IMPROVEMENT OF CONTROL OF A WEB ACCUMULATOR

By

John J. Shelton
Oklahoma State University
USA

ABSTRACT

Previous analysis of the dynamics of a conventional accumulator (no driven rollers within the accumulator, with all rollers identical) was published by Shelton [1]. This 1999 analysis showed that such accumulators commonly suffer from an excessive difference of tension between the entry and exit, often leading to excessive local changes in tension as well as slackness and/or slippage between the web and specific rollers. The analysis also showed that the inertia of the carriage, because of its usually large mechanical advantage, often is negligible in comparison to the total inertia of the rollers. Additional conclusions were that (1) J/R^2, not the inertia J by itself, is a governing parameter, (2) as the operating velocity is increased, the required height (distance of travel) of the accumulator may become excessive, and (3) the force required for counterbalancing the carriage can be calculated for specific goals of optimization of operating conditions. Specific schemes of control were not addressed in the 1999 IWEB paper.

This paper does not repeat derivations or results of the 1999 paper; instead, it examines schemes for control of tension within an accumulator, with emphasis on control of velocities instead of the less desirable control of forces and torques. Equations for velocities during steady-state running, stoppage, constant acceleration or deceleration of the web, and filling or emptying of the accumulator are derived.

Lateral errors, sometimes great enough to cause a failure, are commonly caused by the long multiple spans of an accumulator in combination with excessive flexibility of the locating elements of the carriage and the imperfection of the web; hence, schemes are presented for automatic leveling of the carriage, or tilting of the carriage for incremental correction of errors caused by camber or other imperfections.

Driving all the rollers in an accumulator with precise control of their velocities along with synchronized control of the velocity of the carriage, should almost eliminate the tension variations inherent in current practice, and should almost eliminate the carriage travel now required for the acceleration and deceleration modes of the rollers.

NOMENCLATURE

KE	kinetic energy of a roller
PE	potential energy in spans of a web
E	modulus of elasticity
F	force
J	mass moment of inertia
m	mass
N	identification number of the roller on the carriage farthest from the winder or unwinder
R	radius of the rollers
r	identification number of a roller (0,1,2,…,N)
T	web tension
t	thickness of the web
t_a	time from the start of acceleration after stoppage
t_d	time from the start of deceleration for stoppage
t_{ea}	time from the start of acceleration for emptying a winder accumulator
t_{ed}	time from the start of deceleration for emptying a winder accumulator
t_{fa}	time from the start of acceleration for filling an unwinder accumulator
t_{fd}	time from the start of deceleration for filling an unwinder accumulator
Δ_{te}	time of steady emptying (winder)
Δ_{tf}	time of steady filling (unwinder)
Δ_{ts}	time of stoppage of one end of the web
Δ_{t2}	time of acceleration or deceleration (stoppage)
Δ_{t3}	time for acceleration or deceleration for emptying or filling
V	velocity
V_e	velocity of the web during emptying of a winder accumulator
V_f	velocity of the web during filling of an unwinder accumulator
V_i	steady state velocity of the web
W	width of the web
ε	strain of the web
ω	angular velocity-radians per second
ω_e	angular velocity during emptying of a winder accumulator
ω_f	angular velocity during filling of an unwinder accumulator

Subscripts and other methods of identification

c	pertaining to the carriage
i	“initial”, or normal operating condition
0,1,2,…,N	numbers identifying rollers and spans
{ }	numbers of equations
[ ]	numbers of references

INTRODUCTION

An accumulator, also called a “festoon” or a “looper”, consists of a set of stationary rollers and a set of rollers on a moving carriage, usually one fewer than the number of stationary rollers, with parallel strands of web wrapping each roller by 180 degrees, except perhaps for the first and last stationary rollers, and for cases wherein the web is routed over a guide or other auxiliary equipment. The carriage usually travels vertically, but rail-car carriages with horizontal travel have been used in slow-speed lines in the steel industry. Most vertical accumulators utilize many rollers on the carriage, with the functional and analytical advantage of a usually negligible translational inertia of the carriage.

The primary usage of an accumulator is for “zero-speed splicing”, wherein the function of the processing line requires a constant velocity of the web, but a full-speed flying splice between the expiring roll and the full roll at the unwinder is impractical or prohibitively expensive. Less common, except in processing of metals, is an exit accumulator, where the full winding roll is stopped for cutoff of the web and removal of the roll while the process continues at full speed. Another application of an accumulator is to allow momentary stoppage of a section of a process line while other processes continue at a constant speed.

The three primary modes of operation of an accumulator are (1) stationary carriage, (2) accelerating (or decelerating) carriage and web, and (3) stationary end of the web, with the process being fed out of the unwind accumulator, or with the process feeding into the rewind accumulator. Secondary modes, such as a transition between constant velocity and constant acceleration of the carriage, may be important in some applications. The three primary modes in further detail are:

(1) When the carriage is stationary, the conventional accumulator (control of forces and torques) should function as a series of parallel idler rollers, often with no web guides and within only one zone of tension control. The primary effect of the accumulator on tension then is the drag of the large number of rollers. When the accumulator is full or nearly full, lateral error often is a problem.

(2) The mode of acceleration of the web and the rollers in the accumulator is the primary source of misunderstanding of the behavior of an accumulator, and hence was the primary object of attention in the 1999 analysis by Shelton [1]. Improvement of control during modes of acceleration is further emphasized in this paper.

(3) The mode of a stationary end of the web is analogous to a block and tackle, such as the pulley block with hook for lifting a string of drill pipe in well drilling: The speed ratio and the mechanical advantage (numerically identical) are readily determined by counting the number of strands of web which travel into and out of the carriage assembly, as by counting the strands of cable supporting the pulley block.

Operation in the above second mode (acceleration) requires a judicious choice of control schemes to achieve effective control of tension as well as velocities of the web and the carriage. Control of *forces* (including control of torque) for control of *both* the web and the carriage is generally unsatisfactory, partially because friction is usually unknown and highly variable. Instead, the *velocity* of *either* the web *or* the carriage must be controlled. The velocity of both may be controlled, with proper implementation of controls. If position and velocity of the carriage as well as the velocity of the web are predictably controlled, the behavior of the web and the carriage can be analyzed by superposition of the action as a block and tackle [above item (3)] and as a passive transport system [above item (1)], with the transport velocity equal to the instantaneous velocity of the relevant unwinder or winder.

