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Scope of Study: There is a general awareness among American educators of today that mathematics received by the average high school student is inadequate. It is generally accepted that the desired improvements will largely depend upon the attitudes and preparation of the high school mathematics teacher. This report involved the study of what writers, teachers, and other interested agencies are suggesting to facilitate a speedier and more comprehensive transition from arithmetic to algebra. It discusses some of the suggestions offered for dealing with basic concepts of algebra, some reported findings of groups employing the suggestions, and how the high school mathematics teacher might use some of these ideas.

Findings and Conclusions: With even the most well-writen textbooks the high school mathematics teacher has a vast amount of preparation to do in order to adequately introduce the beginning algebra students to the fundamentals of its structure. Many of the concepts that noted mathematicians feel should be introduced and developed in the pre-college student are not treated in accepted texts. It is necessary that high school mathematics teachers be continuous students of their field, studying at some institution as often as possible and reading their journals carefully at all times.

ADVISER'S APPROVAL James H. Zant

THE TRANSITION FROM ARITHMETIC TO ALGEBRA

By

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CHAPTER I

INTRODUCTION

The purpose of this report is to discuss the methods offered by writers of textbooks and other individuals or agencies for the successful introduction of some basic concepts confronting students moving from arithmetic into algebra. It attempts to offer these suggestions along with some of the implications for the consideration of the new teacher of first year algebra and those teachers of first year algebra who have lacked the time or inclination to give lengthy consideration to the hurdles facing the student making the transition from arithmetic to algebra.

In methods of procedure the report is an outgrowth of certain findings by the writer from a careful scrutiny of a sizable sampling of textbooks and other pertinent literature as they relate to the problems experienced by the student in the transition from Arithmetic to Algebra. The paper is not intended as a summary of the majority reflections of these writers but a careful study of these reflections in the light of eight years of actual teaching experience. Certainly, no claim is made that the opinions and conclusions to be set forth belong exclusively to the writer but it is felt there is yet a vast amount of gainful exploitation possible in connection with these views. Then, this report is conceived and developed to deal with the following assertions:

1. The field of Mathematics has grown more rapidly than is generally known by highschool Mathematics instructors. Consequently, <u>more</u> Mathematics must be learned <u>faster</u> by today's student to attain the merest competency than perhaps any generation before. Thus, the earliest concepts

should be meaningfully instilled at all costs.

2. Very nearly all normal children like Mathematics as long as they understand it and this inclination persists unto and through adulthood.

3. Many would-be Mathematics students are lost at various levels of abstractions (concepts that fail to lend themselves easily to palpable demonstration). Many concepts treated as abstractions can be meaningfully demonstrated otherwise with careful thought and ingenuity on the part of the instructor.

4. The Mathematics of a university student should be no more difficult for a University student than a first grader's mathematics is to a first grader, a student with basic concepts properly instilled along the way will find this true under an instructor as capable of demonstration of his material as most first grade teachers are of theirs.

5. It is extremely difficult for the gifted to understand or sympathize with the less fortunate. Thus, many textbook writers inadvertantly or intentionally write textbooks that preclude the average student's unassisted progress.

It is not intended to convey the idea that higher mathematics is possible for all individuals. This definitely is not the case, nor would it be desirable if possible. However, whether one accepts it or not the technological world has become and is becoming ever more mathematical. This implies that he who would be an intelligent citizen in today's world must of necessity be better informed in the subject than is the average citizen of today. This is possible for coming generations if teachers can check the loss of the many students who are discouraged from mathematics at an early phase of their pursuit of it by difficulties that may be eliminated.

The report involves general ideas that are fortunately receiving consideration from some outstanding mathematicians. In a recent work Professor Fehr of Columbia University says:

"The learning process of going from experience with things, to thinking about things to abstraction and concept formation, and finally to reorganizing the newly learned concept into whole structure is, in the field of mathematics, the one that has the most promise of permanence of learning to the solution of quantitative problems. The building of concepts is so necessary if we are to develop mathematics as a way of thinking that will serve us in our various life careers"[1]

No attempt is made in the report to deal with all the concepts the student will have to master for a successful first year in algebra. However, it is felt that the more basic notions are considered. An attempt is made to consider the idea that mathematically intelligent citizens are gained or lost in this early formative stage of pupil development. Therefore, numerous and appealing experiences should be provided even at the expense of failing to cover an arbitrarily chosen topic somewhere near the back of the book. On this particular notion Breslich said:

"Not enough experiences are provided to develop real understanding of the concept of Algebra. Literal numbers, signed numbers, symbols of operation, exponents, equations follow one another in rapid succession, although each alone offers serious difficulty to the learner. Most textbooks introduce all these concepts within a very few pages, hence no real understanding is attained by the pupil. The result is confusion and dislike of the subject which is in reality simple if the fundamentals are thoroughly mastered."[2]

Textbooks remain in print and in use that fail to escape this criticism. However, there is much satisfaction in finding numerous modern writers who either anticipated or heeded Mr. Breslich's admonitions in their works.

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[1]Howard F. Fehr, "Reorinentation in Mathematics Education" Teachers College Record, Vol. LIV (May, 1953), p. 435.

[2]Earnest R. Breslich, The Administration of Mathematics in Secondary Schools (Chicago, 1933), p. 267.

CHAPTER II

THE FORMULA

Many of the textbooks dealing with a first course in algebra begin with the formula as an introduction to the study. There seems to be a majority opinion that the formula can be made more concrete than the equation when introducing algebraic concepts. It is seemingly agreed by most that formulas might well be studied before directed numbers or equations. The general rule of "one difficulty at time" pervades the structures of nearly all. It is conceded that most students are unable to focus on the meaning of the formula if there is disturbance by another new idea which is not understood or is vaguely understood.

In Modern mathematics one's level of achievement is greatly determined by his ability to master symbolic language. The formula, this writer has found, is a comparatively easy and meaningul introduction to symbolic language. This language of algebra is at least as important for a student to master as the skills in manipulation. It is often an extreme test of patience for the most understanding teacher to constantly remind himself that the pupil is learning a new language and that the pupil's concept of symbolism develops gradually. One must not assume that the average pupil actually has a full understanding of the common symbols +, -, ÷, x. Situations must be created whereby the need and use of symbolism are as nearly obvious as possible. Thorndike has said:

> "Yet observation of the learning of algebra shows that the pupils learn by their concrete experiences with letters, coefficients, exponents, etc., more than by analytically scrutamizing of their

> definitions. They learn by what they do with the algebraic facts, and what the results are, more than what they are told about them."[3]

There would seem to be a general opinion that a good approach to the teaching of the formula is to deal with it as a kind of shorthand for abbreviating practical rules.

Most students of today have grown up in an economy of installment buying and numerous credit plans and hence it is very likely that many will have some understanding of the rule for finding interest. If this is true there will be little difficulty in employing the rule in the form I = prt. However, many highly regarded authorities on the subject suggest that pupils be trained to first state the rule in abbreviated word-form before using symbols. Instead of beginning by expressing the above mentioned rule as I = prt the rule is expressed as Interest = principal x rate x time. It is the opinion of the writer that this practice is a profitable one, for his experiences have taught him that mistakes are often committed when this procedure is not followed. Take the simple case of the student during marketing for the family: Eggs are sold at a certain price per dozen, usually and most students have at one time or another participated in the experience of purchasing eggs. An agreement is immediately possible between student and teacher that cost of eggs equals number of dozens multiplied by price per dozen. It can be suggested that the first letters of those words referring to numberical quantities be used to write a "short hand" rule for this relationship: Thus, C = d x p. Even when steps are taken to assure cost and price agreement in units error is quite often forthcoming. The letter d has a constant value of twelve for the student because d-o-z-e-n spells dozen and is defined as

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[3]Edward L. Thorndike, The Psychology of Algebra, (New York, 1924), p. 242.

