WRINKLING OF WEBS ON ROLLERS AND DRUMS

by

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ABSTRACT

Wrinkles are frequently observed in thin webs wrapping rollers, winding cores, and drums in vacuum coaters. They are the buckling response of the web to compressive deformation in the transverse direction (TD), which may arise from inward steering of the web edges (e.g. on a roller bending under its own weight), or thermal expansion (under the heat load of coating deposition). Analysis of conditions which cause wrinkling starts from the idealised case of a free thin cylindrical shell under end load. This buckles to form a regular pattern of rings, which run round the circumference and are sinusoidal in the TD. However, the analysis fails to account for the rigid roller or drum preventing deformation towards the axis, and friction between the web and roller surface.

A new, approximate, Rayleigh-Ritz variational analysis has been used to find the conditions for wrinkling into a continuous sinewave on a roller or drum. The wavelength is smaller, and the critical strain higher, than the case of the free thin cylindrical shell. Furthermore, wrinkling can be suppressed by imposing a tension in the machine direction. At a particular tension, wrinkles only form above a certain level of TD strain. They require a trigger, such as the presence of a dirt particle, to form. Practically, this continuous solution is expected when the coefficient of friction is extremely high, or the edges of the web are physically restrained.

In other cases with more typical coefficients of friction, the web can adopt a lower energy configuration by forming isolated wrinkles (approximated by a single cycle of a sinewave) and flat areas in between. The TD strain in the web after buckling varies in a sawtooth manner, with minima at the web edges and the wrinkle locations, and maxima midway between them. Once again, wrinkles require an initiating event to form, and the critical TD strain for wrinkling increases with tension. The spacing between wrinkles falls with increasing tension. Wrinkles are approximately equally spaced, typically 100 mm apart, and absent from the web edges, in accordance with observations.

NOMENCLATURE

a	Amplitude of wrinkle sinewave (half of peak to trough height difference)
d	Web thickness
E	Young's modulus of the web
L	Width of web constrained at the edges, or "affected width" with friction contact: it relieves strain by forming a wrinkle at its centre
L_c	Affected width when a wrinkle forms at the critical TD strain
MD	Machine Direction, i.e. along direction of travel
P_{crit}	Critical force to buckle a cylindrical shell
R	Roller radius
TD	Transverse Direction, i.e. perpendicular to direction of travel
U_b	Bending strain energy per unit MD length in a single wrinkle
U_c	Strain energy per unit MD length from TD compression in a single wrinkle
U_s	Strain energy per unit MD length from MD stretching in a single wrinkle
U_w	Total strain energy in the wrinkled web per unit MD and TD length
w	Displacement of web above the roller surface
y	TD coordinate
γ	TD compressive strain at any point after wrinkling
ε	Fractional extra length in the wrinkle sinewave
ε_{crit}	Critical TD compressive strain for buckling into wrinkles
ε_x	Initial MD tensile strain (i.e. before wrinkling)
ε_y	Initial TD compressive strain relative to width after Poisson's ratio contraction (before wrinkling)
ε	Scaled extra length in the wrinkle sinewave, given by $\varepsilon' = \lambda\varepsilon / L$
ε_1	TD compressive strain in the wrinkle sinewave after formation
θ	Ratio $\varepsilon / \varepsilon_y$ in equation 14
λ	Wavelength of the wrinkle sinewave
μ	Coefficient of friction
ν	Poisson's ratio of web
ρ	Factor $(1-\nu^2)^{-1}$

INTRODUCTION

Webs of thin materials exhibit wrinkling during transport through most process equipment. Corrugations with a wavelength of a few cm may appear in free spans between rollers, either along the machine direction (MD) or inclined to it. On wound rolls and soft rubber rolls, corrugations of much shorter wavelength may be observed over most or part of the web width. These can also be seen on high speed lines where entrained air causes the web to lose direct contact with rollers. A different type again is observed over rollers at all speeds: wrinkles are a few mm wide but separated by tens or hundreds of mm. Most corrugations and wrinkles are short-lived, and cause no permanent effect on the material. However, wrinkles on wound rolls may be set in with time, causing unacceptable appearance of the final film product. Wrinkles over rollers may be made permanent by the action of pressure (e.g. in a nip), or heat (e.g. from vapour deposition). These lasting effects may cause defects in downstream processing, such as uncoated streaks, changes in coating appearance, and notched slit edges. In turn these may result in product loss.

Wrinkles are the buckling response of the web to compressive deformation. Transverse direction (TD) compression, which may arise from inward steering of the web edges (e.g. from roller bending under its own weight), or thermal expansion (under the heat load of coating deposition), results in wrinkles aligned along the MD. Inclined wrinkles imply the presence of shear, such as from misaligned rollers [1].

Analysis of conditions which cause wrinkles aligned in the MD starts from the idealised case of a free thin cylindrical shell under end load [2]. This buckles to form a regular pattern of rings, which run round the circumference and are sinusoidal in the TD (figure 1). The shell theory is a reasonable approximation when the web is supported on a compliant cushion, such as an entrained air layer [3], soft rubber or a wound roll [2].

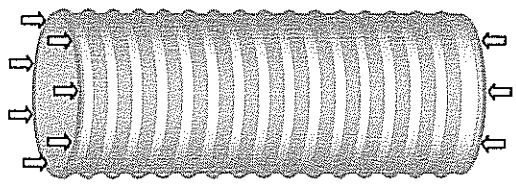


Figure 1 – Schematic diagram of cylindrical shell buckling under end load into a sinusoidal pattern.

However, the previous analysis fails to account for the surface of a rigid roller or drum preventing inward radial displacement, and friction between the web and roller surface. These cases are considered in reference [4] and this paper. In addition, the appearance of wrinkles as single ridges separated by considerable distances [5] does not emerge from the analysis. This paper shows that this arises naturally when friction is included.

