LAMINATE THEORY BASED 2D CURL MODEL

By

Sam Kidane
3M Company
USA

ABSTRACT

The laminate theory provides a method of analyzing the mechanics of multilayer composite laminates. With the appropriate definition of the load vector, the laminate theory can be extended to model curl resulting from different stages of web handling. The most direct application would be for predicting curl resulting from change in temperature or moisture content of a multilayer web. Other curl sources such as web lamination and curing/drying shrinkage can also be modeled if the load vectors are appropriately defined. This paper shows some applications of a laminate theory based 2D curl model for predicting web curl in both Machine Direction and Cross Direction. The results obtained from this model are validated by both Finite Element Analysis and Experimentation.

INTRODUCTION

The Laminate theory is almost identical to the classical plate theory with the same underlying assumptions such as *small out of plane displacement*, *thin plate* (width and length at least ten times larger than thickness dimension) … etc. In laminate theory, the classical plate theory stress-strain relation is extended to accommodate orthotropic properties in a lamina (a single layer orthotropic thin plate).

Web planarity problems such as web curl are usually orthotropic, multilayer and two dimensional in nature. The orthotropic property refers to the difference in material properties between Machine Direction (MD) and Cross Direction (CD) of web materials. This orthotropic nature of webs stems from different web processes such as tentering, lengthwise orientation, film forming processes such as extrusion casting …etc. A web curl problem is also multilayer since most web based products go through processes such as coating/deposition and lamination.

The *orthotropic*, *multilayer* and *2D* aspects of the Laminate Theory make it very attractive for analyzing web planarity issues of webs. In this paper a laminate theory based two dimensional curl model is developed. This model is capable of modeling curl resulting from different web handling processes. The model also provides additional tools for superimposing curl due to cascaded processes, solving inverse curl problems, and a 3D curl visualization method. Finite Elements and experimental work is also presented to validate the 2D model.

An alternative method of solving curl problem is to use Finite Elements (FE) modeling. Some of the shortcomings associated with FE modeling of curl are:

- A model has to be built from scratch for every curl problem.
- Post-processing to obtain the curl radius from raw FE results is cumbersome.
- Inverse curl problem can not be solved using FE.
- A license is required to run commercial FE packages.

The developed 2D curl model overcomes all these shortcoming observed in FE modeling. One advantage of FE modeling over the 2D model is the capability of large displacement nonlinear analysis which is important to capture the curl interaction between of MD and CD.

LAMINATE THEORY

The 2D curl model is an adaptation of the Laminate Theory for the prediction of curl due to different web processes. As a background for the 2D curl model, a brief outline of the Laminate Theory is presented in this section. Most of the material in this section is adapted from the book "Analysis and Performance of Fiber Composites" by B. D. Agarwal, et. al [1].

Orthotropic Plate Mechanics

The stress-strain relationship for an orthotropic thin plate is give by eqn {1}. In this equation *L* denotes the *Longitudinal* direction and *T* denotes the *Transverse* direction. It is also assumed that the orthotropic plate is a *specially orthotropic lamina* meaning the major axis is aligned with the *Longitudinal* axis and the minor axis is aligned with the *Transverse* direction. The major and minor axes respectively correspond to the maximum and minimum stiffness directions. In subsequent section these two directions will correspond to respectively Machine Direction (MD) and Cross Direction (CD).

$$\begin{Bmatrix} \sigma_L \\ \sigma_T \\ \tau_{LT} \end{Bmatrix} = \begin{Bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{Bmatrix} \begin{Bmatrix} \varepsilon_L \\ \varepsilon_T \\ \gamma_{LT} \end{Bmatrix} \qquad \{1\}$$

The quantities Q_{ij} in eqn. {1} are called the stiffness matrix elements and are related to engineering constants as follows:

$$
\begin{aligned}
Q_{11} &= \frac{E_L}{1-\nu_{LT}\nu_{TL}} \\
Q_{22} &= \frac{E_T}{1-\nu_{LT}\nu_{TL}} \\
Q_{12} &= \frac{\nu_{LT}E_T}{1-\nu_{LT}\nu_{TL}} = \frac{\nu_{TL}E_L}{1-\nu_{LT}\nu_{TL}} \\
Q_{66} &= G_{LT}
\end{aligned}
\qquad \{2\}^1
$$

Under the circumstances where the *Longitudinal* and *Transverse* axes are offset at an angle θ from the vertical and horizontal axes, a transformation matrix T can be used to relate the stress and strain in the off axes x and y (eqn. {3}).

