VIBRATION OF TWO AXIALLY TRANSLATING MEDIA INTERCONNECTED BY WINKLER ELASTIC FOUNDATION

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ABSTRACT

Transverse vibrations of two translating strings interconnected by a Winkler elastic foundation, and subjected to axial loading are investigated. The natural frequencies are composed of two infinite sets, representing in-phase and out-of-phase vibrations of the two strings. The effects of the axial tension ratios of the two continuous media, as well as the effects of the elastic foundation stiffness are investigated. In general, it is found that the natural frequencies increase with increasing foundation stiffness. Different mass and tension ratios between the stings alter the critical translation speed, in contrast to presence of the elastic foundation.

NOMENCLATURE

Dimensional variables

F_i	External force per unit length (N/m)
k	Winkler foundation stiffness (N/m)
m_i	Mass per unit length (kg/m)
p_i	Axial tension (N)
t	Time (sec)
wi	Out-of-plane displacement (m)
x	Spatial coordinate (m)
L	Support separation distance (m)
V	Axially translation velocity (m/s)

Non-dimensional variables

$T = t(p_1/m_1L^2)^{1/2}$	Non-dimensional time
$K_s = kL2/p_1$	Foundation stiffness for the string model
$R_m=m_1/m_2$	Mass ratio
$R_p = p_1/p_2$	Axial tension ratio
$\overline{\lambda}_s = \lambda\left(p_1/m_1L^2\right)^{-1/2}$	Eigenvalue, $\overline{\lambda}_s = i\omega$
$v_s=V(m_1/p_1)^{1/2}$	Axial translation speed

ω_{1n}, ω_{2n}	Synchronous and asynchronous natural frequencies
$X = x/L$	Spatial coordinate
$W_i = w/L$	Out of plane displacement

Subscript

$i = 1,2$	Indicate string-1 or -2

INTRODUCTION

Axially translating strings/beams, have applications such as magnetic tape systems, fiber winders, power transmission belts, textile and paper web handling machinery [1]. Dynamics of an axially moving medium has gyroscopic components due to the appearance of convective acceleration components in its governing equations of motion [2]. Such a system can be modeled either as a string, or as a beam depending on the flexural resistance to out-of-plane deformation relative in-plane resistance. The eigenvalues of general discrete gyroscopic systems are purely imaginary, and the corresponding eigenfunctions are complex and speed dependent due to the convective acceleration components [2]. These eigenvectors can be obtained by casting the governing equations in state space, where the orthogonality of the eigenvectors are confirmed, and the solution can be established using the expansion theorem [3,4]. A closed form solution for the general axially moving continua problems, subjected to arbitrary excitations and initial conditions was given by Wickert and Mote [2]. It was shown that at supercritical critical translation speeds the eigenvalues of the system become real and divergence and flutter instabilities co-exist.

The partial (or complete) elastic foundation is a distributed constrained layer, which could represent the effect of external pressure acting on the flexible structure [5,6]. The translating string on elastic foundation models dynamic systems including continuously supported conveyer belts, air-guided magnetic tapes, and translating paper pulp sheets supported by air jets [6,7]. The elastic foundation renders the translating string dispersive, and the propagation speed of traveling harmonic waves become frequency-dependent [8]. Perkins found that an elastic foundation has no effect on the critical speed of the translating string, however, it could alter the vibration mode shapes and thus significantly influence the forced response of the string [9]. Parker showed that the supercritical stability behavior of elastically supported, translating string is considerably different than that of the unsupported case [10]. In general, any elastic foundation (distributed or discrete) leads to multiple critical speeds and a single region of divergence instability above the first critical speed, whereas the unsupported string has one critical speed and stable at all supercritical speeds.

Practical applications of the string on an elastic foundation are in the paper making industry, where the sheets of pulp move between two pulleys and are supported on an elastic foundation formed by a sequence of air jets discharging at the underside along the length dimension [6]. Tan *et al.* used the fluid bearing forces as damping mechanism for the vibration and acoustic control of flexible elements [11]. They studied the vibrations of moving strings coupled with distributed hydrodynamic bearings by transfer function method. Both studies showed that the critical speed of the translating string is not altered by the presence of the bearing forces. Tan *et al.* studied the dynamic characteristics of a constrained string translating across an elastic foundation [12]. They investigated the mode localization, eigenvalue loci veering, and wave propagation aspects. Vibrations of translating string/beam systems guided by a single spring loaded guide have been reported in references [13-19] among others.

