00
(0.05a, 0.65a)	33.25	0.44	1.12	9.04	0.25	25.12
(0.10a, 0.65a)	31.50	0.85	2.20	8.59	0.49	24.76
(0.15a, 0.65a)	29.75	1.24	3.11	7.85	0.76	24.52
(0.20a, 0.65a)	28.00	1.59	4.11	7.04	1.04	24.52
(0.25a, 0.65a)	26.25	1.89	4.88	6.30	1.34	24.28
(0.30a, 0.65a)	24.50	2.12	5.49	5.60	1.68	23.95
(0.35a, 0.65a)	22.75	2.28	5.96	4.92	2.05	23.56
(0.40a, 0.65a)	21.00	2.38	6.28	4.31	2.48	23.07
(0.45a, 0.65a)	19.25	2.41	6.45	3.74	2.96	22.46
(0.50a, 0.65a)	17.50	2.38	6.58	3.18	3.52	21.94
(0.55a, 0.65a)	15.75	2.28	6.37	2.75	4.16	21.24
(0.60a, 0.65a)	14.00	2.14	6.12	2.33	4.90	20.55
(0.65a, 0.65a)	12.25	1.95	5.74	1.95	5.74	19.79
(0.70a, 0.65a)	10.50	1.73	5.23	1.61	6.73	19.13
(0.75a, 0.65a)	8.75	1.48	4.60	1.29	7.80	18.38
(0.80a, 0.65a)	7.00	1.20	3.85	1.00	9.10	17.66
(0.85a, 0.65a)	5.25	0.91	2.89	0.74	10.58	17.07
(0.90a, 0.65a)	3.50	0.61	2.04	0.48	12.23	16.78
(0.95a, 0.65a)	1.75	0.31	1.04	0.24	13.97	16.21
(1.00a, 0.65a)	16.20	0.00	0.00	0.00	0.00	16.20

(x, y)	ϕ_1	$-\phi_2$	ϕ_3	$-\phi_4$	ϕ_5	$\sum_1^5 \phi$
(0.00a, 0.70a)	30.00	0.00	0.00	0.00	0.00	30.00
(0.05a, 0.70a)	28.50	0.38	1.34	6.95	0.22	22.73
(0.10a, 0.70a)	27.00	0.69	2.63	6.77	0.45	22.43
(0.15a, 0.70a)	25.50	1.00	3.82	6.23	0.69	22.78
(0.20a, 0.70a)	24.00	1.28	4.88	5.79	0.94	22.76
(0.25a, 0.70a)	22.50	1.52	5.78	5.26	1.21	22.71
(0.30a, 0.70a)	21.00	1.71	6.50	4.74	1.52	22.57
(0.35a, 0.70a)	19.50	1.85	7.03	4.22	1.86	22.32
(0.40a, 0.70a)	18.00	1.93	7.39	3.72	2.25	21.99
(0.45a, 0.70a)	16.50	1.96	7.58	3.26	2.69	21.55
(0.50a, 0.70a)	15.00	1.94	7.60	2.82	3.20	21.04
(0.55a, 0.70a)	13.50	1.87	7.46	2.42	3.78	20.45
(0.60a, 0.70a)	12.00	1.76	7.17	2.06	4.46	19.82
(0.65a, 0.70a)	10.50	1.61	6.73	1.73	5.23	19.13
(0.70a, 0.70a)	9.00	1.43	6.14	1.43	6.14	18.43
(0.75a, 0.70a)	7.50	1.22	5.41	1.15	7.10	17.64
(0.80a, 0.70a)	6.00	1.00	4.53	0.90	8.37	17.00
(0.85a, 0.70a)	4.50	0.76	3.52	0.66	9.78	16.39
(0.90a, 0.70a)	3.00	0.51	2.41	0.43	11.38	15.86
(0.95a, 0.70a)	1.50	0.26	1.22	0.21	13.07	15.33
(1.00a, 0.70a)	15.30	0.00	0.00	0.00	0.00	15.30

(x, y)	ϕ_1	$-\phi_2$	ϕ_3	$-\phi_4$	ϕ_5	$\sum_1^5 \phi$
(0.00a, 0.75a)	25.00	0.00	0.00	0.00	0.00	25.00
(0.05a, 0.75a)	23.75	0.28	1.64	4.98	0.19	20.33
(0.10a, 0.75a)	22.50	0.54	3.20	5.00	0.39	20.56
(0.15a, 0.75a)	21.25	0.79	4.61	4.65	0.60	21.02
(0.20a, 0.75a)	20.00	1.01	5.81	4.55	0.83	21.08
(0.25a, 0.75a)	18.75	1.20	6.84	4.23	1.06	21.23
(0.30a, 0.75a)	17.50	1.36	7.64	3.86	1.33	21.24
(0.35a, 0.75a)	16.25	1.46	8.22	3.50	1.62	21.12
(0.40a, 0.75a)	15.00	1.53	8.60	1.30	1.97	20.93
(0.45a, 0.75a)	13.75	1.56	8.79	2.74	2.35	20.60
(0.50a, 0.75a)	12.50	1.55	8.80	2.39	2.80	20.17
(0.55a, 0.75a)	11.25	1.49	8.64	2.06	3.31	19.65
(0.60a, 0.75a)	10.00	1.41	8.30	1.75	3.91	19.05
(0.65a, 0.75a)	8.75	1.29	7.80	1.48	4.60	18.38
(0.70a, 0.75a)	7.50	1.15	7.10	1.22	5.41	17.64
(0.75a, 0.75a)	6.25	0.99	6.33	0.99	6.33	16.93
(0.80a, 0.75a)	5.00	0.81	5.35	0.77	7.45	16.22
(0.85a, 0.75a)	3.75	0.62	4.19	0.57	8.78	15.54
(0.90a, 0.75a)	2.50	0.42	2.90	0.37	10.34	14.95
(0.95a, 0.75a)	1.25	0.21	1.48	0.18	12.10	14.44
(1.00a, 0.75a)	14.20	0.00	0.00	0.00	0.00	14.20

