

## **COMPUTATION OF SPAN LENGTH VARIATIONS DUE TO OUT-OF-ROUND MATERIAL ROLLS**

**By**

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### **ABSTRACT**

It is well known that non-ideal elements such as out-of-round/eccentric material rolls affect web tension. However, the mechanism through which these non-ideal components induce tension oscillations was not clear previously. In a companion paper (Modeling and Identification of the Source of Oscillations in Web Tension) it is shown that an out-of-round/eccentric material roll produces length variations in the web span adjacent to the roll. These length variations are the main reason for oscillations in the tension signal; this was experimentally verified in the companion paper. In order to reproduce these tension oscillations in model simulations, it was necessary to include span length variations in the tension dynamics models. Given a generic profile for the out-of-round unwind roll, determination of the length of the adjacent span as a function of time as material is released from the roll is a formidable task. The focus of this paper is on finding a relationship between the shape of the out-of-round material roll and length of the span adjacent to it.

The simplest case to analyze is the length variations due to an eccentric roller. Considering the geometry of the problem, it is possible to find an expression in closed form that gives the length of the web span as function of the angular displacement of the roller. The expression for the rate of change of span length as a function of angular velocity is obtained by direct differentiation of the closed form expression. Finding closed form expressions for length of the span adjacent to an out-of-round material roll even for simple cases, such as an elliptical roll, is not trivial.

As a starting point, an elliptical material roll is taken into consideration. To find the length of the web span between the material roll and the idle roller it is necessary to find the line tangent to both of them. An analytical approach to the problem did not provide any insights into finding a closed form expression for span length as a function of angular displacement of the material roll. To overcome this problem a convex optimization problem is formulated and an efficient numerical approach is developed to obtain the common tangent to the material roll and the first idle roller. Once the common tangent is

obtained, span length and rate of change of span length can be found numerically as well. The algorithm and related pertinent discussions are given. Incorporation of this algorithm into web line model simulation software will enable better correlation of model and experimental tension data.

## NOMENCLATURE

$A$	Area of cross section of the web [m <sup>2</sup> ]
$a$	Ellipse major axis length [m]
$\overline{AB}$	Line segment with ending points $A$ and $B$
$\hat{A}BC$	Angle between line segments $\overline{AB}$ and $\overline{BC}$
$b$	Ellipse minor axis length [m]
$d$	Distance between geometrical centers of rollers [m]
$d(t,C)$	Distance between the line $t$ and the point $C$ [m]
$E$	Modulus of elasticity (Young's modulus) $\left[ \frac{N}{m^2} \right]$
$J$	Optimization problem cost function
$L$	Free span web length [m]
$R$	Radius of the roller [m]
$T_i$	Web tension in the $i$ -th span [N]
$v$	Web velocity (or peripheral velocity of the roller) [m/s]
$X$	Abscissa in the Cartesian framework
$Y$	Ordinates in the Cartesian framework
$\omega$	Roller angular velocity [rad/sec]

## Subscripts

$i$	span or roller number
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## INTRODUCTION

In a companion paper [1] experimental validation of tension dynamic model was discussed. Analysis of the experimental data showed that the models are not able to predict some steady state oscillations. It was shown that these oscillations in web tension are due to non-ideal rollers such as eccentric idle rollers or out-of-round material rolls. The reason why non-ideal rollers cause tension oscillations is because they induce length variations the spans adjacent to them. This was not considered in many of the existing models and there was no analysis to determine the effect of span length variations on web tension. The model for web tension dynamics considering span length variations was derived and analyzed in [1] and is given by:

$$\dot{T}_i(t) = \frac{v_i(t)(EA - T_i(t)) - v_{i-1}(t)(EA - T_{i-1}(t)) + (EA - T_i(t))\dot{L}(t)}{L(t)} \quad \{1\}$$

Equation {1} shows how the tension dynamics is dependent on the derivative of the span length through the  $EA$  term, therefore even small span length variations are expected to induce tension oscillations.

From a dynamic simulation stand point, given a non-ideal roller, in order to simulate the behavior of web tension in a span it would be necessary to obtain an expression for the derivative of the span length,  $\dot{L}$ . The aim of this paper is to present the initial work

towards finding an algorithm that is capable of computing the span length, and its derivative, due to a non-ideal roller or an out of round material roll with different shapes.

The simplest problem to address for a non-ideal roller is the case of an eccentric idler roller. For this case as shown in Section 2 it is possible to find a closed form expression for the length of the span as a function of the angular displacement of the roller. Moreover, assuming that the web velocity is known and in the absence of slippage between the web and roller, it is possible to obtain a time dependent expression for the angular displacement and hence a time dependent expression for the span length.