The use of two non-translating strings, connected by elastic foundation is common in engineering, and a variety of problems adopt it as a model [20]. The basic model uses a Winkler foundation, in which the strings are connected through closely spaced, but non-interconnected linear springs. Oniszczuk studied the free and forced transverse vibration of elastically connected double strings interconnected by Winkler elastic foundation [21-24]. Cabanska-Placzkiewicz studied the transverse vibration of double viscoelastic Voigt- Kelvin strings connected by viscoelastic foundation [20]. Cheng *et al.* studied the vibrations of an optical fiber coupler, used in telecommunications [25]. Oniszczuk studied the transverse vibration of a beam-string system interconnected by an elastic foundation [26].

Transverse vibration of two axially moving media is encountered in web handling applications. Recently, the authors analyzed the moving media as a couple of translating beams [35]. In this paper the transverse vibrations of two translating, tensioned strings interconnected by an elastic foundation are analyzed. The two strings are of different masses and tensions. The model represents the coupled behavior of various bonded, multi-layer webs during manufacturing such as paper, diapers and others.

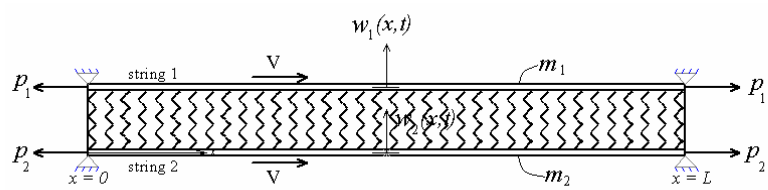


Figure 1 – Double strings connected by elastic foundation.

GOVERNING EQUATIONS

The model of the system consists of two parallel and homogeneous strings joined by a Winkler foundation of stiffness k. The Winkler foundation is a simplified model for the capillary adhesion forces [34]. Both stings have the same length L between the two supports, fixed at their ends, axially translating with velocity V, and axially tensioned to p_1 and p_2. The out of plane acceleration of a medium moving axially with transport velocity V is expressed with the total time derivative of the out-of-plane displacements w, as follows:

$$\frac{D^2 w}{Dt^2} = \frac{D}{Dt}\left(\frac{Dw}{Dt}\right) = \frac{D}{Dt}\left(\frac{\partial w}{\partial t} + V\frac{\partial w}{\partial x}\right) = \frac{\partial^2 w}{\partial t^2} + 2V\frac{\partial^2 w}{\partial x \partial t} + V^2\frac{\partial^2 w}{\partial x^2} \qquad \{1\}$$

The *local* acceleration of the strings is represented by the $\partial^2 w_i / \partial t^2$ term; the *Coriolis* acceleration is represented by the $2V\partial^2 w_i / \partial x \partial t$ term; and, the *centrifugal* acceleration is represented by the $V^2 \partial^2 w_i / \partial x^2$ term. The coupled governing equations of the transverse vibrations of the system are given as written as (e.g. [34]):

$$m_1 \frac{D^2 w_1}{Dt^2} - p_1 \frac{\partial^2 w_1}{\partial x^2} + k(w_1 - w_2) = f_1 \qquad \{2a\}$$

$$m_2 \frac{D^2 w_2}{Dt^2} - p_2 \frac{\partial^2 w_2}{\partial x^2} + k(w_2 - w_2) = f_2 \qquad \{2b\}$$

The fixed support boundary conditions are:

$$w_1(0,t) = w_1(L,t) = 0 \qquad \{3a,b\}$$

$$w_2(0,t) = w_2(L,t) = 0 \qquad \{3c,d\}$$

The two governing equations can be written in the following non-dimensional homogenous form:

$$\frac{\partial^2 W_1}{\partial T^2} - (1 - v_s^2)\frac{\partial^2 W_1}{\partial X^2} + 2v_s \frac{\partial^2 W_1}{\partial X \partial T} + K_s(W_1 - W_2) = 0 \qquad \{4a\}$$

$$\frac{\partial^2 W_2}{\partial T^2} - \left(\frac{R_m}{R_p} - v_s^2\right)\frac{\partial^2 W_2}{\partial X^2} + 2v_s \frac{\partial^2 W_2}{\partial X \partial T} + R_m K_s(W_2 - W_1) = 0 \qquad \{4b\}$$

with the non-dimensional variables defined in the nomenclature. The non-dimensional forms of the boundary conditions become:

$$W_1(0,T) = W_1(1,T) = 0 \qquad \{5a,b\}$$

$$W_2(0,T) = W_2(1,T) = 0 \qquad \{5c,d\}$$

SOLUTION METHOD

The Orthogonality of the Solution

The system of equations given by equations {2} can be written in the form of a system of second order differential equations as:

$$\boldsymbol{MW}_{,TT} + \boldsymbol{GW}_{,T} + \boldsymbol{K^*W} = \boldsymbol{f} \qquad \{6\}$$

where a subscripted comma $_{,T}$ indicates partial differentiation, and ***M, G, K**** are the mass, gyroscopic and stiffness operators, respectively, and

$$\boldsymbol{W} = \begin{Bmatrix} W_1 \\ W_2 \end{Bmatrix}, \ \boldsymbol{f} = \begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} \qquad \{7\}$$