Control of tension in an accumulator, or even the sensing of tension when the web is accelerating or decelerating, is a difficult problem which has not received adequate attention. Unless every roller in an accumulator is driven and velocity-controlled, imperfect control of tension in sections of the accumulator will occur.

The following analysis is primarily concerned with the kinematics of an accumulator, for understanding inherent problems of behavior even though the web is idealized by neglecting its elasticity and though the drives are idealized by neglecting short periods of transition.

All existing accumulators known to the author suffer from inherent problems of variation of tension from one end of the accumulator to the other, and from unwanted changes in tension as the mode of operation of the accumulator changes between a stationary carriage and conditions of filling or emptying. Further, accumulators commonly suffer from lateral errors in web position.

Problems with behavior of accumulators, while evident in most installations, are exaggerated in applications in the metals industries because of the required long period of stoppage, web camber so severe that slackness of the longer edge is often not eliminated by tension, and high speed of operation. Even though accumulators for metals are very expensive because of their size and the required ruggedness of rollers and other components, concepts as well as details of design are often flawed. Stiffness against tilting of the carriage is often inadequate; furthermore, lateral tilting of the carriage may be caused by the tension in a straight, centered web because of unequal stiffness of the carriage positioners on the two sides, such as by unequal lengths of cables from positioning winches.

During zero-speed splicing, the control of tension with a stationary clamped end and an output at the required process tension may present an additional problem of control, as the *type* of a control system generally changes as the web changes between zero and finite velocity.

This paper is aimed at continuing and augmenting the work of Shelton [1] published in 1999. No other basic analysis of accumulators was found. Koc, et al., [2] cited the dangers of high tension (breaks) or low tension (folds), but did not analyze the mechanics of an accumulator, nor did they implement their proposed addition of non-controlling dancers.

ANALYSIS OF DYNAMICS

Velocities

In Figure 1, the number of rollers on each carriage is (N + 1)/2. The number of spans to and from the moving rollers is N + 1, so that the mechanical advantage of the carriage is N + 1; that is, the force on the carriage required to balance the steady-state tension is (N + 1) times the tensile force, and the velocity of the carriage with a stationary web at the unwinder or winder is $V_i/(N + 1)$. In the following analysis, subscripts for tension and velocity in a span correspond to the number of the downstream roller for an unwinder, and the upstream roller for a winder. Odd-numbered rollers are on the carriage, and even-numbered are stationary. The illustration of winding and unwinding on two-drum devices is arbitrary.

Table 1 shows magnitudes of tension between rollers on the carriage and stationary rollers for (A) deceleration of the winder or acceleration of the unwinder, or (B) acceleration of the winder or deceleration of the unwinder, for changes of tension units across the accumulator of ± 20 percent and ± 50 percent of a tension value which is

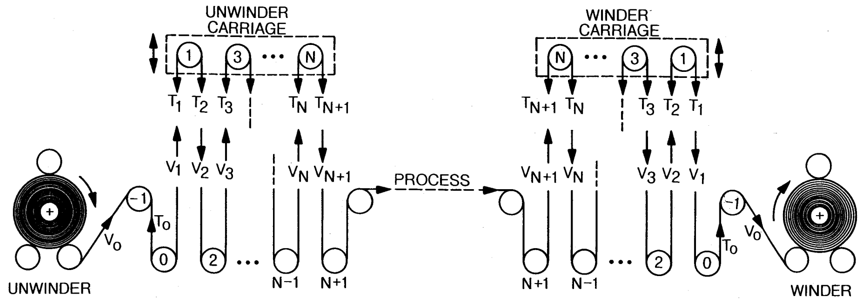


Figure 1 – Accumulators for Unwinder and Winder with Nomenclature (Two-Drum Unwinder and Two-Drum Winder Shown)

controlled to a constant value of 100 percent, as represented by 100 in the tables. The tables are for values of N of 7. Columns labeled (A) were calculated from equations {20} [1], and those labeled (B) were calculated from equations {22} [1], but the same results are obtained from either set of equations, perhaps with a loss of intuitive understanding if negative variables are used.

Table 1 corresponds to Table 3 by Shelton [1], except that Table 3 is for N = 9 instead of N = 7. Please note, however, the reversal of the footnotes, correcting the 1999 table.

	B*		A*		B*		A*	
Column	1	2	3	4	5	6	7	8
$(T_{N+1}-T_0)/T_{N+1}$	−1/5	−1/4	1/5	1/6	−1/2	−1	1/2	1/3
$(T_0-T_{N+1})/T_0$	1/6	1/5	−1/4	−1/5	1/3	1/2	−1	−1/2
T_0	120.00	100.00	80.00	100.00	150.00	100.00	50.00	100.00
T_1	115.56	95.56	84.44	104.44	138.88	88.88	61.11	111.11
T_2	111.67	91.67	88.33	108.33	129.17	79.17	70.83	120.83
T_3	108.33	88.33	91.67	111.67	120.83	70.83	79.17	129.17
T_4	105.56	85.56	94.44	114.44	113.89	63.89	86.11	136.11
T_5	103.33	83.33	96.67	116.67	108.33	58.33	91.67	141.67
T_6	101.67	81.67	98.33	118.33	104.17	54.17	95.83	145.83
T_7	100.56	80.56	99.44	119.44	101.39	51.39	98.61	148.61
T_8	100.00	80.00	100.00	120.00	100.00	50.00	100.00	150.00
F_c	846.67	686.67	753.33	913.33	916.67	516.67	683.33	1083.33

* B acceleration of winder or deceleration of unwinder
A deceleration of winder or acceleration of unwinder

Table 1 – Relative Tensions during Constant Acceleration or Deceleration of an Accumulator (N = 7)

SCHEMES FOR IMPROVED CONTROL OF AN ACCUMULATOR

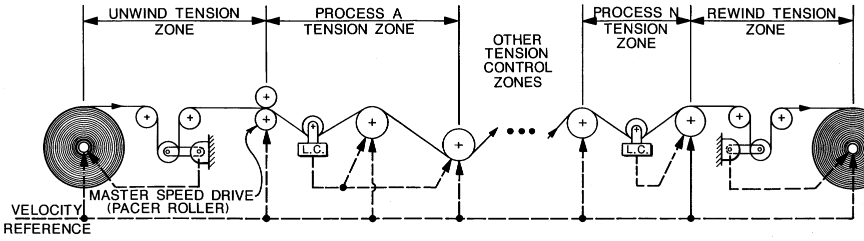
Figure 2 shows the usual web process with multiple zones of automatic tension control, with a single master speed drive, where a single nominal speed V_i for the process is chosen by the operator. Differing tensions in the various process zones obviously result in deviations from the nominal speed, but these deviations are usually somewhat less than 1.0 percent. Control of an accumulator is a web-handling operation wherein major changes of velocity in different portions of the process line are under control of the machine operator; hence, the machine may have several master speed rollers.

