Name: George Thomas Date of Degree: August 2, 1958

Institution: Oklahoma State University

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Title of Study: A GUIDE FOR THE IMPROVEMENT OF MATHEMATICAL INSTRUCTION IN THE PUBLIC SCHOOLS OF GEORGIA

Pages in Study: 70 Candidate for Degree of Master of Science

Major Field: Natural Science

Scope of Study: This study attempts to characterize in broad outline only, through suggested essentials of course content and other media, what schools teach and on what level they teach it. This is mainly determined by three factors:

1. The nature of the society
2. The nature of the school population and the individual learner
3. The values - that is, the goals to be realized by the school.

These factors have been discussed in some detail in each of the areas necessary for a good functional mathematics program.

Findings and Conclusions: The total high school mathematics program should be reorganized for teaching purposes and those who are currently engaged in the mathematical teaching process should be required to know more about content material of the broad area of the field of mathematics than many teachers do at the present time. Prospective mathematics teachers should seek to broaden their general scientific knowledge as a whole by taking courses on the college level that give a rigorous treatment to the applications and principles of the physical sciences. Mathematics teachers as a whole should seek to find out not only how students learn but how they learn most effectively. With this information on hand the usual emphasis which is being given to practical applications without connecting them with the underlying mathematical principles would be reduced.

It should not be overlooked that this guide is intended to serve as a source of information for all high school teachers of the state of Georgia. However, no set pattern can be established. The details should be tailored to fit the local situation.

ADVISER'S APPROVAL James H. Zant

A GUIDE FOR THE IMPROVEMENT OF MATHEMATICAL INSTRUCTION IN THE PUBLIC SCHOOLS OF GEORGIA

By

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Bachelor of Science

Savannah State College

Savannah, Georgia

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Submitted to the faculty of the Graduate School
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fulfillment of the requirements
for the degree of
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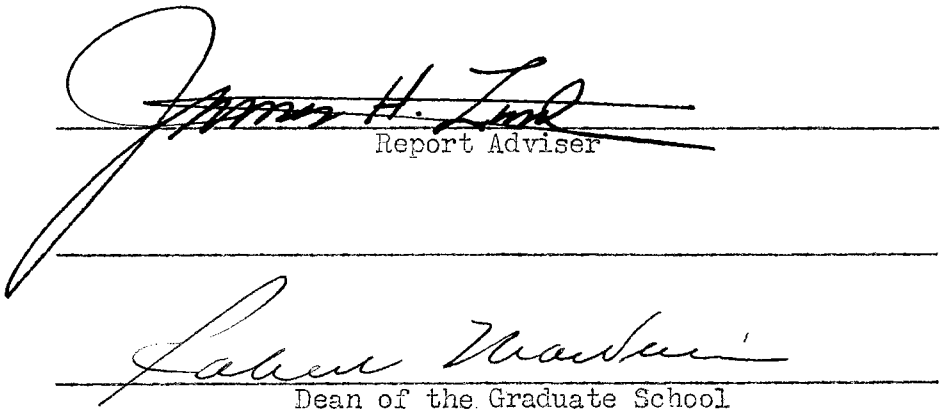
A GUIDE FOR THE IMPROVEMENT OF MATHEMATICAL INSTRUCTION

IN THE PUBLIC SCHOOLS OF GEORGIA

Report Approved:



Report Adviser

Dean of the Graduate School

PREFACE

The majority of the mathematics teachers in the state of Georgia assume the initiative of preparing teaching units for each course taught. Much time and energy are used in this preparation, but the work of the several teachers is disjointed from school system to school system, or from school to school within the same system.

The purpose of this guide is to provide some degree of unity of contents in all the schools of the state so that all students graduating will have been exposed to basic mathematical concepts in the courses completed.

Indebtedness is acknowledged for valuable information received from the College Entrance Examination Board, New York; Dr. Leo G. Provost, University of Utah; Dr. E. C. Kemble, Harvard University; Dr. A. H. Passow, Teachers College, Columbia University; Dr. J. R. Rackley, Pennsylvania State University; Dr. J. Minor Gwynn, University of North Carolina; Dr. Ray H. Simpson, University of Illinois; Dr. K. B. Henderson, University of Illinois, and many others whose names appear elsewhere in this study.

I am particularly indebted to Dr. James H. Zant for his encouragement and continued guidance in the preparation of this report and for making it possible for me to study this school year on a National Science Foundation Grant at the Oklahoma State University.

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CHAPTER I

PLANNING MATHEMATICAL PERSPECTIVES

A. Introduction

An excellent program of mathematical instruction is a factor which sets some schools apart as superior. Yet it is surprising to find that the nature and the quality of the program in mathematics are often overlooked in their efforts to improve instruction. Problems of finance, transportation, and other non-instructional factors receive the major share of attention in a program designed to produce better schools. Perhaps this is because such things as dollars, bricks, and buses are tangible, and people know more about what to do with them than with the intangible elements of classroom instruction. It would therefore seem appropriate to state the problem for this investigation as being to provide suggestive methods of improving mathematical instruction in the public school system of Georgia.

Schools in Georgia need a program in mathematics which will develop the mathematical talents of each pupil. At the present time, many students with outstanding ability do not continue in mathematics, either because courses are not available or because they are not encouraged to continue. Each teacher of mathematics in the junior and senior high schools should have a plan for identifying students with high potential in the field. Courses in mathematics should be provided by all schools for the maximum development of this potential.

One of the purposes of education in a democracy should be to discover students of exceptional ability and to make provision for them in each of the fields of learning. There is therefore a great need for better guidance and improved curriculum offerings in the field of mathematics at the secondary level.

The State Department of Education has felt a need for improving mathematical instruction in the school systems of Georgia for many years and has endeavored to provide encouragement to teachers along these lines by suggesting that each school system work out its own courses of study, providing the assistance of the state instructional supervisor, and by raising the certification requirements of all teachers throughout the state.

Curriculum Framework for Georgia Schools, a guide to curriculum planning, is the only recent publication by the state. It presents a picture of some needs and problems of youth and of American society with which education for social competence has direct concern. The effect of learning experiences on the individual and on groups is portrayed. Up to this time, however, no publication dealing specifically with the improvement of mathematical instruction on the state level has been available.

The writer expects to produce a comprehensive guide of suggested methods for the improvement of mathematical instruction in Georgia schools that may be adopted by the state and issued to any and all mathematics teachers within the state wishing to improve mathematical instruction.

The writer has not given any consideration to the teaching of Modern Mathematics since most of the high school teachers in Georgia

have not been exposed to this type of material.

The writer shall be limited in his presentation since the substance of this study is the result of three short form questionnaires and information from several modern textbook writers and various magazine articles.

The writer proceeded with this study by sending out two distinct questionnaires to specialists in general curriculum planning and mathematics, by reading and citing material from recent textbooks and mathematical magazines, by drawing upon the resources of resident instructors, and by drawing upon his own resources as a scholar, citizen, and teacher of secondary mathematics.

"A Guide for the Improvement of Mathematical Instruction" is a proposed plan, the purpose of which is to introduce mathematics teachers to the most modern concepts of the teaching process by giving some direction to basic concepts.

What the schools teach, when they teach it, and how they teach are determined by three factors:

1. The nature of our society
2. The nature of the school population and the individual learner
3. The values--that is, the goals to be realized by the school

This guide attempts to characterize these in broad outline only. In addition, through units and other media which spell out in some detail each of the areas covered, the guide attempts to bring into proper focus the many factors and inter-relationships influencing the mathematical program. It would be impossible, of course, in a guide of this nature to develop in minute detail all aspects of the total program. However, it is the hope of the writer that local groups in various counties and city systems of Georgia will utilize the guide

to stimulate further thinking and study regarding what seem to be the basic considerations for planning and developing a mathematical curriculum. Some objectives of this guide are summarized as follows:

1. To aid teachers in becoming self-directive
2. To create teacher initiative and resourcefulness
3. To help teachers isolate and analyze their problems
4. To aid teachers in the formulation of a sound educational philosophy
5. To give teachers security and to develop confidence in their ability to solve problems
6. To acquaint teachers with sources of aid in solving problems
7. To provide some criteria for estimating growth in students
8. To introduce mathematics teachers to new material
9. To put the fundamentals of mathematics within reach of every child
10. To help mathematics teachers to define their own problems, to study, to plan, and to act.

From reading and studying the contents of this guide, there should emerge increased discipline as each one connected with the teaching of mathematics in Georgia today realizes the full import of purposes and methods, and envisions new opportunities and duties contributing to life and success of Georgia's youth.

For the inexperienced teacher, this guide can serve to strengthen the organization and enrichment of course content. For the experienced teacher, it can best serve as a point of departure from which, it is hoped, will come more study and experimentation toward more effective and purposeful teaching in various schools throughout Georgia. It is not intended to be a comprehensive treatise on the subject.

No pattern will fit every situation, nor would such a pattern be desirable. This study is not intended as a directive to administrators or to teachers. Such an instrument would be contrary to the democratic principles upon which it is based.

The need to define our educational problems is accentuated more than ever before by current trends in the world and in our own society.

Problems of the aims and values of the American heritage of freedom and their perpetuation need re-emphasis in more valid and functional teaching. The personal adjustment and human welfare of individuals and groups, the vitalizing of moral values, and the solving of problems of living present a challenge to educational workers which traditional educational procedures have been unable to meet. New knowledge of what learning is and how it takes place makes it desirable to restudy our present mathematical curriculum organization. The learner's growth and development as a citizen, as a homemaker, and as a worker are of utmost importance. More emphasis is being placed upon education through democratic processes which achieve the ideals and superior values of American freedom through cooperation and participation.

It is important to note that at present there is a shortage of skilled manpower in all areas which require special talent and training. Greatest publicity has been given to our lack of well-qualified persons in mathematics, science, and the teaching profession. The Interim Report of the Subcommittee on Research and Development of the Congressional Committee on Atomic Energy has accurately stated the most important causal factor of this shortage by pointing out that:

> It is important to recognize at the outset that these serious shortages are not a sign of failure but an indication of startling success. A way of life has been created in which the demands for superior skill in every field have become so tremendous that growth and expansion have become constant elements of the normal pattern. A society has been created in which progress is not simply a matter of occasional spurts and short-time crises. Rapid progress has become a normal part of our national life, and the skilled manpower to keep up with this steadily and rapidly advancing march of progress will be in demand for years to come.[1]

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[1]Bulletin of Atomic Energy. Interim Report on Real Responsibilities of the Scientist. Volume XII (December, 1957). By Sub-committee on Research and Development of the Congressional Committee of Atomic Energy.