$$
\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = [T]^{-T} \begin{Bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & 2Q_{66} \end{Bmatrix} [T] \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \frac{1}{2}\gamma_{sy} \end{Bmatrix} = \begin{Bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} \end{Bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{sy} \end{Bmatrix}
\qquad \{3\}
$$

$$
\text{Where } T = \begin{bmatrix} \cos^2\theta & \sin^2\theta & 2\sin\theta\cos\theta \\ \sin^2\theta & \cos^2\theta & -2\sin\theta\cos\theta \\ -\sin\theta\cos\theta & \sin\theta\cos\theta & \cos^2\theta-\sin^2\theta \end{bmatrix}
$$

For a laminate made of *n* orthotropic layers, the relationship between the mid plane strains and curvatures (ε^o and κ) and *per unit length* applied moments and forces (*N* and *M*) is:

$$
\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} \\ A_{12} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26} \\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26} \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} \Rightarrow \begin{Bmatrix} N \\ M \end{Bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{Bmatrix} \varepsilon^0 \\ \kappa \end{Bmatrix}
\qquad \{4\}
$$

The *A*, *B* and *D* matrices in the above equation are respectively known as *extensional*, *coupling* and *bending* stiffness matrices. The elements of these matrices are computed from the stiffness elements of each layer (eqn. {5}).

$$
\begin{aligned}
A_{ij} &= \sum_{k=1}^{n} (Q_{ij})_k (h_k - h_{k-1}) \\
B_{ij} &= \frac{1}{2}\sum_{k=1}^{n} (Q_{ij})_k (h_k^2 - h_{k-1}^2) \\
D_{ij} &= \frac{1}{3}\sum_{k=1}^{n} (Q_{ij})_k (h_k^3 - h_{k-1}^3)
\end{aligned}
\qquad \{5\}
$$

---

[1] From these equations it is apparent that $\nu_{LT}E_T = \nu_{TL}E_L$

In the above equation h_k is the height of each layer measured from the geometric center of the laminate (Fig. 1).

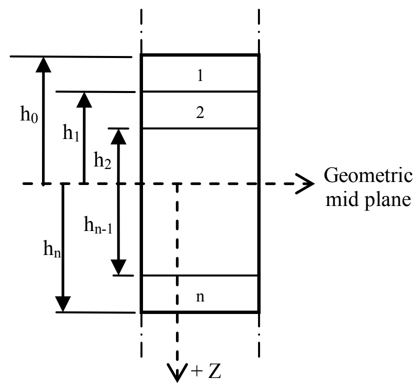


Figure 1 – Layer thickness nomenclature

Equation 4 relates the strain and curvature of the mid plane to the external applied forces and moments (*F* & *M*). The *A*, *B* and *D* matrix elements are a function of the material properties of each layer while the force and moment vectors are a result of external applied loads.

TWO DIMENSIONAL CURL MODEL

Once the stiffness matrix elements (A_{ij}, B_{ij} and D_{ij}) are determined, the mid-plane strains and curvatures of the multilayer web can be calculated by pre multiplying the force and moment vector by the inverse of the stiffness matrix (eqn. {4}). The resulting curvatures κ_x and κ_y will correspond to the MD and CD curvatures of the web if the x-y coordinate system is defined along MD and CD. This section will outline how to calculate the force and moment matrix for different web processes.

<u>Curl Due to Hygro-Thermal Expansion</u>

The effects of change in temperature and humidity on MD and CD curl can be incorporated in the model by defining equivalent forces and moments as shown in eqn. {6} and {7} [1].

$$\begin{Bmatrix} N_x^T \\ N_y^T \\ N_{xy}^T \end{Bmatrix} = \Delta T \sum_{k=1}^{n} \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix}_k \begin{Bmatrix} \alpha_L \\ \alpha_T \\ 0 \end{Bmatrix}_k (h_k - h_{k-1}) \; ,$$

$$\begin{Bmatrix} M_x^T \\ M_y^T \\ M_{xy}^T \end{Bmatrix} = \frac{1}{2} \Delta T \sum_{k=1}^{n} \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix}_k \begin{Bmatrix} \alpha_L \\ \alpha_T \\ 0 \end{Bmatrix}_k (h_k^2 - h_{k-1}^2) \qquad \{6\}$$

$$\begin{Bmatrix} N_x^H \\ N_y^H \\ N_{xy}^H \end{Bmatrix} = \Delta C \sum_{k=1}^{n} \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix}_k \begin{Bmatrix} \beta_L \\ \beta_T \\ 0 \end{Bmatrix}_k (h_k - h_{k-1}) \quad ,$$

$$\begin{Bmatrix} M_x^H \\ M_y^H \\ M_{xy}^H \end{Bmatrix} = \frac{1}{2} \Delta C \sum_{k=1}^{n} \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix}_k \begin{Bmatrix} \beta_L \\ \beta_T \\ 0 \end{Bmatrix}_k (h_k^2 - h_{k-1}^2) \qquad \{7\}$$

Equation {6} represents the equivalent forces and moments due to temperature difference ΔT, while eqn. {7} represents the equivalent forces and moments due to humidity change ΔC. In these equations α and β are the Coefficient of Thermal Expansion (CTE) and Coefficient of Moisture Expansion (CME) respectively. *k* is an index that runs from 1 to n, the outer most layer number.

