Name: Robert Landis Kumler Date of Degree: May 25, 1958

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Scope of Report: A simplified system of Boolean algebra is established, and some modern mathematical terms are introduced in their natural context. Examples are given of applications of Boolean algebra to electrical circuits and language analysis. Emphasis is placed on the concept of a mathematical system and insight into mathematics as a whole.

ADVISER'S APPROVAL James H Zant

INTRODUCING MODERN MATHEMATICAL CONCEPTS THROUGH THE USE OF BOOLEAN ALGEBRA

By

ROBERT LANDIS KUMLER

Bachelor of Science

University of California

Berkeley, California

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Dean of the Graduate School

PREFACE

"Modern mathematics" is, basically, the mathematics that has been developed in the last fifty years. However, no sharp dividing line exists between it and traditional mathematics, for much of the work of modern mathematics is concerned with new uses of old concepts, as well as the development of new ideas. The over-all effect is that mathematics now assumes a unified and structured aspect that it never had in the classical high school approach.

The purpose of this report is to show a natural means of introducing some of these new and extended concepts at the ninth grade level and, at the same time, to provide for insight into mathematics as a whole.

It is expected that the interested reader will have sufficient background in modern mathematics to provide supplementary material in the event that this approach is used in the classroom, for this report is basically an outline of essential ideas and not a teaching unit.

A Boolean algebra, in a simplified form, is used as the framework for this introduction for three reasons: (1) Boolean algebra is an outstanding example of a mathematical system and thus lends itself naturally to the illustration of basic mathematical principles and concepts; (2) it relates mathematics to other fields in a most fascinating manner; and (3) it can reveal new uses for mathematics that are not covered in the usual high school courses.

Until recently, most of the ideas to be introduced here were studied only by graduate students in mathematics. It is not the purpose of this report to justify making the change. However, the change is made with the sincere belief that it will lead to understanding mathematics as a living subject rather than as a series of mechanical manipulations. Furthermore, some of these new concepts should be presented as early as possible in the mathematics curriculum (even the third or fourth grade) in order to relate the individual subjects to the whole system and to avoid the turmoil of having to revise and even discard the limited ideas that otherwise would be established through the traditional approach.

I want to express my appreciation to the National Science Foundation for the opportunity to carry on this advanced work in science and mathematics. I am indebted to the following professors for their help and guidance in the field of modern mathematics: R. V. Andree, University of Oklahoma; E. Walters, William Penn High School, York, Pennsylvania; R. E. Johnson, Smith College; and M. E. Berg, Oklahoma State University. And finally, my sincere thanks go to Dr. James H. Zant not only for his invaluable assistance on the entire program, but also for helping me catch some of his enthusiasm for modern mathematics.

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CHAPTER I

INTRODUCTION

Modern mathematics is an example of mathematical development from an abstract point of view. Now mathematics always has been abstract, but people believed that its axioms and theorems expressed the laws of nature. Mathematicians are not concerned solely with physical truths. Nevertheless, fascinating results are being obtained by the application of symbolic and mathematical logic on problems of medicine, bio-chemistry, bio-physics, sociology, and even philosophy.

A dictionary might define mathematics as the science of space and numbers, but symbolic logic is the creation of a system from these basic materials much as an artist creates a picture from his collection of paints. Perhaps a more modern definition of mathematics would be that it is the creation of a logical structure or mathematical system.

To understand modern mathematics one must have then a notion of a mathematical system. Veblen and Young made it quite clear where one should start:

> The starting point of any strictly logical treatment of any branch of mathematics must then be a set of undefined elements and relations, and a set of unproved propositions

involving them; and from these all other propositions (theorems) are to be derived by the methods of formal logic.[1]

It is a simple truth, but it took men many generations to realize that a definition must depend eventually on words and ideas which have not been defined. In fact, it well might be said that mathematicians do not know what they are talking about, since the primitive ideas have no physical meanings so far as abstract mathematics is concerned; however, the point is that it does not matter as long as the undefined elements and postulates are consistent and independent.

