THE KE MODELS: THEORETICAL DYNAMIC SUBSYSTEMS OF LONGITUDINAL WEB STRAIN

By

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ABSTRACT

Efficient design and optimization of many production processes often require models which predict transient and steady state web strains. To date, much attention has been given to modeling web strains much less than unity. Considerably less attention however, has been given to modeling strains of relatively lower modulus materials. A particular related challenge often involves selection of dancers vs. load cells as feedback devices in tension control systems.

This paper explores derivations of theoretical "Ke" models as primitive functions of roller motions. At a fundamental level, simple linear and nonlinear differential equations exist for each strain component or "subsystem" independent of others. Combinations can determine total strains in web spans including those at inputs and outputs of dancer rollers and within festoons. Validity is retained at any value of strain including zero and negative values (compression). The author demonstrates that mathematical equations of high web strains instead of becoming unwieldy, can be applied with accuracy and with a large degree of natural elegance. Applied classical control theory allows users a natural intuition when interpreting results which are primarily outputs of computer simulations.

The "free web span" has been extensively studied within the web handling community and is again examined here as a 1st section of web under any dynamic strain feeding into a 2nd section of web between two driven rollers. A free web span Ke based model is compared to a first order approximate model of the same physical system while applying step changes to roller velocities. Both models are compared as final values of strain approach extremely high values toward infinity. Using Ke models, all strain-time trajectories in the free web span as a result of step changes to roller velocities are shown to be sections of an S-shaped curve designated "The Universal Strain Time Curve". The output of the first order approximate model, when plotted on the Universal Strain Time Curve (USTC), reveals that the first order approximate model may often be applied with acceptable results for strains from 0 through 25%. Finally, an example model of a tension control system with load cell feedback demonstrates how consecutively higher order subsystems may be included as elements of a Ke Subsystem Library.

A practical and intuitive method of modeling web strains of any value has been developed here and may be applied by scientists and engineers having a basic knowledge of classical control system theory. With relatively accurate input data, effects on strain resulting from various roller inertias, web span lengths, dancers vs. load cells, and many other design decisions can be simulated. For both high and low modulus materials, Ke models provide a high degree of accuracy when simulating web strains during process design and optimization. This research is applicable to a broad spectrum of webs from thin plastics to paper, textiles, flat metals, wires, films, belts, foils, strips, threads, fabrics, and composites which are manufactured in rolling processes. The academic derivation process which has been applied also reinforces a useful framework to solve similar scientific problems.

NOMENCLATURE

A	cross-sectional area of web
$\bar{A}$	normalized cross-sectional area of web $\stackrel{\text{def}}{=} \frac{A}{A_u} = \frac{1}{1+\varepsilon}$
H	height of web
kei	dynamic function of strain due to web span input roller rotation
kel	dynamic function of strain due to change in web span length
keo	dynamic function of strain due to web span output roller rotation
L	length of web in a span
S	LaPlace operator
T	tension of web
t	time in seconds
V	velocity of web at roller surface
$\bar{V}$	qualitative velocity of web at roller surface $\stackrel{\text{def}}{=} V\bar{A}$
W	width of web
ε	strain of web
Λ	velocity of web length
λ	incremental length of web
τ	time constant

Prescripts:

d	differential (or incremental)
Δ	change (from the unstretched condition)

Subscripts:

o	pertaining to the initial or reference state
n	pertaining to the number of the span output roller or the span itself 0,1,2,3,…
ss	pertaining to the steady state or final value
u	pertaining to the unstretched condition

Other notation and definitions:

$\lvert f(x)\rvert$	the absolute value of $f(x)$
$f(x)\rvert_c$	$f(x)$ at the conditions of c

$\lim_{x \to a} f(x)$	limit of $f(x)$ as x approaches a
USTC	Universal Strain Time Curve
base case	A base case model is defined here as a model which includes a selection of parameters and their respective numerical values which mathematically represent an electro-mechanical control system. Base case parameters should be selected by the system designer and normalized so as to reveal only the dominant system characteristics in the time and/or frequency domain. As such, base case parameter values may often be chosen = 1. When employing the base case concept, the control system characteristics of physical real world *Web*(handling) *tech*(nology) systems are explicitly revealed by the normalized distribution of their *Omega*(ω) frequencies relative to the distribution of their associated base case frequencies. In this respect, a base case "model", "function", "subsystem", or "control system" (often used interchangeably) should include only the set of parameters which have been determined to be of interest to the designer for the particular analysis at hand.

INTRODUCTION

Historically, automatic control systems have been known to exist for over two thousand years as cited by Kecura [30]. More recently, the industrial revolution of the 19^{th} century has led to advancements by Maxwell (1868) applying the first control differential equations and by Routh (1874) and Hurwitz (1895) applying advanced stability criteria. Later in the 1930's and 1940's, Nyquist began plotting system poles and Bode developed the concept of root locus. Web handling control theory seems to have emerged later in the 20^{th} century as a specific field of study with Shelton [6].

In his popular web span model it's already well known that Campbell [1] in 1958 did not consider the tension of the web entering the span but more importantly, the rate of change of tension within the span was assumed to be a simple proportional function of roller velocities [1] p140, eq.3.29. Campbell's model would be more valid if the web span input and output rollers were to undergo linear translational motions (such as in a translational dancer) but not for roller rotational motions. Still, Campbell was riding the crest of modern theory at that time. Later in 1969, King [2] p5, eq. (9) developed the differential equation of strain in the free web span which includes the affect of the upstream web strain. King's work was limited to fixed position rollers, and as he continued his development to include roller inertias and friction, the resultant complexity of the mathematical equations grew, and as a result he chose to simplify the development by assuming that the strain of the web is very small [2], p6, eq. (17). In 1976, Brandenburg [3] faced similar challenges with modeling complexity, and in the Appendix he states that "because of the great length of the mathematical calculations, only the methods of derivation of the main results can be explained here". Brandenburg [3] eq.11 shows that changes in strain as a result of changes in roller rotational velocities reveal a type of exponential or "time-constant" type of characteristic and in summary he states that "it would appear that the dynamic behavior of the web is determined by lag elements with transport." Both Shelton [15] and Shin [21] followed with general web transport system modeling in 1986 where like King [2], in order to manage the elaborate equations, the strains in the web were assumed to be small (much less than unity).

In 1991 at the First International Conference on Web Handling, Reid and Shin [4] describe the limitations of previous works when dealing with tension transfer and the "non-ideal affects" which cause tension variation in actual processes. Reid and Shin's development of variable-gain control here is a method to compensate for the effect of the mechanical system inertias which change with roll diameters on unwinds and rewinders. A roll velocity control loop proportional gain which varies in direct proportion to roll inertia yields a more constant through roll control system bandwidth and therefore, a more consistent performance once the system is tuned to a desirable state. The associated dynamic gain function has since found wide successful application in industry. At this same conference in 1991 though, Carlson [10] challenges the web handling industry to go beyond the scope of Reid and Shin's PID related application and into more advanced modern control strategies, but he cites that "one present challenge is the nature of transport rollers coupled by the web." Over the ensuing years, the challenges of tension control continued. In 1997, Shelton [18] p74 in Additional Observations and Conclusions confronts "the complicated relationships between amplitude ratios and phase angles of the changes in tension in spans before and after the sensing roller," and in a separate study Pagilla [7] notes that "the offline methods that are employed for such tuning are often heuristic and fail to provide the required performance, especially in the presence of uncertainties." Inventor and entrepreneur Boulter [9] later asserts in 2001 that "due to their difficulty and importance in industry, tension problems have drawn the attention of many researchers. One problem is the establishment of a proper mathematical model." Also in 2001, while investigating the considerations of dancers vs. load cells, Carlson [14] concedes "I would fully agree with the assertion by Shelton that the dynamics of dancers and load-cell rollers, even with several simplifying assumptions, are quite complicated. In particular, during testing, it was very difficult to arrive at an experimental format that could isolate the response across all frequencies to a very small number of variables." Even as recently as 2002, Pagilla and Shelton [11] unite to proclaim that "the entire system of web transport dynamics along with the dynamics of motor, controller, and mechanical drive must be modeled, but such modeling has not been accomplished". We see that within the general study of tension control and related control problems in the web handling industry, results to date have been challenging if not frustrating. The general simplification by Shelton [15] and Shin [21] that web strain is small has limited related theory to approximate results for many of today's low modulus high strain materials.

