RECENT ADVANCES IN WEB LONGITUDINAL CONTROL

by

P. R. Pagilla[1] and D. Knittel[2]
[1]Oklahoma State University
USA
[2]Louis Pasteur University
FRANCE

ABSTRACT

Stable web transport through processing machinery is critical in the web processing industry. Demands for improved performance under a wide variety of dynamic conditions and web materials are placing additional emphasis on developing new advanced control techniques. Further, technological advances in areas such as drive hardware, microprocessors, and sensors, are opening up new possibilities for implementing advanced control methods that are robust to a number of process and material variations and result in superior performance over existing industrial control methods.

Mathematical models of fundamentals elements in a web process line are presented. A systematic procedure for computing the equilibrium inputs as well as reference velocities of all rollers based on the master speed reference is given. Recently developed robust control methods for web longitudinal control are described. Implementation of the controllers on two experimental platforms is given, and a sample of the experimental results is presented. Finally, some potential new directions and future research topics are discussed.

INTRODUCTION

Considerable amount of work has been done on modeling the web longitudinal dynamic behavior in the last five decades. Models have been continuously refined over the years as a result of better process understanding and new modeling techniques. The developed models for most part have been used in analysis of web dynamic behavior and not for control design. The web processing industry has been using classical control algorithms such as PID-type controllers. In some cases, variations of the PID-type controllers have been used with some feed-forward compensation based on process knowledge to improve performance. The classical methods do not directly take robustness into consideration at the control design stage, whereas recent advanced control techniques have the ability to incorporate measures of robustness at the design stage itself. Robustness is a fundamental requirement in the design of any controller. It reflects the ability of the controller to provide closed-loop stability and adequate performance in the presence of plant variations (both parametric and structural) and disturbances. Recent research in using robust model-based control methods for web longitudinal control has shown the potential for substantial performance improvements.

Considerable research effort has gone into focused studies on longitudinal dynamical behavior of a web; these studies date back to the 1950's. Earliest work discussing control of web tension was reported in [1]. Various process parameters that affect modeling of tension in the web in a printing press were discussed. The importance of regulating web tension to within a few percent of the reference tension in the presence of disturbances and irregularities was emphasized for achieving good printing. A mathematical model for longitudinal dynamics of a web span between a pair of two pinch rolls, which are driven by two motors, is given in [2]. This model does not predict tension transfer and does not consider tension in the entering span. Maintenance of sheet tension by control of electric drives to rolls that nip or pull the sheet was discussed in [3]. A model that considers tension in the entering span was developed in [4]. A summary of the Ph.D. dissertation of G. Brandenburg was reported by D.L. King in [5]. It gives fundamental equations for a moving web and extends the study to the case of a three-roller system under transient conditions. The relationships for the web strain and web velocity in the nip and the unsupported web are derived from four basic equations: stress-strain relations, the equations of motion, the continuity equations, and the requirements for conservation of mass. The steady state and transient behavior of tensile force, stress, and strain in a web as functions of variables such as wrap angle, position and speed of the driven rolls, density, cross-sectional area, modulus of elasticity, and temperature was investigated in [6].

A mathematical model of production machines which process flexible material is given in [7, 8]. Dynamics for various components in the machine such as an idle roller, unwind/winder rolls, and spring-loaded roller are developed. Materials with linear elastic and linear visco-elastic tensile properties have been considered. The effects of friction between material and guide rollers have been included. In [9], equations describing the tension dynamics are derived based on the fundamentals of the web behavior and the dynamics of drives used for web transport; these equations were subsequently linearized. A simple example system was considered to compare torque control versus velocity control of a roll for the regulation of tension in a web. Non-ideal effects such as temperature and moisture change on web tension were studied in [10]. Analysis of a multi-span web system with a passive dancer for minimizing disturbances due to eccentric unwind roll is given. An overview of lateral and longitudinal behavior and control of moving webs was presented in [11]. Effect of web tension changes during start-up and shut-down in a mult-span web transport system was studied in [12]. A decentralized decoupling controller and a fuzzy controller designs were presented in [13]. A non-interaction tension control algorithm was proposed in [14] to reject disturbances due to interaction between neighboring processing sections. A review of problems in tension control of webs at the time was given in [15]. In [16], a worst case analysis procedure is used to determine the minimum bandwidth of the speed regulator, and its effect on tension regulator is investigated. Research related to the choice of load cells and dancers for tension regulation was discussed in [17–23].

Some of the recent work of the authors in robust control of web longitudinal dynamics can be found in [24–33] and [34–41]. The goal of this paper is to present tje mathematical models of basic elements, give a simple procedure to obtain the equilibrium inputs and reference velocities, and discuss recent developments in robust control designs for web handling which give better regulation of tension as well as transport velocity in the presence of disturbances and parameter variations. Implementation procedure and experimental results are also discussed.

Nomenclature

A	: Cross-sectional area of web	b_{fi}	: Viscous friction coefficient
E	: Young's modulus of web material	J_i	: Moment of inertia of roller/roll
L_i	: Web length	R_i	: Radius of roller/roll
t_i	: Web tension	t_{ri}	: Reference web tension
u_i	: Control input of roller/roll	$u_{i,eq}$	: Equilibrium input to roller/roll
v_i	: Velocity of web	v_{ri}	: Reference web velocity
w	: Web width	V_i, T_i, U_i	: Deviations from references
δ	: Web thickness	ε_i	: Strain
ρ	: Web density	τ	: Time
ω_i	: Angular velocity		

MATHEMATICAL MODELS

This section considers the mathematical models of the basic elements (free span, idle/driven roller, unwind roll, and winder roll) in a web process line. Since there may be a number of spans and rollers in a typical web process line, a numbering system is needed. The numbering convention used in this paper is shown in Figure1. The rollers (including the material rolls) are numbered from 0 to N with 0 representing the unwind roll and N representing the winder roll. The peripheral velocities of the rollers are represented by $v_0, v_1, \ldots, v_N$. Similarly the spans are numbered from 1 to N with 1 representing the span immediately next to the unwind roll and N representing the span immediately before the winder roll. The tensions and the strains in i^{th} span are represented by t_i and ε_i.

The web span tension dynamics is obtained by using the conservation of mass principle to the control volume shown in Figure 1. If the web is assumed to be purely elastic, i.e., $t_i = EA\varepsilon_i$, the web tension dynamics in the i-th span is given by

$$L_i\dot{t}_i = v_{i-1}t_{i-1} - v_i t_i + EA(v_i - v_{i-1}). \tag{1}$$

This approximation is widely used in the web processing industry, explicit early appearance of which can be found in [6, 7, 9].

