

**DESIGN OF TENSION CONTROL SYSTEMS TO MINIMIZE
INTERACTION AND DISTURBANCE PROPAGATION IN WEB
PROCESS LINES**

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ABSTRACT

In roll-to-roll systems (R2R), a local controller is designed for each section or tension zone based on measurement of web tension and speed from that section; this is typically referred to as decentralized control. This is particularly suitable for R2R systems because of ease of implementation and their structure. Since the material is transported from the unwind to the rewind through the process sections, the entire machine can be divided into several sections and decentralized controllers may be utilized for each section. It is important to understand the dynamics of web transport from one section to other downstream sections, that is, interaction between different sections, and how disturbances are propagated. Tension control system designs that minimize disturbance propagation will aid in improving process performance.

In this paper, we will first investigate minimization of interaction between subsystems of R2R systems when decentralized feedback and feedforward controllers are employed. In particular, we will consider a new interaction metric for dynamical systems which will quantify the amount of interaction; and show how model-based feedforward action can be gainfully employed to minimize disturbance propagation. We will also discuss control strategies that will minimize disturbance propagation. To evaluate the proposed designs and recommendations, we will show results from experiments conducted on a large experimental web platform.

NOMENCLATURE

A_w	:	Area of cross-section of web
b_{fi}	:	Viscous friction coefficient
d	:	Disturbance
\hat{d}	:	Disturbance estimate

E	:	Modulus of elasticity of web material
G	:	Plant transfer function
G_m	:	Model transfer function
H	:	Closed loop transfer function
J_i	:	Driven roller moment of inertia of i^{th} roller
K_i	:	Decentralized controller
L_i	:	Length of i^{th} web span
L_H	:	Relative error matrix
n_i	:	Gear ratio of i^{th} motor
R_i	:	Radius of i^{th} roller
T_i	:	Web tension variation
t_i	:	Web tension
t_0	:	Wound on tension
t_{ri}	:	Tension reference
u_f	:	Feedforward control input
U_i	:	Torque input variation
u_i	:	Torque input to i^{th} roller
u_{ri}	:	Desired torque input
V_i	:	Velocity variation
v_i	:	Velocity of i^{th} roller
v_{ri}	:	velocity reference
y	:	System output
y_f	:	Desired output

Subscripts:

i	:	Span index, $i = 0, 1, 2, \dots$
r	:	Reference value

INTRODUCTION

A large scale system consists of many actuators, sensors, and process sections. A large scale system can be divided into many interconnected subsystems. The structure of the large scale system plays a key role in minimizing interaction. R2R systems are large scale interconnected systems with multiple subsystems comprising of many tension zones. Each tension zone can be considered as a subsystem with its own set of inputs and outputs. Interaction between tension zones occurs due to web transport, in particular, due to transport of strain. In R2R systems, decentralized controllers are often employed because of the structure of the system and their ease of implementation. It is important to understand the mechanisms for disturbance propagation from each section to other downstream sections, and how disturbances are propagated. It is also necessary to investigate whether correction of tension disturbances in one tension zone will affect tension in other zones. Design of controllers that minimize disturbance propagation will aid in improving the processing of the material in the process sections.

Recently, efforts were made to minimize interaction between sections of an R2R system by designing filters along with existing decentralized PI control strategies [2]. Though the designed control strategy is able to attenuate the disturbance, the attenuation levels are not significant. In the current research, feedforward control action along with interaction minimizing filters are designed and implemented as decentralized control. The feedforward action can aid in minimizing coupling between two sections. In order to mitigate the propagation of disturbances to downstream sections, it is suggested that downstream web tension measurement be used as feedback to the tension control system.

The remainder of the paper describes the interaction minimization procedure, the feedforward control action to improve performance and minimize disturbance propagation, implementation strategy for the feedforward action, and application of the feedforward control action. Experimental results will be presented and discussed.

INTERACTION MINIMIZATION

A schematic of the experimental platform containing three tension zones which is provided in Figure 1 will be utilized as an example to describe the developments in the paper.