(x, y)	ϕ_1	$-\phi_2$	ϕ_3	$-\phi_4$	ϕ_5	$\sum_1^5 \phi$
(0.00a, 0.80a)	20.00	0.00	0.00	0.00	0.00	20.00
(0.05a, 0.80a)	19.00	0.21	2.03	2.88	0.16	18.10
(0.10a, 0.80a)	18.00	0.41	3.93	3.27	0.33	18.57
(0.15a, 0.80a)	17.00	0.60	5.61	3.21	0.50	19.30
(0.20a, 0.80a)	16.00	0.77	7.03	3.39	0.69	19.56
(0.25a, 0.80a)	15.00	0.92	8.16	3.23	0.88	19.89
(0.30a, 0.80a)	14.00	1.04	9.03	3.00	1.10	20.12
(0.35a, 0.80a)	13.00	1.12	9.63	2.78	1.35	20.07
(0.40a, 0.80a)	12.00	1.18	10.03	2.47	1.63	20.02
(0.45a, 0.80a)	11.00	1.20	10.23	2.21	1.96	19.77
(0.50a, 0.80a)	10.00	1.19	10.23	1.93	2.33	19.44
(0.55a, 0.80a)	9.00	1.16	10.04	1.67	2.76	18.97
(0.60a, 0.80a)	8.00	1.09	9.66	1.43	3.26	18.40
(0.65a, 0.80a)	7.00	1.00	9.10	1.20	3.85	17.66
(0.70a, 0.80a)	6.00	0.90	8.37	1.00	4.53	17.00
(0.75a, 0.80a)	5.00	0.77	7.45	0.81	5.35	16.22
(0.80a, 0.80a)	4.00	0.63	6.34	0.63	6.34	15.41
(0.85a, 0.80a)	3.00	0.48	5.01	0.46	7.55	14.61
(0.90a, 0.80a)	2.00	0.32	3.48	0.31	9.04	13.89
(0.95a, 0.80a)	1.00	0.16	1.79	0.15	10.86	13.34
(1.00a, 0.80a)	12.90	0.00	0.00	0.00	0.00	12.90

(x, y)	ϕ_1	$-\phi_2$	ϕ_3	$-\phi_4$	ϕ_5	$\sum_1^5 \phi$
(0.00a, 0.85a)	15.00	0.00	0.00	0.00	0.00	15.00
(0.05a, 0.85a)	14.25	0.15	2.59	1.03	0.13	15.80
(0.10a, 0.85a)	13.50	0.30	4.87	1.80	0.25	16.52
(0.15a, 0.85a)	12.75	0.44	6.88	1.99	0.39	17.60
(0.20a, 0.85a)	12.00	0.56	8.50	2.34	0.53	18.10
(0.25a, 0.85a)	11.25	0.67	9.74	2.31	0.68	18.70
(0.30a, 0.85a)	10.50	0.75	10.65	2.18	0.85	19.06
(0.35a, 0.85a)	9.75	0.82	11.09	2.06	1.04	19.21
(0.40a, 0.85a)	9.00	0.86	11.71	1.84	1.26	19.28
(0.45a, 0.85a)	8.25	0.87	11.91	1.65	1.51	19.15
(0.50a, 0.85a)	7.50	0.87	11.90	1.45	1.80	18.88
(0.55a, 0.85a)	6.75	0.84	11.66	1.27	2.13	18.43
(0.60a, 0.85a)	6.00	0.80	11.22	1.09	2.53	17.86
(0.65a, 0.85a)	5.25	0.74	10.58	0.91	2.89	17.07
(0.70a, 0.85a)	4.50	0.66	9.78	0.76	3.52	16.39
(0.75a, 0.85a)	3.75	0.57	8.78	0.62	4.19	15.54
(0.80a, 0.85a)	3.00	0.46	7.55	0.48	5.01	14.61
(0.85a, 0.85a)	2.25	0.36	6.03	0.36	6.03	13.60
(0.90a, 0.85a)	1.50	0.24	4.23	0.23	7.36	12.62
(0.95a, 0.85a)	0.75	0.12	2.19	0.12	9.10	11.81
(1.00a, 0.85a)	11.10	0.00	0.00	0.00	0.00	11.10

(x, y)	ϕ_1	$-\phi_2$	ϕ_3	$-\phi_4$	ϕ_5	$\sum_1^5 \phi$
(0.00a, 0.90a)	10.00	0.00	0.00	0.00	0.00	10.00
(0.05a, 0.90a)	9.50	0.10	3.23	-0.12	0.09	12.84
(0.10a, 0.90a)	9.00	0.19	6.16	0.79	0.17	14.34
(0.15a, 0.90a)	8.50	0.28	8.56	1.45	0.26	15.79
(0.20a, 0.90a)	8.00	0.37	10.37	1.44	0.36	16.92
(0.25a, 0.90a)	7.50	0.43	11.66	1.47	0.46	17.71
(0.30a, 0.90a)	7.00	0.49	12.55	1.41	0.58	18.22
(0.35a, 0.90a)	6.50	0.53	13.16	1.36	0.71	18.48
(0.40a, 0.90a)	6.00	0.56	13.59	1.22	0.86	18.67
(0.45a, 0.90a)	5.50	0.56	13.79	1.10	1.03	18.66
(0.50a, 0.90a)	5.00	0.57	13.75	0.97	1.23	18.44
(0.55a, 0.90a)	4.50	0.55	13.47	0.85	1.46	18.02
(0.60a, 0.90a)	4.00	0.53	12.94	0.72	0.73	17.42
(0.65a, 0.90a)	3.50	0.48	12.23	0.61	2.41	16.68
(0.70a, 0.90a)	3.00	0.43	11.38	0.51	2.41	15.86
(0.75a, 0.90a)	2.50	0.37	10.34	0.42	2.90	14.95
(0.80a, 0.90a)	2.00	0.31	9.04	0.32	3.48	13.89
(0.85a, 0.90a)	1.50	0.23	7.36	0.24	4.23	12.62
(0.90a, 0.90a)	1.00	0.16	5.25	0.16	5.25	11.19
(0.95a, 0.90a)	0.50	0.08	2.74	0.08	6.66	9.75
(1.00a, 0.90a)	8.80	0.00	0.00	0.00	0.00	8.80

(x, y)	ϕ_1	$-\phi_2$	ϕ_3	$-\phi_4$	ϕ_5	$\sum_1^5 \phi$
(0.00a, 0.95a)	5.00	0.00	0.00	0.00	0.00	5.00
(0.05a, 0.95a)	4.75	0.05	4.27	-0.67	0.04	9.71
(0.10a, 0.95a)	4.50	0.10	8.00	0.21	0.09	12.28
(0.15a, 0.95a)	4.25	0.14	10.86	0.13	0.45	14.65
(0.20a, 0.95a)	4.00	0.18	12.79	0.68	0.18	16.11
(0.25a, 0.95a)	3.75	0.21	13.97	0.69	0.23	16.84
(0.30a, 0.95a)	3.50	0.24	14.70	0.70	0.29	17.55
(0.35a, 0.95a)	3.25	0.26	15.12	0.67	0.36	17.80
(0.40a, 0.95a)	3.00	0.28	15.64	0.61	0.44	18.20
(0.45a, 0.95a)	2.75	0.28	15.89	0.55	0.52	18.33
(0.50a, 0.95a)	2.50	0.28	15.85	0.49	0.62	18.21
(0.55a, 0.95a)	2.25	0.27	15.47	0.43	0.74	17.76
(0.60a, 0.95a)	2.00	0.26	14.81	0.36	0.88	17.06
(0.65a, 0.95a)	1.75	0.24	13.97	0.31	1.07	16.21
(0.70a, 0.95a)	1.50	0.21	13.07	0.26	1.22	15.33
(0.75a, 0.95a)	1.25	0.18	12.10	0.21	1.48	14.44
(0.80a, 0.95a)	1.00	0.15	10.86	0.16	1.79	13.33
(0.85a, 0.95a)	0.75	0.12	9.10	0.12	2.19	11.81
(0.90a, 0.95a)	0.50	0.08	6.66	0.08	2.74	9.75
(0.95a, 0.95a)	0.25	0.04	3.54	0.04	3.54	7.25
(1.00a, 0.95a)	5.50	0.00	0.00	0.00	0.00	5.50

