Name: James Alton Risenhoover Date of Degree: May 24, 1959

Institution: Oklahoma State University Location: Stillwater, Oklahoma

Title of Study: DIFFERENCES IN GEOMETRIC ASSUMPTIONS

Pages in Study: 23

Major Field: Natural Science

Scope of Study: Several books on Foundations of Geometry and Non-Euclidean Geometry were examined. Those that gave a complete set of assumptions were given special attention. The sets appeared to be different. They were examined to see why they were different and if one could be said to be better than another. A composite set of assumptions with a discussion of each one was presented.

Findings and Conclusions: Although the assumptions appear different, and some cannot be checked directly against others, they must be assumed to be eqivalent because they lead to the same theorems. No basis was found for saying that any particular set was better than the others. The different sets seems to be the results each author's attempt to improve some existing set. That is each author had corrected an error or changed something that he did not like in an existing set to get his set. Some authors stated that their set was a combination of two or more sets. The biggest difference is in the assumption that assign a metric property to geometry. That is most of the sets contain the same or equivalent assumptions except for the metric assumptions. The whole argument seems to be the result of a search for elegance instead of simplicity. Until a better set is written students must continue to use and study the existing sets. The set of assumptions presented in this paper would be an aid to studying the nature of assumptions.

ADVISER'S APPROVAL James H. Zant

DIFFERENCES IN GEOMETRIC ASSUMPTIONS

By

JAMES ALTON RISENHOOVER

Bachelor of Arts

East Central State College

Ada, Oklahoma

1948

Submitted to the faculty of the Graduate School of
the Oklahoma State University
in partial fulfillment of the requirements
for the degree of
MASTER OF SCIENCE
May, 1959

DIFFERENCES IN GEOMETRIC ASSUMPTIONS

Report Approved:

Report Advisor

Dean of the Graduate School

TABLE OF CONTENTS

CHAPTER I

INTRODUCTION

In beginning courses in foundations of mathematics the student is told that a mathematical system must have a consistent, independent and sufficient set of assumptions. The existance of such a set is necessary, but it is not necessary that the set be known. Theoretically, there should be a set of assumptions for Euclidean geometry that are consistent, independent and sufficient, but the set has not been written yet. Many of the people who feel that they are qualified to write a book on geometry seem to believe that they are qualified to write a set of assumptions. Perhaps many of them are, but the improvements that each author makes in the assumptions for his geometry creates a problem for the student. The problem being that whenever a student wishes to change books he finds that a different book, about the same geometry, apparently has a different set of assumptions. The student has two possible solutions, he can verify for himself that the assumptions are equivalent or he can assume that they are equivalent. The first solution is so time consuming that the student usually takes the second one. Attempts to solve the overall problem have almost always lead to another set of equivalent assumptions.

The apparent differences in the different sets of assumptions concerns two types of students. Beginning students, if they become aware of the problem, are likely to doubt that geometry is a mathematical system until they are convinced, by some teacher or by doing research on their own. The problem is of most concern to students of Foundations of Geometry. The problem should concern high school geometry teachers, but fortunately most of them never heard of it.

The problem is unique in mathematics in that it has the simple solution; ignore it and it will go away. If a student takes progressively harder courses in geometry, and avoids courses in Foundations of Geometry, the question of assumptions never comes up after two or three courses. College textbooks in Euclidean geometry frequently start out with a statement similar to the following; this is a continuation of the high school course in plane geometry, therefore the student should have in mind the assumptions and propositions contained in a high school course in plane geometry. By such a simple statement the question of assumptions is dispensed with. It can not be said for certain that the author is ignoring the problem. He may not know that the assumptions in most high school geometry books are so poorly stated that even high school students can pick them apart. Ignoring the assumptions is better than writing a new and different set of assumptions.

