THE BEHAVIOR OF WEBS TRANSITTING CROWNED ROLLERS

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ABSTRACT

Crowned rollers are known to center running belts by observation. Crown can be an unintentional artifact of rollers whose surfaces are machined on lathes while supported at their ends. The bending stiffness of the rotating roller allows the roller to deflect away from the cutting tool. If centering is desired the crown may be intentionally cut into the roller surface.

Webs have small finite buckling strength. The same steering forces that will center a belt on a crowned roller may buckle or wrinkle a web. The objective herein is to demonstrate modeling methods that can be used to determine when the crown level will induce buckles or wrinkles in a web or a belt.

INTRODUCTION

The earliest references describing use of a crowned roller are found in literature from the time of the industrial revolution. Crowned rollers were employed to laterally stabilize and center belts transmitting power between line shafts. Reynolds [1] proposed a creep theory for belt behavior on a pulley in 1874. This theory was used in designs and referred in texts such as the 1896 book by Robinson [2] or Schwamb *et al* [3] where the capability of a crown pulley to center a belt was recognized. Guidelines for the amount of crown per the width of pulley were mentioned in 1909 machine design text by Unwin [4]. Swift [5, 6] described the behavior of a belt transiting a pulley by further developing the creep theory. Swift proposed the normal entry rule which contends that under no-slip conditions that the surface velocities of a belt will match the velocities of the surface of the crown roller in terms of magnitude and direction. This results in the belt entering the roller *normal* to the axis of rotation. Swift used the normal entry rule to explain why an off-center belt would climb toward the center of a crowned pulley. In 1970, Firbank [7] proposed the use of shear theory pointing out some inaccuracies represented by creep theory when it is applied to mechanics of belts containing steel reinforcement strands.

Unlike belts, webs are much thinner and have little buckling resistance to compressive cross machine direction (CMD) stresses. Whenever a web transits a crowned roller, CMD compressive stresses result in the web. If these compressive stresses exceed critical values of CMD stress (σ_{ycr}), instabilities in the form of troughs and wrinkles are generated in web. *Troughs* are defined herein as out-of-plane instabilities generated by small compressive CMD stresses in webs in free spans. *Wrinkles* are defined as web instabilities on rollers. The compressive stress required to wrinkle a web on a roller is much larger than that required to induce a web trough in a free span. The web spans upstream and downstream of the crowned roller will be referred to as the entering and exit spans, respectively.

Beisel conducted tests and the initial finite element modeling of a web transiting a crown roller[8]. His tests employed a variable crown roller that he designed. He would increase the crown during a test until troughs would form in web. He also tested a set of fixed crowned rollers in which he would slowly decrease web tension until a wrinkle formed on the roller. Several researchers have spent a great deal of effort to understand web behavior due to roller crown. To date there is no closed form expression that will predict web troughs or wrinkles in webs due to roller crown.

The analyses presented herein have employed the static as well as the explicit dynamic finite element method for studying behavior of webs on crowned rollers. The explicit contact algorithms are capable of modeling the friction forces between web and rollers and determining when the conditions of stick or slip will occur. Similar to previous explicit modeling [9] and [10] of webs only minimal boundary conditions will be set in these analyses. The web velocity and tension, the web/roller friction coefficient, and the web elastic properties are the only conditions prescribed. If the traction between a web and roller is sufficient, normal entry may be observed in the output. Stick and slip behavior between a web and roller will be governed by Coulomb's friction model. It will be shown that the web does not slip over the majority over its wrap on roller surface till the very exit from crowned roller. This finding will then be used to develop static analysis of trough and wrinkles generated due to roller crown for various web line parameters.

ANALYSIS OF A WEB TRANSITING A CROWNED ROLLER

A variation in surface speed across the width of web results from the variation in radius of the roller in the CMD. For the following theoretical development, we assume the web material to conform to the roller surface and a no slip condition between the web and roller. The change in radius across width of the roller causes variation in surface speed across the web width. This results in more tension at the web center than at the edges of web. The geometry of a web moving over a crowned roller is shown in Figure 1. The origin of coordinate system is located at center of roller where the center line of web and roller intersect. It will be assumed briefly that the web can be analyzed as two separate beams of web width *(W/2)*.

Given the properties of the web and roller profile, assuming normal entry of the web to the roller, considering only elastic stiffness, the machine direction stress (σ_{md}) can be related to the strain in the web by using Hooke's law as:

$$\sigma_{md} = E\varepsilon_{md(z)} \qquad \{1\}$$

where *E* is Young's modulus and *z* is the distance of given point from the centroid axis of the web. Expression {1} is the uniaxial form of Hooke's Law.

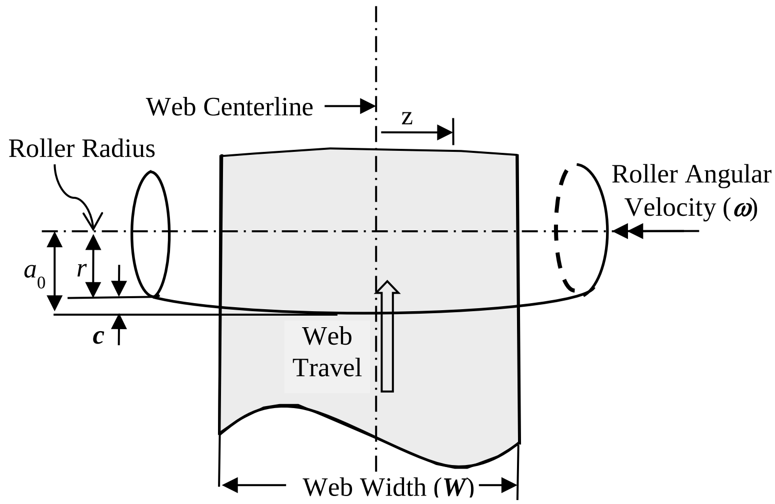


Figure 1 – Geometry and Configuration of Web on Crowned Roller

The application appears to be a state of biaxial stress. The CMD stress in the web in the entering span will be shown to be minor compared to the MD stresses and thus expression {1} is a reasonable assumption. Assuming that the web remains in contact with the roller and moves with the same velocity, the web velocity ($V_{(z)}$) is the roller surface velocity and is the product of the roller radius (r) and the roller angular velocity (ω). The strain imparted to the web is related to the radius profile of the roller per:

$$\varepsilon_{md}(z) = \frac{V_{(z)} - V_{avg}}{V_{avg}} \qquad \{2\}$$

The roller radius and the web velocity can be derived in terms of the roller profile coefficients as:

$$r = a_0 - a_1 z^2 \text{ and the velocity is } V(z) = \left(a_0 - a_1 z^2\right)\omega \qquad \{3\}$$

where a_0 is the crown roller radius at intersection of web-roller centerline as shown in Figure 1 and a_1 is the parabolic roller profile coefficient. The average velocity of the web in contact with the roller is obtained by integrating the velocity given in expression {3} across the web width:

$$V_{avg} = \frac{1}{W}\int_{-W/2}^{W/2}\left(a_0 - a_1 z^2\right)\omega dz = \omega\left(a_0 - \frac{a_1 W^2}{12}\right) \qquad \{4\}$$

Substituting equations {4} and {3} into {2} yields:

$$\varepsilon_{md}(z) = \frac{a_1 W^2 - 12 a_1 z^2}{12 a_0 - a_1 W^2} \qquad \{5\}$$

Substituting expression {5} into equation {1} and adding the stress due to MD tension (T) gives the total MD stress across the web width due to roller crown as:

$$\sigma_{md} = E \frac{a_1 W^2 - 12 a_1 z^2}{12 a_0 - a_1 W^2} + \frac{T}{h} \qquad \{6\}$$

where *h* is the web thickness. To find the moment generated in half the web width, this stress distribution is integrated over *W/2* with the *y* coordinate transformed to the half width coordinate z_c:

$$z = z_c + \frac{W}{4} \qquad \{7\}$$

$$M_{centroid} = \int_{-W/4}^{W/4} \sigma_{md} z_c h dz = \frac{-E a_1 t W^4}{16\left(12 a_0 - a_1 W^2\right)} \qquad \{8\}$$

This moment in combination with normal entry would steer the half web width beam in the entry span toward the CMD center of the crown roller. The opposing half width web beam would also be steered toward the center of the roller. This results in CMD compressive stresses forming in the web in the entry span and on the surface of the crown roller. This provides an explanation of why a crown roller will induce troughs and wrinkles into a web. It does not provide expressions that can be used to estimate when roller crown will induce troughs and wrinkles into a web. That is the focus of this study.