Curl Due to Lamination Strain Mismatch

One of the main causes of curl is strain mismatch during lamination. When more than one layers of web are laminated with non-equal strains web curl results. The laminate theory can be used to predict the resulting MD and CD curl.

Given the lamination tension, the amount of strain in both MD and CD can be calculated for each layer. The MD and CD strain for the i^{th} layer is given by eqn. {8}. In this equation tension on the i^{th} layer (T_i) is given in force per unit length.

$$\varepsilon_i^{MD} = \frac{T_i}{E_i t_i}$$
$$\varepsilon_i^{CD} = -\nu_i \varepsilon_i^{MD} \qquad \{8\}$$

Once the strains in both MD and CD are calculated for all layers, the load eqn. in {6} can be modified to account for the stains introduced by the lamination process. By assigning ΔT = 1, CTE value equal to the computed MD and CD strains can be defined for all the layers. The load vector so obtained is then pre-multiplied by inverse of the stack stiffness matrix to result in the mid-plane strains and curvatures.

Curl Due to Coating Shrinkage

Similarly curl due to coating shrinkage can be modeled. In order to do this, the equivalent strain of the coating due to drying/curing shrinkage needs to be determined experimentally. This strain will be specific to a particular substrate, drying/curing conditions, coating thickness…etc.

Another approach for determining the equivalent strain is to use the curl model to back calculate the equivalent strain. The curl of a substrate after drying/curing can be measured and used in equation {4} to calculate the loads associated with that magnitude of curl. Once the load is calculated, equation {6} is used to calculate the equivalent strain (for ΔT = 1).

Multiprocess Curl Modeling

One advantage of this linear curl model is a sequence of curl causing web handling processes can be modeled by simply applying superposition principle. Consider an example of a two stage lamination process. First web *A* at tension T_A is laminated to web

B at tension T_B. On a second lamination station, the AB laminate at tension T_{AB} is laminated to web C at tension T_C (Fig. 2).

⇒

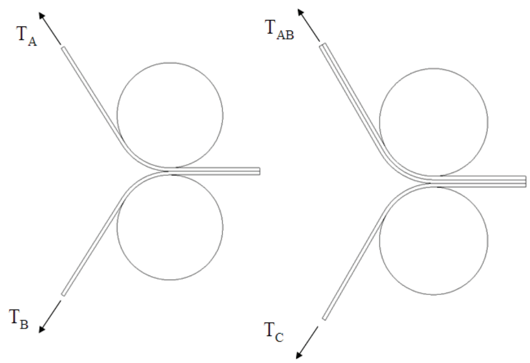


Figure 2 – Two stage lamination process

To determine the final curl of the laminate, first the load due to the first lamination process is determined by the method outline in section 3.2 (the corresponding load vector will be denoted by L_1). Then the load due to the second lamination process is computed. For the second lamination process the strain on the laminate AB due to tension T_{AB} needs to be calculated as shown by eqn. {9}[2]. In this equation T_{AB} is tension per unit length.

$$\varepsilon_{AB}^{MD} = \frac{T_{AB}}{E_A^{MD} t_A + E_B^{MD} t_B} \qquad \{9\}$$

The CD strain for the web AB can be calculated from the MD strain and the *equivalent* Poisson's ratio of laminate AB (ν_{AB}).

$$\varepsilon_{AB}^{CD} = -\nu_{AB} \varepsilon_{AB}^{MD} \qquad \{10\}$$

The equivalent Poisson's ratio of the laminate AB can is in turn computed according to equation {11}.

$$\nu_{AB} = \frac{E_A^{CD} \nu_A t_A + E_B^{CD} \nu_B t_B}{E_A^{CD} t_A + E_B^{CD} t_B} \qquad \{11\}$$

After determining the strains on layers AB and C, the load vector L_2 due to the second lamination process can be determined. To determine the combined effect of the two lamination processes these two load vectors are superimposed (L_1+L_2). This combined load vector is premultiplied by the inverse of the overall stiffness matrix to obtain the final curl configuration.

