CONTROL DESIGN FOR LONGITUDINAL WEB DYNAMICS: BENEFITS AND DRAWBACKS OF ROBUST CONTROL APPROACHES

By

Dominique Knittel, Jonathan Frechard, and Yannick Martz
University of Strasbourg
FRANCE

ABSTRACT

Web tension and speed are two key variables to be monitored and controlled in order to achieve the expected final product quality. One of the main objectives in web handling plants is to reach an expected web speed while maintaining the web tension within an acceptable range around the tension reference in the entire processing line. In the recent years, many works have focused on the topic of web tension control and have proposed various ways to enhance the performance: H_{∞}, optimal state feedback, neural network, etc. But the common practice in industrial web transport systems remains the use of decentralized PI-type controllers.

An improved design methodology of these PI controllers with fixed -order and -structure synthesis approaches has been made. Nevertheless, despite high performances for a nominal working point, it has been noticed that the closed-loop system performances depend on the web elasticity since the dynamic behavior is strongly affected by the Young's modulus. Consequently the emphasis of this contribution is on the automatic tuning of PID (or PI) controllers for web processing plants that guaranty good performances of the closed-loop system.

NOMENCLATURE

V_i	Web linear speed
T_i	Web tension in the span *i*
J_i	Inertia of the roller *i*
K_i	Torque constant of the motor *i*
R_i	Radius of the roller *i*
f_d	Web/roller dynamic friction coefficient
	Strain of the web span *i*
L_i	Length of the web span *i*
E	Young Modulus

S	Web cross-section
u_i	Control signal of the roller *i*

INTRODUCTION

Roll-to-Roll systems handling web material such as papers, polymers, textiles or metals are very common in industry. Printing, coating and drying are examples of operations that can be performed in different sections of a web line. Web handling systems are recently used to produce new technologies such as thin solar panels, printed electronics, etc. Web tension and speed are the two key variables that need to be monitored and controlled in order to achieve the expected final product quality. One of the main objectives in a web handling machinery is to reach an expected web speed while maintaining the web tension within an acceptable range around the tension reference in the entire processing line [1]. In order to set up the speed and tension controllers, requirements have to be fixed. These requirements are the bandwidth of the closed-loop system and overshoot.

The nonlinear model of a web transport system is built from the equations describing web tension behavior between two consecutive rollers and the velocity of each roller. The studied web handling plant, composed of an unwinder, intermediate motor driven rollers, several idle rollers and a rewinder, is divided into several subsections that are controlled independently.

The objective is to synthesize a “robust” tension PI controller for each subsection (except for the master-roll subsection) [4, 5, 16, 21]. The H_∞ problem can be expressed as follows: find a stabilizing controller that minimizes the H_∞ -norm of the transfer function between a set of exogenous inputs (typically the references) and some performance outputs. It turns out that the problem is non-convex for the design of PI controllers. The fitness function (which has to be minimized) is then based on the H_∞ -norm with the constraint that the closed-loop system has to be stable.

The benefits and drawbacks of the automatic controller tuning are discussed.

MODELING

The non-linear model of a web transport system is established from the equations characterizing the speed of each roller and the web tension behavior of each web span (web between two consecutive rollers) [2, 6, 7]

Web Speed Determination

The web linear velocity V_i of a roller *i* is equal to the linear roller (see Fig. 1), which depends on the upstream web tension T_{i-1} and the downstream web tension T_i is given by:

$$J_i \frac{dV_i}{dt} = (T_i - T_{i-1}){R_i}^2 + K_i R_i u_i - f_d V_i \tag{1}$$

This equation assumes that no slippage occurs: the web velocity is equal to the linear roller velocity. In addition, a static friction can be added.

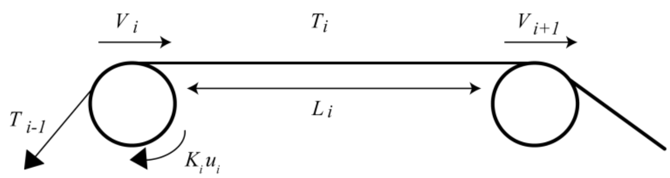

Figure 1 – Generic analyzed web span.

Web Tension Determination

The strain ε_i of the web span *i*, which depends on the upstream web strain ε_{i-1} and the velocities of the two consecutive rollers, is given by the differential equation [2]:

$$\frac{d}{dt}\left(\frac{L_i}{1+\epsilon_i}\right) = \frac{V_{i+1}}{1+\epsilon_i} + \frac{V_i}{1+\epsilon_{i-1}} \qquad \{2\}$$