Many of these historical findings have been intriguingly consistent with the author's own experiences in industry. Today there exist many works concerning the modeling of web strain and tension in the pursuit of better control of web transport systems. Some of these models have relied on the developer's intuitive interpretation of empirical data, [3][7][11][14][34] while others have employed rigorous mathematical theory followed by simplification techniques [2][5][8][15][17][18][22] in order to make the equations more manageable. Still, during the entire industrial automation revolution of the 20th century and to date, while web handling industry experts have kept pace with the tremendous gains in the modeling of the employed motor drive systems, dancers, web guiding, and other web path electro-mechanical components, there has remained limited success in the modeling of these components when coupled with the web material as a whole system. The more accurate dynamic characteristics of web strain seem to lay comfortably hidden and obscure from our intuition in practical application. Consistent with the characteristic time constants noted by Brandenburg [2], today's researchers recognize that magnitudes of derivatives of strain-time trajectories seem to increase proportionally with higher roller velocities and then seem to decrease proportionally with longer web spans. The need still exists though for more exact models which can more readily be applied in tension control, higher order systems, advanced control strategies, and canonical forms. This need is the basis of this paper.

Effective model development begins by first including only the dominant system parameters. With this approach, roller rotational “kei” and “keo” models are first developed individually. They’re then combined into a free web span Ke model to provide for transient strain analysis. The results of the free web span Ke model are found to be unified with both our intuition of web span time constants and with the principles of mass flow. The free web span Ke model is next followed by a 1st order approximation model of the same physical system which provides for frequency analysis of web spans under limited conditions and is needed to design the associated controllers. With the 1st order approximation model, Ke model theory becomes a unified theory which provides for both transient time response and steady state frequency response analyses. Finally, the free web span Ke model and the 1st order approximation model are both enhanced with the roller translational motion “kel” model. The “kel” model, when combined with both of the free web span models, provides for very accurate transient and frequency analyses of dancer and festoon based tension control systems. All web strain time trajectories due to roller motions reside in the world of mathematical exponentials.

BASIC STRAIN RELATIONSHIPS

A typical web transport system is shown in Figure 1.

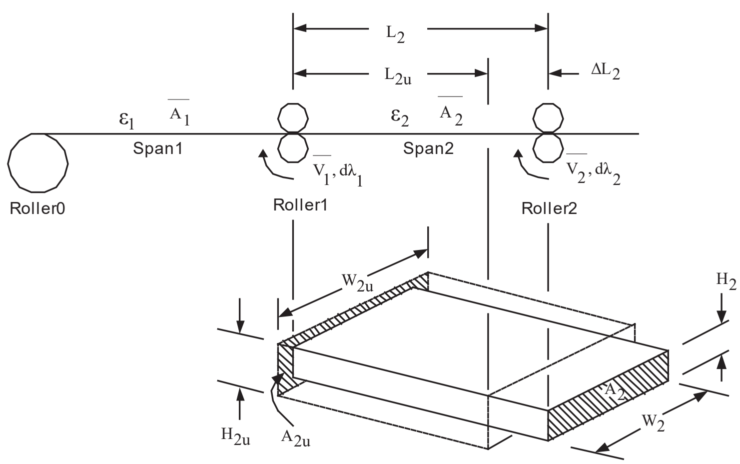

Figure 1 – Web Transport System with Fixed Web Span 2 Length

With respect to “base case” in nomenclature, only the dominant system parameters are considered and some historical assumptions [2][4][[15][21][22] are repeated here:

1. Normal roll wrap is not shown and is assumed negligible, that is, applying the assumptions of Shin [21], “the length of contact region between the web material and

rollers are negligible compared to the length of free web span between the rollers (i.e., the tension variations in contact region are negligible)."

2. Strains are assumed to be uniform within each span.
3. Material slippage and friction on rollers is currently assumed to be zero.
4. The web is assumed to be perfectly elastic.
5. Material density is uniform and normalized to a value of 1 throughout the entire web transport system.
6. Temperature, humidity, and viscoelastic affects are negligible.

Considering the web material in span 2 of Figure 1, some basic relationships useful for this development are:

$$\varepsilon_2 \stackrel{\text{def}}{=} \frac{\Delta L_2}{L_{2u}} \tag{1}$$

$$L_2 = L_{2u} + \Delta L_2 \tag{2}$$

$$L_{2u} = L_2 - \Delta L_2 \tag{3}$$

$$\Delta L_2 = L_2 - L_{2u} \tag{4}$$

$$\varepsilon_2 = \frac{\Delta L_2}{L_{2u}} = \frac{L_2 - L_{2\mathrm{u}}}{L_{2u}} = \frac{L_2}{L_{2\mathrm{u}}} - 1 \tag{5}$$

Substituting from {3} into {5}

$$\varepsilon_2 = \frac{L_2}{L_2 - \Delta L_2} - 1 \tag{6}$$

Solving for L_{2u} in {5}

$$L_{2u} = \frac{L_2}{1 + \varepsilon_2} \tag{7}$$

and substituting from {7} into {4}

$$\Delta L_2 = L_2 - \frac{L_2}{1 + \varepsilon_2} \tag{8}$$

Prior to delivery of material from web span 1, $L_{2\mathrm{u}} = L_{2\mathrm{uo}}$ and $\Delta L_2 = \Delta L_{2\mathrm{o}}$ so that using the form of {6}

$$\varepsilon_{2\mathrm{o}} = \frac{L_2}{L_2 - \Delta L_{\mathbf{2o}}} - 1 \tag{9}$$

WEB SPAN INPUT ROLLER kei FUNCTION

Considering first only the motion of the input roller, span 2 output roller velocity is assumed to be zero $(V_2 = 0)$. As Shelton [18] p43, we similarly consider an unstretched incremental length of material in Figure 1 which passes the point of the entry nip into span 2 such that

$$d\lambda_{1u} = \frac{d\lambda_1}{1+\varepsilon_1} \tag{10}$$

Recognizing this same unstretched length of material being added to the unstretched length of material in span 2

$$dL_{2u} = \frac{d\lambda_1}{1+\varepsilon_1} \tag{11}$$

After an incremental length dL_{2u} of unstretched material has been delivered to span 2, $L_{2u} = L_{2uo} + dL_{2u}$ and $\Delta L_2 = \Delta L_{2o} - dL_{2u}$ so that again substituting into {6} we have

$$\varepsilon_2 = \frac{L_2}{L_2 - \Delta L_2} - 1 = \frac{L_2}{L_2 - (\Delta L_{2o} - dL_{2u})} - 1 = \frac{L_2}{L_2 - \Delta L_{2o} + dL_{2u}} - 1 \tag{12}$$

With acceptance of {9}and {12}, increment of strain $d\varepsilon_2$ caused by incremental length of stretched material delivered into span 2 is

$$\frac{d\varepsilon_2}{d\lambda_1} = \frac{\varepsilon_2 - \varepsilon_{2o}}{d\lambda_1} \tag{13}$$

Please note that in {13} the symbol "d" represents the "incremental" which is normally defined by the symbol "Δ". The symbol "Δ" has been reserved here for use with the change in length of material ΔL and the author apologizes for any inconvenience. Substituting from {12} into {13} for ε_2

$$\frac{d\varepsilon_2}{d\lambda_1} = \frac{\dfrac{L_2}{L_2 - \Delta L_{2o} + dL_{2u}} - 1 - \varepsilon_{2o}}{d\lambda_1} \tag{14}$$

