UNIVERSITY OF OKLAHOMA

GRADUATE COLLEGE

CONCEPTUAL DIFFERENCES OF RUME AND NON-RUME STUDENTS

A THESIS

SUBMITTED TO THE GRADUATE FACULTY

in partial fulfillment of the requirements for the

Degree of

MASTER OF SCIENCE

By

TIA WILLIAMS
Norman, Oklahoma
2019

CONCEPTUAL DIFFERENCES OF RUME AND NON-RUME STUDENTS

A THESIS APPROVED FOR THE
DEPARTMENT OF MATHEMATICS

BY THE COMMITTEE CONSISTING OF

Dr. Milos Savic, Chair

Dr. John Albert, Co-Chair

Dr. Sepideh Stewart

To Matt:

Thank you for always being my rock. Your love and encouragement have given me the strength to make our dreams a reality. The finishing touches have finally been applied.

Love you bunches.

Acknowledgements

Dr. Milos Savic,

Thank you for taking a chance and mentoring me. You have been the most kind and understanding person since my first day of grad school. Thank you for all the support throughout this journey. None of this would have been possible without you and for that I am forever grateful.

Table of Contents

List of Tables

Chapter 1: Introduction

How do people read RUME papers? This may be a difficult question to answer, because "mathematics education research involves the application of the methods of an established discipline that research can come complete with that discipline's jargon and be as opaque as technical discussions often are to 'non-specialists'" (McKnight, 2000. P. viii). In investigating this large question, I wanted to ask a more specific question: Do non-RUME students and RUME students have conceptual differences when reading mathematical education research papers? I define conceptual differences using Tall and Vinner (1981) concept image/concept definition framing. I believe this is important to the RUME community as it is necessary for those individuals to be able to read and understand the ideas that are being portrayed. Mathematics education research, carefully conducted, is something far more fundamental and widely useful than might be implies by its use by the advocates of innovation in undergraduate mathematics education (McKnight, 2000, p. vii). Another value to the RUME community is that it illustrates that not everyone reads and construes research papers identically. What one person may interpret may not be what another person takes away. This should be noted as researchers should be able to present the material in a way that many readers can understand it. Finally, I believe this research is very important to the RUME community because if RUME researchers are the only people that we are reaching, then we are just keeping the community small and not allowing room for growth.

Chapter 2: Background Literature

As students' progress through the college mathematics curriculum, enter graduate school, and eventually become practicing mathematicians, reading mathematics textbooks and journal articles appears to become easier and lead to increased proficiency and understanding (Shepherd et al., 2014). One's level of education could be a contributing factor. Definitions play a pivotal role in mathematics. Research on students' understandings of mathematical definitions reveals that learners encounter different types of obstacles. According to Vinner (1991), serious difficulties in comprehending definitions can be attributed to the dichotomy that exists between the structure of mathematics as conceived by professional mathematicians and the cognitive processes involved in concept acquisition by learners (Parameswaran, 2010, p. 43). In contrast to reading mathematics, many of the words that are used in RUME articles do not have formal definitions attached to them. Definitions in mathematics education do not have, or perhaps cannot have, mathematical precision. Concepts may only be approachable rather than precisely definable (Selden and Selden, 1993, p. 432). This makes some concepts more difficult for the readers to understand.

Parameswaran did a study on twelve mathematicians over their approach to understanding mathematical definitions (2010). In this study, the researcher found that mathematicians said that it is important to have a good amount of examples and theorems when learning a new concept definition. Once they have a clear understanding of the examples and theorems, they then form their own concept images with examples. This is followed by a rigorous concept definition. Mathematics education instructors spend more time thinking about what is going to happen in their classes. Vinner (1991) did a study in an upper division geometry course and he spent

the first twenty minutes of his class discussing concept images versus concept definitions. In this study, his students would come to him and state their concept images and ask how to make it more clear. More clear indicating that the students wanted to learn to form a concept definition. Examples were less of a contributing factor. It can be seen from these two studies that mathematics education and mathematics differs.

When reading, good readers will engage in strategies such as summarizing the paragraph that they just read, relating what they read to their prior knowledge, and predicting the direction in which they think the argument is heading (Weber et al., 2008, p. 4). These strategies serve two purposes. The first is to foster comprehension by allowing the reader to construct an interpretation of the text that is meaningful to them (Weber et al., 2008, p. 4). The second is to monitor comprehension; if the reader is unable to execute a metacognitive strategy (e.g., if they are unable to summarize a paragraph or an argument takes a turn that they did not expect), this serves as a cue to the reader that comprehension is not proceeding smoothly and remedial action, such as re-reading the text or seeking clarification, is necessary (Weber et al., 2008, p. 4).

