WEB TENSION VARIATIONS CAUSED BY TEMPERATURE CHANGES AND SLIP ON ROLLERS

By

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ABSTRACT

Webs are frequently heated or cooled on rollers, where thermal expansion or contraction attempts to reduce or increase tension respectively. In the case of vacuum coating, the web first cools on the chilled drum, then is heated by coating deposition and radiation, and finally cools before exit. Friction between the web and drum surface may be unable to sustain the tension gradient that the temperature profile would produce in the web moving at constant speed. In that case, zones of microslip exist over at least part of the wrap, possibly including the entry region.

A method of calculating the tension profile around the roller or drum for an elastic web in steady state will be presented, using simple friction laws. The speed difference between web and drum surface is determined by iteration, working backwards from the exit tension until the correct entry tension is attained. However, forward integration is necessary if the web is not elastic, or the heat transfer is affected by the tension or the amount of slip.

The calculation shows that the web may be slipping over a large proportion of the contact area. In some cases, the speed matching is only momentary. Practically, this may lead to difficulties in controlling tension or speed. The model also allows the cross-web direction stress to be estimated. In a region of heating, this often becomes compressive, and can be compared with a critical value for buckling. Exceeding this level would lead to wrinkle formation in the hot, soft web, usually a serious quality problem. The model can therefore be used to explore process design and conditions to reduce the likelihood of wrinkling and improve the ability to control tension and speed.

NOMENCLATURE

CLTE	Coefficient of linear thermal expansion, α
MD	Machine Direction, i.e. along direction of travel
TD	Transverse Direction, i.e. across the web
TSF	Traction Safety Factor
E	Young's modulus of the web
E'	Web stiffness (equation 4)
f	Friction force per unit area
g_i	See equation 23
h	Web thickness
i	Step index in the MD
k	Creeping friction parameter (equation 24)
L_n	Characteristic length of exponential temperature change in region n
n	Index for heating or cooling region
p	Nominal contact pressure between web and roller
R	Roller radius
x, x_i	MD coordinate
x_n	MD coordinate at the start of heating or cooling zone n
y	TD coordinate, width of edge microslip zone (equation 25)
α	Coefficient of linear thermal expansion, CLTE
α'	Coefficient of MD thermal expansion with TD constraint (equation 4)
γ	Distance of transition from end of step (equation 21)
Δ	Discretized distance interval
δ	Distance of transition from beginning of step (equation 18)
$\delta\varepsilon_x$, etc	Change in ε_x etc.
ε_x, ε_i	MD strain
ε_y	TD strain
ε_0	Entry MD strain
ε_m	MD "matching strain" when web and roller speeds are equal
θ, θ_i	Temperature
θ_{n1}	Initial temperature in heating region n
θ_{na}	Temperature asymptote in heating region n
μ	Coefficient of friction
ν	Poisson's ratio of web
σ_x	MD stress
σ_y	TD stress
σ_{crit}	Critical TD stress for buckling on the roller

INTRODUCTION

Continuous, flexible web materials are transported through machines using rollers in nearly all cases. These impart a friction force, or "traction" to the web, and hence affect its tension and lateral position. They also influence undesirable deformation normal to the web, such as bagginess, wrinkling, curl and cockle, both on the rollers and in the free spans between them.

In steady state, the effect of traction on tension in an elastic web is well-understood, and was reviewed by the author at an earlier conference [1]. The contact area consists of two zones, as shown in Figure 1. In the first, the tension remains at the value in the preceding span, and the web travels at the same speed as the drum surface without any relative movement. This is often called the "stick" or "adhesion" zone, although the bonding of the surfaces is not required. In the second "microslip" or "creep" zone, the web tension changes slowly with a gradient dependent on the friction properties of the interface. The tension rises or falls continuously so that its value where the web leaves the roller is equal to that in the following span. A tension increase necessarily leads to a strain and hence speed increase in an elastic web. Therefore, each point on the web moves slightly ahead of its original point of contact on the roller surface through the whole of this microslip zone. Correspondingly, a falling tension leads to a lower web speed and a small retardation of the web relative to the surface.

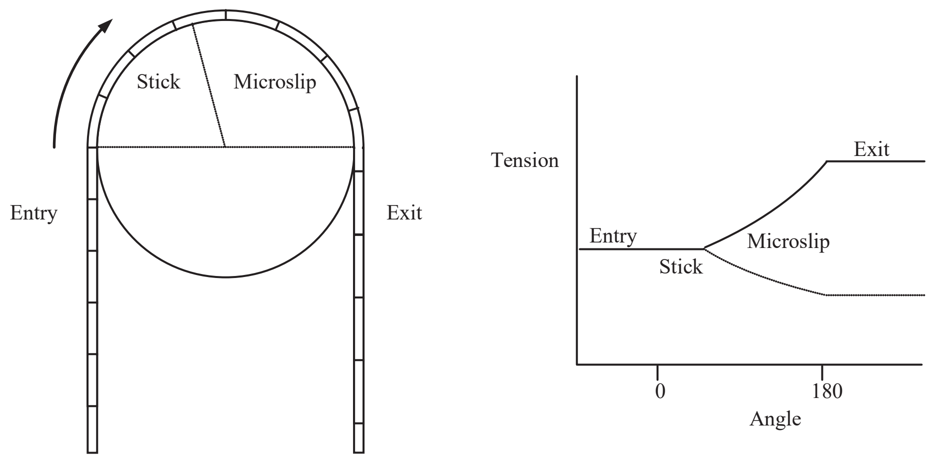

Figure 1 – Elastic web moving over roller showing stick zone and strain increase in the microslip zone. Tension increasing (solid) and decreasing (dotted).

Changes in speed are governed by the tension change and the web elasticity, and are generally below 1% for stiff webs. The relative movement while in contact is small (normally below 1 mm), but can be enough to mark sensitive webs with an inappropriate choice of roller surface and generate wear debris.

Many manufacturing and converting processes run with webs whose behavior is more complex. Heating and cooling an elastic web normally produce expansion and contraction respectively, or a fall or rise in tension if the web is held fixed. Uptake and loss of moisture in paper webs have similar effects.