The ***M, G***, and ***K**** matrices take on the forms given in Appendix A.

The equations of motion can be expressed in state space representation as [28]:

$$\boldsymbol{AU}_{,T} + \boldsymbol{BU} = \boldsymbol{q} \qquad \{8\}$$

where the state and excitation vectors are:

$$\boldsymbol{U} = \{W_{1,T} \ \ W_{2,T} \ \ W_1 \ \ W_2\}^T, \ \boldsymbol{q} = \{f_1 \ \ f_2 \ \ 0 \ \ 0\}^T \tag{9}$$

and the matrix differential operators are:

$$\boldsymbol{A} = \begin{bmatrix} \boldsymbol{M} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{K}^* \end{bmatrix}; \ \boldsymbol{B} = \begin{bmatrix} \boldsymbol{G} & \boldsymbol{K}^* \\ \boldsymbol{K}^* & \boldsymbol{0} \end{bmatrix} \tag{10}$$

Equation {8} is the canonical form of the equation of motion {6}, where ***A*** is a symmetric and ***B*** is a skew symmetric matrix operator. Orthogonality of eigenfunctions with respect to each operator is guaranteed in the canonical form, when ***A*** and ***B*** are symmetric and skew symmetric, respectively [2,3,28,29]. The inner product of two vectors U_1 and U_2 is defined as:

$$\langle U_1, U_2 \rangle = \int_0^1 U_1^T \overline{U}_2 dX \tag{11}$$

where the over bar denotes complex conjugation. The general solution of Eq. {8} is in the form:

$$U(X,T) = \mathrm{Re}\left\{\hat{\phi}_1(X)\overline{\lambda} e^{\overline{\lambda}T} \quad \hat{\phi}_2(X)\overline{\lambda} e^{\overline{\lambda}T} \quad \hat{\phi}_1(X) e^{\overline{\lambda}T} \quad \hat{\phi}_2(X) e^{\overline{\lambda}T}\right\} \tag{12}$$

where the eigenvalues $\overline{\lambda}(= i\omega)$ are purely imaginary, with $i = \sqrt{-1}$, and the eigenfunctions $\hat{\phi}_j$ are complex.

Natural Frequency Analysis

In order to obtain the natural frequencies and the mode shapes of the system, the response of string-2 is expressed in terms of the response of the string-1, from equation {2} as follows:

$$W_2 = \frac{1}{K}\left[-\left(1-v^2\right)\frac{\partial^2 W_1}{\partial X^2} + 2v\frac{\partial^2 W_1}{\partial X \partial T} + \frac{\partial^2 W_1}{\partial T^2} + KW_1\right] \tag{13}$$

Equations {2a} and {2b} are then combined into a single fourth-order partial differential equation:

$$\frac{\partial^4 W_1}{\partial T^4} + A_1\frac{\partial^4 W_1}{\partial T^3 \partial X} + A_2\frac{\partial^4 W_1}{\partial X^2 \partial T^2} + A_3\frac{\partial^4 W_1}{\partial X^3 \partial T} + A_4\frac{\partial^2 W_1}{\partial T^2} + A_5\frac{\partial^2 W_1}{\partial X \partial T} + A_6\frac{\partial^2 W_1}{\partial X^2} + A_7\frac{\partial^4 W_1}{\partial X^4} = 0 \tag{14}$$

The constant coefficients $A_1 - A_7$, given in Appendix B, depend on the system parameters. Considering the solution given in Eq. {12} in the above equation, the eigenfunction for string-1 becomes:

$$\hat{\phi}_1(X) = \sum_{k=1}^{4} c_k e^{i\gamma_k X} \qquad \{15\}$$

where c_k are constant coefficients, and γ_k are the roots of the characteristic equation of Eq. {14}. This characteristic equation is obtained by substituting $e^{i\gamma_k X} e^{\bar{\lambda} T}$ into Eq. {14}. The roots of this equation are obtained using Mathematica^{TM}, but they are omitted here due to space limitations. The eigenfunction for string-2 is found by substituting Eq. {15} into Eq. {13}, and becomes:

$$\hat{\phi}_2(X) = \sum_{k=1}^{4} B_k c_k e^{i\gamma_k X} \qquad \{16\}$$

with,

$$B_2 = \frac{1}{K}\left[\left(1 - v^2\right)\gamma_k^2 + 2v \ i\gamma_k \bar{\lambda} + \bar{\lambda}^2 + K\right] \qquad \{17\}$$