For an elastic web, the web tension T is obtained using Hooke's law:

$$T = ES\epsilon_i \qquad \{3\}$$

The web tension is determined using the non-linear differential Eq. {2}. This equation can be linearized around working points T_0 and V_0. Considering $T_i = T_0 + t_i$, $V_i = V_0 + v_i$, $T_{i-1} = T_0 + t_{i-1}$, and $V_{i+1} = V_0 + v_{i+1}$ the linear equation becomes [2]:

$$L_i \frac{dt_i}{dt} = V_0(t_{i-1} - t_i) + (v_{i+1} - v_i)(ES + T_0) \qquad \{4\}$$

Linear Model

The relations shown in Eqs. {1} and {4} permit to build the state-space representation of the studied roll-to-toll system:

$$\begin{cases} \dot{x}(t) = A\,x(t) + B\,u(t) \\ y(t) = C\,x(t) \end{cases} \qquad \{5\}$$

where *x* is the state vector, *u* is the control vector and *y* is the output vector. *A* is the state matrix, *B* the input matrix and *C* the output matrix.

The system scheme is shown in Fig. 2, the small circles (V_2, V_4, V_6, …) correspond to the idle rollers equipped with load cells and the large circles (V_1, V_3, V_5, …) correspond to the motor driven rollers. The calculation of each roller speed and tension in each span is needed for the dynamical model. Therefore, the state vector is composed of the velocity of each motor driven roller, the speed of each idle roller and the web tension in each web span:

$$x = [V_1\, T_{s1}\, V_2\, T_{s2}\, V_3\, T_{s3}\, \dots\, T_{s12}\, V_{13}]^T \qquad \{6\}$$

The system has 8 inputs: the input web tension T_{in} and the seven motor control signals.

{7}

The system has 13 outputs: the motor driven roller rotational speeds and the web tension located at each idle roller.

$$y = [\omega_1\ \omega_3\ \omega_5\ \omega_7\ \ \omega_9\ \omega_{11}\ \omega_{13} T_1\ T_3\ T_5\ T_7\ \ T_9\ T_{11}]^T \qquad \{8\}$$

where:

$$T_1 = \frac{T_{s1} + T_{s2}}{2} \qquad T_3 = \frac{T_{s3} + T_{s4}}{2} \qquad T_5 = \frac{T_{s5} + T_{s6}}{2}$$
$$\{9\}$$
$$T_7 = \frac{T_{s7} + T_{s8}}{2} \qquad T_9 = \frac{T_{s9} + T_{s10}}{2} \qquad T_{11} = \frac{T_{s11} + T_{s12}}{2}$$

The built linear model permits to synthesize robust controllers. The robustness needed is regarding elasticity variations. The elasticity can vary substantially during a same unwind-rewind process. The influence of the web elasticity on a roll-to-roll system is studied and shown in Fig. 3. This figure represents the maximum singular values of the studied generic plant for different web Young's modulus [21]. One can see that the resonance peaks magnitude and frequency depend on the web elasticity value. The resonances move to lower frequencies for a web elasticity decrease.

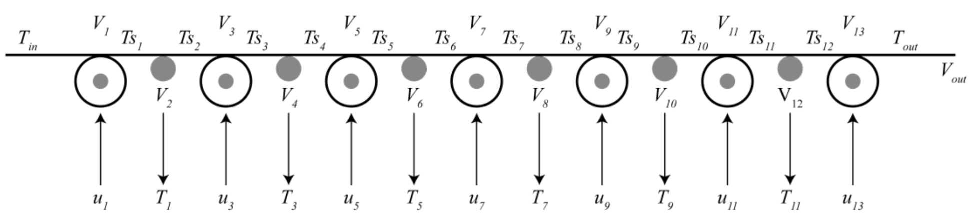

Figure 2 – Global generic studied system.

CONTROL STRATEGY

Motor Velocity Control

The velocity control loop uses an IP controller. The main advantage of the IP controller is that it does not introduce a zero in the closed loop transfer function.

The speed controller structure is depicted in Fig. 4. In order to have a rotational velocity reference, the linear velocity reference is divided by the roller radius. Moreover, in order to simplify the closed loop and to remove the sensibility to inertia variation, the controller output is multiplied by the roller inertia. The system open loop relation, when the loading effects of the web and frictions are compensated by a feedforward (not represented in Fig. 5), becomes:

$$\frac{\Omega_i}{u_i} = \frac{K_i}{J_i s} \qquad \{10\}$$

The closed loop of the torque has been considered very fast and approximated by the gain K_i. The transfer function of closed loop with IP controller gives:

$$\frac{\Omega_i(s)}{\Omega_{ref}(s)} = \frac{1}{1 + \frac{s}{a} + \frac{s^2}{abK_i}} \qquad \{11\}$$

The IP controller parameters a and b are calculated as follows:

$$a = \frac{\omega_V}{2\zeta_V} \qquad b = \frac{{\omega_V}^2}{aK_i} \qquad \{12\}$$

where ω_v is the desired bandwidth of the speed closed loop, ζ_v is the desired damping factor and K_i is the motor torque constant.