TABLE VI

PRESSURE DISTRIBUTION CALCULATIONS - FINAL STAGE $P(x,y) = \sum_1^5 \phi$ *

(x, y)	ϕ_1	ϕ_2	$-\phi_3$	ϕ_4	$-\phi_5$	$\sum_1^5 \phi$
(0.00a, 0.05a)	5.00	0.00	0.00	0.00	0.00	5.00
(0.05a, 0.05a)	9.68	7.13	0.03	7.13	0.03	23.89
(0.10a, 0.05a)	14.36	12.87	0.06	4.88	0.06	31.98
(0.15a, 0.05a)	19.04	16.42	0.10	3.52	0.70	38.81
(0.20a, 0.05a)	23.73	17.89	0.14	2.63	0.12	43.98
(0.25a, 0.05a)	28.41	17.94	0.28	1.20	0.14	48.04
(0.30a, 0.05a)	33.08	17.29	0.23	1.53	0.16	51.51
(0.35a, 0.05a)	37.77	16.33	0.29	1.21	0.18	54.85
(0.40a, 0.05a)	42.45	15.05	0.35	0.97	0.19	57.93
(0.45a, 0.05a)	47.13	13.30	0.43	0.77	0.20	60.58
(0.50a, 0.05a)	51.81	11.16	0.53	0.62	0.20	62.87
(0.55a, 0.05a)	56.49	8.80	0.64	0.49	0.20	64.95
(0.60a, 0.05a)	61.18	6.59	0.77	0.40	0.20	67.20
(0.65a, 0.05a)	65.86	4.60	0.95	0.32	0.18	69.65
(0.70a, 0.05a)	70.54	2.70	1.14	0.26	0.16	72.20
(0.75a, 0.05a)	75.22	0.75	1.45	0.20	0.14	74.58
(0.80a, 0.05a)	79.90	-1.13	1.87	0.15	0.12	76.94
(0.85a, 0.05a)	84.50	-2.49	2.35	0.11	0.07	79.71
(0.90a, 0.05a)	89.26	-2.77	3.11	0.07	0.06	83.40
(0.95a, 0.05a)	93.94	-1.81	4.26	0.04	0.03	87.88
(1.00a, 0.05a)	91.25	0.00	0.00	0.00	0.00	91.25

* ϕ's are percentage of injection pressure

(x, y)	ϕ_1	ϕ_2	$-\phi_3$	ϕ_4	$-\phi_5$	$\sum_1^5 \phi$
(0.00a, 0.10a)	10.00	0.00	0.00	0.00	0.00	10.00
(0.05a, 0.10a)	14.36	4.88	0.06	12.87	0.06	31.98
(0.10a, 0.10a)	18.72	9.01	0.13	9.01	0.13	36.50
(0.15a, 0.10a)	23.09	11.92	0.20	6.60	0.18	41.22
(0.20a, 0.10a)	27.45	13.55	0.28	4.99	0.24	45.47
(0.25a, 0.10a)	31.81	14.11	0.36	3.82	0.29	49.10
(0.30a, 0.10a)	36.18	14.04	0.46	2.96	0.33	52.39
(0.35a, 0.10a)	40.54	13.48	0.57	2.35	0.36	55.43
(0.40a, 0.10a)	44.90	12.55	0.69	1.87	0.38	58.24
(0.45a, 0.10a)	49.26	11.27	0.84	1.50	0.40	60.79
(0.50a, 0.10a)	53.73	9.72	1.03	1.21	0.40	63.22
(0.55a, 0.10a)	57.99	8.42	1.25	0.97	0.40	65.32
(0.60a, 0.10a)	62.35	6.31	1.52	0.78	0.38	67.54
(0.65a, 0.10a)	66.71	4.66	1.85	0.63	0.36	69.79
(0.70a, 0.10a)	71.08	3.07	2.23	0.50	0.33	72.09
(0.75a, 0.10a)	75.44	1.58	2.80	0.39	0.29	74.33
(0.80a, 0.10a)	79.80	0.21	3.53	0.30	0.24	76.55
(0.85a, 0.10a)	84.16	-0.73	4.49	0.22	0.18	78.91
(0.90a, 0.10a)	88.53	-1.06	5.86	0.14	0.13	81.63
(0.95a, 0.10a)	92.89	-0.74	7.89	0.07	0.06	84.26
(1.00a, 0.10a)	86.25	0.00	0.00	0.00	0.00	86.25

(x, y)	ϕ_1	ϕ_2	$-\phi_3$	ϕ_4	$-\phi_5$	$\sum_1^5 \phi$
(0.00a, 0.15a)	15.00	0.00	0.00	0.00	0.00	15.00
(0.05a, 0.15a)	19.04	3.52	0.07	16.42	0.10	38.81
(0.10a, 0.15a)	23.09	6.60	0.18	11.92	0.20	41.22
(0.15a, 0.15a)	27.13	8.95	0.30	8.95	0.30	44.44
(0.20a, 0.15a)	31.18	10.59	0.41	6.82	0.38	47.86
(0.25a, 0.15a)	35.22	11.26	0.53	5.35	0.46	50.84
(0.30a, 0.15a)	39.26	11.44	0.67	4.19	0.52	53.71
(0.35a, 0.15a)	43.31	11.16	0.83	3.34	0.57	56.41
(0.40a, 0.15a)	47.35	10.53	1.01	2.68	0.60	58.94
(0.45a, 0.15a)	51.39	9.59	1.23	2.15	0.62	61.30
(0.50a, 0.15a)	54.44	8.43	1.50	1.74	0.62	63.49
(0.55a, 0.15a)	59.48	7.13	1.81	1.40	0.61	65.59
(0.60a, 0.15a)	63.53	5.77	2.19	1.13	0.58	67.66
(0.65a, 0.15a)	67.57	4.42	2.67	0.91	0.54	69.70
(0.70a, 0.15a)	71.61	3.13	3.21	0.73	0.48	71.78
(0.75a, 0.15a)	75.66	1.93	4.02	0.57	0.42	73.72
(0.80a, 0.15a)	79.70	0.91	4.99	0.44	0.35	75.71
(0.85a, 0.15a)	83.74	0.17	6.26	0.31	0.27	77.70
(0.90a, 0.15a)	87.79	0.20	8.00	0.21	0.18	79.62
(0.95a, 0.15a)	91.83	0.21	10.49	0.10	0.09	81.14
(1.00a, 0.15a)	82.50	0.00	0.00	0.00	0.00	82.50