It is anticipated that a good set of assumptions will be written soon. With the emphasis that is presently being placed on foundations of mathematics someone will surely bring geometry up to date. Until this is done the existing sets must be used. The set of assumptions proposed in this paper is not a workable set therefore they will not add to the problem by adding another set of assumptions. It is intended that the set and the discussion of each assumption should contain enough information about the nature of assumptions to enable a student to evaluate a set of assumptions for himself.

This paper contains information that should be useful to a high school teacher in the selection of a text book, but it is not recommended for use as enrichment material for a high school course. The teaching of high school geometry is difficult enough without mentioning, that the set of assumptions being used may not be perfect.

CHAPTER II

BASIC ASSUMPTIONS

The assumptions presented in this chapter are for the demonstration of the properties of assumptions and nothing else. They are consistent, but not independent or sufficient for Euclidean geometry. Some of the simple properties of sets such as, intersection, union, empty, non-empty, belongs to and does not belong to, will be used. The notation is not standard, but it is simple.

Assumption 1. There exists a set S, such that the elements of S will be the elements of a geometry with the following assumptions. The elements of S shall be denoted by the capital letters of the English Alphabet.

Assumption 2. There exists a relation which determines a particular type of subset of S. This type of subset will be denoted by L( ).

Assumptions 1 and 2 are seldom found among the numbered assumptions of a geometry. They make a nice introductory paragraph, and they are different in nature from the other assumptions. The set S and the relation are usually undefined, and all of the assumptions are about the properties of the set and the relation. All of the following assumptions will be about the set S and the relation which determines subsets.

The second sentence in assumption 1 and 2 are not part of the assumptions. They are put in that place for convenience. The practice of including notes about notation with the assumption has become almost standard procedure. It can be justified only on the grounds that it makes a neater more compact paper, and the meaning of the symbols are not likely to be overlooked.

To see the importance of assumption 1 just assume, there does not exist a set S, such that the elements of S will be the elements of a geometry with the following assumptions. Note that assumption 2 is limiting and describing the set S. It states definitely that there exists a relation which determines subsets of a particular type. It does not say that there are not other relations which determine other types of subsets.

The last two sentences in the paragraph above illustrate a fault that many geometry books have. Most authors seem to think it is their duty or privilege to interpret their assumptions. Perhaps this is necessary, but it would be much better if the assumptions were written so that they could speak for themselves.

<u>Assumption 3</u>. The set S contains at least two elements.

This assumption could have been combined with assumption 1. This is done in many of the books in which assumption 1 is stated as an assumption. Combining two assumptions does not make one assumption, but it does make for compactness, and it sometimes makes a student wonder why one set of assumptions has twenty assumptions and another set has twelve. The possibility of combining assumptions will come up again. It is only a partial answer to the question of apparent differences in assumptions.

Assumption 4. Every A and B that belongs to S determines one and only one subset of the type L( ), such that A and B belongs to the set. To show that A and B determines the set and belongs to the set we write L(a,b).

The second part of this assumption is usually not stated. It seems necessary in view of the fact that sets can be found in which two elements determine a subset and the two elements do not belong to the subset. A set of books on a shelf with the relation being, all books between two given books, will serve as a model. Any two books determine a subset but do not themselves belong to the subset.

In Hilbert's statement of this assumption he did not state that A and B belonged to the set.[1] In his interpretation of the meaning of his assumption he stated that A and B did belong to the set. This was equivalent to putting in another assumption, in a place where it could easily be overlooked. There is no way of knowing just how many assumptions used in geometry are never stated as assumptions.

This assumption obviously could be broken into two assumptions, possibly three. The words, a unique, could be substituted for, one and only one. The assumption is stated in various ways, and various meanings are attributed to it. One important thing to notice here is that it appears in all sets of assumptions for geometries of points and

---

[1] David Hilbert, Foundations of Geometry, tr. E. J. Townsend, (La Salle, 1947) p. 4.

lines. Notice also that it states a sufficient condition and not a necessary one. That is L(a,b) may possibly be determined in other ways, and by other elements of S.

