THE BEHAVIOR OF A FLEXIBLE WEB IN CONTACT WITH A ROLLER

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ABSTRACT

In the contact patch between a web and a roller, there may be regions of stick and microslip or full slip. Within the stick zone, the velocities of web and roller surface match, but in the microslip zone they differ by a small amount as the web deforms. Analysis of the stick zone indicates an upper limit to the shear stress in the web, determined by friction. As the web steering in the incoming span is increased, the shear stress increases to satisfy the condition of velocity matching. Eventually, friction in the stick zone cannot support the shear stress and a microslip zone forms on entry, resulting in a change in the behavior in the span. The influence of web bagginess and roller profile on the shear stress limit is analyzed.

An exit microslip zone will normally be present, as the web changes tension or develops shear stress and bending moment as a result of steering in the outgoing span. Attempts to analyze this with a beam model, as used in a free span, will be described. However, the requirement to match conditions at the start of the span is impossible to meet, so partial width microslip zones are postulated.

Work continues to understand these microslip zones.

NOMENCLATURE

MD	Machine Direction, i.e. along direction of travel
TD	Transverse Direction, i.e. across the web
A	Cross section area of web
e	The base of natural logarithms, 2.718
E	Young's modulus of the web
f_x, f_y	Components of friction force on beam per unit length
F	Shear force on beam
F_{max}	Maximum shear force in stick zone
F_0	Shear force in stick zone
G	Shear modulus of the web

h	Web thickness
I	Second moment of area of web section, $hw^3/12$
m	Friction couple per unit length
M	Bending moment of the web
n	Shear coefficient
R	Roller radius
S	Friction force per unit area
S_x, S_y	Components of friction force
T	Web tension per unit width
T_0	Web tension in the stick zone
T_1,T_2	Tension before and after roller
u	Relative velocity of web on roller
v	Local velocity of web
V	Velocity of roller surface
x, y	Coordinates along roller surface
x', y'	Coordinates relative to deformed web centre line
w	Web width
α, β	Gradients of tension and shear force at start of microslip zone
γ	Shear strain
$\varepsilon_{x'}$	Tensile strain in local web direction
ε_r	MD web strain to match roller surface speed
θ	Slope of web centre line
κ	Web curvature
κ_0	Intrinsic web curvature (camber)
μ	Coefficient of friction
σ_x	MD stress
σ_y	TD stress
τ_{xy}	Shear stress
$\tau_{\max}$	Maximum value of shear stress in a stick zone
	Angle of roller surface movement relative to web

INTRODUCTION

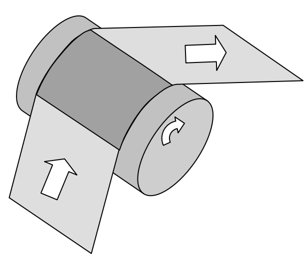


Figure 1 – Contact patch when a web runs over a roller.

When a flexible, elastic web wraps a roller, a contact patch is formed containing a rectangular area of web at any instant (figure 1). Both roller surface and web move through the patch when the line is running. Generally, it is desirable to have the web and roller surface move at the same speed, which implies there is also speed matching in at least part of the patch. This is a “stick zone”, in which each point on the web remains in contact with the same point on the roller surface. Any physical adhesion implied by the term “stick” is normally weak or absent, but there is a resistance to motion over the surface due to friction.

If the web is straight, under tension and entering normal to the axis of a cylindrical roller, it can make and lose contact without any forces acting in its plane. The stick zone then extends over the whole of the contact patch. Attempts to change tension in the web or steer it generate distributed forces over at least part of the contact patch in both dynamic and static situations. If the stress in the web varies as a result, it will also change its strain and hence move relative to the roller surface in a “microslip” zone.

Machine Direction Slip and Tension Change

A web passing over a braked roller in steady state (figure 2) has the strain produced by tension in the previous span as it touches down. This is unchanged over a stick zone in the first part of the wrap. In the second part of the wrap, a microslip zone forms in which the web gradually increases in strain, advancing slowly along the surface [1]. The tension rises until it reaches the value in the following span. The tension change is balanced by friction forces, which in turn drive the roller against the braking torque applied. Friction is generated by the relative motion and the contact pressure *T/R* produced by web tension *T* acting on the curved surface of the roller, radius *R*. The relative movement in the microslip zone is typically 0.1 mm [2].

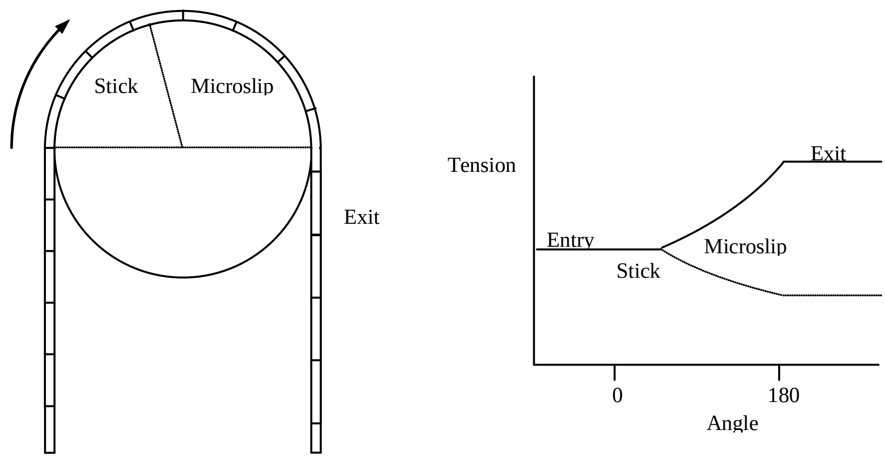


Figure 2 – Elastic web moving over a braked roller showing stick zone and strain increase in the microslip zone. Tension increases (solid), or decreases on a driven roller (dotted).

