

CALCULATING COILSET BASED ON CURVATURE OF A WEB

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ABSTRACT

A web which has passed over a roll can exhibit coilset due to differential yielding of fibers through its thickness. Modeling can provide a radius of curvature, R , the web would achieve if no external forces were applied. However, measuring this radius of curvature is often impractical and can be difficult when the radius of curvature becomes large. For tinplated steel products, a simple method of quantifying coilset is to hang a standard length of web on a wall, and measure the distance, C , the web curls off the wall. In this paper, the non-linear differential equations to determine the coilset measurement, C , from a given radius of curvature, R , are developed and solved numerically.

NOMENCLATURE

C = coilset, m.

$D = \frac{Et^3}{12(1-\nu^2)}$ = flexural rigidity, Nm

E = modulus of elasticity, Pa.

g = acceleration due to gravity, m/s².

L = standard length of web, m.

ℓ = length parameter, m.

M = bending moment per unit width of web, Nm/m.

P = deformed radius of curvature of web, m.

R = undeformed radius of curvature of web, m.

t = web thickness, m.

T = axial tension per unit width of web, N/m.

V = shear force per unit width of web, N/m.

α = angle of web from horizontal, radians.
 θ = rotation due to bending moment, radians.
 ϕ = angle of undeformed web, radians.
 ρ = density of web, kg/m³.
 ν = poisson's ratio.

INTRODUCTION

In flat rolled processes such as rolling [1] and tension leveling [2], the web may be left with a high level of residual stress. One possible manifestation of these stresses is coilset. Mathematical models of these processes can provide an estimate of the radius of curvature the web would achieve if no external straightening forces (i.e. tension and gravity) were applied, R in Figure 1. However for tinplate steel, the usual method of quantifying coilset is to measure the distance a standard length of web curls off a wall when hung vertically in gravity, C in Figure 1. This document shows how C can be calculated from R .

Stresses in the cross material direction of the web are ignored. The web is assumed to obey Hooke's Law. The web deforming under its own weight is assumed to be plane-strain. The radius of curvature R is assume to be large so that bending stresses in the web can be assumed to follow that of elementary straight beam theory. Deflection of the web under its own weight is assumed to be primarily due to bending moment, and not shear or axial forces. The numerical solution is found to agree with experimental results.

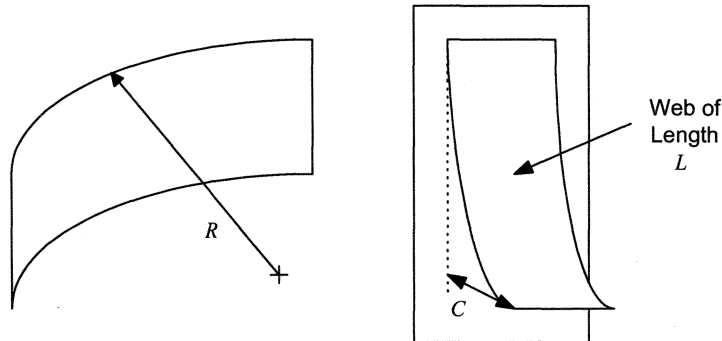


Figure 1: Coilset measurement

THEORY

Two ends of a flat plate will rotate by an angle, θ , when a pure bending moment, M , is applied to the ends (Figure 2). The angle of rotation, and the bending moment are related by:

$$\theta = \frac{M}{D} L, \quad \{1\}$$

where D is the flexural rigidity.

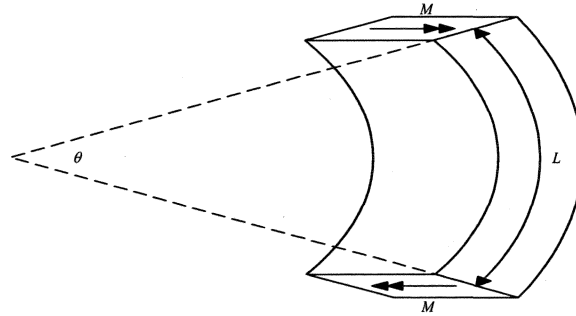


Figure 2: Rotational deflection of web under pure bending

In incremental form this equation becomes:

$$\frac{d\theta}{dl} = \frac{M}{D} \quad \{2\}$$

Although equation {1} assumes an initially flat web and small deflections, this equation still provides sufficient accuracy if the radius of curvature of the plate is large. That is, the curved beam nature of the problem can be ignored.