<u>Assumption</u> 5. If A, B, C and D are four distinct elements of S and C and D belongs to L(a,b), then L(a,b) = L(a,c) = L(a,d) = L(b,c) = L(b,d) = L(c,d).

The meaing of this assumption is sometimes read into assumption 4, but most of the good sets recognize it as an independent assumption. It can be shortened by writing it as; any two elements of a set of the type L( ) completely determine the set.

<u>Assumption</u> 6. Every C that belongs to S divides any L( ) which contains C into two subsets, such that C and only C belongs to both subsets. Since A divides L(a,b) into two subsets, the subset to which B belongs will be called L(a,1,b), and the subset to which B does not belong will be called L(a,2,b).

This assumption is difficult to connect with a particular assumption from some well known set. The necessity of this assumption or an equivalent assumption is shown by noticing that there are geometries in which it is not true. There are two ways in which this assumption can be omitted from a set of assumptions. It is the geometric equivalent of Dedekind's postulate therefore it may be part of a metric assumption.[2] It can be introduced by making a mark alongside a ruler then placing a dot near the middle of the mark so that the reader can see that a point divides a line into two parts. There is nothing wrong with introducing

---

[2] Richard Dedekind, <u>Essay on the Theory of Numbers</u>, tr. W. W. Beman, (La Salle, 1948) p. 11.

the assumption as part of the metric assumption if it is stated so that the reader will know how it is being introduced.

This is a very convenient place to introduce this assumption in this set. It makes possible the definition of subsets of L(a,b) and the proof of the following theorems.

Definition: The intersection of L(a,1,b) and L(b,1,a) will be called AB.

Theorem 5.1 Both A and B belong to the set AB.

Theorem 5.2 If there exists a C in S such that C belongs to AB, $C \neq B$, $C \neq A$, then C does not belong to either L(a,2,b) or L(b,2,a).

Assumption 7. For every A and B in S, $A \neq B$, there exists elements C, D and E, in S, $C \neq A$, $C \neq B$, $D \neq A$ and $E \neq B$, such that C belongs to AB, D belongs to L(a,2,b) and E belongs to L(b,2,a).

This should give a sufficient number of elements to L(a,b), but it can not be proved that there is more than one set of the type L( ). If it is assumed that there is more than one set of the type L( ) then it can not be proved that there is more than one element in each set. If it is assumed that there is more than one element in each set then it can not be proved that there is more than one set.

Assumption 8. Every subset of the type L( ) contains at least two elements of S.

There are good arguments against the necessity of this assumption. It can be argued that sets containing only one element would in no way interfere with the geometry. Since it takes two elements to determine a subset of the type L( ), the existance of sets with less than two elements would never be known. It is independant of the seven preceeding assumptions but its independance of the whole set is not claimed. It is stated here because it is related to assumption 6 and 7, in so far as all three of them could be replaced by assumption 17.

Assumption 9. If C belongs to L(a,b), C belongs to S.

This assumption is not necessary for the non-metric part of Euclidean Geometry. It is necessary when a metric property is assumed for Euclidean Geometry In the non-metric part of Euclidean Geometry it is just nice to know that the subset L( ) are not cluttered up with a lot of elements that do not belong to S.

Assumption 10. For every A and B in S there exists a C in S such that C does not belong to L(a,b).

This assumption is part of a more general assumption, which if ever stated would probably read something like this; The elements of S appear wherever needed and do not appear where they are not needed.

This assumption is necessary if there is to be more than one subset of the type L( ). Since any two elements of S determine a subset of the type L( ), A and C determine a subset L(a,c). The intersection of L(a,b) and L(a,c) can be A and only A. It is A because of the assumption that the two elements that determine a set belong to the set. It can not contain elements other than A. If it did L(a,b) would be identical to L(a,c) by assumption 5, but this would contradict assumption 10. Thus the following theorem is easily established.