PREVIOUS WRINKLING THEORY

Thin shell buckling theory

The standard theory for an isolated, infinitely long cylindrical shell of isotropic elastic material is given in several references [6], [7], and using the energy method adopted here in reference [8]. The shell can buckle into a continuous axisymmetric sinewave, as shown in figure 1, with wavelength given by:

$$\lambda = \pi \sqrt[4]{\frac{4\rho}{3}} \sqrt{Rd} \qquad \{1\}$$

where λ is wavelength, R is cylinder radius, d is shell wall thickness, and ρ is the factor $(1-\nu^2)^{-1}$, with ν being Poisson's ratio. For $\nu = 0.3$, this becomes:

$$\lambda = 3.46\sqrt{Rd} \qquad \{2\}$$

The wavelength is determined by geometry, specifically the shell radius and thickness. It is not influenced by modulus or applied load.

The critical load (force) is:

$$P_{crit} = \frac{2\pi\sqrt{\rho}Ed^2}{\sqrt{3}} = 3.803Ed^2 \qquad \{3\}$$

where E is Young's modulus, again for $\nu = 0.3$. Expressed as a critical strain, this becomes:

$$\varepsilon_{crit} = \sqrt{\frac{\rho}{3}}\frac{d}{R} = 0.605\frac{d}{R} \qquad \{4\}$$

Like the wavelength, the critical strain is dependent on radius and thickness, but not on modulus.

<u>**Application to webs**</u>

In applying the results to webs [2], the shell is identified with the web passing over a roller or drum, defining the values of radius and thickness. On wound rolls, the outer turn only is considered. On process rollers, a portion of the web wrapping the roller is assumed to act in the same way as the full circumference of the cylindrical shell (figure 2). This segment may be the whole of the wrap, or the part where the web has developed a TD thermal compressive stress.

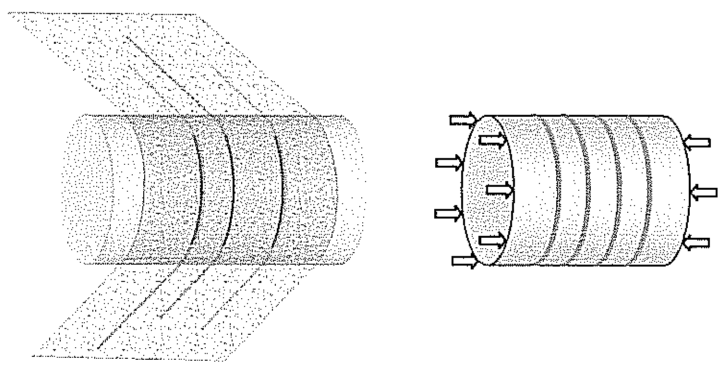


Figure 2 – Equivalence of wrinkles in a web on roller (left) and in an end-loaded free cylindrical shell (right).

The theory assumes that the shell is loaded at the ends, and the buckling transition is accompanied by inward movement of the ends with the load remaining constant. The web is different as it has no lateral load applied to the edges. Instead, the frictional forces act to restrain the outer portions of the web and prevent movement. TD compressive strain builds up in the central part, and may be partially relieved by buckling, but without inward movement of the edges. The two cases are compared in figure 3. Adapting the theory described later show that a free cylindrical shell with fixed ends buckles with the same wavelength and critical strain as equations 2 and 4.

According to the standard theory, the web may move both outwards and inwards in forming a buckle. However, a rigid roller or drum will prevent inwards movement, so all movement must be radially outwards (figure 4). Finally, the effect of initial MD tension is neglected.

Despite these shortcomings, the theory has been applied to buckling of webs. Measurements demonstrate that equation 2 is followed for wound reels [2]. It should also be valid for webs on rollers with significant entrained air, and soft rubber coverings. Direct measurements of TD load are difficult, and equation 4 does not appear to have been tested.

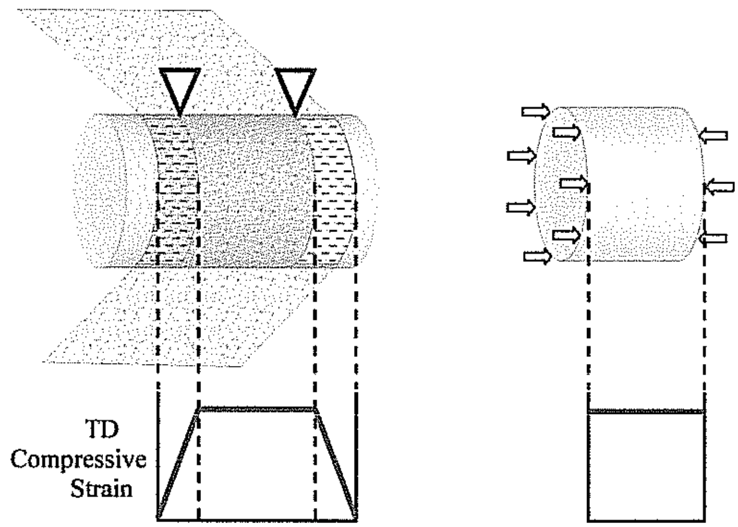


Figure 3 – Loading conditions for buckling into wrinkles.
Left: Web on roller, showing slip zones where TD strain rises from zero at the edge to the value in the web centre, which behaves as if restrained at the slip zone edges (triangles).
Right: Cylindrical shell under constant force loading.

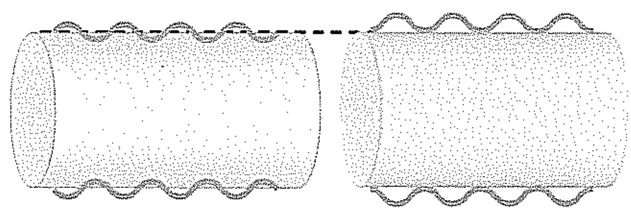


Figure 4 – Schematic diagram of buckling of (left) a free cylindrical shell, and (right) a shell prevented from inward movement by a rigid roller.