This paper examines the behavior of the web-roller system under these conditions, with the goal of calculating the machine direction (MD) tension variation over the length of contact. The web properties and changing conditions may attempt to impose a tension gradient in the stick zone that is larger than the web-roller friction can support, triggering microslip. In addition to the MD tension behavior, the stresses in the transverse direction (TD) are important in forming or suppressing wrinkles on the roller. These can sometimes be seen on heating rollers, and are a frequent problem in vacuum metalizing. The MD and TD stresses in web are linked through Poisson's ratio, so knowledge of the MD tension behavior is needed to predict wrinkling.

THEORY

Web Constitutive Equations

Hooke's Law including thermal expansion in 2 dimensions links small increments in the MD and TD stress $\delta\sigma_x$ and $\delta\sigma_y$, strain $\delta\varepsilon_x$ and $\delta\varepsilon_y$, and temperature $\delta\theta$.

$$\begin{aligned} \delta\varepsilon_x &= \frac{1}{E}\left(\delta\sigma_x - \nu\delta\sigma_y\right) + \alpha\delta\theta \\ \delta\varepsilon_y &= \frac{1}{E}\left(\delta\sigma_y - \nu\delta\sigma_x\right) + \alpha\delta\theta \end{aligned} \qquad \{1\}$$

Young's modulus E, Poisson's ratio ν and coefficient of linear thermal expansion (CLTE) α are taken to be constant; it should be straightforward to include temperature dependence and anisotropy if required.

It is assumed that $\delta\varepsilon_y$ =0 and the web is sufficiently wide that the central region dominates the behavior. Therefore, stress and strain vary only with machine direction coordinate x, and equation 1 reduces to:

$$\frac{d\varepsilon_x}{dx} = \frac{1}{E'}\frac{d\sigma_x}{dx} + \alpha'\frac{d\theta}{dx} \qquad \{2\}$$

$$\frac{d\sigma_y}{dx} = \nu\frac{d\sigma_x}{dx} - \alpha E\frac{d\theta}{dx} \qquad \{3\}$$

The stiffness E' and effective CLTE α' in equation 2 take account of the coupling between MD and TD, and are given by:

$$E' = \frac{E}{1-\nu^2} \qquad \alpha' = \alpha\left(1+\nu\right) \qquad \{4\}$$

Temperature Calculation

The web temperature may change as a result of conduction from a heated or cooled roller, or there may be regions of heating or cooling where the web is exposed to nozzles supplying hot or cold air, infrared heating elements, liquid sprays, vapor or plasma deposition. The calculations described here assume that the temperature varies only in the MD. Although any continuous temperature curve can be input, a particular simple heat transfer model was used.

This assumes constant heat input in each region, temperature-independent specific heat capacity and heat transfer coefficient to the roller at constant temperature, leading to web temperature exponentially approaching an asymptote:

$$\theta = \theta_{n1} + (\theta_{na} - \theta_{n1})\left\{1 - \exp\left(-\frac{x - x_n}{L_n}\right)\right\} \qquad \{5\}$$

θ_{n1} is the temperature at the start of region n, located at MD coordinate x_n. The temperature approaches the asymptotic value θ_{na}, with characteristic length L_n. The final profile is made up from a number of regions in sequence.

Coulomb Friction Behavior

The web-roller contact pressure p is generated by the MD tension in the web of thickness h acting over the curved surface of the roller, radius R, as shown in figure 2.

$$p = \frac{\sigma_x h}{R} \tag{6}$$

Steady state conditions, at practical speeds where inertial forces are negligible, result in a balance between the MD stress gradient and the friction force per unit area f acting on the web in the positive x-direction:

$$f = -h \frac{d\sigma_x}{dx} \tag{7}$$

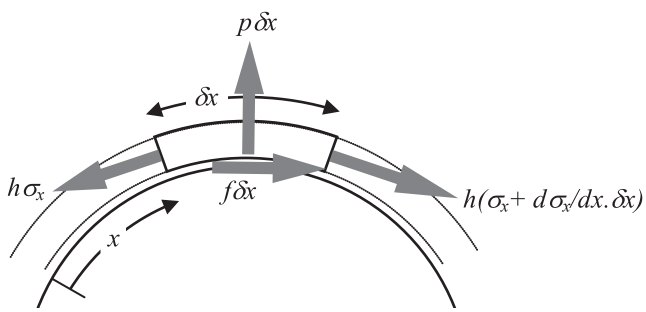

Figure 2 – Forces acting on an element of web length δx.

The Coulomb model assumes a single coefficient of friction μ. In any stick zone regions, the web and roller are travelling at the same speed, and the web has a constant "matching strain" ε_m. The friction force can take any value between limits:

$$-\mu p \le f \le +\mu p \tag{8}$$

In the simpler case of an elastic web without temperature change, f=0 throughout the stick zone.

In the microslip zones, friction always acts to oppose the relative motion between web and roller. If the web is faster than the roller:

$$\varepsilon_x > \varepsilon_m \quad f = -\mu p \tag{9}$$

On the other hand, if it is slower, then:

$$\varepsilon_x < \varepsilon_m \quad f = +\mu p \tag{10}$$

Differential Equations

In any microslip zones, equations 6, 7 and 9 or 10 can be combined to give:

$$\frac{d\sigma_x}{dx} = \pm \frac{\mu \sigma_x}{R} \tag{11}$$

In this and subsequent equations, the upper sign refers to the web moving faster than the roller, and the lower to the web moving slower. Interestingly, a temperature variation has no effect on MD stress in the microslip zone. Equation 11 has the well-known exponential solutions:

$$\sigma_x \propto \exp\left(\pm \frac{\mu x}{R}\right) \tag{12}$$

The coefficient of proportionality in equation 12 is determined by the boundary conditions. When the microslip zone extends over the whole of the wrap, this equation is known as the belt, capstan or Eytenweil equation, and relates the entry and exit tensions.

In the stick zone(s), the MD stress is completely determined by the temperature according to equation 2, with no strain change:

$$\frac{d\sigma_x}{dx} = -E'\alpha' \frac{d\theta}{dx} \tag{13}$$

The friction force is limited by equation 8, which imposes bounds on the stress gradient and hence temperature gradient in a stick zone:

$$-\frac{\mu \sigma_x}{R} \le \frac{d\sigma_x}{dx} \le \frac{\mu \sigma_x}{R} \tag{14}$$

$$-\frac{\mu \sigma_x}{E'\alpha' R} \le \frac{d\theta}{dx} \le \frac{\mu \sigma_x}{E'\alpha' R} \tag{15}$$

Any point where the temperature gradient lies outside these limits must be located in a microslip zone.