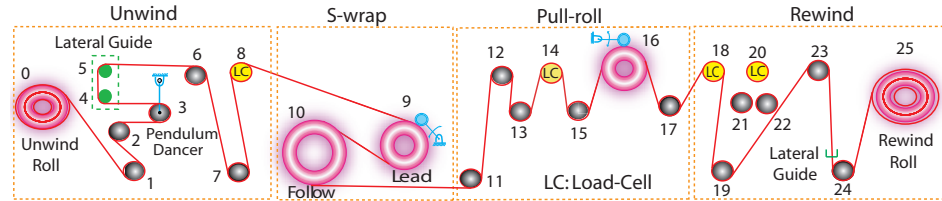


Figure 1 – Schematic of R2R experimental setup

A simplified sketch showing tension zones and driven rollers for a three tension zone web line is provided in Figure 2; the rollers indicated as “LC” are mounted on load-cells to provide tension feedback for tension control systems. The two driven rollers in the S-wrap section are electronically slaved together and are under pure speed control; these are treated as one driven roller (M1) in the simplified sketch and are typically referred to as the Master Speed Roller.

Let the following define variations of speed, tension and motor torque input from their reference values: $V_i = v_{ri} - v_i$, $T_i = t_{ri} - t_i$, and $U_i = u_{ri} - u_i$. The linearized governing equations for each section of the system shown in Figure 2 are given by

Unwind Section:

$$\frac{J_0}{R_0} \dot{V}_0 = T_1 R_0 - n_0 U_0 - \frac{b_f 0}{R_0} V_0 \quad \{1a\}$$

$$L_1 \dot{T}_1 = -T_1 v_{r1} + [A_w E - t_{r1}] V_1 + [t_0 - A_w E] V_0 \quad \{1b\}$$

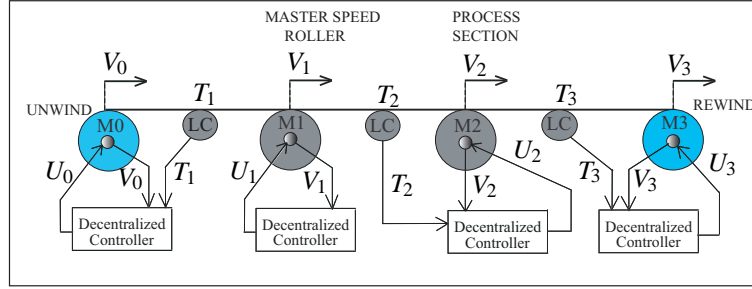


Figure 2 – Simplified line sketch of R2R experimental platform

Lead and Follower Section:

$$\frac{J_1}{R_1} \dot{V}_1 = (T_2 - T_1) R_1 + n_1 U_1 - \frac{b_{f1}}{R_1} V_1 \quad \{2\}$$

Pull Roll Section:

$$\frac{J_2}{R_2} \dot{V}_2 = (T_3 - T_2) R_2 + n_2 U_2 - \frac{b_{f2}}{R_2} V_2 \quad \{3a\}$$

$$L_2 \dot{T}_2 = -T_2 v_{r2} + [A_w E - t_{r2}] V_2 - [A_w E - t_{r1}] V_1 + T_1 v_{r1} \quad \{3b\}$$

Rewind Section:

$$\frac{J_3}{R_3} \dot{V}_3 = -T_3 R_3 + n_3 U_3 - \frac{b_{f3}}{R_3} V_3 \quad \{4a\}$$

$$L_3 \dot{T}_3 = -T_3 v_{r3} + [A_w E - t_{r3}] V_3 - [A_w E - t_{r2}] V_2 + T_2 v_{r2} \quad \{4b\}$$

The coupling between tension zones is evident from Equations {1}–{4} which leads to interaction between tension zones.

A typical tension control system in web handling machines consists of providing a trim to the reference speed of the motor speed feedback loop; this is often referred to as the speed-based tension control system. The speed feedback loop simply consists of a speed controller whose input is the speed error which is the difference between the reference speed and the measured speed. The speed controller is almost always implemented in the drive which allows for closing the speed loop at much smaller sampling periods. The output of the speed controller is the torque input to the motor. The trim to the reference speed is often referred to as the tension trim which is the output of the tension controller whose input is the tension error, the difference between the tension reference and measured tension. Another type of tension control system involves adding the tension trim as a torque correction to the output of the speed controller, which is often referred to as the torque-based tension control system.