Using the form of {8} and substituting for ΔL_{2o} into {14}

$$\frac{d\varepsilon_2}{d\lambda_1} = \frac{\dfrac{L_2}{L_2 - (L_2 - \dfrac{L_2}{1+\varepsilon_{2o}}) + dL_{2u}} - 1 - \varepsilon_{2o}}{d\lambda_1} = \frac{-dL_{2u}(1 + 2\varepsilon_{2o} + \varepsilon_{2o}{}^2)}{d\lambda_1[L_2 + dL_{2u}(1+\varepsilon_{2o})]} \tag{15}$$

Finally, substituting from {11} for both instances of dL_{2u} and letting $d\lambda_1$ approach zero

$$\lim_{d\lambda_1 \to 0}\left(\frac{d\varepsilon_2}{d\lambda_1}\right) = -\frac{1 + 2\varepsilon_2 + \varepsilon_2{}^2}{L_2(1+\varepsilon_1)} = -\frac{(1+\varepsilon_2)^2}{L_2(1+\varepsilon_1)} \tag{16}$$

Note in the final expression of {16} that the change in strain is negative, since material is being added to the web span. The change in strain is also proportional to $\frac{1}{1+\varepsilon_1}$ (or $\overline{A_1}$) since material is being added to span 2 from span 1. The squared term in the numerator may be interpreted as a result that each increment of λ_1 affects both L_{2u} and ΔL_2, relative to {5}. As $d\lambda_1$ approaches zero, $d\varepsilon_2$ also approaches zero. Equation {16} is valid only in the limit as $\varepsilon_2 \rightarrow \varepsilon_{2o}$. The final expression of {16} is defined as the kei function

$$kei \stackrel{\text{def}}{=} -\frac{(1+\varepsilon_2)^2}{L_2(1+\varepsilon_1)} \qquad \{17\}$$

Equation {17} is valid for both positive and negative perturbations of λ_1. Substituting from {17} into {16}

$$\frac{d\varepsilon_2}{d\lambda_1} = kei \qquad \{18\}$$

The differential position strain function of {18} is used to derive the web strain function of input roller velocity. Rearranging {18}

$$d\varepsilon_2 = kei \; d\lambda_1 \qquad \{19\}$$

We write the velocity of web at roller 1

$$V_1 = \frac{d\lambda_1}{dt} \qquad \{20\}$$

Rearranging {20}

$$d\lambda_1 = V_1 dt \qquad \{21\}$$

Substituting from {21} into {19}

$$d\varepsilon_2 = kei \, V_1 dt \qquad \{22\}$$

and rearranging {22} yields the kei strain derivative

$$\frac{d\varepsilon_2}{dt} = kei \, V_1 \qquad \{23\}$$

The kei Model

Rearranging {22} and integrating each side

$$\int d\varepsilon_2 = \int V_1 kei \;\; dt \qquad \{24\}$$

The right side of {24} provides no direct solution since it contains a product of independent variables; however, the left side can be integrated to yield the constant of integration which is the initial strain of material in web span 2. Using incremental or modern control techniques, the solution of strain may be obtained in the time simulation. Integrating the left side of {24} the form of the solution yields the kei model

$$\varepsilon_2 = \varepsilon_{2o} + \int V_1 kei \ dt \qquad \{25\}$$

kei model base case. A normalized base case for the kei model is set with $V_{1o} = 0$, $\varepsilon_1 = 0$, $\varepsilon_{2o} = 1$, $V_2 = 0$, and length $L_2 = 1$. After applying a unit step of input roller velocity V_1 , the resultant time response is shown in Figure 2.

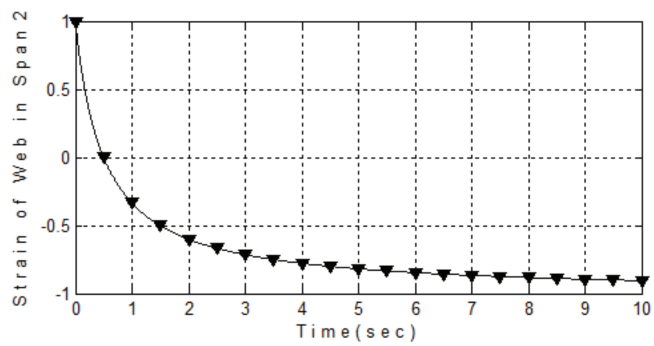

Figure 2 – kei Model Base Case Time Response

With the extended time span in Figure 2, the kei model remains continuous and entirely valid through zero strain and for negative strain (compression). Prior to the zero crossing at ($t \approx 0.46$), $A_2 < A_{2u}$, and $\overline{A_2} < 1$. At the zero crossing, $A_2 = A_{2u}$, and $\overline{A_2} = 1$. After the zero crossing, $A_2 > A_{2u}$ and $\overline{A_2} > 1$. Therefore, webs which are restricted from forming "backlash" type force discontinuities by going slack or buckling, can be conceived as able to be smoothly controlled at an operating strain or tension setpoint of zero, … or even less. As time continues in the kei model base, $\varepsilon_2 \rightarrow -1$ and $\overline{A_2} \rightarrow \infty$.

WEB SPAN OUTPUT ROLLER keo FUNCTION

The derivation process used for the kei model above is repeated for span 2 output roller. In this case, input roller velocity is assumed to be zero ($V_1 = 0$). An incremental length of stretched material $d\lambda_2$ is removed from web span 2 at output roller 2. Since increment of material $d\lambda_2$ is now leaving span 2 instead of entering span 2, {11} is modified as

$$dL_{2u} = \frac{d\lambda_2}{1 + \varepsilon_2} \qquad \{26\}$$

After $d\lambda_2$ is removed from span 2, {12} is modified to represent the new strain

$$\varepsilon_2 = \frac{L_2}{L_2 - \Delta L_{2o} - dL_{2u}} - 1 \qquad \{27\}$$

By derivation similar to {16}

$$\lim_{d\lambda_2 \to 0}\left(\frac{d\varepsilon_2}{d\lambda_2}\right) = \frac{1 + 2\varepsilon_2 + {\varepsilon_2}^2}{L_2(1+\varepsilon_2)} = \frac{(1+\varepsilon_2)^2}{L_2(1+\varepsilon_2)} \qquad \{28\}$$

Note that in {28} relative to {16}, the final expression is now positive instead of negative since strain increases as material is removed from the span. The change in strain is now proportional to $\frac{1}{1+\varepsilon_2}$ or $\overline{A_2}$, since the increment of material is transferred from span 2 instead of from span 1. Also, the squared term in the numerator of {28} may again be intuitively interpreted as a result that each increment of λ_2 removed from the web span affects both L_{2u} and ΔL_2 relative to {5}; however, in this case one of the terms is conveniently cancelled out by the same term in the denominator. The final expression of {28} is defined as the keo function

$$keo \stackrel{\text{def}}{=} \lim_{d\lambda_2 \to 0}\left(\frac{d\varepsilon_2}{d\lambda_2}\right) = \frac{(1+\varepsilon_2)^2}{L_2(1+\varepsilon_2)} = \frac{1+\varepsilon_2}{L_2} \qquad \{29\}$$

Also similarly,

$$\frac{d\varepsilon_2}{d\lambda_2} = keo \qquad \{30\}$$

$$d\varepsilon_2 = keo\ d\lambda_1 \qquad \{31\}$$

$$V_2 = \frac{d\lambda_2}{dt} \qquad \{32\}$$

$$d\lambda_2 = V_2 dt \qquad \{33\}$$

$$d\varepsilon_2 = keo\ V_2\ dt \qquad \{34\}$$

Rearranging {34} yields the keo strain derivative

$$\frac{d\varepsilon_2}{dt} = keo\ V_2 \qquad \{35\}$$

The *keo* Model

Integrating each side of {34}

$$\int d\varepsilon_2 = \int V_2 keo\ \ dt \qquad \{36\}$$

Again performing the integration on the left side yields the keo model

$$\varepsilon_2 = \varepsilon_{2o} + \int V_2 keo\ \ dt \qquad \{37\}$$

***keo* model base case.** A normalized base case for the keo model is set with $V_{2o} = 0$, $\varepsilon_{2o} = 0$, and length $L_2 = 1$. After applying a unit step of output roller velocity V_2 the resultant time response is shown in Figure 3 where again, an extended time plot reveals the exponential nature of the isolated keo model.

