LATERAL MECHANICS OF BAGGY WEBS AT LOW TENSIONS

by

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ABSTRACT

Existing mathematical models describing lateral movement of webs such as paper or plastic film, assume that the entire width of the web is carrying tension or that the web is capable of supporting compressive stresses. For many webs this is not true. Thus a mathematical theory describing lateral movement of baggy webs which do not support compressive stresses have been derived. It is shown that the mechanics of these webs are described by a nonlinear fourth order differential equation for which a numerical solution has been developed. Results show that the lateral deflection of baggy webs is significantly affected by tension at the lower levels of tension.

NOMENCLATURE

E	MD modulus [Pa]
G	shear modulus [Pa]
h	web thickness [m]
K	web coefficient [m^{-1}]
M	bending moment [Nm]
N	shear force [N]
n	shear coefficient
q	lateral load [N/m]
T	axial force [N]
u	lateral deflection [m]
W	web width [m]
x	axial (longitudinal) position [m]
y	widthwise position [m]
α	coefficient
γ	shear angle
ε	strain
κ	curvature [m^{-1}]
σ	stress [Pa]

subscripts

0	upstream roller
b	bending
c	centroid
e	effective
i	initial, frozen-in
s	shear

INTRODUCTION

A web is a thin, continuous, flexible strip of material like paper, metal foil, plastic film or textiles. In production and converting processes the web is transported through a web line such as a coater, annealing line, paper machine or printing press. In many of these processes it is important to have strict control of lateral position. In a four color printing unit, the different colors will not match on top of each other if lateral movement is out of control.

A web moving through a web line is influenced by forces and bending moments transferred from rollers in contact with the web. As can be described by beam theory, the web moves sideways, or laterally, if a bending moment is applied to it [1]. However, it has also been observed that a web may shift sideways even if no bending moment is applied. It is believed that this sideways shift is caused by widthwise variations in material properties. A web with no variations in material properties is often referred to as a perfect web, whereas a web with widthwise variations in material properties is referred to as an imperfect web. These variations may be variations in stiffness or in frozen-in strain. The latter is the cause of baggy webs.

Sideways deflection due to widthwise variations in material properties can not be explained by elementary beam theory. A more general beam theory was thus derived by the author [2][3]. This theory shows that widthwise variations in stiffness or frozen-in strains yield an internal moment in the web that causes it to shift sideways. The theory also shows that the sideways shift increases with increasing web line tension. This is somewhat contradictory to empirical observations. It is believed that this contradiction is due to the assumption of the theory that the entire width of the web is carrying tension or that parts of the web may have negative tension. Observations show that the bagginess is not always stretched out. The baggy parts tend to wrinkle or deflect out of the plane and do not support compressive stresses. Due to this the theory for imperfect or baggy webs have been generalized, so that it is applicable for webs that do not necessarily carry tension across its entire width.

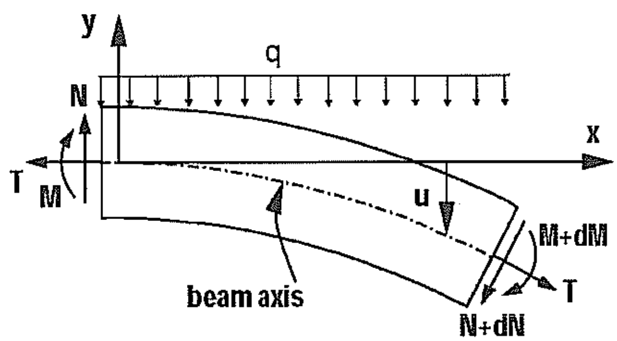

Figure 1 - Deflected beam with applied forces.

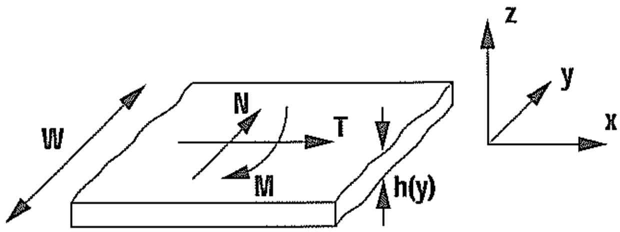

Figure 2 - Web with beam forces.