There is a limit to the ratio of exit to entry tensions that can be supported by friction. Once the limiting condition is reached, the stick zone disappears and slip occurs over the whole of the contact. Assuming a Coulomb friction model leads to the limiting ratio:

$$T_2 / T_1 = e^{\mu\theta} \qquad \{1\}$$

where e is the base of natural logarithms, 2.718…, μ is the kinetic coefficient of friction and θ the angle of wrap in radians. This is commonly termed the belt, capstan or Eytenweil equation.

The Coulomb model predicts the same tension ratio in the full slip condition. In practice, the friction coefficient may depend on the web and roller speeds, for example if entrained air modifies the behavior [3,4]. Speed differences can be large, and the relative movement several centimeters. If the roller is stationary, then the web drags over it continuously.

Lateral Effects

When the forces on the web arise from steering, comparable behavior is to be expected. If the forces are low, an extensive stick zone exists and the entry and exit spans are independent (figure 3a). Attempts to steer by a large amount result in the web skidding on the roller with no stick zone and large relative movement. An intermediate regime of "moment transfer" [5,6] has been found in which the web still passes over the roller normal to its axis, but the bending moment in the exit span is partially transmitted into the entry span, causing steering (figure 3b).

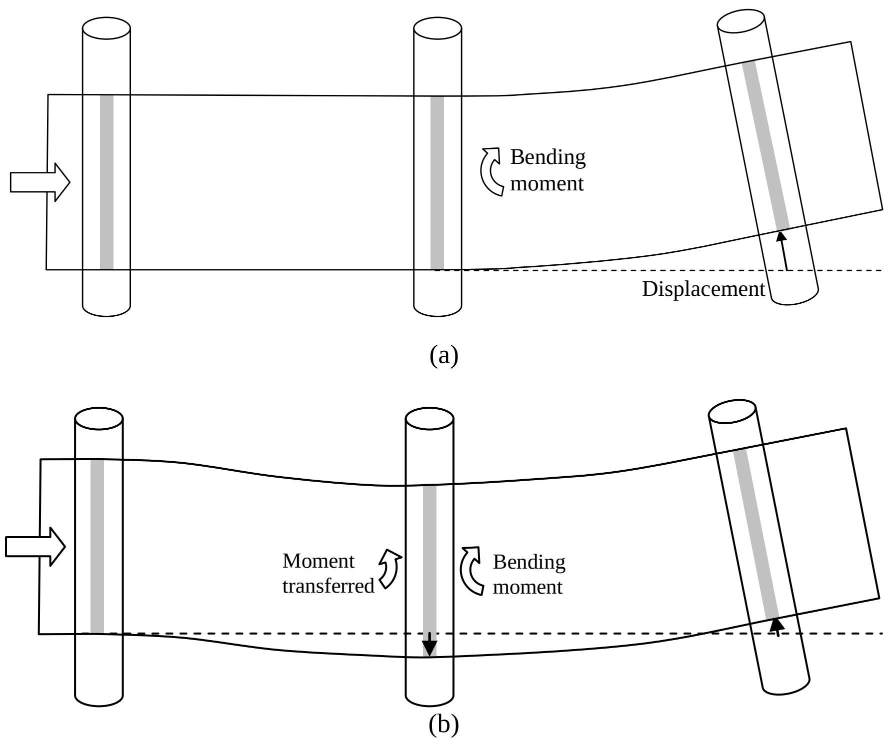


Figure 3 – Steering of a web between rollers inclined in each web plane, (a) without, and (b) with moment transfer. The web path is shown flat for clarity.

Practical Consequences

the end of the guiding span. Gross slip also leads to more rapid wear of the web and roller surfaces. A transfer film of web material can build up, and this can detach as particles. The large relative movement produces long scratches on the web if any contaminant particles or surface defects are present.

Microslip is not a problem in most cases, but in applications requiring damage-free surfaces and no particles on the web, it is a concern. These include flexible displays and computer data storage tape, where all small functional elements must work, and barrier coatings where defects increase the permeability.

A further motivation for modeling the contact patch is to try and understand the mechanisms of steering of a cambered web, where changes to web-roller friction apparently caused a change in steering behavior [7].

Previous and Current Understanding

Although there have been some published papers attempting to analyze the contact patch [8,9], understanding is still incomplete. When the web steers, bending and shear stresses are generated. There will be an interaction with any tension change over the roller: the magnitude of the friction force is limited by local web tension as its direction changes to oppose both steering and tension changes.

This paper attempts further analysis of the contact patch. First, the behavior of web in a free span is reviewed. A beam model leads to links between conditions at the ends of the span that must apply. Then, the stick zone is analyzed and it is demonstrated that friction forces must be transmitted in order to maintain shear stress in the web. Limits to friction impose limits to the shear stress and hence the web steering before a microslip zone forms at the roller entry.

Next, a full width slip or microslip zone is analyzed. Modeling the web as a beam with tension, slope, curvature and shear leads to a complete specification of the friction forces, and hence the change in beam shape in the contact patch.

Considering the web movement just outside the stick zone boundary shows that the initial movement as a beam can be in the machine direction (MD) to change tension and bending moment, or in the transverse direction (TD) to change shear, but not a combination. The implications of this will be deduced.

FREE SPAN

When a web contacts a roller, the surface velocities match as long as there is sufficient friction to oppose any traction forces that develop. If in addition the machine is in steady state, the web is aligned in the machine direction. This is commonly termed the “parallel” (to the surface velocity) or “normal” (to the roller axis) entry rule. If an angle were to develop, the web would track sideways with time, contrary to the steady state assumption.