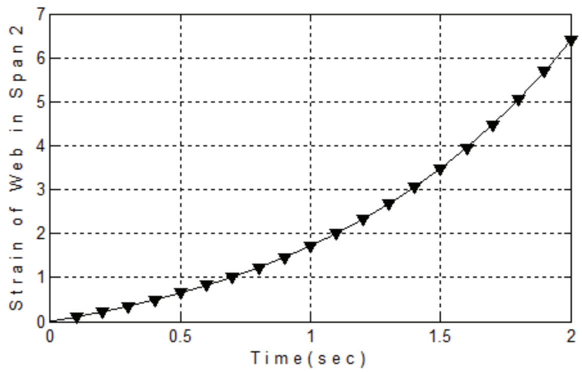

Figure 3 – keo Model Base Case Time Response

WEB SPAN LENGTH kel FUNCTION

The process of derivation used for the kei and keo models is again repeated for the kel model. This model represents the change in strain as a result of a change length of a web span and therefore, as a result of change in length of material in a web span. In this model, $V_1 = V_2 = 0$. Consider again the free web span as shown in Figure 4 where roller 2 is now free to move in the translational direction so as to change the length of web span 2.

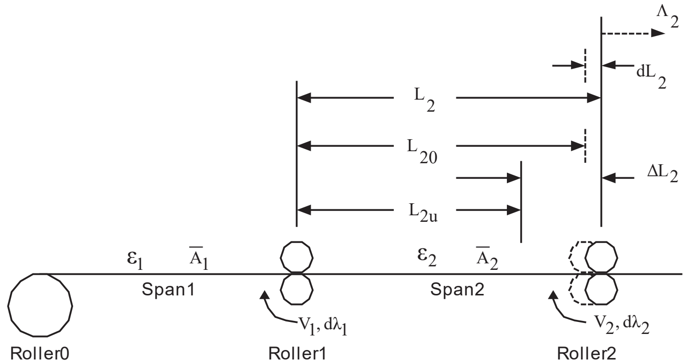

Figure 4 – Web Transport System with Variable Web Span 2 Length

It's again important to note in Figure 6 that ΔL_2 is still the total change in length of material in span 2, from the unstretched condition L_{2u}. It should not be confused with any incremental change in web span length, which is

$$dL_2 = L_2 - L_{2o} \qquad \{38\}$$

The end result of this development is already highly intuitive; however, for the sake of consistency of the model for use in subsystem combinations, a similar Ke model is derived by first recognizing that $L_{2\mathrm{u}}$ has not changed and

$$\varepsilon_2 = \frac{L_2 + dL_2}{L_{2o}} - 1 \qquad \{39\}$$

With a now familiar derivation,

$$\frac{d\varepsilon_2}{dL_2} = \frac{\varepsilon_2 - \varepsilon_{2\mathrm{o}}}{dL_2} = \frac{\left[\frac{(L_{2\mathrm{o}} + dL_2 - L_{2\mathrm{u}}) - (L_{2\mathrm{o}} - L_{\mathrm{u}})}{L_{2\mathrm{u}}}\right]}{dL_2} = \frac{\left(\frac{dL_2}{L_{2\mathrm{u}}}\right)}{dL_2} \qquad \{40\}$$

Simplifying {40} and rearranging

$$d\varepsilon_2 = \frac{dL_2}{L_{2\mathrm{u}}} \qquad \{41\}$$

Substituting from {7} for $L_{2\mathrm{u}}$ in {41}, and letting dL_2 approach zero, the kel function is defined

$$kel \overset{\text{def}}{=} \lim_{dL_2 \to 0} \left(\frac{d\varepsilon_2}{dL_2}\right) = \frac{1 + \varepsilon_2}{L_2} \qquad \{42\}$$

Note that the kel function in {42} is identical to the keo function in {29}. Differences exist only in their definition and application. While keo is a function of strain relative to span output roller rotation, kel is a function of strain relative to a span roller translational motion. Similarly,

$$\frac{d\varepsilon_2}{dL_2} = kel \qquad \{43\}$$

$$d\varepsilon_2 = kel\, dL_2 \qquad \{44\}$$

$$\Lambda_2 \overset{\text{def}}{=} \frac{dL_2}{dt} \qquad \{45\}$$

$$dL_2 = \Lambda_2 dt \qquad \{46\}$$

$$d\varepsilon_2 = kel\, \Lambda_2\, dt \qquad \{47\}$$

Rearranging {47} yields the kel strain derivative

$$\frac{d\varepsilon_2}{dt} = kel\, \Lambda_2 \qquad \{48\}$$

The kel Model

Integrating each side of {47}

$$\int d\varepsilon_2 = \int \Lambda_2 kel \ dt \tag{49}$$

Performing the integration on the left side of {49} yields the kel model:

$$\varepsilon_2 = \varepsilon_{2o} + \int \Lambda_2 kel \ dt \tag{50}$$

***kel* model base case.** A base case for the kel model is set with $V_1 = V_2 = 0$, $\varepsilon_{2o} = 0$, and length $L_{2o} = 1$. After applying a unit step of velocity of web length Λ_2, the resultant time response is shown in Figure 5.

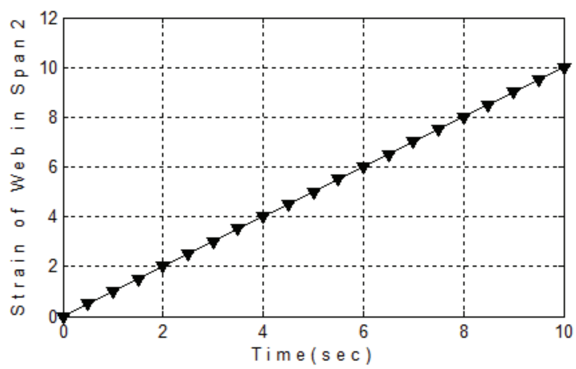

Figure 5 – kel Model Base Case Time Response

Comparing Figure 5 to Figure 3, it's again curious to note that even though $kel = keo$, the time plot is no longer exponential since L_2 is no longer constant. As L_2 increases in the denominator of kel, the exponential nature of ε_2 is negated. Also in the base case, since the original length $L_{2o} = 1$, the strain has a 1:1 correlation with the increase in web span length.

THE FREE WEB SPAN

Having established the Ke strain derivatives relative to three primitive roller motions, we can begin to explore the more practical effects of their combined motions. The most popular of these combinations produces the Ke model of a "free length of the web" [1] which has recently become widely known as "the free web span" [21] and which now combines the kei and keo strain derivatives of {23} and {35} to obtain the free web span strain derivative

$$\frac{d\varepsilon_2}{dt} = V_1 kei + V_2 keo = V_2 \frac{1+\varepsilon_2}{L_2} - V_1 \frac{(1+\varepsilon_2)^2}{L_2(1+\varepsilon_1)} = \frac{V_2}{L_2 \bar{A}_2} - \frac{\overline{V_1}}{L_2 \overline{A_2}^2} \tag{51}$$

We can discern directly from the first two expressions on the right side of {51} that with constant values of V_1 and V_2, during a strain-time trajectory, kei must vary at a rate different than keo in order for the strain to converge to a predictable steady state value.

At convergence

$$V_1 kei = V_2 keo \Big|_{\varepsilon_2 = \varepsilon_{2ss}} \qquad \{52\}$$

The free web span strain derivative form of the internal expressions of {51} is consistent with King [2] eq. 9.

Free Web Span Ke Model

Integrating the first two expressions on the right side of {51}

$$\int d\varepsilon_2 = \int (V_1 kei + V_2 keo)\ dt \qquad \{53\}$$

Performing the integration on the left side of {53} while this time recognizing only the single constant of integration yields the free web span Ke model

$$\varepsilon_2 = \varepsilon_{2o} + \int (V_1 kei + V_2 keo)\ dt \qquad \{54\}$$

Free web span Ke model base case. A base case for the free web span Ke model is set with $\varepsilon_1 = \varepsilon_{2o} = 0$ and $L_2 = 1$. At (t = 0), applying synchronous steps of roller velocities $V_1 = 0.5$ and $V_2 = 1$, yields the resultant time response is shown in Figure 6.