The driven roller velocity dynamics is given by

$$(J_i/R_i)\dot{v}_i = (t_{i+1} - t_i)R_i + u_i - (b_{fi}/R_i)v_i. \tag{2}$$

Since the unwind roll is releasing the material, the inertia of the unwind roll is $J_0(t) = J_{m0} + J_{c0} + J_{w0}(t)$, where J_{m0} is the equivalent inertia of the rotating parts on the motor side, J_{c0} is the inertia of the driven shaft and the core mounted on it, and $J_{w0}(t)$ is the inertia of the web material. Notice that the quantities J_{m0} and J_{c0} are constants whereas $J_{w0}(t)$ is a time-varying quantity since the unwind roll is releasing material. The inertia

J_{w0} at any instant of time is $J_{w0}(t) = (\pi/2)w\rho(R_0^4(t) - R_{c0}^4)$. Applying Newton's second law to the unwind roll,

$$(d/d\tau)(J_0\omega_0) := \dot{J}_0\omega_0 + \dot{\omega}_0 J_0 = t_1 R_0(t) + u_0 - b_{f0}\omega_0 \quad (3)$$

where u_0 is the effective control input to the unwind roll including any transmission ratio. The time rate of $J_0(t)$ is $\dot{J}_0(t) = \dot{J}_{w0}(t) = 2\pi w\rho R_0^3 \dot{R}_0$. Since the linear web-velocity (v_0) and the angular velocity (ω_0) are related by $v_0 = R_0\omega_0$, (3) can be simplified to

$$(J_0/R_0)\dot{v}_0 = t_1 R_0 + u_0 - (b_{f0}/R_0)v_0 + (\dot{R}_0 v_0/R_0^2)J_0 - 2\pi\rho w R_0^2 \dot{R}_0 v_0. \quad (4)$$

Further, the rate of change of radius, $\dot{R}_0$, is a function of the web release velocity v_0 and web thickness, δ, and is approximately given by $\dot{R}_0 \approx -(\delta v_0(t) 2\pi R_0(t))$. Using the rate of change of radius in (4),

$$(J_0/R_0)\dot{v}_0 = t_1 R_0 + u_0 - (b_{f0}/R_0)v_0 - (\delta/2\pi R_0)\left((J_0/R_0^2) - 2\pi\rho w R_0^2\right)v_0^2. \quad (5)$$

The velocity dynamics for the winder roll can be derived in a similar way, and is given by

$$(J_N/R_N)\dot{v}_N = -t_N R_N + u_N - (b_{fN}/R_N)v_N + (\delta/2\pi R_N)\left((J_N/R_N^2) - 2\pi\rho w R_N^2\right)v_N^2. \quad (6)$$

Equations 1, 2, 5, and 6 give the nonlinear dynamics of the span tension, driven roller, unwind roll, winder roll, respectively. It is possible to obtain linear error dynamics by defining a forced equilibrium point, which is described in the following section.

Equilibrium Control and Reference Velocities

The control goal is to regulate web tension in each of the tension zones while maintaining the prescribed web transport velocity. To achieve this, first, systematic calculation of the control input is required to keep the web line at the forced equilibrium of the reference web tension and web velocity in each of the zones. Then, some additional feedback (and feedforward) compensation must be included to provide error convergence in the presence of uncertainties and disturbances. We give a simple procedure for the calculation of equilibrium control inputs that is easy to understand and implement by practising engineers.

Define the following variables: $T_i = t_i - t_{ri}$ and $V_i = v_i - v_{ri}$, where t_{ri} and v_{ri} are tension and velocity references, respectively, T_i and V_i are the variations in tension and velocity, respectively, around their reference values, u_{ieq} as the control input that maintains the forced equilibrium at the reference values, and $U_i = u_i - u_{ieq}$ is the feedback control input that will provide the robustness to disturbances and process variations. To illustrate this concept, consider a four-motor system shown in Fig. 2 (this system is used for the controllers given in Sections 3.1 and 3.2).

Using the derivations in the previous section, the dynamics of span tensions and roller velocities for the example system are

$$(J_0/R_0)\dot{v}_0 = t_1 R_0 + u_0 - (b_{f0}/R_0)v_0 - (\delta/2\pi R_0)\left((J_0/R_0^2) - 2\pi\rho w R_0^2\right)v_0^2, \quad (7a)$$

$$L_1\dot{t}_1 = AE[v_1 - v_0] + t_0 v_0 - t_1 v_1, \quad (7b)$$

$$(J_1/R_1)\dot{v}_1 = (t_2 - t_1)R_1 + u_1 - (b_{f1}/R_1)v_1, \quad (7c)$$

$$L_2\dot{t}_2 = AE[v_2 - v_1] + t_1 v_1 - t_2 v_2, \quad (7d)$$

$$(J_2/R_2)\dot{v}_2 = (t_3 - t_2)R_2 + u_2 - (b_{f2}/R_2)v_2, \quad (7e)$$

$$L_3\dot{t}_3 = AE[v_3 - v_2] + t_2 v_2 - t_3 v_3, \quad (7f)$$

$$(J_3/R_3)\dot{v}_3 = -t_3 R_3 + u_3 - (b_{f3}/R_3)v_3 + (\delta/2\pi R_3)\left((J_3/R_3^2) - 2\pi\rho w R_3^2\right)v_3^2. \quad (7g)$$

The equilibrium control inputs are obtained by setting $t_i = t_{ri}$, $v_i = v_{ri}$ in equations (7a), (7c), (7e), (7g) and solving for u_i. To compute the equilibrium control $u_{0,eq}$, substitute $t_1 = t_{r1}, v_1 = v_{r1}$ in equation (7a) to solve for the control input as

$$u_{0,eq} = (b_{f0}/R_0)v_{r0} - R_0 t_{r1} + (J_0/R_0)\dot{v}_{r0} + (\delta/2\pi R_0)\left((J_0/R_0^2) - 2\pi\rho w R_0^2\right)v_{r0}^2. \quad (8)$$

Also, applying equilibrium condition to the tension dynamics of (8), and simplifying, an equilibrium relationship between the reference velocities and tensions is obtained as

$$v_{r0} = (AE - t_{r1})/(AE - t_0)v_{r1}. \quad (9)$$

Equation (9) indicates that when a reference tension t_{r1} is chosen, the reference velocities have to satisfy a relation to maintain the equilibrium.

Using the definitions of deviations in tensions/velocities and the equilibrium control input, the dynamics of the unwind roll in terms of the velocity variation V_0 is

$$(J_0/R_0)\dot{V}_0 = T_1 R_0 + U_0 - (b_{f0}/R_0)V_0 - \underbrace{(\delta/2\pi R_0)\left((J_0/R_0^2) - 2\pi\rho w R_0^2\right)(V_0^2 + 2v_{r0}V_0)}_{f_0(V_0)}. \quad (10)$$

Similarly, using the definitions of deviations, the tension variation dynamics in the first span is

$$L_1\dot{T}_1 = AE[(V_1 - V_0)] + t_0 V_0 - T_1 v_{r1} - V_1 t_{r1} - T_1 V_1. \quad (11)$$

Equations (10) and (11) represent the velocity and tension error dynamics of unwind section of a web process line.

The equilibrium control and the condition for equilibrium may be obtained for the intermediate roller 1 and span 2 by setting $v_1 = v_{r1}$, $v_2 = v_{r2}$, $t_1 = t_{r1}$, $t_2 = t_{r2}$ into equations (7c) and (7d). These are given by

$$u_{1,eq} = -R_1(t_{r2} - t_{r1}) + (b_{f1}/R_1)v_{r1}, \quad v_{r2} = (AE - t_{r1}/AE - t_{r2})v_{r1}. \quad (12)$$

Using the definitions of deviations and equations (12), and ignoring products of variations $T_i V_i$, the error dynamics for V_1 and T_2 are

$$(J_1/R_1)\dot{V}_1 = R_1(T_2 - T_1) - (b_{f1}/R_1)V_1 + U_1, \quad (13a)$$

$$L_2\dot{T}_2 = EA(V_2 - V_1) + T_1 v_{r1} + t_{r1}V_1 - T_2 v_{r2} - t_{r2}V_2. \quad (13b)$$

Equations (12) and (13) may be written for each intermediate span/roller present in the process line.