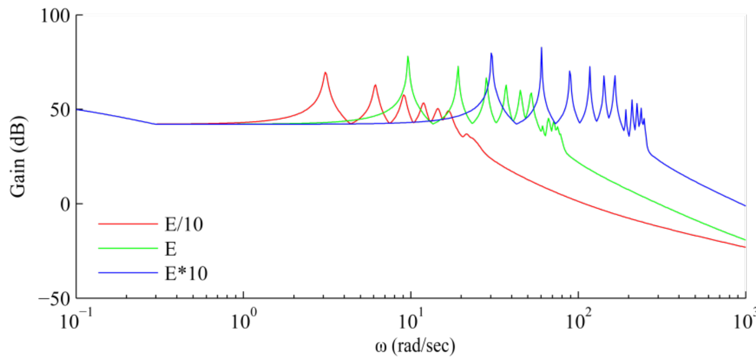

Figure 3 – Maximum singular values of the open-loop system for different web elasticities

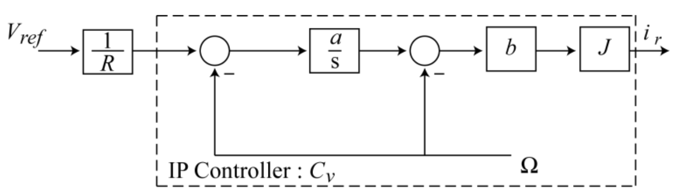

Figure 4 – Velocity IP controller.

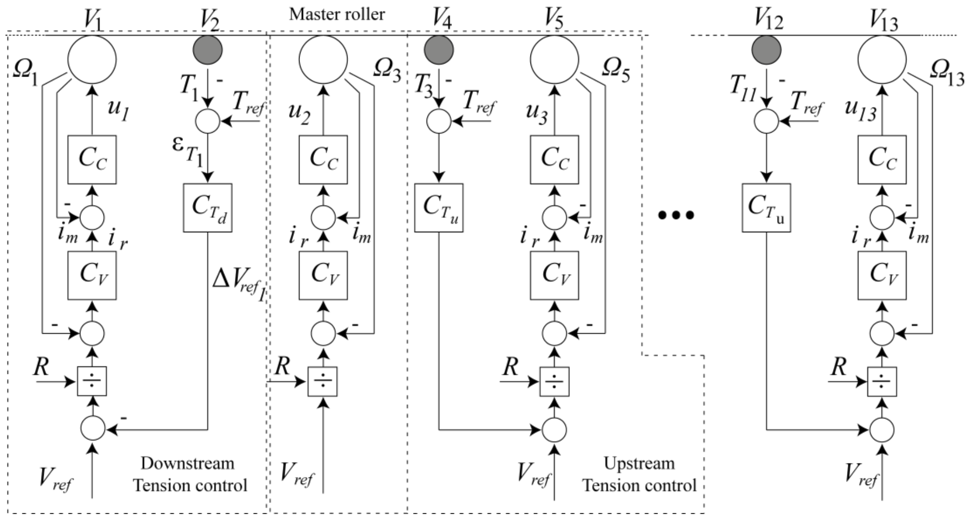

Figure 5 – Generic industrial decentralized control structure.

Web Tension Control

Web tension control has been studied for several years [1-7, 9-16, 21]. In this study, we use an H_∞ approach in order to synthesize each tension controller [5, 12, 16, 21]. In many domains the H_∞ synthesis approach is used [17], it consists in finding a stabilizing controller that minimizes the transfer function H_∞ norm between a set of exogenous inputs r and a set of performance outputs z.

$$\|T_{r\to z}\|_\infty < \gamma \qquad \{13\}$$

H_∞ Fixed Order and Fixed Structure Synthesis

The high order of the obtained controller is the major drawback of the standard H_∞ approach. In fact, the order of the controller is equal to the sum of the weighting functions order and the system order [8]. The controller order cannot always be decreased while performances and stability are assured by using the current model reduction approach. Fixed -structure and -order controllers design algorithms have been developed : they are highly relevant for industrial applications. The mathematical problem seems to be difficult because fixed-order controller synthesis can be formulated as a nonsmooth affine problem in the nonconvex cone of stable matrices. Relevant synthesis tools like *HIFOO* in 2005 [18] and more recently *hinfstruct* [19] have been developed thanks to recent progress in nonsmooth problem solving. In our study, the *hinfstruct* tool is used in order to synthesize the web tension controllers.