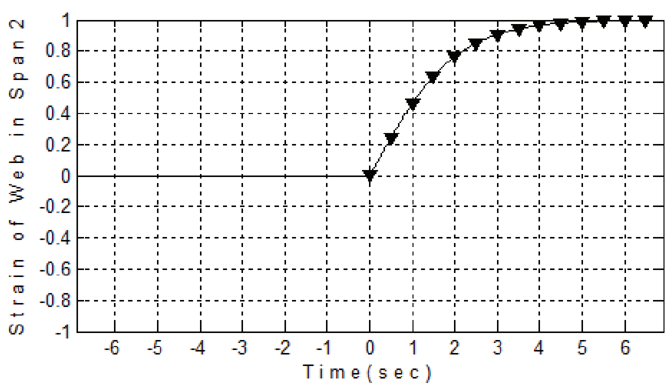

Figure 6 – Free Web Span Ke Model Base Case Time Response

In the free web span Ke model base case, the trajectory of strain from zero to ε_{2ss} approaches a final value of 1.

Free web span Ke model steady state strain. To derive ε_{2ss} , we examine the limit of {54} as $\frac{d\varepsilon_2}{dt}$ approaches zero where we write

$$\lim_{\varepsilon_2 \to \varepsilon_{2ss}} \left(\frac{d\varepsilon_2}{dt}\right) = \lim_{\frac{d\varepsilon_2}{dt} \to 0} \left(\frac{d\varepsilon_2}{dt}\right) = 0 = \frac{V_2}{L_2 \overline{A_{2ss}}} - \frac{\overline{V_1}}{L_2 \overline{A_{2ss}}^2} = V_2 - \frac{\overline{V_1}}{\overline{A_{2ss}}} \qquad \{55\}$$

From the last 2 expressions of {55}

$$\overline{V_1} = V_2\overline{A_{2ss}} \qquad \{56\}$$

Equation {56} represents equal mass flow at the input and output rollers of span 2. By definition of $\overline{A_{2ss}}$

$$1 + \varepsilon_{2ss} = \frac{V_2}{\overline{V_1}} \qquad \{57\}$$

Rearranging

$$\varepsilon_{2ss} = \frac{V_2 - \overline{V_1}}{\overline{V_1}} \qquad \{58\}$$

and in the specific case where $\varepsilon_1 = 0$ ($\overline{A_1} = 1$) the steady state strain in the free web span is given by

$$\varepsilon_{2ss} = \frac{V_2 - V_1}{V_1} \qquad \{59\}$$

Free Web Span Ke Model Alternative Form

By definition of the normalized cross-sectional area of the web in span 2

$$\frac{d\overline{A_2}}{dt} = \frac{d\left(\frac{1}{1+\varepsilon_2}\right)}{dt} = -\frac{\left(\frac{d\varepsilon_2}{dt}\right)}{(1+\varepsilon_2)^2} = -\left(\overline{A_2}\right)^2\left(\frac{d\varepsilon_2}{dt}\right) = -\left(\overline{A_2}\right)^2\left(\frac{V_2}{L_2\overline{A_2}} - \frac{\overline{V_1}}{L_2\overline{A_2}^2}\right) \qquad \{60\}$$

and reducing

$$\frac{d\overline{A_2}}{dt} = \frac{\overline{V_1} - \overline{V_2}}{L_2} \qquad \{61\}$$

Intriguingly, exactly like the Ke form of {51}, the very simple form of {61} completely describes the dynamic strain in free web span including the effects of dynamic strain ε_1 in input web span 1. The form of {61} may conceivably make consecutively higher order cascaded web span modern control theory models and canonical forms more manageable. This could in turn lead to more practical development and implementation of some of the more advanced control strategies proposed by Pagilla [7] and Carlson [10].

Free Web Span 1st Order Approximate Model

Although a complete derivation is beyond the scope of this limited manuscript, a free web span first order approximation model with two parameters may be derived as

$$\frac{d\varepsilon_2}{dt} \cong \frac{\varepsilon_{2ss} - \varepsilon_2}{\tau_2} \cong \frac{V_{2ss}}{L_2}\left[\left(\frac{V_2 - \overline{V_1}}{\overline{V_1}}\right) - \varepsilon_2\right] \qquad \{62\}$$

where the choice for parameter ε_{2ss} is given by {58} and the choice for parameter τ_2 is given by

$$\tau_2 \stackrel{\text{def}}{=} \frac{L_2}{\mathrm{V}_{2ss}} \qquad \{63\}$$

Integrating each side of {62}

$$\int d\varepsilon_2 \cong \int \frac{\mathrm{V}_{2ss}}{L_2}\left[\left(\frac{V_2 - \overline{V_1}}{\overline{V_1}}\right) - \varepsilon_2\right] dt \qquad \{64\}$$

and performing the integration

$$\varepsilon_2 \cong \varepsilon_{2o} + \int \frac{\mathrm{V}_{2ss}}{L_2}\left[\left(\frac{V_2 - \overline{V_1}}{\overline{V_1}}\right) - \varepsilon_2\right] dt \qquad \{65\}$$

Since {65} is a linear first order equation, the time solution can be derived as

$$\varepsilon_2 \cong \varepsilon_{2o} + \frac{V_2 - \overline{V_1}}{\overline{V_1}}\left(1 - \mathrm{e}^{\frac{-V_{2ss}t}{L_2}}\right) \qquad \{66\}$$

and for which the frequency domain transfer function for $\overline{V_1}$ input

$$TF(S) = \frac{\varepsilon_2(\mathrm{S})}{\overline{V_1}(\mathrm{S})} = \frac{1}{\mathrm{S}\tau_2 + 1} = \frac{1}{\mathrm{S}\frac{L_2}{\mathrm{V}_{2ss}} + 1} \qquad \{67\}$$

Both empirically and by mathematical derivation it can be shown that the 1st order approximation model time response is very nearly the exact free web span Ke model time response for either of:

Condition 1: The steady-state web strain is $\varepsilon_{2ss} < 25\%$ <u>or</u>
Condition 2: The web strain ε_2 is near (within 10%) of ε_{2ss}

Condition 1 provides for a complete linear time or frequency domain analysis of many of today's materials which are processed at strains of less than 25%. Condition 2 provides for a linear time or frequency domain analysis of any material near steady state strain, including low modulus materials processed at high strain. Equations {62} through {66} are valid under relatively stable conditions of closed loop tension or strain control where the output is controlled to within 10% of setpoint.

<u>Free web span approximate model base case.</u> A base case for the free web span approximate model is set using the same conditions as the base case for the free web span Ke model with $\varepsilon_1 = \varepsilon_{2o} = 0$ and $L_2 = 1$. At (t = 0), again applying synchronous steps of roller velocities $V_1 = 0.5$ and $V_2 = 1$, the resultant time response is shown in Figure 7. The simple single pole Bode diagram for the approximate model base case is shown in Figure 8.