In this paper we consider the speed-based tension control system with trim to the velocity reference as the input and the measured tension as the output. That is, it is assumed that a speed control system is already designed to provide

satisfactory speed-loop performance for each motor. We consider the three tension-zone example system provided in Figure 2 to illustrate the developments in this paper.

Let y denote the vector of tension variations T_1, T_2, T_3 in the three zones and x denote the vector of speed trims V_1, V_2, V_3 . Let the R2R system dynamics be represented by the input-output relationship: $y(s) = G(s)x(s)$, where $G(s)$ is the transfer function matrix and $x(s)$ is the Laplace transform of the vector x and $y(s)$ is the Laplace transform of the output vector y . The off-diagonal elements of the transfer function matrix $G(s)$ specify the interaction between sections. The system can be represented by the following equations:

$$T_1(s) = G_{11}(s)V_0(s) + G_{12}(s)V_2(s) + G_{13}(s)V_3(s) \quad \{5a\}$$

$$T_2(s) = G_{21}(s)V_0(s) + G_{22}(s)V_2(s) + G_{23}(s)V_3(s) \quad \{5b\}$$

$$T_3(s) = G_{31}(s)V_0(s) + G_{32}(s)V_2(s) + G_{33}(s)V_3(s) \quad \{5c\}$$

The tension equations can be expressed in matrix form as

$$\underbrace{\begin{pmatrix} T_1(s) \\ T_2(s) \\ T_3(s) \end{pmatrix}}_{y(s)} = \underbrace{\begin{pmatrix} G_{11}(s) & G_{12}(s) & G_{13}(s) \\ G_{21}(s) & G_{22}(s) & G_{23}(s) \\ G_{31}(s) & G_{32}(s) & G_{33}(s) \end{pmatrix}}_{G(s)} \underbrace{\begin{pmatrix} V_0(s) \\ V_2(s) \\ V_3(s) \end{pmatrix}}_{x(s)} \quad \{6\}$$

The goal is to design a decentralized controller for each section such that interaction is minimized in the closed-loop transfer function matrix, that is, the off-diagonal elements are minimized.

Let G be separated as $G = \bar{G} + \tilde{G}$ where \bar{G} contains the diagonal part of G and \tilde{G} contains the off-diagonal part of G and can be expressed in matrix form as

$$G(s) = \underbrace{\begin{pmatrix} G_{11}(s) & 0 & 0 \\ 0 & G_{22}(s) & 0 \\ 0 & 0 & G_{33}(s) \end{pmatrix}}_{\bar{G}(s)} + \underbrace{\begin{pmatrix} 0 & G_{12}(s) & G_{13}(s) \\ G_{21}(s) & 0 & G_{23}(s) \\ G_{31}(s) & G_{32}(s) & 0 \end{pmatrix}}_{\tilde{G}(s)} \quad \{7\}$$

The size of \tilde{G} may be used to quantify interaction. In particular, the effect of the off-diagonal elements and corresponding inputs on a particular output may be evaluated by the size of the relative error matrix $L_H \triangleq \tilde{G}\bar{G}^{-1}$ and can be shown in matrix form as

$$L_H(s) = \begin{pmatrix} 0 & \frac{G_{12}(s)}{G_{11}(s)} & \frac{G_{13}(s)}{G_{11}(s)} \\ \frac{G_{21}(s)}{G_{22}(s)} & 0 & \frac{G_{23}(s)}{G_{22}(s)} \\ \frac{G_{31}(s)}{G_{33}(s)} & \frac{G_{32}(s)}{G_{33}(s)} & 0 \end{pmatrix} \quad \{8\}$$

Interaction may be quantified by the size of relative error matrix L_H . A Perron root based interaction metric (PRIM) may be used to quantify the

interaction [2]. For an $n \times n$ transfer function matrix $G(s)$, the Perron-root interaction metric (PRIM) is defined as

$$p_{L_H}(\omega) \triangleq \mathcal{P}(\langle L_H(j\omega) \rangle) \quad \{9\}$$

where $\mathcal{P}(\langle L_H(j\omega) \rangle)$ is the Perron root of the irreducible matrix $\langle L_H(j\omega) \rangle$ at the frequency ω and $j = \sqrt{-1}$.