Mass flow in steady state is constant with time and position, and so if the surface velocity roller on one roller greater than on another, the web must increase its MD strain and hence its tension to preserve the no-slip condition on the chosen roller. This has been termed the “normal strain rule” [10]. As there are no MD forces acting on the web between rollers, the strain must constant along the whole span from the exit of the upstream roller onto the downstream roller. The strain change occurs over the last part of the wrap of each roller. If the coefficient of friction between web and roller is high, the length of the microslip zone over which the strain change occurs is very small. It is frequently assumed that strain changes discontinuously at the line where the web leaves the roller.

Beam Model without Slip

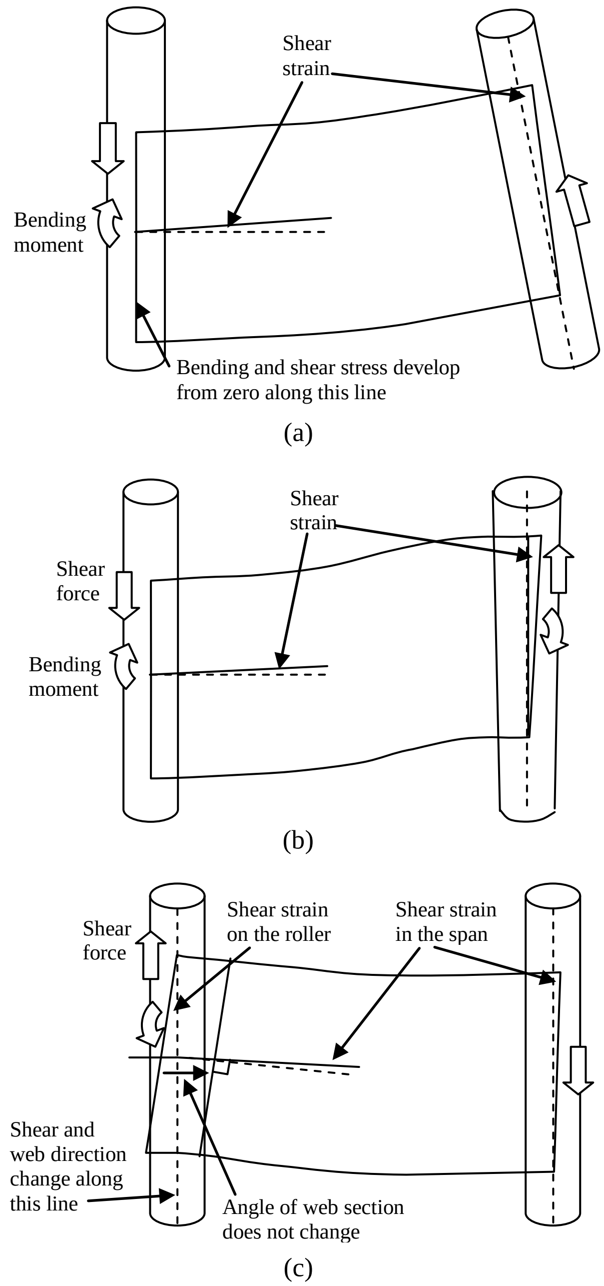


Figure 4 – Web steering from (a) an inclined roller, (b) a tapered diameter roller and (c) shear strain on the upstream roller.

Steady state steering of a web occurs if the downstream roller is either inclined at an angle to the upstream one (figure 4a), or it has a diameter taper from one side to the other (figure 4b). The behavior has been explained by applying the matching velocity condition on entry to the downstream roller, using beam theory for the web in the span, and assuming the web leaves the upstream roller with sections across the undeformed web lying along the TD and at a fixed lateral position [5]. The "beam" is rather unusual in that the role of beam depth is played by the web width, and the beam width is the web thickness. The MD stress is the sum of contributions from tension (uniform across the width in an initially straight web) and bending (varying linearly from negative to positive). At a critical value of steering, the MD stress at one edge reaches zero. Attempting to steer the web further results in a "slack edge" condition, and the beam theory must be modified because the web cannot carry significant compressive stress.

Experimental results on steering and lateral forces have been fitted well by a Timoshenko beam model [5]. This includes the stiffening effect of axial load (MD tension), and shear deformation, important in long and short spans [11] respectively. The boundary conditions at the downstream end are determined by velocity matching with the roller. The direction of movement fixes the beam slope, and its magnitude together with its TD gradient fix the average MD strain and beam curvature. The bending moment is determined not only by the beam curvature, but also by any inherent curvature of the web. Therefore zero bending moment is not the correct boundary condition on a cylindrical downstream roller unless the web is straight.

Any steering is accompanied by lateral forces at the ends of the span: these are transmitted from the adjacent portions of web. At the downstream end, the lateral force is supplied by shear stress in the web, accompanied by shear strain and sections tilted away from the TD. In the absence of slip, that shear propagates around the roller and appears at the entry to the next span, where it can act as an apparent misalignment as shown in figure 4c [11]. Steering is also accompanied by a bending moment at the exit of the upstream roller. In the limit of infinite coefficient of friction, the lateral force and bending moment develop at the line of last contact: for realistic values there must be a finite microslip zone in which the stresses in the web change.

Most previous analyses of steering have assumed the boundary conditions to apply at the lines of contact with the roller. With that assumption, the bending moment, lateral forces and lateral steering in a given web are determined by 3 parameters:

- Angle between downstream roller and web section on upstream roller
- Web curvature coming onto downstream roller
- Tension

Boundary Conditions with Microslip

If the coefficient of friction is infinite on the downstream roller (so velocity matching applies) but finite on the upstream roller, there will be a microslip zone at the exit of the latter. However, it must deliver the web into the span with a combination of section angle, lateral force and curvature that allows it to bend in the span and enter the downstream roller properly. Only one of these 3 can vary independently.