Control of the velocity of the web at the upstream and/or the downstream end(s) of an accumulator must be coordinated with the velocity of the carriage, preferably by automatic control. Coordination of the action of the carriage with the velocity of the web is complicated by the fact that most (if not all) of the rollers in common accumulators are idlers, not powered rollers. The problems of acceleration and deceleration of idlers has evidently been misunderstood and neglected by designers and users of accumulators, with the first known documentation in open literature in the analysis by Shelton [1]. This paper is based on the work reported in 1999.

Complete Control of Velocities

(A) Unwinder Accumulator: The ultimate scheme for control of an accumulator would apparently be control of the velocity of each roller (each roller individually driven, with a command from a computer and with velocity feedback) along with control of the velocity of the carriage (which would have its own feedback of linear velocity). This method of control would allow accelerations and decelerations to be limited only by the torque and temperature limitations of the drive motors, as the inertia of the web is usually negligible. The analysis by Shelton [1] provides a foundation for analysis of a velocity-controlled accumulator.

Complete control of velocities may be achieved by considering the velocities of the surfaces of the rollers as expressed by Shelton [1] in his equations {1} to be superposed on the controlled velocity of the surface of roller 0 of Figure 1. This superposition is illustrated with the specific unwinder with $N = 7$ in Figure 3. The performance of this unwinder is illustrated in the velocity/time sketch of Figure 4, in which seven desired modes of operation are shown. The steady running with an empty accumulator after stoppage is not required, but may result in more reliability than with a nearly full accumulator. The seven above modes of operation are identified with circled numbers at the bottom of Figure 4. The required velocity greater than V_i for refilling the accumulator is an arbitrary tradeoff with the time of filling; furthermore, the acceleration to the velocity of steady filling is arbitrary, not necessarily as great as the time-dictated acceleration after stoppage. Figure 4 arbitrarily shows the maximum velocity of the web as 50 percent greater than V_i, but this great an increase in velocity may cause problems with air entrainment and turning torque in some applications, necessitating a lower velocity of filling.

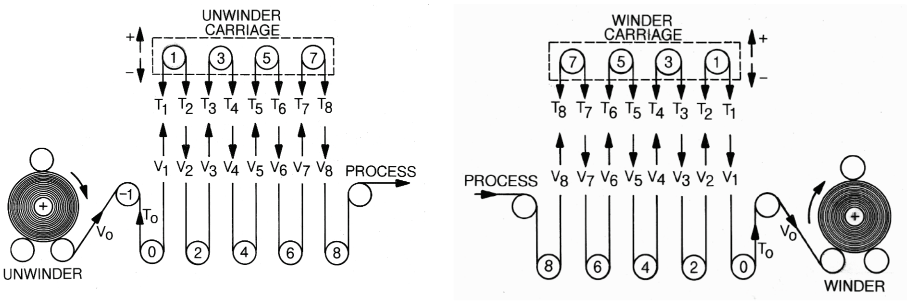


NOTES: (1) DANCER POSITION CONTROLS ANGULAR VELOCITY OF SPINDLE (RADIUS SIGNAL NOT SHOWN) OR LINEAR VELOCITY OF SURFACE OF ROLL.

(2) LOAD CELL SIGNAL CONTROLS VELOCITY OF DRIVE ROLLER.

(3) TACHOMETER FEEDBACK AT DRIVES NOT SHOWN.

Figure 2 – Web Process with Multiple Zones of Automatic Tension Control



(a) Unwinder

(b) Winder

Figure 3 – Accumulators with N = 7

Figure 5 shows the surface velocities of rollers 0, 2, 4, 6, and 8 for a stopped or accelerating accumulator. Equations {1} [1] quantify these velocities of the stationary rollers, and an inspection of Figure 1 reveals that $V_1 = V_0$, $V_3 = V_2$, $V_5 = V_4$, $V_7 = V_6$, and that V_8 is equal to the process velocity V_i. The angular velocities of the stationary (even-numbered) rollers are simply equal to their surface velocities divided by the uniform radius R, as listed in equation {2} [1] for a stopped accumulator.

The angular velocities of the rollers on the carriage are modified by the velocity of the carriage, analogous to the modification of velocities of points on a vehicle wheel or tire by the point of reference and the motion of the vehicle. [Relative to the driver, the velocity of the top of a non-slipping tire is equal to the velocity of the vehicle relative to the ground, and the bottom is equal to the negative of this velocity, while relative to the ground the top of the tire is traveling at twice the velocity of the vehicle while the bottom is instantaneously stationary.] The above analysis of linear velocities did not require radii of rollers to be equal, but the following analysis of angular velocities is simplified by the usual case of all equal radii. During stoppage, the values of ω_1, ω_3, ω_5, and ω_7 are shown in equations {2} [1] to be modified from the angular velocity of a neighboring stationary roller by the term V_c/R, or $(V_i/8)/R$ for this example. For this example of N = 7, the instantaneous difference of angular velocities of sequentially neighboring rollers in the

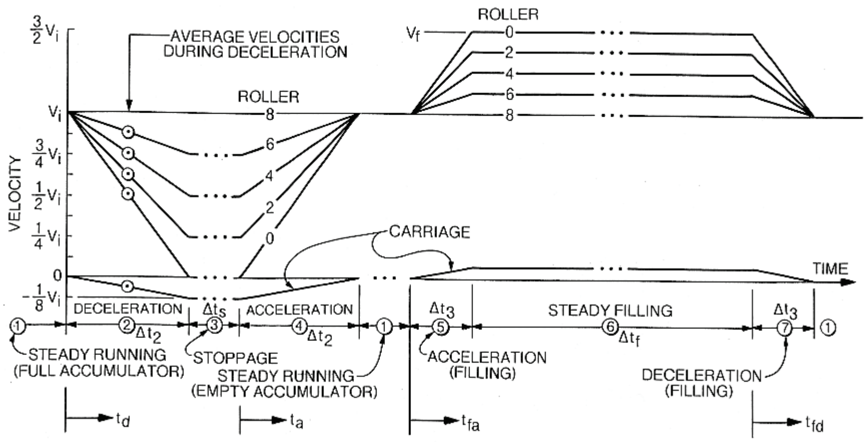


Figure 4 – Example of Modes in Cycle of Stoppage and Filling of Unwinder Accumulator with N = 7 with Filling at 3/2 V_i

stopped accumulator is therefore V_c/R, as shown in Figure 5, which shows angular velocities for the sequence of the modes for stoppage shown in Figure 4. During stoppage of an accumulator, the angular velocity of any roller r (0,1,2,…,N) is thus

$$\left.\omega/\omega_i\right)_{\text{stopped}} = r/8$$

or, in general

$$\left.\omega/\omega_i\right)_{\text{stopped}} = r/(N+1). \qquad \{1\}$$

Equation {1} is a generalization and nondimensionalization of equations {2} [1].