Understanding is an in-the-moment cognitive state of equilibrium that results from (successful) assimilation to a scheme. Understanding is knowing. Knowing-that something is true, knowing-how to do something, knowing-why you do something or why something is true, and knowing-to do something in a particular situation (Ryals and Keene, 2017). Understanding how to create knowledge, maintain it, and put it to use is an issue that is concerning to both students and professors. That is why it is of great importance for the researcher to portray information clearly. Meaning refers to the

actions and schemes that an individual anticipates or enacts in the moment of understanding (Moore, LaForest, and Kim, 2016). Understanding the meaning of certain concepts makes reading scholarly articles and papers easier. Understanding mathematical papers can be very difficult. According to a study done by Alcock and Simpson (2002), students generally do not consult definitions to resolve conflicts because they do not understand wither the relevance or the importance of the definitions (Parameswaran, 2010, p. 45). This has repercussions for both professors and educational researchers. One of the repercussions is that mathematical professors and educational professors need to understand how professional mathematicians view mathematical definitions and what processes they use when they attempt to understand such definitions. They are often very complex and compacted with an unwavering amount of information.

Understanding the theoretical constructs of a mathematics education paper can be very difficult as there are many different ones. Once this understanding has been established by the reader, one can usually have a better insight as to what the author(s) are trying to depict. Students often abandon a particular concept if they don't have a clear understanding. They often feel as if they have already failed at this particular notion so why should they even put forth any more effort. Reading a paragraph over and over and still not having an understanding is often limiting. "Literary understanding is both limitless and constrained – endlessly open to unresolvable interpretative conflicts, but also bounded as legitimate readings can be distinguished from fallacious ones" (Armstrong, 1990, p.1). This is how misconceptions and misinterpretations come about. Making assumptions about definitions of words in any

field creates unique opportunities, but also obstacles, for interpretation. Therefore, concepts without understanding lead the readers to conceptual differences that may also be different from the intended conceptual definition (Tall and Vinner, 1981, p. 152). Therefore, my research questions are:

- What exactly is a RUME conceptual difference according to graduate students?
- What are the ways that graduate students read and interpret RUME?
- What are the differences between RUME and non-RUME students in understanding theoretical constructs?

Chapter 3: Theoretical Framework: Tall and Vinner

In this paper, we will utilize the constructs of Tall and Vinner's (1981) concept image and concept definition to analyze students' notions of the meanings of normative, social norms, sociomathematical norms, and conceptual differences. According to Tall and Vinner, a concept image describes the total cognitive structure that is associated with the concept, which includes all the mental pictures and associated properties and processes. It is built up over the years through experiences of all kinds, changing as the individual meets the new stimuli and matures (Tall and Vinner, 1981). A concept image is something that is developed within the person's mind.

A concept definition is a written description that explains a concept (Tall and Vinner, 1981). A concept definition is the definition that is generally accepted in the mathematical community (Tall and Vinner, 1981, p.152). When a student is given a formal concept definition, the concept image that the student forms might be weak, leading the student to a conceptual difference. Tall and Vinner noted this in their study using the "all" and "some" quantifiers in limits and continuity. They noted that the students had strong mental pictures but weak concept definition images.

The Tall and Vinner (1981) framework of concept definition and concept image have been used in several papers. Wawro, Sweeney, and Rabin used Tall and Vinner's idea of the concept definition to analyze students' notion of a subspace in Linear Algebra. Rösken and Rolka (2007) used the idea to analyze students' conceptual learning regarding the notion of the definite integral. This idea of a concept image and concept definition can be utilized in many different ways. It has been used to analyze the process of the learning of a function in high school, limits and continuity, as well as many other areas.

Chapter 4: Methods

4.1 Participant Information

For this study, six participants were chosen: three RUME students (Pancho, Mary, and Ellie), two non-RUME students (Dakotah and Mary M.), and one student that did not identify as either (Jane). In order to keep anonymity, each participant has chosen their own pseudonym and gender pronoun. Five of the six participants have taken an introductory RUME class or has attended the RUME seminar. The introductory RUME class was not a mandatory class for the non-RUME students.

4.2 Data Collection

The researcher used a paper written by Rasmussen, Wawro, and Zandieh (2015) titled "Examining individual and collective level mathematical progress." The students were asked to read this paper and were given a set of multi-colored highlighters with specific instructions. The instructions for the highlighting were to highlight the research questions in pink, the answers to the specific research questions in blue, definitions/keywords found in the paper in green, and any information they find interesting in yellow. They were given a week to read and highlight the paper. The

participants were also given questions 4-9, listed in Appendix A, to think about while reading the research paper. Once the participants had completed the task, the researcher scheduled the interview to be done. At the time of the interview, the researcher collected the papers with the highlighted information. They were asked a series of questions that are listed in Appendix A. This was done in a room in the Physical Sciences Center on campus. This interview was recorded using a SONY voice recorder. The researcher then transcribed the interviews and promptly de-identified the data to prepare for the data analysis process.

4.3 Data Analysis

Once the data had been transcribed and de-identified, the responses were then coded using Tall and Vinner's theoretical framework of concept image and concept definition along with initial coding. Initial coding (Saldaña, 2013, p.100) was used as there has not been any previous studies done on this subject. This allowed the researcher to see patterns and make her own conclusions using the data rather than having a forced conclusion.