Transitions between Stick and Microslip

As the web moves in steady state, it passes between stick and microslip zones at transition points which are fixed in space. There are 3 possible transitions, considered in the direction of web travel.

Microslip to stick transition. This potentially occurs when the web strain becomes equal to ε_m. For this point of speed matching to be followed by a stick zone, the web must be able to travel at the same speed as the roller with the tension gradient lying within the limits set by equation 14. Thus, equation 15 must also be satisfied immediately after the point of speed matching for a finite length stick zone to form.

Reversal of the microslip direction. If the web strain becomes equal to ε_m but the bounds of equation 15 are exceeded, then a stick zone is impossible. The strain crosses the value ε_m and so the direction of web-roller movement reverses. Therefore, there is an immediate transition between microslip in opposite directions, with a reversal of the tension gradient at the transition point.

Stick to microslip transition. A stick zone can come to an end for one of 2 reasons:

1. The tension must start to change to reach a downstream value. This could be the exit tension, initiating a microslip zone with exponential tension change (as in the case of the elastic, isothermal web). Alternatively, there could be a stick zone later in the contact, again causing a tension change to propagate upstream in a microslip zone. The upstream propagation of tension can occur with a single exponential tension segment, but multiple segments with changes in microslip direction can also occur.
2. Friction is inadequate to support the tension gradient that thermal expansion imposes, i.e. the bounds of equation 15 are exceeded. The sign of the tension gradient reverses at the transition point.

The first zone. If the temperature gradient on entry is within the bounds of equation 15, an initial stick zone is possible but not inevitable. If one is present, there is speed matching and the incoming web strain equals ε_m, which has been termed the "normal strain rule" [2].

An initial microslip zone will be present if the temperature gradient is larger than the bounds, and may be even if it is not. This initial microslip zone may only extend over the initial part of the wrap, as the temperature profile may cause a transition to a stick zone or a reversal of the microslip direction, as described above. This is different from the isothermal case, where initial microslip cannot be arrested later in the contact and total slip occurs. As the later examples show, it is sometimes impossible to satisfy the boundary conditions unless an initial microslip zone exists.

Boundary Conditions

There are several methods of controlling web and roller speeds and tension in the entry and exit spans. Heating rollers are often driven at a set torque, for example to overcome friction of bearings and rotary unions. The web speed is controlled elsewhere, and either entry or exit tension is set, with the other given by a torque balance on the roller. In this case the roller speed finds its own level.

A vacuum metalizing system normally has the drum in speed control, with closed loop control of tension in the entry and exit spans. The torque on the drum is determined by the tension difference. The web speed finds its own level to produce the correct tension change as it moves through the stick and microslip zones on the drum. At constant temperature, an elastic web would match the drum speed on entry, but that may not be the case with temperature changes on the drum.

The boundary conditions in these cases consist of known entry and exit tensions. The matching strain ε_m is unknown and must be determined in the solution. Either web or roller speed is specified; the other is then found using ε_m.

The initial TD stress must also be specified. If the entry span is long, it is expected to be zero. A positive value might be used to check the effect of a spreading device immediately before the roller, whereas a negative value could be used to simulate the effect of troughs or wrinkles in the entry span. The actual value gives the initial levels of strain ε_x and ε_y.

Solution Method

The tension and strain as a function of MD position x can be found by an iterative procedure. First the temperatures are calculated, then the strain components in the exit span. A value of matching strain ε_m is taken, and the tension profile is calculated starting at the exit tension and working back towards the entry. With analytic expressions for temperature and tension, direct integration of the differential equations is possible. However, a numerical procedure allows for more general cases to be treated.

The contact length is discretized into steps of equal length Δ. This must be shorter than the smallest stick or microslip zone. The estimated tension and strain at point i are calculated from those at $i+1$ using the discretized version of equation 2:

$$\varepsilon_i = \varepsilon_{i+1} - \frac{1}{E'}\left(\sigma_{i+1} - \sigma_i\right) - \alpha'\left(\theta_{i+1} - \theta_i\right) \qquad \{16\}$$

This is used to find the stress in a stick zone (with $\varepsilon_i = \varepsilon_{i+1} = \varepsilon_m$), and the strain in a microslip zone where the tension is given by:

$$\sigma_i = \sigma_{i+1} \exp\left(\mp \frac{\mu\Delta}{R}\right) \qquad \{17\}$$

There are two alternative approaches to locating the transitions between zones. The first, adopted here, is to find the exact position by interpolation within a step. The second would be to assume the transition occurs at the end or midpoint of a step; however, this would limit the final solution to an accuracy of order $\mu\Delta/R$ times the entry tension.

Each iteration starts at the exit, then equations 17 and 16 are used to step backwards through the exit microslip zone. This continues as long as each estimated strain (denoted by superscript e) lies the same side of ε_m as the previous one ε_{i+1}.

If it crosses the value ε_m, then web and roller strain match within the step, and the transition location at $x_i+\delta$ is calculated from the strain estimate ε_i^e by linear interpolation:

$$\frac{\delta}{\Delta} = \frac{\varepsilon_m - \varepsilon_i^e}{\varepsilon_{i+1} - \varepsilon_i^e} \qquad \{18\}$$

Next, this transition is tested using equation 15 to check if a stick zone is possible. The temperature gradient is assumed to vary linearly with position between values calculated analytically from equation 5 at points i and $i+1$, and the stress at the transition point is calculated using:

$$\sigma_x = \sigma_i^e \exp\left(\pm \frac{\mu\delta}{R}\right) \qquad \{19\}$$

If the temperature gradient lies within the limits, then a finite stick zone exists, the strain at point i is set equal to ε_m and the stress recalculated with equation 16. If it is outside the limits, there is a change in microslip direction and the stress is replaced by:

$$\sigma_i = \sigma_i^e \exp\left(\pm \frac{2\mu\delta}{R}\right) \qquad \{20\}$$

and the strain recalculated using equation 16.