The theory predicts no effect of MD tension or friction. In contrast, increasing tension is observed to remove wrinkles, and high coefficient of friction surfaces develop more wrinkles. In addition, the only buckling mode predicted is a sinusoid, whereas most wrinkles on rollers are narrow but separated by a few hundred mm [5].

NEW THEORY OF WRINKLING ON RIGID ROLLERS

Sinusoidal buckling of webs on rigid rollers

To overcome the limitations of the previous theory, a new theory has been developed [4]. The assumptions include:

1. The web is modelled as an infinitely long cylindrical shell (as before).
2. An initial MD strain may be present.
3. All radial movement of the shell is outwards.

4. Buckling is accompanied by a reduction in the level of TD compressive strain, but no lateral movement.
5. The buckled shape is a sinewave.
6. A small displacement theory is sufficiently accurate.

The conditions for buckling have been obtained using a Rayleigh-Ritz variational method. The sinusoidal shape is an approximate trial function that should give close to the correct stored strain energy. By calculating the energy as a function of wrinkle amplitude and wavelength, the points of equilibrium can be determined. The wrinkled condition will be stable if it is an energy minimum and has a lower energy than the initial unbuckled state.

The assumed shape of the web is shown in figure 5. The height w above the roller surface varies with TD coordinate y, and is given by:

$$w(y) = a\left[1 - \cos\left(\frac{2\pi y}{\lambda}\right)\right] \qquad \{5\}$$

The half-height is a and the wavelength λ.

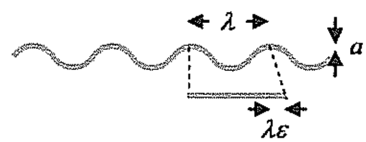


Figure 5 – Sinusoidal buckling pattern, showing amplitude a and wavelength λ. A single wave when flattened has extra length $\lambda\varepsilon$, defining fractional excess length ε.

The fractional extra length ε in the buckled shape is, to a good approximation:

$$\varepsilon = \int_0^{\lambda}\left[\sqrt{1+\left(\frac{dw}{dy}\right)^2} - 1\right]\frac{dy}{\lambda} = \frac{\pi^2 a^2}{\lambda^2} \qquad \{6\}$$

The strain energy in the web is made up of several components. In the unbuckled state, there is strain energy from MD tension and TD compression. In the wrinkled state, there are three components, evaluated below for a single wave:

Bending. The stored energy per unit MD length in a single cycle of the sinewave is given by:

$$U_b = \int_0^{\lambda}\frac{1}{2}\frac{E\rho d^3}{12}\left(\frac{d^2 w(y)}{dy^2}\right)^2 dy \qquad \{7\}$$

where ρ is the factor $(1-\nu^2)^{-1}$. Evaluating equation 7 results in:

$$U_b = \frac{\pi^2 E\rho d^3 \varepsilon}{3\lambda} \qquad \{8\}$$

The bending energy stored is directly proportional to the fractional extra length ε, and inversely proportional to the wavelength for a given ε.

MD Stretching. As the wave rises off the roller, it increases in MD length. The local strain is given by the original MD strain ε_x plus the extra strain w/R. The stretching energy is therefore given by:

$$U_s = \int_0^{\lambda} \frac{1}{2} Ed\left(\varepsilon_x + \frac{w(y)}{R}\right)^2 dy = \frac{Ed\lambda\varepsilon_x^{\,2}}{2} + \frac{Ed\lambda^2\varepsilon_x\sqrt{\varepsilon}}{\pi R} + \frac{3Ed\lambda^3\varepsilon}{4\pi^2R^2} \qquad \{9\}$$

The first two terms are absent from the free cylindrical shell analysis, and the last term is three times larger. These differences arise from the increase in average height above the roller surface.

TD Compression. An unwrinkled web under zero TD stress contracts under the MD tension because of Poisson's ratio. Further compressive strain ε_y relative to this increases the stored energy. Buckling increases the length by ε, and the strain falls, from its original value of ε_y, to a uniform value across the TD of $(\varepsilon_y - \varepsilon)$, resulting in stored compression energy of:

$$U_c = \frac{1}{2} Ed\lambda(\varepsilon_y - \varepsilon)^2 \qquad \{10\}$$

The final expression for energy per unit distance in both MD and TD is:

$$\begin{aligned} U_w &= (U_b + U_s + U_c)/\lambda \\ &= \frac{\pi^2 E\rho d^3\varepsilon}{3\lambda^2} + \frac{Ed\varepsilon_x^{\,2}}{2} + \frac{Ed\lambda\varepsilon_x\sqrt{\varepsilon}}{\pi R} + \frac{3Ed\lambda^2\varepsilon}{4\pi^2R^2} + \frac{1}{2}Ed(\varepsilon_y - \varepsilon)^2 \end{aligned} \qquad \{11\}$$

The unknown wavelength λ and fractional extra length in the wave ε may be found by applying the Rayleigh-Ritz variational principle [8]. At the equilibrium configuration:

$$\frac{\partial U_w}{\partial \lambda} = 0 \quad ; \quad \frac{\partial U_w}{\partial \varepsilon} = 0 \qquad \{12\}$$

The first gives a relationship between wavelength and fractional extra length:

$$\lambda^2 = \frac{2\pi^2\rho d^2}{3(\varepsilon_y - \varepsilon)} \qquad \{13\}$$

Substituting and expressing ε as a fraction θ of the initial strain:

$$\left(\theta(1-\theta)^3\right)^{0.5}\left[1 - \frac{\rho d^2}{R^2\varepsilon_y^{\,2}(1-\vartheta)^2}\right] = \frac{2\sqrt{2\rho}\, d\varepsilon_x}{\sqrt{3}R\varepsilon_y^{\,2}} \qquad \{14\}$$

In general, this must be solved numerically. Interestingly, there is a critical value of MD strain, dependent on ε_y, above which there is no solution. For smaller ε_x, there are two solutions for θ. The lower is an unstable equilibrium, whereas the upper one gives an energy minimum.