(x, y)	ϕ_1	ϕ_2	$-\phi_3$	ϕ_4	$-\phi_5$	$\sum_1^5 \phi$
(0.00a, 0.20a)	20.00	0.00	0.00	0.00	0.00	20.00
(0.05a, 0.20a)	23.73	2.63	0.12	17.89	0.14	43.98
(0.10a, 0.20a)	27.55	4.99	0.24	13.55	0.28	45.47
(0.15a, 0.20a)	31.18	6.88	0.38	10.59	0.41	47.86
(0.20a, 0.20a)	34.90	8.23	0.53	8.23	0.53	50.31
(0.25a, 0.20a)	38.63	9.03	0.68	6.49	0.63	52.83
(0.30a, 0.20a)	42.35	9.34	0.86	5.16	0.71	55.27
(0.35a, 0.20a)	46.08	9.26	1.07	4.14	0.78	57.66
(0.40a, 0.20a)	49.80	8.84	1.30	3.34	0.82	59.86
(0.45a, 0.20a)	53.53	8.16	1.57	2.70	0.85	61.97
(0.50a, 0.20a)	57.25	7.28	1.92	2.19	0.85	63.96
(0.55a, 0.20a)	60.98	6.27	2.31	1.77	0.83	65.88
(0.60a, 0.20a)	64.70	5.20	2.80	1.44	0.79	67.76
(0.65a, 0.20a)	68.43	4.12	3.38	1.16	0.74	69.59
(0.70a, 0.20a)	72.15	3.07	3.94	0.93	0.66	71.55
(0.75a, 0.20a)	75.88	2.11	5.01	0.73	0.57	73.13
(0.80a, 0.20a)	79.60	1.29	6.15	0.56	0.47	74.82
(0.85a, 0.20a)	83.33	0.66	7.58	0.40	0.36	76.40
(0.90a, 0.20a)	87.05	0.26	9.48	0.27	0.25	77.85
(0.95a, 0.20a)	90.78	0.07	12.04	0.13	0.12	78.81
(1.00a, 0.20a)	77.50	0.00	0.00	0.00	0.00	77.50

(x, y)	ϕ_1	ϕ_2	$-\phi_3$	ϕ_4	$-\phi_5$	$\sum_1^5 \phi$
(0.00a, 0.25a)	25.00	0.00	0.00	0.00	0.00	25.00
(0.05a, 0.25a)	28.41	2.00	0.14	17.94	0.18	48.04
(0.10a, 0.25a)	31.81	3.82	0.29	14.11	0.36	49.10
(0.15a, 0.25a)	35.22	5.35	0.45	11.26	0.53	50.84
(0.20a, 0.25a)	38.63	6.49	0.63	9.03	0.68	52.83
(0.25a, 0.25a)	42.03	7.23	0.82	7.23	0.82	54.86
(0.30a, 0.25a)	45.44	7.60	1.03	5.83	0.93	56.91
(0.35a, 0.25a)	48.84	7.64	1.27	4.72	1.01	58.91
(0.40a, 0.25a)	52.25	7.39	1.55	3.84	1.07	60.87
(0.45a, 0.25a)	55.66	6.92	1.88	3.12	1.10	62.73
(0.50a, 0.25a)	59.06	6.27	2.28	2.56	1.10	64.52
(0.55a, 0.25a)	62.47	5.50	2.73	2.08	1.07	66.24
(0.60a, 0.25a)	65.88	4.66	3.29	1.69	1.02	67.91
(0.65a, 0.25a)	69.28	3.79	3.95	1.37	0.95	69.54
(0.70a, 0.25a)	72.69	2.95	4.72	1.10	0.85	71.16
(0.75a, 0.25a)	76.09	2.16	5.77	0.87	0.74	72.61
(0.80a, 0.25a)	79.50	1.47	6.60	0.66	0.61	74.42
(0.85a, 0.25a)	82.91	0.92	8.48	0.48	0.47	75.35
(0.90a, 0.25a)	86.31	0.50	10.37	0.32	0.32	76.45
(0.95a, 0.25a)	89.72	0.22	12.76	0.16	0.16	77.17
(1.00a, 0.25a)	77.50	0.00	0.00	0.00	0.00	77.50

(x, y)	ϕ_1	ϕ_2	$-\phi_3$	ϕ_4	$-\phi_5$	$\sum_1^5 \phi$
(0.00a, 0.30a)	30.00	0.00	0.00	0.00	0.00	30.00
(0.05a, 0.30a)	30.01	1.53	0.16	17.29	0.23	51.51
(0.10a, 0.30a)	36.18	2.96	0.33	14.04	0.46	52.39
(0.15a, 0.30a)	39.26	4.19	0.52	11.44	0.67	53.71
(0.20a, 0.30a)	42.35	5.16	0.71	9.34	0.86	55.27
(0.25a, 0.30a)	45.44	5.83	0.93	7.60	1.03	56.91
(0.30a, 0.30a)	48.53	6.21	1.17	6.21	1.17	58.62
(0.35a, 0.30a)	51.61	6.31	1.44	5.08	1.27	60.77
(0.40a, 0.30a)	54.70	6.19	1.76	4.16	1.34	61.96
(0.45a, 0.30a)	57.70	5.87	2.12	3.41	1.38	63.49
(0.50a, 0.30a)	60.88	5.40	2.55	2.81	1.38	65.15
(0.55a, 0.30a)	63.96	4.81	3.06	2.30	1.34	66.67
(0.60a, 0.30a)	67.05	4.16	3.69	1.88	1.28	68.12
(0.65a, 0.30a)	70.14	3.46	4.38	1.53	1.18	69.56
(0.70a, 0.30a)	73.23	2.77	5.17	1.23	1.06	70.99
(0.75a, 0.30a)	76.31	2.11	6.28	0.97	0.92	72.19
(0.80a, 0.30a)	79.40	1.53	7.50	0.74	0.76	73.42
(0.85a, 0.30a)	82.49	1.03	9.02	0.54	0.58	74.45
(0.90a, 0.30a)	85.58	0.62	10.83	0.36	0.39	75.33
(0.95a, 0.30a)	88.66	0.29	12.99	0.18	0.20	75.94
(1.00a, 0.30a)	76.25	0.00	0.00	0.00	0.00	76.25