In order to obtain the eigenvalues for the double-string system, boundary conditions, in Eq. {5} are evaluated using Eqs. {15,16}. This results in four homogeneous algebraic equations, which are represented in matrix form as:

$$\boldsymbol{D}\left(\bar{\lambda}\right).\boldsymbol{c} = 0 \qquad \{18\}$$

where $\boldsymbol{c} = \{c_1\ c_2\ c_3\ c_4\}^T$ is the coefficient vector, and $\boldsymbol{D}$ is the matrix of coefficients. In order to have a nontrivial solution, the determinant of matrix, det(**D**) = 0. This gives the characteristic equation of the system. The natural frequencies are determined from the solution of the characteristic equation. A computer program using Mathematica^{TM} is developed to determine these complex natural frequencies. The mode shapes are then calculated from Eqs. {15} and {16} and normalized using the real parts of the complex mode shapes with respect to the symmetric matrix operator $\boldsymbol{A}$ as $\left\langle A\phi_m^R, \phi_n^R \right\rangle = \delta_{mm}$.

RESULTS AND DISCUSSION

Mode Shapes and Natural Frequencies for Non-Translating System

It is instructive to investigate the dynamics of the non-translating, $v_s = 0$**,** system [26]. The closed form formulas given below are useful for interpreting the results of the translating string problem. When, $v_s = 0$**,** Eq. {2} reduces to:

$$\frac{\partial^2 W_1}{\partial T^2} - \frac{\partial^2 W_1}{\partial X^2} + K_s\left(W_1 - W_2\right) = 0, \qquad \{19a\}$$

$$\frac{\partial^2 W_2}{\partial T^2} - \frac{R_m}{R_p}\frac{\partial^2 W_2}{\partial X^2} + R_m K_s\left(W_2 - W_1\right) = 0. \qquad \{19b\}$$

This boundary value problem is subjected to the boundary conditions given in Eq. {5}, and it can be solved using the Fourier series method by assuming the mode shape function as [24]:

$$W_j(X,T)=\sum_{n=1}^{\infty} A_{jn}\sin(n\pi X)e^{i\omega_{jn}T} \quad \text{for } j=1,2 \tag{20}$$

This assumes that the natural frequencies of the double string system are divided into odd and even sets of fundamental frequencies, ω_{1n} and ω_{2n}. In general, when the two strings are identical, the free vibrations are described by synchronous and asynchronous vibrations, with ω_{1n} and ω_{2n}, respectively. The natural frequencies of the double string system can be expressed in closed form as:

$$\omega_{1n}=\sqrt{a-b}, \quad \omega_{2n}=\sqrt{a+b} \tag{21}$$

where ω_{1n} and ω_{2n} are the in-phase and out-of-phase natural frequencies, respectively, and,

$$\begin{aligned} a &= \left((n\pi)^2\left(1+\frac{R_m}{R_p}\right)+K_s(1+R_m)\right), \\ b &= \left[\left((n\pi)^2\left(1+\frac{R_m}{R_p}\right)+K_s(1+r_m)\right)^2-4R_m(n\pi)^2\left(\frac{(n\pi)^2}{R_p}+K_s\left(1+\frac{1}{R_p}\right)\right)\right]^{1/2} \end{aligned} \tag{22}$$

If $R_m = R_p = 1$, formulae for natural frequencies are reduced to:

$$\omega_{1n}=n\pi \tag{23}$$

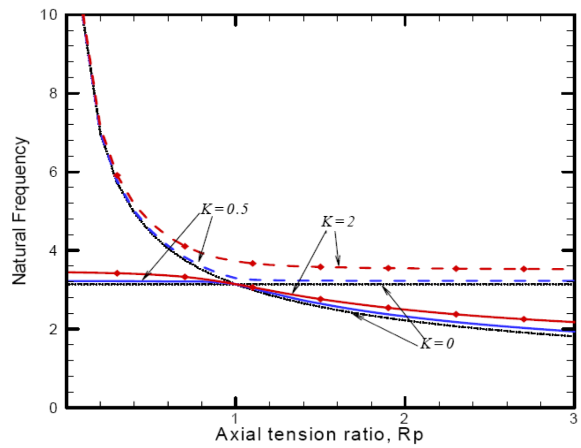
$$\omega_{2n}=\sqrt{(n\pi)^2+2K_s} \tag{24}$$

