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Title of Study: SYLLABUS FOR "BASICS FOR ADVANCED MATHEMATICS"

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Scope and Method of Study: Investigations of arithmetic textbooks were conducted to determine what the authors considered important and essential to a good arithmetic course. An effort was made to discover correlations among the texts. Listed, in no particular sequence, were my beliefs of what was important and essential in such a course. These essentials were: Vocabulary, definitions, self explanatory explanations, practical and understandable examples, varied and numerous practice problems, homework problems, and reliable suggested tests. Due to the recent trend to introduce Modern Mathematics in the Secondary Schools, consideration was given to a more practical course which would include the use of letter symbols, a study of right triangles, and uses of exponents and radicals.

Findings and Conclusions: None of the texts were considered adequate. Excellent illustrations and explanations of certain topics were paraphrased and included in this report. All of the authors agreed that a good course in arithmetic began with an understanding of the four operations of arithmetic. Most of the texts started with a study of fractions. Some of the texts had ample problems and satisfactory methods of explanations. None of them contained a satisfactory amount of material on all the essentials suggested by the author. This conclusion verified a previously studied conclusion and provided the stimulus for the preparation of this report, which the author believes will result in a textbook suitable and adequate for the times.

ADVISER'S APPROVAL James H. Zant

# SYLLABUS FOR "BASICS FOR ADVANCED MATHEMATICS"

By

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SYLLABUS FOR BASICS FOR ADVANCED MATHEMATICS

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## PREFACE

A course in Arithmetic should do two things for the student. It should give him the ability to handle problems rapidly and accurately, and it should give him a knowledge of the problems of business and life. The aim of this syllabus, from which a book will be written, will affect both of these results.

Although accuracy and speed is an objective, understanding of the four fundamental operations, with various number systems, is intended to be a prerequisite. The road to mastery of arithmetic is paved with study and practice. It is hoped that through study and practice, the pupil will be taught to think for himself correctly, and to attain his results by the shortest and best methods.

The author, having taught general mathematics for a number of years, feels as if this book will be more suited to the needs and abilities of High School students: It being prepared by a teacher of the subject, one who is familiar with the study habits and capabilities of students needing this type of subject matter.

It is not the intention of the author to have this book treated as a review course. Better results for both the teacher and the students will be found if the course is approached as a means of better understanding, leading to the ability to handle figures rapidly and accurately thus providing a smoother path in business and life.

It is evident to students that too much emphasis in placed on "covering" the contents of a book and too little emphasis on "uncovering" its contents. A suggested plan of study for a 36 week school year is contained in this syllabus which is intended to "satisfy" both the teacher and student.

Each 6 week period is a topic singled out because of its relative importance to society and the individual within that society. Each week of each 6 week period is devoted to a special part of the 6 week topic. The syllabus further contains suggested daily subjects to be taught, in order to successfully complete the course.

"Basics for Advanced Mathmatics" is intended to be a two semester course for the high school student; specifically designed for the student not planning on a college education.

However, the author is not implying that a prospective college student would not benefit from such a course. The subject matter involves much prior learning by the student and tests his mental ability by complicating previously learned mathematics. An effort is maintained through out the book to give clear examples illustrating the problems he will have to work. Memorization should not be required. A sincere effort on the part of the teacher will easily afford an understanding of mathematics previously avoided in favor of the necessity to cover a given amount of material in a stated time.

It may seem, to a "first user," that the topics outlined present a formidable barrier, but be assured there is ample time and variation of subject matter to interest most any student of mathematics.

The author greatfully acknowledges the efforts of his students who provided the stimulus responsible for the preparation of "Basics, For Advanced Mathematics."

TABLE OF CONTENTS

# CHAPTER I

## INTRODUCTION

Of the books in print today, only a few satisfy all the needs of a high school class in arithmetic. Vocabulary, definitions, practical examples, self indicating explanations, and numerous, varied, enclosed practice problems are the needs to which I am referring. Many college professors who write these texts, for use in the high school, forget that the majority of these students lack much of the ambition and initiative found in their own classes in College. Facilities, personal regulations, more rigorous selection of students and impersonal attitudes are considerations under the supervision of the college professor; all of which allow a more relaxed use of teaching methods.