In the presence of a roll with a generic shape, the problem of finding a time function for the span length is a difficult task. As a starting point, in Section 3 the case of an elliptic material roll was considered. Even for this simple shape there is no known procedure for finding a closed form expression for the length of the span. Instead a numerical algorithm based on the solution of an optimization problem has been developed and implemented. In this case a numerical approximation of the derivative of the span length can also be determined.

### **LENGTH OF THE SPAN ADJACENT TO AN ECCENTRIC IDLE ROLLER**

Derivation of the length of the web span as a function of time due to the presence of an eccentric idle roller is discussed in this section. In particular, a web span with an ideal entry roller and an eccentric exit roller is considered. An idle roller is said to be eccentric whenever its rotational center does not coincide with its geometric center; the distance  $e$  between the geometric center and the center of rotation denotes the amount of eccentricity and is considered to be known. Also, the distance  $d_0$  between the centers of rotation of the two idle rollers is assumed to be known. Finally, for simulation purposes, the initial angular displacement  $\theta_0$  of the eccentric idle roller is assumed to be known. It is also assumed that the line speed is constant and there is no slippage between the web and rollers.

By exploiting the geometry of the roller it is possible to find a closed form expression for the length of the span. The following analysis is divided into two cases (see Fig. 1), the first refers to the under-wrap configuration and the second to the over-wrap configuration.

As a first step in obtaining the closed form expression for the length of the span, it will be shown that, given two circles, the length of the segment tangent to both circles is only a function of the radius of the circles and the distance  $d$  between their geometric centers. Exploiting this property, and considering that the distance  $d$  between the geometric centers of the idle rollers will be time varying because of the eccentricity of the second roller, the closed form expression for the length of the web span is obtained.

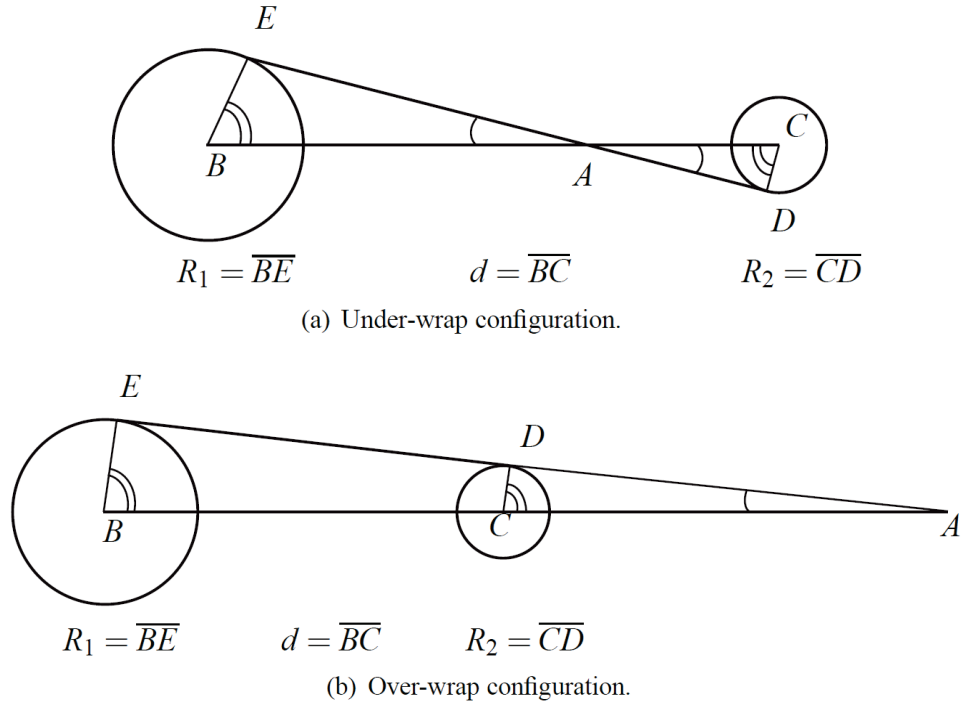


Figure 1 – Idle roller configurations.

The under-wrap configuration shown in Fig. 1(a) is considered first. Since the line segment  $\overline{ED}$  is tangent to both rollers, the two right-angle triangles  $AEB$  and  $ADC$  are similar. Therefore, angles  $\hat{ABE}$  and  $\hat{ACD}$  and angles  $\hat{BAE}$  and  $\hat{CAD}$  are equal. These observations can be used to set up a system of equations to find the length of the segment  $\overline{DE}$ .