Similarly, if only the downstream roller has a finite coefficient of friction so there is a microslip zone at entry, that zone must pick up the web at an acceptable combination of web direction, lateral force and curvature. However, 2 of these can vary independently.

STICK ZONE ON A ROLLER

In an elastic web on a roller, the web stresses and strains are constant in the MD in a stick zone. They may vary in the TD, and friction forces can still act. Consideration of the forces on a small element of web (figure 5) leads to the equilibrium equations [9].

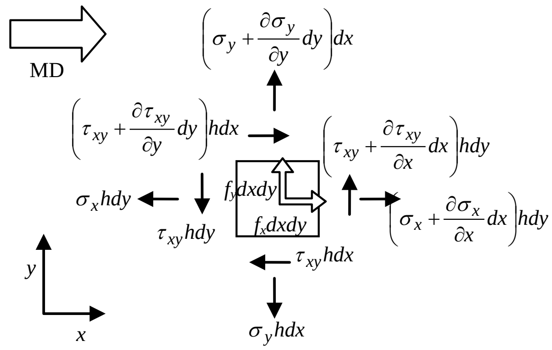


Figure 5 – Forces on an element of web of area *dxdy* in the stick zone on a roller.

$$S_x = -h\left[\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y}\right] \quad \{2\}$$

$$S_y = -h\left[\frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{xy}}{\partial x}\right] \quad \{3\}$$

x and y are MD and TD coordinates in the roller surface, or equivalently in the stretched but straight web. σ_x, σ_y and τ_{xy} are MD, TD and shear components of stress, S_x and S_y are the MD and TD components of the local friction force per unit area, and h the web thickness. The resultant friction force per unit area S is limited by Coulomb's law (with roller radius R):

$$S^2 = S_x^2 + S_y^2 \le \left(\frac{\mu \sigma_x h}{R}\right)^2 \quad \{4\}$$

Integrating equation 2 over y in a full-width stick zone, and using the condition of zero shear stress at the edges gives:

$$f_x = \int S_x dy = -w\frac{dT}{dx} = 0 \quad \{5\}$$

Integrating equation 3 over y, with origin at the web centre line, and using the condition of zero TD stress at the edges gives:

$$f_y = \int S_y dy = 0 \quad \{6\}$$

Multiplying equation 2 by y then integrating by parts gives:

$$m = \int S_x y dy = -\int hy\frac{\partial \sigma_x}{\partial x} dy - \left[hy\tau_{xy}\right] + \int h\tau_{xy} dy \quad \{7\}$$

m is the couple applied by the friction forces acting at all points across the width at a given MD position per unit MD length. The first and second terms are zero, and the third, the integral of shear stress across the web, is the beam shear force:

$$F = \int h \tau_{xy} dy \qquad \{8\}$$

Therefore, any shear force in the web in the stick zone must be balanced by a couple formed by the MD friction forces:

$$F=m \qquad \{9\}$$

In summary, a full width stick zone has a stress profile across the TD which does not vary in the MD. All 3 components may be non-zero, except at the edges where the absence of direct forces at the edges constrains the TD and shear components to be zero. Static friction may be acting. In the TD, it supports any variation of TD stress according to equation 3. In the MD, it supports the variation of shear stress according to equation 2. Averaged over the width, the net friction force is zero but there is a resultant couple per unit length which balances the shear force in the web (equation 9).

Maximum Shear Force in Stick Zone

The maximum shear force that a stick zone can support can be calculated for different distributions of tension across the width. All points are assumed to be on the point of slipping, and the friction force is assumed to be directed predominantly in the MD.

Uniform tension *T*. This is produced by a straight, non-baggy web passing over a cylindrical roller. It can support a symmetric triangular profile of shear stress across the width w (figure 6) with a central maximum. The friction force per unit area is taken as:

$$S_x = \pm \frac{\mu T}{R} \qquad \{10\}$$

It acts in opposite directions either side of the centre line. The maximum shear stress is:

$$\tau_{\max} = \frac{\mu T w}{2Rh} \qquad \{11\}$$

$$F_{\max} = \frac{\mu T w^2}{4R} \qquad \{12\}$$

It is tempting to equate this to the lateral force that can be generated by bending in the entry span to get a criterion for the upper limit at which a stick zone forms on entry. However, the triangular profile does not match the parabolic distribution of shear stress that arises in bending.

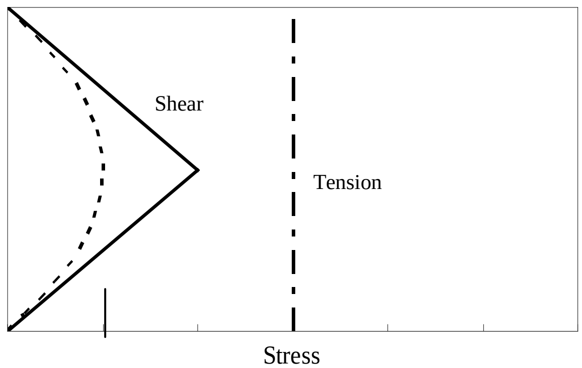


Figure 6 – Distribution of shear stress distribution across the web width (solid line) for a uniform tension (dash-dotted line) in the stick zone. The dashed line shows the largest quadratic variation that can be supported.

If the parabolic distribution is transferred directly to the stick zone as in the dashed curve on figure 6, the points close to the edge are closest to the friction limit. They reach the point of slipping when

$$F_{\max} = \frac{\mu T w^2}{6R} \qquad \{13\}$$

This could be the criterion for the existence of a stick zone at roller entry. Values of F between the values in equations 12 and 13 could result in an adjustment towards the end of the free span and in the first part of the wrap, where the shear stresses redistribute themselves accompanied by microslip.