In considering high school programs of mathematics, it is necessary to think of all students in light of both the general and special educational purposes for which our schools exist. These purposes include provision of assistance to those who still need help in attaining basic competencies needed in ordinary life situations, education for intelligent citizenship, opportunity to explore and test one's ability and interest, the development of those who must assume the technical leadership of our nation, and provision of adequate preparation for subsequent schooling.

Progress of mathematics may vary considerably in the way the achievement of these purposes is approached, but in any program all students should achieve a level of competence in keeping with their needs, abilities and interests. All students should obtain from the program a general understanding and appreciation of mathematics--its nature, complexities, relation to other disciplines, and its relation to the world in which we live.

There should be conscious effort to develop in students the ability to do their own thinking. This involves the use of skillful technique on the part of the teacher to help the students develop originality in discovering solutions to problems.

These ideals and suggestions by which the process of power of democracy may be enlisted for their achievement are presented here as a unit in several related and envolving chapters.

B. Mathematical Guidance

Whenever a school attempts to meet the mathematical needs of pupils with different abilities and goals through different classes, two questions inevitably arise. Which students shall enroll in the specialized courses? When shall a particular pupil begin them? The mathematics teacher should, as a part of his teaching, aid the student in assembling whatever information may facilitate his decisions.

Many students decide prior to entering ninth grade whether to elect algebra, but it should always be possible for a student who may at a later date develop sufficient mathematical interest and ability, perhaps in some general course, to transfer to the specialized courses.

No single factor should be the exclusive basis for guiding a student into algebra, usually the first of these specialized courses. As many of the following criteria should be used as are feasible: The student's past grades in mathematics, his scores in intelligence, algebra aptitude, arithmetic achievement and reading comprehension tests and interest inventories, his work habits, his maturity and readiness, his own or his parents' vocational plans, his total school program, and the personal estimates made of the student by his teachers and counselors.

The number of failures in mathematics under a particular selection system may be used, inversely, as a rough measure of the success of the student.

Some schools, usually the larger ones, section students according to ability within a particular subject. Others attempt to provide for individual differences within a class.

Probably the best single indication of whether a student should enter more advanced mathematics is his record of interest and achievement in algebra and geometry. But again, other factors should be considered. The student's class rank and more test results may be available. The teacher has the further responsibility at this time of providing information relative to mathematics needed as prerequisites for various occupational fields.

The "Guidance Pamphlet in Mathematics" published by the National Council of Teachers of Mathematics provides a general discussion of such requirements and is intelligible to the student. The teacher must remain alert to new sources of information of this kind. Where guidance of students into mathematics courses is not handled by the mathematics department, the mathematics teacher should take the initiative in furnishing the guidance department with any relevant information.

The utilitarian and vocational purposes for studying mathematics should not obscure its purely intellectual rewards. The student who finds aesthetic values and intellectual challenge in mathematics should be encouraged to enroll whether he plans to enter a technical field or not. In this connection, Dr. James H. Zant of the Oklahoma State University stated:

> The custom prevalent in many of our high schools of putting the bright students in the algebra class merely because they can learn the traditional courses may not be the best for them or for the school. Such practices tend to cut down the efficiency and standards of general mathematics courses and the sequential course may not fit the needs of the student unless he expects to study more advanced mathematics or some of the subjects which demand such knowledge. In other words, this does not solve the problem. It may merely penalize the bright student simply because he has more than average intelligence. The thing to do is to strengthen the general mathematics course so that it will actually meet the needs of the non-mathematical scientific students and point out to the students and their parents that the course has

different purposes and goals. The course should not be for "dumbells" and, while it may not be as rigorously difficult as algebra or geometry, it can, if it is well organized and administered, be just as challenging and for many students much more useful.[2]

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[2]James H. Zant, "What Are the Needs of High School Students?" The Mathematics Teacher, February, 1949, 42:75-78.

CHAPTER II

GUIDES TO CURRICULUM AND CONTENT

A. The Curriculum

Efforts to improve the high school mathematics program have in recent years focused primarily upon making it more appropriate to its professional purposes, more significant to youths, more in line with the principles of growth, development and learning, and consequently more efficient in terms of the returns to society for the money expended.

The successes have been in the formulation of statements of philosophy, the educational needs and direction for progress.

So far, attention has been given to some of the shortcomings of today's typical high school program of mathematics. Consideration has also been given to the nature and effectiveness of various efforts toward improvement. It now seems appropriate to set down some of the principal aspects of a modern high school mathematics program as evidenced by questionnaires sent out to educators and specialists in their fields and the writer's study of textual materials. The questionnaires concerned three distinct groups listed as follows:

I. Educational Curriculum
II. Junior High School Mathematics Curriculum
III. Senior High School Mathematics Curriculum

B. General Statement About Group I

The writer summarized the results. The contents of this questionnaire were such that the writer could not make his summary in terms of quantitative measures. Instead, he read the statements and combined the substance of each to the best of his ability.

Experts of Group I expressed their views on questions listed on the questionnaire on page 12. The following persons were consulted:

Dr. Leo G. Provost
University of Utah

Dr. A. H. Passow
Columbia University

Dr. J. Minor Gwyn
University of North Carolina

Dr. J. R. Rackley
Pennsylvania State University

Dr. Ray H. Simpson
University of Illinois

C. Sample Questionnaire for Group I

473 West Bennett
Oklahoma State University
Stillwater, Oklahoma

Dear Educator:

I am engaged in a study of suggested methods for improving instruction in the Georgia public school systems. Will you please elaborate as briefly as possible and to the point on the following?

A. What is learning and how does it take place?

B. What is a curriculum?
   1. Are they fixed, or do they change?
   2. Is a curriculum for the individual or for society or both?

C. What is the role of the school in these various processes, all of which are proposed for meeting human needs?

Very truly yours,

George Thomas, Sr.

D. Tabular Results of Questionnaires

Group I

Number of questionnaires sent 35

Number of questionnaires returned 5

E. Summary of Questionnaire Results for Group I

We learn what we do: the principle of active response. Professors in college use the familiar slogan, "We learn by doing." There is real truth behind it, although a more correct way of saying it is, "We learn only what we have done." Two important psychological principles are illustrated in this common sense idea. First, it illustrates the principle of association. We learn what we have already done, but our world never quite repeats itself; every human situation is new at least in some respect. Even a factory mechanic, lying on his back and tightening a single nut on a machine as it moves above him on the assembly line, is doing something "new" each time, though relatively the same general situation is involved. Thus, as life goes on, our behavior takes on new and different patterns. As tiny increments of learning take place, strange stimuli become familiar to nerve "signs" or signals for the new way of behaving.

The second psychological idea in the principle of learning by doing is that of action. The more progressive schools are sometimes called "activity" schools because as we have seen, activity is one of their most important characteristics. We know now that we learn by acting, by responding. We do not acquire meanings in some mysterious mystical fashion; we respond with them. We understand new meanings only by making practical responses which are appropriate to them.

The purpose of learning is clear; it is to live better--to behave more effectively in the moment-by-moment situation of life. This involves being skillful where skill is required, thinking clearly when problems are encountered, feeling deeply and with sensitivity in moments

of appreciation, and expressing oneself clearly. And this brings us to one of the tasks to which the good teacher is constantly alert. Moment by moment he studies each teaching situation so that he may adapt what he does to the learners' needs. He knows that no fewer than four distinctive kinds of learning situations must be recognized: skill in understanding the first situations requiring effective habits; creative situations, requiring acts of thought; appreciational situations, calling for acts of deep feelings. He knows also that each of these four learning needs must be met by a separate and special kind of teaching, building skills, developing efficiency in thinking and solving problems, drawing out and enhancing creative abilities, and building appreciative feeling for the aesthetic situation.

The good teacher, then, is constantly on the alert to judge which type of learning is involved at any moment. Is practice called for? Is the pupil blocked by lack of facility in recognizing a problem and solving it? Can his aesthetic awareness be extended at this point? Is there some principle of creative work needed that the teacher can give? To be able to distinguish the type of situation confronted is a step in skillful teaching.

The learning phase of habit and skill involves four types of skill: mental skills, as in arithmetical computation, algebraic and geometrical manipulation, and linguistic habits; motor skills, as in handwriting, typewriting, using tools and machines and instruments; expressional skills, the techniques needed in creative acts; social skills, the habits needed in dealing with people.

Thinking and problem solving requires that we recognize the problem; we confront it directly. Dewey calls it the "felt-difficulty"....the "forked-road situation." It has become a problem; impulsive, habitual

behavior will no longer serve. We must confront it directly with a head-on collision. We must appraise the actions which we are to take, rejecting some, accepting the action which we are to take. These processes are almost concurrent as they flash in swift succession, shot through with the mood, feeling, and emotion of the moment, tangled with meanings, desires, or fears. In the thirty years that have followed the publication of How We Think, this analysis has been made the basis of much of the theorizing about thinking that has been going on in the schools of education and teachers colleges of the country.

Learning and the creative process put the pupil into training for spontaneous planning and designing which comes from within. The teacher's task as an artist will be to work with the raw materials of personality in the young egocentric individualists as they come to the school and to guide each child in becoming a unified whole.

In schools, reading, writing, arithmetic, and speaking are extensions and developments of the child's powers of communications. They are best cultivated where communications will serve children's purposes or open up new kinds of vicarious experiences. Stories heard, told, played, read, or written all serve in this way at one time or another. Functional reading ability is achieved to the extent that reading proves useful and gives pleasure and satisfaction. The child's most natural approach to reading is made when this new medium is used to say, express, or communicate what he already knows in firsthand experience. Familiarity is the key which unlocks the riddle of the mysterious new medium, enlisting more effort, more interest and more success.

Good schools often find ways to prevent the disorders which later remedial work must attempt to cure. Attractive booknooks, handy well-

filled bookshelves and tables, and access to good libraries all help reading to fulfill its possibilities by inviting choice, enlisting voluntary effort and building interests. More studious reference reading follows when the needs and purposes which it serves arise. Good secondary school libraries are becoming an exceptional and essential resource.

Number is useful in living. It is involved in the financial and economic life of the pupil. Activities involving banking systems become functional when the boys and girls find out by opening savings accounts that it is wise for every person to keep his or her own account. When they have their own post office, keep stamps available for pupils who correspond with parents, letter writing becomes functional. To plan who shall take certain responsibilities, to provide sufficient food for the various meals, and to do the necessary chores are also among the experiences which require that children shall have a usable knowledge of arithmetic.

