COMBINED STRESSES FOR WEBS GOING OVER ROLLERS

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ABSTRACT

In research it is typical that only one source of stresses is studied at a time. In entry-level mechanics textbooks, you may see the occasional combined stresses taught where a couple of sources are superposed. Yet, seldom do webs experience only a single significant source of stress. Indeed, most webs in the open web span see two or three sources; any or all of which could be significant. More stresses are added when going over a roller. This paper uses superposition to combine stresses commonly found in web handling including the not-so-'simple' web tensioning due to drive systems, bending due to in-plane roller misalignment and bending due to the radius of curvature of a roller. Bending over rollers has become much more important in recent years with thicker materials, such as electronics made as webs, and with more brittle chemistries, such as about half of all vacuum deposition materials. We also consider residual stresses of manufacturing such as bagginess and curl. Finally, we combine all stresses appropriately and use a safety factor to observe how far from yield the most stressed portion of the web is. This safety factor can serve as a proxy to estimate process reliability with regard to tensile failure of the web due to handling.

NOMENCLATURE

ε_C	strain due to residual stresses of curl
ε_M	maximum combined strain due to all factors considered
ε_{RBR}	strain due to roller bend radius
ε_T	strain due to nominal web line tension such as read by a load cell
ε_{YO}	ultimate (or yield) strain of material on the top or bottom of the web
θ_i	in-plane roller misalignment angle
σ_T	nominal stress due to web line tension such as read by a load cell
σ_Y	ultimate (or yield) stress such as measured by tensile testing machines
σ_{YO}	ultimate (or yield) stress of material on the top or bottom of the web

E_1	(effective composite) MD modulus (lb/in^2 or n/m^2) of the web
E_{1O}	MD modulus the material on the top or bottom of the web
d_M	*inplane* component of misalignment of roller (in or m) across web width
D	roller diameter (in or m)
F_B	multiplier accounting for bagginess (unitless)
F_M	multiplier accounting for inplane roller misalignment
F_{TDE}	multiplier accounting for less than perfect electric drive control
F_{TDM}	multiplier accounting for down web tension variation due to drag & inertia
KL	Shelton non-dimensionlization factor for inplane misalignment
L	length of the misaligned span (in or m)
SF	ultimate safety factor of product/process/machine design (unitless)
r_c	curl radius (in or m)
t	composite web thickness (in or m)
T	nominal web line tension (lb/in or N/m) such as read by a load cell
W	web width (in or m)

INTRODUCTION

While studying a single source of web stresses is a necessary first step, this narrow focus approach is far from adequate or safe to be used in the real world of web handling and processing. Webs in the real world seldom see less than three insults of a nominally similar size. Every web sees MD tension such as controlled by drives as well as tension variations due to imperfections of those same drive systems. Every web sees tension at the outside of the bend due to in-plane roller misalignment that could range in size from somewhat less than nominal web tension on a precisely aligned roller to as big or bigger than web tension on a grossly misaligned roller as is commonly found in converting. Finally, every web sees some degree of bagginess that can range in size from nearly negligible to larger than the nominal web tension imposed by drive controls. To this we must also consider the radius of curvature of a web going over a roller and curl due to variations in residual stresses on the top versus the bottom of the web; both of which are quite significant on thick webs, especially if there is a brittle coating applied to the outside.

The previous approach, if these complications were ever even considered, was to apply an overall safety factor. It has been found from uncountable experiences that most webs would not usually run well when tensioned much less than 10% of web strength or tensioned more than 20-25% of web strength [1]. The overall safety factor on strength would thus be 4 (or 5) to 10. It is the upper end of tensioning, a minimum safety factor of 4-5 for this example, that we are concerned with here. True, exceptions exist to simplistic guidelines. One example is creped tissue that can run without undo web breaks or damage when tensioned at up to 50% of yield. Also, in forming operations the web is yielded with relative safety because it is wet or hot and, as a result, quite ductile. While the discussion to follow in this paper does not apply well to tissue or forming operations, the great majority of web handling would lend itself to the approach given here. Our motivation is that using a overall safety factor only on nominal web tension, though easily determined, could be too conservative or too dangerous depending on the details of other stresses not considered. We will discuss the most important of those other stresses here; while still retaining the necessary utility of a safety factor and while maintaining a user-friendly simplicity.

