WIDTHWISE VARIATIONS IN WEB PROPERTIES AND WOUND ROLL STRUCTURE

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ABSTRACT

Widthwise variations in web properties affect wound roll structure. Cole & Hakiel published a two-dimensional model in 1992 which took widthwise variations in web thickness into account. Based on this publication a model accounting for widthwise variations in thickness, MD- and ZD-stiffness and frozen-in strains have been derived. The model enables calculations of wound roll stresses and outer roll radius. The model is presented and results of different effects discussed.

INTRODUCTION

A web is a thin, continuous, flexible strip of material like paper, metal foil, plastic film or textiles. In production and converting processes the web is transported through a web line such as a coater, annealing line, paper machine or magnetic tape recorder. At the end of many of these web lines, the web is wound onto a roll. A wound roll is a convenient format for storing and transporting web material. The quality of the wound roll is important for later processes such as coating, printing and other converting processes. Widthwise variations in web properties may cause degradations in roll quality.

Winding of a web with widthwise variations in web properties yield rolls with widthwise variations in roll structure. Roll structure is a loosely defined term, but in the following we will confine it to in-roll stresses and outer roll radius. Widthwise variations in roll structure may cause vibrations, web flutter, wrinkling and other problems at unwinding.

Mathematical models describe the relations between web properties and roll structure. Some relations are not accounted for in the models. This is especially true in the case of properties with widthwise variations. Thus more work is needed to achieve a better understanding on the effect of widthwise variations in web properties upon widthwise variations in roll structure. In this paper existing models are further developed to account for more web properties of this nature.

THEORY

Mathematical models enabling calculations of in-roll stresses due to web properties and process parameters have been around since the late 1950's[1]. The early models applied to linear isotropic materials. More realistic models was published by Altmann[2] and Hakiel[3] who introduced orthotropic and nonlinear material behavior respectively. Hakiel's model calculates the in-roll stresses from the winding process by adding the contribution from each lap being wound. The contribution from each lap is a function of the material properties of the web and the tension in the lap as it is being wound onto the roll. Radial displacements or compression of the roll is easily calculated from the in-roll stresses.

Cole & Hakiel[4] published a two-dimensional model in 1992 which took widthwise variations in web thickness into account. Based on this publication a model accounting for widthwise variations in thickness and other web properties will is derived in the following text. The other web properties are elastic modulus in the machine direction, elastic modulus in the radial direction (also known as stack modulus), Poisson's ratios and frozen-in strains. Frozen-in strains are deviations in length from the mean length of the web. Due to different aspects of web manufacturing, a web may have dimensional variations across the width of the web. This gives rise to frozen-in strains which often is referred to as bagginess. It is also straightforward to apply a widthwise profile of core properties such as core stiffness and core radius.
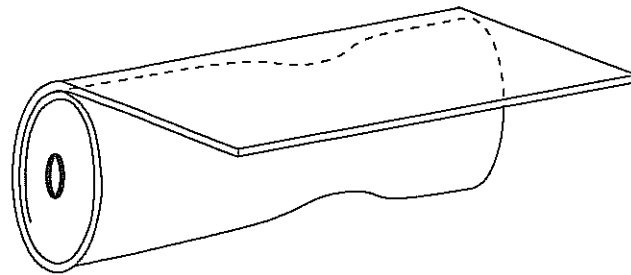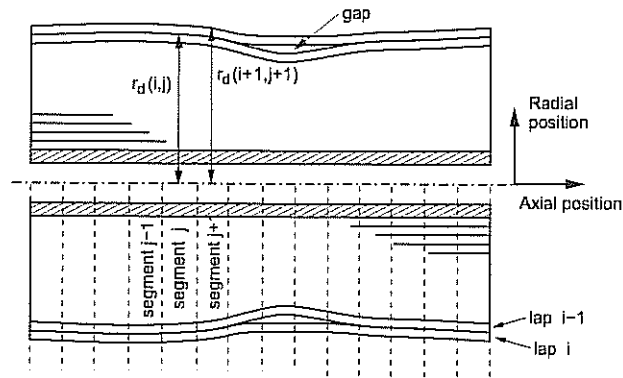

Figure 1 - Winding of roll with nonuniform web.

Figure 2 - Wound roll profile.

