Names: Louis B. Casey and Ben Barber Thaxton

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Institution: Oklahoma State University

Location: Stillwater, Oklahoma

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Scope of Study: Suggestions and ideas for this paper were obtained from the following sources: reports of committees on improving the teaching of physics; articles in educational journals; discussions with experienced physics teachers; analysis of the problems in several high school physics texts and in one first-year college text; and discussions with first-year college physics students. This paper contains several of the basic mathematical principles and skills which the above mentioned sources indicated are essential for one to succeed in high school physics. The principal topics discussed are the following: scientific notation, development of formulas, proportion, variation, and methods of problem solving. Illustrative examples are given to clarify each topic. Included are most of the formulas used in high school physics with pertinent information concerning each formula, and a table of important units of measurement and their equivalents.

Conclusion: Lack of an adequate mathematical preparation is a problem common to most physics classes. This problem can be greatly alleviated by teaching a few of the fundamental principles and skills that are necessary for solving physics problems. Since physics is essentially a problem solving course, it is the responsibility of the physics teacher to teach these principles and skills to his students.

James H. Zant

SUGGESTIONS FOR IMPROVING THE MATHEMATICAL PREPARATION OF HIGH SCHOOL PHYSICS STUDENTS

by

Ben B. Thaxton
Bachelor of Science
Panhandle A. & M. College
Goodwell, Oklahoma
1950

and

Louis B. Casey
Bachelor of Science
Oklahoma State University
Stillwater, Oklahoma
1941

Submitted to the faculty of the Graduate
School of the Oklahoma State University
in partial fulfillment of the require-
ments for the degree of
Master of Science
May, 1958

SUGGESTIONS FOR IMPROVING THE MATHEMATICAL

PREPARATION OF HIGH SCHOOL

PHYSICS STUDENTS

Study Approved:

Study Adviser

Dean of the Graduate School

PREFACE

Several factors have prompted us to attempt a project that would be of direct and practical value to the teacher of physics and/or mathematics. We believe that a definite program of mathematical training for the prospective physics student is badly needed. An inadequate mathematical background leads to failure in physics and a consequent lack of interest.

In this paper are many of the basic formulas that should be an important part of the high school mathematics courses. Included, also, are numerous examples of the various types of verbal problems from physics with a suggested method of analysis. The use of this material in the mathematics courses would generate more interest and make the mathematics courses more meaningful.

Perhaps the most important part of the paper is that which deals with a review of basic mathematical principles and skills. This material should precede the work in physics. A review of this material would emphasize the importance of mathematics in physics and would prepare the students to solve the problems that he will encounter in his study of physics.

We make no claim to originality, except perhaps to the arrangement of materials. We acknowledge the valuable assistance and suggestions from our colleagues in the National Science Foundation Institute and especially to Dr. James H. Zant, the Director. Also, we are grateful to Dr. Frank M. Durbin, Professor of Physics at the Oklahoma State University, from whom we received many important suggestions.

Finally, we express our sincere appreciation for this year of academic training made possible by the National Science Foundation.

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CHAPTER I

INTRODUCTION

It is obvious that students who have an inadequate background in mathematics are not going to succeed in physics. The Joint Committee A.I.P., A.A.P.T., N.S.T.A., on Teaching Materials makes the following statements about high school physics:

> "There should be more use (than at present) of algebraic, trigonometric, and geometric expressions and of other simple mathematical techniques. They (texts on physics) make a fetish of avoiding even the simplest mathematical reasoning."[1]

There is, the writers feel, an urgent need for a definite and purposeful attempt to insure adequate mathematical preparation of potential physics students.

The purpose of this report is to prepare materials that will help to improve the mathematical preparation of high school physics students. The use of this material should facilitate the development of an adequate mathematical background for those students who plan to study the physical science courses in high school, especially physics. An auxiliary result should be an increased interest in mathematics.

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[1]Fred T. Pregger, Chairman, "High School Physics," A Report of the Joint Committee on High School Teaching Materials, Physics Today, Vol. 10, No. 1, (January, 1957), pp. 20-21.

One reason for the lack of interest in mathematics may be attributed, in a large measure, to the inability of students to see any practical use for mathematics. At the high school level, many students must see the purpose for learning something if they are to be interested and to do their best work. Remarks by parents, educators, and other adults to the effect that they have never used any mathematics beyond arithmetic tend to develop attitudes in youngsters that dull their interest in algebra, geometry, and trigonometry. As a consequence, many students fail to take the mathematics courses offered, or if they do take them, they do not try to learn much. Failure to see and to appreciate the relationship between mathematics and science has, no doubt, been a very large factor in the lessening of interest in mathematics. The beginning science courses in high school, that is, general science and biology, have very little mathematics and often the student develops the impression that there is little or no connection between science and mathematics. In the later courses, chemistry and especially physics, which require a reasonably good grasp of high school mathematics, the student finds that he cannot succeed because he is unable to solve the problems. By this time, it is usually too late for him to overcome his deficiency in mathematics, and rather than repeat certain courses or take additional courses in mathematics, he decides to study something requiring less mathematics.

Use of the material suggested here should reduce the number of failures in physics due to inadequate mathematical preparation. Some of the weaknesses in mathematical preparation which the use of this material should correct are described by the staff members of the Department of Physics, University of Nebraska, in their paper, Secondary School Preparation for College Physics Students. They list the following as weaknesses in mathematical preparation for college physics students:

1. Students are afraid of mathematics, so that they react negatively to the mathematical treatment of a physical concept. It is felt that mathematics as a tool in thinking is avoided in the public schools, and that this provides a disservice to the terminal student as well as the college-bound student. The habitual use of mathematics as an analytical tool in the understanding of concepts in junior high school and high school could help overcome this difficulty.

2. Students do not have the ability to analyze a problem. Often a student can memorize and manipulate formulae and substitute numbers in formulae, but hits a stone wall when he is required to use a formula to explain a physical situation. It is felt high school students should be given problems complicated enough to require some analytical thinking, and less emphasis should be put on learning to work a certain type of problem by memorizing rules.

3. Students have little understanding of mathematical manipulations. These students are apt to fail in physics. Others can perform most of the mathematical manipulations required, but have little understanding of these processes. These students make a better showing, but make many errors, and are usually not adept in analyzing problems and situations. The best student is no doubt the one who can perform the manipulations as second nature and, in addition, understands them well.[2]

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[2]"Secondary School Preparation for College Physics Students." An opinion of Staff Members of the Department of Physics, University of Nebraska.

They make the following statement concerning science courses in the high school:

> "Concerning science courses in high school, the department members agree that it is not essential for the physics student to have had high school courses in science--that it is of far more importance that mathematical background be firmly established. However, it is realized that most students develop strong interests in the secondary schools, and the role which science courses play in helping to develop these interests is not to be minimized."[3]

The statements concerning the mathematical preparation of college physics students are equally apropos to the preparation of high school students.

Selection of materials for this paper has been based on a careful analyses of the problems in five typical high school physics texts. Also, the problems in a first-year college physics book were analyzed to determine the mathematics prerequisite for successfully solving the problems.

The writers used their own judgment in the selection of topics; however, they received many suggestions from teachers of high school physics. The writers suggest that the second chapter of this material be covered thoroughly the first few days of the physics course. The other material may be taken up at any appropriate time in the course.

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[3]Ibid.

CHAPTER II

REVIEW OF BASIC MATHEMATICS FOR PHYSICS

The following questions in science must be answered by recrouse to mathematics; How much? How many? How large? How long a time? The mathematical skills and knowledge necessary to find the answers to these questions in physics are often lacking in students beginning a course in high school physics. Either they never did know or they have forgotten.

> "The science teacher must assume some of the responsibility for reteaching and for presenting new material regarding applications of processes and principles as well as symbols and formulas needed."[4]

Mathematical skills and principles dealing with ratio and proportion, scientific notations, etc., that are considered essential for success in a well-taught high school physics course will be discussed in some detail in the following pages.

RATIO AND PROPORTION

Since a great number of problems in physics involve ratio and proportion, it is important that this type of an equation be thoroughly understood.

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[4]James H. Zant, "Mathematics in Science," Science Education, Vol. 25, No. 6, (November, 1941), p. 335.

The ratio of two numbers a and b is the quotient of a divided by b or $\frac{a}{b}$. A statement of the equality of two ratios is called a proportion, thus:

$$\frac{2}{3} = \frac{4}{6}\ ;\ \frac{x}{5} = \frac{8}{15}\ ;\ \frac{a}{b} = \frac{c}{d}$$

All of these are statements of equality of ratios and are, therefore, proportions. Since proportions are algebraic equations, they may be rearranged in accordance with the laws of algebra, that is, we may add, subtract, multiply, or divide both sides by the same quantity without changing the equality. For example, suppose we wish to solve the following proportion for x:

$$\frac{x}{a} = \frac{b}{c}$$

(1) Multiply both sides by a.

$$\frac{\overset{1}{\cancel{a}}x}{\underset{1}{\cancel{a}}} = \frac{ab}{c}$$

The a's in the left member of the equation cancel.

$$x = \frac{ab}{c}$$

Notice that multiplying both sides of the equation by a removed the a from the denominator of the ratio on the left side of the equation and introduced an a into the numerator of the ratio on the right side of the equation. From this, we may deduce the following rule: A factor of the denominator of either ratio may be replaced by one if the factor is written into the numerator of the other ratio, and conversely, a factor of either numerator may be replaced by the number one and written into the denominator of the other ratio.