Figure 5 can be used as a guide for analysis of the deceleration and acceleration modes of an accumulator for determining the required angular velocities of all rollers for complete control of all surface velocities, assuming that all rollers have the same radii. The equation of a straight line in x-y coordinates is y = mx + b, where m is the slope of the line and b is the value of y when x = 0. In coordinates of $t_d/\Delta t_2$ or $t_a/\Delta t_2$ (horizontal axis) versus ω/ω_i (vertical axis), the slope of the deceleration locus for any roller r is $-(8-r)/8$, and all intercepts of the vertical axis by the loci occur at ω/ω_8 of 1.0; hence, the equation for N = 7 is

$$\frac{\omega}{\omega_i} = 1 - \frac{8-r}{8}\frac{t_d}{\Delta t_2},$$

or for a general number of rollers:

$$\frac{\omega}{\omega_i} = 1 - \frac{N+1-r}{N+1}\frac{t_d}{\Delta t_2}. \qquad \{2\}$$

For the acceleration mode, using the independent variable as $t_a/\Delta t_2$ results in slopes of $(8-r)/8$ and intercepts of the ω/ω_8 axis of $r/(N+1)$, resulting in the equation for N = 7:

$$\frac{\omega}{\omega_i} = \frac{r}{8} + \frac{8-r}{8}\frac{t_a}{\Delta t_2} ,$$

or for a general number of rollers:

$$\frac{\omega}{\omega_i} = \frac{r}{N+1} + \frac{N+1-r}{N+1}\frac{t_a}{\Delta t_2} . \qquad \{3\}$$

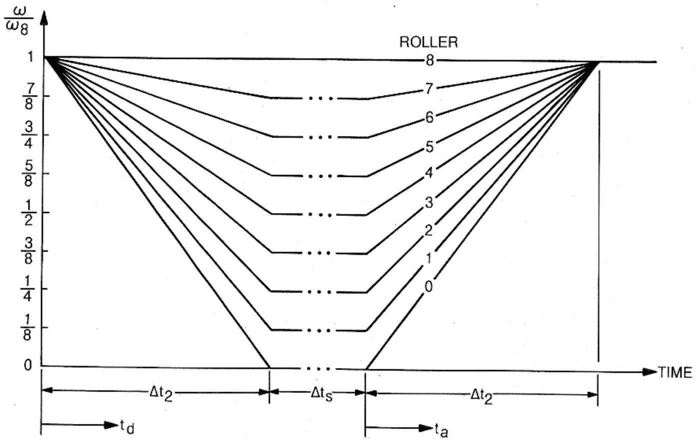


Figure 5 – Example of Angular Velocities for Stoppage of Unwinder Accumulator (N = 7)

The required linearly varying velocity of the carriage during deceleration of the unwinding roll is

$$V_c / V_i = [-1/(N+1)][t_d / \Delta t_2], \qquad \{4\}$$

and during acceleration this velocity is

$$V_c / V_i = [1/(N+1)][-1+(t_a / \Delta t_2)]. \qquad \{5\}$$

During stoppage, the velocity of the carriage is

$$V_c / V_i = -1/(N+1). \qquad \{6\}$$

Filling of the unwinder accumulator in preparation for the next stoppage may be a continuation of the acceleration mode of regaining V_i of the unwinding roll after stoppage, or may be accomplished at any time before the next need for a full accumulator. The maximum velocity of the web is usually arbitrary, after consideration of the economics and technology of high-speed transport of the web. Likewise, the maximum acceleration and deceleration may also be arbitrary. Figures 4 and 6 show these accelerations and decelerations as equal to those for stoppage, and show the maximum velocity V_f as 50 percent higher than V_i.

For a general acceleration and deceleration rate and a general maximum velocity for filling the accumulator, the variables are shown in Figure 4 as the maximum velocity V_f

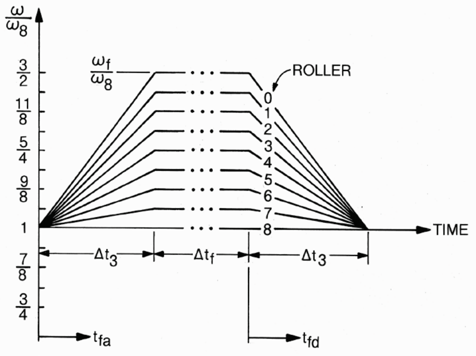


Figure 6 – Example of Angular Velocities for Filling of Unwinder Accumulator with ω/ω_i of 3/2 (N = 7)

and the time period Δt_3 for acceleration and deceleration. The starting time for acceleration for filling is at $t_{fa} = 0$, and for deceleration at $t_{fd} = 0$.

Similarly to the derivation of the velocity equations for stoppage (with the benefit of Figure 6 for establishing relationships between linear web velocities and angular roller velocities), the velocity equations for filling, for chosen values of V_f and Δt_3, may be derived as

$$\frac{\omega}{\omega_i} = 1 + \left[\left(\frac{\omega_f}{\omega_i} - 1\right)\left(\frac{N+1-r}{N+1}\right)\right]\frac{t_{fa}}{\Delta t_3} \qquad \{7\}$$

for acceleration for filling, and

$$\frac{\omega}{\omega_i} = 1 + \left(\frac{\omega_f}{\omega_i} - 1\right)\left(\frac{N+1-r}{N+1}\right)\left(1 - \frac{t_{fd}}{\Delta t_3}\right) \qquad \{8\}$$

for deceleration for achieving a full accumulator when t_{fd} is equal to Δt_3. During the mode of steady filling (with the angular velocity of roller 0 at V_c/R radians per second), the angular velocity of any roller r in the accumulator is

$$\frac{\omega}{\omega_i} = 1 + \left[\left(\frac{\omega_f}{\omega_i} - 1\right)\left(\frac{N+1-r}{N+1}\right)\right]. \qquad \{9\}$$