12 things. Thus instead of a generalized rule for finding the cost for any number of dozens of eggs the student has a fallacious idea that will practically eliminate his chances entirely for getting a correct solution to problems of this nature. Considerable time must be devoted to getting the student started correctly as it is much easier to teach than unteach though both might be difficult.

A formula definitely describes a relationship, a case of determinate corresponce. If this point of view is perceived by one teaching it, there is an opportunity to give the student the necessary background for the function concept. This concept will later determine the extent of his progress in more abstract mathematics, alas it will even determine how far he goes with the formulas. Even the latest textbooks go along with this incidental teaching of a function concept and most suggest the four-fold method of expressing the relationship: the table, the arithmetic rule, the formula, and the graph.

According to Taylor, "a function is the determinate correspondence between two classes of objects,"[4] Richard E. Johnson in his lecture notes at Oklahoma State University, Fall semester, 1957 says "a function is a correspondence that associates with each number of given collection a unique number"[5] or in essence, a refinement of the same thing as Taylor. Dr. James. H. Zant, Professor of Mathematics at Oklahoma State University defines a function as follows:

> "Given any two sets or collections X and Y, by a function of X to Y we mean all possible pairs made up of an element of X and an element of Y which corresponds to it according to some rule or formula. It is understood that an element of

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[4]Angus E. Taylor, Advanced Calculus (New York, 1955) p. 297.

[5]Richard E. Johnson, Lecture Notes of "Calculus for Highschool Teachers", Oklahoma State University, Fall Semester, 1957. p. III-29.

X, like x, appears exactly once as a member of a pair."[6]

Consideration of Professor Zant's helpful amendment to this most pervading but somewhat elusive concept facilitates meaningful consideration of two other notions everywhere associated with the formula. From the beginning it will be helpful if the new algebra student is made conscious of the idea of variables and domain. The collection of numbers over which a function is defined is called the domain of a function. In the formula above, $C = np$, if the price of a dozen eggs is fixed n may take on any one of the set of the positive real numbers and zero. With each value given n, the independent variable, there is associated some value for C, the dependent variable, according to the rule of correspondence linking them. Some effort should be made to include in the list of formulas a number of those that illustrate clearly the limited domains of some functions. For instance, the postage function offers a set of formulas demonstrative of this idea:

> Let C = cost in cents of sending a parcel of weight w ounces by first class mail, then,
>
> C = 3 if w has value greater than 0 and equal or less than 1,
>
> C = 6 for w more than 1, equal or less than 2.
>
> \- -
>
> \- -
>
> C = 960 if w is more than 319 and equal to or less than 320.

This function has as its domain the set of all positive numbers not exceeding 320 ounces.

Certainly, one could hardly expect rigor on the part of a student at this stage of his progress in algebra. However, this idea of mapping or picture relationships can be inculcated by graphing of relations in formulas. Using an arithmetic rule to set up a table exhibiting relation-

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[6]James H. Zant, Analytic Geometry and Calculus, Publication Pending, 1958, p. 1-6

ships will establish a concrete reference point for a student when he later encounters cases of abstraction concerning these ideas. A difficulty in mathematics lies in the fact that any success in it depends upon an unbroken extension from the basic starting point. One might conceivably be an American citizen and learn of British history without any prior knowledge of American history. A study of either might facilitate a more rapid and broader grasp of the other but neither is a prerequisite for the other. So systematic is the structure of mathematics that one seldom finds an analagous situation. It would be unusual to find an individual capable of successfully dealing with the calculus prior to a sound course in algebra to say the least. However, to a lesser extent this is the situation faced by the student confronted with rigorous applications of the function concept and no referrent point in some aspect of his consciousness. One might safely say from all this that where the student is going in mathematics depends to a great extent upon where his teacher has been. The broader the view of the teacher the less likely will be the teaching of concept or topic as an end within itself, neither lending itself as an enrichment of things past nor a link in the chain of things to come.

A great deal of care and time is usually required to teach the student to solve a simple formula for any specified letter. Perhaps here more than any place else the argument for introducing the formulas before the equation as a specified letter necessarily involves the teaching of an understanding of the axioms. Meaningless transposition and baseless numerical operations can hardly occur when the entire problem consists of a relationship of letters to letters.

The work in formulas should provide a generous amount of work in substitution. Not only is this procedure most necessary in the gaining of masterful employment of the formula but it is highly desirable as a foundation upon which one anticipates another most basic idea in mathematics. The concept of equivalence relationships is basic in the study of modern mathematics and that particular system of logic given to the domain of mathematics. When we can apply the same label to every element of a class or set, each element is substitutable for every other. Such a set of things is called an equivalence class.[7] The pupil should also employ whole numbers, common fractions, limited to those used in computation that the world needs to do, and decimals. Dealing with these numbers gives the student an assurance of being in familiar territory plus the fact it provides much needed practice with quantities he should really have consummate skill in using. It is indeed a rarity when a student beginning algebra needs no further practice in common fractions and decimals. Moreover, such a practice is desirable because it is a way of developing the meaning of the formulas.

Finally, Thorndike makes a rather penetrating statement that reads:

> "The pupil builds up or integrates his habits into rules, as well as derives new habits from rules. Learning to compute algebraically is not only, or chiefly, learning rules and how to apply them; it is also building up a hierachy of habits or connection bonds which clarify, reinforce and, in part, create the understanding of what the rules mean and when to apply them."[8]

Though this statement was made some thirty years ago there would probably be very few mathematics instructors who would not agree with it.

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[7]Robert L. Swain, Lecture Notes of "Modern Mathematics for High School Science and Mathematics Teachers" Spring Semester, Oklahoma State University, 1958, pp. 1-28.

[8]Edward L. Thorndike, <u>The Psychology of Algebra</u>. (New York, 1924), p. 246.

CHAPTER III

SIGNED NUMBERS

The concept of signed numbers is one that lends itself to several introductions. Substantiation of this assertion is very easily gained by merely consulting several different elementary algebra texts. By virtue of the fact that there are various ways of introducing this notion the writer holds that some of these methods might generally be better than others. One might contend that it would be most effective to use them all but this would necessitate the spending of undue time on the idea and serve to kill the spirit of certain segments of the class. The following treatment of the notion is a result of careful study of these various presentations in light of personal teaching experience.

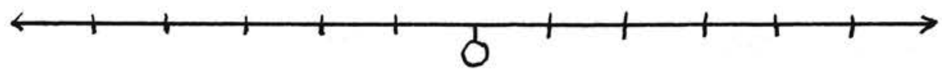
To meaningfully introduce a new concept or even to extend an old one it is necessary to ascertain a good starting place for the students being taught. This place, of course, varies from student to student. Therefore, considerable time and effort must be devoted to its selection. Perhaps there is no best place for what is usually termed the average class. A good one, one which lends itself to broad usage in mathematics to come later, is the idea of ones position and direction in relation to some fixed point or line. In the hills of Tennessee the weather phenomena lends itself to colorful illustration of the idea. Fifty miles south of Dogpatch farmers enjoyed abundant rainfall last year and all those in this region grew fine tobacco crops. A friend who is a tobacco farmer lives fifty miles from Dogpatch. Can one conclude that he grew a good tabacco crop last year? This is hardly possible from this information. However, if one is supplied the information as to whether the farmer lives north or south of Dogpatch, whether it is east of south or west of north notwithstanding, the answer is immediately available.