First, because  $\hat{ABE}$  and  $\hat{ACD}$  are equal their sines are equal, hence:

$$\frac{R_2}{AB} = \frac{R_1}{AC} \Rightarrow \overline{AB} = \overline{AC} \frac{R_2}{R_1} \quad \{2\}$$

Also, the distance between the center of the idle rollers can be written as:

$$d = \overline{AB} + \overline{AC} \quad \{3\}$$

solving {2} and {3} results in:

$$\overline{AC}(d) = \frac{d}{1 + \frac{R_2}{R_1}} \quad \{4\}$$

The angle  $\hat{ACD}$  as function of  $\overline{AC}$  is given by:

$$\hat{A}CD(d) = \text{asin} \frac{R_1}{\overline{AC}(d)} \quad \{5\}$$

Given the Cartesian coordinates for  $B \equiv (X_B, Y_B)$  and  $C \equiv (X_C, Y_C)$ , the coordinates for  $D(d)$  and  $E(d)$  are:

$$\begin{aligned} D(d) &\equiv (X_D, Y_D) = (X_C - R_1 \cos(\hat{A}CD(d)), Y_C - R_1 \sin(\hat{A}CD(d))) \\ E(d) &\equiv (X_E, Y_E) = (X_B + R_2 \cos(\hat{A}CD(d)), Y_B + R_2 \sin(\hat{A}CD(d))) \end{aligned} \quad \{6\}$$

and the span length is:

$$L(d) = \sqrt{(X_D(d) - X_E(d))^2 + (Y_D(d) - Y_E(d))^2}. \quad \{7\}$$

When dealing with an eccentric idle roller the distance  $d$  between the geometric centers of the idle rollers is time varying because the center of the eccentric idler will be rotating. In order to use the previous procedure it is necessary to determine the value of  $d(t)$ . Let  $d_0$  be the distance between the centers of rotation and  $e$  be the eccentricity (see in Fig. 2). The expression for  $d(t)$  is

$$d(t) = \sqrt{(d_0 - e \cos \theta(t))^2 + (e \sin \theta(t))^2} \quad \{8\}$$

Under the assumption of constant web velocity  $v$  and with no slippage between the web and rollers and assuming  $\theta(0) = 0$ , it is possible to write:

$$\theta(t) = \omega t = \frac{v}{R_1} t \quad \{9\}$$

Hence the time dependent equation for  $d(t)$  is

$$d(t) = \sqrt{(d_0 - e \cos \left( \frac{v}{R_1} t \right))^2 + (e \sin \left( \frac{v}{R_1} t \right))^2} \quad \{10\}$$

Clearly, because  $d$  is time varying so is  $L$  (see {7}). The closed form expression for  $L(t)$  can be found by substituting the expression of  $d(t)$  into {7}. Also, once the analytical expression for  $L(t)$  is obtained  $\dot{L}(t)$  is simply obtained by differentiation.

Analogously, in the case of over-wrap it is possible to reach a similar result. As before the first step would be to find an expression for the length  $L$  as function of the distance between the geometric centers of the rollers, refer to Fig. 1(b). In this case the angles  $\hat{BAE}$  and  $\hat{CAD}$  are equal and since the triangles  $AED$  and  $ADC$  are right-angled it is possible to find the cosine of the angle  $\hat{BAE}$ :

$$\cos(\hat{BAE}) = \frac{R_1}{\overline{AC}} = \cos(\hat{CAD}) = \frac{R_2}{\overline{AB}} = \frac{R_2}{d + \overline{AC}} \quad \{11\}$$

Solving the previous equation for  $\overline{AC}$  gives

$$\overline{AC}(d) = \frac{R_1 d}{R_2 - R_1} = \frac{d}{\frac{R_2}{R_1} - 1} \quad \{12\}$$