As illustrated in Fig. 6, the web tension controllers are synthesized with the use of S/KS/T weighting scheme and model matching. The reference model M_0 corresponds to the desired closed-loop system behavior: a second order transfer function is chosen with a cross-over pulsation ω_T and a damping factor ζ_T equals to 1:

$$M_0 = \frac{1}{1 + 2\zeta_T \frac{s}{\omega_T} + \left(\frac{s}{\omega_T}\right)^2} \qquad \{14\}$$

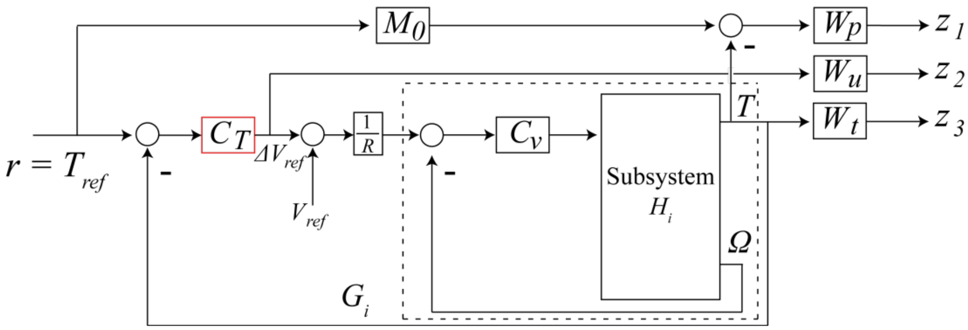

Figure 6 – H_∞ S/KS/T with model matching synthesis scheme.

where s is the Laplace variable. The weighting functions W_p , W_u , and W_t appear in the transfer matrix of the closed loop function $T_{r \to z}$. r is composed of the tension references and z is composed of z_1 , z_2, and z_3 , see Fig. 6.

$$T_{r \to z} = \begin{bmatrix} W_p(M_0 - T_{CL}) \\ W_u C_T S_{CL} \\ W_t T_{CL} \end{bmatrix} \qquad \{15\}$$

where S_{CL} is the sensitivity function defined by:

$$S_{CL} = (1 + G_i G_T)^{-1} \qquad \{16\}$$

G_i contains the subsystem including the speed control loop (see Fig. 7). The web tension controller is C_T and T_{CL} is the complementary sensitivity function defined by:

$$T_{CL} = 1 - S_{CL} \qquad \{17\}$$

$$W_p = \frac{\frac{s}{M} + \omega_B}{s + \omega_B \varepsilon_0} \qquad \{18\}$$

where M is the maximum peak magnitude of S_{CL}. ω_B is the required cross-over frequency ($\omega_B > \omega_T$) and ε_0 is the accepted steady-state error.

The tension controller C_T (see Fig. 7) is a PI controller:

$$\frac{\Delta V_{ref_i}}{\varepsilon_{T_i}} = K_{p_i} \frac{1 + \tau_i}{s} \qquad \{19\}$$

where ΔV_{refi} is the web tension controller output that adjusts the speed reference and ε_{Ti} is the web tension error signal, i is the roller number. K_{pi} and τ_i are the controller parameters to be determined. The optimization problem can be formulated as follows:

$$\begin{aligned} &minimize\ \|T_{r\to z}\|_\infty = \gamma_{opt} \\ &subject\ to\ \lambda < 0 \end{aligned} \quad \{20\}$$

where $\| T_{r\to z} \|_\infty$ is the subsystem closed loop H_∞ norm and λ is the system poles maximum real part also called spectral abscissa. The last condition guarantees the stability of each closed loop subsystem.

The decentralized PI web tension controller is calculated for a generic web tension control case (downstream web tension control or upstream web tension control). C_T is calculated for the model G_i with the speed loop.

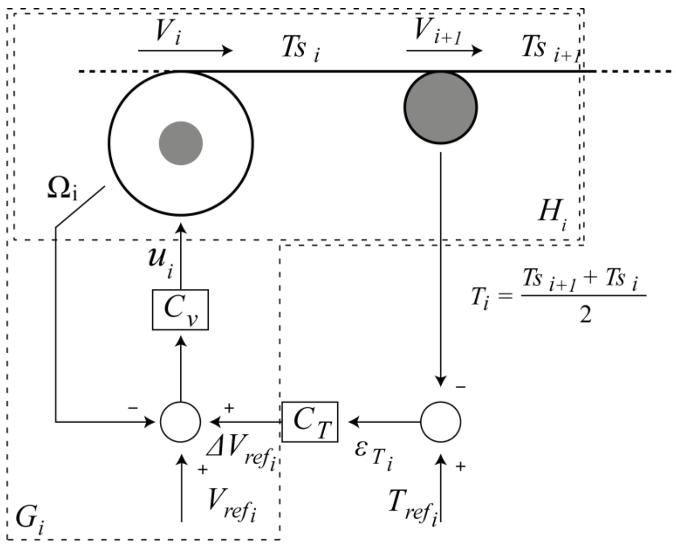

Figure 7 – Downstream tension control

<u>Drive Requirements</u>

The desired cross-over pulsations of web tension control and motor speed loop have to be determined adequately with respect of the whole roll-to-roll plant requirements and size. Klassen [20] advices, for classical large scaled web processing systems, a tension loop cross-over angular frequency of 1.3 rad/s and a 5 rad/s one for the speed loop. But, some applications need higher cross-over angular frequencies to better reject disturbances [3] and it is therefore necessary to also study different ratios of bandwidth between the speed loop and the tension loop.