A similar process is used to step backwards through the stick zone. Equation 15 is used to check whether the stick condition is sustained through the step. If not, the transition point at (x_{i+1}-γ) is found by solving:

$$\frac{\gamma^2 g_i}{2\Delta} + \gamma\left(\frac{d\theta}{dx}\right)_{i+1} = \pm\frac{R}{\mu}\left(\frac{\gamma g_i}{\Delta} + \left(\frac{d\theta}{dx}\right)_{i+1}\right) - \frac{\sigma_{i+1}}{E'\alpha'}$$
$$\text{with} \quad g_i = \left(\frac{d\theta}{dx}\right)_i - \left(\frac{d\theta}{dx}\right)_{i+1} \tag{21}$$

Both sign possibilities must be tested. The next tension value is given by:

$$\sigma_i = \left[\sigma_{i+1} + E'\alpha'\left(\gamma\left(\frac{d\theta}{dx}\right)_{i+1} + \frac{\gamma^2 g_i}{2\Delta}\right)\right]\exp\left(\mp\frac{\mu\gamma}{R}\right) \tag{22}$$

The strain is calculated using equation 16. The next step starts with the assumption of a microslip zone.

The first iteration assumes that the web and roller speeds are matched at the point of entry, i.e. $\varepsilon_m = \varepsilon_0$. If the calculated entry tension does not satisfy the tolerance criterion, a bisection or interval halving method is used to find the solution. A second iteration is carried out with ε_m increased or decreased by 0.01, so that the first two estimates of ε_m bracket the true value. The third is chosen midway between the first two. The required entry tension will lie between the values found with the third and one of the two previous estimates. The other is discarded, and a new ε_m estimate chosen midway between the two retained. The process is repeated, until convergence is achieved.

The numerical method outlined above has been implemented using Visual Basic in Microsoft Excel. Typically, 100 steps along the wrap are used, and around 20 iterations are required for convergence to within 0.1 N/m of the entry tension. Stress values are multiplied by the web thickness to obtain the linear tension in N/m for display.

Transverse Direction Stresses and Wrinkling

Once the MD tension has been found, the TD stress profile is calculated using the discretized form of equation 3. The values can be compared with an estimate of critical stress for buckling. If the calculated stress is more negative (compressive), then it is likely that the web will relieve its strain energy by forming one or more wrinkles, and the compressive stress will fall [3]. The theoretical buckling stress of an end-loaded long, free cylindrical shell is:

$$\sigma_{crit} = -\frac{Eh}{\sqrt{3\left(1-\nu^2\right)}R} \tag{23}$$

This will be used for assessing wrinkling tendency, although it overestimates the buckling load for the shell, but should be increased to allow for the presence of the rigid roller surface and MD web tension [4].

APPLICATION TO HEAT TRANSFER ROLLER

The problem parameters can take a wide range of values. To show some examples of behavior and the scope of the method, examples based on a roller where the web is heated or cooled will be shown.

The roller has radius 0.1 m and angle of wrap 180 degrees. Web of thickness 100 μm, Young's modulus 2 GPa, Poisson's ratio 0.3 and CLTE 17 x 10^{-6} K^{-1} enters at 20 deg. C and 500 N/m tension. The web temperature increases from the entry towards an asymptote with a characteristic length *L* of 0.1 m. Values used are shown in figure 3. The coefficient of friction is taken to be 0.3. The exit tension will be varied, but the belt equation places limits on the maximum and minimum exit tension that can be achieved, of 1283 and 195 N/m respectively.

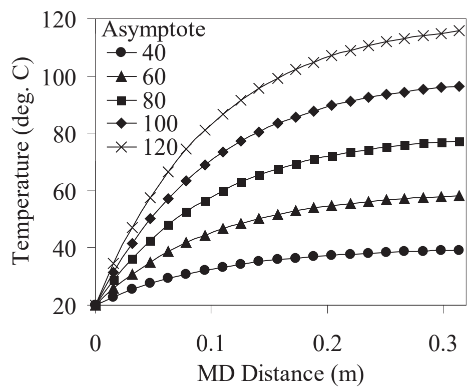

Figure 3: Calculated web temperature profiles for asymptotes (deg. C) in the legend.

<u>Temperature Gradient below Critical.</u>

For a temperature asymptote of 40 deg. C, the maximum temperature gradient is 200 deg./m, and is located at the entry point. The limiting value is 310 deg./m according to equation 15, so the web is able to form a stick zone at the entry.

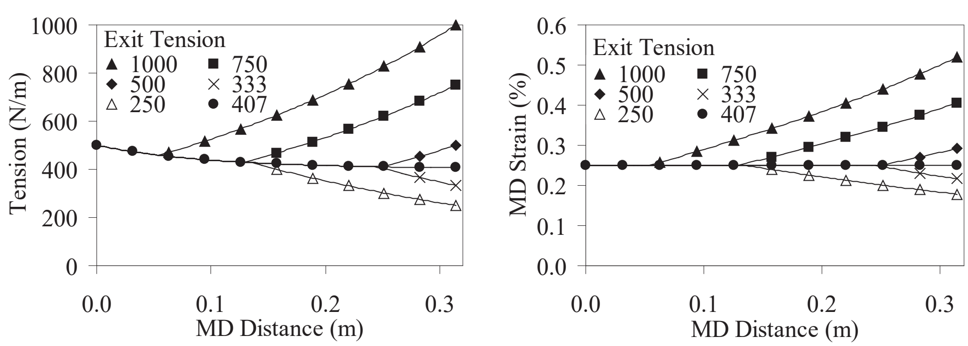

Figure 4 – Variation of tension and MD strain in web entering a roller at 20 deg. C and 500 N/m, and approaching 40 deg. C, for increasing exit tensions in the legend.

The variation of tension and strain around the roller for different values of the exit tension is very similar to an elastic web at constant temperature, as expected. A stick zone starting at the entry is followed by a microslip zone up to the exit. However, there are several differences from the constant temperature case:

1. Tension is no longer constant in the stick zone
2. There is now a microslip zone if the entry and exit tensions are equal, because the web is hotter when it leaves the roller.
3. The stick zone extends over the whole wrap if the exit tension is 407 N/m, not for equal entry and exit tensions.
4. The transition between stick and microslip occurs at different locations, and also is different for tension increasing or decreasing by the same ratio.