A winding roll with widthwise variations in web properties will have widthwise variations in radius and roll structure as illustrated in Fig.1. We divide the roll into laps and axial segments. Each lap is given a number indexed by the letter i. The segments are given a number indexed by the letter j which represents axial position. Adding of laps of web material upon a core is a modeling concept applied to the winding process. The laps added to the roll is wound with a total tension $t(i)$. Due to widthwise variations in roll radius and web properties, the total tension is divided differently into each segment which is given a segment tension $t_s(i, j)$. For each segment we may compute the in-roll stresses by Hakiel's model with the segment tensions as input. In order to do so we need an additional model which enable us to calculate the segment tensions as a function of total tension, widthwise variations in roll radius and web properties.

Upon winding of lap number i, the roll has a widthwise varying radius $r_d(i, j)$. In segments where the lap makes contact with the lap underneath, the lap being wound will cause radial displacements. In segments without contact, no tension exists in the lap and the lap in the segment will be suspended as a straight cylindrical surface above the lap underneath as seen in Fig.2. The radius to the centerline of the unstretched lap of the web is referred to as the relaxation radius. For a web where thickness is the only property varying across the width, the relaxation radius is constant across the width as assumed by Cole & Hakiel[4]. For a web which has frozen-in strains varying across the width of the web, the relaxation radius varies accordingly. Thus, for segments which gap, we have

$$r_d(i+1, j) = r_o(i)\left(1 + \varepsilon_i(j)\right) + \frac{h(j)}{2} \qquad (1)$$

where $r_d(i + 1, j)$ is the roll radius in the jth segment after the addition of the ith lap, $r_o(i)$ is a parameter capturing the variations in relaxation radius due to the new lap being wound, and $\varepsilon_i(j)$ is the frozen-in strain which varies with axial position. For segments which make contact, the roll radius is

$$r_d(i+1, j) = r_d(i, j) + \tilde{U}(i, j) + h(j) \qquad (2)$$

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where $\bar{U}(i,j)$ is the radial displacement defined positively outward of the segment due to the previous lap.

We assume the material response in the plane of the web to be linear elastic and allow for frozen-in strains (initial strains). This yields

$$\sigma_\theta(i,j) = \frac{E(j)}{1 - \nu_w^2(j)}\left(\varepsilon(i,j) - \varepsilon_i(j)\right) \tag{3}$$

Here E is elastic modulus in the machine direction (MD) of the web or in tangential direction in the winding roll, ν_w is in-plane Poisson's ratio and ε is the tangential strain in the outer lap. This is a generalization of Cole & Hakiels's model [4] which assumes constant elastic modulus and Poisson's ratio, and no frozen-in strain. The strain is given by

$$\varepsilon_\theta(i,j) = \frac{r_d(i+1,j) - h(j)/2 - r_o(i)}{r_o(i)} \tag{4}$$

Although the material behavior is nonlinear, the material is assumed to behave linearly during the adding of each single lap. The material properties are updated after each lap. Thus it can be argued, as by Cole & Hakiels[4], that the displacements due to the adding of a single lap is linear with tension. We may introduce

$$\tilde{U}(i,j) = \tilde{U}_o(i,j)\frac{t_s(i,j)}{t_{so}(i,j)} \tag{5}$$

where $\tilde{U}_o(i,j)$ is radial displacements of the jth segment due to any given segment tension $t_{so}(i,j)$. Knowing the relation between winding stress σ_θ and winding tension t_s

$$\sigma_\theta(i,j) = \frac{t_s(i,j)}{h(j)\Delta w(j)} \tag{6}$$

we find the segment tensions from Eqs.(2)-(5)

$$t_s(i,j) = \frac{a_s(i,j) - r_o(i)\left(1 + \varepsilon_i(j)\right)}{\frac{1-\nu_w^2}{E(j)h(j)\Delta w(j)}r_o(i) - b_s(i,j)} \tag{7}$$

for segments making contact with the layer underneath. Here we have

$$a_s(i,j) = r_d(i,j) + \frac{h(j)}{2} \tag{8}$$

and

$$b_s(i,j) = \frac{\tilde{U}_o(i,j)}{t_{so}(i,j)} \tag{9}$$

If any segment tensions, Eq.(7), becomes negative, there is really no contact being made. Those segments are non-contacting and we set $t_s(i,j) = 0$ and apply Eq.(1) for updating the outer roll radius of those segments. The sum of all segment tensions must equal the total winding tension of the lap being wound