Cambered web. A straightened cambered web just at the point of slackness has tension varying linearly from zero at one edge to double the average at the other. Figure 7 shows the profile of tension and limiting shear stress. The friction force direction changes on the high tension side of the centre line in order to preserve zero net force (equation 5). The maximum shear force is now

$$F_{\max} = \frac{\left(4\sqrt{2}-5\right)\mu T w^2}{12R} = 0.055\frac{\mu T w^2}{R}$$

This is less than a quarter of the shear force that can be carried by the straight web (equation 12). As the mean tension is raised, the numerical factor increases as the tension rises at the long edge. A cambered web will therefore form an entry slip zone for less steering than a straight web under the same average tension.

The parabolic shear stress profile in the free span cannot be matched to that on the roller until the minimum tension rises above zero. Therefore, some microslip is likely at the entry.

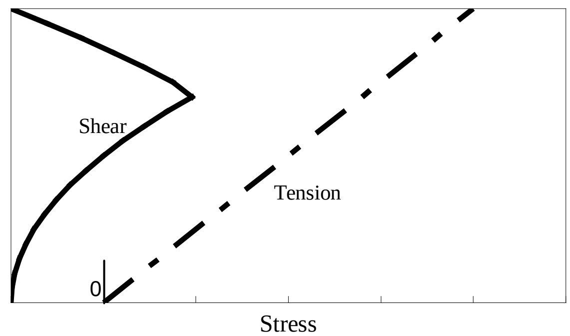
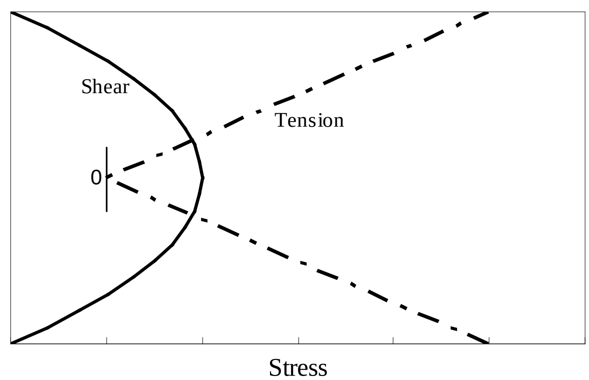
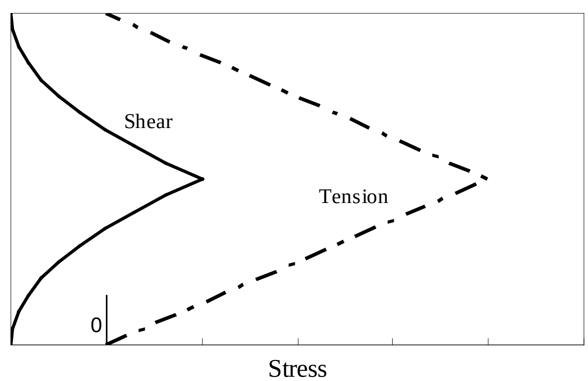


Figure 7 – TD profiles of tension (dash-dotted line) and shear stress (solid) in a baggy web at the critical tension (zero tension on the baggy side).



(a)



(b)

Figure 8 – TD profiles of tension (dash-dotted line) and shear stress (solid) in webs with (a) baggy edges and (b) baggy centre. Each web is on the point of slackness.

Baggy web. Figure 8 shows idealized tension profiles for webs with a baggy centre and baggy edges, on the point of slackness. The shear stress profile for the former is parabolic, and can match exactly the profile in a free span. However, for the baggy edges, there is no match so some microslip must occur at entry. The maximum shear force that can be supported is:

$$F_{\max} = \frac{\mu T w^2}{3R} \qquad \{15\}$$

for the baggy centre web, and

$$F_{\max} = \frac{\mu T w^2}{6R} \qquad \{16\}$$

for the baggy edge web. These are respectively somewhat higher and lower than a uniform web.

In principle, the shear stress profile and maximum shear force can be derived for any tension profile. If part of the web is slack, according to this model it will carry zero tension and constant shear stress (zero shear stress in slack edges).

Profiled diameter rollers. Tension profiles can also be produced by a straight web in the stick zone on rollers with varying diameter. Uniform diameter taper is analogous to a cambered web, a concave roller to baggy centre, and a convex or crowned roller to baggy edges. For the same average tension, the shear force capacity of a concave roller is superior to a plain roller, a convex roller somewhat worse and a tapered diameter much worse, for profiles that bring the web to the point of slackness.

FULL WIDTH SLIP ZONE

When all points across the width of the web are in motion relative to the roller, friction forces act at each point in a direction that opposes the local motion. Using a Coulomb friction model, the magnitude of the friction force is given by:

$$S = \frac{\mu \sigma_x h}{R} \qquad \{17\}$$

σ_x is the web stress component in the circumferential direction of the roller, which may be inclined to the web direction.

Beam Model

The shape of the web in a free span has been successfully modeled as a Timoshenko beam, so it seems sensible to try a similar model for web in contact. The analysis of the stick zone shows that friction can transmit a distributed couple in addition to the more familiar axial and transverse forces. The beam therefore deforms in response to these loads. However, the deformed shape of the beam determines the magnitude and direction of the velocity of all points, and hence the velocity relative to the roller surface and so the direction of the friction force. Furthermore, the deformed beam shape determines the stresses within it, and standard coordinate transformations can be used to find the component acting in the roller circumferential direction. Application of equation 17 then gives the force acting. The magnitude of the relative velocity has no influence on the magnitude of the force, according to the simplest Coulomb model.