The energy at the final buckling wavelength can be calculated using equation 11, and is plotted as function of the fractional excess length in figure 6. As the initial TD strain is increased, the buckled state is first unstable (a), then metastable (b, c, d), and finally stable (e), when its energy falls below that of the initial state.

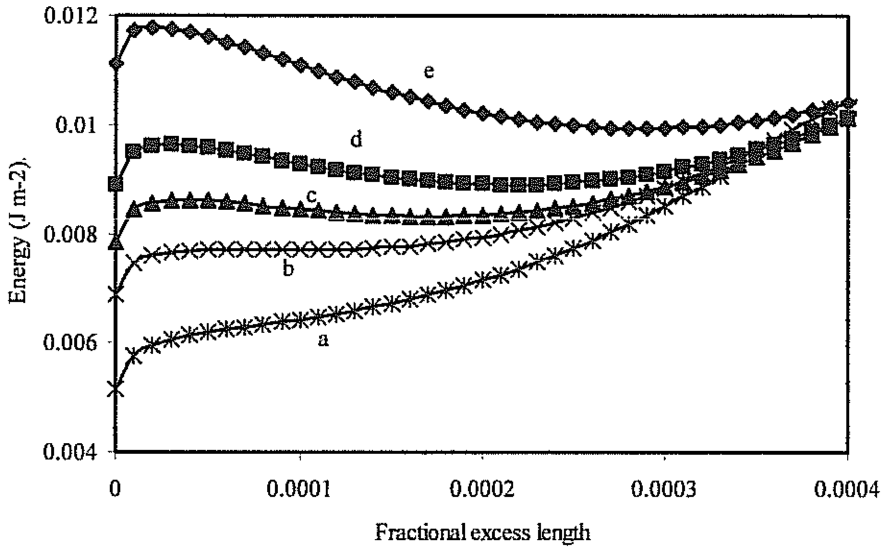


Figure 6 – Plot of energy versus fractional extra length ε in the developing wrinkles. All curves are for 25 μm thick film, Young's modulus 3.56 GPa and Poisson's Ratio 0.38 over a 0.457 m radius drum with an initial MD strain ε_x of 10^{-3}.
(a): TD strain $\varepsilon_y = 3.4 \times 10^{-4}$; only the unbuckled state is an energy minimum[1].
(b): TD strain $\varepsilon_y = 3.94 \times 10^{-4}$; critical condition for wrinkling
(c): TD strain $\varepsilon_y = 4.2 \times 10^{-4}$; a second minimum but with higher energy than the unbuckled state, i.e. metastable wrinkles
(d): TD strain $\varepsilon_y = 4.56 \times 10^{-4}$; two minima have the same energy
(e): TD strain $\varepsilon_y = 5.0 \times 10^{-4}$; stable wrinkles with lower energy than the unbuckled state.

Figure 7 plots the critical values of TD strain against MD strain, showing the zones of metastable and stable wrinkling. Finally, figure 8 shows the variation of wavelength with initial TD strain.

[1] As the wavelength is undefined for TD strain below the critical value, the wavelength at the critical strain was used for this curve.

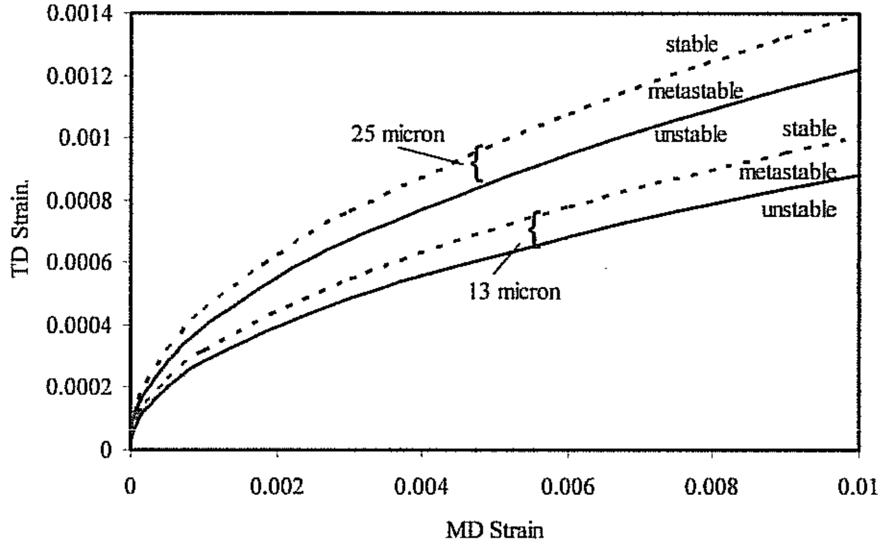


Figure 7 – Plot of the regions of metastable and stable wrinkling for different MD and TD strains, ε_x and ε_y. Curves are calculated for film with Young's modulus 3.56 GPa and Poisson's Ratio 0.38 over a 0.457 m radius drum, of thickness 25 μm (upper) and 13 μm (lower).