Figure 3 shows the geometry and forces of the hanging web. Solution of the problem involves determining how the angle the web makes from horizontal, α , varies along the length of the web. Once this geometry is known, calculation of C is straightforward.

All forces and deformations are calculated as functions of the independent length parameter l running the length of the web. Axial tension T , shear V , bending moment M , and gravitational force $\rho g t d l$, are all per unit width of the web.

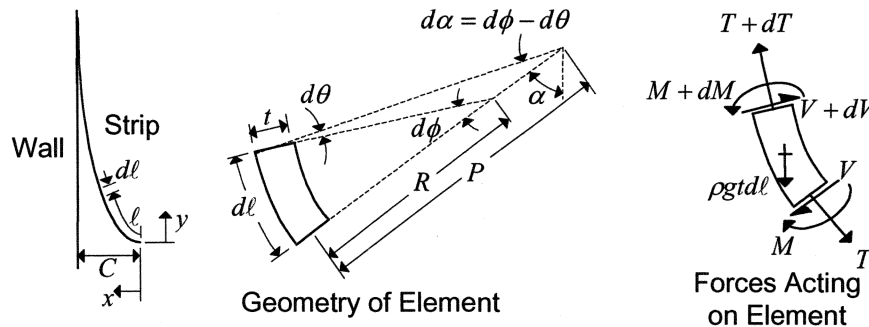


Figure 3: Deformed Web Element dl

When an element of the web, dl , with an undeformed radius of curvature, R , is subjected to a bending moment, M , the element will deform to a new radius of curvature, P . Geometry of the element gives:

$$\frac{d\phi}{d\ell} = \frac{1}{R}, \quad \{3\}$$

$$\frac{d\alpha}{d\ell} = \frac{d\phi}{d\ell} - \frac{d\theta}{d\ell}. \quad \{4\}$$

Substituting {2} and {3} into {4} gives

$$\frac{d\alpha}{d\ell} = \frac{1}{R} - \frac{1}{D}M \quad \{5\}$$

While a force balance on the element gives

$$\frac{dM}{d\ell} = V. \quad \{6\}$$

From a force balance on the free end of the web (Figure 4), the shear, V , and hence the bending moment can be determined as:

$$\frac{dM}{d\ell} = V = \rho g t \ell \cos(\alpha) \quad \{7\}$$

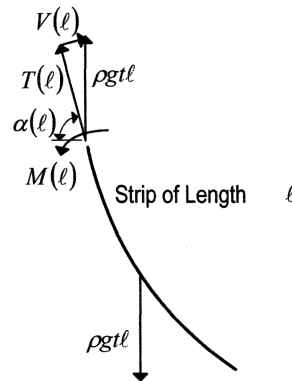


Figure 4: Force Balance on Free end of Web

Assuming that deflections due to shear and axial tension are insignificant, the web will curl off the wall by the incremental distance:

$$\frac{dx}{d\ell} = \cos(\alpha) \quad \{8\}$$

The three differential equations {5}, {7}, and {8} define the problem. With {5} and {7} the angle the web makes from the horizontal, α , can be determined. Once this geometry is known, the distance the web curls off the wall can be calculated using {8}.

Boundary Conditions

The free end of the web has no forces or moments acting on it, therefore:

$$\begin{aligned} M(\ell = 0) &= 0, \\ \alpha(\ell = 0) &= \alpha_c, \\ x(\ell = 0) &= 0, \end{aligned} \quad \{14\}$$

The numerical solution makes an iterative search for the initial angle of the web, α_c , so that the second boundary condition is satisfied.

The second boundary condition involves the web reaction with the wall and is given as:

$$\begin{aligned} M(\ell = L) &= M(\ell = L), \\ \alpha(\ell = L) &= \frac{\pi}{2}, \\ x(\ell = L) &= C, \end{aligned} \quad \{15\}$$

This simply requires that the web be vertical at $\ell = L$. No conditions are needed for the bending moment at $\ell = L$. If the two boundary conditions are satisfied, the horizontal distance traversed by the web $x(\ell = L)$ is assumed to be C .