$$t(i) = \sum_{j=1}^{m} t_s(i,j) \tag{10}$$

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Calculations of the segment tensions are carried out by making a qualified guess on the relaxation radius $r_o(i)$ and iterating the equations above until the condition of Eq.(10) is fulfilled. From the segment tension we compute the in-roll pressures for each segment due the lap being wound by the model of Hakiel[3]. The increment in in-roll pressure is given by the following differential equation

$$r^2 \frac{\partial^2(\delta P)}{\partial r^2} + 3r \frac{\partial(\delta P)}{\partial r} + (1 - E(j)/E_r(j))(\delta P) = 0 \qquad (11)$$

where δP is the pressure increment, r is the in-roll radius, E is the elastic modulus in machine direction of the web and E_r is the elastic modulus in the radial direction. Note that r is an in-roll radius and not the peripheral radius. The boundary condition at the core is given by

$$\frac{\partial(\delta P)}{\partial r} = (E(j)/E_c - 1 + \nu)(\delta P) \qquad \text{at} \qquad r = R_0 \qquad (12)$$

Here E_c is core stiffness and ν is in-roll Poisson's ratio. At the periphery we have

$$\delta P = \frac{1}{r_d(i,j)\Delta w(j)} t_s(i,j) \qquad (13)$$

as a boundary condition. Here Δw is the width of segment j. The pressure increment due to the lap i is then added to the total pressure. From the pressure we calculate displacements and tangential stresses if desired.

Verification of the model have been carried out on thickness profiles on PET films by Cole & Hakiel[4]. Calculations on core pressure and radius profiles were found to be in good agreement with measured values.

RESULTS

In the following we focus on the effect of widthwise variations in thickness, bagginess and radial stiffness of the roll. Initially we assume all web and core properties to be constant across the width of the web. For a typical roll of newsprint we have values as given by Table 1. Upon studying the effect of paper thickness profiles we apply these values with the exception of the paper thickness. The constant value in Table 1 is replaced by a thickness profile which varies across the width of the paper. The mean

TABLE 1 Winding Parameters - Nominal Values

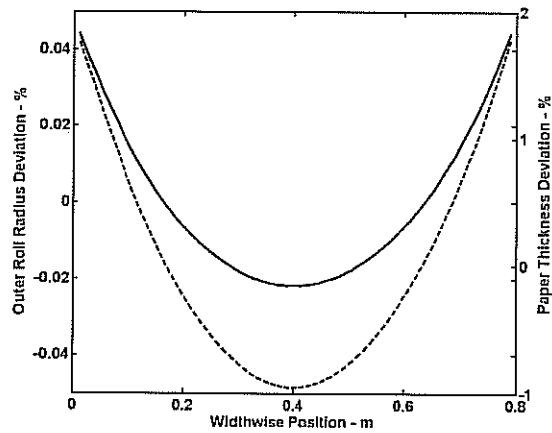| Property	Value	Unit	Property	Value	Unit
Paper Width	0.8	m	In-plane ν	0.2	
Paper Thickness	69.7	μm	In-roll ν	0.01	
MD Modulus	4.13	GPa	Core Outer Radius	0.25	m
K_1	488.349	kPa	Core Stiffness	1.5	GPa
K_2	29.45		Winding Tension	300	N/m
K_3	-0.0427	kPa^{-1}			
K_4	$3.5 \cdot 10^{-4}$	kPa^{-2}			

Figure 3 - Widthwise profiles of outer roll radius deviation
and thickness deviation (dashed line) with respect to mean
values.

value of the profile matches the nominal value of Table 1. The assumed profile is
plotted in Fig.3 and it describes a paper with increased thickness at the edges. The total
variation is about 3%. From calculations with the mathematical model presented above,
we get a widthwise profile of the outer roll radius as given in the same figure, Fig.3.
The relative deviation in radius is not as great as the relative deviation in thickness.
This is due to higher pressure and thus more radial compression in the regions with
higher paper thickness. The calculations also yield an in-roll pressure distribution as
seen in Fig.4. From these results we see that the outer roll radius has its highest values
where the thickness profile has its highest values. This is as expected.