This procedure is illustrated below with a more complicated problem: For example, solve the following equation for v.

$$\frac{mv^2}{4.27 \times 10^5} = \frac{m(6.6 \times 10^8)\ (6.03 \times 10^{27})}{(4.27 \times 10^5)^2}$$

The problem is to isolate the v^2 on the left side of the equation, then take the square root of both sides of the equation. The m from the numerator of the left member is moved to the denominator of the right member. The quantity 4.27×10^5, in the denominator of the left member is moved to the numerator of the right member, thus:

$$v^2 = \frac{\not{m}(6.6 \times 10^8)\ (6.03 \times 10^{27})\ \cancel{(4.27 \times 10^5)}}{\not{m}\cancel{(4.27 \times 10^5)^2}\ (4.27 \times 10^5)}$$

The m's cancel and the 4.27×10^5 cancels with one of the factors of the squared quantity in the denominator. Taking the square root of both sides,

$$v = \sqrt{\frac{(6.6 \times 10^8)\ (6.03 \times 10^{27})}{(4.27 \times 10^5)}} = \sqrt{\frac{6.6 \times 6.03 \times 10^{30}}{4.27}}$$

$$= 10^{15}\sqrt{\frac{6.6 \times 6.03}{4.27}}$$

It is seen that moving quantities from the denominator of one member of the equation to the numerator of the other member of the equation enables one to simplify a problem rather quickly. Once the mechanics of this simple procedure is grasped, one can simplify proportions rapidly. The student should notice that powers of 10, like any other quantity, may be moved from numerator to denominator or vice versa by changing the sign of the exponent. Powers of 10 are combined by adding their exponents. Careful study of the above example will make this process clear.

SCIENTIFIC NOTATION

Many of the problems in physics involve very large numbers or very small numbers. The student must learn to write these numbers in a convenient manner or he will have difficulty manipulating the numbers. A review of the scientific notation is appropriate at this time.

From arithmetic, it is known that division by 10, 100, 1000, etc., moves the decimal one place, two places, or three places respectively, to the left. Whereas multiplication by 10, 100, 1000, etc., moves the decimal to the right in like manner.

$10^0 = 1$	$10^0 = 1$
$10^1 = 10$	$10^{-1} = 1/10 = .1$
$10^2 = 100$	$10^{-2} = 1/100 = .01$
$10^3 = 1000$	$10^{-3} = 1/1000 = .001$
$10^4 = 10,000$	$10^{-4} = 1/10000 = .0001$
etc.	etc.

It is seen from the preceding figures that multiplying by 1/10, 1/100, 1/1000, etc., is the same as dividing by 10, 100, 1000, etc., therefore, multiplying by 10 to any positive power moves the decimal point the number of places to the right indicated by the exponent. The number 660,000,000 may be written as 6.6×10^8 or 66×10^7 or 660×10^6. In each case, the exponent of 10 indicates the number of places the decimal point has been moved to the left. Now every positive number can be written in the form $n \times 10^k$ where n is greater or equal to 1 but less than 10, and k is a positive or negative integer, or zero. To change a positive number given in decimal form, to this form, first write the non-zero digits of the number and place the decimal point after the first such digit counting from the left. A decimal point thus placed is said to be in standard position. Then k is the number of places the decimal point in the original number is removed from the standard position. The exponent k will be positive or zero ($6.6429 = 6.6429 \times 10^0$) if the original number is greater than 1 and negative if the original number is less than 1. Writing a number in this manner enables one to ascertain rather easily the magnitude of the product. When a number is written in this manner, the integer k is the characteristic of the logarithm of the number. Another advantage is that it enables one to locate the decimal point accurately when using the slide rule.

The advantages of this notation are illustrated by several examples.

To calculate the value of the acceleration due to the force of gravity on an object 2000 miles from the earth, this formula is used:

$$mg = \frac{G\ mM}{r^2} \qquad \text{or} \qquad g = \frac{G\ M}{r^2}$$

Given:

$G = 6.66 \times 10^{-8}$ cgs units (G is the constant of proportionali

$r = 6000$ miles $= 6000 \times 5280 \times 12 \times 2.54$ centimeters.

M (mass of earth) $= 6.03 \times 10^{27}$ grams

Find: g (the acceleration of a body due to the force of gravity.

Substituting in the equation and writing the numbers without using the scientific notation gives this equation:

$$\overset{1}{\cancel{m}}g = \frac{(.000000066)\ (\overset{1}{\cancel{m}})\ (6{,}030{,}000{,}000{,}000{,}000{,}000{,}000{,}000{,}000)}{[(6000)\ (5280)\ (12)\ (2.54)]^2}$$

Dividing both sides by m leaves the factor 1 in their places. Now the equation is written using the scientific notation.

$$g = \frac{(6.6 \times 10^{-8})\ (6.03 \times 10^{27})}{[(6 \times 10^3)\ (5.28 \times 10^3)\ (1.2 \times 10^1)\ (2.54)]^2}$$

$$= \frac{3.98 \times 10^{20}}{(9.07)^2 \times 10^{16}}$$

$$= \frac{3.98 \times 10^{20}}{8.23 \times 10^{17}} = \frac{3.98}{8.23} \times 10^3$$

$$= 4.84 \times 10^2 \qquad = 484 \text{ cm/sec.}^2$$

It is obvious that much time and trouble is saved by using the scientific notation. Also, there is much less chance of an error. Using simpler illustrations, the use of the scientific notation will be shown in detail.

One of the very great advantages of using the scientific notation is that it enables one to locate the decimal point accurately. Looking at the problem below, it is rather difficult to tell by inspection where the decimal point would be located in the answer.

$$\frac{.00042 \times 756}{1400}$$

Now this is written using the scientific notation.

$$\frac{(4.2 \times 10^{-4})\ (7.56 \times 10^{2})}{(1.4 \times 10^{3})}$$

It can be seen that 1.4 will go into 4.2 three times and that three times 7.56 is roughly 22 or 23. the answer is roughly 22 times 10 to some power. The actual answer will be $3 \times 7.56 \times 10^{?}$. How may the power of ten be determined? One merely has to remember and apply one of the basic rules of exponents, that is, any factor to a power may be moved from the numerator to the denominator of a fraction by changing the sign of the exponent. $m^2 = \frac{1}{m^{-2}}$ In the above problem, take the 10^3 from the denominator and place it in the numerator and change the sign of the exponent to minus. Then 10^{-4}; 10^2; and 10^{-3} is in the numerator. Adding the exponents -4, $+(-3)$, $+2 = -5$. The answer is 22.68×10^{-5} or $2.268 \times 10^{-4} = .0002268$.

The scientific notation is also very helpful when finding square roots of very large or very small numbers. Suppose one wishes to find the square root of..0000000016. Using the scientific notation this would be written as

$$\sqrt{1.6 \times 10^{-9}}$$

This is = to $\sqrt{16 \times 10^{-10}}$

The latter form is the form needed for this reason: Finding the square root of a number to a power is merely a matter of dividing the exponent by two. If the exponent is an even number, the exponent of the square root will be an integer; thus, fractional exponents are avoided in taking the square root when the exponent is an even number. Going back to the problem of finding the square root of 16×10^{-10} the exponent -10 is divided by two which gives -5 for the exponent of the square root of 10^{-10}. Now all that is necessary to finish the process is to take the square root of 16. The answer, then, is 4×10^{-5}. Had the form 1.6×10^{-9} been used, the answer would have been $1.265 \times 10^{-4\frac{1}{2}}$ which is the same answer, but this answer would have required the use of log tables to evaluate.

CONSTANTS, VARIABLES, AND VARIATION

In the formulas in physics, as well as in other branches of science, some of the symbols are intended to represent fixed numbers, while to other symbols various values may be assigned according to the conditions of the problem.

A symbol which, throughout a discussion, represents a fixed number incapable of change is called a constant. A symbol which, throughout a discussion, may assume different values according to some rule, is called a variable. For example: the area A of a circle in terms of its radius is given by the formula $A = \pi r^2$. The number represented by the symbol π is a constant, but r and A are variables, but must be positive real numbers in this case. π is an absolute constant since it has the same value in all discussions. Arbitrary constants retain the same values throughout any one discussion.

If two quantities, x and y, are related in such manner that however the value of x may change, within its domain, a corresponding value of y is determined and the ratio of y to x is always constant, then y is said to vary directly as x. That is, y varies directly as x if $\frac{y}{x} = k$, or $y = kx$, k being a constant. The constant k is called the constant of variation or the proportionality factor. The amount of energy that is released when matter is converted to energy varies directly with the mass. In equation form, this relationship would be written as:

$$E \propto m \qquad \text{or} \qquad E = km$$

To write the relationship without the variation symbol, a constant has to be introduced. This equation shows that if the mass is doubled, the amount of energy would be doubled, i.e., the energy varies directly as the mass. Another type of variation commonly encountered in physics is called inverse variation. If two quantities, x and y, are so related that however their values may change their product equals a constant, then the two quantities are said to vary inversely. In other words, if the quantity $\underline{y}$ increases as the quantity $\underline{x}$ decreases, then the two quantities are said to vary inversely. The equation expressing the relationship between the pressure and volume of an ideal gas illustrates inverse variation.

$$PV = K \qquad \text{or} \qquad P = \frac{K}{V}$$

If the volume is doubled, the pressure decreases to one-half the original amount. If the volume is reduced by one-half, the pressure must be doubled. Many formulas in physics involve more than two variables, some of which may vary inversely and others which may vary directly. For example, the formula expressing the relationship between the gravitational attraction between two bodies and the distance between them illustrates both direct and inverse variation.

$$F = \frac{Km_1m_2}{d^2}$$

This formula states in mathematical form that the force of attraction between two bodies of masses m_1 and m_2 varies directly as the product of their masses and inversely as the square of the distance between their centers of gravity.

K is the constant of proportionality. It cannot be emphasized too strongly the importance of the ability of the student to translate formulas into a language expression and vice versa, i.e., to translate language expressions of relationships into a mathematical equation. Suppose one knows that the volume of a gas varies directly as the absolute temperature and inversely as the pressure. In mathematical symbols, these expressions could be stated as follows:

$$V \propto t \qquad \text{and} \qquad V \propto \frac{1}{p}$$

Putting the two statements together would give:

$$V \propto \frac{t}{p}$$

The variation symbol may be replaced by an equal sign if a constant is used.

$$V = \frac{kt}{p}$$

K is the constant of proportionality for a given mass of gas. The student will learn later that this constant is equal to $\frac{1}{273}$, for an ideal gas.