For this scheme of complete control of velocities, the time periods for deceleration for stoppage and for acceleration after stoppage are Δt_2, as determined by torque and temperature limitations of the motors, in contrast to the determination based on the total tension difference across an accumulator with non-driven rollers as specified by equation {16} [1]. The rates of acceleration and deceleration for filling, as shown in

Figure 4, are assumed to be the same as those for stoppage; hence, Δt_3 is determined by the chosen maximum velocity V_f :

$$\Delta t_3 = \left[\left(V_f - V_i\right)/ V_i\right]\Delta t_2 . \qquad \{10\}$$

The time Δt_f of steady filling is now completely determined by the above choices, as the average velocity of each roller for a complete cycle of operation of the accumulator must be equal to V_i (with the assumption of negligible strain compared to the original length). Thus, for roller 0:

$$V_i\left(\Delta t_2\right) + V_i\left(\Delta t_s\right) = \left(V_f - V_i\right)\Delta t_3 + \left(V_f - V_i\right)\Delta t_f ,$$

or, solving for Δt_f after substituting equation {9}:

$$\Delta t_f = \frac{\Delta t_s}{\left(V_f / V_i\right) - 1} + \left[\frac{1}{\left(V_f / V_i\right) - 1} - \left(\frac{V_f}{V_i} - 1\right)\right]\Delta t_2 . \qquad \{11\}$$

The velocity of the carriage during acceleration for filling is

$$V_c / V_i = \left[1/\left(N+1\right)\right]\left[\left(V_f / V_i\right) - 1\right]\left(t_{fa} / \Delta t_3\right), \qquad \{12\}$$

and during deceleration to the condition of a full accumulator this velocity is

$$V_c / V_i = \left[1/\left(N+1\right)\right]\left[\left(V_f / V_i\right) - 1\right]\left[1 - \left(t_{fd} / \Delta t_3\right)\right]. \qquad \{13\}$$

During the mode of steady filling, the carriage velocity is

$$V_c / V_i = \left[1/\left(N+1\right)\right]\left[\left(V_f / V_i\right) - 1\right]. \qquad \{14\}$$

During a complete cycle of operation, the positive and negative distances of travel of the carriage must be equal; that is, the sum of each of the average positive velocities multiplied by each corresponding time period must equal the sum of each of the average negative velocities multiplied by each corresponding time period. For the above chosen conditions of operation, from Figure 4 for a general value of N:

$$-\frac{V_i}{N+1}\Delta t_2 - \frac{V_i}{N+1}\Delta t_s + \frac{V_f - V_i}{N+1}\Delta t_3 + \frac{V_f - V_i}{N+1}\Delta t_f = 0 .$$

In the above equation, substitution of equation {10} for Δt_3 and {11} for Δt_f results in the identity 0 = 0, proving the validity of the equation.

Table 2 lists the above equations as they apply to the numbered (circled) modes of Figure 4, for a general number N.

Figure 7 shows schematically the requirement of a dedicated power cable and velocity-feedback cable for each motor. Not shown is the desirable clutch-control cable to each roller.

(B) Winder Accumulator: Figure 3(b) shows a winder accumulator with N = 7. Figure 8 shows that its basic behavior is identical to that of the unwinder accumulator of Figure 4, except that the direction of travel of the carriage is reversed throughout the series of modes of operation. The numbering system for the rollers was chosen for analysis to apply to both accumulators; hence, equations {4} (deceleration) and {5} (acceleration) also apply to the winder.

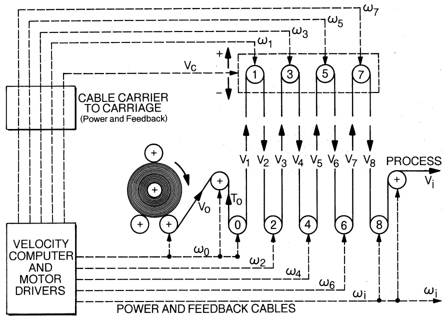


Figure 7 – Example of Control Requirements for Complete Control of Velocities of an Unwinder Accumulator with N = 7

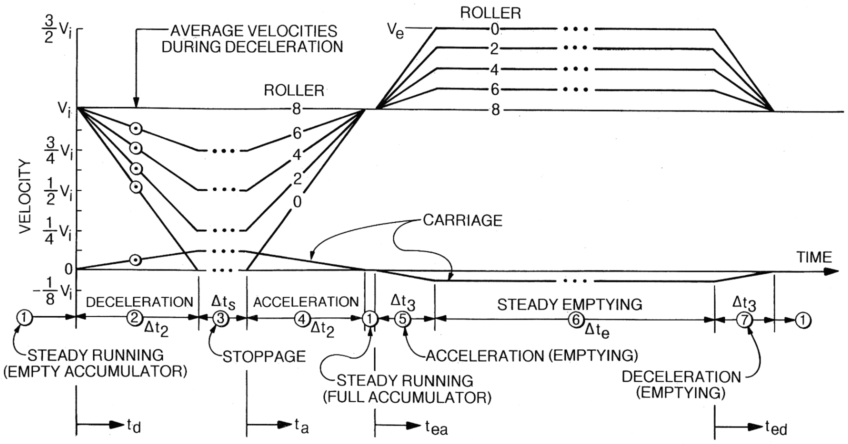


Figure 8 – Example of Modes in Cycle of Stoppage and Emptying of Winder Accumulator with N = 7 with Emptying at 3/2 V_i

Figure 8 shows the acceleration and deceleration for emptying the accumulator in preparation for stoppage as equal to the deceleration and acceleration for stoppage of the unwinder accumulator. The maximum velocity V_e during emptying, however, may be limited by winding conditions (entrainment of air, and balance, cylindricity, and roundness of the roll) and the speed limitations of motors and mechanical transmissions.

Table 3 lists the equations for complete control of velocities of a winder accumulator as they apply to the numbered (circled) modes of operation of Figure 8, for a general number of rollers N. These equations are identical to those in Table 2 except for signs in the carriage equations and subscripts which identify modes of filling and emptying.

The requirements of dedicated cables for power, velocity-feedback, and clutch control for each motor are similar to those for an unwinder as shown in Figure 7.

When the velocities of all rollers as well as the velocity of the carriage are controlled, all drive motors for the accumulator rollers probably should be disconnected.