From the brief suggestions mentioned in textbooks the resourceful teacher is able to place the student in various imaginative positions that might appeal to him and has direct bearing on the notion of positive and negative numbers. The following is an example:

While relaxing in a comfortable American type "igloo" in Northern Alaska the voice of a glib American reporter comes in over the radio announcing the outside temperature as 30°. What coat does a G. I. wear to dinner which will be served in Barracks #10, a quarter of a mile away? A topcoat is worn in weather not severely cold and a specially insulated heavy overcoat for the extremely cold weather. It must be determined whether the reading of 30° was above or below the zero point, or -. Again, a corporation executive has an apartment on the 20th floor of a 42-story skyscraper. Each morning he rides the express elevator ten stories to his office located in the same building. What floor would one go to for an appointment with the executive in his office?

Once the fact is well inculcated that distance from a point is not sufficient to pinpoint the whereabouts of an object it is fairly easy to illustrate the clumsiness encountered in modifying numbers with such terms as up, down, above, below, clockwise, counterclockwise and such opposite notions. Any two opposing systems may be designated as respectively positive and negative. Which shall be called positive and which shall be called negative is simply a matter of choice and convention.

One of the most widely recommended visual aids in teaching the idea of opposites is the number line diagram. Each student should be allowed to make for himself a portable representation of the number line. It will serve well in the teaching of operations with signed numbers.



Inasmuch as this is a somewhat new idea for the student careful attention should be given to using the terms plus, positive, minus, and negative. Many textbooks are careless with these terms, making it quite easy for the teacher to be likewise. The transition from arithmetic to algebra is perhaps easier here if the student is allowed to retain his idea of plus meaning that two or more quantities are to be joined to obtain a sum. There is confusion inreferring to the point five units to the right of zero on the number line as a plus 5 in one instance and positive 5 in the next. A like confusion is encountered when a "minus" 5 and "negative" 5 designate the same point on the number line. This might be eliminated by letting "positive" and "negative" identify opposing natures of a system. For instance, if the direction upward is termed negative the direction downward should be called positive. Thus, one could ever avoid the sound of such sentences as "Take eighty plus a plus ninety," in referring to two like distances to be joined.

The writer has long felt that the symbols designating the nature of the opposing systems should either be different from the conventional addition and subtraction signs or at least placed in different positions. It has been rewarding to finally find a highly regarded mathematician, Dr. Robert L. Swain, who proposes the idea in one of his books. A pupil of one weeks experience in signed numbers is not necessarily a mental case because he writes 5+ +5 in response to the command "find the sum five plus a positive five." Surely, the last 5 of the expression is to be enclosed in parenthesis but why fix the mind on this use of the grouping symbol and invite undue difficulty in teaching grouping? If the use of + and - must be retained to designate positive and negative systems why not place them, say above and slightly to the left of the number they quantify? For instance, ${}^{+}5 + {}^{+}5$ and ${}^{-}5 + {}^{+}5$ could indicate the sum of a positive five plus a positive five and a negative five plus a positive five. This notation should not interfere with the common practice of writing pi as $3.1\overset{+}{4}$ and similar cases. The writer has seen the expression $5+ +5$ carried to its ultimate more than once: $5 + + 5 = 15$.

Consider the problem of operating with signed numbers. In most textbooks there is a nice set of rules for these operations in heavy type. An instructor may require his students to "parrot" them to him until pupil knows them by heart and maybe applies them, or he may lead the pupil in demonstrations whereby the pupil arrives at the rules for himself. Certainly, when a pupil derives these laws of operations for himself they have more meaning for him and he is most likely to apply them correctly and retain them longer.

It is customary to begin with addition. Each child with his number line diagram on cardboard can visualize the conventions to be taught here. First, numbers to the right of zero are designated as being of a positive nature. Secondly, establish the direction of motion to the right from <u>ANY</u> point on the number line, irregardless of its relation to zero, as of a positive nature. Thirdly, get accross the idea that an operation begins <u>AT</u> the operation symbol whether word or symbol. In board drills the instructor might best avoid such commands as "Johnny, find the sum of positive five and a positive six" for a while as this often leads to confusion. A positive five and a positive six have locations on the number line one unit apart. However, a number line without numerals will quite often facilitate a correct response to such a command.

The operation could quite easily be separated from the number nature. The idea should be established that the word "plus" characterizes action taking place, and act of union. The word "positive" is of descriptive essence, characterizing a position right of zero when it is a quantity pointed out or designated. Presently the same word indicates the direction of an operation from aposition. It is the opinion of the writer that clearness of concept could be achieved if the students could find the word used in textbooks as an adjective for designating a position and in adverbal form for indicating direction. For instance, the symbolic command $5+(+3)$ should be written "add positively three to a positive five." If the notation suggested above were adopted the symbolic case would read $\overset{+}{5} + \overset{+}{3}$. Then a positive sign (above the first number) before a plus sign (the operation symbol) refers to position and the positive sign after the operation symbol refers to the direction the operation indicated is to occure. This would hold for a series of sums if position is taken after each operation. For instance $\overset{+}{5} + \overset{+}{3} + \overset{+}{4} + \overset{+}{8} = \overset{+}{20}$ is easy to follow if one takes position at $\overset{+}{5}$ and adds positively three units, takes position at $\overset{+}{8}$ adds positively $\overset{+}{4}$, and on through $\overset{+}{8}$. Numbers written without any accompanying sign should be understood to be of a positive nature.

The general idea holds equally well in the teaching of operations of signed number in the case of negatives. Positions left of zero would be of a negative nature and motion directed left of any point is negatively directed. As an example, take the case of $\overset{-}{5}+\overset{-}{3}$: In conformity with principle already agreed upon the starting position is 5 units left of zero on the number line. Recall that the sign above the number after the operation sign tells how or in what direction you must proceed from the starting position, that is, 3 is to be added negatively. It says in this case that you are to move to the left. Move 3 units left and give the numeral of your position. It is 8 units left of zero and is called $^{-}8$. Take the case of $^{-}8 + {}^{+}5$: This time the starting position is 8 units left of zero and motion is 5 units to the right of this point. Read the numeral of the position. It is $^{-}3$.

$^{-}4 + {}^{+}13$ means the starting postion is 4 units left of zero and 13 is to be added positively. The final position in this case would be 9 units right of zero and this $^{+}9$.

$^{+}3 + {}^{-}3$ indic tes a starting position 3 units right of zero and the direction of motion is to be toward the left and 3 units. This would give a stopping position (the sum) of 0.

The above cases of addition serve to illustrate the application of the idea to all the cases of addition with signed numbers:

$^{+}5 + {}^{+}5$ corresponds to $^{+}a + {}^{+}b$;

$^{-}5 + {}^{-}3$ corresponds to $^{-}a + {}^{-}b$;

$^{-}4 + {}^{+}13$ corresponds to $^{-}a + {}^{+}b$; and

$^{+}3 + {}^{-}3$ corresponds to $^{+}a + {}^{-}b$.