Chapter 5: Results

I split the results into subsections of responses to questions about definitions of five terms in the Rasmussen, Wawro, and Zandieh paper (2015). These questions are in Appendix A.

5.1 Normative

The researcher asked the following question during the interview:

"On page 262 of the paper, it states that classroom mathematical practices refer to the normative ways of reasoning that emerge as learners solve problems, explain their thinking, represent their ideas, and so on. It then goes on to state that normative means that there is empirical evidence that an idea or way of reasoning functions as is it is a mathematical truth in the classroom (Rasmussen, Wawro, and Zandieh, 2015).

How would you determine what normative means?"

The table provides the participants answers given during the interview.

Participant	Response	Example (if given)
Mary (RUME)	If you were looking at the entire population in that particular classroom and you would say that something occurred more frequently than the average.	If 18 out of 20 people use pens instead of pencils every day.
Mary M. (non-RUME)	I think normative refers to the norm in that classroom.	If you go into different classrooms, even within the same course and subject matter, there's going to be different things and different emphasis that each of the instructors are going to have and then also different emphasis that the students are going to have.
Jane (neither)	Normative means there's empirical evidence. It's something they have gone through or through the process of the course, something has evolved in the classroom that has become like a classroom norm. It is a standard or this is the truth for the mathematics they are working on.	No example given.
Pancho (RUME)	How the classroom engages in conversation and once it becomes something adapted by the class.	No example given.
Ellie (RUME)	Things that become norms in the classroom.	Building models in Business Pre-Calc and relating those to what they have learned and will learn with future concepts.
Dakotah (non-RUME)	In the classroom, each student has shown that they have obtained their own version of mastery of material including why it is the way it is as well as the formal mathematical definition of it.	Span of the set of vectors.

Table 1: Normative responses

The formal concept definition was stated in the question here. All of the participants gave their concept images. The formal concept definition stated that normative is that there is empirical evidence that an idea or way of reasoning functions as if it is a mathematical truth in the classroom. As you can see, only two (Dakotah and Jane) of the six participants even cited the word "math" in their response and only two (Dakotah and Ellie) gave a mathematical example with their response. Jane was the only one who somewhat rephrased the given concept definition. Half of the participants (Jane, Ellie, Mary M.) referred to normative as being the norm in a classroom with no mention of mathematical truth. All of the participants stated the word class or classroom in their answer.

The participants were asked to read and highlight their papers and return to the researcher. Upon reviewing the participants highlighting, all of the participants have highlighted this portion of the paper in green which was used to highlight definitions. This tells the researcher that as they were reading the article, that they did in fact recognize this as a concept definition.

5.2 Social Norm

The researcher asked the participants to answer the following question:

What is a social norm?

The concept definition of a social norm was not given in this paper. However, the authors did state "details on how the constructs of social norms, sociomathematical norms, and associated beliefs are operationalized can be found in Yackel and Cobb (1996)" (Rasmussen, Wawro, and Zandieh, 2015, p. 260). As a RUME researcher, it would be expected of the reader to refer back to the citation to expand on these concepts. This would also be the case if you were stuck on a mathematical problem and

needed to refer back to a previous concept. Yackel and Cobb do not give a concept definition of a social norm in their paper either. However, they do give various examples of social norms. Yackel and Cobb do state "that understanding that students are expected to explain their solutions and ways of thinking is a social norm" (Yackel and Cobb, 1996, p. 461). The table below (table 2) are the responses given by the participants when asked "what is a social norm?".

Participant	Response	Example (if given)
Mary (RUME)	The way you behave is consistent with others.	*If I go to the Union, I just know to go to the line because everybody goes to the line. We all go to the line to get food. *In math education or in math, a social norm would be doing my homework because everybody is doing their homework or that everybody is sitting in a classroom and it is really quiet and the professor asks a question and no one else answers, I'm not going to say anything either.
Mary M. (non-RUME)	To me, a social norm is I guess you could say a culture. So, the way people act, the way that people talk, the things that are acceptable. Ways that we have all accepted that this is how we are going to act.	Whenever you go to the beach or a swimming pool, it is accepted that you are going to wear a swimming suit. Whereas, if you are going to work in a professional setting, like if you are teaching a class, you are not gonna wear a swimsuit.
Jane (neither)	A social norm is something that is an agreed upon practice or something that everybody deems "normal". It's just something that a group of people have agreed upon as being as the normal thing or the norm. It's like an agreement between every member of society that we are gonna all think and say	It's a social norm to have men and women's bathrooms and now the change is coming about, and it's a social norm to have non-gender specific bathrooms.