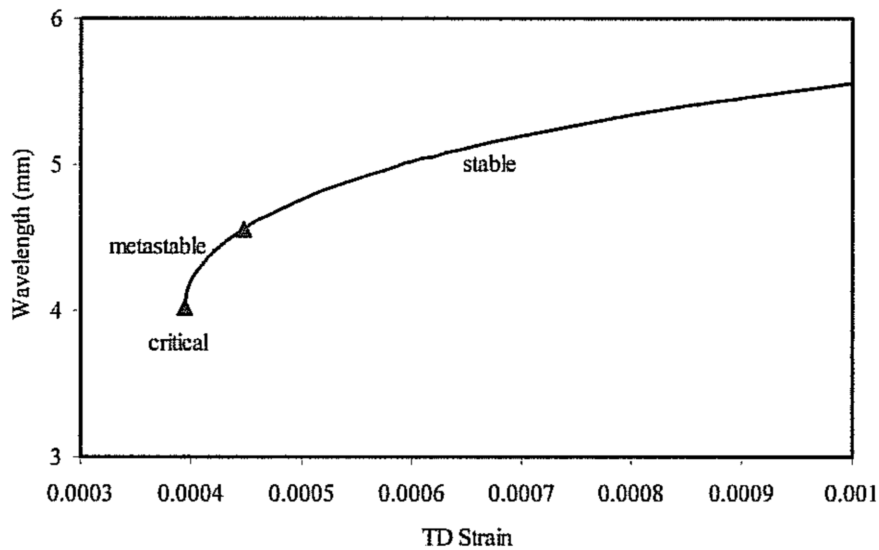


Figure 8 – Calculated variation of wrinkle wavelength with initial TD strain ε_y for 25 μm thick film, Young's modulus 3.56 GPa and Poisson's ratio 0.38, over a 0.457 m radius drum with MD strain ε_x of 10^{-3}.

Limiting Cases

For values of ε_y much larger than d/R, the second term in square brackets is negligible. The left hand side then has a maximum value of 0.1055 when $\theta = 0.25$. This results in a condition for solutions to exist:

$$\frac{\varepsilon_x}{\varepsilon_y^{\,2}} \le 0.40\frac{R}{\sqrt{\rho d}} \qquad \{15\}$$

The wavelength of the buckled state satisfies:

$$\lambda \ge 2\pi d\sqrt{\frac{2\rho}{3\varepsilon_y}} \qquad \{16\}$$

The critical condition is where the equality of equations 15 and 16 are satisfied, i.e.:

$$\varepsilon_{crit} = 1.58\sqrt[4]{\rho}\sqrt{\frac{\varepsilon_x d}{R}} \qquad \{17\}$$

and the TD strain falls from the critical value by a quarter when the wrinkles form. The buckled state has lower energy than the unbuckled state, and is therefore the stable equilibrium condition, provided that:

$$\frac{\varepsilon_x}{\varepsilon_y^{\,2}} \le 0.306\frac{R}{\sqrt{\rho d}} \qquad \{18\}$$

Strain levels between the limits of equations 15 and 18 represent metastable equilibrium.

Another case is small initial MD strain, where the right hand side of equation 14 is negligible. This corresponds to an untensioned thin cylindrical shell which just fits over a rigid former. There is a stable, sinusoidal solution with wavelength

$$\lambda = \pi\sqrt{\frac{2Rd}{3}}\sqrt[4]{\rho} = 2.62\sqrt{Rd} \qquad \{19\}$$

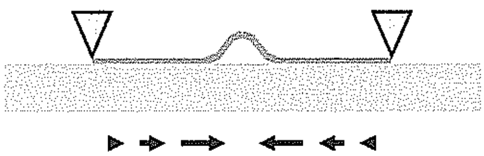
for Poisson's ratio of 0.3. The solution exists provided that the TD strain is large enough, giving the critical condition for buckling, i.e.

$$\varepsilon_{crit} = \sqrt{\rho}\frac{d}{R} = 1.05\frac{d}{R} \qquad \{20\}$$

Comparison with equations 2 and 4 shows that the constraint of the rigid surface increases the critical strain for buckling, and reduces the wavelength. The wavelength is again independent of the TD strain.

Frictionless contact

If the web is able to slide freely over the roller surface, an alternative buckled configuration is a single wrinkle which has relieved the TD compressive strain from the whole width (the portion between the friction zones at the edges). This configuration, shown in figure 9, is more normally observed than the regular sinusoidal pattern analysed above.



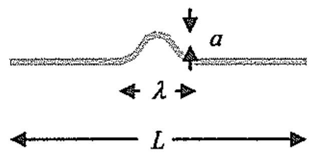


Figure 9 – Left: formation of a single wrinkle between fixed points (triangles) by reduction of TD strain, and inward movement (arrows) towards the wrinkle in the flat portions.
Right: Parameters amplitude a, wavelength λ and constrained width L.

A fixed width L is considered. The TD strain is constant over the whole of L before and after buckling. The energy is made up of the same components as equations 7 to 10, the only difference being the compression energy which becomes:

$$U_c = \frac{1}{2} EdL(\varepsilon_y - \varepsilon')^2 \qquad \{21\}$$

with the scaled extra length ε' defined by:

$$\varepsilon' = \frac{\lambda\varepsilon}{L} \qquad \{22\}$$

The energy per unit length in MD and TD now becomes:

$$U_w = \frac{\pi^2 E\rho d^3 \varepsilon'}{3\lambda^2} + \frac{Ed\varepsilon_x^{\ 2}}{2} + \frac{Ed\lambda^{1.5}\varepsilon_x\sqrt{\varepsilon'}}{\pi R\sqrt{L}} + \frac{3Ed\lambda^2\varepsilon'}{4\pi^2 R^2} + \frac{1}{2}Ed\left(\varepsilon_y - \varepsilon'\right)^2 \qquad \{23\}$$

This has the same form as equation 11, with ε' replacing ε. The second term is constant and plays no part. The third term falls as L increases, which implies that a large width will only form one wrinkle, as that configuration has the lowest energy.

The MD strain ε_x has a smaller effect on inhibiting wrinkles than in the sinusoidal case, because the 3rd term in equation 23 is reduced by a factor $\sqrt{\lambda / L}$. In the limiting case of zero MD strain or infinite width, the wrinkle wavelength and critical TD strain are given by equations 19 and 20.

Solutions for the minima of equation 23 have not been obtained. However, similar behaviour to the sinusoidal case is expected, as indicated schematically in figure 10. The wrinkled condition is unstable at low TD strain, then metastable at intermediate values, and finally stable at the highest values.