Widthwise variations in the elastic modulus of the paper in the machine direction
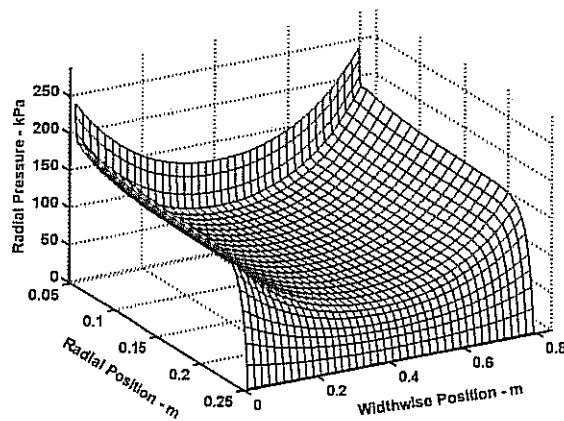will also yield widthwise variations in roll structure. Some believe this has the same

Figure 4 - In-roll radial pressure as function of roll radius
and widthwise position due to thickness profile.
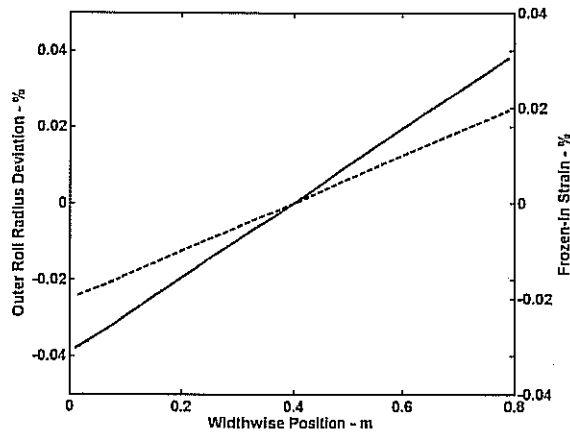
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Figure 5 - Widthwise profiles of outer roll radius deviation with respect to mean value and frozen-in strain (dashed line).

effect as thickness variations since thickness and modulus often act together as a product in many web handling theories. For winding of webs with widthwise variations in web properties this is not true. We see in Eqs.(1)-(2) that thickness acts alone. And the effect of thickness variations is therefore much stronger on wound roll structure, than the effect of similar variations in MD modulus.

We now wish calculate the effect of bagginess upon wound roll structure. Bagginess is a widthwise variation in frozen-in strain which may be caused by nonuniform drying of the paper. In the following example we assume a linear profile of the frozen-in strain as seen in Fig.5. All other properties are as in Table 1. The resulting roll periphery and in-roll pressure distribution are seen in Figs.5 and 6 respectively.
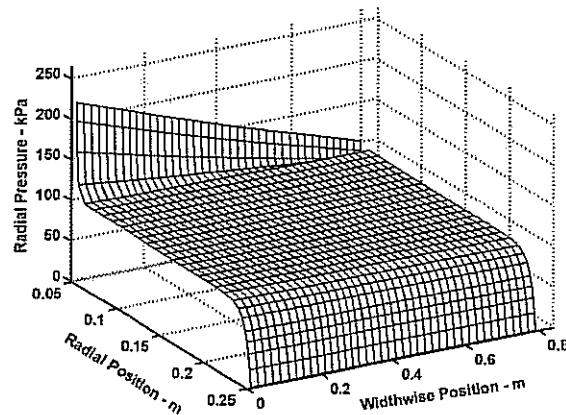

Figure 6 - In-roll radial pressure as function of roll radius and widthwise position due to bagginess.
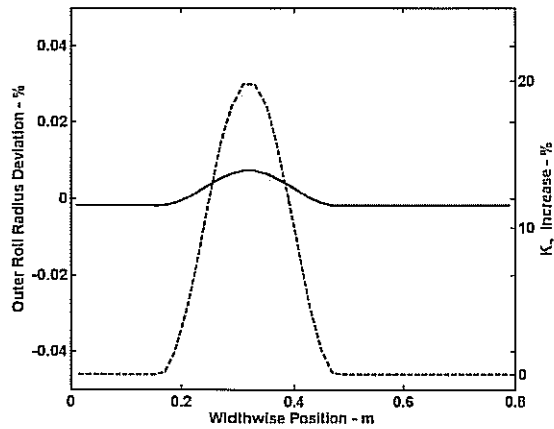
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Figure 7 - Widthwise profiles of outer roll radius deviation
and K_2 increase (dashed line) with respect to mean value
and nominal value.

Regions with positive frozen-in strain receive relatively more paper and thus needs to store more material. Therefore we see an increase in outer roll radius in these regions. These regions also have a lower pressure which results in less radial compression. Since increased paper feeding and reduced compression work together, the relative increase of roll periphery is greater than the relative increase in frozen-in strain.