From  $\overline{AC}(d)$  it is possible to find the angle  $A\hat{C}D$ :

$$A\hat{C}D(d) = \text{acos} \frac{R_1}{\overline{AC}(d)} \quad \{13\}$$

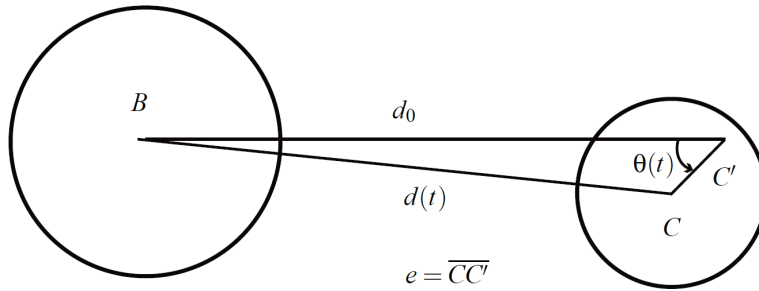


Figure 2 – Eccentric idle roller:  $C$  geometric center of the eccentric roller,  $C'$  center of rotation of the eccentric roller,  $e$  eccentricity of the roller,  $d_0$  distance between the center of rotation of the two rollers,  $d(t)$  distance between the geometric centers of the two rollers.

The angle  $A\hat{C}D$  is also equal to  $B\hat{A}E$  by the triangle similarities, and hence it is possible to find the coordinates of the points  $D(d)$  and  $E(d)$ :

$$\begin{aligned} D(d) &\equiv (X_D, Y_D) = (X_C + R_1 \cos(A\hat{C}D(d)), Y_C + R_1 \sin(A\hat{C}D(d))) \\ E(d) &\equiv (X_E, Y_E) = (X_B + R_2 \cos(A\hat{C}D(d)), Y_B + R_2 \sin(A\hat{C}D(d))) \end{aligned} \quad \{14\}$$

Note that  $d(t)$  in the over-wrap case is also given by {3}. Substitution of  $d(t)$  obtained using {3} into {14} gives the time dependent coordinates of  $D$  and  $E$ . Once these coordinates are obtained, the length can be found using {7}.

For both cases once the closed form expression for  $L(t)$  is obtained its derivative can be analytically obtained and used in the web tension model.

It has to be notice that an eccentric idle roller would cause length variations to both the web span that precedes the roller (as shown in this section) and the one that succeeds the roller. Therefore, for both the web spans preceding and succeeding the eccentric roller the model in {1} has to be used and for each web span the length variations need to be computed as explained in this section.

The material of this section is summarized in Algorithm 1.

### LENGTH OF THE SPAN ADJACENT TO AN OUT-OF-ROUND MATERIAL ROLL

This section considers the case of an out-of-round material roll. Similar to the case of an eccentric idle roller, to simulate the effect of the out-of-round roller on web tension it would be necessary to determine an expression for  $L(t)$  and  $L$ . Given a generic shape for

the roll, finding an analytical expression for  $L(t)$  may not be possible in general. For this reason the case of an elliptical roll has been analyzed first.

Consider the case of an elliptical unwind roll, and suppose that the major axis of the ellipse coincides with the  $y$  axis of the Cartesian reference frame as shown in Fig. 3(a). In order to compute the length of the span, the line tangent to both the ellipse and the first roller must be found. As introduced earlier, when the material roll rotates by an angle  $\phi$  (see Fig. 3(b)), the point of release of the material changes and so does the length of the span, which is a function of  $\phi$ . Finding an analytical expression for  $L(\phi)$  is not a trivial problem since the system of equations that must be solved is nonlinear. In fact, it is difficult to find a solution in closed form even for this simple shape. One way the problem can be approached is as follows: given a point  $P_0$  on the ellipse, find the line  $\mathbf{t}$  tangent to the ellipse at that point. If the distance between the line  $\mathbf{t}$  and the center of the idle roller is equal to the radius of the idle roller, then the line  $\mathbf{t}$  is tangent to both the ellipse and the roller and hence it is the desired line. Once the tangent  $\mathbf{t}$  is found, finding the length  $\mathbf{L}$  is simple. An illustration of how this construction works is shown in Fig. 4. The same figure shows that there are two tangents  $\mathbf{t}_2$  and  $\mathbf{t}_3$  which satisfy the previous condition; this will cause an additional difficulty when the problem is solved numerically. The procedure is explained in more detail in the following.

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**Algorithm 1:** Computation of span length in the presence of an eccentric roller

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**input** : Roller radii  $R_i$ , roller coordinates  $(X_i, Y_i)$ , distance between rotational centers  $d_0$ , eccentricity  $e$ , linear velocity of the web  $v$ , configuration (over-wrap, under-wrap)

**output:** Length of the web span  $L(t)$

```

for  $t \leftarrow 0$  to  $t_{fin}$  do
  compute  $d(t)$  as in {3};
  if Under-wrap then
    compute  $\overline{AC}$  as in {4};
    compute  $\hat{A}CD$  as in {5};
    compute  $E$  and  $D$  as in {6};
  else
    compute  $\overline{AC}$  as in {12};
    compute  $\hat{A}CD$  as in {13};
    compute  $E$  and  $D$  as in {14};
  compute  $L(t)$  as in {7};