Similar procedure may be applied for equations (7f) and (7g) to obtain the condition for equilibrium and the equilibrium input as

$$v_{r2} = \frac{AE - t_{r3}}{AE - t_{r2}}v_{r3}, \quad u_{3,eq} = t_{r3}R_3 + \frac{J_3}{R_3}\dot{v}_{r3} + \frac{b_{f3}}{R_3}v_{r3} - \frac{\delta}{2\pi R_3}\left(\frac{J_3}{R_3^2} - 2\pi\rho w R_3^2\right)v_{r3}^2. \quad (14)$$

Further, the tension and velocity error dynamics of the winder section can be obtained as

$$L_3\dot{T}_3 = AE(V_3 - V_2) + T_2V_2 - T_3V_3 + T_2 v_{r2} + t_{r2}V_2 - T_3 v_{r3} - t_{r3}V_3, \quad (15a)$$

$$\frac{J_3}{R_3}\dot{V}_3 = -R_3T_3 + U_3 - \frac{b_{f3}}{R_3}V_3 + \underbrace{\frac{\delta}{2\pi R_3}\left(\frac{J_3}{R_3^2} - 2\pi\rho w R_3^2\right)(V_3^2 + 2v_{r3}V_3)}_{f_3(V_3)}. \quad (15b)$$

Equations for a general process line can be determined in the same way.

CONTROL DESIGNS

The control goal is to regulate web tension in each of the tension zones while maintaining the prescribed web transport velocity. A brief theoretical description of four advanced robust controllers is given in the following.

Decentralized State Feedback Controller

Design of a stable decentralized state feedback controller will be discussed for the example considered in the previous section. The process line shown in Figure 2 is decomposed into four subsystems: (1) Unwind subsystem, which consists of the unwind roll dynamics and the first web span tension dynamics; define state vector for the unwind subsystem as $x_0^T(t) = [T_0(t), V_0(t)]$. (2) Master speed subsystem, which consists just the web velocity error dynamics at the master speed roller; define $x_1(t) = V_1(t)$. (3) Process subsystem, which consists of the intermediate roller dynamics and the web tension dynamics for the span immediately upstream of the intermediate roller; define $x_2^T(t) = [T_2(t), V_2(t)]$. (4) Winder subsystem, which consists of the winder roll dynamics and the span tension dynamics adjacent to it; define $x_3^T(t) = [T_3(t), V_3(t)]$.

The decentralized state feedback control input for each motor is chosen as

$$U_0 = -K_0^T x_0 + f_0(V_0) \tag{16}$$

$$U_j = -K_j^T x_j, \; j = 2,3 \tag{17}$$

$$U_N = -K_3^T x_3 - f_3(V_3) \tag{18}$$

where $K_i, \; i = 0, \ldots, 3$ are feedback gain vectors. The dynamics of each subsystem under these decentralized control inputs can be simplified to

$$\dot{x}_i = \bar{A}_i x_i + \sum_{j=0, j\neq i}^{N} A_{ij} x_j \tag{19}$$

where A_i is the system matrix for subsystem i and $\bar{A}_i := A_i - B_i K_i^T$. The following statement gives the condition for stability of the closed-loop system: The equilibrium, $x_i = 0$, of the dynamics given by (19) is globally exponentially stable, if the feedback gains K_i are chosen such that

$$\min_{\omega \in R} \sigma_{min}(\bar{A}_i - j\omega I) > \sqrt{N(\xi_i^2 + \varepsilon_i)} > 0 \tag{20}$$

where $\xi_i^2 = \sum_{j=0, j\neq i}^{N} \eta_{ji}^2$, $\eta_{ij} = \sigma_{max}(A_{ij})$. The error convergence rate can be adjusted with a suitable choice of ε_i in the design.

In the following, an implementation procedure for computation of the feedback controller gains for each subsystem is given. Since the pair (A_i, B_i) is controllable, it is possible to adjust the eigenvalues of $\bar{A}_i$ arbitrarily by choosing feedback gains. The following strategy is proposed for choosing the gains systematically. First choose a stable matrix $\bar{A}_i$ for the i-th subsystem that satisfies the sufficient condition, and then compute the feedback controller gain vector K_i. For the master speed section ($i = 1$) choose $\bar{A}_1 = -C_{11}$. For the unwind ($i = 0$), process ($i = 2$), and winder ($i = 3$) choose the closed-loop matrices as $\bar{A}_i = \begin{bmatrix} -v_{ri}/L_i & -(AE - t_i)/L_i \\ C_{i1} & -C_{i2} \end{bmatrix}$, $i = 0, 2, 3$ where C_{i1} and C_{i2} are positive constants chosen such that the matrices $\bar{A}_i$ are asymptotically stable and satisfy the sufficient condition given by (20) and $L_0 = L_1$. Once $\bar{A}_i$ are obtained, the controller gain vectors are computed from the expression $\bar{A}_i = A_i - B_iK_i$. The gain for the master speed subsystem is $K_1 = (J_1/R_1)(-b_{f1}/J_1 - C_{11})$ and the gain vectors for other subsystems is given by $K_i^T = (J_i/R_i)\left[(R_i^2/J_i - C_{i1}), \quad (-b_{fi}/J_i - C_{i2})\right]$. Note that $\bar{A}_i$ is not time-varying because it is not a function of the time-varying parameters R_i and J_i. Hence, the choice of $\bar{A}_i$ is fixed and need not be changed with the change in the radius and inertia of the unwind or rewind roll. But the matrices $\bar{A}_i$ contain reference values v_{ri} and t_{ri}. Since the web processing line runs at predetermined set of reference values, for each pair of operating reference values, (t_{ri}, v_{ri}), one can obtain the corresponding $\bar{A}_i$. Since the quantity AE is much larger than t_{ri} for most webs, the sufficient condition as a function of v_{ri} is of value. It is verified that the sufficient condition is satisfied for all i and for all web speeds in the range of 50 fpm to 2000 fpm. The following values were used: $v_{r1} \in [50, 2000]$ fpm, $t_{ri} = 14.35$ lbf, $\varepsilon_i = 10$ for all i, $C_{01} = 120$, $C_{02} = 200$, $C_{11} = 4000$, $C_{21} = 1500$, $C_{22} = 400$, $C_{31} = 15$, and $C_{32} = 15$.

Model Reference Adaptive Controller

To design the model reference adaptive controller, the web process line is also decomposed into four subsystems: Unwind, Master Speed, Process, and Winder. The state space dynamics of each subsystem, $\mathbb{S}_i$, is described by

$$\mathbb{S}_i: \quad \dot{x}_i(t) = A_i x_i(t) + b_i U_i(t) + \sum_{j=0, j\neq i}^{N} A_{ij} x_j(t), \; i = 0, 1, \ldots, N. \tag{21}$$

It is assumed that b_i and A_{ij} are known. Also, each subsystem matrix, A_i is uncertain but it is assumed that there exist constant vectors k_i such that, for an asymptotically stable matrix A_{mi}, $(A_i - A_{mi}) = b_i k_i^\top$. The reference model for each individual subsystem, $\mathbb{S}_{mi}$, is described by the equations

$$\mathbb{S}_{mi}: \dot{x}_{mi}(t) = A_{mi} x_{mi}(t) + b_i r_i(t) - b_i k_{mi}^\top x_m + \sum_{j=0, j\neq i}^{N} A_{ij} x_{mj}(t). \tag{22}$$

where x_{mi} is the i-th subsystem reference state, k_{mi} is a gain matrix, and $x_m^\top(t) = [x_{m0}^\top, x_{m1}^\top, x_{m2}^\top, x_{m3}^\top]$ is the reference state of the overall system. With the structure for the reference model (22), the condition for existence of solution to the control problem can be specified in terms of the state matrices of the reference model, A_{mi}, as given by equation (25) later. The reason for including the term $b_i k_{mi}^\top x_m$ in (22) becomes clear when the reference model for the entire large-scale system is considered. It is given by

$$\mathbb{S}_m: \dot{x}_m(t) = A_m x_m(t) + Br(t) - BK_m^\top x_m \tag{23}$$

where $r^\top(t) = [r_0(t), r_1(t), r_2(t), r_e(t)]$ and $K_m = [k_{m0}, k_{m1}, k_{m2}, k_{m3}]$. The diagonal elements of the matrix A_m are given by $\text{diag}(A_m) = \text{diag}(A_{m0}, A_{m1}, A_{m2}, A_{m3})$. The off-diagonal elements are given by $A_m(i,j) = A_{i,j}$, $i \neq j$. Note that if A_m is not stable for given A_{mi}, then eigenvalues of $A_m - BK_m^\top$ can be placed at desired locations by choosing K_m. If A_m is asymptotically stable for given A_{mi}, then K_m can be simply chosen as the null matrix.