THEORY

For a perfect or uniform web the stiffness will be constant across the width of the web and no frozen-in strains will be present. Generally this is not the case, and the stiffness may vary with widthwise position. Stiffness is the product of web thickness h and elastic modulus in the machine direction E. Frozen-in strains ε_i are the strains that are present in the absence of stresses. When they vary across the width of the web, they cause parts of the web to be longer than other parts. This is also known as bagginess. Sometimes the axial force T will stretch out the bagginess so that all parts of the web will carry tension. Other times the axial force or tension is too small and parts of the web will remain baggy. In order to describe the latter case, further generalization of the existing theory [2][3] have been made.

In many web line applications the web is in a situation where stresses and strain are below the yielding point. Most webs also show a linear material response. Thus we have apply Hookes law and get

$$\sigma(y) = E\left\{\varepsilon(y) - \varepsilon_i(y)\right\} \tag{1}$$

Most textbooks neglect the frozen-in strains, but we need to account for them since they are the mathematical representation of the bagginess which we wish to study. Most webs can only carry insignificantly small compressive loads in MD and CD. For these webs we assume that the stress is always tensile. Thus we need to modify the expression for stress into the following:

$$\sigma(y) = \begin{cases} E\left\{\varepsilon(y) - \varepsilon_i(y)\right\} & \sigma > 0 \\ 0 & \text{else} \end{cases} \tag{2}$$

This modification is the most significant generalization in this model compared to the previous models[2][3]. Since we wish to focus on the effect of bagginess we have chosen to neglect any widthwise variation in stiffness.

For a web with linear widthwise variations in frozen-in strain, we may describe the frozen-in strain by a frozen-in curvature κ_{web}. The curvature is the inverse of the radius of curvature $\rho = 1/\kappa$. In addition to the frozen-in curvature, the strain of the web will also be influenced by the curvature due to bending κ_b. This is expressed by

$$\varepsilon(x,y) = \varepsilon_c + \left\{\kappa_b(x) + \kappa_{web}\right\}\left\{y - y_c\right\} \tag{3}$$

where y_c is the position of the centroid. Another generalization of the model is the fact that for a web where parts of it do not carry tension, the centroid is not necessarily positioned at the center of the web.The total curvature is defined by

$$\kappa \equiv -\frac{\partial^2 u}{\partial x^2} \tag{4}$$

where u is the deflection of the web. Total curvature gets contributions from bending κ_b, shear κ_s and bagginess κ_{web}:

$$\kappa = \kappa_b + \kappa_s + \kappa_{web} \tag{5}$$

From the definition of the shear coefficient [3] we have

$$\kappa_s = -\frac{nT}{GhW_e}\kappa \tag{6}$$

where n is the shear coefficient, G is the shear modulus and W_e is the effective web width. We chose to define a shear factor

$$\alpha_s = 1 + \frac{nT}{GhW_e} \tag{7}$$

For webs with thickness h the axial force T and the bending moment about the z-axis M are given by

$$T = \int_A \sigma(y)\, dA = \int \sigma(y)\, h\, dy \tag{8}$$

and

$$M = \int_A \sigma(y) y\, dA = \int \sigma(y)\, y\, h\, dy \tag{9}$$

The integrals are carried out over the width of the web. Inserting Eqs.(2) and (3) in Eqs.(8) and (9) yields

$$T = \varepsilon_c E h W_e \tag{10}$$

$$M = \frac{EhW_e^3}{12}\kappa_b \tag{11}$$

where the effective web width W_e is the load carrying part of the total web width. Mathematically it is given by

$$W_e = \begin{cases} \sqrt{\frac{2T}{Eh|\kappa_b|}} & W_e < W \\ W & \text{else} \end{cases} \tag{12}$$

Since bending varies with axial position (i.e. x-coordinate), the effective web width may also vary with axial position.

Equilibrium of forces and moments give rise to the following equation[1][2]

$$\frac{\partial^2 M}{\partial x^2} - T\kappa = -q \tag{13}$$

where q is lateral load. The second derivative of the moment, Eq.(11), is

$$\frac{\partial^2 M}{\partial x^2} = \frac{\alpha_s EhW_e^3}{12}\frac{\partial^2 \kappa}{\partial x^2} + \left(\frac{3\alpha_s}{4\kappa_b}\left(\frac{\partial \kappa}{\partial x}\right)^2 - \frac{3}{2}\alpha_s\frac{\partial^2 \kappa}{\partial x^2}\right)\frac{EhW_e^3}{12} \qquad \text{if } W_e < W$$
$$\frac{\partial^2 M}{\partial x^2} = \frac{EhW_e^3}{12}\alpha_s\frac{\partial^2 \kappa}{\partial x^2} \qquad \text{else} \tag{14}$$