FINITE ELEMENT MODEL OF A WEB ON A CROWNED ROLLER

ABAQUS/Explicit® is a commercial FE program used to conduct simulations. Details related to the explicit modeling of webs and contoured roller interaction is provided in [9]. The rollers in the following analyses are modeled as rigid analytical surfaces in Abaqus. This allows the geometries of the contact surfaces of the cylindrical and crowned roller surfaces to be defined continuously. The web is modeled as an elastic material with shell elements.

The Crowned Roller Model

Every model constructed herein has fixed values of parameters such as roller crown, web line tension and web span lengths. For problems involving lateral disturbances in a web span Fu et al. [10] found that to achieve a steady state solution that a web length three times that of the entry web span length must be allowed to transit the entry span. Similar lengths of web will be allowed to transit the entry span in the current simulations. The simulations were setup with the elastic axis of the web initially aligned with the roller centers. These simulations were setup to model test conditions chosen by Beisel [8]. Beisel developed a variable crown roller shown in Figure 2 by designing a roller with a variable wall thickness. When internally pressurized the radius of the outer surface would deform parabolically per expression {3}. When no internal pressure was provided the roller was cylindrical. The radius of Beisel's roller as a function of CMD location and internal pressure is presented in expression {9}. Beisel could conduct tests in which he would set a span length and web tension. He would then increase the internal pressure in the roller until troughs would appear in the entry span. In this case we will input the pressure at which Beisel saw troughs appear into expression {9} and use that expression to define the radius variation of the crowned roller modeled in Abaqus.

$$
\begin{aligned}
& r(\mathrm{mm}) = a_o - a_1 z^2 && z(\mathrm{mm}) \\
& a_o = 38.7874 + 5.0190 * 10^{-6} P && P(\mathrm{KPa}) \qquad \{9\} \\
& a_1 = 1.5745 * 10^{-7} - 3.4381 * 10^{-2} P
\end{aligned}
$$

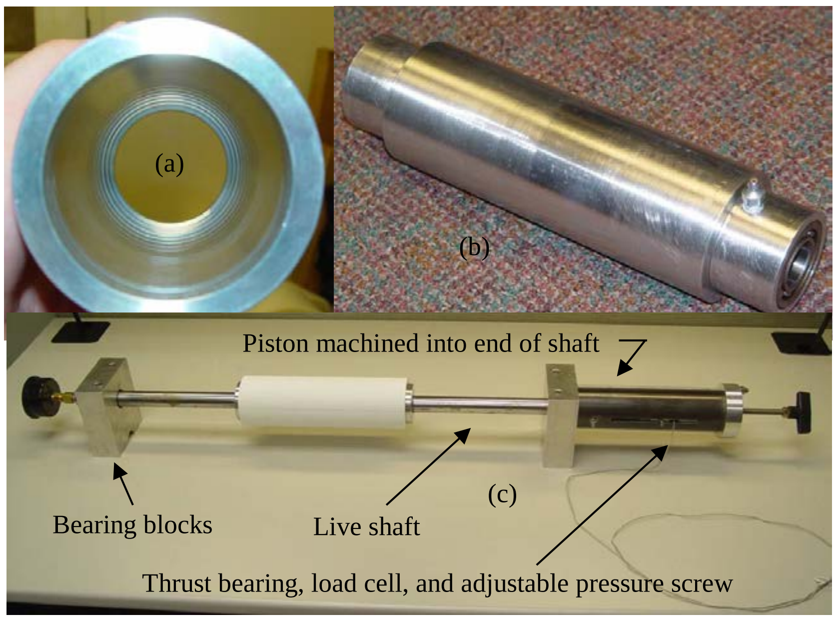


Figure 2 – An Adjustable Crown Roller: (a) a cylindrical roller with an internally stepped wall thickness (b) a version in which crown could be adjusted with the web line stalled and (c) a version in which the crown could be adjusted with the web in motion.

The remaining web line conditions and modeling parameters are given in Table 1.

The crowned roller is modeled in the third position (R3) of the typical four roller setup shown in Figure 3. Rollers R1, R2, and R4 are cylindrical rollers with a nominal radius of 36.8 mm (1.45 in). The results presented first are from a model of Beisel's crowned roller where the crown resulted from an internal pressure of 5512 KPa (800psi) per expression {9}, an entry span length of 254mm (10in) and several web line tension levels.

The computational time and resources needed for executing every successful simulation greatly depends on mesh refinement. Reduction in the size of element used in simulations increases the total number of elements to be handled during given analysis causing additional time for job execution. A sufficiently small element size is needed to accurately define contact mechanics underlying the interacting surfaces. This is particularly important when cylindrical rollers in contact with a web are being modeled, even more so when one of the rollers is non-cylindrical due to crown. Determining the optimum mesh size involves various combinations of material properties, correct dimensioning, types of elements and suitable computational cost analysis. A convergence study was carried out to support the selection of mesh size based on optimum combination of accuracy and computational cost. The shell elements used in analysis for this study are S4R (4-node doubly curved general-purpose shells, reduced integration with hourglass control, finite membrane strain) elements. The least size of shell elements used for this study is 3.8mm (0.15in).

Property	Value
Web Width *(W)*	152.4mm (6 inch)
Web Thickness *(h)*	0.0234mm (0.00092 inch)
Young's Modulus *(E)*	4908.5N/mm^2 (712000 psi)
Poisson's Ratio *(υ)*	0.3
Entering Span to concave roller *(L)*	254mm (10 inch)
Pre-entering span *(L_a)*	152.4mm(6 inch)
Roller Radius *(r)*	36.8mm (1.45 inch)
Wrap Angle *(β)*	1.57rad (90 degrees)
Web Velocity *(v)*	25.4mm/s (1 inch/sec)
Coefficient of Friction *(μ)*	Variable

Table 1 – Parameter Values for Adjustable Crown Roller Model

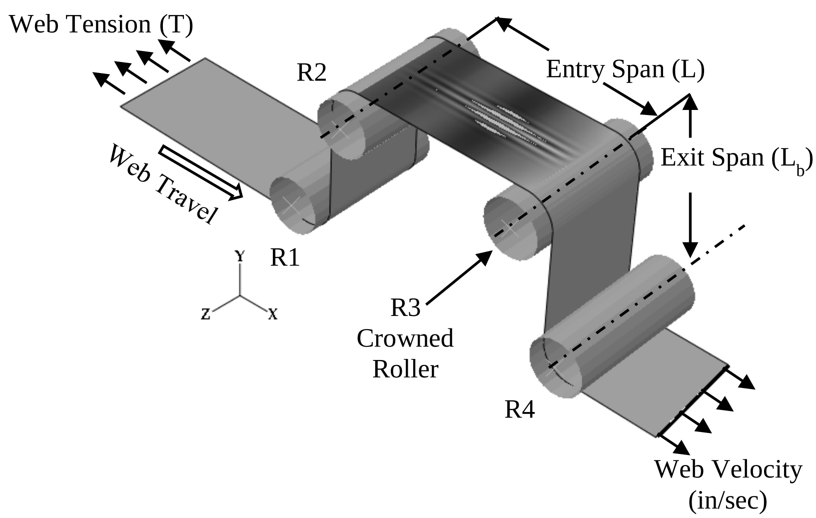


Figure 3 – Crown Roller Model Setup and Trough Formation in Entering Span (L)

<u>Convergence of Stresses</u>

Figure 4 shows the convergence of the MD and CMD stresses through the simulation time when the element size was set at 3.81 mm. The MD stresses converge faster than that of CMD stresses. The MD stresses at the center of web achieve convergence earlier than the edge. Note how well the Abaqus MD stresses at the web center and edge agree with that given by the closed form expression {6}. Though it appeared good accuracy was achieved for calculating the stresses in Figure 4 using all sizes of mesh, the mesh size became of greater concern when attempting to study troughs in the entry span and to model the wavelength of the troughs accurately.