		Equation or Constant Value		
	Mode	Rollers (Equation)	Carriage (Equation)	Variables (in addition to V_i or ω_i, and r)
1.	Steady running at V_i	$\omega_i = V_i/R$	$V_c = 0$	--
2.	Deceleration for stoppage	$\omega = \omega_i\left[1 - \frac{N+1-r}{N+1}\frac{t_d}{\Delta t_2}\right]$ (2)	$V_c = -\frac{V_i}{N+1}\frac{t_d}{\Delta t_2}$ (4)	t_d, Δt_2
3.	Stoppage (Time of stoppage chosen)	$\omega = \omega_i\left[\frac{r}{N+1}\right]$ (1)	$V_c = -\frac{V_i}{N+1}$ (6)	--
4.	Acceleration after stoppage	$\omega = \omega_i\left[\frac{r}{N+1} + \frac{N+1-r}{N+1}\frac{t_a}{\Delta t_2}\right]$ (3)	$V_c = -\frac{V_i}{N+1}\left(1 - \frac{t_a}{\Delta t_2}\right)$ (5)	t_a, Δt_2
5.	Acceleration for filling	$\omega = \omega_i\left\{1 + \left[\left(\frac{\omega_f}{\omega_i} - 1\right)\left(\frac{N+1-r}{N+1}\right)\right]\frac{t_{fa}}{\Delta t_3}\right\}$ (7),(10)	$V_c = \frac{V_i}{N+1}\left(\frac{V_f}{V_i} - 1\right)\frac{t_{fa}}{\Delta t_3}$ (12),(10)	V_f, t_{fa}, Δt_2
6.	Steady filling	$\omega = \omega_i\left\{1 + \left[\left(\frac{\omega_f}{\omega_i} - 1\right)\left(\frac{N+1-r}{N+1}\right)\right]\right\}$ (9)	$V_c = \frac{V_i}{N+1}\left(\frac{V_f}{V_i} - 1\right)$ (14)	V_f
	Time for steady filling	$\Delta t_f = \frac{\Delta t_s}{(V_f/V_i) - 1} + \left[\frac{1}{(V_f/V_i) - 1} - \left(\frac{V_f}{V_i} - 1\right)\right]\Delta t_2$ (11)		
7.	Deceleration (filling)	$\omega = \omega_i\left\{1 + \left(\frac{\omega_f}{\omega_i} - 1\right)\left(\frac{N+1-r}{N+1}\right)\left(1 - \frac{t_{fd}}{\Delta t_3}\right)\right\}$ (8),(10)	$V_c = \frac{V_i}{N+1}\left(\frac{V_f}{V_i} - 1\right)\left(1 - \frac{t_{fd}}{\Delta t_3}\right)$ (13),(10)	V_f, t_{fd}, Δt_2

Table 2 – Control of Velocities of Powered Rollers in an Unwinder Accumulator

from the rollers by means of normally disengaged electric clutches, except during operation of the accumulator, which may be less than 1.0 percent of the time for large rolls of a thin web. The life of the motors would then be expected to be increased greatly, and operation with a failed drive motor should then be satisfactory (with the one roller functioning as an idler) until repairs can be accomplished.

The following articles discuss schemes for control of tensions in accumulators with decreasing complexity but also decreasing performance capability in comparison to the above scheme of complete control of all velocities. A knowledge of the tradeoff between reduced complexity and cost compared to compromised performance should allow more suitable decisions than were previously possible.

Control of only Stopping Rollers and Roll, along with Control of Full-Speed Rollers; Mechanical Design of Hoists for the Carriage

(A) Unwinder. Figure 9 is a sketch of an unwinder accumulator with N = 7, with no drives to rollers which must change ratios relative to neighboring rollers during stoppage and related modes. This scheme represents the original intention of the analysis and writeup by Shelton [1], wherein performance may be limited by (1) torque and temperature limitations of drive motors during acceleration and deceleration (as with the scheme for complete control of velocities), (2) slipping of the web relative to rollers, resulting in scratching which is sometimes objectionable, (3) slack hanging loops of the web, which sometimes are caught by a roller as a doubled web, causing breakage of the web or other requirements for shutdown of the process, (4) poor control of tension, perhaps with disturbances originating from an accumulator propagating into the process zone(s) or even into the accumulator at the other extreme end of the machine, and (5) an inevitable difference in tension from one end of the accumulator to the other because of the inertia of accelerating web-driven rollers.

Figure 9 shows a zero-speed clamp which is necessary for maintaining tension in the web during splicing to the end of the new web. The sketching of the dancer for control of the unwinder (with feedback of dancer position disconnected during stoppage) is not a general recommendation, nor is the location of the load cells for controlling the tension at the entry of the process; these choices depend on many considerations.

The control computer in Figure 9 must generate real-time functions of velocity for all seven modes of control as shown in Figure 4 and as listed in Table 2, but only for roller 0, roller (-1), and other synchronized rollers if they exist. The initiating and

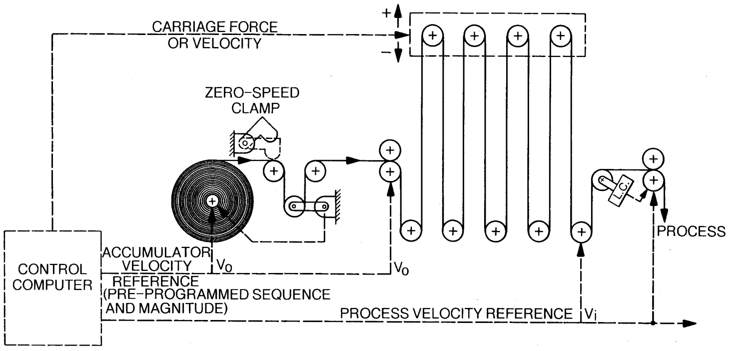


Figure 9 – Control of Entering and Exiting Rollers
and Carriage of Unwinder Accumulator