Eqs.(4),(13),(12) and (14) combined, give us the following differential equation for the lateral mechanics of an imperfect web:

$$\frac{\partial^2 \kappa}{\partial x^2} - \frac{3}{2}\frac{\alpha_s}{\kappa_b}\left[\frac{\partial \kappa}{\partial x}\right]^2 = -\frac{12}{\alpha_s}\sqrt{\frac{Eh}{2T}}\left(\kappa + \frac{q}{T}\right)|\kappa_b|^{3/2} \quad \text{if } W_e < W$$
$$\frac{\partial^2 \kappa}{\partial x^2} - K^2\kappa = -\frac{K^2}{T}q \qquad \text{else} \tag{15}$$

where κ_b is given by Eqs.(5)-(7) as $\kappa_b = \alpha_s\kappa - \kappa_{web}$ and

$$K = \sqrt{\frac{T}{\alpha_s EhW^3/12}} \tag{16}$$

This is a nonlinear second order differential equation. At higher tensions all parts of the web width will carry tension and $W_e = W$ for the entire length of the web. Then we only use the lower part of Eq.(15). By substituting κ with Eq.(4) it can be shown that the lateral deflection u is given by a nonlinear fourth order differential. Since this differential equation is extremely difficult to solve, we chose to work with Eq.(15) and integrate its solution to obtain a solution for lateral displacement. For that we need four boundary conditions.

For a web moving between two rollers in a web line with a distance L apart, we define the coordinate system such that the web leaves the upstream roller with no deflection and an angle of deflection given by the shear angle. This is expressed by

$$u = 0 \qquad \text{at } x = 0 \tag{17}$$

and

$$\frac{\partial u}{\partial x} = \gamma_o \qquad \text{at } x = 0 \tag{18}$$

where γ_o is the shear angle at the upstream roller. The shear angle is proportional to the shear force

$$N = -\frac{T}{K^2}\frac{\partial^3 u}{\partial x^3} \tag{19}$$

At the downstream roller we assume traction between the web and the roller. Thus the surface particles of the web must move in the same direction as the surface particles of the roller. This results is two boundary conditions[1][2]:

$$u' = \theta \qquad \text{at } x = L \tag{20}$$

and

$$u'' = -\kappa_{web} \qquad \text{at } x = L \tag{21}$$

Here θ is the angle of misalignment between the rollers and κ_{web} is the frozen-in curvature of the web.

The differential equation, Eq.(15), together with the definition of curvature, Eq.(4), and the four boundary conditions, Eqs.(17)-(21), is a well posed mathematical description of the lateral mechanics of a web moving between to rollers a distance L apart. Due to the nonlinear nature of the differential equation, no analytical solution have been established.

NUMERICAL SOLUTION

Lateral mechanics for webs at higher tensions is normally analyzed by solving a fourth order differential equation for lateral displacement and four boundary conditions. With two boundary conditions at each boundary, a central difference method is applied. At lower tensions parts of the web do not carry tension and the describing differential equation become highly nonlinear. A central difference technique does not yield a practical method for solving the problem. Instead we use a backward difference method.

By dividing the span length between the rollers into $N-1$ segments, each with an equal length dx, and making a local numerical approximation of the differential equation for the curvature of the web, Eq.(15), we discretisize the mathematical problem into N solvable equations. The differential equation is numerically approximated by the following equations

$$\kappa_i - 2\kappa_{i+1} + \kappa_{i+2} - \frac{3\alpha_s}{2}\frac{(\kappa_{i+1}-\kappa_i)^2}{\alpha_s\kappa_i - \kappa_{web}} + \frac{12}{\alpha_s}\sqrt{\frac{Eh}{2T}}\Delta x^2 \left|\alpha_s\kappa_i - \kappa_{web}\right|^{3/2}\left(\kappa_i + \frac{q}{T}\right) = 0 \tag{22}$$

for positions along the web where parts of the width are not carrying tension ($W_e < W$), and

$$\kappa_i - 2\kappa_{i+1} + \kappa_{i+2} - \Delta x^2 K^2 \left(\kappa_i - \frac{q}{T}\right) = 0 \tag{23}$$

for positions along the web where the entire width is carrying tension ($W_e = W$). Here κ_{i+1} and κ_{i+2} are the assumed known curvatures at the two neighboring nodes downstream. The equations are solved from the downstream roller and backwards to the upstream roller. Two boundary conditions at the downstream roller is needed. From Eq.(21) we have