The goal is to design bounded decentralized control inputs $u_i(t)$ such that $x_i(t)$ are bounded and the error $e_i(t) = x_i(t) - x_{mi}(t)$ converges to zero, that is, $\lim_{t\to\infty} e_i(t) = 0$ for all $i \in \{0, 1, 2, 3\}$. The following statement gives the stability of the closed-loop system.

Given the large scale system (21) and the reference model (22), there exists a positive definite matrix $P_i = P_i^\top$ such that the decentralized control law and the parameter update law given by

$$u_i(t) = r_i(t) - k_{mi}^\top x_m(t) - \hat{k}_i^\top x_i(t) \tag{24a}$$

$$\dot{\hat{k}}_i(t) = -(e_i^\top(t) P_i b_i) x_i(t) \tag{24b}$$

where $\hat{k}_i$ is estimate of k_i, render the closed-loop system asymptotically stable if

$$\delta_s(A_{mi}) > \sqrt{N}\xi_i. \tag{25}$$

The matrices A_{mi} are chosen for each section as follows. Unwind section: $A_{m0} = \begin{bmatrix} -v_{r0}/L_1 & (AE - t_0)/L_1 \\ C_{01} & -C_{02} \end{bmatrix}$ where $C_{01} = 120$ and $C_{02} = 2000$. Master speed section: $A_{m1} = C_{12} = 4000$. Process and rewind sections: $A_{mi} = \begin{bmatrix} -v_{ri}/L_i & (AE - t_{ri})/L_i \\ -C_{i1} & -C_{i2} \end{bmatrix}$ where $i = 2, 3$ and $C_{21} = 1500$, $C_{22} = 400$, $C_{31} = 15$, $C_{32} = 15$. The condition given in (25) is satisfied for the given matrices A_{mi}.

Multivariable H_∞ Robust Controller Synthesis

Robust H_∞ control is a powerful tool to synthesize multivariable controllers with interesting properties of robustness and disturbance rejection. The H_∞ control theory deals with the minimization of the H_∞-norm of the transfer matrix (26) from an exogenous input to a pertinent controlled output of a given plant. The synthesis of the multivariable controller is done using the nominal model; this model should correspond to the starting phase for having an optimal controller in this phase.

The system under study has three motors (cf. Fig. 3) and exhibits the inherent problems of elastic web transport systems. The inputs are the control signals u_u, u_v, u_w which are the motor torques. The outputs are the unwinding web tension T_u, traction motor's linear velocity V, and winding web tension T_w.

Due to the wide-range of variation of the roller radius during the winding process, the dynamic behavior of the system is considerably modified with time. To analyze this modified behavior, the unwinder and winder will be considered separately. With quasi-static assumption on radius variations, the static gains between the control signals and web tensions appear to be proportional to the inverse of the radius [29]. With this assumption, the control signals are multiplied by the corresponding radius measurement or estimation to get compensation for varying radius. The synthesis of the controller is done using the plant which includes the radii multiplication (gain scheduling). This approach allows to reduce web tension variations significantly despite velocity changes during processing [29].

The H_∞ controller can be synthesized using the mixed sensitivity approach, as shown in Fig. 4, where w are the exogenous inputs (like tension and velocity references: T_{uref}, V_{ref}, T_{wref}) and z are the controlled signals. The frequency weighting functions W_p, W_u and W_t appear in the closed-loop transfer function matrix as follows:

$$T_{wz} := \begin{bmatrix} W_p S & W_u K S & W_t T \end{bmatrix}^\top \tag{26}$$

where $S = (I + GK)^{-1}$ is the sensitivity function and $T = I - S$ is the complimentary sensitivity function. The controller K can be computed using the Linear Matrix Inequalities (LMI) or Riccati equations, which can be solved via γ-iteration algorithm. Several commercial softwares exist for computing the controller using the two approaches.

The controller stabilizes the system such that the H_∞-norm of the transfer function between w and z is

$$\|T_{zw}\|_\infty := \sup \sigma_{max}(T_{zw}(j\omega)) \leq \gamma \tag{27}$$

with γ close to γ_{min} (the smallest value of γ). σ_{max} denotes the maximum singular value.

The performance and robustness of the controller depends on the choice of the weighting functions. The frequency weighting function W_p is usually selected with a high gain at low frequency to reject low frequency perturbations and reduce steady-state error. The form of W_p is given by $W_p(s) = ((1/M)s + w_B)/(s + w_B\varepsilon_0)$ where M is the maximum peak magnitude of S, $\|S\|_\infty \leq M$, w_B is the required bandwidth frequency and ε_0 is the allowable steady-state error. The weighting function W_u is used to avoid large control signals and the weighting function W_t increases the roll-off at high frequencies. For example, in [29] the selected weighting functions applied on a 3-motor plant are chosen as $W_u = \mathrm{diag}(0.1, 0.1, 0.1)$, $W_t(s) = \mathrm{diag}(s,s,s)$, and $W_p(s) = \mathrm{diag}((0.7s+10)/(s+0.01), (0.7s+6)/(s+0.01), (0.7s+10)/(s+0.01))$. The poles in the weighting matrix W_p are chosen as almost integrators to avoid numerical problems.

So far, we have presented the basic methodology to synthesize robust control for small scale systems (that means with a low number of inputs and outputs). What do we mean by "robust"? The robustness reflects the ability of the system to maintain adequate performance and, in particular, stability of the closed-loop system when there are variations in the plant dynamics (for e.g., due to parameters variations) and discrepancy between the true plant model and the nominal model which is used for controller design. Robustness of a controlled system is therefore a fundamental requirement in designing any controller. When designing a control system via standard methods (PID, optimal control, etc), robustness is not taken into account directly, and is often checked afterwards. Robustness enhancement at the design stage is the primary motivation for research in the area of robust control, and specifically in H_∞approach. For example, parameter uncertainties and model multiplicative input and output uncertainties can be taken into account by using adequate weighting functions.

Robustness of the controlled system can be analyzed via μ-analysis. A Linear Fractional Representation (LFR) is useful for robustness analysis [31]. To consider parameter variations directly into the synthesis of the controller, different approaches exist, such as for example μ-synthesis and Linear Parameter Varying (LPV) control. The first method enables to obtain a Linear Time Invariant (LTI) controller whereas the second one is more efficient when parameters vary widely.

Multi–model control

In this approach, the sum of four output weighted controllers $K(\Pi)$ (see Fig. 5) are used. The controllers $K(\Pi_i), i \in \{1, \ldots, 4\}$, are computed separately for four H_∞-like problems corresponding to the corners of the radii variation box (Fig. 5). Other control synthesis strategies (e.g. PID) can be used on each corner. The resulting controller $K(\theta)$, with $\theta^\top = [R_u(t), R_w(t)]$, is given by the following relation:

$$K(\theta) = \alpha_1 K(\Pi_1) + \alpha_2 K(\Pi_2) + \alpha_3 K(\Pi_3) + \alpha_4 K(\Pi_4) \tag{28}$$

where α_i are functions of $R_u(t)$ and $R_w(t)$.