Increasing Temperature Gradient

If the temperature gradient exceeds the value for a stick zone on entry, a microslip zone appears there. As the temperature gradient increases, this microslip zone becomes larger, as shown in figure 5. In it, the web is moving slower than the roller, and tension falls exponentially. A stick zone is located partway through the contact for all cases except the highest temperature of 120 deg. C. In that case, there is only a transition in the direction of microslip. The matching strain (and roller speed) is larger for the higher asymptotes.

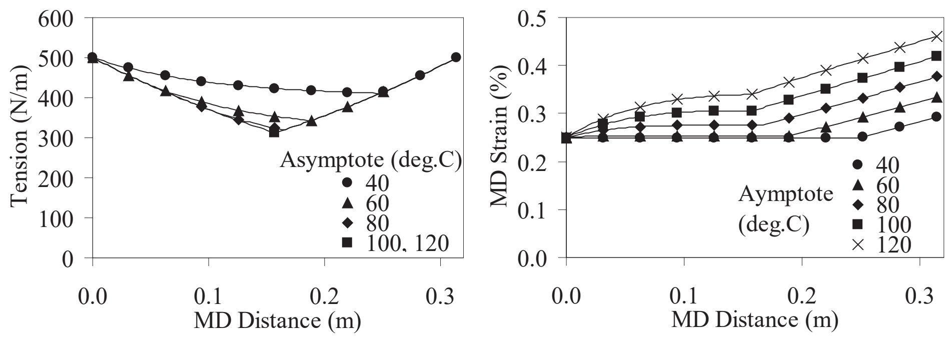

Figure 5 – Tension and strain variation for web on a roller with temperatures rising from 20 deg. C to the asymptotes shown. Entry and exit tensions are set at 500 N/m.

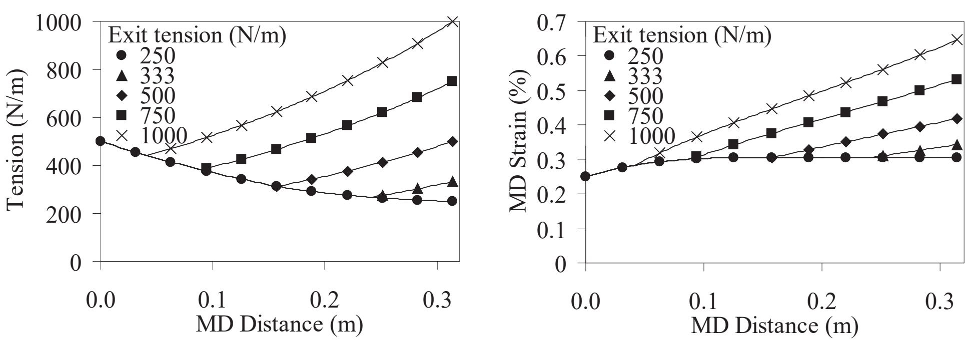

Figure 6 – Tension and strain variation for different exit tensions and a temperature asymptote of 100 deg. C.

Figure 6 shows the effect of varying exit tension for an asymptote of 100 deg. C. The 500 N/m curve is the same as on figure 5. All have the initial microslip zone. At the lowest tension of 250 N/m, the subsequent stick zone extends to the exit. This is a curious case where the order of stick and microslip zones is reversed.

As the exit tension is increased, the exit microslip zone appears and increases in extent. The tension and strain curves are the same for all tensions until the transition into the exit microslip zone. The matching strain value is 0.00305 for tensions of 500 N/m and below, where there is a stick zone.

At higher tensions, there is no stick zone and the transition to microslip in the opposite direction occurs earlier on the roller for higher tension. The matching strain values decrease with increasing tension: 0.00301 for 750 N/m and 0.00283 for 1000 N/m.

Figure 7 shows the effect of changing entry tension for a constant exit tension of 500 N/m. All curves show a microslip zone at the entry. There is a stick zone for entry tensions of 500 N/m and above. The transition to microslip in the opposite direction comes earlier for the lower entry tensions. The matching strain is close to the MD strain on entry, and so is approximately proportional to entry tension. The strain at the exit increases slightly with entry tension, even though the exit tensions are the same. This is because the entry TD stress is set to zero; the TD strain remains constant along the contact but is more negative for the higher entry tensions, and affects the MD strain through equation 1.

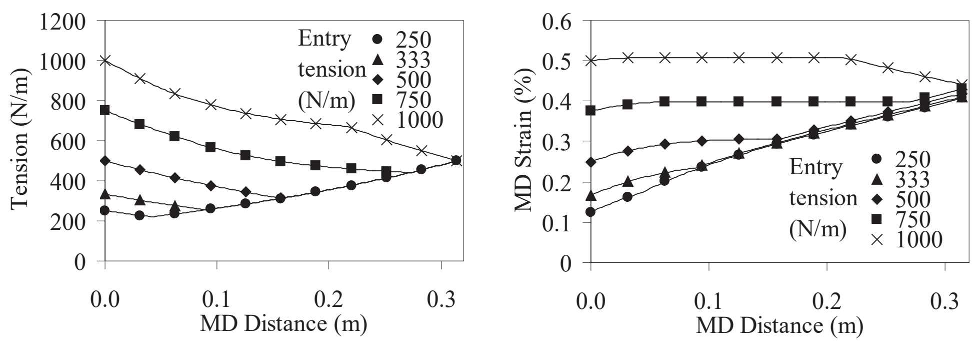

Figure 7 – Effect of changing entry tension for exit tension of 500 N/m and web temperature asymptote of 100 deg. C.

<u>Cooling Roller</u>

Cooling produces an increase in tension over the roller, as shown in figure 8. Entry and exit tensions are fixed at 500 N/m, and the entry temperature is 100 deg. C. At the highest temperature asymptote, 80 deg. C, there is a stick zone at the entry, extending over most of the roller, followed by a small microslip zone at the exit. All the other cases, with greater cooling, show a microslip zone at the roller entry, a stick zone and a final microslip zone. The matching strain falls as temperature is lowered. Comparing with figure 5 shows longer stick zones at the same temperature change, because the higher tension caused by cooling can support a larger temperature gradient in the stick zone.