Note that in this case, the synchronous natural frequencies w_{1n} of the double string system are identical to those of a single string with the same boundary conditions. This is because when the two strings are moving in synchronous mode, the elastic foundation does not experience any deformation; the two strings vibrate with the same amplitude and direction. On the other hand, the asynchronous natural frequencies ω_{2n} are identical to those of a single string on an elastic foundation of stiffness $2Ks$ [9]. The asynchronous frequencies increase with increasing Ks as shown in Eqn {24}.

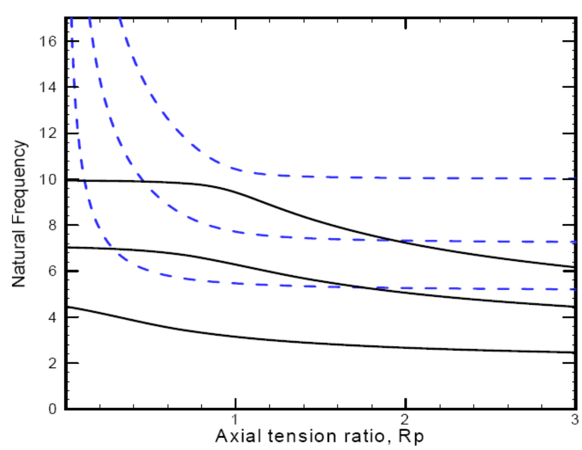
In more general cases, Eqs. {21,22}show that the natural frequencies depend on the ratio R_m/R_p, parameter Ks and the product K_sR_m. Close inspection of the ratio R_m/R_p shows that it is the square of the wave speed ratio in the strings $R_m/R_p = (c_2/c_1)^2$. Next, the effect of the tension parameter R_p will be investigated. Changing R_p is equivalent to changing the tension p_2 while all other non-dimensional parameters are kept constant.

Figure 2a shows the effect of axial tension ratio Rp on the natural frequencies for K_s = 0, 0.5, and 2, $R_m = 1$. Considering that $R_p = p_1/p_2$, increasing R_p values cause a decrease in the natural frequency as expected. Figure 2a shows that in the range $0 \le R_p < 1$, the natural frequencies decrease as p_2 decreases. In this range $p_2 > p_1$ and p_1 dominates the lowest natural frequency. At $R_p = R_m$ (=1), the wave speeds in the two strings are identical, i.e. $c_1 = c_2$. Further decrease in p_2 (increase in R_p) causes a sharp change in the behavior of the curve. In the range $R_p > 1$, $p_1 > p_2$ and p_2 determines the lowest natural frequency. Thus at $R_p = R_m$ the string with the critical frequency switches from string-1 to sting-2, or vice-versa.

Case $K_s = 0$ plotted with dotted lines, represent the limit condition $K_s \rightarrow 0$ for Eqs. {21} and {22}, in Figure 2. On the other hand when $K_s > 0$ the two strings are coupled and the two modes move apart from each other at $R_p = R_m$ region. This figure shows that the coupling due to elastic foundation causes the natural frequencies to increase. Figure 2b shows the first six natural frequencies of the double string system, for $K_s = 10$ as a function of R_p, where the natural frequencies are shifted to higher values, relative to $K_s = 0$ case.



a) $K = 0, 0.5, 2$



b) $K = 10$

Figure 2 – The synchronous (solid) and asynchronous (dashed) natural frequencies versus axial tension ratio, R_p for $R_m = 1$, $v = 0$ a) K_s=0, 0.5, and 2, b) K_s=10.