One of the child's most basic mathematical needs is social orientation. Schools recognize and serve this need in various ways. They introduce the child to a larger social group and orient him in many new social relations. They give children an orientation in their neighborhoods and communities, gradually expanding concepts gained in firsthand contacts and using them to make vicarious experience more meaningful.

Such informal approaches are sometimes made to history or geography. History is not a book, nor is it a chronological train of events to be learned for recitation purposes. It is the story of how the present came to be. History, then, is a resource which preserves man's past achievements and puts them at the disposal of subsequent generations as

a social heritage, a starting point for further human advance. The frequent evidences of interest in current history and world affairs in the series of observations which form the basis of this statement are in striking contrast to the former approach to the study of history.

School life meets health needs. If a child is to live a happy useful life, he must also have a healthy life. The responsibility of the school for the child's health varies, of course, with the extent to which the responsibilities are assumed by the parents and the community.

Science is a field of human endeavor which is transforming modern life. It represents a method and is a body of knowledge which children can explore and appreciate as it relates to their purposes and meets their needs. Elementary school science is not a mass of ready-made generalizations to be demonstrated and learned.

The unique values of art, music, and literature are gained from the ways in which they enrich group living and challenge creative endeavor. The cultural treasures which the world's creative personalities have built up through the ages are a part of each child's heritage. They should find their way into the life of every American child by contact and association in school and community. Opportunities for creative self-expression should be more accessible, not only to persons of high potential, but to all children.

If individuals are to live best themselves, they must also live best for the group. It is one of the major responsibilities of the school to give the opportunity to learn to live in a group. The status of an individual in the group is determined in part by the contribution he gives to it, and the success of the group is determined

in part by the contribution it makes. A good teacher guides his experience to include larger groups.

F. General Statement About Group II

Experts of Group II expressed their views on questions listed on the questionnaire on pages 21 and 22.

The writer compiled and summarized this data. However, answers supplied by all experts in this area were closely related.

Members of Group II included:

College Entrance Examination Board
New York

Dr. H. O. Kemble
Harvard University

Dr. Dan T. Dawson
Stanford University

Dr. Robert Kalin
Florida State University

Dr. Charles H. Butler
Western Michigan College of Education

Dr. K. B. Henderson
University of Illinois

Dr. W. M. Polishook
Temple University

Dr. Francis J. Mueller
Maryland State Teachers College

The results of this questionnaire will be found on the pages that follow.

G. Sample Questionnaire for Group II

473 West Bennett
Oklahoma State University
Stillwater, Oklahoma

Dear Educator:

I am engaged in a study of suggested methods for improving instruction in the Georgia public school systems. Please check the items from the following list that, in your judgment, will represent an adequate high school program.

Very truly yours,

George Thomas, Sr.

****

	Course	Grade level (Circle one)		
A.	General Mathematics	7	8	9
B.	Elementary Applied Mathematics	7	8	9
C.	First Year Algebra	7	8	9

From the following list, please check those topics that should be included:

A. General Mathematics

1. Local and community life ( )
2. Systems of measuration ( )
3. The use of decimal numbers ( )
4. Practice in using decimals ( )
5. Approximate computation ( )
6. Commodity costs to the consumer ( )
7. Consumer income and buying ( )
8. Problems of taxation ( )
9. The fundamental operations of arithmetic ( )
10. Business activities ( )
11. Investments ( )
12. Insurance and annuities ( )
13. Taxes and revenue ( )
14. Practical use of geometry ( )

15. Properties of space figures ( )
16. Solving equations ( )
17. Indirect measurement ( )
18. Introduction to algebra ( )

B. Elementary Applied Mathematics

1. Properties of geometric figures ( )
2. Mechanical drawing ( )
3. Solving problems by algebra ( )
4. Operations with signed numbers ( )
5. Ratio, proportion, variation ( )
6. Experimental projects ( )

C. First Year Algebra

1. Algebra and its applications ( )
2. The properties of equations ( )
3. Positive and negative numbers ( )
4. Multiplication and division of signed numbers ( )
5. Equations of the first degree in one unknown ( )
6. Equations of first degree in two unknowns ( )
7. Special products and factoring ( )
8. Operations with fractions ( )
9. Ratio, proportion, and variation ( )
10. Laws of exponent ( )
11. Square roots and radicals ( )
12. Quadratic equations ( )
13. Numerical trigonometry ( )

H. Tabular Results of Questionnaires

Group II

1. Number of Questionnaires sent 15
2. Number of Questionnaires returned 8
3. Number of persons indicating General Mathematics should be taught 8
4. Number of persons indicating Elementary Applied Mathematics should be taught 8
5. Number of persons indicating First Year Algebra should be taught 8
6. Number of persons indicating General Mathematics should be taught in

Grades	7	8	9	10	11	12
Number	7	1	0	0	0	0

7. Number of persons indicating Elementary Applied Mathematics should be taught in

Grades	7	8	9	10	11	12
Number	1	7	0	0	0	0

8. Number of persons indicating First Year Algebra should be taught in

Grades	7	8	9	10	11	12
Number	0	0	8	0	0	0

From the results of this questionnaire and also from an extended study of modern textbooks and articles on the teaching of mathematics the following course outlines have been developed. They should be considered as suggestive rather than the final word.

I. Desirable Content Material as Indicated by Group II

1. Essentials of Course Contents
General Mathematics
Grade 7

I. Local and Community Life

1. Use of Mathematics in Home and Community
2. Community and Town Planning—Scale Drawing
3. Public Utilities—Gas, Electricity, Telephone, etc.
4. Community Budget and Taxes
5. Cost of Community Projects—Bond Issues
6. Organizations for Community Affairs
7. Investment in Education
8. Investment in Family Health

II. Systems of Measuration

1. Systems of Linear Measure
2. Measurement of Area—Volumes
3. Systems of Weight Measure
4. Measurement of Time—Julian, Gregorian, New World Calendar
5. Measurement of Temperature
6. Experiments in Measurements
7. Surfaces, Weight, Volumes

III. The Use of Decimal Numbers

1. Place Value of Our Number System
2. Place Value for Decimal Numbers
3. Fractions and Decimal Numbers
4. Fundamental Operations with Decimal Numbers
5. Applications of Percentage
6. Short-cuts in Computing with Per Cents
7. Business and Consumer Problems

IV. Practice in Using Decimals

1. Large and Small Numbers
2. The Scientific Notation
3. Computation with Large and Small Numbers
4. Problems in Elementary Science
5. The Use of the Metric System
6. Experimental Problems in Science
7. Problems in Applied Mathematics

V. Approximate Computation

1. Rounding Off Numbers
2. Significant Digits—Accuracy of Data
3. Approximation of Results—Relative Error
4. Fundamental Operations with Approximate Numbers
5. Applications to Practical Problems
6. The Use of Logarithms

VI. Commodity Costs to the Consumer

1. Cost of Food and Clothing
2. Buying or Owning a Home Vs. Renting
3. Cost of Operating a Home
4. Cost of Operating a Car
5. Vacation and Travel Cost
6. Recreation and Hobby Costs
7. Cost of Health Maintenance
8. Cost of Luxuries and Incidentals

VII. Consumer Income and Buying

1. Income of Unskilled Labor
2. Income of Trades and Professions
3. Incomes of Business and Industry
4. Incomes Compared with Investments
5. Budgeting Income to Meet Expenses
6. Social Security Investments
7. Saving and Investing
8. Installment Plan Buying Vs. Cash Payments

VIII. Problems of Taxation

1. Various Sources and Need of Taxes
2. Real Estate and Property Taxes
3. Tariff and Import Duties
4. Sales Taxes—National, State, City
5. License and Operating Taxes
6. Income Taxes—National, State
7. Inheritance and Gift Taxes—National, State
8. Corporation Taxes—National, State, City

IX. The Fundamental Operations of Arithmetic

1. Fundamental Processes with Integers, Fractions, Decimals
2. Measuration—English and Metric Systems
3. Approximate Numbers and Approximations
4. Use of Percentage, Operations with Per Cents

X. Business Activities

1. Problems of Retail and Wholesale Business (Terminology)
2. Commission, Profit and Loss, Real Estate Transactions
3. Banks and Banking Activities
4. Simple Interests--Exact Time, etc.
5. Interest Computation--Ordinary and 6% Method
6. Installment Buying--Interest Rates
7. Personal Finance Companies

XI. Investments

1. Stocks and Bonds: Brokerage, Yield, Effective Rates
2. Government Bonds--National, State, Municipal
3. Compound Interest
4. Savings and Loan Associations
5. Cost of Owning a Home
6. Investment in Education and Training

XII. Insurance and Annuities

1. Life Insurance--Different Forms, Rates, Benefits
2. Insurance on Real Estate--Fire, Storm, Theft, etc.
3. Accident and Health Insurance
4. Automobile Insurance--Various Types
5. Other Forms of Insurance--Crop, Marine, Workman, Group, etc.
6. Annuities, Social Security

XIII. Taxes and Revenue

1. Local and State Taxes--Real Estate, Sale, Income, etc.
2. Federal Taxes--Income Taxes, Internal Revenue
3. Import Duties, Sale Taxes, Luxury Taxes
4. Distribution of Tax Funds

XIV. Practical Use of Geometry

1. Properties of a Triangle--Similar Triangles
2. Properties of Various Quadralaterals, and Regular Polygons
3. Properties of the Circles, Central Angles, Inscribed, etc.
4. Constructions in Geometry--Use of Protractor
5. The use of Formulas in Geometry
6. Indirect measurement with Triangles--Field Protractor
7. Practical Applications of Geometry

XV. Properties of Space Figures

1. Rectangular solids, Spheres, Cylinders--Volume, Area, etc.
2. Volume and Area of a Pyramid, Cone
3. Practical Applications

XVI. Solving Equations

1. Formulas and Equations--Meaning and Interpretation
2. Four Fundamental Operations with Equations
3. Solving a First Degree Equation in One Unknown (Positive Result)
4. Solving Verbal Problems with Equations
5. Ratio and Proportion--Solving Problems

XVII. Indirect Measurement

1. Similar Triangles--Scale Drawing, Ratio, Proportion
2. Measurements Found by Approximate Methods, Experiments
3. Squares and Square Roots--Similar Triangles
4. The Pythagorean Formula and Square Root
5. Practical Experiments

XVIII. Introduction to Algebra

1. The Language of Algebra and Its Use
2. Positive and Negative Numbers
3. Interpretations of the Signs (+) and (-)
4. Fundamental Operations with Signed Numbers (Plus and Minus)
5. Solving Simple Equations--Verbal Problems