A course, using this book as its text, will have no need of outside references, work books, impromptu problems, explanation of terms, and setting aside extra time for extra help. The book is self explanatory and uses the teacher as a means of guidance instead of a means of understanding the contents of the text.

The book will contain 6 sections distributed over 36 chapters enclosing 180 daily topics. Each section will encompass one of the major components of arithmetic. The six chapters in each section will include all of the material necessary to give the best practical application of the main topic. Each of the daily topics will provide sufficient problem practice and understanding, to support a logical step to the next days learning.

The first page of every chapter will be devoted to new vocabularly. It is difficult to read with meaning without previous understanding. Turning to the back of the book or even referring to another text for the meaning of a word, term, or expression is distracting, time consuming, and aggravating to say the least.

Following the vocabulary will be a short summary on objectives and what the chapter will include. When appropriate, application and historic points of interest will be emphasized to stimulant individual effort.

The daily topics will begin with an explanation of the days work. It is here that the experience and training of the teacher will be of most benefit. Elaboration of the contents of the text and illustrations of problems with black board demonstration can only inhance the preparation of the student for problem solving. Examples, not only of the simpler problems, but the complicated ones, as well, will be self explanatory, such that the student, with sufficient effort, will be able to solve the problems of each day.

Daily homework is a factor of learning involving much of the teachers time for which compensation is ridiculously lacking in physical comforts but adequate in personal knowledge of a job well done. Practice, practice, practice. One of the most satisfactory teaching methods is used in helping the teacher to teach this course. Once something is learned, a new something can be learned quicker by applying the old knowledge to the new. The course is dedicated to this type of constructive learnings.

There will be a sufficient number of problems for class work and a reasonable home work assignment. It is suggested by this author, that these problems be checked by the teacher or corrected in class. Delay in returning papers to the students is an indication of laxity which has been found to be contagious. If a class knows what is expected and realizes each of their efforts is under scrutiny each day, their efforts increase and their products of labor are rewarding for all concerned. While correcting problems in class is time saving, the time could be used to better advantage. It is difficult to intentionally make work for ourselves, but, in the begining, the choice was our own. Teaching isn't a job, it is a dedication.

The sixth week of each 6 week period is to be used in solving problems of all types found in the previous 5 weeks. Supervised class work, problem discussion, review and test preparation are all good teaching methods to be applied. The last day of each six week period is set aside for a comprehensive test on the 6 week work. This test is included in the context of the book as a suggested test which may be used as the test or as a starting point for a teacher prepared test.

The author does not intend to restrict the use of tests as a teaching method or measuring stick, by limiting the suggested tests to 6. It is felt that the author's knowledge resulting from his endeavors, the greater period of time for concentration and the possible remuneration for his efforts, might prove to be a greater stimulus for the preparation of a more valid and reliable test than one prepared between classes or late at night by the teacher.

For the purpose of illustrating the context, Chapter II is developed.

## CHAPTER II

The following outline of "Basics For Advanced Mathematics," will illustrate the manner in which the context of the book will be prepared. Each weeks work will begin with a new chapter but each new chapter will use the one preceding as a stepping stone. The daily topics, listed under each chapter, have been chosen so as to present the best possible course in High School arithmetic.

### OUTLINE OF COURSE

Section A. Fractions

Chapter I. Number Systems

1. Brief history and use of symbols.
2. Writing and reading numbers.
3. Number systems.
4. Large numbers and infinity.
5. Four operations of Arithmetic.

Chapter II. Proper Fractions

1. Definition, vocabulary, reducing or simplification.
2. Common multiples and denominators.
3. Addition and subtraction of proper fractions.
4. Multiplication and division of proper fractions.
5. Reduction and cancellation of proper fractions.