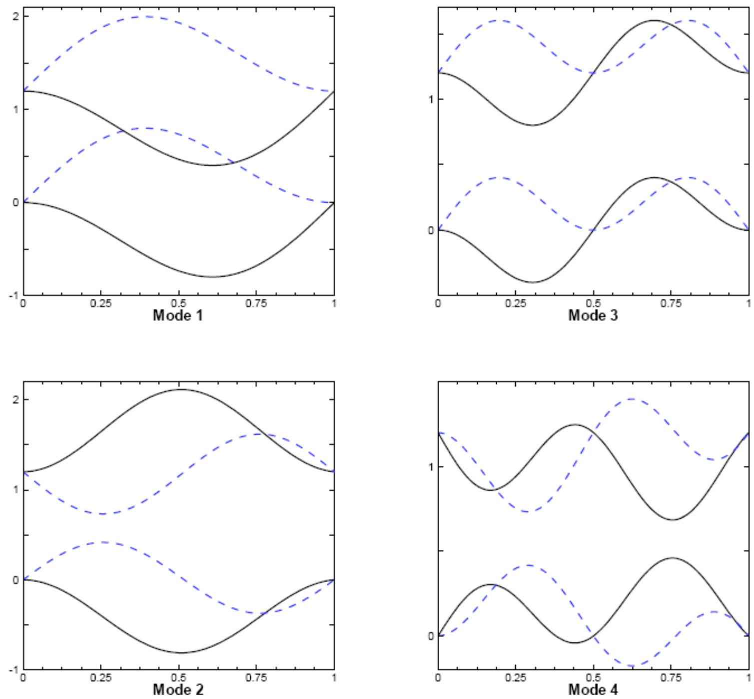


Figure 3 – The first four complex mode shapes; the real (solid) and imaginary parts (dashed) for $K_s = 10$, $v = 0.5$, $R_m = R_p = 1$.

Mode Shapes and Natural Frequencies of Translating System

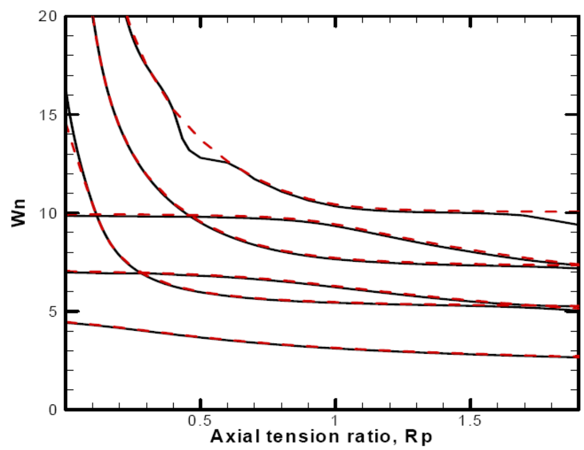
The translating double string system also displays the in-phase and out-of-phase behavior seen in the non-translating system. In case where $R_p = R_m = 1$ closed form formulas for ω_{1n} and ω_{2n} of the translating system can be obtained [33]:

$$\omega_{1n} = n\pi\left(1 - v_s^2\right) \qquad \{25\}$$

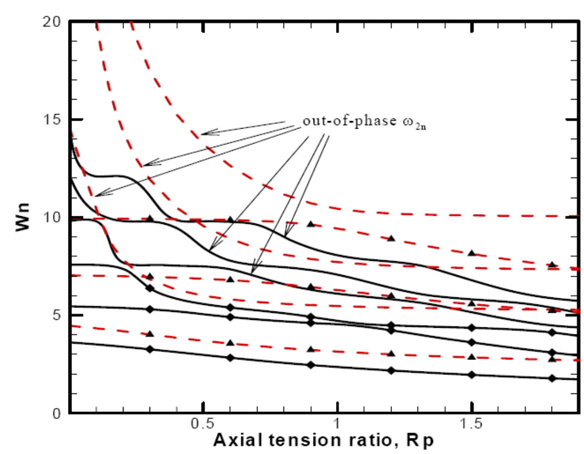
$$\omega_{2n} = \sqrt{\left[(n\pi)^2 + 2K_s\right]\left(1 - v_s^2\right)^2} \qquad \{26\}$$

If the elastic foundation stiffness between the two strings is very small, i.e., $K_s \rightarrow 0$, then the $\omega_{2n} \rightarrow \omega_{1n}$. For the case of non-identical double string system, a Mathematica™ code is developed to solve det($\boldsymbol{D}$) = 0, in Eq. {18}.

The natural frequencies of the translating double string system are distinguished by two fundamental odd and even sets ω_{1n} and ω_{2n}, where the subscript n = 1, 2, … This distinction becomes clear in Fig. 3 where the first for mode shapes, corresponding to the first four natural frequencies are plotted, for the parameters $K_s = 10$, $\nu = 0.5$, $R_m = R_p = 1$. As expected the mode shapes have real and imaginary parts. However, the odd numbered modes ω_1, and ω_3 show synchronous deflection, and even numbered modes ω_2, and ω_4 show asynchronous deflections. It can easily be deduced that the elastic foundation is not stretched for the synchronous modes, and it is stretched for the asynchronous vibration modes. This figure also shows that the mode shapes are not symmetrical with respect to the mid-span of the strings; this distortion is due to the effects of translation, as was also observed by Wickert and Mote [2]. The separation of mode shapes into out-of-phase (asynchronous) and in-phase (synchronous) behaviors is the result of the coupling of the two strings by the Winkler foundation, and it is observed for non-translating string systems (e.g., [21-24,26,30-32]). In case the strings are not identical, the vibration of the two strings still show in-phase and out-of-phase characteristics, for odd- and even-modes, respectively, however the mode shapes are not parallel to each other. This effect is observed for other R_m values as well as R_p values, and symmetry and anti-symmetry of the modes further deteriorate with decreasing values of R_m, and R_p [33,35].