Boolean algebra is a true mathematical system and is used here as the framework for the introduction of modern mathematical terms and thought at the ninth grade level. It is felt that at this level the basic laws of algebra can be demonstrated in a concrete manner in the Boolean system that is not possible when one is concerned with the real number system as a whole. The simple application of Boolean algebra, as well as possible extensions of this modern approach, should provide the student with a new appreciation of the place of mathematics in the system of human knowledge. George Boole, the founder of Boolean algebra, expressed his opinion in 1853:

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[1]Oswald Veblen and John W. Young, Projective Geometry (Boston: Ginn and Company, 1918), I, p. 1.

> But upon the very ground that human thought, traced to its ultimate elements, reveals itself in mathematical forms, we have a presumption that the mathematical sciences occupy, by the constitution of our nature, a fundamental place in human knowledge, and that no system of mental culture can be complete or fundamental, which altogether neglects them.[2]

The modern mathematics student should be freed from mere manipulation of symbols according to rules in an utterly structureless system. Algebra should come to mean the use of variables, expressions, and functions within or over a given number system, and it is hoped that this approach will help one achieve that end.

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[2]George Boole, An Investigation of the Laws of Thought (New York: Dover Publications, Inc., 1951), p. 423.

CHAPTER II

BUILDING THE SYSTEM

Basic Definitions

The study of Boolean algebra can begin as soon as one has established the idea of a set of elements, two operations, an equivalence relationship, and the duality principle.

Some mathematicians prefer not to define a set at all. However, when speaking of sets they invariably use objects of some sort. These objects usually are material. For example, one speaks of the set of people or chairs in a room; but since a set is purely a mental concept, one can speak equally well of the set of "virtues" or "goals" that a person might have. Now, with this connotation of the word "object" in mind, the following definition of a set should not be too restrictive. A *set S*, according to Meserve, is "...a collection into a whole of distinct, perceived, or considered objects called the *elements* of S."[1] The particular sets to be used in most of this discussion can be

[1]Bruce E. Meserve, *Fundamental Concepts of Algebra* (Cambridge, Massachusetts: Addison Wesley Publishing Company, 1953), p. 1.

represented by the elements 0 and 1. A system involving only two digits, specifically 0 and 1, is known as a binary system, dyadic system, or number system to the base two.

The two operations are to be represented by conventional symbols + and $\cdot$, but the conventional meaning is not necessarily intended here. In fact, the symbols for these operations can take on many different meanings within the limits of the postulates to be established shortly.

An <u>equivalence relation</u> is defined to be:

Any relation having the three properties:

reflexive, $a = a$

symmetric, if $a = b$ then $b = a$

transitive, if $a = b$ and $b = c$ then $a = c$.[2]

These properties are implied by the symbol " = " which is to be used in the basic postulates. Other well known symbols such as " $\cong$ " and " $\sim$ " are also equivalence (or equals) relations, but symbols such as " $<$ " (less than) and " / " (divides) do not satisfy all three properties.

The <u>principle of duality</u> is particularly important in Boolean algebra. The dual of a statement is the statement obtained by interchanging certain pairs of words or phrases in the original statement. In geometry a basic postulate states that "Two (non-concurrent) points determine a unique line." The dual of this statement with the interchange,

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[2]Ibid., p. 7.

line $\longleftrightarrow$ point, is "Two (non-concurrent) lines determine a unique point." It should be noticed that in this case the "dual" is not a postulate of ordinary Euclidean geometry since the lines may be parallel, but in projective geometry both postulates are valid. The importance of the <u>frame of reference</u> here should be noted, for this term is to be discussed later.

In a mathematical system much time can be saved by employing the principle of duality, for if every postulate in the system has its dual in the system, then the dual of each theorem is also valid. The next step then is to establish the meaning of a postulate.