Chapter III. Improper Fractions

1. Definition, reducing or simplification.

2. Converting to improper fractions and the reverse operation.

3. Addition and subtraction of mixed numbers.

4. Multiplication and division of mixed numbers.

5. Application with problems involving all types of fractions.

Chapter V. Negative Numbers

1. Introduction, positive and negative signs.

2. Addition and subtraction of signed numbers.

3. Multiplication and division of signed numbers.

4. Use of fractions with negative parts.

5. Application with problems involving positive and negative integers and fractions.

Chapter VI. Application

1. Problems.

2. Problems.

3. Problems.

4. Problems.

5. Suggested test over 6 weeks work in fractions.

Section B. Decimals

Chapter VII. Decimal system

1. Definition, pointing off, writing and reading decimals.

2. Multplying and dividing by 10, 100, . . .

3. Addition and subtraction of decimal numbers.

4. Multiplication of decimal numbers.

5. Division of decimal numbers.

Chapter VIII. Dollars and Cents

1. Problems in cents.

2. Conversion of cents to dollars.

3. Problems in dollars and cents.
4. Fractional equivelents.
5. Use of decimals and fractions.

Chapter IX. Measurements

1. Refresher on common English units.
2. Metric system.
3. Distance.
4. Weights.
5. Volumes.

Chapter X. Common Conversions

1. English to metric.
2. Metric to English.
3. Centigrade and Fahrenheit.
4. Aliquot parts.
5. Aliquot parts.

Chapter XI. Significant Figures and Rounding Off

1. Definition and rule.
2. Problems concerning significant figures.
3. Rounding off before operation.
4. Rounding off answers.
5. Repeating and non-repeating decimals.

Chapter XII. Application

1. Problems.
2. Problems.
3. Problems.
4. Problems.
5. Suggested test over 6 weeks work in decimals.

Section C. Percentages

Chapter XIII. Introduction to Percentages

1. Definition and symbols.
2. Expressing percent 4 ways.
    a. Fraction with 100 as denominator.
    b. Fraction in lowest terms.
    c. Decimal.
    d. Use of sign (%).
3. Elements of percentage.
    a. Base.
    b. Rate.
    c. Percentage.
    d. Amount or difference - Base = Percentage.
4. (a) Given base and rate, find percentage.

    (b) Given base and percentage; find rate.
5. (a) Given rate and percentage; find base.

    (b) Given rate and amount or difference; find base.

Chapter XIV. Decimal Conversion

1. Percent to decimals.
2. Decimals to percent.
3. Problems in Addition and Subtraction.
4. Problems in Multiplication and Division.
5. Complex problems.

Chapter XV. Fraction Conversion

1. Percentage to fractions.
2. Fractions to percentage.
3. Fraction and decimal parts of percentage.

4. Problems involving conversion.

5. Problems involving conversion.

Chapter XVI. Profit and Loss

1. Definitions - profit, loss, cost, rate, selling price.

2. Given cost and rate; find profit or loss.

3. Problems on finding discount.

4. Determining premium or discount.

5. Given premium or discount and cost; find rate.

Chapter XVII. Premium and Discount

1. Definitions.

2. Problems on finding premium.

3. Problems on finding discount.

4. Determine premium or discount.

5. Given premium or discount and cost; find rate.

Chapter XVIII. Application

1. Problems.

2. Problems.

3. Problems.

4. Problems.

5. Suggested test over 6 weeks work in percentage.

Section D. Roots and Powers

Chapter XIX. Square root determination

1. Method of determining square roots.

2. Problems in finding square roots.

3. Use of square root tables.

4. Interpolation.

5. Problems using tables and interpolation.

Chapter XX. Higher roots

1. Use of tables.
2. Problems involving tables.
3. Addition and Subtraction of numbers containing radical sign.
4. Multiplication and division of numbers containing radical sign.
5. Comprehensive root problems.