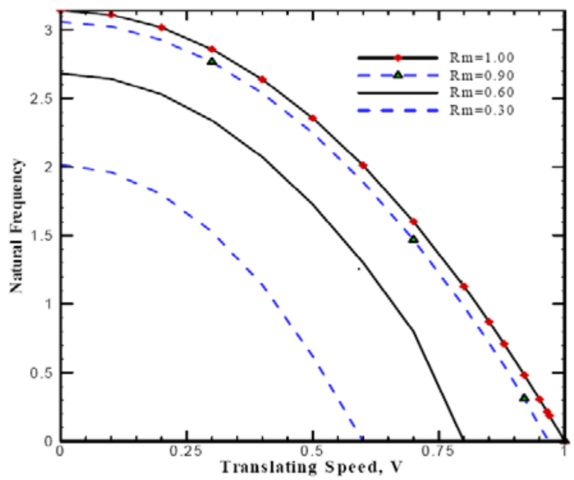
a) v = 0.0 (dashed) and 0.1 (solid)



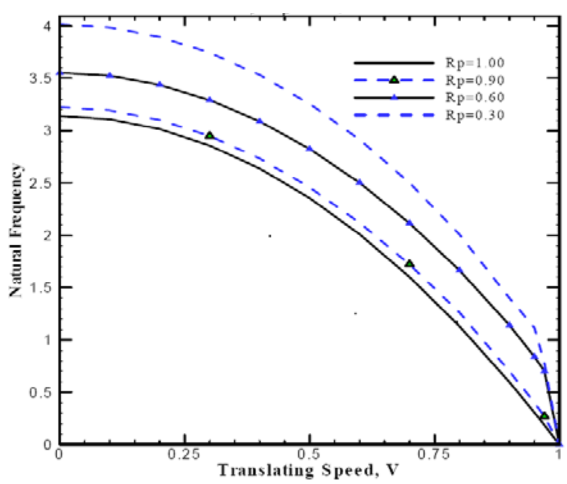
b) v = 0.0 (dashed) and 0.5 (solid)

Figure 4 – Comparison of the effect of axial tension ratio on the natural frequencies for different velocities, for K = 10, R_m = 1.

The effect of translation speed and axial tension R_p on a double string system with $K = 10$ and $R_m = 1$, is investigated in Fig. 4. In particular in Fig. 4a the translation speed values of 0 and 0.1 are compared. This figure shows that at this relatively slow translation speed the natural frequencies are nearly the same as compared to the non-translating string. However, when the case of $\nu = 0.5$ is considered, in Fig. 4b, it is seen that the natural frequencies drop significantly both for in-phase and out-of-phase modes. Frequency clustering is observed on this figure near the intersection points of the in-phase and out-of-phase curves for $\nu = 0$ case.



a) $R_p = 1$, $K = 10$



b) $R_m = 1$, $K = 10$

Figure 5 – The critical speed of the double string system for different a) R_p, and b) R_m

Critical Speeds

The natural frequencies of this system as a function of the non-dimensional translating speed are shown in Fig. 5. In particular Fig. 5a shows the effect of the mass ratio R_m and Fig. 5b shows the effect of the tension ratio R_p on the natural frequencies at different translation speeds. Figs. 5a and 5b show that for $R_m = R_p = 1$, similar to a single axially moving string analyzed in reference [9], the natural frequency vanishes at the critical translation speed ns = 1. This result is expected, as the odd numbered natural frequencies are not affected by the presence of the Winkler foundation. Hence, the onset of divergence instability for the double string system analyzed here is identical to the case of the single string, and the elastic stiffness does not alter the divergence instability [10]. The critical speed curves are plotted for different for R_m and R_p in Figs. 5a and 5b, respectively. As expected, when the mass ratio decreases, or the mass of the second string increases, the natural frequencies and the critical speed drops. On the other hand, when the axial tension ratio decreases the natural frequencies are elevated but still have the same critical speed.