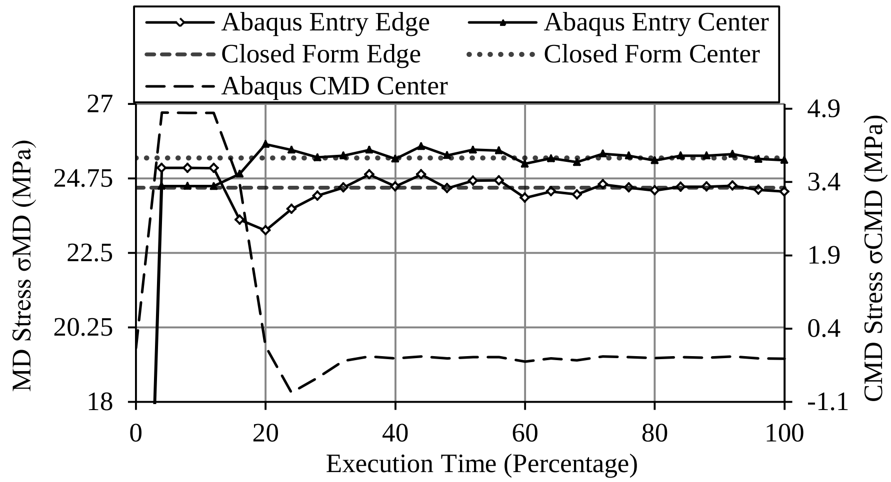


Figure 4 – Stress Convergence through Simulation Time
(5512 KPa (800psi) Crown, L = 254mm (10in), T = 66.7 N (20 lb))

The mesh had to be fine enough so that 4-5 elements would define a trough wavelength. During tests, Beisel determined the onset of trough instability in the entry web span visually for the given combination of web properties and web line parameters. We will study the out-of-plane deformations (U2) from Abaqus of the web across the web width to determine when a trough is formed in the given span or not.

The wavelength (λ) of troughs formed as a result of uniform CMD compressive stress (σ_e) can be predicted using expressions given by Beisel *et al* [8] as:

$$\lambda_{troughs} = \frac{2L}{\sqrt[4]{1+\frac{\sigma_x}{\sigma_e}}} \quad \text{where} \quad \sigma_e = \frac{\pi^2 D}{L^2 h} \quad \text{and} \quad D = \frac{Eh^3}{12(1-\nu^2)} \qquad \{10\}$$

Also in Figure 5, the MD stress at the entry and exit locations also is shown as a function of CMD position. The MD stress profile in the web undergoes a reversal from the entry to the exit of the crown roller. The location of point where the stress reversal occurs is dictated by slippage of the web on the roller surface and will be examined further. The closed form expression {6} predicts the stresses at the entry of crowned roller accurately except at the edges, where some disagreement is seen due to slippage.

FORMATION OF TROUGHS DUE TO ROLLER CROWN

Abaqus Explicit Analysis of Trough Formation

The web behavior at entry, on the wrap and after the web exits a concave roller is explained in detail in [9]. From the results shown thus far (Figure 5) the Abaqus simulations show an opposite behavior for the MD stresses for the crowned roller. Although these behaviors are interesting the current mission is to study web instability due to roller crown.

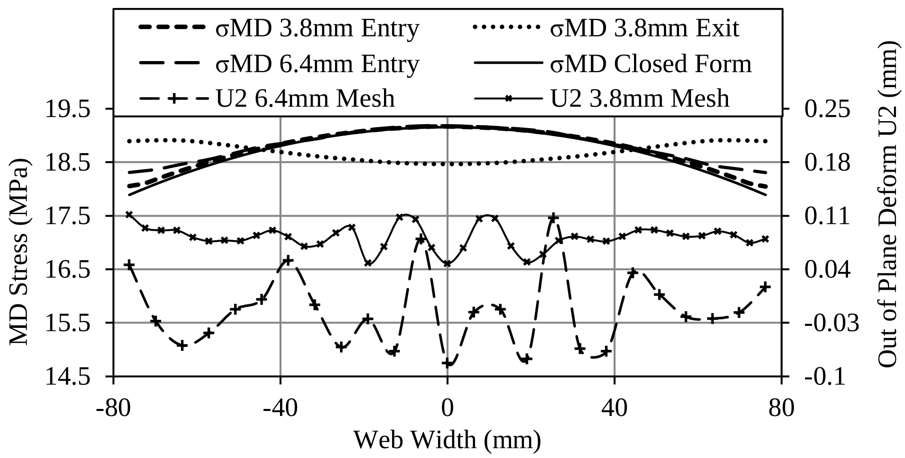


Figure 5 – MD Stress and Trough Formation due to Roller Crown
(5512 KPa (800psi) Crown, L = 254mm (10in), T = 66.7 N (20 lb))

Finite element modeling of web trough instability such as Beisel witnessed in his experiments will be the current focus. Several Abaqus simulations were created and executed in which trough formation was studied due to various levels of roller crown at different web tensions. The geometry and web line configurations for these simulations were taken from Beisel's experiments of troughs generated due to roller crown. We elected not to recreate the exact experimental procedure as documented by Beisel, as it would have been computationally expensive to model the variable crown roller shown in Figure 3 and less precise. Expression {9} was used to determine the variation in crown roller radius that resulted from the internal pressures that Beisel input to his adjustable crown roller. The variation in roller radius associated with the crown was modeled as a rigid analytical surface in Abaqus. This is preferable because the rigid analytical surface defines the contact surface of the roller continuously. The web is discretely modeled with shell elements and thus the discrete surface of the web contacts the continuous analytical surface which describes the crowned roller. This produces less numerical noise as the two surfaces contact than that of two discrete prismatic surfaces in contact.

For an internal pressure of 5512 KPa to the crown roller, Beisel saw troughs had already formed when the web tension was 44.5 N (10lb). Troughs just became visible or initiated when the web tension was adjusted to 66.7 N (15lb). No troughs were visible for the same roller crown for web tensions of 89N (20 lbs) and 111 N(25 lbs). The radius profile due to this internal pressure was modeled into roller R3 in Abaqus and the web tension levels were simulated for the web line conditions given as shown in Figure 6.

The CMD stress behavior of web due to roller crown change modestly with change in web tension. The CMD stresses at the entry of crowned roller are most negative at the center of web. Moreover, the stress reduces and become less negative as the web tension is increased. Similarly the negative CMD stress at the exit of crowned roller also reduces with increase in tension. Note the CMD stresses in the web at the exit of the roller are compressive at the centerline but become positive (tensile) as the web edges are approached where they finally decline to zero. Tensile CMD stresses are witnessed halfway through the exit span (around 76.2mm (3in) downstream of R3) in the web.

The CMD stress contours of the entering spans at each tension level are also shown in Figure 6. In all the cases a CMD compressive ‘stress pocket’ is developed at the end of entering span where the web enters the crowned roller. This behavior was witnessed for all cases simulated at various levels of crown. Beisel [8] also witnessed this concentration of compressive stress near the entry of crowned roller. A closed form expression {11} was developed for web spans with assumed constant levels of MD and CMD stresses applied at boundaries. Beisel reconciled his test results with FE analysis using an ‘effective’ span length, also shown in expression {11}. According to Beisel, the region of the web experiencing compressive CMD stress should be used to determine an effective span length, which was used in the buckling criteria {11} to determine the onset of trough formation. The average CMD compressive stress over this effective length would then be compared to the buckling criteria {11}. When the average stress is equal to or more negative than the buckling stress {11}, troughs would be visible. When the average stress is greater (less negative) than the buckling stress, troughs would not be expected.

$$\sigma_{z,cr\ troughs} = -\frac{\pi h}{L}\sqrt{\frac{\sigma_x E}{3(1-\nu^2)}} \approx -\frac{\pi h}{L_{eff}}\sqrt{\frac{\sigma_x E}{3(1-\nu^2)}} \qquad \{11\}$$

The CMD stresses (z direction in Abaqus) on the web centerline were harvested from the Abaqus results shown in Figure 6. From those stresses the effective length of the entry span in compression was estimated and the CMD stresses over that effective length were averaged. The results for varied web tensions are shown in Table 2. Per Beisel’s criteria troughs would be expected for web tensions of 44.5 and 66.7 N but would not be expected for tensions of 89.0 and 111.2 N. This agreed perfectly with Beisel’s test observations.