METHODOLOGY

We will consider each major web stress source or stress riser in detail and in turn. In addition to size, we must also be clear as to the location of the maximum stress to avoid adding two sources that might not be co-located at the same time or place. Next we will combine stresses and stress risers using superposition. Superposition, though common, is limiting in the sense that nonlinearities are not properly accounted for. These complications would include nonlinear stress-strain curves (below yield), actual yield and buckling of the web in compression to name just a few. However, these limitations are slight and the reward is a tractable, user-friendly approach to process design.

The choice of whether to work in stress or strains is immaterial if linearity is assumed. We will work primarily in strains and incorporate stresses as needed at the end using Hooke's Law.

'SIMPLE' WEB TENSION

We begin by considering the web tension that is necessary to run webs through machines without some supporting device such as a belt. As mentioned earlier, nominal web tension as controlled by drives is usually 10-25% of yield, though it can range from near 0 to 100% in practice. Nominal web tension will be needed in order to even get started. Often the average tension across the width is known via calibrated load cells. While this is 'simple' in concept, there are plenty of complications when you look closer. The first is that drive systems are not perfect and thus do not hold tension perfectly steady with time. An often-quoted standard for ordinary quality drives is to hold tension within 5% of set-point during steady state, 10% during speed changes and perhaps 50% during violent upsets such as a flying splice as read by responsive load cells and data acquisition systems [1, pp 52]. Thus, given these values, the respective tension spikes could thus be a multiplier of 1.05, 1.10 or 1.5X upon nominal drive tension settings. We will give this multiplier, primarily an electrical control consideration, the designation F_{TDE}.

Another complication is that the tension in another span in the very same drive section will be different than the one read by the load cell due to the combined effects of bearing drag and inertia as shown in Figure 1. It has been suggested that the tension not climb more than 10% in a drive zone [2 pp 26]. If the load cells were in the middle of the drive section, that would be a tension multiplier of 1.05 as a ratio of the tension in front of the drive to the tension read by the load cell. We will use the designation F_{TDM} to account for the MD tension being higher in one span of a drive zone than read by the calibrated load cell and is primarily a mechanical design consideration.

A last couple of complications deal with the challenge of getting started by not having any sort of tension reading. For example, nearly all dancers are, inexcusably, uncalibrated [3]. Even more challenging are the many drive sections that have no load cells or other web tension sensing devices whatsoever. The most common examples here are draw controlled machine sections. In any case, we convert nominal web tension, such as perhaps read by calibrated load cells or calibrated dancers, to strain as

$$\varepsilon_T = \frac{T}{t\,E_1} \qquad \{1\}$$

Before we proceed, we must make sure that the electrical control and mechanical design multipliers that we have defined thus far, F_{TDE} and F_{TDm} could occur in the same place *and* at the same time. Since dynamic tension spikes move at the speed of sound in the web and are not largely changed by modest roller inertias and drags, all spans of that drive zone will see similar errors induced by the drive control system. This would, obviously, include the span just upstream of the motor where down-web tension could spike during acceleration.

A NOTE ABOUT MODULUS OF COMPOSITE STRUCTURES

We can also allow for simple composite structures. To find the effective or bulk modulus (as if it were a single material with the equivalent total thickness), we can use the method of equivalent areas for MD tension, in-plane roller misalignment and bagginess. This becomes much more complicated when dealing with bending over a roller and curl because we must first find the neutral axis. Here we can use the method of transformed sections. Unfortunately, that technique is seldom taught for more than two materials in bachelor level engineering classes. At that point, one would probably be inclined to move to FEM or other more versatile method. Brittle coatings are a potentially large and important application area for the method described here. Since many coatings are often quite thin compared to overall composite thickness, their contribution to effective modulus, both tensile and bending, *might* be negligible. In summary, we will use effective modulus of the composite for our calculations. Those with thin brittle coatings can use the failure strain of the material on the top and bottom of the web, whichever is smallest, in the final safety factor calculation.