```

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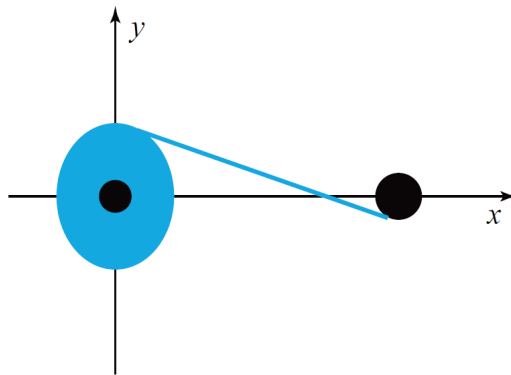
Consider an ellipse  $\mathbf{e}$ , representing the material roll, centered at the origin of the Cartesian reference frame with equation:

$$\mathbf{e} : \frac{x^2}{a^2} + \frac{y^2}{b^2} - 1 = 0 \quad \{15\}$$

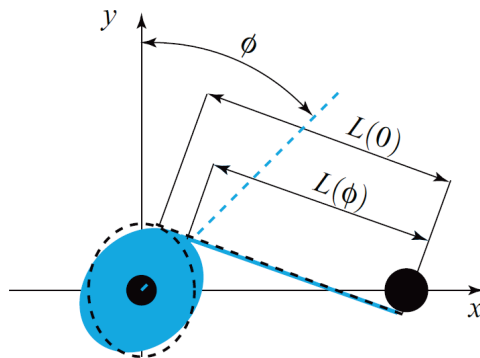
Note that this equation only describes an ellipse centered at the origin with major and minor axes along the axes of the Cartesian coordinate system. Also consider a circle centered at  $C \equiv (x_c, y_c)$  of radius  $R$ , representing the first idle roller. The problem is to find the equation of the line tangent to both the ellipse and the circle, which will represent the first web span. For any given point  $P_e \equiv (x_e, y_e)$  on the ellipse, satisfying the equation in

{15}, it is possible to find the equation for the tangent to the ellipse at the point  $P_e$ , which is

$$\mathbf{t}(x_e, y_e) : \frac{x_e x}{a^2} + \frac{y_e y}{b^2} - 1 = 0 \quad \{16\}$$



(a) Elliptical unwind roll in the initial position.



(b) Elliptical unwind roll rotated by  $\phi$ .

Figure 3 – Example of an out-of-round material roll.



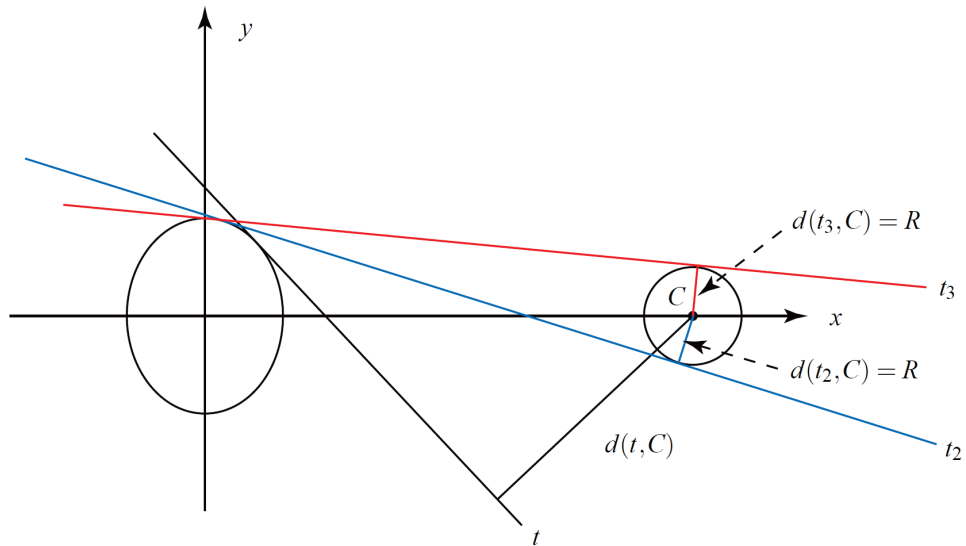


Figure 4 – Procedure to find the length of the span.