T (N)	L_{eff} (mm)	$\sigma_{z,cr}$ {11} (KPa)	$\sigma_{z,avg}$ (KPa)
44.5	171	-64.1	
66.7	164	-82.2	-86.9
89.0	175	-88.7	-77.9
111.2	168	-103.7	-75.2

Table 2 – Beisel’s Trough Prediction (5512 KPa (800psi) Crown, L = 254mm (10in))

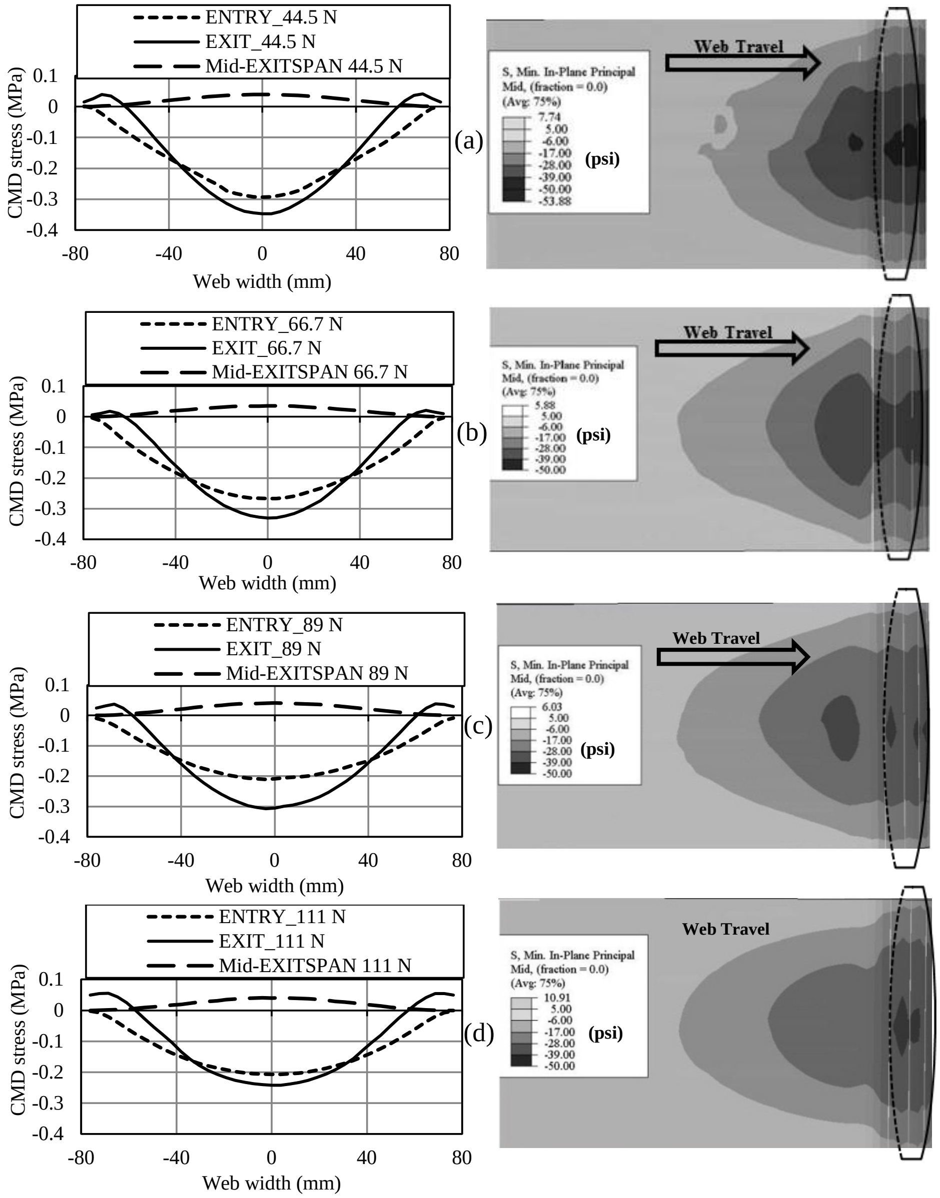


Figure 6 – Compressive CMD Stresses in the Web due to a Fixed Crown and Web Tensions of (a) 44.5 N, (b) 67 N, (c) 89 N and (d) 111 N. (5512 KPa (800psi) Crown, L = 254mm (10in))

The out-of-plane deformations of the web produced by Abaqus for the range of web tension levels were explored next. Beisel rested his conclusions of trough formation in the laboratory based upon visual observations. Figure 7 shows the out-of-plane deformation of the web just upstream of the crowned roller across the web width. Troughs are distinctively formed for web tension levels of 44.5 N (10 lbs) and 66.7 N (15 lb). The out-of-plane deformations associated with web tensions of 89.0 N (20 lb) and 111.2 N (25 lb) are comparatively much smaller. There are CMD compressive stresses present upstream of the crowned roller at all these web tension levels which was shown in Figure 6. Thus some out-plane deformation is to be expected in all cases. The results show amplification of the out-plane-deformation for the two lowest web tensions and this is taken as evidence that troughs have formed.

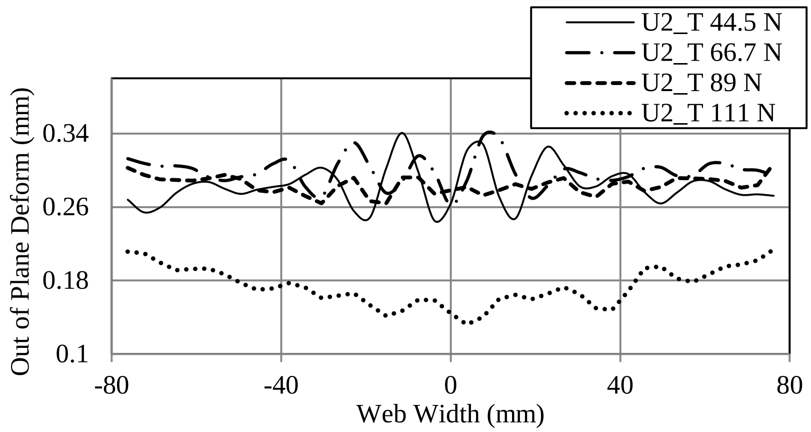


Figure 7 – Trough Formation in Web at Various Tension Levels
(5512 KPa (800psi) Crown, L = 254mm (10in))

Based upon the results shown thus far the expressions for wavelength {10} and the buckling criteria proposed by Beisel {11} agree nicely with the Abaqus Explicit results and Beisel's test observations. Thus the use of Explicit analysis for studying trough instabilities in webs has value.

Friction and Slippage of the Web on the Crown Roller Surface

Web tension is responsible for the development of a normal pressure between the web and roller surfaces. The web, while in contact with the roller can transmit shear as well as normal forces across the contact interface. The relationship between these forces is a cause of the stresses at the interface dictated by friction. Slippage in the Abaqus Explicit models described herein was governed by the Coulomb friction model. According to this model, two contacting surfaces can carry shear stresses up to a certain magnitude across their interface before they start sliding relative to one another.

$$\tau_{crit} = \pm \mu p \qquad \{12\}$$

where μ is the friction coefficient between the web and roller surfaces and p is the contact pressure. A plot of the equivalent shear contact stress against the fraction of contact pressure is shown in Figure 8 where in the contact shear stress is obtained as:

$$\bar{\tau} = \sqrt{\tau_1^2 + \tau_2^2} \qquad \{13\}$$

Here τ_1 and τ_2 are two components of shear stress defined in Abaqus as Cshear1 and CShear2. The values of contact shear stress were extracted for the nodes at the web edge. The abscissa axis begins approximately 20mm (0.75in) before entry of the web to the crown roller, continues on as the web wraps the roller and then proceeds almost 20mm beyond exit of the web from the roller.