Mode	Equation or Constant Value		Variables (in addition to V_i or ω_i, and r)
	Rollers	Carriage	
1. Steady running at V_i	$\omega_i = V_i/R$	$V_c = 0$	--
2. Deceleration for stoppage	$\omega=\omega_i\left[1-\frac{N+1-r}{N+1}\frac{t_d}{\Delta t_2}\right]$	$V_c=\frac{V_i}{N+1}\frac{t_d}{\Delta t_2}$	t_d, Δt_2
3. Stoppage (Time of stoppage chosen)	$\omega=\omega_i\left[\frac{r}{N+1}\right]$	$V_c=\frac{V_i}{N+1}$	--
4. Acceleration after stoppage	$\omega=\omega_i\left[\frac{r}{N+1}+\frac{N+1-r}{N+1}\frac{t_a}{\Delta t_2}\right]$	$V_c=\frac{V_i}{N+1}\left(1-\frac{t_a}{\Delta t_2}\right)$	t_a, Δt_2
5. Acceleration for emptying	$\omega=\omega_i\left\{1+\left[\left(\frac{\omega_e}{\omega_i}-1\right)\left(\frac{N+1-r}{N+1}\right)\right]\frac{t_{ea}}{\Delta t_3}\right\}$	$V_c=-\frac{V_i}{N+1}\left(\frac{V_e}{V_i}-1\right)\frac{t_{ea}}{\Delta t_3}$	V_e, t_{ea}, Δt_2
6. Steady emptying	$\omega=\omega_i\left\{1+\left[\left(\frac{\omega_e}{\omega_i}-1\right)\left(\frac{N+1-r}{N+1}\right)\right]\right\}$	$V_c=-\frac{V_i}{N+1}\left(\frac{V_e}{V_i}-1\right)$	V_e
Time for steady emptying	$\Delta t_e=\frac{\Delta t_s}{(V_e/V_i)-1}+\left[\frac{1}{(V_e/V_i)-1}-\left(\frac{V_e}{V_i}-1\right)\right]\Delta t_2$		
7. Deceleration (emptying)	$\omega=\omega_i\left\{1+\left(\frac{\omega_e}{\omega_i}-1\right)\left(\frac{N+1-r}{N+1}\right)\left(1-\frac{t_{ed}}{\Delta t_3}\right)\right\}$	$V_c=-\frac{V_i}{N+1}\left(\frac{V_e}{V_i}-1\right)\left(1-\frac{t_{ed}}{\Delta t_3}\right)$	V_e, t_{ed}, Δt_2

Table 3 – Control of Velocities of Powered Rollers in a Winder Accumulator

terminating of the modes is unlikely to be chosen to be fully pre-programmed, but probably should also be by a combination of sequencing by limit switches and operator intervention.

Control of forces on the carriage by means of hydraulic cylinders was reported by Pagilla, et al. ([3], [4], and [5]) to be quite unsuccessful, probably primarily because of friction in the cylinders, but with inherent shortcomings of control of hydraulic pressures (decompression surges, friction of valve spools, etc.). Further, the forces caused by the inertia of rollers were not considered. Electric actuators for control of *forces* (probably motors without gear reducers operating small cables or chains) may be practical for small accumulators, but electrical control of *velocity* of an accumulator of any size appears to be both better and easier to design than pneumatic or hydraulic control.

The concept of control of force on the carriage of an accumulator is primarily applicable to small accumulators, such as those for handling webs for infant-care and personal-care products. Because of the variation in tension of the web in the machine direction during acceleration and deceleration, the carriage must be constrained against excessive pitch, and stiff constraint against tilting is essential for minimizing lateral errors. This constraint against pitch and tilting is usually achieved in a vertical accumulator by mechanical locating members, such as cables or chains, instead of by vertically widely spaced bushings, rollers, or other followers on tracks, ways, or rails.

Many accumulators in the steel industry have employed cables at each corner of the tower for supporting the carriage, with travel accomplished with motor-driven drums which are spirally grooved in a precisely machined profile for repeatable winding of the individual cables. Maintaining levelness of the carriage has often been a major problem, however, resulting in large lateral errors in the long spans which are common in a steel-industry accumulator.

Proper practices of mechanical design and maintenance should be applied to hoists for carriages. One known fundamental flaw of design has been unequal lengths of cables on the two sides of the tower, because of the desire to locate all the winches on the back side of the tower, without consideration of the dependence of the spring rate of a cable on its length.

An improvement in lateral behavior of the web in a large, tall accumulator could be accomplished with “servo leveling”, in which a plus or minus deviation from levelness is sensed, and slow-speed wedges under pulley blocks for the suspension cables might correct the out-of-level condition. A further improvement could be sensing of the lateral position of the web at the entry and exit of the accumulator, and commanding an out-of-level condition to compensate for the steering caused by camber of the web or other disturbances. The lateral position sensor near the exit probably should be weighted higher than the one near the entry, and other considerations of the response of the web (the slow response to tilting) would have to be incorporated into the control algorithm.

Chain drives have been employed for hoisting the carriage, using a solid axle across the carriage at each end of the accumulator, with a sprocket at each end of each axle. The vertical travel of the chains on the two axles are synchronized by two right-angle drives with a machine-direction shaft connecting them. Right-angle drives with a third differential input are available for adjustment of the primary input/output shafts relative to each other. Such a differential drive could also be used in each axle across the accumulator for mechanical servo leveling of the carriage and intermittent correcting of the lateral position, as described in the previous paragraph.

The vibration at the frequency of travel of the links of a chain for positioning the carriage may be objectionable. If the design of a carriage hoist does not require the synchronization of a chain and sprocket and does not need to be wound on a drum, a leaf chain might be considered instead of a cable, with the advantages of compactness, durability, and stiffness compared to a wire-rope cable.

Supporting and operating the carriage with a leadscrew at each of its four corners may sometimes be practical. If the pitch of the leadscrews can be consistent along the length and between screws, the position of the four corners could then be measured with encorders on the drive motors. Design for reliability and durability should be possible if the concept is practical, and the stiffness against tilting could be high.

In Figure 9, the control of tension into the process (shown with load-cell sensing) is generally necessary because of the inevitable increase and decrease of tension across the accumulator because of the acceleration and deceleration of the non-driven accumulator rollers.

(B) Winder. Figure 10 for a winder without control of velocities of individual rollers corresponds to the unwinder of Figure 9. The control of tension following the last process zone is generally necessary for versatility of control of the average tension in the accumulator, with the increase and decrease of tension across the accumulator depending on the rate of acceleration and deceleration. The tension of winding, often modified by taper, must generally be controlled by another control station. Design considerations of an accumulator for winding are generally the same as those discussed for unwinding, except for the advantage of the condition of running empty while prepared for stoppage (hence improved lateral behavior) with a winder accumulator.

Attempts at Simple Control of Accumulators

Few accumulators perform really well, with non-detrimental oscillations of tension and velocity and with small differences of tension across the accumulator. The rare cases of good performance probably have resulted from low speed, or from sound mechanical design in combination with a low elastic modulus of the web and large limits to the permissible differences in tension across the accumulator.