2. Essentials of Course Content
Elementary Applied Mathematics
Grade 8

I. Properties of Geometric Figures

1. Lines--Straight, Intersecting, Parallel, Perpendicular, Broken, Curved, Circles
2. Angles--Vertical, Acute, Right, Straight, Obtuse, Equal
3. Triangles--Scalene, Isosceles, Equilateral, Right
4. Circles--Properties of : Central and Inscribed Angles
5. Quadrilaterals--Classification and Definition, Examples
6. Space Figures and Their Properties
7. Symmetry of Geometric Figures

II. Mechanical Drawing

1. Use of Drawing Equipment--Drawing Boards, Ruler, Compasses, Squared Paper, Protractor, T-square, Draftsman Triangle, etc.
2. Geometric Constructions--Draw Figures Listed in Unit I
3. Scale Drawing--Interpretation and Construction of Designs Made with a Given Scale
4. Solution of Problems by Scale Drawing
5. Experimental Problems--Data Gathered by Laboratory Experiment, Problems Solved, Results Verified
6. Problems Involving Vector Quantities
7. Perspective Drawing of Space Figures--Problems

III. Solving Problems by Algebra

1. Simple Equations Involving Only Positive Numbers
2. Interpretation of the Four Fundamental Operations
3. Geometric Problems Solved by Algebra
4. Similar Triangles--Ratio, Proportion
5. Areas of Plane Figures
6. Geometric Formulas and Applications
7. Trigonometric Functions and Similar Right Triangles
8. Use of Trigonometric Functions in Solving Problems
9. The Pythagorean Theorem--Applications

IV. Operations with Signed Numbers

1. Interpretation and Use of the Symbols plus (+) and minus (-)
2. Addition and Subtraction
3. Multiplication and Division
4. Solution of First Degree Equations in One Unknown
5. Simultaneous First Degree Equations in Two Unknowns
6. Problems in Geometry and Science
7. Verbal Problems--Miscellaneous Applications

V. Ratio, Proportion, Variation

1. Ratios and Common Fractions, Definitions and Their Use
2. Proportions and Equations, Definitions and Use
3. Similar Geometric Figures
4. Meaning of Variation--Direct, Inverse, Joint
5. Laws of Science--Problems
6. Miscellaneous Applications--Verbal Problems

VI. Experimental Projects

1. Measurement of Courts and Grounds for Tennis, Basketball, Football, etc. Scale Drawing and Problems
2. Indirect Measurement--Right Triangle Method
3. Measurement of Inaccessible Distances--Scale Drawing, Similar Triangles, Methods of Computation
4. Problems in Science--Parallelogram Law of Vectors, Temperature Scales and Transformation, Atmospheric Pressure, Intensity of Light, Problems in Sound and Light, Problems of Equilibrium and Motion (Statics, dynamics)
5. Solving Problems by Logarithms, Slide Rule, Calculating Machines
6. Mechanical Industrial and Engineering Problems
7. Construction of Graphs Representing Given Data

3. Essentials of Course Content
A First Course in Algebra
Grade 9

I. Algebra and its Applications

1. The Nature of Algebra
2. The Uses of Algebra
3. The Notation of Algebra
4. Algebra and Arithmetic
5. Algebra and Geometry
6. The Use of Formulas
7. The Use of Graphs

II. The Properties of Equations

1. Meaning and Use of Equations
2. Equations Involving the Four Fundamental Operations
3. Solving Equations by Using Addition, Subtraction, Multiplication and Division
4. Root of an Equation, Definition and Meaning
5. Combining Like Terms and Solving Equations
6. Equations Solved by Addition of Equal Quantities
7. Equations Solved by Subtraction
8. Equations Solved by Multiplication
9. Equations Solved by Division--All Operations
10. Solving Verbal Problems by Equations

III. Positive and Negative Numbers

1. Measuring Quantities with Arbitrary Units
2. Meaning and Use of Positive and Negative Numbers
3. Interpretations of the plus (+) and minus (-) signs
4. Need for Computing with Signed Numbers
5. The Number Scale--The Zero Point Arbitrary
6. Addition and Subtraction of Signed Numbers
7. Formulation and Interpretation of Rules of Operation
8. Reduction of Subtraction to Addition
9. Order of Operations with Signed Numbers
10. Addition of Polynomials, Subtraction of
11. Solving Equations--Use of Parentheses, Brackets, etc.
12. Solving Verbal Problems

IV. Multiplication and Division of Signed Numbers

1. Multiplication of Signed Numbers--Rule
2. The Use of Exponents--Terminology
3. The Sign of a Product of Two or More Factors
4. Multiplication of Algebraic Expressions--Arrangements--Procedure
5. Solving Equations--Use of Parentheses, Braces, etc.

6. The Division Problem--Use of Signed Numbers--Rule
7. The Law of Exponents--Use of
8. Division of One Monomial by Another--Rule
9. Division of a Polynomial by a Monomial--Systematic Procedure

V. Equations of the First Degree in One Unknown

1. Solving a First Degree Equation--Directions
2. The Use of "Common Notions" and Fundamental Operations
3. Changing Sign of Each Term--Negative Roots
4. Solving Problems Involving
   a. Time, Rate Distance, Uniform Linear Motion
   b. Properties of Numbers, Percentage, Simple Interest
   c. Geometry and Mechanics

VI. Equations of First Degree in Two Unknowns

1. Solutions of an Equation in Two Unknowns
2. Rectangular Coordinates--Nomenclature
3. Graph of an Equation in Two Unknowns--Directions for Graphing
4. The Graphs of Two Equations in Two Unknowns: Intersecting lines, parallel lines, one line
5. The Common Solution of Two Linear Equations
6. Directions for Solving Two Linear Equations
7. Solving Verbal Problems
   a. Involving Properties of Numbers, Digits
   b. Business, Finance, Economics
   c. Geometry, Mechanics
   d. Mixtures, Chemistry, Physics
8. The Function Idea--Linear Function, mx, mx+k, Graph
9. Properties of the Straight Line Graph, Intercepts, Slope Found from Graph and Equation
10. Line Determined by Point and Slope--Two Points
11. Sets of Collinear Points

VII. Special Products and Factoring

1. Distributive Law--Monomial Times a Polynomial
2. Removing a Monomial Factor
3. Power of a Monomial--Root of
4. Product of Sum and Difference of Two Quantities
5. Factoring the Difference of Two Squares
6. Product of Two Binomials--Square of a Binomial
7. Factoring of a Trinomial--Complete Factorization
8. Solving Equations by Factoring
9. Factoring by Grouping Terms
10. Solving Quadratic Equations by Factoring--Rules
11. Equations with Literal Coefficients
12. Verbal Problems and Applications

VIII. Operations with Fractions

1. Properties of Rational Fractions—Meaning, Terminology, Equivalent Fractions—Lowest Terms—Signed Numbers
2. Reducing Fractions to Lowest Terms—HCF
3. Multiplication and Division of Fractions
4. Equations Involving Fractions
5. Addition and Subtraction of Fractions
6. Equations Containing Fractions
7. Mixed Expressions
8. Complex Fractions—Equations with

IX. Ratio, Proportion, and Variation

1. Meaning and Use of Ratios, Properties
2. Properties of a Proportion
3. Time, Rate, and Distance Problems
4. Work, and Lever Problems
5. Applications

X. Laws of Exponent

1. Meaning and Use of Integral Exponents
2. Laws of Multiplication, Division, Involution, Evolution
3. Zero, Negative and Fractional Exponents
4. Application of the Laws of Exponents

XI. Square Roots and Radicals

1. Meaning of a Root of a Number—Use of Radicals
2. Square Roots—Tables of Square Roots
3. Process of Extracting Square Roots
4. The Square Root of a Fraction
5. Roots of Higher Order—Terminology
6. Simplification of Radicals
7. Operations with Radicals—Rules
8. Equations Containing Radicals

XII. Quadratic Equations

1. Solving by Factoring
2. Graphing a Quadratic Equation
3. Determining the Roots Graphically
4. Solving by Completing the Square
5. The Quadratic Formula—Applications
6. Solving Problems by Quadratic Equations
7. Imaginary Numbers—Complex Numbers
8. Applications to Problems of Science

XIII. Numerical Trigonometry

1. Practical Measurements--Significance of Digits
2. Rounding Off Numbers--Rules--Approximate Computation
3. Tangent of an Angle--Meaning and Application
4. Angle of Elevation, Depression
5. Sine and Cosine of an Angle--Definition, Application
6. Interpolation--Application for Given Angles (Functions)
7. Solving Practical Problems

J. General Statement About Group III

Experts of Group III expressed their views on the questions listed on the enclosed questionnaire.

The writer compiled and summarized the data. Again, answers supplied in this area were closely related. Some members of the group discussed in detail some of the major headings listed and divided many of these into sub-headings as was done with Group II on the reverse side of the questionnaires.

Members of Group III included:

Mr. Robert E. K. Rourke
Kent School
Kent, Connecticut

Dr. J. N. Payne
University of Michigan

Dr. C. H. Fischer
Michigan State University

Mr. John C. Bryan
North High School
Omaha, Nebraska

Dr. E. G. Begle
Yale University

Dr. E. P. Vance
Oberlin College

Dr. F. W. Kokomoor
University of Florida

The results for this group will be found in the pages that follow.

K. Sample Questionnaire for Group III

473 West Bennett
Oklahoma State University
Stillwater, Oklahoma

Dear Educator,

I am engaged in studying suggested methods for improving instruction in the Georgia public school systems. Please check the items from the following list that, in your judgment, will represent an adequate high school program.

Very truly yours,

George Thomas, Sr.