$$\kappa_N = \kappa_{web} \tag{24}$$

Eq.(20) can not be applied to Eqs.(22) and (23). Instead we make a qualified guess on the derivative of the curvature on the downstream roller

$$\kappa_N - \kappa_{N-1} = dx\dot{\kappa}_L^{guess} \tag{25}$$

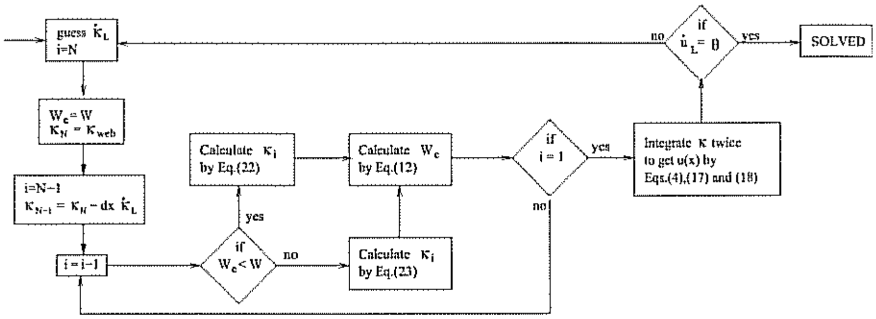

Figure 3 - Flow chart of numerical method.

The solution for the lateral displacement is found by integrating the solution of the curvature by Eq.(4) and the boundary conditions Eqs.(17) and (18). We then check if the third boundary condition, Eq.(20), is matched. If not, we adjust the guess on the derivative of the downstream curvature $\dot{\kappa}_L^{guess}$ until the third boundary condition is matched. The numerical method is described by the flow chart in Fig.3

RESULTS

Lateral behavior of a web of PET film have been studied. The web has properties as given in Table 1. The properties are comparable to the PET film analyzed by Swanson[4]. Although a web curvature is given in the table, the initial example which we will consider is of a perfect web with no curvature traveling between two mis-aligned rollers. The misalignment is 2^o and the web tension is 200 N/m. This is a large misalignment for a web, and we see in Fig.4 that a significant part of the web has zero

TABLE 1 Web Parameters

Elastic Modulus E	4.137	GPa
Width W	0.3	m
Thickness h	23.4	μm
Span Length L	0.67	m
Radius of Curvature κ_{web}	0.0019	m^{-1}
Curvature ρ_{web}	528	m

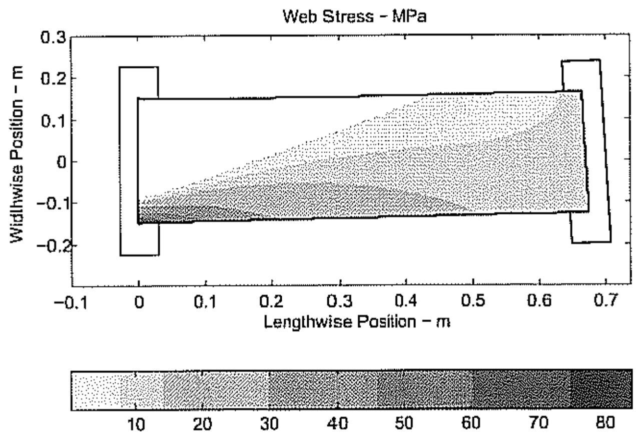

Figure 4 - Deflection and stresses in ideal web at 200 N/m with a misaligned roller ($\theta = 2^o$).

tension. Previous models would have given a result with negative tension in the same part. For many webs which can not support compressive stresses, the results from those models would be wrong. Increasing the tension or reducing the misalignment will reduce the part of the web with zero tension. In Fig.5 we have plotted lateral displacement at the downstream roller due to a more moderate misalignment of 0.1^o as a function of tension. For higher tensions where all parts of the web carry tensile stresses, the lateral displacement increases almost linearly with increasing tension. A wider web is less sensitive to an increase in tension. The results are in agreement with traditional models[1]. At lower tensions where parts of the web have zero stress and traditional models do not apply, the lateral displacement increases with decreasing values of tension. A wide web will start loosing its tensile stresses at lower values of web tension than a narrow web.