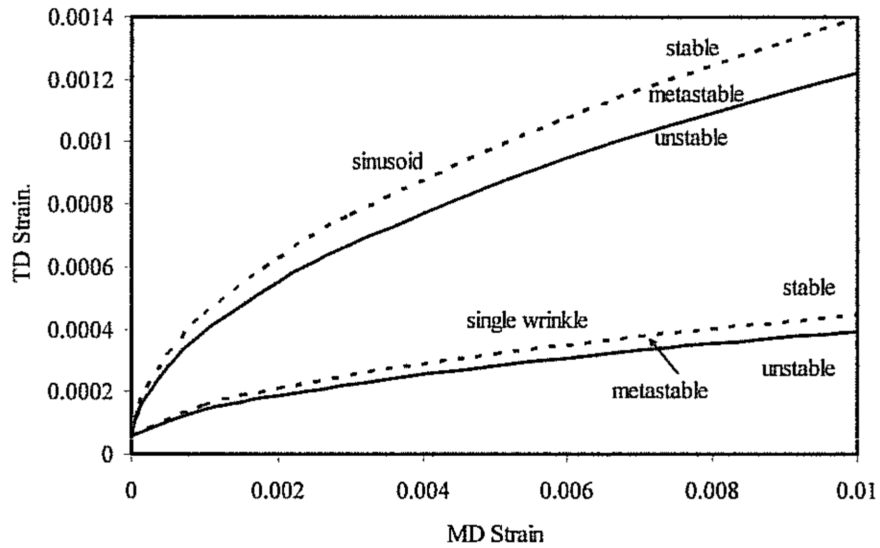


Figure 10 – Schematic regions of stability for single wrinkles on a frictionless surface, compared with the sinusoidal solution for 25 μm film from figure 7.

The reduction of the third term in equation 23 shows that the single wrinkle solution has lower energy than the regular sinusoid solution. Hence the latter will only be observed when sliding is impossible. This is unusual, but may occur for webs which are highly charged or coated with a tacky adhesive. On the other hand, wrinkles will only be observed in frictionless contact if there is some other physical restraint on the web edges such as flanges. The final part of this section addresses wrinkling with a more typical coefficient of friction.

Wrinkling with friction

When a wrinkle forms, it can relieve compressive strain in the web on either side, which slips inwards towards the wrinkle (figure 11). This movement is opposed by frictional forces: the MD tension in the curved web generates a contact pressure between the web and roller, which is able to provide a TD friction force [2],[9]. The material in this region is on the point of slipping, and experiences a frictional force. This sets up a linear variation of TD strain from the reduced value in the wrinkle up to the original value remote from it. The gradient of TD compressive strain γ given by:

$$\frac{d\gamma}{dy} = \pm\frac{\mu\varepsilon_x}{R} \qquad \{24\}$$

where μ is the coefficient of friction between web and roller. Each visible wrinkle is a full cycle of a sinewave with wavelength λ; and there is a width $(L-\lambda)/2$ with reduced compressive strain on either side ("slip zone"). The outermost points do not move when the wrinkle forms, but all other parts move towards the wrinkle as their strain is relieved.

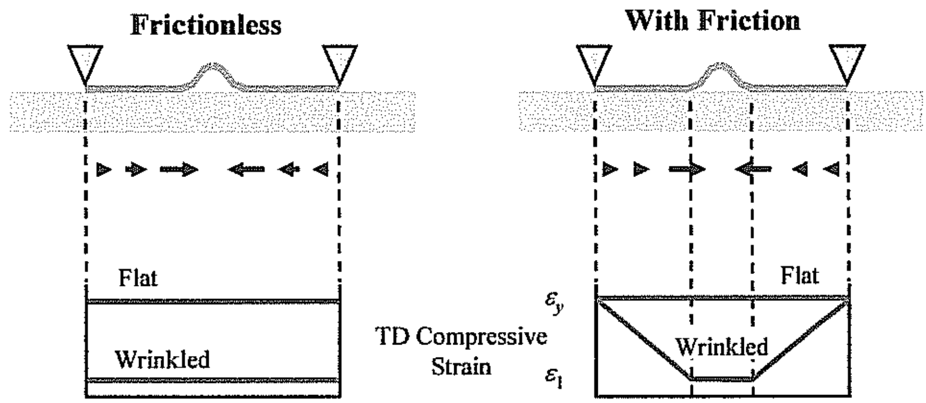


Figure 11 – Comparison of wrinkling for frictionless contact (left) and contact with friction (right). Physical restraint at the triangles is necessary in the first case, but redundant in the second. The arrows show the slip movement, and the graphs beneath show the variation of TD compressive strain with position.

Analysis to find the wrinkle spacing and critical strain can be attempted. Although friction is acting, its contribution can be ignored by considering the stored energy in the final configuration. It is assumed that there are continual small disturbances which act to overcome the frictional resistance, much as vibrations are applied in a creep test to overcome the effect of stiction.

The theoretical problem can be formulated by considering the strain energy in the wrinkle and associated slip region as before. The unstretched TD length between the edges of the slip region is the same in both the flat and wrinkled states. The expression for energy contains two undetermined parameters as before, but application of the variational principle results in equations that cannot be solved easily. Instead, a qualitative discussion will reveal the important aspects of behaviour.

The description of the behaviour of TD strain either side of the wrinkle was proposed by Clow [5]. However, he proposed a critical displacement value for wrinkling, rather than considering the strain energy.

Isolated wrinkles

In a wide web, the first wrinkle to form as the TD strain increases is surrounded by a slip zone (figure 11). If the TD strain in the wrinkle falls to ε_1, the combined "affected width" L of the slip zone and wrinkle is given by:

$$L = \lambda + \left(\frac{2R}{\mu \varepsilon_x} \right) \left(\varepsilon_y - \varepsilon_1 \right) \qquad \{25\}$$

As a first approximation, the wavelength is much less than the width of the slip zone. At the critical TD strain, wrinkling is accompanied by a fall in TD strain by a quarter, independent of other parameters. The affected width at the critical strain, L_c, is therefore strongly influenced by the initial conditions (μ, R, ε_x, and ε_y), and less by the details of the buckled form (λ):

$$L_c = \frac{R\varepsilon_y}{2\mu\varepsilon_x} \quad \{26\}$$

The sinusoid forms by inward movement of its two ends, an increase in the length, and a fall in TD strain. The affected width thus behaves in a very similar way to a somewhat narrower width in the zero friction case (a factor between 1 and 2 – see figure 12).