The value of constants in physics are usually determined from experimental data. Occasionally the student may be given experimental data from which he is to ascertain the value of a constant to solve a similar problem. The following problem will illustrate this procedure:

The centripetal force required to constrain the motion of a particle to a circle varies directly as the square of the velocity V and inversely as the radius r. If the force is 80 when V = 9 and r = 4, find the force when V = 10 and r = 4.5. First an equation is written expressing the relationship between the quantities.

$$f = \frac{KV^2}{r}$$

Here, again, K is the constant of proportionality for the mass involved. Putting in the equation the values given for the various quantities enables one to solve for K.

$$80 = \frac{K \times 9^2}{4}$$

$$4 \times 80 = 81\ K$$

$$K = \frac{4 \times 80}{81} = \frac{320}{81}$$

Having found the value of K from experimental data, this value can now be used to solve other problems of the same nature. The value of K can now be used to find the force required when $V = 10$ and $r = 4.5$.

$$f = \frac{\frac{320}{81} \times 10^2}{4.5} = \frac{32000}{81 \times 4.5} = 87.79 -$$

Later the student will encounter this formula in physics and will find it written like this:

$$f = \frac{MV^2}{r}$$

The student will notice that there is no symbol for a constant. That is because the units have been chosen to make the constant equal to one. In order to have the constant equal to 1, the force has to be in dynes, the mass in grams, the velocity in centimeters per second, and the radius in centimeters.

The procedure for developing a formula will be illustrated by the following:

The factors involved in centrifugal force are the mass, radius, and velocity. The general character of the variation can be determined by three simple experiments. The students whirl weights at the end of strings and note how hard the string pulls on their fingers. The experiment is varied as follows:

1. Rubber stoppers of different weights but with strings of the same length are whirled at the same rate of speed.

2. A single stopper is whirled at different rates of speed but with the string of constant length.

3. A single stopper is whirled with as nearly as possible a constant linear speed but with varying lengths of string.

It will be obvious to the student that as the mass is increased, the centrifugal force is increased, therefore, this relationship may be stated symbolically as,

$C.F. \propto M.$, i.e., centrifugal force varies directly as some power of the mass.

It is also obvious that the centrifugal force increases as the radius decreases provided the other factors remain unchanged. Thus:

$C.F. \propto \frac{1}{r}$ The centrifugal force varies inversely as some power of the radius. Since the centrifugal force increases with increasing velocity, another equation can be written:

$C.F. \propto V$ The centrifugal force varies directly as some power of the velocity. In each of the above equations, it is assumed that the other quantities are constant.

All these statements can be written together as,

$$C.\ F. \propto \frac{MV}{r}$$

The exponents of M, V, and r are not known from the experiments. These exponents could be determined by careful experiments or they may be determined by mathematical processes from known relationships. A method for determining the exponents mathematically will now be explained.

A general equation involving these three variables and centrifugal force might be:

(1) $C.F. = KM^xV^yr^z$

Notice that r does not have to be in the denominator since its exponent may have any value, negative as well as positive. The problem becomes one of assigning values to x, y, and z so that the product of the three quantities may be expressed in terms of three basic quantities or dimensions--length, mass, and time (L,M,T). Each of the quantities M, V, or r can be expressed in terms of the basic dimensions. Because force equals mass times acceleration, centrifugal force has the dimensions:

$$\text{Centrifugal Force} = M \times \frac{L}{T^2} = M^1L^1T^{-2}$$

Mass cannot be expressed more simply, velocity has the dimension $\frac{L}{T}$, and the radius has the dimension L. Hence, since K, a constant, has no physical dimensions, equation (1) $C.F. = KM^xV^yr^z$ may be written,

(2) $M^1L^1T^{-2}$ and this equals $M^x \left(\frac{L}{T}\right)^y L^z$

or

(3) $M^1L^1T^{-2} = M^xL^{y + z}T^{-y}$

By comparing the exponents of M, L, and T, the following equations in x, y, and z may be obtained:

$1 \text{ (unity)} = x$

$1 \text{ (unity)} = y - z$

$-2 = -y$

Hence, solving, $x = 1$, $y = 2$, $z = -1$

Substituting these values of x, y, and z in equation (1) gives

$$(4)\quad C.F. = KM^1V^2r^{-1} \quad \text{or} \quad \frac{KMV^2}{r}$$

Subsequent work can establish the fact that with the proper consistent units, k has the value of 1, or convert the equation into the form more useful for high school use.

$$C.F. = \frac{WV^2}{gr}$$

EQUATIONS

Methods of writing equations have been discussed during the last few pages. To succeed in physics students must be able to translate verbal statements into mathematical statements. He should be able to translate a statement such as, "The volume of a gas varies inversely as the pressure if the temperature remains constant," or, "The volume of two gases is inversely proportional to the pressures," into the algebraic equivalent:

$$V_1 = \frac{K}{P_1} \quad \text{and} \quad V_2 = \frac{K}{P_2}$$

$$\text{then} \quad \frac{V_1}{V_2} = \frac{\frac{K}{P_1}}{\frac{K}{P_2}}$$

$$\text{or} \quad \frac{V_1}{V_2} = \frac{\cancel{K}}{P_1} \times \frac{P_2}{\cancel{K}} = \frac{P_2}{P_1}$$

In general, the student should see that the equation

$$\frac{A_1}{A_2} = \frac{B_1}{B_2}$$

is the algebraic statement of direct proportion while if the order of subscripts of either A or B is changed, the resulting expression such as

$$\frac{A_1}{A_2} = \frac{B_2}{B_1}$$

indicates an inverse proportion, i.e., as one quantity increases, the corresponding quantity decreases. The student should have the ability to go from expressions such as $A \propto B$ and $A \propto C$ to the equation $A = KBC$; or from expressions $A \propto D$ and $A \propto \frac{1}{E}$ to the equation $A = \frac{KD}{E}$. The student should recognize in $A \propto B$ an alternative expression for direct variation and in $A \propto \frac{1}{E}$ an alternative expression for inverse variation.

To develop an understanding of the use of literal numbers, frequent use of symbols other than x for the unknown should be used. In this connection, it is helpful to use the first letter of the word, such as D for dollars, A for area, F for force, V for velocity, etc. All quantities should always be labeled as to units. The writers suggest that the student memorize a particular form of an equation and solve the equation for any particular quantity when the need arises. In memorizing this form, the student should state the equation in words. The equation expressing the relationship between the time for a complete oscillation of a pendulum and its length may be written as

$$T = 2\pi\sqrt{\frac{L}{g}}$$

In words, this equation states that the time in seconds for a complete oscillation of a pendulum varies jointly as the product of 2π and the square root of the length and varies inversely as the square root of the acceleration of gravity. It is understood that *L* and *g* are expressed in the same units.

> "A physics formula is a shorthand notation of the relationship that exists between physical quantities, and this idea should be read into every equation."[5]

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[5]Frank M. Durbin, *Introduction to Physics*, (Prentice-Hall, Englewood Cliffs, N.J., 1955), p. 7.

CHAPTER III

HIGH SCHOOL PHYSICS FORMULAS

Most of the basic formulas in high school physics will be given in this section with comments upon their meaning where the writers feel that more than a mere statement of the formula is needed. The significance of units in which quantities are expressed cannot be over-emphasized. Students should understand that all units of measurement are determined, either directly or indirectly from three fundamental units. The three fundamental units are the units of time, length, and mass. For example, velocity is $\frac{L}{T}$, the L can be in any unit of length and the T can be in any unit of time. With reference to the velocity of an automobile, L is usually expressed in miles and T in hours. In other countries, however, L is in kilometers. As has been pointed out, relationships exist between various quantities studied in physics that can be expressed in mathematical language. A formula is a brief statement of the relationship between the quantities represented in the formula. To use the formulas effectively, the student must be able to state the formula in words and he must know what units the formula is based on. For example the formula $f = ma$ expresses a direct relationship between force, mass and acceleration. This formula is true only when the proper units are used. This formula is true when the force is in dynes, the mass in grams, and the acceleration in centimeters per second per second. It is also true when the force is in poundals, the mass in pounds, and the acceleration in feet per second per second. It is, therefore, extremely important that the student understand that a particular formula requires particular units for the quantities involved. The formulas listed below include nearly all that a student will encounter in physics.

FORMULA	MEANING
(1) $f = ma$	This equation states that the force varies jointly as the mass and acceleration. Where these quantities are expressed in proper units, the force is equal to the product of the mass and acceleration. As has been stated before, units are invented or chosen such as to make the constant equal to one in the formula. Many units of measurements in physics have been invented for this purpose. In this formula, the quantities may be expressed in these units: f in dynes or poundals m in grams or pounds a in cm./Sec.^{2} or ft./Sec.^{2}
(2) $f_c = \frac{mv^2}{r}$	Centripetal force equals the product of the mass and the square of the velocity,

FORMULA	MEANING
	divided by the radius of the circle, where f is in dynes m is in grams v is in centimeters per second r is in centimeters
(3) $f_g = \frac{Gmm'}{d^2}$	This is a statement of the law of gravitational attraction between two bodies. It states that every body in the universe attracts every other body with a force that is proportional to the product of the masses of the bodies and inversely proportional to the square of the distance between their centers of gravity, where m and m' are the masses of the two bodies, d is the distance between their centers of gravity, and G is the constant of proportionality. The value of G has been determined by experimentation.
(4) $\vec{f} = k\overset{\downarrow}{F}$	The force of friction is equal to the coefficient of friction times the force pressing the surfaces together. The arrows near the symbols indicate that f and F are at right angles to one another, f (friction) being parallel to the surface separating the objects and F being at right angles to it.