LPV control

The control can be enhanced by using LPV controllers. Consider the plant, $P(\theta)$, a parameter-dependent system with time-varying vector parameter $\theta^\top = [R_u(t), R_w(t)]$. A parameter-dependent controller is sought with the same vertex property:

$$K(\theta) = \alpha_1 K(\Pi_1) + \alpha_2 K(\Pi_2) + \alpha_3 K(\Pi_3) + \alpha_4 K(\Pi_4) \tag{29}$$

where $K(\theta)$ is now equivalent to $K(\Pi_i) = \begin{bmatrix} A_{Ki} & B_{Ki} \\ C_{Ki} & D_{Ki} \end{bmatrix}$. The resulting controller weights all the state space matrices (and not just the output matrix C as in the case of the multi-model approach). The resulting control strategy has the following properties: the closed-loop system is stable for all admissible trajectories $\theta(t)$ and the L_2-induced gain of the closed-loop is bounded by γ. This synthesis problem can be reduced to solving 3×4 LMI's. An application of this for web handling is proposed in [30]

Semi-decentralized control strategy

In industrial processes, which contain a large number of actuators, it may be inconvenient to use a global multivariable controller. An alternative solution is to use semi-decentralized controllers, which reduce the controller dimensions [27]. For example, in the case of a 9-motor plant, the system is decomposed into three subsystems: each subsystem contains three motors and is controlled independently by its own controller. This choice results from a trade-off between the number of subsystems and their respective size.

Overlapping H_∞Controller

To reduce the coupling between two consecutive subsystems, it may be worthwhile to introduce overlapping [28], i.e., two consecutive controllers can share some inputs and outputs. For instance, control signals of tractors located at the boundary of the two subsystems results from two controllers with output weighting a and b (Fig. 7). Such a decentralized overlapping control strategy has given good results in the case of a vehicle platoon. To illustrate the overlapping principle, decompose a system S into two overlapped subsystems S_1 and S_2 shown by dashed lines in the matrices A,B,C:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & \vdots & A_{13} \\ & \cdots & \cdots & \cdots \\ A_{21} & \vdots & A_{22} & \vdots & A_{23} \\ \cdots & \cdots & \cdots & \\ A_{31} & \vdots & A_{32} & A_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} B_{11} & 0 & \vdots & 0 \\ & \cdots & \cdots & \cdots \\ 0 & \vdots & B_{22} & \vdots & 0 \\ \cdots & \cdots & \cdots & \\ 0 & \vdots & 0 & B_{33} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix},$$

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} C_{11} & 0 & \vdots & 0 \\ & \cdots & \cdots & \cdots \\ 0 & \vdots & C_{22} & \vdots & 0 \\ \cdots & \cdots & \cdots & \\ 0 & \vdots & 0 & C_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}. \tag{30}$$

In the expanded space S_1 and S_2 are disjoint. For subsystems, multivariable H_∞controllers called C_1 and C_2 are synthesized. Each controller C_i is composed of matrices F^i, G^i, H^i, K^i in the state-space representation. The global controller, composed diagonally of controllers C_1 and C_2, is not contractible from expanded space to the initial space. The state space model of the global controller is then rearranged in order to have the contractibility property. This controller is then contracted into the initial space, leading to an implementable controller:

$$\begin{bmatrix}\dot{Z}_1\\ \dot{Z}_2\end{bmatrix}=\begin{bmatrix}F^1 & \vdots & 0\\ \cdots & \cdots & \cdots\\ 0 & \vdots & F^2\end{bmatrix}\begin{bmatrix}Z_1\\ Z_2\end{bmatrix}+\begin{bmatrix}G_1^1 & G_2^1 & \vdots & 0\\ \cdots & \cdots & \cdots & \cdots\\ 0 & \vdots & G_1^2 & G_2^2\end{bmatrix}\begin{bmatrix}y_1\\ y_2\\ y_3\end{bmatrix},$$

$$\begin{bmatrix}u_1\\ u_2\\ u_3\end{bmatrix}=\begin{bmatrix}H_1^1 & \vdots & 0\\ & & \cdots\\ H_2^1 & \vdots & H_1^2\\ \cdots & & \\ 0 & \vdots & H_2^2\end{bmatrix}\begin{bmatrix}Z_1\\ Z_2\end{bmatrix}+\begin{bmatrix}K_{11}^1 & & K_{12}^1 & \vdots & 0\\ & \cdots & \cdots & \cdots & \cdots\\ K_{21}^1 & \vdots & (K_{22}^1+K_{11}^2)/2 & \vdots & K_{12}^2\\ \cdots & \cdots & \cdots & \cdots & \\ 0 & \vdots & K_{21}^2 & & K_{22}^2\end{bmatrix}\begin{bmatrix}y_1\\ y_2\\ y_3\end{bmatrix}. \quad (31)$$

Note that the relation (31) means that the control signal for the overlapped actuator results neither from the average nor from the sum of the outputs of the two controllers. However, the sum obtained with $a = b = 1$ in Fig. 7, is a good approximation of the overlapped control strategy.

EXPERIMENTS

In this section, the two experimental platforms are briefly described followed by a presentation of a representative sample of the experimental results.

Figure 8 shows a sketch of the experimental platform and the web path for conducting experiments using the decentralized state feedback controller, model reference adaptive controller and an industrial PI controller. The line mimics most of the features of an industrial web process line. It is developed with the aim of creating an open-architecture design that allows for modifying the line to conform to research experimentation. The line contains a number of different stations, as shown in Fig. 8, and a number of driven rollers. For conducting the experiments for this work, the web is threaded through four driven rollers M_0 to M_3 as shown, and through many other idle rollers throughout the line to facilitate transport of the web from the unwind to rewind. The nip rollers (denoted by NR), which are pneumatically driven, are used to maintain contact of the web with the driven rollers. The two controlled lateral guides (guides are denoted by DG and the web edge sensors by E), near unwind and rewind sections, respectively, are used to maintain the lateral position of the web on the rollers during web transport.

The web material used in the experiments is Tyvec, which is a product made by Dupont. The product of the elasticity of the web material and its cross-sectional area, EA, is equal to 2000 lbf. For both the master speed and process section driven rollers, the values of the radii and inertia are 0.3 ft and 2 $lb - ft^2$, respectively. For inertia compensation and equilibrium control of unwind and rewind rolls, instantaneous radius of each roll is calculated using the encoder signal. Effective coefficient of friction b_{fi} for all $i = 0, 1, 2, 3$ is taken as 0.5 lbf-ft-sec/rad. The lengths of the three different web spans are $L_1 = 20\ ft$, $L_2 = 33\ ft$ and $L_3 = 67\ ft$.

A schematic of the second experimental platform, which has three motors, is represented on Fig. 3. The inputs to the system are the torque reference signals (u_u, u_v, u_w) of the brushless motors; its measurements are the web tensions T_u and T_w with load cells and the web velocity V. The plant parameters are given in the following: Nominal torques of unwinding/winding motor and traction motor are 6.8 N-m and 2.6 N-m, respectively.

Nominal velocity of all three motors is 3000 rpm. The total web length between the unwinder and winder is 1.9 m. The web width, web thickness, and diameter of the full roll are given by 0.1 m, 0.275 mm, and 0.2 m, respectively. Nominal values of web tension and velocity are 1.5 kg and 100 m/min, respectively. The Young's modulus of the web material is 0.16×10^9 N/m^2.

Extensive experiments with the four motor experimental platform (see Fig. 8) at different web transport speeds were conducted with the currently used industrial decentralized PI controller, decentralized state feedback controller, and adaptive controller. Sampling time for control scanning loop and data acquisition was chosen to be 5 milliseconds. Experimental results for the following case are shown in this paper: Reference velocity $v_{r1} = 1000$ ft/min; $t_{r1} = 24.6$ lbf, $t_{r2} = 20.5$ lbf, and $t_{r3} = 16.4$ lbf; the roll diameter varies from 18 to 13 inches. A schematic of the implementation strategy for the decentralized state feedback controller is shown in Figure 9.