In this work, different tunings are studied, as given in Table 1. The optimal master roller placement is studied in [21]. Is this work, the master roller is placed in second position.

Speed bandwidth ω_V (rad/s)	Tension bandwidth ω_T (rad/s)
20	5
20	10
20	15

Table 1 – Three settings of the speed and tension closed-loop bandwidths

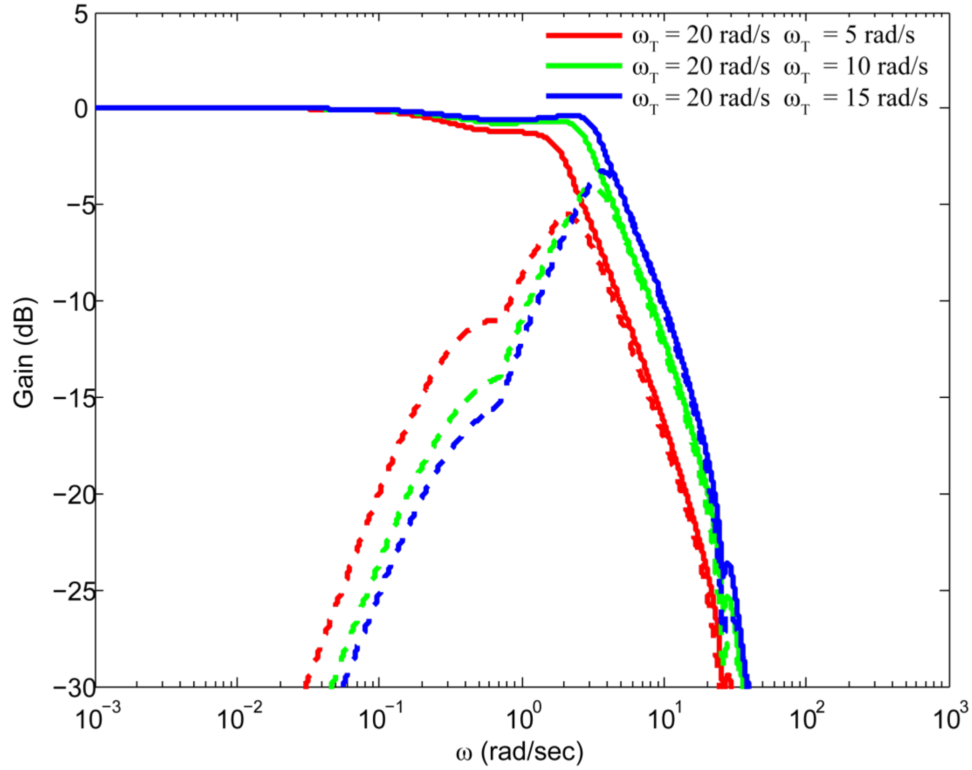

Figure 8 – Maximum of the diagonal transfer functions (solid) and non-diagonal transfer functions (dashed): comparison of 3 bandwidth configurations.

Figure 8 shows the maximum diagonal and non-diagonal transfer functions of the closed loop plant. In fact, the maximum non-diagonal transfer function is used to study the coupling that exists between the web tensions. One can see that, when the tension loop bandwidth decreases, the magnitude of the non-diagonal transfer functions decreases too.

Web tension simulations are presented in figure 9 for a fixed speed bandwidth ω_V (ω_V= 20 rad/s) and three tension bandwidth settings ω_T (ω_T = 5, 10, 15 rad/s) : for slow tension bandwidths, the tensions have slow responses.