	FORMULA	MEANING
(5)	$v = gt$	The velocity in feet per second of a freely falling body is equal to the acceleration due to gravity times the time in seconds.
(6)	$s = \frac{1}{2}gt^2$	The distance a freely falling body will fall in <u>t</u> seconds equals one-half the acceleration due to gravity times the time in seconds squared.
(7)	$v = \sqrt{2gs}$ $= \sqrt{(2g)(\frac{gt^2}{2})}$ $= gt$	This formula for the velocity of a freely falling body in terms of <u>s</u> is derived from (6). Since $gt = v$, or $\frac{ds}{dt} = v = gt$ in terms of <u>t</u>, formula (6) may be written $s = \frac{1}{2}vt$ or solving for <u>v</u> this equation becomes $v = \frac{2s}{t}$. Then if equation (5) or (6) is solved for <u>t</u> and substituted in the formula $v = \frac{2s}{t}$, the resulting formula is $v = \sqrt{2gs}$. NOTE: The acceleration due to gravity is approximately 32 feet/Sec.^{2} or 980 cm./Sec.^{2}
(8)	$a = \frac{v^2}{r}$	A body moving in a circle at constant speed has an acceleration equal to the square of the speed divided by the radius.
(9)	$a_{ave.} = \frac{v_f - v_i}{t}$	The average linear acceleration of an object equals the final velocity minus the

FORMULA	MEANING
	initial velocity divided by the time. It is assumed that the acceleration is uniform over the time interval.
(10) $a_{ave.} = \frac{\Delta v}{\Delta t}$	Equation (9) may be written in this form. The Δ sign means a small change in the quantity before which it is placed. It is called an increment.
(11) $a_{inst.} = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t}$	This equation states that the instantaneous acceleration is equal to the ratio of the increment of the velocity to the corresponding increment of the time, when the increment of the time approaches zero.
(12) $D = \frac{W}{V}$	The density of an object equals the weight divided by the volume.
(13) $SpG = \frac{\text{D of Sub.}}{\text{D of water}}$	The specific gravity of a substance equals the density of the substance divided by the density of water.
(14) $SpG = \frac{\text{Wt. of Sub.}}{\text{Wt. of equal volume of water}}$	Equation (13) may be stated in this form. The specific gravity of a substance equals the weight

FORMULA	MEANING
	of the substance divided by the weight of an equal volume of water.
(15) $P = \frac{F}{A}$	Pressure is force per unit area, therefore it is equal to the force divided by the area over which the force is distributed.
(16) $P = HD$	The pressure of a liquid is equal to the product of the height and density.
(17) $\frac{F}{f} = \frac{A}{a}$ or $\frac{DD^2}{d}$	The ratio of the forces on two pistons equals the ratio of the areas of the pistons (or the diameters squared if they are circular pistons.) This equation state's Pascal's Law. Pressure applied to a confined liquid is transmitted undiminished throughout the entire liquid.
(18) $PV = K$	For a given mass of gas, the pressure times the volume equals a constant, if the temperature remains the same.
(19) $P_1V_1 = P_2V_2$	This is another way of stating (18). The pressure times the volume for a given mass of gas equals the new pressure times the

FORMULA	MEANING
	new volume, if the temperature remains the same.
(20) $\frac{P_1V_1}{K_1} = \frac{P_2V_2}{K_2}$	This equation expresses the relationship between the pressure, volume, and the absolute temperature of an ideal gas.
(21) $\vec{W} = \vec{Fd}$	Work equals the product of the force which is acting and the distance through which it acts. The arrows indicate that that force must act in the direction of the motion.
(22) $P = \frac{W}{t}$ or $\frac{Fd}{t}$	Power is the rate of doing work and is found by dividing the work by the time.
(23) $H.P. = \frac{Ft.\text{-}lb.}{(33,000)(min.)}$	A horsepower equals 33,000 foot pounds of work per minute, thus, horsepower may be found by dividing the foot pounds of work done by 33,000 times the number of minutes involved in doing the work.
(24) $M.A. = \frac{R}{E}$	The mechanical advantage of a machine equals the resistance divided by the effort.

FORMULA	MEANING
(25) $Fa = Ea$	This equation refers to the lever and states that the force times the length of the force arm equals the effort times the length of the effort arm.
(26) $Fd = fD$	The work done by a machine equals the energy supplied to it. The force F the machine exerts, times the distance d through which F operates equals the force f applied to the machine, times the distance D through which the force f operates.
(27) $C_1 = \frac{L_2 - L_1}{L_1(t_2-t_1)}$ or $C_1 = \frac{\Delta L}{L_1 \Delta t}$	The coefficient of linear expansion of a solid is the amount which a unit length of a solid expands per degree rise in temperature. The two formulas given express this relationship mathematically.
(28) $C = \frac{5}{9}(F - 32)$	To change Fahrenheit temperature to centigrade, subtract 32 from the Fahrenheit and multiply by $\frac{5}{9}$.
(29) $v = fL$	The velocity of a sound wave equals the frequency times the wave length.
(30) $B = f_1 - f_2$	The number of beats per second produced by two sounds of different frequencies is equal to the difference of their frequencies.

FORMULA	MEANING
(31) $n' = \frac{nV}{V \pm v}$ (when source moves)	Equations (31) and (32) are mathematical statements of the Doppler effect. In these equations n' is the apparent pitch or observed frequency; n is the true pitch or frequency; V is the speed of sound; and v is the speed of moving body. In equation (31) the negative and positive signs refer respectively to the approach and recession of the sounding body. In equation (32) the positive and negative signs refer respectively to the approach and recession of the observer.
(32) $n' = \frac{n(V \pm v)}{V}$ (when observer moves)	
(33) $W = EI$	Watts equals the potential in volts times the current in amperes.
(34) $W = I^2R$	This equation is derived from (33) by substituting IR for E.
(35) $E = IR$	The potential in volts equals the current in amperes times the resistance in ohms.
(36) $H = .24I^2Rt$ (since I^2R equals watts, this formula may be written $H = .24Wt$)	This formula gives the number of calories of heat generated when I refers to amperes; R to ohms; and t to seconds.
(37) $\frac{1}{R} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}$, etc.	This is the formula for resistances in parallel.

FORMULA	MEANING
(38) $R = r_1 + r_2 + r_3$, etc.	The formula for resistances in series.
(39) $\frac{V_p}{V_s} = \frac{T_p}{T_s}$	This equation states that in a transformer, the ratio of the voltage in the primary winding to the voltage in the secondary winding equals the ratio of the turns of the primary winding to the number of turns in the secondary winding.
(40) $E_p I_p = E_s I_s$	Neglecting losses, the input in watts equals the output in watts.
(41) $I_e = \frac{I_m}{\sqrt{2}} = .707\ I_m$	The effective amperage in an alternating current equals the maximum amperage divided by the square root of two.
(42) $E_e = \frac{E_m}{\sqrt{2}} = .707\ E_m$	The effective potential of an alternating current equals the maximum potential divided by the square root of two.
(43) $X_L = 2\pi fL$	The inductive reactance of an alternating current where <u>f</u> equals frequency and <u>L</u> equals inductance in henrys.
(44) $X_C = \frac{1}{2\pi fC}$	The capacitive reactance of a circuit when f = frequency and C equals the capacitance in farads.

FORMULA	MEANING
(45) $V = I\sqrt{R^2 + \left(2\pi fL - \frac{1}{2\pi fC}\right)}$	When a circuit contains resistance, capacitance, and inductance in series, the voltage is given by this formula.
(46) $E = IZ$	The potential of an alternating current in volts equals the product of the current in amperes and the impedance in ohms.
(47) $\frac{1}{C} = \frac{1}{c_1} + \frac{1}{c_2} + \frac{1}{c_3}$, etc.	This is the formula for calculating the total capacitance of condensers in series.
(48) $C = c_1 + c_2 + c_3$, etc.	This is the formula for calculating the total capacitance of condensers connected in parallel.
(49) $\frac{E_s}{d_s^2} = \frac{E_x}{d_x^2}$	This is the formula for finding the candle power of an unknown lamp by the use of the Bunsen photometer. It states that the candle power of a known lamp divided by the distance squared from the lamp equals the candle power of an unknown lamp divided by the distance from it squared.

FORMULA	MEANING
(50) $\frac{1}{f} = \frac{1}{D_o} + \frac{1}{D_i}$	The relationship between the distances of the object and of the image with the focal length is given by this formula.
(51) $\frac{L_i}{L_o} = \frac{D_i}{D_o}$	The relation between the size of the object and of the image is given by this formula.
(52) $I_r = \frac{\text{sine } i}{\text{sine } r}$	The index of refraction equals the sine of angle of incidence in air divided by the sine of angle of refraction in substance.
(53) $I_r = \frac{S_a}{S_s}$	The index of refraction of light equals the speed of light in air divided by the speed of light in the substance.
(54) $K.E. = \frac{1}{2}MV^2$	The kinetic energy of an object equals one-half the mass times the square of the velocity. The kinetic energy is in ergs or foot poundals depending upon whether or not the mass is expressed in grams or pounds. After finding the kinetic energy in either of these units, the kinetic energy may be converted to gram-centimeters or foot-pounds by dividing by the proper numerical value of g, i.e., 980 or 32.

FORMULA	MEANING
(55) $E = mc^2$	The energy in ergs that a mass m of matter will liberate when transformed to energy equals the mass in grams times the velocity of light in centimeters squared.