Chapter XXI. Exponents

1. Definition and symbols.
2. Practice in removing exponents.
3. Multiplication and division of numbers with similar exponents.
4. Multiplication and division of similar numbers with different exponents.
5. Method of removing numbers with exponents, from the denominator.

Chapter XXII. Indexes

1. Definition and symbols.
2. Converting radical signs to numbers with indexes.
3. Converting numbers with indexes to numbers in radical sign.
4. Simplification of numbers with indexes.
5. Functions of arithmetic and indexes.

Chapter XXIII. Radicals and Pythagorean Theorem

1. Removing radicals from denominator.
2. Right triangles.
3. Pythagorean Theorem.
4. Direct measurements.
5. Indirect measurement using Pythagorean Theorem.

Chapter XXIV. Application

1. Problems.
2. Problems.
3. Problems.
4. Problems.
5. Suggested test over 6 weeks work in Roots and Powers.

Section E. Formulas

Chapter XXV. Use of Letter Symbols

1. Introduction of new symbols.
2. Addition and subtraction of letter symbols.
3. Multiplication and division of letter symbols.
4. Using letter symbols in fractions.
5. Problems involving letter symbols.

Chapter XXVI. Finding Areas

1. Formulas for finding areas.
2. Areas in one plane.
3. Areas in multiple planes.
4. Areas with dimensions given in letter symbols.
5. Problems involving area.

Chapter XXVII. Finding Volumes

1. Formulas for finding volumes.
2. Finding volumes of rectangular solids.
3. Finding volumes of pyramids.
4. Finding volumes of cylinders, spheres, and cones.
5. Mixed problems.

Chapter XXVIII. Graphing Formulas

1. Purpose and use of graphs.
2. Bar graph.

3. Line graph.

4. Graphing functions.

5. Mixed problems.

Chapter XXIX. Equations

1. Equality.

2. Identities and equivelents.

3. Transposition.

4. Finding unknowns.

5. Mixed problems.

Chapter XXX. Application

1. Problems.

2. Problems.

3. Problems.

4. Problems.

5. Suggested test over 6 weeks work in Formulas.

Section F. Practical Applications

Chapter XXXI. Home Calculations

1. Covering surfaces.

2. Water and gas costs.

3. Cost of electricity.

4. Construction costs.

5. Mixed problems.

Chapter XXXII. Traveling

1. Interpreting map distance (scale).

2. Given distance and time; find rate or speed.

3. Given rate and time; find distance.

4. Finding cost of traveling.

5. Mixed problems.

Chapter XXXIII. Money and Interest

1. Savings accounts.
2. Borrowing money.
3. Borrowing money.
4. Installment buying.
5. Mixed problems.

Chapter XXXIV. Taxes

1. Sales tax.
2. Property tax.
3. Income tax.
4. Income tax.
5. Income tax.

Chapter XXV. Insurance

1. Life and accident insurance.
2. Automobile insurance - Comprehensive.
3. Automobile insurance - Liability.
4. Fire insurance.
5. Mixed problems.

Chapter XXXVI. Application

1. Problems.
2. Problems.
3. Problems.
4. Problems.
5. Suggested test over 6 weeks work in Practical Applications.

# CHAPTER III

## TOPIC 1: BRIEF HISTORY AND USE OF SYMBOLS

When the greatness of man's world consisted of the land he could walk over and the distance he could see over, there was little need of a number system. The need for food, shelter and clothing was small as was everything surrounding him. By comparing the fingers of his hand with the number of things he wished to represent, a satisfactory means of communication was developed.

As his environment increased to include the Mediterranean Sea, the need for expressing greater numbers of things became evident and symbols were used to illustrate these greater numbers. A drawing of an animal would mean one animal; the drawing of three animals would represent three of them. As the numbers of things to be recorded increased, short-cuts were taken. If fifteen deer were to be shown, a deer would be drawn and fifteen strokes placed beneath the drawing to illustrate the exact number of them. This type of numeral was most highly developed in early Eygpt about 4000 years ago.