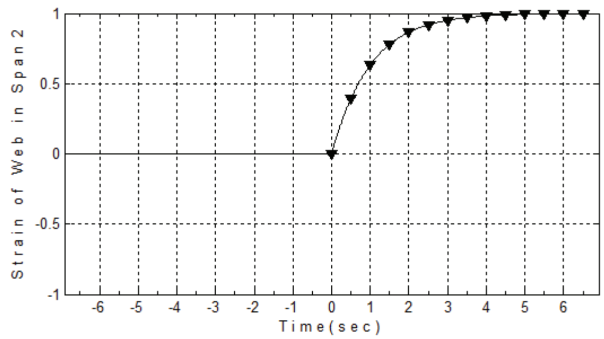

Figure 7 – Free Web Span Approximate Model Base Case Time Response

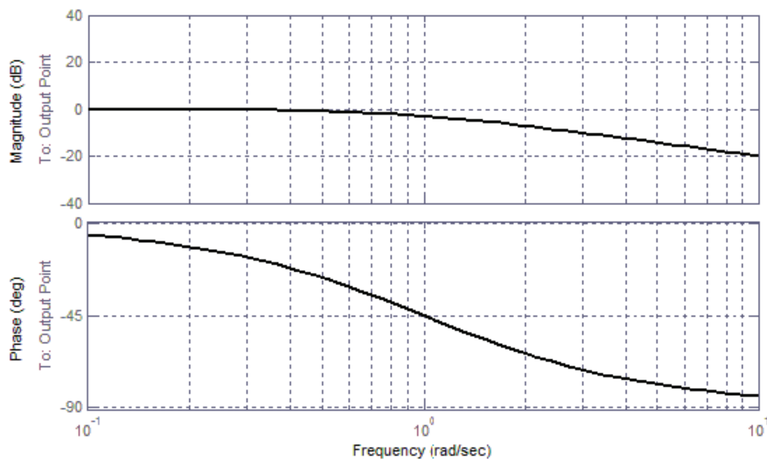

Figure 8 – Free Web Span Approximate Model Base Case Bode Diagram

THE UNIVERSAL STRAIN TIME CURVE

To date, much attention has been given to modeling web strains much less than unity; however, considerably less attention has been given to modeling strains of relatively lower modulus materials. Moving beyond the free web span Ke model base case to strains > 1, an academic study of strain-time trajectories is illustrated by examination of the shape of the trajectories as strains approach extremely high levels toward infinity. Using the free web span Ke model of {54}, with zero strain in the input web span, all strain-time trajectories as a result of step changes to roller velocities V_1 and V_2 can be shown to be sections of an S-shaped curve which has been designated "The Universal Strain Time Curve" and is shown in Figure 9.

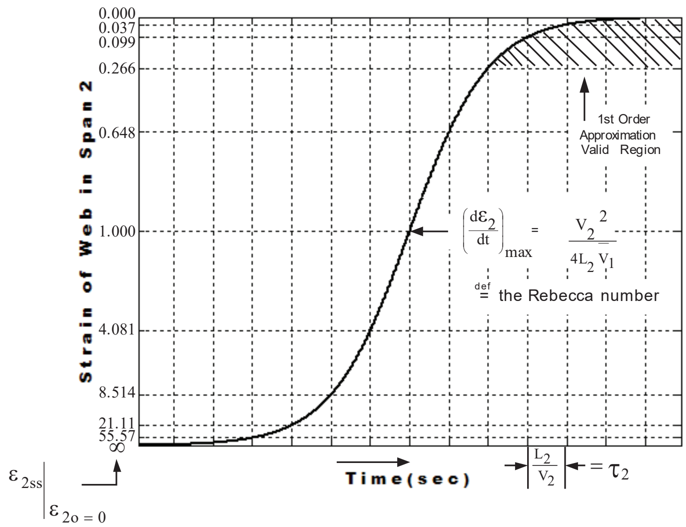

Figure 9 – The Universal Strain Time Curve

The USTC also shows the strain time trajectory for any case in which the initial strain in span 2 is not zero, but is any value less than the steady state strain. The trajectory in each case is an ending section of the curve where the initial strain is zero. In all cases, the steady state strain defines the shape of the curve and all trajectories end at the final top right of the curve in the first order approximation region. In the free web span approximate model base case, the concept of an associated "time constant" τ_2 was introduced. Empirically, this time constant as given by {63} equals $\frac{L_2}{V_{2ss}}$ and defines the X-Axis of the USTC giving the curve its universal nature.

It's important to note that the logarithmic vertical scale indicates only the value of ε_{2ss} for the corresponding strain trajectory starting point on the curve. The vertical scale does not indicate the values of strain ε_2 during the trajectory. The trajectory does however, show the shape of the curve and therefore, it shows how the curve would appear on a linear vertical scale which is not shown but can be revealed for example, as in Figure 6. The actual slope of the curve at any point of course, is always given by {51}. Adding small values of strain to the input web span ($\varepsilon_1 << 1$) has been observed to produce only subtle changes to the shape of the USTC.

The free web span base case time response of Figure 6 is seen to overlay the entire upper half of the USTC of Figure 9 where ε_{2ss}=1. The USTC also elegantly facilitates visualization of results of the first order approximate model (see below). All valid first order approximation model results lie on the top portion of the curve where either ($\varepsilon_{2ss} < 25\%$) or ($\varepsilon_2 \geq 0.9\varepsilon_{2ss}$). This area has been designated the "1st order approximation valid region."

The very lower left section of the curve which forms the start of the USTC as of today may seem purely academic for extremely high values of strain. The nature of mass flow which forms the S-curve however, can be conceptualized intuitively by recognizing that on the lower section of the USTC, strain is dominated by mass flow from the output roller. Here, at the start of the USTC, we see a resemblance to the unbounded keo model base case time response of Figure 3. As strain increases toward the inflection point near the center of the USTC, the existence of at least some mass flow from the input roller begins to manifest itself which restricts the strain from continuing to increase exponentially. After the inflection point, the mass flow at the input roller becomes an ever increasing percentage of the mass flow at the output roller until eventually, they are equal. At that point, on the top of the USTC, the strain is at steady state and $\frac{d\varepsilon_2}{dt} = 0$.

The nature of S-shaped "integral-exponential" characteristic curves resulting from step changes in roller rotational or translational positions have been previously studied by Brandenburg [3] Figures 7.1c and 7.2c and again by Shin [22] Figure 81. The USTC provides a general view of these earlier results.

<u>Compare of Approximate First Order Responses to Universal Strain Time Curve</u>

At this point, it's insightful to compare time trajectories of the 1st order approximate model of {65} to the trajectory of the Ke model of {54} as shown in Figure 10.

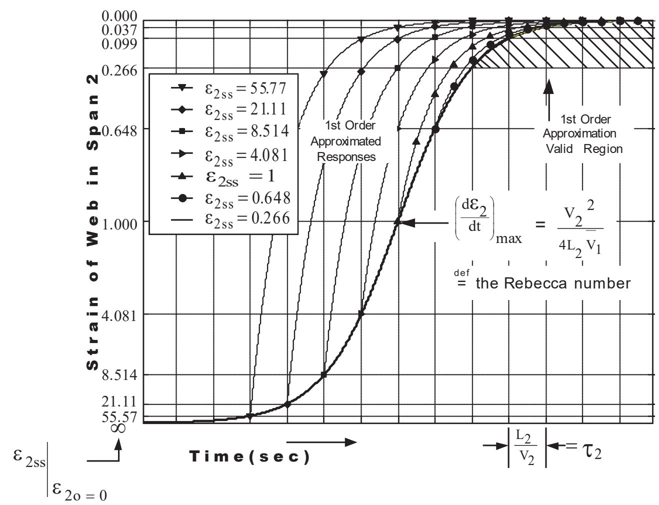

Figure 10 – Comparison of First Order Approximate Model Responses to the Universal Strain Time Curve

The free web span approximate model time response is nearly indistinguishable from the exact Ke model time response for any strain trajectories which begin on the horizontal scale at 2 time constants or more from the start of Ke model base case (center of chart). These trajectories reside in the first order approximation valid region of the USTC.

Ke MODEL BASED COMPLETE TENSION CONTROL SYSTEM WITH LOAD CELL FEEDBACK TO INPUT ROLLER

Consider next the tension control physical system of Figure 11 and the Ke based tension control system model block diagram of Figure 12.