In each of the Figs. 14, 15, and 16, the top plot shows the variation in master speed velocity from its reference (V_1) and the remaining three plots show the tension variations in the three tension zones (T_1, T_2, T_3). Figure 14 shows the results using a well tuned decentralized PI controller (block diagram shown in Fig. 10). Figure 15 shows the results using the decentralized state feedback controller controller. Figure 16 shows results for the adaptive controller. Notice that comparison of the results of the three controllers show that both state feedback and adaptive controllers show much improved regulation of line sped and tension in each of the zones over a well tuned PI controller.

Different synthesized controllers (multi-model H_∞control with and without gain scheduling, H_∞control) are compared on the three motor plant (cf. Fig. 3). The order of the resulting controller is 15. They were implemented in state space representation with a sampling period of 10 ms. The maximum web tension variation is reduced from 9% (with varying gain H_∞controller) to 5% (with multi-model, varying gain H_∞controller) for a step change in velocity. The results of the overlapping control technique are shown in Fig. 13.

DISCUSSIONS AND FUTURE DIRECTIONS

The H_∞ control design approach is relatively easy to use and provides robust performance. The control designer need not completely understand the numerical algorithms. Software is available in commercial packages for calculating multivariable H_∞ controllers, which is easy to use. Therefore, users need not be highly skilled in H_∞ theory to calculate a solution. This is a definite advantage for industrial practitioners.

Although it is not necessary to understand the theory for using commercial packages, it is recommended to try to understand the underlying theory with the help of formal courses. Practitioners should understand the controller design process that involves weighting functions. These functions have to be selected and adjusted to specify the reference tracking properties, the disturbances rejection and the robustness behavior. In a classical one degree of freedom (1-DOF) control synthesis scheme, disturbance rejection and tracking properties are interdependent. In order to consider these two issues separately, a 2-DOF control strategy has to be used. Typically, the two parts of such a controller are designed in two steps: disturbance rejection is optimized with a feedback term K_b and tracking specifications are improved with a feedforward part K_f. However, in the H_∞design of such a controller, these two parts can be computed in a single step. The two-step procedure has the advantage that the design of K_f is completely independent from the one of K_b. As a result, K_b can come from an existing industrial design (with PI controllers), whereas K_f can be synthesized by other modern approaches, for example, the H_∞approach. Several weighting schemes for 1-DOF and 2-DOF H_∞controllers have been described in the literature. The weighting functions can be located at either the input or the output side, and a model matching, with model M_0, can be included. The model M_0 represents the desired transfer function between references and system outputs (in our case: web tensions and velocity). Compared to a 1-DOF strategy, the order of the 2-DOF controller is increased by the order of M_0. Classically, M_0 is a second order Butterworth filter in order to ensure a step response with a smooth start and no overshoot.

The H_∞synthesis problem has well defined stability and robustness properties: for given classes of uncertainties, robustness margins can be guaranteed. The calculated controller is represented with full state space matrices. This controller is then discretized for real-time implementation. The disadvantage of the H_∞approach is that the obtained controller has high-order (equal to the plant model order plus the order of the weighting filters). However, H_∞controller can be, a posteriori, model reduced.

New approaches are under study in H_∞controller synthesis. Most of them consist of calculating a controller with predefined structure and order. For example, one has to optimize the PID parameters of a given decentralized structure in order to minimize the H_∞gain of the transfer function. This problem has been known to be difficult to solve because it requires finding a solution to a bilinear matrix inequality (BMI), which is a non convex problem. For solving such inequalities, commercial software is emerging. Another possible approach that has been suggested is the use of genetic algorithms for solving the BMI problem.

For the model reference control strategy, it was assumed that the modulus of elasticity, area of cross-section and thickness are known. In many cases, only estimates of these quantities are known. An adaptive design that estimates these quantities in real-time is desirable, and would make a good topic for future study. The developments in the paper also rely on the fact that the web is elastic and is composed of a single material. The web materials used in many processes are viscoelastic, and efforts in the literature to model and control viscoelastic webs has been minimal. Application of the adaptive control designs for viscoelastic and composite webs must be investigated in the future.

Acknowledgements

The work of the first author was supported in part by the National Science Foundation under grant no. CMS 9982071. The first author also acknowledges equipment support from the Web Handling Research Center for conducting experiments. Thanks also go to the graduate students, in particular Nilesh Siraskar and Ram Dwivedula, for their help in the development of this paper. The second author thanks the French Ministry of Research for financial support through the project "Winding and high velocity handling of flexible webs" (ERT, contract number 01 B 0395).

REFERENCES

[1] N. Haglov, "Web tension, roll stands and reel changing," in *PATRA News Paper and Rotary Letter Press Conference*, pp. 65–114, 1957.

[2] D. Campbell, *Dynamic Behavior of the Production Process, Process Dynamics*. John Wiley and Sons, Inc., New York, 1958.

[3] K. Grenfell, "Tension control on paper-making and converting machinery," in *Proceedings of the 9th IEEE Annual Conference on Electrical Engineering in the Pulp and Paper Industry*, June 1963.

[4] D. King, "The mathematical model of a news paper press," *Newspaper Techniques*, pp. 3–7, December 1969. (Interim Report).

[5] G. Brandenburg, "The dynamics of elastic webs threading a system of rollers," *Newspaper Techniques*, pp. 12–25, September 1972.

[6] G. Brandenburg, "New mathematical models for web tension and register error," in *Proceedings of the 3rd International IFAC Conference on Instrumentation and Automation in the Paper, Rubber and Plastics Industry*, vol. 1, 1977.

[7] D. Whitworth, *Tension Variations in Pliable Material in Production Machinery*. PhD thesis, Loughborough University of Technology, 1979.

[8] D. Whitworth and M. Harrison, "Tension variations in pliable material in production machinery," *Journal of Applied Mathematical Modeling*, vol. 7, pp. 189–196, June 1983.

[9] J. J. Shelton, "Dynamics of web tension control with velocity or torque control," in *Proceedings of the American Control Conference*, pp. 1/5–5/5, 1986.

[10] K. Shin, *Distributed Control of Tension in Multi-Span Web Transport Systems*. PhD thesis, Oklahoma State University, Stillwater, May 1991.

[11] G. Young and K. Reid, "Lateral and longitudinal dynamic behavior and control of moving webs," *ASME Journal of Dynamic Systems, Measurement, and Control*, vol. 115, pp. 309–317, June 1993.

[12] K. Reid and K. Lin, "Control of longitudinal tension in multi-span web transport systems during start-up and shut-down," in *Proceedings of the Second International Conference on web handling* (J. Good, ed.), pp. 77–95, June 1993.

[13] W. Wolfermann and D. Schroeder, "New decentralized control in processing machines with continuous moving web," in *Proceedings of the Second International Conference on web handling* (J. Good, ed.), pp. 96–117, June 1993.

[14] K. Shin, K. Reid, and S. Kwon, "Non-interacting tension control in a multi-span web transport system," in *Proceedings of the Third International Conference on Web Handling* (J. Good, ed.), pp. 312–326, 1995.

[15] W. Wolfermann, "Tension control of webs - a review of the problems and solutions in the present and future," in *Proceedings of the Third International Conference on Web Handling* (J. Good, ed.), pp. 198–229, June 1995.

[16] B. T. Boulter, "The effect of speed loop bandwidths and line-speed on system natural frequencies in multi-span strip processing systems," in *Proceedings of the IEEE Industry Applications Conference*, pp. 2157–2164, 1997.