Eventually, symbols became somewhat standardized in each society and one man's "quantity symbol" became many other men's symbol for the same quantity. A lotus flower would represent 1000, of which there were so many in Eygptian fields. For 10,000, a pointed finger was drawn; and for 100,000 a tadpole was depicted. There were not more tadpoles than lotus flowers, but probably they seemed like more. For a million, the picture of a man with his hands outstretched, apparantly in amazement at so large a number, was used. These symbols are called hieroglyphics, since numbers are represented by pictures of objects.

Men invented numbers as the need for counting became a necessity. Larger collections of objects and greater quantities had to be represented by simpler symbols since it was no longer feasible, in a trading world, to transport the objects themselves or carry huge writing tablets to illustrate "how many" to someone else. Whereas the number of fingers on both hands met the needs of Stone Age men, billions and numbers larger than billions are familiar and useful to nearly everyone today.

There are tribesmen in Africa today that can count only to three. More than three is many, and difficult to understand. He has no idea of numbers greater than three and has no word for them. He would count one, two, three, many.

It is not easy to define a number. Although we use numbers and have a reasonably good idea of their purpose, defining them is so difficult that more advanced courses in mathematics are required, to gain the understanding necessary for their exact definition.

When we count objects, we arrive at a number. To record this number, a symbol is used. The symbol is not the number but only represents it. Thus, 3 is the symbol for three objects and 11 is the symbol for number eleven. A thousand years ago the Arabs introduced a system similar to this into Europe, and with some revision has become widespread in its use.

## TOPIC 2: WRITING AND READING NUMBERS

It is difficult to believe that a few centuries ago man did not have the vocabulary or need to use words or symbols for large numbers. In our reading and recording of numbers we follow the early Arabs and the later Germans in grouping with reference to thousands and multiples of thousands. The Greeks grouped by myriads and thousands while the Hindu way calls attention to the place, or sequence value. The Romans symbolized large numbers by the repeated use of the same symbol.

A large number, like 8,443,682,155 is read according to these different systems in the English, Sanskrit, Arabic, early German, Greek and Roman languages as follows:

English: 8 billion, 443 million, 682 thousand, 155
Sanskrit or Hindu: 8 padmas, 4 vyarbudas, 4 kōtis, 3 prayutas, 6 laksas, 8 ayutas, 2 sahasra, 1 sata, 5 dasan, 5
Arabic and early German: Eight thousand thousand thousand and four hundred thousand thousand and forty-three thousand thousand and six hundred thousand and eighty-two thousand and one hundred fifty-five
Greek: Eighty four myriads of myriads and four thousand three hundred sixty-eight myriads and two thousand and one hundred fifty-five.
Early Roman: MMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMM MMMM. . . .until 8 million M's had been written after which the specific number would be written.

A Roman would have needed the wall of a football stadium to express the number of dollars spent each year for present day national defense.

The system with which we are familiar is the system in which the symbols 1 through 9 are used, with the symbol, 0, representing a vacant place. The first nine numbers are shown below:

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|
| one | two | three | four | five | six | seven | eight | nine |

All other numbers are represented by combinations of these and another symbol, 0, called zero, naught, or cipher. The zero issused to indicate no value.

The number next higher than nine is named ten and is written with two figures, thus, 10, in which the zero, 0, merely serves to show that the unit one, 1, on its left, is different from the unit 1 standing alone which represents a single thing, while this, 10, represents a group of ten things.

The nine numbers succeeding ten are written and named as follows:

| 11 | 12 | 13 | 14 | 15 | 16 | 17 |
|---|---|---|---|---|---|---|
| eleven | twelve | thirteen | fourteen | fifteen | sixteen | seventeen |

18 19
eighteen nineteen. In each of these, the number on the left represents a group of ten things, while the figure on the right expresses the units or single things additional, required to make up the number.