CHAPTER IV

METHODS OF PROBLEM SOLVING

First, the use of vectors in solving problems in physics will be discussed in this chapter. Vectors are used extensively in solving problems in both elementary and advanced physics. The importance of vectors in physics may be appreciated when one realizes that motion, displacement, velocity, acceleration, force, electric current, magnetic flux, lines of force, stresses and strains, and flow of heat and fluids are all vector quantities. A vector quantity has both magnitude and direction. The RESULTANT of a number of vectors is that vector which would have the same effect as all the original vectors together. The COMPONENT of a vector is its effective value in any given direction. A vector may be considered as the resultant of two or more components, the vector sum of the components being the original vector. Vectors are added by geometric means.

To add vectors, the line segments representing them are placed in a linear series without changing their direction so that each line after the first begins where the one before ends. The line joining the initial point of the first line to the terminal point of the last line is said to be their sum or resultant. Vectors are represented by arrows with the head of the arrow at the terminal end and the tail of the arrow the initial end. Any vector may be resolved into components in any desired direction; however, the sum of the components of a vector added to the vector equals zero while the component of a vector parallel to a vector is equal to the vector itself. This fact enables one to move vectors to any desired location so long as the positive and negative directions are retained and the vector is kept parallel to the original vector. Although vector quantities may be resolved into any number of components and in any direction whatever, usually only two at right angles to each other are useful or needed. To do this, a line is drawn from the initial point of a vector in the desired direction. Then from the terminal end of the vector another line is drawn perpendicular to the first line. The two line segments ending at the point of intersection represent the components of the original vector.

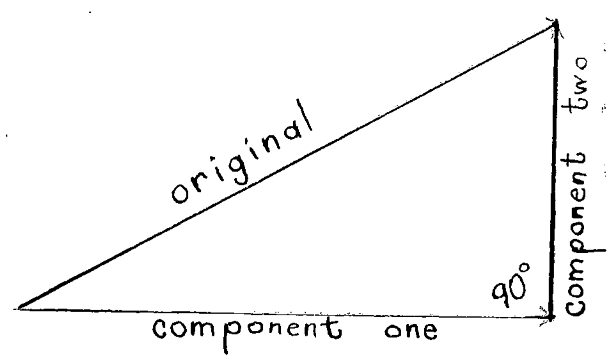


Fig. 1

The following illustrative problems will show this more clearly.

A force of 300 pounds is being exerted on an object in a direction 30 degrees east of north. How much is the force on the object in a due east direction? First draw the vector to any convenient scale, say one inch to 100 pounds. Then from the initial point of the vector, draw a line due east. From the terminal end of the vector draw a line perpendicular to the first line. The length of the first line from the initial end of the vector to the point of intersection with the second line will represent the vector in a due east direction. Measure this in inches and multiply by 100 to get the size of the vector. The following diagram will make this procedure clear.

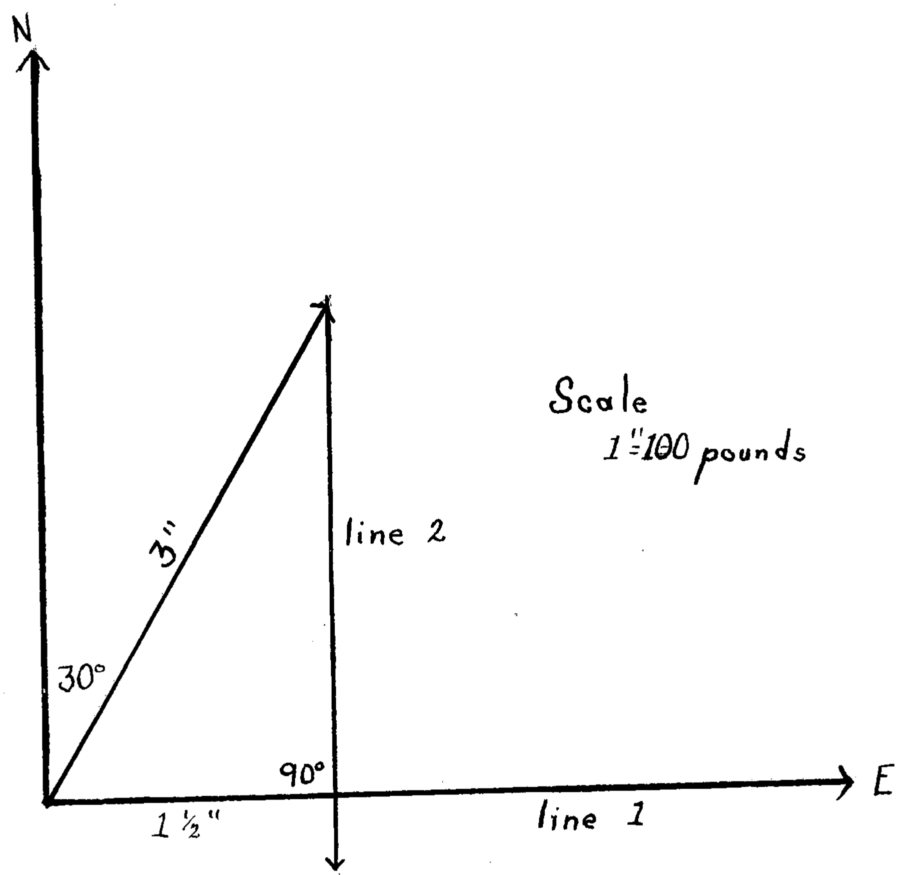


Fig. 2

One and one-half times 100 would give a force of 150 pounds in a due east direction.

The following problem will illustrate the use of vectors in solving problems. Three different solutions are given for the problem.

PROBLEM: A weight of 1000 pounds is supported by two cables which make angles of 30 degrees and 40 degrees, respectively, with the horizontal. Find the tension in the cables, neglecting the weight of the cables?

Given: $w = 1000$ pounds

angle $\theta = 30$ degrees

angle $\phi = 40$ degrees

find f_1 and f_2 the forces in the cables

Solution 1.

There are three forces acting on the weight; the pull of gravity, which is 1000 pounds directed downward, and the two tensions, each acting along the respective cables.

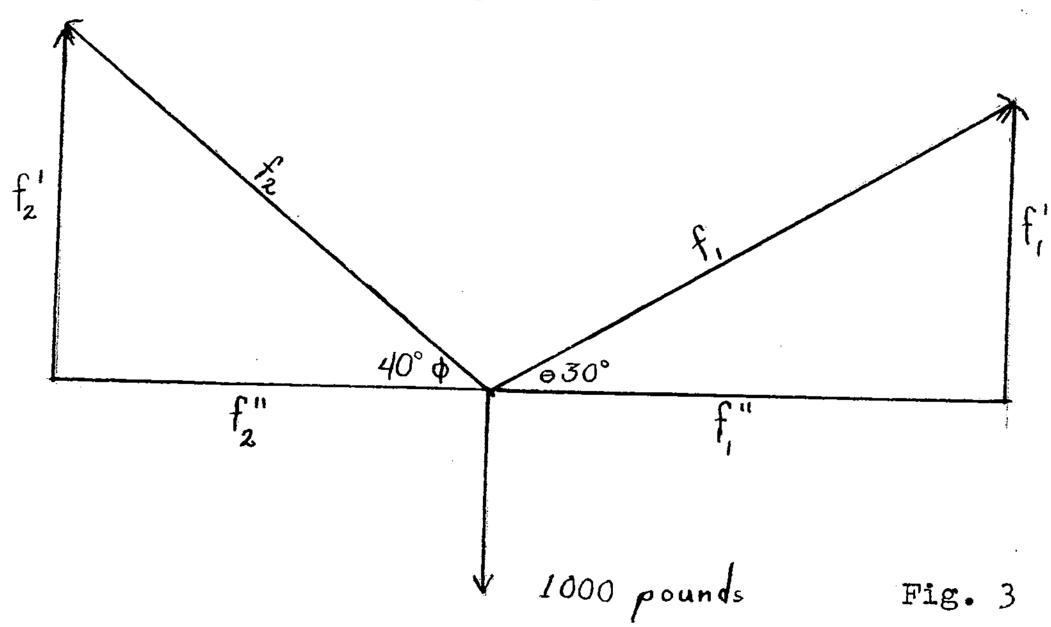


Fig. 3

Since this system of forces is in equilibrium, their sum is zero, the sum of the horizontal components must be zero and the sum of the vertical components must be zero. In other words, the sum of the forces upward equals the sum of the forces downward and the sum of the forces to the left equals the sum of the forces to the right. The forces f_1 and f_2 can be resolved into horizontal components f_1'' and f_2'' respectively, and vertical components f_1' and f_2'. Then

$f_1'' = f_2''$ (forces to right = forces to the left.) and

$f_1' + f_2' = 1000$ (upward forces = downward forces.)

To find the value of the components, it is necessary to express them in terms of f_1 and f_2, thus,

(1) $f_1'' = f_1 \cos 30^\circ$; $f_2'' = f_2 \cos 40^\circ$

and (2) $f_1' = f_1 \sin 30^\circ$; $f_2' = f_2 \sin 40^\circ$

By putting in the values of the trigonometric functions and since $f_1'' = f_2''$, the following equations result:

(3) $.866\ f_1 = .766\ f_2$

and since $f_1' + f_2' = 1000$

(4) $.500\ f_1 + .6428\ f_2 = 1000$

From equation (3) $f_1 = .8845\ f_2$. Substituting this value of f_1 in equation (4) gives

$$.8845\ f_2 + .6428\ f_2 = 1000$$

$f_2 = 922$ pounds to the nearest pound.