Postulates and Theorems

Any mathematical system depends on the assumption of certain statements and undefined terms. Postulates are statements which are assumed to be valid. For the development of a meaningful system, the main requirement is that the postulates contain no contradictory statements. The following three postulates are used in this development of Boolean algebra:

A: $0 \cdot 0 = 0$

$1 + 1 = 1$

B: $1 \cdot 1 = 1$

$0 + 0 = 0$

C: $1 \cdot 0 = 0 \cdot 1 = 0$

$0 + 1 = 1 + 0 = 1$

While two postulates are listed in each case, one of each pair is merely the dual of the other under the interchange $0 \longleftrightarrow 1$ and $+ \longleftrightarrow \cdot$. Yes, $1 + 1 = 1$ is a perfectly valid statement! It should be remembered that both "1" and "+" are undefined. The postulates represent the laws that are assumed true for the particular operations involved. It might help to think of "1" as "everything," and then if one "adds" "everything" to "everything" a logical answer is still "everything." However, it is not necessary that these postulates make "sense." The problem is to see what theorems can be developed for an algebra governed by these basic postulates.

A theorem is a statement which can be proved using the laws of logic, the undefined terms, and the postulates of a given system. Any established theorem may be used in the proof of subsequent theorems. Three more symbols, X, Y, and Z, are needed to simplify the handling of these theorems. The symbols, X, Y, and Z, stand for either of the previous symbols 0 and 1. Thus, it is not necessary to make the interchange $0 \longleftrightarrow 1$ in the theorems, for when it is made there is no change in the statement of the theorem.

Only three basic theorems, and their duals, are necessary to "discover" a great deal of algebra.

Theorem 1a: $X + Y = Y + X$

1b: $X \cdot Y = Y \cdot X$

Theorem 2a: $(X + Y) + Z = X + (Y + Z)$

2b: $(X \cdot Y) \cdot Z = X \cdot (Y \cdot Z)$

Theorem 3a: $X \cdot Y + X \cdot Z = X \cdot (Y + Z)$

3b: $(X + Y) \cdot (X + Z) = X + (Y \cdot Z)$

Now one of the pleasures of binary Boolean algebra is that, since the variables are restricted to the values of 0 and 1, it is feasible to prove these theorems by actually substituting all possible values into the expression and seeing if they work. The proof of theorem 1a is as follows:

Prove: $X + Y = Y + X$

Proof:

X	Y	X+Y	Y+X	
0	0	0	0	by postulate B
0	1	1	1	by postulate C
1	0	1	1	by postulate C
1	1	1	1	by postulate A

The first two columns of the proof represent all possible combinations of values that X and Y can have at any one time. The last two columns are obtained by applying the postulates indicated, and the fact that these two columns are identical is sufficient proof for the theorem. Under the interchange, $+ \longleftrightarrow \cdot$, theorem 1b is valid by the duality principle.

Theorem 3b is proved in a similar manner as follows:

Prove: $(X + Y) \cdot (X + Z) = X + (Y \cdot Z)$

Proof:

X	Y	Z	X+Y	X+Z	Y•Z	(X+Y)•(X+Z)	X+(Y•Z)
0	0	0	0	0	0	0	0
0	0	1	0	1	0	0	0
0	1	0	1	0	0	0	0
0	1	1	1	1	1	1	1
1	0	0	1	1	0	1	1
1	0	1	1	1	0	1	1
1	1	0	1	1	0	1	1
1	1	1	1	1	1	1	1

The first three columns represent all combinations that could occur with the three variables. The next three columns are shown to clarify the final calculations. The last two columns are identical, and the theorem is proved. Theorem 3a is valid by the duality principle. Theorem 2 would be proved in a similar manner.

It should be recognized now that a simple mathematical system has been established through the use of undefined elements and relations, a set of unproved postulates concerning these elements and relations, and theorems derived from these postulates. The next step is to find applications for this system. The obvious extension to ordinary algebra already has been made since five of the fundamental laws have been proved in theorems 1, 2, and 3a. These laws are discussed in detail in the next chapter.

CHAPTER III

APPLICATIONS TO ALGEBRA

If you are not a mathematician, but have had algebra, can you state five basic laws that you used? If you can, you are indeed fortunate, for in all probability you had little difficulty in your mathematical work. These extremely fundamental laws often are ignored in elementary algebra, and as a result it is necessary for the student to memorize many rules and manipulations to replace the basic laws. Further discussion is necessary to apply these theorems to an ordinary algebraic system.