(x, y)	ϕ_1	ϕ_2	$-\phi_3$	ϕ_4	$-\phi_5$	$\sum_1^5 \phi$
(0.00a, 0.35a)	35.00	0.00	0.00	0.00	0.00	35.00
(0.05a, 0.35a)	37.77	1.21	0.18	16.33	0.29	54.85
(0.10a, 0.35a)	40.54	2.35	0.36	13.48	0.57	55.43
(0.15a, 0.35a)	43.31	3.34	0.57	11.16	0.83	56.41
(0.20a, 0.35a)	46.08	4.14	0.78	9.26	1.07	57.66
(0.25a, 0.35a)	48.84	4.72	1.01	7.64	1.27	58.91
(0.30a, 0.35a)	51.61	5.07	1.27	6.31	1.44	60.77
(0.35a, 0.35a)	54.38	5.21	1.57	5.21	1.57	61.68
(0.40a, 0.35a)	57.15	5.16	1.91	4.32	1.65	63.07
(0.45a, 0.35a)	59.92	4.95	2.30	3.59	1.69	64.47
(0.50a, 0.35a)	62.69	4.60	2.76	2.96	1.69	65.88
(0.55a, 0.35a)	65.46	4.16	3.29	2.44	1.65	67.12
(0.60a, 0.35a)	68.23	3.65	3.93	2.01	1.56	68.39
(0.65a, 0.35a)	70.99	3.10	4.66	1.63	1.44	69.62
(0.70a, 0.35a)	73.76	2.54	5.47	1.32	1.30	70.86
(0.75a, 0.35a)	76.53	2.01	6.59	1.04	1.12	71.87
(0.80a, 0.35a)	79.30	1.51	7.80	0.80	0.92	72.89
(0.85a, 0.35a)	82.07	1.06	9.27	0.58	0.71	73.73
(0.90a, 0.35a)	84.84	0.67	10.98	0.39	0.48	74.48
(0.95a, 0.35a)	87.61	0.32	12.95	0.19	0.24	74.93
(1.00a, 0.35a)	75.00	0.00	0.00	0.00	0.00	75.00

(x, y)	ϕ_1	ϕ_2	$-\phi_3$	ϕ_4	$-\phi_5$	$\sum_1^5 \phi$
(0.00a, 0.40a)	40.00	0.00	0.00	0.00	0.00	40.00
(0.05a, 0.40a)	42.45	0.97	0.19	15.05	0.35	57.93
(0.10a, 0.40a)	44.90	1.87	0.38	12.55	0.69	58.24
(0.15a, 0.40a)	47.35	2.68	0.60	10.53	1.01	58.94
(0.20a, 0.40a)	49.80	3.34	0.82	8.84	1.30	59.86
(0.25a, 0.40a)	52.25	3.84	1.07	7.39	1.55	60.87
(0.30a, 0.40a)	54.70	4.17	1.34	6.19	1.76	61.96
(0.35a, 0.40a)	57.15	4.32	1.65	5.16	1.91	63.07
(0.40a, 0.40a)	59.60	4.32	2.01	4.32	2.01	64.22
(0.45a, 0.40a)	62.05	4.18	2.41	3.59	2.05	65.36
(0.50a, 0.40a)	64.50	3.93	2.88	3.01	2.05	66.51
(0.55a, 0.40a)	66.95	3.59	3.43	2.50	1.99	67.63
(0.60a, 0.40a)	69.40	3.19	4.07	2.06	1.88	68.70
(0.65a, 0.40a)	71.85	2.75	4.80	1.69	1.74	69.75
(0.70a, 0.40a)	74.30	2.30	5.60	1.37	1.56	70.81
(0.75a, 0.40a)	76.75	1.85	6.71	1.08	1.34	71.63
(0.80a, 0.40a)	79.20	1.42	7.89	0.83	1.10	72.45
(0.85a, 0.40a)	81.65	1.02	9.28	0.60	0.84	83.16
(0.90a, 0.40a)	84.10	0.66	10.89	0.41	0.57	73.71
(0.95a, 0.40a)	86.55	0.32	12.73	0.20	0.29	74.05
(1.00a, 0.40a)	74.38	0.00	0.00	0.00	0.00	74.38

(x, y)	ϕ_1	ϕ_2	$-\phi_3$	ϕ_4	$-\phi_5$	$\sum_1^5 \phi$
(0.00a, 0.45a)	45.00	0.00	0.00	0.00	0.00	45.00
(0.05a, 0.45a)	47.13	0.77	0.20	13.30	0.43	60.58
(0.10a, 0.45a)	49.26	1.50	0.40	11.27	0.84	60.79
(0.15a, 0.45a)	51.39	2.16	0.62	9.59	1.23	61.30
(0.20a, 0.45a)	53.53	2.70	0.85	8.16	1.57	61.97
(0.25a, 0.45a)	55.66	3.12	1.10	6.92	1.88	62.73
(0.30a, 0.45a)	57.79	3.41	1.38	5.87	2.12	63.49
(0.35a, 0.45a)	59.92	3.59	1.69	4.95	2.30	64.47
(0.40a, 0.45a)	62.05	3.59	2.05	4.18	2.41	65.36
(0.45a, 0.45a)	64.18	3.52	2.46	3.52	2.46	66.28
(0.50a, 0.45a)	66.31	3.34	2.93	2.96	2.45	67.23
(0.55a, 0.45a)	68.44	3.08	3.47	2.48	2.37	68.16
(0.60a, 0.45a)	70.58	2.77	4.11	2.06	2.24	69.06
(0.65a, 0.45a)	72.71	2.42	4.83	1.69	2.07	69.92
(0.70a, 0.45a)	74.84	2.05	5.59	1.37	1.85	70.82
(0.75a, 0.45a)	76.97	1.68	6.66	1.09	1.59	71.50
(0.80a, 0.45a)	79.10	1.31	7.77	0.84	1.31	72.18
(0.85a, 0.45a)	81.23	0.96	9.08	0.61	0.10	72.73
(0.90a, 0.45a)	83.36	0.63	10.57	0.41	0.67	73.16
(0.95a, 0.45a)	85.49	0.31	12.28	0.20	0.34	73.39
(1.00a, 0.45a)	73.75	0.00	0.00	0.00	0.00	73.75