Subtraction should be defined as a special case of addition: a-b means that b added to some number c gives a. In arithmetic, 12-4 means that to 4 some number can be added to get 12. Making this concept clear enables a teacher to use the number line and rapidly teach subtraction. In the experiences in addition above the last position attained was the sum. In subtraction this last position, the sum, is a given quaintity. Given a position somewhere to the right or left of it you merely note what direction and how many units moved to reach the sum. Take the example $^{+}12 - {}^{+}4$, cited above: The number BEFORE the minus sign is the sum and the number AFTER this operation sign the starting position. Thus, $^{+}4 + C = {}^{+}12$. Beginning the count of units a $^{+}4$ and moving to the $^{+}12$ you find you've moved in the direction right of $^+4$ and that you have moved 8 units. In agreement with the above decisions name this directed motion $^+8$. Then, clearly $^+12 - {}^+4 = {}^+8$. This example should serve to demonstrate that all the cases of subtraction could be taken care of since they simply revert to cases of the several combinations of addition already demonstrated.

Take a look at this scheme and see how it lends itself to multiplication and division with signed numbers. You may start with multiplication since it is but a means of addition. To insure an agreement and understanding on the part of the student that multiplication is a type of addition define and illustrate it: "The multiplication of a first number by a second number means to do to the second what was done to unity to obtain the first."[9] Take the problem of $^+3 \times {}^+7$: To obtain the first (the number $^+3$) take unity as an addend three times. Thus $1+1+1 = {}^+3$. Then, by definition the number 7 must be taken as an addend three times: $7+7+7$. The sum is, of course, 21.

To include all cases extend the definition of multiplication further within the limits of its implications to fit the number line idea. Numbers to the right or left of zero may be designated as "zero-differences" of unity. Thus a $^-3 = (0-3) = (0-1) + (0-1) + (0-1) = {}^-1 + {}^-1 + {}^-1 = {}^-3$. Let numbers to the right of zero be the sum of addends $(1-0) + (1-0) \ldots = {}^+n$. To multiply $^-3 \times {}^+5$ make the following observations: The zero-difference of $^-3$ is $(0-3)$. According to the above notion this means there is some number which when added to 3 gives the position 0 on the number line. Beginning at 3 note you move three units left to get 0. Therefore, give the value $^-3$ to the number added to 3 to get 0. Then do to 5 what was done to unity to get $^-3$: $(0-{}^+5) + (0-{}^+5) + (0-{}^+5) = {}^+15$. Try applying this idea to

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9B. R. Buckingham, <u>Elementary</u> <u>Arithmetic</u> (New York, Boston, Chicago, Atlanta, 1947), p. 64.

$^{-}3 \times {}^{-}5$: again $(0-1) + (0-1) + (0-1) = {}^{-}3$. Then, $(0-{}^{-}5) + (0-{}^{-}5) + (0-{}^{-}5) = {}^{+}15$.

Define division of a first number by a second number as the product of the first number and the reciprocal of the second. As an example, ${}^{-}10 \div {}^{-}2 = {}^{-}10 \times {}^{-}1/2 = {}^{+}5$. In short, the definition of division eliminates the need for a different set of rules in division with signed numbers.

After sufficient work has been accomplished by the student with these sensory techniques he will be ready to derive for himself the conventional rules for the basic operations with signed numbers. It is the suggestion of most textbook writers that this is the stage the student might memorize the following rules:

1. If the two numbers to be added have the same sign, find their absolute sum (the arithmetical total), and prefix the same sign before the result.

2. If the two numbers to be added have opposite signs, find their absolute difference (the smaller subtracted from the larger as done in arithmetic), and prefix the sign of the larger number to the result.

3. When two numbers with like signs are multiplied, their product is positive; if their signs are unlike, the product is negative.

Having defined subtraction as a case in which the sum is given and a suitable addend must be found to make this sum correct, no new rule for subtraction is needed. A similar argument applies in case of multiplication and division.

CHAPTER IV

THE COORDINATE SYSTEM

One of the most important concepts needed by any student in first year algebra is that of the coordinate system. It lends itself to pictures of the mapping of a function. Hardly any idea is more fundamental or more widely used than that of the function concept. It pervades the study of mathematics from its most elementary through its most complex structures. So, introducing the coordinate system should command the teacher's fullest attention and maximum resources.

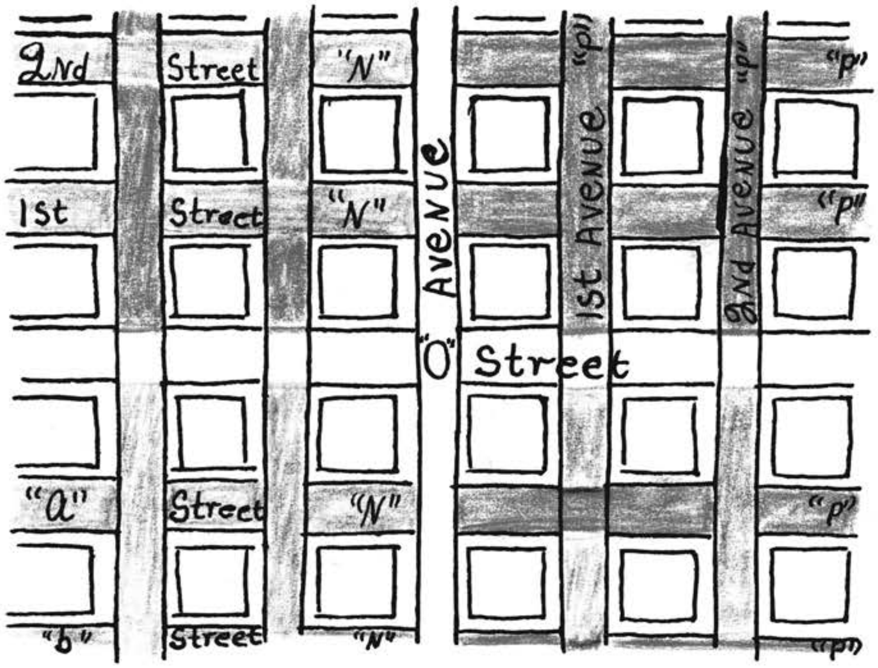
Most students have studied some elementary geography before they reach the point of beginning algebra in their schooling. As a result they have some knowledge of longitude and latitude. For those students so equipped this is a good point of reference for beginning the study of the coordinate system. The two systems incorporate precisely the same idea in that distances east and west of a line extending from north pole to south pole over an arbitrary point are designated as longitudinal distances. Those distance north and south of the equator are designated as latitudinal distances or parallels of latitude. Latitudinal and longitudinal lines are perpendicular to each other and thus intersect each other in specific points. For instance, the intersection of the 36° N paralled of latitude and the 97° W, meridian of longitude locates the town of Stillwater, Oklahoma.

In the event some of the students are unfamiliar with latitude and longitude it is possible that the system of the streets of his town or some town he has visited will furnish an analogy. The actual cases are not numerous but the well planned town might have streets running in one of the cardinal directions and avenues at right angles to them. Suppose yourself in such a town where the streets run in the direction east to west, the avenues extend north and south. There is some street which runs approximately through the center of town dividing it into northern and southern halves. There is some avenue running through the center of town dividing it into an eastern and western half. Where this particular avenue intersects the dividing street a point is neither north, west, south, nor east in the town. A house located on this point should logically have the street and number address "O". The first house in the 10th block north of this point might have the avenue address 10-1 (usually 1001) north Math Avenue. A house located 37 blocks east of the "O" address and second from the corner along the street could logically be referred to as 37-2, E. (likely 3702 in actual cases) Algebra Street. It is easy to see that if one were located at the "O" address he could locate a friend who gave a street address including the number and whether it was east or west. Somewhere along the avenue that ran through the center of town the stree is sure to cross. However, one could lose valuable time if he went all the way to the south city limits without crossing the particular street. Assuming he is walking, he would have to walk all the way back to "O" and proceed North. Surely, somewhere along the way he would find the street sought. This difficulty could easily be avoided if the streets north of "O" had some special number or name code and those south of "O" a different scheme. Then you could directly locate any particular address by using knowledge of two directions simultaneously. You would proceed north or south from "O", whichever the code scheme called for, and upon intersecting the desired street turn off east or west as the number address of the street indicated. A similar idea could be applied to all avenues and it would be equally as easy to locate any given house situated on a named avenue. Unfortunately few towns are so well planned and one usually has to resort to questioning policemen or others for directions, one of the better means of getting one's self completely lost.

