CONTROL DESIGN FOR LONGITUDINAL WEB DYNAMICS: BENEFITS AND DRAWBACKS OF ROBUST CONTROL APPROACHES

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ABSTRACT

Web tension and speed are two key variables to be monitored and controlled in order to achieve the expected final product quality. One of the main objectives in web handling plants is to reach an expected web speed while maintaining the web tension within an acceptable range around the tension reference in the entire processing line. In the recent years, many works have focused on the topic of web tension control and have proposed various ways to enhance the performance: $H_{\infty}$, optimal state feedback, neural network, etc. But the common practice in industrial web transport systems remains the use of decentralized PI-type controllers.

An improved design methodology of these PI controllers with fixed-order and -structure synthesis approaches has been made. Nevertheless, despite high performances for a nominal working point, it has been noticed that the closed-loop system performances depend on the web elasticity since the dynamic behavior is strongly affected by the Young’s modulus. Consequently the emphasis of this contribution is on the automatic tuning of PID (or PI) controllers for web processing plants that guaranty good performances of the closed-loop system.

NOMENCLATURE

$V_i$  Web linear speed
$T_i$  Web tension in the span $i$
$I_i$  Inertia of the roller $i$
$K_i$  Torque constant of the motor $i$
$R_i$  Radius of the roller $i$
$f_d$  Web/roller dynamic friction coefficient
$\epsilon_i$  Strain of the web span $i$
$L_i$  Length of the web span $i$
$E$  Young Modulus
**INTRODUCTION**

Roll-to-Roll systems handling web material such as papers, polymers, textiles or metals are very common in industry. Printing, coating and drying are examples of operations that can be performed in different sections of a web line. Web handling systems are recently used to produce new technologies such as thin solar panels, printed electronics, etc. Web tension and speed are the two key variables that need to be monitored and controlled in order to achieve the expected final product quality. One of the main objectives in a web handling machinery is to reach an expected web speed while maintaining the web tension within an acceptable range around the tension reference in the entire processing line [1]. In order to set up the speed and tension controllers, requirements have to be fixed. These requirements are the bandwidth of the closed-loop system and overshoot.

The nonlinear model of a web transport system is built from the equations describing web tension behavior between two consecutive rollers and the velocity of each roller. The studied web handling plant, composed of an unwinder, intermediate motor driven rollers, several idle rollers and a rewinder, is divided into several subsections that are controlled independently.

The objective is to synthesize a “robust” tension PI controller for each subsection (except for the master-roll subsection) [4, 5, 16, 21]. The $H_{\infty}$ problem can be expressed as follows: find a stabilizing controller that minimizes the $H_{\infty}$-norm of the transfer function between a set of exogenous inputs (typically the references) and some performance outputs. It turns out that the problem is non-convex for the design of PI controllers. The fitness function (which has to be minimized) is then based on the $H_{\infty}$-norm with the constraint that the closed-loop system has to be stable.

The benefits and drawbacks of the automatic controller tuning are discussed.

**MODELING**

The non-linear model of a web transport system is established from the equations characterizing the speed of each roller and the web tension behavior of each web span (web between two consecutive rollers) [2, 6, 7]

**Web Speed Determination**

The web linear velocity $V_i$ of a roller $i$ is equal to the linear roller (see Fig. 1), which depends on the upstream web tension $T_{i-1}$ and the downstream web tension $T_i$ is given by:

$$J_i \frac{dV_i}{dt} = (T_i - T_{i-1})R_i^2 + K_i R_i u_i - f_d V_i \quad \{1\}$$

This equation assumes that no slippage occurs: the web velocity is equal to the linear roller velocity. In addition, a static friction can be added.
Web Tension Determination

The strain $\varepsilon_i$ of the web span $i$, which depends on the upstream web strain $\varepsilon_{i-1}$ and the velocities of the two consecutive rollers, is given by the differential equation [2]:

$$
\frac{d}{dt} \left( \frac{L_i}{1 + \varepsilon_i} \right) = \frac{V_{i+1}}{1 + \varepsilon_i} + \frac{V_i}{1 + \varepsilon_{i-1}}
$$  \hspace{1cm} (2)

For an elastic web, the web tension $T$ is obtained using Hooke’s law:

$$
T = E S \varepsilon_i
$$  \hspace{1cm} (3)