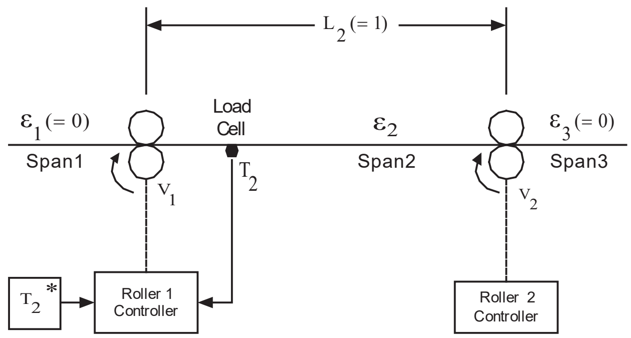

Figure 11 – Tension Control Physical System

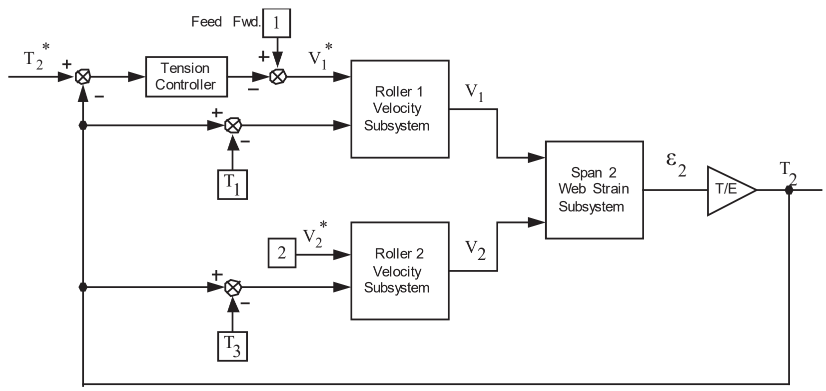

Figure 12 – Ke Based Tension Control System Model Block Diagram

For the sake of base case analysis, the effects of additional web spans normally resulting from the addition of a load cell are neglected. The tension controller is simple proportional with gain = 1. Also in the base case, all fixed parameters (except $V_2{}^*$) are equal to 1. The roller velocity subsystems include complete PI control systems with controllers (gains=1) and plants (motor/roller combined reflected inertias = 1 and friction = 0). The roller velocity subsystems also include torques on rollers from both motors and web tensions. The web strain subsystem includes all components of {51}. Base case conversions from strain to tension (material modulus) $\frac{T}{\varepsilon} = 1$. At $(t = 0)$ the tension command signal $T_{2o}{}^*$ = 1 where the system settles out within 10 seconds (Figure 13). The system again becomes excited at $(t = 15\ )$ when the tension command signal $T_2{}^*$ is step changed to 2. The corresponding roller velocity and tension time plots are shown in Figure 13 .

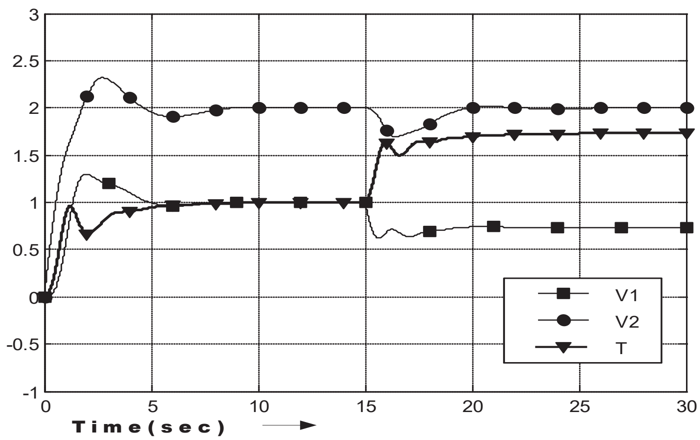

Figure 13 – Tension Controller Time Plots

The steady state tension error at (t = 30) is a result of the simple proportional low gain tension controller used in the normalized base case for analysis purposes only. The disturbances to V_2 are a result of the tension related torque disturbances from web span 2 at roller 2 and the low PI controller gains of the roller velocity subsystem used in the base case. The predicted steady state strain is given by {59} as $\varepsilon_{2ss} = \frac{V_2 - V_1}{V_1}$. Steady state output results of the computer simulation at (t = 30) were measured as $V_{1ss} = 0.732$, $V_{2ss} = 2.000$, and $T_{ss} = 1.732$. Since the tension to strain conversion $\frac{T}{\varepsilon} = 1$, $\varepsilon_{2ss} = T_{ss}$= 1.732, which is exactly the value predicted by {59}.

CONCLUSIONS

Results of function, strain derivative, and model development are summarized in Table 1.

	Ke Function	Strain Derivative	Model
Web Span Input Roller	$kei = -\frac{(1+\varepsilon_2)^2}{L_2(1+\varepsilon_1)}$	$\frac{d\varepsilon_2}{dt} = kei\ V_1$	$\varepsilon_2 = \varepsilon_{2o} + \int V_1 kei \ \ dt$
Web Span Output Roller	$keo = \frac{1+\varepsilon_2}{L_2}$	$\frac{d\varepsilon_2}{dt} = keo\ V_2$	$\varepsilon_2 = \varepsilon_{2o} + \int V_2 keo \ \ dt$
Web Span Length	$kel = \frac{1+\varepsilon_2}{L_2}$	$\frac{d\varepsilon_2}{dt} = kel\ \Lambda_2$	$\varepsilon_2 = \varepsilon_{2o} + \int \Lambda_2 kel \ \ dt$
Free Web Span		$\frac{d\varepsilon_2}{dt} = V_1 kei + V_2 keo$	$\varepsilon_2 = \varepsilon_{2o} + \int (V_1 kei + V_2 keo)\, dt$
Free Web Span Alternative Form		$\frac{d\overline{A_2}}{dt} = \frac{\overline{V_1} - \overline{V_2}}{L_2}$	$\overline{A_2} = \overline{A_{2o}} + \int \left(\frac{\overline{V_1} - \overline{V_2}}{L_2}\right) dt$
Free Web Span Approximation		$\frac{d\varepsilon_2}{dt} \cong \frac{\varepsilon_{2ss} - \varepsilon_2}{\tau_2} \cong \frac{\mathrm{V}_{2ss}}{L_2}\left[\left(\frac{V_2 - \overline{V_1}}{\overline{V_1}}\right) - \varepsilon_2\right]$	$\varepsilon_2 \cong \varepsilon_{2o} + \frac{V_2 - \bar{V}_1}{\bar{V}_1}\left(1 - \mathrm{e}^{\frac{-\mathrm{V}_{2ss}\mathrm{t}}{L_2}}\right)$

Table 1 – Functions, Strain Derivatives, and Models Summary

Application of the Ke models should result in more efficient design and optimization of many production processes which produce transient and steady state web strains. Although dancer and festoon application examples are beyond the scope of this limited manuscript, the kel model provides for the additional subsystem needed for the study of dancers, festoons, and roller translational registration correction systems. Similar to the combined kei and keo subsystems in the free web span, the form of kel model of {50} provides for simple inclusion into the total system to reveal web strains and other dynamic variables throughout the entire web transport system. The free web span 1st order approximation model of {65}, the associated transfer function of {67}, and the kel model of {50} provide for application of the powerful frequency analysis techniques of classical control system theory, and may help researchers in many areas of interest where a few have already included [5][8] [11][12][14][15][20][34].

The author hopes primarily to have provided a new perspective on the analysis of dynamic web strains resulting from both rotational and translational roller motions. With only the Ke models revealed here so far, an understanding of strain time trajectories during process start-ups and splicing, and the associated forward loop gain parameters of steady-state process conditions should help designers minimize undesirable material stress and/or web breaks and minimize printed material registration errors. The control of strain-time trajectories may even suggest clever ways for designers to more efficiently bring the process "up to strain" during machine start-ups [8] or restore the process to the conditions of desirable strain following splices and other process disturbances. Strain-time trajectories within web spans do indeed occur for each roller rotational velocity or position, or translational position correction [3] in printed material registration systems. For frequency domain analysis, it has been shown that the 1st order approximate model of {65} is often valid for values of strain less than 25% or for process conditions within 10% of machine steady state.

The tension control system of Figure 12 is the base case form of a model which represents "the entire system of web transport dynamics" cited by Pagilla and Shelton

[11] in 2002. With respect to base case, additional parameters which create higher order forms should be added only as needed for the particular analysis at hand. Similarly, although beyond the scope of this particular manuscript, the addition model or model combinations needed for analyses of dancer (or festoon) and load cell based tension control systems are included in Table 2.