---

2 This can be easily shown by using springs-in-parallel analogy.

Inverse Curl Problem

The *inverse curl* problem can be defined as: *Determine the forces (subsequently strains) that cause a given curvature vector κ*. Solving the inverse curl problem will enable determination of the strains in a certain layer given a certain curl configuration for the multilayer laminate. Once these strains are determined, the strain distribution on the other layers can be calculated for a flat final laminate.

In this analysis the midplane strain vector ε^o is used as a free variable to satisfy the force *(N)* and the moment *(M)* coupling condition (described below). The superscript '^{o}' is used to differentiate midplane strains (strains of the whole laminate) from strains to be calculated for the k^{th} layer (ε_x and ε_y).

$$\begin{Bmatrix} N \\ M \end{Bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{Bmatrix} \varepsilon^o \\ \kappa \end{Bmatrix} \quad where\ \kappa = \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix}, \quad \varepsilon^o = \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \varepsilon_{xy}^o \end{Bmatrix} \tag{12}$$

If strains ε_x and ε_y are introduced into the k^{th} layer the resulting forces and moments due to these strains is given by eqn. {13}. In this equation h_k is the height of the top face and h_{k-1} is the height of the lower face of the k^{th} layer as measured from the geometric neutral axis.

$$\{N\} = \begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix}_k \begin{Bmatrix} E_x \varepsilon_x (h_k - h_{k-1}) \\ E_y \varepsilon_y (h_k - h_{k-1}) \\ 0 \end{Bmatrix} \qquad (a)$$

$$\{M\} = \begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \frac{1}{2} \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix}_k \begin{Bmatrix} E_x \varepsilon_x (h_k^2 - h_{k-1}^2) \\ E_y \varepsilon_y (h_k^2 - h_{k-1}^2) \\ 0 \end{Bmatrix} \qquad (b) \tag{13}$$

By dividing eqn. {13a} by eqn. {13b} the following relationship between the force and moment vectors (*N* and *M*) vectors can be derived.

$$\frac{N}{M} = 2\frac{h_k - h_{k-1}}{h_k^2 - h_{k-1}^2} \tag{14}$$

From eqn. 12:

$$\begin{aligned} N &= A\varepsilon^0 + B\kappa \\ M &= B\varepsilon^0 + D\kappa \end{aligned} \tag{15}$$

Equations {15} and {14} can be simultaneously solved to obtain an expression for midplane strains ε^o (eqn. {16}).

$$\varepsilon^o = ((2\frac{h_k - h_{k-1}}{h_k^2 - h_{k-1}^2})B - A)^{-1}(B - (2\frac{h_k - h_{k-1}}{h_k^2 - h_{k-1}^2})D)\kappa \qquad \{16\}$$

Once the midplane strains ε^o are calculated, the deformation vector is fully defined as $[(\varepsilon^o)^T \kappa^T]^T$. This deformation vector is pre-multiplied by the stiffness matrix of the laminate to obtain the load vector. Once the load vector is determined the strain on the particular layer can be calculated from either eqn. {13a} or {13b}.

2D Curl Visualization

For visualization purposes a 3D plot of the final curl configuration can be constructed by using the curvatures obtained (κ_x, κ_y and κ_{xy}) and making a few assumptions. Plane curvatures are defined in terms of the out of plane displacement w as follows [1]:

$$\begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} = -\begin{Bmatrix} \dfrac{\partial^2 w}{\partial x^2} \\ \dfrac{\partial^2 w}{\partial y^2} \\ 2\dfrac{\partial^2 w}{\partial x \partial y} \end{Bmatrix} \qquad \{17\}$$

Since the curl model is linear it does not account for large displacement nonlinear effects such as cupping. Cupping is the blocking of curl in one direction due to a strong orthogonal curl. In other words the web becomes very stiff in the one direction due to a strong curl in other. Considering this factor and integrating the above three equation as ordinary differential equations we obtain the following three equations.

$$\begin{aligned} w(x,y) &= \frac{1}{2}\kappa_x x^2 + c_1 x + c_2 \\ w(x,y) &= \frac{1}{2}\kappa_y y^2 + c_3 y + c_4 \\ w(x,y) &= \frac{1}{2}\kappa_{xy} xy + c_5 y + c_6 \end{aligned} \qquad \{18\}$$

Superimposing the three equations we obtain:

$$w(x,y) = \frac{1}{2}\kappa_x x^2 + c_1 x + c_2 + \frac{1}{2}\kappa_y y^2 + c_3 y + c_4 + \frac{1}{2}\kappa_{xy} xy + c_5 y + c_6 \qquad \{19\}$$

To further simplify the out of plane displacement expression we will make the following assumptions which will not affect the curl configuration:

1. The origin of our coordinate system is positioned at the geometric center of the specimen with MD aligned with the x axis and CD aligned with the y axis.
2. The specimen is pinned at the origin (zero out of plane displacement at (0,0)).