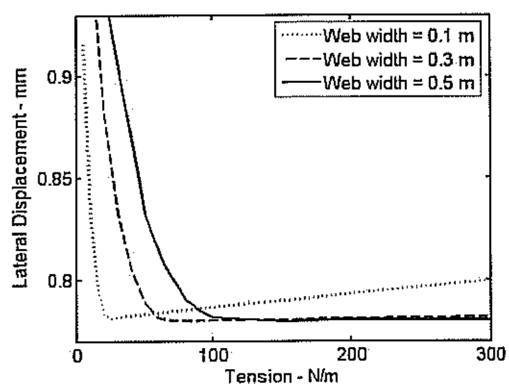

Figure 5 - Lateral displacement at misaligned downstream roller as a function of tension.

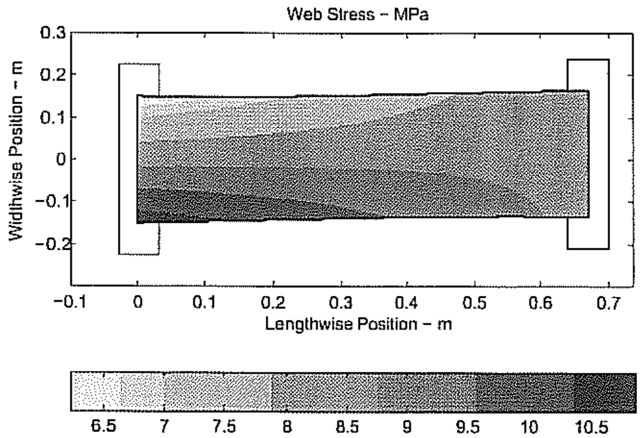

Figure 6 - Deflection and stresses in baggy web at 200 N/m.

We will now focus on baggy webs with a curvature as given i Table 1 traveling between two perfectly aligned rollers. For higher levels of tension the entire web is tensioned as seen in Fig.6 where the tension is 200 N/m. Note that the displacement are magnified by a factor of 100. We see that the baggy web moves laterally to the slack side. There has been some debate on this issue, but previously reported results [2][4] agree with this finding. Two important assumptions are made in these reported studies. It has been assumed that there is traction at the downstream roller and that the web is stretched out so that the entire web carries tension. The above example is consistent with both of these assumptions, but with the model presented here we no longer need to make the second of these assumptions. Thus we may test the lateral behavior at lower tensions.

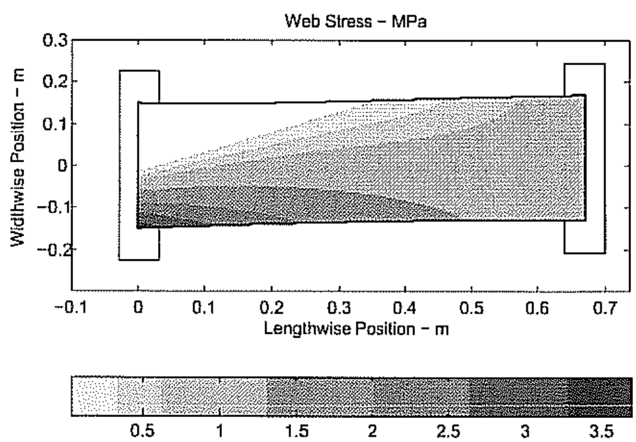

Figure 7 - Deflection and stresses in baggy web at 20 N/m.

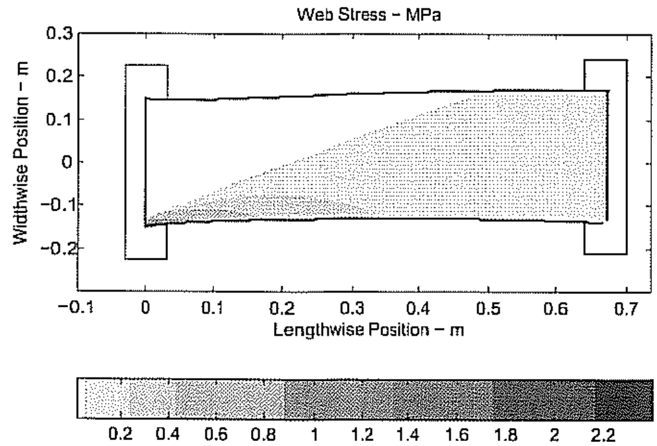

Figure 8 - Deflection and stresses in baggy web at 2 N/m.