TENSION AT THE OUTSIDE OF THE BEND DUE TO IN-PLANE ROLLER MISALIGNMENT

Shelton was the first to thoroughly model stresses in a web span for the 'steering' guide [4]. This approach was later co-opted by Roisum for the much more general and thus perhaps more important problem of in-plane roller misalignment [2, pp 156-157]. Later this was used to define allowable roller misalignment [5]. Stresses and deflections for this mechanics are shown in Figure 2. Here we will calculate a stress riser for this cause, much as we did earlier by finding the peak tension spike caused by less-than-perfect drive response as well as due to idler roller drag and inertia.

We begin by calculating a very useful nondimensionalization factor, KL, where

$$KL = \frac{L}{W}\sqrt{12\,\varepsilon_T} \qquad \{2\}$$

The L/W ratio, the length to width of a span, is a nondimensionalization commonly found in guide formulas. ε_T is the average strain due to nominal web tension and will be defined and used in equation 7. The purpose of calculating a KL factor for this exercise is to use some very convenient approximations. If KL < 0.5, which is often the case except for very extensible webs or very long spans, then we can proceed with the following. Here θ is the angle of *inplane* roller misalignment as the ratio of absolute misalignment divided by roller width. Note that *in-plane* roller misalignment is about two orders of magnitude fussier than *out-of-plane* misalignment so the latter need not be considered here.

$$\theta_i = \frac{d_M}{W} \qquad \{3\}$$

Finally, our strain riser multiplier due to in-plane roller misalignment, F_M, can be calculated as

$$F_M = 1 + \frac{\theta_i}{\varepsilon_T \frac{L}{W}} \qquad \{4\}$$

Again we must check to see if this multiplier, F_M, could exist at the same time and place as the previous two, F_{TDE} and $F_{TDE.}$ The answer is yes, the worst alignment on the entire machine *could* indeed be in the parallelism of the driven roller and the idler upstream. However, the worst alignment could also be elsewhere. At this point we could be conservative and simply multiply the multipliers together or check each roller using actual values for that roller or use statistics knowing something about the distributions of misalignments in a web line. We will take the simple and conservative approach by assuming the worst as is the norm in engineering design. The worst case is where all reasonable and possible stress risers coincide in time and place.

BEND RADIUS OF A WEB GOING OVER A ROLLER

Bending of the web going over a roller is the first of two factors that can be quite important for thick webs. The strain can be calculated as

$$\varepsilon_{RBR} = \frac{half\ thickness}{radius\ of\ curvature} = \frac{t}{D} \qquad \{5\}$$

The strain is tensile on the top, corresponding to the other factors discussed so far. Now we only need to see if this strain addition could coincide in time and space with the other factors. Yes indeed, the roller with the smallest size (most in the line will be of the same or similar size), could be also the place with the highest misalignment and could also see a tension spike due to drive system errors. This location could also be where the downweb tension profile is highest (due to inertia and drag), though not necessarily. We will treat it conservatively as before and say that these stress multipliers can coincide.

CURL DUE TO RESIDUAL STRESSES

Curl can be measured by any number of methods [6]. We can treat it identically to roller bend radius. The strain can be calculated from measured radius as

$$\varepsilon_C = \frac{t}{2r_C} \qquad \{6\}$$

Again we check for alignment of time and place. Here, the odds of the tensile side of the curl, the bottom of the curvature when the web is not constrained, is only 50/50 that it will line up with the critical roller as about half of the rollers will put the compressive side of curl on the outside of the bend radius. Still, as process design engineers we will take the conservative approach saying that this coincidence is possible and even if not aligning there, the next most critical roller might see curl and roller bending adding. We remind the user that this and the previous calculation assume that the neutral axis is near the midpoint of the web thickness which is not a good assumption for composites whose plies have quite varied moduli when we may need to use the method of composite sections or other technique.