The web tension is determined using the non-linear differential Eq. (2). This equation can be linearized around working points $T_0$ and $V_0$. Considering $T_i = T_0 + t_i$, $V_i = V_0 + v_i$, $T_{i-1} = T_0 + t_{i-1}$, and $V_{i+1} = V_0 + v_{i+1}$ the linear equation becomes [2]:

$$
L_i \frac{dt_i}{dt} = V_0 (t_{i-1} - t_i) + (v_{i+1} - v_i) (ES + T_0)
$$  \hspace{1cm} (4)

Linear Model

The relations shown in Eqs. (1) and (4) permit to build the state-space representation of the studied roll-to-toll system:

$$
\begin{align*}
\dot{x}(t) &= A x(t) + B u(t) \\
y(t) &= C x(t)
\end{align*}
$$  \hspace{1cm} (5)

where $x$ is the state vector, $u$ is the control vector and $y$ is the output vector. $A$ is the state matrix, $B$ the input matrix and $C$ the output matrix.

The system scheme is shown in Fig. 2, the small circles ($V_2$, $V_4$, $V_6$, ...) correspond to the idle rollers equipped with load cells and the large circles ($V_1$, $V_3$, $V_5$, ...) correspond to the motor driven rollers. The calculation of each roller speed and tension in each span is needed for the dynamical model. Therefore, the state vector is composed of the velocity of each motor driven roller, the speed of each idle roller and the web tension in each web span:

$$
x = \begin{bmatrix} V_1 & T_{s1} & V_2 & T_{s2} & V_3 & T_{s3} & \ldots & T_{s12} & V_{13} \end{bmatrix}^T
$$  \hspace{1cm} (6)

The system has 8 inputs: the input web tension $T_{in}$ and the seven motor control signals.

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The system has 13 outputs: the motor driven roller rotational speeds and the web tension located at each idle roller.

\[
y = [\omega_1 \omega_3 \omega_5 \omega_7 \omega_9 \omega_{11} T_1 T_3 T_5 T_7 T_9 T_{11}]^T
\]

where:

\[
T_1 = \frac{T_{s1} + T_{s2}}{2} \quad T_3 = \frac{T_{s3} + T_{s4}}{2} \quad T_5 = \frac{T_{s5} + T_{s6}}{2}
\]

\[
T_7 = \frac{T_{s7} + T_{s8}}{2} \quad T_9 = \frac{T_{s9} + T_{s10}}{2} \quad T_{11} = \frac{T_{s11} + T_{s12}}{2}
\]

The built linear model permits to synthesize robust controllers. The robustness needed is regarding elasticity variations. The elasticity can vary substantially during a same unwind-rewind process. The influence of the web elasticity on a roll-to-roll system is studied and shown in Fig. 3. This figure represents the maximum singular values of the studied generic plant for different web Young’s modulus [21]. One can see that the resonance peaks magnitude and frequency depend on the web elasticity value. The resonances move to lower frequencies for a web elasticity decrease.

![Figure 2 – Global generic studied system.](image)

**CONTROL STRATEGY**

**Motor Velocity Control**

The velocity control loop uses an IP controller. The main advantage of the IP controller is that it does not introduce a zero in the closed loop transfer function.

The speed controller structure is depicted in Fig. 4. In order to have a rotational velocity reference, the linear velocity reference is divided by the roller radius. Moreover, in order to simplify the closed loop and to remove the sensibility to inertia variation, the controller output is multiplied by the roller inertia. The system open loop relation, when
the loading effects of the web and frictions are compensated by a feedforward (not represented in Fig. 5), becomes:

\[
\frac{\Omega_i}{u_i} = \frac{K_i}{J_i s} \tag{10}
\]

The closed loop of the torque has been considered very fast and approximated by the gain \(K_i\). The transfer function of closed loop with IP controller gives:

\[
\frac{\Omega_i(s)}{\Omega_{ref}(s)} = \frac{1}{1 + \frac{s}{a} + \frac{s^2}{abK_i}} \tag{11}
\]

The IP controller parameters \(a\) and \(b\) are calculated as follows:

\[
a = \frac{\omega_v}{2\zeta_v} \quad b = \frac{\omega_v^2}{aK_i} \tag{12}
\]

where \(\omega_v\) is the desired bandwidth of the speed closed loop, \(\zeta_v\) is the desired damping factor and \(K_i\) is the motor torque constant.