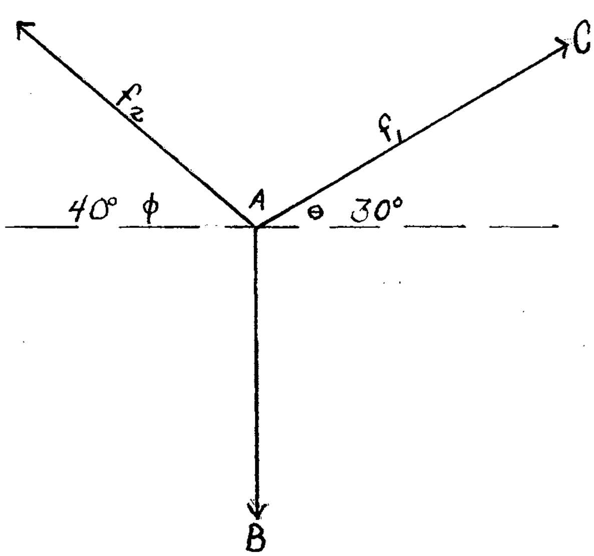
then $f_1 = .8845\ f_2$

$= .8845 \times 922$

$= 815$ pounds to the nearest pound.

Solution 2.

In order that there may be translational equilibrium, the vector sum of the forces acting at a point must be zero. This fact enables one to sketch a diagram that will form a triangle which will represent the forces involved. The diagrams below illustrate how to construct the triangle of forces. First, one should always draw a sketch of the physical situation. From this sketch, one can see the forces that are involved and he can construct an accurate representation.



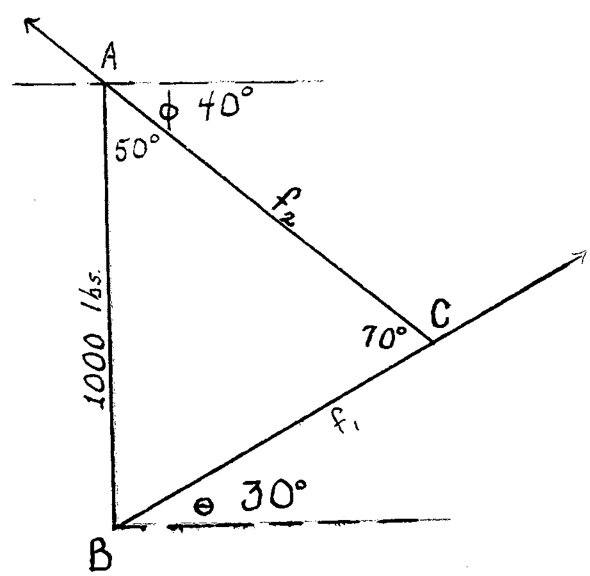


Fig. 4

Line AB is drawn straight downward and this represents the gravitational pull of 1000 pounds. Line BC starts from the terminal end of AB and is drawn to the right at an angle of 30° with the horizontal. The force in the other cable is represented by the third vector CA which begins at the terminal end of BC and ends at the initial end of AB, thus, forming a triangle of forces. From the statement of the problem, all the angles of the triangle are known and one side. This enables one to use the law of sines to solve for f_1 and f_2.

$$\frac{\sin 70^\circ}{1000} = \frac{\sin 50^\circ}{f_1} = \frac{\sin 60^\circ}{f_2}$$

Solution 3.

If the diagram in Solution 2 is drawn to scale, one may calculate the forces quite accurately. If one inch represents 200 pounds, the diagram would look as shown below.

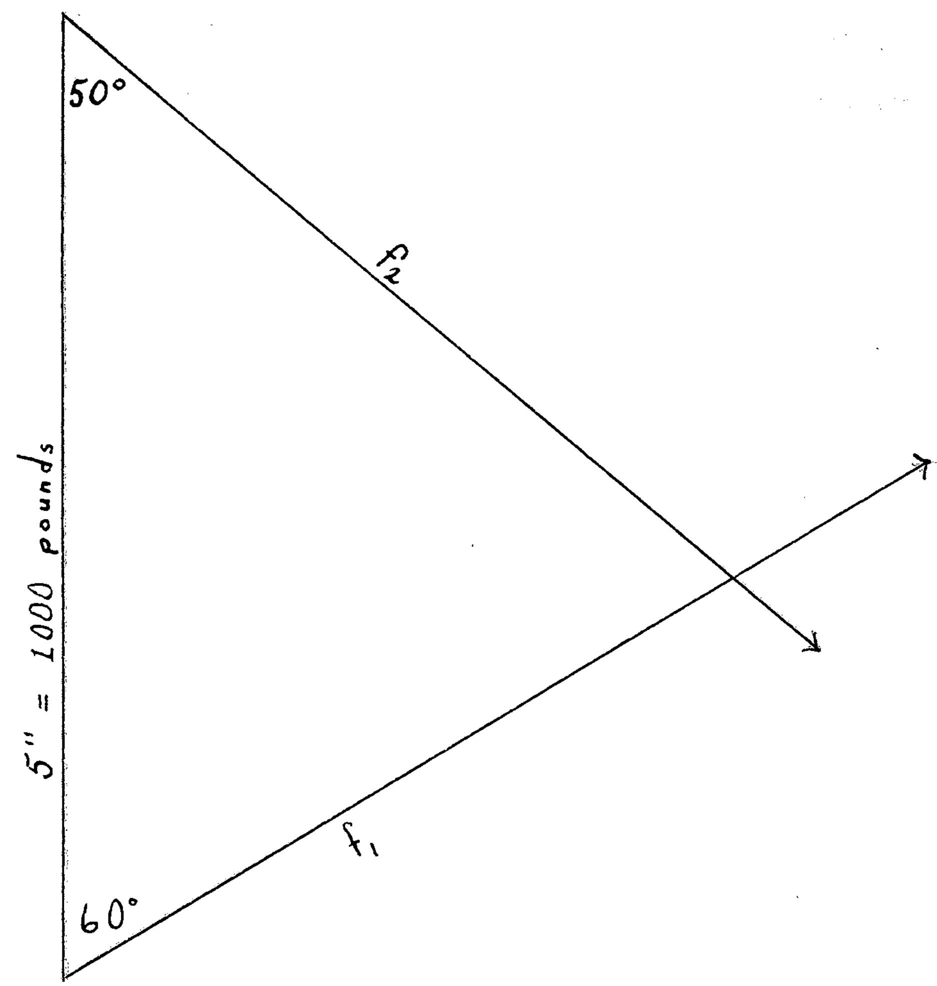


Fig. 5

By careful construction and accurate measurement, one may determine the forces to within two or three pounds.

To facilitate accurate analysis and to avoid errors, the student should develop a method of setting down data and a general procedure that he uses with all problems. In general, the following procedure is recommended:

(1) If one is not given, write a clear statement of the problem.

(2) Put down in tabular form the facts and figures that are given by the statement of the problem.

(3) If possible, draw a sketch of the physical situation.

(4) Write as many equations containing the unknown as possible.

(5) Solve the equations by whatever means possible.

PROBLEM: A lead plummet weighs 1000 grams in air and 912 grams when immersed in water. What is the specific gravity of lead?

Given: $W_a = 1000$ grams — W refers to weight; the subscript a refers to air.

$W_w = 912$ grams — W_w = weight in water

Find: The $S_p\ G_L$ (specific gravity of lead) in gm/cm.^{3}

(1) $S_p\ G_L = \frac{D_L}{D_W}$ D_L (density of lead), D_W (density of water) — Note: this statement is true by definition. See formula no. 13.

(2) $D_L = \frac{W_L}{V_L}$ W_L (weight of lead), V_L (volume of lead) — By definition. See formula no. 12

(3) $V_L = V_{wd}$ (Volume of lead equals the volume of water displaced)

(4) $V_{wd} = BF$ (volume of water displaced equals the bouyant force.)

(5) $BF = W_a - W_w$ (Bouyant force equals the weight in air minus the weight in water.)

The five equations state the relationships of the various quantities involved and solving them enables one to find the answer. To solve equation (1) for the specific gravity requires that the density of lead be known. The real problem here is to find the density of lead. Solving the above equations gives us the following:

$BF = 1000 - 912 = 88$ grams

$V_{wd} = 88$ cm.^{3} (from Archimedes' principle, i.e., the weight of the liquid displaced equals the bouyant force. This weight in grams, when the liquid is water, equals the volume in cm.^{3}

$V_{wd} = V_l = 88$ cm.^{3} (Volume of water displaced equals the volume of lead.)

$D_L = \frac{1000}{88}$ (density of lead equals the weight divided by the volume.)

$= 11.4$ grams cm.^{3}

$S_pG_L = \frac{11.4}{1}$ (the specific gravity of lead equals the density of lead divided by the density of water.)

PROBLEM:

Two perfectly elastic balls, one weighing 1000 grams and the other weighing 400 grams, are suspended by cords so that the distance between the cords and the centers of gravity of the balls is equal to the sum of the radius of the balls. The distance from the support to the centroid of each ball is equal. The large ball is pulled aside so that it has a velocity of 30 cm./sec. when it strikes the smaller ball. What will be the velocity of each ball after the first impact? What will be the velocity after the second impact? What will be the position of the balls at the moment of the second impact? Explain why they will be at this position at the instant of the second impact.

This problem is purely hypothetical, since there are no perfectly elastic balls; however, it does illustrate several important principles in physics and is included here for that reason. This principle may be demonstrated fairly well with proper equipment.