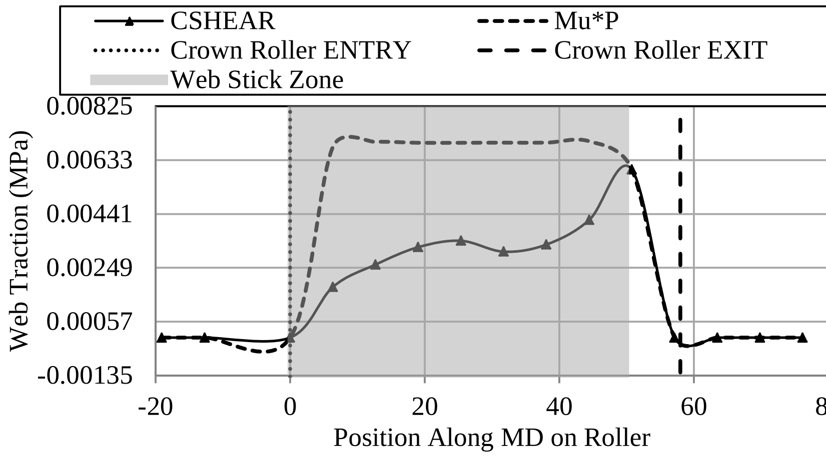


Figure 8 – Stick and Slip of the Web Edge on the Surface of a Crowned Roller (5512 KPa (800psi) Crown, L = 508 mm (20in))

In Figure 8, the shaded area of the web wrap on the crowned roller is in stick conditions, it is not slipping. This is known because is the contact shear stress ($\bar{\tau}$) is less than the critical shear stress (μp). Near the end of the web wrap the contact shear stress and the critical shear stress merge which indicates that slippage is occurring as the web approaches the exit of the crown roller. The behavior of the contact shear stress can be linked to the MD and CMD stress profiles across the web width shown in Figures 5 and 6. The web MD stress profile should be uniform across the web width in the absence of disturbances. The crowned roller is a disturbance and causes the MD stresses to have a crowned profile as shown in Figure 5 and by expression {6}. The web will return to a uniform MD stress profile at some point downstream of the crowned roller. For this to happen web that is travelling faster at the CMD center on the crown roller has to slow near the exit of the roller while the web at the edge must speed up. This affects the MD exit stress profile as seen in Figure 5. As the web at the CMD center slows while slipping on the crown roller surface the MD stress decreases. Due to the Poisson effect the web at the CMD center near the exit will attempt to expand but is restrained by the critical shear contact stresses. As a result the CMD stress at the web center becomes even more compressive in the slip zone. This slippage was responsible for the increases in CMD compressive stress witnessed between the entry and exit of the crown roller in Figure 6.

This analysis of slippage is informative and provides insight regarding the MD and CMD stress behaviors witnessed in the web on the roller. Even more important is that the data in Figure 8 demonstrate that essentially no slippage occurs between the web and the concave roller until the web nears the roller exit. This provides encouragement that static analyses can be used to determine the steady state behavior of a web entering a crown roller. These analysis results can then be used predict the onset of troughs.

Static Analysis of a Web on a Crowned Roller

Explicit finite element analysis has been used to study the details of web behavior in contact with crowned rollers. Research on the lateral behavior of webs approaching contoured and misaligned rollers [9, 10, 12, 13] have confirmed that under steady state conditions the web enters a downstream roller normally. The lateral deformations for a typical simulation are shown in Figure 9. Here again evidence of normal entry is shown. The edge deformation due to Poisson contraction is 0.1163 mm at this web tension which is essentially the deformation witnessed in the upstream portion of the span. As the web approaches the crowned roller further contraction of the web occurs before normal entry is established.

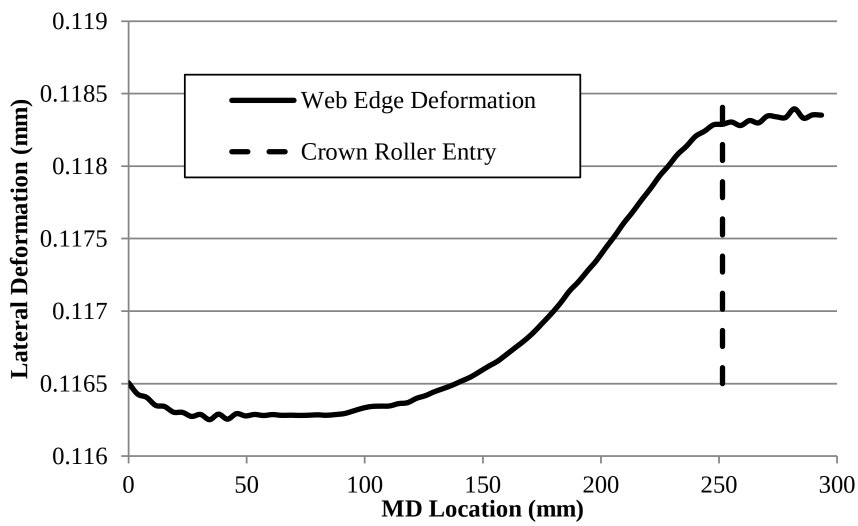


Figure 9 – Typical Web Edge Lateral Deformation.(5512 KPa (800psi) Crown, T = 89 N (20 lb) and L = 254mm (10in))

The explicit analysis results were useful for studying the kinetic and kinematic boundary conditions for a web approaching a crowned roller. Explicit analysis utilizes considerable computational resources in order to produce good results and simulation times can be of the order of a few days to obtain the results shown here. For those who require results in minutes rather than days a static solution was developed for the entry span of a web to a crowned roller. A finite element model of the entry span is developed with a span (L) and width (W) with web properties such as those presented in Table 1. Plane stress two dimensional finite elements were used. From the explicit analysis the parabolic form of the MD stress given by expression {6} was shown to be accurate in Figure 5. It is also known that the web will enter the crown roller normally to the axis of rotation complying with the normal entry law.

In Figure 10 the rectangular coupon is loaded with the parabolic edge traction (σ_{MD}) per expression {6} where the web enters the crowned roller. The web deformations that would result from this edge traction alone will not achieve normal entry of the web to the crowned roller.

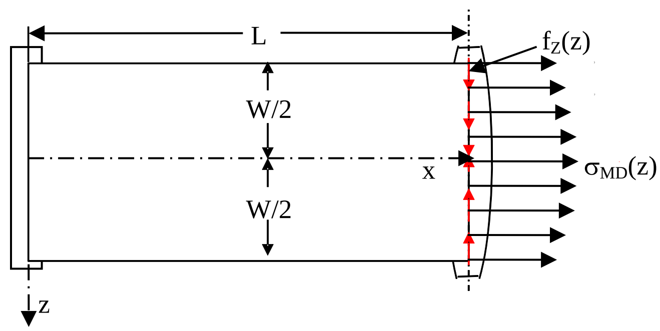


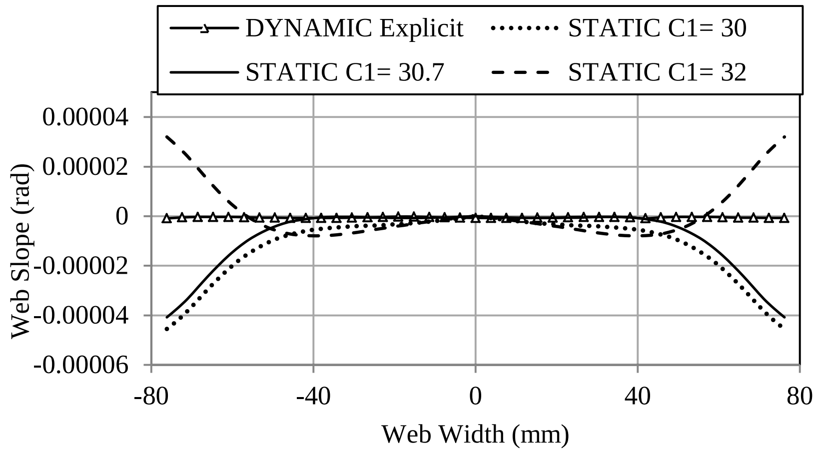
Figure 10 – Static Analysis of a Web approaching a Crowned Roller in Steady State