It is also possible for the ZD stiffness of the paper to vary across the width. In order to create a uniform thickness profile some regions may have been calendered harder than others and thus causing a higher ZD stiffness in the paper in those regions. Widthwise variations in ZD stiffness causes widthwise variations in radial stiffness of the roll. The radial stiffness is equivalent to the stiffness in a stack of paper sheets. The radial stiffness increases with pressure and is responsible for a nonlinear material
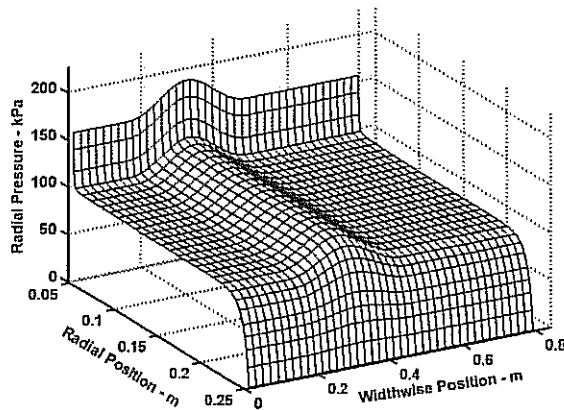

Figure 8 - In-roll radial pressure as function of roll radius
and widthwise position due to K_2 profile.

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behavior in the radial direction. The radial stiffness may be described by the following equation

$$E_r = K_1 + K_2 P + K_3 P^2 + K_4 P^3 \tag{14}$$

where P is the stack pressure or radial pressure. Note that the coefficient K_2 is the most significant parameter for winding. We assume that it varies across the width of the paper as seen in Fig.7. With this profile and other properties as in Table 1, we get a resulting outer radius as in Fig.7 and in-roll radial pressure as in Fig.8.

In the region with a higher value of K_2 the radial stiffness is higher and the radial compression is thus smaller. This results in a relative higher peripheral radius in this region. The increase in pressure has the opposite effect on the peripheral radius, but the effect is not as significant.

CONCLUSIONS

A mathematical model have been presented which enables calculations of wound roll structure for webs with widthwise variations in web properties. We have defined wound roll structure as in-roll stresses and roll radius. The in-roll stresses and the wound roll radius will vary across the width of the roll due to widthwise variations in web properties. This is as expected, and the model allows us to quantify these variations.

Analysis have been carried out on newsprint. We find that the effect of widthwise variations in paper thickness is slightly reduced by the radial compression which is higher in regions with higher thickness. The effect of widthwise variations in frozen-in strain, better known as bagginess, is increased since radial compression is lower in regions with higher paper feeding. Variations in radial stiffness yield a a wound roll structure with increased radius and hardness in regions with higher radial stiffness in the paper.

We have studied variations in paper properties that are smooth. The thickness variations are small, but normal for newsprint. The variations in bagginess and radial stiffness are typical for bad rolls, but not neccessarily worst case scenarios. The effect upon widthwise variations in roll radius is quite small. The effect upon in-roll pressure is more significant. For more special examples where gaping might occur, the effects will be very significant. This could be the case for profiles that are less smooth and have higher variations.

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Name & Affiliation	
	Question
	I have two questions. 1) Which software did you use and 2) How many points will you have over the width of the roller?
Name & Affiliation Jan-Erik Olsen SINTEF	**Answer** It is self developed software. I have run it both in a C program and in MatLab, and calculated times depend on how many laps you want to have there. The run time on a computer is approximately half an hour to one hour. As long as you are working on an ideal case, you can have as many points as you want to, but if you're working on a profile from a paper manufacturer you typically divide it into as many segments as you have measurements. It is typically between 20 and 30.
Name & Affiliation David Pfeiffer JDP Innovations	**Comment** Thank you for a nice presentation. I just wanted to make a comment. Your stiffness profile, which increases the pressure, would result in a rather pronounced hardness profile. If you profiled the hardness of the roll, because of the densification of the material under those pressure peaks, the roll hardness profile would show maybe a higher bump there than the pressure profile does. Plus, our experience with rolls in the field is that are alongside those pressure peaks, you would find a tendency to develop corrugations in the wound roll.
Name & Affiliation Jan-Erik Olsen SINTEF	**Answer** Thank you for your comment. I agree. In this study, we have seen a significant increase in roll hardness but it's hard to correlate directly to the model because roll pressure is not directly or easily transferred to roll hardness.