A smaller value of PRIM less than unity means less interaction. The PRIM provides a form of the overall interaction in a multivariable system and does not provide information about interaction between any two sections in the system. The decentralized pre-filter design given in [2] ensures diagonal dominance (minimize interaction) at all frequencies of the closed-loop system transfer matrix.

A stabilization criteria for multivariable systems employing decentralized controllers is based on the size of the interaction measure and the diagonal part of the closed-loop system transfer matrix which can be found in [3]. Suppose for a rational transfer function matrix $G(s)$ a decentralized tension controller K stabilizes the diagonal part of the transfer function matrix, \bar{G} , i.e., the diagonal part of the closed-loop system \bar{H} is stable where $\bar{H} = \bar{G}K(I + \bar{G}K)^{-1}$. The closed-loop system \bar{H} is,

$$\bar{H}(s) = \begin{pmatrix} \bar{G}_{11}K_1(I + \bar{G}_{11}K_1)^{-1} & 0 & 0 \\ 0 & \bar{G}_{22}K_2(I + \bar{G}_{22}K_2)^{-1} & 0 \\ 0 & 0 & \bar{G}_{33}K_3(I + \bar{G}_{33}K_3)^{-1} \end{pmatrix} \quad \{10\}$$

Then a condition for the stability of the overall system can be obtained based on the Perron root of L_H assuming a diagonal structure of \bar{H} by

$$|\bar{h}_k(j\omega)| < \frac{1}{p_{L_H}(\omega)} \quad \forall k, \omega. \quad \{11\}$$

where p_{L_H} is the perron root interaction metric, j is unit imaginary number, and k is tension zone index, and $\bar{h}_k(j\omega) = \bar{G}_{kk}K_k(I + \bar{G}_{kk}K_k)^{-1}$.

It is evident that when the interaction term $L_H = 0$, any controller structure \bar{H} will satisfy the overall system stability condition given above. It is shown in [2] that using the Perron vector based filter the interaction in the system can be minimized. When the interaction is small, the decentralized controller should be designed such that the diagonal closed-loop system transfer function matrix \bar{H} must be close to identity. In the following two sections, we will show how the performance of the decentralized control system can be improved by utilizing model-based feedforward and the particular structure of feedforward compensation in tension control systems.

FEEDBACK AND FEEDFORWARD CONTROL ACTION

Consider the block diagram showing the control system for one section in Figure 3. Let y_f denote the desired output, d denote the disturbance in the

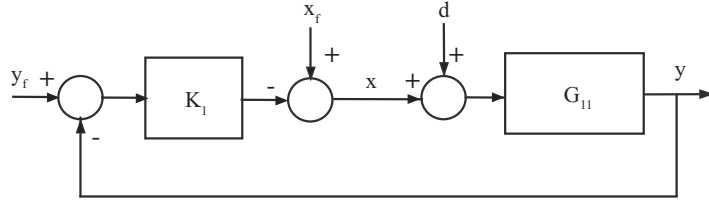


Figure 3 – A section with decentralized controller

system, and $x = (y_f - y)K_1$ with $x_f = 0$ denote the feedback control input. The system output y is expressed as

$$y = \frac{K_1 G_{11}}{1 + K_1 G_{11}} y_f + \frac{G_{11}}{1 + K_1 G_{11}} d \quad \{12\}$$

Consider the control input $x = x_f - (y_f - y)K_1$, where x_f is the feedforward input that gives desired output y_f when applied to a known system model.

$$y_f = G_{11m}(x_f + \hat{d}) \quad \{13\}$$

where G_{11m} is the system model and \hat{d} is the disturbance estimate. With the feedback and feedforward control input, the system output y can be expressed as

$$y = \frac{-K_1 G_{11}}{1 - K_1 G_{11}} y_f + \frac{G_{11}}{1 - K_1 G_{11}} x_f + \frac{G_{11}}{1 - K_1 G_{11}} d \quad \{14\}$$

Substituting equation {13} into equation {14} we get

$$y = \frac{-K_1 G_{11}}{1 - K_1 G_{11}} y_f + \frac{G_{11}}{G_{11m}(1 - K_1 G_{11})} y_f - \frac{G_{11}}{1 - K_1 G_{11}} \hat{d} + \frac{G_{11}}{1 - K_1 G_{11}} d \quad \{15\}$$

This indicates that if $G_{11} \approx G_{11m}$ and with perfect estimation of the disturbance ($\hat{d} = d$), the system output tracks the desired output. Therefore, the use of feedback and feedforward control action will aid in improving the performance of decentralized control systems and minimizing interaction between subsystems. The proposed tension control systems for the three tension zone example are shown in Figure 4.