Theorem 10.1 If A, B and C are any three elements of S such that C does not belong to L(a,b) then A, B and C determine three distinct sets L(a,b), L(a,c) and L(b,c).

The following definition is clumsy, but it will show that such a definition is possible.

Definition. If A, B and C are any three elements of S such that C does not belong to L(a,b) then the set P(a,b,c) shall consist of all elements of S belonging to L(i,j) where I and J are any two elements belonging to L(a,b), L(a,c) or L(b,c). If I = J = A or I = J = B or I = J = C then except for I and J only the elements of L(i,j,x), $X \neq I$, X belonging to P(a,b,c) shall belong to P(a,b,c). If X and Y belong to P(a,b,c) then all elements of L(x,y) shall belong to P(a,b,c).

Some authors consider the plane as another undefined relation. If this definition is accepted there is only one undefined set and one undefined relation in the non-metric part of Euclidean geometry. The metric property may be introduced as an undefined relation. Notice that ever undefined relation requires the assumption of the existance of such a relation. Hilbert assumed one set and two relations.[3] Veblen assumed one set and one relation.[4] They did not number these assumptions.

---

[3]Ibid., p.7

[4]Oswald Veblen, "Foundation of Geometry", Monographs on Topics of Modern Mathematics, ed. J.W.A. Young, (Dover Publishing Co. ed. New York, 1955), p.4.

<u>Definition</u>. If L(m,n) and L(p,q) belong to P(a,b,c) and the intersection of L(m,n) and L(p,q) is empty then L(m,n) and L(p,q) are parallel.

<u>Definition.</u> If L(e,d) does not belong to P(a,b,c,e) and E does not belong to L(a,b) their L(e,d) and L(a,b) are skew.

<u>Assumption 11</u>. Every L(i,j) that belongs to P(a,b,c) divides P(a,b,c) into two subsets, such that their intersection is L(i,j). Since L(a,b) divides P(a,b,c) into two subsets, the set to which C belongs will be denoted by P(a,b,1,c) and the set to which C does not belong by P(a,b,2,c).

This assumption is probably not independent, but its proof would require two much time for the good it would do. It could be made independent by changing assumption 6 to a theorem. The possibility of interchanging assumptions and theorems accounts for a large part of the apparent differences in different sets of assumptions. The assumption is stated here to make the following definitions meaningful.

<u>Definition</u>. When C does not belong to L(a,b) the intersection of P(a,b,1,c) and P(a,c,1,b) shall be called A(a,b,c).

<u>Definition</u>. If C belongs to L(a,1,b), A(a,b,c) = 0, and A(a,b,c) = L(a,1,c,b) = L(a,1,b,c).

<u>Definition</u>. If C belongs to L(a,2,b) and D does not belong to L(a,b) the intersection of P(a,b,1,d) and P(a,c,1,d) shall be called A(a,b,c). A(a,b,c) = 2.

<u>Theorem 11.1</u> A(a,b,c) is never empty.

This theorem was stated to point out the fact that the numbers 0 and 2 as used in the definition have no metric properties whatever. If S was the set of all points in space and L(a,b) was the line containing A and B then A(a,b,c) would be the angle BAC. The zero is not a measure for the angle, but only a name for a particular kind of angle.

Theorem 11.2 If A(a,b,c) = 2, then A(a,b,c) = P(a,b,1,d).

Definition. When C does not belong to L(a,b) the intersection of P(a,b,1,c), P(a,c,1,b) and P(b,c,1,a) will be called T(a,b,c).

Assumption 12. For any C that does not belong to L(a,b) there exists at least one L(c,d) such that L(c,d) is parallel to L(a,b).

Assumption 13. If L(c,d) and L(c,e) are parallel to L(a,b) then L(c,d) = L(c,e).

Assumption 12 and 13 are usually combined to make one assumption. They are separated here because in some geometries one of them is true and the other is false, and in some geometries both are false. Of course assumption 13 can not be true when assumption 12 is false.