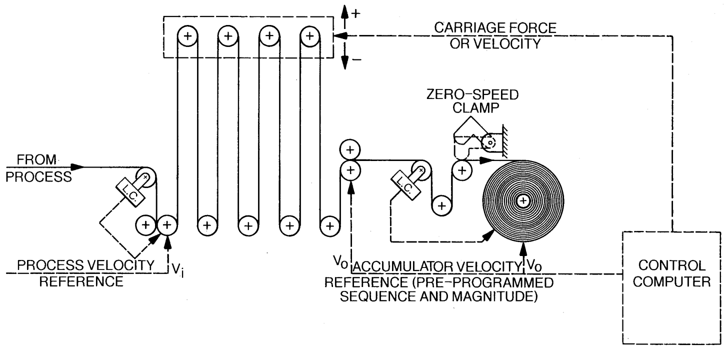


Figure 10 – Control of Entering and Exiting Rollers
and Carriage of Winder Accumulator

Great effort has been expended in the metals industry in striving for satisfactory performance, using simple concepts but complicated implementation of control of accumulators, as documented by Pagilla, et al. ([3], [4], and [5]). These three papers studied the origin and propagation of disturbances, with shocks from the unwind accumulator causing visible flaws in the winding roll. The three papers express suspicion that the poor control of tension is largely caused by friction in the hydraulic cylinders which control the carriage. While this suspicion may be justified, the papers ignored the cumulative tension difference caused by acceleration or deceleration of the accumulator rollers by the web, as expressed in equation {15} [1]. Further, the control of tension in most metal processing lines is fundamentally flawed by the obsolescent practice of control of torque instead of velocity of drives for rollers and rolls. Additionally, control of the velocity of the carriage should be far easier and more dependable than the control of forces, which are highly susceptible to unpredictable disturbances.

Michal [6] verified, based on his experience with hundreds of accumulators, that problems (particularly poor control of tension) are prevalent in small conventional accumulators which control the force on the carriage and which have undriven rollers.

Sudden Starts and Stops. The preceding analysis of dynamics of an accumulator assumed the usual combination of variables, particularly the velocity V_i of the process, which result in the requirement of a finite time period Δt_2 for acceleration or deceleration. It was noted in the Introduction, however, that an ordinary block and tackle does not require such periods of acceleration and deceleration, primarily because of a small value of the effective inertia J/R^2 of the pulleys, in contrast to the inertia of long rollers of most web-handling machines, but also because of the fragility of most webs and the need for avoidance of shocks in processing and winding.

The discussion by Shelton [1] following equation {7} implied that if the value of this equation is small, abrupt starting and stopping of the accumulator may be acceptable. Such an abrupt start of a full unwinding roll or an abrupt stop of a full winding roll must also be acceptable, with surface unwinding and winding more likely to provide acceptable results than center unwinding and winding because of less likelihood of interlayer slippage. The needs for minimizing the cost of slow-speed machines and for avoiding the penalties of poor performance of higher-speed machines justify further study.

All equations for ΔKE, such as (4) through (10) by Shelton [1], show the total change in kinetic energy of the rollers between the stopped condition of the unwinder or winder and the full-speed condition to be proportional to V_i^2 of the web, as might be expected from elementary physics. A lesser-known fact that the stored energy in a spring (the tensioned web) varies with its spring rate (EtW/L_T for a conventional web) and the square of its elongation resulted in equation {6} [1]. The square relationships of V_i in the numerator and ε_i in the denominator, with no other variables except for L_T and the variables established by the web and by the design of the accumulator, seem to indicate that the strain (or tension) of a sudden start or stop is proportional to the process velocity V_i. However, an unwinder accumulator with no excess capacity must start with a small total length L_T (and with a full roll), and a winder accumulator, likewise designed with no excess capacity, must stop a full roll when the value of L_T is small. In either case, little resiliency is provided by the web within the accumulator to mitigate the effects of the respective sudden start and stop.

The above discussion of sudden starts and stops indicates that satisfactory performance of a machine intended for such operation would be extremely rare. It is therefore recommended that modes of operation using conservative values of acceleration and deceleration, based on acceptable tension differences across the accumulator and on limits of performance of drive motors, be the basis of design.

CONCLUSIONS AND RECOMMENDATIONS

Idler-roller accumulators as analyzed by Shelton [1] require a period of acceleration and deceleration for prevention of excessive tension, slackness, and web-to-roller slippage, except for very low-speed lines with low-mass rollers.

In this paper, various schemes for control of accumulators are considered. Control of velocities, both of the web and the carriage, appears to be far more satisfactory than control of forces, as currently practiced. The ultimate control scheme appears to be driving every roller in the accumulator, controlling the velocity of each roller with a pre-programmed command, and achieving precise control with velocity feedback for comparing to the command function. The rates of acceleration and deceleration of an accumulator with control of velocities of all rollers and the carriage could generally be several times greater than those achieved with currently common technology.

REFERENCES

1. Shelton, J.J., “Dynamics of a Web Accumulator”, Proceedings of the Fifth International Conference on Web Handling, Oklahoma State University, June, 1999.
2. Koc, H., Knittel, D., de Mathelin, M., Abba, G., and Cauthier, C., “Web Tension Control in an Industrial Accumulator”, Proceedings of the Fifth International Conference on Web Handling, Oklahoma State University, June, 1999.
3. Pagilla, P.R., King, E.O., Dreinhoefer, L.H., and Garimella, S.S., “Robust Observer-Based Control of an Aluminum Strip Processing Line”, IEEE Transactions on Industry Applications, Vol. 36, No. 3, May/June 2000.
4. Pagilla, P.R., Garimella, S.S., Dreinhoefer, L.H., and King, E.O., “Dynamics and Control of Accumulators in Continuous Strip Processing Lines”, IEEE Transactions on Industry Applications, Vol. 37, No. 3, May/June 2001.
5. Pagilla, P.R., Singh, I., and Dwivedula, R.V., “A Study on Control of Accumulators in Web Processing Lines”, Journal of Dynamic Systems, Measurement, and Control (ASME), Vol. 126, September 2004.
6. Michal, N., “Practical Considerations for Zero Speed Splice Unwinds”, AIMCAL Applied Web Handling Conference, Minneapolis, MN, 2008.

Improvement of Control of a Web Accumulator	J. J. Shelton, Oklahoma State University, USA

Name & Affiliation	Answer
	No Questions until Discussion