****

	Course	Grade level (circle one)		
A.	Plane Geometry	10	11	12
B.	Solid Geometry	10	11	12
C.	Second Year Algebra	10	11	12
D.	Elementary Analysis	10	11	12
E.	Plane Trigonometry	10	11	12

From the following list, please check those topics that should be included:

A. Plane Geometry

1. The Congruency Theorems ( )
2. Theorems on Triangles Proved by the Congruency Theorems ( )
3. Inequalities of Lines and Angles ( )
4. Properties of Parallel and Perpendicular Lines ( )
5. Circles: (Interior Points, Boundary Points) Chords, Central Angles, Arcs ( )
6. Tangents and Secants to a Circle ( )
7. Similar Triangles ( )

B. Solid Geometry

1. How to Represent a Plane by Drawing ( )
2. Perpendicular and Parallel Planes ( )
3. Projection of a Point upon a Plane ( )
4. Surfaces and Secions of Surfaces ( )
5. Lateral Area of a Right Cylinder ( )

6. Volumes of Surfaces and Solids ( )
7. Cavalieri's Theorem ( )
8. Polyhedral Angles ( )
9. Spherical Polygons ( )
10. Inequalities of Space Figures ( )
11. Loci of Space Figures ( )
12. Proportionality of Areas ( )

C. Second Year Algebra

1. Our Number System ( )
2. Application of the Fundamental Operations ( )
3. Special Products and Factors ( )
4. Operations with Fractions ( )
5. Radicals and Fractional Exponents ( )
6. Rectangular Coordinates: Linear Equations ( )
7. Simultaneous Linear Equations ( )
8. Quadratic Equations in One Variable ( )
9. Quadratic Equation in Two Variables ( )
10. Simultaneous Equations in Two Variables ( )
11. Ratio, Proportion, Variation ( )
12. Common Logarithms ( )
13. Sequences and Series ( )
14. The Binomial Theorem ( )

D. Elementary Analysis

1. Elementary Functions of One Variable ( )
2. Sequences and Series ( )
3. Permutations, Combinations, Probability ( )
4. Rational Integral Functions—Polynomials ( )
5. Elementary Statistics ( )
6. Rational Fractional and Irrational Functions ( )
7. Lines and Planes in Space ( )
8. The Conic Sections ( )
9. Simultaneous Equations and Determinants ( )

E. Plane Trigonometry

1. The Right Triangle ( )
2. The Circular Functions ( )
3. Transformation Formulas ( )
4. The Oblique Triangle ( )
5. Graphs of Circular Functions ( )

L. Tabular Results of Questionnaires
Group III

1. Number of Questionnaires sent 15

2. Number of Questionnaires returned 7

3. Number of persons indicating Plane Geometry should be taught 7

4. Number of persons indicating Second Year Algebra should be taught 7

5. Number of persons indicating Plane Trigonometry should be taught 7

6. Number of persons indicating Elementary Analysis should be taught 7

7. Number of persons indicating Solid Geometry should be taught 0

8. Number of persons indicating Plane Geometry should be taught in

Grade	10	11	12
Number	6	1	0

9. Number of persons indicating Second Year Algebra should be taught in

Grade	10	11	12
Number	1	6	0

10. Number of persons indicating Plane Trigonometry should be taught in

Grade	10	11	12
Number	0	6	1

11. Number of persons indicating Elementary Analysis should be taught in

Grade	10	11	12
Number	0	1	6

From the results of this questionnaire and also from an extended study of modern textbooks and articles on the teaching of mathematics the following course outlines have been developed. They should be considered as suggestive rather than the final word.

M. Desirable Content Material As Indicated by Group III

1. Essentials of Course Contents

Plane Geometry
Grade 10

I. The Congruency Theorems

1. Two triangles are congruent if two sides and the included angle of one are equal respectively to two sides and the included angle of the other.
2. Two triangles are congruent if two angles and the included side of one are equal respectively to two angles and the included side of the other.

II. Theorems on Triangles Proved by the Congruency Theorems

1. Two points each equally distant from the extremities of a line segment determine the perpendicular bisector of that segment.
2. Any point on the perpendicular bisector of a line segment is equally distant from the extremities of the segment.

III. Inequalities of Lines and Angles

1. An external angle of a triangle is greater than either of the opposite interior angles.
2. One and only one perpendicular can be drawn from a given external point to a given line.

IV. Properties of Parallel and Perpendicular Lines

1. The parallel postulate is stated and explained.
2. Two lines lying in the same plane either intersect or are parallel.

V. Circles: (Interior Points, Boundary Points) Chords, Central Angles, Arcs

1. Definition of a circle, interior points, boundary points, exterior points of a circle
2. Three concollinear points determine one and only one circle

VI. Tangents and Secants to a Circle

1. Definition of a secant; a tangent to a circle
2. Other properties
3. A line is tangent to a circle if and only if it is perpendicular to a radius at its outer extremity.

VII. Similar Triangles

1. Definition of a Proportion
2. Theorems on Proportions

2. Essentials of Course Content
(One Semester)
A Second Course in Algebra
Grade 11

I. Our Number System

1. The Number Concept
2. Use of Symbols
3. The Four Fundamental Operations
4. Laws of Addition, Subtraction
5. Laws of Multiplication, Division
6. The Symbols 0 and 1 (Identify elements)
7. The Number Scale
8. Addition and Subtraction on the Number Scale
9. Multiplication and Division
10. Raising to a Power
11. Extraction of Roots

II. Application of the Fundamental Operations

1. Addition of Polynomials
2. Subtraction of Polynomials
3. Multiplication of Polynomials
4. Division of Polynomials
5. Solving Equations
6. Deriving Formulas
7. Solving Problems Using Equations

III. Special Products and Factors

1. Product--Prime Factors
2. Monomial Factors
3. Product of the Sum and Difference of Two Quantities
4. Factoring the Difference of Two Squares
5. Applications to Problems
6. Square of a Binomial—Factoring Trinomials Square
7. Product of Two Binomials Having a Common Term
8. Factoring Trinomials
9. Miscellaneous Problems

IV. Operations with Fractions

1. Definition and Properties of a Fraction
2. Reducing Fractions to Lowest Terms—HCF
3. Reducing a Fraction to a Mixed Expression
4. Lowest Common Multiple
5. Reducing Fractions to a Common Denominator
6. Addition and Subtraction of Fractions--Averages
7. Multiplication and Division of Fractions
8. Operations with Mixed Expressions
9. Simplifying Complex Fractions
10. Equations Involving Fractions
11. Solution of Problems and Applications

V. Radicals and Fractional Exponents

1. Use of Radicals--Radical Sign--Index of Root
2. Irrational Numbers
3. Square Root--Product--Simplification
4. Table of Square Roots
5. Process of Finding the Square Root of a Number
6. Approximating Square Roots
7. Geometrical Applications
8. Square Root of a Polynomial
9. Equations Involving Square Roots
10. Fractions with Irrational Denominators
11. Fractional Exponents
12. Laws of Exponents
13. Zero Exponent--Negative Exponents
14. Use of Fractional Exponents
15. General Applications

VI. Rectangular Coordinates: Linear Equations

1. Rectangular Coordinate System--Plotting Points
2. Distance Between Two Points
3. Graphing Linear Equations
4. Intercepts--Slope of a Line
5. Slope--Intercept Form of Equation of Line
6. General Linear Function--Independent, Dependent Variable
7. The General Linear Equation
8. Line Determined by a Point and Its Slope
9. Line Determined by Two Points

VII. Simultaneous Linear Equations

1. Two Linear Equations in Two Unknowns
2. Intersecting Lines
3. Parallel and Coincident Lines
4. Consistent, Inconsistent and Dependent Equations
5. Three Simultaneous Equations in Three Unknowns
6. Determinants of the Second Order--Third Order
7. Linear Equations with Literal Coefficients

VIII. Quadratic Equations in One Variable

1. The General Quadratic Equation
2. Solving by Factoring
3. Solving by Completing the Square
4. Irrational and Complex Roots
5. Solving by the Quadratic Formula
6. The Discriminant--Nature of the Root
7. Graph of the Quadratic Function
8. Relation Between Roots and Coefficients

IX. Quadratic Equation in Two Variables

1. The Circle, Type Form, Graph, Properties
2. The Parabola, Type Form, Graph, Properties
3. The Ellipse, Properties of
4. The Hyperbola, Properties of
5. General Equation of Second Degree--XY Term Missing

X. Simultaneous Equations in Two Variables

1. A Linear and Quadratic Equation--Method of Solving
2. Two Quadratic Equations--Special and General Methods
3. Graphs and Graphical Solutions
4. Simultaneous Equations of Higher Degree--Graphs

XI. Ratio, Proportion, Variation

1. Definitions--Properties
2. Geometrical Applications
3. Continued Proportion--Properties
4. Direct, Inverse, and Joint Variation

XII. Common Logarithms

1. Scientific Notation
2. Definition of a Logarithm--General Characteristics
3. Common Logarithm--Use of Tables
4. Antilogarithms--Characteristic--Mantissa
5. Laws of Logarithms
6. Computations with Logarithms
7. Solving Exponential Equations
8. Logarithms of Trigonometric Functions
9. Applications--Solution of Right Triangles

XIII. Sequences and Series

1. Definitions--Elementary Properties
2. Arithmetic Sequence--Nth Term; Means
3. Arithmetic Series--Formulas
4. Geometric Sequence--Properties of
5. Geometric Series, Finite, Infinite--Formulas for
6. Repeating or Periodic Decimals

XIV. The Binomial Theorem

1. Development of Theorem--Mathematical Induction
2. Properties of the Binomial Formula--Applications
3. The Compound Interest Formula
4. Binomial Formula with Fractional and Negative Exponent
5. Application

3. Essentials of Course Contents
Trigonometry
(One Semester)
Grade 11

I. The Right Triangle

1. Trigonometric Functions of an Acute Angle
2. Functional Relations--Reciprocal Formulas, etc.
3. Construction of Angle with a Given Function
4. Functions of Special Angles--Use of Tables
5. Solution of the Right Triangle--Areas, Logarithms
6. Vector Quantities--Components, Forces
7. Fundamental Formulas--Identities

II. The Circular Functions

1. General Definition of the Trigonometric Functions
2. Functional Evaluation Formulas
3. Measurement of Angles
4. Arc length--Area of Sector, Segment
5. Identities and Conditional Equations

III. Transformation Formulas

1. Functions of the Sum (Difference) of Two Angles
2. Functions of the Double Angle--Half Angle
3. Product Formulas, Sum-and-difference Formulas
4. Equations and Identities

IV. The Oblique Triangle

1. The Law of Sines--Circumcircle, Applications
2. Law of Cosines--Applications, Newton's Parallelogram Law
3. Tangent of Half Angle--Application--Area of Triangle
4. Law of Tangents--Applications
5. Practical Applications

V. Graphs of Circular Functions

1. Graphs of Simple Trigonometric Functions
2. Graphs of Multiple Functions
3. Graphs of Functions of Multiple Angles
4. Period, Frequence, Wavelength
5. Inverse Circular Functions--Formulas
6. Graphs of Inverse Functions

4. Essentials of Course Content
Elementary Analysis
Grade 12

I. Elementary Functions of One Variable

1. The Linear Function—Graph, Slope, Intercepts
2. Equation of the Line if Given (1) Point and Slope, (2) Two Points
3. Distance Between Two Points—Loci
4. The Quadratic Equation—Solution
5. The Graph of the Quadratic Function
6. Simultaneous Equations
7. The Tangent to a Curve

II. Sequences and Series

1. The Arithmetic Sequence—Series (A. S.)
2. Formulas for an A. S.
3. The Geometric Sequence—Series (G. S.)
4. Formulas for a Geometric Series
5. The Binomial Formula. The Number e
6. Compound Interest and Annuity Formulas
7. DeMoivre's Theorem
8. Formulas for Multiple Angles