[17] K. Reid and K. Lin, "Dynamic behavior of dancer subsystems in web transport systems," in *Proceedings of the Second International Conference on web handling* (J. Good, ed.), pp. 135–146, June 1993.

[18] N. A. Ebler, R. Arnason, G. Michaelis, and N. D. Sa, "Tension control: Dancer rolls or load cells," *IEEE Transactions on Industry Applications*, vol. 29, no. 4, pp. 727–739, 1993.

[19] P. Lin and M. Lan, "Effects of PID gains for controller design with dancer mechanism on web tension," in *Proceedings of the Second International Conference on Web Handling* (J. Good, ed.), pp. 66–76, June 1993.

[20] G. Rajala, "Active dancer control for web handling machine," Master's thesis, University of Wisconsin-Madison, August 1995.

[21] B. C. McDow and C. D. Rahn, "Adaptive web-tension control using a dancer arm," *Tappi Journal*, vol. 81, no. 10, pp. 197–205, 1998.

[22] J. J. Shelton, "Limitations to sensing web tension by means of roller reaction forces," in *Proceedings of the Fifth International Conference on Web Handling, Stillwater, Oklahoma*, June 1999.

[23] D. H. Carlson, "Considerations in the selection of a dancer or load cell based tension regulating strategy," in *Proceedings of the Sixth International Conference on Web Handling* (J. Good, ed.), pp. 243–262, June 2001.

[24] H. Koc, D. Knittel, M. de Mathelin, and G. Abba, "Modeling and h_∞ robust control for winding systems," in *Proc. of the Sixth Intl. Conf. on Web Handling*, (Stillwater, OK), June 2001.

[25] E. Laroche, D. Knittel, H. Koc, and M. de Mathelin, "Web winding system robustness analysis via μ-analysis," in *Proc. of the IEEE Conf. on Control Applications*, (Mexico City, Mexico), September 2001.

[26] D. Knittel, D. Gigan, E. Laroche, and M. de Mathelin, "Decentralized H_∞ control of large scale web transport systems," in *15th Triennial World Congress of the International Federation of Automatic Control(IFAC)*, 2002.

[27] D. Knittel, E. Laroche, D. Gigan, and H. Koc, "Tension control for winding systems with two degrees of freedom h_∞ controller," *IEEE Transactions on Control Systems Technology*, vol. 10, no. 2, pp. 197–208, 2002.

[28] D. Knittel, D. Gigan, and E. Laroche, "Robust decentralized overlapping control of large scale winding systems," in *Proceedings of the American Control Conference*, pp. 1805–1810, May 2002.

[29] H. Koc, D. Knittel, M. de Mathelin, and G. Abba, "Modeling and robust control of winding systems for elastic webs," *IEEE Transactions on Control Systems Technology*, vol. 10, no. 2, pp. 197–208, 2002.

[30] Y. Xu, D. Knittel, and M. de Mathelin, "Two-degrees-of-freedom gain scheduled control for quasi-periodic disturbance rejection: application in the unwinding of an eccentric roll," in *IFAC Symposium on Robust Control Design (ROCOND'03)*, 2003.

[31] E. Laroche and D. Knittel, "An improved linear fractional model for robustness analysis of winding systems," *Control Engineering Practice*, vol. 13, pp. 659–666, 2005.

[32] A. Benlatreche, D. Knittel, and E. Ostertag, "State feedback controller synthesis using bmi optimization of large scale web handling systems," in *IFAC World Congress*, (Prague, Czech Republic), 2005.

[33] A. Benlatreche, D. Knittel, and E. Ostertag, "State feedback control with full or partial action for large scale winding systems," in *IEEE IAS Conference*, (Hongkong), 2005.

[34] P. R. Pagilla, E. O. King, L. Dreinhoefer, and S. Garimella, "Robust observer-based control of an aluminum strip processing line," *IEEE Transactions on Industry Applications*, vol. 36, no. 3, pp. 835–840, 2000.

[35] P. R. Pagilla, S. S. Garimella, L. H. Dreinhoefer, and E. O. King, "Dynamics and control of accumulators in continuous strip processing lines," *IEEE Transactions on Industry Applications*, vol. 37, no. 3, pp. 934–940, 2001.

[36] P. R. Pagilla, R. V. Dwivedula, Y. Zhu, and L. P. Perera, "The role of active dancers in tension control of webs," in *Proceedings of the Sixth International Conference on Web Handling, Stillwater, Oklahoma*, June 2001.

[37] P. R. Pagilla, R. V. Dwivedula, Y. Zhu, and L. P. Perera, "Periodic tension disturbance attenuation in web process lines using active dancers," *ASME Journal of Dynamic Systems, Measurement, and Control*, vol. 125, pp. 361–371, September 2003.

[38] P. R. Pagilla, R. V. Dwivedula, and Y. Zhu, "A comparative study on active and passive dancers used for attenuation of web tension disturbances," in *Proceedings of the Seventh International Conference on Web Handling, Stillwater, Oklahoma*, (Stillwater, OK), June 2003.

[39] P. R. Pagilla, I. P. Singh, and R. V. Dwivedula, "A study on control of accumulators in web processing lines," *ASME Journal of Dynamic Systems, Measurement, and Control*, vol. 126, pp. 453–461, September 2004.

[40] P. R. Pagilla, N. B. Siraskar, and R. V. Dwivedula, "Decentralized adaptive control of large-scale systems with applications to web processing lines," in *Proceedings of the 16th IFAC World Congress*, (Prague, Czech Republic), July 2005.

[41] R. V. Dwivedula, Y. Zhu, and P. R. Pagilla, "Characteristics of active and passive dancers: A comparative study," *Control Engineering Practice*, 2005. in press.