Now, consider a generic line  $\ell : \alpha x + \beta y + \gamma = 0$  and a point  $P_0 \equiv (x_0, y_0)$ , the equation that gives the distance between the line and the point is:

$$\mathbf{d}(\ell, P_0) = \sqrt{\frac{\alpha x_0 + \beta y_0 + \gamma}{\alpha^2 + \beta^2}} \quad \{17\}$$

For this line to be a tangent  $\mathbf{t}$  of the ellipse:

$$\begin{aligned} \alpha &= \frac{x_e}{a^2} \\ \beta &= \frac{y_e}{b^2} \\ \gamma &= -1 \end{aligned} \quad \{18\}$$

Therefore, the distance between the tangent  $\mathbf{t}$  and the center of the roller  $C$  is

$$\mathbf{d}(t(x_e, y_e), C) = \sqrt{\frac{|\alpha x_c + \beta y_c + \gamma|}{\alpha^2 + \beta^2}} \quad \{19\}$$

Note that since  $x_e$  and  $y_e$  are on the ellipse, they are not independent. In fact, they can be parameterized in the following way:

$$\begin{aligned} x_e &= a \cos(\theta) \\ y_e &= b \sin(\theta) \end{aligned} \quad \{20\}$$

with  $\theta \in [0, \pi/2]$ ,  $a$  and  $b$  being the length of the minor and major axes of the ellipse, respectively. Hence the distance  $\mathbf{d}(t(x_e(\theta), y_e(\theta)), C) = \mathbf{d}(\theta)$  is a function of the parameter

$\theta$  only. This distance will equal  $R$  only for the two tangents  $t_2$  and  $t_3$  in Fig. 4. Therefore the cost function

$$J = (d(\theta) - R)^2 \quad \{21\}$$

will be positive everywhere except for the two values of  $\theta$  corresponding to the tangents  $t_2$  and  $t_3$  in Fig. 4, for which it will be zero. Hence, solving the optimization problem:

$$\min_{\theta} J(\theta) \quad \{22\}$$

the desired  $\theta$  can be found, which when substituted into {20} and {16} will give the equation for the tangent. In order to avoid the possibility that the numerical algorithm gives the solution corresponding to the tangent  $t_3$  it is possible to constrain the numerical algorithm to search in the interval  $[0, \theta_0]$  instead of  $[0, \pi/2]$ , where  $\theta_0$  is the value of  $\theta$  for which the tangent to the ellipse passes through the center  $C$  of the idle roller. More specifically,  $\theta_0$  solves the equation:

$$d(\theta_0, C) = 0$$

This equation can be easily solved analytically and its solution is

$$\theta_0 = \arcsin \frac{-\frac{y_c}{b} + \sqrt{\frac{x_c^2}{a^2} \left( \frac{x_c^2}{a^2} + \frac{y_c^2}{b^2} - 1 \right)}}{\frac{x_c^2}{a^2} + \frac{y_c^2}{b^2}} \quad \{23\}$$

Note that restricting the search between  $[0, \theta_0]$  not only avoids the problem of getting the undesired solution but it also results in achieving a faster convergence time of the numerical algorithm. In fact, the objective function  $J(\theta)$  is a strictly convex function in the interval  $[0, \theta_0]$  and numerical algorithms for minimum search are extremely efficient when applied to convex functions. Once the equation of the tangent  $t_2$  is obtained, finding the length of the span is trivial.

Note that the equation for the tangent to the ellipse in {16} is valid only if the major and minor axes of the ellipse are aligned with the fixed coordinate system  $\mathcal{F}$ . If the major and minor axes of the ellipse are not aligned with the fixed coordinate axes, one can perform appropriate transformations to resolve this issue. Let the major axes be rotated by an angle  $\omega t$  with respect to the fixed coordinate system  $\mathcal{F}$ . Given a point  $P_0 \equiv (x_0, y_0)$  on the surface of the rotated material roll, the problem of finding the tangent  $\mathbf{t}$  to the material roll at the point  $P_0$  needs to be solved. Now consider a second coordinate system  $\mathcal{F}'$  having its axes along the major and minor axes of the elliptical material roll. The coordinates of point  $P_0$  in  $\mathcal{F}'$  are

$$\begin{pmatrix} x'_0 \\ y'_0 \end{pmatrix} = \begin{pmatrix} \cos \omega t & \sin \omega t \\ -\sin \omega t & \cos \omega t \end{pmatrix} \begin{pmatrix} x_0 \\ y_0 \end{pmatrix} \quad \{24\}$$