3. The deformed configuration has zero slope in the MD and CD direction at the origin. A zero slope ensures symmetry along the x = 0 and y = 0 lines.

Applying the first and second assumptions gets rid of the constants c_2, c_4 and c_6, while applying the third assumption gets rid of constants c_1, c_3 and c_5. The final out of plane deformation can hence be represented by eqn. {20}.

$$w(x,y) = \frac{1}{2}(\kappa_x x^2 + \kappa_y y^2 + \kappa_{xy} xy) \quad \{20\}$$

It should be noted that eqn. {20} represents a solution and is not necessarily the unique solution to the system of partial differential equations given by eqn. {17}. Nonetheless it gives a good understanding of the deformed state of the specimen. This equation can be plotted in Excel or other software for any given specimen size once the curvature terms (κ_x κ_y, and κ_{xy}) are determined.

CURL MODEL VALIDATION

Finite Elements Modeling

For the purpose of validating the 2D curl model outlined in section 3, Finite Elements analysis was carried out in ANSYS. A two layered laminate having the following properties was modeled.

Properties	1st layer	2nd layer
Thickness	127 μm	76 μm
Modulus	1.378 Gpa	2.275 Gpa
Poisson's ratio	0.3	0.3
Lamination tension	175 N/m	378.3 N/m

Table 1 – ANSYS model parameters

A quarter model of a 25 cm x 25 cm laminated web was used. This model was meshed with 20-node SOLID186 elements. Contact elements were defined at the interface of the two layers to model the bonding of the two layers. Four *Load Steps* were defined to simulate a lamination process.

1st load step: The two layers were 'bonded' and tension was applied on each layer according to table 1. This ensured contact between the two layers was initiated[3].

2nd load step: The contact elements are 'killed'. This removes the bond between the layers enabling the two layers to be independently stretched by their respective applied tension applied in step 1 above.

3rd load step: The contact elements are reinstated. This bonds the two layers in the stretched configuration.

4th load step: The applied tensions are removed from both layers.

The lamination tensions given in Table 1 result in 0.001 MD strain and -0.0003 CD strain on the first layer; and 0.00218 MD and -0.000655 CD strain on the second layer. The

[3] This is a dummy load step added to ensure the contact elements on each layer 'see' each other. Since the two layers are bonded, this step will not result in independent stretching on the individual layers. But once the contact elements are killed (2nd load step) the layers will be stretched to the intended strain.

deformed shape obtained from ANSYS is show in Figure 3. Given the above strain it intuitively makes sense that the web is curled towards the bottom layer in the MD while is it curled towards the top layer in the CD direction.

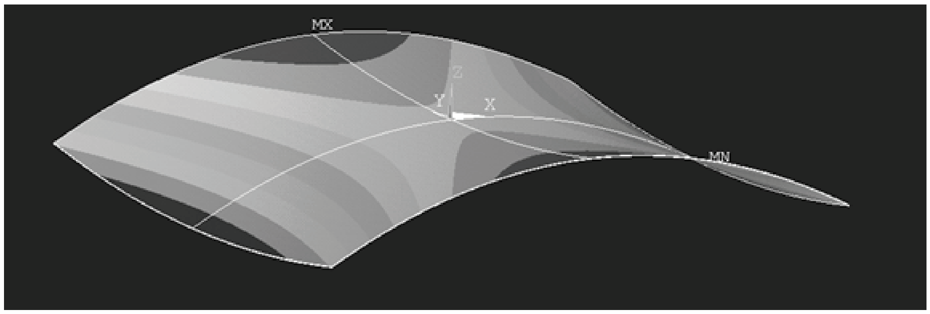


Figure 3 – Deformed shape of laminate (ANSYS model).

Once the solution is obtained the result is post processed to obtain the curl along MD and CD. This is done by extracting the out of plane displacement of the nodes along the center line of the web in both MD and CD. This out of plane displacement is then plotted and a second order curve is fitted to it. The radius of curvature is closely approximated by two times the coefficient of the second order term. The results obtained from ANSYS are compared to the results obtained by using the 2D model in the Table 2 below.

	2D Model	ANSYS
MD curl radius [m]	-0.11648[4]	-0.11662
CD curl radius [m]	0.388298	0.388379

Table 2 – FE vs. 2D model

It can be observed that almost identical results are obtained by both the 2D model and FE analysis. This has been checked for different material properties and lamination tensions and similar degree of correlation was observed.