III. Permutations, Combinations, Probability

1. Meaning of a Permutation—Formulas
2. Definition of Combination—Formulas
3. The Binomial Theorem—Binomial Coefficients
4. Meaning of Probability—Value of an Expectation
5. Mutually Exclusive Events
6. Independent and Dependent Events

IV. Rational Integral Functions—Polynomials

1. The Fundamental Theorem of Algebra
2. The Remainder Theorem—Factor Theorem
3. Synthetic Substitution
4. The Theorem of Rational Roots
5. The Graph of a Polynomial
6. Upper and Lower Limits to the Roots
7. Relation Between the Roots and the Coefficients

V. Elementary Statistics

1. Nature of a Statistical Problem
2. Tabulation of Data—Frequency Distribution
3. Measures of Variability—Elementary Concepts
4. Measures of Central Tendency—Computation, Interpretation
5. Approximate Measure of Linear Correlation
6. Interpretation of the Above Analysis—Applications
7. Elementary Theory of Sampling

VI. Rational Fractional and Irrational Functions

1. Operations with Simple and Complex Fractions
2. Solution of Fractional Equations--Extraneous Roots
3. Differentiation of Fractions
4. Graphs of Fractional Functions
5. Equations Involving Irrational Functions
6. Differentiation of Implicit Functions
7. Graphs of Irrational Functions

VII. Lines and Planes in Space

1. Properties of Perpendicular Lines and Planes
2. Properties of Parallel Lines and Planes

VIII. The Conic Sections

1. Definition and Development of Standard Equations for the Circle, Parabola, Ellipse, Hyperbola
2. The General Equation of the Form $Ax^2+Cy^2+Dx+Ey+F=0$
3. The Most General Equation of the Second Degree in Two Variables
4. Characteristic Properties of Each of the Conic Sections
5. Areas and Volumes
6. Tangent and Normal Lines

IX. Simultaneous Equations and Determinants

1. Determinants of Second and Third Order
2. Solution of First Degree Equations in Two and Three Unknowns
3. Evaluation of Determinants of Order n
4. Solution of Simultaneous Equations by Cramer's rule
5. The Rank of a Determinant
6. Linearly Independent and Dependent Equations

CHAPTER III

CONTRIBUTIONS OF THE TEACHER

A. Note to Teachers

The writer's knowledge of individual differences indicates that mathematics of a more abstract nature is not for all pupils. It would be a serious injustice to some pupils to force them to study some kinds of mathematics. Shall we then counsel them to avoid the study of mathematics? This writer thinks not. Fortunately, a better alternative is suggested.

Under the old system, the teacher was the chief actor. He was occupied in trying to impress his ideas upon the pupils. But to understand the child and to keep pace with his growth the teacher must grow himself, for the same fundamental laws that govern growth prevail on every successive plane.

The true business of school is not to chain students to preconceived ideas, but to set him free to discover his own ideas and to help him bring all his powers to bear on the problem of learning. Under these conditions learning becomes experiencing. Teaching is energizing and stimulating. It is not concerned with "shaking the bones of a dead past out of a sense of academic duty." A school is no longer a "state controlled manufactory of echoes." Achievement in terms of understanding, attitudes and behaviors which boys and girls need for active living in a democracy rather than words and doubtful meaning is ever becoming

more desirable.

In every school there are teachers who have the flame of creative teaching burning within them. Some of these, at times, find that they must work alone, but they work alone secure in the knowledge that such enthusiasm creates like interest in others.

It is desirable that teachers become familiar with the art of successful teaching. In order for a teacher to be successful he should take advantage of the various opportunities as they present themselves.

Many teachers of the state of Georgia have never even seen the code of professional ethics. A copy has been included with the hope that in the future teachers and teachers in training will be alert enough to become ever sincere about what the composition of successful teaching includes.

B. What Makes a Successful Teacher

Teaching: A Highly Skilled Profession

"A teacher affects eternity; he can never tell where his influence stops." This quotation is from the "Education of Henry Adams." It speaks a world of truth. It behooves us over and over again to state sincere appreciation of the responsibility of teachers, the taska that lie before them and the accomplishments of the past. The late Dr. Harvey Cushing expressed Adams' same sentiment in his "Life of Sir William Osler: "No bubble is so irredescent or floats longer than that blown by the successful teacher."

After World War II a brilliant young ex-officer writes to say good-bye as he starts to the university where he has accepted an appointment. He proceeds with a heart-moving confession so complete in itself that one can do no more than set it down admiringly without an added word of comment:

> In spite of the flattering offers from Government and business, in my heart I knew I would return to teaching. At first the war shook my faith in the effectiveness of education. It seemed we had failed if after all those years we could again resort to force of arms to settle human differences. Then as the storm gathered, the leisure to lament the irony itself disappeared. Involved in the struggle, hurried by new responsibilities and pressing adjustments, thinking and feeling were reduced to the tasks of day-by-day survival. Hurt, and confusion can surely breed anger and build up in individuals the same border disputes which causes the spirit, rather than the body, to be disarmed. That's what this summer has been for me—a peace conference between myself and the world. Living through my own reconstruction, I saw more clearly the problems of those who have undergone the same experience in the Army itself, or in that larger army, a militarized world. As the insight sharpened, I knew that I must go back to where there are young people seeking to understand. The Renaissance will come, if ever, not from a miracle of science nor from a brilliant stroke of diplomacy, but through the slow, often tedious, pace of understanding and the brotherhood and moral integrity which clear

thinking can create.[1]

And so it is with all great and successful teachers, the realization of what a spiritual and soul satisfying profession it is.

A teacher must work with magic tools to accomplish the seven cardinal objectives of education which have been generally accepted as the aims of the principles of education. As I enumerate them I realize how they fit into every niche of human life and living: Sound health, worthy home membership, mastery of the tools, techniques and spirit of learning, faithful citizenship, vocational effectiveness, wise use of leisure, ethical character. Along with this, which is achieved by the skill of the teacher, is fulfillment of the function of the school which is: To understand and practice desirable social relationships, to discover and develop his own desirable individual aptitudes, to cultivate the habit of critical thinking, to appreciate and desire worthwhile activities, to gain command of the common and integrating knowledge and skills, to develop a sound body and normal mental attitudes.

The value of professional growth to the teacher is immeasurable. The profession of teaching is one of the most fascinating and at the same time discouraging of all careers. It is hard to see tangible results, since the best results of teaching only develop years later when the teacher is all but forgotten. Day after day the teacher strives to accomplish certain things in his teaching, and one failure follows after another apparently; it is impossible to know the extent of success since the learning process is gradual and cannot be adequately measured. Tests and oral recitations are no measure of the results

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[1]The Institute of Research, Career Series, "Teaching A Highly Skilled Profession" The Carnegie Foundation (New York:1948), p. 1.

of teaching; they are misleading and discouraging at times to both pupil and teacher.

To combat this discouraging nature of the profession it is necessary to feel progress within oneself. A teacher who is teaching for the small salary or who is satisfied to continue year after year with the same methods, the same course of study and the same presentation is not alive to the opportunities of his profession and can hardly be called a teacher. One who is utterly devoted to his profession and alert to its possibilities must feel progress, and this is chiefly possible only through personal growth. There is nothing to compare with the feeling of eager anticipation of the teacher who has spent several months in interesting travel and concentrated study of his particular teaching problems as approaching a new year of teaching. He goes to work not as a slave bound by monotonous routine and tedium but as a messenger carrying an important message to the future leaders of the nation.

This inspiration is the fundamental on which all good teaching rests. Without it teaching is colorless, uninteresting and dull not only to the pupil, but to the teacher also. The monetary return from teaching is as yet pathetically small and inadequate. Progress is being made in most states to correct this. However, compared with other professions, some of which require less specific and specialized training, the teacher has the lowest economic expectancy. And yet there is scarcely a teacher who has given years to the profession who would change his decision if it were possible. The return is in satisfaction and knowledge of service rather than in material compensation. To see a young child's face light up with comprehension is worth years of hard study and hours of tedious preparation. This return is not possible

without professional growth. The teacher's sense of inner satisfaction and realization of service to the community depend upon his personal growth and attitude toward work.

Closely related to this sense of satisfaction is the position of honor in the community who is more respected and loved than the teacher. In many places, he is looked upon as the real leader of the social and intellectual life of the community. Such an accomplishment is perhaps the highest a human being and the greatest achievement to which one can aspire. The teacher who gives himself wholly to his profession and is constantly growing within reaches his goal. The place in the community may be compared with that of the minister or the doctor only.

Perhaps the greatest distinction between teaching and other professions is that the teacher remains young in mind and spirit. Through continued study, constant growth, persistent keeping pace with the tempo of the times, and everyday association with the young, the teacher's own youth is arrested. His face is always in the direction of progress of the future. He thinks and feels the urge of youth to improve things as they are; he constantly visions things about as they might be, a world that could be. The greatest value prolong the mental and intellectual youth of the individual. One who has lost interest in things about him is old; one who reminisces of the past, who loses contact with the movements of the present is old. The teacher who keeps abreast of his profession through study, observation, and intensive thought is daily renewed in spirit. His interests are as progressive as the morning paper and his attitude toward life is fresh and unspoiled by prejudice and conservatism.

In a series on careers published by The Institute for Research,

the one on teaching has an introductory page from "The Unknown Teacher" by Harvey Van Dyke. It is so beautiful and valuable in every word that it is listed here verbatum:

> And What of Teaching? Ah, there you have the worst paid and best rewarded of all the vocations. Dare not to enter it unless you love it. For the vast majority of men and women it has no promise of wealth or fame, but they to whom it is dear for its own sake are among the nobility of mankind. I sing the praise of the unknown teacher. Great generals win campaigns, but it is the unknown soldier who wins the war. Famous educators plan new systems of pedagogy, but it is the unknown teacher who delivers and guides the young. He lives in obscurity and contends with hardships. For him no trumpets blare, no chariots wait, no golden decorations are decreed. He keeps the watch along the borders of darkness and makes the attack on the trenches of ignorance and folly. Patient in his daily duty he strives to conquer the evil powers which are the enemies of youth. He awakens sleeping spirits. He quickens the indolent, encourages the eager, and steadies the unstable. He communicates his own joy in learning and sharing with boys and girls the best treasures of his mind. He lights many candles, which, in later years will shine back to cheer him. This is his reward. Knowledge may be gained from books; but the love of knowledge is transmitted only by personal contact. No one has deserved better of the republic than the unknown teacher. No one is more worthy to be enrolled in a democratic aristocracy, king of himself and servant of mankind![2]

A successful teacher must have a good attitude toward life; one must be sanely optimistic, and this must be founded on faith in humanity. The evil things in life should aid in discovering the good side of it. The teacher cannot teach what he himself cannot feel; if the material does not inspire him, he cannot expect it to inspire others. The personality of the teacher is often the only guarantee of a lasting influence of a subject on the pupil. The teacher who is warm and sympathetic will likely be that teacher the student will never forget. One should ever strive to be that teacher after whom students will wish to pattern their lives rather than the one for whom there is no respect or love.