In  $\mathcal{F}'$  since the major and minor axes of the ellipse are aligned with the coordinate axes, {16} can be used to find the equation of the tangent  $\mathbf{t}$ . So {16} in  $\mathcal{F}'$  can be written as

$$\mathbf{t}(x'_0, y'_0) : \frac{x'_0 x'}{a^2} + \frac{y'_0 y'}{b^2} - 1 = 0 \quad \{25\}$$

To transform  $\mathbf{t}$  back to the coordinate system  $\mathfrak{F}$ , the following change of coordinates must be performed:

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos \omega t & \sin \omega t \\ -\sin \omega t & \cos \omega t \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} \quad \{26\}$$

By substituting {26} into {25} the equation for  $\mathbf{t}$  in  $\mathfrak{F}$  is obtained. This equation can be used instead of {16} to set up the optimization problem when the major and minor axes of the ellipse are not aligned with the axes of the fixed coordinate system  $\mathfrak{F}$ . It should be noted that as a consequence of {24}, {25} and {26},  $\mathbf{d}$  is also function of  $\omega t$ . Therefore, the new optimization problem is

$$\min_{\theta \in [\theta_{min}, \theta_{max}]} J(\theta, \omega t) \quad \{27\}$$

The implementation algorithm that finds the length  $L(t)$  is given in Algorithm 2. Algorithm 3 gives a basic implementation for the cost function  $J(\theta, \omega t)$  that defines the optimization problem.

## CONCLUSION AND FUTUREWORK

Algorithms to compute the length of a web span in the presence of non-ideal rollers such as eccentric idle rollers or elliptically shaped material rolls are developed in this paper. These algorithms are required in tension model simulations which include the effects of the two non-ideal elements. These two algorithms are expected to form a basis for addressing more general shapes for material rolls such as flat spots which will be considered in the future.

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**Algorithm 2:** Computation of the web span length in the presence of an elliptical material roll

---

**input** : Roller coordinates  $(X_i, Y_i)$ , length of major and minor axes  $a$  and  $b$ , roller radius  $R$ , angular velocity of the material roll  $\omega$ , configuration (over-wrap, under-wrap)

**output:** Length of the web span  $L(t)$

```

begin
  compute  $\theta_0$  as in {23};
  if Under-wrap then
     $\theta_{min} \leftarrow 0$ ;
     $\theta_{max} \leftarrow \theta_0$ ;
  else
     $\theta_{min} \leftarrow \theta_0$ ;
     $\theta_{max} \leftarrow \pi/2$ ;
  for  $t \leftarrow 0$  to  $t_{fin}$  do
     $\delta \leftarrow \omega t$ ;
    solve  $\min_{[\theta_{min}, \theta_{max}]} J(\theta, \delta)$ ;
    /* Note: the function given to the optimization solver
       should be implemented in such a way to return not only the
       solution  $\theta$  but also the equation for the tangent
       associated to  $\theta$  (in particular  $\alpha x + \beta y + \gamma = 0$ ) and the
       coordinates of the point  $P$  on the ellipse */
    begin Compute the contact point  $P_{cont} \equiv (X_{cont}, Y_{cont})$  on roller
      AngleOfTangent  $\leftarrow \text{atan2}(\beta, -\alpha)$ ;
      if Under-wrap then
         $X_{cont} \leftarrow x_c + R \cos(\text{AngleOfTangent} - \pi/2)$ ;
         $Y_{cont} \leftarrow y_c + R \sin(\text{AngleOfTangent} - \pi/2)$ ;
      else
         $X_{cont} \leftarrow x_c + R \cos(\text{AngleOfTangent} + \pi/2)$ ;
         $Y_{cont} \leftarrow y_c + R \sin(\text{AngleOfTangent} + \pi/2)$ ;
      end
     $L(t) \leftarrow \text{norm}(P - P_{cont})$ ;
  end

```

---

**Algorithm 3:**  $J(\theta, \delta)$

---

**input** : A point  $P_0(\theta)$  on the surface of the ellipse, the angle of rotation  $\delta = \omega t$  of the ellipse respect to the fixed frame  $\mathfrak{F}$ , the data describing the rollers (radii, coordinates etc.)

**output:** Distance between the tangent at the point  $P_0(\theta)$  and the center  $C$  of the roller

```

begin
  compute the point  $P'_0$  as in {24};
  compute the tangent to  $P'_0$  as in {25};
  compute the equation of the tangent in the frame  $\mathfrak{F}$  using {26};
  compute the distance as in {17} using the coefficient obtained in the previous step;
end

```

---

## **ACKNOWLEDGEMENTS**

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## **REFERENCES**

1. Branca, C., Pagilla, P. R., and Reid, K. N., "Modeling and identification of the source of oscillations in web tension," Proceedings of the Tenth Intl. Web Handling Conference, 2009.