![Figure 3](image-url)  
Figure 3 – Maximum singular values of the open-loop system for different web elasticities

![Figure 4](image-url)  
Figure 4 – Velocity IP controller.
Web Tension Control

Web tension control has been studied for several years [1-7, 9-16, 21]. In this study, we use an $H_\infty$ approach in order to synthesize each tension controller [5, 12, 16, 21]. In many domains the $H_\infty$ synthesis approach is used [17], it consists in finding a stabilizing controller that minimizes the transfer function $H_\infty$ norm between a set of exogenous inputs $r$ and a set of performance outputs $z$.

$$\|T_{r \rightarrow z}\|_\infty < \gamma$$ \tag{13}

$H_\infty$ Fixed Order and Fixed Structure Synthesis

The high order of the obtained controller is the major drawback of the standard $H_\infty$ approach. In fact, the order of the controller is equal to the sum of the weighting functions order and the system order [8]. The controller order cannot always be decreased while performances and stability are assured by using the current model reduction approach. Fixed -structure and -order controllers design algorithms have been developed : they are highly relevant for industrial applications. The mathematical problem seems to be difficult because fixed-order controller synthesis can be formulated as a nonsmooth affine problem in the nonconvex cone of stable matrices. Relevant synthesis tools like HIFOO in 2005 [18] and more recently hinfstruct [19] have been developed thanks to recent progress in nonsmooth problem solving. In our study, the hinfstruct tool is used in order to synthesize the web tension controllers.

As illustrated in Fig. 6, the web tension controllers are synthesized with the use of S/KS/T weighting scheme and model matching. The reference model $M_0$ corresponds to the desired closed-loop system behavior: a second order transfer function is chosen with a cross-over pulsation $\omega_T$ and a damping factor $\zeta_T$ equals to 1:

$$M_0 = \frac{1}{1 + 2\zeta_T \frac{s}{\omega_T} + \left(\frac{s}{\omega_T}\right)^2}$$ \tag{14}
where \( s \) is the Laplace variable. The weighting functions \( W_p, W_u, \) and \( W_t \) appear in the transfer matrix of the closed loop function \( T_{r \rightarrow z} \). \( r \) is composed of the tension references and \( z \) is composed of \( z_1, z_2, \) and \( z_3 \), see Fig. 6.

\[
T_{r \rightarrow z} = \begin{bmatrix} W_p(M_0 - T_{CL}) \\ W_dC_TS_{CL} \\ W_tT_{CL} \end{bmatrix}
\]  \( \{15\} \)

where \( S_{CL} \) is the sensitivity function defined by:

\[
S_{CL} = (1 + G(G_T)^{-1})^{-1} \]  \( \{16\} \)

\( G_i \) contains the subsystem including the speed control loop (see Fig. 7). The web tension controller is \( C_T \) and \( T_{CL} \) is the complementary sensitivity function defined by:

\[
T_{CL} = 1 - S_{CL} \]  \( \{17\} \)

\[
W_p = \frac{s + \omega_B}{s + \omega_B \varepsilon_0} \]  \( \{18\} \)

where \( M \) is the maximum peak magnitude of \( S_{CL} \), \( \omega_B \) is the required cross-over frequency \( (\omega_B > \omega_T) \) and \( \varepsilon_0 \) is the accepted steady-state error.

The tension controller \( C_T \) (see Fig. 7) is a PI controller:

\[
\frac{\Delta V_{ref_i}}{\varepsilon_{T_i}} = K_p \frac{1 + \tau_i}{s} \]  \( \{19\} \)

where \( \Delta V_{ref_i} \) is the web tension controller output that adjusts the speed reference and \( \varepsilon_{T_i} \) is the web tension error signal, \( i \) is the roller number. \( K_p \) and \( \tau_i \) are the controller parameters to be determined. The optimization problem can be formulated as follows:
\[
\begin{align*}
\text{minimize } & \| T_{r \rightarrow z} \|_\infty = \gamma_{opt} \\
\text{subject to } & \lambda < 0
\end{align*}
\]

where \( \| T_{r \rightarrow z} \|_\infty \) is the subsystem closed loop \( H_\infty \) norm and \( \lambda \) is the system poles maximum real part also called spectral abscissa. The last condition guarantees the stability of each closed loop subsystem.