The next number above nineteen, is ten and ten, or two groups of ten, written 20, and called twenty. We begin again by adding one each time, thus, 20 and 1, 21, twenty one; 20 and 2, 22, twenty two; and so on until we reach 20 and 10, 30, thirty. The process repeats itself until we reach the highest number possible to be written with two figures, 90 and 9, 99, ninety nine. 90 and 10, 100, one hundred is written with three figures in which the last two, which are zeros, are vacant spaces with no value in themselves but serve to give greater value to the 1.

The <u>order</u> of a figure is the place it occupies in a number. From what has been said, it is clear that a figure in the 1st place, with no others to the right of it, expresses units or single things; but standing on the left of another figure, that is, in the 2n d place, expresses groups of tens; and standing at the left of two figures, or in 3rd place, expresses 10s of 10s or 100s; and in the 4th place, expresses tens of hundreds or thousands. Therefore, counting from the right,

The order of units . . . . . is in the 1st place, . . . 1

The order of tens . . . . . .is in the 2nd place,. . . 10
The order of hundreds . . . .is in the 3rd place,. . . 100
The order of thousands. . . .is in the 4th place,. . . .1000

By this arrangement, the same figure has different values according to the place, or order, in which it stands. Thus, 8 in the 1st place is eight units; in the second place is 8 tens or eighty; in the 3rd place is 80 tens, 8 hundreds or eight hundred.

For convenience in reading and writing numbers, orders are divided into groups of three each, and each group of three orders is called a period. Each period is composed of one set of units, tens and hundreds. Example:

| of quadrillions | of trillions | of billions | of millions | of thousands | of units |
|---|---|---|---|---|---|
| hundreds tens units | hundreds tens units | hundreds tens units | hundreds tens units | hundreds tens units | hundreds tens units |
| 3 2 3 | 1 7 4 | 9 8 5 | 0 4 5 | 1 1 2 | 7 3 3 |
| 6th period | 5th period | 4th period | 3rd period | 2nd period | 1st period |

The number would be read: 3 hundred twenty three quadrillion, 1 hundred seventy four trillion, 9 hundred eighty five billions, forty five million, 1 hundred twelve thousand 7 hundred thirty three.

Things to remember:

Ten units to any order always make one of the next higher order.

Moving a figure one place to the left increases its value tenfold.

Moving a figure one place to the right decreases it tenfold.

Vacant orders in a number are filled with zeros.

Rules to follow:

1. Beginning at the right end of the number, separate the number into periods of three figures each.

2. Commencing at the left, read in succession each period with its name.

Example: Express in words the number which is represented by 304712021.

304 712 021 is read 3 hundred four million, 7 hundred twelve thousand, twenty one.

Roman numberals are used to a limited degree in our present society. A better understanding of our number system will result from reading the following brief explanation.

In the Roman number system, numbers are represented by seven letters. The letter I represents one; V, five; X, ten; L, fifty; C, one hundred; D, five hundred; and M, one thousand. The other numbers are represented according to the following principles:

1. Every time a symbol is repeated, its value is repeated. Thus, II denotes two; XX denotes twenty. IIII does not appear, nor do symbols consecutively repeat themselves more than 3 times.

2. Where a single symbol of less value is placed before one of greater value, the less is taken from the greater; thus, IX denotes nine, one less than ten. A prefix symbol never repeats itself.

3. Where a symbol of less value is placed after one of greater value, the less is added to the greater; thus VI denotes six, one more than five.

4. Where a single symbol of less value stans between two symbols of greater value, it is taken from the following symbol, not added to the preceding one; thus XIV denotes fourteen, one less than fifteen.

5. A bar ( - ) placed over a symbol increases its value a thousand times. Thus $\bar{V}$ denotes five thousand; $\bar{M}$ denotes one million. (The use of this bar is fairly recent)

Roman Numerals

| | |
|---|---|
| I . . . . . . . 1 | XXVI . . . . . . . 26 |

## TOPIC 3: NUMBER SYSTEMS

The simplest numbers are the natural numbers 1, 2, 3, 4, 5, . . . . . used in counting. These are called positive integers. The negative integers, -1, -2, -3, -4, . . . . are similar to positive integers as far as representing a collection of things, but their values are opposite. The symbol, 0, is called the zero integer, and is considered neither positive nor negative.