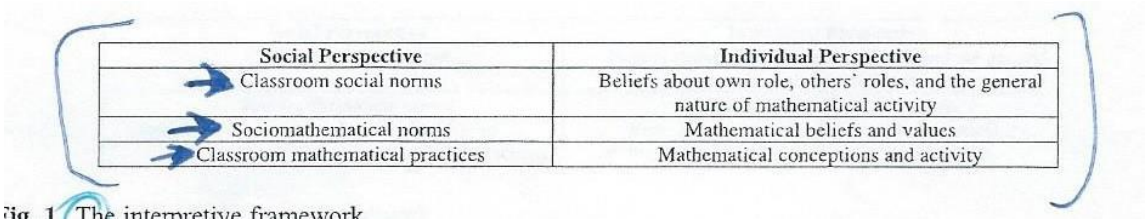
	that this is okay and that we are all going to agree to what the thing is.	
Pancho (RUME)	A norm that relates to how we interact in class. Different roles that people play	Not given.
Ellie (RUME)	Social norms are what the usual social interactions that are happening in your classrooms… well in general, but I am usually thinking about the classroom setting.	A lot of students have a social norm that they sit in the same seat in all of their classes.
Dakotah (non-RUME)	A social norm is to me sort of how society perceives a given topic whether it be like here we are talking about mathematics, but a social norm is sort of the general stigma that's placed on whatever the concept is.	Not given.

Table 2: Social Norm responses

Two students (Pancho and Ellie) refer to a social norm as ways that people interact in classrooms. Pancho then says, "well in general, but I'm usually thinking about the classroom setting." Four of the six participants gave examples with their responses. Two students (Mary and Mary M.) related to a social norm as a type of behavior.

As far as the highlighting for this concept, it would not have a specific concept definition. This would eliminate the use of the green highlighter. None of the participants had highlighter marks or notes around the area where Rasmussen et al. indicated that the detail on how the constructs of social norms, sociomathematical norms, and associated beliefs are operationalized could be found in the Yackel and Cobb (1996) paper. However, the Rasmussen et al. paper did have a small chart at the bottom of page 260 that gave a very brief description of their interpretive framework. It has a two-column chart with the first column being the social perspective column, it had

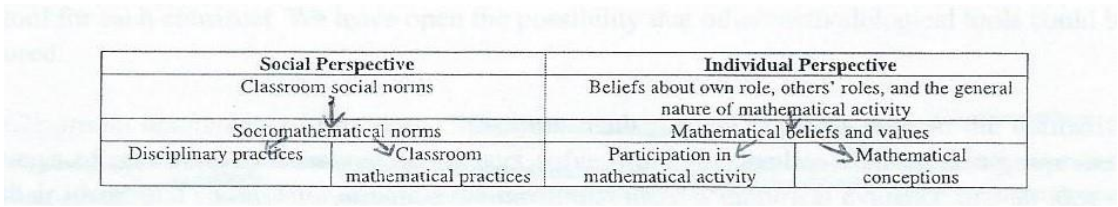
three rows, classroom social norms, sociomathematical norms, and classroom mathematical progress. In the individual perspective column, it had a brief description as to what each one of the social perspectives were. Pancho had a set of brackets drawn around this in blue pen with blue arrows pointing at each of the three social perspectives (shown below).



Social Perspective	Individual Perspective
Classroom social norms	Beliefs about own role, others' roles, and the general nature of mathematical activity
Sociomathematical norms	Mathematical beliefs and values
Classroom mathematical practices	Mathematical conceptions and activity

Fig. 1 The interpretive framework

Dakotah did highlight, in yellow, the individual perspective for classroom social norms. The yellow highlighter was used to highlight items that the reader found interesting. These were the only two participants that had any type of highlighting or notes indicating that they looked at this particular box of concepts. On page 261 of the Rasmussen, Wawro, and Zandieh paper, the authors also broke the first chart (the chart from page 260) down a bit further. The authors separated the classroom mathematical practices into two categories: Disciplinary practices and classroom mathematical practices. Mary M. did highlight this in blue indicating this chart as an answer to a research question. Mary drew arrows in black pen as shown below.



Social Perspective		Individual Perspective	
Classroom social norms		Beliefs about own role, others' roles, and the general nature of mathematical activity	
Sociomathematical norms		Mathematical beliefs and values	
Disciplinary practices	Classroom mathematical practices	Participation in mathematical activity	Mathematical conceptions

Pancho wrote How? above this chart in blue pen. This is an indication that these participants did look at this chart and put some sort of thought into it.

5.3 Sociomathematical Norm

As stated with the social norm, Rasmussen et al. referenced the Yackel and Cobb (1996) paper for sociomathematical norms. Yackel and Cobb once again do not directly give a concept definition of a socio-mathematical norm in their paper. However, they do directly state, "normative understandings of what counts as mathematically different, mathematically sophisticated, mathematically efficient, and mathematically elegant in a classroom are sociomathematical norms" (Yackel and Cobb, 1996, p. 461). They also gave various examples. One of these referred to a sociomathematical norm as normative aspects of mathematical discussions that are specific to students' mathematical activity (Yackel and Cobb, 1996, p. 458).

The researcher asked what is a sociomathematical norm directly after asking what a social norm was. The responses are listed in Table 3.