Theorem 1 symbolizes the commutative laws for addition and multiplication.

Theorem 1a: $X + Y = Y + X$

1b: $X \cdot Y = Y \cdot X$

The students may smile indulgently when this law is stated and groan when they are told to learn the name of it. Perhaps they will feel better when they recognize that there is reason for the law in that it does not always apply. For example, one can pour acid into water $(X + Y)$ or water into acid $(Y + X)$, and the immediate results may be radically different. The algebra of permutations can serve readily as another example.

Theorem 2 symbolizes the <u>associative laws</u> of addition and multiplication.

$(X + Y) + Z = X + (Y + Z)$

$(X \cdot Y) \cdot Z = X \cdot (Y \cdot Z)$

Actually, this law represents a characteristic limitation of the human mind. One might say this is intuitively true, but it can cause difficulty if it is not recognized as a basic law. The concept of closure could well be brought in here for classroom use.

The final and most important law is the <u>distributive law</u>.

Theorem 3a: $X \cdot Y + X \cdot Z = X \cdot (Y + Z)$

Technically, it might be stated that multiplication distributes over addition, but the reverse situation is not true. But why is this law so important? It appears throughout the study of algebra.

If one looks at theorem 3a from left to right, the algebraic operation might be called factoring out the monomial term. However, from right to left the operation would be called multiplication. Many weeks are spent teaching these separate operations in algebra when in reality they are covered by one law! Collecting terms is another application of the distributive law. It is used when fractions are "added." All of the important identities, such as $(a^2 - b^2) = (a - b)(a + b)$, are direct consequences of the distributive law.

Now one way of stressing these laws is to have the students prove them for themselves, and then the laws may "belong" to the students. Theorems 1, 2, and 3 have been proved for Boolean algebra. Are these proofs valid for ordinary algebra? Suppose $X = 1$, $Y = 2$, and $Z = 3$. Then theorem 3b would state $(1 + 2) \cdot (1 + 3) = 1 + (2 \cdot 3)$, but $3 \cdot 4$ is not equal to $1 + 6$! What is the matter? The Boolean system restricted the values of X, Y, and Z to 0 and 1. The symbols 2 and 3 are outside of the frame of reference.

Does $3X = 2$ have a solution? Actually, this is a meaningless question unless the frame of reference is specified, for the answer is "yes" in the system of rational numbers, "no" in the system of integers, "yes" in the mod 10 system, and "no" in the mod 6 system.

The frame of reference, or domain, in which one is working is an important concept in geometry as well. For example, the statement "Two lines which do not intersect are parallel" is valid in plane Euclidean geometry, but it is not valid in solid geometry.

Theorems 1, 2, and 3a hold for the real number system, but the proofs given here are only valid for the Boolean system. Why not prove these laws directly in the frame of reference where they are to be used? The answer is best shown by an example of a proof as it might be handled at the senior high school level.

To give a rigorous proof one would have to develop the number system for positive integers and extend it step by step through the real and complex number systems. However, an outline of the preliminary details should be sufficient and serve to convince the reader that this is not ninth grade material.

An algebraic proof of theorem 1a depends on Peano's postulates[1] and certain lemmas developed from them. The induction principle, which in turn involves the well ordering principle, also is used. For example:

Prove: $X + Y = Y + X$ for X and Y positive integers

Proof: (By use of the induction principle)

Is the statement true for $Y = 1$? I.e., does $X + 1 = 1 + X$? Yes--by a lemma provable from Peano's postulates. Then assume the statement is true for $Y = n$ where n is some positive integer greater than 1, thus $X + n = n + X$ (hypothesis). Then prove it true for $Y = n + 1$, i.e.,

$X + (n + 1) = (n + 1) + X$

$$
\begin{aligned}
X + (n + 1) &= X + n^+ && \text{from Peano's postulates} \\
&= (X + n)^+ && \text{from Peano's postulates} \\
&= (n + X)^+ && \text{by hypothesis} \\
&= (n^+ + X) && \text{from Peano's postulates} \\
&= (n + 1) + X && \text{from Peano's postulates}
\end{aligned}
$$

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[1]Bruce E. Meserve, Fundamental Concepts of Algebra (Cambridge, Massachusetts: Addison Wesley Publishing Company, 1953), p. 9.