*Computation of Span Length Variations  
Due to Out-of-Round Material Rolls*

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K. N. Reid**, Oklahoma State  
University, USA

**Name & Affiliation**

Clarence Klassen,  
KlassENgineering

**Question**

I have a question about something nonsymmetrical, like your elliptical roll. Do you have any idea regarding the change of mass when we raise the heavy side of the roll and the effect on velocity? How would that affect tension?

**Name & Affiliation**

C. Branca, Oklahoma State  
University

**Answer**

We are working on an algorithm for those effects and we have some results. When any roller is not perfectly round, it will produce span length changes. The process to address a general shape is more complicated. The algorithm we have developed will address general shape.

**Name & Affiliation**

Mark Weaver, Rockwell  
Automation

**Comment**

You discussed a case where you may have a non-symmetric mass distribution. You may have more mass on one side of a roller for instance. That will appear as a velocity disturbance and of course there will be a tension disturbance as well.

**Name & Affiliation**

Bob Lucas, Winder  
Science

**Question**

In your earlier example, where you were dealing with an eccentric idler roll, I was wondering if that model was complete. In reality, you have an eccentric flywheel – it's a friction limited flywheel application – where you have web tensions on both sides trying to drive it. There will be a variation of angular velocity of the roller if you have a difference in radius of the roll. This the influence the accelerating and decelerating of the roll as it rotates. Your basic model of the web behavior has to consider that you have some mass in that roll that has to be constantly accelerated and decelerated in the rotation. Your model is fine assuming that the inertia of your idler roll is zero. You have shown this as a fixed speed, a constant speed. There must some interplay between the varying surface velocities of the roll and the web tension. That is not part of your model.

**Name & Affiliation**

Mark Weaver, Rockwell  
Automation

**Comment**

This is a preliminary work. We wanted to explore what happens when you must deal with these flat sided wound rolls. These rolls have been sitting on the floor ahead of the unwinder for hours. We know they're not elliptical, we know they're not egg-shaped, we know that they are strange in shape. If you were to get the signal of the disturbance, could you determine the shape of the roll? Is that possible?

**Name & Affiliation**

P. R. Pagilla, Oklahoma  
State University

**Name & Affiliation**

Tim Walker, T. J. Walker  
& Associates

**Answer**

If the velocity of the web on the roll surface is known then this is reflected in the tension model.

**Question**

I understand how a non-concentric roller has a difference in radius at the entrance and exit. But the web on the roll or roller, if it is not slipping, is unaffected by a change in radius while it is on the elliptical shape. It enters at a certain radius and that radius changes as the non-concentric roll rotates. A change in tension does not occur unless the web slips in the machine direction. I have a question for you, Mark: If you want the machine to tell you how bad the roll is, what method of feedback might be considered? Would you use the tension inferred from a load cell, given the demand on the torque in the motor?

**Answer**

My first choice would be infer the roll shape from the torque signal. That torque signal and the angular position of the roll are available inside the drive.

**Question**

Would you be driving a roller in a speed loop trying to control constant speed?

**Answer**

I would want to decouple this as much as possible. Let us say the objective was to maintain constant surface speed velocity. We would typically have a disturbance observer that would inject a disturbance torque estimate into the torque summing junction so that you would not disturb the velocity regulator.

**Question**

How did you distort the rolls, did you run into them or hit them with a hammer?

**Answer**

I wound a rod in between the roll layers.

**Comment**

So the rod established the angular position of the eccentricity. These eccentricities often occur in pairs because wound rolls are often compressed between clam shell grips when roll trucks lift the rolls to move them from one location to another.

**Question**

In paper mills rolls often set supported by their cores on rails. In this case there are no flat edges but due to the dead weight of the wound roll there can be settlement that results in an eccentric roll. There is an eccentric mass and there will be difficulty in unwinding such rolls at high or even moderate speeds depending upon how long that roll has been sitting and the degree of eccentricity. To deal with that, the angular backlash in couplings and crossover through backlash transfer functions requires that that we

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Science

slow down the machine.

**Name & Affiliation**

Mark Weaver, Rockwell  
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**Answer**

We also slow down for those events usually. But the point is if you have a drive system that has the capability where you are not driving the torque to saturation and the necessary response is available you could maintain perfect angular velocity. The surface velocity is going to vary with the radius and will create a tension disturbance.