Experimentation

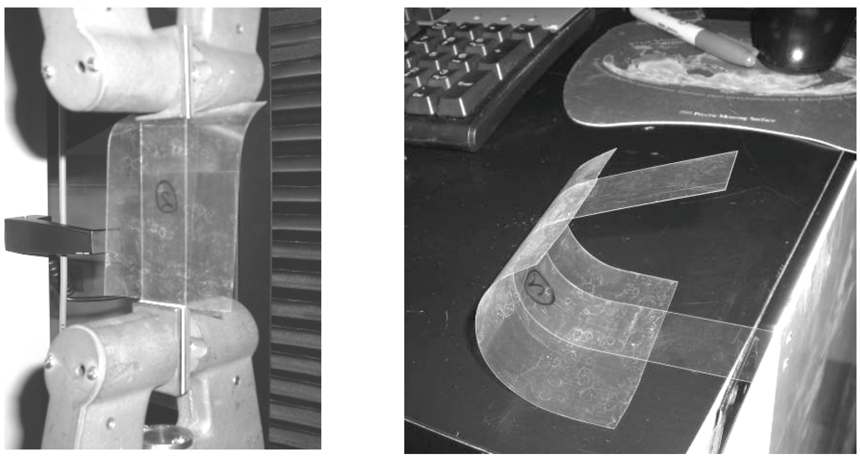
In addition to Finite Elements modeling, lamination experiments were carried out in the lab to validate the 2D curl model. A 50.8 μm thick box sealing tape made of Poly Ethylene Terephthalate (PET) was laminated to a 127μm thick PET substrate at different tensions. The box sealing tape has a 25 μm thick adhesive on one side which was used to laminate the two layers together. The lamination was carried out on an Instron material testing machine. First the material properties of the films used for lamination were measured using DMA (Dynamic Material Analysis). These material properties are listed in Table 3 below. Before the lamination experiments, the films were put in an oven for 24 hrs at 85^{o}C to anneal out any initial curvature (such as due to corset).

[4] Upward curl is assumed positive.

		Thick/Material	MD Modulus	CD Modulus
1st Layer	Substrate	127μm PET	4.447 Gpa	5.046 Gpa
2nd Layer	Box sealing tape	50.8μm PET	4.758 Gpa	3.881 Gpa
	Adhesive	25 μm	207 Kpa	207 Kpa

Table 3 – Material property of films used in lamination experiment

The 127μm thick PET specimen was first fixed between the grips of the Instron. The grips of the Instron are set so that the length of the specimen between the grips equals to 7.5 cms. Then a load cycle that slowly ramps up to a preset load and holds that load for a pre-specified time was used to introduce strain in the 127μm thick specimen. Once the specimen reaches the target load the box sealing tape is laminated to it ensuring no strain is induced in the box sealing tape in the lamination process. The following picture shows the specimen during and after the lamination process.

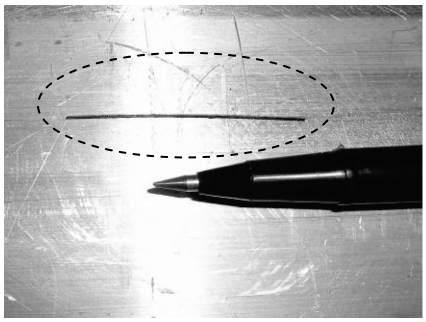


(a) End of lamination process (b) Specimen after lamination

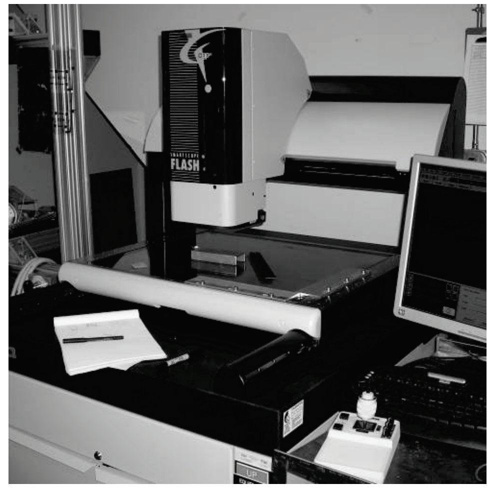
Figure 4 – Lamination experiment

Once the box sealing tape is laminated the specimen is removed from the grips and the excess box sealing tape trimmed. The MD curl is then measured using the Curl Gauge [2].