Thus the statement is true for $Y = 1, 2, 3 \cdots n$ and, therefore, true for all positive integers.

This proof certainly requires far too much background for the ninth grade, while the proof on page 8 should be understood readily by the entire class.

In conclusion, it has been stated that:

> Inadequate mastery of fundamental terminology, concepts, and skills is probably the most outstanding cause for the difficulty encountered by individuals of all ages in dealing with anything of a mathematical nature.[2]

Part of the answer to this problem may be to pay more attention to these basic laws. In addition, it is felt that practical applications, to show the true nature of mathematics (rather than just for drill purposes), would be effective in the learning process. The final step, then, is to introduce some simple but unique applications of Boolean algebra.

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[2]F. L. Wren, "Secondary Mathematics," Encyclopedia of Educational Research, ed. Walter S. Monroe (New York: The Macmillan Company, 1950).

CHAPTER IV

APPLICATIONS FOR FUN AND UNDERSTANDING

It should be recalled now that the symbols 0 and 1 and the operations + and • were undefined in the Boolean system. The beauty of having these elements and operations undefined is that one then can apply different interpretations and arrive at delightful yet significant conclusions.

Electric Circuits

There are two basic electrical circuit arrangements. Two switches connected in series, as shown in Figure 1, will conduct only when both are closed, while parallel circuits will conduct if either switch is closed, as in Figure 2.

Figure 1. Series Switches

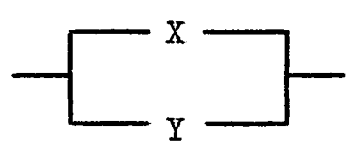


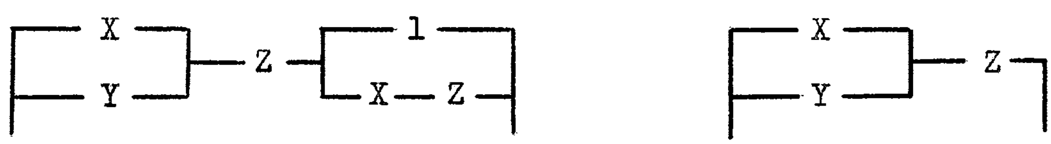
Figure 2. Parallel Switches

Now an isomorphism can be established between binary Boolean algebra and electrical circuits under the following correspondence:

As related to switches		As related to Boolean algebra
open	$\longleftrightarrow$	0
closed	$\longleftrightarrow$	1
series	$\longleftrightarrow$	$\cdot$
parallel	$\longleftrightarrow$	+

The term, isomorphism, means that the systems are abstractly identical under the interchange or correspondence as shown above.

Consider the combination series-parallel circuits shown in Figure 3.



(a) (b)

Figure 3. Equivalent Series Parallel Circuits

These circuits are identical in that they will each conduct electricity when their related switches are closed. The symbol 1 represents a switch that remains closed at all times.

The Boolean polynomial $[(X + Y) \cdot Z] \cdot [1 + X \cdot Z]$ is an algebraic representation of Figure 3(a). Now by applying the commutative laws, the distributive law and theorem 3b as follows:

$$[(X + Y) \cdot Z] \cdot [1 + X \cdot Z] = [X \cdot Z + Y \cdot Z] \cdot [X \cdot Z + 1]$$
$$= X \cdot Z + Y \cdot Z \cdot 1$$
$$= Z \cdot X + Z \cdot Y$$
$$= Z \cdot (X + Y)$$

one arrives at the algebraic symbolization of Figure 3(b).