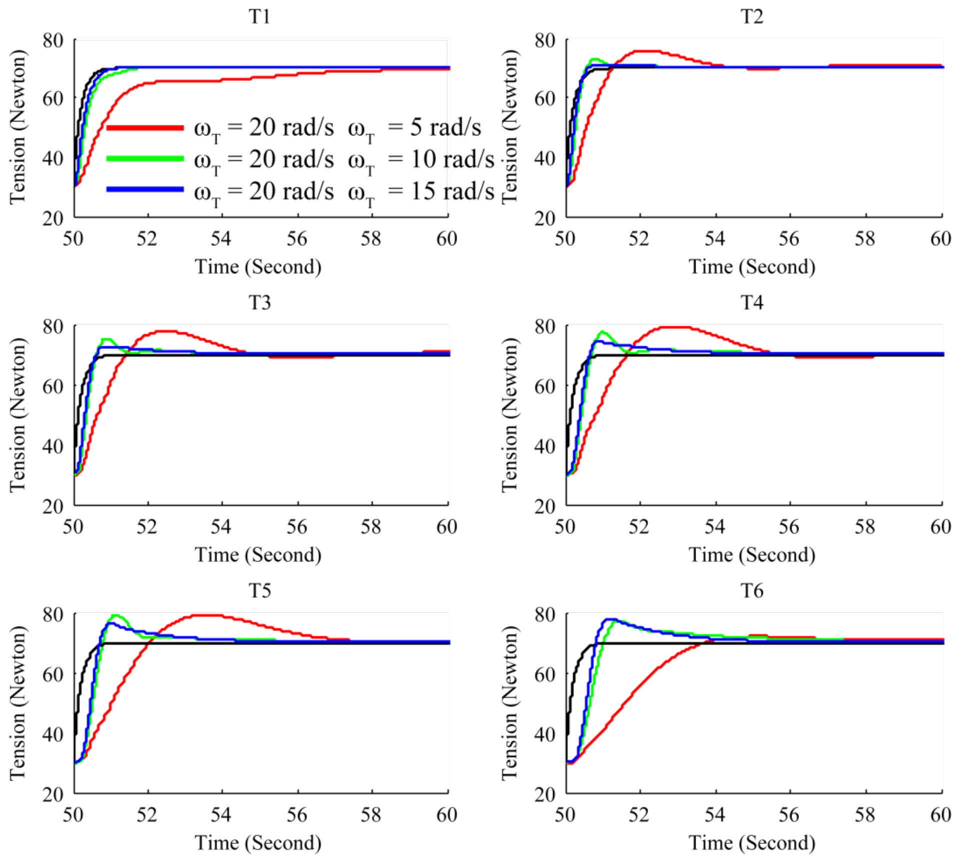

Figure 9 – Simulation of the web tensions in the time domain: comparison of 3 bandwidth configurations in term of reference tracking.

In industrial web handling systems, the web longitudinal dynamics is disturbed by the unwinding of a non-circular roll. This non-circularity leads to a perturbation composed of several sinus with frequencies depending on the roller rotational frequency. In this study, for didactic reasons, the tension disturbance is composed of a fundamental signal and three harmonics. The perturbation signal is represented in figure 10, in the frequency domain.

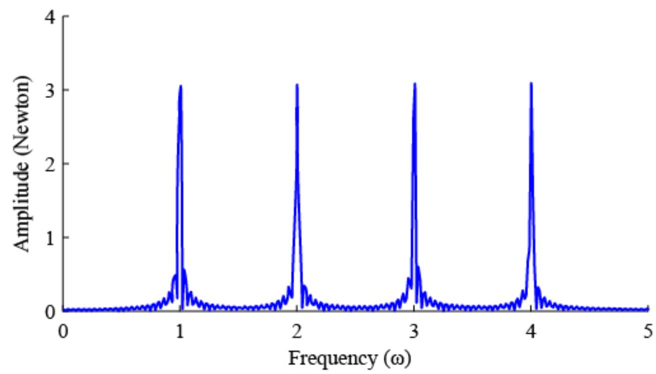

Figure 10 – Perturbation signal in the frequency domain (Fourier Transform): all the sinus have the same amplitude

The simulation results are shown in figure 11. One can see that the perturbation signal is well rejected in the case of higher tension bandwidth.