Assumption 14. For any P(a,b,c) there exists a D belonging to S such that D does not belong to P(a,b,c).

Definition. If the intersection of P(a,b,c) and P(d,e,f) is empty, P(a,b,c) is parallel to P(d,e,f).

Assumption 15. For any D that does not belong to P(a,b,c) there exists at least one P(d,e,f) such that P(d,e,f) is parallel to P(a,b,c).

Assumption 16. If P(d,e,f) and P(d,i,j) are parallel to P(a,b,c) then P(d,e,f) = P(d,i,j).

Assumption 14 is necessary if there is to be more than one set of the type P( ). Assumption 15 and 16 are very similar to assumption 12 and 13. Assumption 15 and 16 are probably not independent in this set. They are stated here because in some geometries 12 and 13 are false and 15 and 16 are true and independent.

<u>Assumption</u> <u>17</u>. There exists a 1 to 1 correspondence between the elements of any set of the type L( ) and the real numbers such that for any A and B in S the elements of L(a,b) are in 1 to 1 correspondence with the real numbers and A corresponds to 0 and B corresponds to 1.

One of the first thing to notice about this assumption is that it does not in any way imply the relation called distance or length. It does imply that the elements of L( ) have many of the properties of the real numbers. It is obvious that if this assumption is used assumptions 6, 7 and 8 may be omitted from this list. Statements could be made about order and useful definitions could be made. Assumption 17 adds very little useful material to this set of assumptions. It was stated here because it will probably become quite popular in the next few years. The main thing against it is that it requires that the student of geometry also be a student of arithmetic, and it makes the foundations of geometry depend upon the foundations of arithmetic.

<u>Assumption</u> <u>18</u>. The set of <u>all points in space</u> has most of the properties of the set S.

This is the final assumption in this series. First a set was assumed to exist, then sixteen descriptive assumptions described the set so that now it has been recognized. The next step is to translate the assumptions and definitions so that they will be about points, lines, planes, angles and other elements of geometry. The translation will not be made here. Any reader who is interested may do so then see how many of the theorems in his high school geometry book can be proved. One such check, gave one theorem proved, three hundred three that could not be proved.[5] In fact in any of the important geometries about points, lines and planes very few theorems can be proved. In each one at least one important, characteristic assumption is missing. The missing characteristic assumption will be the subject of the next chapter.

---

[5]G. A. Wentworth, A Text-book of Geometry, Rev. ed. (Boston, 1898)

CHAPTER III

CHARACTERISTIC ASSUMPTIONS OF THE BASIC GEOMETRIES

In this chapter some of the different types of geometries of points, lines and planes will be discussed. These geometries have been classified in so many different ways and have been know by so many different names that it is almost necessary to quote a few of their assumptions to designate a certain geometry rather than call it by a name. Projective Geometry and Euclidean Geometry are standard terms, but Non-Euclidean Geometry and its brances are sometimes confusing.

Projective Geometry

Projective Geometry is distinguished from the Euclidean and Non-Euclidean by four characteristic assumptions that are not necessary for the other geometries.

Assumption A. Every line has one special point.

Assumption B. Every plane has one special line.

Assumption C. There is one special plane.

The special point on a line is defined to be the point where two parallel lines intersect. The special line is defined to be the line containing all the special points and only the special points in a plane. The special plane contains all the special lines and only the special lines. Notice that it is not necessary to assume that the three assumptions above are false in Euclidean Geometry. They may simply be omitted from the set of assumptions for Euclidean Geometry.

Assumption D. If a projective leaves each of three distinct points of a line invariant, it leaves every point of the line invariant.

Assumption D concerns properties of points and lines that are not usually defined in Euclidean Geometry, therefore it may be omitted from the Euclidean set of assumptions. Notice that the omission does not mean that it is not true, but only that it is not needed.