To achieve normal entry the last row of elements is also loaded with the shear tractions (f_z). Investigations of the shear stresses from the explicit analyses provided evidence that the shear traction would be well represented by a 5th order polynomial:

$$f_z(z) = C_1\left(z - \frac{8}{W^4}z^5 - \frac{2}{W^2}z^3\right) \qquad \{14\}$$

where the constant C_1 depends upon web dimensions and material properties, the roller crown level and the web tension. The value of C_1 is varied until the web attains normal entry over as much of the width as possible. The slope across the web width is shown to be very sensitive to the value of C_1 for the test case shown in Figure 11 (a). Deformations in the x direction are restrained on the left edge (x=0) and deformations in the z direction are restrained on the web centerline (z=0).

The value of C_1 that results in the least web slope was 30.7 for the test case shown. Although the CMD stresses do not match the results from Abaqus Explicit perfectly at this value of C_1 the error may be found to be acceptable when comparison is made to test data.



(a)

(b)

Figure 11 – Iteration of C_1 to achieve Normal Entry in (a) and the Resulting CMD Stresses (b). (5512 KPa (800psi) Crown, T = 89 N (20 lb) and L = 254mm (10in))

Prediction of Troughs Using Static Crowned Roller Model

The buckling criterion for the onset of web troughs was developed earlier. Whenever the average CMD stress in the zone of compressive CMD stress on the web centerline upstream of the crown roller becomes more compressive than the value $\sigma_{z,cr}$ given in expression {11} troughs should be imminent. Now the static model combined with the trough buckling criteria can be used to predict the level of crown in a downstream roller that will produce troughs in a web span of given length, width, thickness, material properties, nominal roller radius (a_0) and web tension. The crown is adjusted through the parameter a_1 until troughs form per the buckling criterion. The finite element code, the iteration of the shear traction forces (f_z) and the iteration of the crown level are combined in one software tool.

Comparison of Trough Predictions with Test Results

Beisel [8] performed tests with the adjustable crowned roller (Figure 2) where he set the web tension and then increased the roller crown until troughs became visible. Abaqus/Explicit simulations and static finite element analyses were conducted at these web tensions and span lengths to also determine what levels of crown would induce troughs. The results are shown for a test case shown in Figure 12. All the results should be interpreted as a lower bound for the crown level that will produce troughs. Thus any level of crown beyond these lower bounds would also produce troughs and if large enough could also produce wrinkles. The results all agree quite nicely.

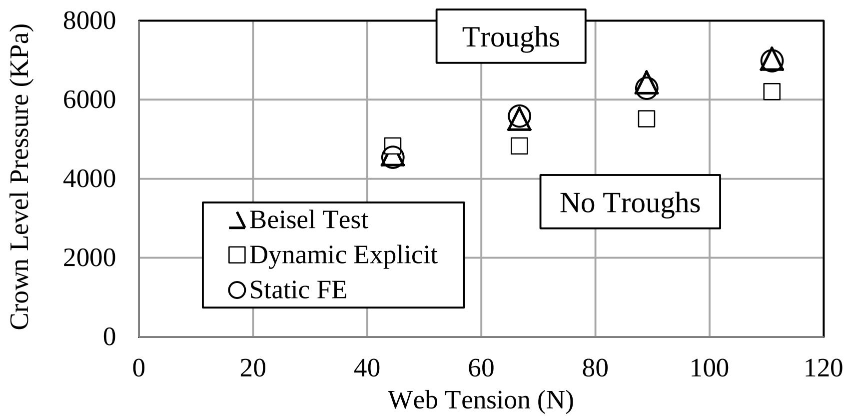


Figure 12 – A Comparison of Trough Test Data with Explicit and Static Predictions, L = 254mm (10in)

The radius variation due to the internal pressurization of the adjustable crown roller can be investigated by substituting the pressure levels in Figure 12 back into expression {9}. In Figure 12 internal pressures in the adjustable roller that produced troughs ranged from 4500 to 7000 KPa. Using expression {9} the difference in the roller radius between the center of the roller (z= 0 mm) and the edge of the roller (z = 102 mm) for this pressure range would be 0.0145 mm (.00057 in) to 0.0232 mm (0.00091 in). Thus very little roller crown is required to induce troughs in this polyester web.

FORMATION OF WRINKLES DUE TO ROLLER CROWN

At a given web tension as crown is increased the compressive CMD stress upstream of the crowned roller will become more negative until troughs occur. The troughs will limit the CMD stresses, the buckling will prevent the CMD stresses from becoming more negative in the upstream span. As the crown becomes yet larger the amplitude of the out-of-plane deformation of the troughs will increase and will result in large negative CMD stresses in the web transiting the crowned roller. This gives rise to the next level of instability in web-lines: *Wrinkles*. Much larger compressive stresses are required to buckle the web (in the form of a shell) on a roller compared to a web span (as a flat plate). The web on a roller behaves similar to a sector of a cylindrical shell that is internally pressurized. The internal pressure is due to web tension. Timoshenko [11], developed the axial buckling stress and the buckled wavelength of a cylindrical shell:

$$\sigma_{z,cr\ wrinkles} = -\frac{Et}{r\sqrt{3(1-\nu^2)}} \text{ and } \lambda_{wrinkles} \approx 3.44\sqrt{rh} \qquad \{15\}$$

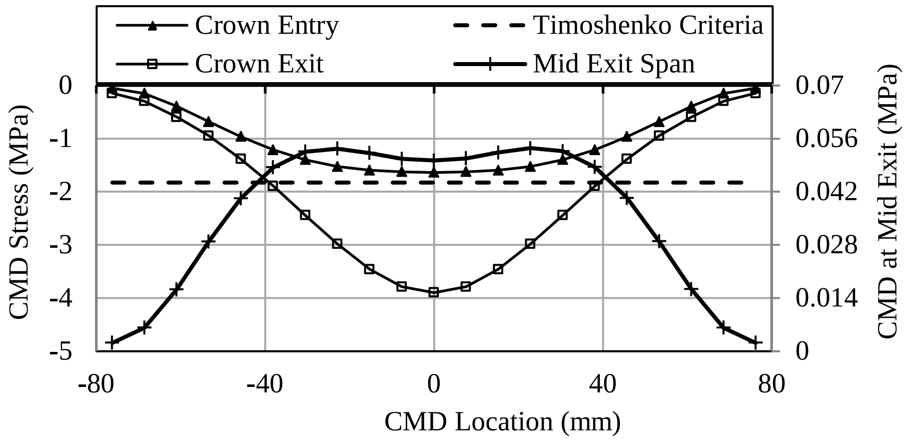
Whenever the CMD stress in the web entering the crowned roller becomes more compressive than that given by expression {15}, wrinkles will form on the crowned roller. This wrinkle failure criterion has been proven valid for wrinkling due to roller misalignment [12] and due to roller taper [13]. This criterion will be used herein to determine when wrinkles will occur on crown rollers. Beisel [8] collected experimental data for wrinkles generated due to a roller crown. Beisel knew that wrinkling was coupled to troughs and that he could eliminate troughs by increasing web tension per expression {11}. In these tests he used rollers with fixed crown level; radius variation was achieved by machining on a lathe. The wrinkling tests began at high tension where no troughs were present. The web tension was then slowly decreased. Troughs would appear. Further decreases in web tension would cause the trough amplitude to increase. Yet further decreases would finally result in a wrinkle appearing on the crowned roller. These experiments were conducted several times and the web tensions that induced wrinkles were recorded.