For a lower tension of 20 N/m we see in Fig.7 that a significant part of the web do not carry tension. Also for this case the web moves laterally to the slack side. Even for very small amounts of tension the web moves to the slack side. This is seen in Fig.8 where stresses are plotted for a web tension of 2 N/m. Calculations for many different values of tension have been carried out. The results are summarized in Fig.9 where lateral displacement at the downstream roller as a function of web tension is plotted. The lateral displacement is almost independent of tension when the entire web is tensioned. This is seen in the part of the plot with high levels of tension. As parts of the web looses its tension, the lateral displacement increases with decreasing tension. For all levels of tension the web moves to the slack side. Note also the effect of web width. A wide web will start loosing its tensile stresses at lower values of web tension than a narrow web. For a specific value of tension a wider web will deflect more laterally than a narrow web at lower tensions.

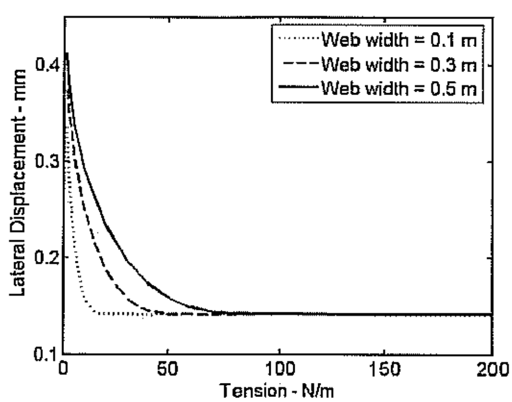

Figure 9 - Lateral displacement at downstream roller due to web curvature as a function of tension.

CONCLUSION

A mathematical theory describing lateral deflections of baggy webs which do not support compressive stresses have been derived. Results show that the lateral deflection of perfect webs and baggy webs is significantly affected by tension at the lower levels of tension. At lower tensions the lateral deflection increases significantly with decreasing tension. For all levels of tensions a baggy web moving between to perfectly aligned rollers will move to the slack side if there is traction between the web and the rollers.

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Lateral Mechanics of Baggy Webs at Low Tensions	**J. E. Olsen**, SINTEF Materials and Chemistry, NORWAY
Name & Affiliation Herong Lei Eastman Kodak	**Comment** While the web is running and the tension is low you will develop a web weave so the web moves left and right. Thus both groups could be right in that circumstance.
Name & Affiliation J. E. Olsen SINTEF	**Answer** I have not taken that into account but you have a point. I agree.
Name & Affiliation Tim Walker TJWalker & Associates	**Question** The lateral motions here are obviously very small, on the order of 1 millimeter and less. What do you think the practical application of your modeling is when we are talking about fractions of a millimeter of motion?
Name & Affiliation J. E. Olsen SINTEF Materials & Chemistry	**Answer** My experience is with newsprint. When you print on newsprint with four color units you need to precisely position the web laterally in the machine. If the web moves laterally in the machine more than 0.2 millimeters you will notice registration errors. However, lateral movement may not be the only problem; wrinkling may also give you registration errors.
Name & Affiliation Ron Swanson 3M	**Question** I have a question on Figure 5, where you have shown the curves deviating from the straight line. Is this the result of part of the upstream web becoming slack?
Name & Affiliation J. E. Olsen SINTEF Materials & Chemistry	**Answer** Yes. The transition is where the web starts going slack. Once the web becomes slack or buckles deflections increase.
Name & Affiliation Ron Swanson 3M	**Question** The conclusion would be the magnitude of its lateral displacement goes up considerably with slackness in the span.
Name & Affiliation J. E. Olsen SINTEF Materials & Chemistry	**Answer** Yes, that is correct.
Name & Affiliation John Shelton Oklahoma State University	**Comment** Regarding the previous comment about web weaving or oscillating: If you have edge slackness of a cambered web and the web must negotiate through a nip, you are really in trouble as far as lateral behavior is concerned.

Name & Affiliation	Answer
J. E. Olsen SINTEF Materials & Chemistry	I agree.
Name & Affiliation	**Question**
David Landskron Quad Graphics Printing	You were discussing registration problems earlier. Do you also see some dot gain problems when this shifting occurs? When you get this movement you had registration problems between the print units. Do you also see a problem with dot gain when you have this problem? The actual dot you print on the paper in newsprint becomes oblong instead of round.
Name & Affiliation	**Answer**
J. E. Olsen SINTEF Materials & Chemistry	I have not studied this.