All of the integers, positive, negative and zero, together with numbers commonly expressed as the quotient of two integers are called rational numbers. For example, 2/3, two thirds; -15/4, minus fifteen fourths; and 3/1, three ones are rational numbers.

There exist many numbers which are not rational or cannot be expressed as the quotient of two integers. These numbers are called irrational numbers. A careful study of the irrational numbers is beyond the scope of this book. It will suffice to say that an important property of any irrational number is that it can be approximated by rational numbers to any degree of accuracy desired. For example, the following rational numbers

3.1 3.14 3.141 3.1416

are successively more accurate approximations of , pi, by rational numbers. (pi is the ratio of the circumference of a circle to its diameter)

The rational numbers together with the irrational numbers constitute the real number system to which we shall devote our time.

Cardinal numbers and ordinal numbers are a further break-down of the real number system. Cardinal numbers are numbers depicting a stated quantity of things and usually names "how many", such as 1, one; 2, two; 3, three; 27, twenty seven; 111, one hundred eleven. Ordinal numbers give the position or succession of a thing and usually answers the question "which one" or "where", such as, 3rd, third; 4th, fourth; 5th fifth; 27th, twenty seventh; 111th, one hundred eleventh.

Most numbers are composed of products of other numbers, such as, 4 is composed of 2 times 2; 12 is composed of 2 times 6 or 2 times 2 times 3 or 3 times 4. These numbers are referred to as compound numbers since they are compounded from other numbers. If we do not consider the number 1 and the number itself as factors of anumber, another group of numbers, called prime numbers, are included in the real number system. Prime numbers are numbers that cannot be represented by a product of two or more numbers, other than itself and one, such as, 1, 2, 3, 5, 7, 11, 13, 19, 23, 29, 31, . . . . .   .

## TOPIC 4: LARGE NUMBERS AND INFINITY

"One victim of overwhelming numbers was King Shirham of India, who, according to an old legend, wanted to reward his grand vizier Sissa Ben Dahir for inventing and presenting to him the game of chess. The desires of the clever vizier seemed very modest. "Majesty," he said kneeling in front of the king, "give me a grain of wheat to put on the first square of this chessboard, and two grains to put on the second square, and four grains to put on the third, and eight grains to put on the fourth. And so, oh King, doubling the number for each succeeding square, give me enough grains to cover all 64 squares of the board."

"You do not ask for much, oh my faithful servant," exclaimed the king, silently enjoying the thought that his liberal proposal of a gift to the inventor of the miraculous game would not cost him much of his treasure. "Your wish will certainly be granted." And he ordered a bag of wheat to be brought to his throne.

But when the counting began, with one grain for the first square, two for the second, four for the third, and so forth, the bag was emptied before the twentieth square was accounted for. More bags of wheat were brought before the king but the number of grains needed for each succeeding square increased so rapidly that it soon became clear that with all the crop of India the king could not fulfill his promise to Sissa Ben. To do so would have required 18,446,744,073,709,551, 615 grains!

Assuming that a bushel of wheat contains about 5,000,000 grains, one would need some 4000 billion bushels to satisfy the demand of Sissa Ben. Since the world production of wheat averages about 2,000, 000,000 bushels a year, the amount requested by the grand vizier was that of the world's wheat production for the period of some two thousand years! [1]

Although such a number as the number of grains of wheat requested by Sissa Ben is unbelievably large, it is still finite, and given enough time, it could be represented by a number down to the last grain. The grains of sand on all the beaches of the earth can be represented by a number since there is a definate number of grains of sand. The problem here, is to count them.

Infinity is an expression for a number having no bounds. It is the number of all numbers. Like the African tribesmen, all we can do is count as far as we can and the next number will be "many", even though infinity

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[1] George Gamow, One Two Three . . . . Infinity (New American Library, 1957), p. 19-20.

is still an infinite distance away.