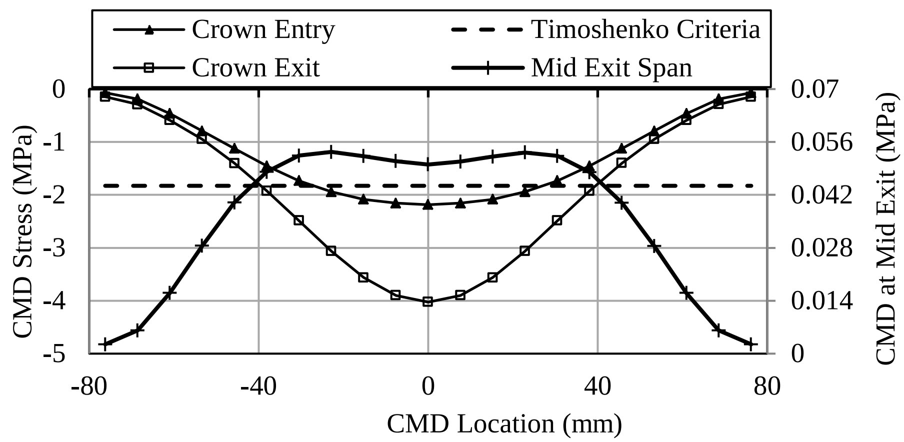
Explicit Modeling of Web Wrinkling on a Crowned Roller

The model setup for studying wrinkling due to roller crown is essentially the same as described in previous section and shown in figure 2. The crowned roller is modeled in the third position (R3) of the typical four rollers with a nominal radius of 38.1 mm (1.5in). The roller used has a diameter variation of 0.15mm (0.006in) over a 203 mm (8in) roller width. The crown roller radius at intersection of web-roller centerline (a_0) is 38.17mm (1.503in) and (a_1) is 0.00476 mm^{-1} (0.000188in^{-1}). Other web and line parameters are the same as shown in Table 1.

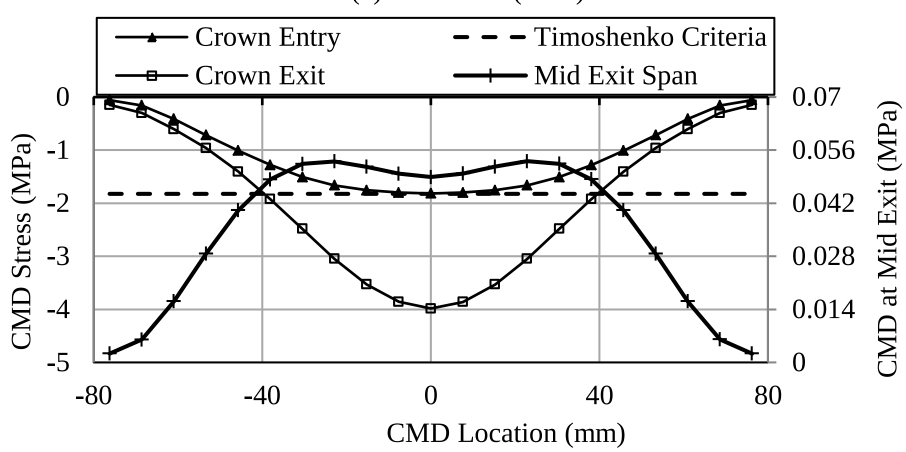
The mesh density for studying wrinkle behavior is a concern in the presented study since it greatly affects the computational cost. For the web being modeled expression {15} predicts a buckled wavelength of 3.24 mm (0.128”). Abaqus Explicit could be used to model the wrinkling directly if a mesh with elements on the order of 0.25 mm could be used. This would result in unacceptable simulation times. Instead an approach adopted previously [12,13] will be employed herein. The mesh size will be dictated by obtaining convergence of the CMD stresses. The converged CMD stresses at the entry of the web to the crown roller will be compared to the axial buckling stress {15}. When those CMD stresses become more negative than the buckling stress, web wrinkling has occurred. The critical buckling stress for the polyester web and the nominal roller radius is -1.82 MPa (-265psi).

The modeling effort began with a case with a 500 mm (20”) web span. This was the shortest web span tested. According to Beisel’s test results for this case, a wrinkle should not be generated at 89 N (20lb) of web tension. On reduction of this tension, a wrinkle will be generated. Test results ranged with wrinkles forming as high as 86.7 N and as low as 81.8 N. In the modeling tension was kept constant during a given simulation.





(b) T = 80.1 N (18 lb)



(c) T = 85.4 N (19.2 lb)

Figure 13 – Abaqus CMD Stress Levels at the Entry and Exit of a Fixed Crown Roller and Mid-way through the Exit Span as a Function of Web Tension (L = 500 mm (10in), a_0 = 38.17mm and a_1 = 0.00476 mm^{-1}).

The first case modeled was case where the web tension was set to 89 N (20 lb). The CMD stresses across the web width at entry and exit of the crowned roller are shown in Figure 13(a). Also shown are the CMD stresses induced in web at half-way through the exit span. Note that the CMD compressive stress in the web is smaller than that given by the wrinkle buckling criteria {15}. Thus wrinkling is not expected. Note that the CMD compressive stress does surpass the criteria as the web exits the crowned roller. This will not result in a wrinkle. The web material that forms a wrinkle must come from some portion of the web width. This occurs easily at the entry to the crowned roller because out-of-plane deformation associated with troughs can become the material which forms the wrinkle. At the exit, friction between the web and roller act to restrain the web from becoming narrower and forming a wrinkle. Thus compressive stresses much more negative than that given by expression {15} can be supported by the web on the roller without wrinkling. This is why the failure criterion is written with regard to the CMD stress in the web *entering* the crown roller.

The next simulation was at a higher web tension of 80.1 N (18 lb) and the results are shown in Figure 13(b). Now the CMD stress from Abaqus at the entry to the crown roller was more negative that given by expression {15} and wrinkles would be expected.

The web tension at which wrinkles will appear for this roller/web combination has been bracketed. The final simulation was run at a web tension of 85.4 N (19.2 lb) and the results are shown in Figure 13(c). Now the CMD compressive stress at the entry of the crown roller matches that given by expression {15}. Thus for this combination of roller crown and web properties the 85.4 N web tension separates wrinkled from non-wrinkled behavior. Lesser web tensions will result in wrinkles too and greater tensions will prevent wrinkles from occurring.

Results for other span lengths are shown in Figure 14 for the same crown roller. For these cases only results for the web tensions which produced CMD compressive stresses at the entry of the crown roller equal to that given by expression {15} are shown.

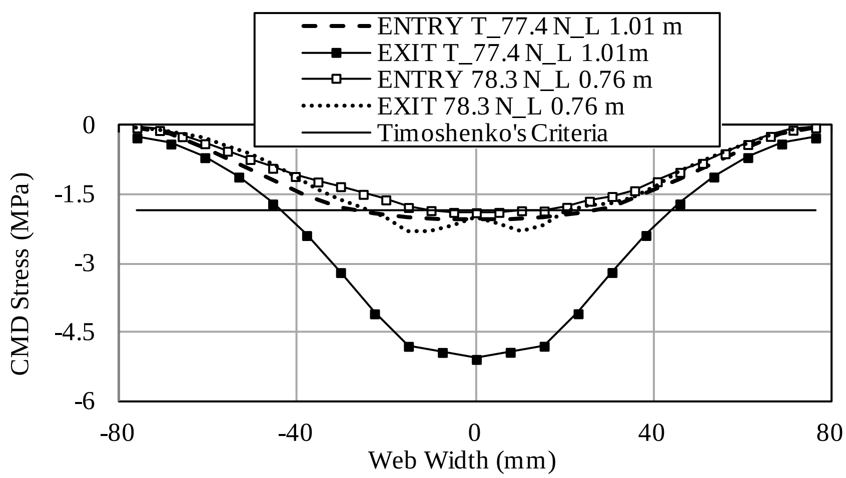


Figure 14 – Simulation Results: Web Tension required to induce Wrinkles for other Span Lengths (a_0 = 38.17mm and a_1 = 0.00476 mm^{-1}).