Curvilinear coordinates (x', y') are defined along and normal to the beam centre line respectively (figure 9). The beam centre line lies at angle θ to the x-axis. Its curvature κ has contributions from bending, shear strain gradient and intrinsic curvature κ_0:

$$\kappa = \frac{d\theta}{dx'} = -\frac{M}{EI} + \frac{d\gamma}{dx'} + \kappa_0 \qquad \{18\}$$

M is the bending moment, E is Young's modulus and I the second moment of area, $w^3h/12$. The shear strain γ is related to the shear force F:

$$\gamma = F / nAG \qquad \{19\}$$

A is the section area, hw, G the shear modulus, and n is the shear coefficient, dependent on the distribution of shear stress in y', and equal to 5/6 for a free span. The tension T acts along the local x' axis and the shear force at right angles to it.

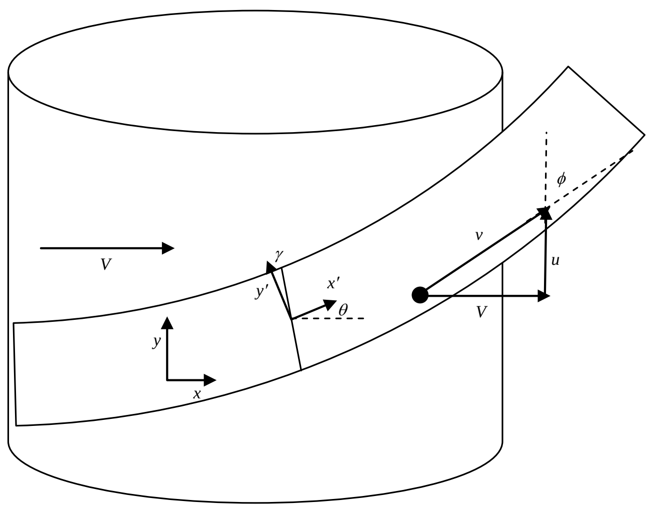


Figure 9 – Web slipping on a roller, showing coordinates on the roller (x,y) and web (x',y'). The web direction is at θ to the roller surface velocity V. Subtracting the roller surface velocity from the local velocity v gives the relative velocity u inclined at ϕ to x'.

The MD strain at distance y' from the centre line is:

$$\varepsilon_{x'} = \frac{T}{Eh} + \frac{My'}{EI} \qquad \{20\}$$

Defining a reference strain as the web MD strain in the stick zone as ε_r, where it is moving at the roller surface speed V, the web velocity at any point is:

$$v = V\frac{1 + \varepsilon_{x'} - \kappa_0 y'}{1 + \varepsilon_r} \qquad \{21\}$$

It is directed parallel to the beam centre line, i.e. at an angle θ to the roller x-axis. The movement relative to the roller surface is determined by a vector subtraction of the roller surface velocity from v, giving the velocity of magnitude u and angle ϕ to the web centre line.

$$
\begin{aligned}
u \sin\phi &= V \sin\theta \\
u \cos\phi &= v - V \cos\theta
\end{aligned}
\qquad \{22\}
$$

The friction force is therefore directed along $-\phi$. Although u/V and θ are both small, the angle ϕ can take any value between 0 and 360°. Its variation with x defines the trajectory of a point on the web relative to a point on the roller. The distributed frictional forces and couple may then be obtained by careful integration over y', similar to equations 5 to 7. A correction may be necessary for the variation in incremental length dx' across the width, and there is a difficulty at the roller entry and exit where the contact lines are inclined to the local y' axis.

Consideration of the forces acting on a small length of the beam (figure 10) leads to equilibrium equations:

$$
\begin{aligned}
f_{x'} &= -w\frac{dT}{dx'} + \kappa F \\
f_{y'} &= -\frac{dF}{dx'} - \kappa w T \\
m &= -\frac{dM}{dx'} + F
\end{aligned}
\qquad \{23\}
$$

The first allows tension to change by MD motion. It is not normally needed in beam analysis. The second relates the transverse load to a change in shear force and the tension direction, and is used in Timoshenko beam theory. The third is a moment balance, and takes the form used in all beam theories with the addition of a distributed couple m.

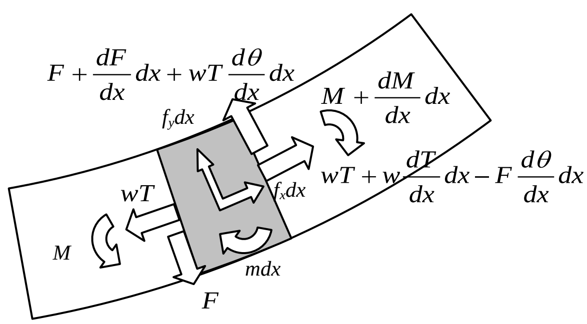


Figure 10 – Forces and moments acting on a small length element of web slipping on a roller.

The set of equations (18 to 23) has to be solved to determine the beam shape and the variation of tension. They are first order, non-linear ordinary differential equations. It would appear that if all the variables are known at one location x, the equations can be integrated exactly either forwards or backwards. This is similar to the behavior of tension in a stick zone without any lateral effects.

Exit Microslip Zone

Changing tension and steering the web in the exit span require the web to change MD tensile and shear strains from their values in the stick zone before it leaves the roller. If this happens in a full width microslip zone, then equations 18 to 23 should apply. In the simple case of a tension increase, the web strain increases and therefore it moves forward relative to the roller surface, generating a friction force acting backwards on the web. This is resisted by a positive gradient of tension, consistent with the initial assumption. Similar arguments hold for the change in bending moment and shear stress: the web deformation associated with them generates movement relative to the roller surface and friction force opposing the change. Therefore the tension, bending moment and shear stress can all move away from the values they held in the stick zone.