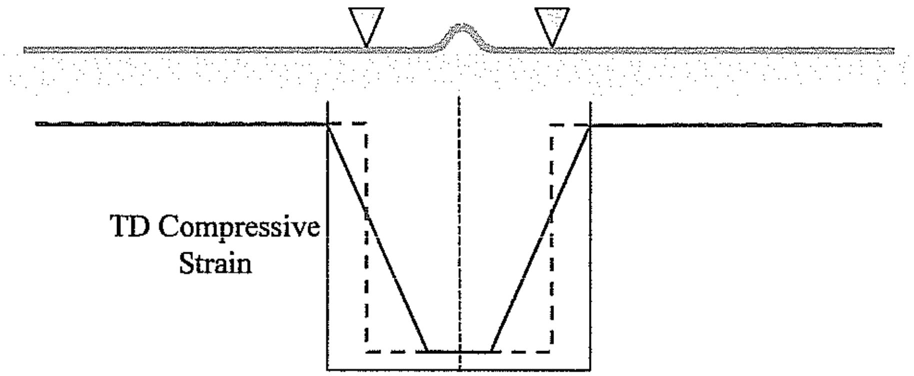


Figure 12 – Approximate equivalence of a wrinkle on a frictionless surface between two fixed points (triangles, dotted line), and on a surface with friction (solid line).

The discussion of the zero friction case can therefore be applied to this situation also. As MD strain, coefficient of friction or both increase, or roller radius decreases, the width of the slip zone will decrease and the critical TD strain will rise, until it approaches the value for sinusoidal buckling given by equation 17. For low coefficient of friction, low MD strain and large roller radius, width of the slip zone will be large, and the critical TD strain approach that predicted by equation 20. The variation of critical TD strain with slip zone length is shown schematically in figure 13.

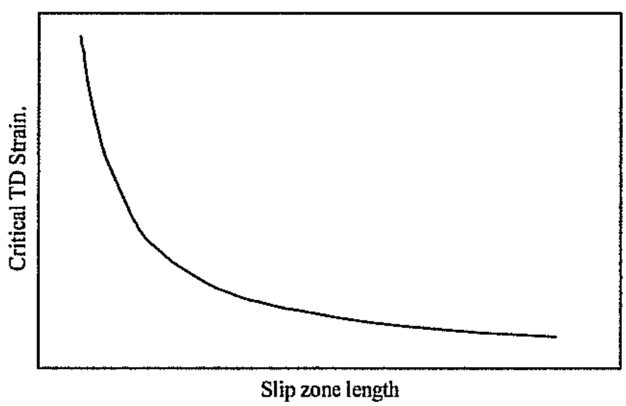


Figure 13 – Schematic plot of critical strain for wrinkling versus slip zone length.

Formation of several wrinkles

If the initial TD strain is equal to the critical value, and the web is wide enough, several isolated wrinkles can form, each with its affected width L_c. However, they can only exist if they are further apart than L_c. If they approach more closely, there is insufficient energy available to support two wrinkles, and one will disappear. If they are further apart than 2 L_c, a third wrinkle is able to form in the gap between them. However, wrinkles formed when the TD strain is equal to or just above the critical value are metastable, so any spacing of wrinkles above the minimum of L_c is possible.

A similar argument can be used to show that a wrinkle can only form a distance $2.5L_c$ or greater from the edge. The TD strain rises linearly from the edge, and must reach the critical TD strain before falling again in the slip zone of the first wrinkle. Any attempt to form a wrinkle close to the edge results in the web pushing outwards and the wrinkle collapsing. As the web has two edges, the total width must exceed 5 L_c for a wrinkle to form.

Figure 14 summarises possible positions of wrinkles at the critical TD strain.

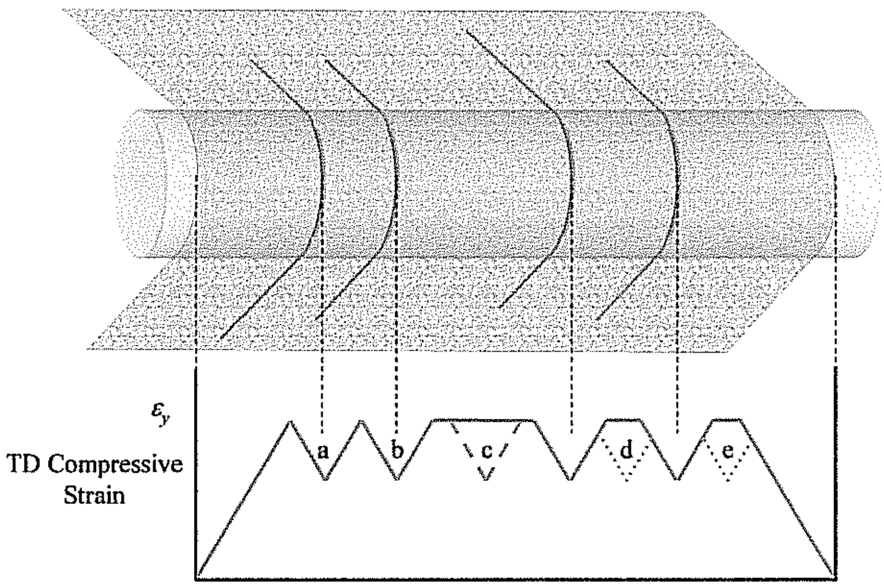


Figure 14 – Possible wrinkle locations at the critical TD compressive strain.
(a): Shortest possible distance from the edge: strain must rise to the critical value then drop by a quarter at the wrinkle.
(b): Shortest distance between wrinkles a and b is L_c.
(c): A new wrinkle can form in an area wider than $2L_c$.
(d): A wrinkle cannot form in an area narrower than $2L_c$.
(e): This wrinkle is too close to the edge to form.