Unlike the street systems in most towns the coordinate system is a precisely defined idea with all points given as an address incorporating the notion of two distance form "O". A point is some distance or no distance right or left of "O" corresponding to an address on a street east to west, or on the avenue that runs through the center of town. Further, the point may lie up or down from "O" corresponding to avenue addresses north or south of the street running through "O" dividing the town into northern and southern halves. To help the pupil understand this idea fully you may wish to make use of an imaginary town with the idealized street system. Better yet let him be apart of the Planning Commission that sets about constructing such a town. After sufficiently careful surveying he builds a system of streets running east and west, each parallel to the other. Perpendicular to these streets is laid off a system of avenues running north and south. The street running through what will be the center of town is named "O" Street and the avenue dividing the town into eastern and western halves named "O" Avenue. All the avenues east of "O" Avenue are numbered consecutively 1, 2, 3, . . . N. Those avenues west of "O" are lettered a, b, c, . . .z, aa, ab, and on through the last avenue. Code the streets similarly. Pretend the employment of two giant sized Construction Companies to pave the avenues and streets. Give company A a contract for paving the streets and company B will pave the avenues with each paving the same number of intersections. Each will receive the same basic sum of money for completion of its project but in typical American haste offer a bonus to the company satisfactorily completing its job first. Further, the Commission divides the town into four separate zones, NE, NW, SW, and SE. Request that both companies begin work in the NE zone for this will be the zone of the biggest tax payers, the rich. So Company A and Company B work furiously in the NE zone and streets and avenues are paved in record time. However, because of the competition for the bonus anticipated friction develops between the two companies. In order to prevent possible conflict the commission agrees to let the companies work in separate zones during the remainder of the project. Company A, the street paving company, chooses to work in the SE zone and Company B, the company paving the avenues chooses the NW zone.

In the course of time it develops that the Commission decides that the main street and main avenue, those respectively running through "O" should be paved. It is finally agreed that since engaging one of companies already employed would give the other unfair adavantage and to enagage both would probably bring them close enough for conflict a third company would be engaged for this job and neither company A nor B would be entitled to earn pay for paving the routes through "O".

When company A has finished with the SE zone Company B has just finished the NW zone and an unusually long rainy season sets in preventing any further work. Before it has ceased a recession occurs and the Commission will not be able to engage either company for further development but will be able to pay them for work they have already finished. The group of Commissioners assigned the task of checking the work before payment to the companies makes a diagram of the town and sets about devising a system for indicating on it the areas of the town for which the companies must be paid.

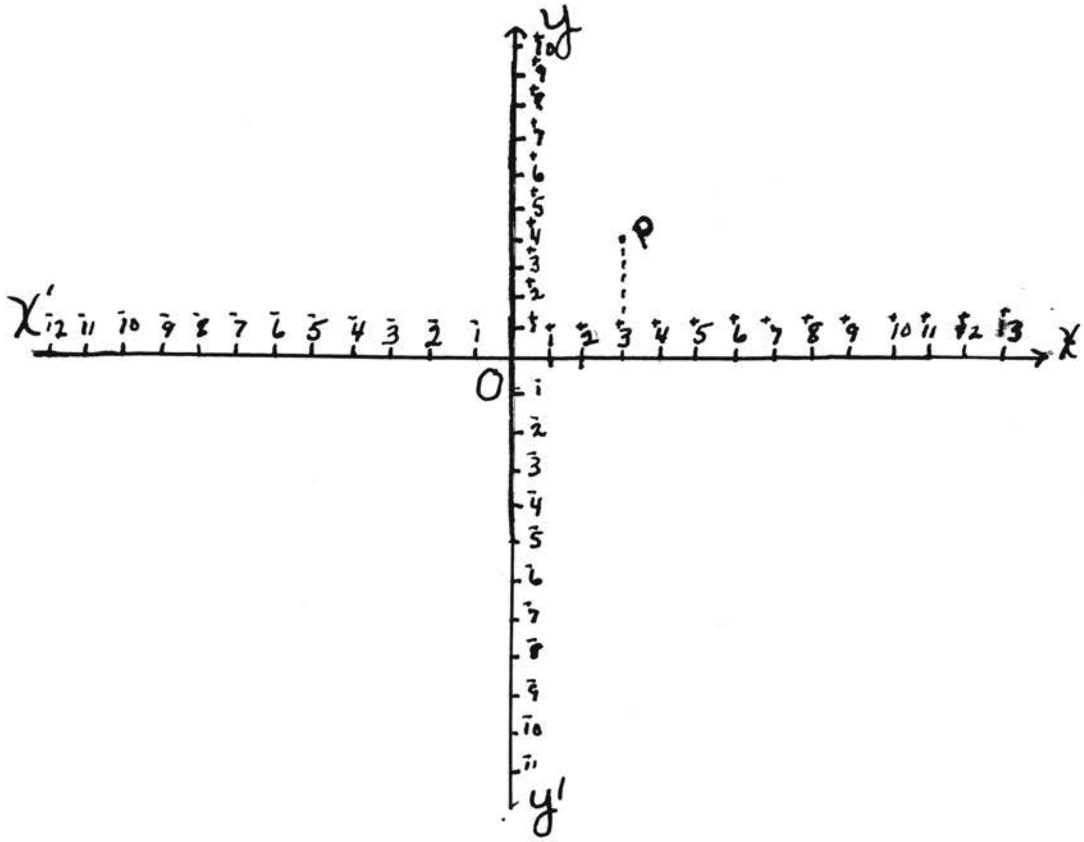


Paved streets and avenues will be colored red and the letter "P" will be placed on them for payment. Unpaved streets and avenues will be colored green and the letter "N" placed on them to indicate non-payment. The avenue and street running through "O" will not be colored at all for neither A nor B will receive payment for them.

When the job is completed on finds streets and avenues are red in the NE zone and the entire zone is marked "P". In the SE zone the streets are colored red and each marked "P". The avenues in this zone are colored green and marked "N". In the NW zone the avenues are red and marked "P" and the streets are green and marked "N". In the SW zone the streets and avenues are green and the entire zone is marked "N". With the aid of this diagram the Commission quite readily finds it must pay each company for one-half the job each was originally hired to perform. Both companies are paid for the NE zone. Company A is paid for finishing the SE zone and Company B for finishing the NW zone. Neither company receives money for the SW zone nor the routes through "O".