In the context of the above discussion, the transfer function of decentralized tension zones is G_{11} with T_1 as output and V_1 as input. Similar variables exist for two downstream sections, i.e. G_{22} and G_{33} . K_1 , K_2 , and K_3 are decentralized feedback tension controllers that provide velocity correction to respective inner velocity loop as shown in Figure 4.

FEEDFORWARD IMPLEMENTATION TO MINIMIZE DISTURBANCE PROPAGATION

In general, control schemes in R2R systems are implemented in the process sections with tension feedback from the upstream zone as shown in Figure 4.

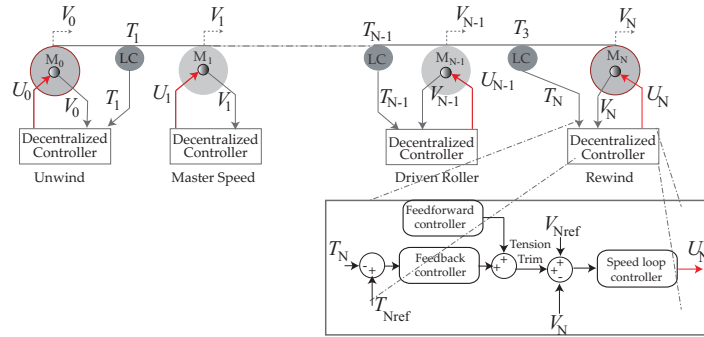


Figure 4 – Decentralized tension control structure for roll-to-roll systems with an inner velocity loop and an outer tension loop

The feedforward action x_f given by equation {13} contains disturbance estimation and is used to attenuate the disturbance in a particular section. However, this feedforward control action x_f also generates a disturbance in the neighboring downstream section, that is, if the feedforward action is implemented with tension measurement from the upstream tension zone, it generates an estimate of disturbance in the downstream section and as a result there is a possibility of disturbance propagation into the downstream sections.

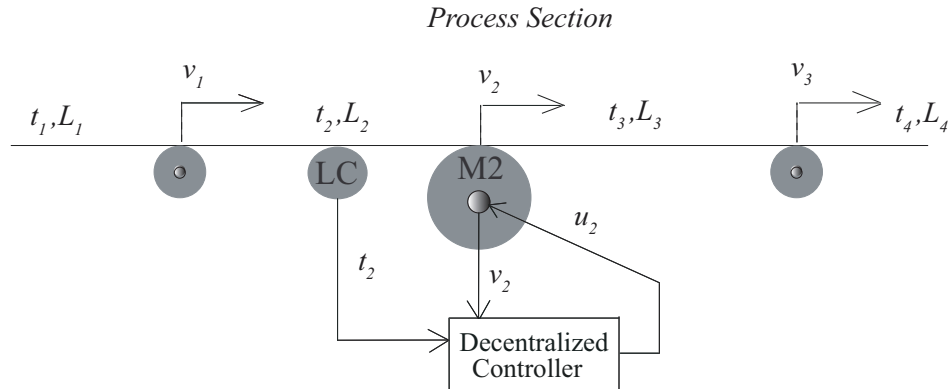


Figure 5 – Decentralized control with upstream tension feedback

The existing decentralized control scheme in the process sections is implemented as shown in Fig. 5; the driven roller is controlled through feedback from upstream span web tension t_2 . Note that u_2 is the torque input to the motor shown in Fig. 5. This feedback strategy is able to reject the disturbance and regulate the tension in the upstream tension zone. However, the control action generated by the driven roller induces disturbances into the downstream tension zone. Hence, although the strategy is able to mitigate disturbances in the upstream tension zone, it acts as a disturbance source to the adjoining

downstream tension zone. This can be deduced from the tension and velocity dynamics corresponding to the section of the web shown in Fig. 5 given by the following equations:

$$\dot{v}_1 = \frac{R_1^2}{J_1}(t_2 - t_1) - \frac{b_{f1}}{J_1}v_1 + \frac{R_1}{J_1}n_1u_1, \quad \{16\}$$