Higher TD strains

For initial TD strain somewhat greater than the critical value, isolated wrinkles become the stable configuration (rather than metastable). By equation 26, the affected width increases. However, wrinkles can now form closer together as their slip zones may overlap. As initial TD strain increases, the strain ε_1 in the wrinkle after it has formed falls. The maximum strain between wrinkles also falls, to below the critical value. The minimum wrinkle spacing (and wrinkle-free distance at web edges) decreases, as indicated in figure 15.

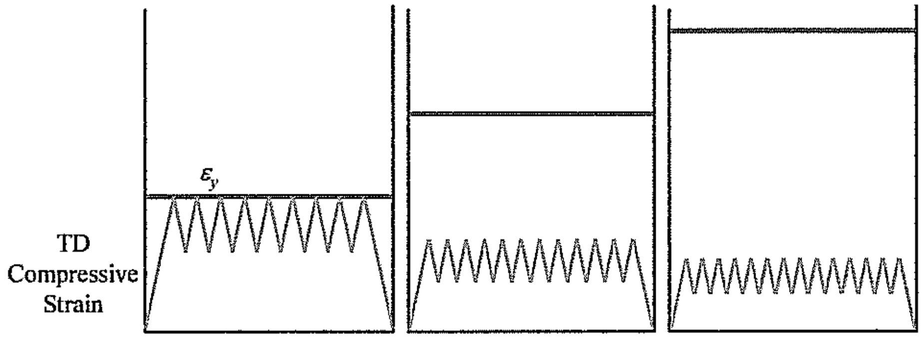


Figure 15 – Schematic diagram of TD compressive strain at initial compressive strain ε_y equal to critical (left) and higher (centre, right); both before and after wrinkles have formed at the minimum spacing.

NUMERICAL EXAMPLE

Figures 6 to 8 have used parameters appropriate to polyester film moving over a drum, for example in a vacuum coater. The web has thickness 25 µm, Young's modulus of 3.56 GPa, and Poisson's ratio of 0.38. The drum radius is taken to be 0.457 m and the coefficient of friction 0.3.

The critical TD strain for wrinkling on the drum in the absence of MD tension (equation 20) is 5.7×10^{-5}. Typical transport or winding tension levels are 100 N/m (0.5 pli), corresponding to an MD strain of 0.001. This raises the critical strain for sinusoidal wrinkling (equation 18) to 3.94×10^{-4}. For formation of isolated wrinkles, the critical strain is expected to be lower, around 10^{-4}. The affected width at critical strain (equation 26) is 76 mm: only webs wider than 380 mm will wrinkle.

For a typical PET film coefficient of linear thermal expansion of 2×10^{-5} K^{-1}, isolated wrinkles are expected for a temperature rise of only 5 deg C. If the film is to be heated by 50 deg C without wrinkling, an MD strain of around 0.01 is required, corresponding to a very high tension of 1000 N/m (6 pli).

The sinusoidal wrinkles have an amplitude of 25 µm and wavelength of 4 mm, whereas the isolated wrinkles have amplitude 160 µm, wavelength 6 mm, and spacing 76 mm, both at the appropriate critical strain

CONCLUSIONS

This paper has extended the previous application of buckling theory to web wrinkling. The radial constraint imposed by the rigid surface of a roller increases the TD critical buckling strain (or load) by a factor of 1.73 over the free cylindrical shell, for zero MD tension.

Including MD tension in the theoretical analysis has confirmed its inhibiting effect on wrinkle formation by raising the critical TD strain. In a practical situation, raising MD strain may reduce TD compressive strain by increasing the Poisson contraction, giving a second influence of tension on wrinkles.

The flat condition of the web under MD tension is always an energy minimum, as any movement off the drum results in an increase in energy. Therefore, even at high values of TD compression, a sufficiently large disturbance is needed to trigger buckling into one or more wrinkles. This could be a particle of grit a few microns in size on the roller, or a ripple that has formed in the previous span. Smaller disturbances cause the web to "snap" back onto the roller.

There is a critical compressive TD strain for wrinkles to form. At values just above this, they are metastable. Once formed, they may disappear again if conditions change so that they momentarily become unstable, or move sideways off the roller.

At higher TD strains, the wrinkles become stable, easier to form and less likely to disappear. Their minimum separation decreases. However, the actual wrinkle pattern observed will not be simple, as it is the end result of several wrinkle formation events. Imperfections may be located at specific positions, triggering wrinkles there but not elsewhere. Arbitrary widths of flat web are also in equilibrium, so wrinkle-free sections are always possible. Once a disturbance occurs, it may take some time for a wrinkle to develop into its final form, as friction must be overcome. Finally, slight roller misalignment or lateral web movement will cause inclined wrinkles in the incoming span: as these convert into wrinkles on the roller, they will track sideways.

The coefficient of friction has an influence. If it is zero, some edge constraint is needed for wrinkles to form at all. If it is very high and the web cannot slide on the roll, sinusoidal wrinkles will form. However, for typical friction values, isolated wrinkles can occur at lower critical strain. They cannot form within a certain distance of the web edge, and this distance rises when MD strain, TD strain in the centre and coefficient of friction decrease, or radius increases. If the web width is less than half this distance, wrinkles cannot form.

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Name & Affiliation	**Question**
Jim Dobbs 3M	In a vacuum coater with the web on a drum the thermal expansion effects are going to affect both TD and MD expansion. There would be a loss of effective MD tension because of the expansion while on the drum. Could you comment on the nature of the wrinkles there?
Name & Affiliation	**Answer**
Dilwyn Jones Emral Ltd.	The MD strain in this theory should be relative to the unstressed MD length, so as the web heats up the MD strain would decrease. It's the current MD strain that's important, not the one you started with. It's not the tension that you're applying as you come onto the drum but the tension that it's fallen to after the MD expansion.