With this diagram and such a fanciful flight of the imagination, the student's knowledge of what a town is like being quite broader than his grasp of imaginary meridians and parallels, the x- and y- axes are easily introduced. The color schemes and the associations of P's and N's aid him in remembering the quadrants in which x or y is positive or negative. The streets and avenues are so numerous in the corrdinate system that there is never more than a single house in a block, thus each one has a corner address. Then, any house in the plan has a street and an avenue running to it. The NE corner of town corresponds to quadrant I, the NW to quadrant II, the SW to quadrant III, and the SE to quadrant IV. In quadrant I both x and y are positive. X, the distance from the y-axis, corresponds to streets in the NE zone of town. All those streets paved are resignated "P" routes. The x-axis corresponds to the street through "O" and the y-axis the avenue through "O". Recall that neither company receives consideration on these routes, or one may say each receives "O" consideration. Yet you are readily able to identify where streets or avenues cross these main routes. A house whose only address is 5th street might be found on the corner of 5th street and "O" avenue or (0.5). A number of exercises dealing with the location of addresses give the student a clear approach to locating any designated points in the plane. Insist that since the distance is the same whether he leaves "O" by way of "O" Street to the desired avenue and proceeds up or down to the proper number or vice versa it is required that he follows the procedure of street distance first and then avenue distance.

As an aid to seeing just how the suggestions above might be instituted the following figure is offered:



XX' is called the horizontal axis; YY' is called the vertical axis; together they are called the axes; the point O is called the origin: PS, perpendicular to the vertical axis, is called the abscissa of P; PR perpendicular to the horizontal axis, is called the ordinate of the point P; PR and PS together are called the coordinates of P. Distances on OX are considered positive, on OX' negative, on OY positive, and on OY' negative. The part of the plane within the angle XOY is called the first quadrant; the part within tha angle YOX' is called the second quadrant; the part within the angle X'OY' is called the third quadrant; the part within the angle Y'OX is called the fourth quadrant. The abscissa of P, according to the indicated scale, is 3, and the ordinate is 4. The point P is called the point (3,4).

CHAPTER V

EQUATIONS

"The algebraic equation, based on the principle of balance and equality, is one of the most important helps ever developed for solving problems," says Smith, Totten, and Douglas in one of their algebra series. These same authors offer a definition for an equation which seems generally in agreement with a variety of others: "An equation is a mathematical statment that two quantities are equal."[10]

The equation concept lends itself to many varieties of introduction. Some textbook authors merely state their definition and offer a group of examples. Others use a definition and beautifully drawn geometrical illustrations. Perhaps one of the most frequent illustrations is that of a pair of scales. The writer has found this one of the methods offering wider appeal to the average student encountering the idea for the first time.

To begin with, no matter what method of introduction is used an effort must be made to assure no language barrier between student and instructor. With this in mind one may discuss the very simple equation growing out of the following problem: If three equal bars of metal have a total weight of 24 ounces, how much does each bar weigh? Let w equal the weight of one bar, then 3 w will represent the weight of three bars, and write the equation $3w = 24$. With a pair of scales to illustrate the problem you are

[10]Smith, Totten, and Douglas. Algebra One, (New York), p. 74.

usually able to get desired answers to some questions asked about the situation. How is balance involved? What things are equal in the equation? Is it possible to tell what w equals? This mode of approach will lead to a satisfactory condition for the unwary teacher at this stage of the game. However, aituations will shortly present themselves that leads one to wonder just what actually happened to the student during or since this easy discussion. Usually, when traced it is a case of what happened during the discussion. The proper steps were not taken to assure no lauguage barrier. In the simple instance cited above with a pair of scales before you and the equation $3w = 24$ have the student learn the following facts:

1. An equation is a mathematical statement that two quantities are equal. Here 3w and 24 are equal quantities.

2. The symbol $=$ is called the equal sign. Think of it as indicating a "balance point" between equal quantities.

3. In an equation, a letter whose value is not known at first and is to be found is called an unknown number or an unknow. In the quation $3w = 24$, w is the unknown quantity.

4. An equation has two distinct parts, called members, one on each side of the equal sign. In the equation $3w = 24$, 3w is the left member and 24, is the right member.

5. The two members of an equation may be thought of as balancing each other, and as corresponding to the opposite sides of a pair of scales. Whenever a change is made in one member of an equation, an equal change must be made in the other member to maintain the balance.

If one assures himself that he and student are thinking the same things in this situation the pair of scales will facilitate the introduction of the axioms needed for the solution of some simple equations. However, an important term which involves a most functional idea is that of "inverse." This idea of opposite processes is one that persists through all mathematics and thus should receive ample stress that it may be well learned. The balance or pair of scales lends itself readily to demonstrations of inverse operations. Moreover, the situation brings but the relationship of addition and multiplication, of subtraction and division, and other incidental learnings which will not be merely accidentals if taken into full account by the instructor.

One may continue with the same equation as an illustration: $3w = 24$. What has been done to the unknown? Usually the student is able to see that w was multiplied by 3. Then, if one performs the inverse operation upon w it is alone or isolated. As the instructor is speaking of this procedure he is demonstrating on the scales and the student actually sees what happens to the scales when w is isolated without bothering the other member of the scales. Upon an average student this makes a lasting impression and he will more often than not remember that you must do the same operation upon both sides of an equation to keep it balanced. The four axioms in use more often than others, addition axiom, subtraction axiom, multiplication axiom, and division axiom are very easily demonstrated with these scales. The transition from arithmetic into this particular aspect of algebra is thus greatly facilitated beaause there is an arithmetical demonstration of an algebraic concept. The arithmetic involves palpable symbols, scales and weights, and this is good for a student on his first mile toward complex instances of a concrete nature and finally pure abstractions. A search of many of the vary latest textbooks reveals that by far the most widely used idea employed in attempt to give the student sensory demonstration of the properties of an equation is that of the scales. Whatever method is used to introduce the equation the task has hardly begun. Certain maniplulation though rapid and correct do not always serve to insure that the student actually understands an equation. The bane of the indifferent algebra student and indeed a high hurdle for the sincere and average student is the formulating of equations. However, one cannot ascertain the degree of mastery of the concept on the part of the student by any other method.

For students going from arithmetic to algebra problems involving equations and their solutions are usually typed. There are sets of problems under the labels of "mixture problems," "rate problems," "digit problems," "age problems," "work problems," and such. A textbook which seems highly conscious of the transition to be made by a student going from arithmetic to algebra is that of Smith and Lankford in their Algebra One-1955. Following is a group of problems proposed by these writers for students going from arithmetic to algebra, each exercise preceded by a similar exercise in arithmetic:

1. a. If the difference between two numbers is 12 and the smaller number is 3, what is the larger?

   b. If the difference between two numbers is 12 and the smaller number is N, what is the larger in terms of N?

2. a. If the difference between two numbers is 12 and the larger number is 16, what is the smaller?

   b. If the difference between two numbers is 12 and the larger number is N, what is the smaller in term of N?

3. a. If one part of 36 is 7, what is the other part?

   b. If one part of 35 is N, what is the other part in terms of N?

4. a. If the length of a rectangle is 8 in. and the width is 5 in, what is the perimeter?

   b. If the length of a rectangle is M inches and the width is N inches, what is the perimeter?

5. a. How far will an airplane travel if it goes 300 miles and hour for 5 hours?

   b. How far will an airplane travel if it goes N miles and hour for 5 hours? 2N miles an hour for 6 hours?

6. a. A girl is 4 years older than her brother. If the boy is 12 years old, how old is the girl?

   b. A girl is 4 years older than her brother. If the boy is p years old, how old is the girl?