To measure the CD curl, a very narrow specimen (~2mm) is cut in the CD direction along the midsection of the laminated specimen. Since this specimen is very small and has a relatively small curvature, the Curl Gauge cannot be used to measure the curl. Instead an optical measuring system (SmartScope® by Optical Gaging Products (OGP), Inc) was used to scan points along the CD specimen. The SmartScope® uses a built in algorithm to fit a curve through the measured points and determine the corresponding curvature of the CD specimen. Figure 5 shows pictures of the CD specimen and the SmartScope by OGP Inc. The specimen is colored black using a sharpie to increase the contrast for the curl measurement.



(a) CD specimen



(b) OGP SmartScope®

Figure 5 – CD curl measurement

The lamination experiments were carried out at three tensions: 66.72 N, 88.96 N and 111.21 N. Five specimens were tested for each tension condition. Both MD and CD curl were measured for all the specimens. The measured curl results are summarized in the plot below. In this plot the experiment results are compared against 2D curl model results and Finite Elements Analysis (both linear and nonlinear FE analysis).

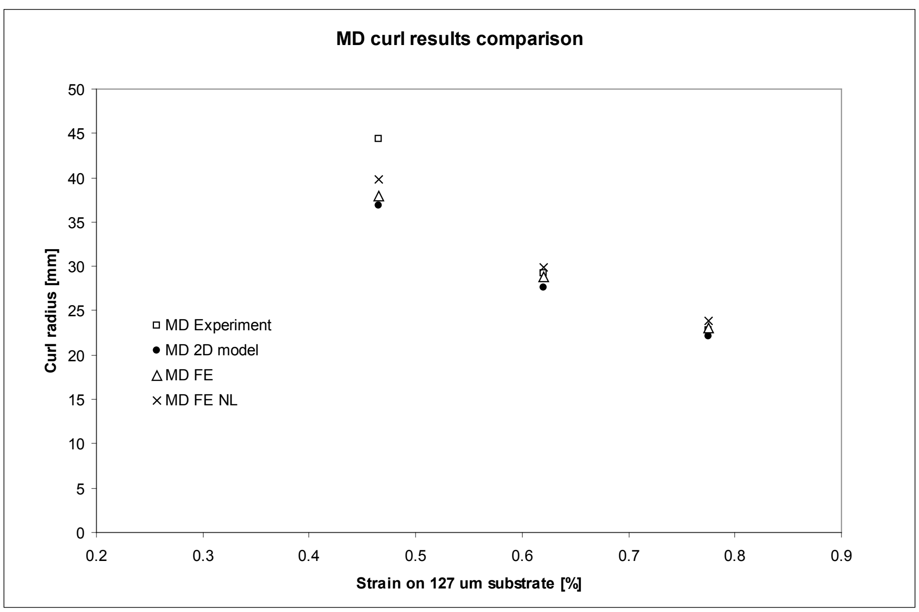


Figure 6 – MD Result comparison (2D model, experiment results, FE results)

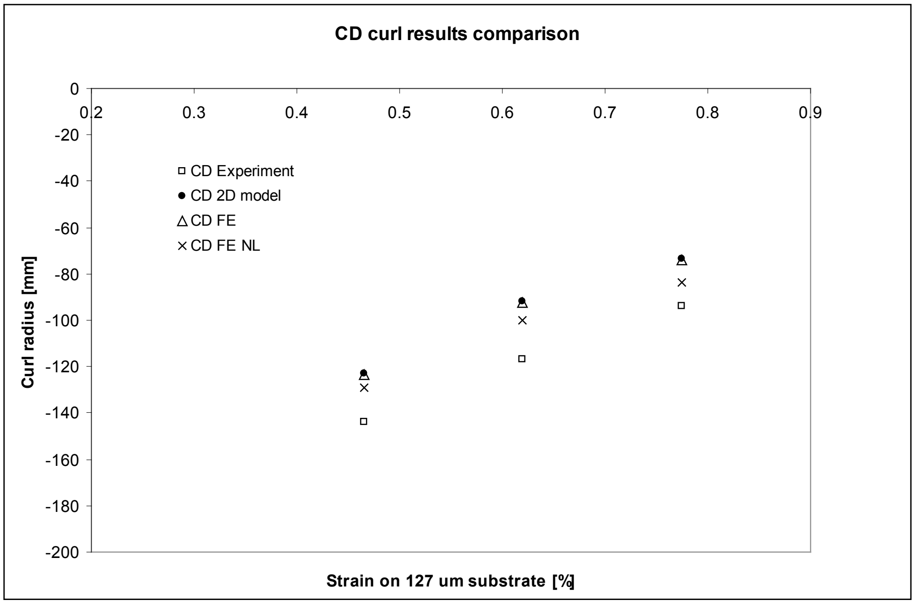


Figure 7 – CD Result comparison (2D model, experiment results, FE results)

In this plot the radius of curvature is plotted on the vertical axis in millimeters, while the horizontal axis represents strain applied on the 127μm specimen. The experiment result points in the plot represent average values of 5 specimens.