An exciting study of infinities will be encountered in more advanced mathematics. At the present time, we are not prepared to understand the immensities involved nor the technical nature with which they must be treated.

The following is stated for the purpose of trying to interest you in infinities and perhaps enable you to carry on a reasonably intelligent but adolescent conversation concerning infinities. If you do not understand why a statement is made as a fact, further research is available in many current books of advanced mathematics, although it is not necessary for the successful completion of this course.

Things to remember:

There is an infinite number of numbers.

There are as many numbers between two numbers as there are numbers.

There are as many points on a line segment (line of stated length) as there are points on a line of infinite length.

There are as many points on a perimeter as there are points in the surface surrounded.

## TOPIC 5: FOUR OPERATIONS OF ARITHMETIC

The study of mathematics becomes increasingly more complicated as we include different number systems and proofs of previously accepted laws and rules. However, in every one of the fields of mathematical endeavor, the operations of arithmentic are applied. The ability to add, subtract, multiply and divide rapidly and accurately is certainly the very foundation of any future in a mathematical world.

To place ourselves in a position to move ahead to more advanced work, the following pages are devoted to these functions in order to refresh our memory on their solutions.

PROBLEMS FOR PRACTICE

Perform the problems, solving for the sum, difference, product and quotient as indicated.

| 1. | 2. | 3. | 4. | 5. | 6. |
|---|---|---|---|---|---|
| 7324 | 4631 | 42 | 8 | 7356 | 17 |
| 1192 | 921 | 5225 | 1903 | 4601 | 117 |
| 403 | 1313 | 1763 | 23 | 6163 | 1117 |
| 3960 | 3112 | 443 | 1776 | 4115 | 117 |
| 33 | 111 | 1899 | 1844 | 55 | 17 |

| 7. | 8. | 9. | 10. | 11. |
|---|---|---|---|---|
| 429790 | 666288 | 734229 | 722224 | 594226 |
| 193509 | 968965 | 109722 | 532449 | 429339 |
| 887505 | 567825 | 356270 | 432869 | 790193 |
| 594974 | 687232 | 870279 | 278943 | 567509 |

| 12. | 13. | 14. | 15. | 16. |
|---|---|---|---|---|
| 5435 | 4233 | 9178 | 3427 | 8032 |
| -2314 | -3926 | -7169 | -2118 | -3497 |

| 17. | 18. | 19. | 20. |
|---|---|---|---|
| 94043063 | 46725021 | 78431005 | 84654329 |
| -65319464 | -19642196 | -41005436 | -24391766 |

| 21. | 22. | 23. | 24. | 25. |
|---|---|---|---|---|
| 6274 | 5768 | 4097 | 8429 | 3035 |
| x989 | x870 | x798 | x906 | x670 |

26. 88909 x 247

27. 76007 x 578

28. 80704 x 231

29. 96780 x 298

30. 47809 x 407

31. 63)2898

32. 72)2592

33. 41)2337

34. 50)2850

35. 63)2391

36. 51)5049

37. 18)1026

38. 69)3864

39. 596)185352

40. 358)100598

41. 739)295985

# CHAPTER IV

## SUMMARY

An attempt is made to outline a good course in High School arithmetic. The syllabus contains a complete outline of a one year course. It is divided into 180 daily topics, each containing sufficient application to keep the students mentally occupied and interested.

To illustrate the daily topics, Chapter I is developed to show the depth of the subject used to prepare the student in the fundamentals of arithmetic. The intention of the author is to emphasis these fundamentals so that the student will not have to depend upon memorization. There is sufficient coverage of practical, business and life problems to give the High School graduate a better than average start in the competition he will have to face.

As far as the prospective college student taking a course of this type, the author has recently and unexpectedly come to realize that the study of Modern Mathematics insists upon the understanding that could be gained from the text prepared from this syllabus.

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