$$\dot{v}_2 = \frac{R_2^2}{J_2}(t_3 - t_2) - \frac{b_{f2}}{J_2}v_2 + \frac{R_2}{J_2}n_2u_2, \quad \{17\}$$

$$\dot{t}_2 = \frac{EA_w}{L_2}(v_2 - v_1) + \frac{1}{L_2}(t_1v_1 - t_2v_2), \quad \{18\}$$

$$\dot{v}_3 = -\frac{R_3^2}{J_3}t_3 - \frac{b_{f3}}{J_3}v_3 + \frac{R_3}{J_3}n_3u_3, \quad \{19\}$$

$$\dot{t}_3 = \frac{EA_w}{L_3}(v_3 - v_2) + \frac{1}{L_3}(t_2v_2 - t_3v_3). \quad \{20\}$$

Now consider the disturbance generated at the roller with web velocity v_1 as shown. The feedback and feedforward control action is applied at driven roller M2 with web velocity v_2 . The control action x_2 has feedforward action x_{f2} in order to reject periodic disturbances. The feedforward action is synthesized with the plant model and disturbance estimation \hat{d} . With tension feedback from upstream span t_2 , the control action x_2 can reject the disturbance by correcting the velocity v_2 . Since the velocity v_2 is also input to the downstream span tension, the corrective action x_2 generates disturbance in the downstream span tension t_3 . The interaction between different subsystems propagates the disturbance further downstream affecting the other processes.

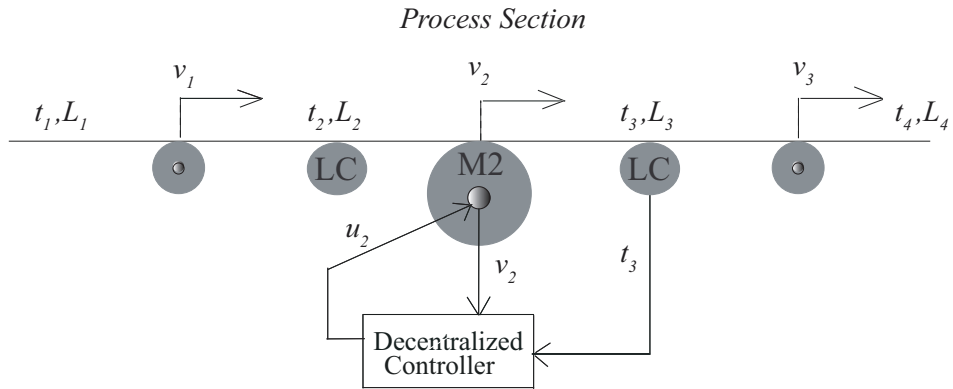


Figure 6 – Decentralized control with downstream tension feedback

The proposed implementation of feedforward control action is shown in Figure 6. In the proposed scheme, web tension feedback is obtained from the downstream span. Although this strategy creates disturbance in the upstream

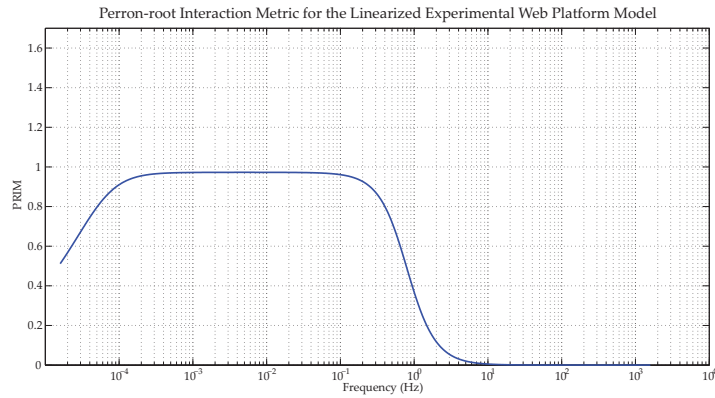


Figure 7 – Perron root interaction metric for the linearized model of the roll-to-roll system

span (within acceptable limits), it is capable of mitigating the interaction and reducing propagation of disturbances into downstream spans. This strategy is useful in reducing tension disturbance propagation into critical downstream process sections.