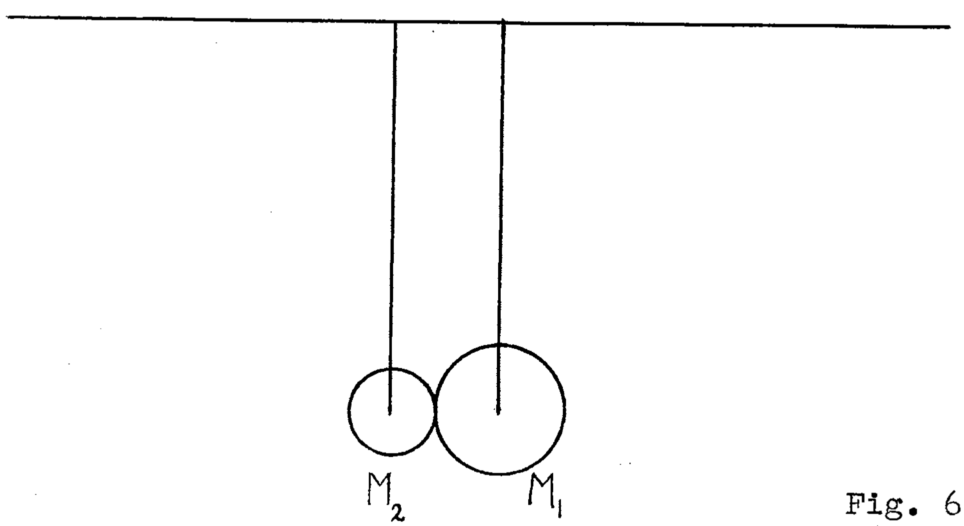


Fig. 6

Given:

$M_1 = 1000$ gms.

$M_2 = 400$ gms.

$V_1 = 30$ cm./sec.

Find:

V_{i1} the velocity of M_1 after the first impact.

V_{i2} the velocity of M_2 after the first impact.

(1) $M_1V_1 = M_1V_{i1} + M_2V_{i2}$ (Law of conservation of momentum)

(2) $M_1V_1^2 = M_1V_{i1}^2 + M_2V_{i2}^2$ (Law of conservation of kinetic energy.)

Substituting the known values in the first equation gives the following:

(a) $(1000)(30) = 1000\ V_{i1} + 400\ V_{i2}$

Solving for V_{i1}:

$$V_{i1} = \frac{30{,}000 - 400\ V_{i2}}{1000}$$

Substituting the known quantities in equation (2) gives:

(b) $(1000)(900) = 1000\ V_{i1}^2 + 400\ V_{i2}^2$

Rewriting this equation with the value of V_{i1} obtained from equation (1) gives an equation in one unknown.

$$9 \times 10^5 = 1000\left(\frac{30{,}000 - 400\ V_{i2}}{1000}\right)^2 + 400\ V_{i2}^2$$

Solving for V_{i2}:

$$9 \times 10^8 = 9 \times 10^8 - 24 \times 10^6 V_{i2} + 16 \times 10^4 V_{i2}^2 + 4 \times 10^5\ V_{i2}^2$$

$$5.6\ V_{i2}^2 - 240\ V_{i2} = 0$$

$$V_{i2}\ (5.6\ V_{i2} - 240) = 0$$

$$V_{i2} = 0$$

or $5.6\ V_{i2} = 240$

$$V_{i2} = 42.7 \text{ cm./sec. (Velocity after first impact.)}$$

Substituting this value for V_{i2}, equation (a) may be solved for V_{i1}.

$$30{,}000 = 1000\ V_{i1} + (400)(42.7)$$

$$10\ V_{i1} = 129.2$$

$$V_{i1} = 12.92 \text{ cm./sec. (Velocity of } M_1 \text{ after first impact.)}$$

In solving the equation for V_{i2}, there are two roots, one of which is zero. Since the physical conditions of the problem makes this answer impossible for the velocity after the first impact, the question might arise as to the significance of the zero root of the equation. It may be found upon further consideration of the problem that this is, in fact, the velocity of the small ball after the second impact. One may see this intuitively by thinking of what must take place at the second impact to fulfill the law of conservation of momentum. In order for the large ball to have the momentum to carry it back to its initial position, the small ball will have to give back all the momentum it received from the large ball; therefore, its velocity will be reduced to zero.

PROBLEM:

A skater weighing 180 pounds has a velocity of 25 feet per second. What must be the coefficient of friction between the ice and the skater such that the force of friction will reduce his velocity to zero in a distance of 200 feet?

Given:

$M = 180$ pounds

$V = 25$ feet per second

$d = 200$ feet

Find: C (the coefficient of friction.)

(1) $C = \frac{F_f}{M}$ (The coefficient of friction equals the force of friction in pounds divided by the mass in pounds.)

(2) $F_f d = \frac{\frac{1}{2}MV^2}{32}$ (The kinetic energy the skater possesses must equal the energy he loses by stopping. This energy is expended in the work done against friction which equals the force of friction times the distance. Since the formula $\frac{1}{2}MV^2$ is the kinetic energy in poundals, it must be divided by 32 to get it into pounds.)

(3) $F_f = \frac{\frac{1}{2}(180)(25)^2}{(32)(200)} = \frac{1125}{128}$

(4) $C = \frac{(1125)}{(180)(128)} = .049$ to the nearest thousandths.

Another solution of this problem, presented on the next page, is given to illustrate other principles of physics.

Solution 2.

The unknown sought in this problem is the coefficient of friction between the skater and the ice. In accordance with the suggestions given in this paper, the first equation one should write is one containing the quantity to be found.

(1) $C = \frac{F_f}{M}$ (The coefficient of friction equals the force of friction in pounds divided by the mass in pounds.)

(2) $F_f t = \frac{Mu - MV}{32}$ (The force of friction in pounds times the time in seconds equals the mass in poundals times the initial velocity in feet per second minus the final velocity in feet per second times the mass in poundals. The right side of the equation is divided by 32 to change poundals to pounds.)

(3) $d = \frac{(u + v)}{2}\, t$ (The linear distance an object will travel equals the average of the initial and final velocity times the time in seconds.)

Substituting the known quantities in the third equation enables one to find the time in seconds.

$$200 = \frac{(25 + 0)}{2}\, t$$

$$t = 16 \text{ seconds}$$

Substituting the known quantities in the second equation enables one to find the force of friction in pounds.

$$F_f(16) = \frac{(180)(25) - (180)(0)}{32}$$

$$F_f = \frac{1125}{128}$$

Substituting the known quantities in the first equation one may solve for the coefficient of friction.

$$C = \frac{\frac{1125}{128}}{180}$$

$C = .049$ to the nearest thousandths

PROBLEM:

A resistance of 8 ohms is in series with a capacitance of 100 microfarads and an inductance of .1 henry. A potential difference of 120 volts is impressed across the terminals of the combination. The frequency is 60 cycles/sec. Compute the power for this circuit.

(1) $P = VIK$ (The power of an alternating current equals the potential in volts times the current in amperes times a power factor.)

(2) $$K = \frac{R}{\sqrt{R^2 + (2\pi fL - \left(\frac{1}{2\pi fC}\right)^2}}$$ (Where R is the resistance in ohms; f is the frequency in cycles per second; L is the inductance in henrys; and C is the capacitance in farads.)

(3) $$I = \frac{V}{R^2 + \left(2\pi fL - \frac{1}{2\pi fC}\right)^2}$$

Given:

$R = 8$ ohms

$L = .1$ henry

$C = 100$ microfarads $= 10^{-4}$ farads

$V = 120$ volts

$f = 60$ cycles/sec.

Find: The power in watts.

Since inductance causes the current to lag behind the E.M.F. and capacitance causes the current to lead the E.M.F., their combined effect is to cancel each other ($X = X_L - X_C$). When they are combined in a circuit with a resistance, the combined impedance will equal their vector sum. Thus, a vector diagram of this problem may be drawn as shown below.

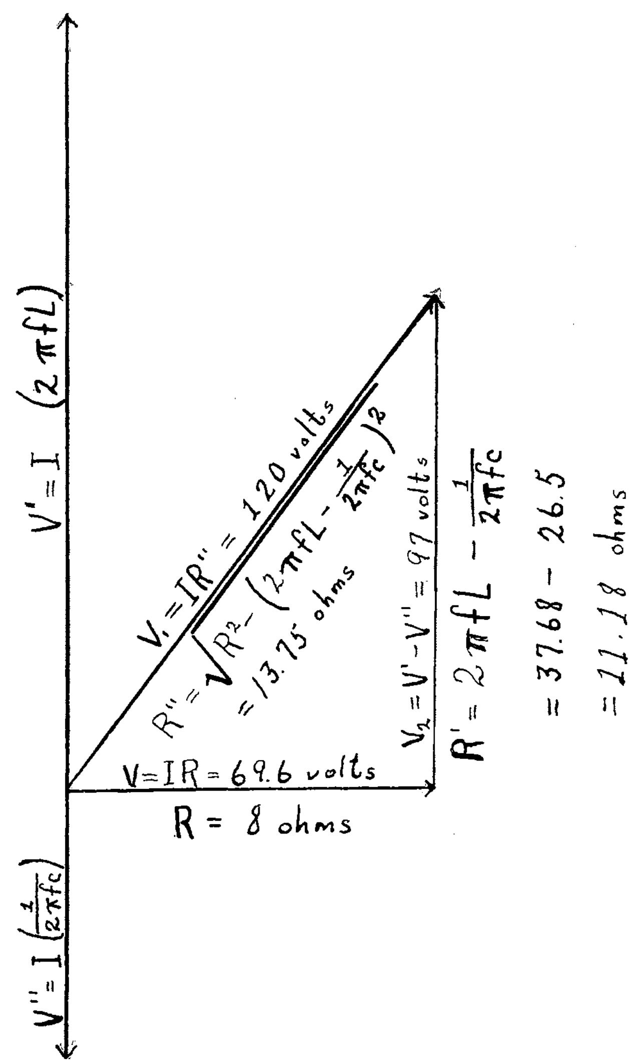


Fig. 7

Substituting the known quantities in equation (2),

$$K = \frac{R}{\sqrt{R^2 + \left(2\pi fL - \frac{1}{2\pi fC}\right)^2}}$$

$$= \frac{8}{\sqrt{64 + \left[2(3.14)(60)(.1) - \left(\frac{1}{2(3.14)(60)(10^{-4})}\right)\right]^2}}$$

$$= .582$$

Substituting the known quantities in equation (3) enables one to determine the current.

$$I = \frac{120}{\sqrt{64 + \left[2(3.14)(60)(.1) - \left(\frac{1}{2(3.14)(60)(10^{-4})}\right)\right]^2}}$$

$$= 8.7 \text{ amperes}$$

Substituting in equation (1):

$$P = (120)(8.7)(.582)$$

$$= 608 \text{ watts}$$

PROBLEM:

The musical note, orchestra A, has a frequency of 440 vib/sec. What is the velocity of sound in a medium in which this note has a wave length of 75 cm?