This entire analysis assumes that the only reaction with the wall occurs at $\ell = L$, so the web does not rest against the wall.

NUMERICAL SOLUTION

Depending on whether the initial choice of $\alpha(\ell = 0)$ is less than, equal to, or greater than α_c , the solution of {5,7,8} can give three different geometries (Figure 5). A MATLAB routine was written to make the iterative search for the α_c which gives the web shape in the centre of Figure 5.

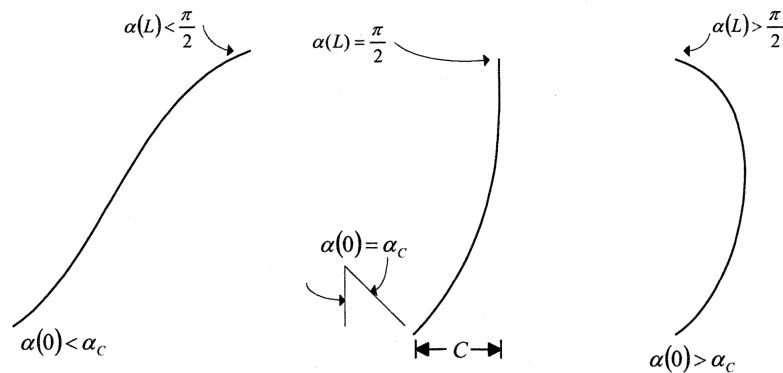


Figure 5: Possible Solutions for Different $\alpha(\ell = 0)$

Figure 6 shows a comparison of the theory presented here with measured values. It is usual to describe the curvature of a web, R , in terms of its inverse. This circumvents the problem of dealing with a flat web when R becomes infinite.

Measurements were made for webs in the thickness range $0.152\text{mm} \leq t \leq 0.183\text{mm}$. The MATLAB routine was run for a thickness of $t = 0.168\text{ mm}$, the midpoint of the measured thickness. The standard length is $L = 0.9144\text{ m}$. Material is steel; $\nu = 0.3$, $E = 200\text{ GPa}$, and $\rho = 7850\text{ kg/m}^3$.

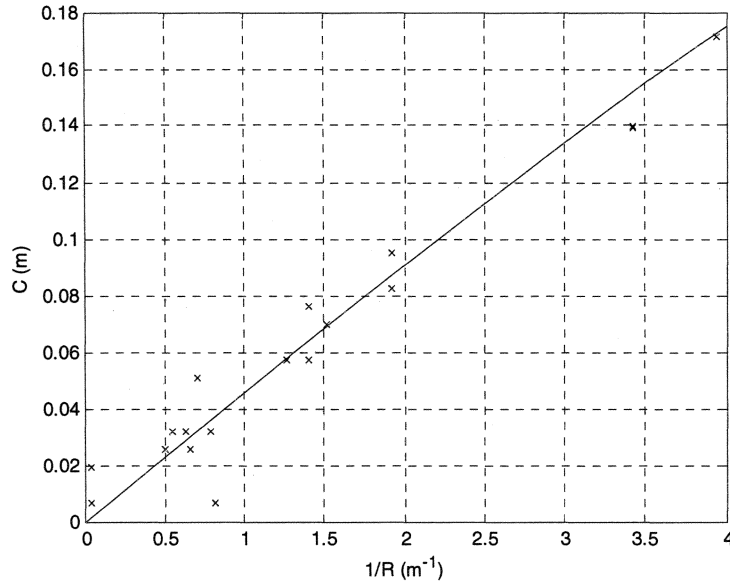


Figure 6: Measured and Calculated Values of R and C

CONCLUSION

For tinplate steel products, coilset is measured as the distance a web will curl off a wall when hung in gravity. This coilset measurement however, cannot be directly related to mathematical models of rolling and tension leveling. These models will instead provide an estimate of the radius of curvature the web would achieve if no straightening forces were applied. Measuring this radius of curvature is typically more difficult than measuring the distance curled off a wall. The preceding discussion shows how the simpler tinplate coilset measurement can be related to the web's curvature, and hence to the residual stresses through mathematical models.

REFERENCES

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