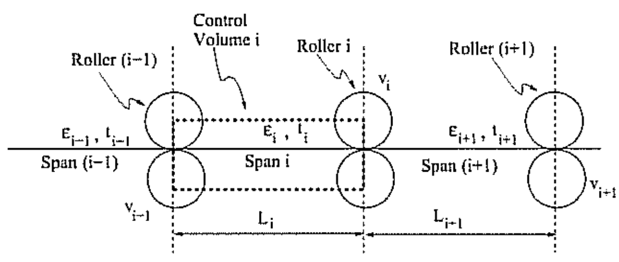

Figure 1: Nomenclature used in the paper

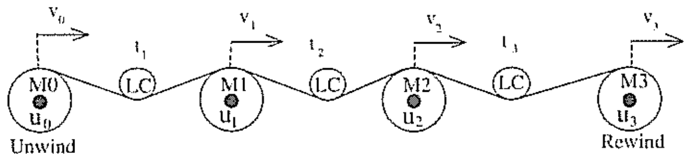

Figure 2: Schematic of a web process line

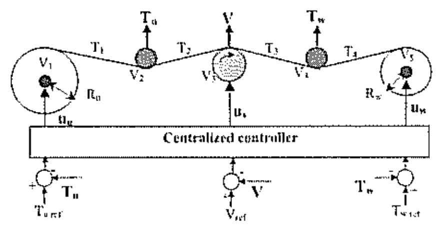

Figure 3: Centralized control of a 3-motor plant

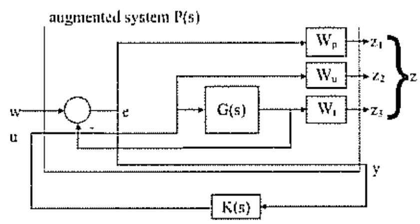

Figure 4: Mixed sensitivity method

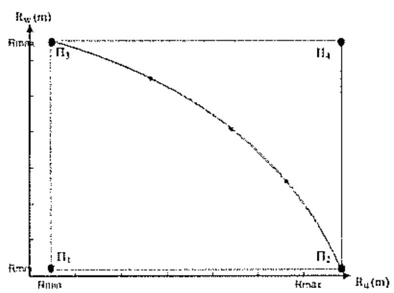

Figure 5: Radii trajectory

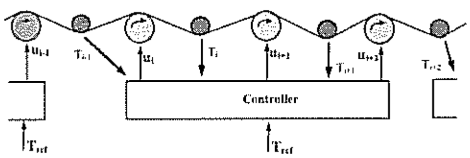

Figure 6: Semi-decentralized control strategy

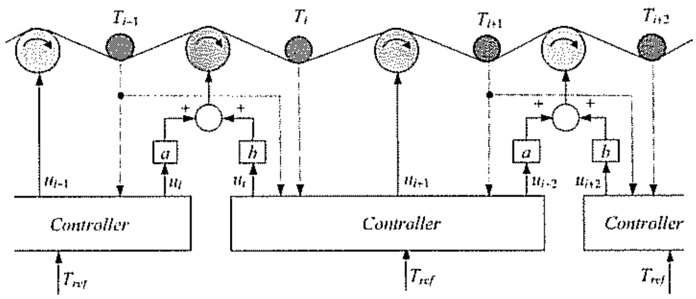

Figure 7: Semi-decentralized weighted overlapping control strategy

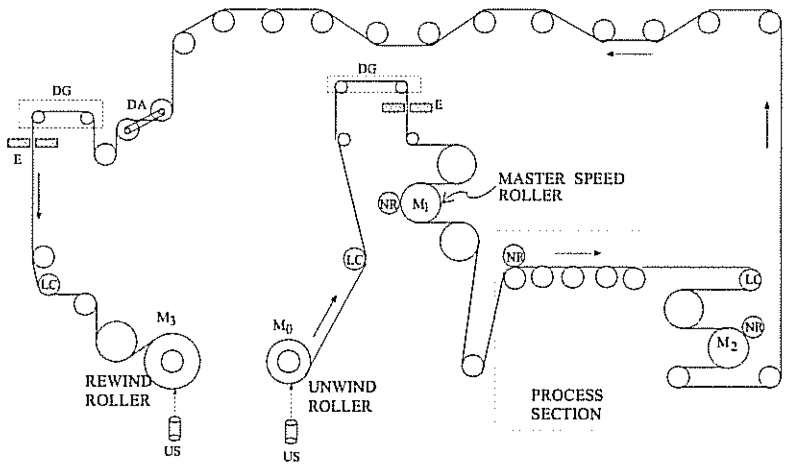

Figure 8: Sketch of the four motor experimental platform

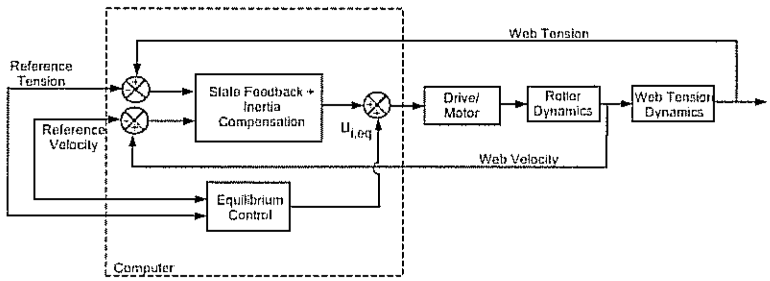

Figure 9: Decentralized state feedback control strategy

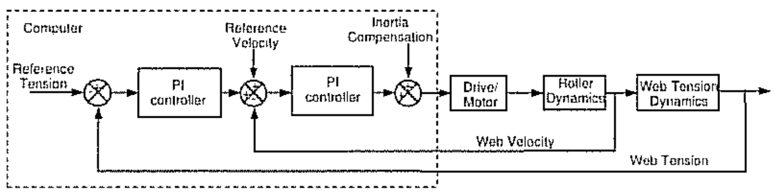

Figure 10: Decentralized control strategy with PI controller

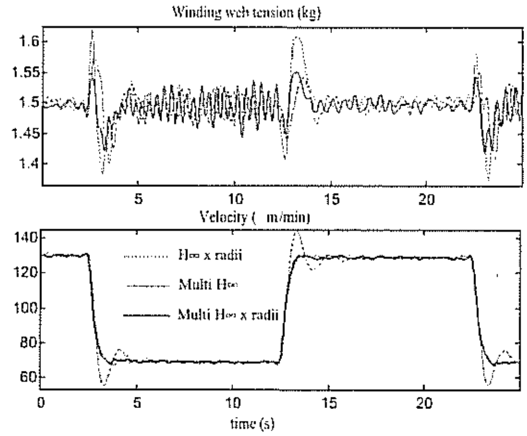

Figure 11: Comparison of multivariable controllers

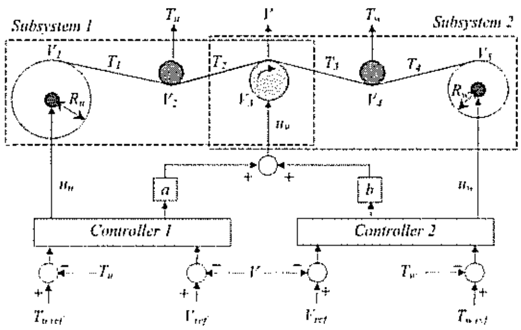

Figure 12: Semi-decentralized overlapped control of the 3-motor platform

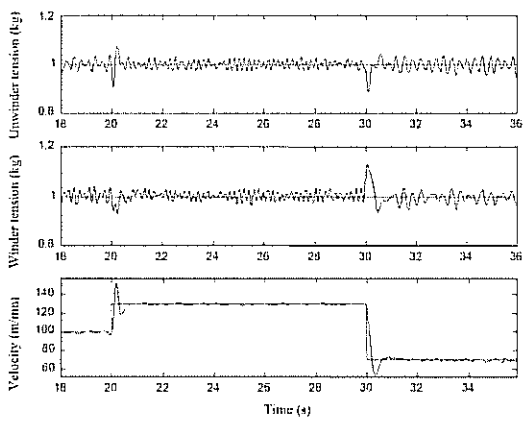

Figure 13: Semi-decentralized overlapped controllers

Figure 14: Decentralized PI controller

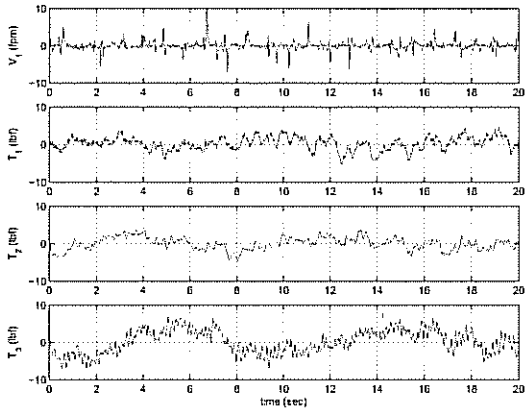

Figure 15: Decentralized state feedback controller

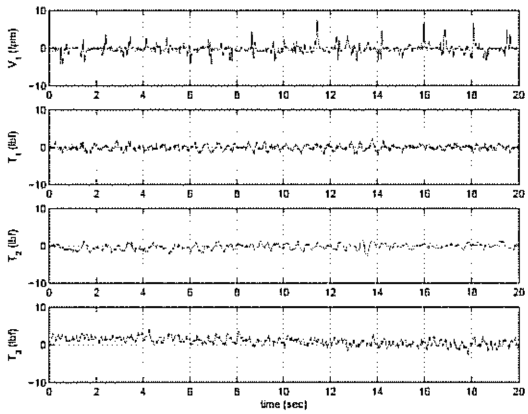

Figure 16: Decentralized adaptive controller