Given:

L = 75 cm

f = 440 vib/sec

Find: V in cm/sec.

(1) V = fL = 75 X 440 = 33000 cm/sec

PROBLEM:

A particle has a displacement of .6 cm at the instant it has completed 5/6 of a cycle starting from rest. What is its amplitude of vibration?

Given:

$w = \frac{5}{6} \times 2\pi$ (angular rotation in radians completed)

$B = \frac{2\pi}{6}$ Radians 60° (from statement of the problem)

d = .6 cm

Amplitude = Radius of reference circle

Find: A in cm.

This problem may be more easily comprehended if this diagram is made showing the conditions of the problem.

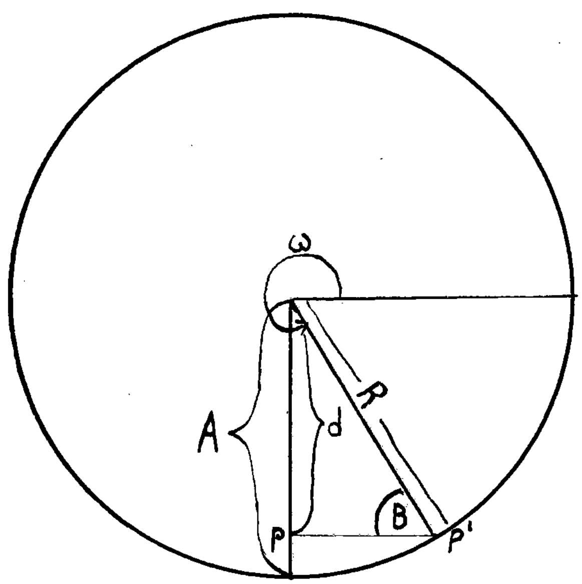


Fig. 8

(1) $A = R$

(2) $\sin B = \frac{d}{R}$

(Substituting in equation 2)

$\sin 60^{\circ} = \frac{d}{A} = \frac{\sqrt{3}}{2} = \frac{.6}{A}$

(3) $A = .6 \times \frac{2}{\sqrt{3}} = .692$

PROBLEM:

A concave mirror has a focal length of 60 cm. An object is placed at a distance of 100 cm from the mirror. Where is the image?

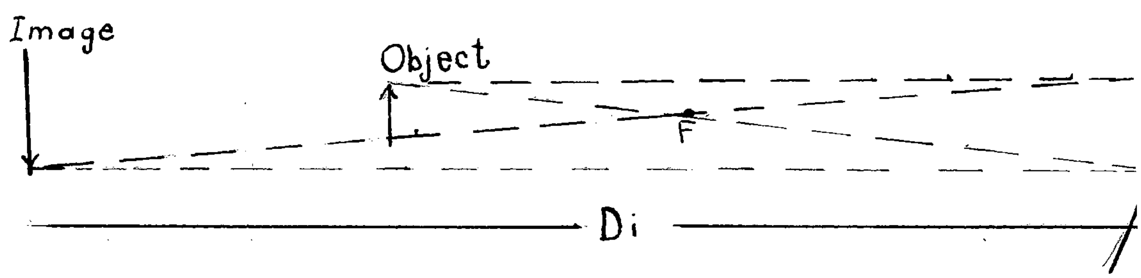
Given:

$f = 60$ cm

$D_o = 100$ cm

Find: D_i in cm

Fig. 9



A Real Image

f is the focal length of mirror or lens;

D_o is the distance to the object from mirror or lens;

D_i is the distance to image from mirror or lens.

(1) $\frac{1}{D_o} - \frac{1}{D_i} = \frac{1}{f}$

(2) $\frac{1}{100} - \frac{1}{D_i} = \frac{1}{60}$

$\frac{1}{D_i} = \frac{1}{60} - \frac{1}{100} = \frac{2}{300}$

$D_i = 150$ cm

In a problem of this nature a scaled drawing should be made using the data of the problem and two of the following rays.

1. A ray parallel to the principal axis passes through the principal focus after passing through a lens and is reflected through the principal focus of a mirror.
2. A ray through the center of curvature is undeviated as it traverses the lens or mirror.
3. A ray passing through the principal focus leaves the lens (or mirror) parallel to the principal axis.

If the diagram is drawn to scale, the image can be located quite accurately by measurement. This serves as a check on computation and vice versa.

CHAPTER V

TABLES IMPORTANT TO PHYSICS

Length

1 in. = 2.54 cm

1 m = 39.37 in.

30.5 cm = 1 ft.

1 mile = 1.61 Km

1 mile = 5280 ft.

1 Km = 0.62 mi.

1 m = 3.28 ft.

1 yd. = .9144 meters

Mass

1 lb. = 454 gm

1 oz. = 2835 gm

1 Kg = 2.2 lb.

Volume

1 m^3 = 264.2 gal.

1 m^3 = 35.3 $ft.^3$

1 gal. = 231 cu. in. = 3785.432 cu. cm

1 $in.^3$ = 16.4 cm^3

1 $ft.^3$ = 0.028 m^3

1 cm^3 = 0.061 $in.^3$

1 m^3 = 1.308 $yd.^3$

Handy Equivalents

1 H.P. = 550 ft-lb/sec = 33,000 ft-lb/min = 746 watts

1 ft-lb = 13.56×10^6 ergs

1 Joule = 0.738 ft-lbs

1 gm f = 980 dynes

60 mph = 88 ft/sec

1 ft^3 = 7.48 gal.

1 cal = 3.97 BTU = 4.19 Joules

1 BTU = 252 cal = 778 ft-lb

24 hrs = 86,400 sec

1 volt = 10^8 emu

1 electron volt = 1.6×10^{-12} erg

1 emu of charge = 3×10^{10} esu

1 emu of charge = 10 coulombs

1 emu of current = 10 amps

1 esu of Pd = 300V

1 watt = 10^7 ergs/sec or 1 Joule/sec or 1 volt X 1 amp

1 Joule = 10^7 ergs

1 Kilowatt hr = 36×10^{12} ergs

1 megacycle = 10^6 cycles

$\sqrt{2} = 1.41$ $\sqrt{3} = 1.73$ $^2 = 9.87$

Velocity of light c = 2.998×10^8 cm/sec or 186,000 miles/sec

Number of molecules of gas/cm^3 where p = 76 cm of hg and T = 0°C. is 2.69×10^{19}

Number of molecules/gm molecule = 6.023×10^{23}

Volume of gram-molecule of gas normal p and t = 22,415 cm^3

Faraday = 96,520 coulombs/gm-mole

Charge on electron e = 4.8×10^{-10} esu

16×10^{-2} emu

Mass of an electron = 9.1×10^{-28} gm

Ratio of charge to mass of electron e/m = 1.76×10^{7} emu/gm

Mass of Hydrogen Atom = 1.67×10^{24} gm

Planck's constant h = 6.62×10^{-27} erg-sec

Boltzman's constant k = 1.38×10^{-16} erg/deg c

One atomic mass unit -- 1.66×10^{-24} gm or 931 mev.

Gravitational constant -- G = 6.66×10^{-8} cgs units

Gas constant R = 8.315×10^{7} ergs/mole/k°

Atomic wt. of H = 1.008

Density of water = 1 gm/cm^3 or 62.4 lb/ft^3

Density of Hg = 13.6 gm/cm^3

Density of Air = 0.00129 gm/cm^3 Normal p and t

Heat of fusion of ice = 80 cal/gm

Heat of vaporization of water = 540 cal/gm

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VITA

Louis B. Casey

Candidate for the Degree of

Master of Science

Title: SUGGESTIONS FOR IMPROVING THE MATHEMATICAL PREPARATION OF HIGH SCHOOL PHYSICS STUDENTS

Major Field: Natural Science

Biographical:

Personal data: Born near Keota, Oklahoma, June 4, 1917.

Education: Was graduated from Keota High School in 1934; received the Bachelor of Science degree from the Oklahoma Agricultural and Mechanical College, with a major in Biology, in May, 1941; attended Gonzaga University 1947-1948; attended an Academic Year Institute (1957-1958) of the National Science Foundation at Oklahoma State University; received the Master of Science degree in Natural Science from Oklahoma State University in May, 1958.

Professional experience: Served in the Navy from 1942 to 1945 as an aircraft instructor and mechanic; taught school at Clarkston, Washington, for seven years and at Modesto, California, for two years.

VITA

Ben Barber Thaxton

Candidate for the Degree of

Master of Science

Title: SUGGESTIONS FOR IMPROVING THE MATHEMATICAL PREPARATION OF HIGH SCHOOL PHYSICS STUDENTS

Major Field: Natural Science

Biographical:

Personal data: Born near Felt, Oklahoma, April 22, 1918, son of Lee and Lilah Ruth Thaxton.

Education: Attended grade and high school at Felt, Oklahoma; graduated from Felt High School in 1936; received the Bachelor of Science degree from Panhandle Agricultural and Mechanical College, Goodwell, Oklahoma, with a major in mathematics, in May, 1950; attended an Academic Year Institute (1957-1958) of the National Science Foundation at Oklahoma State University; completed requirements for the Master of Science degree in May, 1958.

Professional experience: Served from 1941 to 1945 in the U. S. Army Air Force as a weather observer and weather forecaster; taught school as classroom teacher at Vilas, Colorado, three years, at Felt, Oklahoma, five years, and at Yarbrough, Oklahoma, two years; member of Phi Delta Kappa, professional fraternity for men in education.