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[2] Ibid. p. ii.

C. What the World Expects of a Teacher

Teacher Ethics:

We, the members of the National Education Association of the United States, hold these truths to be self-evident--

—that the primary purpose of education in the United States is to develop citizens who will safeguard, strengthen, and improve the democracy obtained through a representative government;

—that the achievement of effective democracy all aspects of American life and the maintenance of our national ideals depend upon making acceptable educational opportunities available to all;

—that the quality of education reflects the ideals, motives, preparation, and conduct of the members of the teaching profession;

—that whoever chooses teaching as a career assumes the obligation to conduct himself in accordance with the ideals of the profession.

As a guide for the teaching profession, the members of the National Education Association have adopted this code of professional ethics. Since all teachers should be members of a united profession, the basic principles herein enumerated form apply to all persons engaged in the professional aspect of education--elementary, secondary and collegiate.

First Principle:

The primary obligation of the teaching profession is to guide children, youth, and adults in the pursuit of knowledge and skills, to prepare them in the way of democracy, and to help them to become happy, useful, self-supporting citizens. The ultimate strength of the nation lies in the social responsibility, economic competence, and moral strength of the individual American.

In fulfilling the obligations of the first principle, the teacher will

1. Deal justly and impartially with students regardless of their physical, mental, emotional, political, economic, social, racial, or religious characteristics.
2. Recognize the differences among students and seek to meet their individual needs.
3. Encourage students to formulate and work for high individual goals in the development of their physical, intellectual, creative, and spiritual endowment.
4. Aid students to develop an understanding and appreciation not only of the opportunities and benefits of American democracy but also of their obligations to it.
5. Respect the right of every student to have confidential information about himself withheld except when its release is to authorized agencies or is required by law.

6. Accept no remuneration for tutoring except in accordance with approved policies of the governing board.

Second Principle:

The members of the teaching profession share with parents the task of shaping each student's purposes and acts toward socially acceptable ends. The effectiveness of many methods of teaching is dependent upon cooperative relationships with the home.

In fulfilling the obligations of this second principle the teacher will

1. Respect the basic responsibility of parents for their children.
2. Seek to establish friendly and cooperative relationships with the home.
3. Help to increase the student's confidence in his own home and avoid disparaging remarks which might undermine that confidence.
4. Provide parents with information that will serve the best interests of their children, and be discreet with information received from parents.
5. Keep parents informed about the progress of their children as interpreted in terms of the purpose of the school.

Third Principle:

The teaching profession occupies a position of public trust involving not only the individual teacher's conduct, but also the interaction of the school and the community. Education is most effective when these many relationships operate in a friendly, cooperative, and constructive manner.

In fulfilling the obligation of this third principle, the teacher will

1. Adhere to any reasonable pattern of behavior accepted by the community for professional persons.
2. Perform the duties of citizenship and participate in community activities with due consideration for his obligations to his students, his family, and himself.
3. Discuss controversial issues from an objective point of view, thereby keeping his class free from partisan opinions.
4. Recognize that the public schools belong to the people of the community, encourage lay participation in shaping the purposes of the school, and strive to keep the public informed of the educational program which is being provided.
5. Respect the community in which he is employed and be loyal to the school system, community, state and nation.

Fourth Principle:

The members of the teaching profession have inescapable obligations with respect to employment. These obligations are nearly always shared employer-employee responsibilities based upon mutual respect and good faith.

In fulfilling the obligations of this fourth principle, the teacher will

1. Conduct professional business through the proper channels.
2. Refrain from discussing confidential and official information with unauthorized persons.
3. Apply for employment on the basis of competence only, and avoid asking for a specific position known to be filled by another teacher.
4. Seek employment in a professional manner, avoiding such practice as the indiscriminate distribution of applications.
5. Refuse to accept a position when the vacancy has been created through unprofessional activity of pending controversy over professional policy or the application of unjust personnel practices and procedures.
6. Adhere to the conditions of a contract until service thereunder has been performed, the contract has been terminated by mutual consent, or the contract has otherwise been legally terminated.
7. Give and expect due notice before a change of position is to be made.
8. Be fair in all recommendations that are given concerning the work of other teachers.
9. Accept no compensation from producers or instructional suppliers when one's recommendations affect the local purchase or use of such teaching aids.
10. Engage in no gainful employment, outside of the contract, where the employment affects adversely the professional status or impairs the standing with students, associates and the community.
11. Cooperate in the development of the school policies and assume one's professional obligations thereby incurred.
12. Accept one's obligation to the employing board for maintaining a professional service.

Fifth Principle:

The teaching profession is distinguished from many other occupations by the uniqueness and quality of the professional relationships among all teachers. Community support and respect are influenced by the standards of teachers and their attitudes toward the teaching profession.

In fulfilling the obligations of this fifth principle, the teacher will

1. Deal with other members of the profession in the same manner as he himself wishes to be treated.
2. Stand by other teachers who have acted on his behalf and at his request.
3. Speak constructively of other teachers, but report honestly to responsible persons in matters involving the welfare of students, the school system and the profession.
4. Maintain active membership in professional organizations, and, through participation, strive to attain the objectives that justify such organized groups.

5. Seek to make professional growth continuous by such procedures as study, research, travel, conferences, and attendance at professional meetings.
6. Make the teaching profession so attractive in ideals and practice that sincere and able young people will want to enter it.[3]

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[3]"The National Education Association's New Code of Ethics," Adopted by the 1952 Representatives Assembly in Detroit. National Education Association Journal. 41:371-372.

CHAPTER IV

EVALUATION

A. Measurement and Evaluation of Student Progress

The scientist insists that he knows only when he can measure the amount of a thing. Thorndike's great slogan was: "Whatever exists, exists in some amount." It was the amount of the thing upon which he centered his attention. When Thorndike and Judd began their work, the meaning of the idea of measurement had already been agreed upon by physical scientists. The laboratory psychologist concurred with the physicist Max Planck that "whatever can be measured is real," and with Sir Francis Galton, the eminent British scientist, that "until the phenomenon of any branch of knowledge has been subjected to measurement and number, it cannot assume the status and dignity of a science."

It was to be expected, therefore, that our educational pioneers should insist upon knowing quantity as well as quality of the things involved in the work of education. And that had been lacking in the teacher's examination and marking systems. As Thorndike said, "The school or the parent who receives a report of 70 as the rating of his child in elementary composition does not know what it was 70 of. . . or whether it was really 60, 65, 70, 75 or 80 of it." Clearly defined units of measure and instruments by which to count them were lacking.

But "clearly defined units" imply measuring scales, and the use of these is the very essence of the scientific way of working. A

modern measuring instrument has two characteristics: First, it is constructed of materials appropriate to the functions to be measured. Second, its dimensions const of a series of equal units arranged in numerical rank-order from a zero point. The intervals must be equal units irrespective of their position on the measuring unit; one unit of linear measure or circular measure, of weight, of electrical resistance, etc., must be equal to another unit. For example, on a handwriting scale sample 80 must be as much better than sample 70 as sample 20 is better than sample 10.

In the moment-by-moment life of every human being the objects of the environment are constantly being evaluated. Most of this process of evaluation is subjective (reveals temperament or personal reaction)—the product of personal experience—and, as we saw with teacher's marks, much of it is grossly unreliable. For example, books like Hugo Musterberg's On the Witness Stand have established beyond question the unreliability of much legal testimony given in our courts by persons who claimed to have been eyewitness observers. It was to be expected, therefore, that as our educational pioneers began to apply more exact methods of observation to the work of education, they would first want to supplant subjective evaluation with more objective kinds of measurement. They would have agreed with the frequent pronouncement of Robert Fletcher, a professor at Dartmouth College, who used to say to his students: "What, after fifty years of effort, can we measure in schoolwork?"

Not everything. This reminds us that we still distinguish between phases of school work which can now be measured by reliable scales and tests and those in which we must resort to general subjective judgment.

While we can measure physique, intelligence, and certain aspects of temperament, some aptitudes and attitudes, and a good many achievements of the pupils, we still are in the subjective evaluating stage when it comes to understanding the Whole Person--the total personality. There we still rely on the specialist--the trained psychoanalyst using such techniques as those of Sigmund Freud and followers--and on the judgment of teachers, parents, and others who do know the student.

But in this evaluating process, besides the psychoanalytic methods, we now have available many instruments to help make our judgments clearer and more balanced--questionnaires, interest inventories, personal history records, Rorschach test results, and rating scales in which the traits of one person can be compared with another. Likewise, in judging teachers and their teaching we can use rating scales to define more precisely traits and phases of work. There is the man-to-man type, in which a teacher is rated by comparing his traits and abilities on a scale with other actual teachers on various traits and aspects of teaching. There are also self-rating scales, so designed that the teacher can measure himself by answering specific questions about himself and his work.

In spite of the gains achieved through the development of those instruments, the best measure of the teacher's work is still the actual behavior of his pupils. How well do they read, write, reckon, solve problems, spell and use language? Such questions can be answered objectively by the use of quantitative scales and tests, as we shall see in a moment. How truly creative are they? Are they sensitive persons, marked by dignity and integrity? For these qualities there are almost no valid and reliable measuring instruments; personal

judgment still dominates the process of evaluation. The more profound question to ask about the influence of the teacher's life and teaching on his students is: What was its long-time effect in their later years? Did his teaching carry over into later years? Definite instruments and techniques to aid in answering such questions we do not have.

Neither can we measure the whole school or school system objectively, although forty years of steady improvement in methods of administering schools have produced instruments which greatly clarify and define the judging process. Hundreds of school systems have been "surveyed" by the new professors of education who have sat in classrooms, appraised teachers' methods and results, and scrutinized printed courses of study, financial and pupil accounting, records and reports and business management. Most of the surveying of what a good school should be and do was done, however, on their own judgment.