Participant	Response	Example (if given)
Mary (RUME)	I do not know. Your behavior is consistent with others behaviors, but in this case, your behavior is more mathematically oriented.	Not given.
Mary M. (non-RUME)	The way that culture in whatever mathematical setting has developed. They way that is acceptable for students to talk about mathematics or the language that they use. A lot of the terminology would be a part of that and then depending on whether the classroom is lecture based or more interactive based.	If students feel like they should speak up or whether they should be active learners or passive learners.
Jane (neither)	The same as a social norm except it is in the context of mathematics.	Not given.
Pancho (RUME)	Related to the discourse of the classroom, how you engage talking about mathematics.	Not given.
Ellie (RUME)	Still a social norm, but now throw in how they need to be	A sociomathematical norm I try to establish in my

	understanding mathematics; How they interact with one another about mathematics.	classroom is that I'm gonna give you guys 60 seconds to try and start this problem.
Dakotah (non-RUME)	How the general public or the general society would perceive some mathematical topic that we are talking about.	Not given.

Table 3: Sociomathematical Norm Responses

Two-thirds of the students above mentioned that a sociomathematical norm was how mathematics was expressed. Only one-third of the students gave an example with their answer. Every participant did mention mathematics in their answer.

This concept highlighting would coincide with the information for a social norm as neither of these concepts had a concept definition given with them. This concept was referenced to a paper written by Yackel and Cobb (1996), but a true concept definition was not supplied in this paper either. No highlighting for this concept is noted.

5.4 Theoretical Construct

Rasmussen, Wawro, and Zandieh talked about four theoretical constructs in their paper. Two constructs are associated with the collective mathematical progress and two constructs are associated with the individual mathematical progress. The concept definition is not directly stated, but the authors do make several inferences as to what it is. They talk about how each construct can be viewed differently and, in each section, and they address the specific tools that they used.

Table 4 provides the participants answers when asked, “what is a theoretical construct?”

Participant	Response	Example (if given)
Mary (RUME)	A field's construct for a particular idea or item.	Anxiety or fear or depression
Mary M. (non-RUME)	The ways in the class that students are coming up with the norms and the way the teachers are enforcing those norms.	Bringing up a question and having a discussion and maybe students disagreeing and all the things in the background that are working to lead students to those norms.
Jane (neither)	Reference the framework and the theory behind it. Theoretical framework behind how they are assessing these specific things. It's the cumulation of knowledge that they have gathered from reading other people's papers and everything to come about like figuring out they are going to make this assessment and how they are going to define what a norm is in the classroom.	Not given.
Pancho (RUME)	Not given.	Not given.
Ellie (RUME)	Ideas that help explain a theory. Some way to explain a phenomena that you think is happening in the world.	Explanation or diagram
Dakotah (non-RUME)	Breaking down a general concept into two sub-categories… the formal definition and you have why…of course they are going to overlap, but I do believe they are separate entities in that construct.	Not given.

Table 4: Theoretical Construct Responses

Fifty percent of the students referred to a theoretical construct as a way to reference a theory, an idea, or a theoretical framework. Mary M. and Jane referred to this concept as being related to a norm in the classroom.

The concept definition for a theoretical construct was not supplied in the Rasmussen, Wawro, and Zandieh paper. On page 262, the authors state that they

provide further background on the theoretical constructs and associated methods for making sense of collective and individual mathematical progress. Rasmussen, Wawro, and Zandieh included the following four constructs in their study: Collective mathematical practices, disciplinary practices, mathematical conceptions, participation in mathematical activity.

The concept definition for classroom mathematical practices was given in the Rasmussen, Wawro, and Zandieh paper. Classroom Mathematical Practices refer to the normative ways of reasoning that emerge as learners solve problems, explain their thinking, represent their ideas, and so on. Pancho, Mary, Dakotah, Mary M., and Jane highlighted this concept definition using the green highlighter. Ellie did not highlight the concept definition of classroom mathematical practices but did highlight the concept definition for normative.

The concept definition for disciplinary practice was given in the paper that the students were asked to read on page 264. Disciplinary practices refer to the ways in which mathematicians go about their profession. All six participants recognized this concept definition and used the appropriate green highlighter to do so.

The concept definition for mathematical conceptions was given on page 265 of the Rasmussen et al. paper. The concept definition of a mathematical conception according to the authors is: “as students solve problems, explain their thinking, represent their ideas, and makes sense of others’ ideas, they necessarily bring forth various conceptions of the ideas being discussed and potentially modify their conceptions.” Fifty percent of the students (Jane, Pancho, Mary M.) highlighted this using green.

The concept definition for the fourth construct, participation in mathematical activity, was not given. The authors made a reference to the Krummheuer (2007, 2011) study. Krummheuer's characterized individual learning as participation within a mathematics classroom using the constructs of production design and recipient design (Rasmussen, Wawro, Zandieh, 2015, p.266). Rasmussen et al. go on to explain Krummheuer's ideals of both production design and the recipient design. Jane highlighted the Krummheuer reference in yellow highlighter which was used to indicate something that the reader found interesting. Dakotah and Mary M. highlighted the reference in green which would indicate that it was a definition. The other three students (Ellie, Pancho, and Mary) did not highlight the reference in any color.