7. a. Pat, Bill, and Ned received a legacy from their grandfather. If Pat received $1000, Bill $100 less than that, and Ned $500 less than Pat and Bill together, how much did Bill and Ned each receive?

   b. Pat, Bill and Ned received a legacy from their grandfather. If Pat received n dollars, Bill $100 less than that, and Ned $500 less than Pat and Bill together, how much did Bill and Ned each receive?[11]

These authors have doubltblessly given considerable thought and study to the problem of the average student making the transition from arithmetic to algebra. However, it is of absolute certainty that no amount of thought and work by those other than the student can assure him of success in setting up equations.

As has been indicated, textbooks found in today's schools offer very little that is actually new toward the introduction and development of the concept of an equation. However, there are persons thinking and writing on the problem. In contrast to what has been given, attention is called to an article by Robert E. K. Rourke appearing in the 1958 February issue of the Mathematics Teacher. Entitled "Some Implications of Twentieth Century Mathematics for High Schools," the article has much to say on the transition from arithmetic to algebra of today's new algebra students.

Mr. Rourke expresses strong feelings on the need of new goals, new curricula, new texts, and new methods of teacher training. He emphasizes that these should be "new" — hot just a permuting of chapters in textbooks, not just a reprint of the old stuff in technicolor. Admitting that he is not the only one to make such a clamor, Mr. Rourke calls to attention that many of his shouting colleagues fail to spell out in detail what they

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[11] Smith and Lankford, Algebra One, 1955, p. 138.

mean and are sorely misunderstood by builders and new curricula. In avoiding this mistake he seeks to show by examples exactly what he has in mind with reference to how sentences in one variable may be treated. The following is taken directly from his article and offered verbatim in most instances:[12]

We commonly encounter sentences of two kinds in high school mathematics:

1. Sentences involving names of numbers, or numerals. These sentences are either true or false. For example:

$$1 + 2 = 3, \quad 5 > 3, \quad 6 + 2 = 17.$$

2. Sentence involving a variable. These sentences are neither true or false; they are "true of" or "false of" certain numeral replacements for x. For example:

$$x + 3 = 8, \quad x > 5, \quad 2x + 1 = 4.$$

In sentences of the second type, the set of possible replacements for x is the whole set of numbers under consideration in a given context. This set -- the totality of numbers under consideration--is sometimes called the <u>universal set</u>, and denoted by U. In a given problem, it is important to keep in mind the universal set. For example, suppose that the universal set consists of all positive integers less less than 20.

$$U = \{1, 2, 2, \ldots, 20\}.$$

Then, in the sentence

$$x + 1 > 12,$$

the possible replacements for x are the 20 integers in U. Of these replacements, the set

$$\{12, 13, 14, \ldots, 20\}.$$

makes the sentence

$$x + 1 > 12$$

true; and the set

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[12] Robert E. K. Rourke, "Some Implications of Twentieth Century Mathematics for High Schools," The Mathematics Teacher, Vol. LI-Number 2, Feb. 1958, p. 75.

$$\{1, 2, 3, \ldots, 11\}$$

makes this sentence false. The set of replacements for x that make the sentence true is called the solution set of the sentence. Hence the sentence,

$$x + 1 > 12$$

divides the universal set, U, into two subsets: One set containing all replacements for x that make the sentence true (the solution set); the other set containing all replacements for x that make the sentence false.

In highschool algebra, the sentences are, for the most part, equations and inequalities. We use the variable as a placeholder for a numeral, which is the name of a number. Now, is not this notion of variable clear and simple? Do we need the additional impedimenta of such expressions as "literal number," "letter-number," "general number," "unknown quantity?" No wonder that many of our students are confused about the meaning of x^4. I do not suggest that we get into the rarefied atmosphere of philosophical discussion of the meaning of "variable": I do say that here is clarification along with simplification.

Thus we have sentences with variables, and these sentences are equations or inequalities. Usually our universal set is the set of real numbers. Here is a fruitful point of view for a high school teacher. Let us think of our sentences (equations or inequalities in x) as selectors of sets: they select from U just that set of numbers that make the sentence true when used as replacements for x. This set of numbers is the solution set of the sentence. Then, if we assume the usual one-to-one correspondence between the real numbers and the points on a line, we can graph the solution set of the sentence. This graph is also called the graph of sentence. Let us consider some examples: Example 1. From the foregoing point of view, let us examine the sentence.

$$3x + 2 = 6.$$

What is the solution set?

The answer depends on U, the unvierse of numbers under consideration. If U is the set of positive integers, then the sentence selects nothing: the solution set is empty since there is no integer that can replace x and yield a true sentence.

We have a convenient notation for denoting solution sets, using the set-builder:

$$\{x \mid \quad \}$$

The braces "$\{\}$" are read "set", the vertical "|" is read "such that." We put the variable on the left-hand side of the vertical bar, and the sentence on the right-hand side. This gives:

$$\{x \mid 3x + 2 = 6\}.$$

Read: "the set of all x such that

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$$3x \quad 2 = 6."$$

If, in Example 1, U is the set of r al numbers, then the sentence selects $4/3$. Using the set-builder notation, we can denote the solution set thus:

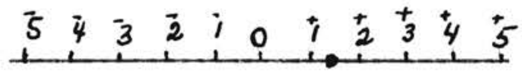

Figure V-1

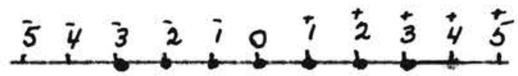

Figure V-2

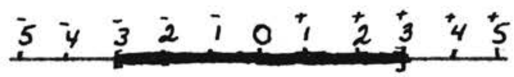

Figure V-3

The graph of the sentence is made up of just one point. (See Figure 1.)

Example 2. Suppose that $x^2 \leq 9$ is our sentence. What is the solution set:

$$\{x \mid x^2 \leq 9\}?$$

Again note the importance of U. If U is the set of integers, then

$$\{x \mid x^2 \leq 9\} = \{-3, -2, -1, 0, 1, 2, 3\},$$

and the graph is Figure 2.

If U is the set of real numbers, then

$$\{x \mid x^2 \leq 9 = \{x \mid -3 \leq x \leq 3\},$$

and the graph becomes an interval. (See Figure 3.)

Example 3. The sentence

$$x^2 \quad -4 = (x+2)\ (x-2)$$

selects everything in the universe:

$$\{x \mid x^2 \quad -4 = (x+2)\ (x-2)\} = U.$$

We call such a sentence an identity. The graph is the entire number scale, if U is the set of real numbers.

The foregoing approach <u>is</u> new and of such an explicit nature that there is no doubt about just what Mr. Rourke means when he suggest modifications in the antiquated methods of most of today's textbooks and teachers. Emphasis is placed on the solution sets of equations and inequalities in one variable.

These solution sets are within the ranges of the new student's guessing, experimenting, or intuition. The approach offeres the following advantages:

1. It enables an approach to equalities and inequalities together.
2. The concept of the graph is broadened by the identification of points corresponding to the solution set of the sentence.
3. Insight is gained by the student before he begins mechanical manipulations of the equations and inequalities.
4. Experience is gained in the language of sets.

These cited advantages render the suggested approach worthy of the most sincere consideration of today's algebra teachers.