(x, y)	ϕ_1	ϕ_2	$-\phi_3$	ϕ_4	$-\phi_5$	$\sum_1^5 \phi$
(0.00a, 0.50a)	50.00	0.00	0.00	0.00	0.00	50.00
(0.05a, 0.50a)	51.81	0.62	0.20	11.16	0.53	62.87
(0.10a, 0.50a)	53.63	1.21	0.40	9.72	1.03	63.22
(0.15a, 0.50a)	55.44	1.74	0.62	8.43	1.50	63.49
(0.20a, 0.50a)	57.25	2.19	0.85	7.28	1.92	63.98
(0.25a, 0.50a)	59.00	2.56	1.10	6.27	2.28	64.52
(0.30a, 0.50a)	60.88	2.81	1.38	5.39	2.55	65.15
(0.35a, 0.50a)	62.69	2.96	1.69	4.60	2.76	65.88
(0.40a, 0.50a)	64.50	3.01	2.05	3.93	2.88	66.51
(0.45a, 0.50a)	66.31	2.96	2.05	3.34	2.93	67.23
(0.50a, 0.50a)	68.13	2.83	2.90	2.83	2.90	67.98
(0.55a, 0.50a)	69.94	2.63	3.43	2.39	2.81	68.72
(0.60a, 0.50a)	71.75	2.38	4.04	2.00	2.65	69.45
(0.65a, 0.50a)	73.56	2.10	4.73	1.65	2.44	70.15
(0.70a, 0.50a)	75.38	1.80	5.54	1.35	2.18	70.80
(0.75a, 0.50a)	77.19	1.50	6.45	1.07	1.88	71.42
(0.80a, 0.50a)	79.00	1.19	7.50	0.83	1.54	71.97
(0.85a, 0.50a)	80.81	0.88	8.70	0.60	1.18	72.43
(0.90a, 0.50a)	82.63	0.58	10.06	0.41	0.80	72.76
(0.95a, 0.50a)	84.44	0.29	11.57	0.20	0.40	72.96
(1.00a, 0.50a)	73.13	0.00	0.00	0.00	0.00	73.13

(x, y)	ϕ_1	ϕ_2	$-\phi_3$	ϕ_4	$-\phi_5$	$\sum_1^5 \phi$
(0.00a, 0.55a)	55.00	0.00	0.00	0.00	0.00	55.00
(0.05a, 0.55a)	56.50	0.50	0.20	8.80	0.64	64.95
(0.10a, 0.55a)	57.99	0.97	0.40	8.02	1.25	65.32
(0.15a, 0.55a)	59.48	1.40	0.61	7.13	1.83	65.58
(0.20a, 0.55a)	60.98	1.77	0.83	6.27	2.31	65.88
(0.25a, 0.55a)	62.47	2.08	1.07	5.50	2.73	66.24
(0.30a, 0.55a)	63.96	2.30	1.35	4.81	3.06	66.67
(0.35a, 0.55a)	65.46	2.44	1.65	4.16	3.29	67.12
(0.40a, 0.55a)	66.95	2.50	1.99	3.59	3.43	67.63
(0.45a, 0.55a)	68.44	2.48	2.37	3.08	3.47	68.16
(0.50a, 0.55a)	69.94	2.39	2.81	2.63	3.43	68.72
(0.55a, 0.55a)	71.43	2.24	3.30	2.24	3.30	69.30
(0.60a, 0.55a)	72.93	2.05	3.88	1.88	3.21	69.77
(0.65a, 0.55a)	74.42	1.82	4.53	1.57	2.86	70.41
(0.70a, 0.55a)	75.91	1.57	5.20	1.28	2.55	71.02
(0.75a, 0.55a)	77.41	1.30	6.13	1.03	2.20	71.41
(0.80a, 0.55a)	78.90	1.04	7.08	0.78	1.80	71.86
(0.85a, 0.55a)	80.40	0.77	8.15	0.58	1.38	72.23
(0.90a, 0.55a)	81.89	0.51	9.36	0.39	0.93	72.50
(0.95a, 0.55a)	83.38	0.25	10.64	0.19	0.47	72.73
(1.00a, 0.55a)	72.50	0.00	0.00	0.00	0.00	72.50

(x, y)	ϕ_1	ϕ_2	$-\phi_3$	ϕ_4	$-\phi_5$	$\sum_1^5 \phi$
(0.00a, 0.60a)	60.00	0.00	0.00	0.00	0.00	60.00
(0.05a, 0.60a)	61.18	0.40	0.19	6.59	0.77	67.20
(0.10a, 0.60a)	62.35	0.78	0.38	6.31	1.52	67.54
(0.15a, 0.60a)	63.53	1.13	0.58	5.77	2.20	67.66
(0.20a, 0.60a)	64.70	1.44	0.79	5.20	2.80	67.76
(0.25a, 0.60a)	65.88	1.69	1.02	4.66	3.29	67.91
(0.30a, 0.60a)	67.05	1.88	1.28	4.16	3.69	68.12
(0.35a, 0.60a)	68.23	2.01	1.56	3.65	3.93	68.39
(0.40a, 0.60a)	69.40	2.06	1.88	3.19	4.07	68.70
(0.45a, 0.60a)	70.58	2.06	2.24	2.77	4.11	69.06
(0.50a, 0.60a)	71.75	2.00	2.65	2.38	4.04	69.45
(0.55a, 0.60a)	72.93	1.88	3.21	2.05	3.88	69.77
(0.60a, 0.60a)	74.10	1.73	3.64	1.73	3.64	70.29
(0.65a, 0.60a)	75.28	1.55	4.23	1.45	3.32	70.72
(0.70a, 0.60a)	76.45	1.35	4.84	1.19	2.98	71.17
(0.75a, 0.60a)	77.63	1.13	5.69	1.00	2.55	71.47
(0.80a, 0.60a)	78.80	0.91	6.54	0.75	2.09	71.82
(0.85a, 0.60a)	79.90	0.68	7.45	0.55	1.59	72.05
(0.90a, 0.60a)	81.10	0.45	8.53	0.37	1.08	72.32
(0.95a, 0.60a)	82.33	0.23	9.58	0.18	0.54	72.61
(1.00a, 0.60a)	72.50	0.00	0.00	0.00	0.00	72.50

(x, y)	ϕ_1	ϕ_2	$-\phi_3$	ϕ_4	$-\phi_5$	$\sum_1^5 \phi$
(0.00a, 0.65a)	65.00	0.00	0.00	0.00	0.00	65.00
(0.05a, 0.65a)	65.86	0.32	0.18	4.60	0.95	69.65
(0.10a, 0.65a)	66.71	0.63	0.36	4.66	1.85	69.79
(0.15a, 0.65a)	67.57	0.91	0.54	4.42	2.67	69.70
(0.20a, 0.65a)	68.43	1.16	0.74	4.12	3.38	69.52
(0.25a, 0.65a)	69.28	1.37	0.95	3.79	3.95	69.54
(0.30a, 0.65a)	70.14	1.53	1.18	3.46	4.38	69.56
(0.35a, 0.65a)	70.99	1.63	1.44	3.10	4.66	69.62
(0.40a, 0.65a)	71.85	1.69	1.74	2.75	4.80	69.75
(0.45a, 0.65a)	72.71	1.69	2.07	2.42	4.83	69.92
(0.50a, 0.65a)	73.56	1.65	2.44	2.10	4.73	70.15
(0.55a, 0.65a)	74.42	1.57	2.86	1.82	4.53	70.41
(0.60a, 0.65a)	75.28	1.45	3.32	1.55	4.23	70.72
(0.65a, 0.65a)	76.13	1.30	3.84	1.30	3.84	71.10
(0.70a, 0.65a)	77.00	1.14	4.40	1.08	3.43	71.38
(0.75a, 0.65a)	77.84	0.96	5.14	0.87	2.94	71.60
(0.80a, 0.65a)	78.70	0.78	5.84	0.68	2.40	71.92
(0.85a, 0.65a)	79.56	0.58	6.71	0.50	1.82	72.11
(0.90a, 0.65a)	80.61	0.39	7.60	0.34	1.23	72.51
(0.95a, 0.65a)	81.27	0.20	8.47	0.17	0.62	72.55
(1.00a, 0.65a)	72.50	0.00	0.00	0.00	0.00	72.50