5.5 Conceptual Differences

This concept definition was not given to the students nor was it available in the article that they were asked to read. The students were given this question to think about while they read the Rasmussen, Wawro, and Zandieh paper.

Table 5 provides the participants responses when the researcher asked, "what is a conceptual difference?"

Participant	Response	Example (if given)
Mary (RUME)	Referenced another author: relational and instrumental learning. *Relational is synonymous with procedural…Knowing what to do and why.	Not given.
Mary M. (non-RUME)	A difference in concept.	Not given.
Jane (neither)	Different ways of thinking about similar topic. I think it is when two people are talking about a similar topic or something, but they have a different understanding of that concept. They are ultimately	In the context of linear algebra, some of the ways they looked at vectors.

	driving towards the same idea or the same thought but their understandings of it are just different.	
Pancho (RUME)	*How you cognitively approach something *How you perceive a concept.	Not given.
Ellie (RUME)	What the difference between two schemas. What are the links between the two schemas are what makes them related but what are those things that are missing that are making them different.	Right now, I am teaching on exponentials and logarithms and I think of those as two different schemas.
Dakotah (non-RUME)	When two people are talking about a given topic and their ways of explaining it to themselves and to others may differ in some capacity whether it be a fundamental difference. Maybe their wording is slightly different, but it characterizes the same thing.	Not given.

Table 5: Conceptual Difference Responses

Fifty percent of the students related a conceptual difference to a type of relation between two things, whether that had been a person or schema.

No highlighting results to be noted as this was just a general question asked by the researcher. However, the participants were given this question to thin about as they read this article. Mary wrote a note on her paper that referenced another author. Pancho wrote, “difference in how you cognitively approach something and difference in how you perceive a concept.”

Chapter 6: Discussion

During the interview, the researcher asked the students what things, such as highlight or making notes in the margins do you do when you read educational articles? Four out of the six students (Jane, Mary M., Pancho, and Ellie) stated that they usually

highlight their papers when they are reading them. However, of these four students, Pancho is the only one that uses the given system regularly. The other three students (Ellie, Jane, and Mary M.) just use one or two colors and make annotations in the margins of their papers. Mary and Dakotah do not use a highlighting technique. Mary uses an electronic device to read her papers, so she usually just makes notes in the margins on the paper on her screen. Dakotah stated that he has never really done any highlighting in any note-taking capacity. He stated that he used a self-explaining technique. He usually read about a paragraph at a time, and then, after each paragraph, he verbally summarizes what he had just read.

The researcher also asked the participants if the found any benefits from highlighting the given paper. Overall, the students thought that highlighting the given paper with specific colors was a useful tool. Five out of the six students thought that it made it easier for them to go back and reference terms. One student (Pancho) stated that it helped him to organize. Pancho is also the only student that uses this technique on a regular basis. One third of the students found highlighting the paper useful as it made then read the paper more carefully and allowed them to reinforce whatever they had just read. Jane stated, "that chances are if I'm highlighting it, I'm reading over it again as I'm highlighting it." Mary stated, "I don't usually highlight in general, but I did find highlighting in this case did help because you know you can go back and pick out the definitions easily because they are all one color."

The responses from table 1, regarding the normative responses, tell the researcher that even though the concept definition of normative is clearly given, that each reader may have a different interpretation of the concept. This is important for

researchers to think about because we need to make sure that we unambiguously portray what we want our readers to get from each concept. As information recipients, we need to make sure that we are carefully reading and understanding what is trying to be shown.

The concept definition of a social norm was not given in the paper the participants were asked to read. However, Rasmussen, Wawro, and Zandieh did give a reference to Yackel and Cobb (1996). Did the students try and go find the Yackel and Cobb (1996) paper to gain a greater understanding of the terms? Did the participants notice that the Yackel and Cobb (1996) paper did not define these terms or really give a clear concept definition either? These are both questions that cannot be answered in this study.

Although the concept definition of a sociomathematical norm was not directly given in the paper they were asked to read, there was a chart that stated that sociomathematical norms are mathematical beliefs and values. The students would have needed to refer back to the citation that was given in the paper that they were reading to try and gain a better understanding of what a sociomathematical norm is. Although the Yackel and Cobb (1996) citation was given, it doesn't appear that the students referred to that paper to gain a greater understanding of a social norm or a sociomathematical norm. As a researcher, how much information should we be required to offload? This would also apply to a mathematical classroom.

John and Annie Selden wrote a paper on unpacking mathematical statements. In their paper the Selden's said, "unpacking (the logical structure of) an informal statement we will mean associating with it a logically equivalent formal statement including the

logical features that are understood by convention, rather than explicitly expressed, in the original statement" (Selden and Selden, 1995, p.128). When students are reading research papers, how much information are they "unpacking?" How much information should a researcher need to "unpack" for the students? Conceptual differences can occur during this "unpacking" process. How do you know when you are finished "unpacking?" Does this indicate the difficulty of the concept?