But as the years passed, the administrators followed the leads of the measurers and developed "score cards" and rating scales. They saw that the process of judging school administration could be greatly improved if the complex total could be broken down into clearly specified aspects and characteristics. These have improved the evaluation of the work of a school or school system in various definite ways. They catalogue detailed and hitherto neglected aspects of school work; they bring new progressive ideas and practices to the attention of large numbers of teachers; they standardize the many items of schoolwork and administration, stress the important role of the philosophy of the school, and give emphasis to major phases of the curriculum.

Take a glance at the total body of descriptive facts about each pupil that can be given to the teacher as she begins her year's work

with a new class. These appear in the cumulative record folder of the pupil if such a file is available in the school.

With respect to the pupil's physique, the teacher should find such routine physical facts as his exact age, his height, and his weight. She should know from experience or consulting the physical education department whether the relationship of the child's height and weight falls within the normal range. She should have the answers to such questions as: Is his vision normal? Is his hearing normal? (Imperfect hearing expressed in loss in decibels.) Is his general health good? Does he suffer from malnutrition, rickets, anemia? Has he diabetes, asthma, enlarged tonsils, or adenoids? What is the condition of his teeth, skin, etc.?

If the child's emotional condition deviates markedly from that of the normal happy child, the teacher should try to understand the whole home and school background. Each pupil's folder should contain descriptive accounts of teacher's visits to the home and other facts needed to give a full picture of the environmental influences surrounding the child. While facial tics (involuntary twitching of the muscles) and cases of acute stammering are easy to detect, there may be facts about the child's home life which should be contained in the records. In the case of the young child, for instance, is there enuresus (bed-wetting)? Does the child suck his thumb on going to sleep past the time when thumb sucking is usual? Does he have severe tantrums? Repeated nightmares? Is he afraid of the dark or of animals? Has he other fears?

In addition to such important descriptive material, the teacher today can draw upon the results of using fairly reliable scales and tests in at least three critical phases of his work:

First: He can know the general mental ability of his pupils as stated by Mental Age (MA) and Intelligence Quotient (IQ) and obtained from fairly reliable tests of verbal intelligence.

Second: He can discover by means of objective tests special aptitudes needed in most kinds of scholastic work and in some vocational fields.

Third: He can measure the skills and knowledge (and with some reliability the social attitudes) of his students in all the commonly used school subjects of study.

Fairly reliable tests are also available for many newer school subjects and curriculum fields, for example, in health, hygiene and physical education, home economics, industrial arts, public speaking, reading and the vocations. The changing curriculum has created a demand for tests of qualities and achievements that are not the direct product of the school study, and these are rapidly being made available. One can now find tests for knowledge of contemporary affairs, for social adjustments and development, comprehensive individual history, record forms, diagnostic child-study records, study habits and skills, guidance tests and inventory, adult profile, occupational interest blanks, interest inventories, vocational interest blanks and vocational interest schedules.

In addition, the teacher should find in the cumulative record folder the descriptions of personality traits, examples of creative achievements and instances of leadership and fellowship in the pupil's daily school life.

While many of the most critical facts about the pupils are verbal descriptions--life history records, home and neighborhood episodes, etc.--

many are based on definite quantitative measures. It is to those that we shall devote brief attention.

So far has the spirit of science come to imbue our attitudes about school work today, that knowledge of the general mental ability of each pupil in a class is regarded as indispensable. "What is his IQ?" is the first question asked about any pupil whose schoolwork in any subject is not satisfactory. "Only 80," comes the answer, perhaps, and with it, a good deal of light on the situation. An IQ of 80 means that teaching will have to be much more elementary, material more carefully selected and limited, and ideas more concretely illustrated than if the answer had been 110, 120, or higher. "He has an IQ of 140, yet he is doing poorly in his work," tells the teacher that the boy certainly has sufficient general mental ability to master any intellectual problems of the school curriculum. The cause of poor work must lie in other factors than intelligence—perhaps in physical condition or in emotional disturbance.

You will frequently need to know the special abilities and disabilities of your pupils. Some children in the elementary school are "ready" for reading long before others in the same class, some have distinctive aptitudes for arithmetic, others for creative expression in music and the other arts, some excel in physical activities on the athletic field, and others in the intellectual abstractions of mathematics or science. Knowledge of these special abilities will help you use the resources of the school in developing the talents of the young people to the very maximum. Knowledge of disabilities will help you concentrate on special difficulties, will prevent you from trying the impossible, and will point the way to needed remedial methods.

The fact of widespread differences in aptitude, and hence, interest, has always been known to thoughtful people, but in our generation scientific psychologists have built on that knowledge a very helpful body of aptitude tests. The history of these tests reaches back more than a generation to the pioneer work in 1890, when J. McKeen Cattrell published ten tests for various specific mental performances. Harvard's Hugo Munsterberg showed that psychological tests could be devised to select girls who would make good telephone operators and men who would become efficient streetcar motormen. He did this by measuring such traits as speed of reaction, attention span, and specific abilities in observation, memory and accuracy.

A few years later, in World War I, the Trade Test Division of the United States Army designed and standardized effective proficiency tests for hundreds of occupations and aptitude tests for various trade skills. Three typical tests were used: verbal trade tests, picture trade tests and performance trade tests. This work speeded up the civilian measuring movement enormously, and between the two world wars scores of trade and proficiency tests were designed and standardized. These proved a great stimulus to the development of aptitude tests in the psychological laboratories.

"Abstract" tests are also used in which specific sensory, motor, or mental efficiencies are measured by psychological laboratory tests. An example is the measurement of reaction time used in the selection of machine operators.

In high school such aptitude tests have been devised as the Iowa Placement Examinations, Aptitude Series and many other tests used to detect special aptitudes in algebra, geometry, foreign languages,

mathematics and the sciences. In the elementary school, "readiness" tests in reading and arithmetic have been developed.

Pupils' progress in some, if not all, schools is being measured on the basis of subject matter achievement tests and apart from the broader desired outcomes necessary for meeting the over-all needs of the pupils.

There is a lack of good appraisal instruments and a lack of record and report forms in the areas other than the academic skills, especially in the fields of personal and social competence and effectiveness.

Parents and the public expect measurement and evaluation to be in terms of specific subject matter areas because they do not understand the broader purposes of education and the research which indicates that pupils, after graduation, retain general concepts and understanding rather than large bodies of factual information.

Pupil progress should be measured in terms of capacities as well as achievements.

Measurement and evaluation of pupils should be continuous, and the recording of the results should be cumulative.

If one faces facts squarely, he realizes that schools too often jeopardize the pupils rather than promote their life adjustment. To fail a retarded child in a course of study geared to normal mentality is to frustrate him. To promote a child in a regular grade when he falls far short of requirements is to give him a false sense of security—to lead him to believe that he is competing successfully with other children. As a teacher, one is not adjusting to life if he continues to brush aside reality.

As a child finds himself, he comes to know and to live with his

limitations as well as his strengths. He should be made aware of his strengths and limitations as he proceeds through school instead of having the rude awakening to the stringent requirements of college entrance or the exacting demands of the employer.

The teacher should study the present systems of grading, promoting and reporting to parents and challenge the underlying principles which tend to restrain and subdue the use of good techniques along these lines and make recommendations for reorganization if necessary.

The efficient school system today can report to the parents the tested proficiency of any child in the several skills and abilities involved in reading, writing, spelling, language, arithmetic, high school mathematics and science, the languages, history, geography and the social sciences. It can appraise with a good deal of objectivity his ability to think through novel problems. It can portray his personal characteristics with illuminating paragraphs describing his conduct in school, his leadership and organizing ability, his cooperation, his creativeness and his appreciation in the arts. And it can compare his abilities definitely with those of comparable pupils in American schools. It can, in short, present a fairly complete and objective profile of the whole pupil in action.

The efficient school can do this by reporting his performances on any one of several moderately reliable and well-validated general achievement tests. For example:

Stanford Achievement Test. World Book Company, Yonkers-on-Hudson, New York.

Cooperative General Achievement Test; for the social studies, the sciences and mathematics. The Cooperative Test Division of the Educational Testing Service.

Iowa Every Pupil Tests of Basic Skills; for reading, language, and arithmetic skills.

Metropolitan Achievement Tests for Reading, Arithmetic, Mathematics and Language. California Test Bureau.

As their names imply, these tests are measures of general achievement, good for over-all appraisal of the work of classes or individuals and for such administrative needs as classification and promotion of pupils, reports to parents, and so forth.

But the measurers have given the teacher more effective devices to help him in teaching than mere scores on tests and comparisons with group and national norms, important though those may be. They have perfected a vast body of diagnostic tests and remedial instruments in the various school subjects. Consider reading as a single example.

In our modern world of words, reading is without doubt the most important tool of learning in the school as well as the most crucial skill for general everyday competence. Within the school itself no progress in history, the social studies, mathematics, the sciences, literature and the languages is possible without at least a moderate proficiency in reading. Outside the school it is the chief means by which the citizen becomes informed of current events, the affairs and problems of the community, state, nation and modern world. It is the basis of success in many of the occupations and is, of course, one of our chief leisure activities.

For these reasons a tremendous amount of creative, scientific energy has gone into the study of reading abilities and the techniques of improving reading instruction in the schools. Elaborate analyses have been made of the major kinds of reading, distinguishing, for example, oral and silent reading of leisure types and of work types, essential

skills in work-study types of reading and methods of developing them, such obstacles to good reading as low intelligence, visual and auditory defects, defects of muscular coordination, speech and glandular disturbances, emotional factors, disturbing home environments, and so forth. As a consequence of these voluminous studies in reading, many diagnostic and remedial tests have been developed, such as those listed below:

The Gates Reading Readiness Test

Diagnostic tests in oral reading, such as the Gray Standardized Oral Reading Paragraphs or the Gray Oral Reading Check Tests.

Iowa Silent Reading Tests, New Edition, Elementary

Corrective exercises in reading, such as the remedial devices by Gates, Gray, Horn, and others.

Reading is without a doubt one of the basic areas contributing to the understanding and development of mathematical understanding. The transfer value should be emphasized and the teacher should thoroughly understand that teaching is most effective and in the matrix of a situation which grips the learner, which is to him vital and worthwhile.

B. Criteria for Estimating Growth in Group Interaction

1. Amount of growth in the quality of participation--growth therein.
2. Growth in the ability of the group to isolate, clarify and agree upon a problem.
3. Growth in the ability to plan and organize the task.
4. Improvement in the quality of group summaries as the job progressed.
5. Growth in the ability of the group to transact the business of the task as defined and planned.
6. Group or "we" feeling:
   a. Growth of facility in communication
   b. Growth in freedom to disagree
   c. Growth in the ability to give criticism courteously and accept it graciously
   d. Growth of the individual in a feeling of greater security from working with a group rather than as an individual

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