(x, y)	ϕ_1	ϕ_2	$-\phi_3$	ϕ_4	$-\phi_5$	$\sum_1^5 \phi$
(0.00a, 0.70a)	70.00	0.00	0.00	0.00	0.00	70.00
(0.05a, 0.70a)	70.54	0.26	0.16	2.70	1.14	72.20
(0.10a, 0.70a)	71.07	0.50	0.33	3.07	2.23	72.10
(0.15a, 0.70a)	71.61	0.73	0.48	3.13	3.20	71.78
(0.20a, 0.70a)	72.15	0.93	0.66	3.07	3.94	71.55
(0.25a, 0.70a)	72.69	1.10	0.85	2.95	4.72	71.16
(0.30a, 0.70a)	73.23	1.23	1.06	2.77	5.17	70.99
(0.35a, 0.70a)	73.76	1.32	1.30	2.54	5.47	70.86
(0.40a, 0.70a)	74.30	1.37	1.56	2.30	5.60	70.81
(0.45a, 0.70a)	74.84	1.37	1.85	2.05	5.60	70.82
(0.50a, 0.70a)	75.38	1.35	2.18	1.80	5.54	70.80
(0.55a, 0.70a)	75.91	1.28	2.55	1.57	5.20	71.02
(0.60a, 0.70a)	76.45	1.19	2.98	1.35	4.84	71.17
(0.65a, 0.70a)	77.00	1.08	3.43	1.08	3.43	71.38
(0.70a, 0.70a)	77.53	0.95	3.88	0.95	3.88	71.67
(0.75a, 0.70a)	78.06	0.80	4.53	0.77	3.30	71.78
(0.80a, 0.70a)	78.60	0.65	5.12	0.60	2.59	72.14
(0.85a, 0.70a)	79.14	0.49	5.87	0.44	2.04	72.17
(0.90a, 0.70a)	79.68	0.33	6.63	0.30	1.37	72.45
(0.95a, 0.70a)	80.21	0.16	7.34	0.15	0.68	72.50
(1.00a, 0.70a)	72.50	0.00	0.00	0.00	0.00	72.50

(x, y)	ϕ_1	ϕ_2	$-\phi_3$	ϕ_4	$-\phi_5$	$\sum_1^5 \phi$
(0.00a, 0.75a)	75.00	0.00	0.00	0.00	0.00	75.00
(0.05a, 0.75a)	75.22	0.20	0.14	0.75	1.45	74.58
(0.10a, 0.75a)	75.44	0.39	0.29	1.58	2.80	74.33
(0.15a, 0.75a)	75.66	0.57	0.42	1.93	4.02	73.72
(0.20a, 0.75a)	75.88	0.73	0.57	2.11	5.01	73.13
(0.25a, 0.75a)	76.09	0.87	0.74	2.16	5.77	72.61
(0.30a, 0.75a)	76.31	0.97	0.92	2.11	6.28	72.19
(0.35a, 0.75a)	76.53	1.04	1.12	2.01	6.59	71.87
(0.40a, 0.75a)	76.75	1.08	1.34	1.85	6.71	71.63
(0.45a, 0.75a)	76.97	1.09	1.59	1.68	6.66	71.50
(0.50a, 0.75a)	77.19	1.07	1.88	1.50	6.45	71.43
(0.55a, 0.75a)	77.41	1.03	2.19	1.30	6.13	71.41
(0.60a, 0.75a)	77.63	0.96	2.55	1.13	5.69	71.47
(0.65a, 0.75a)	77.84	0.87	2.94	0.96	5.14	71.60
(0.70a, 0.75a)	78.06	0.77	3.33	0.80	4.53	71.78
(0.75a, 0.75a)	78.28	0.65	3.87	0.65	3.88	71.85
(0.80a, 0.75a)	78.50	0.53	3.99	0.51	3.14	72.41
(0.85a, 0.75a)	78.72	0.40	4.97	0.38	2.38	72.15
(0.90a, 0.75a)	78.94	0.27	5.60	0.26	1.59	72.30
(0.95a, 0.75a)	79.16	0.14	6.17	0.13	0.81	72.44
(1.00a, 0.75a)	72.50	0.00	0.00	0.00	0.00	72.50

(x, y)	ϕ_1	ϕ_2	$-\phi_3$	ϕ_4	$-\phi_5$	$\sum_1^5 \phi$
(0.00a, 0.80a)	80.00	0.00	0.00	0.00	0.00	80.00
(0.05a, 0.80a)	79.90	0.15	0.12	-1.13	1.87	76.94
(0.10a, 0.80a)	79.80	0.30	0.24	0.21	3.53	76.55
(0.15a, 0.80a)	79.70	0.44	0.35	0.91	4.99	75.71
(0.20a, 0.80a)	79.60	0.56	0.47	1.29	6.15	74.82
(0.25a, 0.80a)	79.50	0.66	0.61	1.47	6.60	74.42
(0.30a, 0.80a)	79.40	0.74	0.76	1.53	7.50	73.42
(0.35a, 0.80a)	79.30	0.80	0.92	1.51	7.80	72.89
(0.40a, 0.80a)	79.20	0.83	1.10	1.42	7.89	72.45
(0.45a, 0.80a)	79.10	0.84	1.31	1.31	7.77	72.18
(0.50a, 0.80a)	79.00	0.83	1.54	1.19	7.50	71.97
(0.55a, 0.80a)	78.90	0.80	1.80	1.04	7.08	71.86
(0.60a, 0.80a)	78.80	0.75	2.09	0.91	6.54	71.82
(0.65a, 0.80a)	78.70	0.68	2.40	0.78	5.84	71.92
(0.70a, 0.80a)	78.60	0.61	2.59	0.65	5.12	72.14
(0.75a, 0.80a)	78.50	0.51	3.14	0.51	3.14	72.41
(0.80a, 0.80a)	78.40	0.42	3.56	0.42	3.56	72.16
(0.85a, 0.80a)	78.30	0.32	4.02	0.31	2.70	72.21
(0.90a, 0.80a)	78.20	0.21	4.50	0.21	1.82	72.40
(0.95a, 0.80a)	78.10	0.11	4.94	0.10	0.95	72.42
(1.00a, 0.80a)	72.50	0.00	0.00	0.00	0.00	72.50