BAGGINESS OF WEBS

Bagginess is an epidemic problem where residual stresses of manufacturing vary noticeably across the width of the product and to a lesser extent down the length of the web (and with time) [7]. Bagginess must be at least considered if not explicitly measured because it could easily be as large, larger or much larger than the effects already considered. We will treat it as a stress multiplier and give it the symbol F_B. This factor is defined as the ratio of the maximum stress across the width to the average stress imposed by simple line tension. While this can be measured with great difficulty, it can also be estimated very approximately in some cases. In particular, if some slackness is observed (on a span whose alignment is good), then the tension there is less than zero. It would not be hard to imagine then that a 100% reduction in tension on one lane, from the average across the width to zero, could easily be accompanied by a 100% increase in tension on some other lane; in which case F_B would be 2. For those of you not satisfied with guesses, go ahead and measure if you like.

NOMINAL STRESS

We will define nominal stress as the average stress due to the tension setpoint as σ_T. Using Hookes law and solving for strain we get

$$\varepsilon_T = \frac{\sigma_T}{E_1} \tag{7}$$

PEAK STRESS AND ULTIMATE SAFETY FACTOR

The peak strain will be found by superposition of the factors thus considered.

$$\varepsilon_M = F_B F_M F_{TDE} F_{TDM}[\varepsilon_T + \varepsilon_C + \varepsilon_{RBR}] \tag{8}$$

We will define an ultimate Safety Factor, SF, as the ultimate stress (or yield stress if you please), σ_Y, divided by the maximum combined stress. Note that for many materials yield and ultimate stress are similar in magnitude though the former is of interest here and the latter is by far easiest to define and measure. Of course, either value would be in the machine direction. Again using Hooke's law, now with ultimate instead of nominal tension and combining it with equation 8 we get.

$$SF = \frac{\sigma_Y}{E_1 \sigma F_B F_M F_{TDE} F_{TDM}[\varepsilon_T + \varepsilon_C + \varepsilon_{RBR}]} \tag{9}$$

Those with thin brittle coatings have a work-around step to use the calculation. Here the SF is defined similarly, but staying with strains already calculated as

$$SF = \frac{\varepsilon_{YO}}{\sigma F_B F_M F_{TDE} F_{TDM}[\varepsilon_T + \varepsilon_C + \varepsilon_{RBR}]} \tag{10a}$$

where

$$\varepsilon_{YO} = \frac{\sigma_{YO}}{E_{1O}} \tag{10}$$

Here we can use either the yield stress of the critical material on the outside (top or bottom) of the web, such as measured by a tensile testing machine, or the yield strain directly, such as measured by a bend radius test. In bookkeeping summary, ε_T is the nominal strain due to web drive tension using a composite modulus while ε_{YO} is the strain at which the outer material yields or breaks.

APPLICATION OF THE NEW OVERALL SAFETY FACTOR

Recall that prior to this more explicit use of safety factors, we often gave nominal web tension guidelines that amounted to something like a minimum safety factor of 4-5. While this might work for many situations, it could be too conservative for a few. Now we can cut the corner more closely with better knowledge of the individual components that would cause stress to peak in some time and place. However, an even more important application would be those many places where the prior safety factor approach was not enough and thus the process design risky. These applications could be where misalignment or bagginess or web thickness were too high. These could also be where the web fails at low strains and thus all factors, no matter how small, need to be considered. All we need to do is to enter starting point values and check the ultimate safety factor. If too low for our taste for risk, we can play 'what if' scenarios by upsizing roller diameters for thick products or increasing alignment precisions for others as examples.