It is important that the students understand theoretical constructs as this leads to a greater understanding of the information that is attempting to be represented. The fourth construct, participation in mathematical activity, showed that there is a conceptual difference between students. Two of the students referred to this as a concept definition, one student thought it was just an interesting piece of information, and three students did not highlight at all. For the concept definition of classroom mathematical practices, Ellie was the only student that did not highlight.

Conceptual differences are important for researchers to think about as they construct their research. Researchers need to be cognizant when they are writing their papers – they must be very clear as to what they would like the reader to take from their articles/papers.

Chapter 7: Response to Research Questions

Possibly reading RUME at a person's beginning stages is similar to learning and reading math. Beginning RUME students can often struggle when reading undergraduate mathematical education papers. The language is different. The concept definitions are not precise as they can be interpreted differently by each reader leading

to conceptual differences. The more RUME papers one reads, the more "in tune" one gets to the language of the RUME community. In mathematics, it is intended that everyone who reads the definition of a concept will have essentially the same basic understanding of the concept definition. Different individuals' concept images are likely to differ, but everyone should be able to agree on whether an example satisfies the concept's definition (Shepherd, Selden, and Selden, 2012, p. 230). It appears that for many students, a major factor in effective reading is sensitivity to their own confusion and errors and appropriate response to them (Shepherd, Selden, and Selden, 2012, p. 242). Hopefully this study is beneficial to educators so they can format their writing style to fit the needs of the reader which may lead to fewer conceptual differences between the different fields of study. For RUME educators, it is important as it allows them to see how other communities of people read and interpret their work.

My goal in doing this research is to show whether there was a difference in comprehension between RUME students and non-RUME students. The data suggests that there is a conceptual difference between this group of RUME and non-RUME students and how they interpreted this article. This was shown using concept definitions and concept images. Every participant had difficulties when describing normative when the concept definition was given, every participant had their own concept images pertaining to social norms, sociomathematical norms, conceptual differences, and theoretical constructs.

Chapter 8: Future Research Questions

Does the difficulty level of the article contribute to conceptual differences? Articles that require extensive outside research may be difficult for many in their

beginning stages of RUME. During the interview process, the participants made the researcher aware of some of their difficulties. For example, Pancho did not like the way Rasmussen et al. had their paper organized and Ellie stated that there were a lot of intimidating words in the paper, and she took a whole hour to read just the theory portion as she was just trying to understand the paper.

In my experience, if a student has difficulty understanding a concept then they are more likely to just skim that portion of the paper and move on. The students' interest or lack of interest may contribute to conceptual differences. If students are forced to read papers, especially those that are not related to their field of study, then I feel as if they are not reading them to their full potential.

The students were asked if they felt as if they read and interpreted educational articles in the same manner that their colleagues do. Their responses were all very similar. Four of the six participants stated that they thought that they interpreted these educational articles differently than their colleagues. Dakotah was the only one that has a split opinion. He stated that he felt he interpreted the articles the same when it was relating to his non-RUME colleagues and differently when he was talking to his RUME colleagues. Pancho stated that he thought that he did interpret things the same way.

Do non-RUME students have a greater difficulty reading these articles because they are not as likely to need them for their dissertations?

If a student is doing research in an area, then they obviously have some interest in the subject matter. A RUME student will possibly have a greater understanding as to why they are being asked to read the articles versus non-RUME.

What can we as researchers do to change these conceptual differences?

What can we do to portray the information so that it is understood by more people in the mathematics community?

At what point of the "unpacking" process do the students start to have conceptual differences?

This last question is a hard question. I asked myself this several time while working on this thesis. At times, papers have a great deal of information in them, especially in the theoretical construct area. So, when the researcher is writing, I feel as if we should always be aware that not everyone has the same background. Is there a certain "unpacking" point that students get lost when reading certain papers?

Chapter 9: Conclusion

This study indicates that there are in fact conceptual differences between RUME and non-RUME students. Although this was a small study, it has results that I believe are important to the RUME community. There were conceptual differences with every term. At most, 66% of the students agreed upon one particular thing at a time. Sixty-six percent of the students agreed that a sociomathematical norm had something to do with math. Albeit this may lead is to believe that this is a difficult concept that may need some further researching that they did not have the appropriate amount of time to do so. When the concept definition was actually given, every student gave their concept image. Only one-third of the participants even mentioned the word math in their response even though the definition clearly stated that normative means that there is empirical evidence that an idea or way of reasoning functions as if it is a mathematical truth in the classroom (Rasmussen, Wawro, and Zandieh, 2015).