If all points across the width start to move at the same MD location, there is a transition from the stick to a full width microslip zone. By analogy with the case of MD slippage only, a transition from zero to a finite gradient of tension, shear and bending moment is expected. Considering tension and shear only, their variations can be written:

$$\begin{aligned} T &= T_0 + \alpha x \\ F &= F_0 + \beta x \end{aligned} \qquad \{24\}$$

where T_0 and F_0 are the values in the stick zone, which lies at x<0. In the very first part of the microslip zone, the slope is zero, and the coordinates x,y and x',y' coincide. The web velocity is

$$v = V\left(1 + \frac{\alpha x}{Eh}\right) \qquad \{25\}$$

and it is directed at angle given by the change in shear strain, i.e.

$$\theta = \frac{\beta x}{nAG} \qquad \{26\}$$

Applying equation 22 for the direction of relative movement gives:

$$\begin{aligned} u \sin\phi &= \frac{V\beta x}{nAG} \\ u \cos\phi &= \frac{V\alpha x}{Eh} \end{aligned} \qquad \{27\}$$

Dividing these two yields:

$$\tan\phi = \frac{\beta Eh}{\alpha nAG} \qquad \{28\}$$

All points across the web move in the same direction, so the friction forces are:

$$f_x = -\frac{\mu T w}{R}\cos\phi$$
$$f_y = -\frac{\mu T w}{R}\sin\phi \qquad \{29\}$$

Equation 23 gives the tension and shear force gradient produced by these friction forces. Substituting in and dividing the two equations gives:

$$\tan\phi = \frac{\beta\left(1 + wT_0 / nAG\right)}{\alpha w\left(1 - \beta F_0 / nAG\alpha w\right)} \qquad \{30\}$$

There are only four values of ϕ that satisfy both equations 28 and 30. For zero shear in the stick zone, $F_0 = 0$ and the solutions are $\phi = 0$, 90, 180 or 270 deg. In other words, the web can only start to move by slipping in the MD or TD but not at an intermediate angle. If it slips at any other angle, the friction force generated is not directed against the motion. A combination of tension change and bending is possible, but a change in shear force must happen without any tension of change or bending.

Once full width movement starts, it will continue, prescribed entirely by equations 18 to 23. If the initial movement is in the MD, part forward and part backward to initiate tension and bending moment changes, then those will grow up to the last contact on the roller. It will be possible to select the point where the initial movement changes direction and the MD length of the microslip zone to match any combination of tension and bending moment at the roller exit. However, for each combination, the web slope and shear are fixed. As discussed earlier, the following span requires the web also to have specific values of slope and shear for a given bending moment. These two requirements are in general incompatible. It demonstrates that a stick zone followed by a full width microslip zone cannot be a full description of the contact patch: the only additional possibility is a partial width microslip zone.

Entry Microslip Zone

Similar reasoning can be applied to a possible full width microslip zone extending from initial contact to the start of the stick zone. If only web tension is changing, slip is in the MD and the friction force tends to increase the deviation from the speed matching tension. The tension will therefore never reach the value in the stick zone, as has been previous argued. A similar argument applies for changes in bending moment and shear force considered alone. It appears that the entry microslip zone will not exist. However, it may be that shear force close to the maximum value F_{max} can combine with the friction couple m to give a bending moment gradient in the same direction as the latter. Again, a partial width microslip zone is a possibility. Further thought is needed here.

PARTIAL WIDTH MICROSLIP ZONE

The reasoning of the previous sections showed that a partial width slip zone must be present just after the stick zone if the web is steered in the following span, and may be needed just before if there is steering coming onto the roller.

Partial width slip zones have been used to explain the phenomenon of "moment transfer". Isolation of steering effects on the roller is lost, and steering in the following span produces steering in the opposite direction in the preceding span. There is no full width stick zone, and the partial width slip zone extends over the whole wrap (figure 11). Once stick conditions occur across part of the width, the whole of the web is constrained to move in the same direction as the roller surface, i.e. normal entry is obeyed. In the analysis of Dobbs and Kedl [6], the friction force has the effect of changing MD stress only. Shear stresses to maintain equilibrium on the roller (equations 2 and 3) are ignored, but shear forces are considered in the free spans. Analyses of Young et al [12] and Good [13] assume the roller imparts a change in bending moment, also ignoring the effect of shear stress. Nevertheless, all 3 studies resulted in models that gave reasonable agreement with experiments. It will be interesting to develop the current work to a stage where its predictions can be tested against the same data.

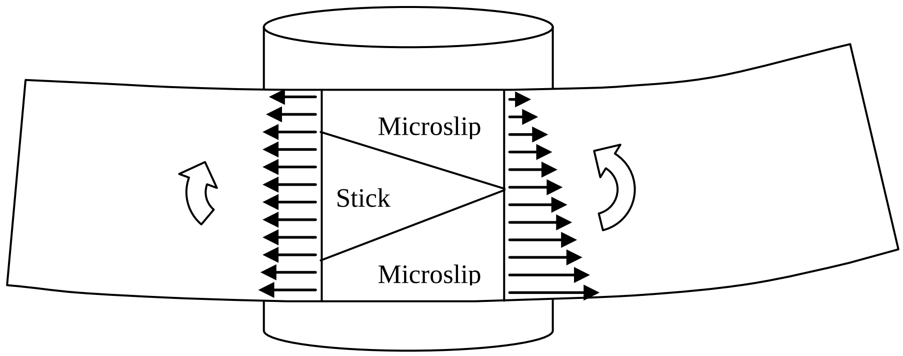


Figure 11 – Proposed tension distributions at the start and end of contact when moment transfer occurs, together with the location of the stick and microslip zones, after [6].