SUMMARY AND CONCLUSIONS

The free transverse vibration of an elastically connected axially loaded, translating double string system is analyzed. In general, the natural frequencies of the system are composed of two infinite sets, w1n and w2n. When the two strings are identical, the free vibrations are described by synchronous and asynchronous vibrations, with ω_{1n} and ω_{2n}, respectively. The vibrations still show in-phase and out-of-phase characteristics, as the parameters of the strings change. The mode shapes are distorted by increasing the translating speed, and become more significant when the two strings are not identical. The synchronous mode shapes stem from the fact that the two continuous media are vibrating with the same amplitude and direction anti-symmetrically, with respect to an axis passing through the mid-thickness of the system. On the other hand, the asynchronous mode shapes are vibrating with the same amplitude but in opposite directions symmetrically with respect to the mid-thickness. It is found that the natural frequencies increase with increasing elastic stiffness K of the foundation. Frequency clustering is observed for the natural frequencies as a function of the tension ratio R_p, when K is not zero. Divergence instability occurs at the same critical speed of a single traveling string; and, the frequency-velocity relationship is similar to that of a single traveling string. The elastic foundation for identical system does not alter the critical speed, and different for each mass ratio.

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APPENDIX A

$$\boldsymbol{K}^* = \begin{bmatrix} (v_s^2 - 1) & 0 \\ 0 & \left(v_s^2 - \dfrac{R_m}{R_p}\right) \end{bmatrix} \frac{\partial^2}{\partial X^2} + \begin{bmatrix} 1 & -1 \\ -R_m & R_m \end{bmatrix} K_s$$
$$\boldsymbol{G} = 2v_s \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \frac{\partial}{\partial X}, \quad \boldsymbol{M} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{A1}$$

APPENDIX B

Coefficients of equation {14} are:

$$A_1 = 4v_s, \quad A_2 = -\left(1 + \frac{R_m}{R_m}\right) + 6v_s^2, \quad A_3 = 2v_s\left(\left(1 + \frac{R_m}{R_m}\right) - 2v_s^2\right), \quad A_4 = K_s(1 + R_m),$$
$$A_5 = 2v_s K_s (1 + R_m), \quad A_6 = -K_s R_m \left(1 + \frac{1}{R_p}\right) + K_s v_s^2 (1 + R_m) \tag{B1}$$

Vibration of Two Axially Translating Media Interconnected by Winkler Elastic Foundation	**M. Gaith & S. Müftü,** Al-Isra Private University and Northeastern University, USA

Name & Affiliation	**Question**
Unknown	You may wish to consider an extension to this problem where the aerodynamic damping is considered. The surface area is large and the forces due to damping could be quite large. The permeability of the web and the wire are going to be quite different.
Name & Affiliation	**Answer**
S. Müftü, Northeastern University	We were inspired by industry on this problem. We decided to work on this problem and demonstrate what we can do and then work towards bigger problems. So that's why we started with the common engineering assumptions I mentioned. Now we understand the solutions based upon the original assumptions and we can start to consider suggestions of the sort you have offered.
Name & Affiliation	**Question**
Unknown	You have shown us these waves which are coupled with a critical speed. I didn't understand how the coupling occurs and what produces the excitation? I also didn't catch how the critical speed impacted operations.
Name & Affiliation	**Answer**
S. Müftü, Northeastern University	I didn't talk about the waves here, these are the mode shapes. So if you were to excite the webs, this is what you would see. The importance of the critical speed is this. As I run the media faster and faster, I am reducing the effect of tension in the system. A motivation for this work is that as webs are transported at higher speeds, the natural frequencies decrease and eventually the system becomes completely unstable.
Name & Affiliation	**Question**
John Shelton, Oklahoma State University	Several years ago Keith Good and Ron Markum put together a linear tape drive system to demonstrate that webs could be transported under no tension. In this setup they pulled 1 inch wide magnetic recording tape from an unwinding roll with a nip. The web exited the nip and was unsupported thereafter. At high speeds the web would travel several feet horizontally and unsupported after it exited the nip. The web then went into a sinusoidal standing wave that increased exponentially in amplitude. The drag forces of course increased with amplitude and the web would drop to the floor. The point was that the web has mass and inertia and could be transported (in this case thrown) from one point to another under no tension. This in fact happens in paper machines where the web is thrown from one felt to the next.

Name & Affiliation	Answer
S. Müftü, Northeastern University	Yes, there are situations where you can transport webs beyond that critical velocity, but those velocities are very high.