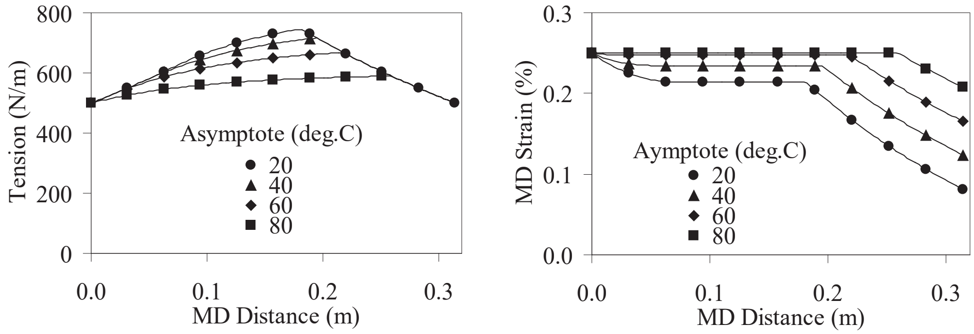

Figure 8 – Tension and strain changes over a roller where the web cools from 100 deg. C to the temperatures indicated, with entry and exit tensions of 500 N/m.

TD Tension

Heating the web on a roller tends to produce compression (negative tension) in the TD, with increased likelihood of wrinkles and creases. For the case of equal entry and exit MD tensions of 500 N/m, and temperature rising from 20 to an asymptote of 100 deg. C, a negative TD tension is produced, with a maximum of 280 N/m (a compressive stress of 2.8 MPa). Figure 9 shows the variation of TD tension through the contact for this case and others chosen to reduce the compression. The compression can be reduced to around 200 N/m by halving the entry tension or doubling the exit tension. However, this change is approaching the friction limit of the roller.

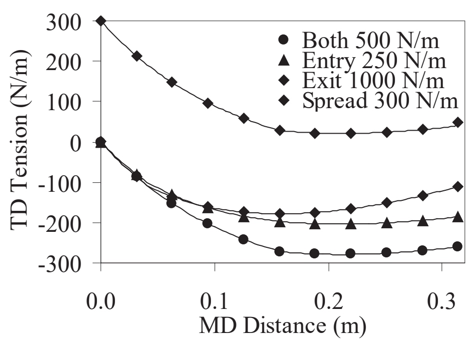

Figure 9 – Variation of TD tension for temperature increasing from 20 to 100 deg. C with both MD entry and exit tensions of 500 N/m; and the individual effects of lowering entry tension, raising exit tension, and applying a TD entry tension by spreading.

The critical stress for buckling is -1.2 MPa (a tension of -120 N/m) according to equation 23. Neither reducing entry tension nor raising exit tension can prevent the critical stress being reached; therefore neither will be effective at preventing wrinkles.

The compression can be removed by applying a TD tension at the roll entry. Applying 300 N/m keeps the TD tension positive throughout. The MD tension profile is unchanged by this, but the MD strain curve is shifted downwards by a constant amount, and the matching strain is reduced accordingly. TD tension may therefore prevent wrinkles; but a sufficient value may be unrealistic, as described in the Discussion.

AN ALTERNATIVE FRICTION MODEL

A "creeping friction" model, where the coefficient is dependent on the sliding velocity, offers some advantages in modeling this problem. The need to locate and apply all the various tests at transition points disappears, and the stepping process can be carried out in either direction. Figure 10 shows the dependence of friction force on the difference between web MD strain and the "matching strain", which is equivalent to the fractional velocity difference.

$$\begin{aligned} f &= \mu p & (\varepsilon_x - \varepsilon_m) &< -k \\ f &= -\mu p(\varepsilon_x - \varepsilon_m)/k & -k < (\varepsilon_x - \varepsilon_m) &< k \\ f &= -\mu p & (\varepsilon_x - \varepsilon_m) &> k \end{aligned} \quad \{24\}$$

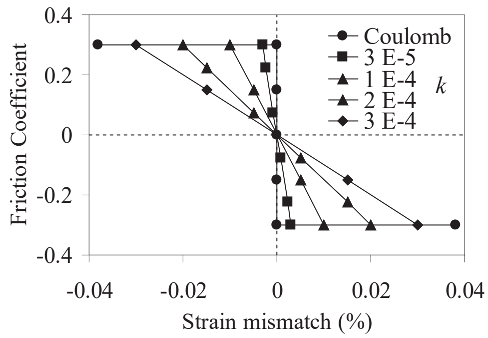

Figure 10 – Creeping friction model showing friction coefficient as a function of strain mismatch ε_x - ε_m for different values of *k*.

Figure 11 shows tension profiles calculated for several *k* values, using the example of web heating from 20 to 60 deg. C. A forward stepping procedure was used. Values of *k* below 2 x 10^{-5} gave convergence problems.

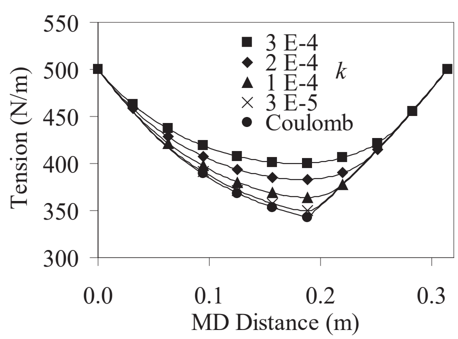

Figure 11 – Tension curves for different values of the creeping friction parameter *k*. Web temperature increases from 20 towards 60 deg. C.

Low values of *k* give tension curves closer to the Coulomb model prediction, at the risk of numerical difficulties. The curves for larger *k* deviate increasingly: the discontinuities in tension gradient are smoothed out. There is no extended stick zone, as small relative motion occurs throughout when there is a friction force transmitted.

APPLICATION TO VACUUM METALIZING DRUM

In a vacuum metalizer, hot metal atoms are directed at the web moving over a chilled drum. The first and last zones of the wrap are normally shielded from the source. The temperature of the web falls in the first region, rises under the heat load of the condensing metal and radiation in the deposition region, and then falls in the final region before leaving the drum. Methods of calculating temperature are described in [5].