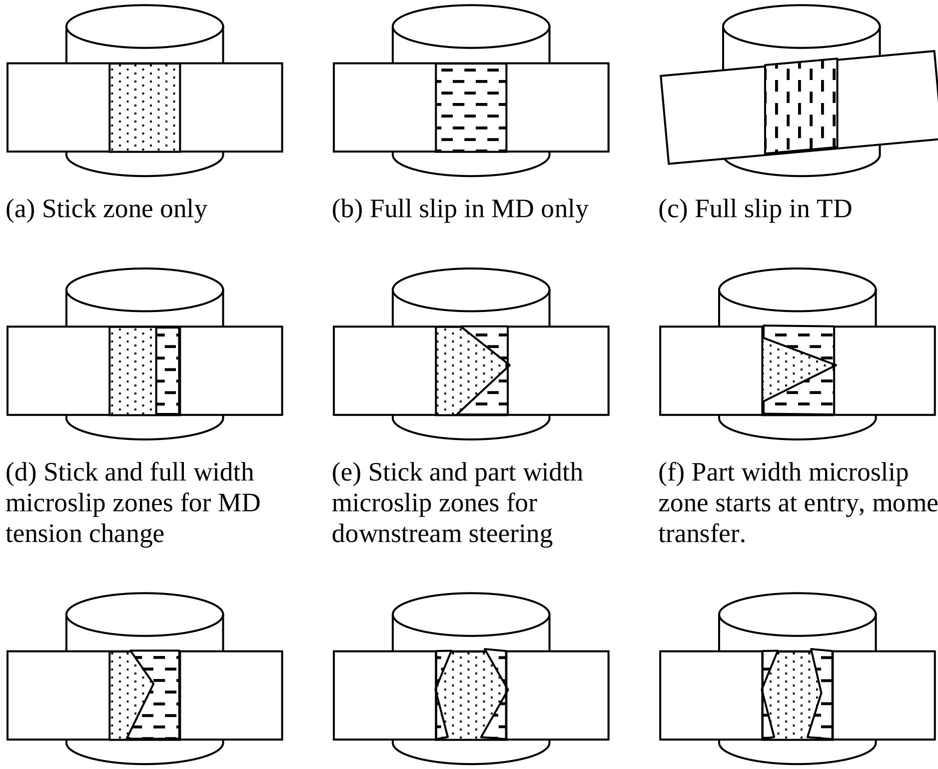
POSSIBLE CONTACT BEHAVIOUR

Based on the above reasoning, figure 12 shows the possible arrangements of stick, microslip and full slip zones in the contact patch.

ANALYSIS OF MULTIPLE ROLLERS AND SPANS

With MD tension variations only in an elastic web, it is possible to predict the variation of tension in free spans and on rollers even when some rollers are in the full slip condition [2]. A search for the correct speed match condition to satisfy tension conditions is needed, similar to on a roller where the web changes temperature [14]. It would be useful to model the web behavior when web steering forces act, alone, or in conjunction with tension changes. However, the situation is more complicated than in 1 dimension.

In addition to web tension, the direction of web travel, shear force and bending moment must match at all boundaries: between free span and web as it touches and leaves the roller, and at boundaries between microslip and stick zones on the roller. The nonlinear nature of the equations for the microslip zones suggest that a numerical solution will be required, either using the beam model or a full 2D model if partial width zones need to be considered. Finding a method to satisfy all the conditions simultaneously appear daunting at this stage.



(a) Stick zone only

(b) Full slip in MD only

(c) Full slip in TD

(d) Stick and full width microslip zones for MD tension change

(e) Stick and part width microslip zones for downstream steering

(f) Part width microslip zone starts at entry, moment transfer.

(g) Stick, part and full width slip zones

(i) Entry microslip, stick, part and full width microslip zones

Figure 12 – Possible configurations of stick, microslip and full slip zones (schematic).

CONCLUSIONS

When a web wraps a roller, there are 3 possible steady state friction behaviors.

1. With a frictionless roller and no steering in either upstream or downstream span, the web and roller velocities match at all points and the stick zone extends over the whole wrap.
2. With low braking or driving of the roller or steering, there is velocity matching over part of the wrap (the stick zone) and microslip in the remainder, where the web gradually changes strain by moving slightly over the roller surface.
3. At sufficiently high roller torque or steering, a slip zone extends over the whole wrap. The relative velocity of web and roller is never zero and can be large.

In a slip or microslip zone, friction force acts in the opposite direction of relative motion. In the stick zone, web stress does not change with position. Friction force may act to maintain the stress distribution, in particular shear stresses generated by steering in the previous span.

There is a limit to the shear that can be sustained in a stick zone. If steering in the upstream span generates less than the maximum shear, the stick zone may commence when the web first touches the roller and the normal entry rule applies. However, if normal entry would lead to a shear stress greater than maximum, then an entry microslip or full wrap slip zone must form. The maximum shear in a stick zone is lower than that for a plain roller for cambered web, baggy edges, or a crowned roller. These may lose traction in steering sooner, especially if the web is close to the point of slackness. On the other hand, the maximum shear is higher for baggy centre or a concave roller, when the web should behave better.

A beam model for the web may be applicable when the full web width is sliding. The angle, curvature, shear force and tension in the beam at any one point around the wrap determine how those variables change with position. The values at the start and end of the wrap must match those in the adjacent spans.

If the web is steered in the downstream span, shear force and bending moment are set up in the web as it leaves the roller. The change from the values in the earlier stick zone must occur by microslip over the last part of the wrap. However, the requirement to match the downstream span leads to the conclusion that a partial width slip zone is normally present over at least part of the wrap, and the beam model will be inadequate.

Further work is needed to understand the microslip zones at entry and exit, and to make some of the conclusions more rigorous.

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