The decentralized PI web tension controller is calculated for a generic web tension control case (downstream web tension control or upstream web tension control). \( C_T \) is calculated for the model \( G_i \) with the speed loop.

\[ T_i = \frac{T_{s_{i+1}} + T_{s_i}}{2} \]

\[ \Omega_i, \; U_i, \; C_V, \; C_T, \; T_{ri} \]

**Drive Requirements**

The desired cross-over pulsations of web tension control and motor speed loop have to be determined adequately with respect of the whole roll-to-roll plant requirements and size. Klassen [20] advices, for classical large scaled web processing systems, a tension loop cross-over angular frequency of 1.3 rad/s and a 5 rad/s one for the speed loop. But, some applications need higher cross-over angular frequencies to better reject disturbances [3] and it is therefore necessary to also study different ratios of bandwidth between the speed loop and the tension loop.

In this work, different tunings are studied, as given in Table 1. The optimal master roller placement is studied in [21]. In this work, the master roller is placed in second position.
Table 1 – Three settings of the speed and tension closed-loop bandwidths

<table>
<thead>
<tr>
<th>Speed bandwidth $\omega_V$ (rad/s)</th>
<th>Tension bandwidth $\omega_T$ (rad/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>5</td>
</tr>
<tr>
<td>20</td>
<td>10</td>
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<td>20</td>
<td>15</td>
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Figure 8 shows the maximum diagonal and non-diagonal transfer functions of the closed loop plant. In fact, the maximum non-diagonal transfer function is used to study the coupling that exists between the web tensions. One can see that, when the tension loop bandwidth decreases, the magnitude of the non-diagonal transfer functions decreases too.

Web tension simulations are presented in figure 9 for a fixed speed bandwidth $\omega_V$ ($\omega_V = 20$ rad/s) and three tension bandwidth settings $\omega_T$ ($\omega_T = 5, 10, 15$ rad/s): for slow tension bandwidths, the tensions have slow responses.
In industrial web handling systems, the web longitudinal dynamics is disturbed by the unwinding of a non-circular roll. This non-circularity leads to a perturbation composed of several sinus with frequencies depending on the roller rotational frequency. In this study, for didactic reasons, the tension disturbance is composed of a fundamental signal and three harmonics. The perturbation signal is represented in figure 10, in the frequency domain.
Figure 10 – Perturbation signal in the frequency domain (Fourier Transform): all the sinus have the same amplitude

The simulation results are shown in figure 11. One can see that the perturbation signal is well rejected in the case of higher tension bandwidth.

Figure 11 – Web tension simulations: comparison of 3 bandwidth configurations in term of disturbance rejection.
Web elasticity can vary with air temperature and moisture. Moreover, it is interesting to use the same control settings for different web materials. In order to evaluate the robustness to elasticity variations, simulations have been made for a Young’s modulus divided by a factor 5. The simulation results are shown in figure 12. One can see that the reference tracking is more robust to web elasticity decreasing for a web tension bandwidth $\omega_T = 10$ rad/s. Therefore web tension bandwidth should have a reduced value.

![Graphs showing web tension simulations for different bandwidth configurations](image)

Figure 12 – Web tension simulations for a Young’s modulus divided by a factor 5: comparison of 3 bandwidth configurations in term of reference tracking.

**Discussions**

As illustrated in the given example, the $H_\infty$ fixed-order and -structure synthesis is a convenient framework to calculate automatically industrial controllers (for example PI or PID controllers) for a given control structure and a given plant model. As the controllers are synthesized in the frequency domain with a reference model, the closed loop has good performances. The synthesis needs to choose firstly the weighting filters which give the frequency shapes. The selection of weighting filters is well described in the literature.

The main drawback of this automatic controller methodology is that the designer needs to establish a dynamic model of the plant (and the controller optimization is made in the frequency domain).
CONCLUSION

In industrial control structures, each motor is firstly torque and velocity controlled. Usually, the torque and velocity controllers are easy to adjust. However, the tension controllers (as well the dancer position controller if dancers are used) are difficult to tune and depend on the web elasticity and speed [3]. The tension controllers can be adjusted automatically in the $H_\infty$ fixed-order and -structure synthesis framework.

REFERENCES