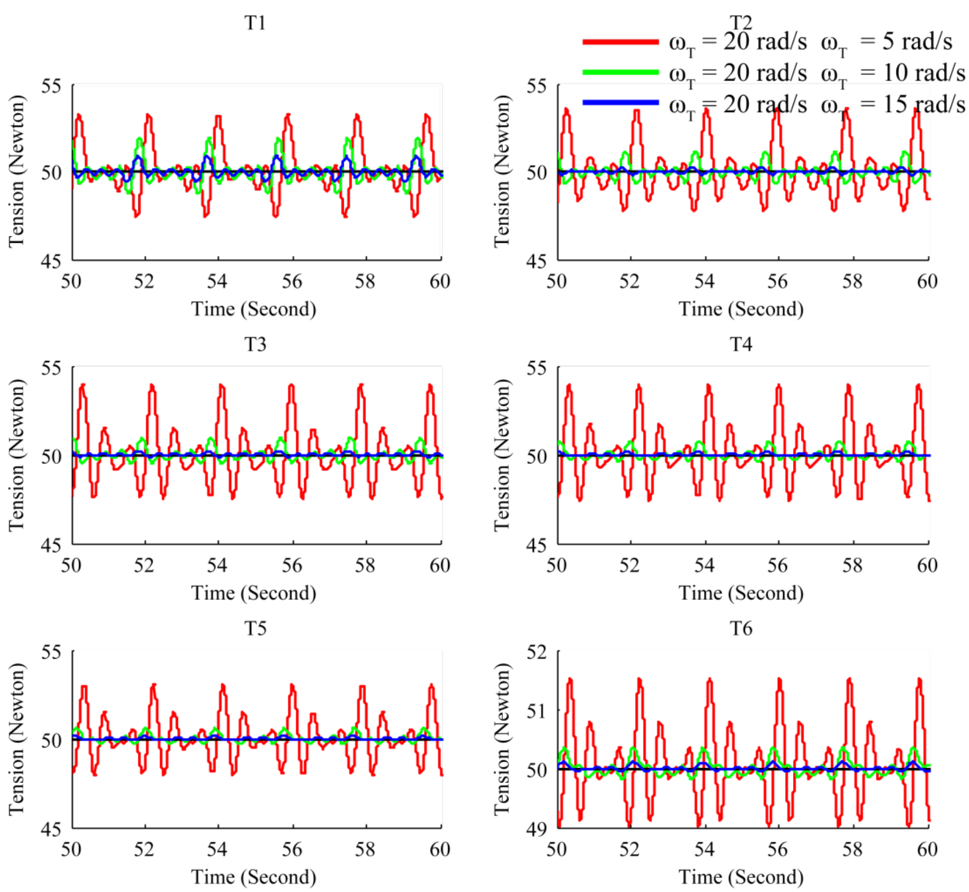

Figure 11 – Web tension simulations: comparison of 3 bandwidth configurations in term of disturbance rejection.

Web elasticity can vary with air temperature and moisture. Moreover, it is interesting to use the same control settings for different web materials. In order to evaluate the robustness to elasticity variations, simulations have been made for a Young's modulus divided by a factor 5. The simulation results are shown in figure 12. On can see that the reference tracking is more robust to web elasticity decreasing for a web tension bandwidth ω_T = 10 rad/s. Therefore web tension bandwidth should have a reduced value.

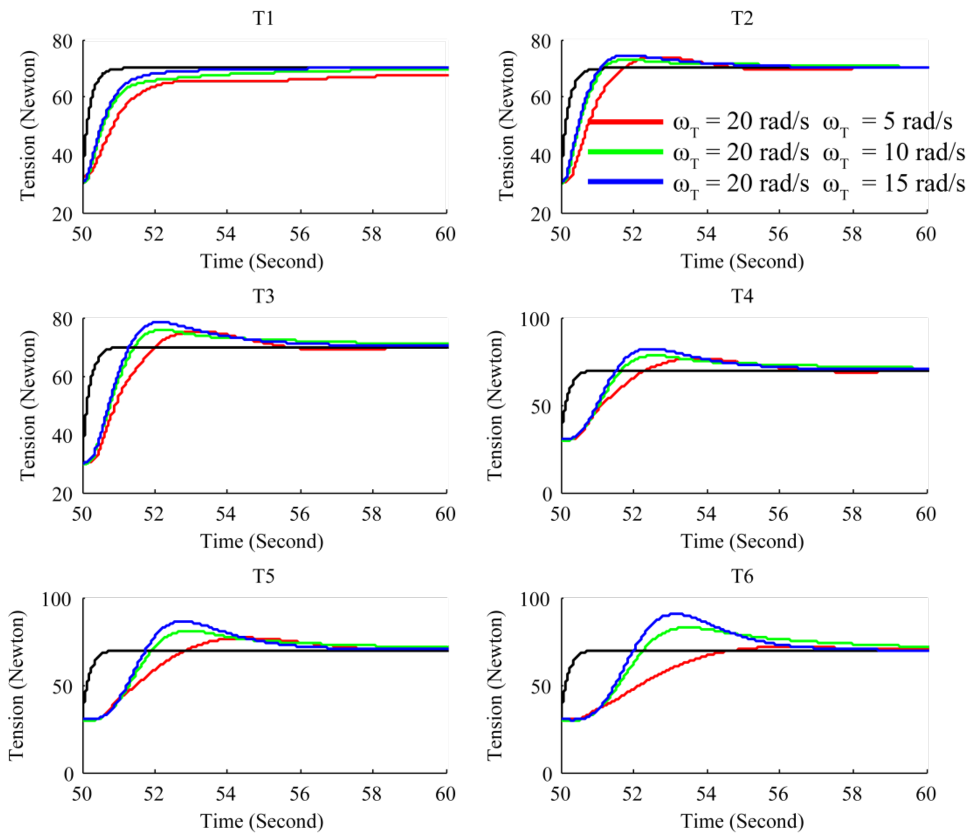

Figure 12 – Web tension simulations for a Young's modulus divided by a factor 5: comparison of 3 bandwidth configurations in term of reference tracking.

Discussions

As illustrated in the given example, the H_∞ fixed-order and -structure synthesis is a convenient framework to calculate automatically industrial controllers (for example PI or PID controllers) for a given control structure and a given plant model. As the controllers are synthesized in the frequency domain with a reference model, the closed loop has good performances. The synthesis needs to choose firstly the weighting filters which give the frequency shapes. The selection of weighting filters is well described in the literature.

The main drawback of this automatic controller methodology is that the designer needs to establish a dynamic model of the plant (and the controller optimization is made in the frequency domain).