Projective Geometry may be developed using the four assumptions above and all of the Euclidean assumptions or by using the four assumptions above and all of the Euclidean assumptions except the metric assumptions. That is, if the set of assumptions in Chapter II were sufficient for the non-metric part of Euclidean geometry, then a sufficient set for Projective Geometry could be made by combining that set and the four assumptions above.

EUCLIDEAN GEOMETRY

The characteristic assumption of Euclidean Geometry is the metric assumption. Projective Geometry can be developed with or without a metric assumption, but most of all Euclidean Geometry depends upon the metric assumption. Without it circles and spheres can not be usefully defined, and there is no way to say that two angles are equal. The set of assumptions in Chapter II do not contain a metric assumption therefore it is not possible to prove that, if two lines intersect, the verticle angles are equal.

The metric assumption has been introduced in various ways. For a long time the concept of <u>superposition</u> worked nicely. It became unpopular and is found now in only a few high school geometry books. The concept of superposition was replaced by the assumption of congruence. Congruence worked nicely, but it required four or five assumptions for completeness. It has been criticized for this, and attempts are being made to replace it by the idea of 1 to 1 correspondence. The idea of 1 to 1 correspondence has been well developed in mathematics. It is a simple concept that can be introduced at the high school level.

> <u>Assumption E</u>. There exists a relation between any two points in space. This relation will be called the distance between the points.

<u>Assumption</u> <u>F</u>. There exists a special 1 to 1 correspondence between the points on a line and the real numbers, such that number differences will be a measure of distance on the line.

The special 1 to 1 correspondence has some advantages. It would be useful in integrating algebra and geometry. If it was used, high school math teachers would no long be found pointing out the 1 to 1 correspondence in their algebra classes and then carefully avoiding it in their geometry classes.

For the student who wishes to check one set of assumptions against another, this can sometimes be done by isolating from each set the assumptions that assign the metric property to geometry. These will be the hardest to check for equivalence. Frequently they can only be checked by showing that each leads to the proof of the same theorems. The non-metric assumptions can usually be checked by calling one set, theorems and using the other set to prove them. If each set can be proved, using the other as assumptions then the two sets are equivalent. If this is not the case then something is wrong, but a general statement of the trouble can not be made. The metric assumptions frequently can not be isolated, because they are combined with non-metric assumptions. In this case it is sometimes possible to check one set against the other. The check that will always work, and for that reason is the best one, is to see if both sets are used to prove the same theorems.

NON-EUCLIDEAN GEOMETRY

The term, Non-Euclidean Geometry is best used to apply to a geometry that assumes a contradiction of the Euclidean parallel postulate. There are at least three non-euclidean geometries, each has a characteristic assumption that is a contradiction of the Euclidean parallel assumption, and certain other Euclidean assumptions are omitted or modified. It has not been considered worthwhile to publish a set of assumptions for any of the Non-Euclidean geometries. The authors of Non-Euclidean text-books say; take the Euclidean set of assumptions and change certain assumptions. Thus Non-Euclidean geometry depends upon Euclidean, but this is not necessary.

GEOMETRY OF LOBATSCHEWSKI

This was the first non-euclidean geometry developed. It assumes that through a given point not on a given line there are two parallels to the given line. If the parallel assumption is modified and the assumptions that extend Euclidean plane geometry to Euclidean solid geometry are omitted all other assumptions of Euclidean geometry may be adopted as the assumptions of Lobatschewskian geometry. Thus this geometry shows the independence of the parallel assumption.

Assumption G. Through any point not on a given line there are two parallels to the given line.

RIEMANN'S GEOMETRY

This geometry is due to an assumption made by Riemann, but the geometry was not developed by him. The characteristic assumption is that, through a given point not on a given line there is no parallel to the given line. This leads to the proposition that all lines intersect. This leads to Riemannian Geometry I and Riemannian Geometry II.

Riemannian Geometry I assumes no parallels and two lines intersect in one point. It is a plane geometry therefore the assumption of parallel planes and the existance of points not on a given plane may be omitted. All the plane assumptions of Euclidean geometry expcept those connected with parallel lines are assumed.