EXPERIMENTAL SETUP AND PROCEDURE

The experimental platform shown in Figure 1 is utilized for experimentation to verify the strategies discussed in the preceding sections. Web tension at its reference value is maintained by regulating the speed of the driven roller in that zone. The velocity correction provided by the outer tension loops influences the web tension in the adjacent zone. Fig. 7 shows the PRIM with the three tension loop velocity corrections as the inputs and the web tensions in the three zones as the outputs. From the PRIM plot it is evident that interaction is dominant in the range of frequencies between 10^{-4} Hz to 1 Hz and is minimal above 1 Hz. The magnitude of interaction is close to 1 indicating that the velocity correction provided by the tension loop will have almost the same influence on one other tension zone. Note that the PRIM provides the worst case scenario for all the three tension zones and provides no information about the effect of any particular input-output pair.

Experiments were conducted on the experimental R2R system to compare three scenarios:

1. characterize the interaction in the actual system with feedback control action,
2. utilize Perron root based filters to minimize the interaction,
3. use Perron root based filters and feedforward action to avoid propagation of disturbances.

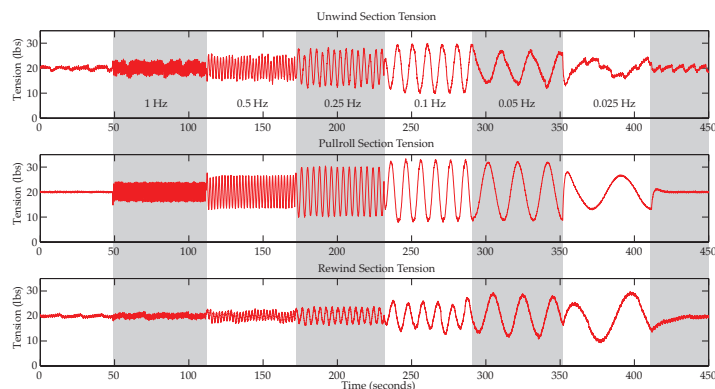


Figure 8 – Tension measurement at the unwind, pull roll and rewind section with sinusoidal velocity disturbances at the S-wrap section.

To illustrate the effect of these strategies, velocity disturbances at the S-wrap section were introduced to create tension disturbances in the unwind and pull roll tension zones and the effect of these disturbances in the rewind section were observed to evaluate the interaction between these two zones. In the experiments, a six inch wide polymer web (called Tyvek) was transported with a web speed of 150 feet per minute (fpm) under a reference web tension of 20 pounds (lbf). A sinusoidal speed disturbance of magnitude 5 fpm was introduced at the S-wrap driven rollers for a duration of one minute; six distinct frequencies were considered from 0.01 to 1 Hz. The feedforward control action was applied to the pull roll with both upstream and downstream web tension feedback to evaluate the different strategies.

EXPERIMENTAL RESULTS

Figure 8 shows the tension signals in the three tension zones when the web is transported in the forward direction (that is from the unwind to the rewind) and represents scenario (1) stated in the preceding section. Note that the tension disturbances observed in the unwind and the pull roll tension zones are due to the direct effect of the S-wrap velocity disturbance. The interaction of the different zones and disturbance propagation into downstream sections is evident from the tension signal measured in the rewind tension zone. From the plots it is evident that the magnitude of interaction is small above 0.25 Hz and increases with decreasing frequency. At low frequencies the tension disturbance observed at the rewind section is as high as the tension disturbances observed in the unwind and the pull roll sections as predicted by the PRIM (see Fig. 7).

Figure 9 shows the experimental results with pre-filters and feedback control action only; this represents scenario (2). The results with the pre-filter show reduction of interaction between zones and minimization of disturbance propagation into the downstream tension zones.

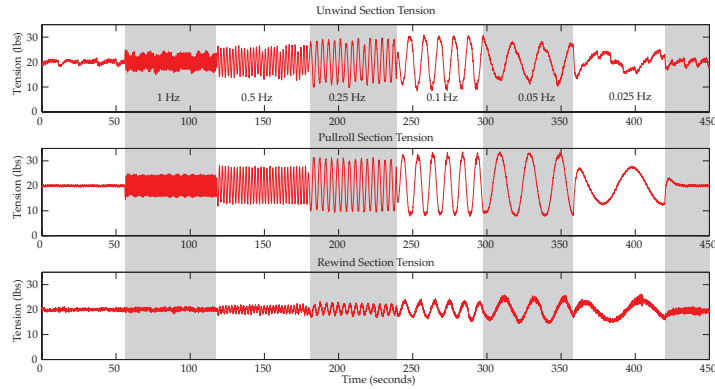


Figure 9 – Interaction between tension zones with pre-filter; tension measurement at the unwind, pull roll and rewind section.