CONCLUSION

In industrial control structures, each motor is firstly torque and velocity controlled. Usually, the torque and velocity controllers are easy to adjust. However, the tension controllers (as well the dancer position controller if dancers are used) are difficult to tune and depend on the web elasticity and speed [3]. The tension controllers can be adjusted automatically in the H_∞ fixed-order and -structure synthesis framework.

REFERENCES

1. Wolfermann, W., "Tension Control of Webs. A Review of the Problems and Solutions in the Present and Future," International Conference on Web Handling, Oklahoma, 1995, 198-229.
2. Koç, H., Knittel, D., de Mathelin, M., and Abba, G., "Modeling and Robust Control of Winding Systems for Elastic Webs," IEEE Transactions on Control Systems Technology, Vol. 10, No. 2, 2002, pp. 197-208.
3. Gassmann, V., "Commande Décentralisée Robuste de Systèmes D'Entrainement de Bandes à Elasticité Variable," Ph.D. Thesis, University of Strasbourg, Strasbourg, France, 2011.
4. Gassmann, V., Knittel, D., Pagilla, P., and Bueno, M., "H_∞ Unwinding Web Tension Control of a Strip Processing Plant Using a Pendulum Dancer," American Control Conference, St. Louis, Missouri, USA, 2009.
5. Gassmann, V., Knittel, D., Pagilla, P., and Bueno, M., "Fixed-Order H_∞ Tension Control in the Unwinding Section of a Web Handling System Using a Pendulum Dancer," IEEE Transactions on Control Systems Technology , Vol. 20, No. 1, 2012, pp. 173-180.
6. Knittel, D., Laroche, E., Gigan, D., and Koç, H., "Tension Control for Winding Systems with Two-Degrees-of-Freedom H_∞ Controllers," IEEE Transactions on Industry Applications, Vol. 39, No. 1, 2003, pp. 113-120.

8. Skogestad, S. and Postlethwaite, I., "Multivariable Feedback Control: Analysis and Design," Wiley-Interscience, 2nd ed., 2005.
9. Knittel, D., Arbogast, A., Vedrines, M., and Pagilla, P., "Decentralized Robust Control Strategies with Model Based Feedforward for Elastic Web Winding Systems," American Control Conference, 2006, pp. 1968-1975.
10. Pagilla, P. R., Siraskar, N. B., and Dwivedula, R. V., "Decentralized Control of Web Processing Lines," IEEE Control Systems Technology, Vol. 15, No. 1, 2007, pp. 106-117.
11. Angermann, A., Aicher, M., and Schroder, D., "Time-Optimal Tension Control for Processing Plants with Continuous Moving Webs," IEEE Industry Applications Conference, Vol. 5, 2000, pp. 3505-3511.
12. Gassmann, V. and Knittel, D., "Robust PI-LPV Tension Control with Elasticity Observer for Roll-to-Roll Systems," IFAC World Congress, Milano, Italy, September 2011.
13. Sakamoto, T. and Kobayashi, T., "Decomposition and Decentralized Controller Design of Web Transfer System," IFAC Symposium on Large Scale Systems, 2004, pp. 149-154.

14. Shin, K. H., "Distributed Control of the Tension in Multi-Span Web Transport Systems," Ph.D. Thesis, Oklahoma State University, Stillwater, USA, 1991.
15. Young, G. E. and Reid, K. N., "Lateral and Longitudinal Dynamic Behavior and Control of Moving Webs," Journal of Dynamic Systems, Measurement, and Control, Vol. 115, No. 2B, 1993, pp. 309-317.
16. Knittel, D., Henrion, D., Millstone, M., and Vedrines, M., "Fixed-Order and Structure H_∞ Control with Model Based Feedforward for Elastic Web Winding Systems," IFAC Conference in Large Scale Systems (invited session), Gdansk, Poland, July 2007.
17. da Silva, M. M., Brüls, O., Swevers, J., Desmet, W., and Brussel, H. V., "Computer-Aided integrated Design for Machines with Varying Dynamics," Mechanism and Machine Theory, Vol. 44, No. 9, 2009, pp. 1733-1745.
18. Burke, J. V., Henrion, D., Lewis, A. S., and Overton, M. L., "HIFOO - A MATLAB Package for Fixed-Order Controller Design and H_∞ Optimization," 5th IFAC Symposium on Robust Control Design, Toulouse, France, 2006.
19. Apkarian, P. and Noll, D., "Nonsmooth H_∞ Synthesis," IEEE Transactions on Automatic Control, Vol. 51, No. 1, 2006, pp. 71-86.
20. Klassen, C., "Drive Response Requirements for Web Handling," AIMCAL Web Handling Conference, Prague, Czech Republic, 2012.
21. Frechard, J. and Knittel, D., "Drive Requirements for Elastic Web Roll-to-Roll Systems," Mechanism and Machine Theory, Elsevier, Vol. 66, August 2013, pp. 14-31.