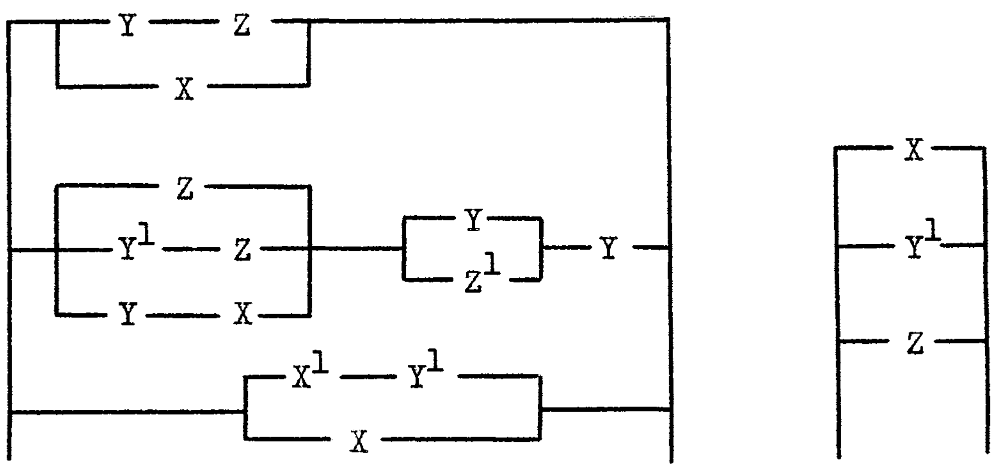
Relays are of tremendous importance electrically and represent a condition where a switch may be either closed or open. The English equivalent of this situation (which will be used later) is that of a "not" or complementary statement. If the symbol X^1 is used, it could be interpreted as meaning that whatever the status of switch X, X^1 is in the opposite position. In other words, if switch X is closed then X^1 is open.

An additional theorem, 4a, can be intuitively established here.

Theorem 4a. $X + X^1 = 1$

Electrically, this means that when a switch and related relay are connected in parallel the circuit always will conduct.

Figure 4(a) represents a little more complex circuit which can, by the methods of Boolean algebra, be reduced to Figure 4(b). Both circuits will do the identical job. Which one would you rather buy and maintain?



(a) (b)

Figure 4. Equivalent Switching Circuits

The interested reader will find a complete development of this electrical circuit application in an article by Franz Hohn.[1]

Contract and Language Analysis

An isomorphism also exists between Boolean algebra and general statements involving "and" and "or." Under the relationships

or $\longleftrightarrow$ + $\longleftrightarrow$ parallel

and $\longleftrightarrow$ $\cdot$ $\longleftrightarrow$ series

many interesting applications can be made to language statements. The "not" relationship is also available as

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[1]Franz Hohn, "Some Mathematical Aspects of Switching," American Mathematical Monthly, CXII (February, 1955), p. 75.

discussed. It should be noted that the "or" in X + Y implies X or Y or both. If just X or Y is intended, the expression would become $(X + Y) \cdot (X \cdot Y)^1$, which implies X or Y and not X and Y. In other words, the possibility of both conditions existing must be excluded by using the "not" relationship.

As an example of the possibilities here, suppose three men, X, Y, and Z, take out an insurance policy that will pay the survivor $50,000 if either X or Y dies providing that either X or Z also die. If one interprets "providing that" as equivalent to "and," then the statement is represented readily by a Boolean polynomial $(X + Y) \cdot (X + Z)$. Theorem 3b immediately simplifies this to $X + Y \cdot Z$, and a much clearer statement of the policy follows: the policy will pay $50,000 if X dies or if Y and Z die. Figure 7 shows an "electrical" picture of the policy conditions.

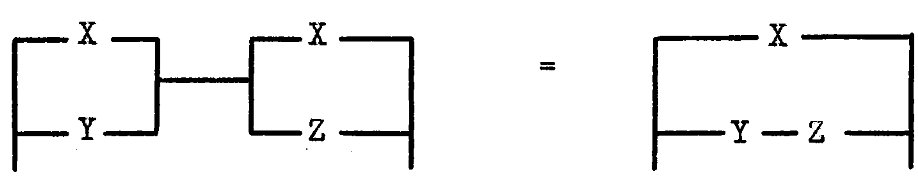

Figure 5. Equivalent Circuits

The applications here are truly unlimited. The chief difficulty is that in many legal contracts the wording becomes so involved that a computer, along with Boolean algebra, may be necessary for clarification.