	Web Span Ke Model	Web Span 1st Order Approximation Model	Span Length kel Model
Load Cell Based Transient Analysis	X		
Load Cell Based Frequency Analysis		X	
Dancer Based Transient Analysis	X		X
Dancer Based Frequency Analysis		X	X

Table 2 – Model Combinations Needed for Analyses of Dancer (or Festoon) and Load Cell Based Tension Control Systems

Related and other continuing research using Ke model based theory could include:

1. Validation of real world parameters by comparison to normalized base cases.
2. Dancer position feedback tension control system followed by exploration of results of dancers vs. load cells as material moduli, mechanical inertias, and other parameters vary from base case.
3. Festoon unwind revealing dynamic strains within each span during festoon motion.
4. Inclusion of web roller drive systems backlash and torsional resonance.
5. Inclusion of affects of roller regions of adhesion, Coulomb friction, and slip.
6. Laminated webs and viscoelastic strain models.
7. Inclusion of temperature and other typically less dominant effects.
8. Related control system design and compensation techniques.
9. Basic Ke model forms of cascaded web span subsystems operating in draw and/or tension control.
10. Using the free web span condensed Ke model of {61} to develop modern control system representation of higher order cascaded web span systems and related canonical forms.
11. Advanced analysis of active vs. passive dancers.

Finally, the author wishes to reinforce the view that "a web often flows through a process like a river, running slower often upstream in those sections where the cross section of web is larger, and running faster often downstream in those sections where the cross section of web is smaller. With the web materials of today, the fluid of the river may also have varying degrees of density and viscoelasticity which create waves and ripples in the current." In this paper, the author hopes to have better exposed some of the most basic primitive "longitudinal dynamics of the web", that the reader has a new perspective on this fascinating technology, and that the reader will be able to add more insight into some future related experiences in web handling, while appreciating the natural elegance in the world of mathematical exponentials.

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REFERENCES

1. Campbell, D.P., Process Dynamics, Wiley, 1958, pp. 113-156.
2. King, D.L., "The Mathematical Model of a Newspaper Press," Newspaper Techniques, Dec., 1969, pp. 3-7.
3. Brandenburg, G., "New Mathematical Model for Web Tension and Register Error," Proceedings: 3rd International IFCA Conference On Instrumentation and Automation in the Paper, Rubber and Plastics Industries, Vol. 1, May 1976, pp. 411-438.
4. Reid, K.N. and Shin, K.H., "Variable-Gain Control of Longitudinal Tension In a Web Transport System," Proceedings of the First International Conference on Web Handling, Oklahoma State University, May 19-22, 1991.
5. Reid, K.N., "Dynamic Behavior of Dancer Subsystems in Web Transport Systems," Proceedings of the Second International Conference on Web Handling, Oklahoma State University, Jun. 6-9, 1993.
6. Shelton, J.J., "Lateral Dynamics of a Moving Web," D.P. Report, Oklahoma State University, 1968.
7. Pagilla, P.R., Cimino, M., and Knittel, D, "Design of Fixed Structure Controllers for Web Tension Control," Proceedings of the Ninth International Conference on Web Handling, Oklahoma State University, USA, and Louis Pasteur University and INSA, France, Jun. 10-13, 1997.
8. Reid, K.N. and Lin, K.C., "Control of Longitudinal Tension in Multi-Span Web Transport Systems during Start Up," Proceedings of the Second International Conference on Web Handling, Oklahoma State University, Jun. 6-9, 1993.
9. Hou, Y., Gao, F. Jiang, and Boulter, B.T., "Active Disturbance Rejection Control for Web Tension Regulation," 2001 IEEE Conference on Decision and Control.
10. Carlson, D. "Advanced Controls For Web Handling: Ready For Production?," Proceedings of the First International Conference on Web Handling, Oklahoma State University, May 19-22, 1991.
11. Pagilla, P.R., Shelton, J.J., and Dwivedula, R., "Dynamics of Motor, Controller, and Mechanical Drive for Tension Control (0200-1)," WHRC Project Report, Oklahoma State University, Oct., 2002.
12. Pagilla, P.R., "The Role of Active Dancers in Tension Control of Webs (9898-1)," WHRC Project Report, Oklahoma State University, Jun., 2002.
13. Ducotey, K. and Good, J.K, "Dynamic Coefficient of Traction between Web Media and Rollers (8687-1), WHRC Project Report, Oklahoma State University, June, 1999.

14. Carlson, D.H., "Considerations in the Selection of a Dancer or Load Cell Based Tension Regulation Strategy", Proceedings of the Sixth International Conference on Web Handling, Oklahoma State University, Jun. 10-13, 2001.
15. Shelton, J.J., "Dynamics of Web Tension Control with Velocity or Torque Control," Proceedings of American Control Conference, Atlanta, Jun., 1986.
16. Asano, K., Yamamoto, K., Kawase, T., and Nomura, N., "Impedance Control for Tension and Looper in Hot Strip Finishing Mills," Ref. United States Patent No. 5,660,066, August 26, 1997.
17. Whitworth, D.P.D. and Harrison, M.C., "Tension Variations in Pliable Material in Production Machinery", Appl. Math. Modeling, Vol. 7, Jun., 1983, pp.189-196.
18. Shelton, J.J., "Sensing of Web Tension by Means of Roller Reaction Forces," WHRC Project 9192-1, Oklahoma State University, Jul., 1997.
19. Bettendorf, R., "Real Time Dynamic Simulation of a Paper Winder", Proceedings of the Seventh International Conference on Web Handling, Oklahoma State University, Jun. 1-4, 2003.
20. Dwivedula, R.V., Zhu, Y.L., and Pagilla, P.R., "A Comparative Study on Active and Passive Dancers Used for Attenuation of Web Tension Disturbances," Proceedings of the Seventh International Conference on Web Handling, Oklahoma State University, Jun. 1-4, 2003.
21. Shin, K.H. "Tension Control in a Moving Web," M.S. Report, Oklahoma State University, Dec., 1986.
22. Shin, K.H., "Distributed Control of Tension in Multi-Span Web Transport Systems," D.P. Report, Oklahoma State University, May, 1991.
23. Rajala, G., "Active Dancer Control for a Web Handling Machine", M.S. Report, University of Wisconsin-Madison, Aug., 1995.
24. Hale, F.J., Introduction to Control System Analysis and Design, Prentice-Hall, Inc., 1973.
25. Washington, A.J., Technical Calculus with Analytic Geometry, The Benjamin/ Cummings Publishing Company, Aug, 1971.
26. Levingston, I.J., Statics and Strength of Materials, Prentice-Hall, Inc., 1971.
27. Eddens, G.R., "A Dancer Primer: Controlling Web Tension," Paper, Film, and Foil Converter, Vol. 61, Nov. 1987, pp. 70-72.
28. Kuribayashi, K. and Nakajima, K., "An Active Dancer Roller System for Tension Control of Wire and Sheet," IFAC 9th World Congress, Budapest, Hungary, 1984.
29. Reid, K.N., "Web Transport Systems (0000-4)," Project Report, Oklahoma State University, Oct., 2001.
30. Kucera, V., "Kucera V.: Control Theory and Forty Years of IFAC: A Personal View," IFAC Newsletter, June 1997, p. 4.
31. Knittel, D., Verdines, M., Henrion, D., and Pagilla, P., "Fixed-Order H. Decentralized Control with Model Based Feedforward for Elastic Web Winding Systems," Proceedings of the IEEE Industry Applications Society Conference, Tampa, Florida, USA, Oct., 2006.
32. Franklin, G.F. and Powell, J.D., Digital Control of Dynamic Systems, Addison-Wesley Publishing Company, Inc., Second Printing, June 1981.
33. Roisum, D., "First Web Handling Equations/Models," Converting Quarterly, Blogs, Web, Web Handling and Converting, Nov. 16, 2010.
34. Liu, Z., "Dynamic Analysis of Center-Driven Web Winder controls," Proceedings IEEE Industrial Application Soiciety IAS, Phoenix, Sep., 2008.