Figure 12 shows 2 representative temperature curves, for a single deposition region and for 6 magnetrons delivering the same total power. The heating starts at 0.067m and ends at 1.21m, 0.133m prior to exit. In the second case, the heat input occurs over 0.133m lengths separated by 0.067m where the web cools. The characteristic length of the exponential rise to steady state is 0.269m. The upper temperature asymptote is 135 deg. C for the single region case and 200 deg. C for each of the multiple regions, the lower temperature asymptote -20 deg. C, and web enters at 20 deg. C.

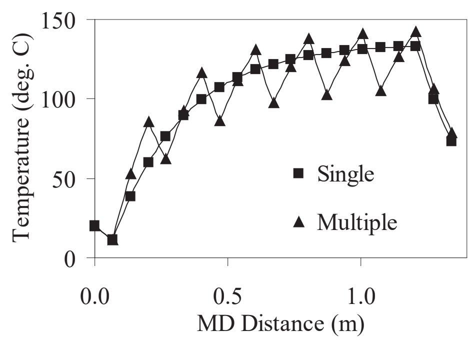

Figure 12 – Temperature profiles for web on drum with single and multiple heating regions

The drum diameter is 0.7 m, with 220 degrees of wrap (1.34m). The 15 µm thick web has modulus 2.06 GPa, Poisson's ratio 0.38, CLTE of 17 x 10^{-6} K^{-1} and coefficient of friction 0.7. Entry and exit tensions are 70 and 100 N/m respectively.

The model results (figure 13) show that the whole of the wrap is in the microslip condition in both cases. The MD tension falls initially, contrary to the expectation of thermal contraction. It is lowest near the midpoint of the wrap, and finally rises to the exit value. The TD tension falls to a minimum value of -70 N/m. The calculated critical stress for buckling is only 0.8 N/m, therefore wrinkles would be expected. In the single heating region case, the MD strain crosses the value matching the drum speed near the midpoint of the wrap.

The effect of the temperature cycling is apparent in the multiple region case, superposed on the same curves as the single region case. The MD strain gradient and TD tension gradient reverse at the same locations as the temperature gradient. However, the reversal of the MD tension gradient and microslip direction happens at fewer locations (five) which do not coincide with the boundaries of the heating regions.

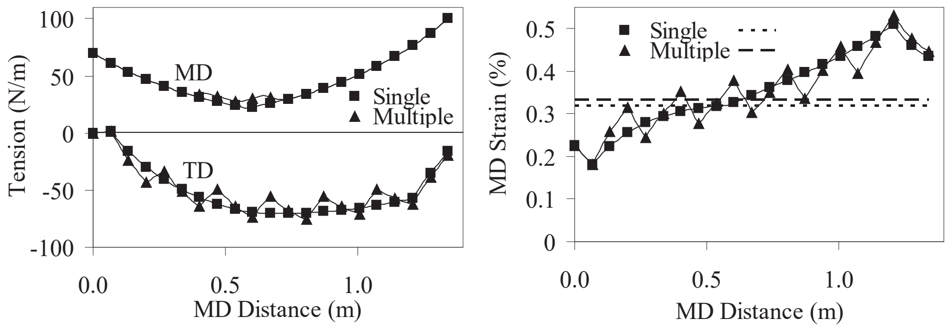

Figure 13 – MD and TD tensions (left) and MD strains (right) for web on a vacuum metalizing drum with single and multiple heating regions. Dashed lines show the matching strain values.

DISCUSSION

MD Tension Profile

The examples show that the traction behavior and tension profile of web on a roller can be altered radically if its temperature changes. Even if the entry and exit tensions are maintained identical, the web can be in a state of microslip over the whole wrap, with speed matching only one MD location.

Control of tension and lateral position of a web is only possible when there is good traction between web and roller: this requires a stick zone to be present. Reducing its extent to a single line may be expected to give control problems, although perhaps not as severe as if there was slip over the whole wrap and no speed matching. Possible problems include greater transmission of tension fluctuations both upstream and downstream, and lateral web movement resulting from tension fluctuations and inconsistent coefficient of friction.

The web-roller friction performance is sometimes assessed with a "traction safety factor", (TSF) [6], an estimate of friction force available divided by the force needed to overcome drag and inertia during speed change. A TSF greater than 2 is acceptable. However, larger TSF's may give unsatisfactory performance if the web temperature changes. A sizeable part of the available friction force is in opposite directions in different parts of the contact, and so not available to overcome drag. This should be considered when designing web lines with heating and cooling rollers.

TD Tension and Wrinkling

The calculations show that large negative TD tensions can develop during heating on the assumption of no lateral expansion or contraction. This compressive stress can be well in excess of the critical value for buckling, and therefore wrinkles are expected.

However, in practice there will be relief of the TD compression near the edges. TD friction forces are required to balance the stress increase from zero at the edge towards the maximum σ_y in a constant width region in the centre, over a distance [3]:

$$y = \left| \frac{R\sigma_y}{\mu\sigma_x} \right| \quad \{25\}$$

The plots of TD tension in figure 9 show a largest negative value of around 300 N/m, at locations where the MD tension has also fallen to a similar value. Hence y is around 0.33 m, and a web of width 0.66 m is required to reach the calculated value at the centre. This is likely to be an underestimate, as the friction force given by equation 8 will be directed at an angle, and only a component will be available to balance stress gradients in the MD and TD. Nevertheless, a significant portion of the web could develop sufficient TD compression for wrinkling.

The "TopWeb" software [7] is being extended to include calculations of heat load on the web from vacuum deposition, the resulting temperature and tension variations, and prediction of wrinkling behavior [8].

Similar Problems

A microslip zone has been shown to form at the entry when an elastic web at constant temperature contacts a roller, if the tension in the previous span is varying rapidly with time [9,10]. In the absence of microslip, this would result in a large tension gradient, resulting from the time variation and transport. Friction cannot sustain this large gradient and an initial microslip zone forms.