LIMITATIONS

We already discussed one limitation of this approach; that is the assumption that there is alignment in time and space for all the stress risers discussed here. While this is absolutely the case for some factors, it may not be for others and thus is *slightly* conservative. We also allowed using an ultimate rather than yield stress criteria, which is *slightly* non-conservative. More importantly, this analysis uses superposition and that requires linearity. If bagginess is severe or the structure complex, this analysis breaks down. Finally, the calculation is only as good as the inputs. Here, the bagginess multiplier will be the hardest to pin down because that will vary some from supply roll to supply roll and especially between suppliers. A separate program may be needed to screen for the bagginess problem so as not to unnecessarily burden product/process/machine design [8]. Future work by others could consider the winding analogue to this analysis for simply getting a web safely through a machine does not mean that the web can be safely wound. Minimum core diameters are just one example where product/process design is also needed.

CONCLUDING REMARKS

Safety factors and probability are rarely considered in research for cultural reasons. Safety factors and probability are rarely considered in engineering in the rush to design and keep on a schedule. However, very little extra effort is needed to do more studied design for web-roller systems as the prior work has already been done long ago. All we have done here is to assemble the ideas and make it accessible [9 abbottapp xxxx]. The user need only measure or estimate the individual sources of stress multiplication.

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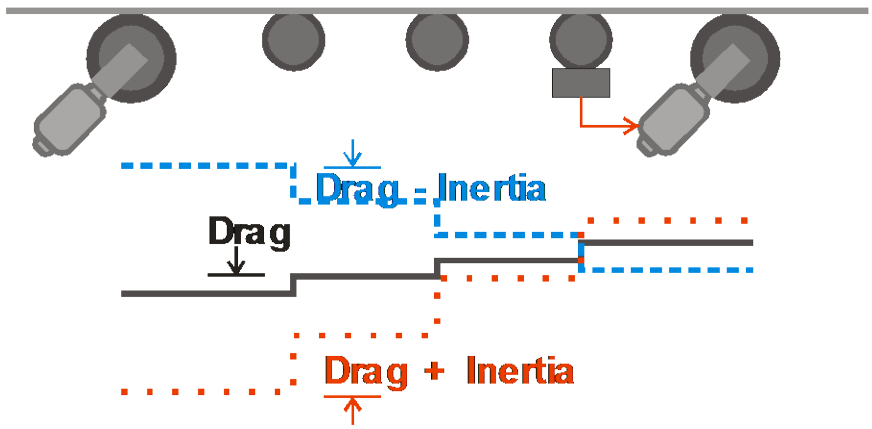

Figure 1 – Downweb stress profile due to bearing drag and inertia

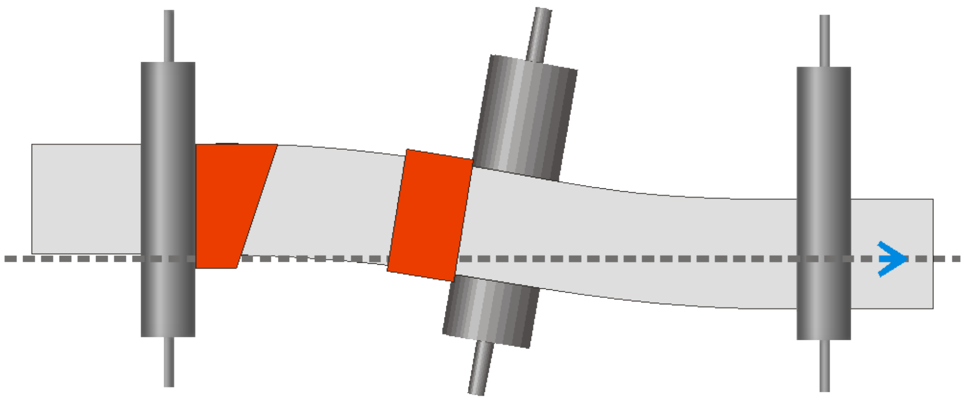

Figure 2 – Stresses on the outside of the bend due to inplane roller misalignment