Figure 10 shows the experimental results with pre-filters and feedforward action obtained by upstream zone tension measurement; this represents scenario (3). Although, the results with this strategy indicate rejection of disturbance in pull roll section but it generates disturbance into the downstream rewind tension zone.

Figure 11 shows the experimental results with pre-filters and feedforward action obtained by downstream zone tension measurement; this represents scenario (3). The results with this strategy indicate significant interaction reduction as well as minimization of disturbance propagation into the downstream rewind tension zone.

CONCLUDING REMARKS

The use of feedforward control action along with interaction minimizing filters is proposed with feedback from downstream tension zone in this paper. The proposed strategy was able to minimize the interaction as well reduce disturbance propagation into downstream tension zones. The proposed scheme was able to attenuate the effect of disturbances as well as restrict disturbance propagation into further downstream tension zones.

ACKNOWLEDGEMENTS

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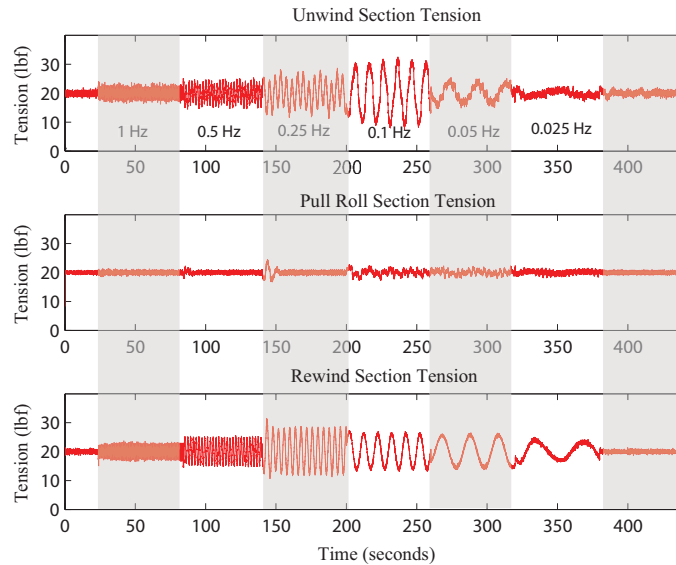


Figure 10 – Interaction between tension zones with pre-filter and feedforward action obtained by upstream zone tension feedback; tension measurement at the unwind, pull roll and rewind section.

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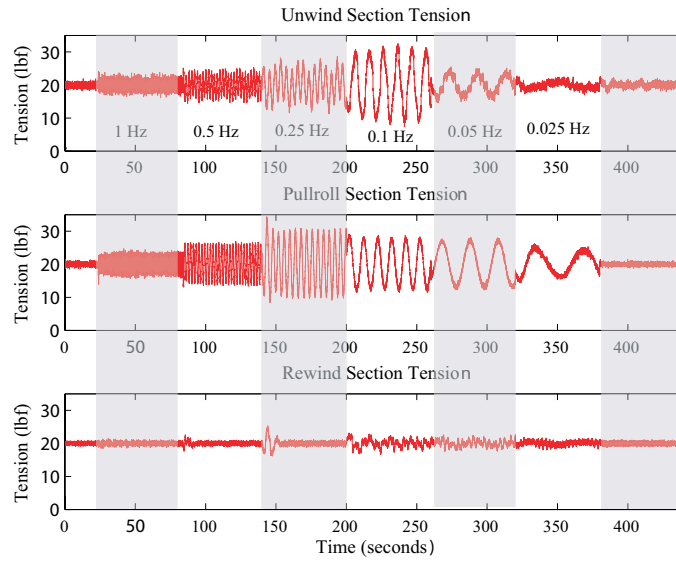


Figure 11 – Interaction between tension zones with pre-filter and feedforward action obtained by downstream zone tension feedback; tension measurement at the unwind, pull roll and rewind section.

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