As another simple example to show the complete adaptability of this abstract mathematical system to the English language, consider an additional theorem.

Theorem 5a: $(X^1)^1 = X$

Now remember how these theorems were developed and proved, and suppose X is equivalent to the statement, "John will study." Then X^1 would mean "John will not study." Theorem 5a then would be interpreted as "John will not not study," and the conclusion is that "John will study."

Boolean algebra is used extensively in point set theory. The direct application to Venn circle diagrams, for instance, would be suitable at the high school level and would give further meaning to the concept of a mathematical system. How far can one go with a system such as this?

> What may be the final estimate of the value of the system, I have neither the wish nor the right to anticipate. The estimation of a theory is not simply determined by its truth. It also depends upon the importance of its subject and the extent of its applications; beyond which something must still be left to the arbitrariness of human Opinion.[2]

These words of George Boole were written in 1847. His reaction is typical of basic research scientists, and perhaps that is the field where modern mathematics ultimately will be of greatest benefit to mankind.

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[2]George Boole, The Mathematical Analysis of Logic (New York: Philosophical Library, Inc., 1948), p. 2.

CHAPTER V

CONCLUSION

There are many more concepts of "modern mathematics" that have not been discussed here. However, in the classroom development of this material, terms such as closure, identity elements, inverse elements, groups, rings, and fields most certainly would be involved. Much insight into mathematics can be obtained through the study of other number systems. For example, when considering numbers modulo 12, it is only natural to talk about the group concept.

The modern ideas are not limited to algebra. The traditional methods of handling geometry and trigonometry are being revised, and new subjects such as topology are being introduced. Thus it should not be felt that the introduction of modern mathematics through a unit on Boolean algebra is the end of the story. Modern concepts should permeate all mathematical work, and many of them should be considered by the students as they venture into new material. It is only in this manner that the full value of such a unit can be realized.

Perhaps the most important concept that can be achieved with the use of Boolean algebra is a sense of the structure of a mathematical system. It is hoped that not only will the student understand this particular system, but also that he may be able to detect similar structures in quite dissimilar systems. This is one of the real essences of mathematics--to be sensitive to structural similarities, when they exist, and to be able to abstract from diverse systems the essential simple structure.

The words of Dantzig seem to epitomize the underlying thoughts of this report as related to modern mathematics and Boolean algebra.[1]

> The mathematician may be compared to a designer of garments, who is utterly oblivious of the creatures whom his garments may fit. To be sure, his art originated in the necessity for clothing such creatures, but this was long ago; to this day a shape will occasionally appear which will fit into the garment as if the garment has been made for it. Then there is no end of surprise and delight!

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[1]Tobias Dantzig, Number, the Language of Science (New York: The Macmillan Company, 1933), p. 231.

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VITA

Robert Landis Kumler

Candidate for the Degree of

Master of Science

Report: INTRODUCING MODERN MATHEMATICAL CONCEPTS THROUGH THE USE OF BOOLEAN ALGEBRA

Major Field: Natural Science

Biographical:

Personal Data: Born at Albany, New York, October 23, 1923, the son of Clifford F. and Marjorie Conklin Kumler.

Education: Graduated from Thacher School, Ojai, California, in 1941; received the Bachelor of Science degree from the University of California, with a major in Electrical Engineering, in June, 1947; member of Tau Beta Pi and Eta Kappa Nu; attended San Jose State College, University of California at Santa Barbara, University of Southern California, and Montana State College; completed requirements for the Master of Science degree at Oklahoma State University in May, 1958.

Professional experience: Seven years of engineering work with the U. S. Bureau of Reclamation, U. S. Department of Naval Ordnance, and the Pacific Gas and Electric Company; three years of private school teaching; member of National Science Teachers Association.

Typist: Elizabeth J. Kerby
KERBY TYPING SERVICE