In general the 2D model shows good correlations with both experimental results and Finite Elements analysis. The MD results in particular show very close correlation. In the CD direction a higher degree of result variation within specimens was observed for the same lamination tension. This could be attributed to a number of reasons. The first being the measured signal to noise ratio. Since the CD curl is caused by Poisson's effect it is at least three times smaller than the MD curl. Assuming the same noise level, this would result in unfavorable signal to noise ratio for the CD measurements. Another possible factor is associated with the size of the CD specimens. The maximum length of CD specimen is limited to 1" due to the width of the laminated specimen. Since the curl measurement scheme used by the SmartScope® relies on the relative position of points on the specimen, the shorter the sample the higher the associated curve-fitting errors.

Another observation is CD curl seems to be over predicted by the 2D curl model. This could be due to a couple of reasons. The first possible explanation is creep in the box sealing tape adhesive due to bending induced shear forces relieving some of the strain introduced during lamination. Compared to the MD specimen, there is a higher likelihood of this happening in the CD specimen since due to the relatively small size of the CD specimen any imperfections (such as edge conditions and voids) will adversely affect bond between the substrate PET and the box sealing tape. This is also observed in the MD direction except to a lesser extent.

A second possible factor for the over prediction of CD curl by the 2D linear curl model is the nonlinear coupling effect between MD curl and CD curl. Due to the presence of strong MD curl, the CD direction curl is diminished due to cupping effect in the MD (this can be minimized by cutting very narrow CD strips up to a certain extent). This effect is also captured in the Nonlinear Finite Elements analysis. It can be observed that the CD curl obtained using nonlinear FE modeling is flatter than the curl obtained using a linear model. Another possible cause of error could be the assumption of the Poisson's ratio to be 0.3. For a more accurate representation of the system the Poisson's ratio needs to be experimentally determined for the specimens used for lamination.

CONCLUSION

A two dimensional curl prediction tool is developed based on Laminate Theory. The curl model is capable of predicting curl in both Machine Direction and Cross Direction for a number of web processes such as lamination, drying/curing, hygrothermal aging … etc. Using the principle of superposition the model is also able to handle cascaded processes. Additional features such as reverse curl problem solution and curl visualization methods are also incorporated in the 2D model.

Both experimental and Finite Elements analyses were carried out to validate the model. The 2D curl model showed almost exact matches to results obtained using Finite Elements analysis. The model also showed good correlation with experimental results. The 2D curl model was observed to somewhat over predict curl compared to experimental results. The possible reasons for this are, creep in adhesive used for lamination, large displacement nonlinear stiffening effect and the uncertainty associated with the Poisson's values used in model. The experimental results were also observed to be slightly noisy for the Cross Direction due to specimen size and measurement noise issues.

In conclusion a reasonably accurate 2D curl modeling tool has been developed. This tool can be useful to investigate sources of curl issues and also to determine optimal values for process parameters such as lamination tension without spending significant resources and time on pilot line and experiments.

REFERENCES

1. Agarwal, D. and Broutman, L. J., Analysis and Performance of Fiber Composites, 2nd edition, Wiley Interscience, 1990.
2. Swanson, R. P., "Measurement of Web Curl," Applied Web Handling Conference, 2006, Charlotte, NC.

Laminate Theory Based 2D Curl Model **S. Kidane,** 3M Corporation, USA

Name & Affiliation	Question
Dilwyn Jones, Emral Ltd.	I think this is very good work and I enjoyed the paper. I wondered if you measured the MD curl in thin strips in the same way you measured the CD curl and whether you got the same results as with the wide specimen.
Name & Affiliation	**Answer**
Sam Kidane, 3M Corporation	In this case, the stronger curl was in the machine direction, so the machine direction specimen did not need to be as narrow. This web has a very weak cross-direction curl. The cross-direction curl would not diminish the machine-direction curl specifically. We used one-inch wide specimens to measure the curl.
Name & Affiliation	**Question**
Dilwyn Jones, Emral Ltd.	If a thin sheet is curled in one direction, it can't also curl in a second direction. Whereas if it is very narrow, it can curl in a second direction.
Name & Affiliation	**Answer**
Sam Kidane, 3M Corporation	We did witness curl in the cross-direction. The cross-direction curl was much smaller magnitude than the machine-direction curl. If you look at the numbers, the machine direction curl is 25 mm while the cross-direction curl is about 3 times that. The CD curl was much milder than the machine direction curl.