Riemannian Geometry II is the best developed of all the Non-Euclidean geometries. This is because it was found that a sphere was a perfect model. For this reason it is usually known as Spherical Geometry. It requires the modification of a great number of the Euclidean assumptions. If the set of assumptions given in Chapter II were to be reworked for spherical geometry, assumptions 4, 5, 6, 7, 12, 13 and 17 would have to be omitted or modified. This is the geometry in which it is possible to have parallel planes but not parallel lines.

CHAPTER IV

CONCLUSION

The apparent differences in geometric assumptions impair only the beauty of geometry. They are the result of the geometry being built before the foundation. Other areas of mathematics have the same problem. The real numbers grew up then the foundations were developed. It is generally agreed that the foundations of real numbers are good. That is that there are a few good developments. Every attempt to develop a good foundation for geometry has been followed by someone who thought they could do better. Thus, there are literally dozens of sets of assumptions for Euclidean Geometry. Each just different enough to enable the author to get it printed.

The beginning student should avoid the problem created by the different sets of assumptions. It is not a problem that is likely to be solved by a beginning student. The only solution is for some respected master to write a set that is better than all the rest. The set must not only be good, but it must be simple so that it can be presented in the first course in geometry. Until such a set has been written the problem must be lived with. There is no better solution at the present time than, to learn one set, use it, and ignore the rest. After a high school course in geometry most students will have assumed for themselves the necessary assumptions to go on to other courses. Meanwhile students of Foundations of Geometry, are working on the overall solution. They may only be making it worse, but it is only by such attempts that the better set will be written.

It is believed that this paper shows the need for the better set and that it will give a beginning student enough information about the general nature of assumptions to enable him to accept whatever set he knows without wasting time looking for the best set. There is only one idea that might be of use in writing a better set of assumptions. That is the using of sets to avoid preconceived ideas about points and lines.

BIBLIOGRAPHY

Bonola, Roberto. Non-Euclidean Geometry. New York: Dover, 1955.

Coolidge, Julian Lowell. The Elements of Non-Euclidean Geometry. Oxford: Clarendon Press, 1909. p.p. 11-48.

Dedeking, Richard. Essays on the Theory of Numbers. tr. W.W. Beman, La Salle; Open Court Publishing Co. 1948.

Hilbert, David. Foundations of Geometry. tr. E. J. Townsend. La Salle, Open Court Publishing Co. 1947.

Robinson, Gilbert de B. The Foundations of Geometry. Toronto: The University of Toronto Press, 1952.

Veblen, Oswald. "Foundation of Geometry", Monographs on Topics of Modern Mathematics. New York: Dover, 1955. p.p. 3-51.

Wentworth, G. A. A Text-book of Geometry. Boston: Ginn and Co. 1898.

VITA

James Alton Risenhoover

Candidate for the Degree of

Master of Science

Report: DIFFERENCES IN GEOMETRIC ASSUMPTIONS

Major Field: Natural Science

Biolgraphical:

Personal Data: Born near St. Louis, Oklahoma, October 19, 1920, the son of Ralph and Ruthia Risenhoover.

Education: Attended grade school at Blanton View, a country school near Maysville, Oklahoma; graduated from St. Louis High School in 1940; received the Bachelor of Arts degree from East Central State College, with a major in Mathematics, in May, 1948: completed requirements for the Master of Science degree in May, 1959.

Professional experience: Taught math and science in Corona High School, Corona, New Mexico for two years 1948-50; left New Mexico and returned to Oklahoma to attend Oklahoma A and M for one year; taught math and science at Wynnewood High School, Wynnewood, Oklahoma, for one year 1951-52: left Wynnewood to teach math and science for four years in the Swink High School, Swink, Colorado, 1952-56; returned to Wynnewood High School to teach math and science for two years, 1956-58; Attended Oklahoma State University in the Academic Year Program of the National Science Foundation during the year of 1958-59.