The steady-state passage of web over rollers driven at different surface speeds, previously modeled by one of the authors [11], is very similar to the problem modeled here. The roller surface speed variation with distance would be equivalent thermal strains produced by a temperature profile consisting of a series of steps. In [11], the strain matching criterion is that the web speed equals the speed of one of the rollers: the iteration process tests each roller in turn.

More Realistic Material Models

The calculation could be extended to orthotropic elasticity and expansion, and temperature dependent properties and friction coefficient. Irreversible shrinkage as a function of temperature and time could also be included. Both the Coulomb and creeping models of friction could continue to be used.

Many polymeric webs have significant viscoelasticity at typical heating conditions for web processing. At any location, the stress is a function of the previous stress and thermal history, which implies that the backward stepping can no longer be used. The Coulomb friction model could be used with forward stepping to cover the cases where there is a single stick zone, either at the entry or midway through the wrap, and no later change of microslip direction. The downstream end of the stick zone would be located where the tension is the start of an exponential rise or fall (equation 12) reaching the required exit value. Forward stepping would also be needed if the friction coefficient or heat transfer depends on the contact pressure. The creeping friction model should continue to work with forward stepping in these cases, but it is more difficult to identify the extent of the stick zones.

CONCLUSIONS

A method of calculating tension variations in a web along the contact with a roller or drum while it responds to other influences on strain has been devised. The particular model developed treats a thermally expanding elastic web reacting to temperature changes. The tension profile and location of stick and microslip zones are affected by the temperature change. If the temperature gradient lies outside the range where friction can support the resulting tension gradient, additional microslip is triggered. In these cases, the microslip region can extend over most or all of the contact, with speed matching over a small or infinitesimal stick zone.

The web constitutive model is 2-dimensional, and this enables the calculation of the TD stress assuming constant width. This exceeds the theoretical buckling stress and therefore wrinkles will probably form for typical heating conditions. However, it is likely that there will be lower compressive stress than this ideal calculation, as a result of lateral slip, and the buckling criterion may be too severe for tensioned web on a rigid roller.

The stress calculation can be linked with a heat transfer model. This is particularly useful in vacuum metalizing, where the web speed and deposition rate can be changed to reduce the risk of wrinkles and control problems. Model results for realistic temperature profiles reveal that there are no finite stick zones, with possible implications for control.

ACKNOWLEDGEMENTS

The authors would like to thank Dr Charles Bishop for guidance on practical conditions and problems in vacuum deposition onto webs.

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Web Tension Variations Caused by Temperature Changes and Slip on Rollers

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Name & Affiliation	**Question**
Jim Dobbs, 3M Company	The entry spreader condition: Is that reality or a wished for thing?
Name & Affiliation	**Answer**
Dilwyn Jones, Emral Ltd.	I believe a spreader can put an entry spreading tension into the web. I think it is very difficult to know how much because the spreading is always upstream of the entry location. There is obviously some decay in the stress before the entry.
Name & Affiliation	**Question**
Jim Dobbs, 3M Company	That is exactly my question. I don't know if the 300 Newtons per meter was realistic or not.
Name & Affiliation	**Question**
Günther Brandenburg, Technische Universität München	I missed the continuity equation. Did you need it?
Name & Affiliation	**Answer**
Dilwyn Jones, Emral Ltd.	The web strain and velocity are related at all points by the continuity equation. It is implicit in the comments I made relating the difference between web strain and matching strain to the relative speeds of the web and roller.
Name & Affiliation	**Comment**
Günther Brandenburg, Technische Universität München	You have mass flow on the surface and there must be conditions that must be satisfied from the input to the output. The stick and slip must be referenced in the nonlinear continuity equation. It should be 2-dimensional because you are calculating in 2 dimensions.
Name & Affiliation	**Answer**
Dilwyn Jones, Emral Ltd.	This is steady-state behavior. There is no time dependence, so the continuity equation isn't necessary to reach a solution. It would be needed if we were looking at dynamic changes.
Name & Affiliation	**Comment**
Günther Brandenburg, Technische Universität München	In the moving web, you also have steady-state solutions of the continuity equation, so the ratio of the output and input velocities should be equal to the ratio of one plus the strain at output over input.

Name & Affiliation	Answer
Dilwyn Jones, Emral Ltd.	I set the entry tension and exit tension independently in the model. This could be done by adjusting the velocities of a 3 roller model. My focus has been on the behavior of the web coming around the drum. The wrinkles are a steady state problem not a dynamic problem.
Name & Affiliation	**Question**
Unknown	Is the adhesion zone different from the microslip zone in relation to temperature?
Name & Affiliation	**Answer**
Dilwyn Jones, Emral Ltd.	The microslip zone starts on entry, where the temperature gradient is largest in the heating roller case. Later on, the stick zone occurs where the gradient is smaller. The model does not include possible effects of relative movement on temperature, but they could exist and be included. For example, heat transfer coefficient could depend on contact pressure and the sliding velocity. Temperature would change differently in stick and microslip zones.
Name & Affiliation	**Question**
Prabhakar Pagilla, Oklahoma State University	I have a question regarding the Hooke's Law in your model. You have separated the thermal strain from the mechanical strain. At what temperature do you calculate the modulus of elasticity?
Name & Affiliation	**Answer**
Dilwyn Jones, Emral Ltd.	We assumed a constant modulus of elasticity. We could extend the model and allow a temperature dependent modulus so that we could predict the temperature at each step and update the modulus.
Name & Affiliation	**Question**
Prabhakar Pagilla, Oklahoma State University	Can you separate the two effects like that? So you have modulus varying with temperature and you input this into your equation, is that what you are saying?
Name & Affiliation	**Answer**
Dilwyn Jones, Emral Ltd.	At the moment, the modulus is a constant. The assumption of a temperature dependent modulus is not in the model currently.
Name & Affiliation	**Question**
Prabhakar Pagilla, Oklahoma State University	Is the modulus taken at room temperature because I take $\delta\theta$, the temperature change to be large? If $\delta\theta$ is large you must consider that in the equation, right?
Name & Affiliation	**Answer**
Dilwyn Jones, Emral Ltd.	Hooke's Law does not depend on the size of $\delta\theta$. If the properties depend on θ then it is only valid for small increments. This leads to the differential equation {2} which is continuous. This can be solved for a modulus dependent on θ, but we have not done this yet.