Prediction of Wrinkles using a Static Crowned Roller Model

The static code developed for studying troughs due to roller crown cannot be used for studying wrinkling. The large CMD compressive stresses which develop in the web at the entry to the crowned roller are due to troughs. The amplitudes of the troughs and the CMD compressive stresses in the web at the entry to the roller increase with further increase of crown level. Thus any finite element code used to predict wrinkles must be capable of nonlinear solutions which can model the out-of-plane deformation due to the troughs and allow the large CMD stresses in the web at the entry to the roller to develop. Beisel [8] was successful in developing a model for wrinkling on crowned rollers using the commercial finite element code COSMOS®. Beisel used the wrinkle membrane material behavior in shell elements (Shell4) to model the entry span and the decrease in web width due to troughs in the entry span. Beisel had previously employed these elements for modeling web wrinkling as result of misaligned and tapered rollers and he verified his model results by comparison to laboratory tests. The wrinkle membrane behavior is incorporated into the element behavior by allowing each element to (1) be taut in the presence of two tensile in-plane principal stresses, (2) be wrinkled in the presence of one tensile principal stress and one null principal stress or (3) be slack with null stresses. The wrinkled behavior requires iteration to convergence as the constitutive equations that couples stress to strain are state dependent on the Cartesian and principal strains [14].

This is a static nonlinear solution. The model constructed for studying wrinkling due to roller crown is shown in Figure 15. The model contains five panels of elements. Three of the five panels model the web on the upstream, crown and downstream rollers. Those panels are separated by two other panels of elements that model the entry and exit spans of web to the crown roller. The elements modeling the web on the rollers and the exiting span are modeled using the ordinary shell elements which can react compressive stress.

The entering web span is modeled with shell elements that have the wrinkle membrane material behavior. These elements are not allowed to react compressive principal stresses and are used to model the z direction width reduction associated with troughs. The model is constrained in y direction at both ends and center line. The web line tension (T) is applied as an edge traction (σ_x) at the ends of the model as shown. The dashed horizontal lines on the panels of elements which model the upstream and downstream rollers denote rows of nodes whose z direction deformations have been coupled. This is done to allow Poisson contraction due to web tension but to also enforce normal entry of the web to these rollers.

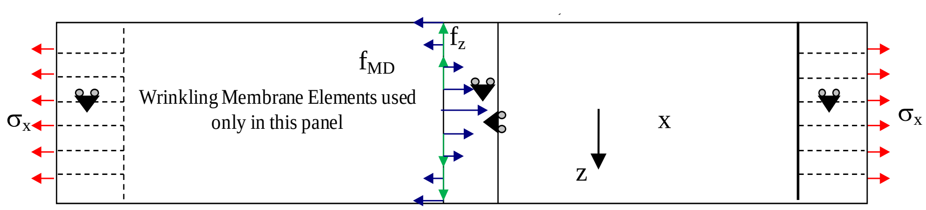


Figure 15 – Static model for Studying Web Wrinkling on Concave Rollers

A given crown level is assumed in combination with span lengths, web width and thickness, material properties, a nominal roller radius and a starting value of web tension. The parabolic loading (in x direction) resulting from the roller geometry and web properties calculated using expression {6} is applied to the middle panel as shown in Figure 15. The MD forces (f_{MD}) applied to the web should create same MD stress profile given by {6}. The shear forces (f_z) calculated from the 5^{th} order polynomial {14} are introduced and the value of C_1 is varied until normal entry is achieved at the entry to the crown roller. Once accomplished the CMD stresses in the web at the entry to the crown roller are compared with the buckling criteria {15}. If the CMD compressive stress in the web is larger than that given by expression {15} the web tension is increased and the computations resume. If the CMD compressive stress in the web is smaller than that given by {15} the web tension is decreased and the computations resume. This procedure continues until the CMD compressive stresses in the web at the entry to the crown roller agree sufficiently with that given by expression {15}. Then the web tension which separates troughed behavior at the entry to the crown roller from wrinkles on the roller is known for a roller of given crown level and nominal radius engaged with a web of given span length and material properties.

<u>Comparison of Wrinkle Predictions with Test Results</u>

A comparison is made with Beisel's test data [8], the Abaqus Explicit results from Figures 13 and 14 and results from the static nonlinear modeling. In the tests a roller with fixed crown was used, (a_0) is 38.17mm (1.503in) and (a_1) is 0.00476 mm^{-1}(0.000188in^{-1}). Web and line parameters are the same as shown in Table 1. Three entry span lengths were tested and modeled. The results are shown in Figure 16. The test data points are the averages of many tests and the error bars show the range of web tension in which wrinkles appeared. The results of the tests and the modeling methods agree well. Web behaviors for a given entry span length are also described. At high levels of tension there will be no troughs but nor will the web be planar as the web is entering a crowned roller. As web tension is decreased troughs will appear. Further decreases will finally result in wrinkles. There appears to be a benefit to shorter entry span lengths.

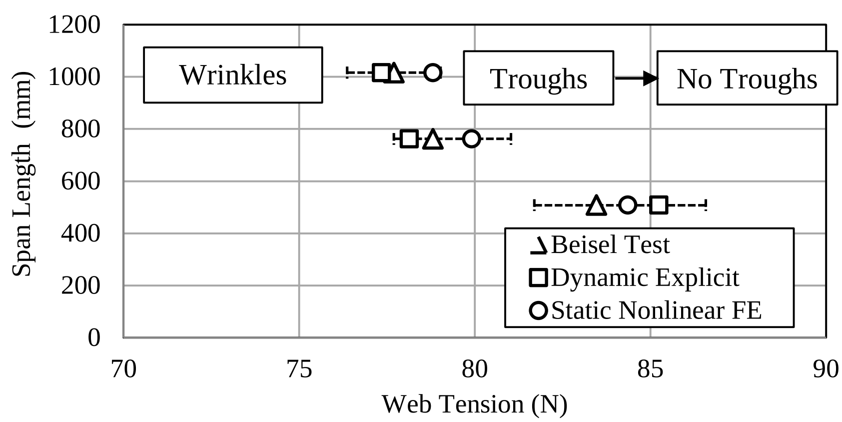


Figure 16 – A Comparison of Wrinkle Test Data with Explicit and Static Predictions (a_0 = 38.17mm and a_1 = 0.00476 mm^{-1})

CONCLUSIONS

This study has shown the value of exploring kinetic and kinematic boundary conditions between a web and rollers using explicit finite element codes. For the case of a crown roller interacting with a web it was found that stick behavior existed from the entry of the web to the roller over much of the contact area (Figure 8). This stick behavior ensured the web and roller surface velocities matched and thus ensured the validity of expression {6} (Figure 5). The explicit analyses also demonstrated that the web entered the crown roller normally (Figure 9). Finally the explicit analyses provided the form of the f_z steering loads {14} that were required to force the web to enter the roller normally. These findings allowed the development of simpler less computationally expensive static models to determine when a crown roller will induce troughs or wrinkles into a web.

The methods of analysis developed herein are applicable to either continuous webs or web belts transiting crowned rollers. By tests and analysis it has been proven that very little crown is needed to induce web troughs and wrinkles. The methods of analysis can be used to set bounds for the level of crown that is acceptable for a particular web being transported at a specified tension.

Troughs and wrinkles due to roller crown can be diagnosed in a web line. Troughs of the appearance shown in Figure 3 are one clue. The troughs are nearly aligned with the machine direction. Wrinkles will form near the CMD center of the roller and not travel laterally. Roller misalignment and taper disturbances both cause troughs that are inclined with respect to the machine direction and wrinkles will travel laterally on the roller. It is possible that roller deflection will induce troughs and wrinkles similar to that induced by the crowned roller but the failure sequence differs. If roller deflection is a problem increasing web tension should make troughs and wrinkles more severe. If roller crown is the problem increasing web tension *can* cause wrinkles to become troughs. Further increases in tension *can* make the troughs disappear. The word *can* is emphasized here because there may be other limits that prevent the web tension from being increased.

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