This study contained some limitations. One of the limitations was the length of the given paper. The Rasmussen et al. (2015) paper might have been rather lengthy (24 pages single-spaced) and full of theoretical constructs. Another limitation was that the participants were all graduate students who volunteered their time for this study. They were asked to read and highlight a paper and then take time to have an interview with the researcher. This was in addition to all of their other duties including their own coursework, teaching classes, grading papers, and tutoring in the math center. If the students had more time, they may have gotten a little deeper into the paper and had a better understanding of some of the terms. Lack of interest was another limitation. Some of the students were not interested in the given material as it did not go along with their current research.

For future research, I strongly believe that the participants would need more time to complete the task to allow for further research of the theoretical aspects. I think it would also be interesting to see if the results differ when the material is more aligned with students' interests.

References

Alcock, Lara, and Adrian Simpson. “Definitions: Dealing with Categories Mathematically.” *For the Learning of Mathematics*, vol. 22, no. 2, 2002, pp. 28–34. *JSTOR*.

Armstrong, Paul B. “Conflicting Readings: Variety and Validity in Interpretation.” *Choice Reviews Online*, vol. 27, no. 10, Jan. 1990

McKnight, Curtis C. *Mathematics Education Research: A Guide for the Research Mathematician*. American Mathematical Society, 2000.

Moore, Kevin C., et al. “Putting the Unit in Pre-Service Secondary Teachers’ Unit Circle.” *Educational Studies in Mathematics*, vol. 92, no. 2, 28 Dec. 2015, pp. 221–241.

Parameswaran, R. Expert mathematicians’ approach to understanding definitions. The Mathematics Educator, vol. 20, no. 1, 2010, p. 43–51.

Rasmussen, Chris, et al. “Examining Individual and Collective Level Mathematical Progress.” *Educational Studies in Mathematics*, vol. 88, no. 2, 2014, pp. 259–281.

Ryals, M., and K. Keene. “A Success Factor Model for Calculus: The Relative Impact of and Connections between Factors Affecting Student Success in College Calculus.” Proceedings of the 20th annual conference on Research in Undergraduate Mathematics Education, San Diego, CA, 2017, pp. 871–878.

Saldaña Johnny. *The Coding Manual for Qualitative Researchers*. SAGE, 2016.

Selden, Annie, and John Selden. “Collegiate Mathematics Education Research: What Would That Be Like?” *The College Mathematics Journal*, vol. 24, no. 5, 1993, pp. 431–445.

Selden, John, and Annie Selden. “Unpacking the Logic of Mathematical Statements.” *Educational Studies in Mathematics*, vol. 29, no. 2, 1995, pp. 123–151.

Shepherd, Mary D., and Carla C. Van De Sande. “Reading Mathematics for Understanding—From Novice to Expert.” *The Journal of Mathematical Behavior*, vol. 35, 2014, pp. 74–86.

Tall, David, and Shlomo Vinner. “Concept Image and Concept Definition in Mathematics with Particular Reference to Limits and Continuity.” *Educational Studies in Mathematics*, vol. 12, no. 2, 1981, pp. 151–169.

Vinner, Shlomo. “The Role of Definitions in the Teaching and Learning of Mathematics.” *Advanced Mathematical Thinking*, 1991, pp. 65–81.

Weber, K., Brophy, A., & Lin, K. “Learning advanced mathematical concepts by reading text.” Proceedings of the 20th annual conference on Research in Undergraduate Mathematics Education, San Diego, CA, 2017.

Yackel, Erna, and Paul Cobb. “Sociomathematical Norms, Argumentation, and Autonomy in Mathematics.” *Journal for Research in Mathematics Education*, vol. 27, no. 4, 1996, pp. 458–477.

Appendix A: Questions from Interview

1. What pseudonym and gender pronoun would you like me to use for you?
2. Speaking in terms of identifying oneself, how would you define identity?
3. How do you identify, RUME or non-RUME?
4. What is a conceptual difference?
5. Do you feel like you interpret educational readings in the same manner the your colleagues do?
    a. If so, what leads you to this conclusion?
    b. If not, what leads you to this conclusion?
6. What types of mathematical educational materials are you most involved with reading?
7. Would you typically read undergraduate mathematical research outside of the college setting?
    a. Why?
8. FOR RUME participants: Have you read research materials outside of mathematics or mathematics education?
    a. If so, which topics are you likely to read papers from?
        i. Why?
9. What things, such as highlighting, making notes in the margin, etc., do you do when you read educational articles?
    a. Did you find any benefits of the highlighting while reading this paper?
        i. If yes, could you give me some reasoning?
        ii. If no, why?
10. What are the characteristics of a methodological practice?
11. On page 262 of the paper, it states that classroom mathematical practices refer to the normative ways of reasoning that emerge as learners solve problems, explain their ways of thinking, represent their ideas, and so on. It then goes on to state that normative means that there is empirical evidence that an idea or way of reasoning functions as if it is a mathematical truth in the classroom.
    How would you determine what normative means?
12. What are theoretical constructs?

13. What is a social norm?
14. What is a sociomathematical norm?
15. Do you feel like there is a difference between a social norm and a sociomathematical norm?
16. Is there anything else you would like to add?