CHAPTER VI

SOME NOTIONS ON THE MECHANICS OF ALGEBRA

Most writers on the subject suggest that algebra be shown as a generalization of arithmetic. The student passes from arithmetical to algebraic reasoning when he replaces numbers by letters, each one of which represents any one of a group or set of numbers. Thus, algebra is more general, or more catagorical, than arithmetic; that is, the conclusions reached in algebra apply to a much greater number of objects than the results obtained in arithmetic, which apply to specific number of objects. However, instead of merely informing the student of this wonderful truth and letting this aspect of the subject be forgotton, continuous effort should be exerted to gain understanding and appreciation for it. It might be a good idea to give a bit of the history and evolution of most of the problems in algebra. For obvious reasons this is not done at great lengths in textbooks. However, presenting a notion in the precise and polished form striven for by some writers may leave the student perplexed. This seems to have been the thinking of Max A. Sobel of New Jersey Public Schools when he performed an experiment. He chose six schools in Newark, New Jersey, one in Paterson, New Jersey. Each school offered the means for two first year algebra classes of more or less equal ability, determined by I. Q. and other tests. Every attempt was made to insure uniform teaching ability. One group, designated the control group and consisting of a class from each school, was taught from the textbook as written. The other group, referred to as the experimental group and made up of one class from each of the seven schools, was taught from a manual of instructions which presented a broad, overall view of the problem, discussed the several methods of approach to be tested and provided numerious illustrative examples. The results of this experiment might be summarized as follows:

> "Students learn and retain certain concepts and skills better through an inductive, concrete, unverbalized teaching approach as opposed to a deductive, abstract, verbalized method of presentation."[13]

Letters For Numbers

In the transition from arithmetic to algebra there are ideas and concepts the student must master in order to accomplish any of the mechanics of algebra. Perhaps the first and most feared by prospective students of algebra is the idea of using letters for numbers. It has been the observation of the writer that many students somehow get the idea that the letters represent a mystic number system which is most uncooperative in revealing its nature. Thus, the teacher's first job is to dispel this fear without minimizing the importance of the idea of letters representing numbers.

In the arithmetical experience of the student it is usually learned that a number multiplied by zero gives a product of zero: $0 \times 1 = 0$, $0 \times 2 = 0$, $0 \times 3 = 0$. . ., $0 \times n = 0$. It is also quite easy to get the student to follow this line of development. One points out that n is no particular number but is a place holder for any real number. If you replace n with any one of the real numbers the product is still 0. A similar case is true if zero is employed properly with the three remaining operations: $0 + k = k$, $s - 0 = x$, $\frac{0}{x} = 0$.

Another easy case of extending arithmetic into this idea is that of buying and selling. One says an article costs $2 and the price of 5 such

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[13] Max A. Sobel, The Mathematics Teacher, Vol. XLIX, Number 6, Oct., 1956, p. 427.

articles is \$10. And article cost \$3 and the price of 5 such articles is \$15. Finally, the price of 5 articles costing d dollars each is \$5d. It is possible at this stage to develop the general idea that n articles at \$d each cost \$n x d. Numerous similar ideas may be employed, all within the aritmetical experiences of the child. When this notion of letting letters represent any number within the domain has been dealt with successfully the student is on his way to mastery of the fundamentals of algebra.

Order Of Operations

The student must be shown that as was true in arithmetic there are certain ideas agreed upon by mathematicians without any particular attention to logical structure or intuitive appeal. In arithmetic its agreed that the integer one greater than ten will have the name "eleven" when actually "twenty-one" is more logical. In algebra an expression involving addition, subtraction, multiplication, and division all at once is evoluated according to the following agreement among mathematicians: the multiplication of the factors comes first, division of resulting adjoined factors next, and last the addition and subtraction of terms from left to right. If there are parentheses present the part of the expression within a parenthesis is evaluated first and the parenthesis is dropped.

Grouping

In even the most recent writings one fails to find an abundance of suggestion concerning the idea of grouping. Yet this a most necessary and troublesome idea to teach to students in beginning algebra. The oneness of a group of terms enclosed in a symbol of grouping does not lend itself easily to arithmetical illustration. Expecially is this true if confronted with groups within groups. From reading Henry Thomas: Mathematics Made Easy the notion is gained that some illustrations of the idea may be expressed as follows:

1. Suppose on Monday you earned \$20 on a certain job, and on Tuesday you worked on two jobs and earned \$10 and \$9 on them. Your total earnings for the two days would be \$20 and (\$10+\$9), or \$20+(\$10+\$9), or \$39.

2. Suppose on Monday you earned \$20, and on Tuesday you bought a hat for \$4 and a pair of shoes for \$6. Your balance would be \$20 - (\$4+\$6), or \$20 - \$4 - \$6, or \$10.

3. Suppose on Monday you earned \$20, and on Tuesday you gave a \$10 bill for a pair of shoes and received \$4 in change. Your balance would be \$20 - (\$10 - \$4), or \$20 - \$10+ \$4, or \$14.[14]

The amounts of money earned and spent might be an experience yet to be gained by beginning algebra students but may be modified to fit. This could aid in leading him to discover the rules for removing parenthesis or the idea of dealing with an expression needing many of the grouping symbols to show its related parts.

Combining Similar Terms

When the student has mastered the idea that letters may represent numbers he is soon involved in the mechanics of adding or subtracting these literal numbers. He is told that these operation on numbers are possible only if the terms are similar. Draw upon the student's experiences in arithmetic in this instance once he is taught all the aspects of similar terms. He is used to adding fractions with similar names (fractions of like denominations), replying that one cannot subtract horses from jets and such statements. However, very recent writers insist that students not be told such unions are not possible but teach that it might be a bit difficult to agree upon a name for these unions. Certainly, this would

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[14]Henry Thomas, Mathematics Made Easy, (New York, 1940), p. 84.

agree with the notion of a b being an indicated sum and save the student the trouble of unlearning a notion if he pursued the study of mathematics into Boolean Algebra.

Factoring

Some idea of factoring and primes should be a part of the students mathematical knowledge long before he is ready for beginning algebra. Factoring is a much used process in the mechanics of algebra. Most writers of textbooks are content to make this statement and proceed with the rules for factoring expressions of a particular type. Perhaps a fuller apprecia- tion by the student might be had if there were several illustration using arithmetic. Refer to any expression to be factored as a product. Recall that products are obtained by multiplication of certain numbers. In fact- oring one seeks to find those numbers which might have been used in this multiplication. The expression is prime where there can be found no numbers which when multiplied give the original expression other than the number itself and one. By the time factoring is introduced the student is famil- iar with such expressions as $a^2 + 2ab + b^2$. The arithmetical number 484 may be broken down into its actural meaning to get a similar number: $484 = 400 + 80 + 4 = 400 + 2 \times 20 \times 2 + 4$. The 400 is the square of 20. The 4 is the square of 2. The product of 20×2 is 40 and $2 \times 40 = 80$. Thus, 484 may be expressed as $(20 + 2)\ (20 + 2)$ for $22 \times 22 = 484$. Note that $(20 + 2)$ is merely the sum of the square roots of the first and last terms. Using this same idea in the algebric expression gives $(a + b)\ (a + b)$ as the fact- ors desired. By choosing a number equal to the difference of two squares it is equally easy to give an arithmetical example of factoring $a^2 - b^2$. Knowing that 48 is such a number one writes it as being equal to $8^2 - 4^2$. From this equation, $8^2 - 4^2 = 48$, show that $(8 - 4)\ (8 + 4) = 4 \times 12$ is the same number. A sufficient amount of like examples give the child confidence and some intuitive appreciation of the fact that $a^2 - b^2 = (a \quad b) \quad (a - b)$. The factoring of the sum or difference of two cubes lends itself to a similar treatment. By the time these simple processes in factoring are accomplished the student should have acquired enough confidence and facility to proceed into those cases of factoring that lend themselves less easily to arithemtical illustrations.

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