(x, y)	ϕ_1	ϕ_2	$-\phi_3$	ϕ_4	$-\phi_5$	$\sum_1^5 \phi$
(0.00a, 0.85a)	85.00	0.00	0.00	0.00	0.00	85.00
(0.05a, 0.85a)	84.58	1.11	0.07	-2.49	2.35	79.71
(0.10a, 0.85a)	84.16	0.22	0.18	-0.73	4.49	78.91
(0.15a, 0.85a)	83.74	0.31	0.27	0.17	6.26	77.70
(0.20a, 0.85a)	83.32	0.40	0.36	0.66	7.58	76.44
(0.25a, 0.85a)	82.91	0.48	0.47	0.92	8.48	75.35
(0.30a, 0.85a)	82.49	0.54	0.58	1.03	9.02	74.45
(0.35a, 0.85a)	82.07	0.58	0.71	1.06	9.27	73.73
(0.40a, 0.85a)	81.65	0.60	0.84	1.02	9.28	73.16
(0.45a, 0.85a)	81.23	0.61	1.00	0.96	9.08	72.73
(0.50a, 0.85a)	80.81	0.60	1.18	0.88	8.69	72.43
(0.55a, 0.85a)	80.39	0.58	1.38	0.77	8.15	72.23
(0.60a, 0.85a)	79.98	0.55	1.59	0.68	7.48	72.05
(0.65a, 0.85a)	79.56	0.50	1.82	0.58	6.71	72.11
(0.70a, 0.85a)	79.14	0.44	2.04	0.49	5.87	72.17
(0.75a, 0.85a)	78.72	0.38	2.38	0.40	4.97	72.15
(0.80a, 0.85a)	78.30	0.31	2.70	0.32	4.01	72.21
(0.85a, 0.85a)	77.88	0.23	3.03	0.23	3.03	72.30
(0.90a, 0.85a)	77.46	0.16	3.37	0.16	1.92	72.48
(0.95a, 0.85a)	77.04	0.08	3.66	0.08	1.01	72.53
(1.00a, 0.85a)	72.50	0.00	0.00	0.00	0.00	72.50

(x, y)	ϕ_1	ϕ_2	$-\phi_3$	ϕ_4	$-\phi_5$	$\sum_1^5 \phi$
(0.00a, 0.90a)	90.00	0.00	0.00	0.00	0.00	90.00
(0.05a, 0.90a)	89.26	0.07	0.06	-2.77	3.11	83.40
(0.10a, 0.90a)	88.53	0.14	0.13	-1.06	5.86	81.63
(0.15a, 0.90a)	87.79	0.21	0.18	-0.20	8.00	79.62
(0.20a, 0.90a)	87.05	0.27	0.25	0.26	9.48	77.85
(0.25a, 0.90a)	86.31	0.32	0.32	0.50	10.37	76.45
(0.30a, 0.90a)	85.58	0.36	0.39	0.62	10.83	75.33
(0.35a, 0.90a)	84.84	0.39	0.48	0.67	10.98	74.44
(0.40a, 0.90a)	84.10	0.41	0.57	0.66	10.88	73.71
(0.45a, 0.90a)	83.36	0.41	0.67	0.63	10.57	73.16
(0.50a, 0.90a)	82.63	0.41	0.80	0.58	10.06	72.76
(0.55a, 0.90a)	81.89	0.39	0.93	0.51	9.36	72.50
(0.60a, 0.90a)	81.15	0.37	1.08	0.45	8.53	72.32
(0.65a, 0.90a)	80.42	0.34	1.23	0.39	7.60	72.51
(0.70a, 0.90a)	79.68	0.30	1.37	0.33	6.63	72.45
(0.75a, 0.90a)	78.94	0.26	1.59	0.27	5.60	72.30
(0.80a, 0.90a)	78.20	0.21	1.82	0.21	4.50	72.40
(0.85a, 0.90a)	77.46	0.16	1.92	0.16	3.37	72.48
(0.90a, 0.90a)	76.73	0.11	2.24	0.11	2.24	72.46
(0.95a, 0.90a)	75.99	0.07	2.39	0.05	1.11	72.61
(1.00a, 0.90a)	72.50	0.00	0.00	0.00	0.00	72.50

(x, y)	ϕ_1	ϕ_2	$-\phi_3$	ϕ_4	$-\phi_5$	$\sum_1^5 \phi$
(0.00a, 0.95a)	95.00	0.00	0.00	0.00	0.00	95.00
(0.05a, 0.95a)	93.94	0.04	0.03	-1.81	4.26	87.88
(0.10a, 0.95a)	92.89	0.07	0.06	-0.74	7.89	84.26
(0.15a, 0.95a)	91.83	0.10	0.09	-0.21	10.49	81.14
(0.20a, 0.95a)	90.78	0.13	0.12	0.07	12.04	78.81
(0.25a, 0.95a)	89.72	0.16	0.16	0.22	12.76	77.17
(0.30a, 0.95a)	88.66	0.18	0.20	0.29	12.99	75.94
(0.35a, 0.95a)	87.61	0.19	0.24	0.32	12.95	74.93
(0.40a, 0.95a)	86.55	0.20	0.29	0.32	12.73	74.05
(0.45a, 0.95a)	85.49	0.20	0.34	0.31	12.28	73.39
(0.50a, 0.95a)	84.44	0.20	0.40	0.29	11.57	72.96
(0.55a, 0.95a)	83.38	0.19	0.47	0.25	10.64	72.72
(0.60a, 0.95a)	82.33	0.18	0.54	0.23	9.58	72.61
(0.65a, 0.95a)	81.27	0.17	0.62	0.20	8.47	72.54
(0.70a, 0.95a)	80.21	0.15	0.68	0.16	7.34	72.50
(0.75a, 0.95a)	79.16	0.13	0.81	0.14	6.17	72.44
(0.80a, 0.95a)	78.10	0.10	0.95	0.11	4.94	72.42
(0.85a, 0.95a)	77.04	0.08	1.01	0.08	3.66	72.53
(0.90a, 0.95a)	75.98	0.05	1.11	0.07	2.39	72.61
(0.95a, 0.95a)	74.93	0.03	1.17	0.03	1.17	72.64
(1.00a, 0.95a)	72.50	0.00	0.00	0.00	0.00	72.50

VITA

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Master of Science

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